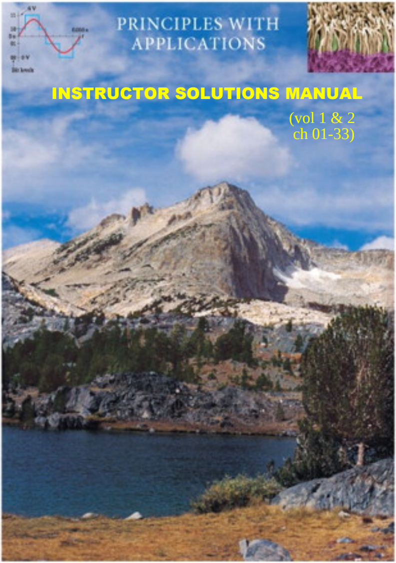
SEVENTH EDITION

PHYSICS

PRINCIPLES WITH APPLICATIONS

INSTRUCTOR SOLUTIONS MANUAL

(vol 1 & 2
ch 01-33)



DOUGLAS C.

GIANCOLI

INSTRUCTOR SOLUTIONS MANUAL

VOLUMES 1 & 2

DOUGLAS C. GIANCOLI'S

PHYSICS PRINCIPLES WITH APPLICATIONS

7TH EDITION

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PREFACE

This *Instructor's Solutions Manual* provides answers and worked-out solutions to all end of chapter questions and problems from chapters 1 – 15 of *Physics: Principles with Applications*, *7th Edition*, by Douglas C. Giancoli. At the end of the manual are grids that correlate the 6th edition questions and problems to the 7th edition questions and problems.

We formulated the solutions so that they are, in most cases, useful both for the student and the instructor. Accordingly, some solutions may seem to have more algebra than necessary for the instructor. Other solutions may seem to take bigger steps than a student would normally take: e.g. simply quoting the solutions from a quadratic equation instead of explicitly solving for them. There has been an emphasis on algebraic solutions, with the substitution of values given as a very last step in most cases. We feel that this helps to keep the physics of the problem foremost in the solution, rather than the numeric evaluation.

Much effort has been put into having clear problem statements, reasonable values, pedagogically sound solutions, and accurate answers/solutions for all of the questions and problems. Working with us was a team of five additional solvers – Karim Diff (Santa Fe College), Thomas Hemmick (Stony Brook University), Lauren Novatne (Reedley College), Michael Ottinger (Missouri Western State University), and Trina VanAusdal (Salt Lake Community College). Between the seven solvers we had four complete solutions for every question and problem. From those solutions we uncovered questions about the wording of the problems, style of the possible solutions, reasonableness of the values and framework of the questions and problems, and then consulted with one another and Doug Giancoli until we reached what we feel is both a good statement and a good solution for each question and problem in the text.

Many people have been involved in the production of this manual. We especially thank Doug Giancoli for his helpful conversations. Karen Karlin at Prentice Hall has been helpful, encouraging, and patient as we have turned our thoughts into a manual. Michael Ottinger provided solutions for every chapter, and helped in the preparation of the final solutions for some of the questions and problems. And the solutions from Karim Diff, Thomas Hemmick, Lauren Novatne, and Trina VanAusdal were often thought-provoking and always appreciated.

Even with all the assistance we have had, the final responsibility for the content of this manual is ours. We would appreciate being notified via e-mail of any errors that are discovered. We hope that you will find this presentation of answers and solutions useful.

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SEVENTH EDITION

VOLUME I

PHYSICS

PRINCIPLES WITH APPLICATIONS

DOUGLAS C.

GIANCOLI

1

INTRODUCTION, MEASUREMENT, ESTIMATING

Responses to Questions

$\boxed{1.}$ (*a*) A particular person's foot. Merits: reproducible. Drawbacks: not accessible to the general public; not invariable (size changes with age, time of day, etc.); not indestructible.

(*b*) Any person's foot. Merits: accessible. Drawbacks: not reproducible (different people have different size feet); not invariable (size changes with age, time of day, etc.); not indestructible.

Neither of these options would make a good standard.

$\boxed{2.}$ The distance in miles is given to one significant figure, and the distance in kilometers is given to five significant figures! The value in kilometers indicates more precision than really exists or than is meaningful. The last digit represents a distance on the same order of magnitude as a car's length! The sign should perhaps read "7.0 mi (11 km)," where each value has the same number of significant figures, or "7 mi (11 km)," where each value has about the same % uncertainty.

3. The number of digits you present in your answer should represent the precision with which you know a measurement; it says very little about the accuracy of the measurement. For example, if you measure the length of a table to great precision, but with a measuring instrument that is not calibrated correctly, you will not measure accurately. Accuracy is a measure of how close a measurement is to the true value.

4. If you measure the length of an object, and you report that it is "4," you haven't given enough information for your answer to be useful. There is a large difference between an object that is 4 meters long and one that is 4 feet long. Units are necessary to give meaning to a numerical answer.

5. You should report a result of 8.32 cm. Your measurement had three significant figures. When you multiply by 2, you are really multiplying by the integer 2, which is an exact value. The number of significant figures is determined by the measurement.

$\boxed{6.}$ The correct number of significant figures is three: $\sin 30.0° = 0.500$.

7. Useful assumptions include the population of the city, the fraction of people who own cars, the average number of visits to a mechanic that each car makes in a year, the average number of weeks a mechanic works in a year, and the average number of cars each mechanic can see in a week.

(*a*) There are about 800,000 people in San Francisco, as estimated in 2009 by the U.S. Census Bureau. Assume that half of them have cars. If each of these 400,000 cars needs servicing twice a year, then there are 800,000 visits to mechanics in a year. If mechanics typically work 50 weeks a year, then about 16,000 cars would need to be seen each week. Assume that on average, a mechanic can work on 4 cars per day, or 20 cars a week. The final estimate, then, is 800 car mechanics in San Francisco.

(*b*) Answers will vary.

Responses to MisConceptual Questions

1. (*d*) One common misconception, as indicated by answers (*b*) and (*c*), is that digital measurements are inherently very accurate. A digital scale is only as accurate as the last digit that it displays.

2. (*a*) The total number of digits present does not determine the accuracy, as the leading zeros in (*c*) and (*d*) are only placeholders. Rewriting the measurements in scientific notation shows that (*d*) has two-digit accuracy, (*b*) and (*c*) have three-digit accuracy, and (*a*) has four-digit accuracy. Note that since the period is shown, the zeros to the right of the numbers are significant.

3. (*b*) The leading zeros are not significant. Rewriting this number in scientific notation shows that it only has two significant digits.

4. (*b*) When you add or subtract numbers, the final answer should contain no more decimal places than the number with the fewest decimal places. Since 25.2 has one decimal place, the answer must be rounded to one decimal place, or to 26.6.

5. (*b*) The word "accuracy" is commonly misused by beginning students. If a student repeats a measurement multiple times and obtains the same answer each time, it is often assumed to be accurate. In fact, students are frequently given an "ideal" number of times to repeat the experiment for "accuracy." However, systematic errors may cause each measurement to be inaccurate. A poorly working instrument may also limit the accuracy of your measurement.

6. (*d*) This addresses misconceptions about squared units and about which factor should be in the numerator of the conversion. This error can be avoided when students treat the units as algebraic symbols that must be cancelled out.

7. (*e*) When making estimates, students frequently believe that their answers are more significant than they actually are. This question helps the student realize what an order-of-magnitude estimation is NOT supposed to accomplish.

8. (*d*) This addresses the fact that the generic unit symbol, like [*L*], does not indicate a specific system of units.

Solutions to Problems

1. (*a*) 214 $\boxed{\text{3 significant figures}}$

   (*b*) 81.60 $\boxed{\text{4 significant figures}}$

   (*c*) 7.03 $\boxed{\text{3 significant figures}}$

(*d*) 0.03 $\boxed{\text{1 significant figure}}$

(*e*) 0.0086 $\boxed{\text{2 significant figures}}$

(*f*) 3236 $\boxed{\text{4 significant figures}}$

(*g*) 8700 $\boxed{\text{2 significant figures}}$

2. (*a*) $1.156 = \boxed{1.156\times10^0}$

(*b*) $21.8 = \boxed{2.18\times10^1}$

(*c*) $0.0068 = \boxed{6.8\times10^{-3}}$

(*d*) $328.65 = \boxed{3.2865\times10^2}$

(*e*) $0.219 = \boxed{2.19\times10^{-1}}$

(*f*) $444 = \boxed{4.44\times10^2}$

3. (*a*) $8.69\times10^4 = \boxed{86,900}$

(*b*) $9.1\times10^3 = \boxed{9100}$

(*c*) $8.8\times10^{-1} = \boxed{0.88}$

(*d*) $4.76\times10^2 = \boxed{476}$

(*e*) $3.62\times10^{-5} = \boxed{0.0000362}$

$\boxed{4.}$ (*a*) $14 \text{ billion years} = \boxed{1.4\times10^{10} \text{ years}}$

(*b*) $$(1.4\times10^{10}\text{ yr})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right) = \boxed{4.4\times10^{17}\text{ s}}$$

$\boxed{5.}$ $$\%\text{ uncertainty} = \frac{0.25\text{ m}}{5.48\text{ m}}\times100\% = \boxed{4.6\%}$$

6. (*a*) $$\%\text{ uncertainty} = \frac{0.2\text{ s}}{5.5\text{ s}}\times100\% = 3.636\% \approx \boxed{4\%}$$

(*b*) $$\%\text{ uncertainty} = \frac{0.2\text{ s}}{55\text{ s}}\times100\% = 0.3636\% \approx \boxed{0.4\%}$$

(*c*) The time of 5.5 minutes is 330 seconds.

$$\%\text{ uncertainty} = \frac{0.2\text{ s}}{330\text{ s}}\times100\% = 0.0606\% \approx \boxed{0.06\%}$$

7. To add values with significant figures, adjust all values to be added so that their exponents are all the same.

$$(9.2\times10^3\ \text{s})+(8.3\times10^4\ \text{s})+(0.008\times10^6\ \text{s})=(9.2\times10^3\ \text{s})+(83\times10^3\ \text{s})+(8\times10^3\ \text{s})$$
$$=(9.2+83+8)\times10^3\ \text{s}=100.2\times10^3\ \text{s}=\boxed{1.00\times10^5\ \text{s}}$$

When you add, keep the least accurate value, so keep to the "ones" place in the last set of parentheses.

8. When you multiply, the result should have as many digits as the number with the least number of significant digits used in the calculation.

$$(3.079\times10^2\ \text{m})(0.068\times10^{-1}\ \text{m})=2.094\ \text{m}^2\approx\boxed{2.1\ \text{m}^2}$$

9. The uncertainty is taken to be 0.01 m.

$$\%\ \text{uncertainty}=\frac{0.01\ \text{m}^2}{1.57\ \text{m}^2}\times100\%=0.637\%\approx\boxed{1\%}$$

10. To find the approximate uncertainty in the volume, calculate the volume for the minimum radius and the volume for the maximum radius. Subtract the extreme volumes. The uncertainty in the volume is then half of this variation in volume.

$$V_{\text{specified}}=\tfrac{4}{3}\pi r_{\text{specified}}^3=\tfrac{4}{3}\pi(0.84\ \text{m})^3=2.483\ \text{m}^3$$
$$V_{\min}=\tfrac{4}{3}\pi r_{\min}^3=\tfrac{4}{3}\pi(0.80\ \text{m})^3=2.145\ \text{m}^3$$
$$V_{\max}=\tfrac{4}{3}\pi r_{\max}^3=\tfrac{4}{3}\pi(0.88\ \text{m})^3=2.855\ \text{m}^3$$
$$\Delta V=\tfrac{1}{2}(V_{\max}-V_{\min})=\tfrac{1}{2}(2.855\ \text{m}^3-2.145\ \text{m}^3)=0.355\ \text{m}^3$$

The percent uncertainty is $\dfrac{\Delta V}{V_{\text{specified}}}=\dfrac{0.355\ \text{m}^3}{2.483\ \text{m}^3}\times100=14.3\approx\boxed{14\%}$.

11. To find the approximate uncertainty in the area, calculate the area for the specified radius, the minimum radius, and the maximum radius. Subtract the extreme areas. The uncertainty in the area is then half this variation in area. The uncertainty in the radius is assumed to be 0.1×10^4 cm.

$$A_{\text{specified}}=\pi r_{\text{specified}}^2=\pi(3.1\times10^4\ \text{cm})^2=3.019\times10^9\ \text{cm}^2$$
$$A_{\min}=\pi r_{\min}^2=\pi(3.0\times10^4\ \text{cm})^2=2.827\times10^9\ \text{cm}^2$$
$$A_{\max}=\pi r_{\max}^2=\pi(3.2\times10^4\ \text{cm})^2=3.217\times10^9\ \text{cm}^2$$
$$\Delta A=\tfrac{1}{2}(A_{\max}-A_{\min})=\tfrac{1}{2}(3.217\times10^9\ \text{cm}^2-2.827\times10^9\ \text{cm}^2)=0.195\times10^9\ \text{cm}^2$$

Thus the area should be quoted as $\boxed{A=(3.0\pm0.2)\times10^9\ \text{cm}^2}$.

12. (*a*) 286.6 mm $\quad 286.6\times10^{-3}$ m $\quad \boxed{0.2866\ \text{m}}$

(*b*) 85 μV $\quad 85\times10^{-6}$ V $\quad \boxed{0.000085\ \text{V}}$

(*c*) 760 mg $\quad 760\times10^{-6}$ kg $\quad \boxed{0.00076\ \text{kg}}$ (if last zero is not significant)

(d) 62.1 ps 62.1×10^{-12} s $\boxed{0.0000000000621\text{ s}}$

(e) 22.5 nm 22.5×10^{-9} m $\boxed{0.0000000225\text{ m}}$

(f) 2.50 gigavolts 2.50×10^{9} volts $\boxed{2{,}500{,}000{,}000\text{ volts}}$

Note that in part (f) in particular, the correct number of significant digits cannot be determined when you write the number in this format.

13. (a) 1×10^{6} volts $\boxed{1\text{ megavolt}} = 1\text{ MV}$

(b) 2×10^{-6} meters $\boxed{2\text{ micrometers}} = 2\ \mu\text{m}$

(c) 6×10^{3} days $\boxed{6\text{ kilodays}} = 6\text{ kdays}$

(d) 18×10^{2} bucks $\boxed{18\text{ hectobucks}} = 18\text{ hbucks}$ or 1.8 kilobucks

(e) 7×10^{-7} seconds $\boxed{700\text{ nanoseconds}} = 700\text{ ns}$ or $0.7\ \mu\text{s}$

14. $$1\text{ hectare} = (1\text{ hectare})\left(\frac{1.000\times10^{4}\text{ m}^2}{1\text{ hectare}}\right)\left(\frac{3.281\text{ ft}}{1\text{ m}}\right)^2\left(\frac{1\text{ acre}}{4.356\times10^{4}\text{ ft}^2}\right) = \boxed{2.471\text{ acres}}$$

15. (a) $93\text{ million miles} = (93\times10^{6}\text{ miles})(1610\text{ m/1 mile}) = \boxed{1.5\times10^{11}\text{ m}}$

(b) $1.5\times10^{11}\text{ m} = (1.5\times10^{11}\text{ m})(1\text{ km}/10^{3}\text{ m}) = \boxed{1.5\times10^{8}\text{ km}}$

16. To add values with significant figures, adjust all values to be added so that their units are all the same.

$$1.80\text{ m} + 142.5\text{ cm} + 5.34\times10^{5}\ \mu\text{m} = 1.80\text{ m} + 1.425\text{ m} + 0.534\text{ m} = 3.759\text{ m} = \boxed{3.76\text{ m}}$$

When you add, the final result is to be no more accurate than the least accurate number used. In this case, that is the first measurement, which is accurate to the hundredths place when expressed in meters.

17. (a) $1.0\times10^{-10}\text{ m} = (1.0\times10^{-10}\text{ m})(39.37\text{ in/1 m}) = \boxed{3.9\times10^{-9}\text{ in}}$

(b) $$(1.0\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ atom}}{1.0\times10^{-10}\text{ m}}\right) = \boxed{1.0\times10^{8}\text{ atoms}}$$

18. (a) $$(1\text{ km/h})\left(\frac{0.621\text{ mi}}{1\text{ km}}\right) = 0.621\text{ mi/h, so the conversion factor is } \boxed{\frac{0.621\text{ mi/h}}{1\text{ km/h}}}.$$

(b) $$(1\text{ m/s})\left(\frac{3.28\text{ ft}}{1\text{ m}}\right) = 3.28\text{ ft/s, so the conversion factor is } \boxed{\frac{3.28\text{ ft/s}}{1\text{ m/s}}}.$$

(c) $$(1\text{ km/h})\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.278\text{ m/s, so the conversion factor is } \boxed{\frac{0.278\text{ m/s}}{1\text{ km/h}}}.$$

Note that if more significant figures were used in the original factors, such as 0.6214 miles per kilometer, more significant figures could have been included in the answers.

$\boxed{19.}$ (*a*) Find the distance by multiplying the speed by the time.

$$1.00 \text{ ly} = (2.998\times10^{8} \text{ m/s})(3.156\times10^{7} \text{ s}) = 9.462\times10^{15} \text{ m} \approx \boxed{9.46\times10^{15} \text{ m}}$$

(*b*) Do a unit conversion from ly to AU.

$$(1.00 \text{ ly})\left(\frac{9.462\times10^{15} \text{ m}}{1.00 \text{ ly}}\right)\left(\frac{1 \text{ AU}}{1.50\times10^{11} \text{ m}}\right) = \boxed{6.31\times10^{4} \text{ AU}}$$

$\boxed{20.}$ One mile is 1609 m, according to the unit conversions in the front of the textbook. Thus it is 109 m longer than a 1500-m race. The percentage difference is calculated here.

$$\frac{109 \text{ m}}{1500 \text{ m}}\times100\% = \boxed{7.3\%}$$

21. Since the meter is longer than the yard, the soccer field is longer than the football field.

$$\ell_{\text{soccer}} - \ell_{\text{football}} = 100.0 \text{ m}\times\frac{1.094 \text{ yd}}{1 \text{ m}} - 100.0 \text{ yd} = \boxed{9.4 \text{ yd}}$$

$$\ell_{\text{soccer}} - \ell_{\text{football}} = 100.0 \text{ m} - 100.0 \text{ yd}\times\frac{1 \text{ m}}{1.094 \text{ yd}} = \boxed{8.6 \text{ m}}$$

Since the soccer field is 109.4 yd compared with the 100.0-yd football field, the soccer field is $\boxed{9.4\%}$ longer than the football field.

22. (*a*) # of seconds in 1.00 yr: $1.00 \text{ yr} = (1.00 \text{ yr})\left(\frac{3.156\times10^{7} \text{ s}}{1 \text{ yr}}\right) = \boxed{3.16\times10^{7} \text{ s}}$

(*b*) # of nanoseconds in 1.00 yr: $1.00 \text{ yr} = (1.00 \text{ yr})\left(\frac{3.156\times10^{7} \text{ s}}{1 \text{ yr}}\right)\left(\frac{1\times10^{9} \text{ ns}}{1 \text{ s}}\right) = \boxed{3.16\times10^{16} \text{ ns}}$

(*c*) # of years in 1.00 s: $1.00 \text{ s} = (1.00 \text{ s})\left(\frac{1 \text{ yr}}{3.156\times10^{7} \text{ s}}\right) = \boxed{3.17\times10^{-8} \text{ yr}}$

23. (*a*) $\left(\frac{10^{-15} \text{ kg}}{1 \text{ bacterium}}\right)\left(\frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}}\right) = \boxed{10^{12} \text{ protons or neutrons}}$

(*b*) $\left(\frac{10^{-17} \text{ kg}}{1 \text{ DNA molecule}}\right)\left(\frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}}\right) = \boxed{10^{10} \text{ protons or neutrons}}$

(*c*) $\left(\frac{10^{2} \text{ kg}}{1 \text{ human}}\right)\left(\frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}}\right) = \boxed{10^{29} \text{ protons or neutrons}}$

(*d*) $\left(\frac{10^{41} \text{ kg}}{1 \text{ Galaxy}}\right)\left(\frac{1 \text{ proton or neutron}}{10^{-27} \text{ kg}}\right) = \boxed{10^{68} \text{ protons or neutrons}}$

24. The radius of the ball can be found from the circumference (represented by "c" in the equations below), and then the volume can be found from the radius. Finally, the mass is found from the volume of the baseball multiplied by the density $(\rho = \text{mass/volume})$ of a nucleon.

$$c_{\text{ball}} = 2\pi r_{\text{ball}} \rightarrow r_{\text{ball}} = \frac{c_{\text{ball}}}{2\pi};\ V_{\text{ball}} = \tfrac{4}{3}\pi r_{\text{ball}}^3 = \tfrac{4}{3}\pi\left(\frac{c_{\text{ball}}}{2\pi}\right)^3$$

$$m_{\text{ball}} = V_{\text{ball}}\rho_{\text{nucleon}} = V_{\text{ball}}\left(\frac{m_{\text{nucleon}}}{V_{\text{nucleon}}}\right) = V_{\text{ball}}\left(\frac{m_{\text{nucleon}}}{\tfrac{4}{3}\pi r_{\text{nucleon}}^3}\right) = V_{\text{ball}}\left(\frac{m_{\text{nucleon}}}{\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_{\text{nucleon}}\right)^3}\right)$$

$$= \tfrac{4}{3}\pi\left(\frac{c_{\text{ball}}}{2\pi}\right)^2\left(\frac{m_{\text{nucleon}}}{\tfrac{4}{3}\pi\left(\tfrac{1}{2}d_{\text{nucleon}}\right)^3}\right) = m_{\text{nucleon}}\left(\frac{c_{\text{ball}}}{\pi d_{\text{nucleon}}}\right)^3 = (10^{-27}\ \text{kg})\left(\frac{0.23\ \text{m}}{\pi(10^{-15}\ \text{m})}\right)^3$$

$$= 3.9\times10^{14}\ \text{kg} \approx \boxed{4\times10^{14}\ \text{kg}}$$

25. (*a*) $2800 = 2.8\times10^3 \approx 1\times10^3 = \boxed{10^3}$

(*b*) $86.30\times10^3 = 8.630\times10^4 \approx 10\times10^4 = \boxed{10^5}$

(*c*) $0.0076 = 7.6\times10^{-3} \approx 10\times10^{-3} = \boxed{10^{-2}}$

(*d*) $15.0\times10^8 = 1.5\times10^9 \approx 1\times10^9 = \boxed{10^9}$

26. The textbook is approximately 25 cm deep and 5 cm wide. With books on both sides of a shelf, the shelf would need to be about 50 cm deep. If the aisle is 1.5 m wide, then about 1/4 of the floor space is covered by shelving. The number of books on a single shelf level is then $\tfrac{1}{4}(3500\ \text{m}^2)\left(\frac{1\ \text{book}}{(0.25\ \text{m})(0.05\ \text{m})}\right) = 7.0\times10^4$ books. With 8 shelves of books, the total number of books stored is as follows:

$$\left(7.0\times10^4\ \frac{\text{books}}{\text{shelf level}}\right)(8\ \text{shelves}) \approx \boxed{6\times10^5\ \text{books}}$$

27. The distance across the U.S. is about 3000 miles.

$$(3000\ \text{mi})(1\ \text{km}/0.621\ \text{mi})(1\ \text{h}/10\ \text{km}) \approx \boxed{500\ \text{h}}$$

Of course, it would take more time on the clock for a runner to run across the U.S. The runner obviously could not run for 500 hours non-stop. If he or she could run for 5 hours a day, then it would take about 100 days to cross the country.

28. A commonly accepted measure is that a person should drink eight 8-oz. glasses of water each day. That is about 2 quarts, or 2 liters of water per day. Approximate the lifetime as 70 years.

$$(70\ \text{yr})(365\ \text{d}/1\ \text{yr})(2\ \text{L}/1\ \text{d}) \approx \boxed{5\times10^4\ \text{L}}$$

$\boxed{29.}$ An NCAA-regulation football field is 360 feet long (including the end zones) and 160 feet wide, which is about 110 meters by 50 meters, or $5500\ \text{m}^2$. We assume the mower has a cutting width of

0.5 meters and that a person mowing can walk at about 4.5 km/h, which is about 3 mi/h. Thus the distance to be walked is as follows:

$$d = \frac{\text{area}}{\text{width}} = \frac{5500\ \text{m}^2}{0.5\ \text{m}} = 11000\ \text{m} = 11\ \text{km}$$

At a speed of 4.5 km/h, it will take about $11\ \text{km} \times \frac{1\ \text{h}}{4.5\ \text{km}} \approx \boxed{2.5\ \text{h}}$ to mow the field.

$\boxed{30.}$ There are about 3×10^8 people in the U.S. Assume that half of them have cars, that they drive an average of 12,000 miles per year, and that their cars get an average of 20 miles per gallon of gasoline.

$$(3\times10^8\ \text{people})\left(\frac{1\ \text{automobile}}{2\ \text{people}}\right)\left(\frac{12{,}000\ \text{mi/auto}}{1\ \text{yr}}\right)\left(\frac{1\ \text{gallon}}{20\ \text{mi}}\right) \approx \boxed{1\times10^{11}\ \text{gal/yr}}$$

31. In estimating the number of dentists, the assumptions and estimates needed are:
- the population of the city
- the number of patients that a dentist sees in a day
- the number of days that a dentist works in a year
- the number of times that each person visits the dentist each year

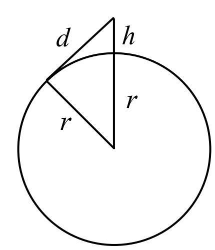
We estimate that a dentist can see 10 patients a day, that a dentist works 225 days a year, and that each person visits the dentist twice per year.

(*a*) For San Francisco, the population as of 2010 was about 800,000 (according to the U.S. Census Bureau). The number of dentists is found by the following calculation:

$$(8\times10^5\ \text{people})\left(\frac{2\ \text{visits/yr}}{1\ \text{person}}\right)\left(\frac{1\ \text{yr}}{225\ \text{workdays}}\right)\left(\frac{1\ \text{dentist}}{10\ \text{visits/workday}}\right) \approx \boxed{700\ \text{dentists}}$$

(*b*) For Marion, Indiana, the population is about 30,000. The number of dentists is found by a calculation similar to that in part (*a*), and would be about $\boxed{30\ \text{dentists}}$. There are about 40 dentists (of all types, including oral surgeons and orthodontists) listed in the 2012 Yellow Pages.

32. Consider the diagram shown (not to scale). The balloon is a distance $h = 200$ m above the surface of the Earth, and the tangent line from the balloon height to the surface of the Earth indicates the location of the horizon, a distance *d* away from the balloon. Use the Pythagorean theorem.



$$(r+h)^2 = r^2 + d^2 \quad\rightarrow\quad r^2 + 2rh + h^2 = r^2 + d^2$$

$$2rh + h^2 = d^2 \quad\rightarrow\quad d = \sqrt{2rh + h^2}$$

$$d = \sqrt{2(6.4\times10^6\ \text{m})(200\ \text{m}) + (200\ \text{m})^2} = 5.1\times10^4\ \text{m} \approx \boxed{5\times10^4\ \text{m}}(\approx 80\ \text{mi})$$

33. At \$1,000 per day, you would earn \$30,000 in the 30 days. With the other pay method, you would get $\$0.01(2^{t-1})$ on the *t*th day. On the first day, you get $\$0.01(2^{1-1}) = \$0.01$. On the second day, you get $\$0.01(2^{2-1}) = \$0.02$. On the third day, you get $\$0.01(2^{3-1}) = \$0.04$. On the 30th day, you get $\$0.01(2^{30-1}) = \$5.4\times10^6$, which is over 5 million dollars. Get paid by the $\boxed{\text{second method}}$.

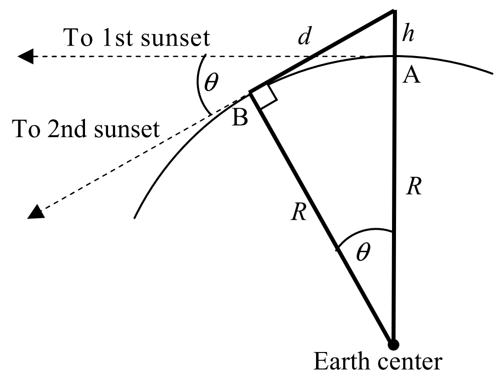
34. In the figure in the textbook, the distance d is perpendicular to the radius that is drawn approximately vertically. Thus there is a right triangle, with legs of d and R, and a hypotenuse of $R+h$. Since $h \ll R,\ h^2 \ll 2Rh$.

$$d^2+R^2=(R+h)^2=R^2+2Rh+h^2 \quad\rightarrow\quad d^2=2Rh+h^2 \quad\rightarrow\quad d^2\approx 2Rh \quad\rightarrow\quad R=\frac{d^2}{2h}$$

$$=\frac{(4400\text{ m})^2}{2(1.5\text{ m})}=\boxed{6.5\times10^6\text{ m}}$$

A better measurement gives $R=6.38\times10^6$ m.

35. For you to see the Sun "disappear," your line of sight to the top of the Sun must be tangent to the Earth's surface. Initially, you are lying down at point A, and you see the first sunset. Then you stand up, elevating your eyes by the height $h=130$ cm. While you stand, your line of sight is tangent to the Earth's surface at point B, so that is the direction to the second sunset. The angle θ is the angle through which the Sun appears to move relative to the Earth during the time to be measured. The distance d is the distance from your eyes when standing to point B.



Use the Pythagorean theorem for the following relationship:

$$d^2+R^2=(R+h)^2=R^2+2Rh+h^2 \quad\rightarrow\quad d^2=2Rh+h^2$$

The distance h is much smaller than the distance R, so $h^2 \ll 2Rh$ which leads to $d^2 \approx 2Rh$. We also have from the same triangle that $d/R=\tan\theta$, so $d=R\tan\theta$. Combining these two relationships gives $d^2\approx 2Rh=R^2\tan^2\theta$, so $R=\dfrac{2h}{\tan^2\theta}$.

The angle θ can be found from the height change and the radius of the Earth. The elapsed time between the two sightings can then be found from the angle, because we know that a full revolution takes 24 hours.

$$R=\frac{2h}{\tan^2\theta} \quad\rightarrow\quad \theta=\tan^{-1}\sqrt{\frac{2h}{R}}=\tan^{-1}\sqrt{\frac{2(1.3\text{ m})}{6.38\times10^6\text{ m}}}=(3.66\times10^{-2})^\text{o}$$

$$\frac{\theta}{360^\text{o}}=\frac{t\text{ sec}}{24\text{ h}\times\dfrac{3600\text{ s}}{1\text{ h}}} \quad\rightarrow$$

$$t=\left(\frac{\theta}{360^\text{o}}\right)\left(24\text{ h}\times\frac{3600\text{ s}}{1\text{ h}}\right)=\left(\frac{(3.66\times10^{-2})^\text{o}}{360^\text{o}}\right)\left(24\text{ h}\times\frac{3600\text{ s}}{1\text{ h}}\right)=\boxed{8.8\text{ s}}$$

36. Density units $=\dfrac{\text{mass units}}{\text{volume units}}=\boxed{\left[\dfrac{M}{L^3}\right]}$

37. (*a*) For the equation $\upsilon = At^3 - Bt$, the units of At^3 must be the same as the units of υ. So the units of *A* must be the same as the units of υ/t^3, which would be $\boxed{L/T^4}$. Also, the units of *Bt* must be the same as the units of υ. So the units of *B* must be the same as the units of υ/t, which would be $\boxed{L/T^2}$.

(*b*) For *A*, the SI units would be $\boxed{\text{m/s}^4}$, and for B, the SI units would be $\boxed{\text{m/s}^2}$.

38. (*a*) The quantity υt^2 has units of $(\text{m/s})(\text{s}^2) = \text{m}\bullet\text{s}$, which do not match with the units of meters for *x*. The quantity 2*at* has units $(\text{m/s}^2)(\text{s}) = \text{m/s}$, which also do not match with the units of meters for *x*. Thus this equation $\boxed{\text{cannot be correct}}$.

(*b*) The quantity $\upsilon_0 t$ has units of $(\text{m/s})(\text{s}) = \text{m}$, and $\frac{1}{2}at^2$ has units of $(\text{m/s}^2)(\text{s}^2) = \text{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

(*c*) The quantity $\upsilon_0 t$ has units of $(\text{m/s})(\text{s}) = \text{m}$, and $2at^2$ has units of $(\text{m/s}^2)(\text{s}^2) = \text{m}$. Thus, since each term has units of meters, this equation $\boxed{\text{can be correct}}$.

39. Using the units on each of the fundamental constants (*c*, *G*, and *h*), we find the dimensions of the Planck length. We use the values given for the fundamental constants to find the value of the Planck length.

$$\ell_P = \sqrt{\frac{Gh}{c^3}} \rightarrow \sqrt{\frac{[L^3/MT^2][ML^2/T]}{[L/T]^3}} = \sqrt{\left[\frac{L^3L^2T^3M}{MT^3L^3}\right]} = \sqrt{\left[\frac{L^5}{L^3}\right]} = \sqrt{\left[L^2\right]} = [L]$$

$$\ell_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\bullet\text{s}^2)(6.63\times10^{-34}\ \text{kg}\bullet\text{m}^2/\text{s})}{(3.00\times10^8\ \text{m/s})^3}} = 4.05\times10^{-35}\ \text{m}$$

Thus the order of magnitude is $\boxed{10^{-35}\ \text{m}}$.

40. The percentage accuracy is $\dfrac{2\ \text{m}}{2\times10^7\ \text{m}}\times100\% = \boxed{1\times10^{-5}\%}$. The distance of 20,000,000 m needs to be distinguishable from 20,000,002 m, which means that $\boxed{\text{8 significant figures}}$ are needed in the distance measurements.

41. Multiply the number of chips per wafer by the number of wafers that can be made from a cylinder. We assume the number of chips per wafer is more accurate than 1 significant figure.

$$\left(400\frac{\text{chips}}{\text{wafer}}\right)\left(\frac{1\ \text{wafer}}{0.300\ \text{mm}}\right)\left(\frac{250\ \text{mm}}{1\ \text{cylinder}}\right) = \boxed{3.3\times10^5\ \frac{\text{chips}}{\text{cylinder}}}$$

42. Assume that the alveoli are spherical and that the volume of a typical human lung is about 2 liters, which is $0.002\ \text{m}^3$. The diameter can be found from the volume of a sphere, $\frac{4}{3}\pi r^3$.

$$\tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(d/2)^3 = \frac{\pi d^3}{6}$$

$$(3\times 10^8)\pi\frac{d^3}{6} = 2\times 10^{-3}\ \text{m}^3 \quad\rightarrow\quad d = \left[\frac{6(2\times 10^{-3})}{3\times 10^8\ \pi}\text{m}^3\right]^{1/3} = \boxed{2\times 10^{-4}\ \text{m}}$$

43. We assume that there are 40 hours of work per week and that the typist works 50 weeks out of the year.

$$(1.0\times 10^{12}\ \text{bytes})\times\frac{1\ \text{char}}{1\ \text{byte}}\times\frac{1\ \text{min}}{180\ \text{char}}\times\frac{1\ \text{hour}}{60\ \text{min}}\times\frac{1\ \text{week}}{40\ \text{hour}}\times\frac{1\ \text{year}}{50\ \text{weeks}} = 4.629\times 10^4\ \text{years}$$

$$\approx \boxed{46{,}000\ \text{years}}$$

44. The volume of water used by the people can be calculated as follows:

$$(4\times 10^4\ \text{people})\left(\frac{1200\ \text{L/day}}{4\ \text{people}}\right)\left(\frac{365\ \text{days}}{1\ \text{yr}}\right)\left(\frac{1000\ \text{cm}^3}{1\ \text{L}}\right)\left(\frac{1\ \text{km}}{10^5\ \text{cm}}\right)^3 = 4.38\times 10^{-3}\ \text{km}^3/\text{yr}$$

The depth of water is found by dividing the volume by the area.

$$d = \frac{V}{A} = \frac{4.38\times 10^{-3}\ \text{km}^3/\text{yr}}{50\ \text{km}^2} = \left(8.76\times 10^{-5}\ \frac{\text{km}}{\text{yr}}\right)\left(\frac{10^5\ \text{cm}}{1\ \text{km}}\right) = 8.76\ \text{cm/yr} \approx \boxed{9\ \text{cm/yr}}$$

45. We approximate the jar as a cylinder with a uniform cross-sectional area. In counting the jelly beans in the top layer, we find about 25 jelly beans. Thus we estimate that one layer contains about 25 jelly beans. In counting vertically, we see that there are about 15 rows. Thus we estimate that there are $25\times 15 = 375 \approx \boxed{400\ \text{jelly beans}}$ in the jar.

46. The volume of a sphere is given by $V = \tfrac{4}{3}\pi r^3$, so the radius is $r = \left(\frac{3V}{4\pi}\right)^{1/3}$. For a 1-ton rock, the volume is calculated from the density, and then the diameter from the volume.

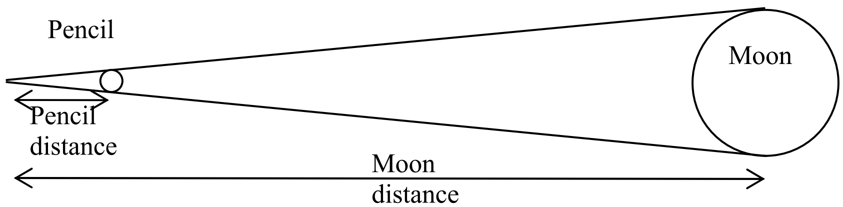
$$V = (1\ \text{T})\left(\frac{2000\ \text{lb}}{1\ \text{T}}\right)\left(\frac{1\ \text{ft}^3}{186\ \text{lb}}\right) = 10.8\ \text{ft}^3$$

$$d = 2r = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 2\left[\frac{3(10.8\ \text{ft}^3)}{4\pi}\right]^{1/3} = 2.74\ \text{ft} \approx \boxed{3\ \text{ft}}$$

47. We do a "units conversion" from bytes to minutes, using the given CD reading rate.

$$(783.216\times 10^6\ \text{bytes})\times\frac{8\ \text{bits}}{1\ \text{byte}}\times\frac{1\ \text{s}}{1.4\times 10^6\ \text{bits}}\times\frac{1\ \text{min}}{60\ \text{s}} = 74.592\ \text{min} \approx \boxed{75\ \text{min}}$$

48. A pencil has a diameter of about 0.7 cm. If held about 0.75 m from the eye, it can just block out the Moon. The ratio of pencil diameter to arm length is the same as the ratio of Moon diameter to Moon distance. From the diagram, we have the following ratios.



$$\frac{\text{Pencil diameter}}{\text{Pencil distance}}=\frac{\text{Moon diameter}}{\text{Moon distance}} \rightarrow$$

$$\text{Moon diameter}=\frac{\text{Pencil diameter}}{\text{Pencil distance}}(\text{Moon distance})=\frac{7\times10^{-3}\text{ m}}{0.75\text{ m}}(3.8\times10^{5}\text{ km})\approx\boxed{3500\text{ km}}$$

The actual value is 3480 km.

49. To calculate the mass of water, we need to find the volume of water and then convert the volume to mass. The volume of water is the area of the city (48 km^2) times the depth of the water (1.0 cm).

$$\left[(48\text{ km}^2)\left(\frac{10^5\text{ cm}}{1\text{ km}}\right)^2\right](1.0\text{ cm})\left(\frac{10^{-3}\text{ kg}}{1\text{ cm}^3}\right)\left(\frac{1\text{ metric ton}}{10^3\text{ kg}}\right)=4.8\times10^5\text{ metric tons}\approx\boxed{5\times10^5\text{ metric tons}}$$

To find the number of gallons, convert the volume to gallons.

$$\left[(48\text{ km}^2)\left(\frac{10^5\text{ cm}}{1\text{ km}}\right)^2\right](1.0\text{ cm})\left(\frac{1\text{ L}}{1\times10^3\text{ cm}^3}\right)\left(\frac{1\text{ gal}}{3.78\text{ L}}\right)=1.27\times10^8\text{ gal}\approx\boxed{1\times10^8\text{ gal}}$$

50. The person walks 4 km/h, 12 hours each day. The radius of the Earth is about 6380 km, and the distance around the Earth at the equator is the circumference, $2\pi R_{\text{Earth}}$. We assume that the person can "walk on water," so ignore the existence of the oceans.

$$2\pi(6380\text{ km})\left(\frac{1\text{ h}}{4\text{ km}}\right)\left(\frac{1\text{ day}}{12\text{ h}}\right)=835\text{ days}\approx\boxed{800\text{ days}}$$

51. The volume of the oil will be the area times the thickness. The area is $\pi r^2=\pi(d/2)^2$.

$$V=\pi(d/2)^2 t \rightarrow d=2\sqrt{\frac{V}{\pi t}}=2\sqrt{\frac{1000\text{ cm}^3\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3}{\pi(2\times10^{-10}\text{ m})}}=\boxed{3\times10^3\text{ m}}$$

This is approximately 2 miles.

52. $$\left(\frac{8\text{ s}}{1\text{ yr}}\right)\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right)\times100\%=\boxed{3\times10^{-5}\%}$$

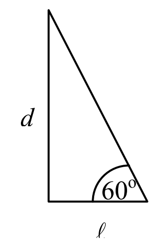
53. (*a*) $$1.0\,\text{Å}=\left(1.0\,\text{Å}\right)\left(\frac{10^{-10}\text{ m}}{1\,\text{Å}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)=\boxed{0.10\text{ nm}}$$

(*b*) $1.0\,\overset{\circ}{\mathrm{A}} = \left(1.0\,\overset{\circ}{\mathrm{A}}\right)\left(\dfrac{10^{-10}\ \mathrm{m}}{1\,\overset{\circ}{\mathrm{A}}}\right)\left(\dfrac{1\ \mathrm{fm}}{10^{-15}\ \mathrm{m}}\right) = \boxed{1.0\times10^{5}\ \mathrm{fm}}$

(*c*) $1.0\ \mathrm{m} = (1.0\ \mathrm{m})\left(\dfrac{1\,\overset{\circ}{\mathrm{A}}}{10^{-10}\ \mathrm{m}}\right) = \boxed{1.0\times10^{10}\ \overset{\circ}{\mathrm{A}}}$

(*d*) $1.0\ \mathrm{ly} = (1.0\ \mathrm{ly})\left(\dfrac{9.46\times10^{15}\ \mathrm{m}}{1\ \mathrm{ly}}\right)\left(\dfrac{1\,\overset{\circ}{\mathrm{A}}}{10^{-10}\ \mathrm{m}}\right) = \boxed{9.5\times10^{25}\ \overset{\circ}{\mathrm{A}}}$

54. Consider the diagram shown. Let ℓ represent the distance he walks upstream. Then from the diagram find the distance across the river.



$$\tan 60^{\circ} = \frac{d}{\ell} \quad\rightarrow\quad d = \ell \tan 60^{\circ} = (65\ \text{strides})\left(\frac{0.8\ \mathrm{m}}{\text{stride}}\right)\tan 60^{\circ} = \boxed{90\ \mathrm{m}}$$

55. (*a*) Note that $\sin 15.0^{\circ} = 0.259$ and $\sin 15.5^{\circ} = 0.267$, so $\Delta\sin\theta = 0.267 - 0.259 = 0.008.$

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^{\circ}}{15.0^{\circ}}\right)100 = \boxed{3\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{8\times10^{-3}}{0.259}\right)100 = \boxed{3\%}$$

(*b*) Note that $\sin 75.0^{\circ} = 0.966$ and $\sin 75.5^{\circ} = 0.968$, so $\Delta\sin\theta = 0.968 - 0.966 = 0.002.$

$$\left(\frac{\Delta\theta}{\theta}\right)100 = \left(\frac{0.5^{\circ}}{75.0^{\circ}}\right)100 = \boxed{0.7\%} \qquad \left(\frac{\Delta\sin\theta}{\sin\theta}\right)100 = \left(\frac{2\times10^{-3}}{0.966}\right)100 = \boxed{0.2\%}$$

A consequence of this result is that when you use a protractor, and you have a fixed uncertainty in the angle ($\pm0.5^{\circ}$ in this case), you should measure the angles from a reference line that gives a large angle measurement rather than a small one. Note above that the angles around 75° had only a 0.2% error in sin *θ*, while the angles around 15° had a 3% error in sin *θ*.

56. Utilize the fact that walking totally around the Earth along the meridian would trace out a circle whose full 360° would equal the circumference of the Earth.

$$(1\ \text{minute})\left(\frac{1^{\circ}}{60\ \text{minute}}\right)\left(\frac{2\pi(6.38\times10^{3}\ \mathrm{km})}{360^{\circ}}\right)\left(\frac{0.621\ \mathrm{mi}}{1\ \mathrm{km}}\right) = \boxed{1.15\ \mathrm{mi}}$$

57. Consider the body to be a cylinder, about 170 cm tall $(\approx 5'7'')$, and about 12 cm in cross-sectional radius (which corresponds to a 30-inch waist). The volume of a cylinder is given by the area of the cross section times the height.

$$V = \pi r^2 h = \pi(0.12\ \mathrm{m})^2(1.7\ \mathrm{m}) = 7.69\times10^{-2}\ \mathrm{m}^3 \approx \boxed{8\times10^{-2}\ \mathrm{m}^3}$$

58. The units for each term must be in liters, since the volume is in liters.

$$[\text{units of } 4.1][\text{m}] = [\text{L}] \quad \rightarrow \boxed{[\text{units of } 4.1] = \frac{\text{L}}{\text{m}}}$$

$$[\text{units of } 0.018][\text{year}] = [\text{L}] \quad \rightarrow \quad \boxed{[\text{units of } 0.018] = \frac{\text{L}}{\text{year}}}$$

$$\boxed{[\text{units of } 2.7] = \text{L}}$$

59. Divide the number of atoms by the Earth's surface area.

$$\frac{\text{number of atoms}}{\text{m}^2} = \frac{6.02\times10^{23}\ \text{atoms}}{4\pi R_{\text{Earth}}^2} = \frac{6.02\times10^{23}\ \text{atoms}}{4\pi(6.38\times10^6\ \text{m})^2} = \boxed{1.18\times10^9\ \frac{\text{atoms}}{\text{m}^2}}$$

This is more than a billion atoms per square meter.

60. The density is the mass divided by volume. There will be only 1 significant figure in the answer.

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{6\ \text{g}}{2.8325\ \text{cm}^3} = 2.118\ \text{g/cm}^3 \approx \boxed{2\ \text{g/cm}^3}$$

61. Multiply the volume of a spherical universe times the density of matter, adjusted to ordinary matter. The volume of a sphere is $\frac{4}{3}\pi r^3$.

$$m = \rho V = (1\times10^{-26}\ \text{kg/m}^3)\tfrac{4}{3}\pi\left((13.7\times10^9\ \text{ly})\times\frac{9.46\times10^{15}\ \text{m}}{1\ \text{ly}}\right)^3(0.04)$$

$$= 3.65\times10^{51}\ \text{kg} \approx \boxed{4\times10^{51}\ \text{kg}}$$

Solutions to Search and Learn Problems

1. Both Galileo and Copernicus built on earlier theories (by Aristotle and Ptolemy), but those new theories explained a greater variety of phenomena. Aristotle and Ptolemy explained motion in basic terms. Aristotle explained the basic motion of objects, and Ptolemy explained the basic motions of astronomical bodies. Both Galileo and Copernicus took those explanations of motion to a new level. Galileo developed explanations that would apply in the absence of friction. Copernicus's Sun-centered theory explained other phenomena that Ptolemy's model did not (such as the phases of Venus).

2. From Example 1–7, the thickness of a page of this book is about 6×10^{-5} m. The wavelength of orange krypton-86 light is found from the fact that 1,650,763.73 wavelengths of that light is the definition of the meter.

$$1\ \text{page}\left(\frac{6\times10^{-5}\ \text{m}}{1\ \text{page}}\right)\left(\frac{1{,}650{,}763.73\ \text{wavelengths}}{1\ \text{m}}\right) = 99\ \text{wavelengths} \approx \boxed{100\ \text{wavelengths}}$$

3. The original definition of the meter was that 1 meter was one ten-millionth of the distance from the Earth's equator to either pole. The distance from the equator to the pole would be one-fourth of the circumference of a perfectly spherical Earth. Thus the circumference would be 40 million meters: $\boxed{C = 4\times10^{7}\text{ m}}$. We use the circumference to find the radius.

$$C = 2\pi r \rightarrow r = \frac{C}{2\pi} = \frac{4\times10^{7}\text{ m}}{2\pi} = \boxed{6.37\times10^{6}\text{ m}}$$

The value in the front of the textbook is 6.38×10^{6} m.

4. We use values from Table 1–3.

$$\frac{m_{\text{human}}}{m_{\substack{\text{DNA}\\ \text{molecule}}}} = \frac{10^{2}\text{ kg}}{10^{-17}\text{ kg}} = \boxed{10^{19}}$$

5. The surface area of a sphere is given by $4\pi r^2$, and the volume of a sphere is given by $\frac{4}{3}\pi r^3$.

(*a*) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{4\pi R_{\text{Earth}}^2}{4\pi R_{\text{Moon}}^2} = \frac{R_{\text{Earth}}^2}{R_{\text{Moon}}^2} = \frac{(6.38\times10^{3}\text{ km})^2}{(1.74\times10^{3}\text{ km})^2} = \boxed{13.4}$$

(*b*) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4}{3}\pi R_{\text{Earth}}^3}{\frac{4}{3}\pi R_{\text{Moon}}^3} = \frac{R_{\text{Earth}}^3}{R_{\text{Moon}}^3} = \frac{(6.38\times10^{3}\text{ km})^3}{(1.74\times10^{3}\text{ km})^3} = \boxed{49.3}$$

2

DESCRIBING MOTION: KINEMATICS IN ONE DIMENSION

Responses to Questions

1. A car speedometer measures only speed. It does not give any information about the direction, so it does not measure velocity.

2. If the velocity of an object is constant, then the speed and the direction of travel must also be constant. If that is the case, then the average velocity is the same as the instantaneous velocity, because nothing about its velocity is changing. The ratio of displacement to elapsed time will not be changing, no matter the actual displacement or time interval used for the measurement.

3. There is no general relationship between the magnitude of speed and the magnitude of acceleration. For example, one object may have a large but constant speed. The acceleration of that object is then zero. Another object may have a small speed but be gaining speed and therefore have a positive acceleration. So in this case the object with the greater speed has the lesser acceleration.

   Consider two objects that are dropped from rest at different times. If we ignore air resistance, then the object dropped first will always have a greater speed than the object dropped second, but both will have the same acceleration of $9.80\ \mathrm{m/s^2}$.

4. The accelerations of the motorcycle and the bicycle are the same, assuming that both objects travel in a straight line. Acceleration is the change in velocity divided by the change in time. The magnitude of the change in velocity in each case is the same, 10 km/h, so over the same time interval the accelerations will be equal.

5. Yes. For example, a car that is traveling northward and slowing down has a northward velocity and a southward acceleration.

6. The velocity of an object can be negative when its acceleration is positive. If we define the positive direction to be to the right, then an object traveling to the left that is having a reduction in speed will have a negative velocity with a positive acceleration.

   If again we define the positive direction to be to the right, then an object traveling to the right that is having a reduction in speed will have a positive velocity and a negative acceleration.

7. If north is defined as the positive direction, then an object traveling to the south and increasing in speed has both a negative velocity and a negative acceleration. Or if up is defined as the positive direction, then an object falling due to gravity has both a negative velocity and a negative acceleration.

8. Yes. Remember that acceleration is a *change in velocity* per unit time, or a *rate of change* in velocity. So velocity can be increasing while the rate of increase goes down. For example, suppose a car is traveling at 40 km/h and one second later is going 50 km/h. One second after that, the car's speed is 55 km/h. The car's speed was increasing the entire time, but its acceleration in the second time interval was lower than in the first time interval. Thus its acceleration was decreasing even as the speed was increasing.

   Another example would be an object falling WITH air resistance. Let the downward direction be positive. As the object falls, it gains speed, and the air resistance increases. As the air resistance increases, the acceleration of the falling object decreases, and it gains speed less quickly the longer it falls.

9. If the two cars emerge side by side, then the one moving faster is passing the other one. Thus car A is passing car B. With the acceleration data given for the problem, the ensuing motion would be that car A would pull away from car B for a time, but eventually car B would catch up to and pass car A.

10. If there were no air resistance, the ball's only acceleration during flight would be the acceleration due to gravity, so the ball would land in the catcher's mitt with the same speed it had when it left the bat, 120 km/h. Since the acceleration is the same through the entire flight, the time for the ball's speed to change from 120 km/h to 0 on the way up is the same as the time for its speed to change from 0 to 120 km/h on the way down. In both cases the ball has the same magnitude of displacement.

$\boxed{11.}$ (*a*) If air resistance is negligible, the acceleration of a freely falling object stays the same as the object falls toward the ground. That acceleration is $9.80\ \text{m/s}^2$. Note that the object's speed increases, but since that speed increases at a constant rate, the acceleration is constant.

(*b*) In the presence of air resistance, the acceleration decreases. Air resistance increases as speed increases. If the object falls far enough, the acceleration will go to zero and the velocity will become constant. That velocity is often called the terminal velocity.

12. Average speed is the displacement divided by the time. Since the distances from A to B and from B to C are equal, you spend more time traveling at 70 km/h than at 90 km/h, so your average speed should be less than 80 km/h. If the distance from A to B (or B to C) is *x* km, then the total distance traveled is 2*x*. The total time required to travel this distance is *x*/70 plus *x*/90. Then

$$\bar{v} = \frac{d}{t} = \frac{2x}{x/70 + x/90} = \frac{2(90)(70)}{90+70} = 78.75\ \text{km/h} \approx 79\ \text{km/h}.$$

$\boxed{13.}$ Yes. For example, a rock thrown straight up in the air has a constant, nonzero acceleration due to gravity for its entire flight. However, at the highest point it momentarily has zero velocity. A car, at the moment it starts moving from rest, has zero velocity and nonzero acceleration.

14. Yes. Any time the velocity is constant, the acceleration is zero. For example, a car traveling at a constant 90 km/h in a straight line has nonzero velocity and zero acceleration.

15. A rock falling from a cliff has a constant acceleration IF we neglect air resistance. An elevator moving from the second floor to the fifth floor making stops along the way does NOT have a constant acceleration. Its acceleration will change in magnitude and direction as the elevator starts and stops. The dish resting on a table has a constant (zero) acceleration.

16. The slope of the position versus time curve is the object's velocity. The object starts at the origin with a constant velocity (and therefore zero acceleration), which it maintains for about 20 s. For the next 10 s, the positive curvature of the graph indicates the object has a positive acceleration; its speed is increasing. From 30 s to 45 s, the graph has a negative curvature; the object uniformly slows to a stop, changes direction, and then moves backwards with increasing speed. During this time interval, the acceleration is negative, since the object is slowing down while traveling in the positive direction and then speeding up while traveling in the negative direction. For the final 5 s shown, the object continues moving in the negative direction but slows down, which gives it a positive acceleration. During the 50 s shown, the object travels from the origin to a point 20 m away, and then back 10 m to end up 10 m from the starting position.

17. Initially, the object moves in the positive direction with a constant acceleration, until about $t = 45$ s, when it has a velocity of about 37 m/s in the positive direction. The acceleration then decreases, reaching an instantaneous acceleration of 0 at about $t = 50$ s, when the object has its maximum speed of about 38 m/s. The object then begins to slow down but continues to move in the positive direction. The object stops moving at $t = 90$ s and stays at rest until about $t = 108$ s. Then the object begins to move in the positive direction again, at first with a larger acceleration, and then with a lesser acceleration. At the end of the recorded motion, the object is still moving to the right and gaining speed.

Responses to MisConceptual Questions

1. (*a*, *b*, *c*, *d*, *e*, *f*, *g*) All of these actions should be a part of solving physics problems.

2. (*d*) It is a common misconception that a positive acceleration always increases the speed, as in (*b*) and (*c*). However, when the velocity and acceleration are in opposite directions, the speed will decrease.

3. (*d*) Since the velocity and acceleration are in opposite directions, the object will slow to a stop. However, since the acceleration remains constant, it will stop only momentarily before moving toward the left.

4. (*c*) Students commonly confuse the concepts of velocity and acceleration in free-fall motion. At the highest point in the trajectory, the velocity is changing from positive (upward) to negative (downward) and therefore passes through zero. This changing velocity is due to a constant downward acceleration.

5. (*a*) Since the distance between the rocks increases with time, a common misconception is that the velocities are increasing at different rates. However, both rocks fall under the influence of gravity, so their velocities increase at the same rate.

6. (*c*) Since the distances are the same, a common error is to assume that the average speed will be halfway between the two speeds, or 40 km/h. However, it takes the car much longer to travel the 4 km at 30 km/h than at 50 km/h. Since more time is spent at 30 km/h, the average speed will be closer to 30 km/h than to 50 km/h.

7. (*c*) A common misconception is that the acceleration of an object in free fall depends upon the motion of the object. If there is no air resistance, the accelerations for the two balls have the same magnitude and direction throughout both of their flights.

8. (*b, c*) Each of the given equations is based on Eqs. 2–11a–d. Answer (*a*) has the acceleration replaced properly with $-g$, but the initial velocity is downward and as such should be negative. Answer (*d*) is

incorrect because the initial velocity has been inserted for the average velocity. Answers (*b*) and (*c*) have the correct signs for each variable and the known values are inserted properly.

9. (*a*) Increasing speed means that the slope must be getting steeper over time. In graphs (*b*) and (*e*), the slope remains constant, so these are cars moving at constant speed. In graph (*c*), as time increases *x* decreases. However, the rate at which it decreases is also decreasing. This is a car slowing down. In graph (*d*), the car is moving away from the origin, but again it is slowing down. The only graph in which the slope is increasing with time is graph (*a*).

Solutions to Problems

1. The distance of travel (displacement) can be found by rearranging Eq. 2–2 for the average velocity. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$\bar{v} = \Delta x/\Delta t \quad \rightarrow \quad \Delta x = \bar{v}\Delta t = (95\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)(2.0\text{ s}) = 0.053\text{ km} = \boxed{53\text{ m}}$$

2. The average speed is given by Eq. 2–1, using d to represent distance traveled.

$$\bar{v} = d/\Delta t = 235\text{ km}/2.75\text{ h} = \boxed{85.5\text{ km/h}}$$

3. The average velocity is given by Eq. 2–2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8.5\text{ cm} - 4.8\text{ cm}}{4.5\text{ s} - (-2.0\text{ s})} = \frac{3.7\text{ cm}}{6.5\text{ s}} = \boxed{0.57\text{ cm/s}}$$

$\boxed{\text{The average speed cannot be calculated.}}$ To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.

4. The average velocity is given by Eq. 2–2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4.2\text{ cm} - 8.4\text{ cm}}{6.1\text{ s} - 3.0\text{ s}} = \frac{-12.6\text{ cm}}{3.1\text{ s}} = \boxed{-4.1\text{ cm/s}}$$

The negative sign indicates the direction.

5. The time of travel can be found by rearranging the average velocity equation.

$$\bar{v} = \Delta x/\Delta t \quad \rightarrow \quad \Delta t = \Delta x/\bar{v} = (3.5\text{ km})/(25\text{ km/h}) = \boxed{0.14\text{ h}} = 8.4\text{ min}$$

6. (*a*) The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated as follows:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left(\frac{1\text{ mi}}{5\text{ s}}\right)\left(\frac{1610\text{ m}}{1\text{ mi}}\right) = 322\text{ m/s} \approx \boxed{300\text{ m/s}}$$

(*b*) The speed of 322 m/s would imply the sound traveling a distance of 966 meters (which is approximately 1 km) in 3 seconds. So the rule could be approximated as $\boxed{\text{1 km every 3 seconds.}}$

7. The time for the first part of the trip is calculated from the initial speed and the first distance, using *d* to represent distance.

$$\bar{v}_1 = \frac{d_1}{\Delta t_1} \quad \rightarrow \quad \Delta t_1 = \frac{d_1}{\bar{v}_1} = \frac{180\text{ km}}{95\text{ km/h}} = 1.895\text{ h} = 113.7\text{ min}$$

The time for the second part of the trip is now calculated.

$$\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 4.5\text{ h} - 1.895\text{ h} = 2.605\text{ h} = 156.3\text{ min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\overline{v}_2 = \frac{d_2}{\Delta t_2} \quad\rightarrow\quad d_2 = \overline{v}_2\Delta t_2 = (65\text{ km/h})(2.605\text{ h}) = 169.3\text{ km} \approx 170\text{ km}$$

(*a*) The total distance is then $d_{\text{total}} = d_1 + d_2 = 180\text{ km} + 169.3\text{ km} = 349.3\text{ km} \approx \boxed{350\text{ km}}$.

(*b*) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2–1.

$$\overline{v} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}} = \frac{349.3\text{ km}}{4.5\text{ h}} = 77.62\text{ km/h} \approx \boxed{78\text{ km/h}}$$

8. The distance traveled is $38\text{ m} + \frac{1}{2}(38\text{ m}) = 57\text{ m}$, and the displacement is $38\text{ m} - \frac{1}{2}(38\text{ m}) = 19\text{ m}$. The total time is $9.0\text{ s} + 1.8\text{ s} = 10.8\text{ s}$.

(*a*) $$\text{Average speed} = \frac{\text{distance}}{\text{time elapsed}} = \frac{57\text{ m}}{10.8\text{ s}} = \boxed{5.3\text{ m/s}}$$

(*b*) $$\text{Average velocity} = v_{\text{avg}} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{19\text{ m}}{10.8\text{ s}} = \boxed{1.8\text{ m/s}}$$

9. The distance traveled is 3200 m $(8\text{ laps} \times 400\text{ m/lap})$. That distance probably has either 3 or 4 significant figures, since the track distance is probably known to at least the nearest meter for competition purposes. The displacement is 0, because the ending point is the same as the starting point.

(*a*) $$\text{Average speed} = \frac{d}{\Delta t} = \frac{3200\text{ m}}{14.5\text{ min}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{3.68\text{ m/s}}$$

(*b*) $$\text{Average velocity} = \overline{v} = \Delta x/\Delta t = \boxed{0\text{ m/s}}$$

10. The average speed is the distance divided by the time.

$$\overline{v} = \frac{d}{t} = \left(\frac{1\times 10^9\text{ km}}{1\text{ yr}}\right)\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right) = 1.141\times 10^5\text{ km/h} \approx \boxed{1\times 10^5\text{ km/h}}$$

$\boxed{11.}$ Both objects will have the same time of travel. If the truck travels a distance d_{truck}, then the distance the car travels will be $d_{\text{car}} = d_{\text{truck}} + 210\text{ m}$. Using the equation for average speed, $\overline{v} = d/\Delta t$, solve for time, and equate the two times.

$$\Delta t = \frac{d_{\text{truck}}}{\overline{v}_{\text{truck}}} = \frac{d_{\text{car}}}{\overline{v}_{\text{car}}} \qquad \frac{d_{\text{truck}}}{75\text{ km/h}} = \frac{d_{\text{truck}} + 210\text{ m}}{95\text{ km/h}}$$

Solving for d_{truck} gives $d_{\text{truck}} = (210\text{ m})\dfrac{(75\text{ km/h})}{(95\text{ km/h} - 75\text{ km/h})} = 787.5\text{ m}$.

The time of travel is

$$\Delta t = \frac{d_{\text{truck}}}{\overline{v}_{\text{truck}}} = \left(\frac{787.5\text{ m}}{75{,}000\text{ m/h}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 0.63\text{ min} = 37.8\text{ s} \approx \boxed{38\text{ s}}$$

Also note that $\Delta t = \dfrac{d_{\text{car}}}{\bar{v}_{\text{car}}} = \left(\dfrac{787.5\text{ m} + 210\text{ m}}{95{,}000\text{ m/h}}\right)\left(\dfrac{60\text{ min}}{1\text{ h}}\right) = 0.63\,\text{min} = 37.8\text{ s}.$

ALTERNATE SOLUTION:

The speed of the car relative to the truck is $95\text{ km/h} - 75\text{ km/h} = 20\text{ km/h}$. In the reference frame of the truck, the car must travel 210 m to catch it.

$$\Delta t = \frac{0.21\text{ km}}{20\text{ km/h}}\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 37.8\text{ s}$$

12. The distance traveled is 500 km (250 km outgoing, 250 km return, keep 2 significant figures). The displacement (Δx) is 0 because the ending point is the same as the starting point.

To find the average speed, we need the distance traveled (500 km) and the total time elapsed.

During the outgoing portion, $\bar{v}_1 = \dfrac{\Delta x_1}{\Delta t_1}$, so $\Delta t_1 = \dfrac{\Delta x_1}{\bar{v}_1} = \dfrac{250\text{ km}}{95\text{ km/h}} = 2.632\text{ h}$. During the return portion, $\bar{v}_2 = \dfrac{\Delta x_2}{\Delta t_2}$, so $\Delta t_2 = \dfrac{\Delta x_2}{\bar{v}_2} = \dfrac{250\text{ km}}{55\text{ km/h}} = 4.545\text{ h}$. Thus the total time, including lunch, is $\Delta t_{\text{total}} = \Delta t_1 + \Delta t_{\text{lunch}} + \Delta t_2 = 8.177\text{ h}$.

$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{500\text{ km}}{8.177\text{ h}} = \boxed{61\text{ km/h}}$$

To find the average velocity, use the displacement and the elapsed time.

$$\boxed{\bar{v} = \Delta x/\Delta t = 0}$$

13. Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\bar{v} = \frac{d}{\Delta t} \quad\rightarrow\quad \Delta t = \frac{d}{\bar{v}} = \frac{4.25\text{ km}}{155\text{ km/h}} = 0.0274\text{ h}\left(\frac{60\text{ min}}{1\text{ h}}\right) = 1.645\text{ min} \approx \boxed{1.6\text{ min}} \approx 99\text{ s}$$

14. (*a*) The area between the concentric circles is equal to the length times the width of the spiral path.

$$\pi R_2^2 - \pi R_1^2 = w\ell \;\rightarrow\; \ell\frac{\pi(R_2^2 - R_1^2)}{w} = \frac{\pi[(0.058\text{ m})^2 - (0.025\text{ m})^2]}{1.6\times10^{-6}\text{ m}} = 5.378\times10^3\text{ m} \approx \boxed{5400\text{ m}}$$

(*b*) $5.378\times10^3\text{ m}\left(\dfrac{1\text{ s}}{1.2\text{ m}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 74.69\text{ min} \approx \boxed{75\text{ min}}$

15. The average speed of sound is given by $v_{\text{sound}} = \Delta x/\Delta t$, so the time for the sound to travel from the end of the lane back to the bowler is $\Delta t_{\text{sound}} = \dfrac{\Delta x}{v_{\text{sound}}} = \dfrac{16.5\text{ m}}{340\text{ m/s}} = 4.85\times10^{-2}\text{ s}$. Thus the time for the ball to travel from the bowler to the end of the lane is given by $\Delta t_{\text{ball}} = \Delta t_{\text{total}} - \Delta t_{\text{sound}} = 2.80\text{ s} - 4.85\times10^{-2}\text{ s} = 2.7515\text{ s}$. The speed of the ball is as follows:

$$v_{\text{ball}} = \frac{\Delta x}{\Delta t_{\text{ball}}} = \frac{16.5\text{ m}}{2.7515\text{ s}} = 5.9967\text{ m/s} \approx \boxed{6.00\text{ m/s}}$$

16. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either $d_{\text{car}} = \upsilon_{\text{car}}t = (95\text{ km/h})t$ or $d_{\text{car}} = \ell_{\text{train}} + \upsilon_{\text{train}}t = 1.30\text{ km} + (75\text{ km/h})t.$ To solve for the time, equate these two expressions for the distance the car travels.

$$(95\text{ km/h})t = 1.30\text{ km} + (75\text{ km/h})t \;\rightarrow\; t = \frac{1.30\text{ km}}{20\text{ km/h}} = 0.065\text{ h} = \boxed{3.9\text{ min}}$$

Note that this is the same as calculating from the reference frame of the train, in which the car is moving at 20 km/h and must travel the length of the train.

The distance the car travels during this time is $d = (95\text{ km/h})(0.065\text{ h}) = 6.175\text{ km} \approx \boxed{6.2\text{ km}}.$

If the train is traveling in the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either $d_{\text{car}} = (95\text{ km/h})t$ or $d_{\text{car}} = 1.30\text{ km} - (75\text{ km/h})t.$ To solve for the time, equate these two expressions for the distance the car travels.

$$(95\text{ km/h})t = 1.30\text{ km} - (75\text{ km/h})t \quad\rightarrow\quad t = \frac{1.30\text{ km}}{170\text{ km/h}} = 7.65\times10^{-3}\text{ h} \approx \boxed{28\text{ s}}$$

The distance the car travels during this time is $d = (95\text{ km/h})(7.65\times10^{-3}\text{ h}) = \boxed{0.73\text{ km}}.$

17. The average acceleration is found from Eq. 2–4.

$$\bar{a} = \frac{\Delta\upsilon}{\Delta t} = \frac{95\text{ km/h} - 0\text{ km/h}}{4.3\text{ s}} = \frac{(95\text{ km/h})\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)}{4.3\text{ s}} = \boxed{6.1\text{ m/s}^2}$$

18. (*a*) The average acceleration of the sprinter is $\bar{a} = \dfrac{\Delta\upsilon}{\Delta t} = \dfrac{9.00\text{ m/s} - 0.00\text{ m/s}}{1.38\text{ s}} = \boxed{6.52\text{ m/s}^2}.$

(*b*) We change the units for the acceleration.

$$\bar{a} = (6.52\text{ m/s}^2)\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)^2 = \boxed{8.45\times10^4\text{ km/h}^2}$$

19. The initial velocity of the car is the average velocity of the car before it accelerates.

$$\bar{\upsilon} = \frac{\Delta x}{\Delta t} = \frac{120\text{ m}}{5.0\text{ s}} = 24\text{ m/s} = \upsilon_0$$

The final velocity is $\upsilon = 0,$ and the time to stop is 4.0 s. Use Eq. 2–11a to find the acceleration.

$$\upsilon = \upsilon_0 + at \quad\rightarrow\quad a = \frac{\upsilon - \upsilon_0}{t} = \frac{0 - 24\text{ m/s}}{4.0\text{ s}} = -6.0\text{ m/s}^2$$

Thus the magnitude of the acceleration is $\boxed{6.0\text{ m/s}^2}$, or $(6.0\text{ m/s}^2)\left(\dfrac{1\,g}{9.80\text{ m/s}^2}\right) = \boxed{0.61\,g\text{'s}}.$

20. We assume that the speedometer can read to the nearest km/h, so the value of 120 km/h has three significant digits. The time can be found from the average acceleration, $\bar{a} = \Delta\upsilon/\Delta t.$

$$\Delta t = \frac{\Delta \upsilon}{\bar{a}} = \frac{120\ \text{km/h} - 65\ \text{km/h}}{1.8\ \text{m/s}^2} = \frac{(55\ \text{km/h})\left(\dfrac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)}{1.8\ \text{m/s}^2} = 8.488\ \text{s} \approx \boxed{8.5\ \text{s}}$$

21. (*a*) $\bar{\upsilon} = \dfrac{\Delta x}{\Delta t} = \dfrac{385\ \text{m} - 25\ \text{m}}{20.0\ \text{s} - 3.0\ \text{s}} = \boxed{21.2\ \text{m/s}}$

(*b*) $\bar{a} = \dfrac{\Delta \upsilon}{\Delta t} = \dfrac{45.0\ \text{m/s} - 11.0\ \text{m/s}}{20.0\ \text{s} - 3.0\ \text{s}} = \boxed{2.00\ \text{m/s}^2}$

22. The acceleration can be found from Eq. 2–11c.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)} = \frac{0 - (28\ \text{m/s})^2}{2(88\ \text{m})} = \boxed{-4.5\ \text{m/s}^2}$$

23. By definition, the acceleration is $a = \dfrac{\upsilon - \upsilon_0}{t} = \dfrac{21\ \text{m/s} - 14\ \text{m/s}}{6.0\ \text{s}} = 1.167\ \text{m/s}^2 \approx \boxed{1\ \text{m/s}^2}$.

The distance of travel can be found from Eq. 2–11b.

$$x - x_0 = \upsilon_0 t + \tfrac{1}{2}at^2 = (14\ \text{m/s})(6.0\ \text{s}) + \tfrac{1}{2}(1.167\ \text{m/s}^2)(6.0\ \text{s})^2 = 105\ \text{m} \approx \boxed{110\ \text{m}}$$

It can also be found from Eq. 2–7 and Eq. 2–8.

$$x - x_0 = \bar{\upsilon}\Delta t = \frac{\upsilon_0 + \upsilon}{2}\ \Delta t = \frac{14\ \text{m/s} + 21\ \text{m/s}}{2}(6.0\ \text{s}) = 105\ \text{m} \approx \boxed{110\ \text{m}}$$

24. Assume that the plane starts from rest. The distance is found by solving Eq. 2–11c for $x - x_0$.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad x - x_0 = \frac{\upsilon^2 - \upsilon_0^2}{2a} = \frac{(35\ \text{m/s})^2 - 0}{2(3.0\ \text{m/s}^2)} = 204.2\ \text{m} \approx \boxed{2.0 \times 10^2\ \text{m}}$$

25. For the baseball, $\upsilon_0 = 0$, $x - x_0 = 3.5\ \text{m}$, and the final speed of the baseball (during the throwing motion) is $\upsilon = 43\ \text{m/s}$. The acceleration is found from Eq. 2–11c.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)} = \frac{(43\ \text{m/s})^2 - 0}{2(3.5\ \text{m})} = 264\ \text{m/s}^2 \approx \boxed{260\ \text{m/s}^2}$$

26. The sprinter starts from rest. The average acceleration is found from Eq. 2–11c.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)} = \frac{(11.5\ \text{m/s})^2 - 0}{2(18.0\ \text{m})} = 3.674\ \text{m/s}^2 \approx \boxed{3.67\ \text{m/s}^2}$$

Her elapsed time is found by solving Eq. 2–11a for time.

$$\upsilon = \upsilon_0 + at \quad \rightarrow \quad t = \frac{\upsilon - \upsilon_0}{a} = \frac{11.5\ \text{m/s} - 0}{3.674\ \text{m/s}^2} = \boxed{3.13\ \text{s}}$$

27. The words "slows down uniformly" imply that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2–7 and 2–8.

$$x - x_0 = \frac{\upsilon_0 + \upsilon}{2}t = \left(\frac{28.0\ \text{m/s} + 0\ \text{m/s}}{2}\right)(8.00\ \text{s}) = \boxed{112\ \text{m}}$$

28. The final velocity of the car is zero. The initial velocity is found from Eq. 2–11c with $\upsilon = 0$ and solving for υ_0. Note that the acceleration is negative.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad \upsilon_0 = \sqrt{\upsilon^2 - 2a(x - x_0)} = \sqrt{0 - 2(-4.00 \text{ m/s}^2)(65 \text{ m})} = \boxed{23 \text{ m/s}}$$

29. The final velocity of the driver is zero. The acceleration is found from Eq. 2–11c with $\upsilon = 0$ and solving for a.

$$a = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)} = \frac{0 - \left[(95 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right]^2}{2(0.80 \text{ m})} = -435.2 \text{ m/s}^2 \quad \rightarrow \quad |a| \approx \boxed{440 \text{ m/s}^2}$$

Converting to "g's": $|a| = \dfrac{435.2 \text{ m/s}^2}{(9.80 \text{ m/s}^2)/g} = \boxed{44\ g\text{'s}}$.

30. (*a*) The final velocity of the car is 0. The distance is found from Eq. 2–11c with an acceleration of $a = -0.50 \text{ m/s}^2$ and an initial velocity of 85 km/h.

$$x - x_0 = \frac{\upsilon^2 - \upsilon_0^2}{2a} = \frac{0 - \left[(75 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right]^2}{2(-0.50 \text{ m/s}^2)} = 434 \text{ m} \approx \boxed{430 \text{ m}}$$

(*b*) The time to stop is found from Eq. 2–11a.

$$t = \frac{\upsilon - \upsilon_0}{a} = \frac{0 - \left[(75 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right]}{(-0.50 \text{ m/s}^2)} = 41.67 \text{ s} \approx \boxed{42 \text{ s}}$$

(*c*) Take $x_0 = x(t = 0) = 0$. Use Eq. 2–11b, with $a = -0.50 \text{ m/s}^2$ and an initial velocity of 75 km/h. The first second is from $t = 0$ s to $t = 1$ s, and the fifth second is from $t = 4$ s to $t = 5$ s.

$$x(0) = 0;\ x(1) = 0 + (75 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)(1 \text{ s}) + \tfrac{1}{2}(-0.50 \text{ m/s}^2)(1 \text{ s})^2 = 20.58 \text{ m} \quad \rightarrow$$

$$x(1) - x(0) = 20.58 \text{ m} \approx \boxed{21 \text{ m}}$$

$$x(4) = 0 + (75 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)(4 \text{ s}) + \tfrac{1}{2}(-0.50 \text{ m/s}^2)(4 \text{ s})^2 = 79.33 \text{ m}$$

$$x(5) = 0 + (75 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)(5 \text{ s}) + \tfrac{1}{2}(-0.50 \text{ m/s}^2)(5 \text{ s})^2 = 97.92 \text{ m}$$

$$x(5) - x(4) = 97.92 \text{ m} - 79.33 \text{ m} = 18.59 \text{ m} \approx \boxed{19 \text{ m}}$$

31. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is $(95 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 26.39 \text{ m/s}$. The location where the brakes are applied is found from the equation for motion at constant velocity.

$$x_0 = \upsilon_0 t_R = (26.39 \text{ m/s})(0.40 \text{ s}) = 10.56 \text{ m}$$

This is now the starting location for the application of the brakes. In each case, the final speed is 0.

(*a*) Solve Eq. 2–11c for the final location.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow$$

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 10.56\ \text{m} + \frac{0 - (26.39\ \text{m/s})^2}{2(-3.0\ \text{m/s}^2)} = 126.63\ \text{m} \approx \boxed{130\ \text{m}}$$

(*b*) Solve Eq. 2–11c for the final location with the second acceleration.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 10.56\ \text{m} + \frac{0 - (26.39\ \text{m/s})^2}{2(-6.0\ \text{m/s}^2)} = \boxed{69\ \text{m}}$$

$\boxed{32.}$ Calculate the distance that the car travels during the reaction time and the deceleration.

$$\Delta x_1 = v_0 \Delta t = (18.0\ \text{m/s})(0.350\ \text{s}) = 6.3\ \text{m}$$

$$v^2 = v_0^2 + 2a\Delta x_2 \quad \rightarrow \quad \Delta x_2 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (18.0\ \text{m/s})^2}{2(-3.65\ \text{m/s}^2)} = 44.4\ \text{m}$$

$$\Delta x = 6.3\ \text{m} + 44.4\ \text{m} = 50.7\ \text{m}$$

$\boxed{\text{Since she is only 20.0 m from the intersection, she will NOT be able to stop in time. She will be 30.7 m past the intersection.}}$

33. Use the information for the first 180 m to find the acceleration and the information for the full motion to find the final velocity. For the first segment, the train has $v_0 = 0$ m/s, $v_1 = 18$ m/s, and a displacement of $x_1 - x_0 = 180$ m. Find the acceleration from Eq. 2–11c.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \quad \rightarrow \quad a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)} = \frac{(18\ \text{m/s})^2 - 0}{2(180\ \text{m})} = 0.90\ \text{m/s}^2$$

Find the speed of the train after it has traveled the total distance (total displacement of $x_2 - x_0 = 255$ m) using Eq. 2–11c.

$$v_2^2 = v_0^2 + 2a(x_2 - x_0) \quad \rightarrow \quad v_2 = \sqrt{v_0^2 + 2a(x_2 - x_0)} = \sqrt{2(0.90\ \text{m/s}^2)(255\ \text{m})} = \boxed{21\ \text{m/s}}$$

34. Calculate the acceleration from the velocity–time data using Eq. 2–11a, and then use Eq. 2–11b to calculate the displacement at $t = 2.0$ s and $t = 6.0$ s. The initial velocity is $v_0 = 65$ m/s.

$$a = \frac{v - v_0}{t} = \frac{162\ \text{m/s} - 85\ \text{m/s}}{10.0\ \text{s}} = 7.7\ \text{m/s}^2 \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$x(6.0\ \text{s}) - x(2.0\ \text{s}) = [(x_0 + v_0(6.0\ \text{s}) + \tfrac{1}{2}a(6.0\ \text{s})^2) - (x_0 + v_0(2.0\ \text{s}) + \tfrac{1}{2}a(2.0\ \text{s})^2)]$$

$$= v_0(6.0\ \text{s} - 2.0\ \text{s}) + \tfrac{1}{2}a[(6.0\ \text{s})^2 - (2.0\ \text{s})^2] = (85\ \text{m/s})(4.0\ \text{s}) + \tfrac{1}{2}(7.7\ \text{m/s}^2)(32\ \text{s}^2)$$

$$= 463.2\ \text{m} \approx \boxed{460\ \text{m}}$$

35. During the final part of the race, the runner must have a displacement of 1200 m in a time of 180 s (3.0 min). Assume that the starting speed for the final part is the same as the average speed thus far.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8800\ \text{m}}{(27 \times 60)\ \text{s}} = 5.432\ \text{m/s} = v_0$$

The runner will accomplish this by accelerating from speed υ_0 to speed υ for t seconds, covering a distance d_1, and then running at a constant speed of υ for $(180-t)$ seconds, covering a distance d_2. We have these relationships from Eq. 2–11a and Eq. 2–11b.

$$\upsilon = \upsilon_0 + at \quad d_1 = \upsilon_0 t + \tfrac{1}{2}at^2 \quad d_2 = \upsilon(180-t) = (\upsilon_0 + at)(180-t)$$

$$1200\text{ m} = d_1 + d_2 = \upsilon_0 t + \tfrac{1}{2}at^2 + (\upsilon_0 + at)(180-t) \quad\rightarrow\quad 1200\text{ m} = 180\upsilon_0 + 180at - \tfrac{1}{2}at^2 \quad\rightarrow$$

$$1200\text{ m} = (180\text{ s})(5.432\text{ m/s}) + (180\text{ s})(0.20\text{ m/s}^2)t - \tfrac{1}{2}(0.20\text{ m/s}^2)t^2 \quad\rightarrow$$

$$0.10t^2 - 36t + 222.24 = 0 \quad\rightarrow\quad t = \frac{36 \pm \sqrt{36^2 - 4(0.10)(222.24)}}{2(0.10)} = 353.7\text{ s}, 6.28\text{ s}$$

Since we must have $t < 180$ s, the solution is $\boxed{t = 6.3\text{ s}}$.

36. (*a*) The train's constant speed is $\upsilon_{\text{train}} = 5.0$ m/s, and the location of the empty box car as a function of time is given by $x_{\text{train}} = \upsilon_{\text{train}}t = (5.0\text{ m/s})t$. The fugitive has $\upsilon_0 = 0$ m/s and $a = 1.4\text{ m/s}^2$ until his final speed is 6.0 m/s. The elapsed time during the acceleration is $t_{\text{acc}} = \dfrac{\upsilon - \upsilon_0}{a} = \dfrac{6.0\text{ m/s}}{1.4\text{ m/s}^2} = 4.286$ s. Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the empty box car before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by Eq. 2–11b, $x_{\text{fugitive}} = x_0 + \upsilon_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}(1.4\text{ m/s}^2)t^2$. For him to catch the train, we must have $x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0\text{ m/s})t = \tfrac{1}{2}(1.4\text{ m/s}^2)t^2$. The solutions are $t = 0$ s, 7.1 s. Thus the fugitive cannot catch the car during his 4.286 s of acceleration.
Now the equation of motion of the fugitive changes. After the 4.286 s of acceleration, he runs with a constant speed of 6.0 m/s. Thus his location is now given (for times $t > 5$ s) by the following:

$$x_{\text{fugitive}} = \tfrac{1}{2}(1.4\text{ m/s}^2)(4.286\text{ s})^2 + (6.0\text{ m/s})(t - 4.286\text{ s}) = (6.0\text{ m/s})t - 12.86\text{ m}$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$x_{\text{train}} = x_{\text{fugitive}} \quad\rightarrow\quad (5.0\text{ m/s})t = (6.0\text{ m/s})t - 12.86\text{ m} \quad\rightarrow\quad t = 12.86\text{ s} \approx \boxed{13\text{ s}}$$

(*b*) The distance traveled to reach the box car is given by the following:

$$x_{\text{fugitive}}(t = 15.0\text{ s}) = (6.0\text{ m/s})(12.86\text{ s}) - 12.86\text{ m} = \boxed{64\text{ m}}$$

37. For the runners to cross the finish line side-by-side, they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2–11b.

For Sally: $22 = 5.0 + 5.0t + \tfrac{1}{2}(-0.40)t^2 \quad\rightarrow\quad t^2 - 25t + 85 = 0 \quad\rightarrow$

$$t = \frac{25 \pm \sqrt{25^2 - 4(85)}}{2} = 4.059\text{ s}, 20.94\text{ s}$$

The first time is the time she first crosses the finish line, so that is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.

For Mary: $22 = 0 + 4t + \tfrac{1}{2}at^2 \quad\rightarrow\quad a = \dfrac{22 - 4t}{\tfrac{1}{2}t^2} = \dfrac{22 - 4(4.059)}{\tfrac{1}{2}(4.059)^2} = \boxed{0.70\text{ m/s}^2}$

38. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car and find another time that both cars have the same displacement from the origin.

For the speeder, traveling with a constant speed, the displacement is given by the following:

$$\Delta x_s = v_s t = (135 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)(t) = (37.5\,t) \text{ m}$$

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1 second of reaction time.

$$\Delta x_{p1} = v_{p1}(1.00 \text{ s}) = (95 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)(1.00 \text{ s}) = 26.39 \text{ m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for $(t-1.00)$ s. So this second part of the police car displacement, using Eq. 2–11b, is given as follows:

$$\Delta x_{p2} = v_{p1}(t-1.00) + \tfrac{1}{2}a_p(t-1.00)^2 = [(26.39 \text{ m/s})(t-1.00) + \tfrac{1}{2}(2.60 \text{ m/s}^2)(t-1.00)^2] \text{ m}$$

So the total police car displacement is the following:

$$\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = (26.39 + 26.39(t-1.00) + 1.30(t-1.00)^2) \text{ m}$$

Now set the two displacements equal and solve for the time.

$$26.39 + 26.39(t-1.00) + 1.30(t-1.00)^2 = 37.5\,t \quad \rightarrow \quad t^2 - 10.55t + 1.00 = 0$$

$$t = \frac{10.55 \pm \sqrt{(10.55)^2 - 4.00}}{2} = 9.57\times 10^{-2} \text{ s}, \boxed{10.5 \text{ s}}$$

The answer that is approximately 0 s corresponds to the fact that both vehicles had the same displacement of zero when the time was 0. The reason it is not exactly zero is rounding of previous values. The answer of 10.5 s is the time for the police car to overtake the speeder.

as a check on the answer, the speeder travels $\Delta x_s = (37.5 \text{ m/s})(10.5 \text{ s}) = 394 \text{ m}$, and the police car travels $\Delta x_p = [26.39 + 26.39(9.5) + 1.30(9.5)^2] \text{ m} = 394 \text{ m}$.

39. Choose downward to be the positive direction, and take $y_0 = 0$ at the top of the cliff. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80 \text{ m/s}^2$. The displacement is found from Eq. 2–11b, with x replaced by y.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y - 0 = 0 + \tfrac{1}{2}(9.80 \text{ m/s}^2)(3.55 \text{ s})^2 \quad \rightarrow \quad y = \boxed{61.8 \text{ m}}$$

40. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the top of the Empire State Building. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80 \text{ m/s}^2$.

(*a*) The elapsed time can be found from Eq. 2–11b, with x replaced by y.

$$y - y_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(380 \text{ m})}{9.80 \text{ m/s}^2}} = 8.806 \text{ s} \approx \boxed{8.8 \text{ s}}$$

(*b*) The final velocity can be found from Eq. 2–11a.

$$v = v_0 + at = 0 + (9.80 \text{ m/s}^2)(8.806 \text{ s}) = \boxed{86 \text{ m/s}}$$

41. Choose upward to be the positive direction, and take $y_0 = 0$ to be the height from which the ball was thrown. The acceleration is $a = -9.80\ \text{m/s}^2$. The displacement upon catching the ball is 0, assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2–11b, with *x* replaced by *y*.

$$y = y_0 + \upsilon_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow$$

$$\upsilon_0 = \frac{y - y_0 - \frac{1}{2}at^2}{t} = -\tfrac{1}{2}at = -\tfrac{1}{2}(-9.80\ \text{m/s}^2)(3.4\ \text{s}) = 16.66\ \text{m/s} \approx \boxed{17\ \text{m/s}}$$

The height can be calculated from Eq. 2–11c, with a final velocity of $\upsilon = 0$ at the top of the path.

$$\upsilon^2 = \upsilon_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{\upsilon^2 - \upsilon_0^2}{2a} = 0 + \frac{0 - (16.66\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{14\ \text{m}}$$

42. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the height where the ball was hit. For the upward path, $\upsilon_0 = 25\ \text{m/s}$, $\upsilon = 0$ at the top of the path, and $a = -9.80\ \text{m/s}^2$.

(*a*) The displacement can be found from Eq. 2–11c, with *x* replaced by *y*.

$$\upsilon^2 = \upsilon_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{\upsilon^2 - \upsilon_0^2}{2a} = 0 + \frac{0 - (25\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{32\ \text{m}}$$

(*b*) The time of flight can be found from Eq. 2–11b, with *x* replaced by *y*, using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + \upsilon_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad t(\upsilon_0 + \tfrac{1}{2}at) = 0 \quad \rightarrow$$

$$t = 0,\ t = \frac{2\upsilon_0}{-a} = \frac{2(25\ \text{m/s})}{-9.80\ \text{m/s}^2} = \boxed{5.1\ \text{s}}$$

The result of $t = 0$ s is the time for the original displacement of zero (when the ball was hit), and the result of $t = 5.1$ s is the time to return to the original displacement. Thus the answer is $t = 5.1$ seconds.

(*c*) This is an estimate primarily because the effects of the air have been ignored. There is a non-trivial amount of air effect on a baseball as it moves through the air—that's why pitches like the "curve ball" work, for example. So ignoring the effects of air makes this an estimate. Another effect is that the problem says "almost" straight up, but the problem was solved as if the initial velocity was perfectly upward. Finally, we assume that the ball was caught at the same height as which it was hit. That was not stated in the problem either, so that is an estimate.

43. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the maximum height of the kangaroo. Consider just the downward motion of the kangaroo. Then the displacement is $y = 1.45$ m, the acceleration is $a = 9.80\ \text{m/s}^2$, and the initial velocity is $\upsilon_0 = 0$. Use Eq. 2–11b to calculate the time for the kangaroo to fall back to the ground. The total time is then twice the falling time.

$$y = y_0 + \upsilon_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad y = \tfrac{1}{2}at^2 \quad \rightarrow \quad t_{\text{fall}} = \sqrt{\frac{2y}{a}} \quad \rightarrow$$

$$t_{\text{total}} = 2\sqrt{\frac{2y}{a}} = 2\sqrt{\frac{2(1.45\ \text{m})}{(9.80\ \text{m/s}^2)}} = \boxed{1.09\ \text{s}}$$

44. Choose upward to be the positive direction, and take $y_0 = 0$ to be at the floor level, where the jump starts. For the upward path, $y = 1.2$ m, $v = 0$ at the top of the path, and $a = -9.80\ \text{m/s}^2$.

(*a*) The initial speed can be found from Eq. 2–11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a(y - y_0)} = \sqrt{-2ay} = \sqrt{-2(-9.80\ \text{m/s}^2)(1.2\ \text{m})} = 4.8497\ \text{m/s} \approx \boxed{4.8\ \text{m/s}}$$

(*b*) The time of flight can be found from Eq. 2–11b, with *x* replaced by *y*, using a displacement of 0 for the displacement of the jumper returning to the original height.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 \quad \rightarrow \quad t(v_0 + \tfrac{1}{2}at) = 0 \quad \rightarrow$$

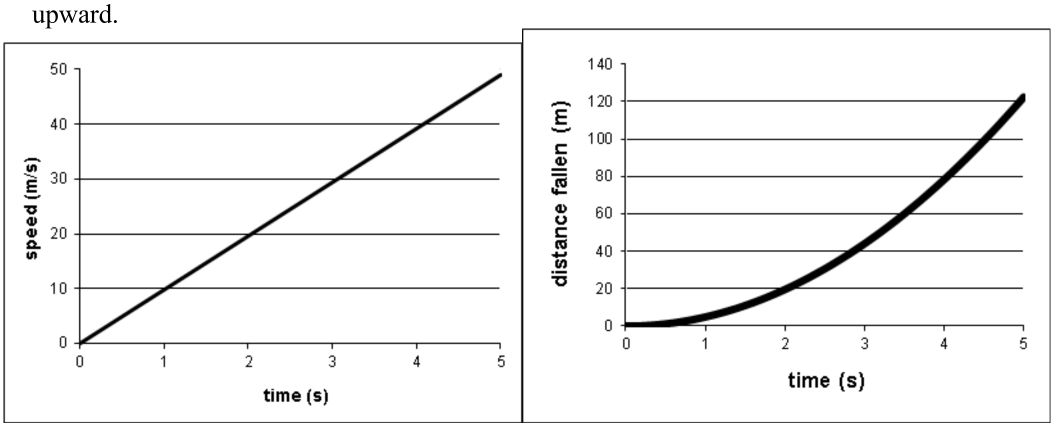
$$t = 0,\ t = \frac{2v_0}{-a} = \frac{2(4.897\ \text{m/s})}{9.80\ \text{m/s}^2} = \boxed{0.99\ \text{s}}$$

The result of $t = 0$ s is the time for the original displacement of zero (when the jumper started to jump), and the result of $t = 0.99$ s is the time to return to the original displacement. Thus the answer is $t = 0.99$ seconds.

45. Choose downward to be the positive direction, and take $y_0 = 0$ to be at the height where the object was released. The initial velocity is $v_0 = 0$, and the acceleration is $a = 9.80\ \text{m/s}^2$.

(*a*) The speed of the object will be given by Eq. 2–11a with $v_0 = 0$, so $v = at = (9.80\ \text{m/s}^2)t$. This is the equation of a straight line passing through the origin with a slope of $9.80\ \text{m/s}^2$.

(*b*) The distance fallen will be given by Eq. 2–11b with $v_0 = 0$, so $y = y_0 + v_0 t + \tfrac{1}{2}at^2 =$ $0 + 0 + (4.90\ \text{m/s}^2)t^2$. This is the equation of a parabola, with its vertex at the origin, opening upward.



46. Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 24.0$ m/s, $a = -9.80\ \text{m/s}^2$, and $y - y_0 = 13.0$ m.

(*a*) The velocity can be found from Eq. 2–11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{(24.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(13.0\ \text{m})} = \pm 17.9\ \text{m/s}$$

Thus the speed is $\boxed{|v| = 17.9\ \text{m/s}}$.

(*b*) The time to reach that height can be found from Eq. 2–11b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2(24.0\text{ m/s})}{-9.80\text{ m/s}^2}t + \frac{2(-13.0\text{ m})}{-9.80\text{ m/s}^2} = 0 \quad \rightarrow$$

$$t^2 - 4.898t + 2.653 = 0 \quad \rightarrow \quad t = \frac{4.898 \pm \sqrt{(4.898)^2 - 4(2.653)}}{2} = \boxed{t = 4.28\text{ s, } 0.620\text{ s}}$$

(*c*) There are two times at which the object reaches that height—once on the way up $(t = 0.620\text{ s})$ and once on the way down $(t = 4.28\text{ s})$.

47. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height from which the object is released. The initial velocity is $v_0 = 0$, and the acceleration is $a = g$. Then we can calculate the position as a function of time from Eq. 2–11b, with *x* replaced by *y*, as $y(t) = \tfrac{1}{2}gt^2$. At the end of each second, the position would be as follows:

$$y(0) = 0; \quad y(1) = \tfrac{1}{2}g; \quad y(2) = \tfrac{1}{2}g(2)^2 = 4y(1); \quad y(3) = \tfrac{1}{2}g(3)^2 = 9y(1)$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$d(1) = y(1) - y(0) = y(1); \quad d(2) = y(2) - y(1) = 3y(1); \quad d(3) = y(3) - y(2) = 5y(1)$$

We could do this in general. Let *n* be a positive integer, starting with 0.

$$y(n) = \tfrac{1}{2}gn^2 \qquad y(n+1) = \tfrac{1}{2}g(n+1)^2$$

$$d(n+1) = y(n+1) - y(n) = \tfrac{1}{2}g(n+1)^2 - \tfrac{1}{2}gn^2 = \tfrac{1}{2}g((n+1)^2 - n^2)$$

$$= \tfrac{1}{2}g(n^2 + 2n + 1 - n^2) = \tfrac{1}{2}g(2n+1)$$

The value of $(2n+1)$ is always odd, in the sequence $\boxed{1, 3, 5, 7, \ldots}$.

48. (*a*) Choose upward to be the positive direction, and $y_0 = 0$ at the ground. The rocket has $v_0 = 0$, $a = 3.2\text{ m/s}^2$, and $y = 775\text{ m}$ when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq. 2–11c, with *x* replaced by *y*.

$$v_{775\text{ m}}^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow$$

$$v_{775\text{ m}} = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{0 + 2(3.2\text{ m/s}^2)(775\text{ m})} = 70.43\text{ m/s} \approx \boxed{7.0\times10^1\text{ m/s}}$$

The positive root is chosen since the rocket is moving upward when it runs out of fuel. Note that the value has 2 significant figures.

(*b*) The time to reach the 775 m location can be found from Eq. 2–11a.

$$v_{775\text{ m}} = v_0 + at_{775\text{ m}} \quad \rightarrow \quad t_{775\text{ m}} = \frac{v_{775\text{ m}} - v_0}{a} = \frac{70.43\text{ m/s} - 0}{3.2\text{ m/s}^2} = 22.01\text{ s} \approx \boxed{22\text{ s}}$$

(*c*) For this part of the problem, the rocket will have an initial velocity $v_0 = 70.43\text{ m/s}$, an acceleration of $a = -9.80\text{ m/s}^2$, and a final velocity of $v = 0$ at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2–11c.

$$v^2 = v_{775\text{ m}}^2 + 2a(y - 775\text{ m}) \quad \rightarrow$$

$$y_{\max} = 775\text{ m} + \frac{0 - v_{775\text{ m}}^2}{2a} = 775\text{ m} + \frac{-(70.43\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 775\text{ m} + 253\text{ m} = 1028\text{ m} \approx \boxed{1030\text{ m}}$$

(*d*) The time for the "coasting" portion of the flight can be found from Eq. 2–11a.

$$v = v_{775\text{ m}} + at_{\text{coast}} \quad \to \quad t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 70.43\text{ m/s}}{-9.80\text{ m/s}^2} = 7.19\text{ s}$$

Thus the total time to reach the maximum altitude is $t = 22.01\text{ s} + 7.19\text{ s} = 29.20\text{ s} \approx \boxed{29\text{ s}}$.

(*e*) For this part of the problem, the rocket has $v_0 = 0$ m/s, $a = -9.80\text{ m/s}^2$, and a displacement of -1028 m (it falls from a height of 1028 m to the ground). Find the velocity upon reaching the Earth from Eq. 2–11c.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \to$$

$$v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{0 + 2(-9.80\text{ m/s}^2)(-1028\text{ m})} = -141.95\text{ m/s} \approx \boxed{-142\text{ m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

(*f*) The time for the rocket to fall back to the Earth is found from Eq. 2–11a.

$$v = v_0 + at \quad \to \quad t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-141.95\text{ m/s} - 0}{-9.80\text{ m/s}^2} = 14.48\text{ s}$$

Thus the total time for the entire flight is $t = 29.20\text{ s} + 14.48\text{ s} = 43.68\text{ s} \approx \boxed{44\text{ s}}$.

49. Choose downward to be the positive direction, and take $y_0 = 0$ to be the height where the object was released. The initial velocity is $v_0 = -5.40$ m/s, the acceleration is $a = 9.80\text{ m/s}^2$, and the displacement of the package will be $y = 105$ m. The time to reach the ground can be found from Eq. 2–11b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \to \quad t^2 + \frac{2v_0}{a}t - \frac{2y}{a} = 0 \quad \to \quad t^2 + \frac{2(-5.40\text{ m/s})}{9.80\text{ m/s}^2}t - \frac{2(105\text{ m})}{9.80\text{ m/s}^2} = 0 \quad \to$$

$$t^2 - 1.102t - 21.43 = 0 \quad \to \quad t = \frac{1.102 \pm \sqrt{(1.102)^2 - 4(-21.43)}}{2} = 5.21\text{ s}, -4.11\text{ s}$$

The correct time is the positive answer, $\boxed{t = 5.21\text{ s}}$.

50. (*a*) Choose $y = 0$ to be the ground level and positive to be upward. Then $y_0 = 15$ m, $a = -g$, and $t = 0.83$ s describe the motion of the balloon. Use Eq. 2–11b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \to$$

$$v_0 = \frac{y - y_0 - \frac{1}{2}at^2}{t} = \frac{0 - 15\text{ m} - \frac{1}{2}(-9.80\text{ m/s}^2)(0.83\text{ s})^2}{(0.83\text{ s})} = -14.01\text{ m/s} \approx \boxed{-14\text{ m/s}}$$

So the speed is $\boxed{14\text{ m/s}}$.

(*b*) Consider the change in velocity from being released to being at Roger's room, using Eq. 2–11c.

$$v^2 = v_0^2 + 2a\Delta y \quad \to \quad \Delta y = \frac{v^2 - v_0^2}{2a} = \frac{-(-14.01\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 10.01\text{ m}$$

Thus the balloons are coming from two floors above Roger, or the $\boxed{\text{fifth floor}}$.

51. Choose upward to be the positive direction and $y_0 = 0$ to be the location of the nozzle. The initial velocity is v_0, the acceleration is $a = -9.80 \text{ m/s}^2$, the final location is $y = -1.8 \text{ m}$, and the time of flight is $t = 2.5 \text{ s}$. Using Eq. 2–11b and substituting *y* for *x* gives the following:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{-1.8 \text{ m} - \frac{1}{2}(-9.80 \text{ m/s}^2)(2.5 \text{ s})^2}{2.5 \text{ s}} = 11.53 \text{ m/s} \approx \boxed{12 \text{ m/s}}$$

52. Choose upward to be the positive direction and $y_0 = 0$ to be the level from which the ball was thrown. The initial velocity is v_0, the instantaneous velocity is $v = 14 \text{ m/s}$, the acceleration is $a = -9.80 \text{ m/s}^2$, and the location of the window is $y = 18 \text{ m}$.

(*a*) Using Eq. 2–11c and substituting *y* for *x*, we have

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow$$

$$v_0 = \pm\sqrt{v^2 - 2a(y - y_0)} = \pm\sqrt{(14 \text{ m/s})^2 - 2(-9.80 \text{ m/s}^2)(18 \text{ m})} = 23.43 \text{ m/s} \approx \boxed{23 \text{ m/s}}$$

Choose the positive value because the initial direction is upward.

(*b*) At the top of its path, the velocity will be 0, so we can use the initial velocity as found above, along with Eq. 2–11c.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (23.43 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{28 \text{ m}}$$

(*c*) We want the time elapsed from throwing (velocity $v_0 = 23.43 \text{ m/s}$) to reaching the window (velocity $v = 14 \text{ m/s}$). Using Eq. 2–11a, we have the following:

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{14 \text{ m/s} - 23.43 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.9622 \text{ s} \approx \boxed{0.96 \text{ s}}$$

(*d*) We want the time elapsed from the window $(v_0 = 14 \text{ m/s})$ to reaching the street $(v = -23.43 \text{ m/s})$. Using Eq. 2–11a, we have:

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{-23.43 \text{ m/s} - 14 \text{ m/s}}{-9.80 \text{ m/s}^2} = 3.819 \text{ s} \approx \boxed{3.8 \text{ s}}$$

The total time from throwing to reaching the street again is $0.9622 \text{ s} + 3.819 \text{ s} = 4.8 \text{ s}$.

53. Choose downward to be the positive direction and $y_0 = 0$ to be the height from which the stone is dropped. Call the location of the top of the window y_w, and the time for the stone to fall from release to the top of the window is t_w. Since the stone is dropped from rest, using Eq. 2–11b with *y* substituting for *x*, we have $y_w = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}gt_w^2$. The location of the bottom of the window is $y_w + 2.2 \text{ m}$, and the time for the stone to fall from release to the bottom of the window is $t_w + 0.31 \text{ s}$. Since the stone is dropped from rest, using Eq. 2–11b, we have the following:

$$y_w + 2.2 \text{ m} = y_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}g(t_w + 0.31 \text{s})^2$$

Substitute the first expression for y_w into the second one and solve for the time.

$$\tfrac{1}{2}gt_w^2 + 2.2\text{ m} = \tfrac{1}{2}g(t_w + 0.31\text{ s})^2 \rightarrow \tfrac{1}{2}gt_w^2 + 2.2 = \tfrac{1}{2}g(t_w^2 + 2t_w(0.31) + (0.31)^2) \rightarrow$$
$$2.2 = \tfrac{1}{2}g(2t_w(0.31) + (0.31)^2) \rightarrow 2.2 = t_w(0.31)g + \tfrac{1}{2}g(0.31)^2 \rightarrow$$
$$t_w = \frac{2.2 - \tfrac{1}{2}g(0.31)^2}{(0.31)g} = 0.569\text{ s}$$

Use this time in the first equation to find the desired distance.

$$y_w = \tfrac{1}{2}gt_w^2 = \tfrac{1}{2}(9.80\text{ m/s}^2)(0.569\text{ s})^2 = 1.587\text{ m} \approx \boxed{1.6\text{ m}}$$

54. For the falling rock, choose downward to be the positive direction and $y_0 = 0$ to be the height from which the stone is dropped. The initial velocity is $v_0 = 0$ m/s, the acceleration is $a = g$, the final position is $y = H$, and the time of fall is t_1. Using Eq. 2–11b with *y* substituting for *x*, we have $H = y_0 + v_0 t + \tfrac{1}{2}t^2 = 0 + 0 + \tfrac{1}{2}gt_1^2$. For the sound wave, use the constant speed equation that $v_s = \frac{\Delta x}{\Delta t} = \frac{H}{T - t_1}$, which can be rearranged to give $t_1 = T - \frac{H}{v_s}$, where $T = 3.4$ s is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for t_1 into the equation for *H* from the stone, and solve for *H*.

$$H = \tfrac{1}{2}g\left(T - \frac{H}{v_s}\right)^2 \rightarrow \frac{g}{2v_s^2}H^2 - \left(\frac{gT}{v_s} + 1\right)H + \tfrac{1}{2}gT^2 = 0 \rightarrow$$
$$4.239\times10^{-5}H^2 - 1.098H + 56.64 = 0 \rightarrow H = 51.7\text{ m},\ 2.59\times10^4\text{ m}$$

If the larger answer is used in $t_1 = T - \frac{H}{v_s}$, a negative time of fall results, so the physically correct answer is $\boxed{H = 52\text{ m}}$.

55. Slightly different answers may be obtained since the data come from reading the graph.

(*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48\text{ s}}$.

(*b*) The indication of a constant velocity on a velocity vs. time graph is a slope of 0, which occurs from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*c*) The indication of a constant acceleration on a velocity vs. time graph is a constant slope, which occurs from $\boxed{t = 0\text{ s to } t \approx 42\text{ s}}$, again from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$, and again from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$.

56. Slightly different answers may be obtained since the data come from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being “in” a certain gear.

(*a*) The average acceleration in 2nd gear is given by $\bar{a}_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{24\text{ m/s} - 14\text{ m/s}}{8\text{ s} - 4\text{ s}} = \boxed{2.5\text{ m/s}^2}$.

(*b*) The average acceleration in 4th gear is given by $\bar{a}_4 = \frac{\Delta v_4}{\Delta t_4} = \frac{44\text{ m/s} - 37\text{ m/s}}{27\text{ s} - 16\text{ s}} = \boxed{0.64\text{ m/s}^2}$.

57. Slightly different answers may be obtained since the data come from reading the graph.

(*a*) The instantaneous velocity is given by the slope of the tangent line to the curve. At $t = 10.0$ s, the slope is approximately $\upsilon(10) \approx \dfrac{3\text{ m} - 0}{10.0\text{ s} - 0} = \boxed{0.3\text{ m/s}}$.

(*b*) At $t = 30.0$ s, the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately $\upsilon(30) \approx \dfrac{20\text{ m} - 8\text{ m}}{35\text{ s} - 25\text{ s}} = \boxed{1.2\text{ m/s}}$.

(*c*) The average velocity is given by $\bar{\upsilon} = \dfrac{x(5) - x(0)}{5.0\text{ s} - 0\text{ s}} = \dfrac{1.5\text{ m} - 0}{5.0\text{ s}} = \boxed{0.30\text{ m/s}}$.

(*d*) The average velocity is given by $\bar{\upsilon} = \dfrac{x(30) - x(25)}{30.0\text{ s} - 25.0\text{ s}} = \dfrac{16\text{ m} - 9\text{ m}}{5.0\text{ s}} = \boxed{1.4\text{ m/s}}$.

(*e*) The average velocity is given by $\bar{\upsilon} = \dfrac{x(50) - x(40)}{50.0\text{ s} - 40.0\text{ s}} = \dfrac{10\text{ m} - 19.5\text{ m}}{10.0\text{ s}} = \boxed{-0.95\text{ m/s}}$.

58. Slightly different answers may be obtained since the data come from reading the graph.

(*a*) The indication of a constant velocity on a position versus time graph is a constant slope, which occurs from $\boxed{t = 0\text{ s to } t \approx 18\text{ s}}$.

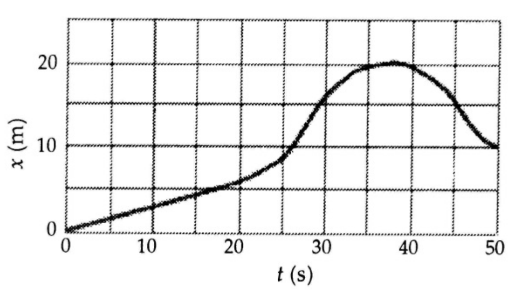
(*b*) The greatest velocity will occur when the slope is the highest positive value, which occurs at about $\boxed{t = 27\text{ s}}$.

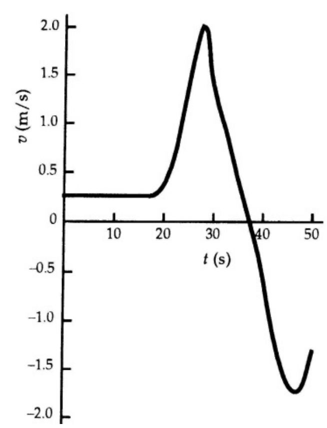
(*c*) The indication of a 0 velocity on a position versus time graph is a slope of 0, which occurs at about $\boxed{t = 38\text{ s}}$.

(*d*) $\boxed{\text{The object moves in both directions.}}$ When the slope is positive, from $t = 0$ s to $t = 38$ s, the object is moving in the positive direction. When the slope is negative, from $t = 38$ s to $t = 50$ s, the object is moving in the negative direction.

59. The υ vs. t graph is found by taking the slope of the x vs. t graph.

Both graphs are shown here.





60. Choose the upward direction to be positive and $y_0 = 0$ to be the level from which the object was thrown. The initial velocity is υ_0 and the velocity at the top of the path is $\upsilon = 0$. The height at the top of the path can be found from Eq. 2–11c with x replaced by y.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y - y_0 = \frac{-v_0^2}{2a}$$

From this we see that the displacement is inversely proportional to the acceleration, so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, then the $\boxed{\text{displacement increases by a factor of 6}}$.

61. We are treating the value of 30 g's as if it had 2 significant figures. The initial velocity of the car is $v_0 = (95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s}$. Choose $x_0 = 0$ to be location at which the deceleration begins. We have $v = 0$ and $a = -30g = -294\text{ m/s}^2$. Find the displacement from Eq. 2–11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad x = x_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (26.39\text{ m/s})^2}{2(-294\text{ m/s}^2)} = 1.18\text{ m} \approx \boxed{1.2\text{ m}}$$

62. (*a*) For the free-falling part of the motion, choose downward to be the positive direction and $y_0 = 0$ to be the height from which the person jumped. The initial velocity is $v_0 = 0$, acceleration is $a = 9.80\text{ m/s}^2$, and the location of the net is $y = 18.0\text{ m}$. Find the speed upon reaching the net from Eq. 2–11c with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow$$

$$v = \pm\sqrt{0 + 2a(y - 0)} = \pm\sqrt{2(9.80\text{ m/s}^2)(18.0\text{ m})} = 18.78\text{ m/s}$$

The positive root is selected since the person is moving downward.

