

A Pedagogical Demonstration/Lecture Plan for the Introduction of Gravitational Wave Astronomy



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This work is dedicated to my parents.

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Chapter 1

Introduction

Gravitational waves are ripples in spacetime and were first predicted by Albert Einstein in 1916 [4]. After decades of technological development, they were finally observed on September 14th, 2015 [4]. Their signal came from the coalescence of two black holes, where the tiny shift in spacetime was measured by the Laser Interferometer Gravitational Wave Observatory (LIGO). Since their initial detection there have been 12 known signals and 56 potential candidates [7].

With more gravitational wave observatories, VIRGO in Europe, KAGRA in Japan, and LIGO India, gravitational wave astronomy is providing a new way to study the universe, and will continue to mature as a sophisticated field of physics. Unfortunately, its seldom taught in early undergraduate physics/astronomy courses. Due to the fact that LIGO is primarily a Michelson interferometer, the introduction of gravitational wave astronomy can easily be paired with a lecture demonstration involving such a device. It is with little doubt that physics demonstrations play an important role in the teaching of science and can drastically improve the way students learn [6][17][21][24][28][29]. However, gravitational waves are mathematically complex for an early undergraduate student, so a simplified method to the introduction of Einsteins equation must be considered. This thesis is a work-in-progress on a pedagogical demonstration/lecture plan for the introduction of gravitational wave astronomy for early undergraduates, but its highlighted that the essential aspects on effective lecture demonstrations and the mathematics of gravitational wave astronomy have been covered thoroughly, where the future work on this project would include the refinement of the device and lecture plan.

Chapter 2

Physics Demonstrations as a Learning Tool

2.1 Failing to Learn from Demonstrations

There are a large quantity of factors that could prevent a student from learning from a demonstration. As an educator its important to understand these aspects before developing a demonstration to either avoid encountering these problems or to address them as they arise.

There are five major dimensions that could play an influence in why a student may fail to learn. The first is the idea that the student may lack the theoretical framework to separate important information from unimportant information [17][24]. For demonstrations to work at all, students must see the intended outcome in conjunction with the explanation. It frequently happens that the experiment may have unintended side effects that could distract and confuse the observer. Scientists and engineers are actively practicing the separation of signal and noise and have the foresight to know how certain situations should occur, but its important to know that this skill isn't as present in a young student.

The second, is the interference of discourse learned from other contexts of the physics course or from outside experiences [24]. Its expected that students will draw on experiences from both inside and outside the classroom in attempt to explain a phenomena. If a demonstration has any surface level similarity to their experiences then the students might speculate a common influence that is otherwise untrue.

The third aspect ties into the fact that students may mistakenly or have problems with piecing together coherent representational frameworks from the information

provided [17][24]. This may mean that the demonstration and explanation proceeded so quickly that the student couldn't put them together, or, that without the theoretical background present the student may not understand what the experiment is supposed to be representative of [8][30]. Another critical point to this is that there might be more physical aspects to the demonstration that are left out or skimmed over that require an additional explanation.

Not having the opportunity to practice scientific discussion and the different levels of scientific literacy between an educator and student is the fourth reason why a student may fail to learn from a demonstration [24][26]. It's easy to explain a concept and use the associated terminology, but this could come out as jargon and be confusing. This is critical because students are being introduced to these concepts for the first time and don't have these key vocabulary words down. If a student doesn't feel comfortable with the terminology then they might fail to comprehend the concept associated with the demonstration, but can also be discouraged from continuing a physics education [26].

The fifth aspect comes from the idea that the demonstration plays no role in a student's grade and can be seen as entertainment [6][8][24]. Studies have shown that if students don't actively participate/engage with the demonstration then a student that didn't see the experiment will test as well as the student that did [6][17].

In general, a student may fail to learn from a physics demonstration because they may not have the level of intuition needed to understand the demonstration and may lack the opportunity or confidence to actively engage with the physics concepts.

2.2 Promoting an Active Learning Environment

A common, but effective, way to help prevent these contingencies from occurring or to address them as they arise is to create an active learning environment. On the contrary to traditional demonstrations and lectures, where the teacher shows and tells, which has shown to be less effective [1][6][10][17][24][29], an active learning

environment is one that allows students to engage with the material, participate in class, and collaborate with others [23].

Introduced here are three methods of creating an active learning environment for a demonstration. The first is where students record their prediction, observe the demonstration, and hear the explanation [6][24]. The second is where students make a prediction, observe the demonstration, discuss it with fellow students, and finally hear the instructors explanation [1][6][24].

The discussion aspect serves to address the majority of concerns from the section 2.1 of this chapter. It allows the instructor to overhear what students may have not understood, act as a check for students to see whether their own talk about the phenomena was shared among others or was viable and fruitful, and to check if what the students constructed as a phenomena was to be of interest or simply noise [24]. One particularly interesting outcome that could occur is that the demonstration doesn't proceed successfully. This opens the door to high potential of learning by discovering what went wrong, through the discussion of the different variables and how they influence the demonstration [1].

The third method to creating an active learning environment is to create a contextual road map of the unit\chapter that is built around the demonstration [1][10][24]. What this entails is that the instructor shows the demonstration before the chapter and has the students engage in discussion to make predictions regarding the underlying physics concepts and variables behind it [1][24]. As the necessary theoretical framework is developed the instructor can then have the students discuss and predict what exactly occurs within the demonstration. At the end of the unit the instructor can demonstrate and thoroughly explain it, where the final discourse could look into more advanced or present day experiments [1]. This sort of conceptual road map discussion is consistent with scientific inquiry through observations and hypothesis, the construction of variables to produce theoretical framework, and lastly, the verification and future outlook of such work [1][24].

A more advanced version of the third method is to split the demonstration into three distinct sections: observe, test, and apply [10]. As before, the demonstration

will be done before the unit begins and students will make predictions/explanations. Once the students have begun to build enough foundational knowledge and feel comfortable with the terminology they can revisit their explanations, be introduced to a new demonstration, but be asked to make a new prediction and convince their neighbors that their prediction is correct, and then be shown the demonstration in conjunction with the instructors explanation. Finally, when the students are more than comfortable with their abilities they can apply it to explain another phenomena or build a new device [10]. The last section plays more of a role in a lab setting, but promotes high engagement and scientific thought by allowing students to explore the physics concept in more depth and apply it in new forms, while simultaneously building off of the in class demonstration.

2.3 Forms of Effective Lecture Demonstrations

There are two forms of effective lecture demonstrations that can be carried out, interactive lecture demonstrations (ILD) [29] or interactive lecture experiments (ILE) [21].

ILDs have been shown to increase learning gains between a control and experimental classroom on standard tests by more than 15% when compared to traditional methods [6][17][28][29]. The formalized procedure for an ILD is [17][29]:

1. Instructor describes the demonstration.
2. Students record their names and individual predictions on a Prediction Sheet, which will be collected. (Predictions will not be graded, but class participation points are usually awarded)
3. Students engage in small-group discussion with their one or two nearest neighbors.
4. Students record their final predictions on the Prediction Sheet.
5. Instructor elicits common student predictions from the whole class.
6. Instructor carries out the demonstration with the use of displays, that give

a more complete view of the experiment (multiple monitors, LCD, panel or computer projector).

7. A few students describe the results and discuss them in the context of the demonstration. Option: Students fill out Results Sheet, identical to the Prediction Sheet, to take with them.
8. Instructor discusses analogous physical situation(s) with different "surface" features - that is, different physical situation(s) based on the same concept(s).

A couple key parts to understand is that after step three, the instructor must carefully observe and pick an appropriate time to move on and steps seven and eight can also be adjusted to the educators agenda, but must guide the discussion towards the important points raised in the demonstration. This form is different than the general forms argued for in section 2.2, in that they collect prediction sheets to entice students to engage through the use of class participation points and that they use real-time displays to give students the confidence and trust in the apparatus and measurement device(s) [29].

ILEs build on ILDs by requiring students to analyze data during and after lecture demonstrations [21][22]. Due to the fact that this form of demonstrations is relatively new and in progression, it has not been shown that ILEs improve understanding of specific topics, potentially because of the fact that other interactive pedagogies were employed in this study [22]. On the contrary, open-ended questions on the exam showed differences on some topics between sections on topics that were addressed by ILEs, giving strong evidence that ILEs have the potential to promote learning of both of physics concepts and analytical skills [22][21]. The procedure for an ILE is [21]:

1. Demonstration stage - Students are shown an experimental setup during lecture. The instructor discusses physics concepts and phenomena illuminated by it without going into specific details. Students are then asked to discuss and make predictions about the experiment.
2. Data Collection and Sharing - The experiment is then demonstrated and the

data from it is collected, uploaded, and shared with the students along with a set of specific data analysis questions to help direct their investigations.

3. Data Analysis - Students are asked to work in small groups outside of class to analyze the data and address the specific questions. These questions are indicative of their conceptual understanding as well as their ability to support qualitative analysis with quantitative results, which is captivated in the submission of a brief written report.
4. In-class Discussion - Students bring results of their analysis to class and share their results through the use of clickers and discussion.
5. Problem Solving - As a follow up homework assignment, students will be asked questions requiring application of the concepts learned in the ILE.

Although the first study found that 78% of students found ILE's to be helpful or very helpful at the end of the semester [21], the second study found that only 32% of students found ILEs to be helpful in understanding laboratory experiments on the same topic [22]. This discrepancy is alarming, but it's noted in the second study that the ILEs were reported as too time intensive, contained major software problems (Logger Pro), and the lack of feedback on their homework sets contributed to this negative response.

If a more careful approach is taken and more research is done on ILE's, then this form could be a strong candidate of creating an active learning environment and addressing the issues on why students may fail to learn from a lecture demonstration. ILDs, however, have been studied extensively and can be considered a very secure way of presenting a physics demonstration.

Chapter 3

Gravitational Wave Theory

The formalism presented here is based primarily based on [2] and [25].

Gravitational waves are one of the many predictions of Einstein's Theory of General Relativity. The assumption is that the reader is familiar with basic concepts and the mathematical tools of general relativity. More specifically this includes: the coordinate distance, ds^2 ; the Minkowski metric $\eta_{\mu\nu}$; the generalized metric, $g_{\mu\nu}$; the Christoffel symbol, $\Gamma_{\mu\nu}^\lambda$; the Riemann Tensor, $R_{\alpha\sigma\beta}^\tau$; Ricci Scalar, R ; and ultimately has been introduced to Einsteins equation:

$$R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (3.1)$$

A great and quick mathematical introduction to these concepts can be found in paper [12] or more thoroughly in Carroll's text. The purpose of this chapter is to expose the reader to gravitational waves, and use this math to supplement the simplified math for the lecture plan.

3.1 Linearized Gravity and Gravitational Waves

When Einstein's equation is typically introduced, is it usually verified by considering the Newtonian limit, thus providing the constants of Equation 3.1. This means that the gravitational field can be expressed as a weak field, static (no time derivative), and that the test particles are moving slowly ($v \ll c$). Gravitational waves are difficult to consider in the weak field limit as sources of gravitational waves have strong fields, are highly energetic, and highly time dependent. Luckily, at a great distance away from the source, the highly energetic gravitational waves have lost much of their strength, which allows the weak field limit to be considered. Negating the restrictions on the motion of test particles, and taking the time dependence into

account, allows the phenomena of gravitational waves, that is otherwise absent or ambiguous in Newtonian Theory, to emerge.

