Cheatsheet of Sagemath in Linear Algebra

Last updates: 2020/05/04

1. Matrix and vectors

There are three ways to define a matrix.

```
A = matrix([[1,5],[2,6]])
A
```

```
[1 5]
[2 6]
```

```
B = matrix(QQ, [[1,-2,1],[0,2,-8],[5,0,-5]])
B
```

```
[ 1 -2  1]
[ 0  2 -8]
[ 5  0 -5]
```

```
C = matrix(3, 3, [[4, -1, 6], [2, 1, 6], [2,-1,8]])
C
```

```
[ 4 -1  6]
[ 2  1  6]
[ 2 -1  8]
```

Sometimes, the following matrix is used very often.

```
D = identity_matrix(QQ, 5)
D
```

```
[1 0 0 0 0]
[0 1 0 0 0]
[0 0 1 0 0]
[0 0 0 1 0]
[0 0 0 0 1]
```

Vector can be defined as

```
v = vector([0,8,10])
u = vector([1,-3,5])
```

solve system of linear equations

$B \mathbf{x}=v$

```
B\v
```

```
(1, 0, -1)
```

If you would like to get the augmented matrix of the linear system Bx=v

```
M=B.augment(v)
M
```

```
[ 1 -2  1  0]
[ 0  2 -8  8]
[ 5  0 -5 10]
```

Reduced row echelon form

To get the reduced row echelon form, use the commend M.rref()

```
M.rref()
```

```
[ 1  0  0  1]
[ 0  1  0  0]
[ 0  0  1 -1]
```

Three Row Operations

There are three commands for the elementary operations

1.(Replacement) add k times row i to row j: M.with_added_multiple_of_row(j,i,k)

2.(Interchange) swap the row i and row j: M.with_swapped_rows(i,j)

3.(Scaling) k times row i: M.with_rescaled_row(i,k)

It is not necessary to memorize the commands, you first type: M.with_, then press Tab. You will be able to select the commands.

```
M0 = M.with_swapped_rows(1,2)
M0
```

```
M1 = M0.with_added_multiple_of_row(1,0,-5)
M1
```

```
[  1  -2   1   0]
[  0  10 -10  10]
[  0   2  -8   8]
```

```
M2 = M1.with_rescaled_row(1, 1/10)
M2
```

```
[ 1 -2  1  0]
[ 0  1 -1  1]
[ 0  2 -8  8]
```

Vector operations

```
u+v, -2*v
```

```
((1, 5, 15), (0, -16, -20))
```

2. Eigenvalues and Eigenvectors

Eigenvalues

```
C.eigenvalues()
```

Characteristic_polynomial

```
cp = C.characteristic_polynomial()
cp
```

```
x^3 - 13*x^2 + 40*x - 36
```

```
cp.factor()
```

```
(x - 9) * (x - 2)^2
```

Eigenvectors_right()

```
C.eigenvectors_right()
```

```
[(9, [
  (1, 1, 1)
  ], 1), (2, [
  (1, 0, -1/3),
  (0, 1, 1/6)
  ], 2)]
```

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```

```