For the net-stretching part of the motion, choose downward to be the positive direction, and $y_0 = 18.0\text{ m}$ to be the height at which the person first contacts the net. The initial velocity is $v_0 = 18.78\text{ m/s}$, the final velocity is $v = 0$, and the location at the stretched position is $y = 19.0\text{ m}$. Find the acceleration from Eq. 2–11c with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0^2 - (18.78\text{ m/s})^2}{2(1.0\text{ m})} = \boxed{-176\text{ m/s}^2}$$

This is about 18 g's.

(*b*) For the acceleration to be smaller, in the above equation we see that the displacement would have to be larger. This means that the net should be $\boxed{\text{loosened}}$.

63. Choose downward to be the positive direction and $y_0 = 0$ to be at the start of the pelican's dive. The pelican has an initial velocity of $v_0 = 0$, an acceleration of $a = g$, and a final location of $y = 14.0\text{ m}$. Find the total time of the pelican's dive from Eq. 2–11b, with *x* replaced by *y*.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}at^2 \quad \rightarrow \quad t_{\text{dive}} = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(14.0\text{ m})}{9.80\text{ m/s}^2}} = 1.69\text{ s}$$

The fish can take evasive action if he sees the pelican at a time of $1.69\text{ s} - 0.20\text{ s} = 1.49\text{ s}$ into the dive. Find the location of the pelican at that time from Eq. 2–11b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at = 0 + 0 + \tfrac{1}{2}(9.80\text{ m/s}^2)(1.49\text{ s})^2 = 10.9\text{ m}$$

Thus the fish must spot the pelican at a minimum height from the surface of the water of $14.0\text{ m} - 10.9\text{ m} = \boxed{3.1\text{ m}}$.

64. The initial velocity is $v_0 = 15\text{ km/h}$, the final velocity is $v = 65\text{ km/h}$, and the displacement is $x - x_0 = 4.0\text{ km} = 4000\text{ m}$. Find the average acceleration from Eq. 2–11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow$$

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{[(65\text{ km/h})^2 - (15\text{ km/h})^2]\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2}{2(4000\text{ m})} = \boxed{3.9\times10^{-2}\text{ m/s}^2}$$

65. The speed limit is $40\text{ km/h}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 11.11\text{ m/s}$.

(*a*) For your motion, you would need to travel $(10+15+50+15+70)\text{ m} = 160\text{ m}$ to get the front of the car to the third stoplight. The time to travel the 160 m is found using the distance and the speed limit.

$$\Delta x = \bar{v}\Delta t \quad \rightarrow \quad \Delta t = \frac{\Delta x}{\bar{v}} = \frac{160\text{ m}}{11.11\text{ m/s}} = 14.40\text{ s}$$

$\boxed{\text{No}}$, you cannot make it to the third light without stopping, since it takes you longer than 13.0 seconds to reach the third light.

(*b*) The second car needs to travel 165 m before the third light turns red. This car accelerates from $v_0 = 0$ to a maximum of $v = 11.11\text{ m/s}$ with $a = 2.00\text{ m/s}^2$. Use Eq. 2–11a to determine the duration of that acceleration.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{acc}} = \frac{v - v_0}{a} = \frac{11.11\text{ m/s} - 0\text{ m/s}}{2.00\text{ m/s}^2} = 5.556\text{ s}$$

The distance traveled during that time is found from Eq. 2–11b.

$$(x - x_0)_{\text{acc}} = v_0 t_{\text{acc}} + \tfrac{1}{2}at_{\text{acc}}^2 = 0 + \tfrac{1}{2}(2.00\text{ m/s}^2)(5.556\text{ s})^2 = 30.87\text{ m}$$

Since 5.556 s have elapsed, there are $13.0 - 5.556 = 7.444\text{ s}$ remaining to clear the intersection. The car travels another 7.444 s at a speed of 11.11 m/s, covering a distance of $\Delta x_{\text{constant speed}} = v_{\text{avg}}t = (11.11\text{ m/s})(7.444\text{ s}) = 82.70\text{ m}$. Thus the total distance is $30.87\text{ m} + 82.70\text{ m} = 113.57\text{ m}$. $\boxed{\text{No}}$, the car cannot make it through all three lights without stopping. The car has to travel another 51.43 m to clear the third intersection and is traveling at a speed of 11.11 m/s. Thus the front of the car would clear the intersection a time $t = \dfrac{\Delta x}{v} = \dfrac{51.43\text{ m}}{11.11\text{ m/s}} = \boxed{4.6\text{ s}}$ after the light turns red.

66. The average speed for each segment of the trip is given by $\bar{v} = \dfrac{d}{\Delta t}$, so $\Delta t = \dfrac{d}{\bar{v}}$ for each segment.

For the first segment, $\Delta t_1 = \dfrac{d_1}{\bar{v}_1} = \dfrac{2100\text{ km}}{720\text{ km/h}} = 2.917\text{ h}$.

For the second segment, $\Delta t_2 = \dfrac{d_2}{\bar{v}_2} = \dfrac{2800\text{ km}}{990\text{ km/h}} = 2.828\text{ h}$.

Thus the total time is $\Delta t_{\text{tot}} = \Delta t_1 + \Delta t_2 = 2.917\text{ h} + 2.828\text{ h} = 5.745\text{ h} \approx \boxed{5.7\text{ h}}$.

The average speed of the plane for the entire trip is

$$\bar{v} = \frac{d_{\text{tot}}}{\Delta t_{\text{tot}}} = \frac{2100\text{ km} + 2800\text{ km}}{5.745\text{ h}} = 852.9\text{ km/h} \approx \boxed{850\text{ km/h}}.$$

Note that Eq. 2–11d does NOT apply in this situation.

67. (*a*) Choose downward to be the positive direction and $y_0 = 0$ to be the level from which the car was dropped. The initial velocity is $v_0 = 0,$ the final location is $y = H,$ and the acceleration is $a = g.$ Find the final velocity from Eq. 2–11c, replacing *x* with *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm\sqrt{2gH}$$

The speed is the magnitude of the velocity, $\boxed{v = \sqrt{2gH}}$.

(*b*) Solving the above equation for the height, we have that $H = \dfrac{v^2}{2g}.$ Thus for a collision of $v = 35\text{ km/h},$ the corresponding height is as follows:

$$H = \frac{v^2}{2g} = \frac{\left[(35\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(9.80\text{ m/s}^2)} = 4.823\text{ m} \approx \boxed{4.8\text{ m}}$$

(*c*) For a collision of $v = 95\text{ km/h},$ the corresponding height is the following:

$$H = \frac{v^2}{2g} = \frac{\left[(95\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(9.80\text{ m/s}^2)} = 35.53\text{ m} \approx \boxed{36\text{ m}}$$

68. Choose downward to be the positive direction and $y_0 = 0$ to be at the roof from which the stones are dropped. The first stone has an initial velocity of $v_0 = 0$ and an acceleration of $a = g.$ Eqs. 2–11a and 2–11b (with *x* replaced by *y*) give the velocity and location, respectively, of the first stone as a function of time.

$$v = v_0 + at \quad \rightarrow \quad v_1 = gt_1 \qquad\qquad y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad y_1 = \tfrac{1}{2}gt_1^2$$

The second stone has the same initial conditions, but its elapsed time is $t_2 = t_1 - 1.30\text{ s},$ so it has velocity and location equations as follows:

$$v_2 = g(t_1 - 1.30\text{ s}) \qquad\qquad y_2 = \tfrac{1}{2}g(t_1 - 1.30\text{ s})^2$$

The second stone reaches a speed of $v_2 = 12.0\text{ m/s}$ at a time given by

$$t_2 = \frac{v_2}{g} = t_1 - 1.30\text{ s} \quad \rightarrow \quad t_1 = 1.30\text{ s} + \frac{v_2}{g} = 1.30\text{ s} + \frac{12.0\text{ m/s}}{9.80\text{ m/s}^2} = 2.524\text{ s}$$

The location of the first stone at that time is

$$y_1 = \tfrac{1}{2}gt_1^2 = \tfrac{1}{2}(9.80\text{ m/s}^2)(2.524\text{ s})^2 = 31.22\text{ m}$$

The location of the second stone at that time is

$$y_2 = \tfrac{1}{2}g(t_1 - 1.30\text{ s})^2 = \tfrac{1}{2}(9.80\text{ m/s}^2)(2.524\text{ s} - 1.30\text{ s})^2 = 7.34\text{ m}$$

Thus the distance between the two stones is $y_1 - y_2 = 31.22\text{ m} - 7.34\text{ m} = 23.88\text{ m} \approx \boxed{23.9\text{ m}}$.

69. For the motion in the air, choose downward to be the positive direction and $y_0 = 0$ to be at the height of the diving board. Then diver has $v_0 = 0$ (assuming the diver does not jump upward or downward), $a = g = 9.80 \text{ m/s}^2$, and $y = 4.0 \text{ m}$ when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. 2–11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0)x \to v = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(4.0 \text{ m})} = 8.85 \text{ m/s}$$

For the motion in the water, again choose down to be positive, but redefine $y_0 = 0$ to be at the surface of the water. For this motion, $v_0 = 8.85 \text{ m/s}$, $v = 0$, and $y - y_0 = 2.0 \text{ m}$. Find the acceleration from Eq. 2–11c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) \quad \to \quad a = \frac{v^2 - v_0^2}{2(y - y_0)x} = \frac{0 - (8.85 \text{ m/s})^2}{2(2.0 \text{ m})} = -19.6 \text{ m/s}^2 \approx \boxed{-20 \text{ m/s}^2}$$

The negative sign indicates that the acceleration is directed upward.

70. First consider the "uphill lie," in which the ball is being putted down the hill. Choose $x_0 = 0$ to be the ball's original location and the direction of the ball's travel as the positive direction. The final velocity of the ball is $v = 0$, the acceleration of the ball is $a = -1.8 \text{ m/s}^2$, and the displacement of the ball will be $x - x_0 = 6.0 \text{ m}$ for the first case and $x - x_0 = 8.0 \text{ m}$ for the second case. Find the initial velocity of the ball from Eq. 2–11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \to \quad v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-1.8 \text{ m/s}^2)(6.0 \text{ m})} = 4.65 \text{ m/s} \\ \sqrt{0 - 2(-1.8 \text{ m/s}^2)(8.0 \text{ m})} = 5.37 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the uphill lie is $\boxed{4.65 \text{ m/s to } 5.37 \text{ m/s}}$, a spread of 0.72 m/s.

Now consider the "downhill lie," in which the ball is being putted up the hill. Use a very similar setup for the problem, with the basic difference being that the acceleration of the ball is now $a = -2.6 \text{ m/s}^2$. Find the initial velocity of the ball from Eq. 2–11c.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \to \quad v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-2.6 \text{ m/s}^2)(6.0 \text{ m})} = 5.59 \text{ m/s} \\ \sqrt{0 - 2(-2.6 \text{ m/s}^2)(8.0 \text{ m})} = 6.45 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the downhill lie is $\boxed{5.59 \text{ m/s to } 6.45 \text{ m/s}}$, a spread of 0.86 m/s. Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, so putting the ball downhill (the "uphill lie") is more difficult.

$\boxed{71.}$ Choose upward to be the positive direction and $y_0 = 0$ to be at the throwing location of the stone. The initial velocity is $v_0 = 15.5 \text{ m/s}$, the acceleration is $a = -9.80 \text{ m/s}^2$, and the final location is $y = -75 \text{ m}$.

(*a*) Using Eq. 2–11b and substituting *y* for *x*, we have the following:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \to \quad (4.9 \text{ m/s}^2)t^2 - (15.5 \text{ m/s})t - 75 \text{ m} = 0 \quad \to$$

$$t = \frac{15.5 \pm \sqrt{(15.5)^2 - 4(4.9)(-75)}}{2(4.9)} = 5.802 \text{ s}, \quad -2.638 \text{ s}$$

The positive answer is the physical answer: $\boxed{t = 5.80 \text{ s}}$.

(*b*) Use Eq. 2–11a to find the velocity just before hitting.

$$v = v_0 + at = 15.5\text{ m/s} + (-9.80\text{ m/s}^2)(5.802\text{ s}) = -41.4\text{ m/s} \quad \rightarrow \quad |v| = \boxed{41.4\text{ m/s}}$$

(*c*) The total distance traveled will be the distance up plus the distance down. The distance down will be 75 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Then using Eq. 2–11c we have the following:

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (15.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 12.26\text{ m}$$

Thus the distance up is 12.26 m, the distance down is 87.26 m, and the total distance traveled is $\boxed{99.5\text{ m}}$.

72. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant-speed phase, and the deceleration phase. The maximum speed of the train is as follows:

$$(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s}$$

In the acceleration phase, the initial velocity is $v_0 = 0$, the acceleration is $a = 1.1\text{ m/s}^2$, and the final velocity is $v = 26.39\text{ m/s}$. Find the elapsed time for the acceleration phase from Eq. 2–11a.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{acc}} = \frac{v - v_0}{a} = \frac{26.39\text{ m/s} - 0}{1.1\text{ m/s}^2} = 23.99\text{ s}$$

Find the displacement during the acceleration phase from Eq. 2–11b.

$$(x - x_0)_{\text{acc}} = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(1.1\text{ m/s}^2)(23.99\text{ s})^2 = 316.5\text{ m}$$

In the deceleration phase, the initial velocity is $v_0 = 26.39\text{ m/s}$, the acceleration is $a = -2.0\text{ m/s}^2$, and the final velocity is $v = 0$. Find the elapsed time for the deceleration phase from Eq. 2–11a.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{dec}} = \frac{v - v_0}{a} = \frac{0 - 26.39\text{ m/s}}{-2.0\text{ m/s}^2} = 13.20\text{ s}$$

Find the distance traveled during the deceleration phase from Eq. 2–11b.

$$(x - x_0)_{\text{dec}} = v_0 t + \tfrac{1}{2}at^2 = (26.39\text{ m/s})(13.20\text{ s}) + \tfrac{1}{2}(-2.0\text{ m/s}^2)(13.20\text{ s})^2 = 174.1\text{ m}$$

The total elapsed time and distance traveled for the acceleration/deceleration phases are:

$$t_{\text{acc}} + t_{\text{dec}} = 23.99\text{ s} + 13.20\text{ s} = 37.19\text{ s}$$
$$(x - x_0)_{\text{acc}} + (x - x_0)_{\text{dec}} = 316.5\text{ m} + 174.1\text{ m} = 491\text{ m}$$

(*a*) If the stations are spaced $3.0\text{ km} = 3000\text{ m}$ apart, then there is a total of $\dfrac{15{,}000\text{ m}}{3000\text{ m}} = 5$ interstation segments. A train making the entire trip would thus have a total of 5 interstation segments and 4 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $3000\text{ m} - 491\text{ m} = 2509\text{ m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39\text{ m/s}$. The time for that 2509 m is given by $\Delta x = \bar{v}\Delta t \quad \rightarrow$

$$\Delta t_{\substack{\text{constant}\\\text{speed}}} = \frac{\Delta x}{\bar{v}} = \frac{2509\text{ m}}{26.39\text{ m/s}} = 95.07\text{ s}.$$ Thus a total interstation segment will take

$37.19\ \text{s} + 95.07\ \text{s} = 132.26\ \text{s}$. With 5 interstation segments of 132.26 s each, and 4 stops of 22 s each, the total time is given by $t_{3.0\ \text{km}} = 5(132.26\ \text{s}) + 4(22\ \text{s}) = 749\ \text{s} = \boxed{12.5\ \text{min}}$.

(*b*) If the stations are spaced $5.0\ \text{km} = 5000\ \text{m}$ apart, then there is a total of $\dfrac{15{,}000\ \text{m}}{5000\ \text{m}} = 3$ interstation segments. A train making the entire trip would thus have a total of 3 interstation segments and 2 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration, $5000\ \text{m} - 491\ \text{m} = 4509\ \text{m}$ of each segment is traveled at an average speed of $\bar{v} = 26.39\ \text{m/s}$. The time for that 4509 m is given by $d = \bar{v}t \quad \rightarrow$

$t = \dfrac{d}{\bar{v}} = \dfrac{4509\ \text{m}}{26.39\ \text{m/s}} = 170.86\ \text{s}$. Thus a total interstation segment will take

$37.19\ \text{s} + 170.86\ \text{s} = 208.05\ \text{s}$. With 3 interstation segments of 208.05 s each, and 2 stops of 22 s each, the total time is given by $t_{5.0\ \text{km}} = 3(208.05\ \text{s}) + 2(22\ \text{s}) = 668\ \text{s} = \boxed{11.1\ \text{min}}$.

73. The car's initial speed is $v_0 = (35\ \text{km/h})\left(\dfrac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 9.722\ \text{m/s}$.

Case I: trying to stop. The constraint is, with the braking deceleration of the car $(a = -5.8\ \text{m/s}^2)$, can the car stop in a 28-m displacement? The 2.0 seconds has no relation to this part of the problem. Using Eq. 2–11c, the distance traveled during braking is as follows:

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (9.722\ \text{m/s})^2}{2(-5.8\ \text{m/s}^2)} = 8.14\ \text{m} \quad \boxed{\text{She can stop the car in time.}}$$

Case II: crossing the intersection. The constraint is, with the acceleration of the car $\left[a = \left(\dfrac{65\ \text{km/h} - 45\ \text{km/h}}{6.0\ \text{s}}\right)\left(\dfrac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 0.9259\ \text{m/s}^2\right]$, can she get through the intersection (travel 43 m) in the 2.0 seconds before the light turns red? Using Eq. 2–11b, the distance traveled during the 2.0 s is

$$(x - x_0) = v_0 t + \tfrac{1}{2}at^2 = (9.722\ \text{m/s})(2.0\ \text{s}) + \tfrac{1}{2}(0.9259\ \text{m/s}^2)(2.0\ \text{s})^2 = 21.3\ \text{m}$$

$\boxed{\text{She should stop.}}$

74. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 500 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are (i) the 10 m of clear room at the rear of the truck, (ii) the 20-m length of the truck, (iii) the 10 m of clear room at the front of the truck, and (iv) the distance the truck travels. Since the truck travels at a speed of $\bar{v} = 18\ \text{m/s}$, the truck will have a displacement of $\Delta x_{\text{truck}} = (18\ \text{m/s})t$. Thus the total displacement of the car during passing is $\Delta x_{\substack{\text{passing}\\ \text{car}}} = 40\ \text{m} + (18\ \text{m/s})t$.

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of $v_0 = 18\ \text{m/s}$ and an acceleration of $a = 1.0\ \text{m/s}^2$. Find $\Delta x_{\substack{\text{passing}\\ \text{car}}}$ from Eq. 2–11b.

$$\Delta x_{\substack{\text{passing}\\ \text{car}}} = x_c - x_0 = v_0 t + \tfrac{1}{2}at = (18\ \text{m/s})t + \tfrac{1}{2}(0.60\ \text{m/s}^2)t^2$$

Set the two expressions for $\Delta x_{\substack{\text{passing}\\\text{car}}}$ equal to each other in order to find the time required to pass.

$$40\text{ m}+(18\text{ m/s})t_{\text{pass}}=(18\text{ m/s})t_{\text{pass}}+\tfrac{1}{2}(0.60\text{ m/s}^2)t_{\text{pass}}^2 \quad\to\quad 40\text{ m}=\tfrac{1}{2}(0.60\text{ m/s}^2)t_{\text{pass}}^2 \quad\to$$

$$t_{\text{pass}}=\sqrt{\frac{80}{0.60}\text{s}^2}=11.55\text{ s}$$

Calculate the displacements of the two cars during this time.

$$\Delta x_{\substack{\text{passing}\\\text{car}}}=40\text{ m}+(18\text{ m/s})(11.55\text{ s})=247.9\text{ m}$$

$$\Delta x_{\substack{\text{approaching}\\\text{car}}}=v_{\substack{\text{approaching}\\\text{car}}}t=(25\text{ m/s})(11.55\text{ s})=288.75\text{ m}$$

Thus the two cars together have covered a total distance of $247.9\text{ m}+288.75\text{ m}=536.65\text{ m},$ which is more than allowed. $\boxed{\text{The car should not pass.}}$

75. Choose downward to be the positive direction and $y_0=0$ to be at the height of the bridge. Agent Bond has an initial velocity of $v_0=0,$ an acceleration of $a=g,$ and will have a displacement of $y=15\text{ m}-3.5\text{ m}=11.5\text{ m}.$ Find the time of fall from Eq. 2–11b with *x* replaced by *y*.

$$y=y_0+v_0t+\tfrac{1}{2}at^2 \quad\to\quad t=\sqrt{\frac{2y}{a}}=\sqrt{\frac{2(11.5\text{ m})}{9.80\text{ m/s}^2}}=1.532\text{ s}$$

If the truck is approaching with $v=25\text{ m/s},$ then he needs to jump when the truck is a distance away given by $d=vt=(25\text{ m/s})(1.532\text{ s})=38.3\text{ m}.$ Convert this distance into “poles.”

$$d=(38.3\text{ m})(1\text{ pole}/25\text{ m})=1.53\text{ poles}$$

So he should jump when the truck is about $\boxed{1.5\text{ poles}}$ away from the bridge.

76. The speed of the conveyor belt is found from Eq. 2–2 for average velocity.

$$\Delta x=\bar{v}\Delta t \quad\to\quad \bar{v}=\frac{\Delta x}{\Delta t}=\frac{1.2\text{ m}}{2.8\text{ min}}=0.4286\text{ m/min}\approx\boxed{0.43\text{ m/min}}$$

The rate of burger production, assuming the spacing given is center to center, can be found as follows:

$$\left(\frac{1\text{ burger}}{0.25\text{ m}}\right)\left(\frac{0.4286\text{ m}}{1\text{ min}}\right)=\boxed{1.7\frac{\text{burgers}}{\text{min}}}$$

77. Choose downward to be the positive direction and the origin to be at the top of the building. The barometer has $y_0=0,$ $v_0=0,$ and $a=g=9.8\text{ m/s}^2.$ Use Eq. 2–11b to find the height of the building, with *x* replaced by *y*.

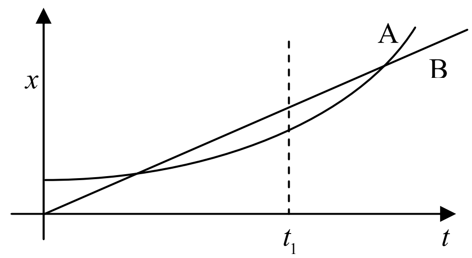
$$y=y_0+v_0t+\tfrac{1}{2}at^2=0+0+\tfrac{1}{2}(9.8\text{ m/s}^2)t^2$$

$$y_{t=2.0}=\tfrac{1}{2}(9.8\text{ m/s}^2)(2.0\text{ s})^2=19.6\text{ m} \qquad y_{t=2.3}=\tfrac{1}{2}(9.8\text{ m/s}^2)(2.3\text{ s})^2=25.9\text{ m}$$

The difference in the estimates is 6.3 m. If we assume the height of the building is the average of the two measurements, then the % difference in the two values is $\dfrac{6.3\text{ m}}{22.75\text{ m}}\times100=27.7\%\approx\boxed{30\%}.$

The intent of the method was probably to use the change in air pressure between the ground level and the top of the building to find the height of the building. The very small difference in time measurements, which could be due to human reaction time, makes a 6.3-m difference in the height. This could be as much as 2 floors in error.

78. (*a*) The two bicycles will have the same velocity at any time when the instantaneous slopes of their *x* vs. *t* graphs are the same. That occurs near the time t_1 as marked on the graph.



(*b*) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.

(*c*) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, so it is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.

(*d*) Bicycle B has the highest instantaneous velocity at all times until the time t_1, where both graphs have the same slope. For all times after t_1, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.

(*e*) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.

79. To find the average speed for the entire race, we must take the total distance divided by the total time. If one lap is a distance of L, then the total distance will be $10L$. The time elapsed at a given constant speed is given by $t = d/v$, so the time for the first 9 laps would be $t_1 = \dfrac{9L}{196.0\text{ km/h}}$, and the time for the last lap would be $t_2 = L/v_2$, where v_2 is the average speed for the last lap. Write an expression for the average speed for the entire race, and then solve for v_2.

$$\bar{v} = \frac{d_{\text{total}}}{t_1 + t_2} = \frac{10L}{\dfrac{9L}{196.0\text{ km/h}} + \dfrac{L}{v_2}} = 200.0\text{ km/h} \quad \rightarrow \quad v_2 = \frac{1}{\dfrac{10}{200.0\text{ km/h}} - \dfrac{9}{196.0\text{ km/h}}} = \boxed{245.0\text{ km/h}}$$

80. Assume that $y_0 = 0$ for each child is the level at which the child loses contact with the trampoline surface. Choose upward to be the positive direction.

(*a*) The second child has $v_{02} = 4.0\text{ m/s}$, $a = -g = -9.80\text{ m/s}^2$, and $v = 0\text{ m/s}$ at the maximum height position. Find the child's maximum height from Eq. 2–11c, with *x* replaced by *y*.

$$v^2 = v_{02}^2 + 2a(y_2 - y_0) \quad \rightarrow$$

$$y_2 = y_0 + \frac{v^2 - v_{02}^2}{2a} = 0 + \frac{0 - (4.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 0.8163\text{ m} \approx \boxed{0.82\text{ m}}$$

(*b*) Since the first child can bounce up to one-and-a-half times higher than the second child, the first child can bounce up to a height of $1.5(0.8163 \text{ m}) = 1.224 \text{ m} = y_1 - y_0$. Eq. 2–11c is again used to find the initial speed of the first child.

$$v^2 = v_{01}^2 + 2a(y_1 - y_0) \quad \rightarrow$$
$$v_{01} = \pm\sqrt{v^2 - 2a(y_1 - y_0)} = \sqrt{0 - 2(-9.80 \text{ m/s}^2)(1.224 \text{ m})} = 4.898 \text{ m/s} \approx \boxed{4.9 \text{ m/s}}$$

The positive root was chosen since the child was initially moving upward.

(*c*) To find the time that the first child was in the air, use Eq. 2–11b with a total displacement of 0, since the child returns to the original position.

$$y = y_0 + v_{01}t_1 + \tfrac{1}{2}at_1^2 \quad \rightarrow \quad 0 = (4.898 \text{ m/s})t_1 + \tfrac{1}{2}(-9.80 \text{ m/s}^2)t_1^2 \quad \rightarrow \quad t_1 = 0 \text{ s}, 0.9996 \text{ s}$$

The time of 0 s corresponds to the time the child started the jump, so the correct answer is $\boxed{1.0 \text{ s}}$.

81. Choose downward to be the positive direction and the origin to be at the location of the plane. The parachutist has $v_0 = 0$, $a = g = 9.80 \text{ m/s}^2$, and will have $y - y_0 = 3200 \text{ m} - 450 \text{ m} = 2750 \text{ m}$ when she pulls the ripcord. Eq. 2–11b, with *x* replaced by *y*, is used to find the time when she pulls the ripcord.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{2(y - y_0)/a} = \sqrt{2(2750 \text{ m})/(9.80 \text{ m/s}^2)} = 23.69 \text{ s} \approx \boxed{23.7 \text{ s}}$$

The speed is found from Eq. 2–11a.

$$v = v_0 + at = 0 + (9.80 \text{ m/s}^2)(23.69 \text{ s}) = 232.16 \text{ m/s} \approx \boxed{230 \text{ m/s}}\left(\frac{3.6 \text{ km/h}}{1 \text{ m/s}}\right) \approx \boxed{840 \text{ km/h}}$$

This is well over 500 miles per hour!

82. As shown in Example 2–15, the speed with which the ball was thrown upward is the same as its speed on returning to the ground. From the symmetry of the two motions (both motions have speed = 0 at top, have same distance traveled, and have same acceleration), the time for the ball to rise is the same as the time for the ball to fall, 1.4 s. Choose upward to be the positive direction and the origin to be at the level where the ball was thrown. For the ball, $v = 0$ at the top of the motion, and $a = -g$. Find the initial velocity from Eq. 2–11a.

$$v = v_0 + at \quad \rightarrow \quad v_0 = v - at = 0 - (-9.80 \text{ m/s}^2)(1.4 \text{ s}) = 13.72 \text{ m/s} \approx \boxed{14 \text{ m/s}}$$

83. (*a*) Multiply the reading rate times the bit density to find the bit reading rate.

$$N = \frac{1.2 \text{ m}}{1 \text{ s}} \times \frac{1 \text{ bit}}{0.28 \times 10^{-6} \text{ m}} = \boxed{4.3 \times 10^6 \text{ bits/s}}$$

(*b*) The number of excess bits is $N - N_0$.

$$N - N_0 = 4.3 \times 10^6 \text{ bits/s} - 1.4 \times 10^6 \text{ bits/s} = 2.9 \times 10^6 \text{ bits/s}$$

$$\frac{N - N_0}{N} = \frac{2.9 \times 10^6 \text{ bits/s}}{4.3 \times 10^6 \text{ bits/s}} = 0.67 = \boxed{67\%}$$

Solutions to Search and Learn Problems

1. The two conditions are that the motion needs to be near the surface of the Earth and that there is no air resistance. An example where the second condition is not even a reasonable approximation is that of parachuting. The air resistance caused by the parachute results in the acceleration not being constant, with values much different than $9.8\ \text{m/s}^2$.

2. The sounds will not occur at equal time intervals because the longer any particular bolt falls, the higher its speed. With equal distances between bolts, each successive bolt, having fallen a longer time when its predecessor reaches the plate, will have a higher average velocity and thus travel the interbolt distance in shorter periods of time. Thus the sounds will occur with smaller and smaller intervals between sounds.

   To hear the sounds at equal intervals, the bolts would have to be tied at distances corresponding to equal time intervals. The first bolt (call it bolt #0) is touching the plate. Since each bolt has an initial speed of 0, the distance of fall and time of fall for each bolt are related to each other by $d_i = \frac{1}{2}gt_i^2$.

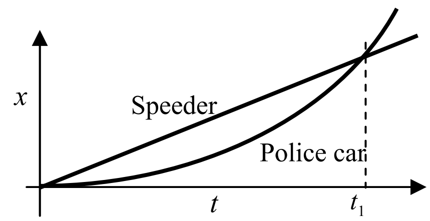
   Thus for bolt #1, $d_1 = \frac{1}{2}gt_1^2$. For bolt #2, we want $t_2 = 2t_1$, so $d_2 = \frac{1}{2}gt_2^2 = \frac{1}{2}g(2t_1)^2 = 4(\frac{1}{2}gt_1^2) = 4d_1$. Likewise, $t_3 = 3t_1$, which leads to $d_3 = 9d_1$; $t_4 = 4t_1$, which leads to $d_4 = 16d_1$, and so on. If the distance from the bolt initially on the pan to the next bolt is d_1, then the distance from that bolt to the next one is $3d_1$, the distance to the next bolt is $5d_1$, and so on. The accompanying table shows these relationships in a simpler format.

Bolt #	Height above floor	Time to fall	Distance between bolts
0	0	0	
1	d_1	t_1	d_1
2	$d_2 = 4\,d_1$	$t_2 = 2\,t_1$	$3\,d_1$
3	$d_3 = 9\,d_1$	$t_3 = 3\,t_1$	$5\,d_1$
4	$d_4 = 16\,d_1$	$t_4 = 4\,t_1$	$7\,d_1$
5	$d_5 = 25\,d_1$	$t_5 = 5\,t_1$	$9\,d_1$

3. Take the origin to be the location where the speeder passes the police car. The speeder's constant speed is $\upsilon_{\text{speeder}} = (140\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 38.89\ \text{m/s}$, and the location of the speeder as a function of time is given by $x_{\text{speeder}} = \upsilon_{\text{speeder}} t_{\text{speeder}} = (38.89\ \text{m/s})t_{\text{speeder}}$. The police car has an initial velocity of $\upsilon_0 = 0\ \text{m/s}$ and a constant acceleration of a_{police}. The location of the police car as a function of time is given by Eq. 2–11b.

$$x_{\text{police}} = \upsilon_0 t + \tfrac{1}{2}at^2 = \tfrac{1}{2}a_{\text{police}}t_{\text{police}}^2$$

   (*a*) The position vs. time graphs would qualitatively look like the graph shown here.



   (*b*) The time to overtake the speeder occurs when the speeder has gone a distance of 850 m. The time is found using the speeder's equation from above.

$$850\ \text{m} = (38.89\ \text{m/s})t_{\text{speeder}} \quad \rightarrow \quad t_{\text{speeder}} = \frac{850\ \text{m}}{38.89\ \text{m/s}} = 21.86\ \text{s} \approx \boxed{22\ \text{s}}$$

(*c*) The police car's acceleration can be calculated knowing that the police car also had gone a distance of 850 m in a time of 21.86 s.

$$850\text{ m} = \tfrac{1}{2}a_{\text{police}}(21.86\text{ s})^2 \quad \rightarrow \quad a_{\text{police}} = \frac{2(850\text{ m})}{(21.86\text{ s})^2} = 3.558\text{ m/s}^2 \approx \boxed{3.6\text{ m/s}^2}$$

(*d*) The speed of the police car at the overtaking point can be found from Eq. 2–11a.

$$v = v_0 + at = 0 + (3.558\text{ m/s}^2)(21.86\text{ s}) = 77.78\text{ m/s} \approx \boxed{78\text{ m/s}}$$

Note that this is exactly twice the speed of the speeder, so it is 280 km/h.

4. (*a*) During the interval from A to B, it is $\boxed{\text{moving in the negative direction}}$, because its displacement is negative.

(*b*) During the interval from A to B, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*c*) During the interval from A to B, $\boxed{\text{the acceleration is negative}}$, because the graph is concave downward, indicating that the slope is getting more negative, and thus the acceleration is negative.

(*d*) During the interval from D to E, it is $\boxed{\text{moving in the positive direction}}$, because the displacement is positive.

(*e*) During the interval from D to E, it is $\boxed{\text{speeding up}}$, because the magnitude of its slope is increasing (changing from less steep to more steep).

(*f*) During the interval from D to E, $\boxed{\text{the acceleration is positive}}$, because the graph is concave upward, indicating the slope is getting more positive, and thus the acceleration is positive.

(*g*) During the interval from C to D, $\boxed{\text{the object is not moving in either direction}}$.

$\boxed{\text{The velocity and acceleration are both 0.}}$

5. We are given that $x(t) = 2.0\text{ m} - (3.6\text{ m/s})t + (1.7\text{ m/s}^2)t^2$, where t is in seconds.

(*a*) The value of 2.0 m is the initial position of the ball. The value of 3.6 m/s is the initial speed of the ball—the speed at $t = 0$. Note that the ball is initially moving in the negative direction, since –3.6 m/s is used. The value of 1.7 m/s^2 is the acceleration of the ball.

(*b*) The units of 2.0 are meters. The units of 3.6 are m/s. The units of 1.7 are m/s^2.

(*c*) $x(1.0\text{ s}) = 2.0\text{ m} - (3.6\text{ m/s})(1.0\text{ s}) + (1.7\text{ m/s}^2)(1.0\text{ s})^2 = \boxed{0.1\text{ m}}$

$x(2.0\text{ s}) = 2.0\text{ m} - (3.6\text{ m/s})(2.0\text{ s}) + (1.7\text{ m/s}^2)(2.0\text{ s})^2 = \boxed{1.6\text{ m}}$

$x(3.0\text{ s}) = 2.0\text{ m} - (3.6\text{ m/s})(3.0\text{ s}) + (1.7\text{ m/s}^2)(3.0\text{ s})^2 = \boxed{6.5\text{ m}}$

(*d*) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{6.5\text{ m} - 0.1\text{ m}}{2.0\text{ s}} = \boxed{3.2\text{ m/s}}$

3

KINEMATICS IN TWO DIMENSIONS; VECTORS

Responses to Questions

1. No, the two velocities are not equal. Velocity is a vector quantity, with a magnitude and direction. If two vectors have different directions, they cannot be equal.

2. No. The car may be traveling at a constant *speed* of 60 km/h and going around a curve, in which case it would be accelerating.

3. (i) During one year, the Earth travels a distance equal to the circumference of its orbit but has a displacement of 0 relative to the Sun.
   (ii) Any kind of cross-country "round trip" air travel would result in a large distance traveled but a displacement of 0.
   (iii) The displacement for a race car from the start to the finish of the Indianapolis 500 auto race is 0.

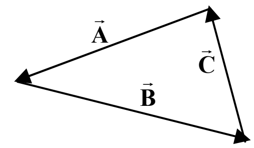
4. The length of the displacement vector is the straight-line distance between the beginning point and the ending point of the trip and therefore the shortest distance between the two points. If the path is a straight line, then the length of the displacement vector is the same as the length of the path. If the path is curved or consists of different straight-line segments, then the distance from beginning to end will be less than the path length. Therefore, the displacement vector can never be longer than the length of the path traveled, but it can be shorter.

5. Since both the batter and the ball started their motion at the same location (where the ball was hit) and ended their motion at the same location (where the ball was caught), the displacement of both was the same.

6. V is the magnitude of the vector $\vec{\mathbf{V}}$; it is not necessarily larger than the magnitudes V_1 and/or V_2. For instance, if $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ have the same magnitude and are in opposite directions, then V is zero. The magnitude of the sum is determined by the angle between the two contributing vectors.

7. If the two vectors are in the same direction, the magnitude of their sum will be a maximum and will be 7.5 km. If the two vectors are in the opposite direction, the magnitude of their sum will be a minimum and will be 0.5 km. If the two vectors are oriented in any other configuration, the magnitude of their sum will be between 0.5 km and 7.5 km.

8. Two vectors of unequal magnitude can never add to give the zero vector. The only way that two vectors can add up to give the zero vector is if they have the same magnitude and point in exactly opposite directions. However, three vectors of unequal magnitude can add to give the zero vector. As a one-dimensional example, a vector 10 units long in the positive *x* direction added to two vectors of 4 and 6 units each in the negative *x* direction will result in the zero vector. In two dimensions, if their sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=0$.



9. (*a*) The magnitude of a vector can equal the length of one of its components if the other components of the vector are all 0; that is, if the vector lies along one of the coordinate axes.
   (*b*) The magnitude of a vector can never be less than one of its components, because each component contributes a positive amount to the overall magnitude, through the Pythagorean relationship. The square root of a sum of squares is never less than the absolute value of any individual term.

10. The odometer and the speedometer of the car both measure scalar quantities (distance and speed, respectively).

11. To find the initial speed, use the slingshot to shoot the rock directly horizontally (no initial vertical speed) from a height of 1 meter (measured with the meter stick). The vertical displacement of the rock can be related to the time of flight by Eq. 2–11b. Take downward to be positive.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}at^2 \quad \to \quad 1\text{ m} = \tfrac{1}{2}gt^2 \quad \to \quad t = \sqrt{2(1\text{ m})/(9.8\text{ m/s}^2)} = 0.45\text{ s}$$

Measure the horizontal range *R* of the rock with the meter stick. Then, if we measure the horizontal range *R*, we know that $R = \upsilon_x t = \upsilon_x(0.45\text{ s})$, so $\upsilon_x = R/0.45\text{ s}$, which is the speed the slingshot imparts to the rock. The only measurements are the height of fall and the range, both of which can be measured with a meter stick.

$\boxed{12.}$ The arrow should be aimed above the target, because gravity will deflect the arrow downward from a horizontal flight path. The angle of aim (above the horizontal) should increase as the distance from the target increases, because gravity will have more time to act in deflecting the arrow from a straight-line path. If we assume that the arrow was shot from the same height as the target, then the "level horizontal range" formula is applicable: $R = \upsilon_0^2 \sin 2\theta_0/g \quad \to \quad \theta = \tfrac{1}{2}\sin^{-1}(Rg/\upsilon_0^2)$. As the range and hence the argument of the inverse sine function increases, the angle increases.

13. If the bullet was fired from the ground, then the *y* component of its velocity slowed considerably by the time it reached an altitude of 2.0 km, because of both the downward acceleration due to gravity and air resistance. The *x* component of its velocity would have slowed due to air resistance as well. Therefore, the bullet could have been traveling slowly enough to be caught.

14. The balloons will hit each other, although not along the line of sight from you to your friend. If there were no acceleration due to gravity, the balloons would hit each other along the line of sight. Gravity causes each balloon to accelerate downward from that "line of sight" path, each with the same acceleration. Thus each balloon falls below the line of sight by the same amount at every instant along their flight, so they collide. This situation is similar to Conceptual Example 3–7 and to Problem 36.

15. The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the

same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $\upsilon_x = \upsilon_0 \cos\theta = (30 \text{ m/s})(\cos 30°) = 26 \text{ m/s}$.

16. A projectile has the least speed at the top of its path. At that point the vertical speed is zero. The horizontal speed remains constant throughout the flight, if we neglect the effects of air resistance.

$\boxed{17.}$ (*a*) Cannonball A, with the larger angle, will reach a higher elevation. It has a larger initial vertical velocity, so by Eq. 2–11c it will rise higher before the vertical component of velocity is 0.

(*b*) Cannonball A, with the larger angle, will stay in the air longer. It has a larger initial vertical velocity, so it takes more time to decelerate to 0 and start to fall.

(*c*) The cannonball with a launch angle closest to 45° will travel the farthest. The range is a maximum for a launch angle of 45° and decreases for angles either larger or smaller than 45°.

18. (*a*) The ball lands at the same point from which it was thrown inside the train car—back in the thrower's hand.

(*b*) If the car accelerates, the ball will land behind the thrower's hand.

(*c*) If the car decelerates, the ball will land in front of the thrower's hand.

(*d*) If the car rounds a curve (assume it curves to the right), the ball will land to the left of the thrower's hand.

(*e*) The ball will be slowed by air resistance and will land behind the thrower's hand.

$\boxed{19.}$ Your reference frame is that of the train you are riding. If you are traveling with a relatively constant velocity (not over a hill or around a curve or drastically changing speed), then you will interpret your reference frame as being at rest. Since you are moving forward faster than the other train, the other train is moving backward relative to you. Seeing the other train go past your window from front to rear makes it look like the other train is going backward.

20. Both rowers need to cover the same "cross-river" distance. The rower with the greatest speed in the "cross-river" direction will be the one that reaches the other side first. The current has no bearing on the time to cross the river because the current doesn't help either of the boats move across the river. Thus the rower heading straight across will reach the other side first. All of his "rowing effort" has gone into crossing the river. For the upstream rower, some of his "rowing effort" goes into battling the current, so his "cross-river" speed will be only a fraction of his rowing speed.

21. When you stand still under an umbrella in a vertical rain, you are in a cylinder-shaped volume in which there is no rain. The rain has no horizontal component of velocity, so the rain cannot move from outside that cylinder into it. You stay dry. But as you run, you have a forward horizontal velocity relative to the rain, so the rain has a backward horizontal velocity relative to you. It is the same as if you were standing still under the umbrella but the rain had some horizontal component of velocity toward you. The perfectly vertical umbrella would not completely shield you.

Responses to MisConceptual Questions

1. (*c*) The shortest possible resultant will be 20 units, which occurs when the vectors point in opposite directions. Since 0 units and 18 units are less than 20 units, (*a*) and (*b*) cannot be correct answers. The largest possible result will be 60 units, which occurs when the vectors point in the same direction. Since 64 units and 100 units are greater than 60 units, (*d*) and (*e*) cannot be correct answers. Answer (*c*) is the only choice that falls between the minimum and maximum vector lengths.

2. (*a*) The components of a vector make up the two legs of a right triangle when the vector is the hypotenuse. The legs of a right triangle cannot be longer than the hypotenuse, therefore (*c*) and (*d*) cannot be correct answers. Only when the vector is parallel to the component is the magnitude of the vector equal to the magnitude of the component, as in (*b*). For all other vectors, the magnitude of the component is less than the magnitude of the vector.

3. (*b*) If you turned 90°, as in (*a*), your path would be that of a right triangle. The distance back would be the hypotenuse of that triangle, which would be longer than 100 m. If you turned by only 30°, as in (*c*), your path would form an obtuse triangle; the distance back would have to be greater than if you had turned 90°, and therefore it too would be greater than 100 m. If you turned 180°, as in (*d*), you would end up back at your starting point, not 100 m away. Three equal distances of 100 m would form an equilateral triangle, so (*b*) is the correct answer.

4. (*a*) The bullet falls due to the influence of gravity, not due to air resistance. Therefore, (*b*) and (*c*) are incorrect. Inside the rifle the barrel prevents the bullet from falling, so the bullet does not begin to fall until it leaves the barrel.

5. (*b*) Assuming that we ignore air resistance, the ball is in free fall after it leaves the bat. If the answer were (*a*), the ball would continue to accelerate forward and would not return to the ground. If the answer were (*c*), the ball would slow to a stop and return backward toward the bat.

6. (*b*) If we ignore air friction, the horizontal and vertical components of the velocity are independent of each other. The vertical components of the two balls will be equal when the balls reach ground level. The ball thrown horizontally will have a horizontal component of velocity in addition to the vertical component. Therefore, it will have the greater speed.

7. (*c*) Both you and the ball have the same constant horizontal velocity. Therefore, in the time it takes the ball to travel up to its highest point and return to ground level, your hand and the ball have traveled the same horizontal distance, and the ball will land back in your hand.

8. (*d*) Both the time of flight and the maximum height are determined by the vertical component of the initial velocity. Since all three kicks reach the same maximum height, they must also have the same time of flight. The horizontal components of the initial velocity are different, which accounts for them traveling different distances.

9. (*c*) The baseball is in projectile motion during the entire flight. In projectile motion the acceleration is downward and constant; it is never zero. Therefore, (*a*) is incorrect. Since the ball was hit high and far, it must have had an initial horizontal component of velocity. For projectile motion the horizontal component of velocity is constant, so at the highest point the magnitude of the velocity cannot be zero, and thus (*b*) is incorrect. However, at the highest point, the vertical component of velocity is zero, so the magnitude of the velocity has a minimum at the highest point. So (*c*) is the correct answer.

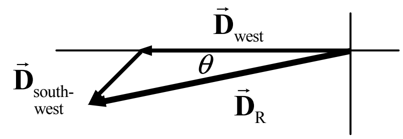
10. (*b*) Both the monkey and bullet fall at the same rate due to gravity. If the gun was pointed directly at the monkey and gravity did not act upon either the monkey or bullet, the bullet would hit the monkey. Since both start falling at the same time and fall at the same rate, the bullet will still hit the monkey.

11. (*b*, *e*) In projectile motion the acceleration is vertical, so the x velocity is constant throughout the motion, so (*a*) is valid. The acceleration is that of gravity, which, when up is positive, is a constant negative value, so (*b*) is not valid and (*c*) is valid. At the highest point in the trajectory the vertical velocity is changing from a positive to a negative value. At this point the y component of velocity is zero, so (*d*) is valid. However, the x component of the velocity is constant, but not necessarily zero, so (*e*) is not valid.

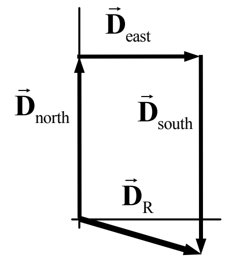
12. (*a*) The maximum relative speed between the two cars occurs when the cars travel in opposite directions. This maximum speed would be the sum of their speeds relative to the ground or 20 m/s. Since the two cars are traveling perpendicular to each other (not in opposite directions), their relative speed must be less than the maximum relative speed.

Solutions to Problems

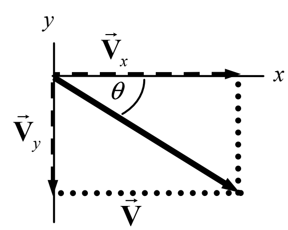
1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_{\mathrm{R}} = \vec{\mathbf{D}}_{\text{west}} + \vec{\mathbf{D}}_{\text{south-west}}$. The westward displacement is $225\text{ km} + (98\text{ km})\cos 45° = 294.3\text{ km}$ and the southward displacement is $(98\text{ km})\sin 45° = 69.3\text{ km}$. The resultant displacement has a magnitude of $\sqrt{294.3^2 + 69.3^2} = \boxed{302\text{ km}}$. The direction is $\theta = \tan^{-1} 69.3/294.3 = \boxed{13° \text{ south of west}}$.



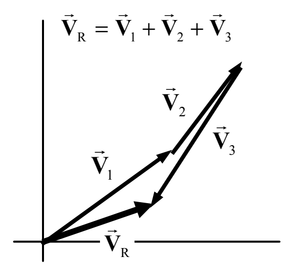
2. The truck has a displacement of $21 + (-26) = -5$ blocks north and 16 blocks east. The resultant has a magnitude of $\sqrt{(-5)^2 + 16^2} = 16.76\text{ blocks} \approx \boxed{17\text{ blocks}}$ and a direction of $\tan^{-1} 5/16 = \boxed{17° \text{ south of east}}$.



3. Given that $V_x = 9.80$ units and $V_y = -6.40$ units, the magnitude of $\vec{\mathbf{V}}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{9.80^2 + (-6.40)^2} = \boxed{11.70\text{ units}}$. The direction is given by $\theta = \tan^{-1}\dfrac{-6.40}{9.80} = \boxed{-33.1°}$, or $33.1°$ below the positive x axis.



4. The vectors for the problem are drawn approximately to scale. The resultant has a length of $\boxed{17.5\text{ m}}$ and a direction $\boxed{19°}$ north of east. If calculations are done, the actual resultant should be 17 m at 23° north of east. Keeping one more significant figure would give 17.4 m at 22.5°.

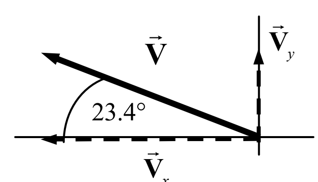


5. (*a*) See the accompanying diagram.

(*b*) $V_x = -24.8 \cos 23.4° = \boxed{-22.8\text{ units}}$ $\quad V_y = 24.8 \sin 23.4° = \boxed{9.85\text{ units}}$

(*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-22.8)^2 + (9.85)^2} = \boxed{24.8\text{ units}}$

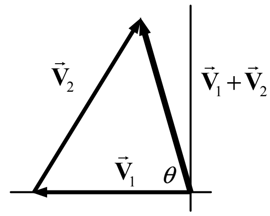
$\theta = \tan^{-1}\dfrac{9.85}{22.8} = \boxed{23.4° \text{ above the } -x \text{ axis}}$



6. (*a*) $V_{1x} = \boxed{-6.6 \text{ units}}$ $\quad V_{1y} = \boxed{0 \text{ units}}$

$V_{2x} = 8.5 \cos 55° = 4.875 \text{ units} \approx \boxed{4.9 \text{ units}}$

$V_{2y} = 8.5 \sin 55° = 6.963 \text{ units} \approx \boxed{7.0 \text{ units}}$



(*b*) To find the components of the sum, add the components of the individual vectors.

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (V_{1x} + V_{2x}, V_{1y} + V_{2y}) = (-1.725, 6.963)$$

$$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{(-1.725)^2 + (6.963)^2} = 7.173 \text{ units} \approx 7.2 \text{ units} \quad \theta = \tan^{-1}\frac{6.963}{1.725} = 76°$$

The sum has a magnitude of $\boxed{7.2 \text{ units}}$ and is $\boxed{76° \text{ clockwise from the negative } x \text{ axis}}$, or 104° counterclockwise from the positive x axis.

7. We see from the diagram that $\vec{\mathbf{A}} = (6.8, 0)$ and $\vec{\mathbf{B}} = (-5.5, 0)$.

(*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (6.8, 0) + (-5.5, 0) = (1.3, 0)$. The magnitude is $\boxed{1.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (6.8, 0) - (-5.5, 0) = (12.3, 0)$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = (-5.5, 0) - (6.8, 0) = (-12.3, 0)$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{-x}$.

8. (*a*) $v_{\text{north}} = (835 \text{ km/h})(\cos 41.5°) = \boxed{625 \text{ km/h}}$ $\quad v_{\text{west}} = (835 \text{ km/h})(\sin 41.5°) = \boxed{553 \text{ km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (625 \text{ km/h})(1.75 \text{ h}) = \boxed{1090 \text{ km}}$

$\Delta d_{\text{west}} = v_{\text{west}} t = (553 \text{ km/h})(1.75 \text{ h}) = \boxed{968 \text{ km}}$

$\boxed{9.}$ $A_x = 44.0 \cos 28.0° = 38.85$ $\quad A_y = 44.0 \sin 28.0° = 20.66$

$B_x = -26.5 \cos 56.0° = -14.82$ $\quad B_y = 26.5 \sin 56.0° = 21.97$

$C_x = 31.0 \cos 270° = 0.0$ $\quad C_y = 31.0 \sin 270° = -31.0$

(*a*) $(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}})_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$

$(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}})_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$

(*b*) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}\right| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7}$ $\quad \theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8°}$

10. $A_x = 44.0 \cos 28.0° = 38.85$ $\quad A_y = 44.0 \sin 28.0° = 20.66$

$B_x = -26.5 \cos 56.0° = -14.82$ $\quad B_y = 26.5 \sin 56.0° = 21.97$

(*a*) $(\vec{\mathbf{B}} - \vec{\mathbf{A}})_x = (-14.82) - 38.85 = -53.67$ $\quad (\vec{\mathbf{B}} - \vec{\mathbf{A}})_y = 21.97 - 20.66 = 1.31$

Note that since the x component is negative and the y component is positive, the vector is in the 2nd quadrant.

$$\left|\vec{\mathbf{B}} - \vec{\mathbf{A}}\right| = \sqrt{(-53.67)^2 + (1.31)^2} = \boxed{53.7} \quad \theta_{B-A} = \tan^{-1}\frac{1.31}{-53.67} = \boxed{1.4° \text{ above} -x \text{ axis}}$$

(*b*) $(\vec{\mathbf{A}}-\vec{\mathbf{B}})_x = 38.85-(-14.82) = 53.67 \quad (\vec{\mathbf{A}}-\vec{\mathbf{B}})_y = 20.66-21.97 = -1.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(53.67)^2+(-1.31)^2} = \boxed{53.7} \quad \theta = \tan^{-1}\frac{-1.31}{53.7} = \boxed{1.4^\circ \text{ below } +x \text{ axis}}$$

Comparing the results shows that $\vec{\mathbf{B}}-\vec{\mathbf{A}}$ is the opposite of $\vec{\mathbf{A}}-\vec{\mathbf{B}}$.

11. $A_x = 44.0 \cos 28.0^\circ = 38.85 \qquad A_y = 44.0 \sin 28.0^\circ = 20.66$

$C_x = 31.0 \cos 270^\circ = 0.0 \qquad C_y = 31.0 \sin 270^\circ = -31.0$

$(\vec{\mathbf{A}}-\vec{\mathbf{C}})_x = 38.85-0.0 = 38.85 \qquad (\vec{\mathbf{A}}-\vec{\mathbf{C}})_y = 20.66-(-31.0) = 51.66$

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{C}}\right| = \sqrt{(38.85)^2+(51.66)^2} = \boxed{64.6} \qquad \theta = \tan^{-1}\frac{51.66}{38.85} = \boxed{53.1^\circ}$$

12. $A_x = 44.0 \cos 28.0^\circ = 38.85 \qquad A_y = 44.0 \sin 28.0^\circ = 20.66$

$B_x = -26.5 \cos 56.0^\circ = -14.82 \qquad B_y = 26.5 \sin 56.0^\circ = 21.97$

$C_x = 31.0 \cos 270^\mathrm{o} = 0.0 \qquad C_y = 31.0 \sin 270^\circ = -31.0$

(*a*) $(\vec{\mathbf{B}}-3\vec{\mathbf{A}})_x = -14.82-3(38.85) = -131.37 \quad (\vec{\mathbf{B}}-3\vec{\mathbf{A}})_y = 21.97-3(20.66) = -40.01$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\left|\vec{\mathbf{B}}-3\vec{\mathbf{A}}\right| = \sqrt{(-131.37)^2+(-40.01)^2} = 137.33 \approx \boxed{137}$$

$$\theta = \tan^{-1}\frac{-40.01}{-131.37} = \boxed{16.9^\circ \text{ below } -x \text{ axis}}$$

(*b*) $(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}})_x = 2(38.85)-3(-14.82)+2(0.0) = 122.16$

$(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}})_y = 2(20.66)-3(21.97)+2(-31.0) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2+(-86.59)^2} = \boxed{149.7} \quad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3^\circ \text{ below } +x \text{ axis}}$$

13. $A_x = 44.0 \cos 28.0^\circ = 38.85 \qquad A_y = 44.0 \sin 28.0^\circ = 20.66$

$B_x = -26.5 \cos 56.0^\circ = -14.82 \qquad B_y = 26.5 \sin 56.0^\circ = 21.97$

$C_x = 31.0 \cos 270^\circ = 0.0 \qquad C_y = 31.0 \sin 270^\circ = -31.0$

(*a*) $(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}})_x = 38.85-(-14.82)+0.0 = 53.67$

$(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}})_y = 20.66-21.97+(-31.0) = -32.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{(53.67)^2+(-32.31)^2} = \boxed{62.6} \quad \theta = \tan^{-1}\frac{-32.31}{53.67} = \boxed{31.0^\circ \text{ below } +x \text{ axis}}$$

(*b*) $(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}})_x = 38.85+(-14.82)-0.0 = 24.03$

$(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}})_y = 20.66+21.97-(-31.0) = 73.63$

$$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right| = \sqrt{(24.03)^2+(73.63)^2} = \boxed{77.5} \quad \theta = \tan^{-1}\frac{73.63}{24.03} = \boxed{71.9^\circ}$$

(*c*) $(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}})_x = 0.0-38.85-(-14.82) = -24.03$

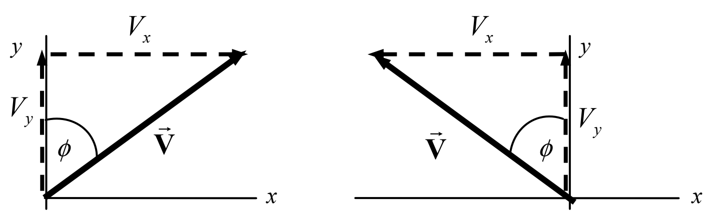
$(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}})_y = -31.0-20.66-21.97 = -73.63$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\left|\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(-24.03)^2+(-73.63)^2} = \boxed{77.5} \quad \theta = \tan^{-1}\frac{-73.63}{-24.03} = \boxed{71.9^\circ \text{ below } -x \text{ axis}}$$

Note that the answer to (*c*) is the exact opposite of the answer to (*b*).

14. If the angle is in the first quadrant, then $V_x = V\sin\phi$ and $V_y = V\cos\phi$. See the first diagram. If the angle is in the second quadrant, then $V_x = V\sin\phi$ and $V_y = -V\cos\phi$. See the second diagram.



15. The x component is negative and the y component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive x axis would be 122.4°. Thus the components are found to be the following:

$x = -4580\sin 38.4^\circ = -2845\text{ m} \qquad y = 4580\cos 38.4^\circ = 3589\text{ m} \qquad z = 2450\text{ m}$

$$\boxed{\vec{\mathbf{r}} = (-2845\text{ m}, 3589\text{ m}, 2450\text{ m})} \qquad |\vec{\mathbf{r}}| = \sqrt{(-2845)^2+(3589)^2+(2450)^2} = \boxed{5190\text{ m}}$$

16. (*a*) Use the Pythagorean theorem to find the possible x components.

$$90.0^2 = x^2+(-65.0)^2 \;\rightarrow\; x^2 = 3875 \;\rightarrow\; x = \boxed{\pm 62.2\text{ units}}$$

(*b*) Express each vector in component form, with $\vec{\mathbf{V}}$ the vector to be determined. The answer is given both as components and in magnitude/direction format.

$(62.2,\ -65.0)+(V_x, V_y) = (-80.0,\ 0) \;\rightarrow$

$V_x = (-80.0-62.2) = -142.2 \qquad V_y = 65.0 \qquad \vec{\mathbf{V}} = \boxed{(-142.2, 65.0)}$

$$\left|\vec{\mathbf{V}}\right| = \sqrt{(-142.2)^2+65.0^2} = \boxed{156\text{ units}} \quad \theta = \tan^{-1}\frac{65.0}{-142.2} = \boxed{24.6^\circ \text{ above } -x \text{ axis}}$$

17. Choose downward to be the positive y direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $v_{x0} = 3.0\text{ m/s}$ and $a_x = 0$. In the vertical direction,

$v_{y0} = 0,\ a_y = 9.80\ \text{m/s}^2,\ y_0 = 0,$ and the final location is $y = 7.5$ m. The time for the tiger to reach the ground is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\ \text{m} = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(7.5\ \text{m})}{9.80\ \text{m/s}^2}} = 1.237\ \text{s}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = v_x t = (3.0\ \text{m/s})(1.237\ \text{s}) = \boxed{3.7\ \text{m}}$$

18. Choose downward to be the positive y direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $v_{x0} = 2.5$ m/s and $a_x = 0$. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\ \text{m/s}^2$, $y_0 = 0$, and the time of flight is $t = 3.0$ s. The height of the cliff is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)(3.0\ \text{s})^2 = \boxed{44\ \text{m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = v_x t = (2.5\ \text{m/s})(3.0\ \text{s}) = \boxed{7.5\ \text{m}}$$

19. Apply the level horizontal range formula derived in the text. If the launching speed and angle are held constant, the range is inversely proportional to the value of g. The acceleration due to gravity on the Moon is one-sixth that on Earth.

$$R_{\text{Earth}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Earth}}} \quad R_{\text{Moon}} = \frac{v_0^2 \sin 2\theta_0}{g_{\text{Moon}}} \quad \rightarrow \quad R_{\text{Earth}}\, g_{\text{Earth}} = R_{\text{Moon}}\, g_{\text{Moon}}$$

$$R_{\text{Moon}} = R_{\text{Earth}} \frac{g_{\text{Earth}}}{g_{\text{Moon}}} = 6R_{\text{Earth}}$$

Thus, on the Moon, the person can jump $\boxed{\text{6 times farther}}$.

20. Choose downward to be the positive y direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $v_{y0} = 0$, $a_y = 9.80\ \text{m/s}^2$, $y_0 = 0$, and the displacement is 7.5 m. The time of flight is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\ \text{m} = \tfrac{1}{2}(9.80\ \text{m/s}^2)\,t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(9.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.237\ \text{s}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 9.5\ \text{m}/1.237\ \text{s} = \boxed{7.7\ \text{m/s}}$$

21. Choose downward to be the positive y direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction $v_{y0} = 0,\ y_0 = 0,$ and $a_y = 9.80\ \text{m/s}^2$. The initial horizontal velocity is 12.2 m/s and the horizontal range is 21.0 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = (21.0\ \text{m})/(12.2\ \text{m/s}) = 1.721\ \text{s}$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2–11b to the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)(1.721\ \text{s})^2 = \boxed{14.5\ \text{m}}$$

22. Choose the point at which the football is kicked as the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $v_{y0} = (18.0 \sin 31.0°)$ m/s, $a_y = -9.80\ \text{m/s}^2$, and the final y velocity will be the opposite of the starting y velocity. Use Eq. 2–11a to find the time of flight.

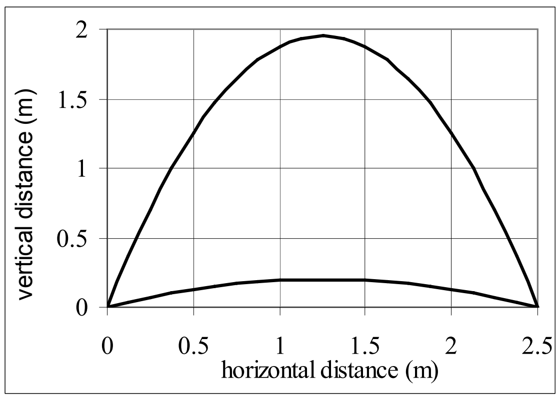
$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{(-18.0 \sin 31.0°)\ \text{m/s} - (18.0 \sin 31.0°)\ \text{m/s}}{-9.80\ \text{m/s}^2} = \boxed{1.89\ \text{s}}$$

23. Apply the level horizontal range formula derived in the text.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow$$

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(2.5\ \text{m})(9.80\ \text{m/s}^2)}{(6.5\ \text{m/s})^2} = 0.5799$$

$$2\theta_0 = \sin^{-1} 0.5799 \quad \rightarrow \quad \theta_0 = \boxed{18°,\ 72°}$$



There are two angles because each angle gives the same range. If one angle is $\theta = 45° + \delta$, then $\theta = 45° - \delta$ is also a solution. The two paths are shown in the graph.

24. When shooting the gun vertically, half the time of flight is spent moving upward. Thus the upward flight takes two seconds. Choose upward as the positive y direction. Since at the top of the flight the vertical velocity is zero, find the launching velocity from Eq. 2–11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad v_{y0} = v_y - at = 0 = (9.80\ \text{m/s}^2)(2.0\ \text{s}) = 19.6\ \text{m/s}$$

Using this initial velocity and an angle of 45° in the level horizontal range formula derived in the text will give the maximum range for the gun.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(19.6\ \text{m/s})^2 \sin (2\times 45°)}{9.80\ \text{m/s}^2} = \boxed{39\ \text{m}}$$

25. The level horizontal range formula derived in the text can be used to find the launching velocity of the grasshopper.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(0.80\ \text{m})(9.80\ \text{m/s}^2)}{\sin 90°}} = 2.8\ \text{m/s}$$

Since there is no time between jumps, the horizontal velocity of the grasshopper is the horizontal component of the launching velocity.

$$v_x = v_0 \cos\theta_0 = (2.8\text{ m/s})\cos 45° = \boxed{2.0\text{ m/s}}$$

26. (*a*) Take the ground to be the $y = 0$ level, with upward as the positive direction. Use Eq. 2–11b to solve for the time, with an initial vertical velocity of 0.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \quad\to\quad 150\text{ m} = 910\text{ m} + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad\to$$

$$t = \sqrt{\frac{2(150-910)}{(-9.80\text{ m/s}^2)}} = 12.45\text{ s} \approx \boxed{12\text{ s}}$$

(*b*) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$\Delta x = v_x t = (4.0\text{ m/s})(12.45\text{ s}) = 49.8\text{ m} \approx \boxed{5.0\times10^1\text{ m}}$$

$\boxed{27.}$ Choose the origin to be where the projectile is launched and upward to be the positive *y* direction. The initial velocity of the projectile is v_0, the launching angle is θ_0, $a_y = -g$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The maximum height is found from Eq. 2–11c with $v_y = 0$ at the maximum height.

$$y_{\text{max}} = 0 + \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{-v_0^2 \sin^2\theta_0}{-2g} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{(36.6\text{ m/s})^2 \sin^2 42.2°}{2(9.80\text{ m/s}^2)} = \boxed{30.8\text{ m}}$$

(*b*) The total time in the air is found from Eq. 2–11b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\to\quad 0 = v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\to$$

$$t = \frac{2v_0 \sin\theta_0}{g} = \frac{2(36.6\text{ m/s})\sin 42.2°}{9.80\text{ m/s}^2} = 5.0173\text{ s} \approx \boxed{5.02\text{ s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos\theta_0)t = (36.6\text{ m/s})(\cos 42.2°)(5.0173\text{ s}) = \boxed{136\text{ m}}$$

(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $v_0 \cos\theta_0 = (36.6\text{ m/s})(\cos 42.2°) = 27.11\text{ m/s}$. The vertical velocity is found from Eq. 2–11a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = (36.6\text{ m/s})\sin 42.2° - (9.80\text{ m/s}^2)(1.50\text{ s}) = 9.885\text{ m/s}$$

Thus the speed of the projectile is as follows:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(27.11\text{ m/s})^2 + (9.885\text{ m/s})^2} = \boxed{28.9\text{ m/s}}$$

28. (*a*) Use the level horizontal range formula derived in the text.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad\to\quad v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(7.80\text{ m})(9.80\text{ m/s}^2)}{\sin 54.0°}} = \boxed{9.72\text{ m/s}}$$

(*b*) Now increase the speed by 5.0% and calculate the new range. The new speed would be $9.72\text{ m/s}\,(1.05) = 10.21\text{ m/s}$ and the new range would be as follows:

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.21\text{ m/s})^2 \sin 54°}{9.80\text{ m/s}^2} = 8.606\text{ m}$$

This is an increase of $\boxed{0.81\text{ m (10\% increase)}}$.

29. Choose the origin to be the point of release of the shot put. Choose upward to be the positive *y* direction. Then $y_0 = 0,\ v_{y0} = (14.4 \sin 34.0°)\text{ m/s} = 8.05\text{ m/s},\ a_y = -9.80\text{ m/s}^2,$ and $y = -2.10\text{ m}$ at the end of the motion. Use Eq. 2–11b to find the time of flight.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}a_y t^2 + v_{y0}t - y = 0 \quad \rightarrow$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(\tfrac{1}{2}a_y\right)(-y)}}{2\tfrac{1}{2}a_y} = \frac{-8.05 \pm \sqrt{(8.05)^2 - 2(-9.80)(2.10)}}{-9.80} = 1.872\text{ s}, -0.2290\text{ s}$$

Choose the positive result since the time must be greater than 0. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = v_x t = [(14.4 \cos 34.0°)](1.872\text{ s}) = \boxed{22.3\text{ m}}$$

30. Choose the origin to be the point on the ground directly below the point where the baseball was hit. Choose upward to be the positive *y* direction. Then $y_0 = 1.0\text{ m},\ y = 13.0\text{ m}$ at the end of the motion, $v_{y0} = (27.0 \sin 45.0°)\text{ m/s} = 19.09\text{ m/s},$ and $a_y = -9.80\text{ m/s}^2.$ Use Eq. 2–11b to find the time of flight.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}a_y t^2 + v_{y0}t + (y_0 - y) = 0 \quad \rightarrow$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(\tfrac{1}{2}a_y\right)(y_0 - y)}}{2\tfrac{1}{2}a_y} = \frac{-19.09 \pm \sqrt{(19.09)^2 - 2(-9.80)(-12.0)}}{-9.80}$$

$$= 0.788\text{ s},\ 3.108\text{ s}$$

The smaller time is the time when the baseball reached the building's height on the way up, and the larger time is the time at which the baseball reached the building's height on the way down. We must choose the larger result, because the baseball cannot land on the roof on the way up. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = v_x t = [(27.0 \cos 45.0°)\text{ m/s}](3.108\text{ s}) = \boxed{59.3\text{ m}}$$

31. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive *y* direction. For the supplies, $y_0 = 235\text{ m},\ v_{y0} = 0,$ $a_y = -g,$ and the final *y* location is $y = 0.$ The initial (and constant) *x* velocity of the supplies is $v_x = 69.4\text{ m/s}.$ First the time for the supplies to reach the ground is found from Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + 0 + \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{-2y_0}{a}} = \sqrt{\frac{-2(235\text{ m})}{(-9.80\text{ m/s}^2)}} = 6.925\text{ s}$$

Then the horizontal distance of travel for the package is found from the horizontal constant velocity.

$$\Delta x = v_x t = (69.4\text{ m/s})(6.925\text{ s}) = \boxed{481\text{ m}}$$

32. We have the same set-up as in Problem 31. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive y direction. For the supplies, $y_0 = 235\text{ m}$, $v_{y0} = 0$, $a_y = -g$, and the final y location is $y = 0$. The initial (and constant) x velocity of the supplies is $v_x = 69.4\text{ m/s}$. The supplies have to travel a horizontal distance of 425 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \to \quad t = \Delta x / v_x = 425\text{ m}/69.4\text{ m/s} = 6.124\text{ s}$$

The y motion must satisfy Eq. 2–11b for this time.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \to$$

$$v_{y0} = \frac{y - y_0 - \frac{1}{2}a_y t^2}{t} = \frac{0 - 235\text{ m} - \frac{1}{2}(-9.80\text{ m/s}^2)(6.124\text{ s})^2}{6.124\text{ s}} = \boxed{-8.37\text{ m/s}}$$

Notice that since this is a negative velocity, the object must be projected DOWN.

The horizontal component of the speed of the supplies upon landing is the constant horizontal speed of 69.4 m/s. The vertical speed is found from Eq. 2–11a.

$$v_y = v_{y0} + a_y t = -8.37\text{ m/s} + (-9.80\text{ m/s}^2)(6.124\text{ s}) = 68.4\text{ m/s}$$

Thus the speed is given by $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(69.4\text{ m/s})^2 + (68.4\text{ m/s})^2} = \boxed{97.4\text{ m/s}}$

33. Choose the origin to be the water level directly underneath the diver when she left the board. Choose upward as the positive y direction. For the diver, $y_0 = 4.0\text{ m}$, the final y position is $y = 0$ (water level), $a_y = -g$, the time of flight is $t = 1.3\text{ s}$, and the horizontal displacement is $\Delta x = 3.0\text{ m}$.

(*a*) The horizontal velocity is determined from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \to \quad v_x = \frac{\Delta x}{t} = \frac{3.0\text{ m}}{1.3\text{ s}} = 2.308\text{ m/s}$$

The initial y velocity is found using Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \to \quad 0 = 4.0\text{ m} + v_{y0}(1.3\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(1.3\text{ s})^2 \quad \to$$
$$v_{y0} = 3.293\text{ m/s}$$

The magnitude and direction of the initial velocity are the following:

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(2.308\text{ m/s})^2 + (3.293\text{ m/s})^2} = \boxed{4.0\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{3.293\text{ m/s}}{2.308\text{ m/s}} = \boxed{55^\circ\text{ above the horizontal}}$$

(*b*) The maximum height will be reached when $v_y = 0$. Use Eq. 2–11c.

$$v_y^2 = v_{y0}^2 + 2a\Delta y \quad \to \quad 0 = (3.293\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(y_{\max} - 4.0\text{ m}) \quad \to$$
$$y_{\max} = \boxed{4.6\text{ m}}$$

(*c*) To find the velocity when she enters the water, the horizontal velocity is the (constant) value of $v_x = 2.308\text{ m/s}$. The vertical velocity is found from Eq. 2–11a.

$$v_y = v_{y0} + at = 3.293\text{ m/s} + (-9.80\text{ m/s}^2)(1.3\text{ s}) = -9.447\text{ m/s}$$

The magnitude and direction of this velocity are found as follows:

$$v=\sqrt{v_x^2+v_y^2}=\sqrt{(2.308\text{ m/s})^2+(-9.447\text{ m/s})^2}=9.725\text{ m/s}\approx\boxed{9.7\text{ m/s}}$$

$$\theta=\tan^{-1}\frac{v_y}{v_x}=\tan^{-1}\frac{-9.447\text{ m/s}}{2.308\text{ m/s}}=\boxed{-76^\circ\text{ (below the horizontal)}}$$

34. Choose the origin to be the point of launch and upward to be the positive y direction. The initial velocity is v_0, the launching angle is θ_0, $a_y=-g$, $y_0=0$, and $v_{y0}=v_0\sin\theta_0$. Eq. 2–11a is used to find the time required to reach the highest point, at which $v_y=0$.

$$v_y=v_{y0}+at_{\text{up}}\quad\rightarrow\quad t_{\text{up}}=\frac{v_y-v_{y0}}{a}=\frac{0-v_0\sin\theta_0}{-g}=\frac{v_0\sin\theta_0}{g}$$

Eq. 2–11c is used to find the height at this highest point.

$$v_y^2=v_{y0}^2+2a_y(y_{\max}-y_0)\quad\rightarrow\quad y_{\max}=y_0+\frac{v_y^2-v_{y0}^2}{2a_y}=0+\frac{-v_0^2\sin^2\theta_0}{-2g}=\frac{v_0^2\sin^2\theta_0}{2g}$$

Eq. 2–11b is used to find the time for the object to fall the same distance with a starting velocity of 0.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_yt^2\quad\rightarrow\quad 0=\frac{v_0^2\sin^2\theta_0}{2g}+0\,(t_{\text{down}})-\tfrac{1}{2}gt_{\text{down}}^2\quad\rightarrow\quad t_{\text{down}}=\frac{v_0\sin\theta_0}{g}$$

A comparison shows that $\boxed{t_{\text{up}}=t_{\text{down}}}$.

35. Choose upward to be the positive y direction. The origin is the point from which the football is kicked. The initial speed of the football is $v_0=20.0\text{ m/s}$. We have $v_{y0}=v_0\sin 37.0^\circ=12.04\text{ m/s}$, $y_0=0$, and $a_y=-9.80\text{ m/s}^2$. In the horizontal direction, $v_x=v_0\cos 37.0^\circ=15.97\text{ m/s}$, and $\Delta x=36.0\text{ m}$. The time of flight to reach the goalposts is found from the horizontal motion at constant speed.

$$\Delta x=v_xt\quad\rightarrow\quad t=\Delta x/v_x=36.0\text{ m}/15.97\text{ m/s}=2.254\text{ s}$$

Now use this time with the vertical motion data and Eq. 2–11b to find the height of the football when it reaches the horizontal location of the goalposts.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_yt^2=0+(12.04\text{ m/s})(2.254\text{ s})+\tfrac{1}{2}(-9.80\text{ m/s}^2)(2.254\text{ s})^2=2.24\text{ m}$$

Since the ball's height is less than 3.05 m, $\boxed{\text{the football does not clear the bar}}$. It is 0.81 m too low when it reaches the horizontal location of the goalposts.

To find the distances from which a score can be made, redo the problem (with the same initial conditions) to find the times at which the ball is exactly 3.05 m above the ground. Those times would correspond with the maximum and minimum distances for making the score. Use Eq. 2–11b.

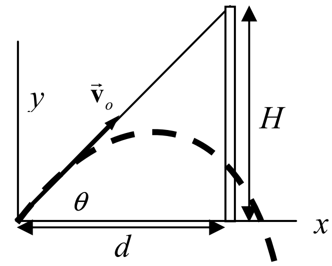
$$y=y_0+v_{y0}t+\tfrac{1}{2}a_yt^2\quad\rightarrow\quad 3.05=0+(12.04\text{ m/s})t+\tfrac{1}{2}(-9.80\text{ m/s}^2)t^2\quad\rightarrow$$

$$4.90t^2-12.04t+3.05=0\quad\rightarrow\quad t=\frac{12.04\pm\sqrt{(12.04)^2-4(4.90)(3.05)}}{2(4.90)}=2.1703\text{ s},\ 0.2868\text{ s}$$

$$\Delta x_1=v_xt=15.97\text{ m/s }(0.2868\text{ s})=4.580\text{ m};\ \Delta x_1=v_xt=15.97\text{ m/s }(2.1703\text{ s})=34.660\text{ m}$$

So the kick must be made in the range from $\boxed{4.6\text{ m to }34.7\text{ m}}$.

36. Choose the origin to be the location from which the balloon is fired, and choose upward as the positive y direction. Assume that the boy in the tree is a distance H up from the point at which the balloon is fired and that the tree is a horizontal distance d from the point at which the balloon is fired. The equations of motion for the balloon and boy are as follows, using constant acceleration relationships:



$$x_{\text{Balloon}} = v_0 \cos\theta_0 t \quad y_{\text{Balloon}} = 0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad y_{\text{Boy}} = H - \tfrac{1}{2}gt^2$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled d to the right.

$$d = v_0 \cos\theta_0 t \quad \rightarrow \quad t = \frac{d}{v_0 \cos\theta_0}$$

Where is the balloon vertically at that time?

$$y_{\text{Balloon}} = v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 = v_0 \sin\theta_0 v \frac{d}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{d}{v_0 \cos\theta_0}\right)^2$$

$$= d \tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{v_0 \cos\theta_0}\right)^2$$

Where is the boy vertically at that time? Note that $H = d \tan\theta_0$.

$$y_{\text{Boy}} = H - \tfrac{1}{2}gt^2 = H - \tfrac{1}{2}g\left(\frac{d}{v_0 \cos\theta_0}\right)^2 = d \tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{v_0 \cos\theta_0}\right)^2$$

Note that $y_{\text{Balloon}} = y_{\text{Boy}}$, so the boy and the balloon are at the same height and the same horizontal location at the same time. Thus they collide!

37. (*a*) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive y direction. At the end of its flight over the 8 cars, the car must be at $y = -1.5$ m. Also for the car, $v_{y0} = 0$, $a_y = -g$, $v_x = v_0$, and $\Delta x = 22$ m. The time of flight is found from the horizontal motion at constant velocity: $\Delta x = v_x t \rightarrow t = \Delta x / v_0$. That expression for the time is used in Eq. 2–11b for the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(-g)(\Delta x / v_0)^2 \quad \rightarrow$$

$$v_0 = \sqrt{\frac{-g(\Delta x)^2}{2(y)}} = \sqrt{\frac{-(9.80\text{ m/s}^2)(22\text{ m})^2}{2(-1.5\text{ m})}} = 39.76\text{ m/s} \approx \boxed{4.0\times10^1\text{ m/s}}$$

(*b*) Again, choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive y direction. The y displacement of the car at the end of its flight over the 8 cars must again be $y = -1.5$ m. For the car, $v_{y0} = v_0 \sin\theta_0$, $a_y = -g$, $v_x = v_0 \cos\theta_0$, and $\Delta x = 22$ m. The launch angle is $\theta_0 = 7.0°$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_0 \cos\theta_0}$$

That expression for the time is used in Eq. 2–11b for the vertical motion.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad y = v_0 \sin\theta_0 \frac{\Delta x}{v_0\cos\theta_0} + \tfrac{1}{2}(-g)\left(\frac{\Delta x}{v_0\cos\theta_0}\right)^2 \quad\rightarrow$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2(\Delta x\tan\theta_0 - y)\cos^2\theta_0}} = \sqrt{\frac{(9.80\text{ m/s}^2)(22\text{ m})^2}{2((22\text{ m})\tan 7.0° + 1.5\text{ m})\cos^2 7.0°}} = \boxed{24\text{ m/s}}$$

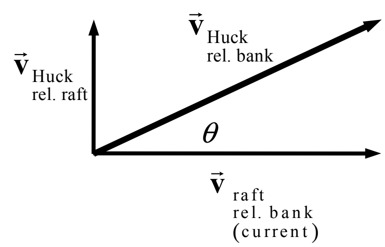
38. Call the direction of the boat relative to the water the positive direction. For the jogger moving toward the bow, we have the following:

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = 2.0\text{ m/s} + 8.5\text{ m/s}$$
$$= \boxed{10.5\text{ m/s in the direction the boat is moving}}$$

For the jogger moving toward the stern, we have the following:

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = -2.0\text{ m/s} + 8.5\text{ m/s}$$
$$= \boxed{6.5\text{ m/s in the direction the boat is moving}}$$

39. Call the direction of the flow of the river the x direction and the direction of Huck walking relative to the raft the y direction.



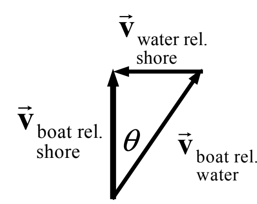
$$\vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.}\\ \text{bank}}} = (0, 0.70)\text{ m/s} + (1.50, 0)\text{ m/s}$$
$$= (1.50, 0.70)\text{ m/s}$$

Magnitude: $v_{\substack{\text{Huck}\\ \text{rel. bank}}} = \sqrt{1.50^2 + 0.70^2} = \boxed{1.66\text{ m/s}}$

Direction: $\theta = \tan^{-1}\dfrac{0.70}{1.50} = \boxed{25° \text{ relative to river}}$

40. From the diagram in Fig. 3–29, it is seen that

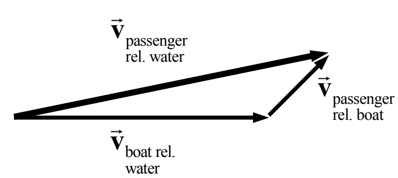
$$v_{\substack{\text{boat rel.}\\ \text{shore}}} = v_{\substack{\text{boat rel.}\\ \text{water}}}\cos\theta = (1.85\text{ m/s})\cos 40.4° = \boxed{1.41\text{ m/s}}.$$



41. If each plane has a speed of 780 km/h, then their relative speed of approach is 1560 km/h. If the planes are 10.0 km apart, then the time for evasive action is found as follows:

$$\Delta d = vt \quad\rightarrow\quad t = \frac{\Delta d}{v} = \left(\frac{10.0\text{ km}}{1560\text{ km/h}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{23.1\text{ s}}$$

42. Call the direction of the boat relative to the water the x direction and upward the y direction. Also see the diagram.



$$\vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{passenger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}}$$
$$= (0.60\cos 45°, 0.60\sin 45°)\text{ m/s}$$
$$+ (1.70, 0)\text{ m/s} = (2.124, 0.424)\text{ m/s}$$

$$v_{\substack{\text{passenger}\\ \text{rel. water}}} = \sqrt{(2.124\text{ m/s})^2 + (0.424\text{ m/s})^2} = \boxed{2.17\text{ m/s}} \qquad \theta_{\substack{\text{passenger}\\ \text{rel. water}}} = \tan^{-1}\frac{0.424}{2.124} = \boxed{11°}$$

43. (a) Call the upward direction positive for the vertical motion. Then the velocity of the ball relative to a person on the ground is the vector sum of the horizontal and vertical motions. The horizontal velocity is $\upsilon_x = 10.0$ m/s and the vertical velocity is $\upsilon_y = 3.0$ m/s.

$$\vec{\mathbf{v}} = (10.0\text{ m/s}, 3.0\text{ m/s}) \quad\rightarrow\quad \upsilon = \sqrt{(10.0\text{ m/s})^2 + (3.0\text{ m/s})^2} = \boxed{10.4\text{ m/s}}$$

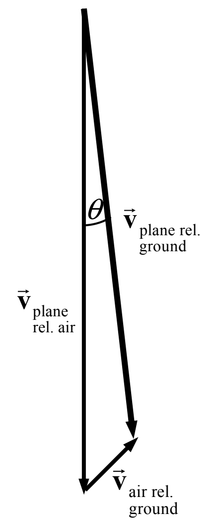
$$\theta = \tan^{-1}\frac{3.0\text{ m/s}}{10.0\text{ m/s}} = \boxed{17^\circ\text{ above the horizontal}}$$

(*b*) The only change is the initial vertical velocity, so $\upsilon_y = -5.0$ m/s.

$$\vec{\mathbf{v}} = (10.0\text{ m/s}, -3.0\text{ m/s}) \quad\rightarrow\quad \upsilon = \sqrt{(10.0\text{ m/s})^2 + (-3.0\text{ m/s})^2} = \boxed{10.4\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{-3.0\text{ m/s}}{10.0\text{ m/s}} = \boxed{17^\circ\text{ below the horizontal}}$$

44. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.



(*a*)
$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\ \text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\ \text{ground}}}$$
$$= (0, -688)\text{ km/h} + (90.0\cos 45.0^\circ, 90.0\sin 45.0^\circ)\text{ km/h}$$
$$= (63.6, -624)\text{ km/h}$$

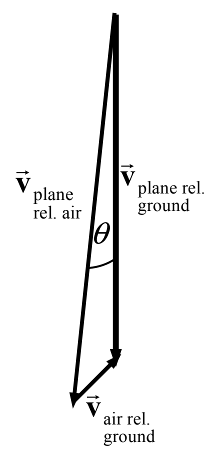
$$\upsilon_{\substack{\text{plane rel.}\\ \text{ground}}} = \sqrt{(63.6\text{ km/h})^2 + (-624\text{ km/h})^2} = \boxed{628\text{ km/h}}$$

$$\theta = \tan^{-1}\frac{63.6}{624} = \boxed{5.82^\circ\text{ east of south}}$$

(*b*) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 90.0 km/h, so after 11.0 min (11/60 h), the plane is off course by this amount.

$$\Delta x = \upsilon_x t = (90.0\text{ km/h})\left(\tfrac{11}{60}\text{ h}\right) = \boxed{16.5\text{ km}}$$

45. Call east the positive *x* direction and north the positive *y* direction. Then the following vector velocity relationship exists.



$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\ \text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\ \text{ground}}} \quad\rightarrow$$

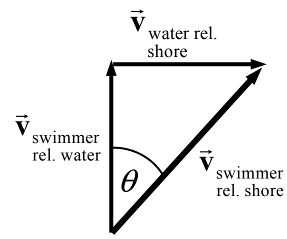
$$\left(0, -\upsilon_{\substack{\text{plane rel.}\\ \text{ground}}}\right) = (-688\sin\theta, 688\cos\theta)\text{ km/h}$$
$$+ (90.0\cos 45.0^\circ, 90.0\sin 45.0^\circ)\text{ km/h}$$

Equate *x* components in the above equation.

$$0 = -688\sin\theta + 90.0\cos 45.0^\circ \quad\rightarrow$$

$$\theta = \sin^{-1}\frac{90.0\cos 45.0^\circ}{688} = \boxed{5.31^\circ\text{, west of south}}$$

46. Call the direction of the flow of the river the x direction and the direction straight across the river the y direction. Call the location of the swimmer's starting point the origin.



$$\vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. shore}}} = \vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} = (0,\ 0.60\ \text{m/s}) + (0.50\ \text{m/s},\ 0)$$
$$= (0.50,\ 0.60)\ \text{m/s}$$

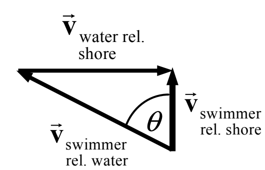
(*a*) Since the swimmer starts from the origin, the distances covered in the x and y directions will be exactly proportional to the speeds in those directions.

$$\frac{\Delta x}{\Delta y} = \frac{\upsilon_x t}{\upsilon_y t} = \frac{\upsilon_x}{\upsilon_y} \quad\rightarrow\quad \frac{\Delta x}{45\ \text{m}} = \frac{0.50\ \text{m/s}}{0.60\ \text{m/s}} \quad\rightarrow\quad \Delta x = 37.5\ \text{m} \approx \boxed{38\ \text{m}}$$

(*b*) The time is found from the constant velocity relationship for either the x or y direction.

$$\Delta y = \upsilon_y t \quad\rightarrow\quad t = \frac{\Delta y}{\upsilon_y} = \frac{45\ \text{m}}{0.60\ \text{m/s}} = \boxed{75\ \text{s}}$$

$\boxed{47.}$ (*a*) Call the direction of the flow of the river the x direction and the direction straight across the river the y direction.



$$\sin\theta = \frac{\upsilon_{\substack{\text{water rel.}\\ \text{shore}}}}{\upsilon_{\substack{\text{swimmer}\\ \text{rel. water}}}} = \frac{0.50\ \text{m/s}}{0.60\ \text{m/s}} \quad\rightarrow\quad \theta = \sin^{-1}\frac{0.50}{0.60} = 56.44^\circ \approx \boxed{56^\circ}$$

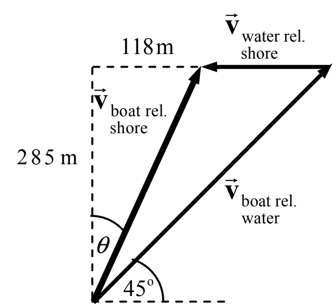
(*b*) From the diagram, her speed with respect to the shore is found as follows:

$$\upsilon_{\substack{\text{swimmer}\\ \text{rel. shore}}} = \upsilon_{\substack{\text{swimmer}\\ \text{rel. water}}} \cos\theta = (0.60\ \text{m/s})\cos 56.44^\circ = 0.332\ \text{m/s}$$

The time to cross the river can be found from the constant velocity relationship.

$$\Delta x = \upsilon t \quad\rightarrow\quad t = \frac{\Delta x}{\upsilon} = \frac{45\ \text{m}}{0.332\ \text{m/s}} = 135.5\ \text{s} \approx \boxed{140\ \text{s} = 2.3\ \text{min}}$$

48. Call the direction of the flow of the river the x direction (to the left in the diagram) and the direction straight across the river the y direction (to the top in the diagram). From the diagram, $\theta = \tan^{-1} 118\ \text{m}/285\ \text{m} = 22.49^\circ$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.



$$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} \cos\theta = \upsilon_{\substack{\text{boat rel.}\\ \text{water}}} \sin 45^\circ \quad\rightarrow$$

$$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} = (2.50\ \text{m/s})\frac{\sin 45^\circ}{\cos 22.49^\circ} = 1.913\ \text{m/s}$$

Equate the horizontal components of the velocities to find the speed of the current.

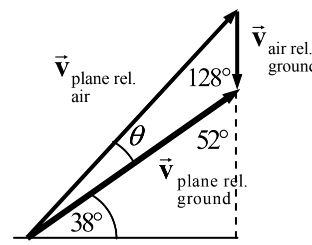
$$v_{\substack{\text{boat rel.}\\\text{shore}}} \sin\theta = v_{\substack{\text{boat rel.}\\\text{water}}} \cos 45^\circ - v_{\substack{\text{water}\\\text{rel. shore}}} \quad \rightarrow$$

$$v_{\substack{\text{water}\\\text{rel. shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}} \cos 45^\circ - v_{\substack{\text{boat rel.}\\\text{shore}}} \sin\theta$$

$$= (2.50\text{ m/s})\cos 45^\circ - (1.913\text{ m/s})\sin 22.49^\circ = 1.036\text{ m/s} \approx \boxed{1.0\text{ m/s}}$$

49. The lifeguard will be carried downstream at the same rate as the child. Thus only the horizontal motion need be considered. To cover 45 m horizontally at a rate of 2 m/s takes $\frac{45\text{ m}}{2\text{ m/s}} = 22.5\text{ s} \approx \boxed{23\text{ s}}$ for the lifeguard to reach the child. During this time they would both be moving downstream at 1.0 m/s, so they would travel $(1.0\text{ m/s})(22.5\text{ s}) = 22.5\text{ m} \approx \boxed{23\text{ m}}$ downstream.

50. Call east the positive x direction and north the positive y direction. The following is seen from the diagram. Apply the law of sines to the triangle formed by the three vectors.

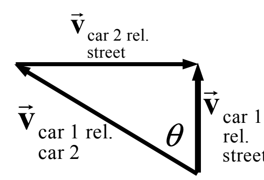


$$\frac{v_{\substack{\text{plane}\\\text{rel. air}}}}{\sin 128^\circ} = \frac{v_{\substack{\text{air rel.}\\\text{ground}}}}{\sin\theta} \quad \rightarrow \quad \sin\theta = \frac{v_{\substack{\text{air rel.}\\\text{ground}}}}{v_{\substack{\text{plane}\\\text{rel. air}}}} \sin 128^\circ \quad \rightarrow$$

$$\theta = \sin^{-1}\left(\frac{v_{\substack{\text{air rel.}\\\text{ground}}}}{v_{\substack{\text{plane}\\\text{rel. air}}}} \sin 128^\circ\right) = \sin^{-1}\left(\frac{82}{580\text{ km/h}} \sin 128^\circ\right) = 6.397^\circ$$

So the plane should head in a direction of $38.0^\circ + 6.4^\circ = \boxed{44.4^\circ\text{ north of east}}$.

51. Call east the positive x direction and north the positive y direction. From the first diagram, this relative velocity relationship is seen.

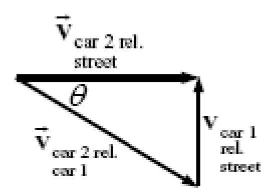


$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}} + \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} \quad \rightarrow \quad v_{\substack{\text{car 1 rel.}\\\text{car 2}}} = \sqrt{(-55)^2 + (35)^2}$$

$$= \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 55/35 = \boxed{58^\circ\text{ west of north}}$$

For the other relative velocity relationship:



$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} + \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{street}}} \quad \rightarrow \quad v_{\substack{\text{car 2 rel.}\\\text{car 1}}} = \sqrt{(55)^2 + (-35)^2}$$

$$= \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 35/55 = \boxed{32^\circ\text{ south of east}}$$

Notice that the two relative velocities are opposites of each other: $\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\\text{car 1}}} = -\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\\text{car 2}}}$

52. (*a*) For the magnitudes to add linearly, the two vectors must be parallel. $\boxed{\vec{\mathbf{V}}_1 \parallel \vec{\mathbf{V}}_2}$

(*b*) For the magnitudes to add according to the Pythagorean theorem, the two vectors must be at right angles to each other. $\boxed{\vec{\mathbf{V}}_1 \perp \vec{\mathbf{V}}_2}$

(*c*) The magnitude of vector 2 must be 0. $\boxed{\vec{\mathbf{V}}_2 = 0}$

53. The deceleration is along a straight line. The starting velocity is $110\text{ km/h}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 30.6\text{ m/s}$, and the ending velocity is 0 m/s. The acceleration is found from Eq. 2–11a.

$$v = v_0 + at \quad \rightarrow \quad 0 = 30.6\text{ m/s} + a(7.0\text{ s}) \quad \rightarrow \quad a = -\frac{30.6\text{ m/s}}{7.0\text{ s}} = -4.37\text{ m/s}^2$$

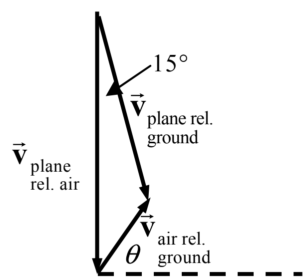
The horizontal acceleration is $a_{\text{horiz}} = a\cos\theta = -4.37\text{ m/s}^2(\cos 26°) = \boxed{-3.9\text{ m/s}^2}$.

The vertical acceleration is $a_{\text{vert}} = a\sin\theta = -4.37\text{ m/s}^2(\sin 26°) = \boxed{-1.9\text{ m/s}^2}$.

The horizontal acceleration is to the left in Fig. 3–48, and the vertical acceleration is down.

54. Call east the *x* direction and north the *y* direction. Then this relative velocity relationship follows (see the accompanying diagram).

$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$



Equate the *x* components of the velocity vectors. The magnitude of $\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}}$ is given as 135 km/h.

$$(135\text{ km/h})\sin 15.0° = 0 + v_{\text{wind }x} \quad \rightarrow \quad v_{\text{wind }x} = 34.94\text{ km/h}.$$

From the *y* components of the relative velocity equation, we find $v_{\text{wind }y}$.

$$-135\cos 15.0° = -185 + v_{\text{wind }y} \quad \rightarrow \quad v_{\text{wind }y} = 185 - 135\cos 15.0° = 54.60\text{ km/h}$$

The magnitude of the wind velocity is as follows:

$$v_{\text{wind}} = \sqrt{v_{\text{wind }x}^2 + v_{\text{wind }y}^2} = \sqrt{(34.94\text{ km/h})^2 + (54.60\text{ km/h})^2} = 64.82\text{ km/h} \approx \boxed{65\text{ km/h}}$$

The direction of the wind is $\theta = \tan^{-1}\dfrac{v_{\text{wind }y}}{v_{\text{wind }x}} = \tan^{-1}\dfrac{54.60}{34.94} = 57.38° \approx \boxed{57° \text{ north of east}}$.

55. The time of flight is found from the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t \quad \rightarrow \quad t = \Delta x / v_x = 8.0\text{ m}/9.1\text{ m/s} = \boxed{0.88\text{ s}}$$

The *y* motion is symmetric in time—it takes half the time of flight to rise, and half to fall. Thus the time for the jumper to fall from his highest point to the ground is 0.44 s. His vertical speed is zero at the highest point. From this time, starting vertical speed, and the acceleration of gravity, the maximum height can be found. Call upward the positive y direction. The point of maximum height is the starting position y_0, the ending position is $y=0$, the starting vertical speed is 0, and $a=-g$. Use Eq. 2–11b to find the height.

$$y=y_0+v_{y0}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0=y_0+0-\tfrac{1}{2}(9.8\text{ m/s}^2)(0.44\text{ s})^2 \quad\rightarrow\quad y_0=\boxed{0.95\text{ m}}$$

56. Choose upward to be the positive y direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y=-9.80\text{ m/s}^2$, the velocity at the window is $v_y=0$, and the vertical displacement is 8.0 m. The initial y velocity is found from Eq. 2–11c.

$$v_y^2=v_{y0}^2+2a_y(y-y_0) \quad\rightarrow$$
$$v_{y0}=\sqrt{v_y^2-2a_y(y-y_0)}=\sqrt{0-2(-9.80\text{ m/s}^2)(8.0\text{ m})}=12.5\text{ m/s}$$

Find the time for the pebbles to travel to the window from Eq. 2–11a.

$$v_y=v_{y0}+at \quad\rightarrow\quad t=\frac{v_y-v_{y0}}{a}=\frac{0-12.5\text{ m/s}}{-9.80\text{ m/s}^2}=1.28\text{ s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x=v_x t \quad\rightarrow\quad v_x=\Delta x/t=8.5\text{ m}/1.28\text{ s}=\boxed{6.6\text{ m/s}}$$

This is the speed of the pebbles when they hit the window.

57. Assume that the golf ball takes off and lands at the same height, so that the level horizontal range formula derived in the text can be applied. The only variable is to be the acceleration due to gravity.

$$R_{\text{Earth}}=v_0^2\sin 2\theta_0/g_{\text{Earth}} \qquad R_{\text{Moon}}=v_0^2\sin 2\theta_0/g_{\text{Moon}}$$
$$\frac{R_{\text{Earth}}}{R_{\text{Moon}}}=\frac{v_0^2\sin 2\theta_0/g_{\text{Earth}}}{v_0^2\sin 2\theta_0/g_{\text{Moon}}}=\frac{1/g_{\text{Earth}}}{1/g_{\text{Moon}}}=\frac{g_{\text{Moon}}}{g_{\text{Earth}}}=\frac{32\text{ m}}{180\text{ m}}=0.1778 \quad\rightarrow$$
$$g_{\text{Moon}}=0.1778\,g_{\text{Earth}}=0.1778(9.80\text{ m/s}^2)=1.742\text{ m/s}^2\approx\boxed{1.7\text{ m/s}^2}$$

58. (*a*) Use the level horizontal range formula from the text to find her takeoff speed.

$$R=\frac{v_0^2\sin 2\theta_0}{g} \quad\rightarrow\quad v_0=\sqrt{\frac{gR}{\sin 2\theta_0}}=\sqrt{\frac{(9.80\text{ m/s}^2)(8.0\text{ m})}{\sin 90^\circ}}=8.854\text{ m/s}\approx\boxed{8.9\text{ m/s}}$$

(*b*) Let the launch point be at the $y=0$ level, and choose upward to be positive. Use Eq. 2–11b to solve for the time to fall to 2.5 m below the starting height, and then calculate the horizontal distance traveled.

$$y=y_0+v_{0y}t+\tfrac{1}{2}a_y t^2 \quad\rightarrow\quad -2.5\text{ m}=(8.854\text{ m/s})\sin 45^\circ t+\tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$
$$4.9t^2-6.261t-2.5\text{ m}=0 \quad\rightarrow$$
$$t=\frac{6.261\pm\sqrt{(6.261)^2-4(4.9)(-2.5)}}{2(4.9)}=\frac{6.261\pm 9.391}{2(4.9)}=-0.319\text{ s},\ 1.597\text{ s}$$

Use the positive time to find the horizontal displacement during the jump.

$$\Delta x=v_{0x}t=v_0\cos 45^\circ t=(8.854\text{ m/s})\cos 45^\circ(1.597\text{ s})=10.0\text{ m}$$

$\boxed{\text{She will land exactly on the opposite bank, neither long nor short.}}$

59. Choose the origin to be at ground level, under the place where the projectile is launched, and upward to be the positive *y* direction. For the projectile, $v_0 = 65.0 \text{ m/s}$, $\theta_0 = 35.0°$, $a_y = -g$, $y_0 = 115 \text{ m}$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The time taken to reach the ground is found from Eq. 2–11b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad\rightarrow$$

$$t = \frac{-v_0 \sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)} = 9.964 \text{ s}, -2.3655 \text{ s} = \boxed{9.96 \text{ s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (v_0 \cos\theta_0)t = (65.0 \text{ m/s})(\cos 35.0°)(9.964 \text{ s}) = \boxed{531 \text{ m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0 \cos\theta_0 = (65.0 \text{ m/s}) \cos 35.0° = \boxed{53.2 \text{ m/s}}$. The vertical component is found from Eq. 2–11a.

$$v_y = v_{y0} + at = v_0 \sin\theta_0 - gt = (65.0 \text{ m/s}) \sin 35.0° - (9.80 \text{ m/s}^2)(9.964 \text{ s})$$
$$= \boxed{-60.4 \text{ m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part (*c*) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(53.2 \text{ m/s})^2 + (-60.4 \text{ m/s})^2} = \boxed{80.5 \text{ m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{-60.4}{53.2} = -48.6°$, so the object is moving $\boxed{48.6° \text{ below the horizontal}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the *y* velocity is 0 and is found from Eq. 2–11c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad\rightarrow\quad 0 = v_0^2 \sin^2\theta_0 - 2gy_{\text{max}}$$

$$y_{\text{max}} = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{(65.0 \text{ m/s})^2 \sin^2 35.0°}{2(9.80 \text{ m/s}^2)} = \boxed{70.9 \text{ m}}$$

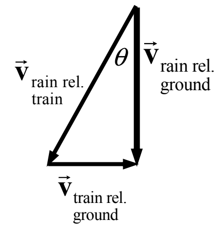
60. Since the arrow will start and end at the same height, use the level horizontal range formula derived in the text. The range is 27 m, and the initial speed of the arrow is 35 m/s.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad\rightarrow\quad \sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(27 \text{ m})(9.80 \text{ m/s}^2)}{(35 \text{ m/s})^2} = 0.216$$

$$\theta_0 = \tfrac{1}{2}\sin^{-1} 0.216 = 6.2°, 83.8°$$

Only the first answer is practical (the arrow might hit the son's head after piercing the apple if it comes in almost straight down), so the result is $\boxed{\theta_0 = 6.2°}$.

61. Choose the x direction to be the direction of train travel (the direction the passenger is facing), and choose the y direction to be up. This relationship exists among the velocities: $\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{rain rel.}\\ \text{train}}} + \vec{\mathbf{v}}_{\substack{\text{train rel.}\\ \text{ground}}}$. From the diagram, find the expression for the speed of the raindrops.



$$\tan\theta = \frac{\upsilon_{\substack{\text{train rel.}\\ \text{ground}}}}{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}}} = \frac{\upsilon_{\text{T}}}{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}}} \rightarrow \boxed{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}} = \frac{\upsilon_{\text{T}}}{\tan\theta}}$$

62. Work in the frame of reference in which the train is at rest. Then, relative to the train, the car is moving at 20 km/h. The car has to travel 1 km in that frame of reference to pass the train, so the time to pass can be found from the constant horizontal velocity relationship.

$$\Delta x = \upsilon_x t \rightarrow t_{\substack{\text{same}\\ \text{direction}}} = \frac{\Delta x}{(\upsilon_x)_{\substack{\text{same}\\ \text{direction}}}} = \frac{1\text{ km}}{20\text{ km/h}} = 0.05\text{ h}\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{180\text{ s}}$$

The car travels 1 km in the frame of reference of the stationary train, but relative to the ground, the car is traveling at 95 km/h so relative to the ground the car travels this distance:

$$\Delta x = \upsilon_x t_{\substack{\text{same}\\ \text{direction}}} = (95\text{ km/h})(0.05\text{ h}) = \boxed{4.8\text{ km}}$$

If the car and train are traveling in opposite directions, then the velocity of the car relative to the train will be 170 km/h. Thus the time to pass will be as follows:

$$t_{\substack{\text{opposite}\\ \text{direction}}} = \frac{\Delta x}{(\upsilon_x)_{\substack{\text{opposite}\\ \text{direction}}}} = \frac{1\text{ km}}{170\text{ km/h}} = \left(\frac{1}{170}\text{ h}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{21.2\text{ s}}$$

The distance traveled by the car relative to the ground is calculated.

$$\Delta x = \upsilon_x t_{\substack{\text{opposite}\\ \text{direction}}} = (95\text{ km/h})\left(\frac{1}{170}\text{ h}\right) = \boxed{0.56\text{ km}}$$

63. (*a*) Choose downward to be the positive y direction. The origin is the point where the bullet leaves the gun. In the vertical direction, $\upsilon_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\text{ m/s}^2$. In the horizontal direction, $\Delta x = 38.0\text{ m}$ and $\upsilon_x = 23.1\text{ m/s}$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \rightarrow t = \Delta x/\upsilon_x = 38.0\text{ m}/23.1\text{ m/s} = 1.645\text{ s}$$

This time can now be used in Eq. 2–11b to find the vertical drop of the bullet.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow y = 0 + 0 + \tfrac{1}{2}(9.80\text{ m/s}^2)(1.645\text{ s})^2 = \boxed{13.3\text{ m}}$$

(*b*) For the bullet to hit the target at the same level, the level horizontal range formula derived in the text applies. The range is 38.0 m, and the initial velocity is 23.1 m/s. Solving for the angle of launch results in the following:

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \rightarrow \sin 2\theta_0 = \frac{Rg}{\upsilon_0^2} \rightarrow \theta_0 = \tfrac{1}{2}\sin^{-1}\frac{(38.0\text{ m})(9.80\text{ m/s}^2)}{(23.1\text{ m/s})^2} = \boxed{22.1^\circ}$$

Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be $67.9°$. That is an unreasonable answer from a practical physical viewpoint—it is pointing the bow nearly straight up.

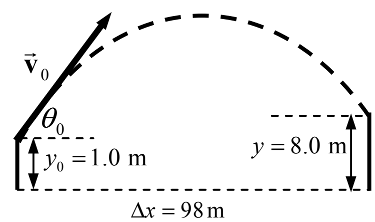
64. Choose downward to be the positive y direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $v_{y0} = 0$, the acceleration is $a_y = 9.80\ \text{m/s}^2$, and the displacement is 35 m. The time of flight is found from Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 35\ \text{m} = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(35\ \text{m})}{9.8\ \text{m/s}^2}} = \boxed{2.7\ \text{s}}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x / t = 5.0\ \text{m}/2.7\ \text{s} = \boxed{1.9\ \text{m/s}}$$

65. The minimum speed will be that for which the ball just clears the fence; that is, the ball has a height of 8.0 m when it is 98 m horizontally from home plate. The origin is at home plate, with upward as the positive y direction. For the ball, $y_0 = 1.0$ m, $y = 8.0$ m, $a_y = -g$, $v_{y0} = v_0 \sin\theta_0$, $v_x = v_0 \cos\theta_0$, and $\theta_0 = 36°$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x = v_x t = v_0 \cos\theta_0 t$, so $t = \dfrac{\Delta x}{v_0 \cos\theta_0}$.



For the vertical motion, apply Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + v_0(\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$y = y_0 + v_0 t \sin\theta_0 - \tfrac{1}{2}gt^2 \quad \rightarrow \quad y = y_0 + v_0 \sin\theta_0 \frac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \sqrt{2\left(\frac{y_0 - y + \Delta x \tan\theta_0}{g}\right)} = \sqrt{2\left(\frac{1.0\ \text{m} - 8.0\ \text{m} + (98\ \text{m})\tan 36°}{9.80\ \text{m/s}^2}\right)} = 3.620\ \text{s}$$

Finally, use the time with the horizontal range to find the initial speed.

$$\Delta x = v_0 \cos\theta_0 t \quad \rightarrow \quad v_0 = \frac{\Delta x}{t\cos\theta_0} = \frac{98\ \text{m}}{(3.620\ \text{s})\cos 36°} = \boxed{33\ \text{m/s}}$$

66. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive y direction. Then for the ball, $y_0 = 2.50$ m, $v_{y0} = 0$, $a_y = -g$, and the y location when the ball just clears the net is $y = 0.90$ m. The time for the ball to reach the net is calculated from Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0.90\text{ m} = 2.50\text{ m} + 0 + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad \rightarrow$$

$$t_{\substack{\text{to}\\\text{net}}} = \sqrt{\frac{2(-1.60\text{ m})}{-9.80\text{ m/s}^2}} = 0.57143\text{ s}$$

The x velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \frac{\Delta x}{t} = \frac{15.0\text{ m}}{0.57143\text{ s}} = 26.25 \approx \boxed{26.3\text{ m/s}}$$

This is the minimum speed required to clear the net.

To find the full time of flight of the ball, set the final y location to be $y = 0$, and again use Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = 2.50\text{ m} + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad \rightarrow$$

$$t_{\text{total}} = \sqrt{\frac{2(-2.50\text{ m})}{-9.80\text{ m/s}^2}} = 0.7143 \approx \boxed{0.714\text{ s}}$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (26.25\text{ m/s})(0.7143\text{ s}) = 18.75 \approx \boxed{18.8\text{ m}}$$

Since this is between 15.0 and 22.0 m, $\boxed{\text{the ball lands in the “good” region}}$.

67. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of $208\text{ km/h} - 156\text{ km/h} = 52\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) =$ 14.44 m/s. For the vertical motion, choose the level of the helicopter to be the origin and downward to be positive. Then the package's y displacement is $y = 78.0\text{ m}$, $v_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2–11b.

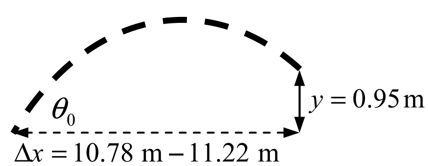
$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 78.0\text{ m} = \tfrac{1}{2}(9.80\text{ m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(78.0\text{ m})}{9.80\text{ m/s}^2}} = 3.99\text{ s}$$

The horizontal distance that the package must move, relative to the “stationary” car, is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = (14.44\text{ m/s})(3.99\text{ s}) = 57.6\text{ m}$$

Thus the angle under the horizontal for the package release will be as follows:

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0\text{ m}}{57.6\text{ m}}\right) = 53.6° \approx \boxed{54°}$$



68. The proper initial speeds will be those for which the ball has traveled a horizontal distance somewhere between 10.78 m and 11.22 m while it changes height from 2.10 m to 3.05 m with a shooting angle of 38.0°. Choose the origin to be at the shooting location of the basketball, with upward as the positive y direction. Then the vertical displacement is $y = 0.95\text{ m}$, $a_y = -9.80\text{ m/s}^2$, $v_{y0} = v_0 \sin\theta_0$, and the

(constant) x velocity is $v_x = v_0 \cos\theta_0$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x = v_x t = v_0 \cos\theta_0 t$, so $t = \dfrac{\Delta x}{v_0 \cos\theta_0}$. For the vertical motion, apply Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Substitute the expression for the time of flight and solve for the initial velocity.