The weakness of the gravitation field allows the metric to be decomposed in the Minkowski metric plus a small perturbation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \quad (3.2)$$

To linearize gravity, start with the Christoffel symbols, which will be given by:

$$\begin{aligned} \Gamma_{\mu\nu}^{\rho} &= \frac{1}{2}g^{\rho\lambda}(g_{\nu\lambda,\mu} + g_{\lambda\mu,\nu} - g_{\mu\nu,\lambda}) \\ &= \frac{1}{2}\eta^{\rho\lambda}(h_{\nu\lambda,\mu} + h_{\lambda\mu,\nu} - h_{\mu\nu,\lambda}). \end{aligned} \quad (3.3)$$

The reason the second line in Equation 3.3 was carried out, is because we are only considering Einsteins equation to the first order. Since the Christoffel symbols are first-order quantities, the only contribution to the Riemann tensor will come from the derivatives of the them. Lowering the index:

$$\begin{aligned} R_{\mu\nu\rho\sigma} &= \eta_{\mu\lambda}\Gamma_{\nu\sigma,\rho}^{\lambda} - \eta_{\mu\lambda,\rho}\Gamma_{\nu\sigma}^{\lambda} \\ &= \frac{1}{2}(h_{\mu\sigma,\nu\rho} + h_{\nu\rho,\mu\sigma} - h_{\mu\rho,\nu\sigma} - h_{\nu\sigma,\mu\rho}) \end{aligned} \quad (3.4)$$

and contracting over μ and ρ gives the Ricci tensor:

$$R_{\mu\nu} = \frac{1}{2}(h_{\mu,\nu\rho}^{\sigma} + h_{\nu,\mu\sigma}^{\sigma} - h_{,\nu\sigma} - \square h_{\mu\nu}) \quad (3.5)$$

contracting again gives the Ricci scalar:

$$R = h_{,\nu\mu}^{\mu\nu} - \square h \quad (3.6)$$

where:

$$h = \eta^{\mu\nu}h_{\mu\nu} = h_{\mu}^{\mu} \quad (3.7)$$

and \square is the d'Alembertian operator where for any arbitrary f [12]:

$$\square f = -\frac{\partial^2 f}{\partial t^2} + \nabla^2 f. \quad (3.8)$$

Rearranging the terms in Equation 3.5 [12]:

$$R_{\mu\nu} = \frac{1}{2}[-\square h_{\mu\nu} + \frac{\partial}{\partial x^{\mu}}(h_{\nu,\sigma}^{\sigma} - \frac{1}{2}h_{,\nu}) + \frac{\partial}{\partial x^{\nu}}(h_{\mu,\sigma}^{\sigma} - \frac{1}{2}h_{,\mu})]. \quad (3.9)$$

The choice of coordinates is completely arbitrary, which can allow us to make the terms in the parentheses vanish if we choose coordinates in which:

$$h_{\mu,\sigma}^\sigma - \frac{1}{2}h_{,\mu} = 0 \quad (3.10)$$

Fortunately, there are four degrees of freedom in Equation 3.9, and the choice of coordinates to make Equation 3.10 true is called the Lorentz gauge [12]. A detailed description of gauge transformations can be found in Chapter 7 of Carroll's text or in paper [20]. Intuitively, the Lorentz gauge transformation describes the set of perturbation metrics that leave the physical space, or curvature, unchanged [3]. The Ricci tensor now takes the form:

$$R_{\mu\nu} = -\frac{1}{2}\square h_{\mu\nu}. \quad (3.11)$$

The linearized Einstein equation then becomes:

$$-\frac{1}{2}\square h_{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}R = -\frac{16\pi G}{c^4}T_{\mu\nu} \quad (3.12)$$

while the vacuum solution assumes $R = 0$ and $T_{\mu\nu} = 0$ to adjust Equation 3.12 to:

$$\square h_{\mu\nu} = 0 \quad (3.13)$$

which is simply the conventional relativistic wave equation. General Relativity predicts the existence of traveling ripples in spacetime itself.

3.2 Gravitational Wave Solutions

The solutions to Equation 3.13 hold for small perturbations $h_{\mu\nu}$ on a background Minkowski metric. One such solution is the plane-wave solution [12]:

$$h_{\mu\nu} = A_{\mu\nu} \exp(ik_\sigma x^\sigma) \quad (3.14)$$

for some (complex) amplitude $A_{\mu\nu}$ and wave number k_β , with $k_\beta = (\omega/c, k_x, k_y, k_z)$. If solution 3.14 is plugged into Equation 3.13 then:

$$\begin{aligned}
0 &= \square h_{\mu\nu} \\
&= \eta^{\rho\sigma} \partial_\rho \partial_\sigma h_{\mu\nu} \\
&= \eta^{\rho\sigma} \partial_\rho (ik_\sigma h_{\mu\nu}) \\
&= -\eta^{\rho\sigma} k_\rho k_\sigma h_{\mu\nu} \\
&= -k_\sigma k^\sigma h_{\mu\nu}
\end{aligned} \tag{3.15}$$

One way for this to be true is if $k_\sigma k^\sigma = 0$, which means that k_σ is a null-vector, or in other words, that its length is zero, implying that gravitational waves must travel at the speed of light [12]. At this point the amplitude $A_{\mu\nu}$ has been neglected, however the Lorentz gauge condition (Equation 3.10) can be employed with our solution (Equation 3.14) to derive properties for this tensor [12]. The choice of indices within this is partially arbitrary, which allows the Lorentz gauge condition to be written as:

$$\eta^{\beta\nu} h_{\alpha\nu,\beta} - \frac{1}{2} \eta^{\beta\alpha} h_{\beta\alpha,\alpha} = 0 \tag{3.16}$$

where the values within the exponent in the first perturbation metric is $k_\beta x^\beta$ and $k_\alpha x^\alpha$ in the second. The final production of Equation 3.16 gives [12]:

$$k_\beta A_\alpha^\beta - \frac{1}{2} k_\alpha A_\beta^\beta = 0 \tag{3.17}$$

Considering the case for $\alpha = 0$:

$$k_\beta A_0^\beta - \frac{1}{2} k_0 A_\beta^\beta = 0 \tag{3.18}$$

$k_0 \neq 0$, because k is a null vector. This means that the only way to satisfy Equation 3.18 is if $A_\mu^\mu = 0$, or for A to be traceless. This simplifies Equation 3.17:

$$k_\beta A_\alpha^\beta = 0 \tag{3.19}$$

This is the dot product of the propagation direction (k) with the wave amplitude (A). This immediately means that these two are perpendicular, imposing that these waves are transverse [12]. Since $k_\beta A_0^\beta = 0$, it must mean that $A_0^\beta = 0$ for all β . The symmetry of h_β^α , from the notion that linearized theory describes a symmetric tensor on a flat background [3], must also mean that A is symmetric, $A_\beta^0 = 0$ for all β [12].

To further reduce the solution, if the wave is traveling in the x^3 direction then $k_\beta = (\omega, 0, 0, \omega)$, from its null vector properties. This then implies that $A_\beta^3 = A_3^\beta = 0$ for all β (radiation in the x^3 direction) [12]. Thus, the amplitude A of the waves has the simple form: $A_{12} = A_{21} \neq 0$ and $A_{11} = -A_{22} \neq 0$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{21} & A_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3.20)$$

In terms of the perturbation, h [12]:

$$h_{\alpha\beta}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(h_{xx} - h_{yy}) & h_{xy} & 0 \\ 0 & h_{xy} & \frac{1}{2}(h_{yy} - h_{xx}) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3.21)$$

This solution for the perturbation is called the transverse traceless gauge, hence the TT.

Polarization

At this point it is important to understand how a passing gravitational wave will affect two particles on nearby geodesics. Imagine two test particles that are essentially at rest, $U = (1, 0, 0, 0)$, and separated by a displacement vector ξ^α . The geodesic deviation equation for these particles is [20]:

$$\nabla_U \nabla_U \xi^\alpha = R_{\mu\nu\beta}^\alpha U^\mu U^\nu \xi^\beta \quad (3.22)$$

The two covariant derivatives ∇_U simplify to being regular derivatives in respect to proper time, from the fact that its the only non-zero part of U . Simplifying Equation 3.22 and lowering the first index:

$$\begin{aligned} \frac{d^2 \xi_\alpha}{d\tau^2} &= R_{\alpha\mu\nu\beta} U^\mu U^\nu \xi^\beta \\ &= R_{\alpha 00\beta} \xi^\beta \end{aligned} \quad (3.23)$$

where only the non-vanishing terms of U were considered in the final step. With the use of Equation 3.4, the Riemann tensor written in terms of the transverse traceless gauge (Equation 3.21) gives:

$$\begin{aligned} R_{\alpha 0 0 \beta} &= \frac{1}{2}(h_{\alpha\beta,00}^{TT} + h_{00,\alpha\beta}^{TT} - h_{\alpha 0,\beta 0}^{TT} - h_{\beta 0,\alpha 0}^{TT}) \\ &= \frac{1}{2}h_{\alpha\beta,00}^{TT} = \frac{1}{2}\frac{\partial^2}{\partial t^2}h_{\alpha\beta}^{TT} \end{aligned} \quad (3.24)$$

where the second line was carried out because $h_{\alpha 0}^{TT} = 0$ for all α . Substituting this back into Equation 3.23:

$$\frac{d^2\xi_\alpha}{d\tau^2} = \frac{1}{2}\xi_\beta \frac{\partial^2}{\partial t^2}h_{\alpha\beta}^{TT} \quad (3.25)$$

If the wave is traveling in the x^3 direction, then $h_{\alpha\beta}^{TT}$ will take on the form in Equation 3.21. It is then advisable to define $h_+ = A_{11}$ and $h_\times = A_{21}$. There are two case that still allow $h_{\alpha\beta}^{TT}$ to be true, (1) $h_+ = 0$ and (2) $h_\times = 0$.

For case (1), Equation 3.25 becomes:

$$\frac{d^2\xi_1}{d\tau^2} = \frac{1}{2}\xi^1 \frac{\partial^2}{\partial t^2}[h_+ \exp(ik_\alpha x^\alpha)] \quad (3.26)$$

$$\frac{d^2\xi_2}{d\tau^2} = -\frac{1}{2}\xi^2 \frac{\partial^2}{\partial t^2}[h_+ \exp(ik_\alpha x^\alpha)] \quad (3.27)$$

where solutions to these equations are [12]:

$$\xi_1 = \xi_1(0)[1 + \frac{1}{2}h_+ \exp(ik_\alpha x^\alpha)] \quad (3.28)$$

$$\xi_2 = \xi_2(0)[1 - \frac{1}{2}h_+ \exp(ik_\alpha x^\alpha)] \quad (3.29)$$

For case (2), Equation 3.25 becomes:

$$\frac{d^2\xi_1}{d\tau^2} = \frac{1}{2}\xi^1 \frac{\partial^2}{\partial t^2}[h_\times \exp(ik_\alpha x^\alpha)] \quad (3.30)$$

$$\frac{d^2\xi_2}{d\tau^2} = \frac{1}{2}\xi^2 \frac{\partial^2}{\partial t^2}[h_\times \exp(ik_\alpha x^\alpha)] \quad (3.31)$$

where solutions to these equations are [12]:

$$\xi_1 = \xi_1(0)[1 + \frac{1}{2}h_\times \exp(ik_\alpha x^\alpha)] \quad (3.32)$$

$$\xi_2 = \xi_2(0)[1 - \frac{1}{2}h_\times \exp(ik_\alpha x^\alpha)] \quad (3.33)$$

From these solutions, gravitational waves have two possible polarizations. One with amplitude h_+ that oscillates the two test bodies along the x and y axis, in a cross

pattern, top of Figure 3.1. The other, h_{\times} , that oscillates 45° from the first, in a x pattern, bottom of Figure 3.1.

