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OPTIMAL QUANTITIES FOR HOSPITAL SUPPLY GROUPINGS

A THESIS

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TABLE OF CONTENTS

		Page
ACKNOWLEDGMENTS		ii
LIST OF ILLUSTRATIONS		iv
SUMMARY		v
Chapter		
I.	INTRODUCTION	1
	The Specific Problem	2
	Assumptions	5
	Method of Procedure	5
	Scope and Limitations	6
II.	LITERATURE SEARCH	8
III.	THE OPTIMIZATION OF SUPPLY GROUPING QUANTITIES	15
	Notation	15
	The Discrete Case	16
	The Poisson Case	22
	The Continuous Case	27
	The Normal Case	30
IV.	COMPUTATIONAL PROCEDURES FOR OPTIMAL SUPPLY GROUPING QUANTITIES	34
	Selection of Supply Groupings for Study	34
	The Probability Distribution of Demand	35
	The Measurement of Relevant Costs	39
	The Development of Computational Procedures	41
	Computational Procedures for the Poisson Case	41
	Computational Procedures for the Normal Case	48
V.	CONCLUSIONS AND RECOMMENDATIONS	55
	Conclusions	55
	Recommendations	58
APPENDIX		60
BIBLIOGRAPHY		63

LIST OF ILLUSTRATIONS

Figure | Page

1. Search Procedure Chart for the Poisson Case. 25
2. Data-Collection Form 37
3. Assembly of Program Card Deck for the Poisson Case 45
4. Results of Data Processing for the Poisson Case. 46
5. Optimal Supply Quantity Worksheet for the Normal Case. . . 50
6. Optimal Supply Quantity Chart for the Normal Case. 52

SUMMARY

The specific aim of this research was the development of a methodology for the determination of economically optimal quantities of items included in hospital supply groupings. The purpose of this study was to provide a basis for the development of practical optimization procedures to be used as tools of scientific decision-making by hospital management with professional assistance.

Any collection or assemblage of one or more kinds of supply items intended for use without replenishment in a given activity during a specified period of time, or for use in a particular procedure or service, whether or not the components are sealed within a container, was referred to as a "supply grouping". The fixed and unit costs of overages and shortages and the probability distributions of demand associated with any item in a multiple-item supply grouping were assumed to be independent of the costs and demands of all other items in the grouping.

An expected total relevant cost model for the discrete case, of the form

$$TC(Q) = E_V \sum_{x=0}^{Q-1} (Q-x)\ P(x) + E_C \sum_{x=0}^{Q-1} P(x) + S_V \sum_{x=Q+1}^{\infty} (x-Q)\ P(x) + S_C \sum_{x=Q+1}^{\infty} P(x)$$

and for the continuous case, of the form

$$TC(Q) = E_V \int_0^Q (Q-x)f(x)dx + E_C \int_0^Q f(x)dx + S_V \int_Q^{\infty}(x-Q)f(x)dx + S_C \int_Q^{\infty} f(x)dx$$

were used to develop a general methodology for the optimization of the supply grouping quantity Q for each case. For the specific cases of the Poisson and normal probability distributions, which were assumed to be typical distributions of demand, more specific optimization procedures were developed and presented.

In the Poisson case, the resulting manual optimization procedure was rather lengthy and complex for practical use and an alternative procedure was designed. A computational routine for the Burroughs BAC-220 digital computer was developed to evaluate TC(Q) for virtually all possible values of the demand for any given value of the mean Poisson demand and to select the grouping quantity, Q, which corresponds to the absolute minimum value of TC(Q).

In the normal case, the resulting optimization equation, of the form

$$(S_V + E_V)\,\Phi\left(\frac{Q-\mu}{\sigma}\right) + (E_C - S_C)\,\phi\left(\frac{Q-\mu}{\sigma}\right) - S_V = 0$$

was used to construct a nomograph and a worksheet for the calculation of the economically optimal supply grouping quantity Q.

It is intended that the results of this research will stimulate interest in the eventual development of a formal body of knowledge in quantitative economics for hospital management systems of the type treated here.

CHAPTER I

INTRODUCTION

The specific aim of this research is the development of a methodology for the determination of economically optimal quantities of items included in hospital supply groupings. The purpose of this study is to provide a basis for the development of practical optimization procedures to be used as tools of scientific decision-making by hospital management.

The proper functioning of the hospital supply system requires that personnel, materials, and equipment be available when and where they are needed. If these resources are not utilized properly, the costs of the resulting deficiencies must be borne by the hospital at the expense of opportunities to improve patient care or to promote other important hospital objectives.

One of the most promising of the various attempts that have been made to improve the operation of hospital supply systems has been the development of standardized prepackaged or preassembled groups of supplies which reduce the time-consuming and costly collection of supply items by professional personnel at the point of use. Some of these groupings are commercially packaged and some are made up in the hospital's central supply service; some are of the disposable type and others are reprocessed in the hospital for repeated use. Although they are not usually thought of as "packages," various other groupings of hospital supplies possess the same characteristics as packaged items. Any collection or assemblage of one or more kinds of supply items intended for use without replenishment in a given activity during a specified period of time, or for use in a particular procedure or service, whether or not the components are sealed within a container, will be referred to as a "supply grouping." For example, the collection of items on a tray, on a cart, in a linen pack, in a bedside stand, or intended for use in a particular nursing procedure may be considered as supply groupings.

The current trend in the design of hospital facilities holds many implications for the supply grouping concept. Mobile patient-care units with packaged supplies, supply requirements in civil defense and disaster planning, increasing emphasis upon self-care and domiciliary accommodations, experiments with preassembled packs for one patient-day, and an accelerating rate of increase in the adoption of disposable supply items are a few of the developments which point up the increasing need for better decision rules in respect to the utilization of hospital supplies. The research reported herein addresses itself to this need.

The Specific Problem

A major problem associated with the use of supply groupings, and one which tends to nullify some of the advantages of this kind of supply system, is the difficulty of ascertaining in advance the number or amount of each supply item that will be needed when the supply grouping is used. Unless the quantities of the items in the supply grouping are equal to the quantities needed at the point of use, either "overages" or "shortages" will result. The former refers to having too many of an item in the supply grouping, while the latter refers to having too few. At best, overages and shortages imply waste and inconvenience. Surplus supplies remaining after the supply grouping is used may be discarded, or, if salvaged, they may require additional handling and processing before they are used again. Supplies which are needed in excess of the number or amount in the supply grouping require either extra time for special procurement, a reduction in the quality of service rendered, or both. In any event, the problems of overages and shortages can result in excessive costs to the hospital and might have an adverse effect upon the quality of patient care.

About two-thirds of the operating costs of hospitals is accounted for by employee wages. Improvements in the utilization of personnel who provide and use supplies should produce a more favorable "results-to-costs ratio" for the hospital. Even though this research deals with supply costs, it should be remembered that all supply costs contain labor costs. The procedures resulting from this study can point the way to better personnel utilization and, hence, either lower labor costs or better services.

Although the demand for each of the items in a supply grouping cannot be known in advance, a sufficiently large number of observations on the current and past demand for the item frequently will allow the construction of a statistical model for the behavior of the demand for an item. Future demand for the item can be known, then, in a probabalistic sense. With this information and with the calculation of certain relevant material and labor costs associated with overages and shortages, it is possible to evaluate alternative quantities of the item on an expected cost basis.

There are two problems of primary interest which must be dealt with if the approach described above is to be used by hospital management. First, none of the traditional nondeterministic order quantity models seems to give an appropriate representation of the expected relevant costs involved when the supply grouping system is used. The costs of overages and shortages typically might include a fixed cost of having too many items in the grouping, a fixed cost of having too few items in the grouping, and a cost per item for each item which is needed but not in the grouping, and for each item in the grouping which is not needed at the time the grouping is used. It is necessary that a model be developed which adequately represents the total expected monetary disadvantage for various grouping quantities in this system.

The other important problem is that of providing hospital management personnel with practical, easily interpreted procedures which can be used successfully in calculating optimal quantities for supply groupings. In general, hospital management personnel are not equipped to deal with nondeterministic phenomena, and it is expected that they will require professional assistance to develop and use the procedures resulting from this study. A demonstration of a scheme for constructing practical procedures from theoretical optimal quantity models should be of significant value to the industrial engineering practitioner or consultant in the hospital and, subsequently, to hospital management.

Assumptions

The unit costs, fixed costs, and demands associated with any item in a multiple-item supply grouping are assumed to be independent of the costs and demands of all other items in the grouping.

The unit costs, fixed costs, and the probability distribution characteristics of demand for each item in a supply grouping are assumed to remain constant unless changes are made in the physical nature of the supply item or supply grouping, or unless changes in certain parameters are deemed necessary as a result of periodic reviews.

It will be assumed that the minimum expected total relevant cost for a multiple-item supply grouping will be the sum of the expected relevant costs of using optimal quantities of the individual items in the grouping. That is, the optimal quantities of the various kinds of items in a supply grouping may be determined by several single-item analyses; the combination of these single-item optimal quantities is assumed to be optimal.

Method of Procedure

The procedure used in achieving the stated objective consisted of two parts. First, the development of mathematical cost-quantity-probability relationships for supply groupings was accomplished. Then, a demonstration of how practical computational procedures can be developed from these relationships was prepared. The approach followed in each of these parts is outlined below:

(1) Development of Mathematical Cost-Quantity-Probability Models.

(a) Construction of an expected total relevant cost model for the discrete case.

(b) Determination of optimal supply grouping quantities for the discrete case.

(c) Determination of optimal supply grouping quantities for the Poisson case.

(d) Construction of an expected total relevant cost model for the continuous case.

(e) Determination of optimal supply grouping quantities for the normal case.

(2) Development of Computational Procedures for the Optimization of Supply Grouping Quantities.

(a) Ascertaining the nature of the probability distribution of demand for items in supply groupings.

(b) Measuring and estimating the relevant costs associated with items in supply groupings.

(c) Development of a computational routine for the Poisson case.

(d) Development of a computational routine for the normal case.

Scope and Limitations

The concepts developed in this paper appear to be of a sufficiently general nature that they can be applied with little or no modification to other nondeterministic systems in which the relevant costs are of the same type. However, the intent of this research was to develop concepts which will be particularly applicable to certain problems of hospital management, and no attempt has been made to illustrate applications in other areas.

The criterion for the optimization of supply quantities is to be monetary cost only. If nonmonetary considerations, such as patient safety, are of significant consequence, the economic optimum may not be the most desirable quantity alternative. The probability of the occurrence of a shortage may be a useful guide in this case, and can be calculated using certain of the procedures that have been developed in this paper.

The methods and procedures which are presented in this thesis have been developed for the analysis of item quantities for existing supply groupings. No attempt has been made to specify the kinds of items which should be included in any supply grouping in the hospital.

In the following chapter, a review of selected literature is presented in order that the progress, the problems, and the contributions of researchers in this area of hospital management systems may be appreciated.