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 = v_0 \sin\theta \frac{\Delta x}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{\Delta x}{v_0 \cos\theta_0}\right)^2 = \Delta x \tan\theta - \frac{g(\Delta x)^2}{2v_0^2 \cos^2\theta_0}$$

$$v_0 = \sqrt{\frac{g(\Delta x)^2}{2\cos^2\theta_0(-y + \Delta x \tan\theta)}}$$

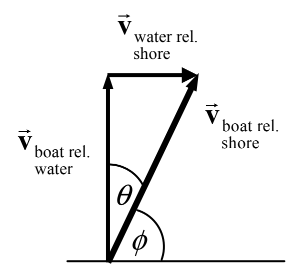
For $\Delta x = 11.00\ \text{m} - 0.22\ \text{m} = 10.78\ \text{m}$, the shortest shot:

$$v_0 = \sqrt{\frac{(9.80\ \text{m/s}^2)(10.78\ \text{m})^2}{2\cos^2 38.0^\circ\,[(-0.95\ \text{m} + (10.78\ \text{m})\tan 38.0^\circ)]}} = 11.078\ \text{m/s} \approx \boxed{11.1\ \text{m/s}}$$

For $\Delta x = 11.00\ \text{m} + 0.22\ \text{m} = 11.22\ \text{m}$, the longest shot:

$$v_0 = \sqrt{\frac{(9.80\ \text{m/s}^2)(11.22\ \text{m})^2}{2\cos^2 38.0^\circ\,[(-0.95\ \text{m} + (11.22\ \text{m})\tan 38.0^\circ)]}} = 11.274\ \text{m} \approx \boxed{11.3\ \text{m/s}}$$

69. Call the direction of the flow of the river the x direction, and the direction the boat is headed (which is different from the direction it is moving) the y direction.



(*a*) $v_{\substack{\text{boat rel.}\\\text{shore}}} = \sqrt{v^2_{\substack{\text{water rel.}\\\text{shore}}} + v^2_{\substack{\text{boat rel.}\\\text{water}}}} = \sqrt{(1.20\ \text{m/s})^2 + (2.20\ \text{m/s})^2}$

$$= 2.506\ \text{m/s} \approx \boxed{2.51\ \text{m/s}}$$

$$\theta = \tan^{-1}\frac{1.20}{2.20} = 28.6^\circ,\ \phi = 90^\circ - \theta = \boxed{61.4^\circ\ \text{relative to shore}}$$

(*b*) The position of the boat after 3.00 seconds is given by the following:

$$\Delta d = v_{\substack{\text{boat rel.}\\\text{shore}}} t = [(1.20\ \text{m/s},\ 2.20\ \text{m/s})](3.00\ \text{s})$$

$$= \boxed{3.60\ \text{m downstream, 6.60 m across the river}}$$

As a magnitude and direction, it would be 7.52 m away from the starting point, at an angle of 61.4° relative to the shore.

70. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive y direction. The y displacement of the projectile is 135 m, and the horizontal range of the projectile is 195 m. The acceleration in the y direction is $a_y = -g$, and the time of flight is 6.6 s.

The horizontal velocity is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad\rightarrow\quad v_x = \frac{\Delta x}{t} = \frac{195\ \text{m}}{6.6\ \text{s}} = 29.55\ \text{m/s}$$

Calculate the initial y velocity from the given data and Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 135\text{ m} = v_{y0}(6.6\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(6.6\text{ s})^2 \rightarrow v_{y0} = 52.79\text{ m/s}$$

Thus the initial velocity and direction of the projectile are as follows:

$$v_0 = \sqrt{v_x^2 + v_{y0}^2} = \sqrt{(29.55\text{ m/s})^2 + (52.79\text{ m/s})^2} = 60.4978\text{ m/s} \approx \boxed{6.0\times 10^1\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{v_{y0}}{v_x} = \tan^{-1}\frac{52.79\text{ m/s}}{29.55\text{ m/s}} = \boxed{61^\circ}$$

71. Find the time of flight from the vertical data, using Eq. 2–11b. Call the floor the $y = 0$ location, and choose upward as positive.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 3.05\text{ m} = 2.40\text{ m} + (12\text{ m/s})\sin 35^\circ t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$4.90t^2 - 6.883t + 0.65\text{ m} = 0 \quad\rightarrow$$

$$t = \frac{6.883 \pm \sqrt{6.883^2 - 4(4.90)(0.65)}}{2(4.90)} = 1.303\text{ s},\ 0.102\text{ s}$$

(*a*) Use the longer time for the time of flight. The shorter time is the time for the ball to rise to the basket height on the way up, while the longer time is the time for the ball to be at the basket height on the way down.

$$x = v_x t = v_0(\cos 35^\circ)t = (12\text{ m/s})(\cos 35^\circ)(1.303\text{ s}) = 12.81\text{ m} \approx \boxed{13\text{ m}}$$

(*b*) The angle to the horizontal is determined by the components of the velocity.

$$v_x = v_0 \cos\theta_0 = 12\cos 35^\circ = 9.830\text{ m/s}$$
$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = 12\sin 35^\circ - 9.80(1.303) = -5.886\text{ m/s}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-5.886}{9.830} = -30.9^\circ \approx \boxed{-31^\circ}$$

The negative angle means it is below the horizontal.

72. Let the launch point be the origin of coordinates, with right and upward as the positive directions. The equation of the line representing the ground is $y_{\text{gnd}} = -x$. The equations representing the motion of the rock are $x_{\text{rock}} = v_0 t$ and $y_{\text{rock}} = -\tfrac{1}{2}gt^2$, which can be combined into $y_{\text{rock}} = -\tfrac{1}{2}\frac{g}{v_0^2}x_{\text{rock}}^2$.

Find the intersection (the landing point of the rock) by equating the two expressions for y, and thereby find where the rock meets the ground.

$$y_{\text{rock}} = y_{\text{gnd}} \quad\rightarrow\quad -\tfrac{1}{2}\frac{g}{v_0^2}x^2 = -x \quad\rightarrow\quad x = \frac{2v_0^2}{g} \quad\rightarrow\quad t = \frac{x}{v_0} = \frac{2v_0}{g} = \frac{2(15\text{ m/s})}{9.80\text{ m/s}^2} = \boxed{3.1\text{ s}}$$

$\boxed{73.}$ Choose the origin to be the point at ground level directly below where the ball was hit. Call upwards the positive y direction. For the ball, we have $v_0 = 28$ m/s, $\theta_0 = 61^\circ$, $a_y = -g$, $y_0 = 0.90$ m, and $y = 0$.

(*a*) To find the horizontal displacement of the ball, the horizontal velocity and the time of flight are needed. The (constant) horizontal velocity is given by $v_x = v_0\cos\theta_0$. The time of flight is found from Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{-v_0 \sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)}$$

$$= \frac{-(28\text{ m/s})\sin 61^\circ \pm \sqrt{(28\text{ m/s})^2\sin^2 61^\circ - 4\left(-\tfrac{1}{2}\right)(9.80\text{ m/s}^2)(0.90\text{ m})}}{2\left(-\tfrac{1}{2}\right)(9.80\text{ m/s}^2)}$$

$$= 5.034\text{ s}, -0.0365\text{ s}$$

Choose the positive time, since the ball was hit at $t = 0$. The horizontal displacement of the ball will be found by the constant velocity relationship for horizontal motion.

$$\Delta x = v_x t = v_0 \cos\theta_0 t = (28\text{ m/s})(\cos 61^\circ)(5.034\text{ s}) = 68.34\text{ m} \approx \boxed{68\text{ m}}$$

(*b*) The centerfielder catches the ball right at ground level. He ran 105 m – 68.34 m = 36.66 m to catch the ball, so his average running speed would be as follows:

$$v_{\text{avg}} = \frac{\Delta d}{t} = \frac{36.66\text{ m}}{5.034\text{ s}} = 7.282\text{ m/s} \approx \boxed{7.3\text{ m/s}}$$

74. Choose the origin to be the point at the top of the building from which the ball is shot, and call upward the positive *y* direction. The initial velocity is $v_0 = 18$ m/s at an angle of $\theta_0 = 42°$. The acceleration due to gravity is $a_y = -g$.

(*a*) $$v_x = v_0 \cos\theta_0 = (18\text{ m/s})\cos 42^\circ = 13.38\text{ m/s} \approx \boxed{13\text{ m/s}}$$

$$v_{y0} = v_0 \sin\theta_0 = (18\text{ m/s})\sin 42^\circ = 12.04\text{ m/s} \approx \boxed{12\text{ m/s}}$$

(*b*) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_x} = \frac{55\text{ m}}{13.38\text{ m/s}} = 4.111\text{ s}$$

With that time of flight, calculate the vertical position of the ball using Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = (12.04\text{ m/s})(4.111\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(4.111\text{ s})^2$$

$$= -33.3\text{ m} = \boxed{-33\text{ m}}$$

So the ball will strike 33 m below the top of the building.

75. First we find the time of flight for the ball. From that time we can calculate the vertical speed of the ball. From that vertical speed we can calculate the total speed of the ball and the % change in the speed. We choose the downward direction to be positive for vertical motion.

$$v_x = v_{0x} = (150\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 41.67\text{ m/s}$$

$$\Delta x = v_x t \quad \rightarrow \quad t = \frac{\Delta x}{v_x} = \frac{18\text{ m}}{41.67\text{ m/s}} = 0.432\text{ s}$$

$$v_y = v_{0y} + at = (9.80\text{ m/s}^2)(0.432\text{ s}) = 4.234\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(41.67\text{ m/s})^2 + (4.234\text{ m/s})^2} = 41.88\text{ m/s}$$

$$\%\text{ change} = \frac{v - v_0}{v_0} \times 100 = \frac{41.88 - 41.67}{41.67} \times 100 = \boxed{0.50\%}$$

Solutions to Search and Learn Problems

1. Consider the downward vertical component of the motion, which will occur in half the total time. Take the starting position to be $y = 0$ and the positive direction to be downward. Use Eq. 2–11b with an initial vertical velocity of 0.

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad h = 0 + 0 + \tfrac{1}{2}gt_{\text{down}}^2 = \tfrac{1}{2}g\left(\frac{t}{2}\right)^2 = \frac{9.80}{8}t^2 = 1.225t^2 \approx \boxed{1.2t^2}$$

As can be seen from the equation, by starting the analysis from the "top" point of the motion, the initial vertical speed is 0. This eliminates the need to know the original launch speed or direction in that calculation. We then also realize that the time for the object to rise is the same as the time for it to fall, so we have to analyze only the downward motion.

2. Consider two balls thrown at different angles θ and different initial velocities v. The initial y component of the velocity can be written as $v_{y0} = v \sin \theta$. Use Eq. 2–11c to write the initial vertical velocity in terms of the maximum height h, given that the vertical velocity is zero at the maximum height. We choose upward as the positive vertical direction.

$$v_y^2 = v_{y0}^2 - 2g(y - y_0) \quad \rightarrow \quad 0 = v_{y0}^2 - 2gh \quad \rightarrow \quad v_{y0} = \sqrt{2gh}$$

The total time of flight is the time it takes for the projectile to return to the ground $(y = 0)$. Insert the initial velocity into Eq. 2–11b to solve for the total time of flight.

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \quad \rightarrow \quad 0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2 = t\left(v_{0y} - \tfrac{1}{2}gt\right) \quad \rightarrow \quad t = \frac{2v_{0y}}{g} = \frac{2\sqrt{2gh}}{g} = 2\sqrt{\frac{2h}{g}}$$

Since both balls reach the same maximum height, they will have the same time of flight. The ball thrown at the shallower angle must have a larger initial velocity in order to have the same initial vertical speed and thus reach the same maximum height. However, it will reach that height in the same time as the ball thrown at the steeper angle.

3. The horizontal component of the speed does not change during the course of the motion, so $v_x = v_{x0}$. The net vertical displacement is 0 if the firing level equals the landing level, so $y - y_0 = 0$. Eq. 2–11c then gives $v_y^2 = v_{y0}^2 + 2a_y(y - y_0) = v_{y0}^2$. Thus $v_y^2 = v_{y0}^2$, and from the horizontal $v_x^2 = v_{x0}^2$. The initial speed is $v_0 = \sqrt{v_{x0}^2 + v_{y0}^2}$. The final speed is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{x0}^2 + v_{y0}^2} = v_0$. Thus $\boxed{v = v_0}$.

4. The ranges can be written in terms of the angles and initial velocities using the level horizontal range equation derived in the text. Then setting the two ranges equal, we can obtain the ratio of the initial velocities.

$$R = \frac{v_0^2 \sin 2\theta}{g} \quad \rightarrow \quad \frac{v_A^2 \sin(2\times 30^\circ)}{g} = \frac{v_B^2 \sin(2\times 60^\circ)}{g} \quad \rightarrow \quad \frac{v_A}{v_B} = \sqrt{\frac{\sin 120^\circ}{\sin 60^\circ}} = \boxed{\frac{v_A}{v_B} = 1}$$

The two projectiles each have the same initial velocity.

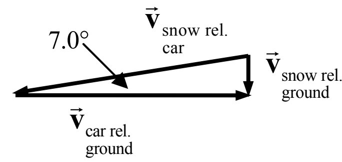
The time of flight is found using Eq. 2–11b with both the initial and final heights equal to 0 and the initial vertical velocities written in terms of the launching angle. We choose the positive vertical direction to be upward.

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \quad \rightarrow \quad 0 = 0 + v_{y0}t - \tfrac{1}{2}gt^2 = t\left(v_{0y} - \tfrac{1}{2}gt\right) \quad \rightarrow \quad t = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\theta}{g}$$

$$\frac{t_B}{t_A} = \frac{2v_0 \sin 60°/g}{2v_0 \sin 30°/g} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$$

The projectile launched at 60° is in the air 1.73 times as long as the projectile launched at 30°.

5. We have $v_{\substack{\text{car rel.}\\ \text{ground}}} = 12\ \text{m/s}.$ Use the diagram, showing the snow falling straight down relative to the ground and the car moving parallel to the ground, and illustrating $\vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{car}}} + \vec{\mathbf{v}}_{\substack{\text{car rel.}\\ \text{ground}}},$ to calculate the other speeds.



$$\tan 7.0° = \frac{v_{\substack{\text{snow rel.}\\ \text{ground}}}}{v_{\substack{\text{car rel.}\\ \text{ground}}}} \quad \rightarrow \quad v_{\substack{\text{snow rel.}\\ \text{ground}}} = (12\ \text{m/s})\tan 7.0° = 1.473\ \text{m/s} \approx \boxed{1.5\ \text{m/s}}$$

4

DYNAMICS: NEWTON'S LAWS OF MOTION

Responses to Questions

1. When you give the wagon a sharp pull forward, the force of friction between the wagon and the child acts on the child to move her forward. But the force of friction acts at the contact point between the child and the wagon—we assume the child is sitting in the wagon. The lower part of the child begins to move forward, while the upper part, following Newton's first law (the law of inertia), remains almost stationary, making it seem as if the child falls backward. The "backward" motion is relative to the wagon, not to the ground.

2. (*a*) Mary, standing on the ground beside the truck, will see the box remain motionless while the truck accelerates out from under it. Since there is no friction, there is no net horizontal force on the box and the box will not speed up. Thus Mary would describe the motion of the box in terms of Newton's first law—there is no force on the box, so it does not accelerate.
   (*b*) Chris, riding on the truck, will see the box appear to accelerate backward with respect to his frame of reference, which is not inertial. He might even say something about the box being "thrown" backward in the truck and try to invoke Newton's second law to explain the motion of the box. But the source of the force would be impossible to specify. (Chris had better hold on, though; if the truck bed is frictionless, he too will slide off if he is just standing!)

3. Yes, the net force can be zero on a moving object. If the net force is zero, then the object's *acceleration* is zero, but its *velocity* is not necessarily zero. [Instead of classifying objects as "moving" and "not moving," Newtonian dynamics classifies them as "accelerating" and "not accelerating." Both zero velocity and constant velocity fall in the "not accelerating" category.]

4. If the acceleration of an object is zero, the vector *sum* of the forces acting on the object is zero (Newton's second law), so there can be forces on an object that has no acceleration. For example, a book resting on a table is acted on by gravity and the normal force, but it has zero acceleration, because the forces are equal in magnitude and opposite in direction.

5. If only one force acts on an object, the net force cannot be zero, so the object cannot have zero acceleration, by Newton's second law. It *is* possible for the object to have zero velocity, but only for an instant. For example (if we neglect air resistance), a ball thrown upward into the air has only the force of gravity acting on it. Its speed will decrease while it travels upward, stops, then begins to fall back to the ground. At the instant the ball is at its highest point, its velocity is zero. However, the ball has a nonzero net force and a nonzero acceleration throughout its flight.

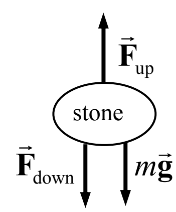
6. (*a*) A force is needed to bounce the ball back up, because the ball changes direction, so the ball accelerates. If the ball accelerates, there must be a force.
   (*b*) The pavement exerts the force on the golf ball.

7. As you take a step on the log, your foot exerts a force on the log in the direction opposite to the direction in which *you* want to move, which pushes the log "backward." (The log exerts an equal and opposite force forward on you, by Newton's third law.) If the log had been on the ground, friction between the ground and the log would have kept the log from moving. However, the log is floating in water, which offers little resistance to the movement of the log as you push it backward.

8. (*a*) When you first start riding a bicycle you need to exert a strong force to accelerate the bike and yourself, as well as to overcome friction. Once you are moving at a constant speed, you need to exert a force that will just equal the opposing forces of friction and air resistance.
   (*b*) When the bike is moving at a constant speed, the *net* force on it is zero. Since friction and air resistance are present, you would slow down if you didn't pedal to keep the net force on the bike (and you) equal to zero.

9. When the person gives a sharp pull, the suddenness of application of the force is key. When a large, sudden force is applied to the bottom string, the bottom string will have a large tension in it. Because of the stone's inertia, the upper string does not immediately experience the large force. The bottom string must have more tension in it and will break first.

   If a slow and steady pull is applied, the tension in the bottom string increases. We approximate that condition as considering the stone to be in equilibrium until the string breaks. The free-body diagram for the stone would look like this diagram. While the stone is in equilibrium, Newton's second law states that $F_{\text{up}} = F_{\text{down}} + mg.$ Thus the tension in the upper string is going to be larger than the tension in the lower string because of the weight of the stone, so the upper string will break first.



10. The acceleration of both rocks is found by dividing their weight (the force of gravity on them) by their mass. The 2-kg rock has a force of gravity on it that is twice as great as the force of gravity on the 1-kg rock, but also twice as great a mass as the 1-kg rock, so the acceleration is the same for both.

11. (*a*) When you pull the rope at an angle, only the horizontal component of the pulling force will be accelerating the box across the table. This is a smaller horizontal force than originally used, so the horizontal acceleration of the box will decrease.
    (*b*) We assume that the rope is angled upward, as in Fig. 4–21a. When there is friction, the problem is much more complicated. As the angle increases, there are two competing effects. The horizontal component of the pulling force gets smaller, as in part (*a*), which reduces the acceleration. But as the angle increases, the upward part of the pulling force gets larger, which reduces the normal force. As the normal force gets smaller, the force of friction also gets smaller, which would increase the acceleration. A detailed analysis shows that the acceleration increases initially, up to a certain angle, and then decreases for higher angles.

    If instead the rope is angled downward, then the normal force increases, which increases the force of friction, and for all angles, the acceleration will decrease.

12. Let us find the acceleration of the Earth, assuming the mass of the freely falling object is $m = 1$ kg. If the mass of the Earth is *M*, then the acceleration of the Earth would be found using Newton's third law and Newton's second law.

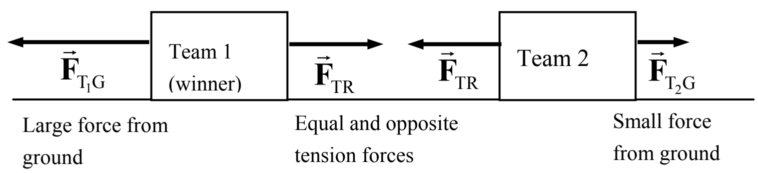
$$F_{\text{Earth}} = F_{\text{object}} \quad \to \quad Ma_{\text{Earth}} = mg \quad \to \quad a_{\text{Earth}} = g\ m/M$$

Since the Earth has a mass that is on the order of 10^{25} kg, the acceleration of the Earth is on the order of $10^{-25}\,g$, or about $10^{-24}\ \text{m/s}^2$. This tiny acceleration is undetectable.

13. Because the acceleration due to gravity on the Moon is less than it is on the Earth, an object with a mass of 10 kg will weigh less on the Moon than it does on the Earth. Therefore, it will be easier to lift on the Moon. (When you lift something, you exert a force to oppose its weight.) However, when throwing the object horizontally, the force needed to accelerate it to the desired horizontal speed is proportional to the object's mass, $F = ma$. Therefore, you would need to exert the same force to throw the 2-kg object with a given speed on the Moon as you would on Earth.

14. In a tug of war, the team that pushes hardest against the ground wins. It is true that both teams have the same force on them due to the tension in the rope. But the winning team pushes harder against the ground and thus the ground pushes harder on the winning team, making a net unbalanced force. The free-body diagram illustrates this.

The forces are $\vec{\mathbf{F}}_{\text{T}_1\text{G}}$, the force on team 1 from the ground, $\vec{\mathbf{F}}_{\text{T}_2\text{G}}$, the force on team 2 from the ground, and $\vec{\mathbf{F}}_{\text{TR}}$, the force on each team from the rope.



Thus the <u>net</u> force on the winning team $(\vec{\mathbf{F}}_{\text{T}_1\text{G}} - \vec{\mathbf{F}}_{\text{TR}})$ is in the "winning" direction.

15. If you are at rest, the net force on you is zero. Hence the ground exerts a force on you exactly equal to your weight. The two forces acting on you sum to zero, so you don't accelerate. If you squat down and then push with a larger force against the ground, the ground then pushes back on you with a larger force by Newton's third law, and you <u>can</u> then rise into the air.

16. The victim's head is not really thrown backward during the car crash. If the victim's car was initially at rest, or even moving forward, the impact from the rear suddenly pushes the car, the seat, and the person's body forward. The head, being attached by the somewhat flexible neck to the body, can momentarily remain where it was (inertia, Newton's first law), thus lagging behind the body. The neck muscles must eventually pull the head forward, and that causes the whiplash. To avoid this, use the car's headrests.

17. (*a*) The reaction force has a magnitude of 40 N.
(*b*) It points downward.
(*c*) It is exerted on Mary's hands and arms.
(*d*) It is exerted by the bag of groceries.

18. Both the father and daughter will have the same magnitude force acting on them as they push each other away, by consideration of Newton's third law. If we assume that the young daughter has less mass than the father, her acceleration should be greater ($a = F/m$). Both forces, and therefore both accelerations, act over the same time interval (while the father and daughter are in contact), so the daughter's final speed will be greater than her father's.

19. Static friction between the crate and the truck bed causes the crate to accelerate.

20. On the way up, there are two forces on the block that are parallel to each other causing the deceleration—the component of weight parallel to the plane and the force of friction on the block. Since the forces are parallel to each other, both pointing down the plane, they add, causing a larger magnitude force and a larger acceleration. On the way down, those same two forces are opposite of each other, because the force of friction is now directed up the plane. With these two forces being opposite of each other, their net force is smaller, so the acceleration is smaller.

21. In a very simple analysis, the net force slowing the moving object is friction. If we consider that the moving object is on a level surface, then the normal force is equal to the weight. Combining these ideas, we get the following:

$$F_{\text{net}} = ma = \mu F_{\text{N}} = \mu mg \to a = \mu g$$

From Table 4–2, the "steel on steel (unlubricated)" coefficient of friction (applicable to the train) is smaller than the "rubber on dry concrete" coefficient of friction (applicable to the truck). Thus the acceleration of the train will be smaller than that of the truck, and therefore the truck's stopping distance will be smaller, from Eq. 2–11c.

22. Assume your weight is W. If you weighed yourself on an inclined plane that is inclined at angle θ, then the bathroom scale would read the magnitude of the normal force between you and the plane, which would be $W \cos \theta$.

Responses to MisConceptual Questions

1. (*a*) The crate does not accelerate up or down, so the net force cannot be vertical. The truck bed is frictionless and the crate is not in contact with any other surface, so there are no horizontal forces. Therefore, no net force acts on the crate. As the truck slows down, the crate continues to move forward at constant speed. (How did the crate stay on the truck in the first place to be able to travel on the truck bed?)

2. (*a*, *b*, *d*) The forces in (*a*), (*b*), and (*d*) are all equal to 400 N in magnitude.
   (*a*) You exert a force of 400 N on the car; by Newton's third law the force exerted by the car on you also has a magnitude of 400 N.
   (*b*) Since the car doesn't move and the only horizontal forces acting on the car are your pushing and the force of friction on the car from the road, Newton's second law requires these forces to have equal magnitudes (400 N) in the opposite direction. Since the road exerts a force of 400 N on the car by friction, Newton's third law requires that the friction force on the road from the car must also be 400 N.
   (*c*) The normal force exerted by the road on you will be equal in magnitude to your weight (assuming you are standing vertically and have no vertical acceleration). This force is not required to be 400 N.

(*d*) The car is exerting a 400-N horizontal force on you, and since you are not accelerating, and the only horizontal forces acting on you are the force from the car and the frictional force from the ground, Newton's second law requires that the ground must be exerting an equal and opposite horizontal force. Therefore, the magnitude of the friction force exerted on you by the road is 400 N.

3. (*d*) For Matt and the truck to move forward from rest, both of them must experience a positive horizontal acceleration. The horizontal forces acting on Matt are the friction force of the ground pushing him forward and the truck pulling him backward. The ground must push Matt forward with a stronger force than the truck is pulling him back. The horizontal forces on the truck are from Matt pulling the truck forward and the friction from the ground pulling the truck backward. For the truck to accelerate forward, the force from Matt must be greater than the backward force of friction from the ground. By Newton's third law, the force of the truck on Matt and the force of Matt on the truck are equal and opposite. Since the force of the ground on Matt is greater than the force of the truck on Matt, the force of the truck on Matt is equal to the force of Matt on the truck, and the force of Matt on the truck is greater than the friction force of the ground on the truck, the ground exerts a greater friction on Matt than on the truck.

4. (*d*) In order to hold the backpack up, the rope must exert a vertical force equal to the backpack's weight, so that the net vertical force on the backpack is zero. The force, F, exerted by the rope on each side of the pack is always along the length of the rope. The vertical component of this force is $F \sin \theta$, where θ is the angle the rope makes with the horizontal. The higher the pack goes, the smaller θ becomes and the larger F must be to hold the pack up there. No matter how hard you pull, the rope can never be horizontal because it must exert an upward (vertical) component of force to balance the pack's weight.

5. (*c*) The boat accelerates forward by horizontal forces acting on the boat. The force that the man exerts on the paddles pushes the paddles forward, but because he is part of the boat this force does not accelerate the boat, so (*a*) is not correct. As the paddle pushes on the water it causes the water to accelerate backward. This force acts to accelerate the water, not the boat, so (*b*) is incorrect. By Newton's third law, as the paddles push the water backward, the water pushes the paddles (and thus the boat) forward. With the force of the water on the paddles pushing the boat forward, the boat would move even when the water was still, so (*d*) is also incorrect.

6. (*c*) The person's apparent weight is equal to the normal force acting on him. When the elevator is at rest or moving at constant velocity, the net force on the person is zero, so the normal force is equal to his weight. When the elevator is accelerating downward, the net force is also downward, so the normal force is less than his weight. When the elevator is accelerating upward, the net force is upward, and the normal force (his apparent weight) is greater than his weight. Since his actual weight does not change, his apparent weight is greatest when he is accelerating upward.

7. (*c*) The weight of the skier can be broken into components parallel to and perpendicular to the slope. The normal force will be equal to the perpendicular component of the skier's weight. For a nonzero slope, this component is always less than the weight of the skier.

8. (*b*) The force of the golf club acting on the ball acts only when the two objects are in contact, not as the ball flies through the air. The force of gravity acts on the ball throughout its flight. Air resistance is to be neglected, so there is no force acting on the ball due to its motion through the air.

9. (*c*) Since the net force is now zero, Newton's first law requires that the object will move in a straight line at constant speed. A net force would be needed to bring the object to rest.

10. (*d*) By Newton's third law, the force you exert on the box must be equal in magnitude to the force the box exerts on you. The box accelerates forward because the force you exert on the box is greater than other forces (such as friction) that are also exerted on the box.

11. (*b*) The maximum static friction force is 25 N. Since the applied force is less than this maximum, the crate will not accelerate, Newton's second law can be used to show that the resulting friction force will be equal in magnitude but opposite in direction to the applied force.

12. (*b*, *d*) The normal force between the skier and the snow is a contact force preventing the skier from passing through the surface of the snow. The normal force requires contact with the surface and an external net force toward the snow. The normal force does not depend upon the speed of the skier. Any slope less than 90° will have a component of gravity that must be overcome by the normal force.

13. (*a*) If the two forces pulled in the same direction, then the net force would be the maximum and equal to the sum of the two individual forces, or 950 N. Since the forces are not parallel, the net force will be less than this maximum.

Solutions to Problems

1. Use Newton's second law to calculate the force.

$$\sum F = ma = (55\text{ kg})(1.4\text{ m/s}^2) = \boxed{77\text{ N}}$$

2. In all cases, $W = mg$, where g changes with location.

(*a*) $W_{\text{Earth}} = mg_{\text{Earth}} = (68\text{ kg})(9.80\text{ m/s}^2) = \boxed{670\text{ N}}$

(*b*) $W_{\text{Moon}} = mg_{\text{Moon}} = (68\text{ kg})(1.7\text{ m/s}^2) = \boxed{120\text{ N}}$

(*c*) $W_{\text{Mars}} = mg_{\text{Mars}} = (68\text{ kg})(3.7\text{ m/s}^2) = \boxed{250\text{ N}}$

(*d*) $W_{\text{space}} = mg_{\text{space}} = (68\text{ kg})(0) = \boxed{0}$

3. Use Newton's second law to calculate the tension.

$$\sum F = F_{\text{T}} = ma = (1210\text{ kg})(1.20\text{ m/s}^2) = 1452\text{ N} \approx \boxed{1450\text{ N}}$$

4. The average acceleration of the blood is given by $a = \dfrac{v - v_0}{t} = \dfrac{0.35\text{ m/s} - 0.25\text{ m/s}}{0.10\text{ s}} = 1.0\text{ m/s}^2$.

The net force on the blood, exerted by the heart, is found from Newton's second law.

$$F = ma = (20 \times 10^{-3}\text{ kg})(1.0\text{ m/s}^2) = \boxed{0.02\text{ N}}$$

5. Find the average acceleration from Eq. 2–11c, and then find the force needed from Newton's second law. We assume the train is moving in the positive direction.

$$v = 0 \qquad v_0 = (120\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s} \qquad a_{\text{avg}} = \frac{v^2 - v_0^2}{2(x - x_0)}$$

$$F_{\text{avg}} = ma_{\text{avg}} = m\frac{v^2 - v_0^2}{2(x - x_0)} = (3.6 \times 10^5\text{ kg})\left[\frac{0 - (33.33\text{ m/s})^2}{2\ (150\text{ m})}\right] = -1.333 \times 10^6\text{ N} \approx \boxed{-1.3 \times 10^6\text{ N}}$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity. We compare the magnitude of this force to the weight of the train.

$$\frac{F_{avg}}{mg}=\frac{1.333\times10^6\,\mathrm{N}}{(3.6\times10^5\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}=0.3886$$

Thus the force is $\boxed{\text{39\% of the weight}}$ of the train.

By Newton's third law, the train exerts the same magnitude of force on Superman that Superman exerts on the train, but in the opposite direction. So the train exerts a force of $\boxed{1.3\times10^6\ \mathrm{N}}$ in the forward direction on Superman.

6. We assume that 30 g's has 2 significant figures. The acceleration of a person having a 30 "g" deceleration is $a=(30g)\left(\frac{9.80\ \mathrm{m/s^2}}{1g}\right)=294\ \mathrm{m/s^2}$. The average force causing that acceleration is $F=ma=(65\ \mathrm{kg})(294\ \mathrm{m/s^2})=\boxed{1.9\times10^4\ \mathrm{N}}$. Since the person is undergoing a deceleration, the acceleration and force would both be directed opposite to the direction of motion. Use Eq. 2–11c to find the distance traveled during the deceleration. Take the initial velocity to be in the positive direction, so that the acceleration will have a negative value, and the final velocity will be 0.

$$v_0=(95\ \mathrm{km/h})\left(\frac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right)=26.39\ \mathrm{m/s}$$

$$v^2-v_0^2=2a(x-x_0)\quad\rightarrow\quad(x-x_0)=\frac{v^2-v_0^2}{2a}=\frac{0-(26.39\ \mathrm{m/s})^2}{2(-294\ \mathrm{m/s^2})}=1.18\ \mathrm{m}\approx\boxed{1.2\ \mathrm{m}}$$

7. Find the average acceleration from Eq. 2–4. The average force on the car is found from Newton's second law.

$$v=0\quad v_0=(95\ \mathrm{km/h})\left(\frac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right)=26.39\ \mathrm{m/s}\quad a_{avg}=\frac{v-v_0}{t}=\frac{0-26.39\ \mathrm{m/s}}{8.0\ \mathrm{s}}=-3.299\ \mathrm{m/s^2}$$

$$F_{avg}=ma_{avg}=(950\ \mathrm{kg})(-3.299\ \mathrm{m/s^2})=-3134\ \mathrm{N}\approx\boxed{-3100\ \mathrm{N}}$$

The negative sign indicates the direction of the force, in the opposite direction to the initial velocity.

8. Find the average acceleration from Eq. 2–11c, and then find the force needed from Newton's second law.

$$a_{avg}=\frac{v^2-v_0^2}{2(x-x_0)}\quad\rightarrow$$

$$F_{avg}=ma_{avg}=m\frac{v^2-v_0^2}{2(x-x_0)}=(7.0\ \mathrm{kg})\left[\frac{(13\ \mathrm{m/s})^2-0}{2\ (2.8\ \mathrm{m})}\right]=211.25\,\mathrm{N}\approx\boxed{210\,\mathrm{N}}$$

9. The problem asks for the average force on the glove, which in a direct calculation would require knowledge about the mass of the glove and the acceleration of the glove. But no information about the glove is given. By Newton's third law, the force exerted by the ball on the glove is equal and opposite

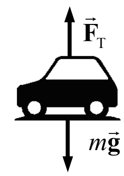
to the force exerted by the glove on the ball. So we calculate the average force on the ball, and then take the opposite of that result to find the average force on the glove. The average force on the ball is its mass times its average acceleration. Use Eq. 2–11c to find the acceleration of the ball, with $\upsilon = 0$, $\upsilon_0 = 35.0$ m/s, and $x - x_0 = 0.110$ m. The initial velocity of the ball is the positive direction.

$$a_{\text{avg}} = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)} = \frac{0 - (35.0\text{ m/s})^2}{2(0.110\text{ m})} = -5568\text{ m/s}^2$$

$$F_{\text{avg}} = ma_{\text{avg}} = (0.140\text{ kg})(-5568\text{ m/s}^2) = -7.80\times10^2\text{ N}$$

Thus the average force on the glove was $\boxed{\text{780 N, in the direction of the initial velocity of the ball.}}$

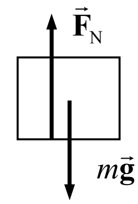
10. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the tension force.



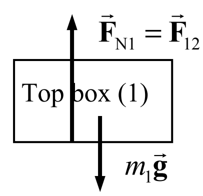
$$\sum F = F_\text{T} - mg = ma \quad \rightarrow \quad F_\text{T} = m(g + a)$$

$$F_\text{T} = (1200\text{ kg})(9.80\text{ m/s}^2 + 0.70\text{ m/s}^2) = \boxed{1.3\times10^4\text{ N}}$$

11. (*a*) The 20.0-kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{196\text{ N}}$. Since the box is at rest, the net force on the box must be 0, so the normal force must also be $\boxed{196\text{ N}}$.



(*b*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's third law. $\vec{\mathbf{F}}_{\text{N}2}$ is the force of the table on box 2. That is the normal force on box 2. Since both boxes are at rest, the net force on each box must be 0. Write Newton's second law in the vertical direction for each box, taking the upward direction to be positive.

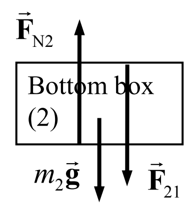


$$\sum F_1 = F_{\text{N}1} - m_1 g = 0$$

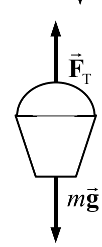
$$F_{\text{N}1} = m_1 g = (10.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{98.0\text{ N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\text{N}2} - F_{21} - m_2 g = 0$$

$$F_{\text{N}2} = F_{21} + m_2 g = 98.0\text{ N} + (20.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{294\text{ N}}$$



12. Choose up to be the positive direction. Write Newton's second law for the vertical direction, and solve for the acceleration.

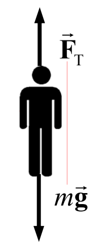


$$\sum F = F_\text{T} - mg = ma$$

$$a = \frac{F_\text{T} - mg}{m} = \frac{163\text{ N} - (14.0\text{ kg})(9.80\text{ m/s}^2)}{14.0\text{ kg}} = \boxed{1.8\text{ m/s}^2}$$

Since the acceleration is positive, the bucket has an $\boxed{\text{upward}}$ acceleration.

13. If the thief were to hang motionless on the sheets, or descend at a constant speed, the sheets would not support him, because they would have to support the full 75 kg. But if he descends with an acceleration, the sheets will not have to support the total mass. A free-body diagram of the thief in descent is shown. If the sheets can support a mass of 58 kg, then the tension force that the sheets can exert is $F_T = (58 \text{ kg})(9.80 \text{ m/s}^2) = 568 \text{ N}$. Assume that is the tension in the sheets. Then write Newton's second law for the thief, taking the upward direction to be positive.

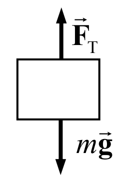


$$\sum F = F_T - mg = ma \quad \rightarrow \quad a = \frac{F_T - mg}{m} = \frac{568 \text{ N} - (75 \text{ kg})(9.80 \text{ m/s}^2)}{75 \text{ kg}} = -2.2 \text{ m/s}^2$$

The negative sign shows that the acceleration is downward.

$\boxed{\text{If the thief descends with an acceleration of } 2.2 \text{ m/s}^2 \text{ or greater, the sheets will support his descent.}}$

14. In both cases, a free-body diagram for the elevator would look like the adjacent diagram. Choose up to be the positive direction. To find the MAXIMUM tension, assume that the acceleration is up. Write Newton's second law for the elevator.



$$\sum F = ma = F_T - mg \quad \rightarrow$$
$$F_T = ma + mg = m(a+g) = m(0.0680g + g) = (4850 \text{ kg})(1.0680)(9.80 \text{ m/s}^2)$$
$$= \boxed{5.08 \times 10^4 \text{ N}}$$

To find the MINIMUM tension, assume that the acceleration is down. Then Newton's second law for the elevator becomes the following.

$$\sum F = ma = F_T - mg \quad \rightarrow \quad F_T = ma + mg = m(a+g) = m(-0.0680g + g)$$
$$= (4850 \text{ kg})(0.9320)(9.80 \text{ m/s}^2) = \boxed{4.43 \times 10^4 \text{ N}}$$

15. Use Eq. 2–11c to find the acceleration. The starting speed is $35 \text{ km/h}\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 9.72 \text{ m/s}$.

$$v^2 = v_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (9.72 \text{ m/s})^2}{2(0.017 \text{ m})} = -2779 \text{ m/s}^2 \approx \boxed{-2800 \text{ m/s}^2}$$

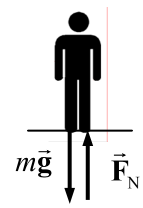
$$2779 \text{ m/s}^2\left(\frac{1\,g}{9.80 \text{ m/s}^2}\right) = 284\ g\text{'s} \approx \boxed{280\ g\text{'s}}$$

The acceleration is negative because the car is slowing down. The required force is found by Newton's second law.

$$F = ma = (68 \text{ kg})(2779 \text{ m/s}^2) = \boxed{1.9 \times 10^5 \text{ N}}$$

This huge acceleration would not be possible unless the car hit some very heavy, stable object.

16. There will be two forces on the woman—her weight, and the normal force of the scales pushing up on her. A free-body diagram for the woman is shown. Choose up to be the positive direction, and use Newton's second law to find the acceleration.

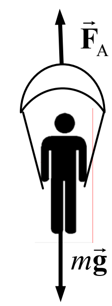


$$\sum F = F_N - mg = ma \quad \rightarrow \quad 0.75\ mg - mg = ma \quad \rightarrow$$

$$a = -0.25\ g = -0.25(9.8\ \text{m/s}^2) = \boxed{-2.5\ \text{m/s}^2}$$

Due to the sign of the result, the direction of the acceleration is $\boxed{\text{down.}}$ Thus the elevator must have started to move down since it had been motionless.

17. (*a*) There will be two forces on the sky divers—their combined weight and the upward force of air resistance, $\vec{F}_A$. Choose up to be the positive direction. Write Newton's second law for the sky divers.



$$\sum F = F_A - mg = ma \quad \rightarrow \quad 0.25\ mg - mg = ma \quad \rightarrow$$

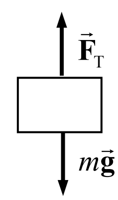
$$a = -0.75\ g = -0.75(9.80\ \text{m/s}^2) = \boxed{-7.35\ \text{m/s}^2}$$

Due to the sign of the result, the direction of the acceleration is down.

(*b*) If they are descending at constant speed, then the net force on them must be zero, so the force of air resistance must be equal to their weight.

$$F_A = mg = (132\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{1290\ \text{N}}$$

18. Choose UP to be the positive direction. Write Newton's second law for the elevator.



$$\sum F = F_T - mg = ma \quad \rightarrow$$

$$a = \frac{F_T - mg}{m} = \frac{21{,}750\ \text{N} - (2125\ \text{kg})(9.80\ \text{m/s}^2)}{2125\ \text{kg}} = 0.4353\ \text{m/s}^2 \approx \boxed{0.44\ \text{m/s}^2}$$

19. (*a*) Use Eq. 2–11c to find the speed of the person just before he strikes the ground. Take down to be the positive direction. For the person, $v_0 = 0$, $y - y_0 = 2.8$ m, and $a = 9.80\ \text{m/s}^2$.

$$v^2 - v_0^2 = 2a(y - y_0) \rightarrow$$

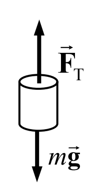
$$v = \sqrt{2a(y - y_0)} = \sqrt{2(9.80\ \text{m/s}^2)(2.8\ \text{m})} = 7.408\ \text{m/s} \approx \boxed{7.4\ \text{m/s}}$$

(*b*) For the deceleration, use Eq. 2–11c to find the average deceleration, choosing down to be positive.

$$v_0 = 8.743\ \text{m/s} \qquad v = 0 \qquad y - y_0 = 0.70\ \text{m} \qquad v^2 - v_0^2 = 2a(y - y_0) \quad \rightarrow$$

$$a = \frac{-v_0^2}{2\Delta y} = \frac{-(7.408\ \text{m/s})^2}{2(0.70\ \text{m})} = -39.2\ \text{m/s}^2$$

The average force on the torso (F_T) due to the legs is found from Newton's second law. See the free-body diagram. Down is positive.

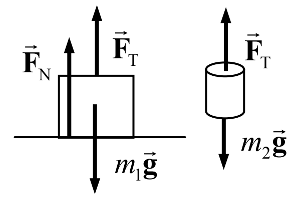


$$F_{net} = mg - F_T = ma \quad \rightarrow$$

$$F_T = mg - ma = m(g - a) = (42\ \text{kg})[9.80\ \text{m/s}^2 - (-39.2\ \text{m/s}^2)] = \boxed{2100\ \text{N}}$$

The force is upward.

20. Free-body diagrams for the box and the weight are shown. The tension exerts the same magnitude of force on both objects.



(*a*) If the weight of the hanging weight is less than the weight of the box, the objects will not move, and the tension will be the same as the weight of the hanging weight. The acceleration of the box will also be zero, so the sum of the forces on it will be zero. For the box,

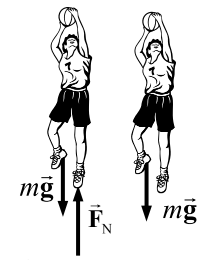
$$F_N + F_T - m_1 g = 0 \quad \rightarrow \quad F_N = m_1 g - F_T = m_1 g - m_2 g = 77.0\text{ N} - 30.0\text{ N} = \boxed{47.0\text{ N}}$$

(*b*) The same analysis as for part (*a*) applies here.

$$F_N = m_1 g - m_2 g = 77.0\text{ N} - 60.0\text{ N} = \boxed{17.0\text{ N}}$$

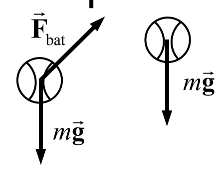
(*c*) Since the hanging weight has more weight than the box on the table, the box on the table will be lifted up off the table, and normal force of the table on the box will be $\boxed{0}$.

21. (*a*) Just before the player leaves the ground, the forces on the player are his weight and the floor pushing up on the player. If the player jumps straight up, then the force of the floor will be straight up—a normal force. See the first diagram. In this case, while touching the floor, $F_N > mg$.



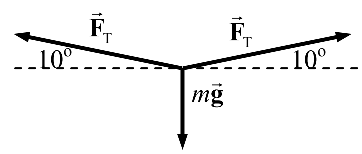
(*b*) While the player is in the air, the only force on the player is his weight. See the second diagram.

22. (*a*) Just as the ball is being hit, if we ignore air resistance, there are two main forces on the ball: the weight of the ball and the force of the bat on the ball.



(*b*) As the ball flies toward the outfield, the only force on it is its weight, if air resistance is ignored.

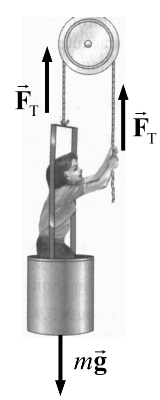
$\boxed{23.}$ Consider the point in the rope directly below Arlene. That point can be analyzed as having three forces on it—Arlene's weight, the tension in the rope toward the right point of connection, and the tension in the rope toward the left point of connection. Assuming the rope is massless, those two tensions will be of the same magnitude. Since the point is not accelerating, the sum of the forces must be zero. In particular, consider the sum of the vertical forces on that point, with UP as the positive direction.



$$\sum F = F_T \sin 10.0^\circ + F_T \sin 10.0^\circ - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2\sin 10.0^\circ} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0^\circ} = \boxed{1410\text{ N}}$$

24. The window washer pulls down on the rope with her hands with a tension force F_T, so the rope pulls up on her hands with a tension force F_T. The tension in the rope is also applied at the other end of the rope, where it attaches to the bucket. Thus there is another force F_T pulling up on the bucket. The bucket–washer combination thus has a net force of $2F_T$ upward. See the adjacent free-body diagram, showing only forces <u>on</u> the bucket–washer combination, not forces exerted <u>by</u> the combination (the pull down on the rope by the person) or internal forces (normal force of bucket on person).



(*a*) Write Newton's second law in the vertical direction, with up as positive. The net force must be zero if the bucket and washer have a constant speed.

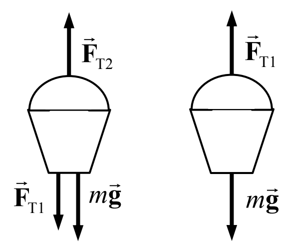
$$\sum F = F_{\mathrm{T}} + F_{\mathrm{T}} - mg = 0 \rightarrow 2F_{\mathrm{T}} = mg \rightarrow$$

$$F_{\mathrm{T}} = \frac{1}{2}mg = \frac{1}{2}(72\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 352.8\,\mathrm{N} \approx \boxed{350\,\mathrm{N}}$$

(*b*) Now the force is increased by 15%, so $F_{\mathrm{T}} = 358.2\ \mathrm{N}(1.15) = 405.72\,\mathrm{N}$. Again write Newton's second law, but with a nonzero acceleration.

$$\sum F = F_{\mathrm{T}} + F_{\mathrm{T}} - mg = ma \quad \rightarrow$$

$$a = \frac{2F_{\mathrm{T}} - mg}{m} = \frac{2(405.72\,\mathrm{N}) - (72\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{72\ \mathrm{kg}} = 1.47\ \mathrm{m/s^2} \approx \boxed{1.5\ \mathrm{m/s^2}}$$



$\boxed{25.}$ We draw free-body diagrams for each bucket.

(*a*) Since the buckets are at rest, their acceleration is 0. Write Newton's second law for each bucket, calling UP the positive direction.

$$\sum F_1 = F_{\mathrm{T1}} - mg = 0 \quad \rightarrow$$

$$F_{\mathrm{T1}} = mg = (3.2\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = \boxed{31\,\mathrm{N}}$$

$$\sum F_2 = F_{\mathrm{T2}} - F_{\mathrm{T1}} - mg = 0 \quad \rightarrow$$

$$F_{\mathrm{T2}} = F_{\mathrm{T1}} + mg = 2\ mg = 2(3.2\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = \boxed{63\ \mathrm{N}}$$

(*b*) Now repeat the analysis, but with a nonzero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{\mathrm{T1}} - mg = ma \quad \rightarrow$$

$$F_{\mathrm{T1}} = mg + ma = (3.2\ \mathrm{kg})(9.80\ \mathrm{m/s^2} + 1.25\ \mathrm{m/s^2}) = 35.36\,\mathrm{N} \approx \boxed{35\,\mathrm{N}}$$

$$\sum F_2 = F_{\mathrm{T2}} - F_{\mathrm{T1}} - mg = ma \quad \rightarrow \quad F_{\mathrm{T2}} = F_{\mathrm{T1}} + mg + ma = 2F_{\mathrm{T1}} = \boxed{71\,\mathrm{N}}$$

26. Choose the *y* direction to be the "forward" direction for the motion of the snowcats and the *x* direction to be to the right on the diagram in the textbook. Since the housing unit moves in the forward direction on a straight line, there is no acceleration in the *x* direction, so the net force in the *x* direction must be 0. Write Newton's second law for the *x* direction.

$$\sum F_x = F_{\mathrm{A}x} + F_{\mathrm{B}x} = 0 \quad \rightarrow \quad -F_{\mathrm{A}}\ \sin 48° + F_{\mathrm{B}}\ \sin 32° = 0 \quad \rightarrow$$

$$F_{\mathrm{B}} = \frac{F_{\mathrm{A}}\ \sin 48°}{\sin 32°} = \frac{(4500\,\mathrm{N})\ \sin 48°}{\sin 32°} = 6311\,\mathrm{N} \approx \boxed{6300\,\mathrm{N}}$$

Since the *x* components add to 0, the magnitude of the vector sum of the two forces will just be the sum of their *y* components.

$$\sum F_y = F_{\mathrm{A}y} + F_{\mathrm{B}y} = F_{\mathrm{A}}\ \cos 48° + F_{\mathrm{B}}\ \cos 32° = (4500\ \mathrm{N})\ \cos 48° + (6311\ \mathrm{N})\ \cos 32°$$

$$= 8363\ \mathrm{N} \approx \boxed{8400\ \mathrm{N}}$$

27. Since all forces of interest in this problem are horizontal, draw the free-body diagram showing only the horizontal forces. $\vec{\mathbf{F}}_{\mathrm{T1}}$ is the tension in the coupling between the locomotive and the first car, and it pulls to the right on the first car. $\vec{\mathbf{F}}_{\mathrm{T2}}$ is the tension in the coupling between the first car and the second

car. It pulls to the right on car 2, labeled $\vec{\mathbf{F}}_{\text{T2R}}$ and to the left on car 1, labeled $\vec{\mathbf{F}}_{\text{T2L}}$. Both cars have the same mass m and the same acceleration a. Note that $|\vec{\mathbf{F}}_{\text{T2R}}| = |\vec{\mathbf{F}}_{\text{T2L}}| = F_{\text{T2}}$ by Newton's third law.

Write a Newton's second law expression for each car.

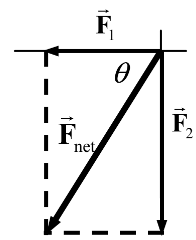
$$\sum F_1 = F_{\text{T1}} - F_{\text{T2}} = ma \qquad \sum F_2 = F_{\text{T2}} = ma$$

Substitute the expression for ma from the second expression into the first one.

$$F_{\text{T1}} - F_{\text{T2}} = ma = F_{\text{T2}} \quad \rightarrow \quad F_{\text{T1}} = 2F_{\text{T2}} \quad \rightarrow \quad \boxed{F_{\text{T1}}/F_{\text{T2}} = 2}$$

This can also be discussed in the sense that the tension between the locomotive and the first car is pulling two cars, while the tension between the cars is only pulling one car.

28. The net force in each case is found by vector addition with components.



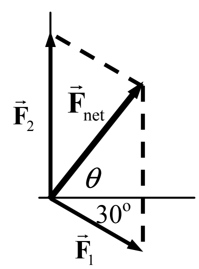
(*a*) $F_{\text{net}\,x} = -F_1 = -10.2\text{ N} \qquad F_{\text{net}\,y} = -F_2 = -16.0\text{ N}$

$$F_{\text{net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = 19.0\text{ N} \qquad \theta = \tan^{-1}\frac{-16.0}{-10.2} = 57.48^\circ$$

The actual angle from the x axis is then 237.48°. Thus the net force is

$$F_{\text{net}} = \boxed{19.0\text{ N at }237^\circ}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{19.0\text{ N}}{18.5\text{ kg}} = \boxed{1.03\text{ m/s}^2\text{ at }237^\circ}$$

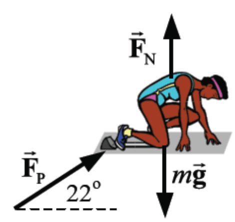


(*b*) $F_{\text{net}\,x} = F_1 \cos 30^\circ = 8.833\text{ N} \qquad F_{\text{net}\,y} = F_2 - F_1 \sin 30^\circ = 10.9\text{ N}$

$$F_{\text{net}} = \sqrt{(8.833\text{ N})^2 + (10.9\text{ N})^2} = 14.03\text{N} \approx \boxed{14.0\text{ N}}$$

$$\theta = \tan^{-1}\frac{10.9}{8.833} = \boxed{51.0^\circ} \qquad a = \frac{F_{\text{net}}}{m} = \frac{14.03\text{ N}}{18.5\text{ kg}} = \boxed{0.758\text{ m/s}^2\text{ at }51.0^\circ}$$

29. Since the sprinter exerts a force of 720 N on the ground at an angle of 22° below the horizontal, by Newton's third law the ground will exert a force of 720 N on the sprinter at an angle of 22° above the horizontal. A free-body diagram for the sprinter is shown.



(*a*) The horizontal acceleration will be found from the net horizontal force. Using Newton's second law, we have the following:

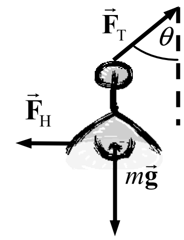
$$\sum F_x = F_{\text{P}} \cos 22^\circ = ma_x \quad \rightarrow$$

$$a_x = \frac{F_{\text{P}} \cos 22^\circ}{m} = \frac{(720\text{ N})\cos 22^\circ}{65\text{ kg}} = 10.27\text{ m/s}^2 \approx \boxed{1.0\times 10^1\text{ m/s}^2}$$

(*b*) Eq. 2–11a is used to find the final speed. The starting speed is 0.

$$v = v_0 + at \quad \rightarrow \quad v = 0 + at = (10.27\text{ m/s}^2)(0.32\text{ s}) = 3.286\text{ m/s} \approx \boxed{3.3\text{ m/s}}$$

30. We use the free-body diagram with Newton's first law for the stationary chandelier to find the forces in question. The angle is found from the horizontal displacement and the length of the wire.



(*a*) $\theta = \sin^{-1}\dfrac{0.15\text{ m}}{4.0\text{ m}} = 2.15°$

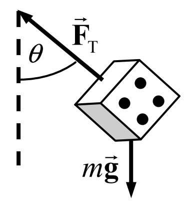
$$F_{\substack{\text{net}\\x}} = F_T \sin\theta - F_H = 0 \quad\to\quad F_H = F_T \sin\theta$$

$$F_{\substack{\text{net}\\y}} = F_T \cos\theta - mg = 0 \quad\to\quad F_T = \frac{mg}{\cos\theta} \quad\to$$

$$F_H = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (27\text{ kg})(9.80\text{ m/s}^2)\tan 2.15° = \boxed{9.9\text{ N}}$$

(*b*) $F_T = \dfrac{mg}{\cos\theta} = \dfrac{(27\text{ kg})(9.80\text{ m/s}^2)}{\cos 2.15°} = \boxed{260\text{ N}}$

31. (*a*) Consider a free-body diagram of the object. The car is moving to the right (the positive direction) and slowing down. Thus the acceleration and the net force are to the left. The acceleration of the object is found from Eq. 2–11a.



$$v = v_0 + = a_x t \quad\to\quad a_x = \frac{v - v_0}{t} = \frac{0 - 25\text{ m/s}}{6.0\text{ s}} = -4.17\text{ m/s}^2$$

Now write Newton's second law for both the vertical (y) and horizontal (x) directions.

$$\sum F_y = F_T\cos\theta - mg = 0 \quad\to\quad F_T = \frac{mg}{\cos\theta} \qquad \sum F_x = -F_T\sin\theta = ma_x$$

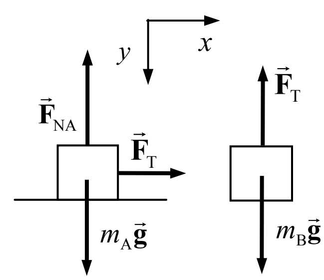
Substitute the expression for the tension from the y equation into the x equation.

$$ma_x = -F_T\sin\theta = -\frac{mg}{\cos\theta}\sin\theta = -mg\tan\theta \quad\to\quad a_x = -g\tan\theta$$

$$\theta = \tan^{-1}\frac{-a_x}{g} = \tan^{-1}\frac{4.17\text{ m/s}^2}{9.80\text{ m/s}^2} = \boxed{23°}$$

(*b*) The angle is $\boxed{\text{toward}}$ the windshield.

32. (*a*) See the free-body diagrams included.



(*b*) For block A, since there is no motion in the vertical direction, we have $F_{NA} = m_A g$. We write Newton's second law for the x direction: $\sum F_{Ax} = F_T = m_A a_{Ax}$. For block B, we only need to consider vertical forces: $\sum F_{By} = m_B g - F_T = m_B a_{By}$. Since the two blocks are connected, the magnitudes of their accelerations will be the same, so let $a_{Ax} = a_{By} = a$. Combine the two force equations from above, and solve for a by substitution.

$$F_T = m_A a \qquad m_B g - F_T = m_B a \quad\to\quad m_B g - m_A a = m_B a \quad\to$$

$$m_A a + m_B a = m_B g \quad\to\quad \boxed{a = g\frac{m_B}{m_A + m_B} \qquad F_T = m_A a = g\frac{m_A m_B}{m_A + m_B}}$$

33. (*a*) From Problem 32, we have the acceleration of each block. Both blocks have the same acceleration.

$$a = g\frac{m_B}{m_A + m_B} = (9.80 \text{ m/s}^2)\frac{5.0 \text{ kg}}{(5.0 \text{ kg} + 13.0 \text{ kg})} = 2.722 \text{ m/s}^2 \approx \boxed{2.7 \text{ m/s}^2}$$

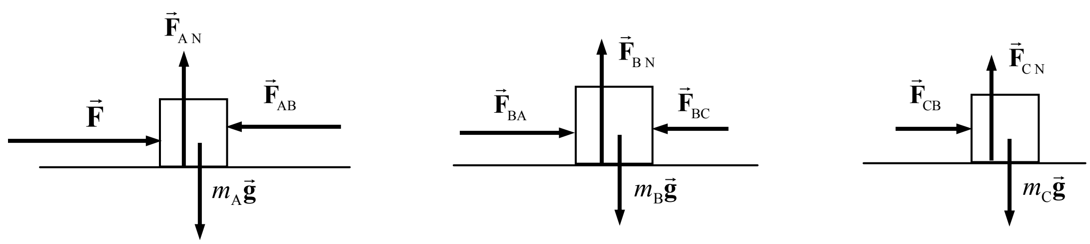
(*b*) Use Eq. 2–11b to find the time.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(1.250 \text{ m})}{(2.722 \text{ m/s}^2)}} = \boxed{0.96 \text{ s}}$$

(*c*) Again use the acceleration from Problem 32.

$$a = g\frac{m_B}{m_A + m_B} = \tfrac{1}{100}g \rightarrow \frac{m_B}{m_A + m_B} = \tfrac{1}{100} \rightarrow m_A = 99m_B = \boxed{99 \text{ kg}}$$

34. (*a*) In the free-body diagrams below, $\vec{\mathbf{F}}_{AB}$ = force on block A exerted by block B, $\vec{\mathbf{F}}_{BA}$ = force on block B exerted by block A, $\vec{\mathbf{F}}_{BC}$ = force on block B exerted by block C, and $\vec{\mathbf{F}}_{CB}$ = force on block C exerted by block B. The magnitudes of $\vec{\mathbf{F}}_{BA}$ and $\vec{\mathbf{F}}_{AB}$ are equal, and the magnitudes of $\vec{\mathbf{F}}_{BC}$ and $\vec{\mathbf{F}}_{CB}$ are equal, by Newton's third law.



(*b*) All of the vertical forces on each block add up to zero, since there is no acceleration in the vertical direction. Thus, for each block, $F_N = mg$. For the horizontal direction, we have the following:

$$\sum F = F - F_{AB} + F_{BA} - F_{BC} + F_{CB} = F = (m_A + m_B + m_C)a \rightarrow \boxed{a = \frac{F}{m_A + m_B + m_C}}$$

(*c*) For each block, the net force must be *ma* by Newton's second law. Each block has the same acceleration since the blocks are in contact with each other.

$$\boxed{F_{A\,\text{net}} = F\frac{m_A}{m_A + m_B + m_C}} \qquad \boxed{F_{B\,\text{net}} = F\frac{m_B}{m_A + m_B + m_C}} \qquad \boxed{F_{C\,\text{net}} = F\frac{m_C}{m_A + m_B + m_C}}$$

(*d*) From the free-body diagram, we see that for m_C, $F_{CB} = F_{C\,\text{net}} = \boxed{F\frac{m_C}{m_A + m_B + m_C}}$. And by Newton's third law, $F_{BC} = F_{CB} = \boxed{F\frac{m_C}{m_A + m_B + m_C}}$. Of course, $\vec{\mathbf{F}}_{23}$ and $\vec{\mathbf{F}}_{32}$ are in opposite directions. Also from the free-body diagram, we use the net force on m_A.

$$F - F_{AB} = F_{A\,\text{net}} = F\frac{m_A}{m_A + m_B + m_C} \rightarrow F_{AB} = F - F\frac{m_A}{m_A + m_B + m_C} \rightarrow$$

$$\boxed{F_{AB} = F\frac{m_B + m_C}{m_A + m_B + m_C}}$$

By Newton's third law, $F_{BA} = F_{AB} = \boxed{F\dfrac{m_B + m_C}{m_A + m_B + m_C}}$.

(*e*) Using the given values, $a = \dfrac{F}{m_A + m_B + m_C} = \dfrac{96.0\text{ N}}{30.0\text{ kg}} = \boxed{3.20\text{ m/s}^2}$. Since all three masses are the same value, the net force on each mass is $F_{net} = ma = (10.0\text{ kg})(3.20\text{ m/s}^2) = 32.0\text{ N}$. This is also the value of F_{CB} and F_{BC}. The value of F_{AB} and F_{BA} is found as follows:

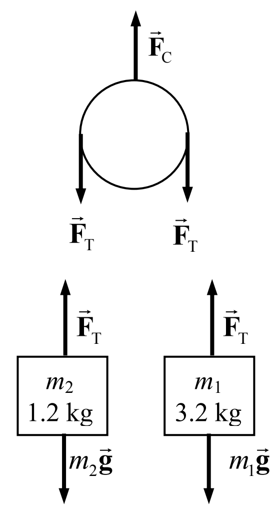
$$F_{AB} = F_{BA} = (m_B + m_C)a = (20.0\text{ kg})(3.20\text{ m/s}^2) = 64.0\text{ N}$$

To summarize:

$$F_{A\text{ net}} = F_{B\text{ net}} = F_{C\text{ net}} = \boxed{32.0\text{ N}} \qquad F_{AB} = F_{BA} = \boxed{64.0\text{ N}} \qquad F_{BC} = F_{CB} = \boxed{32.0\text{ N}}$$

The values make sense in that in order of magnitude, we should have $F > F_{BA} > F_{CB}$, since F is the net force pushing the entire set of blocks, F_{AB} is the net force pushing the right two blocks, and F_{BC} is the net force pushing the right block only.

$\boxed{35.}$ We draw a free-body diagram for each mass. We choose up to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.



$$F_T - m_1 g = m_1 a_1 \qquad F_T - m_2 g = m_2 a_2$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the tension force.

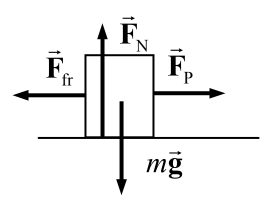
$$F_T - m_1 g = -m_1 a_2 \quad\rightarrow\quad F_T = m_1 g - m_1 a_2 \quad\rightarrow\quad a_2 = \frac{m_1 g - F_T}{m_1}$$

$$F_T - m_2 g = m_2 a_2 = m_2\left(\frac{m_1 g - F_T}{m_1}\right) \quad\rightarrow\quad F_T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

Apply Newton's second law to the stationary pulley.

$$F_C - 2F_T = 0 \quad\rightarrow\quad F_C = 2F_T = \frac{4m_1 m_2 g}{m_1 + m_2} = \frac{4(3.2\text{ kg})(1.2\text{ kg})(9.80\text{ m/s}^2)}{4.4\text{ kg}} = \boxed{34\text{ N}}$$

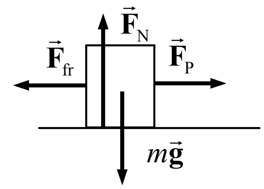
36. A free-body diagram for the crate is shown. The crate does not accelerate vertically, so $F_N = mg$. The crate does not accelerate horizontally, so $F_P = F_{fr}$.



$$F_P = F_{fr} = \mu_k F_N = \mu_k mg = (0.30)(22\text{ kg})(9.80\text{ m/s}^2) = \boxed{65\text{ N}}$$

If the coefficient of kinetic friction is zero, then the horizontal force required is $\boxed{0}$, since there is no friction to counteract. Of course, it would take a force to START the crate moving, but once it was moving, no further horizontal force would be necessary to maintain the motion.

37. A free-body diagram for the box is shown. Since the box does not accelerate vertically, $F_N = mg$.



(*a*) To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of $F_{fr} = \mu_s F_N$. Thus, we have for the starting motion,

$$\sum F_x = F_P - F_{fr} = 0 \quad \to$$

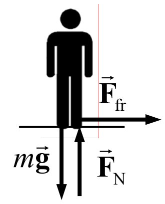
$$F_P = F_{fr} = \mu_s F_N = \mu_s mg \quad \to \quad \mu_s = \frac{F_P}{mg} = \frac{35.0\text{ N}}{(6.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.60}$$

(*b*) The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

$$\sum F = F_P - F_{fr} = ma \quad \to \quad F_P - \mu_k F_N = ma \quad \to \quad F_P - \mu_k mg = ma \quad \to$$

$$\mu_k = \frac{F_P - ma}{mg} = \frac{35.0\text{ N} - (6.0\text{ kg})(0.60\text{ m/s}^2)}{(6.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.53}$$

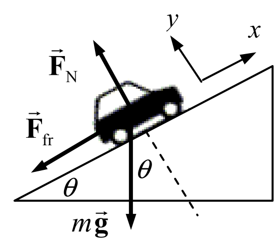
38. A free-body diagram for you as you stand on the train is shown. You do not accelerate vertically, so $F_N = mg$. The maximum static frictional force is $\mu_s F_N$, and that must be greater than or equal to the force needed to accelerate you in order for you not to slip.



$$F_{fr} \ge ma \quad \to \quad \mu_s F_N \ge ma \quad \to \quad \mu_s mg \ge ma \quad \to \quad \mu_s \ge a/g = 0.20g/g = \boxed{0.20}$$

The static coefficient of friction must be at least 0.20 for you not to slide.

39. See the adjacent free-body diagram. To find the maximum angle, assume that the car is just ready to slide, so that the force of static friction is a maximum. Write Newton's second law for both directions. Note that for both directions, the net force must be zero since the car is not accelerating.

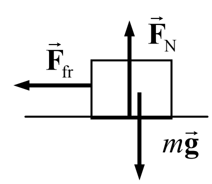


$$\sum F_y = F_N - mg\cos\theta = 0 \quad \to \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = 0 \quad \to \quad mg\sin\theta = F_{fr} = \mu_s F_N = \mu_s mg\cos\theta$$

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = 0.90 \quad \to \quad \theta = \tan^{-1}0.90 = \boxed{42^\circ}$$

40. The force of static friction is what decelerates the crate if it is not sliding on the truck bed. If the crate is not to slide, but the maximum deceleration is desired, then the maximum static frictional force must be exerted, so $F_{fr} = \mu_s F_N$. The direction of travel is to the right. It is apparent that $F_N = mg$ since there is no acceleration in the *y* direction. Write Newton's second law for the truck in the horizontal direction.



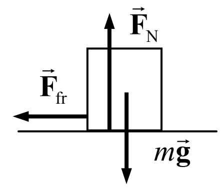
$$\sum F_x = -F_{fr} = ma \quad \to \quad -\mu_s mg = ma \quad \to \quad a = -\mu_s g = -(0.75)(9.80\text{ m/s}^2) = \boxed{-7.4\text{ m/s}^2}$$

The negative sign indicates the direction of the acceleration, as opposite to the direction of motion.

41. Since the drawer moves with the applied force of 9.0 N, we assume that the maximum static frictional force is essentially 9.0 N. This force is equal to the coefficient of static friction times the normal force. The normal force is assumed to be equal to the weight, since the drawer is horizontal.

$$F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}} = \mu_{\text{s}} mg \quad \rightarrow \quad \mu_{\text{s}} = \frac{F_{\text{fr}}}{mg} = \frac{9.0\text{ N}}{(2.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.46}$$

42. A free-body diagram for the box is shown, assuming that it is moving to the right. The "push" is not shown on the free-body diagram because as soon as the box moves away from the source of the pushing force, the push is no longer applied to the box. It is apparent from the diagram that $F_{\text{N}} = mg$ for the vertical direction. We write Newton's second law for the horizontal direction, with positive to the right, to find the acceleration of the box.

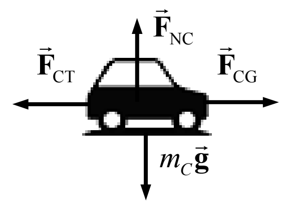


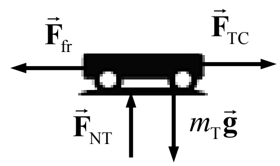
$$\sum F_x = -F_{\text{fr}} = ma \quad \rightarrow \quad ma = -\mu_{\text{k}} F_{\text{N}} = -\mu_{\text{k}} mg \quad \rightarrow$$
$$a = -\mu_{\text{k}} g = -0.15(9.80\text{ m/s}^2) = -1.47\text{ m/s}^2$$

Eq. 2–11c can be used to find the distance that the box moves before stopping. The initial speed is 4.0 m/s, and the final speed will be 0.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (3.5\text{ m/s})^2}{2(-1.47\text{ m/s}^2)} = 4.17\text{ m} \approx \boxed{4.2\text{ m}}$$

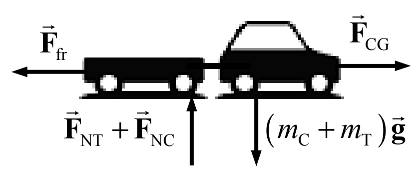
43. We draw three free-body diagrams—one for the car, one for the trailer, and then "add" them for the combination of car and trailer. Note that since the car pushes against the ground, the ground will push against the car with an equal but oppositely directed force. $\vec{\mathbf{F}}_{\text{CG}}$ is the force on the car due to the ground, $\vec{\mathbf{F}}_{\text{TC}}$ is the force on the trailer due to the car, and $\vec{\mathbf{F}}_{\text{CT}}$ is the force on the car due to the trailer. Note that by Newton's third law, $|\vec{\mathbf{F}}_{\text{CT}}| = |\vec{\mathbf{F}}_{\text{TC}}|$.





From consideration of the vertical forces in the individual free-body diagrams, it is apparent that the normal force on each object is equal to its weight. This leads to the conclusion that

$$F_{\text{fr}} = \mu_{\text{k}} F_{\text{NT}} = \mu_{\text{k}} m_{\text{T}} g = (0.15)(350\text{ kg})(9.80\text{ m/s}^2) = 514.5\text{ N}.$$



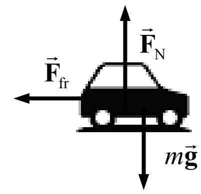
Now consider the combined free-body diagram. Write Newton's second law for the horizontal direction. This allows the calculation of the acceleration of the system.

$$\sum F = F_{\text{CG}} - F_{\text{fr}} = (m_{\text{C}} + m_{\text{T}})a \quad \rightarrow \quad a = \frac{F_{\text{CG}} - F_{\text{fr}}}{m_{\text{C}} + m_{\text{T}}} = \frac{3600\text{ N} - 514.5\text{ N}}{1630\text{ kg}} = 1.893\text{ m/s}^2$$

Finally, consider the free-body diagram for the trailer alone. Again write Newton's second law for the horizontal direction, and solve for F_{TC}.

$$\sum F = F_{\text{TC}} - F_{\text{fr}} = m_{\text{T}} a \quad \rightarrow$$
$$F_{\text{TC}} = F_{\text{fr}} + m_{\text{T}} a = 514.5\text{ N} + (350\text{ kg})(1.893\text{ m/s}^2) = 1177\text{ N} \approx \boxed{1200\text{ N}}$$

44. Assume that kinetic friction is the net force causing the deceleration. See the free-body diagram for the car, assuming that the right is the positive direction and the direction of motion of the skidding car. There is no acceleration in the vertical direction, so $F_N = mg$. Applying Newton's second law to the x direction gives the following.

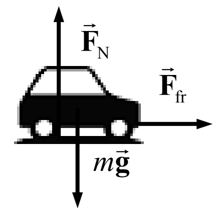


$$\sum F = -F_f = ma \quad \rightarrow \quad -\mu_k F_N = -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g$$

Use Eq. 2–11c to determine the initial speed of the car, with the final speed of the car being zero.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$
$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0 - 2(-\mu_k g)(x - x_0)} = \sqrt{2(0.80)(9.80\text{ m/s}^2)(72\text{ m})} = \boxed{34\text{ m/s}}$$

45. Assume that the static frictional force is the only force accelerating the racer. Then consider the free-body diagram for the racer as shown. It is apparent that the normal force is equal to the weight, since there is no vertical acceleration. It is also assumed that the static frictional force is at its maximum. Thus



$$F_{fr} = ma \quad \rightarrow \quad \mu_s mg = ma \quad \rightarrow \quad \mu_s = a/g.$$

The acceleration of the racer can be calculated from Eq. 2–11b, with an initial speed of 0.

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad a = 2(x - x_0)/t^2$$

$$\mu_s = \frac{a}{g} = \frac{2(x - x_0)}{gt^2} = \frac{2(1000\text{ m})}{(9.80\text{ m/s}^2)(12\text{ s})^2} = \boxed{1.4}$$

46. The analysis of the blocks at rest can be done exactly the same as that presented in Example 4–20, up to the equation for the acceleration, $a = \dfrac{m_B g - F_{fr}}{m_A + m_B}$. Now, for the stationary case, the force of friction is static friction. To find the minimum value of m_A, we assume the maximum static frictional force. Thus $a = \dfrac{m_B g - \mu_s m_A g}{m_A + m_B}$. Finally, for the system to stay at rest, the acceleration must be zero. Thus

$$m_B g - \mu_s m_A g = 0 \quad \rightarrow \quad m_A = m_B/\mu_s = 2.0\text{ kg}/0.30 = \boxed{6.7\text{ kg}}$$

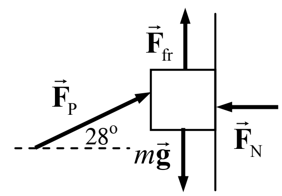
47. (*a*) For m_B not to move, the tension must be equal to $m_B g$, so $m_B g = F_T$. For m_A not to move, the tension must be equal to the force of static friction, so $F_{fr} = F_T$. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_T = F_{fr} \le \mu_s m_A g \quad \rightarrow \quad m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\text{ kg}}{0.40} = 5.0\text{ kg} \quad \rightarrow \quad m_A \ge \boxed{5.0\text{ kg}}$$

(*b*) For m_B to move with constant velocity, the tension must be equal to $m_B g$. For m_A to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_{fr} = \mu_k m_A g \quad \rightarrow \quad m_A = \frac{m_B}{\mu_k} = \frac{2.0\text{ kg}}{0.20} = \boxed{10\text{ kg}}\text{(2 significant figures)}$$

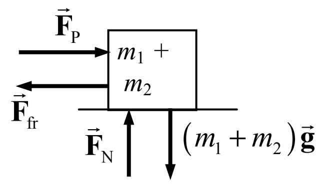
48. Consider a free-body diagram for the box, showing force on the box. When $F_{\text{P}} = 23\,\text{N},$ the block does not move. Thus in that case, the force of friction is static friction, and must be at its maximum value, given by $F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}}.$ Write Newton's second law in both the x and y directions. The net force in each case must be 0, since the block is at rest.



$$\sum F_x = F_{\text{P}} \cos\theta - F_{\text{N}} = 0 \quad \rightarrow \quad F_{\text{N}} = F_{\text{P}} \cos\theta$$
$$\sum F_y = F_{\text{fr}} + F_{\text{P}} \sin\theta - mg = 0 \quad \rightarrow \quad F_{\text{fr}} + F_{\text{P}} \sin\theta = mg$$
$$\mu_{\text{s}} F_{\text{N}} + F_{\text{P}} \sin\theta = mg \quad \rightarrow \quad \mu_{\text{s}} F_{\text{P}} \cos\theta + F_{\text{P}} \sin\theta = mg$$

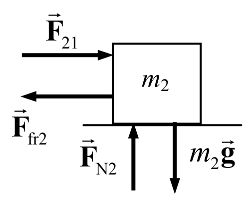
$$m = \frac{F_{\text{P}}}{g}(\mu_{\text{s}} \cos\theta + \sin\theta) = \frac{23\text{ N}}{9.80\text{ m/s}^2}(0.40 \cos 28^\circ + \sin 28^\circ) = \boxed{1.9\text{ kg}}$$

49. (*a*) Since the two blocks are in contact, they can be treated as a single object as long as no information is needed about internal forces (like the force of one block pushing on the other block). Since there is no motion in the vertical direction, it is apparent that $F_{\text{N}} = (m_1 + m_2)g,$ so $F_{\text{fr}} = \mu_{\text{k}} F_{\text{N}} = \mu_{\text{k}}(m_1 + m_2)g.$ Write Newton's second law for the horizontal direction.



$$\sum F_x = F_{\text{P}} - F_{\text{fr}} = (m_1 + m_2)a \quad \rightarrow$$
$$a = \frac{F_{\text{P}} - F_{\text{fr}}}{m_1 + m_2} = \frac{F_{\text{P}} - \mu_{\text{k}}(m_1 + m_2)g}{m_1 + m_2} = \frac{650\text{ N} - (0.18)(190\text{ kg})(9.80\text{ m/s}^2)}{190\text{ kg}}$$
$$= 1.657\text{ m/s}^2 \approx \boxed{1.7\text{ m/s}^2}$$

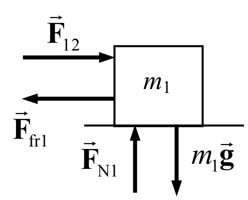
(*b*) To solve for the contact forces between the blocks, an individual block must be analyzed. Look at the free-body diagram for the second block. $\vec{\mathbf{F}}_{21}$ is the force of the first block pushing on the second block. Again, it is apparent that $F_{\text{N}2} = m_2 g,$ so $F_{\text{fr}2} = \mu_{\text{k}} F_{\text{N}2} = \mu_{\text{k}} m_2 g.$ Write Newton's second law for the horizontal direction.



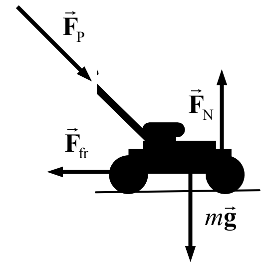
$$\sum F_x = F_{21} - F_{\text{fr}2} = m_2 a \quad \rightarrow$$
$$F_{21} = \mu_{\text{k}} m_2 g + m_2 a = (0.18)(125\text{ kg})(9.80\text{ m/s}^2) + (125\text{ kg})(1.657\text{ m/s}^2) = \boxed{430\text{ N}}$$

By Newton's third law, there will also be a 430-N force to the left on block # 1 due to block # 2.

(*c*) If the crates are reversed, the acceleration of the system will remain the same—the analysis from part (*a*) still applies. We can also repeat the analysis from part (*b*) to find the force of one block on the other, if we simply change m_1 to m_2 in the free-body diagram and the resulting equations.



$$a = \boxed{1.7\text{ m/s}^2}; \quad \sum F_x = F_{12} - F_{\text{fr}1} = m_1 a \quad \rightarrow$$
$$F_{12} = \mu_{\text{k}} m_1 g + m_1 a = (0.18)(65\text{ kg})(9.80\text{ m/s}^2) + (65\text{ kg})(1.657\text{ m/s}^2) = \boxed{220\text{ N}}$$



50. (*a*) We assume that the mower is being pushed to the right. $\vec{\mathbf{F}}_{\text{fr}}$ is the friction force, and $\vec{\mathbf{F}}_{\text{p}}$ is the pushing force along the handle.

(*b*) Write Newton's second law for the horizontal direction. The forces must sum to 0 since the mower is not accelerating.

$$\sum F_x = F_{\text{p}} \cos 45.0° - F_{\text{fr}} = 0 \quad \rightarrow$$

$$F_{\text{fr}} = F_{\text{p}} \cos 45.0° = (88.0 \text{ N}) \cos 45.0° = \boxed{62.2 \text{ N}}$$

(*c*) Write Newton's second law for the vertical direction. The forces must sum to 0 since the mower is not accelerating in the vertical direction.

$$\sum F_y = F_{\text{N}} - mg - F_{\text{p}} \sin 45.0° = 0 \quad \rightarrow$$

$$F_{\text{N}} = mg + F_{\text{p}} \sin 45° = (14.0 \text{ kg})(9.80 \text{ m/s}^2) + (88.0 \text{ N}) \sin 45.0° = \boxed{199 \text{ N}}$$

(*d*) First use Eq. 2–11a to find the acceleration.

$$\upsilon - \upsilon_0 = at \quad \rightarrow \quad a = \frac{\upsilon - \upsilon_0}{t} = \frac{1.5 \text{ m/s} - 0}{2.5 \text{ s}} = 0.60 \text{ m/s}^2$$

Now use Newton's second law for the *x* direction to find the necessary pushing force.

$$\sum F_x = F_{\text{p}} \cos 45.0° - F_{\text{f}} = ma \quad \rightarrow$$

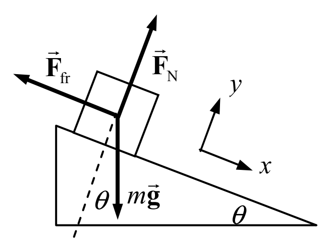
$$F_{\text{p}} = \frac{F_{\text{f}} + ma}{\cos 45.0°} = \frac{62.2 \text{ N} + (14.0 \text{ kg})(0.60 \text{ m/s}^2)}{\cos 45.0°} = \boxed{99.9 \text{ N}}$$

51. The average force can be found from the average acceleration. Use Eq. 2–11c to find the acceleration.

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0) \quad \rightarrow \quad a = \frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)}$$

$$F = ma = m\left(\frac{\upsilon^2 - \upsilon_0^2}{2(x - x_0)}\right) = (60.0 \text{ kg})\left(\frac{0 - (10.0 \text{ m/s})^2}{2(25.0 \text{ m})}\right) = -120 \text{ N}$$

The average retarding force is $\boxed{1.20 \times 10^2 \text{ N}}$, in the direction opposite to the child's velocity.

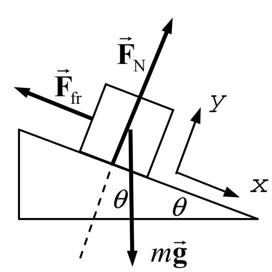


52. (*a*) Here is a free-body diagram for the box at rest on the plane. The force of friction is a STATIC frictional force, since the box is at rest.

(*b*) If the box were sliding down the plane, the only change is that the force of friction would be a KINETIC frictional force.

(*c*) If the box were sliding up the plane, the force of friction would be a KINETIC frictional force, and it would point down the plane, in the opposite direction to that shown in the diagram.

Notice that the angle is not used in this solution.



53. A free-body diagram for the bar of soap is shown. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions, and use those expressions to find the acceleration of the soap.

$$\sum F_y = F_{\text{N}} - mg \cos \theta = 0 \quad \rightarrow \quad F_{\text{N}} = mg \cos \theta$$

$$\sum F_x = mg \sin \theta - F_{\text{fr}} = ma$$

$$ma = mg \sin \theta - \mu_{\text{k}} F_{\text{N}} = mg \sin \theta - \mu_{\text{k}} mg \cos \theta$$

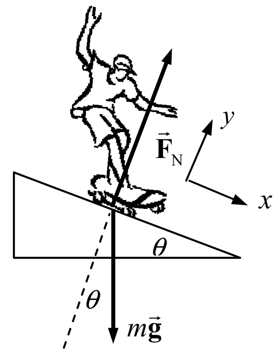
$$a = g(\sin \theta - \mu_{\text{k}} \cos \theta)$$

Now use Eq. 2–11b, with an initial velocity of 0, to find the final velocity.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2x}{g(\sin\theta - \mu_k \cos\theta)}} = \sqrt{\frac{2(9.0\text{ m})}{(9.80\text{ m/s}^2)(\sin 8.0° - (0.060)\cos 8.0°)}} = \boxed{4.8\text{ s}}$$

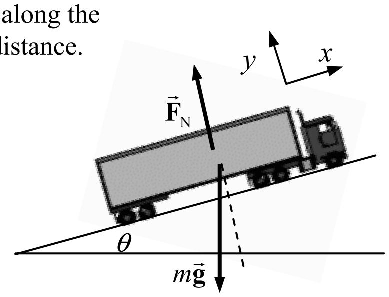
54. From the free-body diagram, the net force along the plane on the skateboarder is $mg \sin\theta$, so the acceleration along the plane is $g \sin\theta$. We use the kinematical data and Eq. 2–11b to write an equation for the acceleration, and then solve for the angle.



$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 = v_0 t + \tfrac{1}{2}gt^2 \sin\theta \quad \rightarrow$$

$$\theta = \sin^{-1}\left(\frac{2\Delta x - v_0 t}{gt^2}\right) = \sin^{-1}\left(\frac{2(18\text{ m}) - 2(2.0\text{ m/s})(3.3\text{ s})}{(9.80\text{ m/s}^2)(3.3\text{ s})^2}\right) = \boxed{12°}$$

55. For a simple ramp, the decelerating force is the component of gravity along the ramp. See the free-body diagram, and use Eq. 2–11c to calculate the distance.

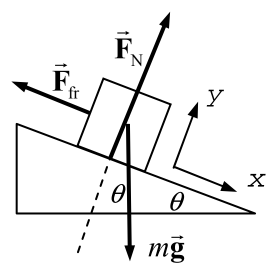


$$\sum F_x = -mg\sin\theta = ma \quad \rightarrow \quad a = -g\sin\theta$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - v_0^2}{2(-g\sin\theta)} = \frac{v_0^2}{2g\sin\theta}$$

$$= \frac{\left[(140\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(9.80\text{ m/s}^2)\sin 11°} = \boxed{4.0\times10^2\text{ m}}$$

56. Consider a free-body diagram of the box. Write Newton's second law for both directions. The net force in the y direction is 0 because there is no acceleration in the y direction.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

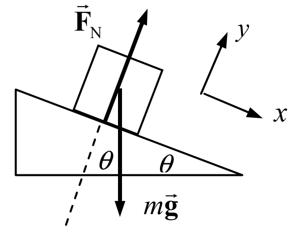
Now solve for the force of friction and the coefficient of friction.

$$F_{fr} = mg\sin\theta - ma = m(g\sin\theta - a) = (25.0\text{ kg})[(9.80\text{ m/s}^2)(\sin 27°) - 0.30\text{ m/s}^2]$$

$$= 103.7\text{ N} \approx \boxed{1.0\times10^2\text{ N}}$$

$$F_{fr} = \mu_k F_N = \mu_k mg\cos\theta \quad \rightarrow \quad \mu_k = \frac{F_{fr}}{mg\cos\theta} = \frac{103.7\text{ N}}{(25.0\text{ kg})(9.80\text{ m/s}^2)\cos 27°} = \boxed{0.48}$$

57. (*a*) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the *y* direction. Use Newton's second law for the *x* direction to find the acceleration.



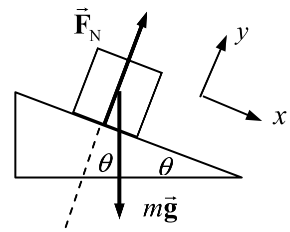
$$\sum F_x = mg \sin\theta = ma \quad \rightarrow$$

$$a = g\sin\theta = (9.80\ \text{m/s}^2)\sin 22.0° = \boxed{3.67\ \text{m/s}^2}$$

(*b*) Use Eq. 2–11c with $v_0 = 0$ to find the final speed.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x-x_0)} = \sqrt{2(3.67\ \text{m/s}^2)(12.0\ \text{m})} = \boxed{9.39\ \text{m/s}}$$

58. (*a*) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the *y* direction. Write Newton's second law for the *x* direction.



$$\sum F_x = mg\sin\theta = ma \quad \rightarrow \quad a = g\sin\theta$$

Use Eq. 2–11c with $v_0 = -4.5$ m/s and $v = 0$ to find the distance that it slides before stopping.

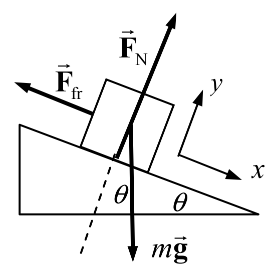
$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (-4.5\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)\sin 22.0°} = -2.758\ \text{m} \approx \boxed{2.8\ \text{m up the plane}}$$

(*b*) The time for a round trip can be found from Eq. 2–11a. The free-body diagram (and thus the acceleration) is the same whether the block is rising or falling. For the entire trip, $v_0 = -4.5$ m/s and $v = +4.5$ m/s.

$$v = v_0 + at \quad \rightarrow \quad t = \frac{v - v_0}{a} = \frac{(4.5\ \text{m/s}) - (-4.5\ \text{m/s})}{(9.80\ \text{m/s}^2)\sin 22°} = 2.452\ \text{s} \approx \boxed{2.5\ \text{s}}$$

59. (*a*) Consider the free-body diagram for the crate on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

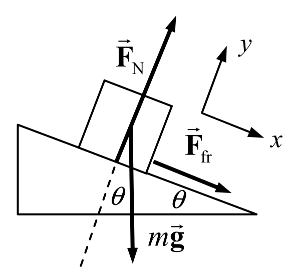
$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80\ \text{m/s}^2)(\sin 25.0° - 0.19\cos 25.0°) = 2.454\ \text{m/s}^2 \approx \boxed{2.5\ \text{m/s}^2}$$

(*b*) Now use Eq. 2–11c, with an initial velocity of 0, to find the final velocity.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x-x_0)} = \sqrt{2(2.454\ \text{m/s}^2)(8.15\ \text{m})} = \boxed{6.3\ \text{m/s}}$$

60. (*a*) Consider the free-body diagram for the crate on the surface. There is no acceleration in the *y* direction. Write Newton's second law for both directions, and find the acceleration.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta + F_{fr} = ma$$

$$ma = mg\sin\theta + \mu_k F_N = mg\sin\theta + \mu_k mg\cos\theta$$

$$a = g(\sin\theta + \mu_k\cos\theta)$$

Now use Eq. 2–11c, with an initial velocity of -3.0 m/s and a final velocity of 0 to find the distance the crate travels up the plane.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$x - x_0 = \frac{-v_0^2}{2a} = \frac{-(-3.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(\sin 25.0° + 0.12\cos 25.0°)} = -0.864\text{ m}$$

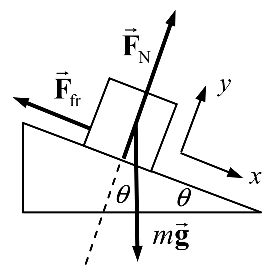
The crate travels $\boxed{0.86\text{ m}}$ up the plane.

(*b*) We use the acceleration found above with the initial velocity in Eq. 2–11a to find the time for the crate to travel up the plane.

$$v = v_0 + at \quad \rightarrow \quad t_{\text{up}} = -\frac{v_0}{a_{\text{up}}} = -\frac{(-3.0\text{ m/s})}{(9.80\text{ m/s}^2)(\sin 25.0° + 0.12\cos 25.0°)} = 0.5761\text{ s}$$

The total time is NOT just twice the time to travel up the plane, because the acceleration of the block is different for the two parts of the motion.

The second free-body diagram applies to the block sliding down the plane. A similar analysis will give the acceleration, and then Eq. 2–11b with an initial velocity of 0 is used to find the time to move down the plane.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\text{fr}} = ma$$

$$ma = mg\sin\theta - \mu_k F_{\text{N}} = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

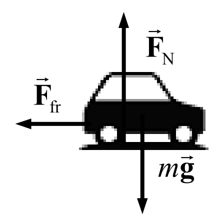
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow$$

$$t_{\text{down}} = \sqrt{\frac{2(x - x_0)}{a_{\text{down}}}} = \sqrt{\frac{2(0.864\text{ m})}{(9.80\text{ m/s}^2)(\sin 25.0° - 0.12\cos 25.0°)}} = 0.7495\text{ s}$$

$$t = t_{\text{up}} + t_{\text{down}} = 0.5761\text{ s} + 0.7495\text{ s} = 1.3256\text{ s} \approx \boxed{1.33\text{ s}}$$

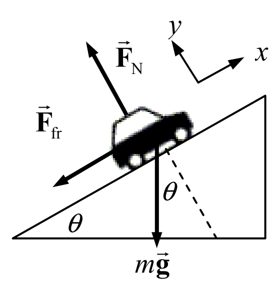
It is worth noting that the final speed is about 2.3 m/s, significantly less than the 3.0 m/s original speed.

61. The direction of travel for the car is to the right, and that is also the positive horizontal direction. Using the free-body diagram, write Newton's second law in the x direction for the car on the level road. We assume that the car is just on the verge of skidding, so that the magnitude of the friction force is $F_{\text{fr}} = \mu_s F_{\text{N}}$.



$$\sum F_x = -F_{\text{fr}} = ma \qquad F_{\text{fr}} = -ma = -\mu_s mg \quad \rightarrow \quad \mu_s = \frac{a}{g} = \frac{3.80\text{ m/s}^2}{9.80\text{ m/s}^2} = 0.3878$$

Now put the car on an inclined plane. Newton's second law in the x-direction for the car on the plane is used to find the acceleration. We again assume the car is on the verge of slipping, so the static frictional force is at its maximum.

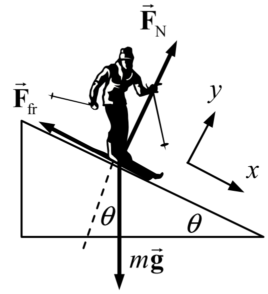


$$\sum F_x = -F_{\text{fr}} - mg\sin\theta = ma \quad \rightarrow$$

$$a = \frac{-F_{\text{fr}} - mg\sin\theta}{m} = \frac{-\mu_s mg\cos\theta - mg\sin\theta}{m} = -g(\mu_s\cos\theta + \sin\theta)$$

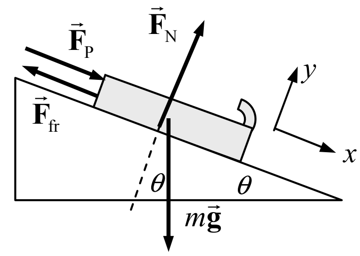
$$= -(9.80\text{ m/s}^2)(0.3878\cos 9.3° + \sin 9.3°) = \boxed{-5.3\text{ m/s}^2}$$

62. Since the skier is moving at a constant speed, the net force on the skier must be 0. See the free-body diagram, and write Newton's second law for both the x and y directions.



$$mg\sin\theta = F_{\mathrm{fr}} = \mu_{\mathrm{s}} F_{\mathrm{N}} = \mu_{\mathrm{s}} mg\cos\theta \quad \rightarrow$$
$$\mu_{\mathrm{s}} = \tan\theta = \tan 12^\circ = \boxed{0.21}$$

63. A free-body diagram for the bobsled is shown. The acceleration of the sled is found from Eq. 2–11c. The final velocity also needs to be converted to m/s.

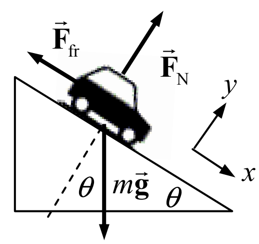


$$v = (60\ \mathrm{km/h})\left(\frac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right) = 16.667\ \mathrm{m/s}$$
$$v^2 - v_0^2 = 2a_x(x - x_0) \quad \rightarrow$$
$$a_x = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(16.667\ \mathrm{m/s})^2 - 0}{2(75\ \mathrm{m})} = 1.852\ \mathrm{m/s^2}$$

Now write Newton's second law for both directions. Since the sled does not accelerate in the y direction, the net force on the y direction must be 0. Then solve for the pushing force.

$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$
$$\sum F_x = mg\sin\theta + F_{\mathrm{P}} - F_{\mathrm{fr}} = ma_x$$
$$F_{\mathrm{P}} = ma_x - mg\sin\theta + F_{\mathrm{fr}} = ma_x - mg\sin\theta + \mu_{\mathrm{k}} F_{\mathrm{N}}$$
$$= ma_x - mg\sin\theta + \mu_{\mathrm{k}} mg\cos\theta = m[a_x + g(\mu_{\mathrm{k}}\cos\theta - \sin\theta)]$$
$$= (22\ \mathrm{kg})[1.852\ \mathrm{m/s^2} + (9.80\ \mathrm{m/s^2})(0.10\cos 6.0^\circ - \sin 6.0^\circ)] = 39.6\ \mathrm{N} \approx \boxed{40\ \mathrm{N}}$$

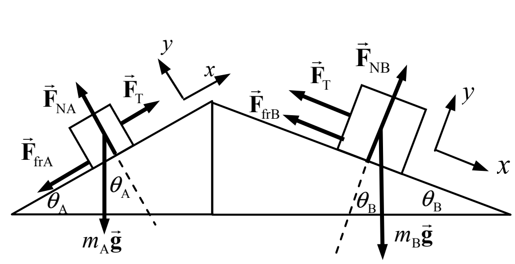
64. Consider a free-body diagram of the car on the icy inclined driveway. Assume that the car is not moving but is just ready to slip, so that the static frictional force has its maximum value of $F_{\mathrm{fr}} = \mu_{\mathrm{s}} F_{\mathrm{N}}$. Write Newton's second law in each direction for the car, with a net force of 0 in each case.



$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$
$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = 0 \quad \rightarrow \quad mg\sin\theta = \mu_{\mathrm{s}} mg\cos\theta$$
$$\mu_{\mathrm{s}} = \sin\theta/\cos\theta = \tan\theta \quad \rightarrow \quad \theta = \tan^{-1}\mu_{\mathrm{s}} = \tan^{-1}0.15 = 8.5^\circ$$

The car will not be able to stay at rest on any slope steeper than 8.5°. $\boxed{\text{Only the driveway across the street is safe for parking.}}$

65. We define the positive x direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two x equations to find the acceleration.



Block A:

$$\sum F_{yA} = F_{NA} - m_A g \cos\theta_A = 0 \to F_{NA} = m_A g \cos\theta_A$$
$$\sum F_{xA} = F_T - m_A g \sin\theta - F_{frA} = m_A a$$

Block B:

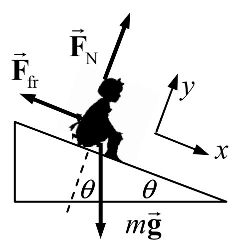
$$\sum F_{yB} = F_{NB} - m_B g \cos\theta_B = 0 \quad \to \quad F_{NB} = m_B g \cos\theta_B$$
$$\sum F_{xB} = m_B g \sin\theta_B - F_{frB} - F_T = m_B a$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{fr} = \mu F_N$.

$$m_A a = F_T - m_A g \sin\theta_A - \mu_A m_A g \cos\theta_A; \quad m_B a = m_B g \sin\theta_B - \mu_B m_B g \cos\theta_B - F_T$$
$$m_A a + m_B a = F_T - m_A g \sin\theta_A - \mu_A m_A g \cos\theta_A + m_B g \sin\theta_B - \mu_B m_B g \cos\theta_B - F_T \quad \to$$

$$\begin{aligned} a &= g\left[\frac{-m_A(\sin\theta_A + \mu_A \cos\theta_A) + m_B(\sin\theta_B - \mu_B \cos\theta_B)}{(m_A + m_B)}\right] \\ &= (9.80\text{ m/s}^2)\left[\frac{-(2.0\text{ kg})(\sin 51° + 0.30\cos 51°) + (5.0\text{ kg})(\sin 21° - 0.30\cos 21°)}{(7.0\text{ kg})}\right] \\ &= \boxed{-2.2\text{ m/s}^2} \end{aligned}$$

66. We assume that the child starts from rest at the top of the slide, and then slides a distance $x - x_0$ along the slide. A force diagram is shown for the child on the slide. First, ignore the frictional force and consider the no-friction case. All of the motion is in the *x* direction, so we will only consider Newton's second law for the *x* direction.



$$\sum F_x = mg\sin\theta = ma \quad \to \quad a = g\sin\theta$$

Use Eq. 2–11c to calculate the speed at the bottom of the slide.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \to \quad v_{(\text{No friction})} = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g\sin\theta(x - x_0)}$$

Now include kinetic friction. We must consider Newton's second law in both the *x* and *y* directions now. The net force in the *y* direction must be 0 since there is no acceleration in the *y* direction.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \to \quad F_N = mg\cos\theta$$
$$\sum F_x = ma = mg\sin\theta - F_{fr} = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$
$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k \cos\theta)$$

With this acceleration, we can again use Eq. 2–11c to find the speed after sliding a certain distance.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \to \quad v_{(\text{friction})} = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g(\sin\theta - \mu_k \cos\theta)(x - x_0)}$$

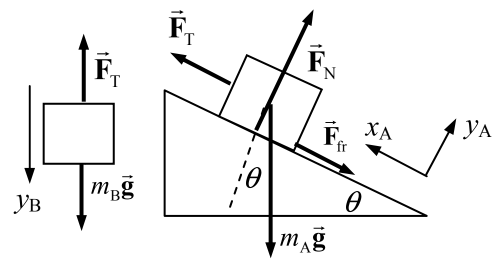
Now let the speed with friction be half the speed without friction, and solve for the coefficient of friction. Square the resulting equation and divide by $g\cos\theta$ to get the result.

$$v_{(\text{friction})} = \frac{1}{2}v_{(\text{No friction})} \quad\rightarrow\quad \sqrt{2g(\sin\theta - \mu_k\cos\theta)(x-x_0)} = \frac{1}{2}\sqrt{2g(\sin\theta)(x-x_0)}$$

$$2g(\sin\theta - \mu_k\cos\theta)(x-x_0) = \frac{1}{4}2g(\sin\theta)(x-x_0)$$

$$\mu_k = \frac{3}{4}\tan\theta = \frac{3}{4}\tan 34^\circ = \boxed{0.51}$$

$\boxed{67.}$ (*a*) Given that m_B is moving down, m_A must be moving up the incline, so the force of kinetic friction on m_A will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, so $a_{yB} = a_{xA} = a.$ Write Newton's second law for each mass.



$$\sum F_{yB} = m_Bg - F_T = m_Ba \quad\rightarrow\quad F_T = m_Bg - m_Ba$$

$$\sum F_{xA} = F_T - m_Ag\sin\theta - F_{fr} = m_Aa$$

$$\sum F_{yA} = F_N - m_Ag\cos\theta = 0 \quad\rightarrow\quad F_N = m_Ag\cos\theta$$

Take the information from the two *y* equations and substitute into the *x* equation to solve for the acceleration.

$$m_Bg - m_Ba - m_Ag\sin\theta - \mu_k m_Ag\cos\theta = m_Aa \quad\rightarrow$$

$$a = \frac{m_Bg - m_Ag\sin\theta - m_Ag\mu_k\cos\theta}{(m_A + m_B)} = \tfrac{1}{2}g(1 - \sin\theta - \mu_k\cos\theta)$$

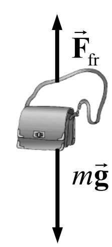
$$= \tfrac{1}{2}(9.80\text{ m/s}^2)(1 - \sin 34^\circ - 0.15\cos 34^\circ) = \boxed{1.6\text{ m/s}^2}$$

(*b*) To have an acceleration of zero, the expression for the acceleration must be zero.

$$a = \tfrac{1}{2}g(1 - \sin\theta - \mu_k\cos\theta) = 0 \quad\rightarrow\quad 1 - \sin\theta - \mu_k\cos\theta = 0 \quad\rightarrow$$

$$\mu_k = \frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin 34^\circ}{\cos 34^\circ} = \boxed{0.53}$$

68. See the free-body diagram for the falling purse. Assume that down is the positive direction, and that the air resistance force $\vec{\mathbf{F}}_{fr}$ is constant. Write Newton's second law for the vertical direction.



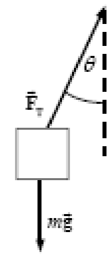
$$\sum F = mg - F_{fr} = ma \quad\rightarrow\quad F_{fr} = m(g - a)$$

Now obtain an expression for the acceleration from Eq. 2–11c with $v_0 = 0,$ and substitute back into the friction force.

$$v^2 - v_0^2 = 2a(x - x_0) \quad\rightarrow\quad a = \frac{v^2}{2(x - x_0)}$$

$$F_{fr} = m\left(g - \frac{v^2}{2(x-x_0)}\right) = (2.0\text{ kg})\left(9.80\text{ m/s}^2 - \frac{(27\text{ m/s})^2}{2(55\text{ m})}\right) = \boxed{6.3\text{ N}}$$

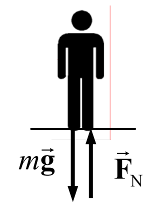
69. See the free-body diagram for the load. The vertical component of the tension force must be equal to the weight of the load, and the horizontal component of the tension accelerates the load. The angle is exaggerated in the picture.



$$F_{\text{net}_x} = F_T \sin\theta = ma \quad\to\quad a = \frac{F_T \sin\theta}{m}; \quad F_{\text{net}_y} = F_T \cos\theta - mg = 0 \quad\to$$

$$F_T = \frac{mg}{\cos\theta} \quad\to\quad a_H = \frac{mg}{\cos\theta}\frac{\sin\theta}{m} = g\tan\theta = (9.80\text{ m/s}^2)\tan 5.0° = \boxed{0.86\text{ m/s}^2}$$

70. A free-body diagram for the person in the elevator is shown. The scale reading is the magnitude of the normal force. Choosing up to be the positive direction, Newton's second law for the person says that $\sum F = F_N - mg = ma \quad\to\quad F_N = m(g+a)$. The kilogram reading of the scale is the apparent weight, F_N, divided by g, which gives $F_{\text{N-kg}} = \dfrac{F_N}{g} = \dfrac{m(g+a)}{g}$.



(*a*) $a = 0 \quad\to\quad F_N = mg = (75.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{7.35\times 10^2\text{ N}}$

$$F_{\text{N-kg}} = \frac{mg}{g} = m = \boxed{75.0\text{ kg}}$$

(*b*) $a = 0 \quad\to\quad F_N = \boxed{7.35\times 10^2\text{ N}},\ F_{\text{N-kg}} = \boxed{75.0\text{ kg}}$

(*c*) $a = 0 \quad\to\quad F_N = \boxed{7.35\times 10^2\text{ N}},\ F_{\text{N-kg}} = \boxed{75.0\text{ kg}}$

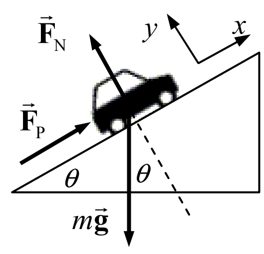
(*d*) $F_N = m(g+a) = (75.0\text{ kg})(9.80\text{ m/s}^2 + 3.0\text{ m/s}^2) = \boxed{9.60\times 10^2\text{ N}}$

$$F_{\text{N-kg}} = \frac{F_N}{g} = \frac{960\text{ N}}{9.80\text{ m/s}^2} = \boxed{98.0\text{ kg}}$$

(*e*) $F_N = m(g+a) = (75.0\text{ kg})(9.80\text{ m/s}^2 - 3.0\text{ m/s}^2) = \boxed{5.1\times 10^2\text{ N}}$

$$F_{\text{N-kg}} = \frac{F_N}{g} = \frac{510\text{ N}}{9.80\text{ m/s}^2} = \boxed{52\text{ kg}}$$

71. The given data can be used to calculate the force with which the road pushes against the car, which in turn is equal in magnitude to the force the car pushes against the road. The acceleration of the car on level ground is found from Eq. 2–11a.



$$v - v_0 = at \quad\to\quad a = \frac{v - v_0}{t} = \frac{21\text{ m/s} - 0}{12.5\text{ s}} = 1.68\text{ m/s}^2$$

The force pushing the car in order to have this acceleration is found from Newton's second law.

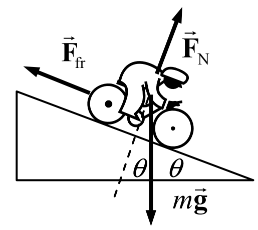
$$F_P = ma = (920\text{ kg})(1.68\text{ m/s}^2) = 1546\text{ N}$$

We assume that this is the force pushing the car on the incline as well. Consider a free-body diagram for the car climbing the hill. We assume that the car will have a constant speed on the maximum incline. Write Newton's second law for the x direction, with a net force of zero since the car is not accelerating.

$$\sum F_x = F_P - mg\sin\theta = 0 \quad\rightarrow\quad \sin\theta = \frac{F_P}{mg}$$

$$\theta = \sin^{-1}\frac{F_P}{mg} = \sin^{-1}\frac{1546\text{ N}}{(920\text{ kg})(9.80\text{ m/s}^2)} = \boxed{9.9^\circ}$$

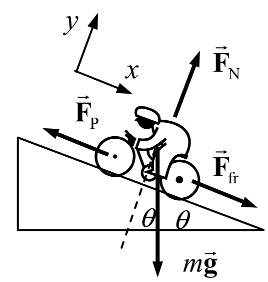
72. Consider a free-body diagram for the cyclist coasting downhill at a constant speed. Since there is no acceleration, the net force in each direction must be zero. Write Newton's second law for the x direction (down the plane).



$$\sum F_x = mg\sin\theta - F_{\text{fr}} = 0 \quad\rightarrow\quad F_{\text{fr}} = mg\sin\theta$$

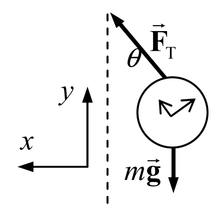
This establishes the size of the air friction force at 6.0 km/h, which can be used in the next part.