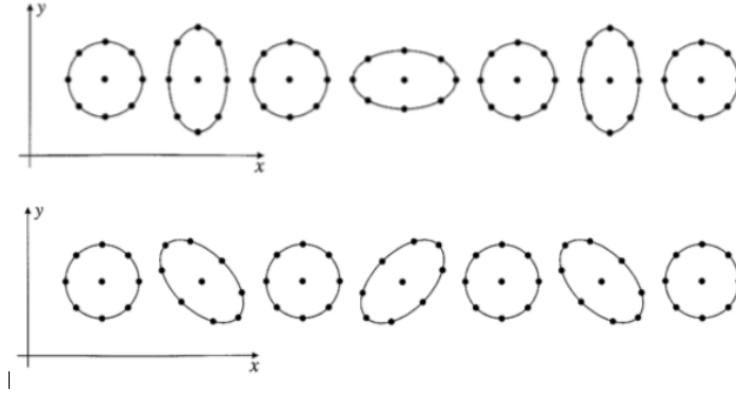


Figure 3.1: Top: Gravitational wave that oscillates test bodies in a cross pattern. Bottom: Gravitational wave that oscillates test bodies in a x pattern. [2]

3.3 Production of Gravitational Waves

Up to this point, plane wave solutions to the linearized vacuum equation have been derived. For the purpose of the later classroom activity that discusses gravitational wave sources, it is necessary to consider Einsteins equation when coupled to matter, $R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$. The stress energy tensor $T_{\mu\nu}$ doesn't vanish, which will cause the metric perturbation to be very complex, but imposing the restraint that the metric perturbation at very large distance from the source will produce the original metric perturbation which will simplify the calculation.

To begin, the trace reversed perturbation is defined as:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} \quad (3.34)$$

where no information has been lost from the original perturbation in Equation 3.2 because:

$$\bar{h} = \eta^{\mu\nu}\bar{h}_{\mu\nu} = -h \quad (3.35)$$

where the property $\eta^{\mu\nu}\eta_{\mu\nu} = 4$ has been used. It is noted that in a vacuum far away from any source the transverse traceless gauge and trace reversed perturbation will be equal:

$$\bar{h}_{\mu\nu}^{TT} = h_{\mu\nu}^{TT} \quad (3.36)$$

By following the same steps in section 3.1, the trace reversed perturbation can be introduced in the expression for the Ricci tensor (Equation 3.9) and the Ricci scalar (Equation 3.6), in which the Lorentz gauge coordinate choice can be utilized to produce the following linearized Einstein equation:

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \quad (3.37)$$

The generalized solution has the generic form [12]:

$$\bar{h}_{\mu\nu} = \frac{4\pi G}{c^4} \int \frac{T_{\mu\nu}(\mathbf{x}', t^{ret})}{|\mathbf{x} - \mathbf{y}|} d^3y \quad (3.38)$$

where the integral is taken over all space, but its effects at position \mathbf{x} are only taken at the retarded time, $t^{ret} = t - |\mathbf{x} - \mathbf{x}'|/c$. The retarded time is the effect that the gravitational field isn't instantaneously changed when the source, $T_{\mu\nu}(\mathbf{x}', t)$, undergoes some change [12]. The derivation of this solution is obtained through the use of a Green function and its identity with the d'Alembertian operator \square :

$$\square_x G(x^\sigma - y^\sigma) = \delta^4(x^\sigma - y^\sigma) \quad (3.39)$$

where \square_x denotes the d'Alembertian with respect to coordinates x^σ . Equation 3.37 can be expressed as:

$$\begin{aligned} \square_x \bar{h}_{\mu\nu}(x^\sigma) &= -\frac{16\pi G}{c^4} T_{\mu\nu}(x^\sigma) \\ &= -\frac{16\pi G}{c^4} \int \square_x G(x^\sigma - y^\sigma) T_{\mu\nu}(y^\sigma) d^4y \end{aligned} \quad (3.40)$$

Since \square_x only acts on the x^σ coordinates on the right hand side of the equation it can be pulled out leaving:

$$\bar{h}_{\mu\nu}(x^\sigma) = -\frac{16\pi G}{c^4} \int G(x^\sigma - y^\sigma) T_{\mu\nu}(y^\sigma) d^4y \quad (3.41)$$

The solutions to the Green function (Equation 3.40) have been worked out to produce waves traveling forward or backward (retarded) in time. In the case of gravitational waves only the retarded solution matters, as it explains the accumulated effect of

signals to the past of the points under consideration. The retarded solution returns Equation 3.38.

This integral now needs to be evaluated to figure out the value of $\bar{h}_{\mu\nu}$. Under the assumption that the gravitational waves are being evaluated at a great distance away from the size of the source Equation 3.38 reduces to:

$$\bar{h}_{\mu\nu} = \frac{4\pi G}{rc^4} \int T_{\mu\nu}(\mathbf{y}, t^{ret}) d^3y \quad (3.42)$$

where $|\mathbf{x} - \mathbf{y}|$ has been replaced by r and pulled out, essentially implying that ($r = \mathbf{x}) \gg \mathbf{y}$, which isn't being integrated over.

If the conservation of mass energy is employed then the right hand side of the equation can be rewritten. It is known:

$$T_{\mu\nu,\nu} = 0 \quad (3.43)$$

and from this:

$$T_{00,0} = -T_{0l,l} \quad (3.44)$$

Differentiating with respect to time, using the symmetry of $T_{\mu\nu}$, and the commutativity of partial derivatives gives:

$$T_{00,00} = -T_{0l,l0} = -T_{l0,0l} \quad (3.45)$$

The conservation of mass energy can again be used to give:

$$T_{m0,0m} = (T_{m0,0})_{,m} = -T_{ml,lm} \quad (3.46)$$

Substituting this into Equation 3.45:

$$(T_{00})_{,00} = T_{ml,lm} \quad (3.47)$$

If Equation 3.47 is multiplied by $x^k x^j$:

$$(T_{00} x^k x^j)_{,00} = T_{ml,lm} x^k x^j \quad (3.48)$$

where $x^k x^j$ are taken inside the derivative because they don't depend on time, and integrated by parts over all space makes the right hand side equal to:

$$\int (T_{ml} x^k x^j)_{,ml} d^3x = T_{ml,m} x^k x^j \Big| - \int T_{ml,m} (\delta_l^k x^j + \delta_l^j x^k) d^3x \quad (3.49)$$

The first term on the right hand side vanishes because a closed system, T_{lm} , is being evaluated at a far distance. Integrating by parts again:

$$(T_{ml}x^kx^j)_{,ml}d^3x = -(T_{mk}x^j| + T_{mj}x^k|) + 2 \int T_{jk}d^3x \quad (3.50)$$

Ignoring surface terms again, leaves only the final term on the right. The integrated Equation 3.48 is now reduced to:

$$2 \int T_{jk}d^3x = \int \frac{\partial^2}{\partial t^2} T_{00}x^kx^j = \frac{\partial^2}{\partial t^2} \int T_{00}x^kx^j \quad (3.51)$$

The term inside the integral on the far right is called the quadrupole moment tensor, I_{jk} , of the mass distribution of the source. This equation then becomes:

$$\frac{1}{2}\ddot{I}_{jk} = \int T_{jk}d^3x \quad (3.52)$$

Finally, substituting this expression back into Equation 3.42 gives:

$$\bar{h}_{jk} = \frac{2\pi G}{rc^4} \frac{1}{2} \ddot{I}_{jk} \quad (3.53)$$

So the lowest order multipole contributing to gravitational waves is the quadrupole, and only if the source has a time varying moment would it be expected to emit gravitational waves [12]. This is equivalent to electromagnetic radiation only being emitted by the change in dipole movement of the charge density.

Binary System

Consider a system of two (neutron) stars of approximately the same mass, in a circular orbit around the x^1-x^2 plane. For simplicity, the Newtonian approximation, where each star has mass M and the radius of orbit is R , the frequency of the orbital motion is:

$$\omega = \frac{2\pi}{\tau} = \frac{v}{R} = \left(\frac{GM}{4R^3}\right)^{1/2} \quad (3.54)$$

At a time t , the two stars are at the positions:

$$(x^1, x^2, x^3) = (R \cos(\omega t), R \sin(\omega t), 0) \text{ and } (-R \cos(\omega t), -R \sin(\omega t), 0) \quad (3.55)$$

The basic definition for the momentum of this binary system will be:

$$I_{jk}(t) = \sum_a m_a x^{aj}(t) x^{ak}(t) = 2M x^j(t) x^k(t) \quad (3.56)$$

where the compenents of the second moment I_{jk} are:

$$I_{jk}(t) = MR^2 \begin{pmatrix} 1 + \cos(2\omega t) & \sin(2\omega t) & 0 \\ \sin(2\omega t) & 1 - \cos(2\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.57)$$

and hence:

$$\ddot{I}_{jk}(t) = -4\omega MR^2 \begin{pmatrix} \cos(2\omega t) & \sin(2\omega t) & 0 \\ \sin(2\omega t) & -\cos(2\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.58)$$

and finally, plugging this back into Equation 3.53, reverting to a 4×4 form, will give a transverse traceless perturbation for radiation in the x^3 direction :

$$h_{jk}(t, r) = \frac{8MGR^2\omega^2}{c^4 r} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\cos(2\omega t) & -\sin(2\omega t) & 0 \\ 0 & -\sin(2\omega t) & \cos(2\omega t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3.59)$$

Chapter 4

Detection of Gravitational Waves

4.1 Indirect Detection

Its clear that since the binary system emits radiation in the form of gravitational waves that it will lose energy. This loss of energy must have observable consequences. If the total energy of the binary system is given by:

$$E = \frac{GM^2}{4R} - \frac{GM^2}{2R} + 2Mc^2 = -\frac{GM^2}{4R} + 2Mc^2 \quad (4.1)$$

where energy from rest masses has been included. As the system emits energy E , R must also decrease (assuming M is unaffected). This system will spin up and eventually the stars coalesce. The frequency of the emitted radiation is [2][25]:

$$L = \frac{2}{5} \left(\frac{GM}{Rc^2} \right)^5 \frac{c^5}{G}. \quad (4.2)$$

As R decreases the frequency of radiation increases, and this is known as a chirp [25]. This equation is more thoroughly derived in [2] and [25], but the frequency of emission is found by solving for the energy density (T_{00}) component of the field equation in terms of the transverse traceless perturbation metric (Equation 3.59). Both sides of the equation can then be integrated through a spherical surface of radius r , which gives an expression for rate of energy loss, Equation 4.2.