CHAPTER II

LITERATURE SEARCH

As recently as the early 1950's, there was a dearth of published information regarding the preparation of standardized groupings of hospital supplies in advance of their being required by professional personnel at the point of use. The preparation and use of supply groupings was a traditional practice in the hospital, nevertheless. One of the difficulties which certainly affected the exchange of information in this area was the reluctance or the inability of hospital management to develop and use objective measures of performance in the evaluation of hospital systems. McGibony, in 1952, attempted to summarize the increasing concern of hospital management in respect to these shortcomings. With respect to needed study and research in the area of administrative services, he said:

> This area includes research, studies, experiments, and demonstration in the administrative and non-clinical phases of hospital activities and related fields such as...cost analysis;...personnel practices, methods, policies and effective utilization; need, use of specialized equipment and supplies...current equipment and supply lists, needs, and consumption rates...and...evaluation of equipment and supply utilization costs.[1]

Although a need for definitive action had been recognized and stated, the philosophy that hospital services do not lend themselves

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[1] McGibony, John R., Principles of Hospital Administration, G.P. Putnam's Sons, New York, 1952, pp. 518 - 520.

to quantitative measurement remained prevalent among hospital management. One of the first significant attempts to remove this obstacle was the publication in 1955 of a series of studies in the University of Pittsburgh Medical Center hospitals by George and Kuehn.[2] The results of these studies indicated that patients assigned to various illness categories required consistently different and measurable kinds of care in terms of nursing services. Data regarding the frequencies of various nursing services rendered were collected and tabulated; the assignment of nursing personnel to areas commensurate with their training was then achieved.

Since 1955, numerous studies have been conducted in attempts to further quantify the care and service needs of the hospital patient. In 1959, Flagle and his colleagues at the Johns Hopkins Hospital reported:

> The daily measurement of patient needs has been developed into a "direct patient care index" - a measure of the number of daily nurse hours of direct patient care required to provide the current standards of care...After standardization of practice through a set of ground rules, the rate of use of a number of items was found to be related to patient condition in much the same manner as nursing time...[3]

In 1960, the Department of Administrative Research of University Hospital, University of Maryland, began a study to establish supply consumption criteria in hospitals. The research team planned to use

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[2]George, Francis L. and Ruth P. Kuehn, Patterns of Patient Care, The Macmillan Company, New York, 1955, 266 pp.

[3]Flagle, Charles D., Robert J. Connor, Richard K. C. Hsieh, Ruth A. Preston, and Sidney Singer, "Optimal Organization and Facility of a Nursing Unit," Progress Report, USPHS No. GN-5537, Operations Research Division, The Johns Hopkins Hospital, Baltimore, Maryland, December 1959, p. i.

a patient classification system developed as a result of Flagle's work at the Johns Hopkins Hospital.[4] In 1964, the final report of the project was published.[5] The investigators determined that the results of the study were generally inconclusive. However, for small samples of the two most frequently recurring patient diagnoses, analyses indicated that age, sex, length of stay, type of accommodation, and physician preference have no significant effect upon supply item usage. These tentative findings lend some credibility to the idea that, for certain procedures, activities, and treatments, the variations in demand will be due to chance alone, and may be well-represented by some theoretical probability distribution with constant parameters.

In 1964, Davis, Parks, and Wickel reported that, through a systems study of surgical pack making, the hospital under study was able to eliminate the problem of shortages in surgical linen packs.[6] The two criteria used in the study were the elimination of shortages and the effective utilization of the personnel involved in the pack-making function. The total number of surgical packs was reduced and an ordering chart for the new system was developed. However, no cost data were reported, nor was there any indication that monetary costs had been considered in developing the new system.

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[4]Flagle, et al., op. cit., 29 pp.

[5]Dow, Wallace M., "Establishment of Supply Consumption Criteria," Final Report, USPHS No. HM-00181-0251, University Hospital, University of Maryland, Baltimore, Maryland, 1964.

[6]Davis, Louis E., George M. Parks, and Samuel R. Wickel, Jr., "A Systems Study of Surgical Pack Making," Hospitals, February 16, 1964, Vol. 38, No. 4, pp. 124 - 132.

In recognition of the need for determining the economic consequences of hospital supply decisions, Freeman, Smalley, Emerzian, and Irwin, in 1964, developed a procedure for measuring the monetary costs associated with the use of both disposable and reusable hospital supply items.[7] This work provided hospital management personnel with the capability of quantitatively measuring the elemental and unit costs of the materials and labor for supply items. The development of this new management capability was a significant contribution to the advancement of objective decision-making in the hospital.

Traditionally, hospital management has dealt with hospital systems as if they were deterministic systems. The most commonly used indexes of performance and requirements have been "averages." These numbers have been accepted generally by hospital management as constants in a deterministic system, and important management decisions are often made with no more information than the average daily census, the average per cent bed occupancy, or the average demand for a certain supply item.

The impact of this approach to decision-making has been discussed frequently by investigators in the field of hospital management systems. For example, Flagle and his colleagues stated in 1959 that:

> The large staff and inventory which insure against the heavy day are wasteful of resources at other times. The assignments which are planned for the "average" day are at some times inadequate and at other times excessive. We must look for flexibility and sensitive guides to allocation of resources if the goals of economy and consistent standards of care to be achieved simultaneously.[8]

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[7] Freeman, John R., Harold E. Smalley, A.D. Joseph Emerzian, and Pamela H. Irwin, "Hospital Supply Decisions: Monetary Costs," Hospital Management, June 1964, Vol. 97, No. 6, pp. 98 - 111.

[8] Flagle, op. cit., p. i.

As the result of a study to predict hospital bed requirements, Blumberg designed a decision display from which the probability of "sufficiency" for a given average census could be read directly for various quantities of beds; these procedures, he reported, will:

> ...clarify much that relates to bed needs and will permit meaningful comparisons of relative capacities to be made between various hospitals and communities...too few beds result in increased health hazards while too many beds lead to higher dollar costs. Although there is no simple way to balance health costs against dollar costs so that an optimal number of beds can be planned, it is believed that such a procedure can and will be developed since it is basic to effective health planning.[9]

Other researchers in the area of hospital management systems such as Balintfy,[10,11] Young,[12] Middlehoven,[13] and Hsieh[14] have recognized the need for management decision mechanisms which incorporate both the nondeterministic nature of hospital systems and the economic consequences of various courses of action. Although these and others have shown that certain quantitative techniques may be

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[9]Blumberg, Mark S., " 'DPF' Concept Helps Predict Bed Needs," The Modern Hospital, December 1961, Vol. 97, No. 6, pp. 75 - 79.

[10]Balintfy, J.L., "Mathematical Models and Analysis of Certain Stochastic Processes in General Hospitals," Doctoral Dissertation, The Johns Hopkins University, Industrial Engineering Department, 1962.

[11]Balintfy, J.L., "A Stochastic Model for the Analysis and Prediction of Admissions and Discharges in Hospitals," Management Sciences: Models and Techniques, Pergamon Press, New York, 1960, pp. 288-299.

[12]Young, John P., "Information Nexus Guides Decision System," The Modern Hospital, Vol. 106, No. 2, February 1966, pp. 101-105.

[13]Middlehoven, W., "Analysis and Reorganization of a Central Supply Delivery System," Master's Essay, The Johns Hopkins University, Department of Operation Research and Industrial Engineering, 1964.

[14]Hsieh, Richard K.C., "A Study of Linen Processing and Distribution in a Hospital," Master's Essay, The Johns Hopkins University, Industrial Engineering Department, 1961.

applicable in the evaluation of hospital systems, little has been done to develop practical tools that can be used by hospital management actually to produce information which is useful in making decisions.

Certain of the well-known inventory theory approaches were investigated by the author in respect to their ability to represent the demand and cost characteristics of the supply grouping concept in hospitals. The expected total relevant cost models of the "newspaper boy" problem[15, 16] and the "spare-parts" problem[17] were found to be of the general type which might be required for an analysis of the supply grouping scheme. It was observed, however, that these models do not adequately represent the costs involved when supply groupings are used. Only a cost-per-item for each item of shortage and overage was accounted for in these models. Chang,[18] in 1963, considered a "shortage lump sum" cost which represented a constant total cost penalty for the occurrence of a shortage, regardless of the number of items of shortage. This parameter of the expected total relevant cost model is of the type which might occur in the supply grouping system. However, in addition to the constant shortage cost, a constant overage cost also might be incurred when the supply grouping system is utilized. Thus, a need was recognized for the development of a mathematical model with appropriate parameters which can be used to

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[15]Sasieni, Maurice, Arthur Yaspan, Lawrence Friedman, Operations Research-Methods and Problems, John Wiley and Sons, Inc., New York, 1964 p. 100.

[16]Naddor Eliezer, Inventory Systems, John Wiley and Sons, Inc., New York 1966, pp. 135-138, 144.

[17]Ibid.

[18]Chang, Sang Hoon, "Determination of Optimum Reject Allowances in Manufacturing," Master's Thesis, Georgia Institute of Technology, School of Industrial Engineering, 1963.

evaluate quantity alternatives for supply groupings in the hospital.

It is intended that the research reported herein will be of value not only in understanding the evolution of the decision-making process in hospital management, but also that it will promote the eventual development of a formal body of knowledge in quantitative economics for hospital management systems.

CHAPTER III

THE OPTIMIZATION OF SUPPLY GROUPING QUANTITIES

There are certain measurements which must be taken prior to the utilization of an optimal quantity model. The probability distribution of the demand for each item in a supply grouping must be ascertained. The costs of overages and shortages must be measured or estimated for each supply item in the grouping. There are four relevant cost parameters which must be considered; these parameters are explained in the development of the expected total relevant cost models to follow. The measurement of the above parameters is discussed in Chapter IV.

It is recognized that, for most familiar hospital supply items such as linens, syringes, etc., the demand is a discrete random phenomenon; for certain items, however, the demand may be for quantities or "amounts" of a continuous nature. Certain foods or liquids may possess demand characteristics of the latter kind. There also may be circumstances under which either a discrete demand distribution or a continuous demand distribution will serve as an adequate approximation to the other to facilitate measurements or calculations. A model of the expected total relevant cost has been developed for each case.

Notation

The following definitions and symbols will be used in the development of the expected total relevant cost models.

x : The random variable which is the actual demand for x quantity units of the supply item.

P(x) : The probability mass function of x for the discrete case only. The probability that exactly x quantity units of the supply item will be required at the point of use.

f(x) : The probability density function of x for the continuous case only. The probability that the demand takes on a value between x and x + dx is given by f(x)dx.

F(x) : The probability distribution function of x in either the discrete case of the continuous case. The probability that no more than x quantity units of the supply item will be required at the point of use.

Q : The number of quantity units of the supply item which will be placed in a supply grouping.

TC(Q) : The expected total relevant cost consequence of using Q quantity units of the supply item in a supply grouping.

S_v : The cost per quantity unit of the excess of the demand, x, over the supply grouping quantity, Q.

S_c : The fixed cost which is incurred when the demand, x, exceeds the supply grouping quantity, Q, by any amount.

E_v : The cost per quantity unit of the excess of the supply grouping quantity, Q, over the demand, x.

E_c : The fixed cost which is incurred when the supply grouping quantity, Q, exceeds the demand, x, by any amount.