Now consider a free-body diagram for the cyclist climbing the hill. F_P is the force pushing the cyclist uphill. Again, write Newton's second law for the x direction, with a net force of 0.



$$\sum F_x = F_{\text{fr}} + mg\sin\theta - F_P = 0 \quad\rightarrow$$
$$F_P = F_{\text{fr}} + mg\sin\theta = 2mg\sin\theta$$
$$= 2(65\text{ kg})(9.80\text{ m/s}^2)(\sin 6.5^\circ) = \boxed{140\text{ N}}$$

73. Consider the free-body diagram for the watch. Write Newton's second law for both the x and y directions. Note that the net force in the y direction is 0 because there is no acceleration in the y direction.



$$\sum F_y = F_T\cos\theta - mg = 0 \quad\rightarrow\quad F_T = \frac{mg}{\cos\theta}$$

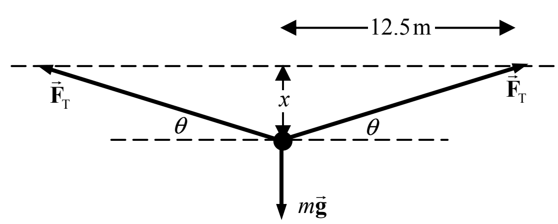
$$\sum F_x = F_T\sin\theta = ma \quad\rightarrow\quad \frac{mg}{\cos\theta}\sin\theta = ma$$

$$a = g\tan\theta = (9.80\text{ m/s}^2)\tan 25^\circ = 4.57\text{ m/s}^2$$

Use Eq. 2–11a with $v_0 = 0$ to find the final velocity (takeoff speed).

$$v - v_0 = at \quad\rightarrow\quad v = v_0 + at = 0 + (4.57\text{ m/s}^2)(16\text{ s}) = \boxed{73\text{ m/s}}$$

74. (*a*) We draw a free-body diagram for the piece of the rope that is directly above the person. That piece of rope should be in equilibrium. The person's weight will be pulling down on that spot, and the rope tension will be pulling away from that spot toward the points of attachment. Write Newton's second law for that small piece of the rope.



$$\sum F_y = 2F_T\sin\theta - mg = 0 \quad\rightarrow\quad \theta = \sin^{-1}\frac{mg}{2F_T} = \sin^{-1}\frac{(72.0\text{ kg})(9.80\text{ m/s}^2)}{2(2900\text{ N})} = 6.988^\circ$$

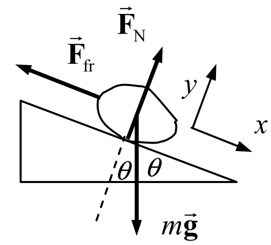
$$\tan\theta = \frac{x}{12.5\text{ m}} \quad\rightarrow\quad x = (12.5\text{ m})\tan 6.988^\circ = 1.532\text{ m} \approx \boxed{1.5\text{ m}}$$

(*b*) Use the same equation to solve for the tension force with a sag of only ¼ of that found above.

$$x = \tfrac{1}{4}(1.532\text{ m}) = 0.383\text{ m};\quad \theta = \tan^{-1}\frac{0.383\text{ m}}{12.5\text{ m}} = 1.755^\circ$$

$$F_T = \frac{mg}{2\sin\theta} = \frac{(72.0\text{ kg})(9.80\text{ m/s}^2)}{2(\sin 1.755^\circ)} = 11{,}519\text{ N} \approx \boxed{12\text{ kN}}$$

The $\boxed{\text{rope will not break}}$, but it exceeds the recommended tension by a factor of about 4.



75. (*a*) Consider the free-body diagram for the snow on the roof. If the snow is just ready to slip, then the static frictional force is at its maximum value, $F_{fr} = \mu_s F_N$. Write Newton's second law in both directions, with the net force equal to zero since the snow is not accelerating.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad\rightarrow\quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = 0 \quad\rightarrow$$

$$mg\sin\theta = F_{fr} = \mu_s F_N = \mu_s mg\cos\theta \quad\rightarrow\quad \mu_s = \tan\theta = \tan 34^\circ = \boxed{0.67}$$

If $\mu_s > 0.67$, then the snow would not be on the verge of slipping.

(*b*) The same free-body diagram applies for the sliding snow. But now the force of friction is kinetic, so $F_{fr} = \mu_k F_N$, and the net force in the *x* direction is not zero. Write Newton's second law for the *x* direction again, and solve for the acceleration.

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$a = \frac{mg\sin\theta - F_{fr}}{m} = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

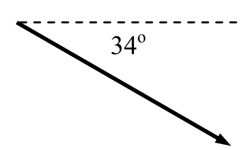
Use Eq. 2–11c with $v_0 = 0$ to find the speed at the end of the roof.

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{2g(\sin\theta - \mu_k\cos\theta)(x - x_0)}$$

$$= \sqrt{2(9.80\text{ m/s}^2)(\sin 34^\circ - (0.10)\cos 34^\circ)(4.0\text{ m})} = 6.111\text{ m/s} \approx \boxed{6.1\text{ m/s}}$$

(*c*) Now the problem becomes a projectile motion problem. The projectile has an initial speed of 6.111 m/s, directed at an angle of 34° below the horizontal. The horizontal component of the speed, (6.111 m/s) cos 34° = 5.066 m/s, will stay constant. The vertical component will change due to gravity. Define the positive direction to be downward. Then the starting vertical velocity is $(6.111\text{ m/s})\sin 34^\circ = 3.417\text{ m/s}$, the vertical acceleration is 9.80 m/s^2, and the vertical displacement is 10.0 m. Use Eq. 2–11c to find the final vertical speed.



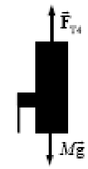
$$v_y^2 - v_{y0}^2 = 2a(y - y_0)$$

$$v_y = \sqrt{v_{y0}^2 + 2a(y - y_0)} = \sqrt{(3.417\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(10.0\text{ m})} = 14.41\text{ m/s}$$

To find the speed when it hits the ground, the horizontal and vertical components of velocity must again be combined, according to the Pythagorean theorem.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.066\text{ m/s})^2 + (14.41\text{ m/s})^2} = 15.27\text{ m/s} \approx \boxed{15\text{ m/s}}$$

76. (*a*) To find the minimum force, assume that the piano is moving with a constant velocity. Since the piano is not accelerating, $F_{T4} = Mg$. For the lower pulley, since the tension in a rope is the same throughout, and since the pulley is not accelerating, it is seen that $F_{T1} + F_{T2} = 2F_{T1} = Mg \rightarrow F_{T1} = F_{T2} = Mg/2$. It also can be seen that since $F = F_{T2}$, $\boxed{F = Mg/2}$.

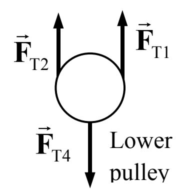


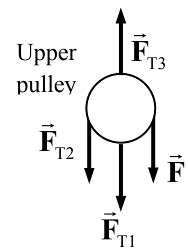
(*b*) Draw a free-body diagram for the upper pulley. From that diagram, we see that

$$F_{T3} = F_{T1} + F_{T2} + F = \frac{3Mg}{2}.$$

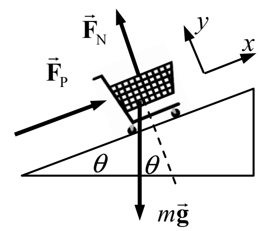
To summarize:

$$\boxed{F_{T1} = F_{T2} = Mg/2 \qquad F_{T3} = 3Mg/2 \qquad F_{T4} = Mg}$$





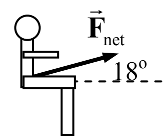
77. Consider a free-body diagram for a grocery cart being pushed up an incline. Assuming that the cart is not accelerating, we write Newton's second law for the *x* direction.

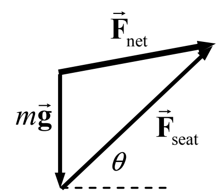


$$\sum F_x = F_P - mg \sin\theta = 0 \quad \rightarrow \quad \sin\theta = \frac{F_P}{mg}$$

$$\theta = \sin^{-1}\frac{F_P}{mg} = \sin^{-1}\frac{18\text{ N}}{(25\text{ kg})(9.80\text{ m/s}^2)} = \boxed{4.2^\circ}$$

78. The acceleration of the pilot will be the same as that of the plane, since the pilot is at rest with respect to the plane. Consider first a free-body diagram of the pilot, showing only the net force. By Newton's second law, the net force MUST point in the direction of the acceleration, and its magnitude is ma. That net force is the sum of ALL forces on the pilot. If we assume that the force of gravity and the force of the cockpit seat on the pilot are the only forces on the pilot, then in terms of vectors, $\vec{\mathbf{F}}_{net} = m\vec{\mathbf{g}} + \vec{\mathbf{F}}_{seat} = m\vec{\mathbf{a}}$. Solve this equation for the force of the seat to find $\vec{\mathbf{F}}_{seat} = \vec{\mathbf{F}}_{net} - m\vec{\mathbf{g}} = m\vec{\mathbf{a}} - m\vec{\mathbf{g}}$. A vector diagram of that equation is shown. Solve for the force of the seat on the pilot using components.





$$F_{x\text{ seat}} = F_{x\text{ net}} = ma\cos 18^\circ = (75\,\text{kg})(3.8\text{ m/s}^2)\cos 18^\circ = 271.1\text{ N}$$

$$F_{y\text{ seat}} = mg + F_{y\text{ net}} = mg + ma\sin 18^\circ$$

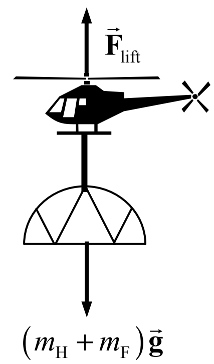
$$= (75\text{ kg})(9.80\text{ m/s}^2) + (75\text{ kg})(3.8\text{ m/s}^2)\sin 18^\circ = 823.2\text{ N}$$

The magnitude of the cockpit seat force is as follows:

$$F = \sqrt{F_{x\text{ seat}}^2 + F_{y\text{ seat}}^2} = \sqrt{(271.1\text{ N})^2 + (823.2\text{ N})^2} = 866.7\text{ N} \approx \boxed{870\text{ N}}$$

The angle of the cockpit seat force is as follows:

$$\theta = \tan^{-1}\frac{F_{y\text{ seat}}}{F_{x\text{ seat}}} = \tan^{-1}\frac{823.2\text{ N}}{271.1\text{ N}} = \boxed{72^\circ} \text{ above the horizontal}$$



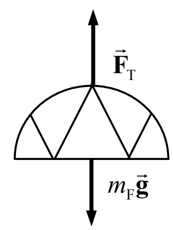
79. (*a*) Both the helicopter and frame will have the same acceleration and can be treated as one object if no information about internal forces (like the cable tension) is needed. A free-body diagram for the helicopter–frame combination is shown. Write Newton's second law for the combination, calling UP the positive direction.

$$\sum F = F_{\text{lift}} - (m_{\text{H}} + m_{\text{F}})g = (m_{\text{H}} + m_{\text{F}})a \quad \rightarrow$$

$$F_{\text{lift}} = (m_{\text{H}} + m_{\text{F}})(g + a) = (7180\ \text{kg} + 1080\ \text{kg})(9.80\ \text{m/s}^2 + 0.80\ \text{m/s}^2)$$

$$= 87{,}556\ \text{N} \approx \boxed{8.76 \times 10^4\ \text{N}}$$

(*b*) Now draw a free-body diagram for the frame alone, in order to find the tension in the cable. Again use Newton's second law.



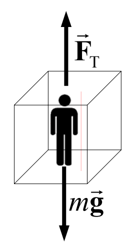
$$\sum F = F_{\text{T}} - m_{\text{F}}g = m_{\text{F}}a \quad \rightarrow$$

$$F_{\text{T}} = m_{\text{F}}(g + a) = (1080\ \text{kg})(9.80\ \text{m/s}^2 + 0.80\ \text{m/s}^2)$$

$$= 11{,}448\ \text{N} \approx \boxed{1.14 \times 10^4\ \text{N}}$$

(*c*) The tension in the cable is the same at both ends, so the cable exerts a force of $\boxed{1.14 \times 10^4\ \text{N}}$ downward on the helicopter.

80. Choose downward to be positive. The elevator's acceleration is calculated by Eq. 2–11c.



$$v^2 - v_0^2 = 2a(y - y_0) \quad \rightarrow \quad a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0 - (3.5\ \text{m/s})^2}{2(2.6\ \text{m})} = -2.356\ \text{m/s}^2$$

See the free-body diagram of the elevator/occupant combination. Write Newton's second law for the elevator.

$$\sum F_y = mg - F_{\text{T}} = ma$$

$$F_{\text{T}} = m(g - a) = (1450\ \text{kg})(9.80\ \text{m/s}^2 - -2.356\ \text{m/s}^2) = \boxed{1.76 \times 10^4\ \text{N}}$$

81. See the free-body diagram for the fish being pulled upward vertically. From Newton's second law, calling the upward direction positive, we have this relationship.

$$\sum F_y = F_{\text{T}} - mg = ma \quad \rightarrow \quad F_{\text{T}} = m(g + a)$$

(*a*) If the fish has a constant speed, then its acceleration is zero, so $F_{\text{T}} = mg$. Thus the heaviest fish that could be pulled from the water in this case is $\boxed{45\ \text{N}\ (10\ \text{lb})}$.

(*b*) If the fish has an acceleration of $2.0\ \text{m/s}^2$, and F_{T} is at its maximum of 45 N, then solve the equation for the mass of the fish.

$$m = \frac{F_{\text{T}}}{g + a} = \frac{45\ \text{N}}{9.8\ \text{m/s}^2 + 2.0\ \text{m/s}^2} = 3.8\ \text{kg} \quad \rightarrow$$

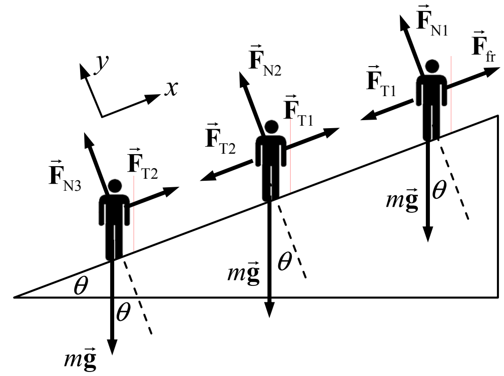
$$mg = (3.8\ \text{kg})(9.8\ \text{m/s}^2) = \boxed{37\ \text{N}\ (\approx 8.4\ \text{lb})}$$

(*c*) It is not possible to land a 15-lb fish using 10-lb line, if you have to lift the fish vertically. If the fish were reeled in while still in the water and then a net was used to remove the fish from the water, it might still be caught with the 10-lb line.

82. Use Newton's second law.

$$F = ma = m\frac{\Delta v}{\Delta t} \rightarrow \Delta t = \frac{m\Delta v}{F} = \frac{(1.0\times10^{10}\ \text{kg})(2.0\times10^{-3}\ \text{m/s})}{(2.5\ \text{N})} = \boxed{8.0\times10^{6}\ \text{s}} = 93\ \text{d}$$

83. Since the climbers are on ice, the frictional force for the lower two climbers is negligible. Consider the free-body diagram as shown. Note that all the masses are the same. Write Newton's second law in the x direction for the lowest climber, assuming he is at rest.

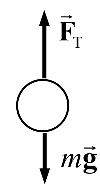


$$\sum F_x = F_{T2} - mg\sin\theta = 0$$
$$F_{T2} = mg\sin\theta = (75\ \text{kg})(9.80\ \text{m/s}^2)\sin 31.0^\circ = \boxed{380\ \text{N}}$$

Write Newton's second law in the x direction for the middle climber, assuming he is at rest.

$$\sum F_x = F_{T1} - F_{T2} - mg\sin\theta = 0 \rightarrow F_{T1} = F_{T2} + mg\sin\theta = 2F_{T2}g\sin\theta = \boxed{760\ \text{N}}$$

84. For each object, we have the free-body diagram shown, assuming that the string doesn't break. Newton's second law is used to get an expression for the tension. Since the string broke for the 2.10 kg mass, we know that the required tension to accelerate that mass was more than 22.2 N. Likewise, since the string didn't break for the 2.05-kg mass, we know that the required tension to accelerate that mass was less than 22.2 N. These relationships can be used to get the range of accelerations.



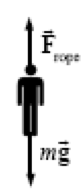
$$\sum F = F_T - mg = ma \rightarrow F_T = m(a+g)$$

$$F_{T_{max}} < m_{2.10}(a+g);\quad F_{T_{max}} > m_{2.05}(a+g) \rightarrow \frac{F_{T_{max}}}{m_{2.10}} - g < a;\quad \frac{F_{T_{max}}}{m_{2.05}} - g > a \rightarrow$$

$$\frac{F_{T_{max}}}{m_{2.10}} - g < a < \frac{F_{T_{max}}}{m_{2.05}} - g \rightarrow \frac{22.2\ \text{N}}{2.10\ \text{kg}} - 9.80\ \text{m/s}^2 < a < \frac{22.2\ \text{N}}{2.05\ \text{kg}} - 9.80\ \text{m/s}^2 \rightarrow$$

$$0.77\ \text{m/s}^2 < a < 1.03\ \text{m/s}^2 \rightarrow \boxed{0.8\ \text{m/s}^2 < a < 1.0\ \text{m/s}^2}$$

85. (*a*) First calculate Karen's speed from falling. Let the downward direction be positive, and use Eq. 2–11c with $v_0 = 0$.



$$v^2 - v_0^2 = 2a(y-y_0) \rightarrow v = \sqrt{0 + 2a(y-y_0)} = \sqrt{2(9.8\ \text{m/s}^2)(2.0\ \text{m})} = 6.26\ \text{m/s}$$

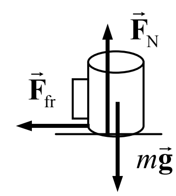
Now calculate the average acceleration as the rope stops Karen, again using Eq. 2–11c, with down as positive.

$$v^2 - v_0^2 = 2a(y-y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y-y_0)} = \frac{0-(6.26\ \text{m/s})^2}{2(1.0\ \text{m})} = -19.6\ \text{m/s}^2$$

The negative sign indicates that the acceleration is upward. Since this is her acceleration, the net force on Karen is given by Newton's second law, $F_{net} = ma$. That net force will also be upward. Now consider the free-body diagram of Karen as she decelerates. Call DOWN the positive direction. Newton's second law says that $F_{\text{net}} = ma = mg - F_{\text{rope}} \quad \to \quad F_{\text{rope}} = mg - ma.$ The ratio of this force to Karen's weight is $\dfrac{F_{\text{rope}}}{mg} = \dfrac{mg - ma}{mg} = 1.0 - \dfrac{a}{g} = 1.0 - \dfrac{-19.6\text{ m/s}^2}{9.8\text{ m/s}^2} = 3.0.$ Thus the rope pulls upward on Karen with an average force of $\boxed{\text{3.0 times her weight}}$.

(*b*) A completely analogous calculation for Jim gives the same speed after the 2.0-m fall, but since he stops over a distance of 0.30 m, his acceleration is -65 m/s^2, and the rope pulls upward on Jim with an average force of $\boxed{\text{7.7 times his weight}}$. Thus, $\boxed{\text{Jim is more likely to get hurt.}}$

86. A free-body diagram for the coffee cup is shown. Assume that the car is moving to the right, so the acceleration of the car (and cup) will be to the left. The deceleration of the cup is caused by friction between the cup and the dashboard. For the cup not to slide on the dash and to have the minimum deceleration time means the largest possible static frictional force is acting, so $F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}}$. The normal force on the cup is equal to its weight, since there is no vertical acceleration. The horizontal acceleration of the cup is found from Eq. 2–11a, with a final velocity of zero.



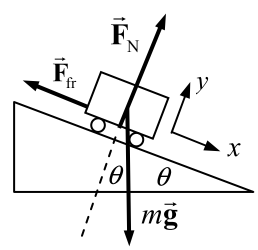
$$\upsilon_0 = (45\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 12.5\text{ m/s}$$

$$\upsilon - \upsilon_0 = at \quad \to \quad a = \frac{\upsilon - \upsilon_0}{t} = \frac{0 - 12.5\text{ m/s}}{3.5\text{ s}} = -3.57\text{ m/s}^2$$

Write Newton's second law for the horizontal forces, considering to the right to be positive.

$$\sum F_x = -F_{\text{fr}} = ma \quad \to \quad ma = -\mu_{\text{s}} F_{\text{N}} = -\mu_{\text{s}} mg \quad \to \quad \mu_{\text{s}} = -\frac{a}{g} = -\frac{(-3.57\text{ m/s}^2)}{9.80\text{ m/s}^2} = \boxed{0.36}$$

87. See the free-body diagram for the descending roller coaster. It starts its descent with $\upsilon_0 = (6.0\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 1.667\text{ m/s}.$ The total displacement in the *x* direction is $x - x_0 = 45.0\text{ m}.$ Write Newton's second law for both the *x* and *y* directions.



$$\sum F_y = F_{\text{N}} - mg\cos\theta = 0 \quad \to \quad F_{\text{N}} = mg\cos\theta$$

$$\sum F_x = ma = mg\sin\theta - F_{\text{fr}} = mg\sin\theta - \mu_{\text{k}} F_{\text{N}} = mg\sin\theta - \mu_{\text{k}} mg\cos\theta$$

$$a = \frac{mg\sin\theta - \mu_{\text{k}} mg\cos\theta}{m} = g(\sin\theta - \mu_{\text{k}}\cos\theta)$$

Now use Eq. 2–11c to solve for the final velocity.

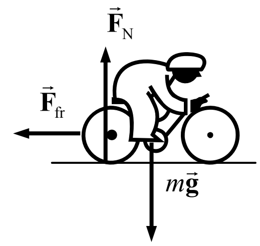
$$\upsilon^2 - \upsilon_0^2 = 2a(x - x_0) \quad \to$$

$$\upsilon = \sqrt{\upsilon_0^2 + 2a(x - x_0)} = \sqrt{\upsilon_0^2 + 2g(\sin\theta - \mu_{\text{k}}\cos\theta)(x - x_0)}$$

$$= \sqrt{(1.667\text{ m/s})^2 + 2(9.80\text{ m/s}^2)[\sin 45° - (0.12)\cos 45°](45.0\text{ m})}$$

$$= 23.49\text{ m/s} \approx \boxed{23\text{ m/s}} \approx 85\text{ km/h}$$

88. Consider the free-body diagram for the cyclist in the sand, assuming that the cyclist is traveling to the right. It is apparent that $F_N = mg$ since there is no vertical acceleration. Write Newton's second law for the horizontal direction, positive to the right.



$$\sum F_x = -F_{fr} = ma \quad \rightarrow \quad -\mu_k mg = ma \quad \rightarrow \quad a = -\mu_k g$$

Use Eq. 2–11c to determine the distance the cyclist could travel in the sand before coming to rest.

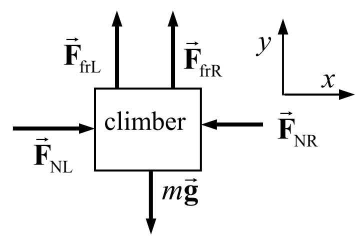
$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{-v_0^2}{-2\mu_k g} = \frac{(20.0\ \text{m/s})^2}{2(0.70)(9.80\ \text{m/s}^2)} = 29\ \text{m}$$

Since there is only 15 m of sand, $\boxed{\text{the cyclist will emerge from the sand}}$. The speed upon emerging is found from Eq. 2–11c.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{v_0^2 - 2\mu_k g(x - x_0)} = \sqrt{(20.0\ \text{m/s})^2 - 2(0.70)(9.80\ \text{m/s}^2)(15\ \text{m})}$$
$$= \boxed{14\ \text{m/s}}$$

89. Since the walls are vertical, the normal forces are horizontal, away from the wall faces. We assume that the frictional forces are at their maximum values, so $F_{fr} = \mu_s F_N$ applies at each wall. We assume that the rope in the diagram is not under any tension and does not exert any forces. Consider the free-body diagram for the climber. F_{NR} is the normal force on the climber from the right wall, and F_{NL} is the normal force on the climber from the left wall. The static frictional forces are $F_{frL} = \mu_{sL} F_{NL}$ and $F_{frR} = \mu_{sR} F_{NR}$. Write Newton's second law for both the x and y directions. The net force in each direction must be zero if the climber is stationary.



$$\sum F_x = F_{NL} - F_{NR} = 0 \quad \rightarrow \quad F_{NL} = F_{NR} \qquad \sum F_y = F_{frL} + F_{frR} - mg = 0$$

Substitute the information from the x equation into the y equation.

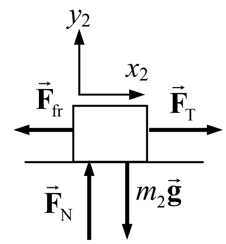
$$F_{frL} + F_{frR} = mg \quad \rightarrow \quad \mu_{sL} F_{NL} + \mu_{sR} F_{NR} = mg \quad \rightarrow \quad (\mu_{sL} + \mu_{sR}) F_{NL} = mg$$

$$F_{NL} = \frac{mg}{(\mu_{sL} + \mu_{sR})} = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)}{1.40} = 4.90 \times 10^2\ \text{N}$$

So $\boxed{F_{NL} = F_{NR} = 4.90 \times 10^2\ \text{N}}$. These normal forces arise as Newton's third law reaction forces to the climber pushing on the walls. Thus the climber must exert a force of at least 490 N against each wall.

$\boxed{90.}$ (*a*) Consider the free-body diagrams for both objects, initially stationary. As sand is added, the tension will increase, and the force of static friction on the block will increase until it reaches its maximum, $F_{fr} = \mu_s F_N$. Then the system will start to move. Write Newton's second law for each object, when the static frictional force is at its maximum, but the objects are still stationary.

$$\sum F_{y\ \text{bucket}} = m_1 g - F_T = 0 \quad \rightarrow \quad F_T = m_1 g$$

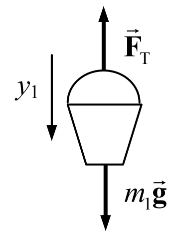


$$\sum F_{y\ \text{block}} = F_N - m_2 g = 0 \quad \rightarrow \quad F_N = m_2 g$$
$$\sum F_{x\ \text{block}} = F_T - F_{fr} = 0 \quad \rightarrow \quad F_T = F_{fr}$$

Equate the two expressions for tension, and substitute in the expression for the normal force to find the masses.

$$m_1 g = F_{fr} \quad \rightarrow \quad m_1 g = \mu_s F_N = \mu_s m_2 g \quad \rightarrow$$
$$m_1 = \mu_s m_2 = (0.45)(28.0\ \text{kg}) = 12.6\ \text{kg}$$

Thus $12.6\ \text{kg} - 2.00\ \text{kg} = 10.6\ \text{kg} \approx \boxed{11\ \text{kg}}$ of sand was added.



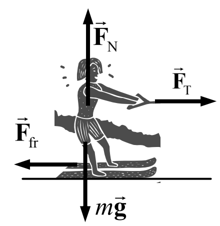
(*b*) The same free-body diagrams can be used, but now the objects will accelerate. Since they are tied together, $a_{y1} = a_{x2} = a$. The frictional force is now kinetic friction, given by $F_{fr} = \mu_k F_N = \mu_k m_2 g$. Write Newton's second law for the objects in the direction of their acceleration.

$$\sum F_{y\ \text{bucket}} = m_1 g - F_T = m_1 a \quad \rightarrow \quad F_T = m_1 g - m_1 a$$
$$\sum F_{x\ \text{block}} = F_T - F_{fr} = m_2 a \quad \rightarrow \quad F_T = F_{fr} + m_2 a$$

Equate the two expressions for tension, and solve for the acceleration.

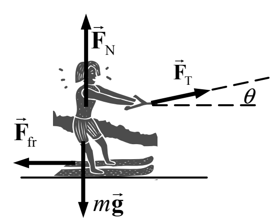
$$m_1 g - m_1 a = \mu_k m_2 g + m_2 a \quad \rightarrow$$
$$a = g\frac{(m_1 - \mu_k m_2)}{(m_1 + m_2)} = (9.80\ \text{m/s}^2)\frac{(12.6\ \text{kg} - (0.32)(28.0\ \text{kg}))}{(12.6\ \text{kg} + 28.0\ \text{kg})} = \boxed{0.88\ \text{m/s}^2}$$

91. (*a*) See the free-body diagram for the skier when the tow rope is horizontal. Use Newton's second law for both the vertical and horizontal directions in order to find the acceleration.



$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg$$
$$\sum F_x = F_T - F_{fr} = F_T - \mu_k F_N = F_T - \mu_k mg = ma$$
$$a = \frac{F_T - \mu_k mg}{m} = \frac{(240\ \text{N}) - 0.25(72\ \text{kg})(9.80\ \text{m/s}^2)}{(72\ \text{kg})} = 0.8833\ \text{m/s}^2 \approx \boxed{0.9\ \text{m/s}^2}$$

(*b*) Now see the free-body diagram for the skier when the tow rope has an upward component.

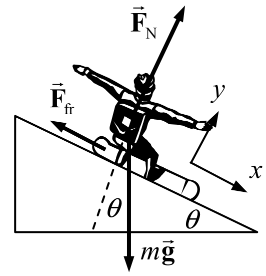


$$\sum F_y = F_N + F_T \sin\theta - mg = 0 \quad \rightarrow \quad F_N = mg - F_T \sin\theta$$
$$\sum F_x = F_T \cos\theta - F_{fr} = F_T \cos\theta - \mu_k F_N$$
$$= F_T \cos\theta - \mu_k (mg - F_T \sin\theta) = ma$$

$$a = \frac{F_T(\cos\theta + \mu_k \sin\theta) - \mu_k mg}{m}$$
$$= \frac{(240\ \text{N})(\cos 12^\circ + 0.25 \sin 12^\circ) - 0.25(72\ \text{kg})(9.80\ \text{m/s}^2)}{(72\ \text{kg})} = \boxed{0.98\ \text{m/s}^2}$$

(*c*) The acceleration is greater in part (*b*) because the upward tilt of the tow rope reduces the normal force, which then reduces the friction. The reduction in friction is greater than the reduction in horizontal applied force, so the horizontal acceleration increases.

92. First consider the free-body diagram for the snowboarder on the incline. Write Newton's second law for both directions, and find the acceleration.



$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$

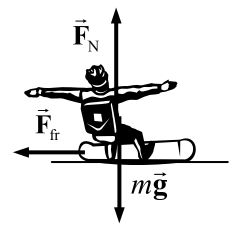
$$\sum F_x = mg\sin\theta - F_{\mathrm{fr}} = ma$$

$$ma = mg\sin\theta - \mu_{\mathrm{k1}}F_{\mathrm{N}} = mg\sin\theta - \mu_{\mathrm{k1}}mg\cos\theta$$

$$a_{\mathrm{slope}} = g(\sin\theta - \mu_{\mathrm{k1}}\cos\theta) = (9.80\ \mathrm{m/s^2})(\sin 28^\circ - 0.18\cos 28^\circ)$$

$$= 3.043\ \mathrm{m/s^2} \approx \boxed{3.0\ \mathrm{m/s^2}}$$

Now consider the free-body diagram for the snowboarder on the flat surface. Again use Newton's second law to find the acceleration. Note that the normal force and the frictional force are different in this part of the problem, even though the same symbol is used.



$$\sum F_y = F_{\mathrm{N}} - mg = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg \qquad \sum F_x = -F_{\mathrm{fr}} = ma$$

$$ma_{\mathrm{flat}} = -F_{\mathrm{fr}} = -\mu_{\mathrm{k2}}F_{\mathrm{N}} = -\mu_{\mathrm{k1}}mg \quad \rightarrow$$

$$a_{\mathrm{flat}} = -\mu_{\mathrm{k2}}g = -(0.15)(9.80\ \mathrm{m/s^2}) = -1.47\ \mathrm{m/s^2} \approx \boxed{-1.5\ \mathrm{m/s^2}}$$

Use Eq. 2–11c to find the speed at the bottom of the slope. This is the speed at the start of the flat section. Eq. 2–11c can be used again to find the distance x.

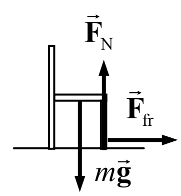
$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$v_{\substack{\text{end of}\\ \text{slope}}} = \sqrt{v_0^2 + 2a_{\mathrm{slope}}(x - x_0)} = \sqrt{(5.0\ \mathrm{m/s})^2 + 2(3.043\ \mathrm{m/s^2})(110\ \mathrm{m})} = 26.35\ \mathrm{m/s}$$

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow$$

$$(x - x_0) = \frac{v^2 - v_0^2}{2a_{\mathrm{flat}}} = \frac{0 - (26.35\ \mathrm{m/s})^2}{2\left(-1.47\ \mathrm{m/s^2}\right)} = 236\ \mathrm{m} \approx \boxed{240\ \mathrm{m}}$$

93. (*a*) Assume that the earthquake is moving the Earth to the right. If an object is to "hold its place," then the object must also be accelerating to the right with the Earth. The force that will accelerate that object will be the static frictional force, which would also have to be to the right. If the force were not large enough, the Earth would move out from under the chair somewhat, giving the appearance that the chair were being "thrown" to the left. Consider the free-body diagram shown for a chair on the floor. It is apparent that the normal force is equal to the weight since there is no motion in the vertical direction. Newton's second law says that $F_{\mathrm{fr}} = ma.$ We also assume that the chair is just on the verge of slipping, which means that the static frictional force has its maximum value of $F_{\mathrm{fr}} = \mu_{\mathrm{s}}F_{\mathrm{N}} = \mu_{\mathrm{s}}mg.$ Equate the two expressions for the frictional force to find the coefficient of friction.

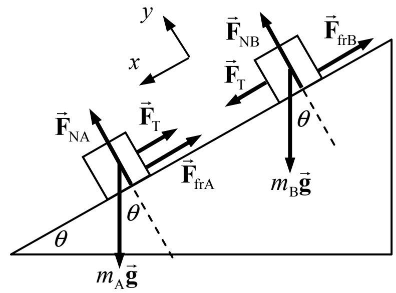


$$ma = \mu_{\mathrm{s}}mg \quad \rightarrow \quad \boxed{\mu_{\mathrm{s}} = a/g}$$

If the static coefficient is larger than this, then there will be a larger maximum frictional force, and the static frictional force will be more than sufficient to hold the chair in place on the floor.

(*b*) For the 1989 quake, $\dfrac{a}{g} = \dfrac{4.0\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} = 0.41.$ Since $\mu_{\mathrm{s}} = 0.25,$ $\boxed{\text{the chair would slide.}}$

94. Since the upper block has a higher coefficient of friction, that block will "drag behind" the lower block. Thus there will be tension in the cord, and the blocks will have the same acceleration. From the free-body diagrams for each block, we write Newton's second law for both the *x* and *y* directions for each block, and then combine those equations to find the acceleration and tension.



(*a*) Block A:

$$\sum F_{yA} = F_{NA} - m_A g \cos\theta = 0 \quad\rightarrow\quad F_{NA} = m_A g \cos\theta$$

$$\sum F_{xA} = m_A g \sin\theta - F_{frA} - F_T = m_A a$$

$$m_A a = m_A g \sin\theta - \mu_A F_{NA} - F_T = m_A g \sin\theta - \mu_A m_A g \cos\theta - F_T$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g \cos\theta = 0 \quad\rightarrow\quad F_{NB} = m_B g \cos\theta$$

$$\sum F_{xB} = m_A g \sin\theta - F_{frA} + F_T = m_B a$$

$$m_B a = m_B g \sin\theta - \mu_B F_{NB} + F_T = m_B g \sin\theta - \mu_B m_B g \cos\theta + F_T$$

Add the final equations together from both analyses and solve for the acceleration.

$$m_A a = m_A g \sin\theta - \mu_A m_A g \cos\theta - F_T;\quad m_B a = m_B g \sin\theta - \mu_B m_B g \cos\theta + F_T$$

$$m_A a + m_B a = m_A g \sin\theta - \mu_A m_A g \cos\theta - F_T + m_B g \sin\theta - \mu_B m_B g \cos\theta + F_T \quad\rightarrow$$

$$a = g\left[\frac{m_A(\sin\theta - \mu_A \cos\theta) + m_B(\sin\theta - \mu_B \cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\ \text{m/s}^2)\left[\frac{(5.0\ \text{kg})(\sin 32^\circ - 0.20\cos 32^\circ) + (5.0\ \text{kg})(\sin 32^\circ - 0.30\cos 32^\circ)}{(10.0\ \text{kg})}\right]$$

$$= 3.1155\ \text{m/s}^2 \approx \boxed{3.1\ \text{m/s}^2}$$

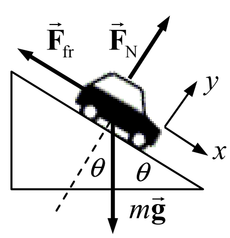
(*b*) Solve one of the equations for the tension force.

$$m_A a = m_A g \sin\theta - \mu_A m_A g \cos\theta - F_T \quad\rightarrow$$

$$F_T = m_A(g \sin\theta - \mu_A g \cos\theta - a)$$

$$= (5.0\ \text{kg})[(9.80\ \text{m/s}^2)(\sin 32^\circ - 0.20\cos 32^\circ) - 3.1155\ \text{m/s}^2] = \boxed{2.1\ \text{N}}$$

95. We include friction from the start, and then for the no-friction result, set the coefficient of friction equal to 0. Consider a free-body diagram for the car on the hill. Write Newton's second law for both directions. Note that the net force on the *y* direction will be zero, since there is no acceleration in the *y* direction.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad\rightarrow\quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma \quad\rightarrow$$

$$a = g\sin\theta - \frac{F_{fr}}{m} = g\sin\theta - \frac{\mu_k mg\cos\theta}{m} = g(\sin\theta - \mu_k\cos\theta)$$

Use Eq. 2–11c to determine the final velocity, assuming that the car starts from rest.

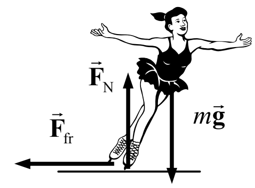
$$v^2 - v_0^2 = 2a(x - x_0) \quad\rightarrow\quad v = \sqrt{0 + 2a(x - x_0)} = \sqrt{2g(x - x_0)(\sin\theta - \mu_k\cos\theta)}$$

The angle is given by $\sin\theta = 1/4 \quad\rightarrow\quad \theta = \sin^{-1}0.25 = 14.5^\circ$.

(*a*) $\mu_k = 0 \quad\rightarrow\quad v = \sqrt{2g(x - x_0)x\sin\theta} = \sqrt{2(9.80\ \text{m/s}^2)(55\ \text{m})\sin 14.5^\circ} = \boxed{16\ \text{m/s}}$

(*b*) $\mu_k = 0.10 \quad\rightarrow\quad v = \sqrt{2(9.80\ \text{m/s}^2)(55\ \text{m})(\sin 14.5^\circ - 0.10\cos 14.5^\circ)} = \boxed{13\ \text{m/s}}$

96. Consider the free-body diagram for the decelerating skater, moving to the right. It is apparent that $F_N = mg$ since there is no acceleration in the vertical direction. From Newton's second law in the horizontal direction, we have



$$\sum F = F_{fr} = ma \quad \to \quad -\mu_k mg = ma \quad \to \quad a = -\mu_k g.$$

Now use Eq. 2–11c to find the starting speed.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \to$$

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0 + 2\mu_k g(x - x_0)} = \sqrt{2(0.10)(9.80 \text{ m/s}^2)(75 \text{ m})} = \boxed{12 \text{ m/s}}$$

$\boxed{97.}$ The initial speed is $v_0 = (45 \text{ km/h})\left(\dfrac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 12.5 \text{ m/s}$. Use Eq. 2–11a to find the deceleration of the child.

$$v - v_0 = at \quad \to \quad a = \frac{v - v_0}{t} = \frac{0 - 12.5 \text{ m/s}}{0.20 \text{ s}} = -62.5 \text{ m/s}^2$$

The net force on the child is given by Newton's second law.

$$F_{net} = ma = (18 \text{ kg})(-62.5 \text{ m/s}^2) = -1116 \text{ N} \approx \boxed{-1100 \text{ N}}, \text{ opposite to the velocity}$$

This force is about 250 lb. We also assumed that friction between the seat and child is zero, and we assumed that the bottom of the seat is horizontal. If friction existed or if the seat was tilted back, then the force that the straps would have to apply would be less.

Solutions to Search and Learn Problems

1. (*a*) The forces acting on the small segment of the rope where $\vec{\mathbf{F}}_P$ acts are F_P in the *y* direction and the two equal tensions acting along the direction of the rope at an angle θ below the *x* axis. By Newton's third law, the tension in the rope is equal to the force that the rope applies to the car. We use Newton's second law in the *y* direction to determine the force on the car.

$$\sum F_y = 0 \to F_P - F_{BR} \sin\theta - F_{CR} \sin\theta = 0 \quad \to \quad F_P - 2F_{CR} \sin\theta = 0$$

$$F_{CR} = \frac{F_P}{2 \sin\theta} = \frac{300 \text{ N}}{2 \sin 5^\circ} = 1720 \text{ N} \approx \boxed{2000 \text{ N}}$$

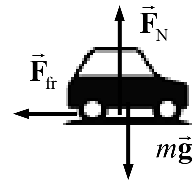
(*b*) The mechanical advantage is the ratio of the force on the car to the force she is applying.

$$\frac{F_{CR}}{F_P} = \frac{1720 \text{ N}}{300 \text{ N}} = 5.73 \approx \boxed{6}$$

(*c*) This method is counterproductive when the mechanical advantage drops below one, which happens when the force on the car is equal to the force she applies to the rope, $F_{CR} = F_P$.

$$F_{CR} = \frac{F_P}{2 \sin\theta} \quad \to \quad \theta = \sin^{-1}\left(\frac{F_P}{2F_{CR}}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{30^\circ}$$

2. (*a*) A free-body diagram for the car is shown, assuming that it is moving to the right. It is apparent from the diagram that $F_N = mg$ for the vertical direction. Write Newton's second law for the horizontal direction, with positive to the right, to find the acceleration of the car. Since the car is assumed NOT to be sliding, use the maximum force of static friction.



$$\sum F_x = -F_{fr} = ma \quad \rightarrow \quad ma = -\mu_s F_N = -\mu_s mg \quad \rightarrow \quad a = -\mu_s g$$

Eq. 2–11c can be used to find the distance that the car moves before stopping. The initial speed is given as v, and the final speed will be 0.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad (x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - v^2}{2(-\mu_s g)} = \boxed{\frac{v^2}{2\mu_s g}}$$

(*b*) Using the given values:

$$v = (95 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 26.38 \text{ m/s} \qquad (x - x_0) = \frac{v^2}{2\mu_s g} = \frac{(26.38 \text{ m/s})^2}{2(0.65)(9.80 \text{ m/s}^2)} = \boxed{55 \text{ m}}$$

(*c*) From part (*a*), we see that the distance is inversely proportional to *g*, so if *g* is reduced by a factor of 6, the distance is increased by a factor of 6 to $\boxed{330 \text{ m}}$.

3. The static friction force is the force that prevents two objects from moving relative to one another. The "less than" sign (<) in the static friction force tells you that the actual force may be any value less than that given by the equation. Typically you use Newton's second law to determine the value of the static friction force. You then verify that the force calculated is in the allowed range given by the static friction equation. The equals sign in the equation is used when you are searching for the maximum force of static friction. For example, if an object is "on the verge" of moving away from a static configuration, you would use the equals sign.

4. As the skier travels down the slope at constant speed, her acceleration parallel to the slope must be zero. Newton's second law can then be written in component form as:

$$\sum F_x = mg \sin\theta - \mu_k F_N = 0 \qquad \sum F_y = F_N - mg \cos\theta = 0$$

The vertical equation can be solved for the normal force, which can then be inserted into the horizontal equation. The horizontal equation can then be solved for the coefficient of kinetic friction.

$$F_N = mg \cos\theta; \quad mg \sin\theta - \mu_k(mg \cos\theta) = 0 \quad \rightarrow \quad \mu_k = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta$$

5

CIRCULAR MOTION; GRAVITATION

Responses to Questions

1. The three major "accelerators" are the accelerator pedal, the brake pedal, and the steering wheel. The accelerator pedal (or gas pedal) can be used to increase speed (by depressing the pedal) or to decrease speed in combination with friction (by releasing the pedal). The brake pedal can be used to decrease speed by depressing it. The steering wheel is used to change direction, which also is an acceleration. There are some other controls that could also be considered accelerators. The parking brake can be used to decrease speed by depressing it. The gear shift lever can be used to decrease speed by downshifting. If the car has a manual transmission, then the clutch can be used to decrease speed by depressing it (friction will slow the car), or, if on a steep downward incline, depressing the clutch can allow the car to increase speed. Finally, shutting the engine off can be used to decrease the car's speed. Any change in speed or direction means that an object is accelerating.

2. Yes, the centripetal acceleration will be greater when the speed is greater since centripetal acceleration is proportional to the square of the speed (when the radius is constant): $a_R = \frac{v^2}{r}$. When the speed is higher, the acceleration has a larger magnitude.

3. No, the acceleration will not be the same. The centripetal acceleration is inversely proportion to the radius (when the speed is constant): $a_R = \frac{v^2}{r}$. Traveling around a sharp curve, with a smaller radius, will require a larger centripetal acceleration than traveling around a gentle curve, with a larger radius.

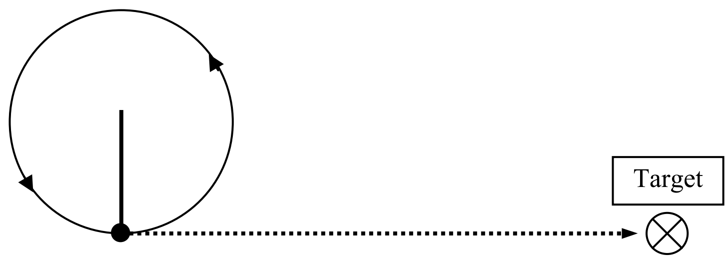
4. The three main forces on the child are the downward force of gravity (the child's weight), the normal force up on the child from the horse, and the static frictional force on the child from the surface of the horse. The frictional force provides the centripetal acceleration. If there are other forces, such as contact forces between the child's hands or legs and the horse, which have a radial component, they will contribute to the centripetal acceleration.

5. On level ground, the normal force on the child would be the same magnitude as his weight. This is the "typical" situation. But as the child and sled come over the crest of the hill, they are moving in a curved path, which can at least be approximated by a circle. There must be a centripetal force, pointing inward toward the center of the arc. The combination of gravity (acting downward) and the normal force on his body (acting upward when the sled is at the top of the hill) provides this centripetal force,

which must be greater than zero. At the top of the hill, if downward is the positive direction, Newton's second law says $F_y = mg - F_N = m\frac{v^2}{r}$. Thus the normal force must be less than the child's weight.

6. No. The barrel of the dryer provides a centripetal force on the clothes to keep them moving in a circular path. A water droplet on the solid surface of the drum will also experience this centripetal force and move in a circle. However, as soon as the water droplet is at the location of a hole in the drum there will be no centripetal force on it and it will therefore continue moving in a path in the direction of its tangential velocity, which will take it out of the drum. There is no centrifugal force throwing the water outward; there is rather a lack of centripetal force to keep the water moving in a circular path.

7. She should let go of the string at the moment that the tangential velocity vector is directed exactly at the target. This would also be when the string is perpendicular to the desired direction of motion of the ball. See the "top view" diagram. Also see Fig. 5–6 in the textbook.



8. At the top of the bucket's arc, the gravitational force and normal forces from the bucket, both pointing downward, must provide the centripetal force needed to keep the water moving in a circle. In the limiting case of no normal force, Newton's second law would give $F_{net} = mg = m\frac{v^2}{r}$, which means that the bucket must be moving with a tangential speed of $v \ge \sqrt{gr}$ or the water will spill out of the bucket. At the top of the arc, the water has a horizontal velocity. As the bucket passes the top of the arc, the velocity of the water develops a vertical component. But the bucket is traveling with the water, with the same velocity, and contains the water as it falls through the rest of its path.

9. For objects (including astronauts) on the inner surface of the cylinder, the normal force provides a centripetal force, which points inward toward the center of the cylinder. This normal force simulates the normal force we feel when on the surface of Earth.

(*a*) Falling objects are not in contact with the floor, so when released they will continue to move with constant velocity until they reach the shell. From the frame of reference of the astronaut inside the cylinder, it will appear that the object "falls" in a curve, rather than straight down.

(*b*) The magnitude of the normal force on the astronaut's feet will depend on the radius and speed of the cylinder. If these are such that $\frac{v^2}{r} = g$ (so that $m\frac{v^2}{r} = mg$ for all objects), then the normal force will feel just like it does on the surface of Earth.

(*c*) Because of the large size of Earth compared to humans, we cannot tell any difference between the gravitational force at our heads and at our feet. In a rotating space colony, the difference in the simulated gravity at different distances from the axis of rotation could be significant, perhaps producing dizziness or other adverse effects. Also, playing "catch" with a ball could be difficult since the normal parabolic paths as experienced on Earth would not occur in the rotating cylinder.

10. (*a*) The normal force on the car is largest at point C. In this case, the centripetal force keeping the car in a circular path of radius R is directed upward, so the normal force must be greater than the weight to provide this net upward force.

(*b*) The normal force is smallest at point A, the crest of the hill. At this point the centripetal force must be directed downward (toward the center of the circle), so the normal force must be less than the weight. (Notice that the normal force is equal to the weight at point B.)

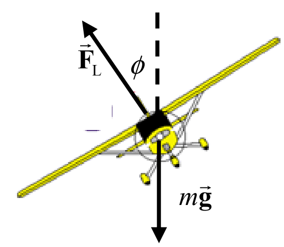
(*c*) The driver will feel heaviest where the normal force is greatest, or at point C.

(*d*) The driver will feel lightest at point A, where the normal force is the least.

(*e*) At point A, the centripetal force is weight minus normal force, or $mg - F_{\mathrm{N}} = \dfrac{mv^2}{R}$. The point at which the car just loses contact with the road corresponds to a normal force of zero, which is the maximum speed without losing contact. Setting $F_{\mathrm{N}} = 0$ gives $mg = \dfrac{mv_{\max}^2}{R} \rightarrow v_{\max} = \sqrt{Rg}$.

11. Yes, a particle with constant speed can be accelerating. A particle traveling around a curve while maintaining a constant speed is accelerating because its direction is changing. However, a particle with a constant velocity cannot be accelerating, since the velocity is not changing in magnitude or direction, and to have an acceleration the velocity must be changing.

12. When an airplane is in level flight, the downward force of gravity is counteracted by the upward lift force, analogous to the upward normal force on a car driving on a level road. The lift on an airplane is perpendicular to the plane of the airplane's wings, so when the airplane banks, the lift vector has both vertical and horizontal components (similar to the vertical and horizontal components of the normal force on a car on a banked turn). Assuming that the plane has no vertical acceleration, then the vertical component of the lift balances the weight and the horizontal component of the lift provides the centripetal force. If F_{L} is the total lift and $\phi =$ the banking angle, measured from the vertical, then $F_{\mathrm{L}} \cos\phi = mg$ and $F_{\mathrm{L}} \sin\phi = m\dfrac{v^2}{r}$, so $\phi = \tan^{-1}(v^2/gr)$.



13. Whether the apple is (*a*) attached to a tree or (*b*) falling, it exerts a gravitational force on the Earth equal to the force the Earth exerts on it, which is the weight of the apple (Newton's third law). That force is independent of the motion of the apple.

14. Since the Earth's mass is much greater than the Moon's mass, the point at which the net gravitational pull on the spaceship is zero is closer to the Moon. It is shown in Problem 30 that this occurs at about 90% of the way from the Earth to the Moon. So, a spaceship traveling from the Earth toward the Moon must therefore use fuel to overcome the net pull backward for 90% of the trip. Once it passes that point, the Moon will exert a stronger pull than the Earth and accelerate the spacecraft toward the Moon. However, when the spaceship is returning to the Earth, it reaches the zero point at only 10% of the way from the Moon to the Earth. Therefore, for most of the trip toward the Earth, the spacecraft is "helped" by the net gravitational pull in the direction of travel, so less fuel is used.

15. The satellite needs a certain speed with respect to the center of the Earth to achieve orbit. The Earth rotates toward the east so it would require less speed (with respect to the Earth's surface) to launch a satellite (*a*) toward the east. Before launch, the satellite is moving with the surface of the Earth so already has a "boost" in the right direction.

16. If the antenna becomes detached from a satellite in orbit, the antenna will continue in orbit around the Earth with the satellite. If the antenna were given a component of velocity toward the Earth (even a very small one), it would eventually spiral in and hit the Earth. If the antenna were somehow slowed down, it would also fall toward the Earth.

17. Yes, we are heavier at midnight. At noon, the gravitational force on a person due to the Sun and the gravitational force due to the Earth are in the opposite directions. At midnight, the two forces point in the same direction. Therefore, your apparent weight at midnight is greater than your apparent weight at noon.

18. Your apparent weight will be greatest in case (*b*), when the elevator is accelerating upward. The scale reading (your apparent weight) indicates your force on the scale, which, by Newton's third law, is the same as the normal force of the scale on you. If the elevator is accelerating upward, then the net force must be upward, so the normal force (up) must be greater than your actual weight (down). When in an elevator accelerating upward, you "feel heavy."

    Your apparent weight will be least in case (*c*), when the elevator is in free fall. In this situation your apparent weight is zero since you and the elevator are both accelerating downward at the same rate and the normal force is zero.

    Your apparent weight will be the same as when you are on the ground in case (*d*), when the elevator is moving upward at a constant speed. If the velocity is constant, acceleration is zero and $N = mg$. (Note that it doesn't matter if the elevator is moving up or down or even at rest, as long as the velocity is constant.)

19. If the Earth were a perfect, nonrotating sphere, then the gravitational force on each droplet of water in the Mississippi would be the same at the headwaters and at the outlet, and the river wouldn't flow. Since the Earth is rotating, the droplets of water experience a centripetal force provided by a part of the component of the gravitational force perpendicular to the Earth's axis of rotation. The centripetal force is smaller for the headwaters, which are closer to the North Pole, than for the outlet, which is closer to the equator. Since the centripetal force is equal to $mg - N$ (apparent weight) for each droplet, N is smaller at the outlet, and the river will flow. This effect is large enough to overcome smaller effects on the flow of water due to the bulge of the Earth near the equator.

20. The satellite remains in orbit because it has a velocity. The instantaneous velocity of the satellite is tangent to the orbit. The gravitational force provides the centripetal force needed to keep the satellite in orbit, acting like the tension in a string when twirling a rock on a string. A force is not needed to keep the satellite "up"; a force is needed to bend the velocity vector around in a circle. The satellite can't just have any speed at any radius, though. For a perfectly circular orbit, the speed is determined by the orbit radius, or vice versa, through the relationship $v_{\text{orbit}} = \sqrt{rg}$, where r is the radius of the orbit and g is the acceleration due to gravity at the orbit position.

21. The centripetal acceleration of Mars in its orbit around the Sun is smaller than that of the Earth. For both planets, the centripetal force is provided by gravity, so the centripetal acceleration is inversely proportional to the square of the distance from the planet to the Sun:

$$\frac{m_{\text{p}}v^2}{r} = \frac{Gm_{\text{s}}m_{\text{p}}}{r^2} \quad \text{so} \quad a_{\text{R}} = \frac{v^2}{r} = \frac{Gm_{\text{s}}}{r^2}$$

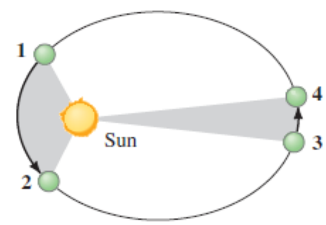
    Since Mars is at a greater distance from the Sun than is Earth, it has a smaller centripetal acceleration. Note that the mass of the planet does not appear in the equation for the centripetal acceleration.

22. For Pluto's moon, we can equate the gravitational force from Pluto on the moon to the centripetal force needed to keep the moon in orbit:

$$\frac{m_{\mathrm{m}}v^2}{r}=\frac{Gm_{\mathrm{p}}m_{\mathrm{m}}}{r^2}\rightarrow m_{\mathrm{p}}=\frac{v^2r}{G}=\frac{4\pi^2r^3}{GT^2}$$

This allows us to solve for the mass of Pluto (m_{p}) if we know G, the radius of the moon's orbit, and the velocity of the moon, which can be determined from the period T and orbital radius. Note that the mass of the moon cancels out.

23. The Earth is closer to the Sun in January. See Fig. 5–29 and the accompanying discussion about Kepler's second law. The caption in the textbook says: "Planets move fastest when closest to the Sun." So in the (greatly exaggerated) figure, the time between points 1 and 2 would be during January, and the time between points 3 and 4 would be July.



Responses to MisConceptual Questions

1. (*b*) As you turn, you feel the force between yourself and the car door. A common misconception is that a centrifugal force is pushing you into the door (answer (*a*)). Actually, your inertia tries to keep you moving in a straight line. As the car (and door) turn right, the door accelerates into you, pushing you away from your straight-line motion and toward the right.

2. (*e*) In circular motion, the velocity is always perpendicular to the radius of the circle, so (*b*), (*c*), and (*d*) are incorrect. The net force is always in the same direction as the acceleration, so if the acceleration points toward the center, the net force must also. Therefore, (*e*) is a better choice than (*a*).

3. (*c*) A common misconception is that the ball will continue to move in a curved path after it exits the tube (answers (*d*) or (*e*)). However, for the ball to move in a curved path, a net force must be acting on the ball. When it is inside the tube, the normal force from the tube wall provides the centripetal force. After the ball exits the tube, there is no net force, so the ball must travel in a straight-line path in the same direction it was traveling as it exited the tube.

4. (*d*) The phrase "steady speed" is not the same as "constant velocity," as velocity also includes direction. A common misconception is that if a car moves at steady speed, the acceleration and net force are zero (answers (*a*) or (*b*)). However, since the path is circular, a radially inward force must cause the centripetal acceleration. If this force (friction between the tires and road) were not present, the car would move in a straight line. It would not accelerate outward.

5. (*b*) A common error in this problem is to ignore the contribution of gravity in the centripetal force. At the top of the loop gravity assists the tension in providing the centripetal force, so the tension is less than the centripetal force. At the bottom of the loop gravity opposes the tension, so the tension is greater than the centripetal force. At all other points in the loop the tension is between the maximum at the bottom and the minimum at the top.

6. (*a*) The forces acting on the child are gravity (downward), the normal force (away from the wall), and the force of friction (parallel to the wall and in this case opposing gravity). In particular, there is nothing "pushing" outward on the rider, so answers (*b*), (*d*), and (*e*) cannot be correct.

7. (*d*) If the net force on the Moon were zero (answer (*a*)), the Moon would move in a straight line and not orbit about the Earth. Gravity pulls the Moon away from the straight-line motion. The large tangential velocity is what keeps the Moon from crashing into the Earth. The gravitational force of the Sun also acts on the Moon, but this force causes the Earth and Moon to orbit the Sun.

8. (*f*) A common misconception is that since the Earth is more massive than the Moon, it must exert more force. However, the force is an interaction between the Earth and Moon, so by Newton's third law, the forces must be equal. Since the Moon is less massive than the Earth and the forces are equal, the Moon has the greater acceleration.

9. (*c*) The nonzero gravitational force on the ISS is responsible for it orbiting the Earth instead of moving is a straight line through space. Astronauts aboard the ISS experience the same centripetal acceleration (free fall toward the Earth) as the station and as a result do not experience a normal force (apparent weightlessness).

10. (*b*) A common misconception is that the mass of an object affects its orbital speed. However, as with all objects in free fall, when calculating the acceleration the object's mass is divided out of the gravitational force. All objects at the same radial distance from the Earth experience the same centripetal acceleration, and by Eq. 5–1 they have the same orbital speed.

11. (*c*) Each of the incorrect answers assumes the presence of an external force to change the orbital motion of the payload. When the payload is attached to the arm, it is orbiting the Earth at the same distance and speed as the shuttle. When it is released, the only force acting on the payload is the force of gravity, which due to the speed of the payload keeps it in orbital motion. For the payload to fall straight down or to follow a curved path that hits the Earth, a force would need to slow down the payload's speed, but no such force is present. To drift out into deep space a force would be needed to overcome the gravity that is keeping it in orbit, but no such force is present.

12. (*d*) Since the penny is rotating around the turntable it experiences a centripetal force toward the center of the turntable, as in (*c*). The rotation is also slowing down, so the penny experiences a decelerating force opposite its velocity, as in (*a*). These two forces are vectors and must be added together to give a net force in the direction of (*d*).

Solutions to Problems

1. (*a*) Find the centripetal acceleration from Eq. 5–1.

$$a_R = \frac{v^2}{r} = (1.10\text{ m/s})^2/1.20\text{ m} = 1.008\text{ m/s}^2 \approx \boxed{1.01\text{ m/s}^2}$$

(*b*) The net horizontal force is causing the centripetal motion, so it will be the centripetal force.

$$F_R = ma_R = (22.5\text{ kg})(1.008\text{ m/s}^2) = 22.68\text{ N} \approx \boxed{22.7\text{ N}}$$

2. Find the centripetal acceleration from Eq. 5–1.

$$a_R = \frac{v^2}{r} = \frac{(525\text{ m/s})^2}{5.20\times10^3\text{ m}} = (53.00\text{ m/s}^2)\left(\frac{1\ g}{9.80\text{ m/s}^2}\right) = \boxed{5.41\ g\text{'s}}$$

3. Find the speed from Eq. 5–3.

$$F_R = \frac{mv^2}{r} \quad\rightarrow\quad v = \sqrt{\frac{F_R r}{\text{m}}} = \sqrt{\frac{(310\text{ N})(0.90\text{ m})}{2.0\text{ kg}}} = 11.81\text{ m/s} \approx \boxed{12\text{ m/s}}$$

4. To find the period, take the reciprocal of the rotational speed (in rev/min) to get min/rev, and then convert to s/rev. Use the period to find the speed, and then the centripetal acceleration.

$$T=\left(\frac{1\ \text{min}}{45\ \text{rev}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)=1.333\frac{\text{s}}{\text{rev}}\qquad r=0.175\ \text{m}\qquad \upsilon=\frac{2\pi r}{T}=\frac{2\pi(0.175\ \text{m})}{1.333\ \text{s}}=0.8249\ \text{m/s}$$

$$a_{\text{R}}=\frac{\upsilon^2}{r}=\frac{(0.8249\ \text{m/s})^2}{0.175\ \text{m}}=3.888\ \text{m/s}^2\approx\boxed{3.9\ \text{m/s}^2}$$

5. The centripetal force that the tension provides is given by Eq. 5–3. Solve that for the speed.

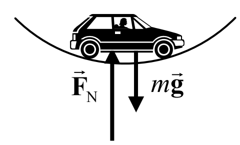
$$F_{\text{R}}=\frac{m\upsilon^2}{r}\quad\rightarrow\quad \upsilon=\sqrt{\frac{F_{\text{R}}r}{m}}=\sqrt{\frac{(75\ \text{N})(1.3\ \text{m})}{0.55\ \text{kg}}}=13.31\ \text{m/s}\approx\boxed{13\ \text{m/s}}$$

6. The centripetal acceleration of a rotating object is given by Eq. 5–1. Solve that for the velocity.

$$\upsilon=\sqrt{a_{\text{R}}r}=\sqrt{(1.25\times10^5\,g)r}=\sqrt{(1.25\times10^5)(9.80\ \text{m/s}^2)(7.00\times10^{-2}\ \text{m})}=2.928\times10^2\ \text{m/s}$$

$$(2.928\times10^2\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi(7.00\times10^{-2}\ \text{m})}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)=\boxed{3.99\times10^4\ \text{rpm}}$$

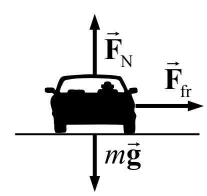
7. A free-body diagram for the car is shown. Write Newton's second law for the car in the vertical direction, assuming that up is positive. The normal force is twice the weight.



$$\sum F=F_{\text{N}}-mg=ma\quad\rightarrow\quad 2mg-mg=m\frac{\upsilon^2}{r}\quad\rightarrow$$

$$\upsilon=\sqrt{rg}=\sqrt{(115\ \text{m})(9.80\ \text{m/s}^2)}=33.57\ \text{m/s}\approx\boxed{34\ \text{m/s}}$$

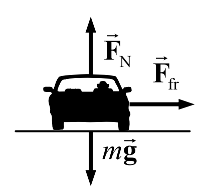
8. In the free-body diagram, the car is coming out of the page, and the center of the circular path is to the right of the car, in the plane of the page. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that friction is the force causing the circular motion. At maximum speed, the car would be on the verge of slipping, and static friction would be at its maximum value.



$$F_{\text{R}}=F_{\text{fr}}\quad\rightarrow\quad m\frac{\upsilon^2}{r}=\mu_{\text{s}}F_{\text{N}}=\mu_{\text{s}}mg\quad\rightarrow\quad \mu_{\text{s}}=\frac{\upsilon^2}{rg}=\frac{\left[(95\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(125\ \text{m})(9.80\ \text{m/s}^2)}=\boxed{0.57}$$

Notice that the result is independent of the car's mass.

9. A free-body diagram for the car at one instant is shown, as though the car is coming out of the page. The center of the circular path is to the right of the car, in the plane of the page. At maximum speed, the car would be on the verge of slipping, and static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that friction is the force causing the circular motion.

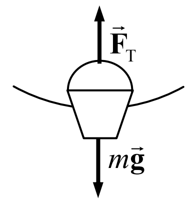


$$F_{\text{R}}=F_{\text{fr}}\quad\rightarrow\quad m\frac{\upsilon^2}{r}=\mu_{\text{s}}F_{\text{N}}=\mu_{\text{s}}mg\quad\rightarrow$$

$$\upsilon=\sqrt{\mu_{\text{s}}rg}=\sqrt{(0.65)(90.0\ \text{m})(9.80\ \text{m/s}^2)}=23.94\ \text{m/s}\approx\boxed{24\ \text{m/s}}$$

Notice that the result is $\boxed{\text{independent of the car's mass}}$.

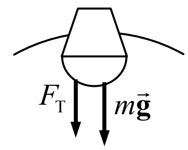
10. (a) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it toward the center of the circle and a centripetal acceleration. Write Newton's second law for the bucket, Eq. 5–3, with up as the positive direction.



$$\sum F_{\mathrm{R}} = F_{\mathrm{T}} - mg = ma = mv^2/r \rightarrow$$

$$v = \sqrt{\frac{r(F_{\mathrm{T}} - mg)}{m}} = \sqrt{\frac{(1.20\text{ m})[25.0\text{ N} - (2.00\text{ kg})(9.80\text{ m/s}^2)]}{2.00\text{ kg}}} = \boxed{1.8\text{ m/s}}$$

(*b*) A free-body diagram of the bucket at the top of the motion is shown. The bucket is moving in a circle, so there must be a net force on it toward the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, Eq. 5–3, with down as the positive direction.



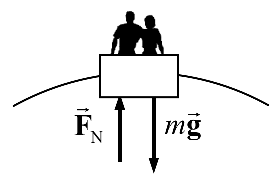
$$\sum F_{\mathrm{R}} = F_{\mathrm{T}} + mg = ma = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{r(F_{\mathrm{T}} + mg)}{m}}$$

If the tension is to be zero, then

$$v = \sqrt{\frac{r(0 + mg)}{m}} = \sqrt{rg} = \sqrt{(1.20\text{ m})(9.80\text{ m/s}^2)} = \boxed{3.43\text{ m/s}}$$

The bucket must move faster than 3.43 m/s in order for the rope not to go slack.

11. The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_{N} is the normal force of the seat pushing up on the passengers. The sum of the forces on the passengers is producing the centripetal motion and must be a centripetal force. Call the downward direction positive, and write Newton's second law for the passengers, Eq. 5–3.



$$\sum F_{\mathrm{R}} = mg - F_{\mathrm{N}} = ma = m\frac{v^2}{r}$$

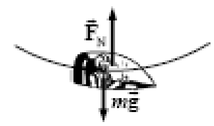
Since the passengers are to feel "weightless," they must lose contact with their seat, and the normal force will be 0. The diameter is 25 m, so the radius is 12.5 m.

$$mg = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(12.5\text{ m})} = 11.07\text{ m/s}$$

$$(11.07\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(12.5\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 8.457\text{ rpm} \approx \boxed{8.5\text{ rpm}}$$

12. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop.

We have $a_{\mathrm{R}} = \frac{v^2}{r} = 6.0\ g$.



$$\frac{v^2}{r} = 6.0\ g \quad \rightarrow \quad r = \frac{v^2}{6.0\ g} = \frac{\left[(840\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{6.0(9.80\text{ m/s}^2)} = 925.9\text{ m} \approx \boxed{930\text{ m}}$$

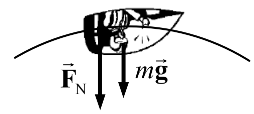
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_{\mathrm{R}} = F_{\mathrm{N}} - mg = m\frac{v^2}{r}$$

The centripetal acceleration is to be $\frac{v^2}{r} = 6.0g.$

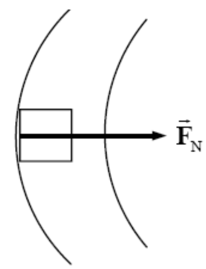
$$F_N = mg + m\frac{v^2}{r} = 7mg = 7(78\text{ kg})(9.80\text{ m/s}^2) = 5350\text{ N} = \boxed{5400\text{ N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. The normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.



$$\sum F_R = F_N + mg = mv^2/r = 6mg \quad \rightarrow \quad F_N = 5mg = \boxed{3800\text{ N}}$$

13. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's second law for the radial direction.



$$\sum F_R = F_N = ma = m\frac{v^2}{r}$$

This is to have nearly the same effect as Earth gravity, with $F_N = 0.90\ mg.$ Equate the two expressions for normal force and solve for the speed.

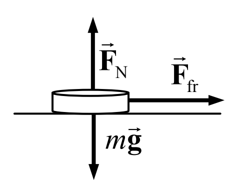
$$F_N = m\frac{v^2}{r} = 0.90mg \quad \rightarrow \quad v = \sqrt{0.90gr} = \sqrt{(0.90)(9.80\text{ m/s}^2)(550\text{ m})} = 69.65\text{ m/s}$$

$$(69.65\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(550\text{ m})}\right)\left(\frac{86{,}400\text{ s}}{1\text{ day}}\right) = 1741\text{ rev/day} \approx \boxed{1700\text{ rev/day}}$$

14. The radius of either skater's motion is 0.80 m, and the period is 2.5 s. Thus their speed is given by $v = 2\pi r/T = \frac{2\pi(0.80\text{ m})}{2.5\text{ s}} = 2.0\text{ m/s}.$ Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5–3.

$$F_R = m\frac{v^2}{r} = \frac{(55.0\text{ kg})(2.0\text{ m/s})^2}{0.80\text{ m}} = 275\text{ N} \approx \boxed{2.8\times10^2\text{ N}}$$

15. The force of static friction is causing the circular motion—it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical acceleration. In the free-body diagram, the coin is coming out of the page and the center of the circle is to the right of the coin, in the plane of the page.

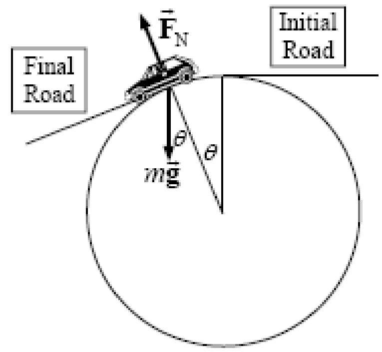


The rotational speed must be changed into a linear speed.

$$v = \left(38.0\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(0.130\text{ m})}{1\text{ rev}}\right) = 0.5173\text{ m/s}$$

$$F_R = F_{fr} \quad \rightarrow \quad m\frac{v^2}{r} = \mu_s F_N = \mu_s mg \quad \rightarrow \quad \mu_s = \frac{v^2}{rg} = \frac{(0.5173\text{ m/s})^2}{(0.130\text{ m})(9.80\text{ m/s}^2)} = \boxed{0.210}$$

16. For the car to stay on the road, the normal force must be greater than 0. See the free-body diagram, write the net radial force, and solve for the radius.

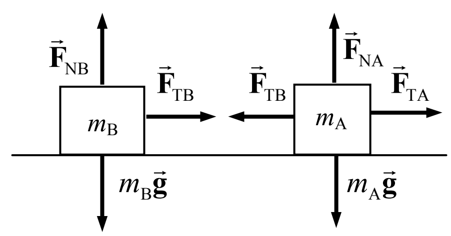


$$F_R = mg\cos\theta - F_N = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv^2}{mg\cos\theta - F_N}$$

For the car to be on the verge of leaving the road, the normal force would be 0, so $r_{\text{critical}} = \frac{mv^2}{mg\cos\theta} = \frac{v^2}{g\cos\theta}$. This expression gets larger as the angle increases, so we must evaluate at the largest angle to find a radius that is good for all angles in the range.

$$r_{\substack{\text{critical}\\ \text{maximum}}} = \frac{v^2}{g\cos\theta_{\max}} = \frac{\left[95\ \text{km/h}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(9.80\ \text{m/s}^2)\cos 18^\circ} = 74.7\ \text{m} \approx \boxed{75\ \text{m}}$$

17. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.



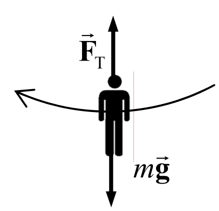
$$\sum F_{RA} = F_{TA} - F_{TB} = m_A a_A = m_A \frac{v_A^2}{r_A} \qquad \sum F_{RB} = F_{TB} = m_B a_B = m_B \frac{v_B^2}{r_B}$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi r}{1\ \text{rev}}\right) = 2\pi r f$.

$$F_{TB} = m_B \frac{v_B^2}{r_B} = m_B (2\pi r_B f)^2 / r_B = \boxed{4\pi^2 m_B r_B f^2}$$

$$F_{TA} = F_{TB} + m_A \frac{v_A^2}{r_A} = 4\pi m_B r_B f^2 + m_A (2\pi r_A f)^2 / r_A = \boxed{4\pi^2 f^2 (m_A r_A + m_B r_B)}$$

18. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal and will point upward when he is at the bottom.



$$\sum F = F_T - mg = ma = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{(F_T - mg)r}{m}}$$

The maximum speed will be obtained with the maximum tension.

$$v_{\max} = \sqrt{\frac{(\vec{\mathbf{F}}_{T\,\max} - mg)r}{m}} = \sqrt{\frac{[1150\ \text{N} - (78\ \text{kg})(9.80\ \text{m/s}^2)](4.7\ \text{m})}{78\ \text{kg}}} = \boxed{4.8\ \text{m/s}}$$

19. (*a*) A free-body diagram of the car at the instant it is on the top of the hill is shown. Since the car is moving in a circular path, there must be a net centripetal force downward. Write Newton's second law for the car, with down as the positive direction.

$$\sum F_R = mg - F_N = ma = m\frac{v^2}{r} \rightarrow$$

$$F_N = m\left(g - \frac{v^2}{r}\right) = (975\text{ kg})\left(9.80\text{ m/s}^2 - \frac{(18.0\text{ m/s})^2}{88.0\text{ m}}\right) = 5965.2\text{ N} \approx \boxed{5970\text{ N}}$$

(*b*) The free-body diagram for the driver would be the same as the one for the car, leading to the same equation for the normal force on the driver.

$$F_N = m\left(g - \frac{v^2}{r}\right) = (62.0\text{ kg})\left(9.80\text{ m/s}^2 - \frac{(18.0\text{ m/s})^2}{88.0\text{ m}}\right) = \boxed{379\text{ N}}$$

Notice that this is significantly less than the 608-N weight of the driver. Thus the driver will feel "light" while driving over the hill.

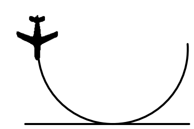
(*c*) For the normal force to be zero, we must have the following:

$$F_N = m\left(g - \frac{v^2}{r}\right) = 0 \rightarrow g = v^2/r \rightarrow v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(88.0\text{ m})} = \boxed{29.4\text{ m/s}}$$

20. The speed is 50 km/h, the curve is unbanked, and the static friction coefficient for rubber on wet concrete is 0.7. If the car is just at the point of slipping, the static frictional force, which is providing the acceleration, would be at its maximum.

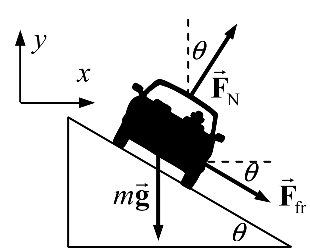
$$\frac{mv^2}{r} = \mu_s mg \rightarrow r = \frac{v^2}{\mu_s g} = \frac{\left[50\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(0.7)(9.80\text{ m/s}^2)} = 28.12\text{ m} \approx \boxed{30\text{ m}}$$

21. The fact that the pilot can withstand 8.0 g's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.



$$a_R = \frac{v^2}{r} = 8.0\,g \rightarrow r = \frac{v^2}{8.0\,g} = \frac{(270\text{ m/s})^2}{8.0(9.80\text{ m/s}^2)} = \boxed{930\text{ m}}$$

22. Since the curve is designed for 65 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5–7 in the textbook, the no-friction banking angle is given by the following:



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(65\text{ km/h})\left(\frac{1.0\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(95\text{ m})(9.80\text{ m/s}^2)} = 19.3°$$

Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_N$. Solve each equation for the normal force.

$$\sum F_y = F_N \cos\theta - mg - F_{\text{fr}} \sin\theta = 0 \;\rightarrow\; F_N \cos\theta - \mu_s F_N \sin\theta = mg \;\rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)}$$

$$\sum F_x = F_N \sin\theta + F_{\text{fr}} \cos\theta = F_R = m\frac{v^2}{r} \;\rightarrow\; F_N \sin\theta + \mu_s F_N \cos\theta + = m\frac{v^2}{r} \;\rightarrow$$

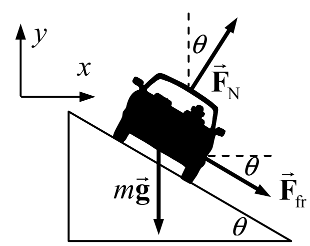
$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)}$$

Equate the two expressions for F_N and solve for the coefficient of friction. The speed of rounding the curve is given by $v = (95\text{ km/h})\left(\frac{1.0\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s}$.

$$\frac{mg}{(\cos\theta - \mu_s \sin\theta)} = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)} \;\rightarrow$$

$$\mu_s = \frac{\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)} = \frac{\left(\frac{v^2}{r} - g\tan\theta\right)}{\left(g + \frac{v^2}{r}\tan\theta\right)} = \frac{\left(\frac{(26.39\text{ m/s})^2}{95\text{ m}} - (9.80\text{ m/s}^2)\tan 19.3^\circ\right)}{\left(9.80\text{ m/s}^2 + \frac{(26.39\text{ m/s})^2}{95\text{ m}}\tan 19.3^\circ\right)} = \boxed{0.32}$$

23. Since the curve is designed for a speed of 85 km/h, traveling at that speed would mean no friction is needed to round the curve. From Example 5–7 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(85\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(78\text{ m})(9.80\text{ m/s}^2)} = 36.10^\circ$$

Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_N$.

$$\sum F_y = F_N \cos\theta - mg - F_{\text{fr}} \sin\theta = 0 \;\rightarrow\; F_N \cos\theta - \mu_s F_N \sin\theta = mg \;\rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)}$$

$$\sum F_x = F_N \sin\theta + F_{\text{fr}} \cos\theta = m\frac{v^2}{r} \;\rightarrow\; F_N \sin\theta + \mu_s F_N \cos\theta = m\frac{v^2}{r} \;\rightarrow$$

$$F_N = \frac{mv^2/r}{(\sin\theta + \mu_s \cos\theta)}$$

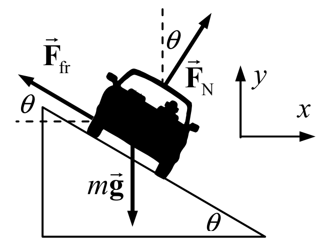
Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta+\mu_s\cos\theta)}=\frac{mg}{(\cos\theta-\mu_s\sin\theta)} \rightarrow$$

$$v_{max}=\sqrt{rg\frac{(\sin\theta+\mu_s\cos\theta)}{(\cos\theta-\mu_s\sin\theta)}}=\sqrt{(78\text{ m})(9.80\text{ m/s}^2)\frac{(\sin 36.10°+0.30\cos 36.10°)}{(\cos 36.10°-0.30\sin 36.10°)}}$$

$$=31.73\text{ m/s}\approx 32\text{ m/s} \qquad \left[31.73\text{ m/s}\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right)=114\text{ km/h}\approx 110\text{ km/h}\right]$$

Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction and centripetal acceleration in the x direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr}=\mu_s F_N$.



$$\sum F_y=F_N\cos\theta-mg+F_{fr}\sin\theta=0 \rightarrow$$

$$F_N\cos\theta+\mu_s F_N\sin\theta=mg \rightarrow F_N=\frac{mg}{(\cos\theta+\mu_s\sin\theta)}$$

$$\sum F_x=F_N\sin\theta-F_{fr}\cos\theta=m\frac{v^2}{r} \rightarrow F_N\sin\theta-\mu_s F_N\cos\theta=m\frac{v^2}{r}\rightarrow$$

$$F_N=\frac{mv^2/r}{(\sin\theta-\mu_s\cos\theta)}$$

Equate the two expressions for the normal force and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta-\mu_s\cos\theta)}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)} \rightarrow$$

$$v=\sqrt{rg\frac{(\sin\theta-\mu_s\cos\theta)}{(\cos\theta+\mu_s\sin\theta)}}=\sqrt{(78\text{ m})(9.80\text{ m/s}^2)\frac{(\sin 36.10°-0.30\cos 36.10°)}{(\cos 36.10°+0.30\sin 36.10°)}}$$

$$=16.41\text{ m/s}\approx 16\text{ m/s} \qquad \left[16.41\text{ m/s}\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right)=59.08\text{ km/h}\right]$$

Thus the range is $\boxed{16\text{ m/s}\le v\le 32\text{ m/s}}$, which is $\boxed{59\text{ km/h}\le v\le 110\text{ km/h}}$.

24. From Example 5–8, we are given that the track radius is 500 m (assumed to have two significant figures), and the tangential acceleration is 3.2 m/s^2. Thus the tangential force is

$$F_{tan}=ma_{tan}=(950\text{ kg})(3.2\text{ m/s}^2)=3040\text{ N}\approx\boxed{3.0\times10^3\text{ N}}$$

The centripetal force is given by Eq. 5–3.

$$F_R=m\frac{v^2}{r}=(950\text{ kg})(15\text{ m/s})^2/(500\text{ m})=427.5\text{ N}\approx\boxed{430\text{ N}}$$

25. The car has constant tangential acceleration, which is the acceleration that causes the speed to change. Thus use constant-acceleration equations to calculate the tangential acceleration. The initial speed is 0, the final speed is $270\text{ km/h}\left(\dfrac{1.0\text{ m/s}}{3.6\text{ km/h}}\right)=75\text{ m/s},$ and the distance traveled is one-half of a circular arc of radius 220 m, so $\Delta x_{\tan}=220\pi\text{ m}.$ Find the tangential acceleration using Eq. 2–11c.

$$v_{\tan}^2-v_{0\tan}^2=2a_{\tan}\Delta x_{\tan}\quad\rightarrow\quad a_{\tan}=\frac{v_{\tan}^2-v_{0\tan}^2}{2\Delta x_{\tan}}=\frac{(75\text{ m/s})^2}{2(220\pi\text{ m})}=4.069\text{ m/s}^2\approx\boxed{4.1\text{ m/s}^2}$$

With this tangential acceleration, we can find the speed that the car has halfway through the turn, using Eq. 2–11c, and then calculate the radial acceleration.

$$v_{\tan}^2-v_{0\tan}^2=2a_{\tan}\Delta x_{\tan}\quad\rightarrow\quad v_{\tan}=\sqrt{v_{0\tan}^2+2a_{\tan}\Delta x_{\tan}}=\sqrt{2(4.069\text{ m/s}^2)(110\pi\text{ m})}=53.03\text{ m/s}$$

$$a_R=\frac{v^2}{r}=\frac{(53.03\text{ m/s})^2}{220\text{ m}}=12.78\text{ m/s}^2\approx\boxed{13\text{ m/s}^2}$$

The total acceleration is given by the Pythagorean combination of the tangential and centripetal accelerations, $a_{\text{total}}=\sqrt{a_R^2+a_{\tan}^2}$. If static friction is to provide the total acceleration, then $F_{\text{fr}}=ma_{\text{total}}=m\sqrt{a_R^2+a_{\tan}^2}$. We assume that the car is on the verge of slipping and is on a level surface, so the static frictional force has its maximum value of $F_{\text{fr}}=\mu_s F_N=\mu_s mg.$ If we equate these two expressions for the frictional force, we can solve for the coefficient of static friction.

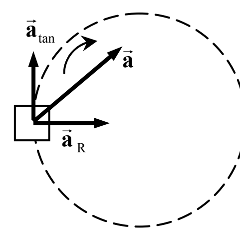
$$F_{\text{fr}}=ma_{\text{total}}=m\sqrt{a_R^2+a_{\tan}^2}=\mu_s mg\quad\rightarrow$$

$$\mu_s=\frac{\sqrt{a_R^2+a_{\tan}^2}}{g}=\frac{\sqrt{(12.78\text{ m/s}^2)^2+(4.069\text{ m/s}^2)^2}}{9.80\text{ m/s}^2}=1.37\approx\boxed{1.4}$$

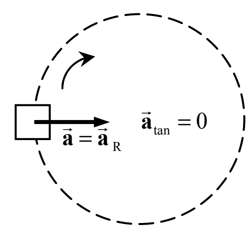
This is an exceptionally large coefficient of friction, so the curve had better be banked.

26. In all cases, we draw a view from above, and the car is moving clockwise around the circular path.

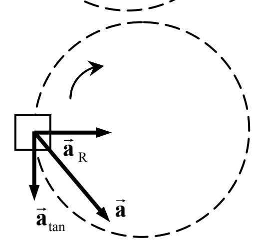
(*a*) In this case, the car is gaining speed, so it has a tangential acceleration in the direction of its velocity, as well as a centripetal acceleration. The total acceleration vector is somewhat "forward."



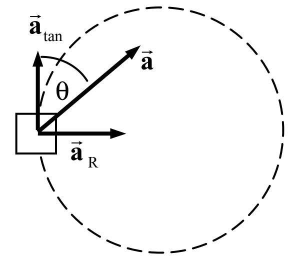
(*b*) In this case, the car has a constant speed, so there is no tangential acceleration. The total acceleration is equal to the radial acceleration.



(*c*) In this case, the car is slowing down, so its tangential acceleration is in the opposite direction as the velocity. It also has a centripetal acceleration. The total acceleration vector is somewhat "backward."



27. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially inward component of the acceleration.



(*a*) $a_R = a\sin\theta = \dfrac{v^2}{r} \quad \rightarrow$

$$v = \sqrt{ar\sin\theta} = \sqrt{(1.05\text{ m/s}^2)(1.95\text{ m})\sin 25.0^{\circ}} = \boxed{0.930\text{ m/s}}$$

(*b*) The particle's speed change comes from the tangential acceleration, which is given by $a_{tan} = a\cos\theta$. Since the tangential acceleration is constant, we use Eq. 2–11a.

$$v_{tan} - v_{0\,tan} = a_{tan}t \quad \rightarrow$$

$$v_{tan} = v_{0\,tan} + a_{tan}t = 0.930\text{ m/s} + (1.05\text{ m/s}^2)(\cos 25.0°)(2.00\text{ s}) = \boxed{2.83\text{ m/s}}$$

28. The spacecraft is at 3.00 Earth radii from the center of the Earth, or three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9}mg_{\substack{\text{Earth's}\\ \text{surface}}} = \frac{(1850\text{ kg})(9.80\text{ m/s}^2)}{9} = \boxed{2010\text{ N}}$$

This could also have been found using Eq. 5–4, Newton's law of universal gravitation.

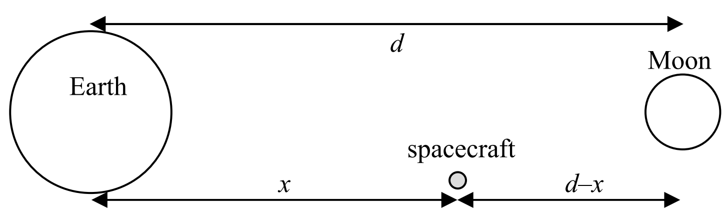
29. (*a*) Mass is independent of location, so the mass of the ball is $\boxed{24.0\text{ kg}}$ on both the Earth and the planet.

(*b*) The weight is found by using $W = mg$.

$$W_{\text{Earth}} = mg_{\text{Earth}} = (24.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{235\text{ N}}$$

$$W_{\text{Planet}} = mg_{\text{Planet}} = (24.0\text{ kg})(12.0\text{ m/s}^2) = \boxed{288\text{ N}}$$

30. For the net force to be zero means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve for the distance x. We measure from the center of the bodies.



$$F_{\substack{\text{Earth-}\\ \text{spacecraft}}} = G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2}; \quad F_{\substack{\text{Moon}\\ \text{spacecraft}}} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2}$$

$$G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2} \quad \rightarrow \quad \frac{x^2}{M_{\text{Earth}}} = \frac{(d-x)^2}{M_{\text{Moon}}} \quad \rightarrow \quad \frac{x}{\sqrt{M_{\text{Earth}}}} = \frac{d-x}{\sqrt{M_{\text{Moon}}}}$$

$$x = d\frac{\sqrt{M_{\text{Earth}}}}{\left(\sqrt{M_{\text{Moon}}} + \sqrt{M_{\text{Earth}}}\right)} = (3.84\times10^8\text{ m})\frac{\sqrt{5.97\times10^{24}\text{ kg}}}{\left(\sqrt{7.35\times10^{22}\text{ kg}} + \sqrt{5.97\times10^{24}\text{ kg}}\right)} = \boxed{3.46\times10^8\text{ m}}$$

This is only about 22 Moon radii away from the Moon. Alternatively, it is about 90% of the distance from the center of the Earth to the center of the Moon.

31. Assume that the two objects can be treated as point masses, with $m_1 = m$ and $m_2 = 4.00\text{ kg} - m$. The gravitational force between the two masses is given by the following:

$$F = G\frac{m_1 m_2}{r^2} = G\frac{m(4.00-m)}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{4.00m - m^2}{(0.25\text{ m})^2} = 2.5\times10^{-10}\ \text{N}$$

This can be rearranged into a quadratic form of $m^2 - 4.00m + 0.234 = 0$. Use the quadratic formula to solve for *m*, resulting in two values, which are the two masses.

$$\boxed{m_1 = 3.94\text{ kg},\ m_2 = 0.06\text{ kg}}$$

32. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text{planet}} = GM_{\text{planet}}/r^2$, where *r* is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{M_{\text{Earth}}}{(2.0\ R_{\text{Earth}})^2} = \frac{1}{(2.0)^2}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{1}{4.0}g_{\text{Earth}} = \frac{9.80\text{ m/s}^2}{4.0} = \boxed{2.5\text{ m/s}^2}$$

33. The force of gravity on an object at the surface of a planet is given by Newton's law of universal gravitation, Eq. 5–4, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}}m}{r_{\text{Moon}}^2} = mg_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r_{\text{Moon}}^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\ \text{kg})}{(1.74\times10^6\ \text{m})^2} = \boxed{1.62\text{ m/s}^2}$$

34. With the assumption that the density of Europa is the same as Earth's, the radius of Europa can be calculated.

$$\rho_{\text{Europa}} = \rho_{\text{Earth}} \quad \rightarrow \quad \frac{M_{\text{Europa}}}{\frac{4}{3}\pi r_{\text{Europa}}^3} = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi r_{\text{Earth}}^3} \quad \rightarrow \quad r_{\text{Europa}} = r_{\text{Earth}}\left(\frac{M_{\text{Europa}}}{M_{\text{Earth}}}\right)^{1/3}$$

$$g_{\text{Europa}} = \frac{GM_{\text{Europa}}}{r_{\text{Europa}}^2} = \frac{GM_{\text{Europa}}}{\left(r_{\text{Earth}}\left(\frac{M_{\text{Europa}}}{M_{\text{Earth}}}\right)^{1/3}\right)^2} = \frac{GM_{\text{Europa}}^{1/3}M_{\text{Earth}}^{2/3}}{r_{\text{Earth}}^2} = \frac{GM_{\text{Earth}}}{r_{\text{Earth}}^2}\frac{M_{\text{Europa}}^{1/3}}{M_{\text{Earth}}^{1/3}}$$

$$= g_{\text{Earth}}\left(\frac{M_{\text{Europa}}}{M_{\text{Earth}}}\right)^{1/3}$$

$$= (9.80\text{ m/s}^2)\left(\frac{4.9\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)^{1/3} = 1.98\text{ m/s}^2 \approx \boxed{2.0\text{ m/s}^2}$$

35. The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\frac{M_{\text{body}}}{R_{\text{body}}^2}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38g_{\text{Earth}}$.

$$G\frac{M_{\text{Mars}}}{R_{\text{Mars}}^2} = 0.38G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} \quad \rightarrow$$

$$M_{\text{Mars}} = 0.38M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{\text{R}_{\text{Earth}}}\right)^2 = 0.38(5.98\times10^{24}\ \text{kg})\left(\frac{3400\text{ km}}{6380\text{ km}}\right)^2 = \boxed{6.5\times10^{23}\ \text{kg}}$$

36. We assume that the distance from the Moon to the Sun is the same as the distance from the Earth to the Sun.

$$F_{ME} = F_x = G\frac{M_{Moon}M_{Earth}}{r^2_{\text{Moon-Earth}}} \qquad F_{MS} = F_y = G\frac{M_{Moon}M_{Sun}}{r^2_{\text{Moon-Sun}}}$$

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(G\frac{M_{Moon}M_{Earth}}{r^2_{\text{Moon-Earth}}}\right)^2 + \left(G\frac{M_{Moon}M_{Sun}}{r^2_{\text{Moon-Sun}}}\right)^2} = GM_{Moon}\sqrt{\left(\frac{M_{Earth}}{r^2_{\text{Moon-Earth}}}\right)^2 + \left(\frac{M_{Sun}}{r^2_{\text{Moon-Sun}}}\right)^2}$$

$$= (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\ \text{kg})\sqrt{\left(\frac{(5.98\times10^{24}\ \text{kg})}{(384\times10^6\ \text{m})^2}\right)^2 + \left(\frac{(1.99\times10^{30}\ \text{kg})}{(149.6\times10^9\ \text{m})^2}\right)^2}$$

$$= \boxed{4.79\times10^{20}\ \text{N}}$$

$$\theta = \tan^{-1}\frac{F_x}{F_y} = \tan^{-1}\frac{\left(G\frac{M_{Moon}M_{Earth}}{r^2_{\text{Moon-Earth}}}\right)}{\left(G\frac{M_{Moon}M_{Sun}}{r^2_{\text{Moon-Sun}}}\right)} = \tan^{-1}\frac{\left(\frac{M_{Earth}}{r^2_{\text{Moon-Earth}}}\right)}{\left(\frac{M_{Sun}}{r^2_{\text{Moon-Sun}}}\right)} = \tan^{-1}\left(\frac{M_{Earth}}{r^2_{\text{Moon-Earth}}}\frac{r^2_{\text{Moon-Sun}}}{M_{Sun}}\right)$$

$$= \tan^{-1}\left[\frac{(5.98\times10^{24}\ \text{kg})}{(384\times10^6\ \text{m})^2}\frac{(149.6\times10^9\ \text{m})^2}{(1.99\times10^{30}\ \text{kg})}\right] = \tan^{-1}\left[\frac{(5.98\times10^{24}\ \text{kg})}{(384\times10^6\ \text{m})^2}\frac{(149.6\times10^9\ \text{m})^2}{(1.99\times10^{30}\ \text{kg})}\right]$$

$$= \tan^{-1}0.456 = \boxed{24.5^\circ}$$

37. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{planet} = GM_{Planet}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{planet} = G\frac{M_{Planet}}{r^2} = G\frac{2.80M_{Earth}}{R^2_{Earth}} = 2.80\left(G\frac{M_{Earth}}{R^2_{Earth}}\right) = 2.80g_{Earth} = 2.80(9.80\ \text{m/s}^2) = \boxed{27.4\ \text{m/s}^2}$$

38. The acceleration due to gravity is determined by the mass of the Earth and the radius of the Earth.

$$g_0 = \frac{GM_0}{r_0^2} \qquad g_{new} = \frac{GM_{new}}{r^2_{new}} = \frac{G2M_0}{(3r_0)^2} = \frac{2}{9}\frac{GM_0}{r_0^2} = \tfrac{2}{9}g_0$$

So g is multiplied by a factor of $\boxed{2/9}$.

39. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{planet} = GM_{Planet}/r^2$, where r is the distance from the center of the planet to the location in question. For this problem, $M_{Planet} = M_{Earth} = 5.97\times10^{24}$ kg.

(*a*) $r = R_{Earth} + 6400\ \text{m} = 6.38\times10^6\ \text{m} + 6400\ \text{m}$

$$g = G\frac{M_{Earth}}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\ \text{kg})}{(6.38\times10^6\ \text{m} + 6400\ \text{m})^2} = \boxed{9.78\ \text{m/s}^2}$$

(*b*) $r = R_{\text{Earth}} + 6400\text{ km} = 6.38\times10^{6}\text{ m} + 6.4\times10^{6}\text{ m} = 12.78\times10^{6}\text{ m}$ (3 significant figures)

$$g = G\frac{M_{\text{Earth}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\text{ kg})}{(12.78\times10^{6}\text{ m})^2} = \boxed{2.44\text{ m/s}^2}$$

40. The distance from the Earth's center is $r = R_{\text{Earth}} + 380\text{ km} = 6.38\times10^{6}\text{ m} + 3.8\times10^{5}\text{ m} = 6.76\times10^{6}\text{ m}$. Calculate the acceleration due to gravity at that location.

$$g = G\frac{M_{\text{Earth}}}{r^2} = G\frac{M_{\text{Earth}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{5.97\times10^{24}\text{ kg}}{(6.76\times10^{6}\text{ m})^2} = 8.714\text{ m/s}^2$$

$$= 8.714\text{ m/s}^2\left(\frac{1\text{ “}g\text{”}}{9.80\text{ m/s}^2}\right) = \boxed{0.889\ g\text{'s}}$$

This is only about an 11% reduction from the value of *g* at the surface of the Earth.

41. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\text{Earth}\atop\text{Venus}} = (150-108)\times10^{6}\text{ km} = 4.2\times10^{10}\text{ m} \qquad r_{\text{Earth}\atop\text{Jupiter}} = (778-150)\times10^{6}\text{ km} = 6.28\times10^{11}\text{ m}$$

$$r_{\text{Earth}\atop\text{Saturn}} = (1430-150)\times10^{6}\text{ km} = 1.28\times10^{12}\text{ m}$$

Jupiter and Saturn will exert a rightward force, and Venus will exert a leftward force. Take the right direction as positive.

$$F_{\text{Earth-}\atop\text{planets}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\text{Earth}\atop\text{Jupiter}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\text{Earth}\atop\text{Saturn}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\text{Earth}\atop\text{Venus}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{(6.28\times10^{11}\text{ m})^2} + \frac{95.1}{(1.28\times10^{12}\text{ m})^2} - \frac{0.815}{(4.2\times10^{10}\text{ m})^2}\right)$$

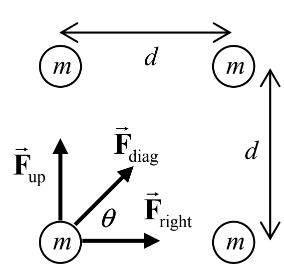
$$= (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})^2(4.02\times10^{-22}\text{ m}^{-2}) = 9.56\times10^{17}\text{ N} \approx 9.6\times10^{17}\text{ N}$$

The force of the Sun on the Earth is as follows:

$$F_{\text{Earth}-\atop\text{Sun}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\text{Earth}\atop\text{Sun}}} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.97\times10^{24}\text{ kg})(1.99\times10^{30}\text{ kg})}{(1.50\times10^{11}\text{ m})^2} = 3.52\times10^{22}\text{ N}$$

So the ratio is $F_{\text{Earth-}\atop\text{planets}}\Big/F_{\text{Earth-}\atop\text{Sun}} = 9.56\times10^{17}\text{ N}/3.52\times10^{22}\text{ N} = \boxed{2.7\times10^{-5}}$, which is 27 millionths.

42. Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the *x* and *y* directions will be the same. Note $\theta = 45°$.



$$F_x = F_{\text{right}} + F_{\text{diag}}\cos\theta = G\frac{m^2}{d^2} + G\frac{m^2}{(\sqrt{2}d)^2}\frac{1}{\sqrt{2}} = G\frac{m^2}{d^2}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

Thus $F_y = F_x = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$. The net force can be found by the Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{2F_x^2} = F_x\sqrt{2} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\sqrt{2} = G\frac{m^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.5\ \text{kg})^2}{(0.80\ \text{m})^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{1.1\times10^{-8}\ \text{N at }45^\circ}$$

The force points toward the center of the square.

43. In general, the acceleration due to gravity of the Earth is given by $g = GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. So, for the location in question,

$$g = \tfrac{1}{10}g_{\text{surface}} \rightarrow G\frac{M_{\text{Earth}}}{r^2} = \tfrac{1}{10}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} \rightarrow r^2 = 10\,R_{\text{Earth}}^2$$

$$r = \sqrt{10}R_{\text{Earth}} = \sqrt{10}(6.38\times10^6\ \text{m}) = \boxed{2.02\times10^7\ \text{m}}$$

44. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = GM_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{star}}}{r^2} = G\frac{5M_{\text{Sun}}}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{5(1.99\times10^{30}\ \text{kg})}{(1\times10^4\ \text{m})^2} = \boxed{7\times10^{12}\ \text{m/s}^2}$$

45. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is given in Example 5–12.

$$v_{\text{orbit}} = \sqrt{G\frac{M_{\text{Earth}}}{r}}$$

$$v = \sqrt{G\frac{M_{\text{Earth}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{(R_{\text{Earth}}+780\ \text{km})}} = \sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\ \text{kg})}{(6.38\times10^6\ \text{m}+7.8\times10^5\ \text{m})}}$$

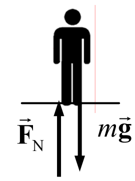
$$= \boxed{7.46\times10^3\ \text{m/s}}$$

46. The speed of a satellite in a circular orbit around a body is given in Example 5–12 as $v_{\text{orbit}} = \sqrt{GM_{\text{body}}/r}$, where r is the distance from the satellite to the center of the body.

$$v = \sqrt{G\frac{M_{\text{body}}}{r}} = \sqrt{G\frac{M_{\text{Earth}}}{R_{\text{Earth}}+4.8\times10^6\ \text{m}}} = \sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\ \text{kg})}{(6.38\times10^6\ \text{m}+4.8\times10^6\ \text{m})}}$$

$$= \boxed{5.97\times10^3\ \text{m/s}}$$

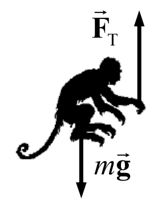
47. Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_N$ is the force of the scale pushing up on you and reads the normal force. Since the scale reads 77 kg, if it were calibrated in newtons, the normal force would be $F_N = (77\text{ kg})(9.80\text{ m/s}^2) = 754.6\text{ N}$.
Write Newton's second law in the vertical direction, with upward as positive.



$$\sum F = F_N - mg = ma \quad \rightarrow \quad a = \frac{F_N - mg}{m} = \frac{754.6\text{ N} - (62\text{ kg})(9.80\text{ m/s}^2)}{62\text{ kg}} = \boxed{2.4\text{ m/s}^2\text{ upward}}$$

Since the acceleration is positive, the acceleration is upward.

48. Draw a free-body diagram of the monkey. Then write Newton's second law for the vertical direction, with up as positive.



$$\sum F = F_T - mg = ma \quad \rightarrow \quad a = \frac{F_T - mg}{m}$$

For the maximum tension of 185 N,

$$a = \frac{185\text{ N} - (12.0\text{ kg})(9.80\text{ m/s}^2)}{(12.0\text{ kg})} = 5.62\text{ m/s}^2 \approx 5.6\text{ m/s}^2$$

Thus the elevator must have an $\boxed{\text{upward acceleration greater than } a = 5.6\text{ m/s}^2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.

49. The speed of an object in a circular orbit of radius r around mass M is given in Example 5–12 by $v = \sqrt{GM/r}$ and is also given by $v = 2\pi r/T$, where T is the period of the orbiting object. Equate the two expressions for the speed and solve for T.

$$\sqrt{G\frac{M}{r}} = \frac{2\pi r}{T} \quad \rightarrow$$

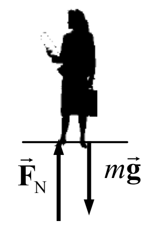
$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(1.74\times10^6\text{ m} + 9.5\times10^4\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ m})}} = \boxed{7.05\times10^3\text{ s} \approx 118\text{ min}}$$

50. The speed of a satellite in circular orbit around the Earth is shown in Example 5–12 to be $v_{\text{orbit}} = \sqrt{G\frac{M_{\text{Earth}}}{r}}$. Thus the velocity is inversely related to the radius, so the closer satellite will be orbiting faster.

$$\frac{v_{\text{close}}}{v_{\text{far}}} = \frac{\sqrt{\frac{GM_{\text{Earth}}}{r_{\text{close}}}}}{\sqrt{\frac{GM_{\text{Earth}}}{r_{\text{far}}}}} = \sqrt{\frac{r_{\text{far}}}{r_{\text{close}}}} = \sqrt{\frac{R_{\text{Earth}} + 1.5\times10^7\text{ m}}{R_{\text{Earth}} + 7.5\times10^6\text{ m}}} = \sqrt{\frac{6.38\times10^6\text{ m} + 1.5\times10^7\text{ m}}{6.38\times10^6\text{ m} + 7.5\times10^6\text{ m}}} = 1.24$$

So $\boxed{\text{the close satellite is moving 1.2 times faster}}$ than the far satellite.

51. Consider a free-body diagram for the woman in the elevator. $\vec{\mathbf{F}}_N$ is the upward force the spring scale exerts, providing a normal force. Write Newton's second law for the vertical direction, with up as positive.



$$\sum F = F_N - mg = ma \quad \to \quad F_N = m(g+a)$$

(*a*, *b*) For constant-speed motion in a straight line, the acceleration is 0, so the normal force is equal to the weight.

$$F_N = mg = (58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{568\text{ N}}$$

(*c*) Here $a = +0.23\,g$, so $F_N = 1.23\,mg = 1.23(58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{699\text{ N}}$.

(*d*) Here $a = -0.23\,g$, so $F_N = 0.77\,mg = 0.77(58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{440\text{ N}}$.

(*e*) Here $a = -g$, so $F_N = \boxed{0}$.

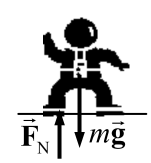
52. The speed of an object in an orbit of radius *r* around the Earth is given in Example 5–12 by $v = \sqrt{GM_{\text{Earth}}/r}$ and is also given by $v = 2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*. Also, for a "near-Earth" orbit, $r = R_{\text{Earth}}$.

$$\sqrt{G\frac{M_{\text{Earth}}}{r}} = \frac{2\pi r}{T} \quad \to \quad T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Earth}}}}$$

$$T = 2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}} = 2\pi\sqrt{\frac{(6.38\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ m})}} = \boxed{5070\text{ s} = 84.5\text{ min}}$$

$\boxed{\text{No}}$, the result does not depend on the mass of the satellite.

53. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\vec{\mathbf{F}}_N$. The magnitude of that force is the apparent weight of the astronaut. Take down as the positive direction.



(*a*) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0, so the net force on the astronaut is 0.

$$\sum F = mg - F_N = 0 \quad \to$$

$$F_N = mg = G\frac{mM_{\text{Moon}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(75\text{ kg})(7.4\times10^{22}\text{ kg})}{(2.5\times10^6\text{ m})^2} = 59.23\text{ N}$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The apparent weight is $\boxed{59\text{ N, away from the Moon}}$.

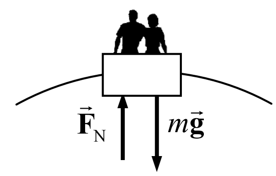
(*b*) Now the astronaut has an acceleration toward the Moon. Write Newton's second law for the astronaut, with down as the positive direction.

$$\sum F = mg - F_N = ma \quad \to \quad F_N = mg - ma = 59.23\text{ N} - (75\text{ kg})(1.8\text{ m/s}^2) = -76\text{ N}$$

Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram—it is pointing toward the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The apparent weight is $\boxed{76\text{ N, toward the Moon}}$.

54. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $v = 2\pi r/T = 2\pi(11.0\text{ m})/12.5\text{ s} = 5.529\text{ m/s}$.

(*a*) See the free-body diagram for the highest point of the motion. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

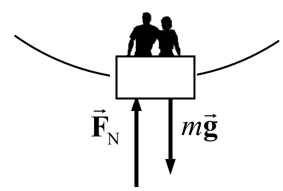


$$\sum F = F_R = mg - F_N = ma = m\frac{v^2}{r} \rightarrow F_N = mg - m\frac{v^2}{r}$$

The ratio of apparent weight to real weight is given by the following:

$$\frac{mg - m\dfrac{v^2}{r}}{mg} = \frac{g - \dfrac{v^2}{r}}{g} = 1 - \frac{v^2}{rg} = 1 - \frac{(5.529\text{ m/s})^2}{(11.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{0.716}$$

(*b*) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.



$$\sum F = F_R = F_N - mg = ma = m\frac{v^2}{r} \rightarrow F_N = mg + m\frac{v^2}{r}$$

The ratio of apparent weight to real weight is given by the following:

$$\frac{mg + m\dfrac{v^2}{r}}{mg} = 1 + \frac{v^2}{rg} = 1 + \frac{(5.529\text{ m/s})^2}{(11.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{1.284}$$

$\boxed{55.}$ The centripetal acceleration will simulate gravity. Thus $\dfrac{v^2}{r} = 0.70\,g \rightarrow v = \sqrt{0.70gr}$. Also for a rotating object, the speed is given by $v = 2\pi r/T$. Equate the two expressions for the speed and solve for the period.

$$v = \sqrt{0.70gr} = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{\sqrt{0.70gr}} = \frac{2\pi(16\text{ m})}{\sqrt{(0.70)(9.80\text{ m/s}^2)(16\text{ m})}} = \boxed{9.6\text{ s}}$$

56. (*a*) The speed of an object in near-surface orbit around a planet is given in Example 5–12 to be $v = \sqrt{GM/R}$, where M is the planet mass and R is the planet radius. The speed is also given by $v = 2\pi R/T$, where T is the period of the object in orbit. Equate the two expressions for the speed.

$$\sqrt{G\frac{M}{R}} = \frac{2\pi R}{T} \rightarrow G\frac{M}{R} = \frac{4\pi^2 R^2}{T^2} \rightarrow \frac{M}{R^3} = \frac{4\pi^2}{GT^2}$$

The density of a uniform spherical planet is given by $\rho = \dfrac{M}{\text{Volume}} = \dfrac{M}{\frac{4}{3}\pi R^3}$. Thus

$$\rho = \frac{3M}{4\pi R^3} = \frac{3}{4\pi}\frac{4\pi^2}{GT^2} = \boxed{\frac{3\pi}{GT^2}}$$

(*b*) For Earth, we have the following:

$$\rho = \frac{3\pi}{GT^2} = \frac{3\pi}{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})[(85\ \mathrm{min})(60\ \mathrm{s/min})]^2} = \boxed{5.4\times10^3\ \mathrm{kg/m^3}}$$

57. Use Kepler's third law for objects orbiting the Sun.

$$\left(T_{\text{Neptune}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{Neptune}}/r_{\text{Earth}}\right)^3 \quad \rightarrow$$

$$T_{\text{Neptune}} = T_{\text{Earth}}\left(\frac{r_{\text{Neptune}}}{r_{\text{Earth}}}\right)^{3/2} = (1\ \text{year})\left(\frac{4.5\times10^9\ \text{km}}{1.50\times10^8\ \text{km}}\right)^{3/2} = \boxed{160\ \text{years}}$$

58. Use Kepler's third law for objects orbiting the Sun.

$$\left(\frac{r_{\text{Icarus}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^2 \quad \rightarrow \quad r_{\text{Icarus}} = r_{\text{Earth}}\left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^{2/3} = (1.50\times10^{11}\ \text{m})\left(\frac{410\ \text{d}}{365\ \text{d}}\right)^{2/3} = \boxed{1.6\times10^{11}\ \text{m}}$$

59. Use Kepler's third law for objects orbiting the Earth. The following are given:

$$T_2 = \text{period of Moon} = (27.4\ \text{day})\left(\frac{86{,}400\ \text{s}}{1\ \text{day}}\right) = 2.367\times10^6\ \text{s}$$

$$r_2 = \text{radius of Moon's orbit} = 3.84\times10^8\ \text{m}$$

$$r_1 = \text{radius of near} - \text{Earth orbit} = R_{\text{Earth}} = 6.38\times10^6\ \text{m}$$

$$\left(T_1/T_2\right)^2 = \left(r_1/r_2\right)^3 \quad \rightarrow$$

$$T_1 = T_2\left(r_1/r_2\right)^{3/2} = (2.367\times10^6\ \text{s})\left(\frac{6.38\times10^6\ \text{m}}{3.84\times10^8\ \text{m}}\right)^{3/2} = \boxed{5.07\times10^3\ \text{s}}(= 84.5\ \text{min})$$

60. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $v = 2\pi r/T$. But the speed can also be calculated for any Earth satellite by $v = \sqrt{GM_{\text{Earth}}/r}$, as derived in Example 5–12. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$\sqrt{GM_{\text{Earth}}/r} = 2\pi r/T \quad \rightarrow$$

$$M_{\text{Earth}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})[(27.4\ \text{d})(86{,}400\ \text{s/d})]^2} = \boxed{5.98\times10^{24}\ \text{kg}}$$

61. There are two expressions for the velocity of an object in circular motion around a mass M: $v = \sqrt{GM/r}$ and $v = 2\pi r/T$. Equate the two expressions and solve for T.

$$\sqrt{GM/r} = 2\pi r/T \quad \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{\left((3\times10^4\ \text{ly})\dfrac{(3\times10^8\ \text{m/s})(3.16\times10^7\ \text{s})}{1\ \text{ly}}\right)^3}{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(4\times10^{41}\ \text{kg})}} = 5.8\times10^{15}\ \text{s} = 1.8\times10^8\ \text{yr}$$

$$\approx \boxed{2\times10^8\ \text{yr}}$$

62. (*a*) The relationship between satellite period *T*, mean satellite distance *r*, and planet mass *M* can be derived from the two expressions for satellite speed: $v=\sqrt{GM/r}$ and $v=2\pi r/T$. Equate the two expressions and solve for *M*.

$$\sqrt{GM/r}=2\pi r/T \quad\rightarrow\quad M=\frac{4\pi^2 r^3}{GT^2}$$

Substitute the values for Io to get the mass of Jupiter.

$$M_{\substack{\text{Jupiter-}\\ \text{Io}}}=\frac{4\pi^2(4.22\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(1.77\ \text{d}\times\frac{24\ \text{h}}{1\ \text{d}}\times\frac{3600\ \text{s}}{1\ \text{h}}\right)^2}=\boxed{1.90\times10^{27}\ \text{kg}}$$

(*b*) For the other moons, we have the following:

$$M_{\substack{\text{Jupiter-}\\ \text{Europa}}}=\frac{4\pi^2(6.71\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.55\times24\times3600\ \text{s})^2}=\boxed{1.90\times10^{27}\ \text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\ \text{Ganymede}}}=\frac{4\pi^2(1.07\times10^9\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.16\times24\times3600\ \text{s})^2}=\boxed{1.89\times10^{27}\ \text{kg}}$$

$$M_{\substack{\text{Jupiter-}\\ \text{Callisto}}}=\frac{4\pi^2(1.883\times10^9\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(16.7\times24\times3600\ \text{s})^2}=\boxed{1.90\times10^{27}\ \text{kg}}$$

$\boxed{\text{Yes}}$, the results are consistent—only about 0.5% difference between them.