A measurable quantity that is a result of a shrinking orbit, is the change in the period of an orbit, from Equation 3.54:

$$\tau = 4\pi \left(\frac{R^3}{GM} \right)^{1/2} \propto R^{3/2} \quad (4.3)$$

as R changes τ will also,

$$\frac{d\tau}{\tau} = \frac{3}{2} \frac{dR}{R} \quad (4.4)$$

from Equation 4.1 $dE/E = -dR/R$ so:

$$\frac{d\tau}{\tau} = -\frac{3}{2} \frac{dE}{E} \quad (4.5)$$

The rate of change of the period is given by $d\tau/dt$, and from the equations above:

$$\frac{d\tau/dt}{\tau} = -\frac{3}{2} \frac{dE/dt}{E} = -\frac{3}{2} \frac{L}{E} = -\frac{12}{5} \frac{G^3 M^3}{R^4 c^5} \quad (4.6)$$

hence:

$$\frac{d\tau}{dt} = -\frac{48\pi}{5} \left(\frac{GM}{Rc^2} \right)^{5/2} \quad (4.7)$$

If $M = 1.4M_\odot$ and $R = 10^6 km$, then $d\tau/dt = -1.84 \times 10^{-13}$, in which this prediction was verified in 1974 by Russell Hulse and Joseph Taylor, who observed the decrease in the period of the pulsar PSR 1913+16 in a binary system to be $d\tau/dt = -(2.422 \pm 0.006) \times 10^{-12}$. Which is in agreement with the general relativistic prediction (Equation 4.7 when adjusted for eccentricity) to an accuracy of 0.3%, as seen in the figure below [25].

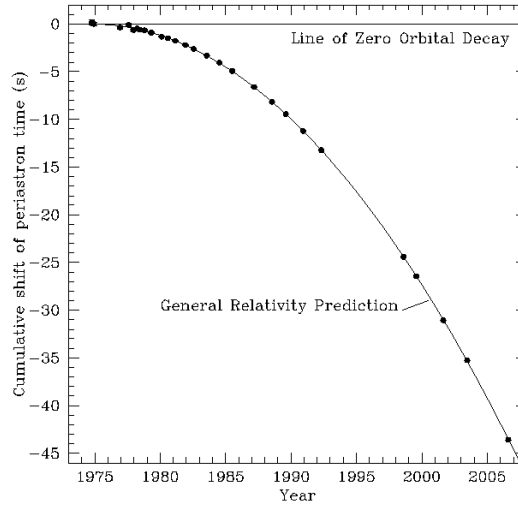


Figure 4.1: The evidence that Binary Pulsar B1913+16 emits gravitational radiation. As gravitational radiation carries energy away from the binary system, the orbit decreases [32].

4.2 Direct Detection with Interferometry

In contrast to the indirect effect of gravitation wave radiation, its even more important to explore how they act on test bodies. An important realization to make is that gravitational radiation is produced by the bulk motion of a large mass [2].

In order to put this into context, consider the binary system metric perturbation in Equation 3.59 and the frequency of Equation 3.54 to get an approximation of:

$$h = \frac{2G^2 M^2}{c^4 r R} \quad (4.8)$$

where h is called the strain, which describes the spacetime distorting effects of the gravitational wave. If the binary system consists of two black holes of 10 solar masses each, separated by ten times their Schwarzschild radius, and at a distance of 100 Mpc, then:

$$h \sim 10^{-21} \quad (4.9)$$

The hope of directly detecting the coalescence of a binary system would take an instrument sensitive enough to measure a strain of order 10^{-21} or less [2]. For example, if there's a bar with a length of 10 m, the change of length would be 10^{-20} m, that's approximately one one-hundred-thousandth the size of a nucleus [25]. It takes a very sensitive instrument to detect such minute changes.

Michelson Interferometer

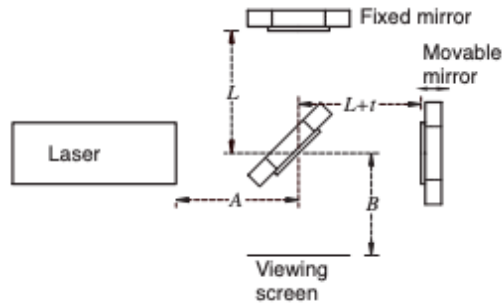


Figure 4.2: Michelson interferometer. [16]

Laser interferometers provide a way to overcome the difficulty of measuring the minuscule perturbations. Interferometers work by merging together two or more sources of light to create an interference pattern. This pattern can be measured and analyzed to give insight on the object or phenomenon being studied.

The interference pattern is a very simple concept. Light waves consists of electric fields. If two waves of the same frequency were to propagate in the same region of space then the resultant electric field at any point in that region is the vector sum of the electric field of each wave. If the phase difference $\Delta\phi$ between the two waves is $\Delta\phi = 2\pi m$, $m = 0, \pm 1, \pm 2, \pm 3...$, their interaction is said to be in phase resulting in constructive interference and a bright spot. If the phase difference between the two waves is $\Delta\phi = \pm(2m+1)\pi$, $m = 0, 1, 2, 3...$, their interaction is said to be out of phase resulting in destructive interference and a dark spot. Another key point to make is that the phase difference can be characterized by wavelength λ and optical path difference ΔL by:

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L \quad (4.10)$$

The most basic interferometer set up is a Michelson interferometer, seen in Figure 4.2. The basic elements are a beam splitter, a monochromatic laser, and two mirrors. Light enters from the left, E . It hits the beam splitter, which transmits and reflects half of the beam in two perpendicular paths, $E_1 = E_2 = (1/\sqrt{2})E_{\text{mich}}$. These light waves transverse the arms lengths, L_1 and L_2 , where they are reflected back by the mirrors, with reflectively r_1 and r_2 , to recombine at the beam splitter to form the fields [5]:

$$E_{\text{symm}} = \frac{1}{2}E[r_2 \exp(-i2kL_2) + r_1 \exp(-i2kL_1)] \quad (4.11)$$

and

$$E_{\text{anti}} = \frac{1}{2}E_{\text{mich}}[r_2 \exp(-i2kL_2) - r_1 \exp(-i2kL_1)] \quad (4.12)$$

where $\exp(-2ikL)$ is the phase shift after the reflection, E_{symm} is the light going back towards the laser and E_{anti} is the light being outputted on the viewing screen. With $\bar{L} = \frac{1}{2}(L_1 + L_2)$ and $\Delta L = L_1 - L_2$, the effective reflectivity (symm part) and transmissivity (anti part) for the Michelson interferometer can be written as [5]:

$$r = \frac{E_{\text{symm}}}{E_{\text{mich}}} = \frac{1}{2} \exp(-2ik\bar{L})[r_2 \exp(ik\Delta L) + r_1 \exp(-ik\Delta L)] \quad (4.13)$$

and

$$t = \frac{E_{\text{anti}}}{E_{\text{mich}}} = \frac{1}{2} \exp(-2ik\bar{L})[r_2 \exp(ik\Delta L) - r_1 \exp(-ik\Delta L)] \quad (4.14)$$

if $r_1 = r_2 = 1$ (perfect mirrors), then [5]:

$$r = \exp(-2ik\bar{L}) \cos(k\Delta L) \quad (4.15)$$

and

$$t = i \exp(-2ik\bar{L}) \sin(k\Delta L) \quad (4.16)$$

If the arms are the same length then $|r| = 1$ and $t = 0$, which means that there is no light transmitted to the viewing screen. This type of precision is very difficult to achieve, so its more common to see an interference pattern on the viewing screen. An example of this pattern can be seen in Figure 6.9a. The fringing effect is due to the wavelike properties of light, or assuming that light is an expanding spherical waveform.

At this point it's partially clear that an interferometer can be used to detect gravitational waves. The intensity of the light measured by a photodiode on the viewing screen depends on how in phase the two beams/fields are, $I \propto |E_{\text{anti}}|^2$ [5]. The strain h is related to t , Equation 4.14, because if there is no gravitational wave present then $\Delta L = 0$, but when a wave passes the strain causes the length of each arm to change ($\Delta L \neq 0$), see Subsection 3.2, which then allows light to pass to the photodiode and produce a photocurrent. Strain is then related to the change in length by $\Delta L/L = h$.

4.3 LIGO

At its heart, LIGO is a well aligned large scale Michelson interferometer where $L = 4\text{km}$. However, it's a little more complicated than that. For example, the length L is still not sensitive enough to detect micro changes caused by the strain, the intensity of light has a large range of uncertainty because the photons of light that make up the beam are randomly distributed, and the laser doesn't emit perfectly coherent light at a single phase [19].

Fabry Perot Cavity

Simplistically, there are two additional aspects to the Michelson interferometer that help negate those complications. The first and main addition are the use of Fabry Perot cavities, shown in Figure 4.3.

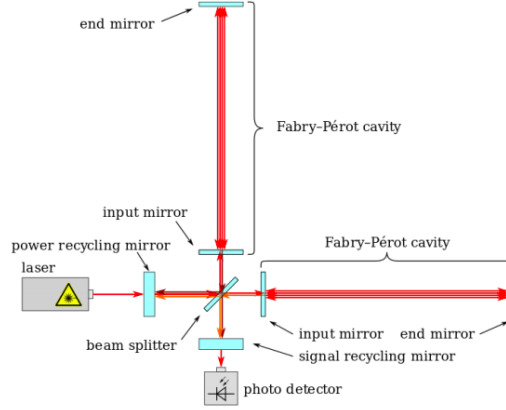


Figure 4.3: LIGO includes two Fabry-Perit cavities in each arm [19]

An additional mirror is placed in each arm near the beam splitter, where the distance of this cavity is 4 km. After entering the cavity from the beam splitter, the laser in each arm bounces between the two mirrors about 300 times before being combined with the other beam [33]. These cavities help increase the distance traveled by each laser to 1200 km, which makes the instrument more sensitive. This addition also boosts the power of the beam. Consider the cavity in Figure 4.4:

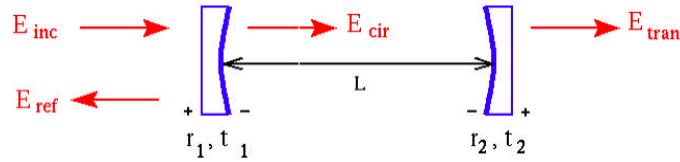


Figure 4.4: Optical configuration of Fabry Perot cavity [31]

Light from the beam splitter is labeled as E_{inc} . From this diagram the equations for the electric fields E_{cir} and E_{tran} are [31]:

$$\begin{aligned} E_{\text{cir}} &= t_1 E_{\text{inc}} + r_1 r_2 \exp(-2ikL) E_{\text{cir}} \\ &= \frac{t_1}{1 - r_1 r_2 \exp(-2ikL)} E_{\text{inc}} \end{aligned} \quad (4.17)$$

and

$$\begin{aligned} E_{\text{tran}} &= t_2 \exp(-2ikL) E_{\text{cir}} \\ &= \frac{t_1 t_2 \exp(-2ikL)}{1 - r_1 r_2 \exp(-2ikL)} E_{\text{inc}} \end{aligned} \quad (4.18)$$

when $2kL = n(2\pi)$, ie, $L = n\lambda/2$, and $r_2 \approx 1$, E_{cir} is then maximized and $E_{\text{tran}} = 0$ which creates a resonating circulating field, and through Poissonian statistics, a more intense beam reduces the random light distribution fluctuations [31]. The reflective field that gets sent back to the beam splitter can now be computed:

$$\begin{aligned} E_{\text{ref}} &= r_1 E_{\text{inc}} - t_1 r_2 \exp(-2ikL) E_{\text{cir, res}} \\ &= \left[\frac{r_1 - \exp(-2ikL)}{1 - r_1 \exp(-2ikL)} \right] E_{\text{inc}} \end{aligned} \quad (4.19)$$