The Discrete Case

The expected total relevant cost model when the demand for the supply item is in discrete quantity units is given by

$$TC(Q) = E_v \sum_{x=0}^{Q-1} (Q-x)P(x) + E_c \sum_{x=0}^{Q-1} P(x) + S_v \sum_{x=Q+1}^{\infty} (x-Q)P(x) + S_c \sum_{x=Q+1}^{\infty} P(x) \qquad (1)$$

in which the terms

$$E_v \sum_{x=0}^{Q-1} (Q-x)\, P(x) + E_c \sum_{x=0}^{Q-1} P(x)$$

represent the expected overage costs incurred when the grouping quantity Q exceeds the demand x, and the terms

$$S_v \sum_{x=Q+1}^{\infty} (x-Q)\ P(x) + S_c \sum_{x=Q+1}^{\infty} P(x)$$

represent the expected shortage costs incurred when the demand x exceeds the grouping quantity Q.

The function TC(Q) will have a local minimum value $TC(Q_o)$ at Q_o if both

$$TC(Q_o+1) \geq TC(Q_o), \text{ and}$$

$$TC(Q_o-1) \geq TC(Q_o).$$

The finite difference operator Δ will be used to simplify the notation to follow.* The sufficient conditions for TC(Q) to have a local minimum value $TC(Q_o)$ at Q_o now may be restated as

$$\Delta TC(Q_o-1) < 0 \leq \Delta TC(Q_o). \qquad (2)$$

It may be verified from (1) that

$$TC(Q+1) = E_v \sum_{x=0}^{Q} (Q+1-x)P(x) + E_c \sum_{x=0}^{Q} P(x) + S_v \sum_{x=Q+2}^{\infty} (x-Q-1)\ P(x) + S_c \sum_{x=Q+2}^{\infty} P(x)$$

Thus the first finite difference of TC(Q) is given by

---

*The first finite forward difference of TC(Q) is denoted by ΔTC(Q) and is defined by

$$\Delta TC(Q) = TC(Q+1) - TC(Q)$$

$$\Delta TC(Q)=E_V \sum_{x=0}^{Q} (Q+1-x)P(x)+E_C \sum_{x=0}^{Q} P(x)+S_V \sum_{x=Q+2}^{\infty} (x-Q-1P(x)+S_C \sum_{x=Q+2}^{\infty} P(x)$$

$$-E_V \sum_{x=0}^{Q-1} (Q-x)P(x)-E_C \sum_{x=0}^{Q-1} P(x)-S_V \sum_{x=Q+1}^{\infty} (x-Q)P(x)-S_C \sum_{x=Q+1}^{\infty} P(x)$$

Upon collecting terms and using the relationships

$$F(Q) = \sum_{x=0}^{Q} P(x) = 1 - \sum_{x=Q+1}^{\infty} P(x)$$

the first finite difference of TC(Q) becomes

$$\Delta TC(Q) = (E_V+E_C+S_V+S_C)F(Q) - S_C F(Q+1) - E_C F(Q-1) - S_V \qquad (3)$$

Similarly, the second finite difference of TC(Q) is found to be

$$\Delta^2 TC(Q) = \Delta[\Delta TC(Q)] = (E_V+E_C+S_V+S_C)P(Q+1) - S_C P(Q+2) - E_C P(Q) \qquad (4)$$

Equation (3) may be used to locate that quantity Q_o for which (2) is satisfied and for which the function TC(Q) takes on a local minimum value. Sufficient conditions for TC(Q) to take on an absolute minimum value at Q_o are that (2) is satisfied and that, for all Q,

$$\Delta^2 TC(Q) \geq 0. \qquad (5)$$

That is, if $TC(Q_o)$ is a local minimum value of TC(Q) and if TC(Q) is a convex function, then Q_o is an absolute economic optimum. Inequality (4) may be used to ascertain whether or not TC(Q) is convex for a particular set of cost elements and a specific discrete probability distribution of demand.

If $TC(Q)$ is found to be convex, then the economically optimal grouping quantity will be the smallest integer Q for which

$$\Delta TC(Q) = (E_V+E_C+S_V+S_C)F(Q) - S_C F(Q+1) - E_C F(Q-1) - S_V \geq 0$$

It may be found that, for certain discrete probability distributions of demand, $\Delta^2 TC(Q)$ is not everywhere nonnegative. Thus the location of a local minimum value of $TC(Q)$ will not insure that other local minima with smaller values of $TC(Q)$ do not exist. It should be reemphasized that (5) is a sufficient condition, but is not necessary, for the existence of an absolute minimum value of $TC(Q)$; that is, if (2) is satisfied, it is necessary to show only that $TC(Q_o)$ is smaller than any other value of the function $TC(Q)$. Thus if every local minimum value of $TC(Q)$ can be located, it is possible to compare those values of $TC(Q)$ and to ascertain the absolute economically optimal grouping quantity Q.

From (2) it can be seen that all local minima of $TC(Q)$ will occur at values of Q for which the algebraic sign of $\Delta TC(Q)$ changes from negative to nonnegative; that is, the local minima of $TC(Q)$ may be said to occur theoretically at certain "zeros" of (3). If the number of relative extreme values of (3) can be ascertained, then the maximum number of theoretical "zeros" of (3), and thus the maximum number of relative minima of $TC(Q)$, can be determined since the conditions

$$\lim_{Q \to \infty} \Delta TC(Q) = E_V,$$

where E_V is a positive parameter, and

$$\Delta TC(0) = (E_V+E_C+S_V+S_C)F(0) - S_CF(1) - S_V$$

can be evaluated for any particular discrete probability distribution. That is, if the algebraic signs of $\Delta TC(0)$ and $\Delta TC(\infty)$ and the maximum number of relative extreme values of $\Delta TC(Q)$ are known, then the maximum number of theoretical "zeros" of $\Delta TC(Q)$ and the maximum number of local minima of $TC(Q)$ are determined.

The functions $\Delta TC(Q)$ and $\Delta^2 TC(Q)$ have a common domain which consists of all nonnegative integers. In order to facilitate further analyses only, let $E'(Q)$ and $E''(Q)$ be defined as extensions of $\Delta TC(Q)$ and $\Delta^2 TC(Q)$, respectively. Let these extensions be defined such that they take on the same values as the original functions at the integers and have a common domain which consists of all nonnegative real numbers. These extensions may be written as follows.

$$E'(Q) = (E_V+E_C+S_V+S_C)F(Q) - S_CF(Q+1) - E_CF(Q-1) -S_V$$

$$E''(Q) = E_V+E_C+S_V+S_C)P(Q+1) - S_CP(Q+2) - E_CP(Q)$$

If $E''(Q)$ is set equal to zero, then the maximum number of relative extreme values of $E'(Q)$ and the corresponding real values of Q are given by the number and values of the roots of

$$E''(Q) = (E_V+E_C+S_V+S_C)P(Q+1) - S_CP(Q+2) - E_CP(Q) \doteq 0$$

Since the limiting values $E'(0)$ and $E'(\infty)$ can be evaluated, the maximum number of points at which the algebraic sign of $E'(Q)$ changes from negative to positive can be ascertained. Since $E'(Q)$ is a continuous extension of $\Delta TC(Q)$, the maximum number of points at which the algebraic sign of $\Delta TC(Q)$ changes from negative to positive also is determined and, subsequently, the maximum number of local minima of $TC(Q)$ is known.

Thus, the search for specific integer values of Q which correspond to the local minima of $TC(Q)$ may be limited to a search for a specific maximum number of points whose general locations are known. Those integer values of Q which correspond to the local minima of $TC(Q)$ now may be ascertained using (3) and (2). When the critical points of $TC(Q)$ have been determined, they may be compared in order to determine the absolute minimum value of $TC(Q)$ and the corresponding absolute economically optimal grouping quantity Q_o.

It has been assumed that the costs E_V, E_C, S_V, and S_C are measurable and, hence, known. It has been assumed also that the discrete probability distribution of the demand, x, can be ascertained. Although the above optimization methodology theoretically can be utilized for any discrete empirical probability distribution, frequently it may be convenient to use some appropriate theoretical distribution from whose population the empirical data reasonably might have come. One of the primary advantages of the latter alternative is that values of the distribution functions and the probability mass functions for various values of the parameters involved have been tabulated and published for several theoretical distributions.

The Poisson probability distribution has been shown to be an appropriate theoretical probability distribution for certain random phenomena in the hospital.[19,20,21,22] It is anticipated that the probability distributions of the demands for small discrete quantities of many items in hospital supply groupings will be found to be Poisson in nature. Thus, more specific techniques have been developed for the optimization of grouping quantities for items for whose demand probabilities are Poisson-distributed.

The Poisson Case

When the probability distribution of demand is Poisson with mean λ and variance λ, the probability mass function of the demand x is given by

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x \geq 0$$

$$P(x) = 0 \quad , \qquad x < 0$$

and the distribution function F(Q) is given by

$$F(Q) = \sum_{x=0}^{Q} \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad Q \geq 0$$

[19] Blumberg, op.cit.

[20] Pakzaban, Mahmood, "Probability Distribution of Demand for Selected Hospital Supply Items," Master's Research Paper; Georgia Institute of Technology, Atlanta, Georgia, Preliminary Findings, May 1966.

[21] Gue, Ronald L., "A Stochastic Description of Direct Patient Care and Its Relation to Communication in a Hospital," Doctoral Dissertation, John Hopkins University, 1964.

[22] Smalley, Harold E. and John R. Freeman, Hospital Industrial Engineering, Reinhold Publishing Corporation, New York, 1966, pp. 342-356.

It follows from the definition of P(x) above that

$$P(x+1) = \frac{\lambda}{x+1} \cdot P(x) \qquad , \qquad x \geq 0$$

$$P(x+2) = \frac{\lambda^2}{(x+1)(x+2)} \cdot P(x), \qquad x \geq 0$$

Thus equation (4) now may be written as

$$\Delta^2 TC(Q) = \frac{a\lambda}{Q+1} P(Q) - \frac{b\lambda^2}{(Q+1)(Q+2)} P(Q) - cP(Q)$$

where

$$a = (E_V + E_C + S_V + S_C) \qquad c = E_C$$

$$b = S_C \qquad d = S_V.$$

As in the previous section, let $E'(Q)$ and $E''(Q)$ be defined as continuous extensions of the discrete functions $\Delta TC(Q)$ and $\Delta^2 TC(Q)$, respectively. Setting $E''(Q)$ equal to zero and simplifying the resulting expression yields

$$cQ^2 + (3c - a\lambda)Q + (2c - 2a + b\lambda^2) = 0 \qquad (7)$$

which can be solved for Q by means of the quadratic formula for any specific values of λ, a, b, and c. In terms of the original notation for the relevant cost parameters, (7) can be written as

$$E_C Q^2 + \left[3E_C - (E_V + E_C + S_V + S_C)\lambda\right]Q + \left[2E_C - 2(E_V + E_C + S_V + S_C)\lambda + S_C\lambda^2\right] = 0 \qquad (8)$$

Equation (8) has not more than two roots, hence $E'(Q)$ can have not more than two critical points. Since $E'(Q)$ must become positive as Q becomes large, the algebraic sign of $E'(Q)$, and thus of $\Delta TC(Q)$, can change from negative to positive at not more than two values of Q. It follows that there can not be more than two local minima for TC(Q). If $\Delta TC(Q)$ is positive when Q is zero, TC(Q) must be compared with one other local minimum of TC(Q), if it exists, in order that the absolutely optimal quantity Q can be ascertained.

Based upon computational experience and other insights, a procedure for determining the optimal supply grouping quantity of an item for which the relevant costs have been measured and for which the demand is Poisson-distributed would seem to consist of three basic steps.