63. Use Kepler's third law, Eq. 5–7b, to find the radius of each moon, using Io's data for r_2 and T_2.

$$(r_1/r_2)^3=(T_1/T_2)^2 \quad\rightarrow\quad r_1=r_2(T_1/T_2)^{2/3}$$

$$r_{\text{Europa}}=r_{\text{Io}}\left(T_{\text{Europa}}/T_{\text{Io}}\right)^{2/3}=(422\times10^3\ \text{km})(3.55\ \text{d}/1.77\ \text{d})^{2/3}=\boxed{671\times10^3\ \text{km}}$$

$$r_{\text{Ganymede}}=(422\times10^3\ \text{km})(7.16\ \text{d}/1.77\ \text{d})^{2/3}=\boxed{1070\times10^3\ \text{km}}$$

$$r_{\text{Callisto}}=(422\times10^3\ \text{km})(16.7\ \text{d}/1.77\ \text{d})^{2/3}=\boxed{1880\times10^3\ \text{km}}$$

The agreement with the data in the table is excellent.

64. As found in Example 5–12, the speed for an object orbiting a distance *r* around a mass *M* is given by $v=\sqrt{GM/r}$.

$$\frac{v_A}{v_B}=\frac{\sqrt{\dfrac{GM_{\text{star}}}{r_A}}}{\sqrt{\dfrac{GM_{\text{star}}}{r_B}}}=\sqrt{\frac{r_B}{r_A}}=\sqrt{\frac{1}{7.0}}=\boxed{0.38}$$

65. Use Kepler's third law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3=\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \quad\rightarrow$$

$$r_{\text{Halley}}=r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3}=(150\times10^6\ \text{km})(76\ \text{yr}/1\ \text{yr})^{2/3}=2690\times10^6\ \text{km}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^6\text{ km} = \boxed{5.4\times10^{12}\text{ m}}$. This distance approaches the mean orbit distance of Pluto, which is 5.9×10^{12} m. $\boxed{\text{It is still in the solar system, nearest to Pluto's orbit.}}$

66. (*a*) Use Kepler's third law, Eq. 5–7b, to relate the orbits of the Earth and the comet around the Sun.

$$\left(\frac{r_{\text{comet}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^2 \rightarrow$$

$$r_{\text{comet}} = r_{\text{Earth}}\left(\frac{T_{\text{comet}}}{T_{\text{Earth}}}\right)^{2/3} = (1\text{ AU})\left(\frac{2400\text{ yr}}{1\text{ yr}}\right)^{2/3} = 179.3\text{ AU} \approx \boxed{180\text{ AU}}$$

(*b*) The mean distance is the numeric average of the closest and farthest distances.

$$179.3\text{ AU} = \frac{1.00\text{ AU} + r_{\max}}{2} \rightarrow r_{\max} = 357.6\text{ AU} \approx \boxed{360\text{ AU}}$$

(*c*) Refer to Fig. 5–29, which illustrates Kepler's second law. If the time for each shaded region is made much shorter, then the area of each region can be approximated as a triangle. The area of each triangle is half the "base" (speed of comet multiplied by the amount of time) times the "height" (distance from Sun). So we have the following:

$$\text{Area}_{\min} = \text{Area}_{\max} \rightarrow \tfrac{1}{2}(v_{\min}t)r_{\min} = \tfrac{1}{2}(v_{\max}t)r_{\max} \rightarrow$$

$$v_{\min}/v_{\max} = r_{\max}/r_{\min} = \boxed{360/1}$$

67. The centripetal acceleration is $a_R = \dfrac{v^2}{R_{\substack{\text{Earth}\\\text{orbit}}}} = \dfrac{\left(2\pi R_{\substack{\text{Earth}\\\text{orbit}}}/T\right)^2}{R_{\substack{\text{Earth}\\\text{orbit}}}} = \dfrac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2}$. The force (from Newton's second law) is $F_R = m_{\text{Earth}}a_R$. The period is one year, converted into seconds.

$$a_R = \frac{4\pi^2 R_{\substack{\text{Earth}\\\text{orbit}}}}{T^2} = \frac{4\pi^2(1.50\times10^{11}\text{ m})}{(3.15\times10^7\text{ s})^2} = \boxed{5.97\times10^{-3}\text{ m/s}^2}$$

$$F_R = ma = (5.97\times10^{24}\text{ kg})(5.97\times10^{-3}\text{ m/s}^2) = \boxed{3.56\times10^{22}\text{ N}}$$

$\boxed{\text{The Sun}}$ exerts this force on the Earth. It is a gravitational force.

68. Since mass *m* is dangling, the tension in the cord must be equal to the weight of mass *m*, so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass *M*, so $F_T = F_R = Ma_R = M\dfrac{v^2}{r}$. Equate the expressions for tension and solve for the velocity.

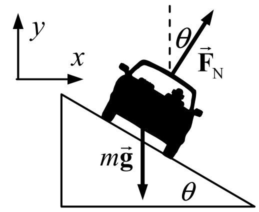
$$M\frac{v^2}{r} = mg \rightarrow v = \boxed{\sqrt{mgR/M}}$$

69. The force is a centripetal force, and is of magnitude $7.45\,mg$. Use Eq. 5–3 for centripetal force.

$$F = m\frac{v^2}{r} = 7.45\,mg \rightarrow v = \sqrt{7.45\,rg} = \sqrt{7.45(11.0\text{ m})(9.80\text{ m/s}^2)} = 28.34\text{ m/s} \approx \boxed{28.3\text{ m/s}}$$

$$(28.34\text{ m/s})\times\frac{1\text{ rev}}{2\pi(11.0\text{ m})} = \boxed{0.410\text{ rev/s}}$$

70. The car moves in a horizontal circle, so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's second law for both the x and y directions.



$$\sum F_y = F_N \cos\theta - mg = 0 \quad \rightarrow \quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_R = F_N \sin\theta = ma_x$$

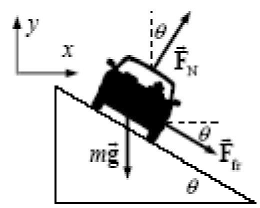
The amount of centripetal force needed for the car to round the curve is as follows:

$$F_R = m\frac{v^2}{r} = (1050 \text{ kg})\frac{\left[(85 \text{ km/h})\left(\dfrac{1.0 \text{ m/s}}{3.6 \text{ km/h}}\right)\right]^2}{72 \text{ m}} = 8.130\times10^3 \text{ N}$$

The actual horizontal force available from the normal force is as follows:

$$F_N \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1050 \text{ kg})(9.80 \text{ m/s}^2)\tan 14^\circ = 2.566\times10^3 \text{ N}$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's second law for both directions, and again the y acceleration is zero.

$$\sum F_y = F_N \cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = m\frac{v^2}{r}$$

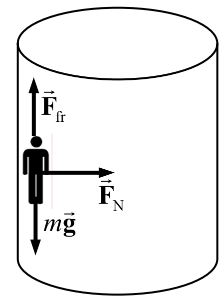
Substitute the expression for the normal force from the y equation into the x equation, and solve for the friction force.

$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = m\frac{v^2}{r} \quad \rightarrow \quad (mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = (8.130\times10^3 \text{ N})\cos 14^\circ - (1050 \text{ kg})(9.80 \text{ m/s}^2)\sin 14^\circ$$
$$= 5.399\times10^3 \text{ N}$$

So a frictional force of $\boxed{5.4\times10^3 \text{ N down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

$\boxed{71.}$ Consider the free-body diagram for a person in the "Rotor-ride." $\vec{\mathbf{F}}_N$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's second law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{fr} - mg = 0 \quad \rightarrow \quad F_{fr} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{fr} = \mu_s F_N = mg \quad \rightarrow \quad F_N = mg/\mu_s$$

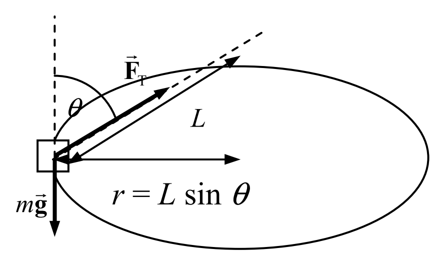
But the normal force must be the force causing the centripetal motion—it is the only force pointing to the center of rotation. Thus $F_R = F_N = m\dfrac{v^2}{r}$. Using $v = 2\pi r/T$, we have $F_N = \dfrac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.50 revolutions per second, the period is 2.0 s.

$$F_N = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \quad \rightarrow \quad \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{(9.80\text{ m/s}^2)(2.0\text{ s})^2}{4\pi^2(5.5\text{ m})} = \boxed{0.18}$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, so the period could be longer or the cylinder radius smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders. By Newton's third law, the riders therefore push against the wall. This gives the sensation of being pressed into the wall.

72. A free-body diagram for the sinker weight is shown. L is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r = L\sin\theta$. Write Newton's second law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.



$$\sum F_y = F_T\cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

The radial force is the horizontal portion of the tension. Write Newton's second law for the radial motion.

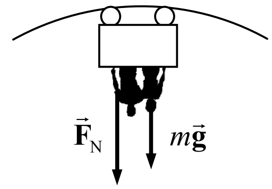
$$\sum F_R = F_T\sin\theta = ma_R = m\frac{v^2}{r}$$

Substitute the tension from the vertical equation and the relationships $r = L\sin\theta$ and $v = 2\pi r/T$.

$$F_T\sin\theta = m\frac{v^2}{r} \quad \rightarrow \quad \frac{mg}{\cos\theta}\sin\theta = \frac{4\pi^2 mL\sin\theta}{T^2} \quad \rightarrow \quad \cos\theta = \frac{gT^2}{4\pi^2 L}$$

$$\theta = \cos^{-1}\frac{gT^2}{4\pi^2 L} = \cos^{-1}\frac{(9.80\text{ m/s}^2)(0.75\text{ s})^2}{4\pi^2(0.25\text{ m})} = \boxed{56^\circ}$$

73. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.



$$\sum F = F_N + mg = ma = m\frac{v^2}{r} \rightarrow F_N = m\left(\frac{v^2}{r} - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car—they are in free fall. The limiting condition is as follows:

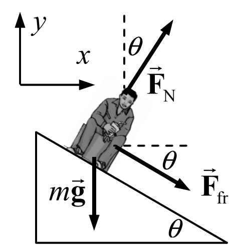
$$\frac{v_{min}^2}{r} - g = 0 \rightarrow v_{min} = \sqrt{rg} = \sqrt{(9.80\text{ m/s}^2)(8.6\text{ m})} = \boxed{9.2\text{ m/s}}$$

74. The speed of the train is $(160\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 44.44\text{ m/s}$.

(*a*) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_R = m\frac{v^2}{R} = \frac{(55\text{ kg})(44.44\text{ m/s})^2}{(570\text{ m})} = 190.6\text{ N} \approx \boxed{1.9\times10^2\text{ N}}$$

(*b*) For the banked case, the normal force will contribute to the radial force needed. Write Newton's second law for both the *x* and *y* directions. The *y* acceleration is zero, and the *x* acceleration is radial.



$$\sum F_y = F_N \cos\theta - mg - F_{fr}\sin\theta = 0 \rightarrow F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = m\frac{v^2}{r}$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

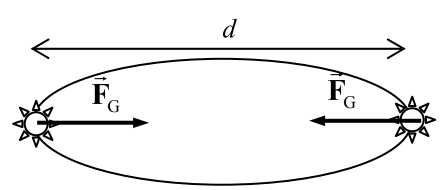
$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = m\frac{v^2}{r} \rightarrow$$

$$(mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta \rightarrow$$

$$F_{fr} = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

$$= (55\text{ kg})\left[\frac{(44.44\text{ m/s})^2}{570\text{ m}}\cos 8.0° - (9.80\text{ m/s}^2)\sin 8.0°\right] = 113.7\text{ N} \approx \boxed{1.1\times10^2\text{ N}}$$

75. See the diagram for the two stars.



(*a*) The two stars don't crash into each other because of their circular motion. The force on them is centripetal and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.

(*b*) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $\upsilon = 2\pi r/T = \pi d/T$, and solve for the mass.

$$F_G = G\frac{M^2}{d^2} = F_R = M\frac{\upsilon^2}{d/2} = M\frac{2(\pi d/T)^2}{d} = \frac{2\pi^2 Md}{T^2} \rightarrow G\frac{M^2}{d^2} = \frac{2\pi^2 Md}{T^2} \rightarrow$$

$$M = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2(8.0\times10^{11}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left(12.6\text{ yr}\times\frac{3.15\times10^7\text{ s}}{1\text{ yr}}\right)^2} = \boxed{9.6\times10^{29}\text{ kg}}$$

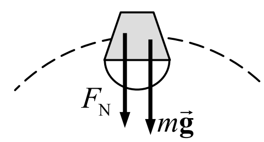
76. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g = GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. Find the location where $g = \frac{1}{2}g_{\text{surface}}$.

$$\frac{GM_{\text{Earth}}}{r^2} = \frac{1}{2}\frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} \rightarrow r^2 = 2R_{\text{Earth}}^2 \rightarrow r = \sqrt{2}R_{\text{Earth}}$$

The distance above the Earth's surface is as follows:

$$r - R_{\text{Earth}} = \left(\sqrt{2}-1\right)R_{\text{Earth}} = \left(\sqrt{2}-1\right)(6.38\times10^6\text{ m}) = \boxed{2.64\times10^6\text{ m}} = 0.414\,R_{\text{Earth}}$$

77. We assume the water is rotating in a vertical circle of radius r. When the bucket is at the top of its motion, there would be two forces on the water (considering the water as a single mass). The weight of the water would be directed down, and the normal force of the bottom of the bucket pushing on the water would also be down. See the free-body diagram. If the water is moving in a circle, then the net downward force would be a centripetal force.



$$\sum F = F_N + mg = ma = m\frac{\upsilon^2}{r} \rightarrow F_N = m\left(\frac{\upsilon^2}{r} - g\right)$$

The limiting condition of the water falling out of the bucket means that the water loses contact with the bucket, so the normal force becomes 0.

$$F_N = m\left(\frac{\upsilon^2}{r} - g\right) \rightarrow m\left(\frac{\upsilon_{\text{critical}}^2}{r} - g\right) = 0 \rightarrow \upsilon_{\text{critical}} = \sqrt{rg}$$

From this, we see that $\boxed{\text{yes}}$, it is possible to whirl the bucket of water fast enough. The minimum speed is $\boxed{\sqrt{rg}}$. All you really need to know is the radius of the circle in which you will be swinging the bucket. It would be approximately the length of your arm, plus the height of the bucket.

78. For an object to be apparently weightless, the object would have a centripetal acceleration equal to g. This is the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_R = \frac{\upsilon^2}{R_{\text{Earth}}}$, along with $\upsilon = 2\pi R_{\text{Earth}}/T$, and solve for T.

$$g = \frac{\upsilon^2}{R_{\text{Earth}}} = \frac{4\pi^2 R_{\text{Earth}}}{T^2} \rightarrow T = 2\pi\sqrt{\frac{R_{\text{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\text{ m}}{9.80\text{ m/s}^2}} = \boxed{5.07\times10^3\text{ s}}(\approx 84.5\text{ min})$$

79. The speed of an object in an orbit of radius *r* around a planet is given in Example 5–12 as $v=\sqrt{GM_{\text{planet}}/r}$, and is also given by $v=2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G\frac{M_{\text{Planet}}}{r}}=\frac{2\pi r}{T} \quad\rightarrow\quad T=2\pi\sqrt{\frac{r^3}{GM_{\text{Planet}}}}$$

For this problem, the inner orbit has radius $r_{\text{inner}}=7.3\times10^7\ \text{m}$, and the outer orbit has radius $r_{\text{outer}}=1.7\times10^8\ \text{m}$. Use these values to calculate the periods.

$$T_{\text{inner}}=2\pi\sqrt{\frac{(7.3\times10^7\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.7\times10^{26}\ \text{kg})}}=\boxed{2.0\times10^4\ \text{s}}$$

$$T_{\text{outer}}=2\pi\sqrt{\frac{(1.7\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.7\times10^{26}\ \text{kg})}}=\boxed{7.1\times10^4\text{s}}$$

Saturn's rotation period (day) is 10 h 39 min, which is about 3.8×10^4 s. Thus the inner ring will appear to move across the sky faster than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky slower than the Sun (about once every two Saturn days).

80. The speed of an object in an orbit of radius *r* around the Moon is given by $v=\sqrt{GM_{\text{Moon}}/r}$, and is also given by $v=2\pi r/T$, where *T* is the period of the object in orbit. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{GM_{\text{Moon}}/r}=2\pi r/T \quad\rightarrow$$

$$T=2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}}=2\pi\sqrt{\frac{\left(R_{\text{Moon}}+100\ \text{km}\right)^3}{GM_{\text{Moon}}}}=2\pi\sqrt{\frac{(1.74\times10^6\ \text{m}+1\times10^5\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\ \text{kg})}}$$

$$=\boxed{7.1\times10^3\ \text{s}\ \ (\approx 2.0\ \text{h})}$$

$\boxed{81.}$ (*a*) The speed of a satellite orbiting the Earth is given by $v=\sqrt{GM_{\text{Earth}}/r}$. For the GPS satellites, $r=R_{\text{Earth}}+(11{,}000)(1.852\ \text{km})=2.68\times10^7\ \text{m}$.

$$v=\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.97\times10^{24}\ \text{kg})}{2.68\times10^7\ \text{m}}}=3.86\times10^3\ \text{m/s}\approx\boxed{3.9\times10^3\ \text{m/s}}$$

(*b*) The period can be found from the speed and the radius.

$$v=2\pi r/T \quad\rightarrow\quad T=\frac{2\pi r}{v}=\frac{2\pi(2.68\times10^7\ \text{m})}{3.86\times10^3\ \text{m/s}}=\boxed{4.4\times10^4\ \text{s}\ \approx 12\ \text{h}}$$

82. (*a*) If the asteroid were a sphere, then the mass would be given by $M=\rho V=\frac{4}{3}\pi\rho r^3$. We first find the mass by multiplying the density and the volume, and then use that mass to solve for the radius as if it were a sphere.

$$M=\rho V=\left(2.3\times10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(40000\times6000\times6000\ \text{m}^3\right)=3.312\times10^{15}\ \text{kg}$$

$$r=\left(\frac{3M}{4\pi\rho}\right)^{1/3}=\left(\frac{3(3.312\times10^{15}\ \text{kg})}{4\pi\left(2.3\times10^{3}\ \dfrac{\text{kg}}{\text{m}^3}\right)}\right)^{1/3}=7005\ \text{m}\approx\boxed{7\times10^{3}\ \text{m}}$$

(*b*) The acceleration due to gravity is found from the mass and the radius of the hypothetical sphere.

$$g=GM/r^2=(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(3.312\times10^{15}\ \text{kg})}{(7005\ \text{m})^2}=4.502\times10^{-3}\ \text{m/s}^2$$

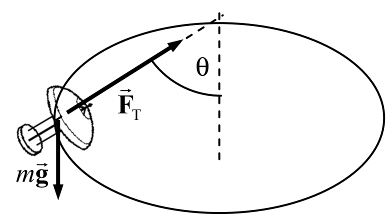
$$\approx\boxed{5\times10^{-3}\ \text{m/s}^2}$$

(*c*) The speed of an object orbiting a mass M is given by $v=\sqrt{GM/r}$. The period of an object moving in a circle path is given by $T=2\pi r/v$. Combine these relationships to find the period.

$$T=\frac{2\pi r}{\sqrt{G\dfrac{M}{r}}}=\frac{2\pi(2\times10^{4}\ \text{m})}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(3.312\times10^{15}\ \text{kg})}{(2\times10^{4}\ \text{m})}}}=3.781\times10^{4}\ \text{s}$$

$$\approx\boxed{4\times10^{4}\ \text{s}}\approx 11\ \text{h}$$

83. The lamp must have the same speed and acceleration as the train. The forces on the lamp as the train rounds the corner are shown in the free-body diagram included. The tension in the suspending cord must not only hold the lamp up, but also provide the centripetal force needed to make the lamp move in a circle. Write Newton's second law for the vertical direction, noting that the lamp is not accelerating vertically.



$$\sum F_y=F_{\text{T}}\cos\theta-mg=0\quad\rightarrow\quad F_{\text{T}}=\frac{mg}{\cos\theta}$$

The force moving the lamp in a circle is the horizontal portion of the tension. Write Newton's second law for that radial motion.

$$\sum F_{\text{R}}=F_{\text{T}}\sin\theta=ma_{\text{R}}=m\frac{v^2}{r}$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the speed.

$$F_{\text{T}}\sin\theta=\frac{mg}{\cos\theta}\sin\theta=mg\tan\theta=m\frac{v^2}{r}\quad\rightarrow$$

$$v=\sqrt{rg\tan\theta}=\sqrt{(215\ \text{m})(9.80\ \text{m/s}^2)\tan16.5^\circ}=\boxed{25.0\ \text{m/s}}$$

84. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $v=\sqrt{G\dfrac{M_{\text{galaxy}}}{r_{\text{Sun orbit}}}}$, so $M_{\text{galaxy}}=\dfrac{r_{\text{Sun orbit}}v^2}{G}$. Substitute in the relationship that $v=2\pi r_{\text{Sun orbit}}/T$.

$$M_{\text{galaxy}}=\frac{4\pi^2(r_{\text{Sun orbit}})^3}{GT^2}=\frac{4\pi^2[(30{,}000)(9.5\times10^{15}\ \text{m})]^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left[(200\times10^{6}\ \text{yr})\left(\dfrac{3.15\times10^{7}\ \text{s}}{1\ \text{yr}}\right)\right]^2}$$

$$=3.452\times10^{41}\ \text{kg}\approx\boxed{3\times10^{41}\ \text{kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\# \text{ stars} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{3.452\times10^{41}\text{ kg}}{2.0\times10^{30}\text{ kg}} = 1.726\times10^{11} \approx \boxed{2\times10^{11}\text{ stars}}$$

85. (*a*) The gravitational force on the satellite is given by $F_{\text{grav}} = G\dfrac{M_{\text{Earth}}m}{r^2}$, where r is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text{net}} = m\dfrac{v^2}{r}$. By substituting $v = 2\pi r/T$ for a circular orbit, we have $F_{\text{net}} = \dfrac{4\pi^2 mr}{T^2}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated and solved for the orbit radius.

$$G\frac{M_{\text{Earth}}m}{r^2} = \frac{4\pi^2 mr}{T^2} \quad \rightarrow$$

$$r = \left(\frac{GM_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0\times10^{24}\text{ kg})(6600\text{ s})^2}{4\pi^2}\right]^{1/3}$$

$$= 7.615\times10^6\text{ m} \approx \boxed{7.6\times10^6\text{ m}}$$

(*b*) From this value the gravitational force on the satellite can be calculated.

$$F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.0\times10^{24}\text{ kg})(5500\text{ kg})}{(7.615\times10^6\text{ m})^2} = 3.796\times10^4\text{ N}$$

$$\approx \boxed{3.8\times10^4\text{ N}}$$

(*c*) The altitude of the satellite above the Earth's surface is given by the following:

$$r - R_{\text{Earth}} = 7.615\times10^6\text{ m} - 6.38\times10^6\text{ m} = \boxed{1.2\times10^6\text{ m}}$$

86. The speed of an orbiting object is given in Example 5–12 as $v = \sqrt{GM/r}$, where r is the radius of the orbit, and M is the mass around which the object is orbiting. Solve the equation for M.

$$v = \sqrt{GM/r} \quad \rightarrow \quad M = \frac{rv^2}{G} = \frac{(5.7\times10^{17}\text{ m})(7.8\times10^5\text{ m/s})^2}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = \boxed{5.2\times10^{39}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ solar masses} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{5.2\times10^{39}\text{ kg}}{2\times10^{30}\text{ kg}} = \boxed{2.6\times10^9\text{ solar masses}}$$

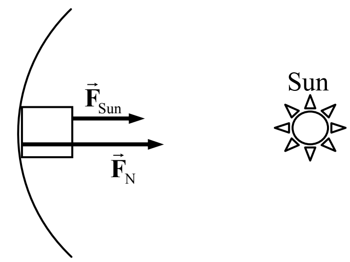
87. Find the "new" Earth radius by setting the acceleration due to gravity at the Sun's surface equal to the acceleration due to gravity at the "new" Earth's surface.

$$g_{\substack{\text{Earth}\\\text{new}}} = g_{\text{Sun}} \quad \rightarrow \quad \frac{GM_{\text{Earth}}}{r^2_{\substack{\text{Earth}\\\text{new}}}} = \frac{GM_{\text{Sun}}}{r^2_{\text{Sun}}} \quad \rightarrow$$

$$r_{\substack{\text{Earth}\\\text{new}}} = r_{\text{Sun}}\sqrt{\frac{M_{\text{Earth}}}{M_{\text{Sun}}}} = (6.96\times10^8\text{ m})\sqrt{\frac{5.98\times10^{24}\text{ kg}}{1.99\times10^{30}\text{ kg}}} = \boxed{1.21\times10^6\text{ m}}$$

This is about 1/5 the actual Earth radius.

88. If the ring is to produce an apparent gravity equivalent to that of Earth, then the normal force of the ring on objects must be given by $F_N = mg$. The Sun will also exert a force on objects on the ring. See the free-body diagram. Write Newton's second law for the object, with the fact that the acceleration is centripetal.



$$\sum F = F_R = F_{Sun} + F_N = m\frac{v^2}{r}$$

Substitute in the relationships $v = 2\pi r/T$, $F_N = mg$, and $F_{Sun} = G\frac{M_{Sun}m}{r^2}$, and solve for the period of the rotation.

$$F_{Sun} + F_N = m\frac{v^2}{r} \quad \rightarrow \quad G\frac{M_{Sun}m}{r^2} + mg = \frac{4\pi^2 mr}{T^2} \quad \rightarrow \quad G\frac{M_{Sun}}{r^2} + g = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r}{G\frac{M_{Sun}}{r^2} + g}} = \sqrt{\frac{4\pi^2(1.50\times10^{11}\text{ m})}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.99\times10^{30}\text{ kg})}{(1.50\times10^{11}\text{ m})^2} + 9.80\text{ m/s}^2}}$$

$$= 7.77\times10^5\text{ s} = \boxed{8.99\text{ d}}$$

Note that the force of the Sun is only about 1/1600 the size of the normal force. The force of the Sun could have been ignored in the calculation with no significant change in the result given above.

89. The speed of an object orbiting a mass is given in Example 5–12 as $v = \sqrt{\frac{GM_{Sun}}{r}}$.

$$v_{new} = 1.5v \text{ and } v_{new} = \sqrt{\frac{GM_{Sun}}{r_{new}}} \quad \rightarrow \quad 1.5v = \sqrt{\frac{GM_{Sun}}{r_{new}}} \quad \rightarrow \quad 1.5\sqrt{\frac{GM_{Sun}}{r}} = \sqrt{\frac{GM_{Sun}}{r_{new}}} \quad \rightarrow$$

$$r_{new} = \frac{r}{1.5^2} = \boxed{0.44r}$$

Note that the answer doesn't depend on either of the asteroid masses.

90. The goal is to form a quantity that has acceleration units, from the speed of the radius of an object in circular motion. Speed has dimensions $\left[\frac{L}{T}\right]$, radius has dimensions $[L]$, and acceleration has dimensions $\left[\frac{L}{T^2}\right]$. To get time units squared in the denominator, the speed must be squared. But the dimensions of speed squared are $\left[\frac{L^2}{T^2}\right]$. This has one too many powers of length, so to reduce that, divide by the radius.

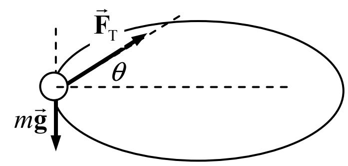
$$\frac{v^2}{r} \rightarrow \frac{\left[\frac{L^2}{T^2}\right]}{[L]} = \left[\frac{L}{T^2}\right] \rightarrow a$$

Factors such as 2 or π, if needed for the final formula, cannot be determined with dimensional analysis.

Solutions to Search and Learn Problems

1. Ex. 5–1: (i) The ball; (ii) tension in the string acting on the ball.
   Ex. 5–2: (i) The Moon; (ii) gravitational force on the Moon from the Earth.
   Ex. 5–3: (i) The ball; (ii) tension in the string acting on the ball.
   Ex. 5–4: (i) The ball; (ii) at the top it is the sum of the tension and force of gravity; at the bottom it is the difference between tension and gravity.
   Ex. 5–5: (i) The tetherball; (ii) the horizontal component of the tension.
   Ex. 5–6: (i) The car; (ii) the force of static friction between the tires and the road.
   Ex. 5–7: (i) The car; (ii) the horizontal component of the normal force.
   Ex. 5–8: (i) The race car; (ii) the radial component of the static friction.
   Ex. 5–9: No centripetal acceleration
   Ex. 5–10: (i) The spacecraft; (ii) the force of gravity on the spacecraft from the Earth.
   Ex. 5–11: No centripetal acceleration.
   Ex. 5–12: (i) The satellite; (ii) the force of gravity on the satellite from the Earth
   Ex. 5–13: (i) Mars; (ii) the force of gravity on Mars from the Sun.
   Ex. 5–14: (i) Earth; (ii) the force of gravity on Earth from the Sun.

2. A free-body diagram for the ball is shown, similar to Fig. 5–7. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's second law for the vertical direction, noting that the ball is not accelerating vertically.



$$\sum F_y = F_{\mathrm{T}} \sin\theta - mg = 0 \quad\rightarrow\quad F_{\mathrm{T}} = \frac{mg}{\sin\theta}$$

The force moving the ball in a circle is the horizontal component of the tension. Write Newton's second law for that radial motion.

$$\sum F_{\mathrm{R}} = F_{\mathrm{T}} \cos\theta = ma_{\mathrm{R}} = m\frac{v^2}{r}$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $v = 2\pi r/T$. Finally, we recognize that if the string is of length L, then the radius of the circle is $r = L\cos\theta$.

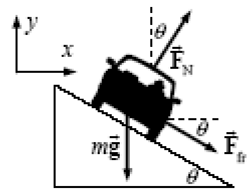
$$F_{\mathrm{T}} \cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 mL\cos\theta}{T^2} \quad\rightarrow$$

$$\sin\theta = \frac{gT^2}{4\pi^2 L} \quad\rightarrow\quad \theta = \sin^{-1}\frac{gT^2}{4\pi^2 L} = \sin^{-1}\frac{(9.80\ \mathrm{m/s^2})(0.500\ \mathrm{s})^2}{4\pi^2(0.600\ \mathrm{m})} = \boxed{5.94^\circ}$$

The tension is then given by $F_{\mathrm{T}} = \dfrac{mg}{\sin\theta} = \dfrac{(0.150\ \mathrm{kg})(9.80\ \mathrm{m\ s^2})}{\sin 5.94^\circ} = \boxed{14.2\ \mathrm{N}}$

3. From Example 5–7 in the textbook, the no-friction banking angle is given by $\theta = \tan^{-1}\dfrac{v_0^2}{Rg}$, or

$v_0^2 = Rg\tan\theta$. The centripetal force in this case is provided by a component of the normal force. Driving at a higher speed with the same radius requires more centripetal force than that provided by the normal force alone. The additional centripetal force is supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. The center of the circle of the car's motion is to the right of the car in the diagram. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg - F_{\text{fr}}\sin\theta = 0 \;\rightarrow\; F_{\text{N}}\cos\theta - \mu_{\text{s}} F_{\text{N}}\sin\theta = mg \;\rightarrow$$

$$F_{\text{N}} = \frac{mg}{(\cos\theta - \mu_{\text{s}}\sin\theta)}$$

$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = m\frac{v^2}{R} \;\rightarrow\; F_{\text{N}}\sin\theta + \mu_{\text{s}} F_{\text{N}}\cos\theta = m\frac{v^2}{R} \rightarrow$$

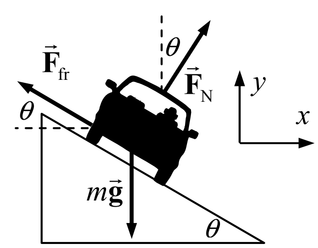
$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta + \mu_{\text{s}}\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed, which is the maximum speed that the car can have.

$$\frac{mv^2/R}{(\sin\theta + \mu_{\text{s}}\cos\theta)} = \frac{mg}{(\cos\theta - \mu_{\text{s}}\sin\theta)} \;\rightarrow$$

$$v_{\max} = \sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1 + \mu_{\text{s}}/\tan\theta)}{(1 - \mu_{\text{s}}\tan\theta)}} = v_0\sqrt{\frac{(1 + Rg\mu_{\text{s}}/v_0^2)}{(1 - \mu_{\text{s}} v_0^2/Rg)}}$$

Driving at a slower speed with the same radius requires less centripetal force than that provided by the normal force alone. The decrease in centripetal force is supplied by a force of static friction, upward along the incline. See the free-body diagram for the car on the incline. Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction and centripetal acceleration in the x direction. Assume that the car is on the verge of skidding, so that the static frictional force is given by $F_{\text{fr}} = \mu_{\text{s}} F_{\text{N}}$.



$$\sum F_y = F_{\text{N}}\cos\theta - mg + F_{\text{fr}}\sin\theta = 0 \;\rightarrow$$

$$F_{\text{N}}\cos\theta + \mu_{\text{s}} F_{\text{N}}\sin\theta = mg \;\rightarrow\; F_{\text{N}} = \frac{mg}{(\cos\theta + \mu_{\text{s}}\sin\theta)}$$

$$\sum F_x = F_{\text{R}} = F_{\text{N}}\sin\theta - F_{\text{fr}}\cos\theta = m\frac{v^2}{R} \rightarrow F_{\text{N}}\sin\theta - \mu_{\text{s}} F_{\text{N}}\cos\theta = m\frac{v^2}{R} \rightarrow$$

$$F_{\text{N}} = \frac{mv^2/R}{(\sin\theta - \mu_{\text{s}}\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/R}{(\sin\theta-\mu_s\cos\theta)}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)} \rightarrow$$

$$v_{min}=\sqrt{Rg\frac{\sin\theta}{\cos\theta}\frac{(1-\mu_s/\tan\theta)}{(1+\mu_s\tan\theta)}}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}$$

Thus $\boxed{v_{min}=v_0\sqrt{\frac{(1-\mu_s Rg/v_0^2)}{(1+\mu_s v_0^2/Rg)}}}$ and $\boxed{v_{max}=v_0\sqrt{\frac{(1+\mu_s Rg/v_0^2)}{(1-\mu_s v_0^2/Rg)}}}$.

4. An object at the Earth's equator is rotating in a circle with a radius equal to the radius of the Earth and a period equal to one day. Use that data to find the centripetal acceleration and then compare it with g.

$$a_R=\frac{v^2}{r}=\frac{\left(\frac{2\pi r}{T}\right)^2}{r}=\frac{4\pi^2 r}{T^2} \rightarrow \frac{a_R}{g}=\frac{\frac{4\pi^2(6.38\times10^6\text{ m})}{(86,400\text{ s})^2}}{(9.80\text{ m/s}^2)}=0.00344\approx\boxed{\frac{3}{1000}}$$

So, for example, if we were to calculate the normal force on an object at the Earth's equator, we could not say $\sum F=F_N-mg=0$. Instead, we would have the following:

$$\sum F=F_N-mg=-m\frac{v^2}{r} \rightarrow F_N=mg-m\frac{v^2}{r}$$

If we then assumed that $F_N=mg_{eff}=mg-m\frac{v^2}{r}$, then we see that the effective value of g is

$$g_{eff}=g-\frac{v^2}{r}=g-0.003g=\boxed{0.997g}.$$

5. (*a*) The acceleration due to gravity at any location at or above the surface of a star is given by $g_{star}=GM_{star}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{star}=G\frac{M_{sun}}{R_{Moon}^2}=(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.99\times10^{30}\text{ kg})}{(1.74\times10^6\text{ m})^2}=\boxed{4.38\times10^7\text{ m/s}^2}$$

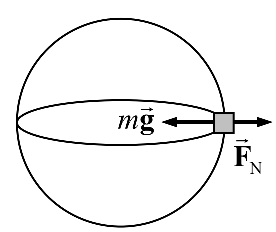
(*b*) $W=mg_{star}=(65\text{ kg})(4.38\times10^7\text{ m/s}^2)=\boxed{2.8\times10^9\text{ N}}$

(c) Use Eq. 2–11c, with an initial velocity of 0.

$$v^2=v_0^2+2a(x-x_0) \rightarrow$$

$$=\sqrt{2a(x-x_0)}=\sqrt{2(4.38\times10^7\text{ m/s}^2)(1.0\text{ m})}=\boxed{9.4\times10^3\text{ m/s}}$$

6. For a body on the equator, the net motion is circular. Consider the free-body diagram as shown. F_N is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's second law with the inward direction as positive.



$$\sum F_R = mg_{\text{Jupiter}} - F_N = m\frac{v^2}{R_{\text{Jupiter}}} \quad \rightarrow$$

$$F_N = m\left(g_{\text{Jupiter}} - \frac{v^2}{R_{\text{Jupiter}}}\right) = m\left(G\frac{M_{\text{Jupiter}}}{R_{\text{Jupiter}}^2} - \frac{v^2}{R_{\text{Jupiter}}}\right)$$

Use the fact that for a rotating object, $v = 2\pi r/T$.

$$F_N = m\left(G\frac{M_{\text{Jupiter}}}{R_{\text{Jupiter}}^2} - \frac{4\pi^2 R_{\text{Jupiter}}}{T_{\text{Jupiter}}^2}\right)$$

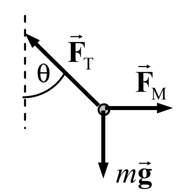
Thus the perceived acceleration of the object on the surface of Jupiter is

$$G\frac{M_{\text{Jupiter}}}{R_{\text{Jupiter}}^2} - \frac{4\pi^2 R_{\text{Jupiter}}}{T_{\text{Jupiter}}^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.9\times10^{27}\ \text{kg})}{(7.1\times10^7\ \text{m})^2} - \frac{4\pi^2(7.1\times10^7\ \text{m})}{\left[(595\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\right]^2}$$

$$= 22.94\ \text{m/s}^2\left(\frac{1\,g}{9.80\ \text{m/s}^2}\right) = \boxed{2.3\ g\text{'s}}$$

Thus you would not be crushed at all. You would certainly feel "heavy" and quite uncomfortable, but not at all crushed.

7. (*a*) See the free-body diagram for the plumb bob. The attractive gravitational force on the plumb bob is $F_M = G\frac{mm_M}{D_M^2}$. Since the bob is not accelerating, the net force in any direction will be zero. Write the net force for both vertical and horizontal directions. Use $g = G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}$.



$$\sum F_{\text{vertical}} = F_T\cos\theta - mg = 0 \quad\rightarrow\quad F_T = \frac{mg}{\cos\theta}$$

$$\sum F_{\text{horizontal}} = F_M - F_T\sin\theta = 0 \quad\rightarrow\quad F_M = F_T\sin\theta = mg\tan\theta$$

$$G\frac{mm_M}{D_M^2} = mg\tan\theta \quad\rightarrow\quad \theta = \tan^{-1}G\frac{m_M}{gD_M^2} = \boxed{\tan^{-1}\frac{m_M R_{\text{Earth}}^2}{M_{\text{Earth}}D_M^2}}$$

(*b*) We estimate the mass of Mt. Everest by taking its volume times its mass density. If we approximate Mt. Everest as a cone with the same size diameter as height, then its volume is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2000\ \text{m})^2(4000\ \text{m}) = 1.7\times10^{10}\ \text{m}^3$. The density is $\rho = 3\times10^3\ \text{kg/m}^3$. Find the mass by multiplying the volume times the density.

$$M = \rho V = (3\times10^3\ \text{kg/m}^3)(1.7\times10^{10}\ \text{m}^3) = \boxed{5\times10^{13}\ \text{kg}}$$

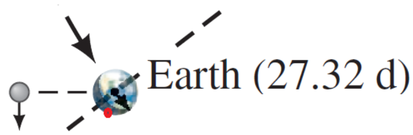
(*c*) With $D_M = 5000$ m, use the relationship derived in part (*a*).

$$\theta = \tan^{-1}\frac{M_M R_{\text{Earth}}^2}{M_{\text{Earth}}D_M^2} = \tan^{-1}\frac{(5\times10^{13}\ \text{kg})(6.38\times10^6\ \text{m})^2}{(5.97\times10^{24}\ \text{kg})(5000\ \text{m})^2} = \boxed{8\times10^{-4}\ \text{degrees}}$$

8. (*a*) The Moon is Full when the Sun and Moon are on opposite sides of the Earth. In this position, a person on the Earth will only be able to see either the Sun or the Moon in the sky at any given time. Therefore, as the Sun sets, the Moon rises and as the Moon sets, the Sun rises.

(*b*) As the Moon orbits the Earth it moves toward the east 1/29.53 of a synodic orbit, or about 12° every day. Therefore, if the Moon was just rising at 6 PM on the day of the Full Moon, it would be about 12° below the horizon at 6 PM the next day, and therefore would not be visible.

(*c*) The red dot represents the location of a person on the Earth who sees the Full Moon rise at 6 PM on the day shown as figure (a). A day later (b) the Earth has completed one full rotation and for the person at that location it is again 6 PM. When the next Full Moon arrives, 29.53 days have elapsed. That means the red dot has revolved around the Earth about 29 and a half times. Because of the half revolution, the dot is on the other side of the Earth. To the observer it is now about 6 AM and the Full Moon is setting as the Sun rises. In part (d) the Earth will have completed about 27 and a third revolutions, so the red dot should be about one-third of a counter-clockwise rotation from 6 PM, or about 2 AM.



(*d*) The Earth completes one full revolution, or 360°, around the Sun every year, or 365.25 days. The angle of the Moon in Fig. 5–31e relative to the "horizontal" (the dashed line in part (a)) is equal to the angle that the Earth moves between consecutive Full Moons:

$$\theta = \left(\frac{29.53 \text{ days}}{365.25 \text{ days}}\right)360° = 29.11°$$

So in 29.53 days the Moon has orbited 360° + 29.11° = 389.11°. The angular speed of the Moon is constant and can be written as the ratio of the orbital angle to orbital period for either sidereal or synodic orbits. Setting the ratios equal, solve for the sidereal period.

$$\omega = \frac{\theta_{\text{sidereal}}}{T_{\text{sidereal}}} = \frac{\theta_{\text{synoptic}}}{T_{\text{synoptic}}} \rightarrow$$

$$T_{\text{sidereal}} = T_{\text{synoptic}}\left(\frac{\theta_{\text{sidereal}}}{\theta_{\text{synoptic}}}\right) = (29.53 \text{ days})\left(\frac{360°}{389.11°}\right) = \boxed{27.32 \text{ days}}$$

6

WORK AND ENERGY

Responses to Questions

1. "Work" as used in everyday language generally means "energy expended," which is similar to the way "work" is defined in physics. However, in everyday language, "work" can involve mental or physical energy expended and is not necessarily connected with displacement, as it is in physics. So a student could say she "worked" hard carrying boxes up the stairs to her dorm room (similar in meaning to the physics usage), or that she "worked" hard on a problem set (different in meaning from the physics usage).

2. No, not if the object is moving in a circle. Work is the product of force and the displacement in the direction of the force. Therefore, a centripetal force, which is perpendicular to the direction of motion, cannot do work on an object moving in a circle.

3. No work is done *on the wall* (since the wall does not undergo displacement), but internally your muscles are converting chemical energy to other forms of energy, which makes you tired.

4. Yes. The normal force is the force perpendicular to the surface an object is resting on. If the object moves with a component of its displacement perpendicular to this surface, the normal force will do work. For instance, when you jump, the normal force does work on you in accelerating you vertically. And it is the normal force of the elevator floor on you that accelerates you in an elevator.

5. (*a*) If the force is the same, then $F = k_1 x_1 = k_2 x_2$, so $x_2 = k_1 x_1 / k_2$. The work done on spring 1 will be $W_1 = \frac{1}{2} k_1 x_1^2$. The work done on spring 2 will be $W_2 = \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_2 (k_1^2 x_1^2 / k_2^2) = W_1 (k_1 / k_2)$. Since $k_1 > k_2$, work $W_2 > W_1$, so more work is done on spring 2.

   (*b*) If the displacement is the same, then $W_1 = \frac{1}{2} k_1 x^2$ and $W_2 = \frac{1}{2} k_2 x^2$. Since $k_1 > k_2$, work $W_1 > W_2$, and more work is done on spring 1.

6. The kinetic energy increases by a factor of 9, since the kinetic energy is proportional to the square of the speed.

7. Friction is not conservative; it dissipates energy in the form of heat, sound, and light. Air resistance is not conservative; it dissipates energy in the form of heat and the kinetic energy of fluids. "Human"

forces, for example, the forces produced by your muscles, such as pushing a box across the floor, are also not conservative. They dissipate energy in the form of heat and also through chemical processes.

8. The speed at point C will be less than twice the speed at point B. The force is constant and the displacements are the same, so the same *work* is done on the block from A to B as from B to C. Since there is no friction, the same work results in the same *change in kinetic energy*. But kinetic energy depends on the square of the speed, so the speed at point C will be greater than the speed at point B by a factor of $\sqrt{2}$, not a factor of 2.

9. The two forces on the book are the applied force upward (nonconservative) and the downward force of gravity (conservative). If air resistance is not negligible, it is nonconservative.

10. (*a*) The speed at the bottom of the hill does not depend on the angle of the hill if there is no friction. If there is no friction, then gravity is the only force doing work on the sled, and the system is conservative. All of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy. The speed at the bottom of the hill depends on only the initial height *h*, not on the angle of the hill. $\mathrm{KE_f} = \frac{1}{2}mv^2 = mgh$, and $v = (2gh)^{1/2}$.

    (*b*) The speed at the bottom of the hill does depend on the angle of the hill if there is friction. If friction is present, then the net force doing work on the sled is not conservative. Only part of the gravitational potential energy of the sled at the top of the hill will be converted into kinetic energy; the rest will be dissipated by the frictional force. The frictional force is proportional to the normal force on the sled, which will depend on the angle θ of the hill. $\mathrm{KE_f} = \frac{1}{2}mv^2$ $= mgh - fx = mgh - \mu mgh\cos\theta/\sin\theta = mgh(1-\mu/\tan\theta)$, so $v = [2gh(1-\mu/\tan\theta)]^{1/2}$, which does depend on the angle of the hill and will be smaller for smaller angles.

11. At the top of the pendulum's swing, all of its energy is gravitational potential energy; at the bottom of the swing, all of the energy is kinetic.

    (*a*) If we can ignore friction, then energy is transformed back and forth between potential energy and kinetic energy as the pendulum swings.

    (*b*) If friction is present, then during each swing energy is lost to friction at the pivot point and also to air resistance. During each swing, the kinetic energy and the potential energy decrease, and the pendulum's amplitude decreases. When a grandfather clock is "wound up," the amount of energy that will eventually be lost to friction and air resistance is stored as potential energy (either elastic or gravitational, depending on the clock mechanism), and part of the workings of the clock is to put that stored energy back into the pendulum at the same rate that friction is dissipating the energy.

12. For each of the balloons, the initial energy (kinetic plus potential) equals the final energy (all kinetic). Since the initial energy depends on only the speed and not on the direction of the initial velocity, and all balloons have the same initial speed and height, the final speeds are all the same.

13. The initial potential energy of the water is converted first into the kinetic energy of the water as it falls. When the falling water hits the pool, it does work on the water already in the pool, creating splashes and waves. Additionally, some energy is converted into heat and sound.

14. Stepping on top of a log and jumping down the other side requires you to raise your center of mass farther than just stepping over a log does. Raising your center of mass farther requires you to do more work and thereby use more energy.

15. (*a*) The golfer raises the club, giving the club potential energy. In swinging, the golfer does work on the club. This work and the change in potential energy increase the kinetic energy of the club. At the lowest point in the swing (when the club's kinetic energy is a maximum), the club hits the ball, converting some of the kinetic energy of the club into kinetic energy of the ball.

    (*b*) The tennis player throws the ball upward, giving the ball some initial kinetic energy. As the ball rises to its highest point, the kinetic energy is converted to potential energy. The player then does work on the racket to increase the racket's kinetic energy. When the racket collides with the ball, the racket does work on the ball. The racket loses kinetic energy, and the ball gains kinetic energy, accelerating the ball forward.

    (*c*) The player pushes the ball upward, doing work on the ball, which gives the ball an initial kinetic energy. As the ball rises, it slows down as its kinetic energy is converted to potential energy. At the highest point the potential energy is a maximum and the kinetic energy is a minimum. As the ball descends the kinetic energy increases.

16. The drawing shows water falling over a waterfall and then flowing back to the top of the waterfall. The top of the waterfall is above the bottom, with greater gravitational potential energy. The optical illusion of the diagram implies that water is flowing freely from the bottom of the waterfall back to the top. Since water won't move uphill unless work is done on it to increase its gravitational potential energy (for example, work done by a pump), the water from the bottom of the waterfall would NOT be able to make it back to the top.

17. The faster arrow has the same mass and twice the speed of the slower arrow, so the faster arrow will have four times the kinetic energy $\left(\mathrm{KE}=\frac{1}{2}mv^2\right)$. Therefore, four times as much work must be done on the faster arrow to bring it to rest. If the force on the arrows is constant, the faster arrow will travel four times the distance of the slower arrow into the hay.

$\boxed{18.}$ When the ball is released, its potential energy will be converted into kinetic energy and then back into potential energy as the ball swings. If the ball is not pushed, it will lose a little energy to friction and air resistance. It will return almost to the initial position but will not hit the instructor. If the ball is pushed, it will have an initial kinetic energy, and will, when it returns, still have some kinetic energy when it reaches the initial position, so it will hit the instructor on the chin. (Ouch!)

19. When a child hops around on a pogo stick, gravitational potential energy (at the top of the hop) is transformed into kinetic energy as the child moves downward, and then stored as spring potential energy as the spring in the pogo stick compresses. As the spring begins to expand, the energy is converted back to kinetic and gravitational potential energy, and the cycle repeats. Since energy is lost due to friction, the child must add energy to the system by pushing down on the pogo stick while it is on the ground to get more spring compression.

20. At the top of the hill, the skier has gravitational potential energy. If the friction between her skis and the snow is negligible, the gravitational potential energy is changed into kinetic energy as she glides down the hill, and she gains speed as she loses elevation. When she runs into the snowdrift, work is done by the contact force between her and the snow. The energy changes from kinetic energy of the skier to kinetic energy of the snow as it moves and to thermal energy from the friction between the skier and the snow.

21. The work done on the suitcase depends on only (*c*) the height of the table and (*d*) the weight of the suitcase.

22. Power is the rate of doing work. Both (*c*) and (*d*) will affect the total amount of work needed, and hence the power. The time the lifting takes, (*b*), will also affect the power. The length of the path (*a*) will affect only the power if different paths take different times to traverse.

23. When you climb a mountain by going straight up, the force needed is large (and the distance traveled is small), and the power needed (work per unit time) is also large. If you take a zigzag trail, you will use a smaller force (over a longer distance, so that the work done is the same) and less power, since the time to climb the mountain will be longer. A smaller force and smaller power output make the climb seem easier.

Responses to MisConceptual Questions

1. (*b*) A common misconception is that all forces do work. However, work requires that the object on which the force is acting has a component of motion in the direction of the force.

2. (*c*) Work is done when the force acting on the object has a component in the direction of motion. Gravity, which provides the centripetal force, is not zero but is always perpendicular to the motion. A common error is the notion that an object moving in a circle has no work done on it. This is true only if the object is moving at constant speed.

3. (*c*) The kinetic energy is proportional to the square of the speed. Therefore, doubling the speed quadruples the kinetic energy.

4. (*d*) A common misconception is that the stopping distance is proportional to the speed. However, for a constant stopping force, the stopping distance is proportional to the initial kinetic energy, which is proportional to the square of the speed. Doubling the initial speed will quadruple the initial kinetic energy and therefore quadruple the stopping distance.

5. (*c*) As the ball falls, gravitational potential energy is converted to kinetic energy. As the ball hits the trampoline, kinetic energy is converted to elastic potential energy. This energy is then transferred back to kinetic energy of the ball and finally gravitational potential energy. No energy is added to the ball during the motion, so it can't rise higher than it started. Some energy may be lost to heat during the motion, so the ball may not rise as high as it initially started.

6. (*e*) The term "energy" is commonly misunderstood. In this problem energy refers to the sum of the kinetic and potential energies. Initially the kinetic energy is a maximum. During the flight kinetic energy is converted to potential energy, with the potential energy a maximum at the highest point. As the ball falls back down the potential energy is converted back into kinetic energy. Since no nonconservative forces act on the ball (there is no air resistance), the total energy remains constant throughout the flight.

7. (*b*) Since the changes in speed are equal, many students think that the change in energy will also be equal. However, the energy is proportional to the square of the speed. It takes four times as much energy to accelerate the car from rest to 60 km/h as it takes to accelerate the car from rest to 30 km/h. Therefore, it takes three times the energy to accelerate the car from 30 km/h to 60 km/h as it takes to accelerate it from rest to 30 km/h.

8. (*d*) Horsepower is not a unit of energy nor of force but a measure of the rate at which work is done.

9. (*b*) The two balls have the same initial kinetic energy and the same initial potential energy. When they hit the ground they will have the same final potential energy, so their final kinetic energies,

and therefore speeds, will be the same. Even though they have the same initial and final speeds, it is a misconception to think they will spend the same time in the air. The ball thrown directly upward travels to a higher point, as all of the kinetic energy can be converted into potential energy, and therefore will spend a longer time in the air.

10. (*e*) A common misconception is that the steeper the slope, the faster the skier will be traveling at the bottom. Without friction, all of the skier's initial gravitational potential energy is converted into kinetic energy. The skier starting from a higher initial position will have the greater speed at the bottom. On the steeper slope, the skier accelerates faster but over a shorter time period. On the flatter slope, the skier accelerates slower but over more time.

11. (*c*) Friction is a nonconservative force, which removes energy from the system. The work done by friction is related to the product of the force of friction and the distance traveled. For a given coefficient of friction, the force of friction on the steeper slope is smaller than on the flatter slope, as it has a smaller normal force. Also, on the steeper slope, the skier travels a shorter distance.

12. (*c*) The kinetic energy depends on the speed and not the position of the block. Since the block moves with constant speed, the kinetic energy remains constant. As the block moves up the incline its elevation increases, so its potential energy also increases.

13. (*a*) The speed of the crate is constant, so the net (total) work done on the crate is zero. The normal force is perpendicular to the direction of motion, so it does no work. Your applied force and a component of gravity are in the direction of motion, so both do positive work. The force of friction opposes the direction of motion and does negative work. For the total work to be zero, the work done by friction must equal the sum of the work done by gravity and by you.

14. (*a*) The kinetic energy is proportional to the square of the ball's speed, and the potential energy is proportional to the height of the ball. As the ball rises, the speed and kinetic energy decrease while the potential energy increases. As the ball falls, the speed and kinetic energy increase while the potential energy decreases.

Solutions to Problems

1. The minimum force required to lift the firefighter is equal to his weight. The force and the displacement are both upward, so the angle between them is $0°$. Use Eq. 6–1.

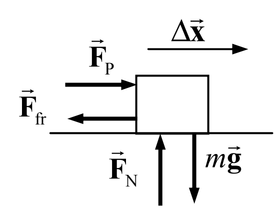
$$W_{\text{climb}} = F_{\text{climb}} d\cos\theta = mgd\cos\theta = (75.0\text{ kg})(9.80\text{ m/s}^2)(28.0\text{ m})\cos 0° = \boxed{2.06\times10^4\text{ J}}$$

2. The maximum amount of work would be the work done by gravity. Both the force and the displacement are downward, so the angle between them is $0°$. Use Eq. 6–1.

$$W_{\text{G}} = mgd\cos\theta = (1.2\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m})\cos 0° = \boxed{5.9\text{ J}}$$

This is a small amount of energy. If the person adds a larger force to the hammer during the fall, then the hammer will have a larger amount of energy to give to the nail.

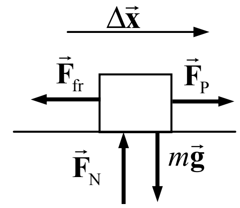
3. Draw a free-body diagram for the crate as it is being pushed across the floor. Since it is not accelerating vertically, $F_{\text{N}} = mg$. Since it is not accelerating horizontally, $F_{\text{P}} = F_{\text{fr}} = \mu_{\text{k}} F_{\text{N}} = \mu_{\text{k}} mg$. The work done to move it across the floor is the work done by the pushing force. The angle between the



pushing force and the direction of motion is $0°$.

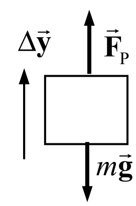
$$W_{\text{push}} = F_{\text{push}} d \cos 0° = \mu_{\text{k}} mgd(1) = (0.50)(46.0 \text{ kg})(9.80 \text{ m/s}^2)(10.3 \text{ m}) = \boxed{2300 \text{ J}}$$

4. (*a*) See the free-body diagram for the crate as it is being pulled. Since the crate is not accelerating horizontally, $F_{\text{P}} = F_{\text{fr}} = 230$ N. The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is $0°$. Use Eq. 6–1.



$$W_{\text{P}} = F_{\text{P}} d \cos 0° = (230 \text{ N})(5.0 \text{ m})(1) = 1150 \text{ J} \approx \boxed{1200 \text{ J}}$$

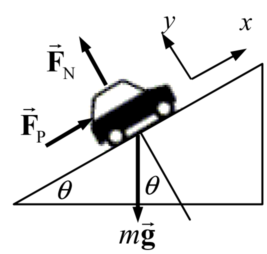
(*b*) See the free-body diagram for the crate as it is being lifted. Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.



$$W_{\text{P}} = F_{\text{P}} d \cos 0° = mgd = (1200 \text{ N})(5.0 \text{ m}) = \boxed{6.0 \times 10^3 \text{ J}}$$

5. Draw a free-body diagram of the car on the incline. The minimum work will occur when the car is moved at a constant velocity. Write Newton's second law in the *x* direction, noting that the car is not accelerated. Only the forces parallel to the plane do work.

$$\sum F_x = F_{\text{P}} - mg \sin\theta = 0 \quad \rightarrow \quad F_{\text{P}} = mg \sin\theta$$



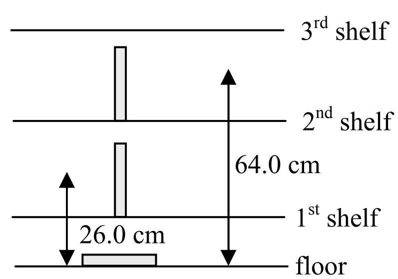
The work done by $\vec{\mathbf{F}}_{\text{P}}$ in moving the car a distance *d* along the plane (parallel to $\vec{\mathbf{F}}_{\text{P}}$) is given by Eq. 6–1.

$$W_{\text{P}} = F_{\text{P}} d \cos 0° = mgd \sin\theta = (950 \text{ kg})(9.80 \text{ m/s}^2)(710 \text{ m}) \sin 9.0° = \boxed{1.0 \times 10^6 \text{ J}}$$

6. The distance over which the force acts is the area to be mowed divided by the width of the mower. The force is parallel to the displacement, so the angle between them is $0°$. Use Eq. 6–1.

$$W = Fd \cos\theta = F \frac{A}{w} \cos\theta = (15 \text{ N}) \frac{200 \text{ m}^2}{0.50 \text{ m}} = \boxed{6000 \text{ J}}$$

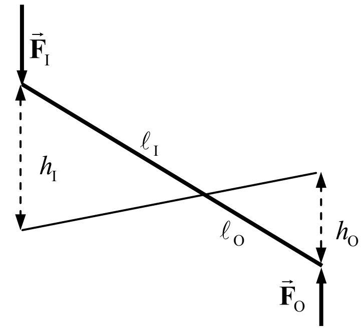
7. The minimum work required to shelve a book is equal to the weight of the book times the vertical distance the book is moved. See the diagram. Each book that is placed on the lowest shelf has its center of mass moved upward by $15.0 \text{ cm} + 11.0 \text{ cm} = 26.0 \text{ cm}$. So the work done to move 28 books to the lowest shelf is $W_1 = 28mg(0.260 \text{ m})$. Each book that is placed on the second shelf has its center of mass moved upward by $15.0 \text{ cm} + 38.0 \text{ cm} \;\; + 11.0 \text{ cm} = 64.0 \text{ cm}$,



so the work done to move 28 books to the second shelf is $W_2 = 28mg(0.640 \text{ m})$. Similarly, $W_3 = 28mg(1.020 \text{ m})$, $W_4 = 28mg(1.400 \text{ m})$, and $W_5 = 28mg(1.780 \text{ m})$. The total work done is the sum of the five work expressions.

$$\begin{aligned} W &= 28mg(0.260 \text{ m} + 0.640 \text{ m} + 1.020 \text{ m} + 1.400 \text{ m} + 1.780 \text{ m}) \\ &= 28(1.40 \text{ kg})(9.80 \text{ m/s}^2)(5.100 \text{ m}) = 1959 \text{ J} \approx \boxed{1960 \text{ J}} \end{aligned}$$

8. Consider the diagram shown. If we assume that the man pushes straight down on the end of the lever, then the work done by the man (the "input" work) is given by $W_{\mathrm{I}} = F_{\mathrm{I}} h_{\mathrm{I}}$. The object moves a shorter distance, as seen from the diagram, so $W_{\mathrm{O}} = F_{\mathrm{O}} h_{\mathrm{O}}$. Equate the two amounts of work.



$$W_{\mathrm{O}} = W_{\mathrm{I}} \quad \rightarrow \quad F_{\mathrm{O}} h_{\mathrm{O}} = F_{\mathrm{I}} h_{\mathrm{I}} \quad \rightarrow \quad \frac{F_{\mathrm{O}}}{F_{\mathrm{I}}} = \frac{h_{\mathrm{I}}}{h_{\mathrm{O}}}$$

But by similar triangles, we see that $\dfrac{h_{\mathrm{I}}}{h_{\mathrm{O}}} = \dfrac{\ell_{\mathrm{I}}}{\ell_{\mathrm{O}}}$, so $\boxed{\dfrac{F_{\mathrm{O}}}{F_{\mathrm{I}}} = \dfrac{\ell_{\mathrm{I}}}{\ell_{\mathrm{O}}}}$.

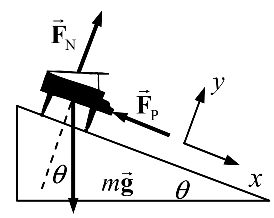
9. Since the acceleration of the box is constant, use Eq. 2–11b to find the distance moved. Assume that the box starts from rest.

$$d = x - x_0 = v_0 t + \tfrac{1}{2} a t^2 = 0 + \tfrac{1}{2}(2.0\ \mathrm{m/s^2})(7.0\ \mathrm{s})^2 = 49\ \mathrm{m}$$

Then the work done in moving the crate is found using Eq. 6–1.

$$W = Fd\cos 0° = mad = (4.0\ \mathrm{kg})(2.0\ \mathrm{m/s^2})(49\ \mathrm{m}) = \boxed{390\ \mathrm{J}}$$

10. The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_{\mathrm{P}}$ is the force of the man pushing on the piano.



(*a*) Write Newton's second law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_{\mathrm{N}} - mg\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{N}} = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{\mathrm{P}} = 0 \quad \rightarrow$$

$$F_{\mathrm{P}} = mg\sin\theta = mg\sin\theta$$

$$= (380\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(\sin 25°) = 1574\ \mathrm{N} \approx \boxed{1600\ \mathrm{N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_{\mathrm{P}}$. The angle between $\vec{\mathbf{F}}_{\mathrm{P}}$ and the direction of motion is $180°$. Use Eq. 6–1.

$$W_{\mathrm{P}} = F_{\mathrm{P}} d\cos 180° = -(1574\ \mathrm{N})(2.9\ \mathrm{m}) = -4565\ \mathrm{J} \approx \boxed{-4600\ \mathrm{J}}$$

(*c*) The angle between the force of gravity and the direction of motion is $65°$. Calculate the work done by gravity.

$$W_{\mathrm{G}} = F_{\mathrm{G}} d\cos 63° = mgd\cos 63° = (380\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(2.9\ \mathrm{m})\cos 65°$$

$$= 4564\ \mathrm{N} \approx \boxed{4600\ \mathrm{J}}$$

(*d*) Since the piano is not accelerating, the net force on the piano is 0, so the net work done on the piano is also 0. This can also be seen by adding the two work amounts calculated.

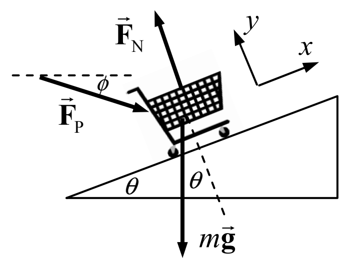
$$W_{\mathrm{net}} = W_{\mathrm{P}} + W_{\mathrm{G}} = -4600\ \mathrm{J} + 4600\ \mathrm{J} = \boxed{0}$$

11. If the person pulls 2 m of rope through his hands, the rope holding the piano will get shorter by 2 m. But that means the rope on the right side of the pulley will get shorter by 1 m, and the rope on the left side will also get shorter by 1 m. Thus for each meter the load is raised, $\boxed{2\ \mathrm{m}}$ of rope must be pulled up.

In terms of energy (assuming that no work is lost to friction), the work done by the man pulling on the rope must be equal to the work done on the piano. If the piano has weight *mg*, and it moves upward a distance d, then the work done on the piano is mgd. The person pulls the rope a distance $2d$, and therefore must exert a force of $\frac{1}{2}mg$ to do the same amount of work.

$$W_{\substack{\text{done by}\\\text{man}}} = W_{\substack{\text{done on}\\\text{piano}}} \quad \to \quad F_{\text{pull}}(2d) = mgd \quad \to \quad F_{\text{pull}} = \tfrac{1}{2}mg$$

12. Consider a free-body diagram for the grocery cart being pushed up the ramp. If the cart is not accelerating, then the net force is 0 in all directions. This can be used to find the size of the pushing force. The angles are $\phi = 17°$ and $\theta = 12°$. The displacement is in the x direction. The work done by the normal force is 0 since the normal force is perpendicular to the displacement. The angle between the force of gravity and the displacement is $90° + \theta = 102°$. The angle between the normal force and the displacement is $90°$. The angle between the pushing force and the displacement is $\phi + \theta = 29°$.



$$\sum F_x = F_\text{P} \cos(\phi + \theta) - mg \sin\theta = 0 \quad \to \quad F_\text{P} = \frac{mg \sin\theta}{\cos(\phi+\theta)}$$

$$W_{mg} = mgd \cos 112° = (16\text{ kg})(9.80\text{ m/s}^2)(7.5\text{ m}) \cos 102° = -244.5\text{ J} \approx \boxed{-240\text{ J}}$$

$$W_{\text{normal}} = F_\text{N} d \cos 90° = \boxed{0}$$

$$W_\text{P} = F_\text{P} d \cos 29° = \left(\frac{mg \sin 12°}{\cos 29°}\right) d \cos 29° = mgd \sin 12°$$

$$= (16\text{ kg})(9.80\text{ m/s}^2)(7.5\text{ m}) \sin 12° = 244.5\text{ J} \approx \boxed{240\text{ J}}$$

13. The work done will be the area under the F_x vs. x graph.

(*a*) From $x = 0.0$ to $x = 10.0$ m, the shape under the graph is trapezoidal. The area is

$$W = (400\text{ N})\tfrac{1}{2}(10\text{ m} + 4\text{ m}) = \boxed{2800\text{ J}}$$

(*b*) From $x = 10.0$ m to $x = 15.0$ m, the force is in the opposite direction from the direction of motion, so the work will be negative. Again, since the shape is trapezoidal, we find

$$W = (-200\text{ N})\tfrac{1}{2}(5\text{ m} + 2\text{ m}) = -700\text{ J}$$

Thus the total work from $x = 0.0$ to $x = 15.0$ m is $2800\text{ J} - 700\text{ J} = \boxed{2100\text{ J}}$.

14. (*a*) The gases exert a force on the jet in the same direction as the displacement of the jet. From the graph we see the displacement of the jet during launch is 85 m. Use Eq. 6–1 to find the work.

$$W_{\text{gas}} = F_{\text{gas}} d \cos 0° = (130\times10^3\text{ N})(85\text{ m}) = \boxed{1.1\times10^7\text{ J}}$$

(*b*) The work done by catapult is the area underneath the graph in Fig. 6–39b. That area is a trapezoid.

$$W_{\text{catapult}} = \tfrac{1}{2}(1100\times10^3\text{ N} + 65\times10^3\text{ N})(85\text{ m}) = \boxed{5.0\times10^7\text{ J}}$$

15. Find the velocity from the kinetic energy, using Eq. 6–3.

$$\text{KE} = \tfrac{1}{2}mv^2 \quad \to \quad v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(6.21\times10^{-21}\text{ J})}{5.31\times10^{-26}}} = \boxed{484\text{ m/s}}$$

16. (*a*) Since $\text{KE} = \frac{1}{2}mv^2$, $v = \sqrt{2\text{KE}/m}$ and $v \propto \sqrt{\text{KE}}$. Thus if the kinetic energy is tripled, the speed will be multiplied by a factor of $\boxed{\sqrt{3}}$.

(*b*) Since $\text{KE} = \frac{1}{2}mv^2$, $\text{KE} \propto v^2$. Thus if the speed is halved, the kinetic energy will be multiplied by a factor of $\boxed{1/4}$.

17. The work done on the electron is equal to the change in its kinetic energy, Eq. 6–4.

$$W = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(9.11\times10^{-31}\text{ kg})(1.10\times10^6\text{ m/s})^2 = \boxed{-5.51\times10^{-19}\text{ J}}$$

Note that the work is negative since the electron is slowing down.

18. The work done on the car is equal to the change in its kinetic energy, Eq. 6–3.

$$W = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(925\text{ kg})\left[(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2 = \boxed{-3.2\times10^5\text{ J}}$$

Note that the work is negative since the car is slowing down.

19. The kinetic energies of both bullets are the same. Bullet 1 is the heavier bullet.

$$m_1 = 2m_2 \quad \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1^2 \quad\rightarrow\quad m_2v_2^2 = 2m_2v_1^2 \quad\rightarrow\quad v_2^2 = 2v_1^2 \quad\rightarrow\quad v_2 = v_1\sqrt{2}$$

The lighter bullet has the higher speed, by a factor of the square root of 2. Both bullets can do the same amount of work.

20. The force of the ball on the glove will be the opposite of the force of the glove on the ball, by Newton's third law. The objects have the same displacement, so the work done on the glove is opposite the work done on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball, Eq. 6–4.

$$W_{\text{on ball}} = (\text{KE}_2 - \text{KE}_1)_{\text{ball}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(0.145\text{ kg})(32\text{ m/s})^2 = -74.24\text{ J}$$

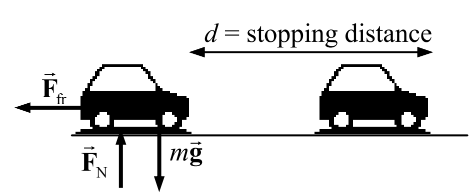
So $W_{\text{on glove}} = 74.24\text{ J}$. But $W_{\text{on glove}} = F_{\text{on glove}}d\cos 0°$, because the force on the glove is in the same direction as the motion of the glove.

$$74.24\text{ J} = F_{\text{on glove}}(0.25\text{ m}) \quad\rightarrow\quad F_{\text{on glove}} = \frac{74.24\text{ J}}{0.25\text{ m}} = \boxed{3.0\times10^2\text{ N}},$$ in the direction of the original velocity of the ball.

21. The force exerted by the bow on the arrow is in the same direction as the displacement of the arrow. Thus $W = Fd\cos 0° = Fd = (105\text{ N})(0.75\text{ m}) = 78.75\text{ J}$. But that work changes the kinetic energy of the arrow, by the work-energy theorem. Thus

$$Fd = W = \text{KE}_2 - \text{KE}_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad\rightarrow\quad v_2 = \sqrt{\frac{2Fd}{m} + v_1^2} = \sqrt{\frac{2(78.75\text{ J})}{0.085\text{ kg}} + 0} = \boxed{43\text{ m/s}}$$

22. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car. Assume the maximum possible frictional force, which results in the minimum braking distance. Thus $F_{\text{fr}} = \mu_s F_N$. The normal force is equal to

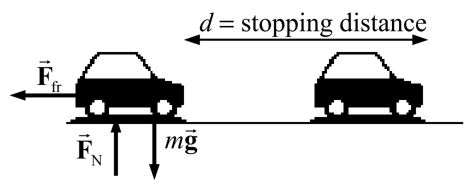


the car's weight if it is on a level surface, so $F_{\text{fr}} = \mu_{\text{s}} mg$. In the diagram, the car is traveling to the right.

$$W = \Delta\text{KE} \quad \rightarrow \quad F_{\text{fr}} d \cos 180° = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 \quad \rightarrow \quad -\mu_{\text{s}} mgd = -\tfrac{1}{2} m v_1^2 \quad \rightarrow \quad d = \frac{v_1^2}{2g\mu_{\text{s}}}$$

Since $d \propto v_1^2$, if v_1 increases by 50%, or is multiplied by 1.5, then d will be multiplied by a factor of $(1.5)^2$, or $\boxed{2.25}$.

$\boxed{23.}$ The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be kinetic (sliding) since the driver locked the brakes. Thus $F_{\text{fr}} = \mu_{\text{k}} F_{\text{N}}$. Since the car is on a level surface, the normal force is equal to the car's weight, so $F_{\text{fr}} = \mu_{\text{k}} mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.



$$W = \Delta\text{KE} \quad \rightarrow \quad F_{\text{fr}} d \cos 180° = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 \quad \rightarrow \quad -\mu_{\text{k}} mgd = 0 - \tfrac{1}{2} m v_1^2 \quad \rightarrow$$

$$v_1 = \sqrt{2\mu_{\text{k}} gd} = \sqrt{2(0.30)(9.80\text{ m/s}^2)(78\text{ m})} = \boxed{21\text{ m/s}}$$

The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

24. The first car mentioned will be called car 1. So we have these statements:

$$\text{KE}_1 = \tfrac{1}{2}\text{KE}_2 \quad \rightarrow \quad \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_2 v_2^2\right) \quad ;$$

$$\text{KE}_{1,\text{fast}} = \text{KE}_{2,\text{fast}} \quad \rightarrow \quad \tfrac{1}{2} m_1 (v_1 + 8.0)^2 = \tfrac{1}{2} m_2 (v_2 + 8.0)^2$$

Now use the mass information, that $m_1 = 2m_2$.

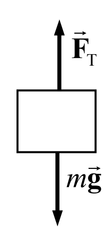
$$\tfrac{1}{2} 2m_2 v_1^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m_2 v_2^2\right); \quad \tfrac{1}{2} 2m_2 (v_1 + 8.0)^2 = \tfrac{1}{2} m_2 (v_2 + 8.0)^2 \quad \rightarrow$$

$$2v_1 = v_2; \quad 2(v_1 + 8.0)^2 = (v_2 + 8.0)^2 \quad \rightarrow \quad 2(v_1 + 8.0)^2 = (2v_1 + 8.0)^2 \quad \rightarrow$$

$$\sqrt{2}(v_1 + 8.0) = (2v_1 + 8.0) \quad \rightarrow \quad v_1 = \frac{8.0}{\sqrt{2}} = 5.657\text{ m/s}; \quad v_2 = 2v_1 = 11.314\text{ m/s}$$

$$\boxed{v_1 = 5.7\text{ m/s}; \; v_2 = 11.3\text{ m/s}}$$

25. (*a*) From the free-body diagram for the load being lifted, write Newton's second law for the vertical direction, with up being positive.



$$\sum F = F_{\text{T}} - mg = ma = 0.160\, mg \quad \rightarrow$$

$$F_{\text{T}} = 1.160\, mg = 1.160(265\text{ kg})(9.80\text{ m/s}^2) = \boxed{3010\text{ N}}$$

(*b*) The net work done on the load is found from the net force.

$$W_{\text{net}} = F_{\text{net}} d \cos 0° = (0.160\, mg)d = 0.160(265\text{ kg})(9.80\text{ m/s}^2)(18.0\text{ m}) = \boxed{7480\text{ J}}$$

(*c*) The work done by the cable on the load is as follows:

$$W_{\text{cable}} = F_{\text{T}} d \cos 0° = (1.160\, mg)d = 1.160(265\text{ kg})(9.80\text{ m/s}^2)(18.0\text{ m}) = \boxed{5.42 \times 10^4\text{ J}}$$

(*d*) The work done by gravity on the load is as follows:

$$W_{\mathrm{G}} = mgd\cos 180° = -mgd = -(265\text{ kg})(9.80\text{ m/s}^2)(18.0\text{ m}) = \boxed{-4.67\times 10^4\text{ J}}$$

(*e*) Use the work-energy theorem to find the final speed, with an initial speed of 0.

$$W_{\text{net}} = \mathrm{KE}_2 - \mathrm{KE}_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad \rightarrow$$

$$v_2 = \sqrt{\frac{2W_{\text{net}}}{m} + v_1^2} = \sqrt{\frac{2(7.48\times 10^3\text{ J})}{265\text{ kg}} + 0} = \boxed{7.51\text{ m/s}}$$

26. Subtract the initial gravitational PE from the final gravitational PE.

$$\Delta\mathrm{PE}_{\mathrm{G}} = mgy_2 - mgy_1 = mg(y_2 - y_1) = (54\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m}) = \boxed{2100\text{ J}}$$

27. The potential energy of the spring is given by $\mathrm{PE}_{\text{el}} = \frac{1}{2}kx^2$, where x is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2\mathrm{PE}_{\text{el}}}{k}} = \sqrt{\frac{2(45.0\text{ J})}{88.0\text{ N/m}}} = \boxed{1.01\text{ m}}$$

28. The initial stretching is from the equilibrium position, $x = 0$. Use that to find the spring constant.

$$\mathrm{PE}_{\text{initial}} = \tfrac{1}{2}kx^2 \quad \rightarrow \quad 6.0\text{ J} = \tfrac{1}{2}k(2.0\text{ cm})^2 \quad \rightarrow \quad k = 3.0\text{ J/cm}^2$$

$$\mathrm{PE}_{\text{final}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(3)(6)^2 = 54\text{ J}; \quad \mathrm{PE}_{\text{final}} - \mathrm{PE}_{\text{initial}} = \boxed{48\text{ J}}$$

29. (*a*) The change in gravitational potential energy is given by the following:

$$\Delta\mathrm{PE}_{\mathrm{G}} = mg(y_2 - y_1) = (66.5\text{ kg})(9.80\text{ m/s}^2)(2660\text{ m} - 1270\text{ m}) = \boxed{9.06\times 10^5\text{ J}}$$

(*b*) The minimum work required by the hiker would equal the change in potential energy, which is $\boxed{9.06\times 10^5\text{ J}}$.

(*c*) $\boxed{\text{Yes}}$. The actual work may be more than this, because the hiker almost certainly had to overcome some dissipative forces such as air friction. Also, while stepping up and down, the hiker does not get the full amount of work back from each up-down event. For example, there will be friction in the joints and muscles.

30. (*a*) Relative to the ground, the potential energy is given by the following:

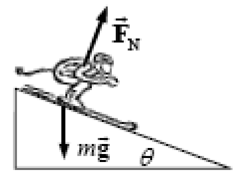
$$\mathrm{PE}_{\mathrm{G}} = mg(y_{\text{book}} - y_{\text{ground}}) = (1.65\text{ kg})(9.80\text{ m/s}^2)(2.20\text{ m}) = \boxed{35.6\text{ J}}$$

(*b*) Relative to the top of the person's head, the potential energy is given by the following:

$$\mathrm{PE}_{\mathrm{G}} = mg(y_{\text{book}} - y_{\text{head}}) = (1.65\text{ kg})(9.80\text{ m/s}^2)(2.20\text{ m} - 1.60\text{ m}) = \boxed{9.7\text{ J}}$$

(*c*) The work done by the person in lifting the book from the ground to the final height is the same as the answer to part (*a*), $\boxed{35.6\text{ J}}$. In part (*a*), the potential energy is calculated relative to the starting location of the application of the force on the book. The work done by the person is not related to the answer to part (*b*), because the potential energy is not calculated relative to the starting location of the application of the force on the book.

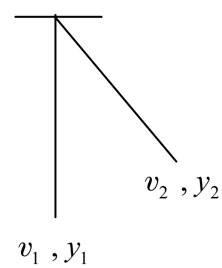
31. The forces on the skier are gravity and the normal force. The normal force is perpendicular to the direction of motion, so she does no work. Thus the skier's mechanical energy is conserved. Subscript 1 represents the skier at the top of the hill, and subscript 2 represents the skier at the bottom of the hill. The ground is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = 285\text{ m}$, and $y_2 = 0$ (bottom of the hill). Solve for v_2, the speed at the bottom, using Eq. 6–13.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad 0 + mgy_1 = \tfrac{1}{2}mv_2^2 + 0 \quad \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(105\text{ m})} = \boxed{45.4\text{ m/s}}$$

32. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, so can do no work—the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 5.0\text{ m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2, the height of her swing. Use Eq. 6–13.

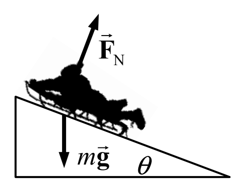


$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad \rightarrow$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(5.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.276\text{ m} \approx \boxed{1.3\text{ m}}$$

$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.65 m long. If that were the case, she could not rise 1.3 m high.

33. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, so that force does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for gravitational potential energy $(y = 0)$. We have $y_1 = 0$, $v_2 = 0$, and $y_2 = 1.12\text{ m}$. Solve for v_1, the speed at the bottom, using Eq. 6–13. Note that the angle is not used.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80\text{ m/s}^2)(1.22\text{ m})} = \boxed{4.89\text{ m/s}}$$

34. We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for gravitational potential energy $(y = 0)$. We have $y_1 = 0$, $v_2 = 0.70\text{ m/s}$, and $y_2 = 2.10\text{ m}$. Solve for v_1, the speed at the bottom. Use Eq. 6–13.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(0.50\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.10\text{ m})} = 6.435\text{ m/s} \approx \boxed{6.4\text{ m/s}}$$

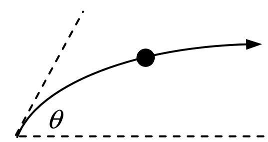
35. The mass is allowed to fall rather than "easing" it down with the hand (which introduces another force into the problem). Under these conditions, mechanical energy is conserved. The unstretched spring, corresponding to the initial position, is the location of zero elastic potential energy. There is no kinetic energy to consider since the mass is at rest at both positions. The change in height is the same as the amount of stretch of the spring. Mechanical energy is conserved.

$$E_1 = E_2 \quad \to \quad \underset{\text{gravity}}{\text{PE}_1} + \underset{\text{elastic}}{\text{PE}_1} = \underset{\text{gravity}}{\text{PE}_2} + \underset{\text{elastic}}{\text{PE}_2} \quad \to \quad mgy_1 + 0 = mgy_2 + \tfrac{1}{2}k(y_2 - y_1)^2 \quad \to$$

$$mg(y_1 - y_2) = \tfrac{1}{2}k(y_2 - y_1)^2 \quad \to \quad \frac{(y_2 - y_1)^2}{(y_1 - y_2)} = y_1 - y_2 = \frac{2mg}{k} = \frac{2(2.5\text{ kg})(9.80\text{ m/s}^2)}{83\text{ N/m}} = 0.59\text{ m}$$

The final position is 59 cm lower than the initial position. If we assume that the ruler is oriented so that higher numbers are nearer the floor than the lower numbers, then we add 59 cm to the original 15 cm and get $\boxed{74\text{ cm}}$.

36. (*a*) See the diagram for the thrown ball. The speed at the top of the path will be the horizontal component of the original velocity.



$$v_{\text{top}} = v_0 \cos\theta = (8.8\text{ m/s})\cos 36° = \boxed{7.1\text{ m/s}}$$

(*b*) Since there are no dissipative forces in the problem, the mechanical energy of the ball is conserved. Subscript 1 represents the ball at the release point, and subscript 2 represents the ball at the top of the path. The ball's release point is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 8.8\text{ m/s}$, $y_1 = 0$, and $v_2 = v_1\cos\theta$. Solve for y_2.

$$E_1 = E_2 \quad \to \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \to \quad \tfrac{1}{2}mv_1^2 + 0 = \tfrac{1}{2}mv_1^2\cos^2\theta + mgy_2 \quad \to$$

$$y_2 = \frac{v_1^2(1-\cos^2\theta)}{2g} = \frac{(8.8\text{ m/s})^2(1-\cos^2 36°)}{2(9.80\text{ m/s}^2)} = 1.365\text{ m} \approx \boxed{1.4\text{ m}}$$

This is the height above its throwing level.

37. Assume that all of the kinetic energy of the car becomes potential energy of the compressed spring.

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kx_{\text{final}}^2 \quad \to \quad k = \frac{mv_0^2}{x_{\text{final}}^2} = \frac{(1200\text{ kg})\left[(85\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(2.2\text{ m})^2} = \boxed{1.4\times10^5\text{ N/m}}$$

$\boxed{38.}$ (*a*) Since there are no dissipative forces present, the mechanical energy of the person–trampoline–Earth combination will be conserved. We take the level of the unstretched trampoline as the zero level for both elastic and gravitational potential energy. Call up the positive direction. Subscript 1 represents the jumper at the start of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic potential energy involved in this part of the problem. We have $v_1 = 4.5\text{m/s}$, $y_1 = 2.0\text{ m}$, and $y_2 = 0$. Solve for v_2, the jumper's speed when he arrives at the trampoline.

$$E_1 = E_2 \quad \to \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \to \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mgv_2^2 + 0 \quad \to$$

$$v_2 = \pm\sqrt{v_1^2 + 2gy_1} = \pm\sqrt{(4.5\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.0\text{ m})} = \pm\,7.710\text{ m/s} \approx \boxed{7.7\text{ m/s}}$$

The speed is the absolute value of v_2.

(*b*) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline and x represent the amount of stretch of the trampoline. We have $v_2 = -7.710\text{ m/s}$, $y_2 = 0$, $x_2 = 0$,

$\upsilon_3 = 0,$ and $x_3 = y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational potential energy at position 3 is negative, so $y_3 < 0$. A quadratic relationship results from the conservation of energy condition.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}m\upsilon_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad \rightarrow$$

$$\tfrac{1}{2}m\upsilon_2^2 + 0 + 0 = 0 + mgy_3 + \tfrac{1}{2}ky_3^2 \quad \rightarrow \quad \tfrac{1}{2}ky_3^2 + mgy_3 - \tfrac{1}{2}m\upsilon_2^2 = 0 \quad \rightarrow$$

$$y_3 = \frac{-mg \pm \sqrt{m^2g^2 - 4\left(\tfrac{1}{2}k\right)\left(-\tfrac{1}{2}m\upsilon_2^2\right)}}{2\left(\tfrac{1}{2}k\right)} = \frac{-mg \pm \sqrt{m^2g^2 + km\upsilon_2^2}}{k}$$

$$= \frac{-(62\text{ kg})(9.80\text{ m/s}^2) \pm \sqrt{(62\text{ kg})^2(9.80\text{ m/s}^2)^2 + (5.8\times10^4\text{ N/m})(62\text{ kg})(7.71\text{ m/s})^2}}{(5.8\times10^4\text{ N/m})}$$

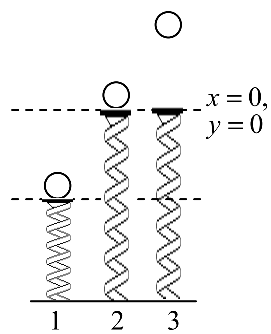
$$= -0.263\text{ m},\ 0.242\text{ m}$$

Since $y_3 < 0, y_3 = -0.26$ m. So he depresses the trampoline $\boxed{0.26\text{ m}}$.

The second term under the quadratic is almost 600 times larger than the first term, indicating that the problem could have been approximated by not including gravitational potential energy for the final position. If that approximation were made, the result would have been found by taking the negative result from the following solution:

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_2^2 = \tfrac{1}{2}ky_3^2 \quad \rightarrow \quad y_3 = \upsilon_2\sqrt{\frac{m}{k}} = (7.71\text{ m/s})\sqrt{\frac{62\text{ kg}}{5.8\times10^4\text{ N/m}}} = \pm\,0.25\text{ m}$$

39. Use conservation of energy. The level of the ball on the uncompressed spring is taken as the zero location for both gravitational potential energy $(y = 0)$ and elastic potential energy $(x = 0)$. It is diagram 2 in the figure. Take "up" to be positive for both x and y.



(*a*) Subscript 1 represents the ball at the launch point, and subscript 2 represents the ball at the location where it just leaves the spring, at the uncompressed length. We have $\upsilon_1 = 0,$ $x_1 = y_1 = -0.160$ m, and $x_2 = y_2 = 0$. Solve for υ_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m\upsilon_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m\upsilon_2^2 + 0 + 0 \quad \rightarrow \quad \upsilon_2 = \sqrt{\frac{kx_1^2 + 2mgy_1}{m}}$$

$$\upsilon_2 = \sqrt{\frac{(875\text{ N/m})(0.160\text{ m})^2 + 2(0.380\text{ kg})(9.80\text{ m/s}^2)(-0.160\text{ m})}{(0.380\text{ kg})}} = \boxed{7.47\text{ m/s}}$$

(*b*) Subscript 3 represents the ball at its highest point. We have $\upsilon_1 = 0,$ $x_1 = y_1 = -0.160$ m, $\upsilon_3 = 0,$ and $x_3 = 0$. Solve for y_3.

$$E_1 = E_3 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m\upsilon_3^2 + mgy_3 + \tfrac{1}{2}kx_3^2 \quad \rightarrow$$

$$0 + mgy_1 + \tfrac{1}{2}kx_1^2 = 0 + mgy_2 + 0 \quad \rightarrow \quad y_2 - y_1 = \frac{kx_1^2}{2mg} = \frac{(875\text{ N/m})(0.160\text{ m})^2}{2(0.380\text{ kg})(9.80\text{ m/s}^2)} = \boxed{3.01\text{ m}}$$

$\boxed{40.}$ Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_1 = 0$ and $y_1 = 32$ m.

$$\text{Point 2: } \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2; \quad y_2 = 0 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(32 \text{ m})} = \boxed{25 \text{ m/s}}$$

$$\text{Point 3: } \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3; \quad y_3 = 26 \text{ m} \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow$$

$$v_3 = \sqrt{2g(y_1 - y_3)} = \sqrt{2(9.80 \text{ m/s}^2)(6 \text{ m})} = \boxed{11 \text{ m/s}}$$

$$\text{Point 4: } \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_4; \quad y_4 = 14 \text{ m} \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_4 \quad \rightarrow$$

$$v_4 = \sqrt{2g(y_1 - y_4)} = \sqrt{2(9.80 \text{ m/s}^2)(18 \text{ m})} = \boxed{19 \text{ m/s}}$$

41. The only forces acting on the bungee jumper are gravity and the elastic force from the bungee cord, so the jumper's mechanical energy is conserved. Subscript 1 represents the jumper at the bridge, and subscript 2 represents the jumper at the bottom of the jump. Let the lowest point of the jumper's motion be the zero location for gravitational potential energy $(y = 0)$. The zero location for elastic potential energy is the point at which the bungee cord begins to stretch. See the diagram in the textbook. We have $v_1 = v_2 = 0$, $y_1 = d$, $y_2 = 0$, and the amount of stretch of the cord $x_2 = d - 15$. Solve for *d*. Note that we ignore Chris's height, since his center of mass falls farther than *d*.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad mgd = \tfrac{1}{2}k(d-15)^2 \quad \rightarrow$$

$$d^2 - \left(30 + 2\frac{mg}{k}\right)d + 225 = 0 \quad \rightarrow \quad d^2 - 56.7d + 225 = 0 \quad \rightarrow$$

$$d = \frac{56.7 \pm \sqrt{56.7^2 - 4(225)}}{2} = 52.4 \text{ m}, 4.29 \text{ m} \quad \rightarrow \quad d = \boxed{52 \text{ m}}$$

The larger answer must be taken because $d > 15$ m.

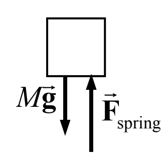
42. The spring must be compressed a distance such that the work done by the spring is equal to the change in kinetic energy of the car. The distance of compression can then be used to find the spring constant. Note that the work done by the spring will be negative, since the force exerted by the spring is in the opposite direction to the displacement of the spring. The maximum acceleration occurs at the point of maximum force by the spring, or at maximum compression.

$$W_{\text{spring}} = \Delta \text{KE} = \quad \rightarrow \quad -\tfrac{1}{2}kx_{\max}^2 = 0 - \tfrac{1}{2}mv_0^2 \quad \rightarrow \quad x_{\max} = v_0\sqrt{\frac{m}{k}}$$

$$F_{\max} = ma_{\max} = -kx_{\max} \quad \rightarrow \quad m(-4.0g) = -kv_0\sqrt{\frac{m}{k}} \quad \rightarrow$$

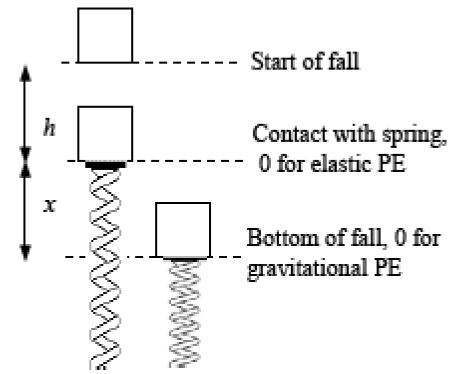
$$k = m\left(\frac{4.0g}{v_0}\right)^2 = (1200 \text{ kg})(16)\frac{(9.80 \text{ m/s}^2)^2}{\left[95 \text{ km/h}\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)\right]^2} = 2648 \text{ N/m} \approx \boxed{2600 \text{ N/m}}$$

43. The maximum acceleration of 5.0 *g* occurs where the force is at a maximum. The maximum force occurs at the bottom of the motion, where the spring is at its maximum compression. Write Newton's second law for the elevator at the bottom of the motion, with up as the positive direction.



$$F_{\text{net}} = F_{\text{spring}} - Mg = Ma = 5.0Mg \quad \rightarrow \quad F_{\text{spring}} = 6.0Mg$$

Now consider the diagram for the elevator at various points in its motion. If there are no nonconservative forces, then mechanical energy is conserved. Subscript 1 represents the elevator at the start of its fall, and subscript 2 represents the elevator at the bottom of its fall. The bottom of the fall is the zero location for gravitational potential energy $(y=0)$. There is also a point at the top of the spring that is the zero location for elastic potential energy $(x=0)$. We have $v_1=0$, $y_1=x+h$, $x_1=0$, $v_2=0$, $y_2=0$, and $x_2=x$. Apply conservation of energy.



$$E_1=E_2 \quad\rightarrow\quad \tfrac{1}{2}Mv_1^2+Mgy_1+\tfrac{1}{2}kx_1^2=\tfrac{1}{2}Mv_2^2+Mgy_2+\tfrac{1}{2}kx_2^2 \quad\rightarrow$$
$$0+Mg(x+h)+0=0+0+\tfrac{1}{2}kx^2 \quad\rightarrow\quad Mg(x+h)=\tfrac{1}{2}kx^2$$
$$F_{\text{spring}}=6.0Mg=kx \quad\rightarrow\quad x=\frac{6.0Mg}{k} \quad\rightarrow\quad Mg\left(\frac{6Mg}{k}+h\right)=\tfrac{1}{2}k\left(\frac{6Mg}{k}\right)^2 \quad\rightarrow\quad \boxed{k=\frac{12Mg}{h}}$$

44. At the release point the mass has both kinetic energy and elastic potential energy. The total energy is $\tfrac{1}{2}mv_0^2+\tfrac{1}{2}kx_0^2$. If friction is to be ignored, then that total energy is constant.

(*a*) The mass has its maximum speed at a displacement of 0, so it has only kinetic energy at that point.

$$\tfrac{1}{2}mv_0^2+\tfrac{1}{2}kx_0^2=\tfrac{1}{2}mv_{\max}^2 \quad\rightarrow\quad v_{\max}=\boxed{\sqrt{v_0^2+\frac{k}{m}x_0^2}}$$

(*b*) The mass has a speed of 0 at its maximum stretch from equilibrium, so it has only potential energy at that point.

$$\tfrac{1}{2}mv_0^2+\tfrac{1}{2}kx_0^2=\tfrac{1}{2}kx_{\max}^2 \quad\rightarrow\quad x_{\max}=\boxed{\sqrt{x_0^2+\frac{m}{k}v_0^2}}$$

45. (*a*) The work done against gravity is the change in potential energy.

$$W_{\substack{\text{against}\\\text{gravity}}}=\Delta\text{PE}=mg(y_2-y_1)=(75.0\text{ kg})(9.80\text{ m/s}^2)(125\text{ m})=\boxed{9.19\times10^4\text{ J}}$$

(*b*) The work done by the force on the pedals in one revolution is equal to the average tangential force times the circumference of the circular path of the pedals. That work is also equal to the potential energy change of the bicycle during that revolution, assuming that the speed of the bicycle is constant. Note that a vertical rise on the incline is related to the distance along the incline by $\text{rise}=\text{distance}\times(\sin\theta)$.

$$W_{\substack{\text{pedal}\\\text{force}}}=F_{\tan}2\pi r=\Delta\text{PE}_{\substack{\text{grav}\\\text{1 rev}}}=mg(\Delta y)_{\text{1 rev}}=mgd_{\text{1 rev}}\sin\theta \quad\rightarrow$$
$$F_{\tan}=\frac{mgd_{\text{1 rev}}\sin\theta}{2\pi r}=\frac{(75.0\text{ kg})(9.80\text{ m/s}^2)(5.10\text{ m})\sin 7.50^\circ}{2\pi(0.180\text{ m})}=\boxed{433\text{ N}}$$

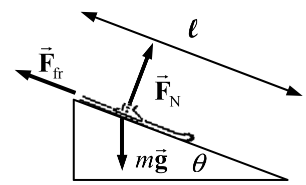
46. Use conservation of energy, where all of the kinetic energy is transformed to thermal energy.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 = E_{\text{thermal}} = \tfrac{1}{2}(2)(66{,}000\text{ kg})\left[(85\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2 = \boxed{3.7\times10^7\text{ J}}$$

47. Apply the conservation of energy to the child, considering work done by gravity and thermal energy. Subscript 1 represents the child at the top of the slide, and subscript 2 represents the child at the bottom of the slide. The ground is the zero location for potential energy $(y = 0)$. We have $v_1 = 0,\ y_1 = 2.2$ m, $v_2 = 1.25$ m/s, and $y_2 = 0$. Solve for the work changed into thermal energy.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + E_{\text{thermal}} \quad \rightarrow$$
$$E_{\text{thermal}} = mgy_1 - \tfrac{1}{2}mv_2^2 = (16.0\text{ kg})(9.80\text{ m/s}^2)(2.20\text{ m}) - \tfrac{1}{2}(16.0\text{ kg})(1.25\text{ m/s})^2 = \boxed{332\text{ J}}$$

48. (*a*) See the free-body diagram for the ski. Write Newton's second law for forces perpendicular to the direction of motion, noting that there is no acceleration perpendicular to the plane.



$$\sum F_\perp = F_N - mg\cos\theta \quad \rightarrow \quad F_N = mg\cos\theta \quad \rightarrow$$
$$F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$$

Now use conservation of energy, including the nonconservative friction force. Subscript 1 represents the ski at the top of the slope, and subscript 2 represents the ski at the bottom of the slope. The location of the ski at the bottom of the incline is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 0,\ y_1 = \ell\sin\theta,$ and $y_2 = 0$. Write the conservation of energy condition, and solve for the final speed. Note that $F_{fr} = \mu_k F_N = \mu_k mg\cos\theta$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 - F_{fr}\ell = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mg\ell\sin\theta - \mu_k mg\ell\cos\theta = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$
$$v_2 = \sqrt{2g\ell(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(85\text{ m})(\sin 28^\circ - 0.090\cos 28^\circ)}$$
$$= 25.49\text{ m/s} \approx \boxed{25\text{ m/s}}$$

(*b*) Now, on the level ground, $F_{fr} = \mu_k mg$, and there is no change in potential energy. We again use conservation of energy, including the nonconservative friction force, to relate position 2 with position 3. Subscript 3 represents the ski at the end of the travel on the level, having traveled a distance ℓ_3 on the level. We have $v_2 = 25.49$ m/s, $y_2 = 0,\ v_3 = 0,$ and $y_3 = 0$.

$$\tfrac{1}{2}mv_2^2 + mgy_2 - F_{fr}\ell_3 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 = \mu_k mg\ell_3 \quad \rightarrow$$
$$\ell_3 = \frac{v_2^2}{2g\mu_k} = \frac{(25.49\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.090)} = 368.3\text{ m} \approx \boxed{370\text{ m}}$$

49. (*a*) Apply energy conservation with no nonconservative work. Subscript 1 represents the ball as it is dropped, and subscript 2 the ball as it reaches the ground. The ground is the zero location for gravitational potential energy. We have $v_1 = 0,\ y_1 = 12.0$ m, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$
$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(12.0\text{ m})} = \boxed{15.3\text{ m/s}}$$

(*b*) Apply energy conservation, but with nonconservative work due to friction included. The energy dissipated will be given by $F_{fr}d$. The distance d over which the frictional force acts will be the

12.0-m distance of fall. With the same parameters as above, and $v_2 = 8.00\text{ m/s}$, solve for the force of friction.

$$\tfrac{1}{2}mv_1^2 + mgy_1 - F_{\text{fr}}d = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad mgy_1 - F_{\text{fr}}d = \tfrac{1}{2}mv_2^2 \quad\rightarrow$$

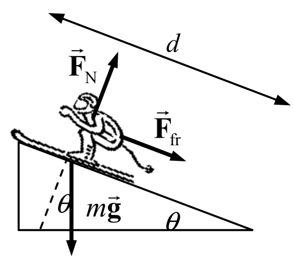
$$F_{\text{fr}} = m\left(g\frac{y_1}{d} - \frac{v_2^2}{2d}\right) = (0.145\text{ kg})\left(9.80\text{ m/s}^2 - \frac{(8.00\text{ m/s})^2}{2(12.0\text{ m})}\right) = \boxed{1.03\text{ N, upward}}$$

50. Since there is a nonconservative force, apply energy conservation with the dissipative friction term. Subscript 1 represents the roller coaster at point 1, and subscript 2 represents the roller coaster at point 2. Point 2 is taken as the zero location for gravitational potential energy. We have $v_1 = 1.30\text{ m/s}$, $y_1 = 32\text{ m}$, and $y_2 = 0$. Solve for v_2. Note that the dissipated energy is given by $F_{\text{fr}}d = 0.23mgd$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 - 0.23mgd = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad v_2 = \sqrt{-0.46gd + v_1^2 + 2gy_1}$$

$$= \sqrt{-0.46(9.80\text{ m/s}^2)(45.0\text{ m}) + (1.70\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(32\text{ m})} = 20.67\text{ m/s} \approx \boxed{21\text{ m/s}}$$

51. Consider the free-body diagram for the skier in the midst of the motion. Write Newton's second law for the direction perpendicular to the plane, with an acceleration of 0.



$$\sum F_\perp = F_{\text{N}} - mg\cos\theta = 0 \quad\rightarrow\quad F_{\text{N}} = mg\cos\theta \quad\rightarrow$$

$$F_{\text{fr}} = \mu_{\text{k}}F_{\text{N}} = \mu_{\text{k}}mg\cos\theta$$

Apply conservation of energy to the skier, including the dissipative friction force. Subscript 1 represents the skier at the bottom of the slope, and subscript 2 represents the skier at the point farthest up the slope. The location of the skier at the bottom of the incline is the zero location for gravitational potential energy $(y = 0)$. We have $v_1 = 9.0\text{ m/s}$, $y_1 = 0$, $v_2 = 0$, and $y_2 = d\sin\theta$.

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}d \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgd\sin\theta + \mu_{\text{k}}mgd\cos\theta \quad\rightarrow$$

$$\mu_{\text{k}} = \frac{\tfrac{1}{2}v_1^2 - gd\sin\theta}{gd\cos\theta} = \frac{v_1^2}{2gd\cos\theta} - \tan\theta = \frac{(11.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(15\text{ m})\cos 19^\circ} - \tan 19^\circ = \boxed{0.091}$$

52. (*a*) Calculate the energy of the ball at the two maximum heights, and subtract to find the amount of energy lost. The energy at the two heights is all gravitational potential energy, since the ball has no kinetic energy at those maximum heights.

$$E_{\text{lost}} = E_{\text{initial}} - E_{\text{final}} = mgy_{\text{initial}} - mgy_{\text{final}}$$

$$\frac{E_{\text{lost}}}{E_{\text{initial}}} = \frac{mgy_{\text{initial}} - mgy_{\text{final}}}{mgy_{\text{initial}}} = \frac{y_{\text{initial}} - y_{\text{final}}}{y_{\text{initial}}} = \frac{2.0\text{ m} - 1.6\text{ m}}{2.0\text{ m}} = 0.20 = \boxed{20\%}$$

(*b*) The ball's speed just before the bounce is found from the initial gravitational potential energy, and the ball's speed just after the bounce is found from the ball's final gravitational potential energy.

$$\text{PE}_{\text{initial}} = \text{KE}_{\text{before}} \quad\rightarrow\quad mgy_{\text{initial}} = \tfrac{1}{2}mv_{\text{before}}^2 \quad\rightarrow$$

$$v_{\text{before}} = \sqrt{2gy_{\text{initial}}} = \sqrt{2(9.80\text{ m/s}^2)(2.0\text{ m})} = \boxed{6.3\text{ m/s}}$$

$$\text{PE}_{\text{final}} = \text{KE}_{\text{after}} \rightarrow mgy_{\text{final}} = \tfrac{1}{2}mv_{\text{after}}^2 \rightarrow$$
$$v_{\text{after}} = \sqrt{2gy_{\text{final}}} = \sqrt{2(9.80\text{ m/s}^2)(1.6\text{ m})} = \boxed{5.6\text{ m/s}}$$

(*c*) The energy "lost" was changed $\boxed{\text{primarily into heat energy}}$ —the temperature of the ball and the ground would have increased slightly after the bounce. Some of the energy may have been changed into acoustic energy (sound waves).

53. Since there is friction in this problem, there will be energy dissipated by friction.

$$E_{\text{friction}} + \Delta\text{KE} + \Delta\text{PE} = 0 \rightarrow E_{\text{friction}} = -\Delta\text{KE} - \Delta\text{PE} = \tfrac{1}{2}m(v_1^2 - v_2^2) + mg(y_1 - y_2)$$
$$= \tfrac{1}{2}(66\text{ kg})[0 - (11.0\text{ m/s})^2] + (66\text{ kg})(9.80\text{ m/s}^2)(230\text{ m}) = \boxed{1.4\times10^5\text{ J}}$$

54. Since there are no nonconservative forces, the mechanical energy of the projectile will be conserved. Subscript 1 represents the projectile at launch and subscript 2 represents the projectile as it strikes the ground. The ground is the zero location for potential energy $(y = 0)$. We have $v_1 = 165$ m/s, $y_1 = 135$ m, and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 \rightarrow$$
$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(165\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(135\text{ m})} = \boxed{173\text{ m/s}}$$

Notice that the launch angle does not enter the problem, so it does not influence the final speed.

55. We apply conservation of mechanical energy. We take the surface of the Moon to be the 0 level for gravitational potential energy. Subscript 1 refers to the location where the engine is shut off, and subscript 2 refers to the surface of the Moon. Up is the positive *y* direction.

(*a*) We have $v_1 = 0$, $y_1 = h$, $v_2 = 3.0$ m/s, and $y_2 = 0$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow mgh = \tfrac{1}{2}mv_2^2 \rightarrow$$
$$h = \frac{v_2^2}{2g} = \frac{(3.0\text{ m/s})^2}{2(1.62\text{ m/s}^2)} = \boxed{2.8\text{ m}}$$

(*b*) We have the same conditions except $v_1 = -2.0$ m/s.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgh = \tfrac{1}{2}mv_2^2 \rightarrow$$
$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{(3.0\text{ m/s})^2 - (-2.0\text{ m/s})^2}{2(1.62\text{ m/s}^2)} = \boxed{1.5\text{ m}}$$

(*c*) We have the same conditions except $v_1 = 2.0$ m/s. And since the speeds, not the velocities, are used in the energy conservation calculation, this is the same as part (*b*), so $h = \boxed{1.5\text{ m}}$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgh = \tfrac{1}{2}mv_2^2 \rightarrow$$
$$h = \frac{v_2^2 - v_1^2}{2g} = \frac{(3.0\text{ m/s})^2 - (-2.0\text{ m/s})^2}{2(1.62\text{ m/s}^2)} = \boxed{1.5\text{ m}}$$

56. (*a*) If there is no air resistance, then conservation of mechanical energy can be used. Subscript 1 represents the glider when at launch, and subscript 2 represents the glider at landing.

