ACT Countdown Answer Keys

ACT Countdown #1
1. B Rewrite the logarithm in exponential form: $10^4 = x$ $x = 10{,}000$
2. G Let x = the number of two point questions and y = the number of five point questions. A system of equations can be used to solve for the number of five point questions (y). Total number of questions: $x + y = 50$ Total number of points on the exam: $2x + 5y = 235$ Solving for y: $-2(x + y = 50)$ Multiply the first equation by -2 to have opposite leading coefficients for x $2x + 5y = 235$ $-2x - 2y = -100$ Add the two equations to eliminate x $\underline{+\ 2x + 5y = 235}$ $3y = 135$ Solve for y (divide both sides by 3) $y = 45$
3. E Use the substitution rule for composition of functions: $f(g(x)) = 3(g(x)) + 1$ $= 3(\frac{x-1}{3}) + 1$ $= x - 1 + 1$ $= x$

ACT Countdown #2

1. D
   Write the equations in slope-intercept form and compare the slopes. Parallel Lines will have the same slope, but a different y-intercept.
   $4x + 4y = 4$
   $4y = -4x + 4$
   $y = -x + 1$

2. H
   Start by distributing both the b and the -2 to their respective parenthesis to get $5b - b^2 - 2b - 16$. Combine like terms and reorder terms by highest degree to end with the expression in answer choice H.

3. E
   The altitude of the triangle forms two 30^o-60^o-90^o triangles. Using that relationship or the Pythagorean Theorem, the height of the triangle is $5\sqrt{3}$. Then using the formula for the area of a triangle, the final area is $\frac{1}{2}bh = \frac{1}{2}(10)(5\sqrt{3}) = 25\sqrt{3}$.

ACT Countdown #3

1. D
   $x + (x + 1) + (x + 2) + (x + 3) = 226$
   $4x + 6 = 226$
   $4x = 220$
   $x = 55$
   The tallest student is $55 + 3 = 58\ inches\ tall$.

2. K
   Squaring each term to reach the desired value of x yields: $25 < x < 64$.

3. D
   The marbles that are not blue are red and green for a quantity of 12 out of 20 marbles. The probability $\frac{12}{20}$ can be simplified to $\frac{3}{5}$.

ACT Countdown #4
1. C To find the average of the original scores, calculate the sum then divide by 5: $\frac{220+22+264+241+280}{5}=\frac{1230}{5}=246.$ In order for Jane to maintain the exact average after a sixth game, she needs to bowl a score that matches the average of the first five games, therefore she needs to bowl a 246 in the last game.
2. K 30% of 280 is 84. Students can set up a proportion $\frac{84}{x}=\frac{70}{100}$ then cross multiply to solve for the value. The result is 120. Alternate method: $0.3(280)=0.7x$ and solve for x to get 120.
3. E The sum of the measures of the interior angles of a quadrilateral are 360^{o}. The sum of the 3 angles have a measure of 300^{o}, so the last angle must have a measure of 60^{o}.
ACT Countdown #5
1. C The ladder and the wall form a right triangle. The hypotenuse is 15 ft. and one of the legs is 12 ft. Using the Pythagorean Theorem or the Pythagorean Triples (the triangle formed is a multiple of a 3-4-5 right triangle), we establish that the missing side is 9 ft.
2. K $x^2 + 20 = 4$ — Subtract 20 from both sides (isolate x^2) $x^2 = -16$ — Take the square root of both sides $x = \pm\sqrt{-16}$ — The square root of a negative number yields an imaginary solution $x = \pm 4i$
3. D Since the sine of an angle is the $\frac{opposite}{hyptenuse}=\frac{8}{17}$, then the right triangle that is formed has a third length of 15 (by the Pythagorean Theorem or using the Pythagorean Triples). The cosine of an angle is the $\frac{adjacent}{hyptenuse}=\frac{15}{17}$ but because the angle is located in the second quadrant ($90° < x < 180°$), then the cosine of the angle must be negative.