The effective reflectivity of the Fabry Perot cavity is defined as $r_{\text{fp}} = E_{\text{ref}}/E_{\text{inc}}$, and if the cavity, in resonance L_{res} , is perturbed by a small amount ΔL [5]:

$$\begin{aligned} r_{\text{fp}}(L_{\text{res}} + \Delta L) &\simeq r_{\text{fp}}(L_{\text{res}}) + \frac{dr_{\text{fp}}}{dL} \Big|_{L=L_{\text{res}}} \Delta L \\ &= -1 + (2ik\Delta L) \left[\frac{\exp(-2ikL)(1 - r_1^2)}{(1 - r_1 \exp(-2ikL))^2} \right]_{L=L_{\text{res}}} \\ &= -1 - (2ik\Delta L) \frac{1 + r_1}{1 - r_1} \\ &= -\exp \left[2ik \left(\frac{1 + r_1}{1 - r_1} \right) \Delta L \right] \end{aligned} \quad (4.20)$$

This shows that a small change in the cavity length ΔL will cause a large change in the phase of this reflected light due to the arm cavity gain G , which is hidden in the parenthesis in the final line of Equation 4.20. For LIGO, $G \sim 130$ [31], which means that the Fabry Perot Cavities makes LIGO 130 times more sensitive than if it only transversed the arm once [5]. This configuration also only allows a very narrow range of frequencies where power can be built up, limiting the incoherent property of light.

Fabry Perot Michelson interferometer

Although the length of the arms have been treated as equal, LIGO actually has them at an imbalance [5], L_1 and L_2 . The addition of the cavities also changes the effective

reflection and transmission coefficients of the Michelson interferometer in Equations 4.13 and 4.14 [5]:

$$\begin{aligned} r_{\text{fpmi}}(L_1, L_2) &= \frac{E_{\text{symm}}}{E_{\text{mich}}} \\ &= \frac{1}{2} \exp(-2ik\bar{L}) [r_{fp}(L_2) \exp(ik\Delta L) + r_{fp}(L_1) \exp(-ik\Delta L)] \end{aligned} \quad (4.21)$$

and

$$\begin{aligned} t_{\text{fpmi}}(L_1, L_2) &= \frac{E_{\text{anti}}}{E_{\text{mich}}} \\ &= \frac{1}{2} \exp(-2ik\bar{L}) [r_{fp}(L_2) \exp(ik\Delta L) - r_{fp}(L_1) \exp(-ik\Delta L)] \end{aligned} \quad (4.22)$$

When the light is at resonance in both respective cavities:

$$r_{\text{fpmi, res}} = -\exp(-2ik\bar{L}) \cos(k\Delta L) \quad (4.23)$$

and

$$t_{\text{fpmi, res}} = -i \exp(-2ik\bar{L}) \sin(k\Delta L) \quad (4.24)$$

When they are slightly displaced from resonance:

$$\begin{aligned} r_{\text{fpmi}}(L_{1, \text{res}} + \Delta L_1, L_{2, \text{res}} + \Delta L_2) &= \\ r_{\text{fpmi, res}} + \frac{1}{2} \exp(-2ik\bar{L}) G[2ik\Delta L_2 \exp(-2ik\bar{L}) + & \\ 2ik\Delta L_1 \exp(-2ik\bar{L})] & \end{aligned} \quad (4.25)$$

and

$$\begin{aligned} t_{\text{fpmi}}(L_{1, \text{res}} + \Delta L_1, L_{2, \text{res}} + \Delta L_2) &= \\ t_{\text{fpmi, res}} + \frac{1}{2} \exp(-2ik\bar{L}) G[2ik\Delta L_2 \exp(-2ik\bar{L}) - & \\ 2ik\Delta L_1 \exp(-2ik\bar{L})] & \end{aligned} \quad (4.26)$$

These reflective and transmission coefficients are thus the result of the compound connection between the Michelson interferometer and Fabry Perot cavities.

Power Recycling

The next addition is the use of a power recycling mirror (prm). This is used to further increase the light power and increase sensitivity. The mirror is placed between the laser and beam splitter, which effectively creates another cavity between it and the compound mirror of the Fabry Perot Michelson interferometer, r_{fpmi} and t_{fpmi} . In

this set, E_{inc} is now entering the power recycling cavity, E_{ref} is the light reflecting off of the mirror and back to the laser, and E_{mich} and E_{symm} are the same ideas from Equation 4.11. The following relations then hold [5]:

$$E_{\text{mich}} = t_{\text{prm}} E_{\text{inc}} - r_{\text{prm}} E_{\text{symm}} \quad (4.27)$$

$$E_{\text{ref}} = r_{\text{prm}} E_{\text{inc}} + t_{\text{prm}} E_{\text{symm}} \quad (4.28)$$

$$E_{\text{symm}} = r_{\text{fpmi}} E_{\text{mich}} \quad (4.29)$$

$$E_{\text{anti}} = t_{\text{fpmi}} E_{\text{mich}} \quad (4.30)$$

It can then be shown that:

$$\frac{E_{\text{mich}}}{E_{\text{inc}}} = \frac{t_{\text{prm}}}{1 + r_{\text{prm}} r_{\text{fpmi}}} \quad (4.31)$$

The effective reflectivity and transmissivity are then:

$$r_{\text{prfpmi}} = \frac{E_{\text{ref}}}{E_{\text{inc}}} = \frac{r_{\text{prm}} + r_{\text{fpmi}}}{1 + r_{\text{prm}} r_{\text{fpmi}}} \quad (4.32)$$

and

$$t_{\text{prfpmi}} = \frac{E_{\text{anti}}}{E_{\text{inc}}} = \frac{r_{\text{prm}} t_{\text{fpmi}}}{1 + r_{\text{prm}} r_{\text{fpmi}}} \quad (4.33)$$

To achieve a power build up within the recycling cavity, $E_{\text{mich}}/E_{\text{inc}}$ needs to be maximized. The gain is then defined as [5]:

$$G_{\text{prm}} = \left| \frac{E_{\text{mich}}}{E_{\text{inc}}} \right|^2 \quad (4.34)$$

where for LIGO, $G \sim 50$.

Signal Readout

The antisymmetric port of the power recycled Fabry Perot Michelson interferometer is dark when the light is in resonance. When there is a signal present the photodiode measures the intensity of light, which produces a photocurrent [5]:

$$I \propto |E_{\text{anti}}|^2 \quad (4.35)$$

where:

$$E_{\text{anti}} = E_{\text{inc}} G_{\text{prm}}^{1/2} t_{\text{fpmi}}(L_1, L_2) \quad (4.36)$$

For the choice of \bar{L} and ΔL , where $t_{\text{fpmi, res}} = 0$, $\exp(-2k\bar{L}) = 1$, and $\cos(k\Delta L) = 1$, Equation 4.26 becomes:

$$t_{\text{fpmi}}(L_{1, \text{res}} + \Delta L_1, L_{2, \text{res}} + \Delta L_2) = -iGk\Delta L' \quad (4.37)$$

and

$$E_{\text{anti}} = -iE_{\text{inc}}G_{\text{prm}}^{1/2}Gk\Delta L' \quad (4.38)$$

where $\Delta L' = \Delta L_1 - \Delta L_2$. This is the displacement of the arms from their resonant length. The photocurrent can then be expressed as:

$$I = I_0G_{\text{prm}}G^2(k\Delta L')^2 \quad (4.39)$$

where I_0 is the power of the laser light from its original source, $|E_{\text{inc}}|^2$. The change in length by a gravitational wave will then produce a detectable signal.

Extraction of Signal

However, there is a lot of noise that is measured by LIGO. These are due to fundamentals noises, like quantum and Brownian noise, but also from seismic and electronic noise [18]. These noises drown out the signal caused by the gravitational wave, which makes it very difficult to pull the signal out. In order to pull signals from the noisy data, LIGO has created templates of the types of sources that are expected to produce gravitational waves which can then be compared to the experimental data.

The models that are created are built by solving the Einstein field equation through numerical simulations. There are a large number of solutions of interest, and the production of them on supercomputers is expensive [18], which makes it not possible to simulate every solution. Instead, a small number of models are made where freely adjustable parameters that links the models are used to match all available numerical simulations.

To get a feel of how much data analysis and template matching it takes, its important to understand the sheer amount of data that's produced by LIGO. The nominal sampling rate of the data is 16.384 kHz, which produces 16,384 samples per second [18]. Its then resampled at 2048 kHz, thus reducing the amount of data by 8, where its then compared to approximately 250,00 template waveforms, which

produces around 500 million samples of signal to noise ration (SNR) for every second of data collected [18]. The signal extraction process then passes through various thresholds of SNR filters to quickly reduce the amount of data. This data is then verified with the partner instrument to further reduces the sample.

The log likelihood ratio (LLR) for each candidate is then calculated by taking the log of the ratio of the probability that a true signal will produce a candidate with some exact set of properties to the probability that noise alone would produce a candidate with the exact same properties [18]. The larger the LLR the stronger the candidate. When a real signal is present, the data is usually surrounded by a large number of candidates. A sort of Monte Carlo method is then used to reduce the sample to have only very high LLRs. In the most recent run this reduced the amount to 10,000 [18]. Each candidate is then modeled as a trial in the noise. This allows the determination of how many candidates would be expected above some certain LLR value [18]. The criterion that LIGO uses to determine if the signal is real is that if it takes 1 million repetitions of the same experiment to obtain the results just due to chance and noise, then the signal is real and a gravitational wave has been detected [18].

Chapter 5

Related Work

Up to this point there are a couple factors that are essential in bringing gravitational wave astronomy into the undergraduate classroom. These include: the effective procedures of a successful classroom demonstration, the understanding of what a gravitational wave is and how it warps spacetime, the associated strain, and the process it takes to detect them. These factors can be scaled to various age ranges, with each range becoming increasingly more mathematically complex. There are a few resources that are already published, which will be analyzed to address their age range, relative time frame for conducting the activity, effectiveness in addressing the factors essential for gravitational wave astronomy, costs, and how well they follow the procedures of a classroom demonstration.

5.1 LIGOs Build an Interferometer

LIGO currently has two "build your own interferometer" projects, one with glue [15], the other with magnets [9]. The only functional difference between the two being that the magnetic project is designed to be taken apart and rebuilt by new sets of students, compared to permanent setup of the glue one. It is also implied in the introduction paragraph of [9] that the overview and learning objectives can be found in [15]. Beyond that, the magnetic one copies the student handout of [15] directly.