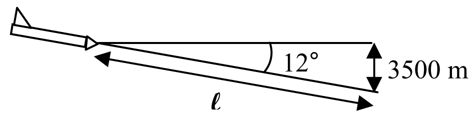
The landing location is the zero location for elastic potential energy (y = 0). We have $y_1 = 3500\text{ m},\ y_2 = 0,$ and $v_1 = 480\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 133.3\text{ m/s}.$ Solve for v_2.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow$$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(133.3\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3500\text{ m})} = 293.8\text{ m/s}\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right)$$

$$= 1058\text{ km/h} \approx \boxed{1100\text{ km/h}}$$

(*b*) Now include the work done by the nonconservative frictional force. Consider the diagram of the glider. The distance over which the friction acts is given by $\ell = \frac{3500\text{ m}}{\sin 12°}$. Use the same subscript representations as above, with $y_1,\ v_1,$ and y_2 as before, and $v_2 = 210\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 58.33\text{ m/s}.$ Write the energy conservation equation and solve for the frictional force.



$$E_1 = E_2 + F_{\text{fr}}\ell \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}\ell \rightarrow F_f = \frac{m(v_1^2 - v_2^2 + 2gy_1)}{2\ell}$$

$$= \frac{(980\text{ kg})[(133.3\text{ m/s})^2 - (58.33\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3500\text{ m})]}{2\left(\frac{3500\text{ m}}{\sin 12°}\right)} = 2415\text{ N} \approx \boxed{2400\text{ N}}$$

57. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W = Fd\cos 0° = mgh.$ The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P = \frac{W}{t} = \frac{mgh}{t} \rightarrow t = \frac{mgh}{P} = \frac{(385\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})}{2750\text{ W}} = \boxed{22.0\text{ s}}$$

58. (*a*) $1\text{ hp} = (1\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right)\left(\frac{4.45\text{ N}}{1\text{ lb}}\right)\left(\frac{1\text{ m}}{3.28\text{ ft}}\right) = 746\text{ N}\cdot\text{m/s} = \boxed{746\text{ W}}$

(*b*) $75\text{ W} = (75\text{ W})\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.10\text{ hp}}$

59. (*a*) $\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(85\text{ kg})(5.0\text{ m/s})^2 = 1062.5\text{ J} \approx \boxed{1100\text{ J}}$

(*b*) The power required to stop him is the change in energy of the player, divided by the time to carry out the energy change.

$$P = \frac{1062.5\text{ J}}{1.0\text{ s}} = 1062.5\text{ W} \approx \boxed{1100\text{ W}}$$

60. The 18 hp is the power generated by the engine in creating a force on the ground to propel the car forward. The relationship between the power and the force is Eq. 6–18, $P = Fv.$ Thus, the force to

propel the car forward is found by $F = P/\upsilon$. If the car has a constant velocity, then the total resistive force must be of the same magnitude as the engine force, so that the net force is zero. Thus, the total resistive force is also found by $F = P/\upsilon$.

$$F = \frac{P}{\upsilon} = \frac{(18\text{ hp})(746\text{ W/1 hp})}{(95\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)} = \boxed{510\text{ N}}$$

61. The power is the force that the motor can provide times the velocity, as given in Eq. 6–18. The force provided by the motor is parallel to the velocity of the boat. The force resisting the boat will be the same magnitude as the force provided by the motor, since the boat is not accelerating, but in the opposite direction to the velocity.

$$P = F\upsilon \quad \rightarrow \quad F = \frac{P}{\upsilon} = \frac{(35\text{ hp})(746\text{ W/1 hp})}{(35\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)} = 2686\text{ N} \approx 2700\text{ N}$$

So the force resisting the boat is $\boxed{\text{2700 N, opposing the velocity}}$.

62. The work done in accelerating the shot put is given by its change in kinetic energy: The power is the energy change per unit time.

$$P = \frac{W}{t} = \frac{\text{KE}_2 - \text{KE}_1}{t} = \frac{\frac{1}{2}m(\upsilon_2^2 - \upsilon_1^2)}{t} = \frac{\frac{1}{2}(7.3\text{ kg})[(14\text{ m/s})^2 - 0]}{1.5\text{ s}} = 476.9\text{ W} \approx \boxed{480\text{ W}}$$

63. The energy transfer from the engine must replace the lost kinetic energy. From the two speeds, calculate the average rate of loss in kinetic energy while in neutral.

$$\upsilon_1 = 95\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 26.39\text{ m/s} \quad \upsilon_2 = 65\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 18.06\text{ m/s}$$

$$\Delta\text{KE} = \tfrac{1}{2}m\upsilon_2^2 - \tfrac{1}{2}m\upsilon_1^2 = \tfrac{1}{2}(1080\text{ kg})[(18.06\text{ m/s})^2 - (26.39\text{ m/s})^2] = -1.999\times10^5\text{ J}$$

$$P = \frac{W}{t} = \frac{1.999\times10^5\text{ J}}{7.0\text{ s}} = 2.856\times10^4\text{ W, or } (2.856\times10^4\text{ W})\frac{1\text{ hp}}{746\text{ W}} = 38.29\text{ hp}$$

So $\boxed{2.9\times10^4\text{ W}}$, or $\boxed{38\text{ hp}}$, is needed from the engine.

64. Since $P = \dfrac{W}{t}$, we have $W = Pt = 2.0\text{ hp}\left(\dfrac{746\text{ W}}{1\text{ hp}}\right)(1\text{ h})\left(\dfrac{3600\text{ s}}{1\text{ h}}\right) = \boxed{5.4\times10^6\text{ J}}$.

65. The average power is the energy transformed per unit time. The energy transformed is the change in kinetic energy of the car.

$$P = \frac{\text{energy transformed}}{\text{time}} = \frac{\Delta\text{KE}}{t} = \frac{\frac{1}{2}m(\upsilon_2^2 - \upsilon_1^2)}{t} = \frac{(975\text{ kg})\left[(95\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(6.4\text{ s})}$$

$$= \boxed{5.3\times10^4\text{ W}} \approx 71\text{ hp}$$

66. The minimum force needed to lift the football player vertically is equal to his weight, mg. The distance over which that force would do work would be the change in height, $\Delta y = (78\text{ m})\sin 33°$. So the work done in raising the player is $W = mg\Delta y$ and the power output required is the work done per unit time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{(82\text{ kg})(9.80\text{ m/s}^2)(83\text{ m})\sin 33°}{75\text{ s}} = 484.4\text{ W} \approx \boxed{480\text{ W}} \approx 0.65\text{ hp}$$

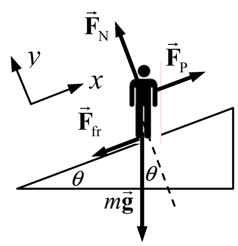
67. The force to lift the water is equal to its weight, so the work to lift the water is equal to the weight times the vertical displacement. The power is the work done per unit time.

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{(27.0\text{ kg})(9.80\text{ m/s}^2)(3.50\text{ m})}{60\text{ s}} = \boxed{15.4\text{ W}}$$

68. The force to lift a person is equal to the person's weight, so the work to lift a person up a vertical distance h is equal to mgh. The work needed to lift N people is $Nmgh$, so the power needed is the total work divided by the total time. We assume the mass of the average person to be 70 kg.

$$P = \frac{W}{t} = \frac{Nmgh}{t} = \frac{47{,}000(70\text{ kg})(9.80\text{ m/s}^2)(200\text{ m})}{3600\text{ s}} = 1.79\times10^6\text{ W} \approx \boxed{2\times10^6\text{ W}} \approx 2400\text{ hp}$$

69. We represent all 30 skiers as one person on the free-body diagram. The engine must supply the pulling force. The skiers are moving with constant velocity, so their net force must be 0.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = F_P - mg\sin\theta - F_{fr} = 0 \quad \rightarrow$$

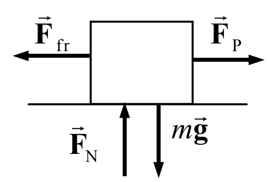
$$F_P = mg\sin\theta + F_{fr} = mg\sin\theta + \mu_k mg\cos\theta$$

The work done by F_P in pulling the skiers a distance d is $F_P d$ since the force is parallel to the displacement. Finally, the power needed is the work done divided by the time to move the skiers up the incline.

$$P = \frac{W}{t} = \frac{F_P d}{t} = \frac{mg(\sin\theta + \mu_k\cos\theta)d}{t}$$

$$= \frac{30(65\text{ kg})(9.80\text{ m/s}^2)(\sin 23° + 0.10\cos 23°)(320\text{ m})}{120\text{ s}} = 24600\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{33\text{ hp}}$$

70. Draw a free-body diagram for the box being dragged along the floor. The box has a constant speed, so the acceleration is 0 in all directions. Write Newton's second law for both the x (horizontal) and y (vertical) directions.



$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg$$

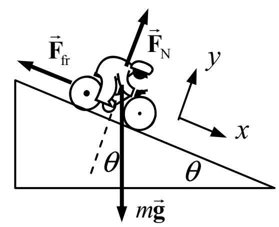
$$\sum F_x = F_P - F_{fr} = 0 \quad \rightarrow \quad F_P = F_{fr} = \mu_k F_N = \mu_k mg$$

The work done by F_P in moving the crate a distance Δx is given by $W = F_P\Delta x\cos 0° = \mu_k mg\Delta x$. The power required is the work done per unit time.

$$P = \frac{W}{t} = \frac{\mu_k mg\Delta x}{t} = \mu_k mg\frac{\Delta x}{t} = \mu_k mg v_x = (0.45)(370\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m/s}) = 1958\text{ W}$$

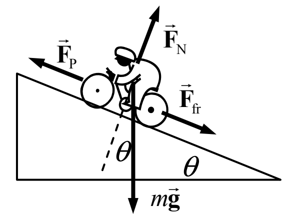
$$= 1958\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{2.6\text{ hp}}$$

71. First, consider a free-body diagram for the cyclist going downhill. Write Newton's second law for the x direction, with an acceleration of 0 since the cyclist has a constant speed.



$$\sum F_x = mg\sin\theta - F_{fr} = 0 \quad \rightarrow \quad F_{fr} = mg\sin\theta$$

Now consider the diagram for the cyclist going up the hill. Again, write Newton's second law for the x direction, with an acceleration of 0. The coordinate axes are the same, but not shown in the second diagram.



$$\sum F_x = F_{fr} - F_P + mg\sin\theta = 0 \quad \rightarrow \quad F_P = F_{fr} + mg\sin\theta$$

Assume that the friction force is the same when the speed is the same, so the friction force when going uphill is the same magnitude as when going downhill.

$$F_P = F_{fr} + mg\sin\theta = 2mg\sin\theta$$

The power output due to this force is given by Eq. 6–18.

$$P = F_P v = 2mgv\sin\theta = 2(75\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m/s})\sin 6.0° = \boxed{610\text{ W}} \approx 0.82\text{ hp}$$

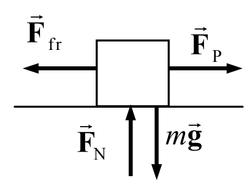
72. First find the kinetic energy of the train, and then find out how much work the web must do to stop the train. Note that the web does negative work, since the force is in the OPPOSITE direction of the displacement.

$$W_{\substack{\text{to stop}\\\text{train}}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(10^4\text{ kg})\left[60\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2 = -1.39\times10^6\text{ J}$$

$$W_{\text{web}} = -\tfrac{1}{2}kx^2 = -1.39\times10^6\text{ J} \quad \rightarrow \quad k = \frac{2(1.39\times10^6\text{ J})}{(500\text{ m})^2} = 11.1\text{ N/m} \approx \boxed{10\text{ N/m}}$$

Note that this is not a very stiff "spring," but it does stretch a long distance.

73. We apply the work-energy theorem. There is no need to use potential energy since the crate moves along the level floor, and there are no springs in the problem. There are two forces doing work in this problem—the pulling force and friction. The starting speed is $v_0 = 0$. Note that the two forces do work over different distances.



$$W_{\text{net}} = W_P + W_{fr} = F_P d_P\cos 0° + F_{fr}d_{fr}\cos 180° = \Delta\text{KE} = \tfrac{1}{2}m(v_f^2 - v_i^2) \quad \rightarrow$$

$$F_P d_P - \mu_k mgd_{fr} = \tfrac{1}{2}mv_f^2 \quad \rightarrow \quad v_f = \sqrt{\frac{2}{m}(F_P d_P - \mu_k mgd_{fr})}$$

$$= \sqrt{\frac{2}{(36.0\text{ kg})}[(225\text{ N})(21.0\text{ m}) - (0.20)(36.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m})]} = \boxed{14.9\text{ m/s}}$$

74. The rock will rise until gravity does –80.0 J of work on the rock. The displacement is upward, but the force is downward, so the angle between them is $180°$. Use Eq. 6–1.

$$W_G = mgd\cos\theta \quad \rightarrow \quad d = \frac{W_G}{mg\cos\theta} = \frac{-80.0\text{ J}}{(1.85\text{ kg})(9.80\text{ m/s}^2)(-1)} = \boxed{4.41\text{ m}}$$

75. (*a*) The spring constant is found by the magnitudes of the initial force and displacement, so $k = F/x$. As the spring compresses, it will do the same amount of work on the block as was done on the spring to stretch it. The work done is positive because the force of the spring is parallel to the displacement of the block. Use the work-energy theorem to determine the speed of the block.

$$W_{\substack{\text{on block}\\ \text{during}\\ \text{compression}}} = \Delta\text{KE}_{\text{block}} = W_{\substack{\text{on spring}\\ \text{during}\\ \text{stretching}}} \rightarrow \tfrac{1}{2}mv_{\text{f}}^2 = \tfrac{1}{2}kx^2 = \tfrac{1}{2}\frac{F}{x}x^2 \rightarrow v_{\text{f}} = \boxed{\sqrt{\frac{Fx}{m}}}$$

(*b*) Use conservation of energy, equating the energy at the original extension with the energy at half the original extension. Let position 1 be the original extension, and position 2 be at half the original extension.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}k\left(\tfrac{1}{2}x\right)^2 \rightarrow mv^2 = kx^2 - k\left(\tfrac{1}{2}x\right)^2 = \tfrac{3}{4}kx^2 \rightarrow$$

$$v^2 = \frac{3kx^2}{4m} = \frac{3Fx}{4m} \rightarrow v = \boxed{\sqrt{\frac{3Fx}{4m}}}$$

76. (*a*) The work done by gravity as the elevator falls is the weight times the displacement. They are in the same direction.

$$W_{\text{G}} = mgd\cos 0° = (925\text{ kg})(9.80\text{ m/s}^2)(28.5\text{ m}) = 2.5835\times10^5\text{ J} \approx \boxed{2.58\times10^5\text{ J}}$$

(*b*) The work done by gravity on the elevator is the net work done on the elevator while falling, so the work done by gravity is equal to the change in kinetic energy.

$$W_{\text{G}} = \Delta\text{KE} = \tfrac{1}{2}mv^2 - 0 \rightarrow v = \sqrt{\frac{2W_{\text{G}}}{m}} = \sqrt{\frac{2(2.5835\times10^5\text{ J})}{(925\text{ kg})}} = \boxed{23.6\text{ m/s}}$$

(*c*) The elevator starts and ends at rest. Therefore, by the work-energy theorem, the net work done must be 0. Gravity does positive work as it falls a distance of $(28.5 + x)$ m (assuming that $x > 0$), and the spring will do negative work at the spring is compressed. The work done on the spring is $\tfrac{1}{2}kx^2$, so the work done by the spring is $-\tfrac{1}{2}kx^2$.

$$W = W_{\text{G}} + W_{\text{spring}} = mg(d + x) - \tfrac{1}{2}kx^2 = 0 \rightarrow \tfrac{1}{2}kx^2 - mgx - mgd = 0 \rightarrow$$

$$x = \frac{mg \pm \sqrt{m^2g^2 - 4\left(\tfrac{1}{2}k\right)(-mgd)}}{2\left(\tfrac{1}{2}k\right)}$$

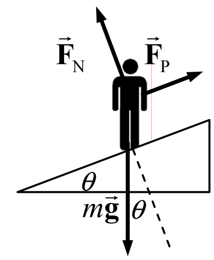
$$= \frac{(925\text{ kg})(9.80\text{ m/s}^2) \pm \sqrt{\begin{array}{l}(925\text{ kg})^2(9.80\text{ m/s}^2)^2 + \\ 2(8.0\times10^4\text{ N/m})(925\text{ kg})(9.80\text{ m/s}^2)(28.5\text{ m})\end{array}}}{8.0\times10^4\text{ N/m}}$$

$$= 2.65\text{ m}, -2.43\text{ m}$$

The positive root must be taken since we have assumed $x > 0$ in calculating the work done by gravity. Using the values given in the problem gives $x = \boxed{2.65\text{ m}}$.

77. (*a*) $\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(3.0\times10^{-3}\text{ kg})(3.0\text{ m/s})^2 = 1.35\times10^{-2}\text{ J} \approx \boxed{1.4\times10^{-2}\text{ J}}$

(*b*) $\text{KE}_{\text{actual}} = 0.35E_{\text{required}} \rightarrow E_{\text{required}} = \dfrac{\text{KE}_{\text{actual}}}{0.35} = \dfrac{1.35\times10^{-2}\text{ J}}{0.35} = \boxed{3.9\times10^{-2}\text{ J}}$



$\boxed{78.}$ See the free-body diagram for the patient on the treadmill. We assume that there are no dissipative forces. Since the patient has a constant velocity, the net force parallel to the plane must be 0. Write Newton's second law for forces parallel to the plane, and then calculate the power output of force $\vec{\mathbf{F}}_P$.

$$\sum F_{\text{parallel}} = F_P - mg\sin\theta = 0 \quad \rightarrow \quad F_P = mg\sin\theta$$

$$P = F_P v = mgv\sin\theta = (75\text{ kg})(9.8\text{ m/s}^2)(3.1\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\sin 12^\circ$$

$$= 131.6\text{ W} \approx \boxed{130\text{ W}}$$

This is about 2 times the wattage of typical household lightbulbs (60–75 W).

$\boxed{79.}$ (*a*) The pilot's initial speed when he hit the snow was 45 m/s. The work done on him as he fell the 1.1 m into the snow changed his kinetic energy. Both gravity and the snow did work on the pilot during that 1.1-m motion. Gravity did positive work (the force was in the same direction as the displacement), and the snow did negative work (the force was in the opposite direction as the displacement).

$$W_{\text{gravity}} + W_{\text{snow}} = \Delta\text{KE} \quad \rightarrow \quad mgd + W_{\text{snow}} = -\tfrac{1}{2}mv_0^2 \quad \rightarrow$$

$$W_{\text{snow}} = -\tfrac{1}{2}mv_0^2 - mgd = -m\left(\tfrac{1}{2}v_i^2 + gd\right) = -(88\text{ kg})\left[\tfrac{1}{2}(45\text{ m/s})^2 + (9.80\text{ m/s}^2)(1.1\text{ m})\right]$$

$$= -9.005\times10^4\text{ J} \approx \boxed{-9.0\times10^4\text{ J}}$$

(*b*) The work done by the snow is done by an upward force, while the pilot moves down.

$$W_{\text{snow}} = F_{\text{snow}}d\cos180^\circ = -F_{\text{snow}}d \quad \rightarrow$$

$$F_{\text{snow}} = -\frac{W_{\text{snow}}}{d} = -\frac{-9.005\times10^4\text{ J}}{1.1\text{ m}} = 8.186\times10^4\text{ N} \approx \boxed{8.2\times10^4\text{ N}}$$

(*c*) During the pilot's fall in the air, positive work was done by gravity and negative work by air resistance. The net work was equal to his change in kinetic energy while he fell. We assume he started from rest when he jumped from the aircraft.

$$W_{\text{gravity}} + W_{\text{air}} = \Delta\text{KE} \quad \rightarrow \quad mgh + W_{\text{air}} = \tfrac{1}{2}mv_f^2 - 0 \quad \rightarrow$$

$$W_{\text{air}} = \tfrac{1}{2}mv_f^2 - mgh = m\left(\tfrac{1}{2}v_f^2 - gh\right) = (88\text{ kg})\left[\tfrac{1}{2}(45\text{ m/s})^2 - (9.80\text{ m/s}^2)(370\text{ m})\right]$$

$$= \boxed{-2.3\times10^5\text{ J}}$$

80. The (negative) work done by the bumper on the rest of the car must equal the change in the car's kinetic energy. The work is negative because the force on the car is in the opposite direction to the car's displacement.

$$W_{\text{bumper}} = \Delta\text{KE} = \quad \rightarrow \quad -\tfrac{1}{2}kx^2 = 0 - \tfrac{1}{2}mv_0^2 \quad \rightarrow$$

$$k = m\frac{v_0^2}{x^2} = (1050\text{ kg})\frac{\left[(8\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(0.015\text{ m})^2} = \boxed{2\times10^7\text{ N/m}}$$

81. The minimum vertical force needed to raise the athlete is equal to the athlete's weight. If the athlete moves upward a distance Δy, then the work done by the lifting force is $W = Fd\cos0^\circ = mg\Delta y$, the

change in PE. The power output needed to accomplish this work in a certain time t is the work divided by the time.

$$P = \frac{W}{t} = \frac{mg\Delta y}{t} = \frac{(62\text{ kg})(9.80\text{ m/s}^2)(5.0\text{ m})}{9.0\text{ s}} = \boxed{340\text{ W}} \approx 0.45\text{ hp}$$

82. The power output for either scenario is given by the change in kinetic energy, divided by the time required to change the kinetic energy. Subscripts of "f" and "i" are used for final and initial values of speed and kinetic energy. Subscript 1 represents the acceleration from 35 km/h to 65 km/h, and subscript 2 represents the acceleration from 55 km/h to 95 km/h.

$$P_1 = \frac{\text{KE}_{1\text{f}} - \text{KE}_{1\text{i}}}{t_1} = \frac{\frac{1}{2}m(v_{1\text{f}}^2 - v_{1\text{i}}^2)}{t_1} \qquad P_2 = \frac{\text{KE}_{2\text{f}} - \text{KE}_{2\text{i}}}{t_2} = \frac{\frac{1}{2}m(v_{2\text{f}}^2 - v_{2\text{i}}^2)}{t_2}$$

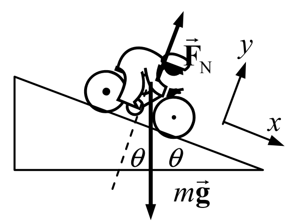
Equate the two expressions for power, and solve for t_2.

$$\frac{\frac{1}{2}m(v_{1\text{f}}^2 - v_{1\text{i}}^2)}{t_1} = \frac{\frac{1}{2}m(v_{2\text{f}}^2 - v_{2\text{i}}^2)}{t_2} \quad \rightarrow \quad t_2 = t_1\frac{(v_{2\text{f}}^2 - v_{2\text{i}}^2)}{(v_{1\text{f}}^2 - v_{1\text{i}}^2)}$$

Since the velocities are included as a ratio, any consistent set of units may be used for the velocities. Thus no conversion from km/h to some other units is needed.

$$t_2 = t_1\frac{(v_{2\text{f}}^2 - v_{2\text{i}}^2)}{(v_{1\text{f}}^2 - v_{1\text{i}}^2)} = (3.8\text{ s})\frac{(95\text{ km/h})^2 - (55\text{ km/h})^2}{(65\text{ km/h})^2 - (35\text{ km/h})^2} = \boxed{7.6\text{ s}}$$

83. (*a*) The work done by gravity is given by Eq. 6–1.



$$W_G = mgd\cos(90 - \theta) = (85\text{ kg})(9.80\text{ m/s}^2)(180\text{ m})\cos 86.0°$$
$$= 1.046\times10^4\text{ J} \approx \boxed{1.0\times10^4\text{ J}}$$

(*b*) The work is the change in kinetic energy. The initial kinetic energy is 0.

$$W_G = \Delta\text{KE} = \text{KE}_\text{f} - \text{KE}_\text{i} = \tfrac{1}{2}mv_\text{f}^2 \quad \rightarrow$$
$$v_f = \sqrt{\frac{2W_G}{m}} = \sqrt{\frac{2(1.046\times10^4\text{ J})}{85\text{ kg}}} = 15.69\text{ m/s} \approx \boxed{16\text{ m/s}}$$

84. Assume that there are no nonconservative forces doing work, so the mechanical energy of the jumper will be conserved. Subscript 1 represents the jumper at the launch point of the jump, and subscript 2 represents the jumper at the highest point. The starting height of the jump is the zero location for potential energy $(y = 0)$. We have $y_1 = 0,$ $y_2 = 1.1\text{ m},$ and $v_2 = 6.5\text{ m/s}.$ Solve for v_1.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$
$$v_1 = \sqrt{v_2^2 + 2gy_2} = \sqrt{(6.5\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(1.1\text{ m})} = \boxed{8.0\text{ m/s}}$$

85. (*a*) Use conservation of mechanical energy, assuming there are no nonconservative forces. Subscript 1 represents the water at the top of the dam, and subscript 2 represents the water as it strikes the turbine blades. The level of the turbine blades is the zero location for potential energy $(y = 0)$. Assume that the water goes over the dam with an approximate speed of 0. We have $v_1 = 0,$ $y_1 = 80\text{ m},$ and $y_2 = 0.$ Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mgy_1 = \tfrac{1}{2}mv_2^2 \quad \rightarrow$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(88\text{ m})} = 41.53\text{ m/s} \approx \boxed{42\text{ m/s}}$$

(*b*) The energy of the water at the level of the turbine blades is all kinetic energy and is given by $\tfrac{1}{2}mv_2^2$. We know that 55% of that energy gets transferred to the turbine blades. The rate of energy transfer to the turbine blades is the power developed by the water.

$$P = 0.55\left(\tfrac{1}{2}\frac{m}{t}v_2^2\right) = \frac{(0.55)(680\text{ kg/s})(41.53\text{ m/s})^2}{2} = \boxed{3.2\times10^5\text{ W}}$$

86. (a) The speed v_B can be found from conservation of mechanical energy. Subscript A represents the skier at the top of the jump, and subscript B represents the skier at the end of the ramp. Point B is taken as the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = 40.6$ m, and $y_2 = 0$. Solve for v_2.

$$E_A = E_B \quad \rightarrow \quad \tfrac{1}{2}mv_A^2 + mgy_A = \tfrac{1}{2}mv_B^2 + mgy_B \quad \rightarrow \quad mgy_A = \tfrac{1}{2}mv_B^2 \quad \rightarrow$$

$$v_B = \sqrt{2gy_A} = \sqrt{2(9.80\text{ m/s}^2)(40.6\text{ m})} = 28.209\text{ m/s} \approx \boxed{28.2\text{ m/s}}$$

(*b*) Now we use projectile motion. We take the origin of coordinates to be the point on the ground directly under the end of the ramp. Then an equation to describe the slope is $y_{\text{slope}} = -x\tan 30°$. The equations of projectile motion can be used to find an expression for the parabolic path that the skier follows after leaving the ramp. We take up to be the positive vertical direction. The initial *y* velocity is 0, and the *x* velocity is v_B as found above.

$$x = v_B t; \quad y_{\text{proj}} = y_0 - \tfrac{1}{2}gt^2 = y_0 - \tfrac{1}{2}g(x/v_B)^2$$

The skier lands at the intersection of the two paths, so $y_{\text{slope}} = y_{\text{proj}}$.

$$y_{\text{slope}} = y_{\text{proj}} \quad \rightarrow \quad -x\tan 30° = y_0 - \tfrac{1}{2}g\left(\frac{x}{v_B}\right)^2 \quad \rightarrow \quad gx^2 - x(2v_B^2\tan 30°) - 2y_0v_B^2 = 0 \quad \rightarrow$$

$$x = \frac{(2v_B^2\tan 30°) \pm \sqrt{(2v_B^2\tan 30°)^2 + 8gy_0v_B^2}}{2g} = \frac{(v_B^2\tan 30°) \pm \sqrt{(v_B^2\tan 30°)^2 + 2gy_0v_B^2}}{g}$$

Solving this with the given values gives $x = -7.09$ m, 100.8 m. The positive root is taken.

Finally, $s\cos 30.0° = x \quad \rightarrow \quad s = \dfrac{x}{\cos 30.0°} = \dfrac{100.8\text{ m}}{\cos 30.0°} = \boxed{116\text{ m}}$.

87. (*a*) $$1\text{ kW}\cdot\text{h} = 1\text{ kW}\cdot\text{h}\left(\frac{1000\text{ W}}{1\text{ kW}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ J/s}}{1\text{ W}}\right) = \boxed{3.6\times10^6\text{ J}}$$

(*b*) $$(580\text{ W})(1\text{ month}) = (580\text{ W})(1\text{ month})\left(\frac{1\text{ kW}}{1000\text{ W}}\right)\left(\frac{30\text{ d}}{1\text{ month}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right) = 417.6\text{ kW}\cdot\text{h}$$

$$\approx \boxed{420\text{ kW}\cdot\text{h}}$$

(*c*) $$417.6\text{ kW}\cdot\text{h} = 417.6\text{ kW}\cdot\text{h}\left(\frac{3.6\times10^6\text{ J}}{1\text{ kW}\cdot\text{h}}\right) = 1.503\times10^9\text{ J} \approx \boxed{1.5\times10^9\text{ J}}$$

(*d*) $(417.6\text{ kW}\cdot\text{h})\left(\dfrac{\$0.12}{1\text{ kW}\cdot\text{h}}\right) = \$50.11 \approx \boxed{\$50}$ (2 significant figures)

Kilowatt-hours is a measure of energy, not power, so $\boxed{\text{no}}$, the actual rates at which the energy is being used at various times does not figure into the bill. They could use the energy at a constant rate, or at widely varying rates; as long as the total used is about 420 kilowatt-hours, the price would be about \$50. If the family's average rate of energy usage increased, then their bill would increase. And there are some power companies that do charge more per kWh for using energy during certain times of the day, which might be seen as a rate-dependent fee.

88. The spring constant for the scale can be found from the 0.60-mm compression due to the 760-N force.

$$k = \frac{F}{x_c} = \frac{760\text{ N}}{6.0\times10^{-4}\text{ m}} = 1.27\times10^6\text{ N/m}$$

Next, use conservation of energy for the jump. Subscript 1 represents the initial location, and subscript 2 represents the location at maximum compression of the scale spring. Assume that the location of the uncompressed scale spring is the 0 location for gravitational potential energy. We have $v_1 = v_2 = 0$ and $y_1 = 1.0\text{ m}$. Solve for y_2, which must be negative.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 \quad\rightarrow\quad mgy_1 = mgy_2 + \tfrac{1}{2}ky_2^2 \quad\rightarrow$$

$$y_2^2 + 2\frac{mg}{k}y_2 - 2\frac{mg}{k}y_1 = y_2^2 + 2x_c y_2 - 2x_c y_1 = y_2^2 + 1.20\times10^{-3}y_2 - 1.20\times10^{-3} = 0$$

Use the quadratic formula to solve for y_2.

$$y_2 = \frac{-1.20\times10^{-3} \pm \sqrt{(1.20\times10^{-3})^2 - 4(-1.20\times10^{-3})}}{2} = -3.52\times10^{-2}\text{ m}, 3.40\times10^{-2}\text{ m}$$

$$F_{\text{scale}} = k|x| = (1.27\times10^6\text{ N/m})(3.52\times10^{-2}\text{ m}) = \boxed{4.5\times10^4\text{ N}}$$

89. (*a*) The work done by the hiker against gravity is the change in gravitational potential energy.

$$W_G = mg\Delta y = (65\text{ kg})(9.80\text{ m/s}^2)(4200\text{ m} - 2800\text{ m}) = 8.918\times10^5\text{ J} \approx \boxed{8.9\times10^5\text{ J}}$$

(*b*) The average power output is found by dividing the work by the time taken.

$$P_{\text{output}} = \frac{W_{\text{grav}}}{t} = \frac{8.918\times10^5\text{ J}}{(4.6\text{ h})(3600\text{ s/1 h})} = 53.85\text{ W} \approx \boxed{54\text{ W}}$$

$$53.85\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{7.2\times10^{-2}\text{ hp}}$$

(*c*) The output power is the efficiency times the input power.

$$P_{\text{output}} = 0.15P_{\text{input}} \quad\rightarrow\quad P_{\text{input}} = \frac{P_{\text{output}}}{0.15} = \frac{53.85\text{ W}}{0.15} = \boxed{360\text{ W}} = \boxed{0.48\text{ hp}}$$

$\boxed{90.}$ (*a*) The tension in the cord is perpendicular to the path at all times, so the tension in the cord doesn't do work on the ball. Only gravity does work on the ball, so the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for potential energy ($y = 0$). We have $v_1 = 0$, $y_1 = \ell$, and $y_2 = 0$. Solve for v_2.

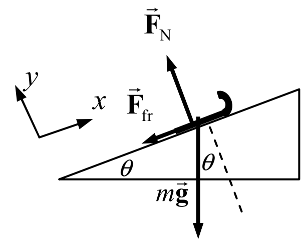
$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mg\ell = \tfrac{1}{2}mv_2^2 \quad \rightarrow \quad v_2 = \boxed{\sqrt{2g\ell}}$$

(*b*) Use conservation of energy to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_2 = \sqrt{2g\ell}$, $y_2 = 0$, and $y_3 = 2(\ell - h) = 2(\ell - 0.80\ell) = 0.40\ell$. Solve for v_3.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow \quad \tfrac{1}{2}m(2g\ell) = \tfrac{1}{2}mv_3^2 + mg(0.40\ell) \quad \rightarrow$$

$$\boxed{v_3 = \sqrt{1.2g\ell}}$$

91. A free-body diagram for the sled is shown as it moves up the hill. From this we get an expression for the friction force.



$$\sum F_y = F_\text{N} - mg\cos\theta = 0 \quad \rightarrow \quad F_\text{N} = mg\cos\theta \quad \rightarrow \quad F_\text{fr} = \mu_\text{k} mg\cos\theta$$

(*a*) We apply conservation of energy with a frictional force as given in Eq. 6–16b. Subscript 1 refers to the sled at the start of its motion, and subscript 2 refers to the sled at the top of its motion. Take the starting position of the sled to be the 0 for gravitational potential energy. We have $v_1 = 2.4$ m/s, $y_1 = 0$, and $v_2 = 0$. The relationship between the distance traveled along the incline (*d*) and the height the sled rises is $y_2 = d \sin\theta$. Solve for *d*.

$$E_1 = E_2 + F_\text{fr}d \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_\text{fr}d \quad \rightarrow$$

$$\tfrac{1}{2}mv_1^2 = mgd\sin\theta + \mu_\text{k} mgd\cos\theta \quad \rightarrow$$

$$d = \frac{v_1^2}{2g(\sin\theta + \mu_\text{k}\cos\theta)} = \frac{(2.3\text{ m/s})^2}{2(9.80\text{ m/s}^2)(\sin 28° + 0.25\cos 28°)} = 0.3910\text{ m} \approx \boxed{0.39\text{ m}}$$

(*b*) For the sled to slide back down, the friction force will now point UP the hill in the free-body diagram. In order for the sled to slide down, the component of gravity along the hill must be larger than the maximum force of static friction.

$$mg\sin\theta > F_\text{fr} \quad \rightarrow \quad mg\sin\theta > \mu_\text{s} mg\cos\theta \quad \rightarrow \quad \mu_\text{s} < \tan 28° \quad \rightarrow \quad \boxed{\mu_\text{s} < 0.53}$$

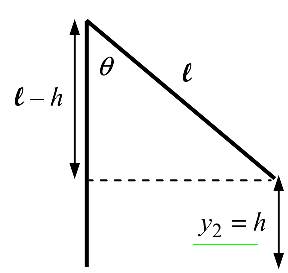
(*c*) We again apply conservation of energy including work done by friction. Subscript 1 refers to the sled at the top of the incline, and subscript 2 refers to the sled at the bottom of the incline. We have $v_1 = 0$, $y_1 = d\sin\theta$, and $y_2 = 0$.

$$E_1 = E_2 + F_\text{fr}d \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_\text{fr}d \quad \rightarrow$$

$$mgd\sin\theta = \tfrac{1}{2}mv_2^2 + \mu_\text{k} mgd\cos\theta \quad \rightarrow$$

$$v_2 = \sqrt{2gd(\sin\theta - \mu_\text{k}\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.3910\text{ m})(\sin 28° - 0.25\cos 28°)}$$
$$= 1.381\text{ m/s} \approx \boxed{1.4\text{ m/s}}$$

92. (*a*) Use conservation of energy for the swinging motion. Subscript 1 represents the student initially grabbing the rope, and subscript 2 represents the student at the top of the swing. The location where the student initially grabs the rope is the zero location for potential energy $(y = 0)$. We have $v_1 = 5.0$ m/s, $y_1 = 0$, and $v_2 = 0$. Solve for y_2.



$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow$$

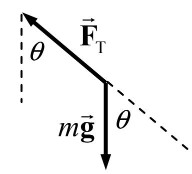
$$\tfrac{1}{2}mv_1^2 = mgy_2 \quad \rightarrow \quad y_2 = \frac{v_1^2}{2g} = h$$

Calculate the angle from the relationship in the diagram.

$$\cos\theta = \frac{\ell - h}{\ell} = 1 - \frac{h}{\ell} = 1 - \frac{v_1^2}{2g\ell} \quad \rightarrow$$

$$\theta = \cos^{-1}\left(1 - \frac{v_1^2}{2g\ell}\right) = \cos^{-1}\left(1 - \frac{(6.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(10.0\text{ m})}\right) = 35.28° \approx \boxed{35°}$$

(*b*) At the release point, the speed is 0, so there is no radial acceleration, since $a_R = \dfrac{v^2}{r}$. Thus the centripetal force must be 0. Use the free-body diagram to write Newton's second law for the radial direction.



$$\sum F_R = F_T - mg\cos\theta = 0 \quad \rightarrow$$

$$F_T = mg\cos\theta = (56\text{ kg})(9.80\text{ m/s}^2)\cos 35.28° = 448\text{ N} \approx \boxed{450\text{ N}}$$

(*c*) Write Newton's second law for the radial direction for any angle, and solve for the tension.

$$\sum F_R = F_T - mg\cos\theta = m\frac{v^2}{r} \quad \rightarrow \quad F_T = mg\cos\theta + m\frac{v^2}{r}$$

As the angle decreases, the tension increases, and as the speed increases, the tension increases. Both effects are greatest at the bottom of the swing, so that is where the tension will be at its maximum.

$$F_{T_{\max}} = mg\cos 0 + m\frac{v_1^2}{r} = (56\text{ kg})(9.80\text{ m/s}^2) + \frac{(56\text{ kg})(6.0\text{ m/s})^2}{10.0\text{ m}} = \boxed{750\text{ N}}$$

93. The energy to be stored is the power multiplied by the time: $E = Pt$. The energy will be stored as the gravitational potential energy increase in the water: $E = \Delta\text{PE} = mg\Delta y = \rho Vg\Delta y$, where ρ is the density of the water and V is the volume of the water.

$$Pt = \rho Vg\Delta y \quad \rightarrow \quad V = \frac{Pt}{\rho g\Delta y} = \frac{(180\times10^6\text{ W})(3600\text{ s})}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(380\text{ m})} = \boxed{1.7\times10^5\text{ m}^3}$$

94. The original speed of the softball is $(120\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s}$. The final speed is 90% of this, or 30 m/s. The work done by air friction causes a change in the kinetic energy of the ball, and thus the speed change. In calculating the work, notice that the direction of the force of friction is opposite to the direction of motion of the ball.

$$W_{fr} = F_{fr}d\cos 180° = \text{KE}_2 - \text{KE}_1 = \tfrac{1}{2}m(v_2^2 - v_1^2) \quad \rightarrow$$

$$F_{fr} = \frac{m(v_2^2 - v_1^2)}{-2d} = \frac{mv_1^2(0.9^2 - 1)}{-2d} = \frac{(0.25\text{ kg})(33.33\text{ m/s})^2(0.9^2 - 1)}{-2(15\text{ m})} = 1.759\text{ N} \approx \boxed{1.8\text{ N}}$$

Solutions to Search and Learn Problems

1. (*a*) (1) When using energy, the actual path followed is not important—only the initial and final positions are needed. (2) The energy terms are scalars, not vectors, so the math of adding the terms is simpler. (3) Forces perpendicular to the motion do no work and therefore can be ignored in the energy equations.

   (*b*) It is never absolutely necessary that energy be used to solve a problem. Force equations can always be used to determine the motion of an object. However, if the path is curved or the forces vary with position, the force equations become too complex to solve algebraically. In those cases it is easier to use energy equations.

   (*c*) The force equations must be used if you need to know the directions of motion, such as the components of the velocity or the horizontal and vertical displacements. Also, the energy equations are not able to provide the time elapsed during motion or calculate acceleration.

   (*d*) The energy equations do not provide any information on the direction of motion, the components of the velocity or centripetal motion, the time elapsed, or the acceleration.

2. (*a*) To maintain a constant speed as the truck goes down the hill, the brakes must convert the change in gravitational potential energy into thermal energy. On a steep hill, the rate of energy conversion (power) is high. The energy is dissipated over time by air passing over the brakes. If the brakes absorb more thermal power than they can dissipate to the air, the brakes heat up and can spontaneously catch fire.

   (*b*) It takes the truck longer to go down a gradual hill. The total amount of energy that is converted to heat is the same, but since it takes longer the power absorbed by the brakes is smaller, so more of the energy can be dissipated to the air. As such, the brakes do not heat up as much.

   (*c*) Shifting to a lower gear makes the engine work to dissipate some of the energy (it forces many of the moving parts of the engine to increase their kinetic energy). The engine has the radiator and cooling system to help dissipate the heat more efficiently.

   (*d*) The heat dissipated is the change in mechanical energy between the initial (indicated by 1) and final (indicated by 2) positions. We set the gravitational potential energy equal to zero at the second position. The initial vertical height is then the product of the distance traveled and the sine of the angle of incline. We use Eq. 6–15, and we treat the mass as if it had 2 significant figures.

$$
\begin{aligned}
W_{\text{NC}} &= (\text{KE}_2 + \text{PE}_2) - (\text{KE}_1 + \text{PE}_1) = \left(\tfrac{1}{2}mv_2^2 + mgy_2\right) - \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) \\
&= \tfrac{1}{2}m(v_2^2 - v_1^2) - mgy_1 \\
&= \tfrac{1}{2}(8000\text{ kg})\left[(35\text{ km/h})^2\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) - (95\text{ km/h})^2\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2\right] \\
&\qquad - (8000\text{ kg})(9.8\text{ m/s}^2)(360\text{ m})\sin 12^\circ \\
&= -8.28\times10^6\text{ J}
\end{aligned}
$$

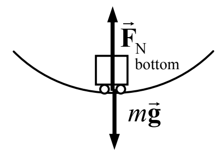
   The heat dissipated is the opposite of this: $\boxed{8.3\times10^6\text{ J}}$.

3. (*a*) The two conservative forces are the force of gravity and the elastic force of a spring. The force of gravity is accounted for in the gravitational potential energy. The spring force is accounted for in the elastic potential energy.

   (*b*) The force of water on a swimmer is a nonconservative, or dissipative, force. As with friction, the force is always against the motion of the swimmer relative to the water.

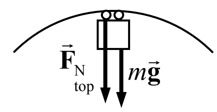
4. As a car accelerates forward, the engine causes the tires to rotate. The force of static friction between the tires and the road accelerates the car forward. Since the force on the car and the car's motion are in the same direction, the force of friction does positive work on the car. A second example would be the positive work done on a crate sitting in the bed of an accelerating truck. As the truck accelerates forward, the force of friction between the crate and the bed accelerates the box forward.

5. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.



$$\sum F_{\text{bottom}} = F_{\text{N bottom}} - mg = m\frac{v_{\text{bottom}}^2}{R} \rightarrow F_{\text{N bottom}} = mg + m\frac{v_{\text{bottom}}^2}{R}$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal and must be downward.



$$\sum F_{\text{top}} = F_{\text{N top}} + mg = m\frac{v_{\text{top}}^2}{R} \rightarrow F_{\text{N top}} = m\frac{v_{\text{top}}^2}{R} - mg$$

Assume that the speed at the top is large enough that $F_{\text{N top}} > 0$, so $v_{\text{top}} > \sqrt{Rg}$.

Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for potential energy $(y = 0)$. We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR$$

The difference in apparent weights is the difference in the normal forces.

$$F_{\text{N bottom}} - F_{\text{N top}} = \left(mg + m\frac{v_{\text{bottom}}^2}{R}\right) - \left(m\frac{v_{\text{top}}^2}{R} - mg\right) = 2mg + m\frac{(v_{\text{bottom}}^2 - v_{\text{top}}^2)}{R}$$
$$= 2mg + m(4gR)/R = \boxed{6mg}$$

Notice that the result does not depend on either v or R.

6. (*a*) As the roller coaster moves between the initial and final positions, three forces act on the coaster: gravity, the normal force, and the force of friction. The work done by gravity is accounted for in the change in potential energy. The normal force is perpendicular to the track and therefore does no work. Since friction is the only other force acting on the coaster, the work done by friction can be calculated as the change in mechanical energy between the initial and final positions.

   (*b*) The average force of friction is the work done by friction divided by the distance traveled. In this problem we would need to know the total distance traveled to calculate the average force of friction.

7

LINEAR MOMENTUM

Responses to Questions

1. For momentum to be conserved, the system under analysis must be "closed"—not have any forces on it from outside the system. A coasting car has air friction and road friction on it, for example, which are "outside" or "external" forces and thus reduce the momentum of the car. If the ground and the air were considered part of the system and their velocities analyzed, then the momentum of the entire system would be conserved, but not necessarily the momentum of any single component, like the car.

2. The momentum of an object can be expressed in terms of its kinetic energy, as follows:

$$p = m\upsilon = \sqrt{m^2\upsilon^2} = \sqrt{m(m\upsilon^2)} = \sqrt{2m\left(\tfrac{1}{2}m\upsilon^2\right)} = \sqrt{2m\text{KE}}$$

   Thus if two objects have the same kinetic energy, then the one with more mass has the greater momentum.

3. Consider this problem as a very light object hitting and sticking to a very heavy object. The large object–small object combination (Earth + jumper) would have some momentum after the collision, but due to the very large mass of the Earth, the velocity of the combination is so small that it is not measurable. Thus the jumper lands on the Earth, and nothing more happens.

4. When you release an inflated but untied balloon at rest, the gas inside the balloon (at high pressure) rushes out the open end of the balloon. That escaping gas and the balloon form a closed system, so the momentum of the system is conserved. The balloon and remaining gas acquire a momentum equal and opposite to the momentum of the escaping gas, so they move in the opposite direction to the escaping gas.

5. As the fish swishes its tail back and forth, it moves some water backward, away from the fish. If we consider the system to be the fish and the water, then, from conservation of momentum, the fish must move forward.

6. (*d*) The girl moves in the opposite direction at 2.0 m/s. Since there are no external forces on the pair, momentum is conserved. The initial momentum of the system (boy and girl) is zero. The final momentum of the girl must be the same in magnitude and opposite in direction to the final momentum of the boy so that the net final momentum is also zero.

7. The air bag greatly increases the amount of time over which the stopping force acts on the driver. If a hard object like a steering wheel or windshield is what stops the driver, then a large force is exerted over a very short time. If a soft object like an air bag stops the driver, then a much smaller force is exerted over a much longer time. For instance, if the air bag is able to increase the time of stopping by a factor of 10, then the average force on the person will be decreased by a factor of 10. This greatly reduces the possibility of serious injury or death.

8. Yes. In a perfectly elastic collision, kinetic energy is conserved. In the Earth–ball system, the kinetic energy of the Earth after the collision is negligible, so the ball has the same kinetic energy leaving the floor as it had hitting the floor. The height from which the ball is released determines its potential energy, which is converted to kinetic energy as the ball falls. If it leaves the floor with this same amount of kinetic energy and a velocity upward, it will rise to the same height as it originally had as the kinetic energy is converted back into potential energy.

9. In order to conserve momentum, when the boy dives off the back of the rowboat the boat will move forward.

10. He could have thrown the coins in the direction opposite the shore he was trying to reach. Since the lake is frictionless, momentum would be conserved and he would "recoil" from the throw with a momentum equal in magnitude and opposite in direction to the coins. Since his mass is greater than the mass of the coins, his speed would be less than the speed of the coins, but, since there is no friction, he would maintain this small speed until he hit the shore.

11. When the tennis ball rebounds from a stationary racket, it reverses its component of velocity perpendicular to the racket with very little energy loss. If the ball is hit straight on, and the racket is actually moving forward, the ball can be returned with an energy (and a speed) equal to (or even greater than) the energy it had when it was served.

12. Yes. Impulse is the product of the force and the time over which it acts. A small force acting over a longer time could impart a greater impulse than a large force acting over a shorter time.

13. The collision in which the two cars rebound would probably be more damaging. In the case of the cars rebounding, the change in momentum of each car is greater than in the case in which they stick together, because each car is not only brought to rest but also sent back in the direction from which it came. A greater impulse results from a greater force, so most likely more damage would occur.

14. (*a*) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times—the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, so its momentum is not conserved.
    (*b*) With this definition of the system, all of the forces are internal, so the momentum of the Earth–ball system is conserved during the entire process.
    (*c*) For a piece of putty falling and sticking to a steel plate, if the system is the putty and the Earth, momentum is conserved for the entire path.

15. "Crumple zones" are similar to air bags in that they increase the time of interaction during a collision, and therefore lower the average force required for the change in momentum that the car undergoes in the collision.

16. For maximum power, the turbine blades should be designed so that the water rebounds. The water has a greater change in momentum if it rebounds than if it just stops at the turbine blade. If the water has a greater change in momentum, then, by conservation of momentum, the turbine blades also have a greater change in momentum and will therefore spin faster.

17. (*a*) The direction of the change in momentum of the ball is perpendicular to the wall and away from it, or to the left in the figure.

    (*b*) Since the force on the wall is opposite that on the ball, the force on the wall is to the right.

18. From Eq. 7–7 for a 1-D elastic collision, $v_A - v_B = v'_B - v'_A$. Let A represent the bat, and let B represent the ball. The positive direction will be the (assumed horizontal) direction that the bat is moving when the ball is hit. We assume that the batter can swing the bat with equal strength in either case, so that v_A is the same in both pitching situations. Because the bat is so much heavier than the ball, we assume that $v'_A \approx v_A$ —the speed of the bat doesn't change significantly during the collision. Then the velocity of the baseball after being hit is $v'_B = v'_A + v_A - v_B \approx 2v_A - v_B$. If $v_B = 0$, the ball tossed up into the air by the batter, then $v'_B \approx 2v_A$ —the ball moves away with twice the speed of the bat. But if $v_B < 0$, the pitched ball situation, we see that the magnitude of $v'_B > 2v_A$, so the ball moves away with greater speed. If, for example, the pitching speed of the ball was about twice the speed at which the batter could swing the bat, then we would have $v'_B \approx 4v_A$. Thus the ball has greater speed after being struck, so the ball will travel farther after being hit. This is similar to the "gravitational slingshot" effect discussed in Search and Learn 4.

$\boxed{19.}$ A perfectly inelastic collision between two objects that initially had momenta equal in magnitude but opposite in direction would result in all the kinetic energy being lost. For instance, imagine sliding two clay balls with equal masses and speeds toward each other across a frictionless surface. Since the initial momentum of the system is zero, the final momentum must be zero as well. The balls stick together, so the only way the final momentum can be zero is if they are brought to rest. In this case, all the kinetic energy would be lost. A simpler situation is dropping a ball of clay onto the floor. The clay doesn't rebound after the collision with the floor, and all of the kinetic energy is lost.

20. Passengers may be told to sit in certain seats in order to balance the plane. If they move during the flight, they could change the position of the center of mass of the plane and affect its stability in flight.

21. In order to maintain balance, your CM must be located directly above your feet. If you have a heavy load in your arms, your CM will be out in front of your body and not above your feet. So you lean backward to get your CM directly above your feet. Otherwise, you might fall over forward.

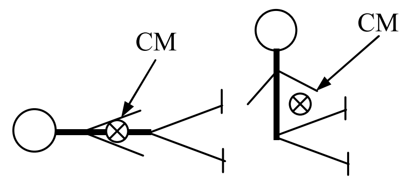
22. The 1-m length of pipe is uniform—it has the same density throughout, so its CM is at its geometric center, which is its midpoint. The arm and leg are not uniform—they are more dense where there is muscle, primarily in the parts that are closest to the body. Thus the CM of the arm or leg is closer to the body than the geometric center. The CM is located closer to the more massive part of the arm or leg.

23. When a rocket expels gas in a given direction, it puts a force on that gas. The momentum of the gas-rocket system stays constant, so if the gas is pushed to the left, the rocket will be pushed to the right due to Newton's third law. So the rocket must carry some kind of material to be ejected (it could be exhaust from some kind of engine, or it could be compressed gas) in order to change direction.

24. Consider Bob, Jim, and the rope as a system. The center of mass of the system is closer to Bob, because he has more mass. Because there is no net external force on the system, the center of mass will stay stationary. As the two men pull hand-over-hand on the rope they will move toward each other, eventually colliding at the center of mass. Since the CM is on Bob's side of the midline, Jim will cross the midline and lose.

25. If there were only two particles as decay products, then by conservation of momentum, the momenta of the two decay products would have to be equal in magnitude and opposite in direction, so that the momenta would be required to lie along a line. If the momenta of the recoil nucleus and the electron do not lie along a line, then some other particle (the neutrino) must have some of the momentum.

26. When you are lying flat on the floor, your CM is inside of the volume of your body. When you sit up on the floor with your legs extended, your CM is outside of the volume of your body. The CM is higher when you sit up, and is slightly in front of your midsection.



27. The engine does not directly accelerate the car. The engine puts a force on the driving wheels, making them rotate. The wheels then push backward on the roadway as they spin. The Newton's third law reaction to this force is the forward pushing of the roadway on the wheels, which accelerates the car. So it is the (external) road surface that accelerates the car.

28. The motion of the center of mass of the rocket will follow the original parabolic path, both before and after explosion. Each individual piece of the rocket will follow a separate path after the explosion, but since the explosion was internal to the system (consisting of the rocket), the center of mass of all the exploded pieces will follow the original path.

Responses to MisConceptual Questions

1. (*d*) Students frequently have one of two common misconceptions. One idea is that since the truck has more mass, it has more momentum and will have a greater momentum change. Alternatively, some students think that since the smaller object has a greater change in speed, it will have the greater change in momentum. In the absence of external net forces, momentum is a conserved quantity. Therefore, momentum lost by one of the vehicles is gained by the other, and the magnitude of the change in momentum is the same for both vehicles.

2. (*b*) A common misconception in this problem is the belief that since the sand is dropped onto the boat, it does not exert a force on the boat and therefore does not accelerate the boat. However, when dropped, the sand has no initial horizontal velocity. For the sand to be at rest on the deck of the boat it must be accelerated from rest to the final speed of the boat. This acceleration is provided by the force of friction between the boat and sand. By Newton's third law, the sand exerts an equal but opposite force on the boat, which will cause the boat to slow down.

3. (*c*) Students may have the misconception that by doubling the mass the final speed will decrease. However, the momentum and kinetic energy are proportional to the mass. So, if each mass is doubled, then every term in the conservation of momentum and conservation of kinetic energy equations is doubled. This factor of two can be divided out to return to the initial equation. Therefore, doubling the masses will have no effect on the final motion.

4. (*a*) Since the net momentum of the astronaut and wrench is zero, the only way for the astronaut to move toward the space station is for the wrench to move away from the station. If the astronaut throws the wrench in any other direction, the astronaut will move away from the wrench but not toward the station. If the astronaut throws the wrench toward the station but does not let go of it, neither the wrench nor the astronaut will move.

5. (*a*) Since the asteroid ends up in the shuttle storage bay, the asteroid and shuttle have the same final speed. This is a completely inelastic collision, so only momentum is conserved.

6. (*a*) A common error is to ignore the vector nature of momentum and impulse. The bean bag and golf ball have the same momentum just before they hit the ground. The bean bag comes to rest when it hits the ground, so the ground has exerted an upward impulse equal to the magnitude of bean bag's momentum. The golf ball rebounds upward with the same magnitude momentum, but in the opposite direction. The ground therefore exerted an upward impulse equal to twice the magnitude of the momentum.

7. (*a*) Students may consider that the superball and clay have the same momentum and as such would be equally effective. However, since the clay and superball interact with the door differently, this is incorrect. The clay sticks to the door, exerting an impulse on the door equal to its momentum. The superball bounces off of the door, exerting an impulse about equal to twice its momentum. Since the superball imparts a greater impulse to the door, it will be more effective.

8. (*c*) This problem requires the student to understand the vector nature of momentum. The ball initially has a momentum toward the batter. If the ball is stopped by the catcher, the change in momentum has the same magnitude as the initial momentum. If the ball is hit straight back to the pitcher, the magnitude of the change in momentum is equal to twice the initial momentum. If the ball is hit straight up at the same speed, the change in momentum has a horizontal and a vertical vector component with the magnitude of each component equal to the initial momentum. Since the two components are perpendicular to each other, the magnitude of the change in momentum will be less than the sum of their magnitudes. As such, the greatest change in momentum occurs when the ball is hit straight back toward the pitcher.

9. (*d*) To solve this question a student should understand the relationships between force, time, momentum, work, and kinetic energy. Impulse is the product of the force and the time over which the force acts. For an object starting at rest, the impulse is also equal to the final momentum. Since the same force acts over the same time on both vehicles, they will have the same momentum. The lighter vehicle will have the greater speed and will therefore have traveled a greater distance in the same time. Since both vehicles start from rest with the same force acting on them, the work-energy theorem shows that the vehicle that travels the greater distance will have the greater final kinetic energy.

10. (*e*) Since the same force acts on both vehicles over the same distance, the work done on both vehicles is the same. From the work-energy theorem both vehicles will have the same final kinetic energy. The lighter vehicle will travel the distance in a shorter amount of time and will therefore experience a smaller impulse and have a smaller final momentum.

11. (*c*) A common misconception is that as the milk drains from the tank car and its mass decreases, the tank car's speed increases. For the tank car's speed to change, a horizontal force would have to act on the car. As the milk drains, it falls vertically, so no horizontal force exits, and the tank car travels at constant speed. As the mass of the tank car decreases, the momentum decreases proportionately, as the milk carries its momentum with it.

12. (*c*) The height to which the bowling ball rises depends upon the impulse exerted on it by the putty and by the rubber ball. The putty sticks to the bowling ball and therefore continues to move forward at the new speed of the bowling ball $(\Delta \upsilon < 5.0 \text{ m/s})$. The rubber bounces backward and therefore has a greater change in velocity $(\Delta \upsilon \approx 10.0 \text{ m/s})$. Since the putty and rubber have the same mass, the rubber exerts a greater impulse onto the bowling ball, causing the bowling ball to travel higher than when it is hit by the putty.

Solutions to Problems

1. Momentum is defined in Eq. 7–1. We use the magnitude.

$$p = mv = (0.028\text{ kg})(8.4\text{ m/s}) = \boxed{0.24\text{ kg}\cdot\text{m/s}}$$

2. From Eq. 7–2 for a single force, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t.$ For an object of constant mass, $\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}.$ Equate the two expressions for $\Delta\vec{\mathbf{p}}.$

$$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}} \quad\rightarrow\quad \Delta\vec{\mathbf{v}} = \frac{\vec{\mathbf{F}}\Delta t}{m}$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$\Delta v = -\frac{F\Delta t}{m} = -\frac{(25\text{ N})(15\text{ s})}{65\text{ kg}} = \boxed{-5.8\text{ m/s}}$$

The skier loses 7.7 m/s of speed.

$\boxed{3.}$ Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let A represent the car and B represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = (m_{\text{A}} + m_{\text{B}})v' \quad\rightarrow$$
$$v' = \frac{m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \frac{(7150\text{ kg})(15.0\text{ m/s}) + 0}{7150\text{ kg} + 3350\text{ kg}} = \boxed{10.2\text{ m/s}}$$

4. The tackle will be analyzed as a one-dimensional momentum-conserving situation. Let A represent the halfback and B represent the tackler. We take the direction of the halfback to be the positive direction, so $v_{\text{A}} > 0$ and $v_{\text{B}} < 0.$

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = (m_{\text{A}} + m_{\text{B}})v' \quad\rightarrow$$
$$v' = \frac{m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = \frac{(82\text{ kg})(5.0\text{ m/s}) + (110\text{ kg})(-2.5\text{ m/s})}{82\text{ kg} + 110\text{ kg}} = 0.703\text{ m/s} \approx \boxed{0.70\text{ m/s}}$$

They will be moving it the direction that the halfback was running before the tackle.

5. The force on the gas can be found from its change in momentum. The speed of 1300 kg of the gas changes from rest to $4.5\times10^{4}\text{ m/s},$ over the course of one second. Use Eq. 7–2.

$$F_{\text{gas}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \Delta v\frac{m}{\Delta t} = (4.5\times10^{4}\text{ m/s})(1300\text{ kg/s})$$
$$= 5.9\times10^{7}\text{ N, in the direction of the velocity of the gas}$$

The force on the rocket is the Newton's third law pair (equal and opposite) to the force on the gas, so the force on the rocket is $\boxed{5.9\times10^{7}\text{ N in the opposite direction of the velocity of the gas}}.$

6. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let A represent the first car and B represent the second car. Momentum will be conserved in the collision. Note that $\upsilon_B = 0$. Use Eq. 7–3.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A\upsilon_A + m_B\upsilon_B = (m_A + m_B)\upsilon' \rightarrow$$
$$m_B = \frac{m_A(\upsilon_A - \upsilon')}{\upsilon'} = \frac{(7700\text{ kg})(14\text{ m/s} - 5.0\text{ m/s})}{5.0\text{ m/s}} = 13{,}860\text{ kg} \approx \boxed{14{,}000\text{ kg}}$$

$\boxed{7.}$ The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0. Use Eq. 7–3 in one dimension.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)\upsilon = m_A\upsilon_A' + m_B\upsilon_B' = 0 \rightarrow$$
$$\upsilon_A' = -\frac{m_B\upsilon_B'}{m_A} = -\frac{(5.30\text{ kg})(10.0\text{ m/s})}{(24.0\text{ kg} + 35.0\text{ kg})} = \boxed{-0.898\text{ m/s}}$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.

8. Consider the motion in one dimension, with the positive direction being the direction of motion of the alpha particle. Let A represent the alpha particle, with a mass of m_A, and let B represent the daughter nucleus, with a mass of $57m_A$. The total momentum must be 0 since the nucleus decayed at rest. Use Eq. 7–3, in one dimension.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A\upsilon_A' + m_B\upsilon_B' \rightarrow$$
$$\upsilon_B' = -\frac{m_A\upsilon_A'}{m_B} = -\frac{m_A(2.8\times10^5\text{ m/s})}{57m_A} \rightarrow |\upsilon_B'| = \boxed{4900\text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

9. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let A represent the alpha particle, with a mass of 4 u, and let B represent the new nucleus, with a mass of 218 u. Use Eq. 7–3 for momentum conservation.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)\upsilon = m_A\upsilon_A' + m_B\upsilon_B' \rightarrow$$
$$\upsilon_A' = \frac{(m_A + m_B)\upsilon - m_B\upsilon_B'}{m_A} = \frac{(222\text{ u})(320\text{ m/s}) - (218\text{ u})(280\text{ m/s})}{4.0\text{ u}} = \boxed{2500\text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

10. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy. Use Eq. 7–3.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A\upsilon_A' + m_B\upsilon_B' \rightarrow \upsilon_B' = -\frac{m_A\upsilon_A'}{m_B}$$
$$\text{KE}_A = 2\text{KE}_B \rightarrow \tfrac{1}{2}m_A\upsilon_A'^2 = 2\left(\tfrac{1}{2}m_B\upsilon_B'^2\right) = m_B\left(-\frac{m_A\upsilon_A'}{m_B}\right)^2 \rightarrow \frac{m_A}{m_B} = \boxed{\frac{1}{2}}$$

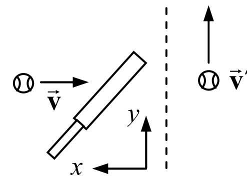
The fragment with the larger kinetic energy has half the mass of the other fragment.

11. Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let A represent the bullet and B represent the block. Since there is no net force outside of the block–bullet system (like friction with the table), the momentum of the block and bullet combination is conserved. Use Eq. 7–3, and note that $\upsilon_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon_A' + m_B\upsilon_B' \quad \rightarrow$$

$$\upsilon_B' = \frac{m_A\upsilon_A - m_A\upsilon_A'}{m_B} = \frac{(0.022\text{ kg})(240\text{ m/s}) - (0.022\text{ kg})(150\text{ m/s})}{2.0\text{ kg}} = \boxed{0.99\text{ m/s}}$$

12. To find the average force, we use Eq. 7–2 and divide the change in momentum by the time over which the momentum changes. Choose the x direction to be the opposite of the baseball's incoming direction, so to the left in the diagram. The velocity with which the ball is moving after hitting the bat can be found from conservation of energy and from knowing the height to which the ball rises.



$$(\text{KE}_{\text{initial}} = \text{PE}_{\text{final}})_{\substack{\text{after}\\ \text{collision}}} \quad \rightarrow \quad \tfrac{1}{2}m\upsilon'^2 = mg\Delta y \quad \rightarrow$$

$$\upsilon' = \sqrt{2g\Delta y} = \sqrt{2(9.80\text{ m/s}^2)(31.5\text{ m})} = 24.85\text{ m/s}$$

The average force can be calculated from the change in momentum and the time of contact.

$$\overline{F}_x = \frac{\Delta p_x}{\Delta t} = \frac{m(\upsilon_x' - \upsilon_x)}{\Delta t} = \frac{(0.145\text{ kg})(0 - -27.0\text{ m/s})}{2.5\times10^{-3}\text{ s}} = 1566\text{ N}$$

$$\overline{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{m(\upsilon_y' - \upsilon_y)}{\Delta t} = \frac{(0.145\text{ kg})(24.85\text{ m/s} - 0)}{2.5\times10^{-3}\text{ s}} = 1441\text{ N}$$

$$\overline{F} = \sqrt{\overline{F}_x^2 + \overline{F}_y^2} = \sqrt{(1566\text{ N})^2 + (1441\text{ N})^2} = 2128\text{ N} \approx \boxed{2100\text{ N}}$$

$$\theta = \tan^{-1}\frac{\overline{F}_y}{\overline{F}_x} = \tan^{-1}\frac{1441}{1566} = 42.6^\circ \approx \boxed{43^\circ}$$

13. The air is moving with an initial speed of $120\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s}$. Thus, in one second, a volume of air measuring $45\text{ m}\times75\text{ m}\times33.33\text{ m}$ will have been brought to rest. By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air. The mass of the stopped air is its volume times its density.

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}\upsilon = \frac{V\rho\Delta\upsilon}{\Delta t} = \frac{(45\text{ m})(75\text{ m})(33.33\text{ m})(1.3\text{ kg/m}^3)(33.33\text{ m/s} - 0)}{1\text{ s}}$$

$$= \boxed{4.9\times10^6\text{ N}}$$

14. (*a*) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let A represent the upper stage (that moves away faster) and B represent the lower stage. It is given that $m_A = m_B$, $\upsilon_A = \upsilon_B = \upsilon$, and $\upsilon_B' = \upsilon_A' - \upsilon_{\text{rel}}$. Use Eq. 7–3 for momentum conservation.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_{\text{A}} + m_{\text{B}})\upsilon = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}(\upsilon'_{\text{A}} - \upsilon_{\text{rel}}) \quad \rightarrow$$

$$\upsilon'_{\text{A}} = \frac{(m_{\text{A}} + m_{\text{B}})\upsilon + m_{\text{B}}\upsilon_{\text{rel}}}{m_{\text{A}} + m_{\text{B}}} = \frac{(725\text{ kg})(6.60\times10^3\text{ m/s}) + \frac{1}{2}(725\text{ kg})(2.80\times10^3\text{ m/s})}{725\text{ kg}}$$

$$= \boxed{8.00\times10^3\text{ m/s, away from Earth}}$$

$$\upsilon'_{\text{B}} = \upsilon'_{\text{A}} - \upsilon_{\text{rel}} = 8.007\times10^3\text{ m/s} - 2.80\times10^3\text{ m/s} = \boxed{5.20\times10^3\text{ m/s, away from Earth}}$$

(*b*) The change in kinetic energy was supplied by the explosion.

$$\Delta\text{KE} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} = \left(\tfrac{1}{2}m_{\text{A}}\upsilon'^2_{\text{A}} + \tfrac{1}{2}m_{\text{B}}\upsilon'^2_{\text{B}}\right) - \tfrac{1}{2}(m_{\text{A}} + m_{\text{B}})\upsilon^2$$

$$= \tfrac{1}{2}[\tfrac{1}{2}(725\text{ kg})][(8.00\times10^3\text{ m/s})^2 + (5.20\times10^3\text{ m/s})^2] - \tfrac{1}{2}(725\text{ kg})(6.60\times10^3\text{ m/s})^2$$

$$= \boxed{7.11\times10^8\text{ J}}$$

15. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$\Delta p = F\Delta t = m\Delta\upsilon \quad \rightarrow$$

$$F = m\frac{\Delta\upsilon}{\Delta t} = (0.145\text{ kg})\left(\frac{46.0\text{ m/s} - -31.0\text{ m/s}}{5.00\times10^{-3}\text{ s}}\right) = \boxed{2230\text{ N, toward the pitcher}}$$

16. (*a*) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta\upsilon = (4.5\times10^{-2}\text{ kg})(38\text{ m/s} - 0) = 1.71\text{ kg}\cdot\text{m/s} \approx \boxed{1.7\text{ kg}\cdot\text{m/s}}$$

(*b*) The average force is the impulse divided by the interaction time.

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{1.71\text{ kg}\cdot\text{m/s}}{3.5\times10^{-3}\text{ s}} = \boxed{490\text{ N}}$$

17. (*a*) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} = [m\upsilon_{\text{initial}} - m\upsilon_{\text{final}}]_{\text{hammer}} = (12\text{ kg})(7.5\text{ m/s}) - 0 = \boxed{9.0\times10^1\text{ kg}\cdot\text{m/s}}$$

(*b*) The average force is the impulse divided by the time of contact.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{9.0\times10^1\text{ kg}\cdot\text{m/s}}{8.0\times10^{-3}\text{ s}} = \boxed{1.1\times10^4\text{ N}}$$

18. The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta_{p_\perp} = m\upsilon_{\perp_{\text{final}}} - m\upsilon_{\perp_{\text{initial}}} = m(\upsilon\sin 45° - \upsilon\sin 45°) = 2m\upsilon\sin 45°$$

$$= 2(6.0\times10^{-2}\text{ km})(28\text{ m/s})\sin 45° = \boxed{2.4\text{ kg}\cdot\text{m/s, to the left}}$$

19. (*a*) The momentum of the astronaut–space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $\upsilon_A = \upsilon_B = 0$. We also have $\upsilon'_A = 2.50$ m/s.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A\upsilon_A + m_B\upsilon_B = 0 = m_A\upsilon'_A + m_B\upsilon'_B \quad\rightarrow$$

$$\upsilon'_B = -\upsilon'_A\frac{m_A}{m_B} = -(2.50\text{ m/s})\frac{125\text{ kg}}{1900\text{ kg}} = -0.1645\text{ m/s} \approx \boxed{-0.16\text{ m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m(\upsilon'_A - \upsilon_A)}{\Delta t} = \frac{(125\text{ kg})(2.50\text{ m/s} - 0)}{0.600\text{ s}} = \boxed{521\text{ N}}$$

(*c*) $\text{KE}_{\text{astronaut}} = \frac{1}{2}(125\text{ kg})(2.50\text{ m/s})^2 = \boxed{391\text{ J}}$, $\text{KE}_{\text{capsule}} = \frac{1}{2}(1900\text{ kg})(-0.1645\text{ m/s})^2 = \boxed{26\text{ J}}$

20. If the rain does not rebound, then the final speed of the rain is 0. By Newton's third law, the force on the pan due to the rain is equal in magnitude to the force on the rain due to the pan. The force on the rain can be found from the change in momentum of the rain. The mass striking the pan is calculated as volume times density.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{(m\upsilon_f - m\upsilon_0)}{\Delta t} = -\frac{m}{\Delta t}(\upsilon_f - \upsilon_0) = \frac{\rho V}{\Delta t}\upsilon_0 = \frac{\rho A h}{\Delta t}\upsilon_0 = \frac{h}{\Delta t}\rho A\upsilon_0$$

$$= \frac{(2.5\times 10^{-2}\text{ m})}{1\text{ h}\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)}(1.00\times 10^3\text{ kg/m}^3)(1.0\text{ m}^2)(8.0\text{ m/s}) = \boxed{0.056\text{ N}}$$

$\boxed{21.}$ Call east the positive direction.

(*a*) $p_{\substack{\text{original}\\\text{fullback}}} = m\upsilon_{\substack{\text{original}\\\text{fullback}}} = (95\text{ kg})(3.0\text{ m/s}) = 285\text{ kg}\cdot\text{m/s} \approx \boxed{290\text{ kg}\cdot\text{m/s, to the east}}$

(*b*) The impulse on the fullback is the change in the fullback's momentum.

$$\Delta p_{\text{fullback}} = m(\upsilon_{\substack{\text{final}\\\text{fullback}}} - \upsilon_{\substack{\text{final}\\\text{fullback}}}) = (95\text{ kg})(0 - 3.0\text{ m/s}) = -285\text{ kg}\cdot\text{m/s} \approx \boxed{-290\text{ kg}\cdot\text{m/s}}$$

The negative sign indicates the impulse is to the west.

(*c*) The impulse on the tackler is the opposite of the impulse on the fullback.

$\boxed{290\text{ kg}\cdot\text{m/s, to the east}}$

(*d*) The average force on the tackler is the impulse on the tackler divided by the time of interaction.

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{285\text{ kg}\cdot\text{m/s}}{0.85\text{ s}} = \boxed{340\text{ N, to the east}}$$

22. Impulse is the change of momentum, Eq. 7–5. This is a one-dimensional configuration.

$$\Delta_p = m(\upsilon_{\text{final}} - \upsilon_0) = (0.50\text{ kg})(3.0\text{ m/s}) = \boxed{1.5\text{ kg}\cdot\text{m/s}}$$

23. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.04 s.

$$\Delta p = \tfrac{1}{2}(250\text{ N})(0.04\text{ s}) \approx \boxed{5\text{ N}\cdot\text{s}}$$

We could also count "boxes" under the graph, where each "box" has an "area" of $(50\text{ N})(0.01\text{ s}) = 0.5\text{ N}\cdot\text{s}$. There are almost seven whole boxes and the equivalent of about three whole boxes in the partial boxes. Ten boxes would be about $\boxed{5\text{ N}\cdot\text{s}}$.

(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta v = m(v_\text{f} - v_\text{i}) \quad \rightarrow \quad v_\text{f} = v_\text{i} + \frac{\Delta p}{m} = 0 + \frac{5\text{ N}\cdot\text{s}}{6.0\times10^{-2}\text{ kg}} \approx \boxed{80\text{ m/s}}$$

24. (*a*) The impulse is the change in momentum. Take upward to be the positive direction. The velocity just before reaching the ground is found from conservation of mechanical energy.

$$E_\text{initial} = E_\text{final} \quad \rightarrow \quad mgh = \tfrac{1}{2}mv_y^2 \quad \rightarrow$$

$$v_y = \pm\sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(2.8\text{ m})} = 7.408\text{ m/s, down}$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = m(\vec{\mathbf{v}}_\text{f} - \vec{\mathbf{v}}_0) = (55\text{ kg})(0 - -7.408\text{ m/s}) = 407\text{ kg}\cdot\text{m/s} \approx \boxed{410\text{ kg}\cdot\text{m/s, upward}}$$

(*b*) The net force on the person is the sum of the upward force from the ground, plus the downward force of gravity.

$$F_\text{net} = F_\text{ground} - mg = ma \quad \rightarrow$$

$$F_\text{ground} = m(g+a) = m\left(g + \frac{(v_\text{f}^2 - v_0^2)}{2\Delta x}\right) = (55\text{ kg})\left((9.80\text{ m/s}^2) + \frac{0-(-7.408\text{ m/s})^2}{2(-0.010\text{ m})}\right)$$

$$= \boxed{1.5\times10^5\text{ N, upward}}$$

This is about 280 times the jumper's weight.

(*c*) We do this the same as part (*b*), but for the longer distance.

$$F_\text{ground} = m\left(g + \frac{(v_\text{f}^2 - v_0^2)}{2\Delta x}\right) = (55\text{ kg})\left((9.80\text{ m/s}^2) + \frac{0-(-7.408\text{ m/s})^2}{2(-0.5\text{ m})}\right)$$

$$= 3557\text{ N} \approx \boxed{4000\text{ N, upward}}$$

This is about 6.5 times the jumper's weight.

25. Let A represent the 0.440-kg ball and B represent the 0.220-kg ball. We have $v_\text{A} = 3.80\text{ m/s}$ and $v_\text{B} = 0.$ Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_\text{A} - v_\text{B} = -(v'_\text{A} - v'_\text{B}) \quad \rightarrow \quad v'_\text{B} = v_\text{A} + v'_\text{A}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_\text{A}v_\text{A} + m_\text{B}v_\text{B} = m_\text{A}v'_\text{A} + m_\text{B}v'_\text{B} \quad \rightarrow \quad m_\text{A}v_\text{A} = m_\text{A}v'_\text{A} + m_\text{B}(v_\text{A} + v'_\text{A}) \quad \rightarrow$$

$$v'_\text{A} = \frac{(m_\text{A} - m_\text{B})}{(m_\text{A} + m_\text{B})}v_\text{A} = \frac{0.220\text{ kg}}{0.660\text{ kg}}(3.80\text{ m/s}) = 1.267\text{ m/s} \approx \boxed{1.27\text{ m/s (east)}}$$

$$v'_\text{B} = v_\text{A} + v'_\text{A} = 3.80\text{ m/s} + 1.27\text{ m/s} = \boxed{5.07\text{ m/s (east)}}$$

26. Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $v_A = 5.80$ m/s and $v_B = 0$. Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad m_A v_A = m_A v'_A (m_A v'_A) \quad \rightarrow$$

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)} v_A = \frac{-0.450\text{ kg}}{1.350\text{ kg}}(5.80\text{ m/s}) = -1.933\text{ m/s} \approx \boxed{1.93\text{ m/s (west)}}$$

$$v'_B = v_A + v'_A = 5.80\text{ m/s} - 1.93\text{ m/s} = \boxed{3.87\text{ m/s (east)}}$$

27. Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $v_A = 5.50$ m/s and $v_B = 3.00$ m/s. Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = 2.50\text{ m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v'_A + m_B(2.50\text{ m/s} + v'_A) \quad \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B(v_B - 2.50\text{ m/s})}{m_A + m_B} = \frac{(0.060\text{ kg})(5.50\text{ m/s}) + (0.090\text{ kg})(3.00\text{ m/s} - 2.50\text{ m/s})}{0.150\text{ kg}}$$

$$= \boxed{2.50\text{ m/s}}$$

$$v'_B = 2.50\text{ m/s} + v'_A = \boxed{5.00\text{ m/s}}$$

Both balls move in the direction of the tennis ball's initial motion.

28. Let A represent the ball moving at 2.00 m/s, and call that direction the positive direction. Let B represent the ball moving at 3.60 m/s in the opposite direction. Thus, $v_A = 2.00$ m/s and $v_B = -3.60$ m/s. Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = 5.60\text{ m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision, noting that $m_A = m_B$.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad v_A + v_B = v'_A + v'_B \quad \rightarrow$$

$$-1.60\text{ m/s} = v'_A + (v'_A + 5.60\text{ m/s}) \quad \rightarrow \quad 2v'_A = -7.20\text{ m/s} \quad \rightarrow \quad v'_A = \boxed{-3.60\text{ m/s}}$$

$$v'_B = 5.60\text{ m/s} + v'_A = \boxed{2.00\text{ m/s}}$$

The two balls have exchanged velocities. This will always be true for 1-D elastic collisions of objects of equal mass.

29. (*a*) Momentum will be conserved in one dimension. Call the direction of the first ball the positive direction. Let A represent the first ball and B represent the second ball. We have $v_B = 0$ and $v'_B = \frac{1}{2}v_A$. Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_A = -\tfrac{1}{2}v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$m_A v_A = -\tfrac{1}{2} m_A v_A + m_B \tfrac{1}{2} v_A \quad \rightarrow \quad m_B = 3m_A = 3(0.280 \text{ kg}) = \boxed{0.840 \text{ kg}}$$

(*b*) The fraction of the kinetic energy given to the second ball is as follows:

$$\frac{\text{KE}'_B}{\text{KE}_A} = \frac{\frac{1}{2} m_B v_B'^2}{\frac{1}{2} m_A v_A^2} = \frac{3m_A \left(\frac{1}{2} v_A\right)^2}{m_A v_A^2} = \boxed{0.75}$$

30. Let A represent the incoming ball and B represent the target ball. We have $v_B = 0$ and $v'_A = -0.450 v_A$. Use Eq. 7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A + v'_A = 0.550 v_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A = m_A v'_A + m_B v'_B$$

$$= m_A(-0.450 v_A) + m_B(0.550 v_A) \quad \rightarrow \quad \boxed{m_B = 2.64 m_A}$$

31. Let A represent the moving ball, and let B represent the ball initially at rest. The initial direction of the ball is the positive direction. We have $v_A = 5.5$ m/s, $v_B = 0$, and $v'_A = -3.8$ m/s.

(*a*) Use Eq.7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A - v_B + v'_A = 5.5 \text{ m/s} - 0 - 3.8 \text{ m/s} = \boxed{1.7 \text{ m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$m_B = m_A \frac{(v_A - v'_A)}{(v'_B - v_B)} = (0.220 \text{ kg}) \frac{(5.5 \text{ m/s} - -3.8 \text{ m/s})}{1.7 \text{ m/s}} = \boxed{1.2 \text{ kg}}$$

32. The one-dimensional stationary target elastic collision is analyzed in Search and Learn 5.
The algebraic details can be found there, and also in Example 7–8. The kinetic energy lost by the neutron is equal to the kinetic energy gained by the target particle. The fraction of kinetic energy lost is found as follows:

$$\frac{\text{KE}_{A_{\text{initial}}} - \text{KE}_{A_{\text{final}}}}{\text{KE}_{A_{\text{initial}}}} = \frac{\text{KE}_{B_{\text{final}}}}{\text{KE}_{A_{\text{initial}}}} = \frac{\frac{1}{2} m_B v_B'^2}{\frac{1}{2} m_A v_A^2} = \frac{m_B \left[v_A \left(\frac{2m_A}{m_A + m_B} \right)^2 \right]}{m_A v_A^2} = \frac{4 m_A m_B}{(m_A + m_B)^2}$$

(*a*) $$\frac{4 m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(1.01)}{(1.01 + 1.01)^2} = \boxed{1.00}$$

All of the initial kinetic energy is lost by the neutron, as expected for the target mass equal to the incoming mass.

(*b*) $$\frac{4 m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(2.01)}{(1.01 + 2.01)^2} = \boxed{0.890}$$

(*c*) $$\frac{4m_{\text{A}}m_{\text{B}}}{(m_{\text{A}}+m_{\text{B}})^2}=\frac{4(1.01)(12.00)}{(1.01+12.00)^2}=\boxed{0.286}$$

(*d*) $$\frac{4m_{\text{A}}m_{\text{B}}}{(m_{\text{A}}+m_{\text{B}})^2}=\frac{4(1.01)(208)}{(1.01+208)^2}=\boxed{0.0192}$$

Since the target is quite heavy, almost no kinetic energy is lost. The incoming particle "bounces off" the heavy target, much as a rubber ball bounces off a wall with approximately no loss in speed.

33. From the analysis in Example 7–9, the initial projectile speed is given by $\upsilon=\frac{m+M}{m}\sqrt{2gh}$.

Compare the two speeds with the same masses.

$$\frac{\upsilon_2}{\upsilon_1}=\frac{\frac{m+M}{m}\sqrt{2gh_2}}{\frac{m+M}{m}\sqrt{2gh_1}}=\frac{\sqrt{h_2}}{\sqrt{h_1}}=\sqrt{\frac{h_2}{h_1}}=\sqrt{\frac{5.2}{2.6}}=\sqrt{2}\quad\rightarrow\quad\boxed{\upsilon_2=\sqrt{2}\upsilon_1}$$

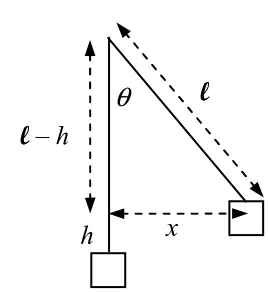
34. (*a*) In Example 7–9, $\text{KE}_{\text{i}}=\frac{1}{2}m\upsilon^2$ and $\text{KE}_{\text{f}}=\frac{1}{2}(m+M)\upsilon'^2$. The speeds are related by $\upsilon'=\frac{m}{m+M}\upsilon.$

$$\frac{\Delta\text{KE}}{\text{KE}_{\text{i}}}=\frac{\text{KE}_{\text{f}}-\text{KE}_{\text{i}}}{\text{KE}_{\text{i}}}=\frac{\frac{1}{2}(m+M)\upsilon'^2-\frac{1}{2}m\upsilon^2}{\frac{1}{2}m\upsilon^2}=\frac{(m+M)\left(\frac{m}{m+M}\upsilon\right)^2-m\upsilon^2}{m\upsilon^2}$$
$$=\frac{\frac{m^2\upsilon^2}{m+M}-m\upsilon^2}{m\upsilon^2}=\frac{m}{m+M}-1=\boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M}=\frac{-380\text{ g}}{398\text{ g}}=-0.95.$ Thus 95% of the energy is lost.

$\boxed{35.}$ From the analysis in the Example 7–9, we know the following:

$$h=\frac{1}{2g}\left(\frac{m\upsilon}{m+M}\right)^2=\frac{1}{2(9.80\text{ m/s}^2)}\left(\frac{(0.028\text{ kg})(190\text{ m/s})}{0.028\text{ kg}+3.1\text{ kg}}\right)^2$$
$$=0.1476\text{ m}\approx\boxed{0.15\text{ m}}$$



From the diagram we see the following:

$$\ell^2=(\ell-h)^2+x^2$$
$$x=\sqrt{\ell^2-(\ell-h)^2}=\sqrt{(2.8\text{ m})^2-(2.8\text{ m}-0.1476\text{ m})^2}=\boxed{0.90\text{ m}}$$

36. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle and B represent the lighter particle. We have $m_A = 1.5m_B$, and $v_A = v_B = 0$.

$$p_{initial} = p_{final} \quad \rightarrow \quad 0 = m_A v_A' + m_B v_B' \quad \rightarrow \quad v_A' = -\frac{m_B v_B'}{m_A} = -\tfrac{2}{3}v_B'$$

The negative sign indicates direction. Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy released.

$$E_{released} = \mathrm{KE}_A' + \mathrm{KE}_B' = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 = \tfrac{1}{2}(1.5m_B)(\tfrac{2}{3}v_B')^2 + \tfrac{1}{2}m_B v_B'^2 = \tfrac{5}{3}(\tfrac{1}{2}m_B v_B'^2) = \tfrac{5}{3}\mathrm{KE}_B'$$

$$\mathrm{KE}_B' = \tfrac{3}{5}E_{released} = \tfrac{3}{5}(5500\text{ J}) = 3300\text{ J} \quad \mathrm{KE}_A' = E_{released} - \mathrm{KE}_B' = 5500\text{ J} - 3300\text{ J} = 2200\text{ J}$$

Thus $\boxed{\mathrm{KE}_A' = 2200\text{ J};\quad \mathrm{KE}_B = 3300\text{ J}}$.

37. Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive *x* direction. Let A represent the sports car and B represent the SUV. We have $v_B = 0$ and $v_A' = v_B'$. Solve for v_A.

$$p_{initial} = p_{final} \quad \rightarrow \quad m_A v_A + 0 = (m_A + m_B)v_A' \quad \rightarrow \quad v_A = \frac{m_A + m_B}{m_A}v_A'$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for v_A', and use that to find v_A.

$$W_{fr} = (\mathrm{KE}_{final} - \mathrm{KE}_{initial})_{\substack{after\\collision}} = 0 - \tfrac{1}{2}(m_A + m_B)v_A'^2$$

$$W_{fr} = F_{fr}\Delta x \cos 180° = -\mu_k(m_A + m_B)g\Delta x$$

$$-\tfrac{1}{2}(m_A + m_B)v_A'^2 = -\mu_k(m_A + m_B)g\Delta x \quad \rightarrow \quad v_A' = \sqrt{2\mu_k g\Delta x}$$

$$v_A = \frac{m_A + m_B}{m_A}v_A' = \frac{m_A + m_B}{m_A}\sqrt{2\mu_k g\Delta x} = \frac{980\text{ kg} + 2300\text{ kg}}{980\text{ kg}}\sqrt{2(0.80)(9.80\text{ m/s}^2)(2.6\text{ m})}$$

$$= 21.37\text{ m/s} \approx \boxed{21\text{ m/s}}$$

38. The impulse on the ball is its change in momentum. Call upward the positive direction, so that the final velocity is positive and the initial velocity is negative. The speeds immediately before and immediately after the collision can be found from conservation of energy. Take the floor to be the zero level for gravitational potential energy.

Falling: $\mathrm{KE}_{bottom} = \mathrm{PE}_{top} \quad \rightarrow \quad \tfrac{1}{2}mv_{down}^2 = mgh_{down} \quad \rightarrow \quad v_{down} = -\sqrt{2gh_{down}}$

Rising: $\mathrm{KE}_{bottom} = \mathrm{PE}_{top} \quad \rightarrow \quad \tfrac{1}{2}mv_{up}^2 = mgh_{up} \quad \rightarrow \quad v_{up} = \sqrt{2gh_{up}}$

$$\text{Impulse} = \Delta p = m\Delta v = m(v_{up} - v_{down}) = m\left(\sqrt{2gh_{up}} - -\sqrt{2gh_{down}}\right) = m\sqrt{2g}\left(\sqrt{h_{up}} + \sqrt{h_{down}}\right)$$

$$= (0.014\text{ kg})\sqrt{2(9.80\text{ m/s}^2)}\left(\sqrt{0.85\text{ m}} + \sqrt{1.5\text{ m}}\right) = 0.13\text{ kg}\cdot\text{m/s}$$

The direction of the impulse is upwards, so the complete specification of the impulse is $\boxed{0.13\text{ kg}\cdot\text{m/s, upward}}$.

39. Fraction KE lost $= \dfrac{\text{KE}_{\text{initial}} - \text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}} = \dfrac{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2 - \frac{1}{2}m_{\text{B}}v_{\text{B}}'^2}{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2} = \dfrac{v_{\text{A}}^2 - v_{\text{B}}'^2}{v_{\text{A}}^2} = \dfrac{(38\text{ m/s})^2 - (15\text{ m/s})^2}{(38\text{ m/s})^2} = \boxed{0.84}$

40. Let A represent the more massive piece and B the less massive piece. Thus $m_{\text{A}} = 3m_{\text{B}}$. In the explosion, momentum is conserved. We have $v_{\text{A}} = v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad 0 = m_{\text{A}}v_{\text{A}}' + m_{\text{B}}v_{\text{B}}' = 3m_{\text{B}}v_{\text{A}}' + m_{\text{B}}v_{\text{B}}' \quad \rightarrow \quad v_{\text{A}}' = -\tfrac{1}{3}v_{\text{B}}'$$

For each block, the kinetic energy gained during the explosion is lost to negative work done by friction on the block.

$$W_{\text{fr}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} = -\tfrac{1}{2}mv^2$$

But work is also calculated in terms of the force doing the work and the distance traveled.

$$W_{\text{fr}} = F_{\text{fr}}\Delta x \cos 180° = -\mu_{\text{k}}F_{\text{N}}\Delta x = -\mu_{\text{k}}mg\Delta x$$

Equate the two work expressions, solve for the distance traveled, and find the ratio of distances.

$$-\tfrac{1}{2}mv^2 = -\mu_{\text{k}}mg\Delta x \quad \rightarrow \quad \Delta x = \frac{v^2}{g\mu_{\text{k}}} \qquad \frac{(\Delta x)_{\text{A}}}{(\Delta x)_{\text{B}}} = \frac{v_{\text{A}}'^2/g\mu_{\text{k}}}{v_{\text{B}}'^2/g\mu_{\text{k}}} = \frac{v_{\text{A}}'^2}{v_{\text{B}}'^2} = \frac{(-\frac{1}{3}v_{\text{B}}')^2}{v_{\text{B}}'^2} = \frac{1}{9}$$

So $\boxed{(\Delta x)_{\text{heavy}}/(\Delta x)_{\text{light}} = 1/9}$.

41. (*a*) Momentum is conserved in the one-dimensional collision. Let A represent the baseball and let B represent the brick.

$$m_{\text{A}}v_{\text{A}} = m_{\text{A}}v_{\text{A}}' + m_{\text{B}}v_{\text{B}}' \quad \rightarrow$$
$$v_{\text{A}}' = \frac{m_{\text{A}}v_{\text{A}} - m_{\text{B}}v_{\text{B}}'}{m_{\text{A}}} = \frac{(0.144\text{ kg})(28.0\text{ m/s}) - (5.25\text{ kg})(1.10\text{ m/s})}{0.144\text{ kg}} = -12.10\text{ m/s}$$

So the baseball's speed in the reverse direction is $\boxed{12.1\text{ m/s}}$.

(*b*) $\text{KE}_{\text{before}} = \frac{1}{2}m_{\text{A}}v_{\text{A}}^2 = \frac{1}{2}(0.144\text{ kg})(28.0\text{ m/s})^2 = \boxed{56.4\text{ J}}$

$\text{KE}_{\text{after}} = \frac{1}{2}m_{\text{A}}v_{\text{A}}'^2 + \frac{1}{2}m_{\text{A}}v_{\text{B}}'^2 = \frac{1}{2}(0.144\text{ kg})(1.21\text{ m/s})^2 + \frac{1}{2}(5.25\text{ kg})(1.10\text{ m/s})^2 = \boxed{13.7\text{ J}}$

42. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$\text{KE}_{\text{bottom}} = \text{PE}_{\text{top}} \quad \rightarrow \quad \tfrac{1}{2}(m+M)v_{\text{bottom}}^2 = (m+M)g(2L) \quad \rightarrow \quad v_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad mv = (m+M)v_{\text{bottom}} \quad \rightarrow \quad v = \frac{m+M}{m}v_{\text{bottom}} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

43. (*a*) The collision is assumed to happen fast enough that the bullet–block system does not move during the collision. So the totally inelastic collision is described by momentum conservation. The conservation of energy (including the nonconservative work done by friction) can be used to relate the initial kinetic energy of the bullet–block system to the spring compression and the dissipated energy. Let *m* represent the mass of the bullet, *M* represent the mass of the block, and *x* represent the distance the combination moves after the collision

$$\text{Collision:}\quad mv = (m+M)v' \quad\rightarrow\quad v = \frac{m+M}{m}v'$$

$$\text{After collision:}\quad \tfrac{1}{2}(m+M)v'^2 = \tfrac{1}{2}kx^2 + \mu(m+M)gx \quad\rightarrow\quad v' = \sqrt{\frac{kx^2}{m+M} + 2\mu gx}$$

$$v = \frac{m+M}{m}\sqrt{\frac{kx^2}{m+M} + 2\mu gx}$$

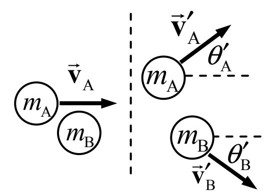
$$= \frac{1.000\text{ kg}}{1.0\times10^{-3}\text{ kg}}\sqrt{\frac{(140\text{ N/m})(0.050\text{ m})^2}{1.000\text{ kg}} + 2(0.50)(9.80\text{ m/s}^2)(0.050\text{ m})} \approx \boxed{920\text{ m/s}}$$

(*b*) The fraction of kinetic energy dissipated in the collision is $\frac{\text{KE}_{\text{initial}} - \text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}}$, where the kinetic energies are calculated immediately before and after the collision.

$$\frac{\text{KE}_{\text{initial}} - \text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}} = \frac{\tfrac{1}{2}mv^2 - \tfrac{1}{2}(m+M)v'^2}{\tfrac{1}{2}mv^2} = 1 - \frac{(m+M)v'^2}{mv^2} = 1 - \frac{(m+M)v'^2}{m\left(\frac{m+M}{m}v'\right)^2}$$

$$= 1 - \frac{m}{m+M} = 1 - \frac{0.0010\text{ kg}}{1.00\text{ kg}} = \boxed{0.999}$$

44. (*a*) See the diagram.



$$p_x:\quad m_A v_A = m_A v'_A \cos\theta'_A + m_B v'_B \cos\theta'_B$$
$$p_y:\quad 0 = m_A v'_A \sin\theta'_A - m_B v'_B \sin\theta'_B$$

(*b*) Solve the *x* equation for $\cos\theta'_B$ and the *y* equation for $\sin\theta'_B$, and then find the angle from the tangent function.

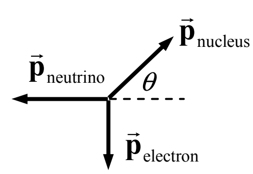
$$\tan\theta'_B = \frac{\sin\theta'_B}{\cos\theta'_B} = \frac{\dfrac{m_A v'_A \sin\theta'_A}{m_B v'_B}}{\dfrac{m_A(v_A - v'_A\cos\theta'_A)}{m_B v'_B}} = \frac{v'_A \sin\theta'_A}{(v_A - v'_A\cos\theta'_A)}$$

$$\theta'_B = \tan^{-1}\frac{v'_A\sin\theta'_A}{v_A - v'_A\cos\theta'_A} = \tan^{-1}\frac{(2.10\text{ m/s})\sin 30.0^\circ}{2.80\text{ m/s} - (2.10\text{ m/s})\cos 30.0^\circ} = \boxed{46.9^\circ}$$

With the value of the angle, solve the *y* equation for the velocity.

$$v'_B = \frac{m_A v'_A \sin\theta'_A}{m_B \sin\theta'_B} = \frac{(0.120\text{ kg})(2.10\text{ m/s})\sin 30.0^\circ}{(0.140\text{ kg})\sin 46.9^\circ} = \boxed{1.23\text{ m/s}}$$

45. Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the *x* and *y* directions.



$$(p_{\text{nucleus}})_x = p_{\text{neutrino}} \qquad (p_{\text{nucleus}})_y = p_{\text{electron}}$$

$$p_{\text{nucleus}} = \sqrt{(p_{\text{nucleus}})_x^2 + (p_{\text{nucleus}})_y^2} = \sqrt{(p_{\text{neutrino}})^2 + (p_{\text{electron}})^2}$$

$$= \sqrt{(6.2\times10^{-23}\ \text{kg}\cdot\text{m/s})^2 + (9.6\times10^{-23}\ \text{kg}\cdot\text{m/s})^2} = \boxed{1.14\times10^{-22}\ \text{kg}\cdot\text{m/s}}$$

$$\theta = \tan^{-1}\frac{(p_{\text{nucleus}})_y}{(p_{\text{nucleus}})_x} = \tan^{-1}\frac{(p_{\text{electron}})}{(p_{\text{neutrino}})} = \tan^{-1}\frac{(9.6\times10^{-23}\ \text{kg}\cdot\text{m/s})}{(6.2\times10^{-23}\ \text{kg}\cdot\text{m/s})} = 57°$$

The momentum of the second nucleus is directed $\boxed{\text{147° from the electron's momentum}}$ and is directed $\boxed{\text{123° from the neutrino's momentum}}$.

46. Write momentum conservation in the x and y directions and KE conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}_{\text{A}}$ to have both x and y components.

$$p_x:\ \ mv_{\text{B}} = mv'_{\text{A}x} \ \rightarrow\ v_{\text{B}} = v'_{\text{A}x}$$
$$p_y:\ \ mv_{\text{A}} = mv'_{\text{A}y} + mv'_{\text{B}} \ \rightarrow\ v_{\text{A}} = v'_{\text{A}y} + v'_{\text{B}}$$
$$\text{KE:}\ \ \tfrac{1}{2}mv_{\text{A}}^2 + \tfrac{1}{2}mv_{\text{B}}^2 = \tfrac{1}{2}mv'^2_{\text{A}} + \tfrac{1}{2}mv'^2_{\text{B}} \ \rightarrow\ v_{\text{A}}^2 + v_{\text{B}}^2 = v'^2_{\text{A}} + v'^2_{\text{B}}$$

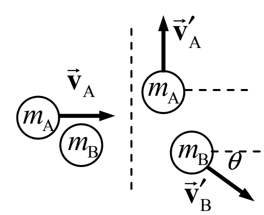
Substitute the results from the momentum equations into the KE equation.

$$(v'_{\text{A}y} + v'_{\text{B}})^2 + (v'_{\text{A}x})^2 = v'^2_{\text{A}} + v'^2_{\text{B}} \ \rightarrow\ v'^2_{\text{A}y} + 2v'^2_{\text{A}y}v'_{\text{B}} + v'^2_{\text{B}} + v'^2_{\text{A}y} = v'^2_{\text{A}} + v'^2_{\text{B}} \ \rightarrow$$
$$v'^2_{\text{A}} + 2v'^2_{\text{A}y}v'_{\text{B}} + v'^2_{\text{B}} = v'^2_{\text{A}} + v'^2_{\text{B}} \ \rightarrow\ 2v'^2_{\text{A}y}v'_{\text{B}} = 0 \ \rightarrow\ v'_{\text{A}y} = 0 \text{ or } v'_{\text{B}} = 0$$

Since we are given that $v'_{\text{B}} \neq 0,$ we must have $v'_{\text{A}y} = 0.$ This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$v'_{\text{A}} = v'_{\text{A}x} = v_{\text{B}} = \boxed{3.7\ \text{m/s}} \qquad v'_{\text{B}} = v_{\text{A}} = \boxed{2.0\ \text{m/s}}$$

47. (*a*) Let A represent the incoming nucleus and B represent the target particle. Take the x direction to be in the direction of the initial velocity of mass A (to the right in the diagram) and the y direction to be up in the diagram. Momentum is conserved in two dimensions and gives the following relationships:



$$p_x:\ \ m_{\text{A}}v_{\text{A}} = m_{\text{B}}v'_{\text{B}}\cos\theta \ \rightarrow\ v = 2v'_{\text{B}}\cos\theta$$
$$p_y:\ \ 0 = m_{\text{A}}v'_{\text{A}} - m_{\text{B}}v'_{\text{B}}\sin\theta \ \rightarrow\ v'_{\text{A}} = 2v'_{\text{B}}\sin\theta$$

The collision is elastic, so kinetic energy is also conserved.

$$\text{KE:}\ \ \tfrac{1}{2}m_{\text{A}}v_{\text{A}}^2 = \tfrac{1}{2}m_{\text{A}}v'^2_{\text{A}} + \tfrac{1}{2}m_{\text{B}}v'^2_{\text{B}} \ \rightarrow\ v^2 = v'^2_{\text{A}} + 2v'^2_{\text{B}} \ \rightarrow\ v^2 - v'^2_{\text{A}} = 2v'^2_{\text{B}}$$

Square the two momentum equations and add them together.

$$v = 2v'_{\text{B}}\cos\theta;\ v'_{\text{A}} = 2v'_{\text{B}}\sin\theta \ \rightarrow\ v^2 = 4v'^2_{\text{B}}\cos^2\theta;$$
$$v'^2_{\text{A}} = 4v'^2_{\text{B}}\sin^2\theta \ \rightarrow\ v^2 + v'^2_{\text{A}} = 4v'^2_{\text{B}}$$

Add these two results together and use them in the x momentum expression to find the angle.

$$v^2 - v'^2_{\text{A}} = 2v'^2_{\text{B}};\ \ v^2 + v'^2_{\text{A}} = 4v'^2_{\text{B}} \ \rightarrow\ 2v^2 = 6v'^2_{\text{B}} \ \rightarrow\ v'_{\text{B}} = \frac{v}{\sqrt{3}}$$

$$\cos\theta = \frac{v}{2v'_{\text{B}}} = \frac{v}{2\dfrac{v}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \ \rightarrow\ \boxed{\theta = 30°}$$

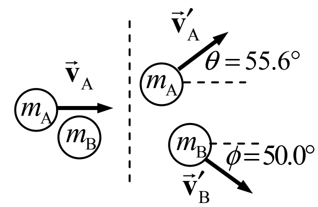
(*b*) From above, we already have $\boxed{v'_B = \dfrac{v}{\sqrt{3}}}$. Use that in the *y* momentum equation.

$$v'_A = 2v'_B \sin\theta = 2\frac{v}{\sqrt{3}}\sin 30° = \boxed{v'_A = \frac{v}{\sqrt{3}}}$$

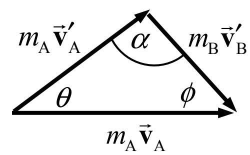
(*c*) The fraction transferred is the final energy of the target particle divided by the original kinetic energy.

$$\frac{\mathrm{KE}_{\text{target}}}{\mathrm{KE}_{\text{original}}} = \frac{\frac{1}{2}m_B v_B'^2}{\frac{1}{2}m_A v_A^2} = \frac{\frac{1}{2}(2m_A)(v^2/3)}{\frac{1}{2}m_A v^2} = \boxed{\frac{2}{3}}$$

48. Let A represent the incoming neon atom and B represent the target atom. A momentum diagram of the collision looks like the first figure.



The figure can be redrawn as a triangle, the second figure, since $m_A\vec{\mathbf{v}}_A = m_A\vec{\mathbf{v}}'_A + m_B\vec{\mathbf{v}}'_B$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.



$$\frac{m_A v'_A}{m_A v_A} = \frac{\sin\phi}{\sin\alpha} \quad\to\quad v'_A = v_A\frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_B v'_B}{m_A v_A} = \frac{\sin\theta}{\sin\alpha} \quad\to\quad v'_B = v_A\frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}$$

The collision is elastic, so write the KE conservation equation, and substitute the results from above. Also note that $\alpha = 180.0 - 55.6° - 50.0° = 74.4°$.

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 \quad\to\quad m_A v_A^2 = m_A\left(v_A\frac{\sin\phi}{\sin\alpha}\right)^2 + m_B\left(v_A\frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}\right)^2 \quad\to$$

$$m_B = \frac{m_A\sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\ \text{u})\sin^2 55.6°}{\sin^2 74.4° - \sin^2 50.0°} = \boxed{39.9\ \text{u}}$$

49. Choose the carbon atom as the origin of coordinates. Use Eq. 7–9a.

$$x_{CM} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\ \text{u})(0) + (16\ \text{u})(1.13\times10^{-10}\ \text{m})}{12\ \text{u} + 16\ \text{u}} = \boxed{6.5\times10^{-11}\ \text{m}}\ \text{from the C atom}$$

50. Use Eq. 7–9a, extended to three particles.

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(1.00\ \text{kg})(0) + (1.50\ \text{kg})(0.50\ \text{m}) + (1.10\ \text{kg})(0.75\ \text{m})}{1.00\ \text{kg} + 1.50\ \text{kg} + 1.10\ \text{kg}}$$
$$= \boxed{0.438\ \text{m}}$$

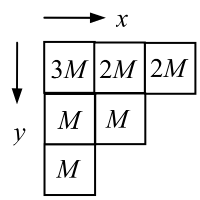
51. Find the CM relative to the front of the car. Use Eq. 7–9a.

$$x_{CM} = \frac{m_{\text{car}}x_{\text{car}} + m_{\text{front}}x_{\text{front}} + m_{\text{back}}x_{\text{back}}}{m_{\text{car}} + m_{\text{front}} + m_{\text{back}}}$$
$$= \frac{(1250\ \text{kg})(2.40\ \text{m}) + 2(65.0\ \text{kg})(2.80\ \text{m}) + 3(65.0\ \text{kg})(3.90\ \text{m})}{1250\ \text{kg} + 5(65.0\ \text{kg})} = \boxed{2.62\ \text{m}}$$

52. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, so $m_1 = \rho(\ell_0)^3$, $m_2 = \rho(2\ell_0)^3$, $m_3 = \rho(3\ell_0)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}\ell_0, x_2 = 2\ell_0, x_3 = 4.5\ell_0$. Use Eq. 7–9a to calculate the CM of the system.

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho\ell_0^3\left(\frac{1}{2}\ell_0\right) + 8\rho\ell_0^3(2\ell_0) + 27\rho\ell_0^3(4.5\ell_0)}{\rho\ell_0^3 + 8\rho\ell_0^3 + 27\rho\ell_0^3} = \frac{138}{36}\ell_0 = \frac{23}{6}\ell_0$$
$$= \boxed{3.8\ell_0 \text{ from the left edge of the smallest cube}}$$

53. Let each case have a mass M. A top view of the pallet is shown, with the total mass of each stack listed. Take the origin to be the back left corner of the pallet. Use Eqs. 7–9a and 7–9b.

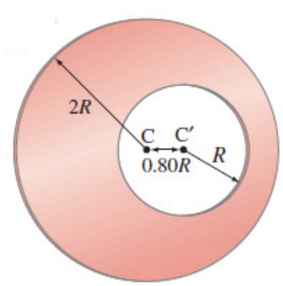


$$x_{\text{CM}} = \frac{(5M)(\ell/2) + (3M)(3\ell/2) + (2M)(5\ell/2)}{10M} = \boxed{1.2\ell}$$
$$y_{\text{CM}} = \frac{(7M)(\ell/2) + (2M)(3\ell/2) + (1M)(5\ell/2)}{10M} = \boxed{0.9\ell}$$

54. Because the brace is uniform, the mass of each "leg" is proportional to its area. Since each "leg" has the same width of 0.20 m, each leg's mass is proportional to its length. We calculate the center of mass relative to the origin of coordinates as given in the diagram. Let the total mass be M.

$$m_{\substack{\text{horizontal}\\\text{leg}}} = \frac{2.06}{2.06+1.48}M = 0.5819M; \quad m_{\substack{\text{vertical}\\\text{leg}}} = \frac{1.48}{2.06+1.48}M = 0.4181M$$
$$x_{\text{CM}} = \frac{m_{\text{horiz}}x_{\text{horiz}} + m_{\text{vert}}x_{\text{vert}}}{m_{\text{horiz}} + m_{\text{vert}}} = \frac{(0.5819M)(1.03\text{ m}) + (0.4181M)(1.96\text{ m})}{M} = \boxed{1.42\text{ m}}$$
$$y_{\text{CM}} = \frac{m_{\text{horiz}}y_{\text{horiz}} + m_{\text{vert}}y_{\text{vert}}}{m_{\text{horiz}} + m_{\text{vert}}} = \frac{(0.5819M)(0.10\text{ m}) + (0.4181M)(-0.74\text{ m})}{M} = \boxed{-0.25\text{ m}}$$

55. Consider the following: We start with a full circle of radius $2R$, with its CM at the origin. Then we draw a circle of radius R, with its CM at the coordinates $(0.80R, 0)$. The full circle can now be labeled as a "shaded" part and a "white" part. The y coordinate of the CM of the entire circle, the CM of the shaded part, and the CM of the white part are all at $y = 0$ by the symmetry of the system. The x coordinate of the entire circle is at $x_{\text{CM}} = 0$, and can be calculated by $x_{\text{CM}} = \frac{m_{\text{shaded}}x_{\text{shaded}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}}$. Solve this equation to solve for the CM of the shaded part.



$$x_{\text{CM}} = \frac{m_{\text{shaded}}x_{\text{shaded}} + m_{\text{white}}x_{\text{white}}}{m_{\text{total}}} \rightarrow$$
$$x_{\text{shaded}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{shaded}}} = \frac{m_{\text{total}}x_{\text{CM}} - m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}}$$

This is functionally the same as treating the white part of the figure as a hole of negative mass. The mass of each part can be found by multiplying the area of the part times the uniform density of the plate.

$$x_{\text{shaded}} = \frac{-m_{\text{white}}x_{\text{white}}}{m_{\text{total}} - m_{\text{white}}} = \frac{-\rho\pi R^2(0.80R)}{\rho\pi(2R)^2 - \rho\pi R^2} = \frac{-0.80R}{3} = \boxed{-0.27R}$$

The negative sign indicates that the CM of the shaded part is to the left of the center of the circle of radius $2R$.

56. Take the upper leg and lower leg together. Note that Table 7–1 gives the relative mass of BOTH legs, so a factor of 1/2 is needed. Assume a person of mass 70 kg.

$$(70\text{ kg})\frac{(21.5+9.6)}{100}\tfrac{1}{2} = 10.885\text{ kg} \approx \boxed{11\text{ kg}}$$

57. With the shoulder as the origin of coordinates for measuring the center of mass, we have the following relative locations from Table 7–1 for the arm components, as percentages of the height. Down is positive.

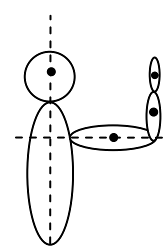
$$x_{\substack{\text{upper}\\\text{arm}}} = 81.2 - 71.7 = 9.5 \quad x_{\substack{\text{lower}\\\text{arm}}} = 81.2 - 55.3 = 25.9 \quad x_{\text{hand}} = 81.2 - 43.1 = 38.1$$

To find the CM, we can also use relative mass percentages. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.

$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}}m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}}m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}} = \frac{(9.5)(6.6)+(25.9)(4.2)+(38.1)(1.7)}{6.6+4.2+1.7}$$

$$= \boxed{\text{19\% of the person's height along the line from the shoulder to the hand}}$$

58. Take the shoulder to be the origin of coordinates. We assume that the arm is held with the upper arm parallel to the floor and the lower arm and hand extended upward. Measure x horizontally from the shoulder and y vertically. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Use the relative mass percentages given in the table.



$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\\text{arm}}}m_{\substack{\text{upper}\\\text{arm}}} + x_{\substack{\text{lower}\\\text{arm}}}m_{\substack{\text{lower}\\\text{arm}}} + x_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(81.2-71.7)(6.6)+(81.2-62.2)(4.2+1.7)}{6.6+4.2+1.7} = 14.0$$

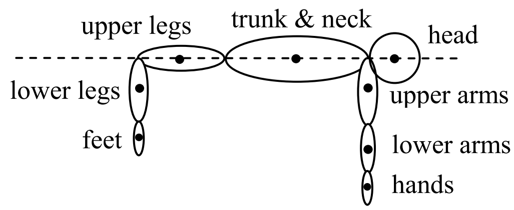
$$y_{\text{CM}} = \frac{y_{\substack{\text{upper}\\\text{arm}}}m_{\substack{\text{upper}\\\text{arm}}} + y_{\substack{\text{lower}\\\text{arm}}}m_{\substack{\text{lower}\\\text{arm}}} + y_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\\text{arm}}} + m_{\substack{\text{lower}\\\text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(0)(6.6)+(62.2-55.3)(4.2)+(62.2-43.1)(1.7)}{6.6+4.2+1.7} = 4.92$$

Convert the distance percentages to actual distance by using the person's height.

$$x_{\mathrm{CM}} = (14.0\%)(155\ \mathrm{cm}) = \boxed{21.7\ \mathrm{cm}} \qquad y_{\mathrm{CM}} = (4.92\%)(155\ \mathrm{cm}) = \boxed{7.6\ \mathrm{cm}}$$

59. See the diagram of the person. The head, trunk, neck, and thighs are all lined up so that their CMs are on the torso's median line. Call down the positive y direction. The y displacements of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass.