First, evaluate (3) and (4) at Q equals zero and solve equation (8) for Q. Equations (3) and (4) evaluate at Q equals zero becomes

$$\Delta TC(0) = (E_V+E_C+S_V+S_C)P(0) - S_C P(1) - S_V \qquad (9)$$

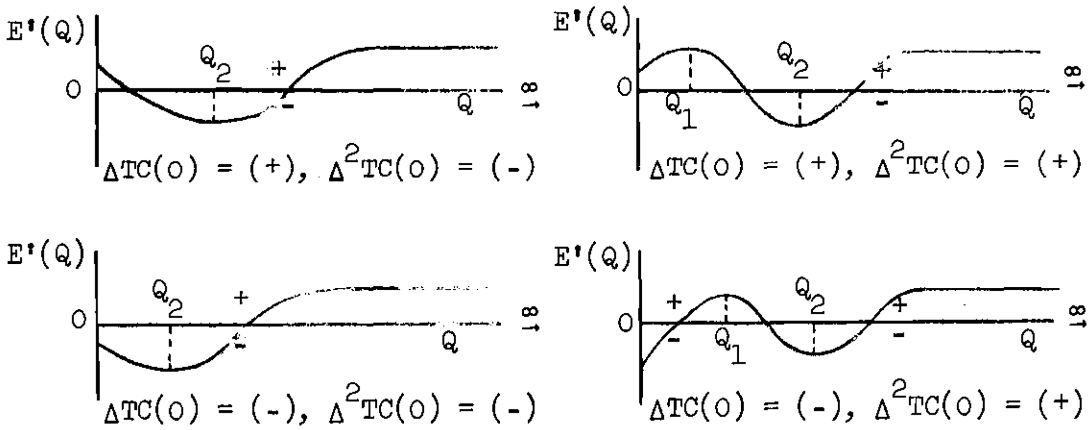
$$\Delta^2 TC(0) = (E_V+E_C+S_V+S_C)P(1) - S_C P(2) - E_C P(0) \qquad (10)$$

Let the two roots of equation (8) be designated as Q_1 for the smaller root and Q_2 for the larger root. The following empirically-developed chart now may serve as a guide in proceeding to step two of the optimization procedure.

Algebraic Sign of $\Delta TC(Q)$	$\Delta^2 TC(0)$	Local Minima of TC(Q) to be Compared: TC(0)	Other TC(Q)	Begin Search for Local Minima at: (from (8) above)
+	-	yes	one	$Q = Q_2$ (nearest integer)
+	+	yes	one	$Q = Q_2$ (nearest integer)
-	-	no	one	$Q = Q_2$ (nearest integer)
-	+	no	two	$Q = Q_1$, $Q = Q_2$ (nearest integer)

Figure 1. Search Procedure Chart for the Poisson Case.

Using the function E'(Q) as a continuous extension of $\Delta TC(Q)$, the approximate conditions used to develop Figure 1 may be shown graphically as



Following the calculation of the number and general location of possible local optimal values of Q, the search for those specific values of Q becomes the second basic step of the optimization procedure. The search procedure will utilize equation (3) and only discrete values of Q.

If the suggested starting points of Figure 1 are used, then local minimum values of $TC(Q)$ will occur whenever Q is the smallest integer value near the starting point for which (3) is nonnegative; that is, for which

$$\Delta TC(Q) = (E_V+E_C+S_V+S_C)F(Q) - S_C F(Q+1) - E_C F(Q-1) - S_V \geq 0 \qquad (11)$$

A suggested approach using a starting point of Figure 1 is to evaluate $\Delta TC(Q)$ at the starting value of Q. If the algebraic sign of $\Delta TC(Q)$ is positive, decrease the starting value of Q by one and use this new value of Q to evaluate $\Delta TC(Q)$, and so on until the algebraic sign of $\Delta TC(Q)$ becomes negative; the next larger value of Q is then a local optimum. If the algebraic sign of $\Delta TC(Q)$ is negative at the starting value of Q, increase Q by one in successive evaluations of $\Delta TC(Q)$ until the algebraic sign of $\Delta TC(Q)$ becomes nonnegative. At that point, Q is a local optimum.

When all of the local optimum values of Q have been found, it is necessary to compare the values of $TC(Q)$ at those points in order to ascertain the absolute optimum quantity Q_0. This is the third basic step of the optimization procedure. $TC(0)$ may be calculated easily from the expected total relevant cost model of (1). If the local optima are relatively small values of Q, (1) also may be used to evaluate $TC(Q)$ at points other than zero. If the values of Q are relatively close together, however, it may be easier to add the finite differences to $TC(Q)$ between these values. For example, if the local minima of $TC(Q)$ are found to be $TC(Q_a)$ and $TC(Q_b)$, the difference between $TC(Q_a)$ and $TC(Q_b)$ is given by

$$TC(Q_b) - TC(Q_a) = \Delta TC(Q_a) + \Delta TC(Q_a+1) + \ldots + \Delta TC(Q_b-1) \quad (12)$$

where $Q_b > Q_a$.

Thus, if (12) is positive, Q_a is optimal; if (12) is negative, Q_b is optimal.

Tables of values for the Poisson distribution function and probability mass function will facilitate the use of the foregoing optimization procedure.

The method of search for optimal quantities which is outlined above, althouth technically feasible, may become rather lengthy and complex, even for small values of the mean demand for a supply item. It is suspected that the routine use of this particular search procedure may involve systemic socts which could have the effect of neutralizing much of the monetary advantage to be achieved through "economically optimal" solutions. Thus, specific manual computational routines for the above search procedure are not presented in this paper. Instead, the expected total relevant cost model given by Equation (1) was used to prepare a simple routine for use with a digital computer in the rapid calculation of optimal quantities for supply items for which the probability distribution of demand is Poisson. This alternative procedure is discussed in Chapter IV.

The Continuous Case

The expected total relevant cost model when the demand for the supply item is in continuous quantity units is given by

$$TC(Q) = E_V \int_0^Q (Q-x)f(x)dx + E_C \int_0^Q f(x)dx + S_V \int_Q^\infty (x-Q)f(x)dx + S_C \int_Q^\infty f(x)dx \quad (13)$$

It will now be assumed that the probability density function, $f(x)$, possesses a continuous derivative with respect to x throughout the region of integration. The sufficient conditions for the function $TC(Q)$ in (13) to have an absolute minimum value $TC(Q)$ at Q_0 are that

$$\frac{d}{dQ} TC(Q) = 0 \qquad \text{for } Q = Q_0,$$

$$\frac{d^2}{dQ^2} TC(Q) > 0 \qquad \text{for all } Q.$$

The first derivative of the expected cost function (13) with respect to Q is found to be

$$\frac{d}{dQ} TC(Q) = E_V \int_0^Q f(x)dx + E_C f(Q) - S_V \int_Q^\infty f(x)dx - S_C f(Q)$$

Upon collecting terms and using the relationships

$$F(Q) = \int_0^Q f(x)dx = 1 - \int_Q^\infty f(x)dx$$

the first derivative of the expected cost function may be expressed as

$$\frac{d}{dQ} TC(Q) = (S_V+E_V)F(Q) + (E_C-S_C)f(Q) - S_V \tag{14}$$

It follows directly that

$$\frac{d^2}{dQ^2} TC(Q) = (S_V+E_V f(Q) + (E_C-S_C) \frac{d}{dQ} f(Q) \tag{15}$$

The sufficient conditions for $TC(Q_0)$ to be the absolute minimum value of TC(Q) now may be stated as

$$\frac{d}{dQ} TC(Q) = (S_V+E_V)F(Q) + (E_C-S_C)f(Q) - S_V = 0 \qquad (16)$$

for $Q = Q_0$, and

$$\frac{d^2}{dQ^2} TC(Q) = (S_V+E_V)f(Q) + (E_C-S_C)f'(Q) > 0 \qquad (17)$$

for all Q, where $f'(Q) = \frac{d}{dQ} f(Q)$.

That is, if TC(Q) is a convex function, its minimum value occurs at that value of Q for which the slope of TC(Q) is zero. Conditions (16) and (17) are sufficient, but not necessary, for $TC(Q_0)$ to be an absolute minimum value of TC(Q). It is necessary to show only that

$$TC(Q_0) \leq TC(Q) \text{ for all } Q.$$

As in the discrete case, it will be necessary to investigate the particular continuous probability distribution of interest to determine whether or not (13) is generally convex for that distribution. If (13) is found to be not convex, (16) and (17) may be used to ascertain the number and locations of local minima of TC(Q) that exist, thus simplifying the search for the absolute minimum and the corresponding absolutely optimal value of Q.

Since it is expected that there are few continuous probability distributions which will appropriately represent the behavior of demand for most items in hospital supply groupings, the continuous case, in general, will not be treated further here. The normal distribution, however, may be an appropriate probability distribution, particularly when the demand for a supply item is for large quantities of the item. The normal distribution also may be especially useful as an approximation to the Poisson distribution when the mean demand for an item whose demand is Poisson-distributed is large. Thus the special case of optimizing the grouping quantities for supply items whose demands are normally distributed will be treated here.

The Normal Case

When the probability distribution of demand is normal with mean μ and variance σ^2, the probability density function of the demand x is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

and the distribution function F(Q) is given by

$$F(Q) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{Q} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Although the functions described above are defined for all real values of x, it can be seen that the actual demand for a supply item will never become negative except for certain well-defined supply systems in which returns are allowed. Since negative demands will not be allowed for the purposes of this study, the problem of truncating the normal distribution function at zero arises. The consequence of truncation at zero may become significant when the variance of the normal distribution is large relative to the magnitude of the mean of the distribution. The magnitude of the error, ε, which is made in calculating the normal distribution function if the distribution function is truncated at zero is given by

$$\varepsilon = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{0} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

That is, the cumulative probability from $-\infty$ to 0 is omitted from the calculations. This error of ommission through truncation at zero can be reduced to a cumulative probability error of approximately 0.001 if it is required that

$$\frac{\mu-x}{\sigma} \geq 3 \qquad\qquad \text{for } x = 0$$

Thus it will be required that the truncation of the normal distribution at zero occur at least three standard deviations from the mean. This requirement is more than satisfied when the normal distribution is used as an approximation to the Poisson distribution for which the mean demand is not less than about ten units. It will be assumed that truncation error is insignificant in the following development.

Equation (17) for the normal distribution may be written as

$$\frac{d^2}{dQ^2}\,TC(Q) = (S_V+E_V)\frac{1}{\sqrt{2\pi}\,\sigma}\cdot e^{-1/2\left(\frac{Q-\mu}{\sigma}\right)^2} - (E_C-S_C)\,\frac{1}{\sqrt{2\pi}\,\sigma}\cdot e^{-1/2\left(\frac{Q-\mu}{\sigma}\right)^2}\cdot\left(\frac{Q-\mu}{\sigma}\right)$$

Since the normal density function is always positive, the above equation may be set equal to zero and simplified to yield

$$(S_V+E_V) - (E_C-S_C)\frac{Q-\mu}{\sigma} = 0 \qquad (18)$$

It can be seen by inspection of (14) that, for the normal case, the slope of TC(Q) is negative at Q equals zero and positive as Q becomes very large. It is clear also that (18) has only one root; hence (16) can have only one root and it must be that value of Q for which TC(Q) takes on an absolute minimum value. Although (13) is not clearly convex, it has been shown to have one and only one local minimum value which is the absolute minimum when the normal distribution is used; the solution of (16) for a given set of parameters for the normal probability distribution will insure that the root obtained is that optimal value of Q for which the function TC(Q) is minimized.