At the top of paper [15], it states that the target age group is from 9th to 12th graders. The learning objectives then proceeded to only cover the skill it takes to build a precision device, the basic explanation of a Michelson interferometer, and the relationship between moving a mirror and the fringing patterns. Both papers then dive into the construction of each device, each model costing roughly \$150, which is

primarily due to the use of expensive optics (beam splitter, mirror mounts, mirrors). It was stated that expensive optics were needed to have quality fringing patterns. Both papers then explain various demonstrations to do: create vibrations, heat the air on one arm, or heat the surface of one arm. They then conclude with a handout that states that the use of outside resources is necessary in order to answer questions about interferometers. Two questions for example were [15]:

1. Use the concept of interference to explain how the interferometer works.
2. Interferometers can be used to measure movements that are much, much smaller that you would measure with tools such as rulers. Using your experience of pulling on the mirror, explain how this is possible.

Its obvious that these "build your own interferometers" are geared towards a high school classroom setting because no mathematical theory was introduced or tested and a handout that might take about an hour to complete wouldn't be a reasonable use of time in a college classroom setting. These papers also don't sell the demonstration device as an aspect that can be used to supplement mathematical theory. This automatically rules out analysis that can be taken on its use as an effective demonstration device. These papers also fail to mention anywhere that the Michelson interferometer is the foundational element to gravitational wave detection, and only ask students to search for what a Michelson interferometer is used for in real life.

The basis of the work in this thesis imitates these devices, however these devices are fairly expensive, aren't formatted to support mathematical theory in the form of a demonstration, and, surprisingly, doesn't mention any use on how its used to detect gravitational waves.

5.2 Gravitational Wave Open Science Center

The LIGO and Virgo collaboration has a very sophisticated website, the Gravitational Wave Open Science Center [13], that provides access to their data and hands on tutorials on how to use software tools to analyze the data sets.

On the website it clearly states that the tutorials are geared towards graduate students or senior scientists that wish to learn about this data and software in order to conduct research in the field of gravitational-wave astronomy [13]. It also states that there are approximately 5 hours of lectures and roughly 10-30 hours of data analysis programming exercises, where the prerequisites are a working knowledge of python, Fourier transforms, the fundamentals of what a gravitational wave is in terms of the strain, the theory behind templates or models that can be tested against the experimental data, and how the device measures the signal. The tutorials very nicely provide access to the 25 cases of a gravitational wave signals, 11 of which are confirmed, and essentially walks the learner through each process of first looking at the signal, second, the creation of a template, third, template matching, and finally the parameter estimation of compact object mergers.

This project is not geared towards early undergraduates. Although it's free and would be nice to show to undergraduate students what working with actual data is like, this project assumes a wide range of knowledge. This project provides the idea to include a more sensor driven approach to the demonstration device in order to give a live data stream, that might be used to quickly calculate factors associated with gravitational wave astronomy.

5.3 Penn State - Searching for Gravitational Waves in Noisy Data

Pennsylvania State University has created two hands on on classroom activities, that are used in conjunction with the video titled "Einstein's Messengers" [11]. This is a 20 minute long video that explains what gravitational waves are and the various challenges on how they are detected. The video is very basic and only requires an understanding of what a black hole and laser are.

One of these classroom activities is called "Searching for Gravitational Waves in Noisy Data" [27]. This is a website that contains: an introduction about LIGO's data, learning objectives, connections to science themes and concepts, connections

to the video, a materials list, a student handout, and a teaching guide. This activity includes a set of four simulated noisy data sets, and 24 gravitational wave templates. The students are then asked to complete a worksheet where they find the correct wave form template for the noisy data and then answer questions related to the activity.

This activity wasn't specific for any age group, but its simplicity makes it suitable for an hour to an hour and half classroom activity for late elementary to middle school students (not including the movie). The great part to this activity is that it ensures that the students understand what gravitational waves are and the different signals they produce, and how the activity is a representation of what gravitational wave astronomers are doing. Its structure as a hands on classroom activity helps promotes active engagement.

5.4 Penn State - Hands-on Gravitational Wave Astronomy

The other activity is called "Hands-on Gravitational Wave Astronomy: Extraction of Astrophysical Information from Simulated Signals" [14]. The website contains the exact same topics as before, except the introduction is more geared towards what gravitational waves are and how they change in a coalescing binary system.

The activity passes out four different data plots of strain vs time for four different time frames before coalescing (far, the total last second, at 1 second, and at 0.1 second). It then has the students measure the period, frequency, and strain. It then gives the students the formulas for the orbital period of the system at any time in terms of the mass of the system [14]:

$$P_{orb}(t) = \left(P_0^{3/8} - \frac{8}{3}kt \right)^{3/8} \quad (5.1)$$

where

$$k = \frac{96}{5c^5} (2\pi)^{3/5} (GM_{tot})^{5/3} \quad (5.2)$$

and

$$M_{tot} = (M_1 M_2)^{3/5} (M_1 M_2)^{-1/5} \quad (5.3)$$

They state that for a circular system the period of orbit is twice the period of orbit of the gravitational wave $P_{orb}(t) = 2P_{gw}(t)$ [14]. It then has the students calculate the orbital separation with Kepler’s third law, the tangential velocity, and finally the total mass from Equation 5.3, given M_1 and M_2 . The activity then gives the gravitational waveform formula for a circular binary system as $h(t) = A(t)\cos(\phi(t))$. where $A(t)$ is defined as the time varying amplitude and $\phi(t)$ is the phase of the wave:

$$A(t) = \frac{2(GM_{tot})^{5/3}}{c^4 r} \left(\frac{\pi}{P_{gw}(t)} \right)^{2/3} \quad (5.4)$$

It then has the students calculate the distance r from the amplitude when the system is far from coalescence, by just looking at the height on the graph and the fact that its period is $P_{gw}(0) = 2P_0$, and P_0 is from the very first part. It then has the student calculate the constant k given the time and period Equation 5.1, and then the total mass from Equation 5.2.

Mathematically, this free worksheet only requires an understanding of algebra, cosine waves, and fractional exponents. This math level is typical of early to middle high school students, and could take an hour to an hour and half to complete (not including the movie). This isn’t typical of a college level classroom activity because each question tells the student what formulas to use, which becomes a non-intuitive algebra problem. The worksheet also just throws formulas onto the paper without any derivation, but on the basis that the students at least know what the formulas are associated with from the movie. This activity does a good job at showing what signals are produced as a binary system coalesces, and how that can lead to the determination of various parameters.

Both of these activities rely on the movie as the “demonstratio” of gravitational astronomy, which isn’t engaging and is mathematically lacking for a early undergraduate student, but serves to show how templates are used to extract signal, and then how these templates then tell us about the physical parameters of the system, which could be a good idea to use in the classroom activity.

Chapter 6

Construction of Michelson Interferometer

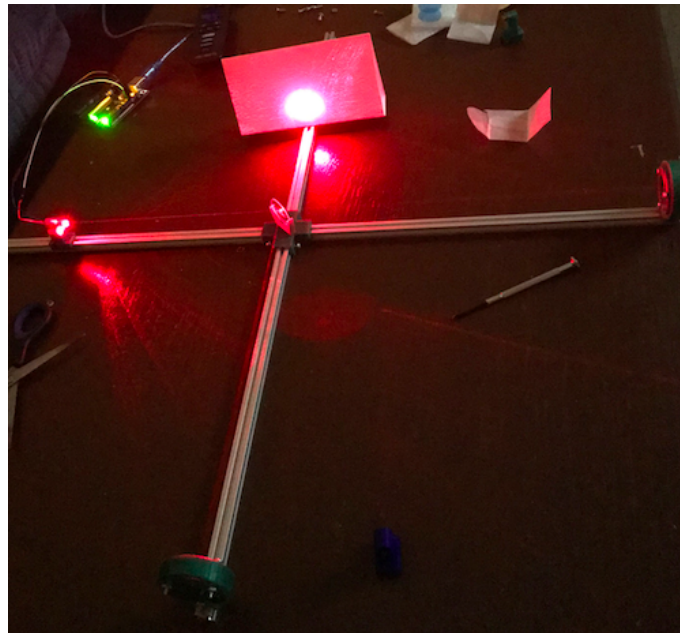


Figure 6.1: Interferometer in Action

The construction of this interferometer (Fig 6.1) is relatively cheap (115\$) compared to the ones discussed in section 5.1. The lower cost is due to the use of 3D printed parts and a laser diode. As before, to obtain decent looking fringes the use of expensive extremely high quality optical pieces for the two end mirrors and the beam splitter were used.

6.1 Materials

1. Edmund Optics - Enhanced Aluminum Mirror 25 mm - 2 x \$20.50
2. Edmund Optics - 50:50 Plate Beam Splitter 25 mm - 1 x \$42.00

3. MakerBeam T-Slot Aluminum Beams 4 Pack - 300 x 10 x 10 mm - 1 x \$12.56
4. GeeBat Laser Dot Diode - 10 pack, 650 nm, 5 mW - 1 x \$5.29
5. Arduino - Elegoo UNO R3 - 1 x \$11.98
6. M2.5x6 or M3x6 Screws* - 10 x \$0.00
7. M2.5x20 or M3x20 Screws* and Associated Bolts - 6 x \$0.00
8. 3D Printed Connector/Beam Splitter Mount - Part file can be found in Appendix A: 3D Parts - 1 x \$0.00
9. 3D Printed Mirror Mount - Part file can be found in Appendix A: 3D Parts - 2 x \$0.00
10. 3D Printed Laser Diode Mount - Part file can be found in Appendix A: 3D Parts - 1 x \$0.00

* Either Screw diameter will work, but the M3 will take a little more effort to screw into the hole. The holes in the CAD file are 2.75 mm but the 3D printer that was used in this build was a Makerbot Repliator+ which doesn't have the capacity for this precision, but its production was suitable for M2.5. In this build, M2.5x8 screws were used which limited the use of the screws in the Connector/Beam Splitter Mount, which was not as secure, but still functional. Screws longer than M2.5x20 can be used.

6.2 Construction Procedures

SAFETY WARNING: Take precaution when working with the laser and arduino. Avoid eye contact with the laser when building and using the interferometer.

Steps:

1. In order to initiate the process all parts must be 3D printed (Fig 6.2). The location of these files can be found in Appendix A. These parts were printed with a 0.1 mm layer height, 10% infill, and 2 shells. A lot of post production

work is needed to ensure all parts are clean, smooth, and are able to house the necessary components. **Note:** It is recommended to test the mirror mounts with an object of similar size to ensure proper fitment, otherwise one might risk scratching the surface.

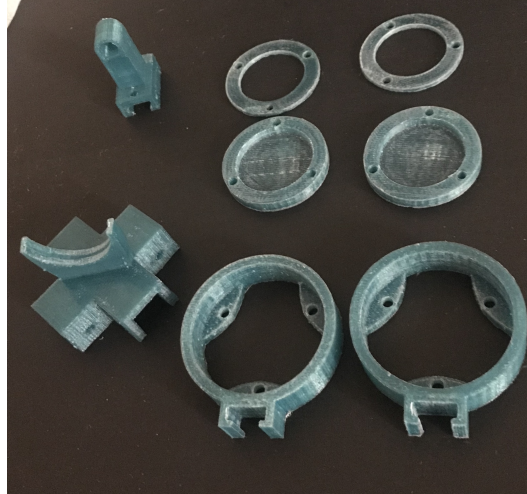


Figure 6.2: 3D parts after post production work

2. Attach each t-slot aluminum beams to the connector/beam splitter with 8 M2.5X6 screws (Fig 6.3). **Note:** This construction uses 4 M2.5X8 screws.