On median line:	head (h):	0	6.9% body mass
	trunk & neck (t n):	0	46.1% body mass
	upper legs (u l):	0	21.5% body mass
From shoulder hinge point:	upper arms (u a):	$81.2 - 71.7 = 9.5$	6.6% body mass
	lower arms (l a):	$81.2 - 55.3 = 25.9$	4.2% body mass
	hands (ha):	$81.2 - 43.1 = 38.1$	1.7% body mass
From knee hinge point:	lower legs (l l):	$28.5 - 18.2 = 10.3$	9.6% body mass
	feet (f):	$28.5 - 1.8 = 26.7$	3.4% body mass

Using this data, calculate the vertical location of the CM.

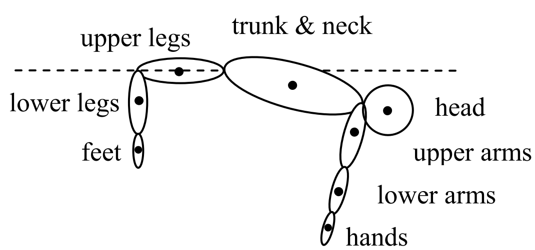
$$y_{\mathrm{CM}} = \frac{y_{\mathrm{h}} m_{\mathrm{h}} + y_{\mathrm{t\,n}} m_{\mathrm{t\,n}} + y_{\mathrm{u\,l}} m_{\mathrm{u\,l}} + y_{\mathrm{u\,a}} m_{\mathrm{u\,a}} + y_{\mathrm{l\,a}} m_{\mathrm{l\,a}} + y_{\mathrm{ha}} m_{\mathrm{ha}} + y_{\mathrm{l\,l}} m_{\mathrm{l\,l}} + y_{\mathrm{f}} m_{\mathrm{f}}}{m_{\substack{\mathrm{full}\\ \mathrm{body}}}}$$

$$= \frac{0 + 0 + 0 + (9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7) + (10.3)(9.6) + (26.7)(3.4)}{100}$$

$$= 4.2591 \approx 4.3$$

So the center of mass is $\boxed{\text{4.3\% of the full body height below the torso's median line}}$. For a person of height 1.7 m, this is about 7.2 cm, which is less than 3 inches. That is most likely $\boxed{\text{inside the body}}$.

60. Based on Fig. 7–27, we place the upper legs parallel to the bar, the lower legs and feet hanging vertically, and the trunk and neck, head, arms, and hands all tilted down by 15°. Call down the positive y direction. The y distances of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass. The calculations for the lower legs and feet are the same as for Problem 59. Here are the calculations for the angled parts of the body.



Trunk & neck: Hip joint: 52.1% from the floor, center of trunk at 71.1%, difference = 19.0%. CM of trunk & neck $= 19.0(\sin 15.0°) = 4.92$

Head: Hip joint: 52.1%, center of head at 93.5%, difference = 41.4% CM of head $= 41.4(\sin 15.0°) = 10.72$

Shoulder: Hip joint: 52.1%, shoulder at 81.2%, difference = 29.1% $= 29.1(\sin 15.0°) = 7.53$

Upper arms:	Shoulder: 81.2%, center of upper arms at 71.7%, difference = 9.5% CM of upper arms = 7.53 (due to shoulder) $+9.5(\cos 15.0°) = 16.71$
Lower arms:	Shoulder: 81.2%, center of lower arms at 55.3%, difference = 25.9% CM of lower arms = 7.53 (due to shoulder) $+25.9(\cos\ 15.0°) = 32.55$
Hands:	Shoulder: 81.2%, center of hands at 43.1%, difference = 38.1% CM of hands = 7.53 (due to shoulder) $+38.1(\cos 15.0°) = 44.33$

On horizontal line:	upper legs (u l):	0	21.5% body mass
From waist hinge point:	trunk & neck (t n):	4.92	46.1% body mass
	head (h):	10.72	6.9% body mass
From shoulder hinge point:	upper arms (u a):	16.71	6.6% body mass
	lower arms (l a):	32.55	4.2% body mass
	hands (ha):	44.33	1.7% body mass
From knee hinge point:	lower legs (l l):	10.3	9.6% body mass
	feet (f):	26.7	3.4% body mass

Using this data, calculate the vertical location of the CM.

$$y_{\text{CM}} = \frac{y_{\text{h}}m_{\text{h}} + y_{\text{t n}}m_{\text{t n}} + y_{\text{u l}}m_{\text{u l}} + y_{\text{u a}}m_{\text{u a}} + y_{\text{l a}}m_{\text{l a}} + y_{\text{ha}}m_{\text{ha}} + y_{\text{l l}}m_{\text{l l}} + y_{\text{f}}m_{\text{f}}}{m_{\substack{\text{full}\\ \text{body}}}}$$

$$= \frac{\begin{bmatrix} 10.72(6.9) + 4.92(46.1) + 0 + (16.71)(6.6) + (32.55)(4.2) \\ + (44.33)(1.7) + (10.3)(9.6) + (26.7)(3.4) \end{bmatrix}}{100}$$

$$= 8.128 \approx 8.1$$

Thus the center of mass is $\boxed{\text{8.1\% of the full body height below the torso's median line}}$. For a person of height 1.7 m, this is about 14 cm. That is about 5.5 inches, and it is most likely slightly $\boxed{\text{outside the body}}$.

$\boxed{61.}$ (*a*) Find the CM relative to the center of the Earth.

$$x_{\text{CM}} = \frac{m_{\text{E}}x_{\text{E}} + m_{\text{M}}x_{M}}{m_{\text{E}} + m_{\text{M}}} = \frac{(5.98\times10^{24}\ \text{kg})(0) + (7.35\times10^{22}\ \text{kg})(3.84\times10^{8}\ \text{m})}{5.98\times10^{24}\ \text{kg} + 7.35\times10^{22}\ \text{kg}}$$

$$= \boxed{4.66\times10^{6}\ \text{m from the center of the Earth}}$$

This is actually inside the volume of the Earth, since $R_{\text{E}} = 6.38\times10^{6}$ m.

(*b*) It is this Earth–Moon CM location that actually traces out the orbit, as discussed in Chapter 5. The Earth and Moon will orbit about this location in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: (i) the motion of the Moon about the Earth–Moon CM and (ii) the motion of the Earth–Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth–Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun).

62. The point that will follow a parabolic trajectory is the center of mass of the mallet. Find the CM relative to the bottom of the mallet. Each part of the hammer (handle and head) can be treated as a point mass located at the CM of the respective piece. So the CM of the handle is 12.0 cm from the bottom of the handle, and the CM of the head is 28.0 cm from the bottom of the handle.

$$x_{\text{CM}} = \frac{m_{\text{handle}}x_{\text{handle}} + m_{\text{head}}x_{\text{head}}}{m_{\text{handle}} + m_{\text{head}}} = \frac{(0.500\text{ kg})(12.0\text{ cm}) + (2.30\text{ kg})(28.0\text{ cm})}{2.80\text{ kg}} = \boxed{25.1\text{ cm}}$$

Note that this is inside the head of the mallet. The mallet will rotate about this point as it flies through the air, giving it a wobbling kind of motion.

63. (*a*) Measure all distances from the original position of the woman.

$$x_{\text{CM}} = \frac{m_{\text{W}}x_{\text{W}} + m_{\text{M}}x_{\text{M}}}{m_{\text{W}} + m_{\text{M}}} = \frac{(52\text{ kg})(0) + (72\text{ kg})(10.0\text{ m})}{124\text{ kg}} = 5.806\text{ m}$$
$$\approx \boxed{5.8\text{ m from the woman}}$$

(*b*) Since there is no force external to the man–woman system, the CM will not move, relative to the original position of the woman. The woman's distance will no longer be 0, and the man's distance has changed to 7.5 m.

$$x_{\text{CM}} = \frac{m_{\text{W}}x_{\text{W}} + m_{\text{M}}x_{\text{M}}}{m_{\text{W}} + m_{\text{M}}} = \frac{(52\text{ kg})x_{\text{W}} + (72\text{ kg})(7.5\text{ m})}{124\text{ kg}} = 5.806\text{ m} \quad \rightarrow$$

$$x_{\text{W}} = \frac{(5.806\text{ m})(124\text{ kg}) - (72\text{ kg})(7.5\text{ m})}{52\text{ kg}} = 3.460\text{ m}$$

$$x_{\text{M}} - x_{\text{W}} = 7.5\text{ m} - 3.460\text{ m} = 4.040\text{ m} \approx \boxed{4.0\text{ m}}$$

(*c*) When the man collides with the woman, he will be at the original location of the center of mass.

$$\underset{\text{final}}{x_{\text{M}}} - \underset{\text{initial}}{x_{\text{M}}} = 5.806\text{ m} - 10.0\text{ m} = -4.194\text{ m}$$

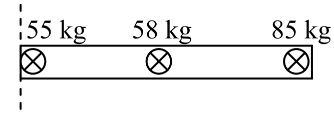
He has moved $\boxed{4.2\text{ m}}$ from his original position.

64. The CM of the system will follow the same path regardless of the way the mass splits and will still be $2d$ from the launch point when the parts land. Assume that the explosion is designed so that m_{I} still is stopped in midair and falls straight down.

(*a*) $$x_{\text{CM}} = \frac{m_{\text{I}}x_{\text{I}} + m_{\text{II}}x_{\text{II}}}{m_{\text{I}} + m_{\text{II}}} \rightarrow 2d = \frac{m_{\text{I}}d + 3m_{\text{I}}x_{\text{II}}}{4m_{\text{I}}} = \frac{d + 3x_{\text{II}}}{4} \rightarrow x_{\text{II}} = \boxed{\tfrac{7}{3}d}$$

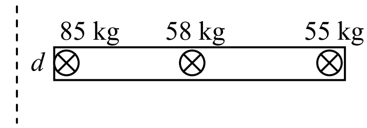
(*b*) $$x_{\text{CM}} = \frac{m_{\text{I}}x_{\text{I}} + m_{\text{II}}x_{\text{II}}}{m_{\text{I}} + m_{\text{II}}} \rightarrow 2d = \frac{3m_{\text{II}}d + m_{\text{II}}x_{\text{II}}}{4m_{\text{II}}} = \frac{3d + x_{\text{II}}}{4} \rightarrow x_{\text{II}} = \boxed{5d}$$

65. Calculate the CM relative to the 55-kg person's seat, at one end of the boat. See the first diagram. Be sure to include the boat's mass.



$$x_{\text{CM}} = \frac{m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}}{m_{\text{A}} + m_{\text{B}} + m_{\text{C}}}$$
$$= \frac{(55\text{ kg})(0) + (58\text{ kg})(1.5\text{ m}) + (85\text{ kg})(3.0\text{ m})}{198\text{ kg}} = 1.727\text{ m}$$

Now, when the passengers exchange positions, the boat will move some distance d as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.



$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$1.727 \text{ m} = \frac{(85 \text{ kg})(d) + (58 \text{ kg})(1.5 \text{ m} + d) + (55 \text{ kg})(3.0 \text{ m} + d)}{218 \text{ kg}} = \frac{198d \text{ kg}\cdot\text{m} + 252 \text{ kg}\cdot\text{m}}{198 \text{ kg}}$$

$$d = \frac{(1.727 \text{ m})(198 \text{ kg}) - 252 \text{ kg}\cdot\text{m}}{198 \text{ kg}} = 0.4543 \text{ m}$$

Thus the boat will move $\boxed{\text{0.45 m toward the initial position of the 85-kg person}}$.

66. Call the origin of coordinates the CM of the balloon, gondola, and passenger at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0. The passenger sliding down the rope cannot change the total momentum of the system, so the CM must stay at rest. Call the upward direction positive. Then the velocity of the passenger with respect to the balloon is $-v$. Call the velocity of the balloon with respect to the ground v_{BG}. Then the velocity of the passenger with respect to the ground is $v_{MG} = -v + v_{BG}$. Apply Eq. 7–10.

$$0 = m v_{MG} + M v_{BG} = m(-v + v_{BG}) + M v_{BG} \quad \rightarrow \quad \boxed{v_{BG} = v\frac{m}{m+M}, \text{ upward}}$$

If the passenger stops, $\boxed{\text{the balloon also stops}}$, and the CM of the system remains at rest.

67. The only forces on the astronauts are internal to the two-astronaut system, so their CM will not change. Call the CM location the origin of coordinates. That is also the initial location of the astronauts.

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} \quad \rightarrow \quad 0 = \frac{(55 \text{ kg})(12 \text{ m}) + (85 \text{ kg})x_B}{140 \text{ kg}} \quad \rightarrow \quad x = -7.76 \text{ m}$$

Their distance apart is $x_A - x_B = 12 \text{ m} - (-7.76 \text{ m}) = \boxed{2.0\times10^1 \text{ m}}$.

68. This is a totally inelastic collision in one dimension. Call the direction of asteroid A the positive direction, and use conservation of momentum.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = (m_A + m_B)v' \quad \rightarrow$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(7.5\times10^{12} \text{ kg})(3.3 \text{ km/s}) + (1.45\times10^{13} \text{ kg})(-1.4 \text{ km/s})}{7.5\times10^{12} \text{ kg} + 1.45\times10^{13} \text{ kg}}$$

$$= 0.0203 \text{ km/s} \approx \boxed{\text{0.2 km/s, in the original direction of asteroid A}}$$

69. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $mgh = \frac{1}{2}mv^2$ for any particular bounce cycle. Thus for an interaction with the floor, the ratio of the energies before and after the bounce is

$\frac{\text{KE}_{\text{after}}}{\text{KE}_{\text{before}}} = \frac{mgh'}{mgh} = \frac{1.20\text{ m}}{1.60\text{ m}} = 0.75.$ We assume that each bounce will further reduce the energy to 75% of its pre-bounce amount. The number of bounces to lose 90% of the energy can be expressed as follows:

$$(0.75)^n = 0.1 \quad \rightarrow \quad n = \frac{\log 0.1}{\log 0.75} = 8.004$$

Thus after $\boxed{\text{8 bounces}}$, 90% of the energy is lost.

As an alternate method, after each bounce, 75% of the available energy is left. So after 1 bounce, 75% of the original energy is left. After the second bounce, only 75% of 75%, or 56%, is left. After the third bounce, 42%. After the fourth bounce, 32%. After the fifth bounce, 24%. After the sixth bounce, 18%. After the seventh bounce, 13%. After the eight bounce, 10%. So it takes 8 bounces.

70. Momentum will be conserved in the horizontal direction. Let A represent the railroad car and B represent the snow. For the horizontal motion, $\upsilon_{\text{B}} = 0$ and $\upsilon'_{\text{B}} = \upsilon'_{\text{A}}$. Momentum conservation in the horizontal direction gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}\upsilon_{\text{A}} = (m_{\text{A}} + m_{\text{B}})\upsilon'_{\text{A}}$$

$$\upsilon'_{\text{A}} = \frac{m_{\text{A}}\upsilon_{\text{A}}}{m_{\text{A}} + m_{\text{B}}} = \frac{(4800\text{ kg})(7.60\text{ m/s})}{4800\text{ kg} + \left(\frac{3.80\text{ kg}}{\text{min}}\right)(60.0\text{ min})} = 7.255\text{ m/s} \approx \boxed{7.3\text{ m/s}}$$

71. Let the original direction of the cars be the positive direction. We have $\upsilon_{\text{A}} = 4.50\text{ m/s}$ and $\upsilon_{\text{B}} = 3.70\text{ m/s}$.

(*a*) Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_{\text{A}} - \upsilon_{\text{B}} = -(\upsilon'_{\text{A}} - \upsilon'_{\text{B}}) \quad \rightarrow \quad \upsilon'_{\text{B}} = \upsilon_{\text{A}} - \upsilon_{\text{B}} + \upsilon'_{\text{A}} = 0.80\text{ m/s} + \upsilon'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}} \quad \rightarrow \quad m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}(0.80\text{ m/s} + \upsilon'_{\text{A}}) \quad \rightarrow$$

$$\upsilon'_{\text{A}} = \frac{m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}(\upsilon_{\text{B}} - 0.80\text{ m/s})}{m_{\text{A}} + m_{\text{B}}} = \frac{(435\text{ kg})(4.50\text{ m/s}) + (495\text{ kg})(2.90\text{ m/s})}{930\text{ kg}} = 3.648\text{ m/s}$$

$$\approx \boxed{3.65\text{ m/s}}; \quad \upsilon'_{\text{B}} = 0.80\text{ m/s} + \upsilon'_{\text{A}} = 4.448\text{ m/s} \approx \boxed{4.45\text{ m/s}}$$

(*b*) Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_{\text{A}} = m_{\text{A}}\upsilon'_{\text{A}} - m_{\text{A}}\upsilon_{\text{A}} = (435\text{ kg})(3.648\text{ m/s} - 4.50\text{ m/s}) = -370.62\text{ kg}\cdot\text{m/s} \approx \boxed{-370\text{ kg}\cdot\text{m/s}}$$

$$\Delta p_{\text{B}} = m_{\text{B}}\upsilon'_{\text{B}} - m_{\text{B}}\upsilon_{\text{B}} = (495\text{ kg})(4.448\text{ m/s} - 3.70\text{ m/s}) = 370.26\text{ kg}\cdot\text{m/s} \approx \boxed{370\text{ kg}\cdot\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved. The slight difference is due to round-off error on the calculations.

72. This is a ballistic "pendulum" of sorts, similar to Example 7–9. The only difference is that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and

conserves momentum, and the energy is still conserved in the rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$\upsilon = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$
$$h = \frac{1}{2g}\left(\frac{m\upsilon}{m+M}\right)^2 = \frac{1}{2(9.80\ \text{m/s}^2)}\left(\frac{(0.0250\ \text{kg})(230\ \text{m/s})}{0.0250\ \text{kg}+1.40\ \text{kg}}\right)^2 = 0.8307\ \text{m} \approx \boxed{0.83\ \text{m}}$$

73. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows:

$$W_{\text{fr}} = \Delta\text{KE} = \tfrac{1}{2}m(\upsilon_{\text{f}}^2 - \upsilon_{\text{i}}^2) \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x \cos 180°\theta = -\mu_{\text{k}}F_{\text{N}}\Delta x = -\mu_{\text{k}}mg\Delta x \quad \rightarrow$$
$$-\mu_{\text{k}}g\Delta x = \tfrac{1}{2}(\upsilon_{\text{f}}^2 - \upsilon_{\text{i}}^2)$$

For post-collision sliding, $\upsilon_{\text{f}} = 0$ and υ_{i} is the speed immediately after the collision, υ'. Use this relationship to find the speed of each car immediately after the collision.

$$\text{Car A: } -\mu_{\text{k}}g\Delta x_{\text{A}}' = -\tfrac{1}{2}\upsilon_{\text{A}}'^2 \quad \rightarrow \quad \upsilon_{\text{A}}' = \sqrt{2\mu_{\text{k}}g\Delta x_{\text{A}}'} = \sqrt{2(0.60)(9.80\ \text{m/s}^2)(18\ \text{m})} = 14.55\ \text{m/s}$$

$$\text{Car B: } -\mu_{\text{k}}g\Delta x_{\text{B}}' = -\tfrac{1}{2}\upsilon_{\text{B}}'^2 \quad \rightarrow \quad \upsilon_{\text{B}}' = \sqrt{2\mu_{\text{k}}g\Delta x_{\text{B}}'} = \sqrt{2(0.60)(9.80\ \text{m/s}^2)(30\ \text{m})} = 18.78\ \text{m/s}$$

During the collision, momentum is conserved in one dimension. Note that $\upsilon_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}\upsilon_{\text{A}} = m_{\text{A}}\upsilon_{\text{A}}' + m_{\text{B}}\upsilon_{\text{B}}'$$
$$\upsilon_{\text{A}} = \frac{m_{\text{A}}\upsilon_{\text{A}}' + m_{\text{B}}\upsilon_{\text{B}}'}{m_{\text{A}}} = \frac{(1500\ \text{kg})(14.55\ \text{m/s}) + (1100\ \text{kg})(18.78\ \text{m/s})}{1500\ \text{kg}} = 28.32\ \text{m/s}$$

For pre-collision sliding, again apply the friction–energy relationship, with $\upsilon_{\text{f}} = \upsilon_{\text{A}}$ and υ_{i} the speed when the brakes were first applied.

$$-\mu_{\text{k}}g\Delta x_{\text{A}} = \tfrac{1}{2}(\upsilon_{\text{A}}^2 - \upsilon_i^2) \quad \rightarrow \quad \upsilon_{\text{i}} = \sqrt{\upsilon_{\text{A}}^2 + 2\mu_{\text{k}}g\Delta x_{\text{A}}} = \sqrt{(28.32\ \text{m/s})^2 + 2(0.60)(9.80\ \text{m/s}^2)(15\ \text{m})}$$
$$= 31.28\ \text{m/s}\left(\frac{1\ \text{mi/h}}{0.447\ \text{m/s}}\right) = \boxed{70\ \text{mi/h}}$$

Car A was definitely over the speed limit.

74. (*a*) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, so $\upsilon_{\text{A}} = 0$. Write momentum conservation for the collision.

$$m_{\text{B}}\upsilon_{\text{B}} = (m_{\text{A}} + m_{\text{B}})\upsilon' \quad \rightarrow$$
$$\upsilon' = \upsilon_{\text{B}}\frac{m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = (2.5\times10^4\ \text{m/s})\frac{1.5\times10^8\ \text{kg}}{6.0\times10^{24}\ \text{kg} + 1.5\times10^8\ \text{kg}} = 6.25\times10^{-13}\ \text{m/s}$$
$$\approx \boxed{6.3\times10^{-13}\ \text{m/s}}$$

(*b*) The fraction of the meteor's KE transferred to the Earth is the final KE of the Earth divided by the initial KE of the meteor.

$$\frac{\mathrm{KE}_{\substack{\text{final}\\\text{Earth}}}}{\mathrm{KE}_{\substack{\text{initial}\\\text{meteor}}}}=\frac{\frac{1}{2}m_{\mathrm{A}}v'^2}{\frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}}^2}=\frac{\frac{1}{2}(6.0\times10^{24}\ \mathrm{kg})(6.25\times10^{-13}\ \mathrm{m/s})^2}{\frac{1}{2}(1.5\times10^{8}\ \mathrm{kg})(2.5\times10^{4}\ \mathrm{m/s})^2}=\boxed{2.5\times10^{-17}}$$

(*c*) The Earth's change in KE can be calculated directly.

$$\Delta\mathrm{KE}_{\text{Earth}}=\mathrm{KE}_{\substack{\text{final}\\\text{Earth}}}-\mathrm{KE}_{\substack{\text{initial}\\\text{Earth}}}=\tfrac{1}{2}m_{\mathrm{A}}v'^2-0=\tfrac{1}{2}(6.0\times10^{24}\ \mathrm{kg})(6.25\times10^{-13}\ \mathrm{m/s})^2=\boxed{1.2\ \mathrm{J}}$$

75. This is a ballistic "pendulum" of sorts, similar to Example 7–9. The mass of the bullet is m, and the mass of the block of wood is M. The speed of the bullet before the collision is v, and the speed of the combination after the collision is v'. Momentum is conserved in the totally inelastic collision, so $mv=(m+M)v'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}}=\Delta\mathrm{KE}=\tfrac{1}{2}m(v_{\mathrm{f}}^2-v_{\mathrm{i}}^2)_{\substack{\text{after}\\\text{collision}}}\qquad W_{\text{fr}}=F_{\text{fr}}\Delta x\cos180°\theta=-\mu_{\mathrm{k}}F_{\mathrm{N}}\Delta x=-\mu_{\mathrm{k}}mg\Delta x\ \rightarrow$$

$$-\mu_{\mathrm{k}}g\Delta x=\tfrac{1}{2}(v_{\mathrm{f}}^2-v_{\mathrm{i}}^2)=-\tfrac{1}{2}v'^2\ \rightarrow\ v'=\sqrt{2\mu_{\mathrm{k}}g\Delta x}$$

Use this expression for v' in the momentum equation in order to solve for v.

$$mv=(m+M)v'=(m+M)\sqrt{2\mu_{\mathrm{k}}g\Delta x}\ \rightarrow$$

$$v=\left(\frac{m+M}{m}\right)\sqrt{2\mu_{\mathrm{k}}g\Delta x}=\left(\frac{0.028\ \mathrm{kg}+1.35\ \mathrm{kg}}{0.028\ \mathrm{kg}}\right)\sqrt{2(0.28)(9.80\ \mathrm{m/s^2})(8.5\ \mathrm{m})}=\boxed{340\ \mathrm{m/s}}$$

76. (*a*) The average force is the momentum change divided by the elapsed time.

$$F_{\text{avg}}=\frac{\Delta p}{\Delta t}=\frac{m\Delta v}{\Delta t}=\frac{(1500\ \mathrm{kg})(0-45\ \mathrm{km/h})\left(\dfrac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right)}{0.15\ \mathrm{s}}=-1.25\times10^5\ \mathrm{N}\approx\boxed{-1.3\times10^5\ \mathrm{N}}$$

The negative sign indicates direction—that the force is in the opposite direction to the original direction of motion.

(*b*) Use Newton's second law. We use the absolute value of the force because the problem asked for the deceleration.

$$F_{\text{avg}}=ma_{\text{avg}}\ \rightarrow\ a_{\text{avg}}=\frac{F_{\text{avg}}}{m}=\frac{1.25\times10^5\ \mathrm{N}}{1500\ \mathrm{kg}}=83.33\ \mathrm{m/s^2}\left(\frac{1\ g}{9.80\ \mathrm{m/s^2}}\right)\approx\boxed{8.5\ g\text{'s}}$$

77. The original horizontal distance can be found from the level horizontal range formula in Chapter 3.

$$R=v_0^2\sin2\theta_0/g=(25\ \mathrm{m/s})^2(\sin56°)/(9.80\ \mathrm{m/s^2})=52.88\ \mathrm{m}$$

The height at which the objects collide can be found from Eq. 2–11c for the vertical motion, with $v_y=0$ at the top of the path. Take up to be positive.

$$v_y^2=v_{y0}^2+2ah\ \rightarrow\ h=\frac{v_y^2-v_{y0}^2}{2a}=\frac{0-[(25\ \mathrm{m/s})\sin28°]^2}{2(-9.80\ \mathrm{m/s^2})}=7.028\ \mathrm{m}$$

Let m represent the bullet and M the skeet. When the objects collide, the skeet is moving horizontally at $v_0 \cos\theta = (25\text{ m/s})\cos 28° = 22.07\text{ m/s} = v_x$, and the bullet is moving vertically at $v_y = 230$ m/s. Write momentum conservation in both directions to find the velocities after the totally inelastic collision.

$$p_x:\quad Mv_x = (M+m)v'_x \quad\rightarrow\quad v'_x = \frac{Mv'_x}{M+m} = \frac{(0.25\text{ kg})(22.07\text{ m/s})}{(0.25+0.015)\text{ kg}} = 20.82\text{ m/s}$$

$$p_y:\quad mv_y = (M+m)v'_y \quad\rightarrow\quad v'_y = \frac{mv'_y}{M+m} = \frac{(0.015\text{ kg})(230\text{ m/s})}{(0.25+0.015)\text{ kg}} = 13.02\text{ m/s}$$

(*a*) The speed v'_y can be used as the starting vertical speed in Eq. 2–11c to find the height that the skeet–bullet combination rises above the point of collision.

$$v_y^2 = v_{y0}^2 + 2ah' \quad\rightarrow\quad h' = \frac{v_y^2 - v_{y0}^2}{2a} = \frac{0-(13.02\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 8.649\text{ m} \approx \boxed{8.6\text{ m}}$$

(*b*) From Eq. 2–11b applied to the vertical motion after the collision, we can find the time for the skeet–bullet combination to reach the ground.

$$y = y_0 + v'_y t + \tfrac{1}{2}at^2 \quad\rightarrow\quad 0 = 7.028\text{ m} + (13.02\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad\rightarrow$$

$$4.9t^2 - 13.02t - 7.028 = 0 \quad\rightarrow$$

$$t = \frac{13.02 \pm \sqrt{(13.02)^2 + 4(4.9)(7.028)}}{9.80} = 3.117\text{ s}, -0.460\text{ s}$$

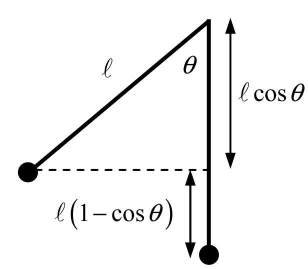
The positive result is used to find the horizontal distance traveled by the combination after the collision.

$$x_{\text{after}} = v'_x t = (20.82\text{ m/s})(3.117\text{ s}) = 64.90\text{ m}$$

If the collision had not happened, the skeet would have gone $\tfrac{1}{2}R$ horizontally.

$$\Delta x = x_{\text{after}} - \tfrac{1}{2}R = 64.90\text{ m} - \tfrac{1}{2}(52.88\text{ m}) = 38.46\text{ m} \approx \boxed{38\text{ m}}$$

78. For the swinging balls, their velocity at the bottom of the swing and the height to which they rise are related by conservation of energy. If the zero of gravitational potential energy is taken to be the lowest point of the swing, then the kinetic energy at the low point is equal to the potential energy at the highest point of the swing, where the speed is zero. Thus we have $\tfrac{1}{2}mv_{\text{bottom}}^2 = mgh$ for any swinging ball, so the relationship between speed and height is $v_{\text{bottom}}^2 = 2gh$. From the diagram we see that $h = \ell(1-\cos\theta)$.



(*a*) Calculate the speed of the lighter ball at the bottom of its swing.

$$v_A = \sqrt{2gh_A} = \sqrt{2(9.80\text{ m/s}^2)(0.35\text{ m})(1-\cos 66°)} = 2.017\text{ m/s} \approx \boxed{2.0\text{ m/s}}$$

(*b*) Assume that the collision is elastic, and use the results of Search and Learn 5. Take the direction that ball A is moving just before the collision as the positive direction.

$$v'_A = \frac{(m_A - m_B)}{(m_A + m_B)}v_A = \frac{(0.045\text{ kg} - 0.065\text{ kg})}{(0.045\text{ kg} + 0.065\text{ kg})}(2.017\text{ m/s}) = -0.3667\text{ m/s} \approx \boxed{-0.37\text{ m/s}}$$

$$v'_B = \frac{2m_A}{(m_A + m_B)}v_A = \frac{2(0.045\text{ kg})}{(0.045\text{ kg} + 0.065\text{ kg})}(2.017\text{ m/s}) = 1.650\text{ m/s} \approx \boxed{1.7\text{ m/s}}$$

Notice that ball A has rebounded backward.

(*c*) After each collision, use the conservation of energy relationship again.

$$h'_A = \frac{v'^2_A}{2g} = \frac{(-0.3667 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{6.9\times10^{-3} \text{ m}} \quad h'_B = \frac{v'^2_B}{2g} = \frac{(1.650 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{1.4\times10^{-1} \text{ m}}$$

79. (*a*) Use conservation of energy to find the speed of mass *m* before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$mgh_A = \tfrac{1}{2}mv_A^2 \quad\rightarrow\quad v_A = \sqrt{2gh_A} = \sqrt{2(9.80 \text{ m/s}^2)(3.60 \text{ m})} = 8.40 \text{ m/s}$$

Use Eq. 7–7 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_A - v_B = v'_B - v'_A \quad\rightarrow\quad v'_B = v'_A + v_A$$

Apply momentum conservation for the collision, and substitute the result from Eq. 7–7.

$$mv_A = mv'_A + Mv'_B = mv'_A + M(v_A + v'_A) \quad\rightarrow$$

$$v'_A = \frac{m-M}{m+M}v_A = \left(\frac{2.50 \text{ kg} - 7.00 \text{ kg}}{9.50 \text{ kg}}\right)(8.40 \text{ m/s}) = \boxed{-3.98 \text{ m/s}}$$

$$v'_B = v'_A + v_A = -3.98 \text{ m/s} + 8.40 \text{ m/s} = \boxed{4.42 \text{ m/s}}$$

(*b*) Again use energy conservation to find the height to which mass *m* rises after the collision. The kinetic energy of *m* immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$\tfrac{1}{2}mv'^2_A = mgh'_A \quad\rightarrow\quad h'_A = \frac{v'^2_A}{2g}$$

$$d'_A = \frac{h'_A}{\sin 30} = \frac{v'^2_A}{2g \sin 30} = \frac{(-3.98 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)\sin 30.0^\circ} = \boxed{1.62 \text{ m}}$$

80. (*a*) Momentum is conserved in the *x* direction. The initial *x* momentum is 0.

$$\underset{\text{before}}{p_x} = \underset{\text{after}}{p_x} \quad\rightarrow\quad 0 = m_{\text{satellite}}v_{x \text{ satellite}} + m_{\text{shuttle}}v_{x \text{ shuttle}} \quad\rightarrow$$

$$v_{x \text{ shuttle}} = -\frac{m_{\text{satellite}}v_{x \text{ satellite}}}{m_{\text{shuttle}}} = -\frac{850 \text{ kg}}{92{,}000 \text{ kg}}(0.30 \text{ m/s}) = -2.8\times10^{-3} \text{ m/s}$$

So the component in the minus *x* direction is $\boxed{2.8\times10^{-3} \text{ m/s}}$.

(*b*) The average force is the change in momentum per unit time. The force on the satellite is in the positive *x* direction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(850 \text{ kg})(0.30 \text{ m/s})}{4.8 \text{ s}} = \boxed{53 \text{ N}}$$

81. (*a*) In the reference frame of the Earth, the final speed of the Earth–asteroid system is essentially 0, because the mass of the Earth is so much greater than the mass of the asteroid. It is like throwing a ball of mud at the wall of a large building—the smaller mass stops, and the larger mass doesn't move appreciably. Thus all of the asteroid's original kinetic energy can be released as destructive energy.

$$\text{KE}_{\text{orig}} = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}[(3200 \text{ kg/m}^3)\tfrac{4}{3}\pi(1.0\times10^3 \text{ m})^3](1.5\times10^4 \text{ m/s})^2 = 1.507\times10^{21} \text{ J}$$

$$\approx \boxed{1.5\times10^{21} \text{ J}}$$

(*b*) $$1.507\times10^{21} \text{ J}\left(\frac{1 \text{ bomb}}{4.0\times10^{16} \text{ J}}\right) = \boxed{38{,}000 \text{ bombs}}$$

82. The initial momentum of the astronaut and the gas in the jet pack is 0, so the final momentum of the astronaut and the gas ejected from his jet pack must also be 0. We let A refer to the astronaut and B refer to the gas. The velocity of the astronaut is taken to be the positive direction. We also assume that gas is ejected very quickly, so that the 35 m/s is relative to the astronaut's rest frame.

$$0 = m_A v'_A + m_B v'_B = (210\text{ kg} - m_B)(2.0\text{ m/s}) + m_B(-35\text{ m/s}) \quad \to$$

$$m_B = \frac{210\text{ kg}(2.0\text{ m/s})}{-37\text{ m/s}} = 11.35\text{ kg} \approx \boxed{11\text{ kg}}$$

83. (*a*) $\boxed{\text{No}}$, there is no net external force on the system. In particular, the spring force is internal to the system.

(*b*) Use conservation of momentum to determine the ratio of speeds. Note that the two masses will be moving in opposite directions. The initial momentum, when the masses are released, is 0.

$$p_{\text{initial}} = p_{\text{later}} \quad \to \quad 0 = m_A v_A - m_B v_B \quad \to \quad v_A / v_B = \boxed{m_B/m_A}$$

(*c*) $$\frac{KE_A}{KE_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{m_A}{m_B}\left(\frac{v_A}{v_B}\right)^2 = \frac{m_A}{m_B}\left(\frac{m_B}{m_A}\right)^2 = \boxed{m_B/m_A}$$

(*d*) The center of mass was initially at rest. Since there is no net external force on the system, the center of mass does not move, and the system $\boxed{\text{stays at rest}}$.

$\boxed{84.}$ Because all of the collisions are perfectly elastic, no energy is lost in the collisions. With each collision, the horizontal velocity is constant, and the vertical velocity reverses direction. So, after each collision, the ball rises again to the same height from which it dropped. Thus, after five bounces, the bounce height will be $\boxed{4.00\text{ m}}$, the same as the starting height.

85. In this interaction, energy is conserved (initial potential energy of mass–compressed spring system = final kinetic energy of moving blocks) and momentum is conserved, since the net external force is 0. Use these two relationships to find the final speeds.

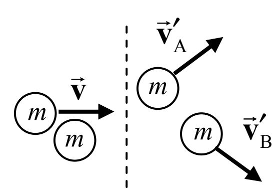
$$p_{\text{initial}} = p_{\text{final}} \quad \to \quad 0 = mv_m - 3mv_{3m} \quad \to \quad v_m = 3v_{3m}$$

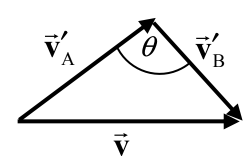
$$E_{\text{initial}} = E_{\text{final}} \quad \to \quad PE_{\substack{\text{spring}\\\text{initial}}} = KE_{\text{final}} \quad \to$$

$$\tfrac{1}{2}kD^2 = \tfrac{1}{2}mv_m^2 + \tfrac{1}{2}3mv_{3m}^2 = \tfrac{1}{2}m(3v_{3m})^2 + \tfrac{1}{2}3mv_{3m}^2 = 6mv_{3m}^2$$

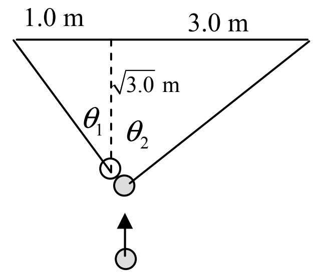
$$\tfrac{1}{2}kD^2 = 6mv_{3m}^2 \quad \to \quad \boxed{v_{3m} = D\sqrt{\frac{k}{12m}};\ v_m = 3D\sqrt{\frac{k}{12m}}}$$

86. In an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. Here is a proof of that fact. Momentum conservation as a vector relationship says $m\vec{\mathbf{v}} = m\vec{\mathbf{v}}'_A + m\vec{\mathbf{v}}'_B \quad \to \quad \vec{\mathbf{v}} = \vec{\mathbf{v}}'_A + \vec{\mathbf{v}}'_B$. Kinetic energy conservation says $\frac{1}{2}mv^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B \quad \to \quad v^2 = v'^2_A + v'^2_B$. The vector equation resulting from momentum conservation can be illustrated by the second diagram. Apply the law of cosines to that triangle of vectors, and then equate the two expressions for v^2.





$$v^2 = v'^2_A + v'^2_B - 2v'_A v'_B \cos\theta$$



Equating the two expressions for v^2 gives

$$v_A'^2 + v_B'^2 - 2v_A'v_B' \cos\theta = v_A'^2 + v_B'^2 \quad \to \quad \cos\theta = 0 \quad \to \quad \theta = 90°$$

For this specific circumstance, see the third diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left" triangle: $\theta_1 = \tan^{-1}\dfrac{1.0}{\sqrt{3.0}} = 30°$. Find θ_2 from the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\dfrac{3.0}{\sqrt{3.0}} = 60°$. Since the balls will separate at a 90° angle, if the target ball goes in the pocket, this does appear to be a $\boxed{\text{good possibility of a scratch shot}}$.

Solutions to Search and Learn Problems

1. It is best to use $\sum \vec{\mathbf{F}}_{\text{ext}} = 0$ and $\sum \vec{\mathbf{p}}_i = \sum \vec{\mathbf{p}}_f$ when the system can be broken up into two or more objects for which only the forces between the objects are significant. The principle of impulse, $\sum \vec{\mathbf{F}}_{\text{ext}} \Delta t = \Delta \vec{\mathbf{p}}$, is useful in cases where the time over which the force acts is known and when examining the change in momentum of a single object due to external forces.

2. In each case, use momentum conservation. Let A represent the 6.0-kg object, and let B represent the 8.0-kg object. Then we have $m_A = 6.0$ kg, $v_A = 6.5$ m/s, $m_B = 8.0$ kg, and $v_B = -4.0$ m/s.

(*a*) In this case, the objects stick together, so $v_A' = v_B'$.

$$m_A v_A + m_B v_B = (m_A + m_B)v_A' \quad \to$$

$$v_B' = v_A' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(6.0\text{ kg})(6.5\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{14.0\text{ kg}} = \boxed{0.5\text{ m/s}}$$

(*b*) In this case, use Eq. 7–7 to find a relationship between the velocities.

$$v_A - v_B = -(v_A' - v_B') \quad \to \quad v_B' = v_A - v_B + v_A'$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' = m_A v_A' + m_B(v_A - v_B + v_A') \quad \to$$

$$v_A' = \frac{(m_A - m_B)v_A + 2m_B v_B}{m_A + m_B} = \frac{(-2.0\text{ kg})(6.5\text{ m/s}) + 2(8.0\text{ kg})(-4.0\text{ m/s})}{14.0\text{ kg}} = \boxed{-5.5\text{ m/s}}$$

$$v_B' = v_A - v_B + v_A' = 6.5\text{ m/s} - (-4.0\text{ m/s}) - 5.5\text{ m/s} = \boxed{5.0\text{ m/s}}$$

(*c*) In this case, $v_A' = 0$.

$$m_A v_A + m_B v_B = m_B v_B' \quad \to$$

$$v_B' = \frac{m_A v_A + m_B v_B}{m_B} = \frac{(6.0\text{ kg})(6.5\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{8.0\text{ kg}} = 0.875\text{ m/s} \approx \boxed{0.9\text{ m/s}}$$

To check for "reasonableness," first note the final directions of motion. A has stopped, and B has reversed direction. This is reasonable. Secondly, calculate the change in kinetic energy.

$$\Delta\text{KE} = \tfrac{1}{2}m_B v_B'^2 - \left(\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2\right)$$

$$= \tfrac{1}{2}(8.0\text{ kg})(0.875\text{ m/s})^2 - \left[\tfrac{1}{2}(6.0\text{ kg})(6.5\text{ m/s})^2 + \tfrac{1}{2}(8.0\text{ kg})(-4.0\text{ m/s})^2\right] = -188\text{ J}$$

Since the system has lost kinetic energy and the directions are possible, this interaction is $\boxed{\text{reasonable}}$.

(*d*) In this case, $v'_B = 0$.

$$m_A v_A + m_B v_B = m_A v'_A \quad \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B v_B}{m_A} = \frac{(6.0\ \text{kg})(6.5\ \text{m/s}) + (8.0\ \text{kg})(-4.0\ \text{m/s})}{6.0\ \text{kg}} = 1.167\ \text{m/s} \approx \boxed{1\ \text{m/s}}$$

This answer is $\boxed{\text{not reasonable}}$, because A continues to move in its original direction while B has stopped. Thus A has somehow "passed through" B. If B has stopped, A should have rebounded and would have had a negative velocity.

(*e*) In this case, $v'_A = -4.0$ m/s.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_B = \frac{(6.0\ \text{kg})(6.5\ \text{m/s} - -4.0\ \text{m/s}) + (8.0\ \text{kg})(-4.0\ \text{m/s})}{8.0\ \text{kg}} = 3.875\ \text{m/s} \approx \boxed{4\ \text{m/s}}$$

The directions are reasonable, in that each object rebounds. Since the speed of both objects is smaller than in the perfectly elastic case (*b*), the system has lost kinetic energy. This interaction is $\boxed{\text{reasonable}}$.

(*f*) As quoted above, $\boxed{\text{the results for } (c) \text{ and } (e) \text{ are reasonable, but } (d) \text{ is not reasonable}}$.

3. Let A represent the cube of mass M and B represent the cube of mass m. Thus $m_A = M = 2m$ and $m_B = m$. Find the speed of A immediately before the collision, v_A, by using energy conservation.

$$Mgh = \tfrac{1}{2} M v_A^2 \quad \rightarrow \quad v_A = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(0.35\ \text{m})} = 2.619\ \text{m/s}$$

Use Eq. 7–7 for elastic collisions to obtain a relationship between the velocities in the collision. We have $v_B = 0$.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = v_A + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad m_A v_A = m_A v'_A + m_B (v_A + v'_A) \quad \rightarrow$$

$$2m v_A = 2m v'_A + m(v_A + v'_A) \quad \rightarrow \quad v'_A = \frac{v_A}{3} = \frac{\sqrt{2gh}}{3} = \frac{\sqrt{2(9.80\ \text{m/s}^2)(0.35\ \text{m})}}{3} = 0.873\ \text{m/s}$$

$$v'_B = v_A + v'_A = \tfrac{4}{3} v_A = 3.492\ \text{m/s}$$

Each mass is moving horizontally initially after the collision, so each has a vertical velocity of 0 as they start to fall. Use constant acceleration Eq. 2–11b with down as positive and the table top as the vertical origin to find the time of fall.

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2 \quad \rightarrow \quad H = 0 + 0 + \tfrac{1}{2} g t^2 \quad \rightarrow \quad t = \sqrt{2H/g}$$

Each cube then travels a horizontal distance found by $\Delta x = v_x \Delta t$.

$$\Delta x_M = v'_A \Delta t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{2}{3}\sqrt{hH} = \tfrac{2}{3}\sqrt{(0.35\ \text{m})(0.95\ \text{m})} = 0.3844\ \text{m} \approx \boxed{0.38\ \text{m}}$$

$$\Delta x_m = v'_B \Delta t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} = \tfrac{8}{3}\sqrt{hH} = \tfrac{8}{3}\sqrt{(0.35\ \text{m})(0.95\ \text{m})} = 1.538\ \text{m} \approx \boxed{1.5\ \text{m}}$$

4. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a one-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 7–7, with $\upsilon_A = 10.4\text{ km/s}$ and $\upsilon_B = \upsilon'_B = -9.6\text{ km/s}$.

$$\upsilon_A - \upsilon_B = -\upsilon'_A + \upsilon'_B \quad \rightarrow \quad \upsilon'_A = 2\upsilon_B - \upsilon_A = 2(-9.6\text{ km/s}) - 10.4\text{ km/s} = \boxed{-29.6\text{ km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed.

5. (*a*) Use Eq. 7–7, along with $\upsilon_B = 0$, to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \quad \rightarrow \quad \upsilon'_B = \upsilon_A + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B = m_A\upsilon'_A + m_B(\upsilon_A + \upsilon'_A) = m_A\upsilon'_A + m_B\upsilon_A + m_B\upsilon'_A \quad \rightarrow$$

$$m_A\upsilon_A - m_B\upsilon_A = m_A\upsilon'_A + m_B\upsilon'_A \quad \rightarrow \quad (m_A - m_B)\upsilon_A = (m_A + m_B)\upsilon'_A \quad \rightarrow \quad \upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A$$

Substitute this result into the result of Eq. 7–7.

$$\upsilon'_B = \upsilon_A + \upsilon'_A = \upsilon_A + \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \upsilon_A\frac{(m_A + m_B)}{(m_A + m_B)} + \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \upsilon_A\frac{2m_A}{(m_A + m_B)}$$

Thus we have $\boxed{\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A}$ and $\boxed{\upsilon'_B = \upsilon_A\frac{2m_A}{(m_A + m_B)}}$.

(*b*) If $m_A \ll m_B$, then approximate $m_A = 0$ when added to or subtracted from m_B.

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{(-m_B)}{(+m_B)}\upsilon_A = -\upsilon_A \qquad \upsilon'_B = \frac{2m_A\upsilon_A}{(m_A + m_B)} = 0$$

The result is $\boxed{\upsilon'_A = -\upsilon_A;\ \upsilon'_B = 0}$. An example of this is a ball bouncing off of the floor. The massive floor has no speed after the collision, and the velocity of the ball is reversed (if dissipative forces are not present).

(*c*) If $m_A \gg m_B$, then approximate $m_B = 0$ when added to or subtracted from m_A.

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{(m_A)}{(m_A)}\upsilon_A = \upsilon_A \qquad \upsilon'_B = \frac{2m_A\upsilon_A}{(m_A + m_B)} = \frac{2m_A\upsilon_A}{(m_A)} = 2\upsilon_A$$

The result is $\boxed{\upsilon'_A = \upsilon_A;\ \upsilon'_B = 2\upsilon_A}$. An example of this would be a golf club hitting a golf ball. The speed of the club immediately after the collision is essentially the same as its speed before the collision, and the golf ball takes off with twice the speed of the club.

(*d*) If $m_A = m_B$, then set $m_A = m_B = m$.

$$\upsilon'_A = \frac{(m - m)}{(m + m)}\upsilon_A = 0 \qquad \upsilon'_B = \frac{2m\upsilon_A}{(m + m)} = \frac{2m\upsilon_A}{2m} = \upsilon_A$$

The result is $\boxed{\upsilon'_A = 0;\ \upsilon'_B = \upsilon_A}$. An example of this is one billiard ball making a head-on collision with another. The first ball stops, and the second ball takes off with the same speed that the first one had.

8

ROTATIONAL MOTION

Responses to Questions

1. The reading on an odometer designed for 27-inch wheels increases by the circumference of a 27-inch wheel $(27\pi'')$ for every revolution of the wheel. If a 24-inch wheel is used, the odometer will still register $(27\pi'')$ for every revolution, but only $24\pi''$ of linear distance will have been traveled. Thus the odometer will read a distance that is farther than you actually traveled, by a factor of $27/24 = 1.125$. The odometer will read 12.5% too high.

2. (*a*) A point on the rim of a disk rotating with constant angular velocity has no tangential acceleration since the tangential speed is constant. It does have radial acceleration. Although the point's speed is not changing, its velocity is, since the velocity vector is changing direction. The point has a centripetal acceleration, which is directed radially inward.

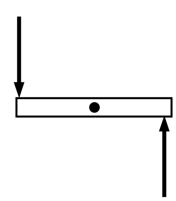
   (*b*) If the disk's angular velocity increases uniformly, the point on the rim will have both radial and tangential acceleration, since it is both moving in a circle and speeding up.

   (*c*) The magnitude of the radial component of acceleration will increase in case (*b*), but the tangential component will be constant. In case (*a*), neither component of linear acceleration will change.

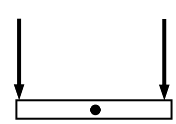
3. Since the torque involves the product of a force times its lever arm, a small force can exert a greater torque than a larger force if the small force has a large enough lever arm.

4. When you do a sit-up from a laying-down position, torque from your abdominal muscles must rotate the upper half of the body. The larger the moment of inertia of the upper half of the body, the more torque is needed, and thus the harder the sit-up is to do. With the hands behind the head, the moment of inertia of the upper half of the body is larger than with the hands out in front.

5. If the net force on a system is zero, the net torque need not be zero. Consider a uniform object with two equal forces on it, as shown in the first diagram. The net force on the object is zero (it would not start to translate under the action of these forces), but there is a net counterclockwise torque about the center of the rod (it would start to rotate under the action of these forces).

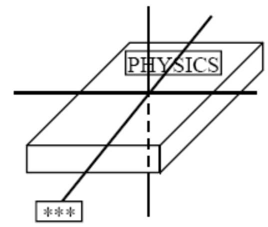


   If the net torque on a system is zero, the net force need not be zero. Consider an object with two equal forces on it, as shown in the second diagram. The net torque on the object is zero (it would not start to rotate under the action of these forces), but there is a net downward force on the rod (it would start to translate under the action of these forces).



6. Running involves rotating the leg about the point where it is attached to the rest of the body. Therefore, running fast requires the ability to change the leg's rotation easily. The smaller the moment of inertia of an object, the smaller the resistance to a change in its rotational motion. The closer the mass is to the axis of rotation, the smaller the moment of inertia. Concentrating flesh and muscle high and close to the body minimizes the moment of inertia and increases the angular acceleration possible for a given torque, improving the ability to run fast.

7. Refer to the diagram of the book laying on a table. The moment of inertia about the "starred" axis (the axis parallel to the longest dimension of the book) will be the smallest. Relative to this axis, more of the mass is concentrated close to the axis.



8. No, the mass cannot be considered as concentrated at the CM when considering rotational motion. If all of the mass were at the CM, then the object would have a rotational inertia of 0. That means it could not have any rotational kinetic energy or angular momentum, for example. The distribution of the mass is fundamental when describing rotational motion.

9. The moment of inertia will be larger when considering an axis through a point on the edge of the disk, because most the mass of the disk will be farther from the axis of rotation than it was with the original axis position.

10. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For a solid ball, $I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = v/R$, so

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\tfrac{2}{5}MR^2v^2/R^2 \quad \rightarrow \quad v = \sqrt{10gh/7}$$

This speed is independent of the angle of the incline, so both balls will have the same speed at the bottom. The ball on the incline with the smaller angle will take more time to reach the bottom than the ball on the incline with the larger angle.

11. The two spheres have different rotational inertias. The sphere that is hollow will have a larger rotational inertia than the solid sphere. If the two spheres are allowed to roll down an incline without slipping, the sphere with the smaller moment of inertia (the solid one) will reach the bottom of the ramp first. See Question 12 below for a detailed explanation of why this happens.

12. (*a*) The sphere will reach the bottom first because it has a smaller rotational inertial. A detailed analysis of that is given below.

(*b*) The sphere will have the greater speed at the bottom, so it will have more translational kinetic energy than the cylinder.

(*c*) Both will have the same energy at the bottom, because they both started with the same potential energy at the top of the incline.

(*d*) The cylinder will have the greater rotational kinetic energy at the bottom, because it has less translational kinetic energy than the sphere.

Here is a detailed analysis of the motion:

Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. If the objects roll without slipping, then $\omega = \upsilon/R$, so $Mgh = \frac{1}{2}M\upsilon^2 + \frac{1}{2}I(\upsilon/R)^2 \quad \rightarrow \quad \upsilon = \sqrt{\dfrac{2Mgh}{M + I/R^2}}$. For a solid ball, $I = \frac{2}{5}MR^2$, and for a cylinder, $I = \frac{1}{2}MR^2$. Thus $\upsilon_{\text{sphere}} = \sqrt{10gh/7}$ and $\upsilon_{\text{cyl}} = \sqrt{4gh/3}$. Since $\upsilon_{\text{sphere}} > \upsilon_{\text{cyl}}$, the sphere has the greater speed at the bottom. That is true for any amount of height change, so the sphere is always moving faster than the cylinder after they start to move. Thus the sphere will reach the bottom first. Since both objects started with the same potential energy, both have the same total kinetic energy at the bottom. But since both objects have the same mass and the cylinder is moving slower, the cylinder has the smaller translational KE and thus the greater rotational KE. Since rotational kinetic energy is $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$, then $\underset{\text{sphere}}{\text{KE}_{\text{rot}}} = \frac{2}{7}mgh$ and $\underset{\text{cylinder}}{\text{KE}_{\text{rot}}} = \frac{1}{3}mgh$.

13. The long rod increases the rotational inertia of the walkers. If a walker gets off-center from the tightrope, gravity will exert a torque on the walker, causing the walker to rotate with their feet as a pivot point. With a larger rotational inertia, the angular acceleration caused by that gravitational torque will be smaller, and the walker will therefore have more time to compensate.

    The long rod also allows the walkers to make small shifts in their center of mass to bring themselves back to being centered on the tightrope. It is much easier for a walker to move a long, narrow object with the precision needed for small adjustments than a short, heavy object like a barbell.

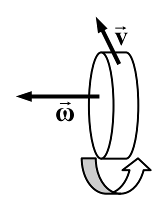
14. Momentum and angular momentum are conserved for closed systems—systems in which there are no external forces or torques applied to the system. Probably no macroscopic systems on Earth are truly closed, so external forces and torques (like those applied by air friction, for example) affect the systems over time.

15. In order to do a somersault, the diver needs some initial angular momentum when she leaves the diving board, because angular momentum will be conserved during the free-fall motion of the dive. She cannot exert a torque about her CM on herself in isolation, so if there is no angular momentum initially, there will be no rotation during the rest of the dive.

16. Once the motorcycle leaves the ground, there is no net torque on it and angular momentum must be conserved. If the throttle is on, the rear wheel will spin faster as it leaves the ground because there is no torque from the ground acting on it. The front of the motorcycle must rise up, or rotate in the direction opposite the rear wheel, in order to conserve angular momentum.

17. While in mid-air, the shortstop cannot exert a torque on himself, so his angular momentum will be conserved. If the upper half of his body (including his hips) rotates in a certain direction during the throwing motion, then to conserve angular momentum, the lower half of his body (including his legs) will rotate in the opposite direction.

18. See the diagram. To the left is west, the direction of the angular velocity. The direction of the linear velocity of a point on the top of the wheel would be north, into the page. If the angular acceleration is east, which is opposite the angular velocity, the wheel is slowing down—its angular speed is decreasing. The tangential linear acceleration of the point on top will be opposite to its linear velocity—it will point south.



19. Using the right-hand rule, point the fingers in the direction of the Earth's rotation, from west to east. Then the thumb points north. Thus the Earth's angular velocity points along its axis of rotation, toward the North Star.

20. Consider a helicopter in the air with the rotor spinning. To change the rotor's angular speed, a torque must be applied to the rotor. That torque has to come from the helicopter. By Newton's third law, an equal and opposite torque will be applied by the rotor to the helicopter. Any change in rotor speed would therefore cause the body of the helicopter to spin in a direction opposite to the change in the rotor's angular velocity.

Some large helicopters have two rotor systems, spinning in opposite directions. That makes any change in the speed of the rotor pair require a net torque of zero, so the helicopter body would not tend to spin. Smaller helicopters have a tail rotor that rotates in a vertical plane, causing a sideways force (thrust) on the tail of the helicopter in the opposite direction of the tendency of the tail to spin.

Responses to MisConceptual Questions

1. (*c*) A common misconception is that if the riders complete the revolution at the same time, they must have the same linear velocities. The time for a rotation is the same for both riders, but Bonnie, at the outer edge, travels in a larger circle than Jill. Bonnie therefore has a greater linear velocity.

2. (*b*) Students may think that the rider would travel half the distance in half the time. This would be true if the object had constant angular speed. However, it is accelerating, so it will travel a shorter distance, $\frac{1}{4}\theta,$ in the first half of the time.

3. (*b*) A common error is to think that increasing the radius of the tires would increase the speed measured by the speedometer. This is actually backward. Increasing the size of the tires will cause the car to travel faster than it would with smaller tires, when the wheels have the same angular speed. Therefore, the speed of the car will be greater than the speed measured by the speedometer.

4. (*c*) Torque is the product of the lever arm and the component of the force perpendicular to the arm. Although the 1000-N force has the greatest magnitude, it acts at the pivot. Thus, the lever arm is zero, and the torque is also zero. The 800-N force is parallel to the lever arm and also exerts no torque. Of the three 500-N forces, (*c*) is both perpendicular to the lever arm and farthest from the pivot.

5. (*c, e, f*) Equations 8–10 show that there are three ways in which the torque can be written. It can be the product of the force, the lever arm, and the sine of the angle between them as in answer (*c*). It can be the product of the force and the component of the lever arm perpendicular to the force, as in answer (*e*). It can also be written as the product of the lever arm and the force perpendicular to the lever arm, as in answer (*f*). Doing the calculations shows that all three torques are equal.

6. (*b*) The location of the mass is very important. Imagine taking the material from the solid sphere and compressing it outward to turn the solid sphere into a hollow sphere of the same mass and radius. As you do this, you would be moving mass farther from the axis of rotation, which would increase the moment of inertia. Therefore, the hollow sphere has a greater moment of inertia than the solid sphere.

7. (*b*) If you don't consider how the location of the mass affects the moment of inertia, you might think that the two kinetic energies are nearly the same. However, a hollow cylinder has twice the moment of inertia as a solid cylinder of the same mass and radius. The kinetic energy is proportional to the moment of inertia, so at the same angular speed the wheel with the spokes will have nearly double the kinetic energy of the solid cylinder. It is only "nearly double" because some of the mass is in the spokes, so the moment of inertia is not exactly double.

8. (*b*) It takes energy to rotate the ball. If some of the 1000 J goes into rotation, less is available for linear kinetic energy, so the rotating ball will travel slower.

9. (*b*) If you do not take into account the energy of rotation, you would answer that the two objects would rise to the same height. Another common misconception is that the mass and/or diameter of the objects will affect how high they travel. When using conservation of energy to relate the total initial kinetic energy (translational and rotational) to the final potential energy, the mass and radius of the objects cancel out. The thin hoop has a larger moment of inertia (for a given mass and radius) than the solid sphere. It will therefore have a greater total initial kinetic energy and will travel to a greater height on the ramp.

10. (*a*) Because there is no external torque, students might think that the angular speed would remain constant. But with no external torque, the angular momentum must remain constant. The angular momentum is the product of the moment of inertia and the angular speed. As the string is shortened, the moment of inertia of the block decreases. Thus, the angular speed increases.

11. (*a*) Work is done on the object, so its kinetic energy increases. Thus the tangential velocity has to increase. Another way to consider the problem is that $\text{KE} = \sqrt{L^2/2I}$. As in Question 10, the angular momentum is constant and the rotation inertial decreases. Thus the kinetic energy (and the speed) has to increase.

12. (*a*) No net torque acts on the Earth, so the angular momentum is conserved. As people move toward the equator their distance from the Earth's axis increases. This increases the moment of inertia of the Earth. For angular momentum to be conserved, the angular speed must decrease, and it will take longer for the Earth to complete a full rotation.

13. (*c*) Students might mistakenly reason that since no net torque acts on you and your moment of inertia decreases as the masses are released, your angular speed should increase. This reasoning is erroneous because the angular momentum of the system of you and the masses is conserved. As the masses fall, they carry angular momentum with them. If you consider you and the masses as two separate systems, each with angular momentum from their moments of inertia and angular speed, it is easy to see that by dropping the masses, no net external torque acts on you and your moment of inertia does not change, so your angular speed will not change. The angular momentum of the masses also does not change until they hit the ground and friction (external torque) stops their motion.

Solutions to Problems

1. (*a*) $(45.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/4 \text{ rad}} = \boxed{0.785 \text{ rad}}$

(*b*) $(60.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/3 \text{ rad}} = \boxed{1.05 \text{ rad}}$

(*c*) $(90.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/2 \text{ rad}} = \boxed{1.57 \text{ rad}}$

(*d*) $(360.0°)(2\pi \text{ rad}/360°) = \boxed{2\pi \text{ rad}} = \boxed{6.283 \text{ rad}}$

(*e*) $(445°)(2\pi \text{ rad}/360°) = \boxed{89\pi/36 \text{ rad}} = \boxed{7.77 \text{ rad}}$

2. The subtended angle (in radians) is the diameter of the Sun divided by the Earth–Sun distance.

$$\theta = \frac{\text{diameter of Sun}}{r_{\text{Earth-Sun}}} \rightarrow$$

$$\text{radius of Sun} = \tfrac{1}{2}\theta\, r_{\text{Earth-Sun}} = \tfrac{1}{2}(0.5°)\left(\frac{\pi \text{ rad}}{180°}\right)(1.5\times10^{11} \text{ m}) = 6.545\times10^{8} \text{ m} \approx \boxed{7\times10^{8} \text{ m}}$$

3. We find the diameter of the spot from the definition of radian angle measure.

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \rightarrow \text{diameter} = \theta\, r_{\text{Earth-Moon}} = (1.4\times10^{-5} \text{ rad})(3.8\times10^{8} \text{ m}) = \boxed{5300 \text{ m}}$$

4. The initial angular velocity is $\omega_0 = \left(6500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 681$ rad/s. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681 \text{ rad/s}}{4.0 \text{ s}} = \boxed{-170 \text{ rad/s}^2}$$

5. (*a*) We convert rpm to rad/s.

$$\omega = \left(\frac{7200 \text{ rev}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 753.98 \text{ rad/s} \approx \boxed{750 \text{ rad/s}}$$

(*b*) To find the speed, we use the radius of the reading head location along with Eq. 8–4.

$$v = r\omega = (3.00\times10^{-2} \text{ m})(753.98 \text{ rad/s}) = 22.62 \text{ m/s} \approx \boxed{23 \text{ m/s}}$$

(*c*) We convert the speed of the point on the platter from m/s to bits/s, using the distance per bit.

$$(22.62 \text{ m/s})\left(\frac{1 \text{ bit}}{0.50\times10^{-6} \text{ m}}\right) = \boxed{4.5\times10^{7} \text{ bits/s}}$$

6. The ball rolls $2\pi r = \pi d$ of linear distance with each revolution.

$$12.0 \text{ rev}\left(\frac{\pi d \text{ m}}{1 \text{ rev}}\right) = 3.5 \text{ m} \rightarrow d = \frac{3.5 \text{ m}}{12.0\pi} = \boxed{9.3\times10^{-2} \text{ m}}$$

7. (*a*) We convert rpm to rad/s.

$$\omega = \left(\frac{2200 \text{ rev}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 230.4 \text{ rad/s} \approx \boxed{230 \text{ rad/s}}$$

(*b*) To find the speed and acceleration, we use the full radius of the wheel, along with Eqs. 8–4 and 8–6.

$$v = \omega r = (230.4 \text{ rad/s})\left(\frac{0.35 \text{ m}}{2}\right) = \boxed{4.0\times10^{1} \text{ m/s}}$$

$$a_{\text{R}} = \omega^2 r = (230.4 \text{ rad/s})^2\left(\frac{0.35 \text{ m}}{2}\right) = \boxed{9300 \text{ m/s}^2}$$

8. In each revolution, the wheel moves forward a distance equal to its circumference, πd.

$$\Delta x = N_{\text{rev}}(\pi d) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{9200 \text{ m}}{\pi(0.68 \text{ m})} = \boxed{4300 \text{ rev}}$$

9. The angular velocity is expressed in radians per second. The second hand makes 1 revolution every 60 seconds, the minute hand makes 1 revolution every 60 minutes, and the hour hand makes 1 revolution every 12 hours.

(*a*) Second hand: $\omega = \left(\dfrac{1\text{ rev}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{\dfrac{\pi}{30}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.05\times10^{-1}\dfrac{\text{rad}}{\text{s}}}$

(*b*) Minute hand: $\omega = \left(\dfrac{1\text{ rev}}{60\text{ min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = \boxed{\dfrac{\pi}{1800}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.75\times10^{-3}\dfrac{\text{rad}}{\text{s}}}$

(*c*) Hour hand: $\omega = \left(\dfrac{1\text{ rev}}{12\text{ h}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = \boxed{\dfrac{\pi}{21{,}600}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.45\times10^{-4}\dfrac{\text{rad}}{\text{s}}}$

(*d*) The angular acceleration in each case is $\boxed{0}$, since the angular velocity is constant.

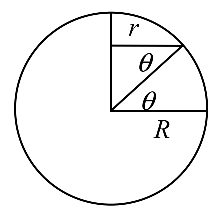
10. The angular speed of the merry-go-round is $2\pi\text{ rad}/4.0\text{ s} = 1.57\text{ rad/s}$.

(*a*) $v = \omega r = (1.57\text{ rad/s})(1.2\text{ m}) = \boxed{1.9\text{ m/s}}$

(*b*) The acceleration is radial. There is no tangential acceleration.

$$a_R = \omega^2 r = (1.57\text{ rad/s})^2(1.2\text{ m}) = \boxed{3.0\text{ m/s}^2\text{ toward the center}}$$

11. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R\cos\theta$, where R is the radius of the Earth, and r is the radius at latitude θ.



(*a*) $v = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times10^6\text{ m}) = \boxed{464\text{ m/s}}$

(*b*) $v = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times10^6\text{ m})\cos 66.5^\circ = \boxed{185\text{ m/s}}$

(*c*) $v = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times10^6\text{ m})\cos 42.0^\circ = \boxed{345\text{ m/s}}$

12. (*a*) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\text{ rad}}{1\text{ year}}\right)\left(\frac{1\text{ year}}{3.16\times10^7\text{ s}}\right) = \boxed{1.99\times10^{-7}\text{ rad/s}}$$

(*b*) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right) = \boxed{7.27\times10^{-5}\text{ rad/s}}$$

13. The centripetal acceleration is given by $a_R = \omega^2 r$. Solve for the angular velocity.

$$\omega = \sqrt{\frac{a_R}{r}} = \sqrt{\frac{(100{,}000)(9.80\text{ m/s}^2)}{0.080\text{ m}}} = 3500\frac{\text{rad}}{\text{s}}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{3.3\times10^4\text{ rpm}}$$

14. Convert the rpm values to angular velocities.

$$\omega_0 = \left(120\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 12.57\text{ rad/s}$$

$$\omega = \left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 29.32\text{ rad/s}$$

(*a*) The angular acceleration is found from Eq. 8–9a.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{29.32\text{ rad/s} - 12.57\text{ rad/s}}{4.0\text{ s}} = 4.188\text{ rad/s}^2 \approx \boxed{4.2\text{ rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 12.57\text{ rad/s} + (4.188\text{ rad/s}^2)(2.0\text{ s}) = 20.95\text{ rad/s}$$

The instantaneous radial acceleration is given by $a_R = \omega^2 r$.

$$a_R = \omega^2 r = (20.95\text{ rad/s})^2\left(\frac{0.61\text{ m}}{2}\right) = \boxed{130\text{ m/s}^2}$$

The tangential acceleration is given by $a_{tan} = \alpha r$.

$$a_{tan} = \alpha r = (4.188\text{ rad/s}^2)\left(\frac{0.61\text{ m}}{2}\right) = \boxed{1.3\text{ m/s}^2}$$

15. (*a*) The angular acceleration can be found from Eq. 8–3a. The initial angular frequency is 0 and the final frequency is 1 rpm.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(1.0\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1.0\text{ min}}{60\text{ s}}\right) - 0}{(12\text{ min})\left(\frac{60\text{ s}}{1.0\text{ min}}\right)} = 1.454\times10^{-4}\text{ rad/s}^2 \approx \boxed{1.5\times10^{-4}\text{ rad/s}^2}$$

(*b*) After 6.0 min (360 s), the angular speed is as follows:

$$\omega = \omega_0 + \alpha t = 0 + (1.454\times10^{-4}\text{ rad/s}^2)(360\text{ s}) = 5.234\times10^{-2}\text{ rad/s}$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$a_{tan} = \alpha R = (1.454\times10^{-4}\text{ rad/s}^2)(4.25\text{ m}) = \boxed{6.2\times10^{-4}\text{ m/s}^2}$$

$$a_R = \omega^2 R = (5.234\times10^{-2}\text{ rad/s})^2(4.25\text{ m}) = \boxed{1.2\times10^{-2}\text{ m/s}^2}$$

16. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$v_1 = v_2 \quad\rightarrow\quad \omega_1 R_1 = \omega_2 R_2 \quad\rightarrow\quad \boxed{\omega_1/\omega_2 = R_2/R_1}$$

17. (*a*) For constant angular acceleration:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{1200\text{ rev/min} - 3500\text{ rev/min}}{2.5\text{ s}} = \frac{-2300\text{ rev/min}}{2.5\text{ s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= -96.34\text{ rad/s}^2 \approx \boxed{-96\text{ rad/s}^2}$$

(*b*) For the angular displacement, we assume constant angular acceleration.

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(3500 \text{ rev/min} + 1200 \text{ rev/min})(2.5 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{98 \text{ rev}}$$

18. The angular displacement can be found from Eq. 8–9d.

$$\theta = \bar{\omega}t = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(0 + 15{,}000 \text{ rev/min})(240 \text{ s})(1 \text{ min}/60 \text{ s}) = \boxed{3.0\times 10^4 \text{ rev}}$$

19. (*a*) The angular acceleration can be found from Eq. 8–9b with $\omega_0 = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2(23 \text{ rev})}{(1.0 \text{ min})^2} = \boxed{46 \text{ rev/min}^2}$$

(*b*) The final angular speed can be found from $\theta = \tfrac{1}{2}(\omega_0 + \omega)t$, with $\omega_0 = 0$.

$$\omega = \frac{2\theta}{t} - \omega_0 = \frac{2(23 \text{ rev})}{1.0 \text{ min}} = \boxed{46 \text{ rpm}}$$

20. (*a*) The angular acceleration can be found from Eq. 8–9c.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (850 \text{ rev/min})^2}{2(1250 \text{ rev})} = \left(-289 \frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 = \boxed{-0.50 \frac{\text{rad}}{\text{s}^2}}$$

(*b*) The time to come to a stop can be found from $\theta = \tfrac{1}{2}(\omega_0 + \omega)t$.

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2(1250 \text{ rev})}{850 \text{ rev/min}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 176.5 \text{ s} \approx \boxed{180 \text{ s}}$$

21. Use Eq. 8–9d combined with Eq. 8–2a.

$$\bar{\omega} = \frac{\omega + \omega_0}{2} = \frac{240 \text{ rpm} + 360 \text{ rpm}}{2} = 300 \text{ rpm}$$

$$\theta = \bar{\omega}\, t = \left(300 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)(6.8 \text{ s}) = 34 \text{ rev}$$

Each revolution corresponds to a circumference of travel distance.

$$34 \text{ rev}\left[\frac{\pi(0.31 \text{ m})}{1 \text{ rev}}\right] = \boxed{33 \text{ m}}$$

$\boxed{22}$. (*a*) The angular acceleration can be found from $\omega^2 = \omega_0^2 + 2\alpha\theta$, with the angular velocities being found from $\omega = v/r$.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{(v^2 - v_0^2)}{2r^2\theta} = \frac{\left[(55 \text{ km/h})^2 - (95 \text{ km/h})^2\right]\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)^2}{2(0.40 \text{ m})^2(75 \text{ rev})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)}$$

$$= -3.070 \text{ rad/s}^2 \approx \boxed{-3.1 \text{ rad/s}^2}$$

(*b*) The time to stop can be found from $\omega = \omega_0 + \alpha t$, with a final angular velocity of 0.

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{v - v_0}{r\alpha} = \frac{-(55\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{(0.40\text{ m})(-3.070\text{ rad/s}^2)} = 12.44\text{ s} \approx \boxed{12\text{ s}}$$

(*c*) We first find the total angular displacement of the tires as they slow from 55 km/h to rest, and then convert the angular displacement to a linear displacement, assuming that the tires are rolling without slipping.

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \quad \rightarrow$$

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{0 - \left(\dfrac{v_0}{r}\right)^2}{2\alpha} = -\frac{\left[\dfrac{(55\text{ km/h})}{0.40\text{ m}}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(-3.070\text{ rad/s}^2)} = 237.6\text{ rad}$$

$$\Delta x = r\Delta\theta = (0.40\text{ m})(237.6\text{ rad}) = \boxed{95\text{ m}}$$

For the total distance, add the distance moved during the time the car slows from 95 km/h to 55 km/h. The tires made 75 revolutions, so that distance is as follows:

$$\Delta x = r\Delta\theta = (0.40\text{ m})(75\text{ rev})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 188\text{ m}$$

The total distance would be the sum of the two distances, 283 m.

23. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.

(*a*) $a_{\substack{\text{tan}\\ \text{small}}} = a_{\substack{\text{tan}\\ \text{large}}} \quad \rightarrow \quad \alpha_{\text{small}} r_{\text{small}} = \alpha_{\text{large}} r_{\text{large}} \quad \rightarrow$

$$\alpha_{\text{large}} = \alpha_{\text{small}} \frac{r_{\text{small}}}{r_{\text{large}}} = (7.2\text{ rad/s}^2)\left(\frac{2.0\text{ cm}}{27.0\text{ cm}}\right) = 0.5333\text{ rad/s}^2 \approx \boxed{0.53\text{ rad/s}^2}$$

(*b*) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 8–9a.

$$\omega = \left(65\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 6.807\text{ rad/s}$$

$$\omega = \omega_0 + \alpha t \quad \rightarrow \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{6.807\text{ rad/s}}{0.5333\text{ rad/s}^2} = 12.76\text{ s} \approx \boxed{13\text{ s}}$$

24. (*a*) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$\tau = r_\perp F = r_\perp mg = (0.17\text{ m})(52\text{ kg})(9.80\text{ m/s}^2) = 86.6\text{ m}\cdot\text{N} \approx \boxed{87\text{ m}\cdot\text{N}}$$

(*b*) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upward on the handle bars as she pedals, which will increase the downward force of her legs.

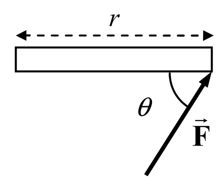
$\boxed{25.}$ Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by the following:

$$\tau_{\substack{\text{applied}\\ \text{forces}}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) = -1.8\text{ m}\cdot\text{N}$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, so the frictional torque is counterclockwise. Thus the net torque is as follows:

$$\tau_{\text{net}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) + 0.60\text{ m}\cdot\text{N} = -1.2\text{ m}\cdot\text{N}$$
$$= \boxed{1.2\text{ m}\cdot\text{N, clockwise}}$$

26. The torque is calculated by $\tau = rF\sin\theta$. See the diagram, from the top view.



(*a*) For the first case, $\theta = 90°$.

$$\tau = rF\sin\theta = (0.96\text{ m})(42\text{ N})\sin 90° = 40.32\text{ m}\cdot\text{N} \approx \boxed{4.0\times10^{1}\text{ m}\cdot\text{N}}$$

(*b*) For the second case, $\theta = 60.0°$.

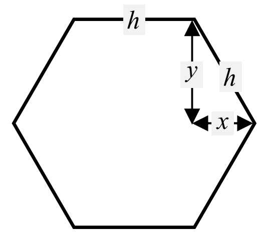
$$\tau = rF\sin\theta = (0.96\text{ m})(42\text{ N})\sin 60.0° = 34.92\text{ m}\cdot\text{N} \approx \boxed{35\text{ m}\cdot\text{N}}$$

27. There is a counterclockwise torque due to the force of gravity on the left block and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$\sum\tau = mg\ell_2 - mg\ell_1 = \boxed{mg(\ell_2 - \ell_1)\text{, clockwise}}$$

28. The force required to produce the torque can be found from $\tau = rF\sin\theta$. The force is applied perpendicularly to the wrench, so $\theta = 90°$.

$$F = \frac{\tau}{r} = \frac{95\text{ m}\cdot\text{N}}{0.28\text{ m}} = 339.3\text{ N} \approx \boxed{340\text{ N}}$$



The net torque still must be $95\text{ m}\cdot\text{N}$. This is produced by six forces, one at each of the six points. We assume for our estimate that those forces are also perpendicular to their lever arms. From the diagram, we estimate the lever arm as follows, and then calculate the force at each point:

$$\text{Lever arm} = r = \tfrac{1}{2}h + x = \tfrac{1}{2}\left(\frac{y}{\cos 30°}\right) + y\tan 30°$$
$$= y\left(\frac{1}{2\cos 30°} + \tan 30°\right) = (7.5\times10^{-3}\text{ m})(1.15)$$

$$\tau_{\text{net}} = (6F_{\text{point}})r \quad\rightarrow\quad F_{\text{point}} = \frac{\tau}{6r} = \frac{95\text{ m}\cdot\text{N}}{6(7.5\times10^{-3}\text{ m})(1.15)} = 1835.7\text{ N} \approx \boxed{1800\text{ N}}$$

29. For each torque, use Eq. 8–10c. Take counterclockwise torques to be positive.

(*a*) Each force has a lever arm of 1.0 m.

$$\tau_{\substack{\text{about}\\ \text{C}}} = -(1.0\text{ m})(56\text{ N})\sin 32° + (1.0\text{ m})(52\text{ N})\sin 58° = 14.42\text{ m}\cdot\text{N} \approx \boxed{14\text{ m}\cdot\text{N}}$$

(*b*) The force at C has a lever arm of 1.0 m, and the force at the top has a lever arm of 2.0 m.

$$\tau_{\substack{\text{about}\\ \text{P}}} = -(2.0\text{ m})(56\text{ N})\sin 32° + (1.0\text{ m})(65\text{ N})\sin 45° = -13.39\text{ m}\cdot\text{N} \approx \boxed{-13\text{ m}\cdot\text{N}}$$

The negative sign indicates a clockwise torque.

30. For a sphere rotating about an axis through its center, the moment of inertia is as follows:

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(10.8\text{ kg})(0.648\text{ m})^2 = \boxed{1.81\text{ kg}\cdot\text{m}^2}$$

31. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by $I = MR^2$.

$$I = MR^2 = (1.1\text{ kg})\left(\tfrac{1}{2}(0.67\text{ m})\right)^2 = \boxed{0.12\text{ kg}\cdot\text{m}^2}$$

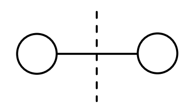
The hub mass can be ignored because its distance from the axis of rotation is very small, so it has a very small rotational inertia.

32. The torque required is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 8–9a. Use the moment of inertia of a solid cylinder.

$$\omega = \omega_0 + \alpha t \quad\rightarrow\quad \alpha = \omega/t$$

$$\tau = I\alpha = \left(\tfrac{1}{2}MR_0^2\right)\left(\frac{\omega}{t}\right) = \frac{MR_0^2\omega}{2t} = \frac{(31{,}000\text{ kg})(7.0\text{ m})^2(0.68\text{ rad/s})}{2(34\text{ s})} = \boxed{1.5\times10^4\text{ m}\cdot\text{N}}$$

33. The oxygen molecule has a “dumbbell” geometry, as though it rotates about the dashed line shown in the diagram. If the total mass is M, then each atom has a mass of $M/2$. If the distance between them is d, then the distance from the axis of rotation to each atom is $d/2$. Treat each atom as a particle for calculating the moment of inertia.



$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \tfrac{1}{4}Md^2 \quad\rightarrow$$

$$d = \sqrt{4I/M} = \sqrt{4(1.9\times10^{-46}\text{ kg}\cdot\text{m}^2)/(5.3\times10^{-26}\text{ kg})} = \boxed{1.2\times10^{-10}\text{ m}}$$

34. (*a*) The moment of inertia of a cylinder is $\tfrac{1}{2}MR^2$.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.380\text{ kg})(0.0850\text{ m})^2 = 1.373\times10^{-3}\text{ kg}\cdot\text{m}^2 \approx \boxed{1.37\times10^{-3}\text{ kg}\cdot\text{m}^2}$$

(*b*) The wheel slows down “on its own” from 1500 rpm to rest in 55.0 s. This is used to calculate the frictional torque.

$$\tau_{\text{fr}} = I\alpha_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} = (1.373\times10^{-3}\text{ kg}\cdot\text{m}^2)\frac{(0-1500\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})}{55.0\text{ s}}$$

$$= -3.921\times10^{-3}\text{ m}\cdot\text{N}$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$\sum\tau = \tau_{\text{applied}} + \tau_{\text{fr}} = I\alpha \quad\rightarrow\quad \tau_{\text{applied}} = I\alpha - \tau_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} - \tau_{\text{fr}}$$

$$= (1.373\times10^{-3}\text{ kg}\cdot\text{m}^2)\frac{(1750\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})}{5.00\text{ s}} - (-3.921\times10^{-3}\text{ m}\cdot\text{N})$$

$$= \boxed{5.42\times10^{-2}\text{ m}\cdot\text{N}}$$

35. (*a*) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\tau = I\alpha = MR^2\alpha = MR^2\frac{a_{\tan}}{R} = MRa_{\tan} = (3.6\text{ kg})(0.31\text{ m})(7.0\text{ m/s}^2)$$

$$= 7.812\text{ m}\cdot\text{N} \approx \boxed{7.8\text{ m}\cdot\text{N}}$$

(*b*) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_{\perp} \quad \to \quad F = \tau/r_{\perp} = 7.812 \text{ m}\cdot\text{N}/(2.5\times10^{-2}\text{ m}) = \boxed{310 \text{ N}}$$

36. (*a*) The angular acceleration can be found from the following:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = \frac{v/r}{t} = \frac{(8.5 \text{ m/s})/(0.31 \text{ m})}{0.38 \text{ s}} = 72.16 \text{ rad/s}^2 \approx \boxed{72 \text{ rad/s}^2}$$

(*b*) The force required can be found from the torque, since $\tau = Fr\sin\theta$. In this situation the force is perpendicular to the lever arm, so $\theta = 90°$. The torque is also given by $\tau = I\alpha$, where I is the moment of inertia of the arm–ball combination. Equate the two expressions for the torque, and solve for the force.

$$Fr\sin\theta = I\alpha$$

$$F = \frac{I\alpha}{r\sin\theta} = \frac{m_{\text{ball}}d_{\text{ball}}^2 + \frac{1}{3}m_{\text{arm}}\ell_{\text{arm}}^2}{r\sin 90°}\alpha$$

$$= \frac{(1.00 \text{ kg})(0.31 \text{ m})^2 + \frac{1}{3}(3.7 \text{ kg})(0.31 \text{ m})^2}{(0.025 \text{ m})}(72.16 \text{ rad/s}^2) = 619.5 \text{ N} \approx \boxed{620 \text{ N}}$$

37. The torque is calculated from $\tau = I\alpha$. The rotational inertia of a rod about its end is $I = \frac{1}{3}M\ell^2$.

$$\tau = I\alpha = \frac{1}{3}M\ell^2\frac{\Delta\omega}{\Delta t} = \frac{1}{3}(0.90 \text{ kg})(0.95 \text{ m})^2\frac{(2.6 \text{ rev/s})(2\pi \text{ rad/rev})}{0.20 \text{ s}} = 22.12 \text{ m}\cdot\text{N} \approx \boxed{22 \text{ m}\cdot\text{N}}$$

38. (*a*) The small ball can be treated as a particle for calculating its moment of inertia.

$$I = MR^2 = (0.350 \text{ kg})(1.2 \text{ m})^2 = 0.504 \text{ kg}\cdot\text{m}^2 \approx \boxed{0.50 \text{ kg}\cdot\text{m}^2}$$

(*b*) To keep a constant angular velocity, the net torque must be zero, so the torque needed is the same magnitude as the torque caused by friction.

$$\sum\tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \quad \to \quad \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}}r = (0.020 \text{ N})(1.2 \text{ m}) = \boxed{2.4\times10^{-2} \text{ m}\cdot\text{N}}$$

39. (*a*) To calculate the moment of inertia about the *y* axis (vertical), use the following:

$$I = \sum M_{\text{i}}R_{\text{ix}}^2 = m(0.50 \text{ m})^2 + M(0.50 \text{ m})^2 + m(1.00 \text{ m})^2 + M(1.00 \text{ m})^2$$

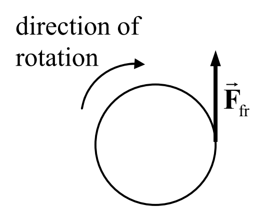
$$= (m+M)\left[(0.50 \text{ m})^2 + (1.00 \text{ m})^2\right] = (5.6 \text{ kg})\left[(0.50 \text{ m})^2 + (1.00 \text{ m})^2\right] = \boxed{7.0 \text{ kg}\cdot\text{m}^2}$$

(*b*) To calculate the moment of inertia about the *x* axis (horizontal), use the following:

$$I = \sum M_{\text{i}}R_{\text{iy}}^2 = (2m + 2M)(0.25 \text{ m})^2 = 2(5.6 \text{ kg})(0.25 \text{ m})^2 = \boxed{0.70 \text{ kg}\cdot\text{m}^2}$$

(*c*) Because of the larger *I* value, it is ten times harder to accelerate the array about the $\boxed{\text{vertical axis}}$.

40. (*a*) The torque exerted by the frictional force is $\tau = rF_{\text{fr}}\sin\theta$. The force of friction is assumed to be tangential to the clay, so $\theta = 90°$.



$$\tau_{\text{total}} = rF_{\text{fr}}\sin\theta = \left(\tfrac{1}{2}(0.090 \text{ m})\right)(1.5 \text{ N})\sin 90° = 0.0675 \text{ m}\cdot\text{N}$$

$$\approx \boxed{0.068 \text{ m}\cdot\text{N}}$$

(*b*) The time to stop is found from $\omega = \omega_0 + \alpha t$, with a final angular velocity of 0. The angular acceleration can be found from $\tau_{\text{total}} = I\alpha$. The net torque (and angular acceleration) is negative since the object is slowing.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{\omega - \omega_o}{\tau / I} = \frac{0 - (1.6 \text{ rev/s})(2\pi \text{ rad/rev})}{(-0.0675 \text{ m}\cdot\text{N})/(0.11 \text{ kg}\cdot\text{m}^2)} = 16.38 \text{ s} \approx \boxed{16 \text{ s}}$$

41. The torque needed is the moment of inertia of the system (merry-go-round and children) times the angular acceleration of the system. Let the subscript "mgr" represent the merry-go-round.

$$\tau = I\alpha = (I_{\text{mgr}} + I_{\text{children}})\frac{\Delta\omega}{\Delta t} = (\tfrac{1}{2}M_{\text{mgr}}R^2 + 2m_{\text{child}}R^2)\frac{\omega - \omega_0}{t}$$

$$= \left[\tfrac{1}{2}(560 \text{ kg}) + 2(25 \text{ kg})\right](2.5 \text{ m})^2 \frac{(15 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{10.0 \text{ s}}$$

$$= 323.98 \text{ m}\cdot\text{N} \approx \boxed{320 \text{ m}\cdot\text{N}}$$

The force needed is calculated from the torque and the radius. We are told that the force is directed perpendicularly to the radius (force is applied "tangentially").

$$\tau = F_\perp R \sin\theta \quad \rightarrow \quad F_\perp = \tau/R = 323.98422.15 \text{ m}\cdot\text{N}/2.5 \text{ m} = \boxed{130 \text{ N}}$$

42. The torque supplied is equal to the angular acceleration times the moment of inertia. The angular acceleration is found by using Eq. 8–9b, with $\omega_0 = 0$. Use the moment of inertia of a sphere.

$$\theta = \omega_0 + \tfrac{1}{2}\alpha t^2 \quad \rightarrow \quad \alpha = \frac{2\theta}{t^2}; \quad \tau = I\alpha = \left(\tfrac{2}{5}Mr_0^2\right)\left(\frac{2\theta}{t^2}\right) \quad \rightarrow$$

$$M = \frac{5\tau t^2}{4r_0^2\theta} = \frac{5(10.8 \text{ m}\cdot\text{N})(15.0 \text{ s})^2}{4(0.36 \text{ m})^2(320\pi \text{ rad})} = 23.31 \text{ kg} \approx \boxed{23 \text{ kg}}$$

43. (*a*) The moment of inertia of a thin rod, rotating about its end, is $\frac{1}{3}M\ell^2$. There are three blades to add together.

$$I_{\text{total}} = 3\left(\tfrac{1}{3}M\ell^2\right) = M\ell^2 = (135 \text{ kg})(3.75 \text{ m})^2 = 1898 \text{ kg}\cdot\text{m}^2 \approx \boxed{1.90\times10^3 \text{ kg}\cdot\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}}\frac{\omega - \omega_0}{t} = (1898 \text{ kg}\cdot\text{m}^2)\frac{(6.0 \text{ rev/s})(2\pi \text{ rad/rev})}{8.0 \text{ s}} = \boxed{8900 \text{ m}\cdot\text{N}}$$

44. The torque on the rotor causes an angular acceleration, $\alpha = \tau/I$. The torque and angular acceleration have the opposite sign as the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into $\omega^2 = \omega_0^2 + 2\alpha\theta$, and solve for the angular displacement.

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \rightarrow \quad \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{0 - \omega_0^2}{2(\tau/I)} = \frac{-\omega_0^2}{2\left(\tau/\tfrac{1}{2}MR^2\right)} = \frac{-MR^2\omega_0^2}{4\tau}$$

$$= \frac{-(3.10 \text{ kg})(0.0710 \text{ m})^2\left[\left(9200\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right]^2}{4(-1.20 \text{ N}\cdot\text{m})} = 3021.8 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)$$

$$= 480.9 \text{ rev} \approx \boxed{480 \text{ rev}}$$

The time can be found from $\theta = \frac{1}{2}(\omega_0 + \omega)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(480.9 \text{ rev})}{9200 \text{ rev/min}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{6.3 \text{ s}}$$

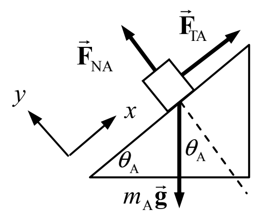
45. The firing force of the rockets will create a net torque but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus, $\tau_{\text{net}} = 4FR$. This torque will cause an angular acceleration according to $\tau = I\alpha$, where $I = \frac{1}{2}MR^2 + 4mR^2$, combining a cylinder of mass M and radius R with four point masses of mass m and lever arm R each. The angular acceleration can be found from the kinematics by $\alpha = \dfrac{\Delta\omega}{\Delta t}$. Equating the two expressions for the torque and substituting enables us to solve for the force.

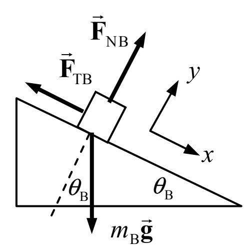
$$4FR = I\alpha = \left(\tfrac{1}{2}M + 4m\right)R^2\frac{\Delta\omega}{\Delta\tau} \quad\rightarrow\quad F = \frac{\left(\tfrac{1}{2}M + 4m\right)R\Delta\omega}{4\Delta t}$$

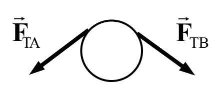
$$= \frac{\left(\tfrac{1}{2}(3600 \text{ kg}) + 4(250 \text{ kg})\right)(4.0 \text{ m})(32 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60 \text{ s})}{4(5.0 \text{ min})(60 \text{ s/min})} = 31.28 \text{ N}$$

$$\approx \boxed{31 \text{ N}}$$

46. (*a*) The free-body diagrams are shown. Note that only the forces producing torque are shown on the pulley. There would also be a gravity force on the pulley (since it has mass) and a normal force from the pulley's suspension, but they are not shown since they do not enter into the solution.







(*b*) Write Newton's second law for the two blocks, taking the positive x direction as shown in the free-body diagrams.

$$m_A: \quad \sum F_x = F_{TA} - m_A g \sin\theta_A = m_A a \quad\rightarrow$$

$$F_{TA} = m_A(g \sin\theta_A + a)$$

$$= (8.0 \text{ kg})\left[(9.80 \text{ m/s}^2)\sin 32° + 1.00 \text{ m/s}^2\right] = 49.55 \text{ N}$$

$$\approx \boxed{50 \text{ N}} \text{ (2 significant figures)}$$

$$m_B: \quad \sum F_x = m_B g \sin\theta_B - F_{TB} = m_B a \quad\rightarrow$$

$$F_{TB} = m_B(g \sin\theta_B - a)$$

$$= (10.0 \text{ kg})\left[(9.80 \text{ m/s}^2)\sin 61° - 1.00 \text{ m/s}^2\right] = 75.71 \text{ N}$$

$$\approx \boxed{76 \text{ N}}$$

(*c*) The net torque on the pulley is caused by the two tensions. We take clockwise torques as positive.

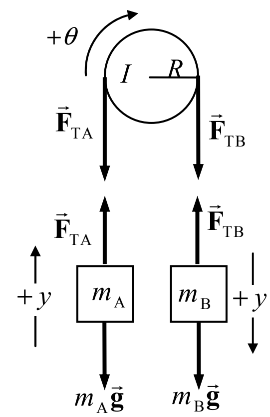
$$\sum\tau = (F_{TB} - F_{TA})R = (75.71 \text{ N} - 49.55 \text{ N})(0.15 \text{ m}) = 3.924 \text{ m}\cdot\text{N} \approx \boxed{3.9 \text{ m}\cdot\text{N}}$$

Use Newton's second law to find the rotational inertia of the pulley. The tangential acceleration of the pulley's rim is the same as the linear acceleration of the blocks, assuming that the string doesn't slip.

$$\sum\tau = I\alpha = I\frac{a}{R} = (F_{TB} - F_{TA})R \quad\rightarrow$$

$$I = \frac{(F_{TB} - F_{TA})R^2}{a} = \frac{(75.71 \text{ N} - 49.55 \text{ N})(0.15 \text{ m})^2}{1.00 \text{ m/s}^2} = \boxed{0.59 \text{ kg}\cdot\text{m}^2}$$

47. (*a*) Since $m_B > m_A$, m_B will accelerate down, m_A will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley has that same acceleration since the cord makes it rotate, so $\alpha_{\text{pulley}} = a/R$. From the free-body diagrams, we have the following:



$$\sum F_{yA} = F_{TA} - m_A g = m_A a \quad \rightarrow \quad F_{TA} = m_A g + m_A a$$

$$\sum F_{yB} = m_B g - F_{TB} = m_B a \quad \rightarrow \quad F_{TB} = m_B g - m_B a$$

$$\sum \tau = F_{TB} r - F_{TA} r = I\alpha = I\frac{a}{R}$$

We have to assume that the tensions are unequal in order to have a net torque to accelerate the pulley. Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{TB}R - F_{TA}R = I\frac{a}{R} \quad \rightarrow \quad (m_B g - m_B a)R - (m_A g + m_A a)R = I\frac{a}{R} \quad \rightarrow$$

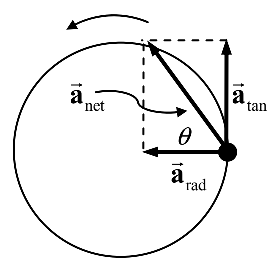
$$a = \frac{(m_B - m_A)}{(m_A + m_B + I/R^2)} g = \frac{(m_B - m_A)}{\left(m_A + m_B + \frac{1}{2} m_P R^2/R^2\right)} g$$

$$= \frac{(75\text{ kg} - 65\text{ kg})}{\left[75\text{ kg} + 65\text{ kg} + \frac{1}{2}(6.0\text{ kg})\right]}(9.80\text{ m/s}^2) = 0.6853\text{ m/s}^2 \approx \boxed{0.69\text{ m/s}^2}$$

(*b*) If the moment of inertia is ignored, then from the torque equation we see that $F_{TB} = F_{TA}$, and the acceleration will be $a_{I=0} = \dfrac{(m_B - m_A)}{(m_A + m_B)} g = \dfrac{(75\text{ kg} - 65\text{ kg})}{75\text{ kg} + 65\text{ kg}}(9.80\text{ m/s}^2) = 0.7000\text{ m/s}^2$. We calculate the percent difference, which is small because of the relatively small mass of the pulley.

$$\%\text{ error} = \left(\frac{0.7000\text{ m/s}^2 - 0.6853\text{ m/s}^2}{0.6853\text{ m/s}^2}\right) \times 100 = 2.145\% \approx \boxed{2\%}$$

48. A top view diagram of the hammer is shown, just at the instant of release, along with the acceleration vectors.



(*a*) The angular acceleration is found from Eq. 8–9c.

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \quad \rightarrow \quad \alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = \frac{(v/r)^2 - 0}{2\Delta\theta}$$

$$= \frac{[(26.5\text{ m/s})/(1.20\text{ m})]^2}{2(8\pi\text{ rad})} = 9.702\text{ rad/s}^2 \approx \boxed{9.70\text{ rad/s}^2}$$

(*b*) The tangential acceleration is found from the angular acceleration and the radius.

$$a_{\text{tan}} = \alpha r = (9.702\text{ rad/s}^2)(1.20\text{ m}) = 11.64\text{ m/s}^2 \approx \boxed{11.6\text{ m/s}^2}$$

(*c*) The centripetal acceleration is found from the speed and the radius.

$$a_{\text{rad}} = v^2/r = (26.5\text{ m/s})^2/(1.20\text{ m}) = 585.2\text{ m/s}^2 \approx \boxed{585\text{ m/s}^2}$$

(*d*) The net force is the mass times the net acceleration. It is in the same direction as the net acceleration.

$$F_{\text{net}} = ma_{\text{net}} = m\sqrt{a_{\tan}^2 + a_{\text{rad}}^2} = (7.30\text{ kg})\sqrt{(11.64\text{ m/s}^2)^2 + (585.2\text{ m/s}^2)^2} = \boxed{4270\text{ N}}$$

(*e*) Find the angle from the two acceleration vectors.

$$\theta = \tan^{-1}\frac{a_{\tan}}{a_{\text{rad}}} = \tan^{-1}\frac{11.64\text{ m/s}^2}{585.2\text{ m/s}^2} = \boxed{1.14°}$$

49. Work can be expressed in rotational quantities as $W = \tau\,\Delta\theta,$ so power can be expressed in rotational quantities as $P = \dfrac{W}{\Delta t} = \tau\dfrac{\Delta\theta}{\Delta t} = \tau\omega.$

$$P = \tau\omega = (265\text{ m}\cdot\text{N})\left(3350\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{125\text{ hp}}$$

50. The energy required to bring the rotor up to speed from rest is equal to the final rotational kinetic energy of the rotor.

$$\text{KE}_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(3.25\times10^{-2}\text{ kg}\cdot\text{m}^2)\left[8750\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 = \boxed{1.36\times10^4\text{ J}}$$

51. Apply conservation of mechanical energy. Take the bottom of the incline to be the zero location for gravitational potential energy. The energy at the top of the incline is then all gravitational potential energy, and at the bottom of the incline, there is both rotational and translational kinetic energy. Since the cylinder rolls without slipping, the angular velocity is given by $\omega = v/R.$

$$E_{\text{top}} = E_{\text{bottom}} \quad\rightarrow\quad Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\tfrac{1}{2}MR^2\frac{v^2}{R^2} = \tfrac{3}{4}Mv^2 \quad\rightarrow$$

$$v = \sqrt{\tfrac{4}{3}gh} = \sqrt{\tfrac{4}{3}(9.80\text{ m/s}^2)(7.20\text{ m})} = \boxed{9.70\text{ m/s}}$$

52. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = v/R.$ The rotational inertia of a sphere about an axis through its center is $I = \tfrac{2}{5}mR^2.$

$$\text{KE}_{\text{total}} = \text{KE}_{\text{trans}} + \text{KE}_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= 0.7(7.25\text{ kg})(3.10\text{ m/s})^2 = \boxed{48.8\text{ J}}$$

53. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$\text{KE}_{\text{daily}} = \tfrac{1}{2}I\omega_{\text{daily}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}^2$$

$$= \tfrac{1}{5}(6.0\times10^{24}\text{ kg})(6.4\times10^6\text{ m})^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right]^2 = \boxed{2.6\times10^{29}\text{ J}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$\mathrm{KE}_{\text{yearly}} = \tfrac{1}{2} I\omega^2_{\text{yearly}} = \tfrac{1}{2}\left(MR^2_{\substack{\text{Sun-}\\\text{Earth}}}\right)\omega^2_{\text{yearly}}$$

$$= \tfrac{1}{2}(6.0\times10^{24}\ \text{kg})(1.5\times10^{11}\ \text{m})^2\left[\left(\frac{2\pi\ \text{rad}}{365\ \text{day}}\right)\left(\frac{1\ \text{day}}{86{,}400\ \text{s}}\right)\right]^2 = \boxed{2.7\times10^{33}\ \text{J}}$$

Thus the total kinetic energy is $\mathrm{KE}_{\text{daily}} + \mathrm{KE}_{\text{yearly}} = 2.6\times10^{29}\ \text{J} + 2.7\times10^{33}\ \text{J} = \boxed{2.7\times10^{33}\ \text{J}}$. The kinetic energy due to the daily motion is about 10,000 times smaller than that due to the yearly motion.

54. Maintaining a constant angular speed ω_{steady} will require a torque τ_{motor} to oppose the frictional torque. The power required by the motor is $P = \tau_{\text{motor}}\omega_{\text{steady}} = -\tau_{\text{friction}}\omega_{\text{steady}}$.

$$\tau_{\text{friction}} = I\alpha_{\text{friction}} = \tfrac{1}{2}MR^2\left(\frac{\omega_{\text{f}}-\omega_0}{t}\right) \rightarrow$$

$$P_{\text{motor}} = \tfrac{1}{2}MR^2\left(\frac{\omega_0-\omega_{\text{f}}}{t}\right)\omega_{\text{steady}} = \tfrac{1}{2}(220\ \text{kg})(5.5\ \text{m})^2\frac{\left[(3.8\ \text{rev/s})\left(\dfrac{2\pi\ \text{rad}}{\text{rev}}\right)\right]^2}{16\ \text{s}} = 1.186\times10^5\ \text{W}$$

$$= 1.186\times10^5\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 158.9\ \text{hp} \approx \boxed{160\ \text{hp}}$$

55. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta\mathrm{KE}_{\text{rot}} = \tfrac{1}{2}I\omega_{\text{f}}^2 - \tfrac{1}{2}I\omega_0^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_{\text{f}}^2 = \tfrac{1}{4}(1440\ \text{kg})(7.50\ \text{m})^2\left(\frac{2\pi\ \text{rad}}{7.00\ \text{s}}\right)^2 = \boxed{1.63\times10^4\ \text{J}}$$

$\boxed{56.}$ Apply conservation of energy to the sphere, as done in Example 8–12.

(*a*) The work of Example 8–12 is exactly applicable here. The symbol *d* represents the distance the sphere rolls along the plane. The sphere is rolling without slipping, so $v_{\text{CM}} = \omega R$.

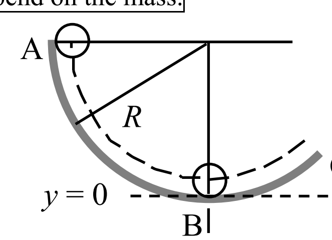
$$v_{\text{CM}} = \sqrt{\tfrac{10}{7}gH} = \sqrt{\tfrac{10}{7}gd\sin\theta} = \sqrt{\tfrac{10}{7}(9.80\ \text{m/s}^2)(10.0\ \text{m})(\sin 30.0)} = 8.367$$
$$= \boxed{8.37\ \text{m/s}}$$

$$\omega = v_{\text{CM}}/R = 8.367\ \text{m/s}/(0.345\ \text{m}) = \boxed{24.3\ \text{rad/s}}$$

(*b*) $$\frac{\mathrm{KE}_{\text{trans}}}{\mathrm{KE}_{\text{rot}}} = \frac{\tfrac{1}{2}Mv_{\text{CM}}^2}{\tfrac{1}{2}I_{\text{CM}}\omega^2} = \frac{\tfrac{1}{2}Mv_{\text{CM}}^2}{\tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\dfrac{v_{\text{CM}}^2}{R^2}} = \boxed{\frac{5}{2}}$$

(*c*) $\boxed{\text{Only the angular speed depends on the radius. None of the results depend on the mass.}}$

57. Use conservation of mechanical energy to equate the energy at points A and B. Call the zero level for gravitational potential energy the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega = v/r$.

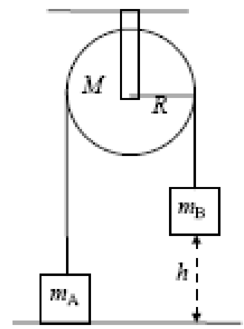


$$E_A = E_B \quad \rightarrow \quad \mathrm{PE}_A = \mathrm{PE}_B + \mathrm{KE}_B = \mathrm{PE}_B + \mathrm{KE}_{B\,\text{CM}} + \mathrm{KE}_{B\,\text{rot}} \quad \rightarrow$$

$$mgR = mgr + \tfrac{1}{2}mv_B^2 + \tfrac{1}{2}I\omega_B^2$$

$$= mgr + \tfrac{1}{2}mv_B^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{v_B}{r}\right)^2 \quad \rightarrow \quad v_B = \boxed{\sqrt{\tfrac{10}{7}g(R-r)}}$$

58. The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with m_A on the ground and all objects at rest. The final state of the system has m_B just reaching the ground and all objects in motion. Call the zero level of gravitational potential energy the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/r$. All objects have an initial speed of 0.



$$E_i = E_f \quad \rightarrow$$

$$\tfrac{1}{2}m_A v_i^2 + \tfrac{1}{2}m_B v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_A g\, y_{1i} + m_B g\, y_{2i} = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_A g\, y_{1f} + m_B g\, y_{2f}$$

$$m_B gh = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_A gh$$

$$v_f = \sqrt{\frac{2(m_B - m_A)gh}{\left(m_A + m_B + \tfrac{1}{2}M\right)}} = \sqrt{\frac{2(38.0\text{ kg} - 32.0\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m})}{\left(38.0\text{ kg} + 32.0\text{ kg} + \left(\tfrac{1}{2}\right)3.1\text{ kg}\right)}} = \boxed{2.0\text{ m/s}}$$

59. Since the lower end of the pole does not slip on the ground, the friction does no work, and mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as though it were at the CM. The final energy is rotational kinetic energy, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity divided by the length.

$$E_{\text{intial}} = E_{\text{final}} \quad \rightarrow \quad \mathrm{PE}_{\text{initial}} = \mathrm{KE}_{\text{final}} \quad \rightarrow \quad mgh = \tfrac{1}{2}I\omega^2 \quad \rightarrow \quad mg\ell/2 = \tfrac{1}{2}\left(\tfrac{1}{3}m\ell^2\right)\left(v_{\text{end}}/\ell\right)^2 \quad \rightarrow$$

$$v_{\text{end}} = \sqrt{3g\ell} = \sqrt{3(9.80\text{ m/s}^2)(1.80\text{ m})} = \boxed{7.27\text{ m/s}}$$

60. The angular momentum is given by Eq. 8–18.

$$L = I\omega = MR^2\omega = (0.270\text{ kg})(1.35\text{ m})^2(10.4\text{ rad/s}) = \boxed{5.12\text{ kg}\cdot\text{m}^2/\text{s}}$$

61. (*a*) The angular momentum is given by Eq. 8–18.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.8\text{ kg})(0.28\text{ m})^2\left[\left(\frac{1300\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 14.94\text{ kg}\cdot\text{m}^2/\text{s} \approx \boxed{15\text{ kg}\cdot\text{m}^2/\text{s}}$$

(*b*) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 14.94\text{ kg}\cdot\text{m}^2/\text{s}}{6.0\text{ s}} = \boxed{-2.5\text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

62. (*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms are internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $$L_\text{i} = L_\text{f} \quad \to \quad I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f} \quad \to \quad I_\text{f} = I_\text{i}\frac{\omega_\text{i}}{\omega_\text{f}} = I_\text{i}\frac{0.90\text{ rev/s}}{0.60\text{ rev/s}} = 1.5\,I_\text{i}$$

The rotational inertia has increased by a factor of $\boxed{1.5}$.

63. Since there are no external torques on the system, the angular momentum of the two-disk system is conserved. The two disks have the same final angular velocity.

$$L_\text{i} = L_\text{f} \quad \to \quad I\omega + I(0) = 2I\omega_\text{f} \quad \to \quad \boxed{\omega_\text{f} = \tfrac{1}{2}\omega}$$

64. There is no net torque on the diver, because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \quad \to \quad I_1\omega_1 = I_2\omega_2 \quad \to \quad \omega_2 = \omega_1\frac{I_1}{I_2} = \left(\frac{2\text{ rev}}{1.5\text{ s}}\right)\left(\frac{1}{3.5}\right) = \boxed{0.38\text{ rev/s}}$$

65. The skater's angular momentum is constant, since no external torques are applied to her.

$$L_\text{i} = L_\text{f} \quad \to \quad I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f} \quad \to \quad I_\text{f} = I_\text{i}\frac{\omega_\text{i}}{\omega_\text{f}} = (4.6\text{ kg}\cdot\text{m}^2)\frac{1.0\text{ rev/1.5 s}}{2.5\text{ rev/s}} = \boxed{1.2\text{ kg}\cdot\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then pulling her arms in to the center of her body (final angular velocity).

