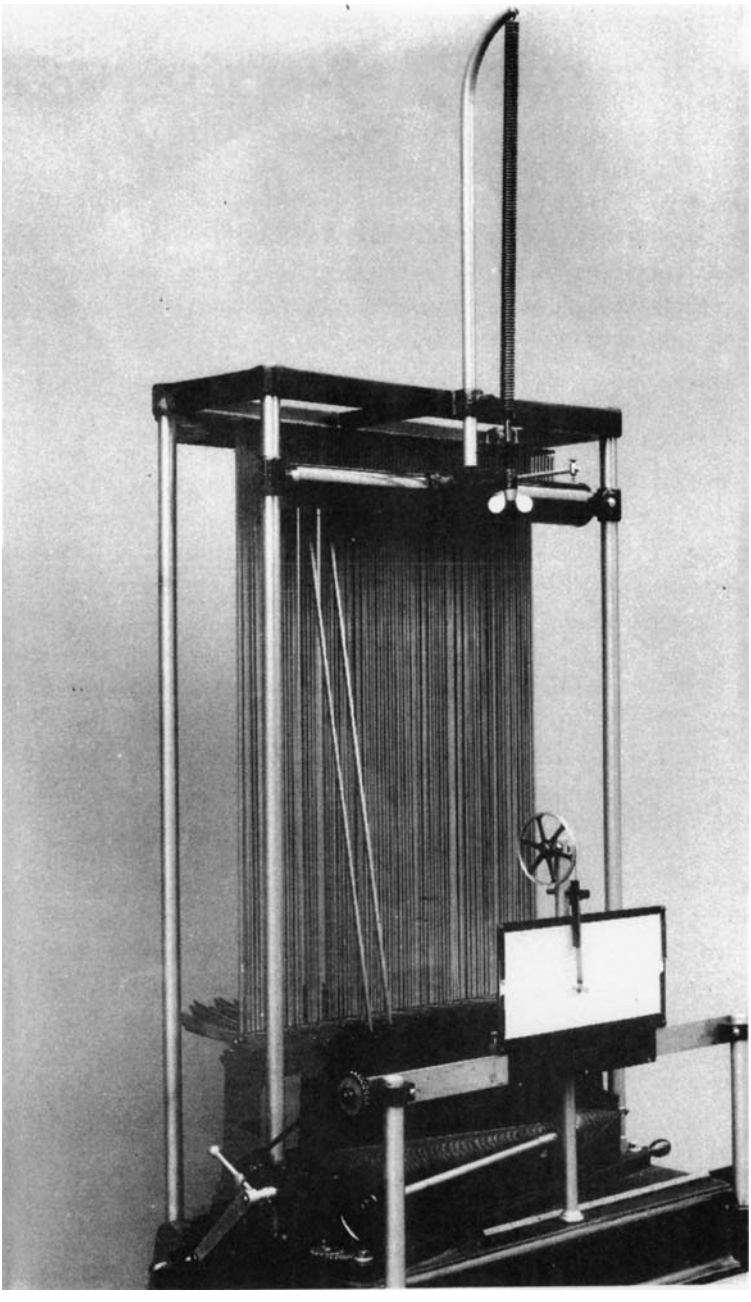
A Student's Guide to Fourier Transforms

Fourier transform theory is of central importance in a vast range of applications in physical science, engineering, and applied mathematics. This new edition of a successful undergraduate text provides a concise introduction to the theory and practice of Fourier transforms, using qualitative arguments wherever possible and avoiding unnecessary mathematics.

After a brief description of the basic ideas and theorems, the power of the technique is then illustrated by referring to particular applications in optics, spectroscopy, electronics and telecommunications. The rarely discussed but important field of multi-dimensional Fourier theory is covered, including a description of computer-aided tomography (CAT-scanning). The final chapter discusses digital methods, with particular attention to the fast Fourier transform. Throughout, discussion of these applications is reinforced by the inclusion of worked examples.

The book assumes no previous knowledge of the subject, and will be invaluable to students of physics, electrical and electronic engineering, and computer science.

