M646 Advanced Partial Differential Equations II

Theory of SemiGroups

Unbounded Operators
closed, densely defined operators
accretive operators and uniqueness for the IVP
the linear space, L(H), of bounded linear operators on H

Abstract IVP's
Solution operators and semigroups
infinitesimal generator of semigroup
properties of S(t)
properties of the generator
existence-uniqueness for the abstract IVP

Semigroups
exponential of a bounded operator
Hille-Yosida theorem
existence-uniqueness for the abstract IVP
Lumer-Phillips theorem- examples
Groups of solution operators- hyperbolic examples
sectorial operators
analytic semigroups
existence-uniqueness for parabolic IVP's

Applications of S/G's to nonlinear IVP's
existence-uniqueness for the abstract semilinear IVP
a nonlinear diffusion equation on R^n
an IBVP in 1-d
a more difficult semilinear example in 1-d
a semilinear problem in 2,3-d: $F(u) = f(u),\ F(u) = f(u) - u\nabla u$

The Navier-Stokes Equations

Equations of fluid flow
    the conservations equations
    special cases: N-S eqs; Euler eqs; Steady N-S eqs; Stokes eqs

Stokes' problem
    Settings: R^n; $U \subset R^n$ open, bounded; $Q_n = (0,L)^n$ with periodic BC's
    Stokes problem in $Q_n = (0,L)^n$ with periodic BC's

Weak Formulation of Stokes Problem (part I)
    The function space E(U); trace theorem in E(U)
    The function space $V^o, H,$ and V
    ∇p for distributions, p
    Characterization of H, V
    The Stokes Operator A, orthogonal projections

Weak Formulation of Stokes Problem (part II)
    weak and variational formulation of the problem
    Existence theorem using Riesz theorem

Stationary N-S equations
    weak formulation and the trilinear form
    Embedding results
    Properties of the trilinear form
    Fixed Point theorems
    existence using the Brouwer FP theorem
    existence using the Leray-Schauder theorem
    Uniqueness of the weak solution
    a non-uniqueness result
    inhomogeneous boundary data

Evolution N-S equations
    Application of semigroups to prove local existence
    Linearized N-S equations
    Some more results from fctl analysis
    Existence of weak solutions to the N-S eqs
        existence of approximate solutions
        a-priori estimates and convergent subseqences
        passing to the limit

uniqueness when $n = 2$ and results for $n > 2$
Existence proof using semidiscretization

Bifurcation of Solutions to NL problems
Bifurcation examples- Taylor and Benard Problems
Derivatives
    Gateaux derivative, Frechet derivative
    examples
Implicit Function theorem on B-spaces
Bifurcation from a simple eigenvalue
    theorem and proof
    examples
Lyapunov-Schmidt method