Since values of both the distribution function and the density function for the normal distribution with zero mean and unit variance have been tabulated and published, it may be helpful for computational purposes to transform equation (16) as follows.

Let

$$F(Q) = \frac{1}{\sqrt{2\pi}\,\sigma}\int_0^Q e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_0^{\frac{Q-\mu}{\sigma}} e^{-1/2\,x^2}\,dx$$

$$= \Phi\left(\frac{Q-\mu}{\sigma}\right)$$

and let

$$f(Q) = \frac{1}{\sqrt{2\pi}\,\sigma} \; e^{-1/2\left(\frac{Q-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{\sigma}\,\phi\left(\frac{Q-\mu}{\sigma}\right)$$

so that (16) how becomes

$$\frac{d}{dQ}\,TC(Q) = (S_V+E_V)\;\Phi\left(\frac{Q-\mu}{\sigma}\right) + \left(\frac{E_C-S_C}{\sigma}\right)\phi\left(\frac{Q-\mu}{\sigma}\right) - S_V = 0 \qquad (19)$$

The abscissa, $t = \frac{Q-\mu}{\sigma}$ of the standard normal distribution function Φ and the standard normal density function ϕ which satisfies (19) may be transformed easily into values of Q by use of the relationship

$$Q = \mu + t\,\sigma.$$

It has been assumed here, as for the discrete case, that the costs, E_V, E_C, S_V, and S_C are measurable and, hence, known, and that the parameters of the normal probability distribution of the demand x can be ascertained. It is anticipated that, because of the relative ease of calculations, the optimal quantity model (19) for the continuous case frequently will be used as an approximate solution to problems involving discrete demand quantities. Computational procedures for use by hospital management personnel for the optimization of grouping quantities whose demands are normally, or approximately normally, distributed are given in Chapter IV.

CHAPTER IV

COMPUTATIONAL PROCEDURES FOR OPTIMAL SUPPLY GROUPING QUANTITIES

In this chapter, the conceptual optimization procedures which were developed in Chapter III will be used to illustrate a methodology for the construction of computational procedures for use by hospital management personnel. It is again emphasized here that hospital management probably will require some professional assistance in the development and proper utilization of such procedures.

Selection of Supply Groupings for Study

The first step in the formulation of procedures regarding the quantities of various items to be contained in certain supply groupings is, of course, the careful design and selection of those supply groupings and the designation of the activities for which they are intended. For those supply groupings whose items are to be used for specific nursing procedures or medical activities, the basic responsibility for specifying the contents lies with those professional personnel who are aware of the requirements of those procedures and activities. When the types of supply items required have been designated for the various treatments and activities, a study of the type accomplished by Davis, Parks, and Wickel[23] may be of significant value in reducing both the degree of duplication of supply items and the number of supply groupings to be

[23] Davis, et.al., op.cit., pp. 124-132.

prepared.

Hospital management staff personnel such as management systems analysts[24] normally will be competent to ascertain the supply item requirements of nonmedical activities and procedures. These supply items may include, but are not limited to, such things as bed linens, the items on maintenance carts, tray utensils and accessories for patient meal service, and central supply service delivery carts.

When the system of supply groupings has been established, it is then possible to ascertain the probability distribution of the demand for each item in each supply grouping.

The Probability Distribution of Demand

Under the assumptions which were stated earlier, the demands, unit costs, and fixed costs associated with any item in a multiple-item supply grouping are assumed to be independent of the costs and demands of all other items in the grouping. Thus, the probability distribution of the demand for each item in a multiple-item supply grouping may be determined without regard for any other item if the characteristics of the supply grouping satisfy the foregoing assumptions. In most cases, this constraint requires that either shortages of two or more different supply items will not occur simultaneously or that the items are prepared at independent locations. (i.e., only one item may be replenished on a trip), and that the disposal and handling processes associated with different types of excess items are independent functions. Naddor[25]

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[24] Smalley, Harold E., "Hospital Management Systems Analyst Training Program," (Final Report), Hospital Systems Research Group, Georgia Institute of Technology, August 1966, 67 pp.

[25] Naddor, op.cit., p. 320.

has assumed that, in systems with several items of inventory, it is unlikely that two or more items will be replenished at the same time; thus, an analysis of a complete multiple-item supply grouping may consist of a series of single-item analyses.

The determination of the probability distribution of demand for a supply item consists of collecting and analyzing data on the actual usage rate of the item in the activity or treatment for which the supply grouping is intended. An example of a type of form for data collection which has been used successfully is shown in Figure 2. Certain controls such as records of stock replenishment may be of assistance in verifying the vaildity and reliability of the results of the data-collection procedure. It should be emphasized here that a sufficient number of observations must be taken to insure the desired level of confidence in the relative or absolute accuracy of the results obtained. Most standard texts in applied statistics present procedures which can be used to deal with this problem.

The nature of the probability distribution of the demand for a supply item can now be determined from a statistical analysis of the results of the data-collection procedure. The empirical probability distribution can be constructed directly from the observed data by tabulating the relative frequencies of occurrence of the observed demand quantities. Various forms of charts, tables, and graphs may be developed from the empirical distribution to facilitate computations involving the relative frequencies of demand.

Often, it may be found that the empirical probability distribution is not significantly different from some well-known theoretical prob-

DEMAND DATA FOR LINEN ITEMS

INSTRUCTIONS

Each time you take a linen item make a tally mark under your shift for that specific item for the bed number involved.

Date..................
Initial stock..........
Added stock............

Bed Number	Day					Evening					Night					Diagnoses
	Sheet	Pillow Case	Spread	Towel	Wash Cloth	Sheet	Pillow Case	Spread	Towel	Wash Cloth	Sheet	Pillow Case	Spread	Towel	Wash Cloth	
1a																
1b																
2a																
2b																
3a																
3b																
4a																
4b																
5																
6																
7																
8																
9																
10																
11																
12																
13																
14																

Figure 2. Data-Collection Form

ability distribution. Tests such as the chi-square test for "goodness of fit"[26] may be used to establish statistically whether or not the empirical data might reasonably have come from the population of a certain theoretical distribution. One of the advantages inherent in using an appropriate theoretical distribution is that the results of many useful calculations for various values of important parameters have been published in the forms of tables, charts, and graphs. The proper utilization of such available resources can reduce significantly the amount of time required to perform certain necessary computations.

In order that the probability distributions of the demand for the items in a supply grouping can represent more accurately the actual random demands for the items, it may be desirable to attempt to identify certain factors which might influence the demand. Although the kinds of items in a supply grouping for a particular activity or treatment will not vary, the quantities which are actually used may be significantly different for different levels of the activity or treatment. For example, the items required in making a particular kind of bed are specified by the hospital; the frequency of bed changes, however, may be significantly greater for surgical patients with drainage problems than for other categories of patients in the same ward. The factorial experimental design[27] and certain tests of statistical hypotheses can be used to detect those levels of the

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[26] Moroney, M.J., Facts from Figures, Penguin Books, Inc., Baltimore, Maryland, 1956, pp. 246-270.

[27] Hicks, Charles R., Fundamental Concepts in the Design of Experiments, Holt, Rinehart, and Winston, New York, 1964, pp. 75-93.

activity or treatment for which the probability distributions of demand are significantly different. Those levels of the activity or treatment for which the demand characteristics are found to be different may require either special forms of the basic supply grouping or supplementary groupings which might be used in addition to a basic supply grouping. The models and procedures reported herein can be applied to ascertain economically optimal supply quantities for whatever grouping forms and probability distributions are found to be appropriate.

The Measurement of Relevant Costs

Four types of relevant monetary costs associated with the supply grouping concept were defined in Chapter III. These costs include a fixed cost of having too few units of a supply item in a grouping (S_C), a fixed cost of having too many units in the grouping (E_C), and a cost per unit for each unit which is needed but is not in the grouping (S_V) and for each unit in the grouping which is not needed (E_V) at the time the grouping is used.

The fixed cost of having too few units of a supply item in a supply grouping (S_C) may be thought of as a constant procurement cost or ordering cost which is incurred regardless of the number of units of shortage. For example, if a shortage of a particular item occurs during the use of a treatment tray, the time and subsequent labor cost of travelling to and from the source of replenishment will not depend upon the number of units of shortage which must be procured.

The fixed cost of having too many units of the supply item in the grouping (E_C) typically might include such costs as the labor and equipment costs of the transportation of any number of unused, contaminated disposable items to a collection point, the materials, equipment, and labor costs associated with the washing, drying, and sterilization procedure for any number of reusable supply items which have been contaminated but not used, or the costs associated with returning any number of unused, uncontaminated units of the item to an open stock storage location. None of these types of costs depends directly upon the number of units involved.

The cost per unit for each unit which is needed but is not in the grouping at the point of use (S_V) may include such costs as the additional materials and labor costs of preparing a replenishment unit of each item of open stock for use in a particular procedure.

The cost per unit for each unit in the grouping which is not used (E_V) typically will consist of such costs as the materials cost per item of contaminated, unused, disposable items and the labor and materials associated with special rehandling or reprocessing of each surplus reusable item.

The components of each of the four types of monetary costs discussed above can be ascertained through the use of the procedures and computations developed by Freeman and Smalley[28] specifically for application in the hospital.

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[28] Freemand, John R. and Harold E. Smalley, "Cost Prediction Manual for Supply Decision in Hospitals," Project Bulletin No. 17, USPHS No. GN-5968, Hospital Systems Research Group, Georgia Institute of Technology, Atlanta, February 1964, 59 pp.

The Development of Computational Procedures

It is recognized that the solution of a series of problems using the optimization procedures developed in Chapter III for the Poisson distribution and the normal distribution may be time-consuming and difficult. In view of these considerations, an attempt was made to simplify the forms of these procedures and to develop practical step-by-step computational routines for use by hospital management personnel with professional assistance.

Computational Procedure for the Poisson Case

As was discussed in Chapter III, the calculations involved in a manual search for optimal grouping quantities for the general Poisson case may become rather lengthy and complex. Thus, an alternative scheme for calculating optimal grouping quantities for the Poisson case was developed. An Algorithmic Language (ALGOL) program was prepared for and validated with the Burroughs Algebraic Compiler for the 200 Electronic Data Processing System at the Rich Electronic Computer Center, Georgia Institute of Technology. This program is based upon the expected total relevant cost model for the discrete case given by Equation (1) in Chapter III. The computer routine evaluates the expected total relevant cost corresponding to each grouping quantity within a range of quantities which virtually exhausts the range of possible demand quantities for a given value of the mean demand for a supply item. A printout of this program is shown in Appendix A. Using this approach to determine optimal supply grouping quantities, it is necessary only to ascertain the mean demand and the fixed and unit costs for each supply item of interest and punch the information onto data cards in a prescribed manner. The data cards may then be placed with the deck of program routine cards, represented by the printout in Appendix A, and the resulting deck of punched cards can be processed using the data processing equipment described above. In order that the preparation of program cards and the interpretation of printed results can be understood and accomplished more easily, an illustrated explanation of each phase of the computational procedure for the Poisson case follows.