ACT Countdown #6
1. A $(-1) + 2^2 \times -3$ $(-1) + 4 \times -3$ $(-1) + -12$ -13
2. J The description of the tangent points reveals a circle with a radius of 4 centered at the point $(-4, 4)$. Using the standard form equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$, and replacing the h and k with the x and y values of the center result in the equation in choice J.
3. A A reflection over the y-axis would cause the shape to move from the first quadrant to the second quadrant (image flip horizontally) and therefore have the y-value stay the same while the x-value becomes the opposite.
ACT Countdown #7
1. E Using the fundamental counting principle, multiply the number of options for each category: $4 \times 2 \times 2 \times 5 = 80$ possible combinations.
2. J You can use Prime Factorization to find the Greatest Common Factor of the 3 numbers. $21 = 3 \times 7$ $84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$ $357 = 3 \times 7 \times 17$ Since 3 and 7 divide evenly into all three numbers, GCF = 3 × 7 = 21
3. B The total volume of the cylinder is $\pi r^2 h = \pi(3^2)(10) = 90\pi$. The total volume of the cone is $\frac{\pi r^2 h}{3} = \frac{\pi(3^2)(10)}{3} = 30\pi$. The volume of the empty space is found by subtracting the total volume of the cone from the total volume of the cylinder.

ACT Countdown #8

1. C

Since $x > 2$ and $y < -2$, x is always positive and y is always negative.

a. Cannot be true because a positive divided by a negative always yields a negative and that is never greater than 1.

b. Cannot be true given the following counter example: $x = 3$ and $y = -10$.
9 > 10 is a false statement.

c. Is the true statement.

d. Cannot be true given the following counter example: $x = 3$ and $y = -4$.
9 + 2 > 16 + 2 is a false statement.

e. Cannot be true given the following counter example: $x = 4$ and $y = -3$.
1/16 > 1/9 is a false statement.

2. K

$2(x - 1) > 3(x + 2)$	Distribute
$2x - 2 > 3x + 6$	Subtract $2x$ from both sides (combine x's)
$-2 > x + 6$	Subtract 6 from both sides (isolate x)
$-8 > x$	Reorder the inequality
$x < -8$	

3. B

To find the perimeter of the roof truss, separate the roof truss into two congruent right triangles by drawing the vertical height from the top to the center of the bottom (bisecting the 30 feet into two 15 foot pieces). The right triangle created would have a base of 15 feet which would be opposite of the 60° angle. Using the ratios of the 30°-60°-90° special right triangle, the hypotenuse of the right triangle can be found to be $10\sqrt{3}$. Adding the base of the roof truss to both sets of hypotenuses, the total perimeter would be $30 + 10\sqrt{3} + 10\sqrt{3} = 30 + 20\sqrt{3}$.

ACT Countdown #9
1. E There are infinitely many irrational numbers between any two real numbers.
2. H $\frac{84}{6} = \frac{210}{x}$ $84x = 6(210)$ $84x = 1260$ $x = 15$ The maximum number of guests is 15 guests.
3. D Simplify using the exponent rules. $\left(\frac{3x^2y}{6x^4}\right)^{-2}\left(\frac{x^{-3}y}{2xy^4}\right)$ Simplify within the parenthesis subtracting exponents $= \left(\frac{y}{2x^2}\right)^{-2}\left(\frac{1}{2x^4y^3}\right)$ Distribute the exponent on the outside of the parenthesis $= \left(\frac{y^{-2}}{2^{-2}x^{-4}}\right)\left(\frac{1}{2x^4y^3}\right)$ Multiply the fractions (combine same bases by adding exponents) $= \left(\frac{y^{-2}}{2^{-1}y^3}\right)$ Reciprocate negative exponents and combine $= \frac{2}{y^5}$
ACT Countdown #10
1. A $\sqrt[3]{-64x^{36}y^{18}}$ Take the cube root of each term (divide exponents by 3) $-4x^{12}y^6$
2. F Substitute the location of the given ordered pair into each equation. $y = 3x$ $3 = 3(1)$ $3 = 3$ (1,3) is contained on the equation of the line.
3. D $\left(\frac{1}{9}+\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)$ $= \left(\frac{1}{9}+\frac{3}{9}\right)+\left(\frac{2}{4}-\frac{1}{4}\right)$ $= \left(\frac{4}{9}\right)+\left(\frac{1}{4}\right)$ $= \left(\frac{16}{36}\right)+\left(\frac{9}{36}\right) = \frac{25}{36}$