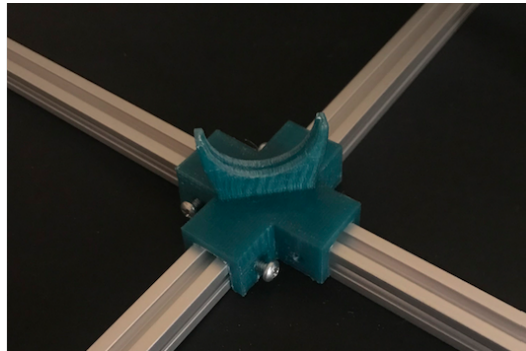


Figure 6.3: T-slot aluminum beams attached to connector/beam splitter mount

3. Feed a laser diode through the laser diode mount and attach it on the end of one of the aluminum beams with two M2.5x6 screws (Fig 6.4). The distance from the end is arbitrary, but in this model its approximately 1 1/2 inch. Attach the red/hot wire to the 13 port on the digital side of arduino and the blue/ground wire to the GND port next to it. Plug the arduino into a computer, and the

code to send to the arduino to power the laser can be found in Appendix A: Code. **Note:** It's highly recommended to test each laser to find the laser with the least amount of defects. This is done by powering it on, unfocusing it by unscrewing the cap some, and seeing which laser has the least amount of spots, seen in figure 6.9b and 6.9a, or by purchasing higher quality optics.

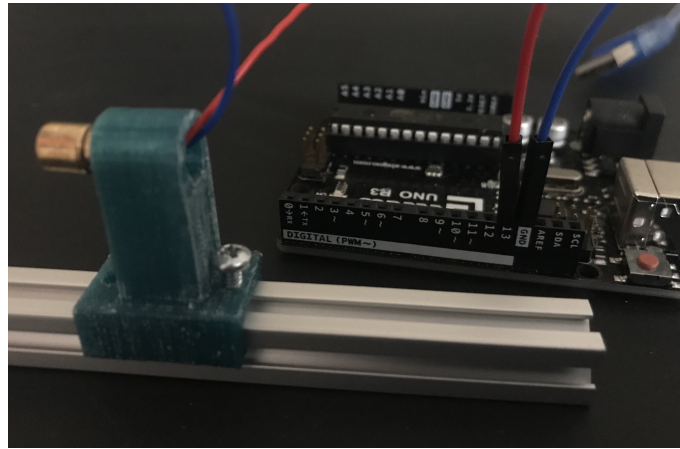
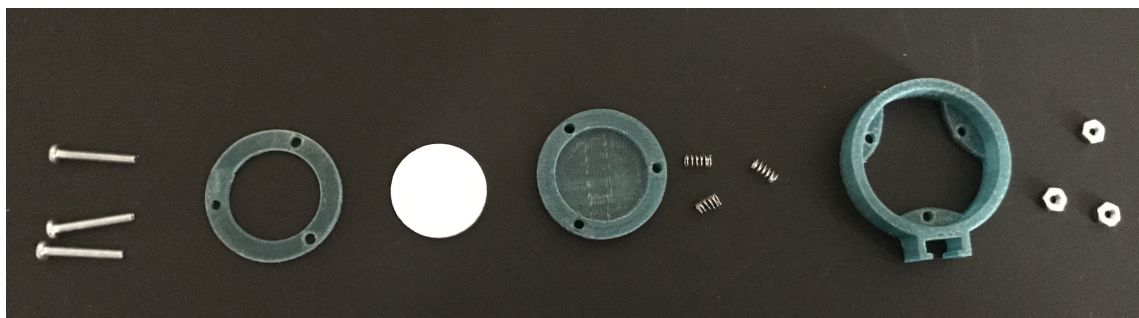
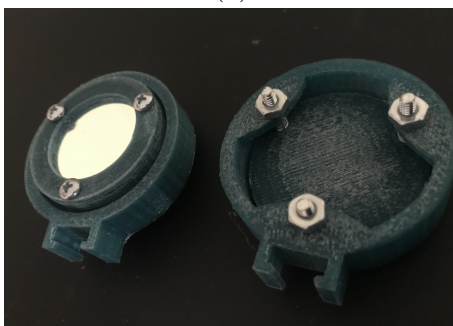


Figure 6.4: Laser diode mount attached and wired

4. Take 6 small springs from the laser diodes by unscrewing the cap entirely. The front to back order list on how to set up the mirror mount is seen in figure 6.5a. First take the mirror and place into the holder (fourth from left in Fig 6.5a). Take the cover (second from left in Fig 6.5a) and place that on top, and then screw the M2.5x20 screws through. With this object in place flip it over and place a spring on each prong. Feed this into the final piece and screw the bolts onto the ends, as seen in figure 6.5b.



(a)



(b)

Figure 6.5: (a) Order setup for mirror mount (b) Final 2 mirror mounts

5. Attach the mirror mounts to the aluminum beams approximately 1 inch from the end and in the orientation seen figure 6.6, paying close attention to the angle the beam splitter holder makes with the laser, as it may be opposite. The beam splitter can now be positioned. **Note:** Be careful moving or adjusting the device as the mirrors and beam splitter can fall.

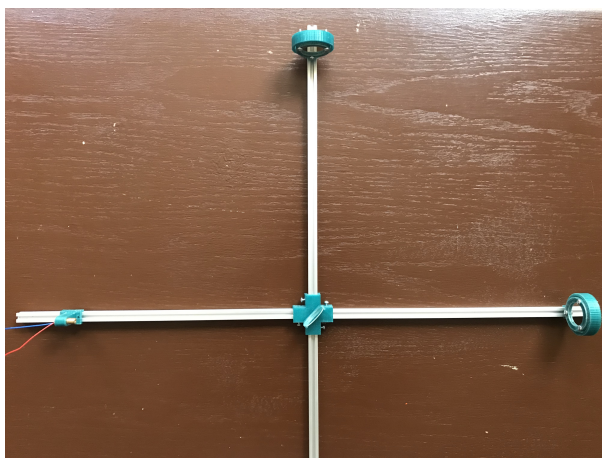


Figure 6.6: Top view setup with mirror mounts, laser diode, and beam splitter

6. In a dark room, power the laser on. Adjust how the laser diode sits in the

mount to have the diode's light pass directly through the center of the beam splitter and hit the center of the opposite mirror. Adjust the beam splitter to have it direct light towards the center of the adjacent mirror.

7. With light blocked from the adjacent mirror, use the bolts in the back of the mirror opposite from the diode to adjust the light to reflect as close as possible to the diode without overlapping the source. This is seen by the bright spot in figure 6.7. **Note:** Use notecards or folded paper to block the light and to see the reflection.

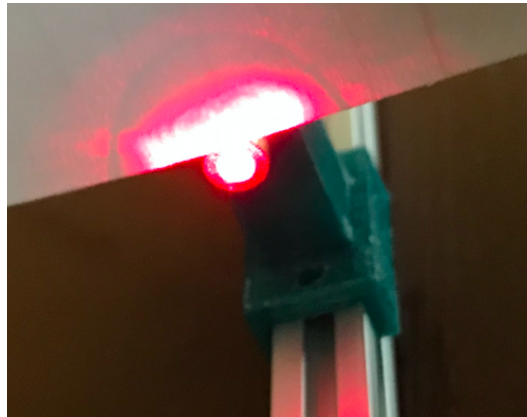


Figure 6.7: Reflection of the opposite mirror mount at the laser diode

8. Remove the paper blocking the light from adjacent mirror and then place it onto the aluminum beam that has no parts. The beams should not be aligned, as represented in figure 6.8a or 6.8b, the difference in these figures depends how focused the diode is. If only one beam is present, this might mean that the adjacent beam is off focus from the paper and the bolts will need adjusting until its present. Its recommended that the beams in figure 6.8a be used to better see the fringing effects.

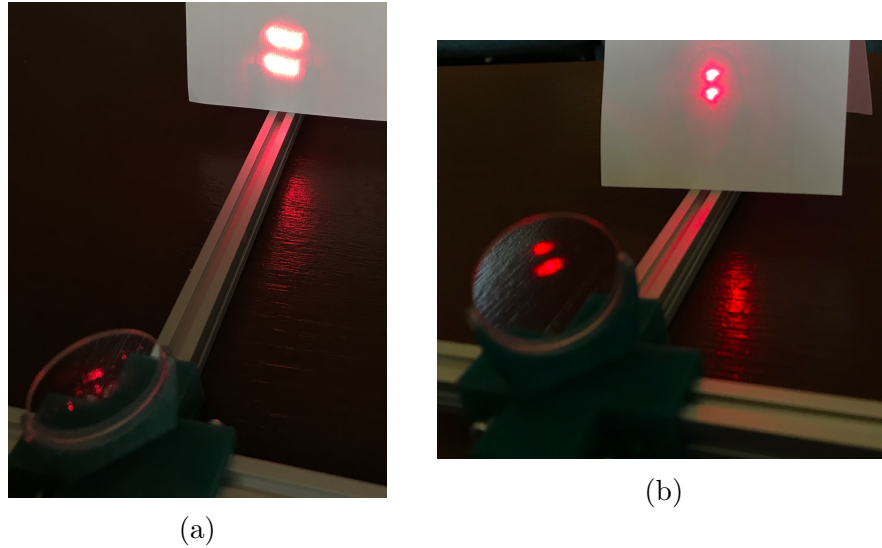


Figure 6.8: (a) Unfocused and (b) More focused out of alignment beams

Only adjust the the mirror that's adjacent from the laser until the the two sources overlap to produce the fringing patterns in figure 6.9a. When the table is tapped with a closed fist then the fringing patterns disappear, figure 6.9b.

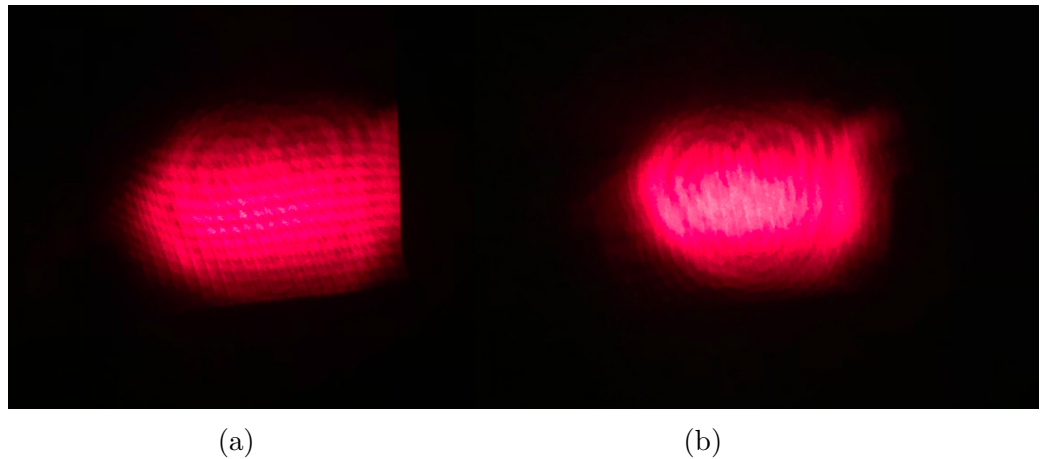


Figure 6.9: (a) Horizontal fringing patterns and (b) No fringing patterns when beams are aligned

Chapter 7

Classroom Lecture/Demonstration

Up to this point, the most important aspects to gravitational wave astronomy have been covered, effective methods of lecture demonstrations have been explored, and the creation of a Michelson interferometer has been shown. With all of these ideas present, a lecture can be formulated to effectively introduce gravitational wave astronomy to an early undergraduate physics/astronomy course. When speaking about early undergraduate this comes with a few assumptions and prerequisites. First and foremost, the student has to at least have taken, or is taking, a calculus class; second, the student should at least be aware that light is a wave and can be expressed as such; finally, a basic understanding of optics or special relativity could be helpful, but not necessary.