A. Preparation of the Computer Program Card Deck

1. For each supply item in each supply grouping of interest:

   a. Assign an identifying number to each supply item of interest, beginning with the integer "1" and proceeding in sequence until each supply item has been assigned a different number. Call the last, and largest, number "N".

   b. Calculate the average demand for each supply item for the period for which its supply grouping was designed. Designate each of these average demands by the letter "D", followed by the identifying subscript for the supply item as specified in 1-a above. (For example: D_1 = 5.900 items per day; D_2 = 2.691 items per day; etc.)

   c. For each supply item, calculate the four relevant costs discussed earlier in this chapter. To simplify matters, designate each of these costs as follows:

E_V = Cost per item when the grouping quantity exceeds the demand. (Overage unit cost)

S_V = Cost per item when the demand exceeds the grouping quantity. (Shortage unit cost)

E_C = The fixed cost incurred when the grouping quantity exceeds demand by any amount. (Fixed overage cost)

S_C = The fixed cost incurred when the demand exceeds the grouping quantity by any amount. (Fixed shortage cost.)

It may be convenient to add the supply item number subscript to each of the letter designations to facilitate reference to each of these costs. (For example: E_{V_1} = \$0.90, S_{V_1} = \$0.72, E_{C_1} = \$0.12, S_{C_1} = \$0.03; E_{V_2} = \$0.62, S_{V_2} = \$0.33, E_{C_2} = \$0.01, S_{C_2} = \$0.42; etc.).

2. Key-punching the Data Cards

   a. On one card only, punch the following information:

      (1) In column "1", punch the numeral "5".

      (2) Beginning in column "3", punch the number "N" (total number of supply items under consideration) from 1-a above.

   b. The average demand, D, for each supply item from 1-b above must be punched onto a separate data card as follows:

      (1) In column "1", punch the numeral "5".

      (2) Beginning in column "3", punch the value of the average demand for the supply item.

      (3) Care must be taken to keep the cards in order. Pencil notation of the appropriate supply item number on each card might be of value in this

respect.

c. A complete set of the four relevant costs E_V, S_V, E_C, and S_C from 1-c above may now be punched onto a single data card for each supply item in the following manner:

(1) In column "1", punch the numeral "5".

(2) Beginning in column "3", punch the calculated values corresponding to each of the four relevant costs E_V, S_V, E_C, and S_C, in that order, allowing two column spaces between each value.

3. The computer program card deck must be assembled as shown in Figure 3. It should be emphasized here that calculated numerical values, not the letter-number designations, must appear on the appropriate data cards. The program routine cards other than the data cards described above will be the same for any number of supply items and any values of the relevant costs.

4. When the complete card deck has been assembled as shown in Figure 3, the deck may be processed on the Burroughs Algebraic Compiler-220.

B. Interpretation of Results of Data Processing

Figure 4 is an example of the data and format which will be printed out for each supply item in part A when the computer card deck has been processed. The interpretation of these results under two different sets of circumstances

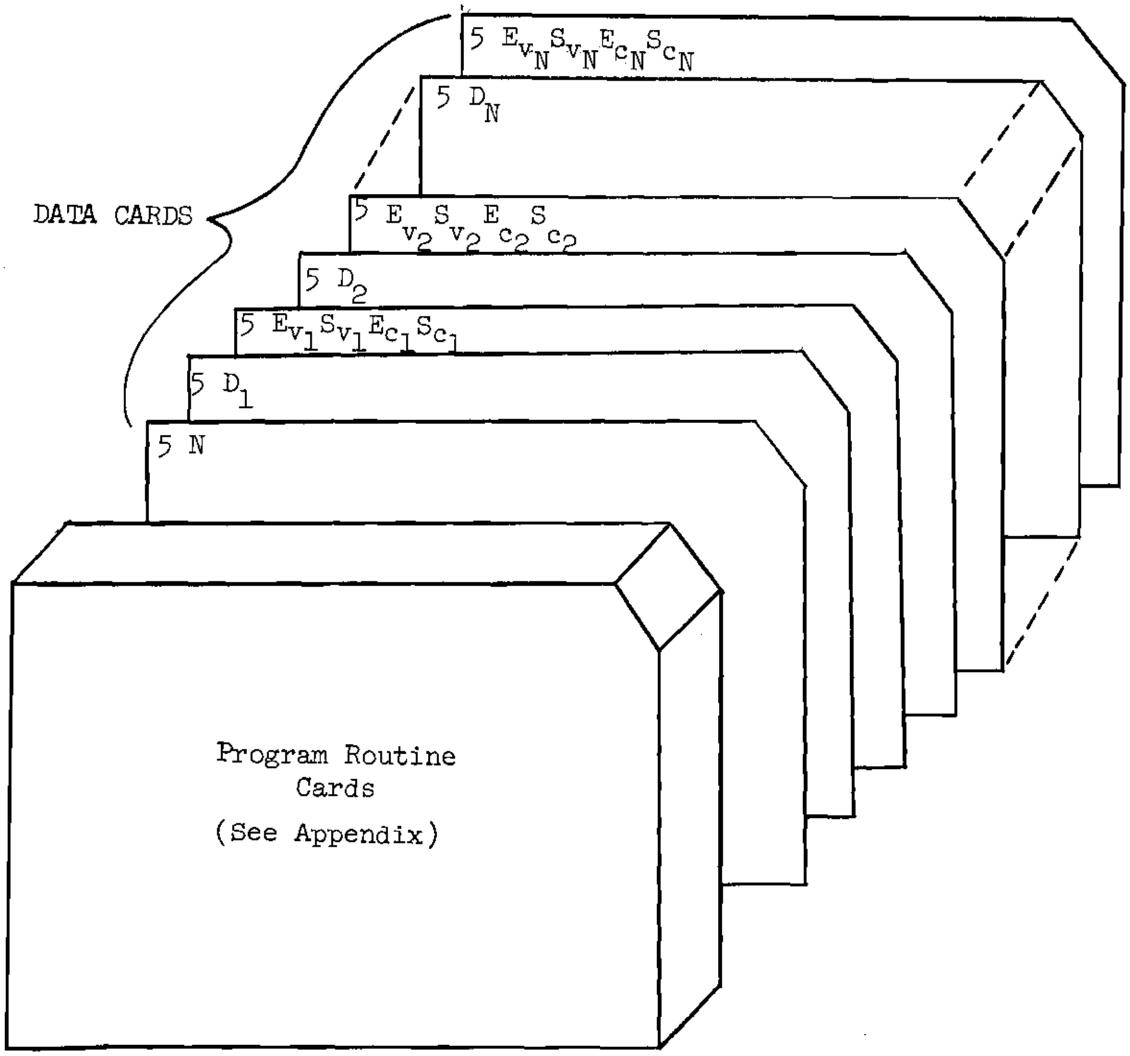

Figure 3. Assembly of Program Card Deck for the Poisson Case.

SUPPLY ITEM NUMBER 3

$E_V = .350 \quad E_C = .920 \quad S_V = .100 \quad S_C = .750$

AVERAGE DEMAND FOR THIS SUPPLY ITEM = 8.620

GROUPING QUANTITY, Q	TC(Q)	F(Q)
0	1.611	.000
1	1.510	.001
2	1.408	.008
3	1.303	.027
4	1.202	.069
5	1.118	.140
6	1.070	.243
7	1.079	.370
8	1.160	.506
9	1.315	.637
10	1.538	.749
11	1.813	.838
12	2.124	.901
13	2.456	.943
14	2.800	.969
15	3.149	.984
16	3.500	.992
17	3.851	.996
18	4.202	.998

OPTIMAL SUPPLY GROUPING QUANTITY = 6

Figure 4. Data Processing Results
For the Poisson Case

will be discussed in the following paragraphs.

When the primary criterion of interest is monetary cost, it is necessary only to read the "Optimal Supply Grouping Quantity" from the printout which will appear for each supply item. This economically optimal quantity is the grouping quantity for a specific supply item which corresponds to the minimum expected relevant cost for one use of that portion of a supply grouping which contains the supply item of interest. That is, if a supply grouping contains only optimal quantities of each of its component supply items, then the sum of the expected relevant costs for each type of supply item in the grouping is the minimum expected relevant cost per use of the supply grouping.

When the criterion of primary interest is other than monetary cost, the results of the data processing still may be of value in making a quantity decision about some item in a supply grouping. In Figure 4, for example, if the economically optimal quantity of six units of supply item number four is used, it can be seen that the probability $F(Q)$ that the demand will not exceed six units is only 0.243. Economically, of course, such a result is not alarming, since the costs of shortages in the case involved were significantly less than the costs of overages. However, if it is desired that the probability of "sufficiency" be increased to, say, 0.998, it can be seen that eighteen units of the supply item must be included in the supply grouping at an increase in the expected cost-per-period of $3.132 above the expected cost-per-period for the economically optimal grouping quantity. Such information may be of some value to hospital management personnel in imputing monetary costs to such factors as patient safety, worker preference, etc.

The procedures described above for the case of hospital supply items whose probability distributions of demand are Poisson were designed to be used routinely by hospital management personnel. It must be emphasized that the economically optimal solutions which result from the application of these procedures are solutions for specific values of the mean demand and relevant costs for supply items whose demand characteristics satisfy the assumptions stated earlier.

Computational Procedures for the Normal Case

When the probability distribution of the demand for a supply item is normally, or approximately normally, distributed, equation (19), which was developed for the normal case in Chapter III, may be used for the computation of the economically optimal grouping quantity for the supply item. Equation (19) may be written as

$$(S_V + E_V)\ \Phi\left(\frac{Q-\mu}{\sigma}\right) + \left(\frac{E_C - S_C}{\sigma}\right)\ \phi\left(\frac{Q-\mu}{\sigma}\right) = S_V,$$

in which Φ is the standard normal distribution function and ϕ is its derived function, the standard normal density. In order to solve equation (19) for Q, let

$$F = \Phi\left(\frac{Q-\mu}{\sigma}\right)$$

and

$$f = \phi\left(\frac{Q-\mu}{\sigma}\right).$$

Then

$$f = \phi \left[\Phi^{-1} (F) \right]$$

in which Φ^{-1} is the inverse of the increasing function Φ. It is now possible to find the pair (F,f) which satisfies both

$$f = \phi \left[\Phi^{-1} (F) \right] \tag{20}$$

and

$$(S_V + E_V) F + \left(\frac{E_C - S_C}{\sigma} \right) f = S_V \tag{21}$$

A graph of equation (20) in given in Figure 5. The graph of equation (21) is a straight line whose vertical and horizontal axis intercepts are given by

$$f_o = \frac{S_V \sigma}{E_C - S_C}$$

and

$$F_o = \frac{S_V}{S_V + E_V} ,$$

respectively, for a particular set of costs and demand parameters.