JOHN JAMES has held teaching positions at the University of Minnesota, the Queen's University Belfast and the University of Manchester, retiring as Senior Lecturer in 1996. He is currently an Honorary Research Fellow at the University of Glasgow, a Fellow of the Royal Astronomical Society and Member of the Optical Society of America. His research interests include the invention, design and construction of astronomical instruments and their use in astronomy, cosmology and upper-atmosphere. Dr James has led eclipse expeditions to Central America, the Central Sahara, Java and the South Pacific islands. He is the author of about 40 academic papers and co-author with R. S. Sternberg of *The Design of Optical Spectrometers* (Chapman & Hall, 1969).



The Harmonic integrator, designed by Michelson and Stratton (see p. 72). This was the earliest mechanical Fourier transformer, built by Gaertner & Co. of Chicago in 1898. (Reproduced by permission of The Science Museum/Science & Society Picture Library.)

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with applications in physics and engineering

Second Edition

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1995
Second edition 2002
Reprinted 2004

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10/13 pt. *System* LaTeX 2$_\varepsilon$ [TB]

A catalogue record for this book is available from the British Library

ISBN 0 521 80826 X hardback
ISBN 0 521 00428 4 paperback

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Preface to the first edition

Showing a Fourier transform to a physics student generally produces the same reaction as showing a crucifix to Count Dracula. This may be because the subject tends to be taught by theorists who themselves use Fourier methods to solve otherwise intractable differential equations. The result is often a heavy load of mathematical analysis.

This need not be so. Engineers and practical physicists use Fourier theory in quite another way: to treat experimental data, to extract information from noisy signals, to design electrical filters, to 'clean' TV pictures and for many similar practical tasks. The transforms are done digitally and there is a minimum of mathematics involved.

The chief tools of the trade are the theorems in Chapter 2, and an easy familiarity with these is the way to mastery of the subject. In spite of the forest of integration signs throughout the book there is in fact very little integration done and most of that is at high-school level. There are one or two excursions in places to show the breadth of power that the method can give. These are not pursued to any length but are intended to whet the appetite of those who want to follow more theoretical paths.

The book is deliberately incomplete. Many topics are missing and there is no attempt to explain everything: but I have left, here and there, what I hope are tempting clues to stimulate the reader into looking further; and of course, there is a bibliography at the end.

Practical scientists sometimes treat mathematics in general and Fourier theory in particular, in ways quite different from those for which it was invented[1]. The late E. T. Bell, mathematician and writer on mathematics, once described mathematics in a famous book title as 'The Queen and Servant of Science'.

[1] It is a matter of philosophical disputation whether mathematics is invented or discovered. Let us compromise by saying that theorems are discovered; proofs are invented.

The queen appears here in her role as servant and is sometimes treated quite roughly in that role, and furthermore, without apology. We are fairly safe in the knowledge that mathematical functions which describe phenomena in the real world are 'well-behaved' in the mathematical sense. Nature abhors singularities as much as she does a vacuum.

When an equation has several solutions, some are discarded in a most cavalier fashion as 'unphysical'. This is usally quite right[2]. Mathematics is after all only a concise shorthand description of the world and if a position-finding calculation based, say, on trigonometry and stellar observations, gives two results, equally valid, that you are either in Greenland or Barbados, you are entitled to discard one of the solutions if it is snowing outside. So we use Fourier transforms as a guide to what is happening or what to do next, but we remember that for solving practical problems the blackboard-and-chalk diagram, the computer screen and the simple theorems described here are to be preferred to the precise tedious calculations of integrals.

Manchester, January 1994 J. F. James

[2] But Dirac's Equation, with its positive and negative roots, predicted the positron.

Preface to the second edition

This edition follows much advice and constructive criticism which the author has received from all quarters of globe, in consequence of which various typos and misprints have been corrected and some ambiguous statements and anfractuosities have been replaced by more clear and direct derivations. Chapter 7 has been largely rewritten to demonstrate the way in which Fourier transforms are used in CAT-scanning, an application of more than usual ingenuity and importance: but overall this edition represents a renewed effort to rescue Fourier transforms from the clutches of the pure mathematicians and present them as a working tool to the horny-handed toilers who strive in the fields of electronic engineering and experimental physics.

Glasgow, January 2001 J. F. James