For the University of Rhode Island, STEM-track physics students take a sequence of three calculus based physics courses (PHY-203, PHY-204, PHY-205 — Elementary Physics I, II and III, respectively) with electricity/magnetism covered in 204 and optics covered in 205. By the time students arrive in 205 they will be concurrently enrolled in differential equations, so that would be the earliest place this demonstration might be appropriate. However, this particular demonstration would be more suitable for inclusion in PHY-306 (Elementary Modern Physics). The first unit of that course covers relativity, and has a brief unit dedicated to GR. Additionally, this content could be used (with more mathematical rigor) in PHY-492 (General Relativity).

For Brown University, physics students take one introductory calculus physics course (PHYS0070), an introduction to astronomy course (PHYS0270) and then an optics/special relativity course in 0160. By the time students enroll in Electricity/magnetism 0470, they will also be enrolled in a differential equations course.

At this point the student may also be enrolled in an cosmology or general relativity astrophysics course where this demonstration could be introduced.

A typical lesson plan has four components: learning objectives, materials, summary of tasks, and potential homework assignments. As such, the lecture plan can be found in Appendix A: Lecture Plan. Its difficult to create a effective lecture demonstration with this setup but an attempt is made that still has the students make predictions, engage in discussion, and ultimately has a final explanation. To simplify the formula for strain of a binary system only the xx and yy components are included in the lecture plan. This prevents the formal definitions and the most intimidating parts of tensor algebra/tensor calculus. These formulas are also only introduced as solutions for the wave equation, with no formal derivations. There are two instances where active participation is required, steps 8 and 11. In step 8, the instructor has the students speak with their neighbors to determine a method on how gravitational waves can be measured and to write that down and then have a discussion on the various ideas. After the mathematics of interferometry have been shown, step 11 then has students predict how an interferometer can be used to detect gravitational waves and what problems may arise, with a discussion to follow in conjunction with the interferometer.

Chapter 8

Future Work

This thesis explored the concepts that make up an effective classroom demonstration, derived the analytical form of gravitational wave emission from a binary system, and how a Michelson Fabry Perot interferometer can be used to detect such waves. Instructions on how to make a basic Michelson interferometer were then provided in order to use such a device as a demonstration tool. This knowledge then cultivated into a lecture plan that can be used to introduce gravitational wave astronomy to early undergraduate students.

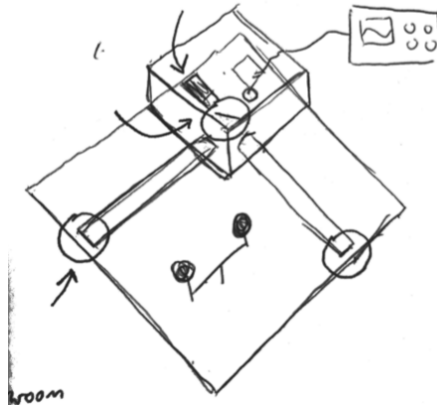


Figure 8.1: Initial concept sketch of the demonstration device

Unfortunately, due to COVID-19, this project didn't conclude with its expected outcome and a considerable amount of work on the demonstration device must be done in the future. As can be seen in Figure 8.1, it was intended that the output signal would have been detected by a photodiode and projected as a reading on a display. This enhances the demonstration by adding the additional element of displays, as is required for the interactive lecture demonstration. Servo motors were going to be introduced to the ends of the mirrors, and to the rotating binary system in Figure 8.1. The servos can be programmed along with the strength of the laser

to give a simulated effect on how the rotating black holes change the mirrors and produce the signal. Once these additions have been made, the lecture plan can then be revamped to create a more interactive lecture demonstration that ultimately asks the students to predict how the rotating masses will create a signal. Once completed the pedagogical demonstration/lecture plan can then be implemented and tested within a classroom, and future micro adjustments can be made.

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Appendix A

3D Parts

Location for 3D CAD models. Either link will work, all files end in .stl.

1. DropBox:

```
https://www.dropbox.com/sh/qibj3qp2merqm7s/AAC390ZVfnTCeMwkV6fiN8t0a?  
dl=0
```

2. Thingiverse:

```
https://www.thingiverse.com/thing:4263553
```

Code

Copy this code directly into the Arduino text editor:

```
//Creating the environment  
void setup() {  
    // initialize digital pin 13 as an output.  
    pinMode(13, OUTPUT);  
}  
  
// the loop function for continuous laser beam  
void loop() {  
    digitalWrite(13, HIGH);  
}
```

Lecture Plan

Lecture Title: Gravitational Wave Astronomy

Lecture Duration: 50 min

Learning Objectives

1. Develop an understanding of what a gravitational wave is
2. Discover how a gravitational wave warps spacetime, which is represented by the dimensionless strain, h
3. Understand the basics of a Michelson-Fabry Perot interferometer and how its used to detect gravitational waves

Materials

The Michelson interferometer from Chapter 6, or any generic Michelson interferometer.

Summary of Tasks

1. Introduce the concept of spacetime manifold, three spacial dimensions and one temporal dimension. **Board:** Draw a potato looking shape, or three dimensional grid.
2. Explain that if one were to choose a section of the manifold that Einsteins Equations will describe its space. **Board:** $R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$. Describe each part of the equation saying that the right side is the mass component and the left side describes the curvature of the manifold (mass bends spacetime). The Ricci Tensor describes the small deviations from flat space. The Ricci scalar represents how the manifold curves locally. Remember to be precise with ones terminology and to avoid the detailed nuances of tensors.
3. It could now be explained that gravitational waves are produced when large mass objects move within this manifold. This could be related to the idea that

if you're underwater and quickly move your hand that someone close to you will feel an effect. **Board:** Draw a binary system with constants M , R , ω . Explain that two rotating black holes are one of these types of systems.

4. Describe the idea that the Einstein's equation that explains this system is complex. Explain that we can study this system very far away in vacuum/flat space. Relate this back to the underwater example and say that up close you feel the effects more, but that someone that's far away could also feel the effects. **Board:** Cross out the stress energy tensor and the Ricci scalar, and write $R_{\mu\nu} = 0$. Explain that those two quantities go to zero because no mass is present, and there isn't an effective local curvature, but that there is still a slight perturbation from the binary system, which doesn't allow the Ricci Tensor to disappear.
5. Explain that the Ricci tensor can be described by flat space, η , and a small perturbation, h . **Board:** $R_{\mu\nu}(\eta, h)$. Explain that the calculation of $R_{\mu\nu}$ involves derivatives, and that the derivative of flat space goes away, and that this small perturbation will only be left. **Board:** $-\frac{\partial^2 h}{\partial t^2} + \nabla^2 h = 0$. **Note:** At this point it might be good to discuss with students what this formula means, since they might have only recently have been introduced to differential equations.
6. Describe that the solutions to this equation are wave equations, thus gravitational waves exist. In terms of the binary system explain that the solutions are given as **Board:** $h = -\frac{8MGR^2\omega^2}{c^4r} \cos(2\omega t)\hat{x}$ and $h = \frac{8MGR^2\omega^2}{c^4r} \cos(2\omega t)\hat{y}$. Explain each part of the formula and that this quantity is known as strain and has no dimensions. **Important:** explain that this formula is when a gravitational wave is traveling in the z direction and that as it passes that spacetime is physically warped. Emphasize that as one axis stretches that the other compresses, **Board:** draw a circle, then draw a horizontal ellipse, then draw a vertical ellipse. **Bonus:** Create a 3D model for this.
7. Provide a quick calculation that if **Board:** $M = 10M_\odot$, $R = 10R_{sch}$, $r = 100 \text{ Mpc}$, then $h \sim 10^{-21}$. Explain that if there's a bar with a length of 10 m,

the change of length would be 10^{-20} m, that's approximately one one-hundred-thousandth the size of a nucleus. **Board:** Relate this back to the circles and explain that the diameter is equal to L and the change to the ellipse is $L + \Delta L$, and then explain that $h = \Delta L/L$.

8. **Question:** Ask the students to speak with their neighbors to determine a method of measuring this strain and have them write down their prediction. Solicit a couple of responses and generate discussion around them, don't explain interferometry unless it was explicitly mentioned.

9. Now introduce the basic idea of a Michelson interferometer. **Demonstrate:** Power on the device and explain the basics of the interferometer. **Board:** Draw the schematics of the Michelson interferometer, light from the source is $E_{\text{mich}} = E \exp(ik\vec{r})$, it hits the beam splitter where 50% is reflected and 50% is transmitted, its sent down to each mirror with reflectively constants r_1 and r_2 at lengths L_1 and L_2 , its reflected back and comes back to the beam splitter where it recombines but again half of the recombined beam goes back towards the laser and the other half is what gets detected, $E_{\text{symm}} = \frac{1}{2}E_{\text{mich}}[r_2 \exp(-i2kL_2) + r_1 \exp(-i2kL_1)]$, $E_{\text{anti}} = \frac{1}{2}E_{\text{mich}}[r_2 \exp(-i2kL_2) - r_1 \exp(-i2kL_1)]$. Explain where the $-2L$ comes from.

10. Describe that these formulas are better expressed as effective reflectivity and transmissivity parts after recombination. **Board:** if the mirrors have perfect reflectively constants, $r_1, r_2 = 1$, and $\bar{L} = \frac{1}{2}(L_1 + L_2)$ and $\Delta L = L_1 - L_2$, then

$$r = \frac{E_{\text{symm}}}{E_{\text{mich}}} = \exp(-2ik\bar{L}) \cos(k\Delta L)$$

and

$$t = \frac{E_{\text{anti}}}{E_{\text{mich}}} = i \exp(-2ik\bar{L}) \sin(k\Delta L)$$

11. **Question:** Now ask students to discuss with their peers how gravitational waves can be detected and what problems might arise, have them write this down on the same prediction sheet. Solicit responses and discuss them in detail. Relate ΔL back to strain h and how a long L makes the detection easier, and explain how if $\Delta L = 0$ no light gets passed through to be detected.

12. If not already discussed, explain the problems associated with this detection.
Demonstration: tap on table to show sensitivity, explain that one needs a powerful light source due to the loss of power, etc.
13. **Optional:** if time permits, explain that Fabry Perot and power recycling cavities are introduced into the system that increase the power of the beam and the effective length. **Board:** Write that the incident beam on the detector is $I = E_{mich}^2 G G_{prm} (k \Delta L)^2$.

Hypothetical Problem(s)

1. A problem can be written that covers how gravitational waves are actually radiation and that the rotational velocity ω increases with time. The students can then determine what the signal readout will eventually look like.
2. Another problem can be written that explains how noisy the data is and how difficult it is to see a signal. A written response can then be required that asks for possible methods of how the signal can be extracted.