Thus, if the line representing equation (21) is plotted on the axes given in Figure 5, the pair (F, f) which satisfies equations (20) and (21) may be read directly. That value t of the standard normal abscissa which corresponds to the pair (F, f) also may be read directly

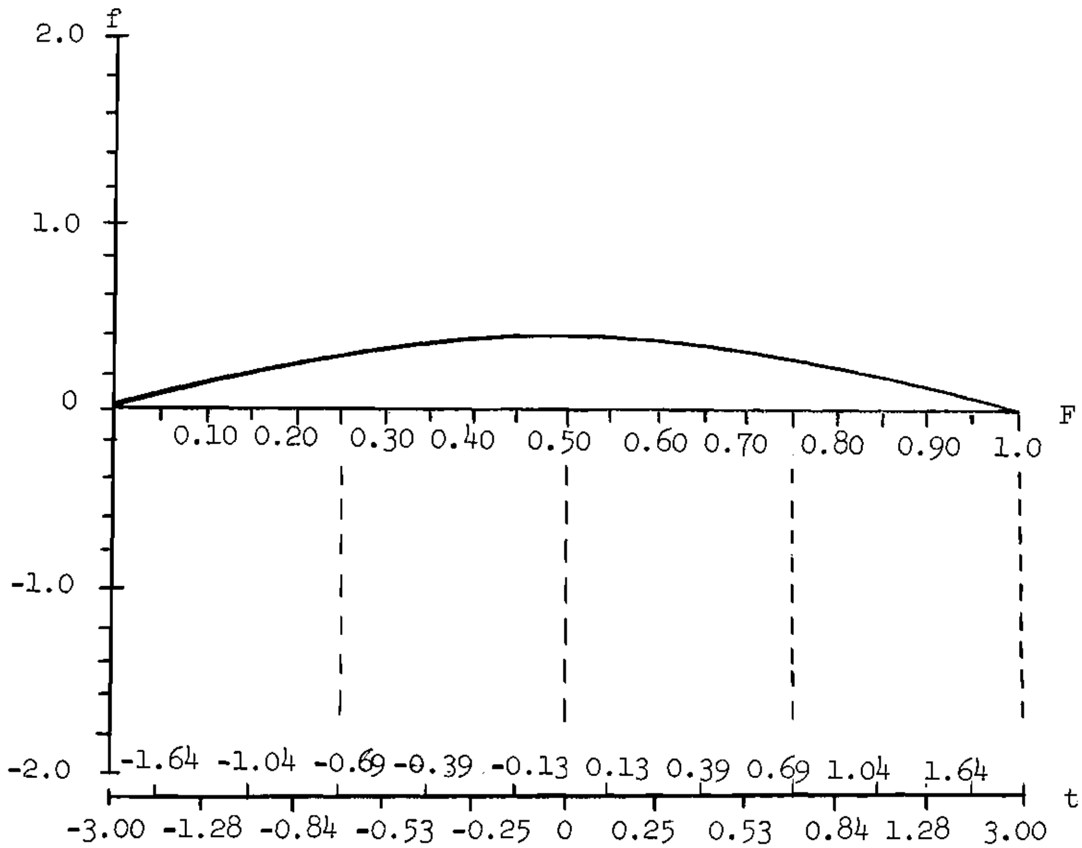

Figure 5. Optimal Supply Quantity Chart for the Normal Case

from Figure 5.[29,30] In order to ascertain the optimal supply grouping quantity Q it is necessary only to calculate

$$Q = \mu + t\sigma.$$

It is noted here that range of values of f on the vertical axis in Figure 5 is arbitrary and may be extended linearly in order to accommodate larger values of f which may be encountered. The computational procedures described above are presented in a simplified form on the worksheet in Figure 6.

The "Optimal Supply Quantity Worksheet" of Figure 6 may be used to calculate economically optimal grouping quantities for supply items whose demands are normally distributed or can be well-represented by the normal probability distribution. It is anticipated that professional assistance will be required by hospital management to ascertain the values on lines (1) through (9) on Figure 9. Such assistance should not be necessary to carry out the routine calculations of lines (10) through (18) which require no knowledge of either the supply characteristics or the nondeterministic nature of the demand function.

As in the discrete case, if nomonetary considerations such as patient safety are of significant consequence for supply items whose demands are normally distributed, the "Optimal Supply Quantity Chart" of Figure 5 may be of value to hospital management in ascertaining supply grouping quantities which meet specific criteria of "sufficiency."

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[29] Wine, R. Lowell, Statistics for Scientists and Engineers, Prentice-Hall Inc., Englewood Cliffs, N.J., 1964, pp. 624-625.

[30] Burington, Richard Stevens, Handbook of Mathematical Tables and Formulas, Handbook Publishers, Inc., Sandusky, Ohio, 1947, pp. 257-260.

Optimal Supply Quantity Worksheet

______ Supply Item	______ Department	______ Supply Grouping
______ Date	______ Analyst	

(1) Average number of units of the item when the grouping is used: _____ units

(2) Standard deviation of the demand for the item: _____ units

(3) Material and labor cost per unit for a normal use of the item: $_____

(4) Cost per unit of handling or processing each unit not used: $_____

(5) Salvage value per unit for unused units: $_____

(6) Line (3) + Line (4) - Line (5):(Cost per unit of excess units): $_____

(7) Fixed cost of handling, disposing of, or reprocessing any number of excess units: $_____

(8) Material and labor cost per unit of shortage in addition to Line (3): $_____

(9) Fixed cost of obtaining any number of shortage units from the item source: $_____

Use the above costs and the chart in Figure 5 in the following steps:

(10) Average demand, D, from Line (1):_________

(11) Standard deviation, S, from Line (2):_________

(12) Unit shortage cost, C, from Line (7):_________

(13) A = Line (6) + Line (8) = _________

(14) B = Line (7) - Line (9) = _________ (Note:B may be a negative number)

(15) Calculate:

(a) $f = \frac{S \times C}{B}$ = _________ (b) $F = \frac{C}{A}$ = _________

(16) On the chart, Figure 5, plot the value of f on the vertical axis labeled "f" and plot the value of F on the horizontal axis labeled "F". Connect these two plotted points with a straight line and mark its intersection with the curve.

(17) Read down from the intersection on the curve to the horizontal scale labeled "t" and record that value of t. t = _________

(18) Calculate optimal supply quantity, Q:

$$Q = D + (t \times S) = \_\_\_\_\_\_\_\_\_ \text{ units.}$$

Figure 6. Optimal Supply Quantity Worksheet for the Normal Case

For example, if it is desired that the "probability of sufficiency" for the grouping quantity of a supply item be 0.95, it can be seen in Figure 5 that, corresponding to an F value of 0.95, the value of t is 1.64. Thus for any supply item whose demand distribution is normal, the grouping quantity which will be "sufficient" about ninety-fine per cent of the time is given by

$$Q = \mu + 1.64\sigma,$$

where μ and σ are the mean and standard deviation, respectively, per period for the supply item.

The foregoing procedures for the optimization of grouping quantities for supply items whose demands are normally distributed also may be of value in the calculation of optimal grouping quantities for the Poisson case when the mean demand is approximately ten units or greater. If these procedures are to be used for the Poisson case, it is necessary only to make the following changes in order to use Figures 5 and 6. (See "The Poisson Case" in Chapter III for definitions of the Poisson notation.)

Let

$$\mu = \lambda$$

$$\sigma = \sqrt{\lambda}$$

When step (17) of Figure 6 has been completed, the economically optimal grouping quantity Q for a supply item whose demand is Poisson-distributed is given approximately by

$$Q = t\sqrt{\lambda} + \lambda$$

since the procedure will not be used for values of λ smaller than ten units.*

---

*This constraint was introduced earlier in Chapter III in respect to reducing the magnitude of the error involved in truncating the normal distribution at zero. In other applications of the normal approximation to the Poisson distribution, a correction for continuity for smaller values of λ is introduced, and for those cases, $Q = t\sqrt{\lambda} + \lambda + 1/2$ is a more satisfactory approximation.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The approach to the optimization of supply quantities for multiple-item hospital supply groupings which is treated in this thesis is applicable only to those supply groupings which possess the following characteristics:

1. The unit costs, fixed costs, and probability distribution of any item in a multiple-item supply grouping are independent of the costs and demand characteristics of all other supply items in the grouping.
2. The supply items contained in each supply grouping are intended for use without replenishment for a particular procedure, treatment, or activity during a specified period of time.
3. The criterion for the optimization of supply item quantities is monetary cost only.
4. The relevant fixed and unit monetary costs associated with the use of a supply item in a supply grouping are constant regardless of the quantity of the supply item included in the grouping.

Under the above conditions, the expected total relevant cost associated with the use of Q units of a supply item in a supply grouping and a methodology for ascertaining the economically optimal value of Q are given by the following relationships.

The Discrete Case

The expected total relevant cost associated with using Q units of a supply item whose demand is expressed in discrete units is given by

$$TC(Q) = E_V \sum_{x=0}^{Q-1} (Q-x)P(x) + E_C \sum_{x=0}^{Q-1} P(x) + S_V \sum_{x=Q+1}^{\infty} (x-Q)P(x) + S_C \sum_{x=Q+1}^{\infty} P(x) \quad (1)$$

If, for a particular discrete probability distribution of demand, the function TC(Q) is found to be convex, that is

$$\Delta^2 TC(Q) \geq 0 \qquad \text{for all } Q,$$

then Q_O is the smallest integer value of Q for which

$$\Delta TC(Q) = (E_V + E_C + S_V + S_C)F(Q) - S_C F(Q+1) - E_C F(Q-1) - S_V \geq 0.$$

If TC(Q) is not a convex function, it is necessary to locate all local minimum values of TC(Q) in order to compare those values of TC(Q) and to ascertain the absolute economically optimal quantity grouping Q_O. If, for a particular probability distribution, the equation

$$\Delta^2 TC(Q) = (E_V + E_C + S_V + S_C)P(Q+1) - S_C P(Q+2) - E_C P(Q) = 0 \qquad (6)$$

is solved for Q, and if the limiting values $\Delta TC(0)$ and $\Delta TC(\infty)$ are calculated, the number of roots, Q, and their values may be utilized to locate all local minima of TC(Q), and subsequently, Q_O.

Such a procedure was developed for the case of the Poisson distribution, but was found to be lengthy and complex. An alternative procedure was developed for the Poisson case in which a data processing routine for the Burroughs BAC-220, based upon Equation (1), was used to ascertain optimal supply grouping quantities. This routine is given in Appendix A and its use is outlined in detail in Chapter III.

The Continuous Case

The expected total relevant cost associated with using Q units of a supply item whose demand is expressed in continuous units is given by

$$TC(Q) = E_V \int_0^Q (Q-x)f(x)dx + E_C \int_0^Q f(x)dx + S_V \int_Q^\infty (x-Q)f(x) + S_C \int_Q^\infty f(x)dx \quad (13)$$

If Q_0 is the optimal quantity of the item to be used in the supply grouping of interest, then

$$TC(Q_0) \le TC(Q) \text{ for all } Q.$$

If, for a particular probability distribution of demand, the function TC(Q) is found to be convex, that is

$$\frac{d^2}{dQ^2} TC(Q) = (S_V+E_V)f(Q) + (E_C-S_C)f'(Q) > 0 \text{ for all } Q, \quad (17)$$

then Q_0 is that quantity Q for which

$$\frac{d}{dQ} TC(Q) = (S_V+E_V)F(Q) + (E_C-S_C)f(Q) - S_V = 0 \quad (16)$$

If TC(Q) is found to be not convex, it is necessary to locate all local minimum values of TC(Q) in order to compare those values of TC(Q) and to ascertain the absolute economically optimal grouping quantity Q_0. If, for a particular probability distribution, (17) is set equal to zero and solved for Q, and if the limiting values $\frac{d}{dQ}$ TC(0) and $\frac{d}{dQ}$ TC(∞) are calculated, the number and the nature of the roots of (16) and, subsequently, the number of local minima of TC(Q) may be ascertained.

Such a procedure was developed for the case of the normal distribution, and it was found that the optimal supply grouping quantity Q_0 for a supply item whose demand is normally distributed is given by that quantity Q which satisfies

$$\frac{d}{dQ} TC(Q_0) = (S_V+E_C)\ \Phi\left(\frac{Q-\mu}{\sigma}\right) + \left(\frac{E_C-S_C}{\sigma}\right)\ \phi\left(\frac{Q-\mu}{\sigma}\right) - S_V = 0 \qquad (19)$$

where Φ and ϕ are tabulated values of the standard normal distribution function and density function, respectively. A manual computational procedure for the optimization of supply grouping quantities for the normal case was prepared.

In both the discrete case and the continuous case, a nondeterministic expected cost model was used successfully to illustrate the preparation of computational routines to be used by hospital management to ascertain economically optimal supply grouping quantities.

Recommendations

The general approach and methodology for the design of computational procedures which are illustrated in Chapters III and IV may be used to develop computational mechanisms for any specific probability distribution of demand for an item in a supply grouping. It may not be possible always to design simple or efficient manual procedures as in the case of the normal distribution. The digital computer routine for the case of the Poisson distribution is an example of an alternative approach to the problem of designing computational procedures.

Although additional research into such problems as the optimization of supply grouping quantities for items whose demand functions are strongly interdependent might prove to be interesting, a more immediate problem in respect to the optimization concept in general would seem to have priority. The nature of the net economic advantage or disadvantage to be gained through "optimal" solutions when all systemic costs are considered has not been investigated here. It is the intention of the author to pursue certain aspects of this problem in future research.

It is intended, however, that the quantitative procedures formulated in this thesis will promote the eventual development of a formal body of knowledge in quantitative economics for hospital management systems. It is expected that this demonstration of the feasibility of constructing useful computational procedures from theoretical models will result in a greater appreciation of the ability of certain engineering approaches to measure the consequences of management decisions in respect to certain nondeterministic phenomena in the hospital.

APPENDIX A

```
2COMMENT V2267 BRAMBLETT,R.M. OPTIMAL QUANTITIES FOR SUPPLY GROUPINGS -
2                   FOR SUPPLY ITEMS WITH POISSON DEMAND                $
2REAL C,TC,J,K,M,L,EV,SV,EC,SC,P,Z,LSTAR,S,F,LFIX,D                     $
2INTEGER Q,I,N,T,LIM,A                                                  $
2ARRAY Q(400),C(400),D(400)                                             $
2INPUT NUM(N)                                                           $
2INPUT MEAN(L)                                                          $
2INPUT COSTS (EV,SV,EC,SC)                                              $
2OUTPUT NOMIN(T,A)                                                      $
2OUTPUT ANS(Q(I),C(I),D(I))                                             $
2OUTPUT COST(EV,EC,SV,SC,L)                                             $
2OUTPUT NOMIN(A)                                                        $
2OUTPUT SEQ(T)                                                          $
2FORMAT TITLEA(W3,B6,*SUPPLY ITEM NUMBER*,B2,I2)                        $
2FORMAT TITLE(W2,B6,*EV=*,x6.3,B3,*EC=*,x6.3,B3,*SV=*,x6.3,B3,*SC=*,x6.3
2      ,W0,B6,*AVERAGE DEMAND FOR THIS SUPPLY ITEM =*,x8.3)             $
2FORMAT TITLE 2(W2,B6,*GROUPING QUANTITY, Q*,B3,*TC(Q)*,B16,*F(Q)* )    $
2FORMAT FORM(B14,12B10,x8.3,B13,x5.3,W0)                                $
2FORMAT OPTIPAK(W2,B28,*OPTIMAL SUPPLY GROUPING QUANTITY = *,12,W2)     $
2         T=0                                                           $
2         READ($$NUM)                                                   $
2ORIGIN..READ($$MEAN)                                                   $
2         READ($$COSTS)                                                 $
2         LFIx=L                                                        $
2         FIx(LFIx)                                                     $
2         LSTAR=SQRT(L)                                                 $
2         FIx(LSTAR)                                                    $
2         LIM=3.(LSTAR)+1+LFIx                                          $
2         J=EC-SC                                                       $
2         K=SV+EV                                                       $
2         M=SC+(L).(SV)                                                 $
2         T=T+1                                                         $
2         Z=ExP(-L)                                                     $
2         F=0.0                                                         $
2         Q(1)=0                                                        $
2         C(1)=M-SC.Z                                                   $
2         F=Z                                                           $
```

(Continued)

```
2         D(1)=F                                                   $
2         Q(2)=1                                                   $
2         C(2)=(K+J).F-SC.Z.L-SV+M                                 $
2         D(2)=Z.(1+L)                                             $
2         I=2                                                      $
2         S=0.0                                                    $
2START..  I=I+1                                                    $
2         Z=Z.(L/Q(I-1))                                           $
2         S=S+Q(I-1).Z                                             $
2         Q(I)=Q(I-1)+1                                            $
2         F=F+Z                                                    $
2         P=Z.(L/Q(I))                                             $
2         D(I)=D(I-1)+P                                            $
2         C(I)=(K.Q(I)+J).F-K.S-SC.P-Q(I).SV+M                     $
2         IF Q(I) EQL LIM                                          $
2         GO TO OPT                                                $
2         GO TO START                                              $
2OPT..    TC=C(1)                                                  $
2         A=0                                                      $
2         FOR I=(1,1,(LIM+1))                                      $
2BEGIN                                                             $
2         IF C(I) LSS TC                                           $
2BEGIN                                                             $
2         TC=C(I)                                                  $
2         A=Q(I)                                                   $
2END                                                               $
2END                                                               $
2         WRITE($$SEQ,TITLEA)                                      $
2         WRITE($$COST,TITLE)                                      $
2         FOR I=(1,1,(LIM+1)                                       $
2         WRITE($$ANS,FORM)                                        $
2         WRITE($$NOMIN,OPTIPAK)                                   $
2         IF T EQL N                                               $
2         STOP                                                     $
2         GO TO ORIGIN                                             $
2FINISH
```

BIBLIOGRAPHY

Literature Cited

Balintfy, J.L., "A Stochastic Model for the Analysis and Prediction of Admissions and Discharges in Hospitals," Management Sciences: Models and Techniques, Pergamon Press, New York, 1960, pp. 288-299.

Balintfy, J.L., "Mathematical Models and Analysis of Certain Stochastic Processes in General Hospitals," Doctoral Dissertation, The Johns Hopkins University, Industrial Engineering Department, 1962.

Blumberg, Mark S., "'DPF' Concept Helps Predict Bed Needs," The Modern Hospital, December 1961, Vol. 97, No. 6, pp. 75-79.

Burington, Richard Stevens, Handbook of Mathematical Tables and Formulas, Handbook Publishers, Inc., Sandusky, Ohio, 1947, pp. 257-260.

Chang, Sang Hoon, "Determination of Optimum Reject Allowances in Manufacturing," Master's Thesis, Georgia Institute of Technology, School of Industrial Engineering, 1963, 39 pp.

Davis, Louis E., George M. Parks, and Samuel R. Wickel, Jr., "A Systems Study of Surgical Pack Making," Hospitals, February 16, 1964, Vol. 38, No. 4, pp. 124-132.

Dow, Wallace M., "Establishment of Supply Consumption Criteria," Final Report, USPHS #HM-00181-0251, University Hospital, University of Maryland, Baltimore, Maryland, 1964.

Flagle, Charles D., Robert J. Connor, Richard K.C. Hsich, Ruth A. Preston, and Sidney Singer, "Optimal Organization and Facility of a Nursing Unit," Progress Report, USPHS #GN-5537, Operations Research Division, The Johns Hopkins Hospital, Baltimore, Maryland, December 1959, 29 pp.

Freeman, John R., Harold E. Smalley, A.D. Joseph Emerzian, and Pamela H. Irwin, "Hospital Supply Decisions: Monetary Costs," Hospital Management, June 1964, Vol. 97, No. 6, pp. 98-111.

Freeman, John R. and Harold E. Smalley, "Cost Prediction Manual for Supply Decisions in Hospitals," Project Bulletin No. 17, USPHS #GN-5968, Hospital Systems Research Group, Georgia Institute of Technology, 1964, 59 pp.

George, Francis L. and Ruth P. Kuehn, Patterns of Patient Care, the Macmillan Company, New York, 1955, 266 pp.

Gue, Ronald L., "A Stochastic Description of Direct Patient Care and its Relation to Communication in a Hospital," Doctoral Dissertation, Johns Hopkins University, 1964.

Hicks, Charles R., Fundamental Concepts in the Design of Experiments, Holt, Rinehart, and Winston, New York, 1964, pp. 75-93.

Hsieh, Richard K.C., "A Study of Linen Processing and Distribution in a Hospital," Master's Essay, The Johns Hopkins University, Industrial Engineering Department, 1961.

McGibony, John R., Principles of Hospital Administration, G.P. Putnam's Sons, New York, 1952, pp. 518-520.

Middlehoven, W., "Analysis and Reorganization of a Central Supply Delivery System," Master's Essay, The Johns Hopkins University, Department of Operation Research and Industrial Engineering, 1964.

Moroney, M.J., Facts from Figures, Penguin Books, Inc., Baltimore, Maryland, 1956, pp. 246-270.

Naddor, Eliezer, Inventory Systems, John Wiley and Sons, Inc. New York, 1966, pp. 135-138, 144.

Pakzaban, Mahmood, "Probability Distributions of Demand for Selected Hospital Supply Items," Master's Research, Georgia Institute of Technology, School of Industrial Engineering, Preliminary Findings, May 1966.

Sasieni, Maurice, Arthur Yaspan, and Lawrence Friedman, Operations Research--Methods and Problems, John Wiley and Sons, Inc., New York, 1964, p. 100.

Smalley, Harold E., "Hospital Management Systems Analyst Training Program," (Final Report), Hospital Systems Research Group, Georgia Institute of Technology, Atlanta, August 1966, 67 pp.

Smalley, Harold E. and John R. Freeman, Hospital Industrial Engineering, Reinhold Publishing Corporation, New York, 1966, 460 pp.

Wine, R. Lowell, Statistics for Scientists and Engineers, Prentice-Hall, Inc., Englewood Cliffs, New Jersye, 1964, pp. 624, 625.

Young, John P., "Information Nexus Guides Decision System," The Modern Hospital, Vol. 106, No. 2, February 1962, pp. 101-105.

Other References

Boas, Arnold, H., Modern Mathematical Tools for Optimization, Chemical Engineering, Reprint, McGraw-Hill Publishing Co., Inc., New York, 1963, 26 pp.

Burroughs Corporation, Extended Algol Reference Manual, Detroit, Michigan, 1964.

Burroughs Corporation, Burroughs Algebraic Compiler, Revised Edition, Detroit, Michigan, 1964.

Goldberg, Samuel, Introduction to Difference Equations, John Wiley and Sons, Inc., New York, 1958, 260 pp.

Goodman, Richard, Modern Statistics, Arc Books, Inc., New York, 1964, pp. 66-71.

Parzen, Emanuel, Modern Probability Theory and Its Applications, John Wiley and Sons, Inc., New York, 1960, pp. 188-190, 206, 444, 445.