66. (*a*) The angular momentum is the moment of inertia (modeling the skater as a cylinder) times the angular velocity.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(48\text{ kg})(0.15\text{ m})^2\left(3.0\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10.18\text{ kg}\cdot\text{m}^2/\text{s}$$

$$\approx \boxed{1.0\times10^1\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I\omega_\text{final} - I\omega_0}{\Delta t} = \frac{0 - 10.18\text{ kg}\cdot\text{m}^2/\text{s}}{4.0\text{ s}} = \boxed{-2.5\text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

67. Since the person is walking radially, no torques will be exerted on the person–platform system, and angular momentum will be conserved. The person is treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(*a*) $$L_\text{i} = L_\text{f} \quad \to \quad I_\text{platform}\omega_\text{i} = (I_\text{platform} + I_\text{person})\omega_\text{f}$$

$$\omega_\text{f} = \frac{I_\text{platform}}{I_\text{platform} + mR^2}\omega_\text{i} = \frac{820\text{ kg}\cdot\text{m}^2}{820\text{ kg}\cdot\text{m}^2 + (75\text{ kg})(3.0\text{ m})^2}(0.95\text{ rad/s}) = 0.5211\text{ rad/s} \approx \boxed{0.52\text{ rad/s}}$$

(*b*) $\text{KE}_\text{i} = \tfrac{1}{2} I_\text{platform} \omega_\text{i}^2 = \tfrac{1}{2}(820\ \text{kg}\cdot\text{m}^2)(0.95\ \text{rad/s})^2 = \boxed{370\ \text{J}}$

$$\text{KE}_\text{f} = \tfrac{1}{2}(I_\text{platform} + I_\text{person})\omega_\text{f}^2 = \tfrac{1}{2}(I_\text{platform} + m_\text{person} r_\text{person}^2)\omega_\text{f}^2$$

$$= \tfrac{1}{2}[820\ \text{kg}\cdot\text{m}^2 + (75\ \text{kg})(3.0\ \text{m})^2](0.5211\ \text{rad/s})^2 = 203\ \text{J} \approx \boxed{2.0\times10^2\ \text{J}}$$

68. Because there is no external torque applied to the wheel–clay system, the angular momentum will be conserved. We assume that the clay is thrown with no angular momentum, so its initial angular momentum is 0. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the clay and the wheel. Subscript 1 represents before the clay is thrown, and subscript 2 represents after the clay is thrown.

$$L_1 = L_2 \quad\rightarrow\quad I_1\omega_1 = I_2\omega_2 \quad\rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \frac{I_\text{wheel}}{I_\text{wheel} + I_\text{clay}} = \omega_1 \left(\frac{\tfrac{1}{2} M_\text{wheel} R_\text{wheel}^2}{\tfrac{1}{2} M_\text{wheel} R_\text{wheel}^2 + \tfrac{1}{2} M_\text{clay} R_\text{clay}^2} \right) = \omega_1 \left(\frac{M_\text{wheel} R_\text{wheel}^2}{M_\text{wheel} R_\text{wheel}^2 + M_\text{clay} R_\text{clay}^2} \right)$$

$$= (1.5\ \text{rev/s}) \left[\frac{(5.0\ \text{kg})(0.20\ \text{m})^2}{(5.0\ \text{kg})(0.20\ \text{m})^2 + (2.6\ \text{kg})(7.0\times10^{-2}\ \text{m})^2} \right] = 1.410\ \text{rev/s} \approx \boxed{1.4\ \text{rev/s}}$$

69. (*a*) The angular momentum of the combination of merry-go-round (abbreviate mgr) and people will be conserved, because there are no external torques on the combination. This situation is a totally inelastic collision in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$L_1 = L_2 \quad\rightarrow\quad I_1\omega_1 = I_2\omega_2 \quad\rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_\text{mgr}}{I_\text{mgr} + I_\text{people}} = \omega_1 \left[\frac{I_\text{mgr}}{I_\text{mgr} + 4M_\text{person} R^2} \right]$$

$$= (0.80\ \text{rad/s}) \left[\frac{1360\ \text{kg}\cdot\text{m}^2}{1360\ \text{kg}\cdot\text{m}^2 + 4(65\ \text{kg})(2.1\ \text{m})^2} \right] = 0.4341\ \text{rad/s} \approx \boxed{0.43\ \text{rad/s}}$$

(*b*) If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $\boxed{0.80\ \text{rad/s}}$.

70. All parts of the object have the same angular velocity. The moment of inertia is the sum of the rod's moment of inertia and the mass's moment of inertia.

$$L = I\omega = \left[\tfrac{1}{12} M\ell^2 + 2m\left(\tfrac{1}{2}\ell\right)^2 \right]\omega = \tfrac{1}{2}\left(\tfrac{1}{6}M + m\right)\ell^2\omega$$

71. (*a*) Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_\text{i} = L_\text{f} \quad\rightarrow\quad I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f} \quad\rightarrow \frac{\omega_\text{f}}{\omega_\text{i}} = \frac{I_\text{i}}{I_\text{f}} = \frac{\tfrac{2}{5} M_\text{i} R_\text{i}^2}{\tfrac{2}{5} M_\text{f} R_\text{f}^2} = \frac{M_\text{i} R_\text{i}^2}{(0.5\ M_\text{i})(0.01\ R_\text{f})^2} = 2.0\times10^4$$

$$\omega_\text{f} = 2.0\times10^4\ \ \omega_i = 2.0\times10^4 \left(\frac{2\pi\ \text{rad}}{30\ \text{day}} \right)\left(\frac{1\ \text{day}}{86{,}400\ \text{s}} \right) = 4.848\times10^{-2}\ \text{rad/s} \approx \boxed{5\times10^{-2}\ \text{rad/s}}$$

The period would be a factor of 20,000 smaller, which would make it about 130 seconds.

(*b*) The ratio of angular kinetic energies of the spinning mass would be as follows:

$$\frac{\mathrm{KE_f}}{\mathrm{KE_i}}=\frac{\frac{1}{2}I_\mathrm{f}\omega_\mathrm{f}^2}{\frac{1}{2}I_\mathrm{i}\omega_\mathrm{i}^2}=\frac{\frac{1}{2}\left[\frac{2}{5}(0.5\,M_\mathrm{i})(0.01\,R_\mathrm{i})^2\right](2.0\times10^4\;\omega_\mathrm{i})^2}{\frac{1}{2}\left(\frac{2}{5}M_\mathrm{i}R_\mathrm{i}^2\right)\omega_\mathrm{i}^2}=2.0\times10^4\;\rightarrow\;\boxed{\mathrm{KE_f}=2\times10^4\;\mathrm{KE_i}}$$

72. The angular momentum of the disk–rod combination will be conserved, because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1=L_2\;\rightarrow\;I_1\omega_1=I_2\omega_2\;\rightarrow$$

$$\omega_2=\omega_1\frac{I_1}{I_2}=\omega_1\frac{I_\mathrm{disk}}{I_\mathrm{disk}+I_\mathrm{rod}}=\omega_1\left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2+\frac{1}{12}M(2R)^2}\right]=(3.3\text{ rev/s})\left(\frac{3}{5}\right)=\boxed{2.0\text{ rev/s}}$$

73. Angular momentum will be conserved in the Earth–asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_\mathrm{asteroid}=\upsilon_\mathrm{asteroid}/R_\mathrm{Earth}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere and the asteroid as a point mass.

$$L_\mathrm{i}=L_\mathrm{f}\;\rightarrow\;I_\mathrm{Earth}\omega_\mathrm{Earth}+I_\mathrm{asteroid}\omega_\mathrm{asteroid}=(I_\mathrm{Earth}+I_\mathrm{asteroid})\omega_\mathrm{f}$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, so the percent change in Earth's angular velocity is found as follows:

$$I_\mathrm{Earth}\omega_\mathrm{Earth}+I_\mathrm{asteroid}\omega_\mathrm{asteroid}=I_\mathrm{Earth}\omega_\mathrm{f}\;\rightarrow\;\frac{(\omega_\mathrm{f}-\omega_\mathrm{Earth})}{\omega_\mathrm{Earth}}=\frac{I_\mathrm{asteroid}}{I_\mathrm{Earth}}\frac{\omega_\mathrm{asteroid}}{\omega_\mathrm{Earth}}$$

$$\%\text{ change}=\frac{(\omega_\mathrm{f}-\omega_\mathrm{Earth})}{\omega_\mathrm{Earth}}(100)=\frac{m_\mathrm{asteroid}R_\mathrm{Earth}^2}{\frac{2}{5}M_\mathrm{Earth}R_\mathrm{Earth}^2}\frac{\frac{\upsilon_\mathrm{asteroid}}{R_\mathrm{Earth}}}{\omega_\mathrm{Earth}}=\frac{m_\mathrm{asteroid}}{\frac{2}{5}M_\mathrm{Earth}}\frac{\upsilon_\mathrm{asteroid}}{\omega_\mathrm{Earth}R_\mathrm{Earth}}(100)$$

$$=\frac{(1.0\times10^5\text{ kg})(3.5\times10^4\text{ m/s})}{(0.4)(5.97\times10^{24}\text{ kg})\left(\frac{2\pi\text{ rad}}{86{,}400\text{ s}}\right)(6.38\times10^6\text{ m})}(100)=\boxed{(3.2\times10^{-16})\%}$$

74. The angular momentum of the person–turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $\upsilon+\upsilon_\mathrm{T}$, where υ is the person's speed relative to the turntable, and υ_T is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_\mathrm{T}=\upsilon_\mathrm{T}/R$, and the person's angular speed relative to the ground is $\omega_\mathrm{P}=\dfrac{\upsilon+\upsilon_\mathrm{T}}{R}=\dfrac{\upsilon}{R}+\omega_\mathrm{T}$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_\mathrm{i}=L_\mathrm{f}\;\rightarrow\;0=I_\mathrm{T}\omega_\mathrm{T}+I_\mathrm{P}\omega_\mathrm{P}=I_\mathrm{T}\omega_\mathrm{T}+mR^2\left(\omega_\mathrm{T}+\frac{\upsilon}{R}\right)\;\rightarrow$$

$$\omega_\mathrm{T}=-\frac{mR\upsilon}{I_\mathrm{T}+mR^2}=-\frac{(65\text{ kg})(2.75\text{ m})(4.0\text{ m/s})}{1850\text{ kg}\cdot\text{m}^2+(65\text{ kg})(2.75\text{ m})^2}=-0.3054\text{ rad/s}\approx\boxed{-0.31\text{ rad/s}}$$

75. Angular momentum is conserved in the interaction between the child and the merry-go-round.

$$L_{\text{initial}} = L_{\text{final}} \rightarrow L_{0\,\text{mgr}} = L_{\text{f child}} + L_{\text{f mgr}} \rightarrow I_{\text{mgr}}\omega_0 = (I_{\text{mgr}} + I_{\text{child}})\omega = (I_{\text{mgr}} + m_{\text{child}}R_{\text{mgr}}^2)\omega \rightarrow$$

$$m_{\text{child}} = \frac{I_{\text{mgr}}(\omega_0 - \omega)}{R_{\text{mgr}}^2\omega} = \frac{(1260\text{ kg}\cdot\text{m}^2)(0.35\text{ rad/s})}{(2.5\text{ m})^2(1.35\text{ rad/s})} = 52.27\text{ kg} \approx \boxed{52\text{ kg}}$$

76. The torque is found from $\tau = I\alpha$. The angular acceleration can be found from $\omega = \omega_0 + \alpha t$, and the initial angular velocity is 0. The rotational inertia is that of a cylinder.

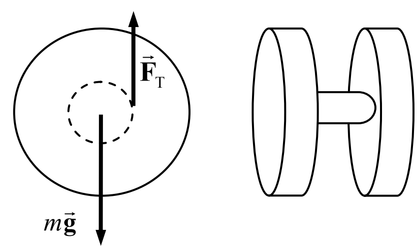
$$\tau = I\alpha = \tfrac{1}{2}MR^2\left(\frac{\omega - \omega_0}{t}\right) = 0.5(1.6\text{ kg})(0.20\text{ m})^2\,\frac{(24\text{ rev/s})(2\pi\text{ rad/rev})}{6.0\text{ s}} = \boxed{0.80\text{ m}\cdot\text{N}}$$

77. The linear speed is related to the angular velocity by $v = \omega R$, and the angular velocity (rad/s) is related to the frequency (rev/s) by $\omega = 2\pi f$. Combine these relationships to find values for the frequency.

$$\omega = 2\pi f = \frac{v}{R} \rightarrow f = \frac{v}{2\pi R};\quad f_1 = \frac{v}{2\pi R_1} = \frac{1.25\text{ m/s}}{2\pi(0.025\text{ m})}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{480\text{ rpm}}$$

$$f_2 = \frac{v}{2\pi R_2} = \frac{1.25\text{ m/s}}{2\pi(0.058\text{ m})}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{210\text{ rpm}}$$

78. (*a*) There are two forces on the yo-yo: gravity and the string tension. If we assume that the top of the string is held in a fixed position, then the tension does no work and mechanical energy is conserved. The initial gravitational PE is converted into rotational and translational KE. Since the yo-yo rolls without slipping at the point of contact of the string, the velocity of the CM is simply related to the angular velocity of the yo-yo: $v_{\text{CM}} = r\omega$, where r is the radius of the inner hub. Let m be the mass of the inner hub and M and R be the mass and radius of each outer disk. Calculate the rotational inertia of the yo-yo about its CM, and then use conservation of energy to find the linear speed of the CM. We take the 0 of gravitational PE to be at the bottom of its fall.



$$I_{\text{CM}} = \tfrac{1}{2}mr^2 + 2\left(\tfrac{1}{2}MR^2\right) = \tfrac{1}{2}mr^2 + MR^2$$

$$= \tfrac{1}{2}(5.0\times10^{-3}\text{ kg})(6.5\times10^{-3}\text{ m})^2 + (5.0\times10^{-2}\text{ kg})(3.75\times10^{-2}\text{ m})^2 = 7.042\times10^{-5}\text{ kg}\cdot\text{m}^2$$

$$m_{\text{total}} = m + 2M = 5.0\times10^{-3}\text{ kg} + 2(5.0\times10^{-2}\text{ kg}) = 0.105\text{ kg}$$

$$\text{PE}_{\text{i}} = \text{KE}_{\text{f}} \rightarrow$$

$$m_{\text{total}}gh = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}m_{\text{total}}v_{\text{CM}}^2 + \tfrac{1}{2}\frac{I_{\text{CM}}}{r^2}v_{\text{CM}}^2 = \tfrac{1}{2}\left(m_{\text{total}} + \frac{I_{\text{CM}}}{r^2}\right)v_{\text{CM}}^2 \rightarrow$$

$$v_{\text{CM}} = \sqrt{\frac{m_{\text{total}}gh}{\tfrac{1}{2}\left(m_{\text{total}} + \frac{I_{\text{CM}}}{r^2}\right)}} = \sqrt{\frac{(0.105\text{ kg})(9.80\text{ m/s}^2)(1.0\text{ m})}{\tfrac{1}{2}\left[(0.105\text{ kg}) + \frac{(7.042\times10^{-5}\text{ kg}\cdot\text{m}^2)}{(6.5\times10^{-3}\text{ m})^2}\right]}} = 1.078\text{ m/s} = \boxed{1.1\text{ m/s}}$$

(*b*) Calculate the ratio $\mathrm{KE_{rot}}/\mathrm{KE_{tot}}$.

$$\frac{\mathrm{KE_{rot}}}{\mathrm{KE_{tot}}}=\frac{\mathrm{KE_{rot}}}{\mathrm{PE_{tot}}}=\frac{\frac{1}{2}I_{\mathrm{CM}}\omega^2}{m_{\mathrm{total}}gh}=\frac{\frac{1}{2}\frac{I_{\mathrm{CM}}}{r^2}v_{\mathrm{CM}}^2}{m_{\mathrm{total}}gh}=\frac{I_{\mathrm{CM}}v_{\mathrm{CM}}^2}{2r^2m_{\mathrm{total}}gh}$$

$$=\frac{(7.042\times10^{-5}\ \mathrm{kg\cdot m^2})(1.078\ \mathrm{m/s})^2}{2(6.5\times10^{-3}\ \mathrm{m})^2(0.105\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(1.0\ \mathrm{m})}=0.9412=\boxed{94\%}$$

79. As discussed in Section 8–3 of the textbook, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $v=r\omega$, where v is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of v to the right relative to the axle, so it has a velocity of $2v$ to the right relative to the ground.

$$\vec{\mathbf{v}}_{\substack{\text{top rel}\\ \text{ground}}}=\vec{\mathbf{v}}_{\substack{\text{top rel}\\ \text{center}}}+\vec{\mathbf{v}}_{\substack{\text{center rel}\\ \text{ground}}}=(v\text{ to the right})+(v\text{ to the right})=2v\text{ to the right}$$

$$v_{\substack{\text{top rel}\\ \text{ground}}}=2v=2(v_0+at)=2at=2(1.00\ \mathrm{m/s^2})(2.25\ \mathrm{s})=\boxed{4.50\ \mathrm{m/s}}$$

80. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$\tau=I\alpha=(mr^2)\left(\frac{\omega-\omega_0}{t}\right)=\frac{(0.60\ \mathrm{kg})(1.5\ \mathrm{m})^2}{5.0\ \mathrm{s}}\left[\left(75\frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{2\pi\ \mathrm{rad}}{\mathrm{rev}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)\right]=\boxed{2.1\ \mathrm{m\cdot N}}$$

That torque comes from the arm swinging the sling and is generated by the arm muscles.

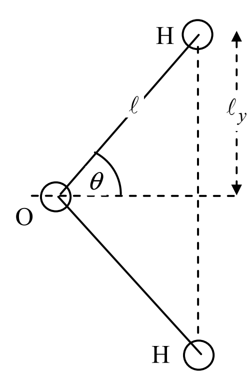
81. (*a*) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $v=\omega R$, so $\omega_{\mathrm{R}}R_{\mathrm{R}}=\omega_{\mathrm{F}}R_{\mathrm{F}}$. If the spacing of the teeth on the sprockets is a distance *d*, then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$Nd=2\pi R\ \text{ so }\ R=\frac{Nd}{2\pi}.\ \text{Thus}\ \omega_{\mathrm{R}}\frac{N_{\mathrm{R}}d}{2\pi}=\omega_{\mathrm{F}}\frac{N_{\mathrm{F}}d}{2\pi}\ \rightarrow\ \boxed{\frac{\omega_{\mathrm{R}}}{\omega_{\mathrm{F}}}=\frac{N_{\mathrm{F}}}{N_{\mathrm{R}}}}.$$

(*b*) $\boxed{\omega_{\mathrm{R}}/\omega_{\mathrm{F}}=52/13=4.0}$

(*c*) $\boxed{\omega_{\mathrm{R}}/\omega_{\mathrm{F}}=42/28=1.5}$

82. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is 1.66×10^{-27} kg. Since the axis passes through the oxygen atom, the oxygen atom will have no rotational inertia.



(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance ℓ from the axis of rotation.

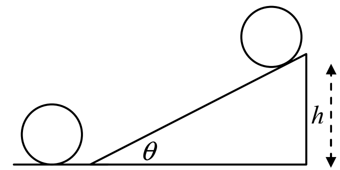
$$I_{\mathrm{perp}}=2m_{\mathrm{H}}\ell^2=2(1.01)(1.66\times10^{-27}\ \mathrm{kg})(0.096\times10^{-9}\ \mathrm{m})^2$$

$$=\boxed{3.1\times10^{-47}\ \mathrm{kg\cdot m^2}}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H—O—H bonds, each hydrogen atom is a distance of $\ell_y = \ell \sin\theta = (9.6\times10^{-11}\text{ m})\sin 52° = 7.564\times10^{-11}\text{ m}$. Thus the moment of inertia is as follows:

$$I_{\text{plane}} = 2m_H\ell_y^2 = 2(1.01)(1.66\times10^{-27}\text{ kg})(7.564\times10^{-11}\text{ m})^2 = \boxed{1.9\times10^{-47}\text{ kg}\cdot\text{m}^2}$$

83. (*a*) Assuming that there are no dissipative forces doing work, conservation of mechanical energy may be used to find the final height *h* of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = v/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \quad\rightarrow\quad \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh \quad\rightarrow\quad \tfrac{1}{2}mv^2 + \tfrac{1}{2}mR^2\frac{v^2}{R^2} = mgh \quad\rightarrow$$

$$h = \frac{v^2}{g} = \frac{(3.0\text{ m/s})^2}{9.80\text{ m/s}^2} = 0.9184\text{ m}$$

The distance along the plane is given by $d = \dfrac{h}{\sin\theta} = \dfrac{0.9184\text{ m}}{\sin 15°} = 3.548\text{ m} \approx \boxed{3.5\text{ m}}$

(*b*) The time can be found from the constant acceleration of the linear motion.

$$\Delta x = \tfrac{1}{2}(v + v_0)t \quad\rightarrow\quad t = \frac{2\Delta x}{v + v_0} = \frac{2(3.548\text{ m})}{0 + 3.0\text{ m/s}} = 2.365\text{ s}$$

This is the time to go up the plane. The time to come back down the plane is the same, so the total time is $\boxed{4.7\text{ s}}$.

84. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}$$

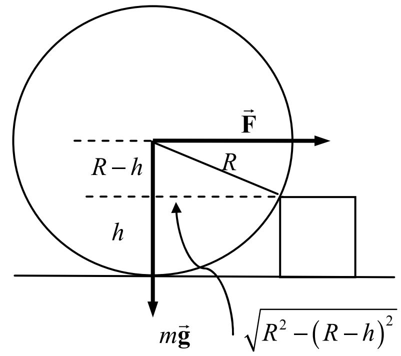
$$= \tfrac{2}{5}(5.98\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{7.08\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(MR_{\substack{\text{Sun-}\\\text{Earth}}}^2\right)\omega_{\text{daily}}$$

$$= (5.98\times10^{24}\text{ kg})(1.496\times10^{11}\text{ m})^2\left[\left(\frac{2\pi\text{ rad}}{365\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{2.67\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}}$$

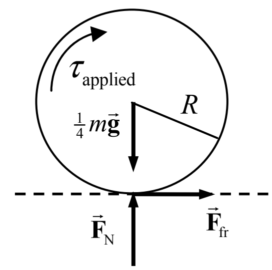
85. The wheel is rolling about the point of contact with the step, so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel: the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum \tau = F(R-h) - mg\sqrt{R^2-(R-h)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2-(R-h)^2}}{R-h} = \boxed{\frac{Mg\sqrt{2Rh-h^2}}{R-h}}$$

86. Each wheel supports ¼ of the weight of the car. For rolling without slipping, there will be static friction between the wheel and the pavement. For the wheel to be on the verge of slipping, there must be an applied torque that is equal to the torque supplied by the static frictional force. We take counterclockwise torques to the right in the diagram. The bottom wheel would be moving to the left relative to the pavement if it started to slip, so the frictional force is to the right. See the free-body diagram.



$$\underset{\text{min}}{\tau_{\text{applied}}} = \underset{\text{friction}}{\tau_{\text{static}}} = RF_{\text{fr}} = R\mu_{\text{s}}F_{\text{N}} = R\mu_{\text{s}}\tfrac{1}{4}mg$$

$$= \tfrac{1}{4}(0.33\text{ m})(0.65)(1080\text{ kg})(9.80\text{ m/s}^2) = \boxed{570\text{ m}\cdot\text{N}}$$

87. (*a*) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass and B the lighter mass.

$$\text{KE} = \tfrac{1}{2}I_{\text{A}}\omega_{\text{A}}^2 + \tfrac{1}{2}I_{\text{B}}\omega_{\text{B}}^2 = \tfrac{1}{2}m_{\text{A}}r_{\text{A}}^2\omega_{\text{A}}^2 + \tfrac{1}{2}m_{\text{B}}r_{\text{B}}^2\omega_{\text{A}}^2 = \tfrac{1}{2}r^2\omega^2(m_{\text{A}}+m_{\text{B}})$$

$$= \tfrac{1}{2}(0.210\text{ m})^2(5.60\text{ rad/s})^2(7.00\text{ kg}) = \boxed{4.84\text{ J}}$$

(*b*) The net force on each object produces centripetal motion so can be expressed as $mr\omega^2$.

$$F_{\text{A}} = m_{\text{A}}r_{\text{A}}\omega_{\text{A}}^2 = (4.00\text{ kg})(0.210\text{ m})(5.60\text{ rad/s})^2 = \boxed{26.3\text{ N}}$$

$$F_{\text{B}} = m_{\text{B}}r_{\text{B}}\omega_{\text{B}}^2 = (3.00\text{ kg})(0.210\text{ m})(5.60\text{ rad/s})^2 = \boxed{19.8\text{ N}}$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod so that there is no net vertical force on either mass.

88. Note the similarity between this problem and MisConceptual Questions 10 and 11. There is no torque applied to the block, so its angular momentum would remain constant. The angular velocity is the speed of the block divided by the radius of the string. The moment of inertia of the block about the center of its motion is $\tfrac{1}{2}mr^2$.

$$I_1\omega_1 = I_2\omega_2 \quad\rightarrow\quad \tfrac{1}{2}mr_1^2\frac{v_1}{r_1} = \tfrac{1}{2}mr_2^2\frac{v_2}{r_2} \quad\rightarrow\quad r_2v_2 = r_1v_1 \quad\rightarrow$$

$$v_2 = v_1\frac{r_1}{r_2} = (2.4\text{ m/s})\left(\frac{0.80\text{ m}}{0.48\text{ m}}\right) = \boxed{4.0\text{ m/s}}$$

89. (*a*) The force of gravity acting through the CM will cause a clockwise torque, which produces an angular acceleration. At the moment of release, the force of gravity is perpendicular to the lever arm from the hinge to the CM.

$$\tau = I\alpha \quad\rightarrow\quad \alpha = \frac{\tau_{\text{gravity}}}{I_{\text{rod about end}}} = \frac{Mg\ell/2}{\frac{1}{3}M\ell^2} = \boxed{\frac{3g}{2\ell}}$$

(*b*) At the end of the rod, there is a tangential acceleration equal to the angular acceleration times the distance from the hinge. There is no radial acceleration, because at the moment of release, the speed of the end of the rod is 0. Thus, the tangential acceleration is the entire linear acceleration.

$$a_{\text{linear}} = a_{\tan} = \alpha\ell = \boxed{\tfrac{3}{2}g}$$

Note that this is bigger than the free-fall acceleration of g.

90. (*a*) We assume that no angular momentum is in the thrown-off mass, so the final angular momentum of the neutron star is equal to the angular momentum before collapse.

$$L_0 = L_{\text{f}} \quad\rightarrow\quad I_0\omega_0 = I_{\text{f}}\omega_{\text{f}} \quad\rightarrow\quad \left[\tfrac{2}{5}(8.0M_{\text{Sun}})R_{\text{Sun}}^2\right]\omega_0 = \left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_{\text{f}}^2\right]\omega_{\text{f}} \quad\rightarrow$$

$$\omega_{\text{f}} = \frac{\left[\tfrac{2}{5}(8.0M_{\text{Sun}})R_{\text{Sun}}^2\right]}{\left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_{\text{f}}^2\right]}\omega_0 = \frac{4R_{\text{Sun}}^2}{R_{\text{f}}^2}\omega_0 = \frac{4(6.96\times10^8\ \text{m})^2}{(12\times10^3\ \text{m})^2}\left(\frac{1.0\ \text{rev}}{9.0\ \text{days}}\right)$$

$$= (1.495\times10^9\ \text{rev/day})\left(\frac{1\ \text{day}}{86{,}400\ \text{s}}\right) = 1.730\times10^4\ \text{rev/s} \approx \boxed{17{,}000\ \text{rev/s}}$$

(*b*) Now we assume that the final angular momentum of the neutron star is only $\frac{1}{4}$ of the angular momentum before collapse. Since the rotation speed is directly proportional to angular momentum, the final rotation speed will be $\frac{1}{4}$ of that found in part (*a*).

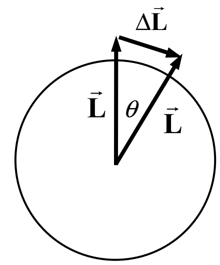
$$\omega_{\text{f}} = \tfrac{1}{4}(1.730\times10^4\ \text{rev/s}) = \boxed{4300\ \text{rev/s}}$$

91. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $v = r\omega = v_{\text{CM}}$ relative to the center of the spool, where v_{CM} is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2v_{\text{CM}}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance ℓ, in the same time the center of the spool, traveling with half the speed, moves forward a distance $\boxed{\ell/2}$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\boxed{\ell/2}$ also.

92. The spin angular momentum of the Moon can be calculated by $L_{\text{spin}} = I_{\text{spin}}\omega_{\text{spin}} = \frac{2}{5}MR_{\text{Moon}}^2\omega_{\text{spin}}$. The orbital angular momentum can be calculated by $L_{\text{orbit}} = I_{\text{orbit}}\omega_{\text{orbit}} = MR_{\text{orbit}}^2\omega_{\text{orbit}}$. Because the same side of the Moon always faces the Earth, $\omega_{\text{spin}} = \omega_{\text{orbit}}$.

$$\frac{L_{\text{spin}}}{L_{\text{orbit}}} = \frac{\frac{2}{5}MR_{\text{Moon}}^2\omega_{\text{spin}}}{MR_{\text{orbit}}^2\omega_{\text{orbit}}} = \frac{2}{5}\left(\frac{R_{\text{Moon}}}{R_{\text{orbit}}}\right)^2 = 0.4\left(\frac{1.74\times10^6\ \text{m}}{3.84\times10^8\ \text{m}}\right)^2 = \boxed{8.21\times10^{-6}}$$

93. The force applied by the spaceship puts a torque on the asteroid, changes the asteroid's angular momentum. We assume that the spaceship's direction is adjusted to always be tangential to the surface. Thus the torque is always perpendicular to the angular momentum and will not change the magnitude of the angular momentum Only the direction of the angular momentum will change, similar to the action of a centripetal force on an object in circular motion. From the diagram, we make an approximation.



$$\tau \approx \frac{\Delta L}{\Delta t} \approx \frac{L\Delta\theta}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{L\Delta\theta}{\tau} = \frac{I\omega\Delta\theta}{Fr} = \frac{\frac{2}{5}mr^2\omega\Delta\theta}{Fr} = \frac{2mr\omega\Delta\theta}{5F}$$

$$= \frac{2(2.25\times10^{10}\text{ kg})(123\text{ m})\left[\left(\frac{4\text{ rev}}{1\text{ day}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ day}}{86,400\text{ s}}\right)\right]\left[5.0^\circ\left(\frac{2\pi\text{ rad}}{360^\circ}\right)\right]}{5(285\text{ N})}$$

$$= (9.860\times10^4\text{ s})\frac{1\text{ h}}{3600\text{ s}} = \boxed{27\text{ h}}$$

Note that in the diagram in the book, the original angular momentum is "up" and the torque is into the page. Thus the planet's axis would tilt backward into the page, not rotate clockwise as it would if it were not rotating.

94. We calculate spin angular momentum for the Sun and orbital angular momentum for the planets, treating them as particles relative to the size of their orbits. The angular velocities are calculated from $\omega = \frac{2\pi}{T}$.

$$L_{\text{Sun}} = I_{\text{Sun}}\omega_{\text{Sun}} = \tfrac{2}{5}M_{\text{Sun}}R_{\text{Sun}}^2\frac{2\pi}{T_{\text{Sun}}} = \tfrac{2}{5}(1.99\times10^{30}\text{ kg})(6.96\times10^8\text{ m})^2\frac{2\pi}{(25\text{ days})}\left(\frac{1\text{ day}}{86,400\text{ s}}\right)$$

$$= 1.1217\times10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Jupiter}} = M_{\text{Jupiter}}R_{\substack{\text{Jupiter}\\ \text{orbit}}}^2\frac{2\pi}{T_{\text{Jupiter}}} = (190\times10^{25}\text{ kg})(778\times10^9\text{ m})^2\frac{2\pi}{(11.9\text{ yr})}\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right)$$

$$= 1.9240\times10^{43}\text{ kg}\cdot\text{m/s}$$

In a similar fashion, we calculate the other planetary orbital angular momenta.

$$L_{\text{Saturn}} = M_{\text{Saturn}}R_{\substack{\text{Saturn}\\ \text{orbit}}}^2\frac{2\pi}{T_{\text{Saturn}}} = 7.806\times10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Uranus}} = M_{\text{Uranus}}R_{\substack{\text{Uranus}\\ \text{orbit}}}^2\frac{2\pi}{T_{\text{Uranus}}} = 1.695\times10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Neptune}} = M_{\text{Neptune}}R_{\substack{\text{Neptune}\\ \text{orbit}}}^2\frac{2\pi}{T_{\text{Neptune}}} = 2.492\times10^{42}\text{ kg}\cdot\text{m/s}$$

$$f = \frac{L_{\text{planets}}}{L_{\text{planets}} + L_{\text{Sun}}} = \frac{(19.240 + 7.806 + 1.695 + 2.492)\times10^{42}\text{ kg}\cdot\text{m/s}}{(19.240 + 7.806 + 1.695 + 2.492 + 1.122)\times10^{42}\text{ kg}\cdot\text{m/s}} = \boxed{0.965}$$

95. (*a*) The angular momentum delivered to the waterwheel is that lost by the water.

$$\Delta L_{\text{wheel}} = -\Delta L_{\text{water}} = L_{\substack{\text{initial}\\\text{water}}} - L_{\substack{\text{final}\\\text{water}}} = mv_1R - mv_2R \rightarrow$$

$$\frac{\Delta L_{\text{wheel}}}{\Delta t} = \frac{mv_1R - mv_2R}{\Delta t} = \frac{mR}{\Delta t}(v_1 - v_2) = (85\text{ kg/s})(3.0\text{ m})(3.2\text{ m/s}) = 816\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\approx \boxed{820\text{ kg}\cdot\text{m}^2/\text{s}^2}$$

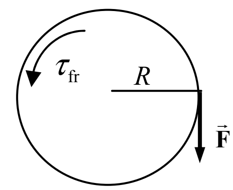
(*b*) The torque is the rate of change of angular momentum, from Eq. 8–19.

$$\tau_{\substack{\text{on}\\\text{wheel}}} = \frac{\Delta L_{\text{wheel}}}{\Delta t} = 816\text{ kg}\cdot\text{m}^2/\text{s}^2 = 816\text{ m}\cdot\text{N} \approx \boxed{820\text{ m}\cdot\text{N}}$$

(*c*) Power is given by $P = \tau\omega$. See the text immediately after Eq. 8–17.

$$P = \tau\omega = (816\text{ m}\cdot\text{N})\left(\frac{2\pi\text{ rev}}{5.5\text{ s}}\right) = \boxed{930\text{ W}}$$

96. (*a*) See the free-body diagram. Take clockwise torques as positive. Write Newton's second law for the rotational motion. The angular acceleration is constant, so constant acceleration relationships can be used. We also use the definition of radian angles, $\Delta\theta = \frac{\Delta s}{R}$.



$$\sum\tau = FR - \tau_{\text{fr}} = I\alpha_1;\ \Delta\theta_1 = \omega_0 t_1 + \tfrac{1}{2}\alpha_1 t_1^2 = \tfrac{1}{2}\alpha_1 t_1^2;\ \Delta s_1 = R\Delta\theta_1$$

Combine the relationships to find the length unrolled, Δs_1.

$$\Delta s_1 = R\Delta\theta_1 = R\left(\tfrac{1}{2}\alpha_1 t_1^2\right) = \frac{Rt_1^2}{2I}(FR - \tau_{\text{fr}})$$

$$= \frac{(0.076\text{ m})(1.3\text{ s})^2}{2(3.3\times10^{-3}\text{ kg}\cdot\text{m}^2)}[(3.5\text{ N})(0.076\text{ m}) - (0.11\text{ m}\cdot\text{N})] = 3.036\text{ m} \approx \boxed{3.0\text{ m}}$$

(*b*) Now the external force is removed, but the frictional torque is still present. The analysis is very similar to that in part (*a*), except that the initial angular velocity is needed. That angular velocity is the final angular velocity from the motion in part (*a*).

$$\omega_1 = \omega_0 + \alpha_1 t_1 = \left(\frac{FR - \tau_{\text{fr}}}{I}\right)t_1 = \frac{[(3.5\text{ N})(0.076\text{ m}) - (0.11\text{ m}\cdot\text{N})]}{(3.3\times10^{-3}\text{ kg}\cdot\text{m}^2)}(1.3\text{ s}) = 61.45\text{ rad/s}$$

$$\sum\tau = -\tau_{\text{fr}} = I\alpha_2;\ \omega_2^2 - \omega_1^2 = 2\alpha_2\Delta\theta_2 = -\omega_1^2;\ \Delta s_2 = R\Delta\theta_2$$

Combine the relationships to find the length unrolled, Δs_2.

$$\Delta s_2 = R\Delta\theta_2 = R\left(\frac{-\omega_1^2}{2\alpha_2}\right) = R\left(\frac{-\omega_1^2 I}{-2\tau_{\text{fr}}}\right) = \frac{(0.076\text{ m})(61.45\text{ rad/s})^2(3.3\times10^{-3}\text{ kg}\cdot\text{m}^2)}{2(0.11\text{ m}\cdot\text{N})}$$

$$= 4.30\text{ m} \approx \boxed{4.3\text{ m}}$$

Solutions to Search and Learn Problems

1. The radian is defined as the ratio of the distance traveled along an arc divided by the radius of the arc. When an angle in radians is multiplied by the radius the result is a distance. Therefore, when angular speed (which is angular displacement divided by time) is multiplied by the radius the result is the displacement along the arc divided by time, which is a linear speed. Degrees and revolutions are not defined in terms of arc lengths and cannot be used in the same way.

2. The angle in radians is the diameter of the object divided by the distance to the object.

$$\Delta\theta_{\text{Sun}} = \frac{2R_{\text{Sun}}}{r_{\text{Earth-Sun}}} = \frac{2(6.96\times10^5\ \text{km})}{149.6\times10^6\ \text{km}} = \boxed{9.30\times10^{-3}\ \text{rad}}$$

$$\Delta\theta_{\text{Moon}} = \frac{2R_{\text{Moon}}}{r_{\text{Earth-Moon}}} = \frac{2(1.74\times10^3\ \text{km})}{384\times10^3\ \text{km}} = \boxed{9.06\times10^{-3}\ \text{rad}}$$

Since these angles are practically the same (only a 2.6% difference), solar eclipses can occur. Based on these values, the Sun would never be completely obscured. But since the orbits are not perfect circles but are ellipses, the above values are just averages. Full (total) solar eclipses do occur.

3. (*a*) We use conservation of energy to determine the speed of each sphere as a function of position on the incline. The sphere with the greater speed would reach the bottom of the incline first. Potential energy will be zero at the base of the incline ($y = 0$) and the initial height will be H. We take position 1 to be at the top of the incline and position 2 to be at a generic location along the incline.

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2 \quad\rightarrow\quad 0 + mgH = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$

$$mg(H-y) = \tfrac{1}{2}\left(mv^2 + \tfrac{1}{2}mr^2\omega^2\right) = \tfrac{1}{2}\left(mv^2 + \tfrac{2}{5}mr^2\frac{v^2}{r^2}\right) = \tfrac{1}{2}\left(\tfrac{7}{5}mv^2\right) \rightarrow$$

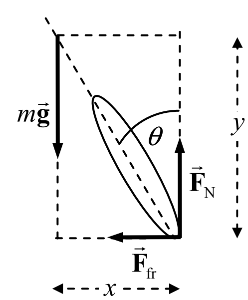
$$v = \sqrt{\tfrac{10}{7}g(H-y)}$$

The velocity along the incline does not depend upon either the mass or the radius of the sphere. Therefore, both spheres have the same speed at each point along the incline, and $\boxed{\text{both will reach the bottom of the incline at the same time.}}$

(*b*) As shown in part (*a*), both spheres will have the same speed at each point along the incline, so $\boxed{\text{both will have the same speed at the bottom of the incline.}}$

(*c*) By conservation of energy, the total kinetic energy at the bottom of the incline will equal the potential energy at the top of the incline. The initial potential energy is proportional to the mass of each sphere, $\boxed{\text{so the more massive sphere will have the greater kinetic energy}}$. The total kinetic energy is independent of the spheres' radii.

4. (*a*) In order not to fall over, the net torque on the cyclist about an axis through the CM and parallel to the ground must be zero. Consider the free-body diagram shown. Sum torques about the CM, with counterclockwise as positive, and set the sum equal to zero.



$$\sum\tau = F_N x - F_{fr} y = 0 \quad\rightarrow\quad \frac{F_{fr}}{F_N} = \frac{x}{y} = \tan\theta \rightarrow \boxed{\tan\theta = \frac{F_{fr}}{F_N}}$$

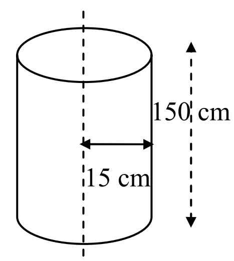
(*b*) The cyclist is not accelerating vertically, so $F_N = mg$. The cyclist is accelerating horizontally due to traveling in a circle. Thus the frictional force must be supplying the centripetal force, so $F_{fr} = mv^2/r$.

$$\tan\theta = \frac{F_{fr}}{F_N} = \frac{mv^2/r}{mg} = \frac{v^2}{rg} \rightarrow \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{(8.2\text{ m/s})^2}{(13\text{ m})(9.80\text{ m/s}^2)} = 27.82° \approx \boxed{28°}$$

(*c*) From $F_{fr} = mv^2/r$, the smallest turning radius results in the maximum force. The maximum static frictional force is $F_{fr} = \mu F_N$. Use this to calculate the radius.

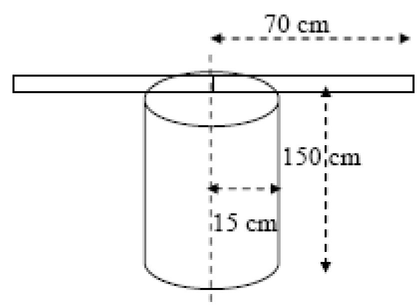
$$mv^2/r_{min} = \mu_s F_N = \mu_s mg \rightarrow r_{min} = \frac{v^2}{\mu_s g} = \frac{(8.2\text{ m/s})^2}{(0.65)(9.80\text{ m/s}^2)} = 10.56\text{ m} \approx \boxed{11\text{ m}}$$

5. Assume a mass of 50 kg, corresponding to a weight of about 110 lb. From Table 7–1, we find that the total arm and hand mass is about 12.5% of the total mass, so the rest of the body is about 87.5% of the total mass. Model the skater as a cylinder of mass 44 kg, and model each arm as a thin rod of mass 3 kg. Estimate the body as 150 cm tall with a radius of 15 cm. Estimate the arm dimension as 70 cm long.



With the arms held tightly, we approximate that the arms are part of the body cylinder. A sketch of the skater in this configuration is then as shown in the first diagram (not to scale). In this configuration, the rotational inertia is

$$I_{in} = I_{cylinder} = \tfrac{1}{2}M_{\substack{total\\body}} R_{body}^2$$



With the skater's arms extended, the second diagram applies. In this configuration, the rotational inertia is

$$I_{out} = I_{body} + I_{arms} = \tfrac{1}{2}M_{body}R_{body}^2 + 2\left(\tfrac{1}{3}M_{arm}\right)L_{arm}^2$$

The forces and torques involved in changing the configuration of the skater are internal to the skater, so the skater's angular momentum is conserved during a configuration change. Thus,

$$L_{in} = L_{out} \rightarrow I_{in}\omega_{in} = I_{out}\omega_{out} \rightarrow$$

$$\frac{\omega_{out}}{\omega_{in}} = \frac{I_{in}}{I_{out}} = \frac{\tfrac{1}{2}M_{\substack{total\\body}} R_{body}^2}{\tfrac{1}{2}M_{body}R_{body}^2 + 2\left(\tfrac{1}{3}M_{arm}\right)L_{arm}^2} = \frac{\tfrac{1}{2}(50\text{ kg})(0.15\text{ m})^2}{\tfrac{1}{2}(44\text{ kg})(0.15\text{ m})^2 + 2\left(\tfrac{1}{3}\right)(3\text{ kg})(0.70\text{ m})^2}$$

$$= 0.381 \approx \boxed{0.4}$$

Alternatively, we would have that $\omega_{in}/\omega_{out} = (0.381)^{-1} = 2.6$, so the skater spins about 2.6 × faster with the arms pulled in.

6. (*a*) The initial energy of the flywheel is used for two purposes: to give the car translational kinetic energy 30 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$W_{\text{fr}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \quad \rightarrow \quad F_{\text{fr}}\Delta x \cos 180^\circ = \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2 - \text{KE}_{\text{flywheel}} \quad \rightarrow$$

$$\text{KE}_{\text{flywheel}} = F_{\text{fr}}\Delta x + \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2$$

$$= (450\text{ N})(3.5\times10^5\text{ m}) + (30)\tfrac{1}{2}(1100\text{ kg})\left[(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2$$

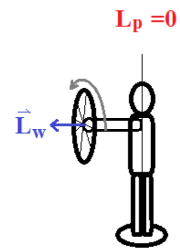
$$= 1.690\times10^8\text{ J} \approx \boxed{1.7\times10^8\text{ J}}$$

(*b*) $\text{KE}_{\text{flywheel}} = \tfrac{1}{2}I\omega^2$, so

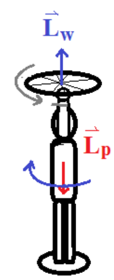
$$\omega = \sqrt{\frac{2\,\text{KE}}{\text{I}}} = \sqrt{\frac{2\,\text{KE}}{\tfrac{1}{2}M_{\text{flywheel}}R_{\text{flywheel}}^2}} = \sqrt{\frac{2(1.690\times10^8\text{ J})}{\tfrac{1}{2}(270\text{ kg})(0.75\text{ m})^2}} = 2110\text{ rad/s} \approx \boxed{2100\text{ rad/s}}$$

(*c*) To find the time, use the relationship that $\text{power} = \dfrac{\text{work}}{t}$, where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$P = \frac{W}{t} \quad \rightarrow \quad t = \frac{W}{P} = \frac{(1.690\times10^8\text{ J})}{(150\text{ hp})(746\text{ W/hp})} = 1.510\times10^3\text{ s}\left(\frac{1\text{ min}}{60\text{ s}}\right) \approx \boxed{25\text{ min}}$$



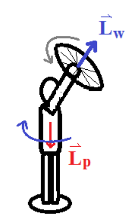
7. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, so any change that the person–wheel–platform undergoes must result in no net angular momentum along the vertical axis. The first diagram shows this condition.



(*a*) Now consider the next diagram. If the wheel is moved so that its angular momentum points upward, then the person and platform must get an equal but opposite angular momentum, which will point downward. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

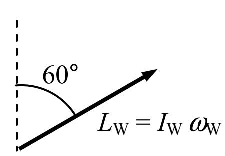
$$L_{\text{i}} = L_{\text{f}} \quad \rightarrow \quad 0 = I_{\text{W}}\omega_{\text{W}} + I_{\text{P}}\omega_{\text{P}} \quad \rightarrow \quad \boxed{\omega_{\text{P}} = -\frac{I_{\text{W}}}{I_{\text{P}}}\omega_{\text{W}}}$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.



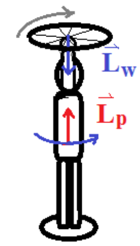
(*b*) Now consider the next diagram. If the wheel is pointing at a 60° angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_{\text{W}}\omega_{\text{W}}\cos 60^\circ$. Also see the simple vector diagram below the adjacent diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_{\text{i}} = L_{\text{f}} \quad \rightarrow \quad 0 = I_{\text{W}}\omega_{\text{W}}\cos 60^\circ + I_{\text{P}}\omega_{\text{P}} \quad \rightarrow \quad \boxed{\omega_{\text{P}} = -\frac{I_{\text{W}}}{2I_{\text{P}}}\omega_{\text{W}}}$$



Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.

(*c*) Consider the final diagram. If the wheel is moved so that its angular momentum points downward, then the person and platform must get an equal but opposite angular momentum, which will point upward. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.



$$L_i = L_f \quad \rightarrow \quad 0 = I_W\omega_W + I_P\omega_P \quad \rightarrow \quad \boxed{\omega_P = -\omega_W I_W / I_P}$$

The platform is again rotating in the opposite direction of the wheel. If the wheel is now spinning clockwise when viewed from above, the platform is spinning counterclockwise.

(*d*) Since the total angular momentum is 0, if the wheel is stopped from rotating, the platform will also stop. Thus $\boxed{\omega_P = 0}$.

9

STATIC EQUILIBRIUM; ELASTICITY AND FRACTURE

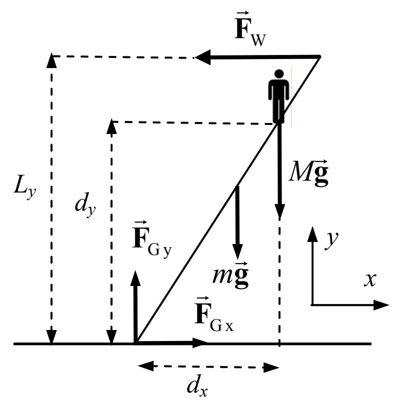
Responses to Questions

1. If the object has a net force on it of zero, then its center of mass does not accelerate. But since it is not in equilibrium, it must have a net torque and therefore an angular acceleration. Some examples are:
   - A compact disk in a player as it comes up to speed, after it has just been inserted.
   - A hard drive on a computer when the computer is first turned on.
   - A window fan immediately after the power to it has been shut off.
   - The drum of a washing machine while it is speeding up or slowing down.
2. The bungee jumper is not in equilibrium, because the net force on the jumper is not zero. If the jumper were at rest and the net force were zero, then the jumper would stay at rest by Newton's first law. The jumper has a net upward force when at the bottom of the dive, and that is why the jumper is then pulled back upward.
3. The meter stick is originally supported by both fingers. As you start to slide your fingers together, more of the weight of the meter stick is supported by the finger that is closest to the center of gravity, so the torques produced by the fingers are equal and the stick is in equilibrium. The other finger feels a smaller normal force, and therefore a smaller frictional force, so the stick slides more easily and moves closer to the center of gravity. The roles switch back and forth between the fingers as they alternately move closer to the center of gravity. Your fingers will eventually meet at the center of gravity.
4. Like almost any beam balance, the movable weights are connected to the fulcrum point by relatively long lever arms, while the platform on which you stand is connected to the fulcrum point by a very short lever arm. The scale "balances" when the torque provided by your weight (large mass, small lever arm) is equal to that provided by the sliding weights (small mass, large lever arm).
5. (*a*) If we assume that the pivot point of rotation is the lower left corner of the wall in the picture, then the gravity force acting through the CM provides the torque to keep the wall upright. Note that the gravity force would have a relatively small lever arm (about half the width of the wall). Thus, the sideways force would not have to be particularly large to start to move the wall.

   (*b*) With the horizontal extension, there are factors that make the wall less likely to overturn:
   - The mass of the second wall is larger, so the torque caused by gravity (helping to keep the wall upright) will be larger for the second wall.
   - The center of gravity of the second wall is farther to the right of the pivot point, so gravity exerts a larger torque to counteract the torque due to $\vec{\mathbf{F}}$.
   - The weight of the ground above the new part of the wall provides a large clockwise torque that helps counteract the torque due to $\vec{\mathbf{F}}$.

6. If the sum of the forces on an object is not zero, then the CM of the object will accelerate in the direction of the net force. If the sum of the torques on the object is zero, then the object has no angular acceleration. Some examples are:
   - A satellite in a circular orbit around the Earth.
   - A block sliding down an inclined plane.
   - An object that is in projectile motion but not rotating.
   - The startup motion of an elevator, changing from rest to having a nonzero velocity.

7. When the person stands near the top, the ladder is more likely to slip. In the accompanying diagram, the force of the person pushing down on the ladder $(M\vec{\mathbf{g}})$ causes a clockwise torque about the contact point with the ground, with lever arm d_x. The only force causing a counterclockwise torque about that same point is the reaction force of the wall on the ladder, $\vec{\mathbf{F}}_W$. While the ladder is in equilibrium, $\vec{\mathbf{F}}_W$ will be the same magnitude as the frictional force at the ground, $\vec{\mathbf{F}}_{Gx}$. Since $\vec{\mathbf{F}}_{Gx}$ has a maximum value, $\vec{\mathbf{F}}_W$ will have the same maximum value, and $\vec{\mathbf{F}}_W$ will have a maximum counterclockwise torque that it can exert. As the person climbs the ladder, his lever arm gets longer, so the torque due to his weight gets larger. Eventually, if the torque caused by the person is larger than the maximum torque caused by $\vec{\mathbf{F}}_W$, the ladder will start to slip—it will not stay in equilibrium.



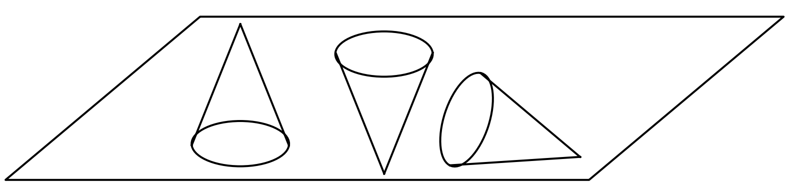
8. The mass of the meter stick is equal to the mass of the rock. Since the meter stick is uniform, its center of mass is at the 50-cm mark. In terms of rotational motion about a pivot at the 25-cm mark, we can treat the stick as though its entire mass is concentrated at the center of mass. The meter stick's mass at the 50-cm mark (25 cm from the pivot) balances the rock at the 0 mark (also 25 cm from the pivot), so the masses must be equal.

9. You lean backward in order to keep your center of mass over your feet. If, due to the heavy load, your center of mass is in front of your feet, you will fall forward.

10. (*a*) The cone will be in stable equilibrium if it is placed flat on its base. If it is tilted slightly from this position and then released, it will return to the original position.

    (*b*) The cone will be in unstable equilibrium if it is balanced on its tip. A slight displacement in this case will cause the cone to topple over.

    (*c*) If the cone is placed on its side, it will be in neutral equilibrium. If the cone is displaced slightly while on its side, it will remain in its new position.



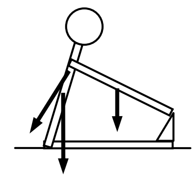
stable equilib. unstable equilib. neutral equilib.

11. When you rise on your tiptoes, your CM shifts forward. Since you are already standing with your nose and abdomen against the door, your CM cannot shift forward. Thus gravity exerts a torque on you and you are unable to stay on your tiptoes—you will return to being flat-footed on the floor.

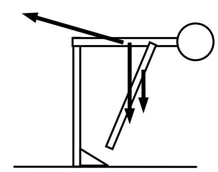
12. When you start to stand up from a normal sitting position, your CM is not over your point of support (your feet), so gravity will exert a torque about your feet that rotates you back down into the chair. You must lean forward in order that your CM is over your feet so that you can stand up.

13. While you are doing a sit-up, your abdomen muscles provide a torque to rotate you up away from the floor. The force of gravity on your upper half-body tends to pull you back down to the floor, which makes doing sit-ups difficult. The force of gravity on your lower half-body provides a torque that opposes the torque caused by the force of gravity on your upper half-body, making the sit-up a little easier. When your legs are bent, the lever arm for the lower half-body is shorter, so less counter-torque is available.

14. For rotating the upper half-body, the pivot point is near the waist and hips. In that position, the arms have a relatively small torque, even when extended, due to their smaller mass. The more massive trunk–head combination has a very short lever arm, so it also has a relatively small torque. Thus, the force of gravity on the upper body causes relatively little torque about the hips, tending to rotate you forward, and the back muscles need to produce little torque to keep you from rotating forward. The force on the upper half-body due to the back muscles is small, so the (partially rightward) force at the base of the spinal column (not shown in the diagram), to keep the spine in equilibrium, will be small.



When you stand and bend over, the lever arm for the upper body is much larger than while you are sitting, which causes a much larger torque. The CM of the arms is also farther from the support point and causes more torque. The back muscles, assumed to act at the center of the back, do not have a very long lever arm. Thus the back muscles will have to exert a large force to cause a counter-torque that keeps you from falling over. Accordingly, there will have to be a large force (mostly to the right, and not drawn in the diagram) at the base of the spine to keep the spine in equilibrium.



15. Configuration (b) is more likely to be stable. In configuration (a), the CG of the bottom brick is at the edge of the table, and the CG of the top brick is to the right of the edge of the table. Thus the CG of the two-brick system is not above the base of support, and gravity will exert a torque to roll the bricks clockwise off the table. Another way to see this is that more than 50% of the brick mass is not above the base of support—50% of the bottom brick and 75% of the top brick are to the right of the edge of the table. It is not in equilibrium.

In configuration (b), exactly half of the mass (75% of the top brick and 25% of the bottom brick) is over the edge of the table. Thus the CG of the pair is at the edge of the table—it is in unstable equilibrium.

16. A is a point of unstable equilibrium, B is a point of stable equilibrium, and C is a point of neutral equilibrium.

17. The Young's modulus for a bungee cord is much smaller than that for ordinary rope. We know that a bungee cord stretches more easily than ordinary rope. From Eq. 9–4, we have $E=\dfrac{F/A}{\Delta\ell/\ell_0}$. The value of Young's modulus is inversely proportional to the change in length of a material under a tension. Since the change in length of a bungee cord is much larger than that of an ordinary rope if other conditions are identical (stressing force, unstretched length, cross-sectional area of rope or cord), it must have a smaller Young's modulus.

18. An object under shear stress has equal and opposite forces applied across its opposite faces. This is exactly what happens with a pair of scissors. One blade of the scissors pushes down on the cardboard, while the other blade pushes up with an equal and opposite force, at a slight displacement. This produces a shear stress in the cardboard, which causes it to fail.

19. Concrete or stone should definitely *not* be used for the support on the left. The left-hand support pulls downward on the beam, so the beam must pull upward on the support. Therefore, the support will be under tension and should not be made of ordinary concrete or stone, since these materials are weak under tension. The right-hand support pushes up on the beam, so the beam pushes down on it; it will therefore be under a compression force. Making this support of concrete or stone would be acceptable.

Responses to MisConceptual Questions

1. (*d*) In attempting to solve this problem, students frequently try to divide the beam into multiple parts to calculate the torque due to the weight of the beam. The beam should be considered as a single object with its weight acting at its center of mass ($\frac{1}{4}\ell$ from the pivot). Since the woman is on the opposite side of the pivot and at the same distance as the beam's center of mass, their forces of gravity and masses must be equal.

2. (*d*) A common misconception is that a nonrotating object has an axis of rotation. If an object is not rotating, it is not rotating about any arbitrary point. When solving an equilibrium problem with no rotation, the student can select any axis for the torques that facilitates solving the problem.

3. (*a*) Students might think that for the net force on the beam to be zero, the tension would equal the weight of the beam. However, this does not take into account the force that the wall exerts on the hinged end. Students might assume that the tension is equal to half of the beam's weight. However, this does not take into account the vector nature of the tension. The vertical component of the tension is equal to half of the weight, but there is also a horizontal component. Adding these two components yields a tension at least half of the weight of the beam.

4. (*c*) Drawing a free-body diagram for this problem will resolve student misconceptions. When the ball is pulled to the side, there are three forces acting on the ball: the vertical weight, the horizontal applied force, and the tension along the direction of the cable. Resolving the tension into horizontal and vertical parts and applying Newton's second law in equilibrium, we can see that the applied force is equal to the horizontal component of the tension.

5. (*a*) As the child leans forward, her center of mass moves closer to the pivot point, which decreases her lever arm. The seesaw is no longer in equilibrium. Since the torque on her side has decreased, she will rise.

6. (*c*) A common misconception is that each cord will support one-half of the weight regardless of the angle. An analysis of the forces using Newton's second law in equilibrium shows that the horizontal components of the tension are equal. Since cord A makes a larger angle with the horizontal, it has a greater total tension and therefore supports more than half the suspended weight.

7. (*c*) The applied force is proportional to the stress, so increasing the force will affect the stress. The strain is how the rope responds to the stress. Increasing the force will then affect the strain. Young's modulus is the constant of proportionality between the stress and strain. It is determined by the properties of the material, so it is not affected by pulling on the rope.

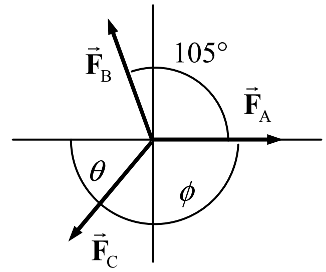
8. (*e*) Students may consider the tension equal to the woman's weight, or half of the woman's weight, if they do not consider the vector nature of the forces. A free-body diagram for the point at the bottom of the woman's foot shows three forces acting: the weight of the woman and the diagonal tensions in the wire on each side of her foot. Applying Newton's second law in equilibrium in the vertical direction shows that the vertical component of the tension must equal half of her weight. Since vertical displacement is small compared to the horizontal length of the wire, the total tension is much greater than the vertical component of the tension.

9. (*d*) When the length, width, and number of floors are doubled, the weight of the garage increases by a factor of eight. To keep the stress on the columns unchanged, the area of the columns should also increase by a factor of eight.

10. (*d*) The stress (applied force) is proportional to the strain (change in length). Doubling the stress will cause the strain to double also.

Solutions to Problems

1. If the tree is not accelerating, then the net force in all directions is 0.



$$\sum F_x = F_A + F_B \cos 105° + F_{Cx} = 0 \quad \rightarrow$$

$$F_{Cx} = -F_A - F_B \cos 105° = -385 \text{ N} - (475 \text{ N}) \cos 105° = -262.1 \text{ N}$$

$$\sum F_y = F_B \sin 105° + F_{Cy} = 0 \quad \rightarrow$$

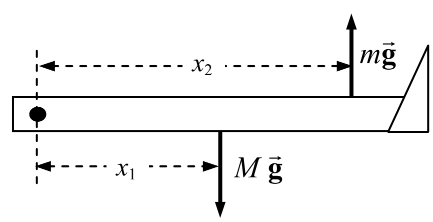
$$F_{Cy} = -F_B \sin 105° = -(475 \text{ N}) \sin 105° = -458.8 \text{ N}$$

$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = \sqrt{(-262.1 \text{ N})^2 + (-458.8 \text{ N})^2} = 528.4 \text{ N} \approx \boxed{528 \text{ N}}$$

$$\theta = \tan^{-1}\frac{F_{Cy}}{F_{Cx}} = \tan^{-1}\frac{-458.8 \text{ N}}{-262.1 \text{ N}} = 60.3°,\ \phi = 180° - 60.3° = \boxed{120°}$$

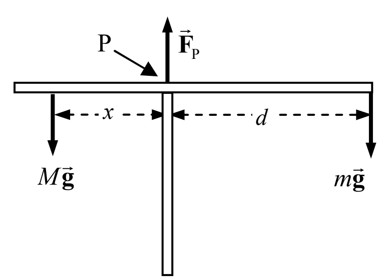
So $\vec{\mathbf{F}}_C$ is 528 N, at an angle of 120° clockwise from $\vec{\mathbf{F}}_A$. The angle has 3 significant figures.

2. Because the mass *m* is stationary, the tension in the rope pulling up on the sling must be *mg*, and the force of the sling on the leg must be *mg*, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.



$$\sum \tau = mgx_2 - Mgx_1 = 0 \quad \rightarrow \quad m = M\frac{x_1}{x_2} = (15.0 \text{ kg})\frac{(35.0 \text{ cm})}{(78.0 \text{ cm})} = \boxed{6.73 \text{ kg}}$$

3. (*a*) See the free-body diagram. Calculate torques about the pivot point P labeled in the diagram. The upward force at the pivot will not have any torque. The total torque is zero, since the crane is in equilibrium.



$$\sum \tau = Mgx - mgd = 0 \quad \rightarrow$$

$$x = \frac{md}{M} = \frac{(2800 \text{ kg})(7.7 \text{ m})}{(9500 \text{ kg})} = \boxed{2.3 \text{ m}}$$

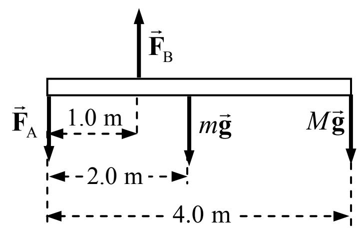
(*b*) Again, we sum torques about the pivot point. Mass *m* is the unknown in this case, and the counterweight is at its maximum distance from the pivot.

$$\sum \tau = Mgx_{\text{max}} - m_{\text{max}} gd = 0 \quad \rightarrow \quad m_{\text{max}} = \frac{Mx_{\text{max}}}{d} = \frac{(9500\ \text{kg})(3.4\ \text{m})}{(7.7\ \text{kg})} = \boxed{4200\ \text{kg}}$$

4. Her torque is her weight times the distance *x* between the diver and the left support post.

$$\tau = mgx \ \rightarrow \ m = \frac{\tau}{gx} = \frac{1800\ \text{m}\cdot\text{N}}{(9.80\ \text{m/s}^2)(4.0\ \text{m})} = \boxed{46\ \text{kg}}$$

5. (*a*) Let $m = 0$. Calculate the net torque about the left end of the diving board, with counterclockwise torques positive. Since the board is in equilibrium, the net torque is zero.



$$\sum \tau = F_\text{B}(1.0\ \text{m}) - Mg(4.0\ \text{m}) = 0 \quad \rightarrow$$

$$F_\text{B} = 4Mg = 4(52\ \text{kg})(9.80\ \text{m/s}^2) = 2038\ \text{N}$$

$$\approx \boxed{2.0\times 10^3\ \text{N, up}}$$

Use Newton's second law in the vertical direction to find F_A.

$$\sum F_y = F_\text{B} - Mg - F_\text{A} = 0 \quad \rightarrow$$

$$F_\text{A} = F_\text{B} - Mg = 4Mg - Mg = 3Mg = 3(52\ \text{kg})(9.80\ \text{m/s}^2) = 1529\ \text{N} \approx \boxed{1500\ \text{N, down}}$$

(*b*) Repeat the basic process, but with $m = 28$ kg. The weight of the board will add more clockwise torque.

$$\sum \tau = F_\text{B}(1.0\ \text{m}) - mg(2.0\ \text{m}) - Mg(4.0\ \text{m}) = 0 \quad \rightarrow$$

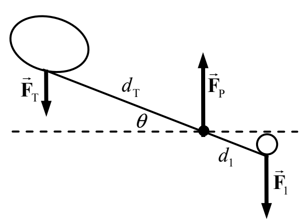
$$F_\text{B} = 4Mg + 2mg = [4(52\ \text{kg}) + 2(28\ \text{kg})](9.80\ \text{m/s}^2) = 2587\ \text{N} \approx \boxed{2600\ \text{N, up}}$$

$$\sum F_y = F_\text{B} - Mg - mg - F_\text{A} \quad \rightarrow$$

$$F_\text{A} = F_\text{B} - Mg - mg = 4Mg + 2mg - Mg - mg = 3Mg + mg$$

$$= [3(52\ \text{kg}) + 28\ \text{kg}](9.80\ \text{m/s}^2) = 1803\ \text{N} \approx \boxed{1800\ \text{N, down}}$$

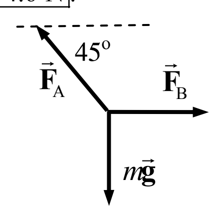
6. Since each half of the forceps is in equilibrium, the net torque on each half of the forceps is zero. Calculate torques with respect to an axis perpendicular to the plane of the forceps, through point P, counterclockwise being positive. Consider a force diagram for one-half of the forceps. $\vec{\mathbf{F}}_1$ is the force on the half-forceps due to the plastic rod, and force $\vec{\mathbf{F}}_\text{P}$ is the force on the half-forceps from the pin joint. $\vec{\mathbf{F}}_\text{P}$ exerts no torque about point P.



$$\sum \tau = F_\text{T} d_\text{T} \cos\theta - F_1 d_1 \cos\theta = 0 \quad \rightarrow \quad F_1 = F_\text{T}\frac{d_\text{T}}{d_1} = (11.0\ \text{N})\frac{8.50\ \text{cm}}{2.70\ \text{cm}} = 34.6\ \text{N}$$

The force that the forceps exerts on the rod is the opposite of $\vec{\mathbf{F}}_1$, so it is also $\boxed{34.6\ \text{N}}$.

7. Write Newton's second law for the junction, in both the *x* and *y* directions.



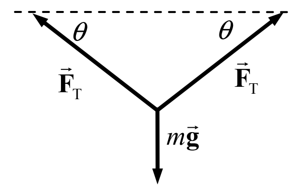
$$\sum F_x = F_\text{B} - F_\text{A}\cos 45° = 0$$

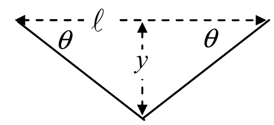
From this, we see that $F_\text{A} > F_\text{B}$. Thus set $F_\text{A} = 1660$ N.

$$\sum F_y = F_\text{A}\sin 45° - mg = 0$$

$$mg = F_\text{A}\sin 45° = (1660\ \text{N})\sin 45° = 1174\ \text{N} \approx \boxed{1200\ \text{N}}$$

8. Since the backpack is midway between the two trees, the angles in the diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the downward vertical force. The angle is determined by the distance between the trees and the amount of sag at the midpoint, as illustrated in the second diagram.





(*a*) $\theta = \tan^{-1}\dfrac{y}{\ell/2} = \tan^{-1}\dfrac{1.5\text{ m}}{3.3\text{ m}} = 24.4^\circ$

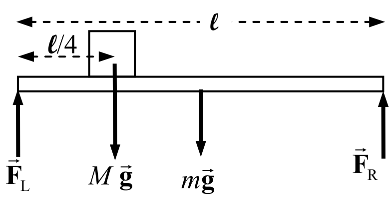
$$\sum F_y = 2F_T \sin\theta_1 - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2\sin\theta_1} = \frac{(19\text{ kg})(9.80\text{ m/s}^2)}{2\sin 24.4^\circ} = 225.4\text{ N} \approx \boxed{230\text{ N}}$$

(*b*) $\theta = \tan^{-1}\dfrac{y}{\ell/2} = \tan^{-1}\dfrac{0.15\text{ m}}{3.3\text{ m}} = 2.60^\circ$

$$F_T = \frac{mg}{2\sin\theta_1} = \frac{(19\text{ kg})(9.80\text{ m/s}^2)}{2\sin 2.60^\circ} = 2052\text{ N} \approx \boxed{2100\text{ N}}$$

9. Let m be the mass of the beam, and M be the mass of the piano. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum \tau = F_R\ell - mg\left(\tfrac{1}{2}\ell\right) - Mg\left(\tfrac{1}{4}\ell\right) = 0$$

$$F_R = \left(\tfrac{1}{2}m + \tfrac{1}{4}M\right)g = \left[\tfrac{1}{2}(110\text{ kg}) + \tfrac{1}{4}(320\text{ kg})\right](9.80\text{ m/s}^2) = 1320\text{ N}$$

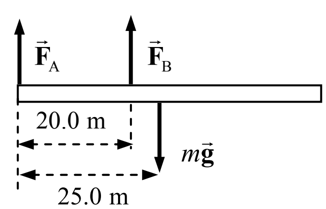
$$\sum F_y = F_L + F_R - mg - Mg = 0$$

$$F_L = (m+M)g - F_R = (430\text{ kg})(9.80\text{ m/s}^2) - 1.32\times10^3\text{ N} = 2890\text{ N}$$

The forces on the supports are equal in magnitude and opposite in direction to the above two results.

$\boxed{F_R = 1300\text{ N down}}$ $\boxed{F_L = 2900\text{ N down}}$

10. Calculate torques about the left end of the beam, with counterclockwise torques positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum \tau = F_B(20.0\text{ m}) - mg(25.0\text{ m}) = 0 \quad \rightarrow$$

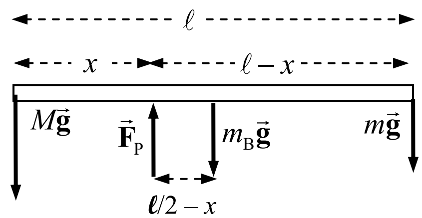
$$F_B = \frac{25.0}{20.0}mg = (1.25)(1200\text{ kg})(9.80\text{ m/s}^2) = \boxed{1.5\times10^4\text{ N}}$$

$$\sum F_y = F_A + F_B - mg = 0$$

$$F_A = mg - F_B = mg - 1.25mg = -0.25mg = -(0.25)(1200\text{ kg})(9.80\text{ m/s}^2) = \boxed{-2900\text{ N}}$$

Notice that $\vec{\mathbf{F}}_A$ points down.

11. The pivot should be placed so that the net torque on the board is zero. We calculate torques about the pivot point, with counterclockwise torques positive. The upward force $\vec{\mathbf{F}}_{\mathrm{P}}$ at the pivot point is shown, but it exerts no torque about the pivot point. The mass of the child is m, the mass of the adult is M, the mass of the board is m_{B}, and the center of gravity is at the middle of the board.



(*a*) Ignore the force $m_{\mathrm{B}}g$.

$$\sum \tau = Mgx - mg(\ell - x) = 0 \quad \rightarrow$$

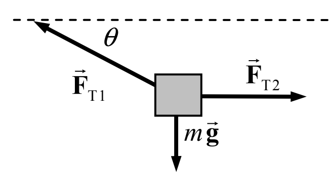
$$x = \frac{m}{m+M}\ell = \frac{(25\text{ kg})}{(25\text{ kg} + 75\text{ kg})}(9.0\text{ m}) = 2.25\text{ m} \approx \boxed{2.3\text{ m from adult}}$$

(*b*) Include the force $m_{\mathrm{B}}g$.

$$\sum \tau = Mgx - mg(\ell - x) - m_{\mathrm{B}}g(\ell/2 - x) = 0$$

$$x = \frac{(m + m_{\mathrm{B}}/2)}{(M + m + m_{\mathrm{B}})}\ell = \frac{(25\text{ kg} + 7.5\text{ kg})}{(75\text{ kg} + 25\text{ kg} + 15\text{ kg})}(9.0\text{ m}) = 2.54\text{ m} \approx \boxed{2.5\text{ m from adult}}$$

12. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.



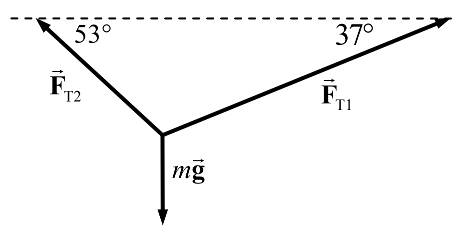
$$\sum F_x = F_{\mathrm{T2}} - F_{\mathrm{T1}}\cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{T2}} = F_{\mathrm{T1}}\cos\theta$$

$$\sum F_y = F_{\mathrm{T1}}\sin\theta - mg = 0 \quad \rightarrow \quad F_{\mathrm{T1}} = \frac{mg}{\sin\theta}$$

$$F_{\mathrm{T2}} = F_{\mathrm{T1}}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \frac{(190\text{ kg})(9.80\text{ m/s}^2)}{\tan 33^\circ} = 2867\text{ N} \approx \boxed{2900\text{ N}}$$

$$F_{\mathrm{T1}} = \frac{mg}{\sin\theta} = \frac{(190\text{ kg})(9.80\text{ m/s}^2)}{\sin 33^\circ} = 3418\text{ N} \approx \boxed{3400\text{ N}}$$

$\boxed{13.}$ Draw a free-body diagram of the junction of the three wires. The tensions can be found from the conditions for force equilibrium.



$$\sum F_x = F_{\mathrm{T1}}\cos 37^\circ - F_{\mathrm{T2}}\cos 53^\circ = 0 \quad \rightarrow \quad F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}$$

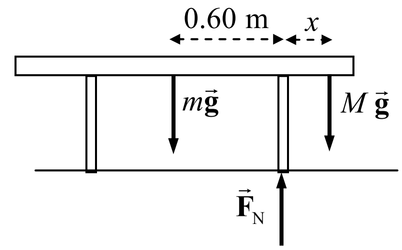
$$\sum F_y = F_{\mathrm{T1}}\sin 37^\circ + F_{\mathrm{T2}}\sin 53^\circ - mg = 0$$

$$F_{\mathrm{T1}}\sin 37^\circ + \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}}\sin 53^\circ - mg = 0 \quad \rightarrow$$

$$F_{\mathrm{T1}} = \frac{(33\text{ kg})(9.80\text{ m/s}^2)}{\sin 37^\circ + \dfrac{\cos 37^\circ}{\cos 53^\circ}\sin 53^\circ} = 194.6\text{ N} \approx \boxed{190\text{ N}}$$

$$F_{\mathrm{T2}} = \frac{\cos 37^\circ}{\cos 53^\circ}F_{\mathrm{T1}} = \frac{\cos 37^\circ}{\cos 53^\circ}(1.946\times 10^2\text{ N}) = 258.3\text{ N} \approx \boxed{260\text{ N}}$$

14. The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass M) is on the right side of the table and that the table (mass m) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table such that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.



$$\sum \tau = mg(0.60\text{ m}) - Mgx = 0 \quad \rightarrow \quad x = (0.60\text{ m})\frac{m}{M} = (0.60\text{ m})\frac{24.0\text{ kg}}{66.0\text{ kg}} = 0.218\text{ m}$$

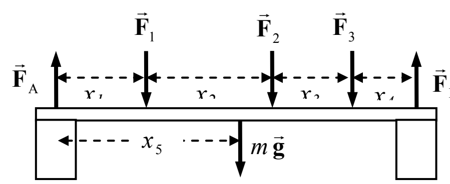
Thus the distance from the edge of the table is $0.50\text{ m} - 0.218\text{ m} = \boxed{0.28\text{ m}}$.

15. The bottle opener will pull upward on the cork with a force of magnitude F_{cork}, so there is a downward force on the opener of magnitude F_{cork}. We assume that there is no net torque on the opener, so it has no angular acceleration. Calculate torques about the rim of the bottle where the opener is resting on the rim.

$$\sum \tau = F(79\text{ mm}) - F_{\text{cork}}(9\text{ mm}) = 0 \quad \rightarrow$$

$$F = \frac{9}{70}F_{\text{cork}} = \frac{9}{79}(200\text{ N}) \text{ to } \frac{9}{79}(400\text{ N}) = 22.8\text{ N to }45.6\text{ N} \approx \boxed{20\text{ N to }50\text{ N}}$$

16. The beam is in equilibrium, so both the net torque and net force on it must be zero. From the free-body diagram, calculate the net torque about the center of the left support, with counterclockwise torques as positive. Calculate the net force, with upward as positive. Use those two equations to find F_A and F_B.



$$\sum \tau = F_B(x_1 + x_2 + x_3 + x_4) - F_1x_1 - F_2(x_1 + x_2) - F_3(x_1 + x_2 + x_3) - mgx_5$$

$$F_B = \frac{F_1x_1 + F_2(x_1 + x_2) + F_3(x_1 + x_2 + x_3) + mgx_5}{(x_1 + x_2 + x_3 + x_4)}$$

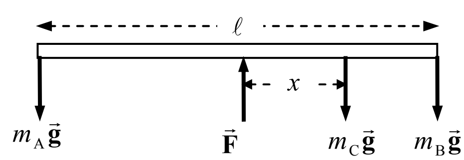
$$= \frac{(4300\text{ N})(2.0\text{ m}) + (3100\text{ N})(6.0\text{ m}) + (2200\text{ N})(9.0\text{ m}) + (280\text{ kg})(9.80\text{ m/s}^2)(5.0\text{ m})}{10.0\text{ m}}$$

$$= 6072\text{ N} \approx \boxed{6100\text{ N}}$$

$$\sum F = F_A + F_B - F_1 - F_2 - F_3 - mg = 0$$

$$F_A = F_1 + F_2 + F_3 + mg - F_B = 9600\text{ N} + (280\text{ kg})(9.80\text{ m/s}^2) - 6072\text{ N} = 6272\text{ N} \approx \boxed{6300\text{ N}}$$

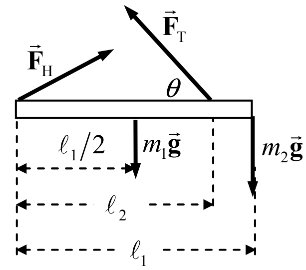
17. From the free-body diagram, the conditions of equilibrium are used to find the location of the girl (mass m_C). The 45-kg boy is represented by m_A and the 35-kg boy by m_B. Calculate torques about the center of the seesaw, and take counterclockwise torques to be positive. The upward force of the fulcrum on the seesaw $(\vec{\mathbf{F}})$ causes no torque about the center.



$$\sum \tau = m_A g\left(\tfrac{1}{2}\ell\right) - m_C gx - m_B g\left(\tfrac{1}{2}\ell\right) = 0$$

$$x = \frac{(m_A - m_B)}{m_C}\left(\tfrac{1}{2}\ell\right) = \frac{(45\text{ kg} - 35\text{ kg})}{25\text{ kg}}\tfrac{1}{2}(3.2\text{ m}) = \boxed{0.64\text{ m}}$$

18. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.



$$\sum\tau = (F_T \sin\theta)\ell_2 - m_1 g\ell_1/2 - m_2 g\ell_1 = 0 \quad\rightarrow$$

$$F_T = \frac{\frac{1}{2}m_1 g\ell_1 + m_2 g\ell_1}{\ell_2 \sin\theta} = \frac{\frac{1}{2}(155\text{ N})(1.70\text{ m}) + (215\text{ N})(1.70\text{ m})}{(1.35\text{ m})(\sin 35.0°)}$$

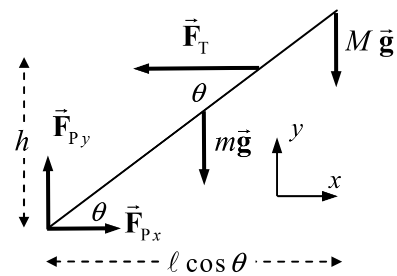
$$= 642.2\text{ N} \approx \boxed{642\text{ N}}$$

$$\sum F_x = F_{Hx} - F_T\cos\theta = 0 \quad\rightarrow\quad F_{Hx} = F_T\cos\theta = (642.2\text{ N})\cos 35.0° = 526.1\text{ N} \approx \boxed{526\text{ N}}$$

$$\sum F_y = F_{Hy} + F_T\sin\theta - m_1 g - m_2 g = 0 \quad\rightarrow$$

$$F_{Hy} = m_1 g + m_2 g - F_T\sin\theta = 155\text{ N} + 215\text{ N} - (642.2\text{ N})\sin 35.0° = 1.649\text{ N} \approx \boxed{2\text{ N}}$$

19. (*a*) The pole is in equilibrium, so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is ℓ.



$$\sum\tau = F_T h - mg(\ell/2)\cos\theta - Mg\ell\cos\theta = 0$$

$$F_T = \frac{(m/2 + M)\,g\ell\cos\theta}{h}$$

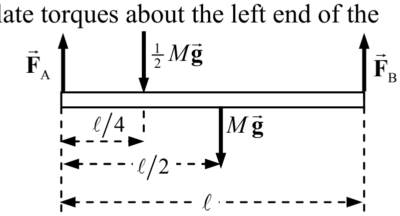
$$= \frac{(6.0\text{ kg} + 21.5\text{ kg})(9.80\text{ m/s}^2)(7.20\text{ m})\cos 37°}{3.80\text{ m}} = 407.8\text{ N} \approx \boxed{410\text{ N}}$$

(*b*) The net force on the pole is also zero since it is in equilibrium. Write Newton's second law in both the *x* and *y* directions to solve for the forces at the pivot.

$$\sum F_x = F_{Px} - F_T = 0 \quad\rightarrow\quad F_{Px} = F_T = \boxed{410\text{ N}}$$

$$\sum F_y = F_{Py} - mg - Mg = 0 \quad\rightarrow\quad F_{Py} = (m+M)g = (33.5\text{ kg})(9.80\text{ m/s}^2) = \boxed{328\text{ N}}$$

20. The center of gravity of each beam is at its geometric center. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



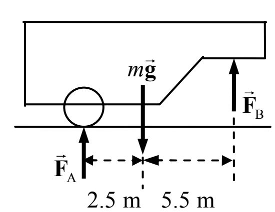
$$\sum\tau = F_B\ell - Mg(\ell/2) - \tfrac{1}{2}Mg(\ell/4) = 0 \quad\rightarrow$$

$$F_B = \tfrac{5}{8}Mg = \tfrac{5}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 5758\text{ N} \approx \boxed{5800\text{ N}}$$

$$\sum F_y = F_A + F_B - Mg - \tfrac{1}{2}Mg = 0 \quad\rightarrow$$

$$F_A = \tfrac{3}{2}Mg - F_B = \tfrac{7}{8}Mg = \tfrac{7}{8}(940\text{ kg})(9.80\text{ m/s}^2) = 8061\text{ N} \approx \boxed{8100\text{ N}}$$

21. To find the normal force exerted on the road by the trailer tires, take the torques about point B, with counterclockwise torques as positive.



$$\sum\tau = mg(5.5\text{ m}) - F_A(8.0\text{ m}) = 0 \quad\rightarrow$$

$$F_A = mg\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = (2500\text{ kg})(9.80\text{ m/s}^2)\left(\frac{5.5\text{ m}}{8.0\text{ m}}\right) = 16{,}844\text{ N}$$

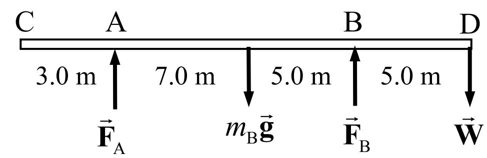
$$\approx \boxed{1.7\times10^4\text{ N}}$$

The net force in the vertical direction must be zero.

$$\sum F_y = F_B + F_A - mg = 0 \quad \rightarrow$$

$$F_B = mg - F_A = (2500\text{ kg})(9.80\text{ m/s}^2) - 16{,}844\text{ N} = 7656\text{ N} \approx \boxed{7.7\times10^3\text{ N}}$$

22. (*a*) For the extreme case of the beam being ready to tip, there would be no normal force at point A from the support. Use the free-body diagram to write the equation of rotational equilibrium under that condition to find the weight of the person, with $F_A = 0$. Take torques about the location of support B, and call counterclockwise torques positive. $\vec{\mathbf{W}}$ is the weight of the person, and m_B is the mass of the beam.



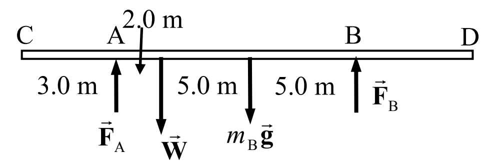
$$\sum \tau = m_B g(5.0\text{ m}) - W(5.0\text{ m}) = 0 \quad \rightarrow$$

$$W = m_B g = \boxed{650\text{ N}}$$

(*b*) With the person standing at point D, we have already assumed that $\boxed{F_A = 0}$. The net force in the vertical direction must also be zero.

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W = 650\text{ N} + 650\text{ N} = \boxed{1.30\times10^3\text{ N}}$$

(*c*) The person moves to a different spot, so the free-body diagram changes again as shown. Again use the net torque about support B and then use the net vertical force.

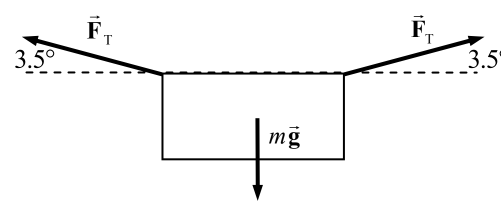


$$\sum \tau = m_B g(5.0\text{ m}) + W(10.0\text{ m}) - F_A(12.0\text{ m}) = 0$$

$$F_A = \frac{m_B g(5.0\text{ m}) + W(10.0\text{ m})}{12.0\text{ m}} = \frac{(650\text{ N})(5.0\text{ m}) + (650\text{ N})(10.0\text{ m})}{12.0\text{ m}} = \boxed{810\text{ N}}$$

$$\sum F_y = F_A + F_B - m_B g - W = 0 \quad \rightarrow \quad F_B = m_B g + W - F_A = 1300\text{ N} - 810\text{ N} = \boxed{490\text{ N}}$$

23. Draw the free-body diagram for the sheet, and write Newton's second law for the vertical direction. Note that the tension is the same in both parts of the clothesline.



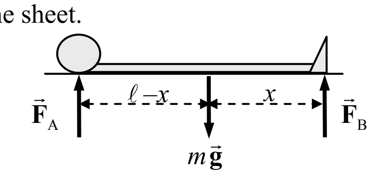
$$\sum F_y = F_T \sin 3.5° + F_T \sin 3.5° - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2(\sin 3.5°)} = \frac{(0.75\text{ kg})(9.80\text{ m/s}^2)}{2\,(\sin 3.5°)}$$

$$= \boxed{60\text{ N}} \text{ (2 significant figures)}$$

The 60-N tension is much higher than the ~7.5-N weight of the sheet because of the small angle. Only the vertical components of the tension are supporting the sheet. Since the angle is small, the tension has to be large to have a large enough vertical component to hold up the sheet.

$\boxed{24.}$ The person is in equilibrium, so both the net torque and net force must be zero. From the free-body diagram, calculate the net torque about the center of gravity, with counterclockwise torques as positive. Use that calculation to find the location of the center of gravity, a distance x from the feet.

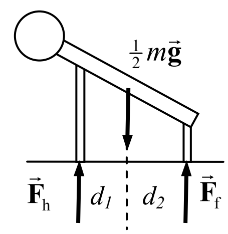


$$\sum \tau = F_{\text{B}}x - F_{\text{A}}(\ell - x) = 0$$

$$x = \frac{F_{\text{A}}}{F_{\text{A}} + F_{\text{B}}}\ell = \frac{m_{\text{A}}g}{m_{\text{A}}g + m_{\text{B}}g}\ell = \frac{m_{\text{A}}}{m_{\text{A}} + m_{\text{B}}}\ell = \frac{35.1\text{ kg}}{31.6\text{ kg} + 35.1\text{ kg}}(1.72\text{ m}) = \boxed{9.05\times10^{-1}\text{ m}}$$

The center of gravity is about 90.5 cm from the feet.

25. (*a*) The man is in equilibrium, so the net force and the net torque on him must be zero. We use half of his weight and then consider the force just on one hand and one foot, assuming that he is symmetrical. Take torques about the point where the foot touches the ground, with counterclockwise as positive.



$$\sum \tau = \tfrac{1}{2}mgd_2 - F_{\text{h}}(d_1 + d_2) = 0$$

$$F_{\text{h}} = \frac{mgd_2}{2(d_1 + d_2)} = \frac{(68\text{ kg})(9.80\text{ m/s}^2)(0.95\text{ m})}{2(1.37\text{ m})} = 231\text{ N} \approx \boxed{230\text{ N}}$$

(*b*) Use Newton's second law for vertical forces to find the force on the feet.

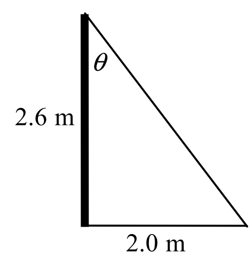
$$\sum F_y = 2F_{\text{h}} + 2F_{\text{f}} - mg = 0$$

$$F_{\text{f}} = \tfrac{1}{2}mg - F_{\text{h}} = \tfrac{1}{2}(68\text{ kg})(9.80\text{ m/s}^2) - 231\text{ N} = 103\text{ N} \approx \boxed{100\text{ N}}$$

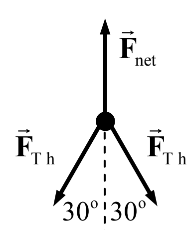
The value of 100 N has 2 significant figures.

26. First consider the triangle made by the pole and one of the wires (first diagram). It has a vertical leg of 2.6 m and a horizontal leg of 2.0 m. The angle that the tension (along the wire) makes with the vertical is

$\theta = \tan^{-1}\dfrac{2.0}{2.6} = 37.6°.$ The part of the tension that is parallel to the ground is therefore $F_{\text{T h}} = F_{\text{T}}\sin\theta.$



Now consider a top view of the pole, showing only force parallel to the ground (second diagram). The horizontal parts of the tension lie as the sides of an equilateral triangle, so each makes a 30° angle with the tension force of the net. Write the equilibrium equation for the forces along the direction of the tension in the net.



$$\sum F = F_{\text{net}} - 2F_{\text{T h}}\cos 30° = 0 \quad \rightarrow$$

$$F_{\text{net}} = 2F_{\text{T}}\sin\theta\cos 30° = 2(115\text{ N})\sin 37.6°\cos 30° = 121.5\text{ N} \approx \boxed{120\text{ N}}$$

27. (*a*) Choose the coordinates as shown in the free-body diagram.

(*b*) Write the equilibrium conditions for the horizontal and vertical forces.

$$\sum F_x = F_{\text{rope}}\sin\phi - F_{\substack{\text{hinge}\\\text{horiz}}} = 0 \quad \rightarrow$$

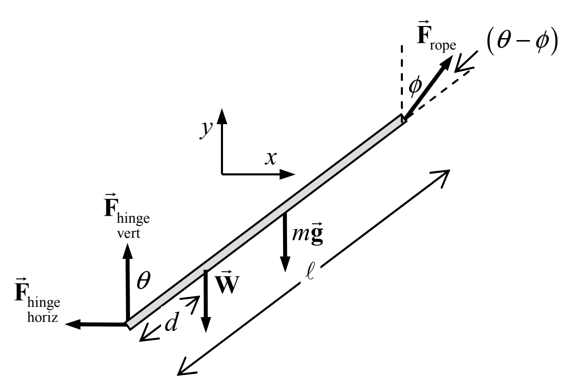
$$F_{\substack{\text{hinge}\\\text{horiz}}} = F_{\text{rope}}\sin\phi = (85\text{ N})\sin 37° = \boxed{51\text{ N}}$$

$$\sum F_y = F_{\text{rope}}\cos\phi + F_{\substack{\text{hinge}\\\text{vert}}} - mg - W = 0 \quad \rightarrow$$

$$F_{\substack{\text{hinge}\\\text{vert}}} = mg + W - F_{\text{rope}}\cos\phi = (3.8\text{ kg})(9.80\text{ m/s}^2)$$

$$+22\text{ N} - (85\text{ N})\cos 37° = -8.6\text{ N} \approx \boxed{-9\text{ N}}$$

So the vertical hinge force actually points downward.



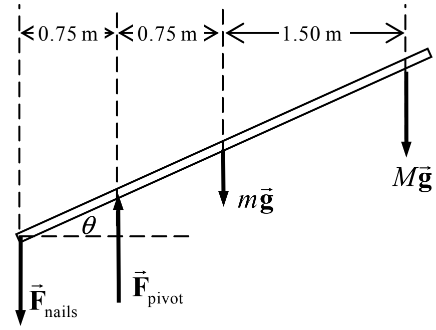
(*c*) We take torques about the hinge point, with clockwise torques as positive.

$$\sum \tau = Wd \sin\theta + mg\left(\tfrac{1}{2}\ell\right)\sin\theta - F_{\text{rope}}\,\ell \sin(\theta-\phi) = 0 \quad \rightarrow$$

$$d = \frac{F_{\text{rope}}\ell \sin(\theta-\phi) - mg\left(\tfrac{1}{2}\ell\right)\sin\theta}{W\sin\theta}$$

$$= \frac{(85\ \text{N})(5.0\ \text{m})\sin 16^\circ - (3.8\ \text{kg})(9.80\ \text{m/s}^2)(2.5\ \text{m})\sin 53^\circ}{(22\ \text{N})\sin 53^\circ} = 2.436\ \text{m} \approx \boxed{2.4\ \text{m}}$$

28. See the free-body diagram. Take torques about the pivot point, with clockwise torques as positive. The plank is in equilibrium. Let *m* represent the mass of the plank and *M* represent the mass of the person. The minimum nail force would occur if there was no normal force pushing up on the left end of the board.

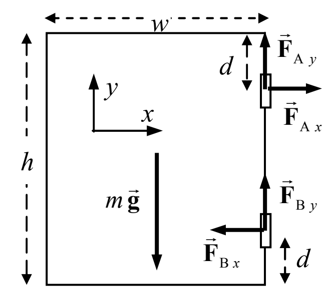


$$\sum \tau = mg(0.75\ \text{m})\cos\theta + Mg(2.25\ \text{m})\cos\theta - F_{\text{nails}}(0.75\ \text{m})\cos\theta = 0 \quad \rightarrow$$

$$F_{\text{nails}} = \frac{mg(0.75\ \text{m}) + Mg(2.25\ \text{m})}{(0.75\ \text{m})} = mg + 3Mg$$

$$= (45\ \text{kg} + 3\,(65\ \text{kg}))(9.80\ \text{m/s}^2) = 2352\ \text{N} \approx \boxed{2400\ \text{N}}$$

29. The forces on the door are due to gravity and the hinges. Since the door is in equilibrium, the net torque and net force must be zero. Write the three equations of equilibrium. Calculate torques about the bottom hinge, with counterclockwise torques as positive. From the statement of the problem, $F_{\text{A}y} = F_{\text{B}y} = \frac{1}{2}mg$.



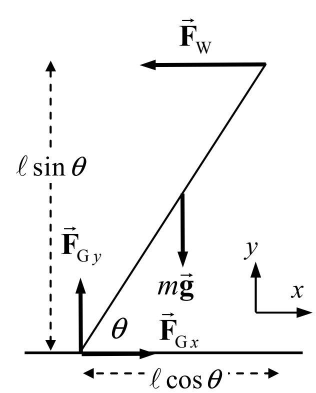
$$\sum \tau = mg\frac{w}{2} - F_{\text{A}x}(h-2d) = 0$$

$$F_{\text{A}x} = \frac{mgw}{2(h-2d)} = \frac{(13.0\ \text{kg})(9.80\ \text{m/s}^2)(1.30\ \text{m})}{2(2.30\ \text{m} - 0.80\ \text{m})} = \boxed{55.2\ \text{N}}$$

$$\sum F_x = F_{\text{A}x} - F_{\text{B}x} = 0 \quad \rightarrow \quad F_{\text{B}x} = F_{\text{A}x} = \boxed{55.2\ \text{N}}$$

$$\sum F_y = F_{\text{A}y} + F_{\text{B}y} - mg = 0 \quad \rightarrow \quad F_{\text{A}y} = F_{\text{B}y} = \tfrac{1}{2}mg = \tfrac{1}{2}(13.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{63.7\ \text{N}}$$

30. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder and counterclockwise torques as positive.



$$\sum \tau = F_{\text{W}}\ell\sin\theta - mg\left(\tfrac{1}{2}\ell\cos\theta\right) = 0 \quad \rightarrow \quad F_{\text{W}} = \frac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{\text{G}x} - F_{\text{W}} = 0 \quad \rightarrow \quad F_{\text{G}x} = F_{\text{W}} = \frac{1}{2}\frac{mg}{\tan\theta}$$

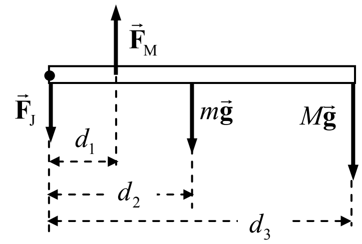
$$\sum F_y = F_{\text{G}y} - mg = 0 \quad \rightarrow \quad F_{\text{G}y} = mg$$

For the ladder not to slip, the force at the ground F_{Gx} must be less than or equal to the maximum force of static friction.

$$F_{Gx} \le \mu_s F_N = \mu_s F_{Gy} \quad \to \quad \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu_s mg \quad \to \quad \frac{1}{2\mu_s} \le \tan\theta \quad \to \quad \theta \ge \tan^{-1}\left(\frac{1}{2\mu_s}\right)$$

Thus the minimum angle is $\boxed{\theta_{\text{min}} = \tan^{-1}(1/2\mu_s)}$.

31. The arm is in equilibrium. Take torques about the elbow joint (the dot in the free-body diagram), so that the force at the elbow joint does not enter the calculation. Counterclockwise torques are positive. The mass of the lower arm is $m = 2.0$ kg, and the mass of the load is M.

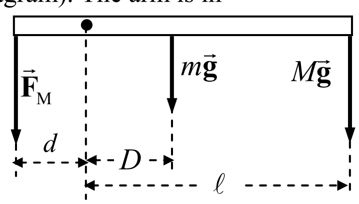


It is given that $F_M = 450$ N.

$$\sum \tau = F_M d_1 - mgd_2 - Mgd_3 = 0 \quad \to$$

$$M = \frac{F_M d_1 - mgd_2}{gd_3} = \frac{(450\text{ N})(0.060\text{ m}) - (2.0\text{ kg})(9.80\text{ m/s}^2)(0.15\text{ m})}{(9.80\text{ m/s}^2)(0.35\text{ m})} = \boxed{7.0\text{ kg}}$$

32. Calculate the torques about the elbow joint (the dot in the free-body diagram). The arm is in equilibrium. Counterclockwise torques are positive.

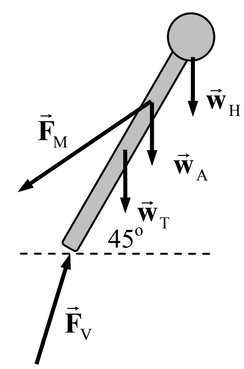


$$\sum \tau = F_M d - mgD - Mg\ell = 0$$

$$F_M = \frac{mD + M\ell}{d} g$$

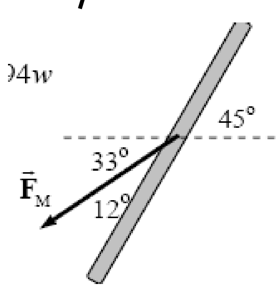
$$= \left[\frac{(2.3\text{ kg})(0.12\text{ m}) + (7.3\text{ kg})(0.300\text{ m})}{0.025\text{ m}}\right](9.80\text{ m/s}^2) = \boxed{970\text{ N}}$$

33. We redraw Figs. 9–14b and 9–14c with the person 45° from the horizontal, instead of the original 30°. All distances are as in the original problem. We still assume that the back muscles pull at a 12° angle to the spine. The 18° angle from the original problem becomes 33°. Torques are taken about the same point at the base of the spine, with counterclockwise torques as positive.



$$\sum \tau = (0.48\text{ m})\, F_M \sin 12° - (0.72\text{ m})(w_H) \sin 45°$$
$$- (0.48\text{ m})(w_A) \sin 45° - (0.36\text{ m})(w_T) \sin 45° = 0$$

As in the original problem, $w_H = 0.07w$, $w_A = 0.12w$, $w_T = 0.46w$. The torque equation then gives the following result:



$$F_M = \frac{[(0.72\text{ m})(0.07) + (0.48\text{ m})(0.12) + (0.36\text{ m})(0.46)]}{(0.48\text{ m}) \sin 12°} w \sin 45° = 1.94w$$

Take the sum of the forces in the vertical direction, set equal to zero.

$$\sum F_y = F_{Vy} - F_M \sin 33° - 0.07w - 0.12w - 0.46w = 0 \quad \to \quad F_{Vy} = 1.71w$$

Take the sum of the forces in the horizontal direction, set equal to zero.

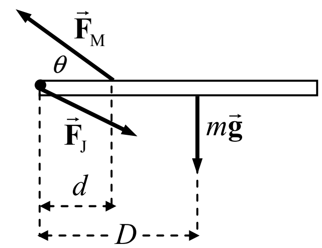
$$\sum F_x = F_{\mathrm{V}x} - F_{\mathrm{M}} \cos 33^\circ = 0 \quad \rightarrow \quad F_{\mathrm{V}y} = 1.63w$$

The final result is

$$F_{\mathrm{V}} = \sqrt{F_{\mathrm{V}x}^2 + F_{\mathrm{V}y}^2} = \boxed{2.4w}$$

This compares with 2.5w for the more bent position.

34. (*a*) Calculate the torques about the elbow joint (the dot in the free-body diagram). The arm is in equilibrium. Take counterclockwise torques as positive.



$$\sum \tau = (F_{\mathrm{M}} \sin\theta)d - mgD = 0 \quad \rightarrow$$

$$F_{\mathrm{M}} = \frac{mgD}{d\sin\theta} = \frac{(3.3\text{ kg})(9.80\text{ m/s}^2)(0.24\text{ m})}{(0.12\text{ m})\sin 15^\circ} = 249.9\text{ N} \approx \boxed{250\text{ N}}$$

(*b*) To find the components of F_{J}, write Newton's second law for both the x and y directions. Then combine them to find the magnitude.

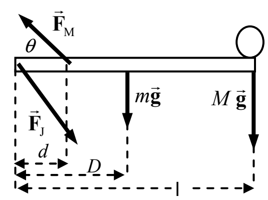
$$\sum F_x = F_{\mathrm{J}x} - F_{\mathrm{M}} \cos\theta = 0 \quad \rightarrow \quad F_{\mathrm{J}x} = F_{\mathrm{M}} \cos\theta = (249.9\text{ N})\cos 15^\circ = 241.4\text{ N}$$

$$\sum F_y = F_{\mathrm{M}} \sin\theta - mg - F_{\mathrm{J}y} = 0 \quad \rightarrow$$

$$F_{\mathrm{J}y} = F_{\mathrm{M}} \sin\theta - mg = (249.9\text{ N})\sin 15^\circ - (3.3\text{ kg})(9.80\text{ m/s}^2) = 32.3\text{ N}$$

$$\mathrm{F}_{\mathrm{J}} = \sqrt{F_{\mathrm{J}x}^2 + F_{\mathrm{J}y}^2} = \sqrt{(241.4\text{ N})^2 + (32.3\text{ N})^2} = 243.6\text{ N} \approx \boxed{240\text{ N}}$$

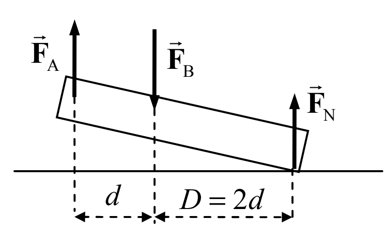
35. Calculate the torques about the shoulder joint, which is at the left end of the free-body diagram of the arm. Since the arm is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques to be positive. The force due to the shoulder joint is drawn, but it exerts no torque about the shoulder joint.



$$\sum \tau = F_{\mathrm{M}} d\sin\theta - mgD - Mg\ell = 0$$

$$F_{\mathrm{M}} = \frac{mD + ML}{d\sin\theta} g = \frac{(3.3\text{ kg})(0.24\text{ cm}) + (8.5\text{ kg})(0.52\text{ m})}{(0.12\text{ m})\sin 15^\circ}(9.80\text{ m/s}^2) = \boxed{1600\text{ N}}$$

36. There will be a normal force upward at the ball of the foot, equal to the person's weight ($F_{\mathrm{N}} = mg$). Calculate torques about a point on the floor directly below the leg bone (and in line with the leg bone force, $\vec{\mathbf{F}}_{\mathrm{B}}$). Since the foot is in equilibrium, the sum of the torques will be zero. Take counterclockwise torques as positive.



$$\sum \tau = F_{\mathrm{N}}(2d) - F_{\mathrm{A}} d = 0 \quad \rightarrow$$

$$F_{\mathrm{A}} = 2F_{\mathrm{N}} = 2mg = 2(72\text{ kg})(9.80\text{ m/s}^2) = \boxed{1400\text{ N}}$$

The net force in the y direction must be zero. Use that to find F_{B}.

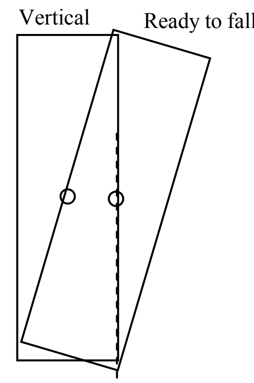
$$\sum F_y = F_{\mathrm{N}} + F_{\mathrm{A}} - F_{\mathrm{B}} = 0 \quad \rightarrow \quad F_{\mathrm{B}} = F_{\mathrm{N}} + F_{\mathrm{A}} = 2mg + mg = 3mg = \boxed{2100\text{ N}}$$

37. Take torques about the elbow joint. Let clockwise torques be positive. Since the arm is in equilibrium, the total torque will be 0.

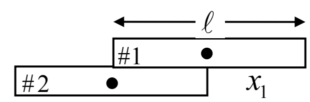
$$\sum \tau = (2.0\text{ kg})g(0.15\text{ m}) + (25\text{ kg})g(0.35\text{ m}) - F_{\max}(0.050\text{ m})\sin 105^\circ = 0 \quad \rightarrow$$

$$F_{\max} = \frac{(2.0\text{ kg})g(0.15\text{ m}) + (25\text{ kg})g(0.35\text{ m})}{(0.050\text{ m})\sin 105^\circ} = 1836\text{ N} \approx \boxed{1800\text{ N}}$$

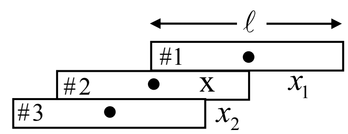
38. From Section 9–4: “An object whose CG is above its base of support will be stable if a vertical line projected downward from the CG falls within the base of support.” For the tower, the base of support is a circle of radius 7.7 m. If the top is 4.5 m off center, then the CG will be 2.25 m off center, and a vertical line downward from the CG will be 2.25 m from the center of the base. As long as that vertical line is less than 7.7 m from the center of the base, the tower will be in $\boxed{\text{stable equilibrium}}$. To be unstable, the CG has to be more than 7.7 m off center, so the top must be more than 2 × (7.7 m) = 15.4 m off center. Thus the top will have to lean 15.4 m – 4.5 m = $\boxed{10.9\,\text{m}}$ farther to reach the verge of instability.



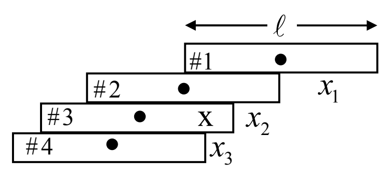
39. (*a*) The maximum distance for brick #1 to remain on brick #2 will be reached when the CM of brick #1 is directly over the edge of brick #2. Thus brick #1 will overhang brick #2 by $x_1 = \ell/2$.



The maximum distance for the top two bricks to remain on brick #3 will be reached when the center of mass of the top two bricks is directly over the edge of brick #3. The CM of the top two bricks is (obviously) at the point labeled X on brick #2, a distance of $\ell/4$ from the right edge of brick #2. Thus $x_2 = \ell/4$.



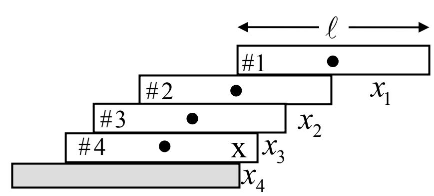
The maximum distance for the top three bricks to remain on brick #4 will be reached when the center of mass of the top three bricks is directly over the edge of brick #4. The CM of the top three bricks is at the point labeled X on brick #3 and is found relative to the center of brick # 3 by $\text{CM} = \dfrac{m(0) + 2m(\ell/2)}{3m} = \ell/3$, or $\ell/6$ from the right edge of brick #3. Thus $x_3 = \ell/6$.



The maximum distance for the four bricks to remain on a tabletop will be reached when the center of mass of the four bricks is directly over the edge of the table. The CM of all four bricks is at the point labeled X on brick #4 and is found relative to the center of brick #4 by

$$\text{CM} = \frac{m(0) + 3m(\ell/2)}{4m} = 3\ell/8, \text{ or } \ell/8 \text{ from the right edge}$$

of brick #4. Thus $x_4 = \ell/8$.



(*b*) From the last diagram, the distance from the edge of the tabletop to the right edge of brick #1 is

$$x_4 + x_3 + x_2 + x_1 = (\ell/8) + (\ell/6) + (\ell/4) + (\ell/2) = 25\ell/24 > \ell$$

Since this distance is greater than ℓ, the answer is $\boxed{\text{yes}}$, the first brick is completely beyond the edge of the table.

(*c*) From the work in part (a), we see that the general formula for the total distance spanned by n bricks is

$$x_1 + x_2 + x_3 + \cdots x_n = (\ell/2) + (\ell/4) + (\ell/6) + \cdots + (\ell/2n) = \boxed{\sum_{i=1}^{n} \frac{\ell}{2i}}$$

(*d*) The arch is to span 1.0 m, so the span from one side will be 0.50 m. Thus, we must solve $\sum_{i=1}^{n} \frac{0.30\text{ m}}{2i} \geq 0.50\text{ m}$. Evaluation of this expression for various values of n shows that 15 bricks will span a distance of 0.498 m and that 16 bricks will span a distance of 0.507 m. Thus, it takes 16 bricks for each half-span, plus 1 brick on top and 1 brick as the base on each side (as in Fig. 9–67b), for a total of $\boxed{\text{35 bricks}}$.

40. The amount of stretch can be found using the elastic modulus in Eq. 9–4.

$$\Delta\ell = \frac{1}{E}\frac{F}{A}\ell_0 = \frac{1}{3\times10^9\text{ N/m}^2}\frac{275\text{ N}}{\pi(5.00\times10^{-4})^2}(0.300\text{ m}) = \boxed{3.50\times10^{-2}\text{ m}}$$

41. (*a*) $$\text{stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(25{,}000\text{ kg})(9.80\text{ m/s}^2)}{1.4\text{ m}^2} = 175{,}000\text{ N/m}^2 \approx \boxed{1.8\times10^5\text{ N/m}^2}$$

(*b*) $$\text{strain} = \frac{\text{stress}}{\text{Young's modulus}} = \frac{175{,}000\times10^5\text{ N/m}^2}{50\times10^9\text{ N/m}^2} = \boxed{3.5\times10^{-6}}$$

42. The change in length is found from the strain.

$$\text{strain} = \frac{\Delta\ell}{\ell_0} \rightarrow \Delta\ell = \ell_0(\text{strain}) = (8.6\text{ m})(3.5\times10^{-6}) = \boxed{3.0\times10^{-5}\text{ m}}$$

$\boxed{43.}$ (*a*) $$\text{stress} = \frac{F}{A} = \frac{mg}{A} = \frac{(1700\text{ kg})(9.80\text{ m/s}^2)}{0.012\text{ m}^2} = 1.388\times10^6\text{ N/m}^2 \approx \boxed{1.4\times10^6\text{ N/m}^2}$$

(*b*) $$\text{strain} = \frac{\text{stress}}{\text{Young's modulus}} = \frac{1.388\times10^6\text{ N/m}^2}{200\times10^9\text{ N/m}^2} = 6.94\times10^{-6} \approx \boxed{6.9\times10^{-6}}$$

(c) $$\Delta\ell = (\text{strain})(\ell_0) = (6.94\times10^{-6})(9.50\text{ m}) = 6.593\times10^{-5}\text{ m} \approx \boxed{6.6\times10^{-5}\text{ m}}$$

44. The change in volume is given by Eq. 9–7. We assume the original pressure is atmospheric pressure, $1.0\times10^5\text{ N/m}^2$.

$$\Delta V = -V_0\frac{\Delta P}{B} = -(1000\text{ cm}^3)\frac{(2.6\times10^6\text{ N/m}^2 - 1.0\times10^5\text{ N/m}^2)}{1.0\times10^9\text{ N/m}^2} = -2.5\text{ cm}^3$$

$$V = V_0 + \Delta V = 1000\text{ cm}^3 - 2.5\text{ cm}^3 = \boxed{997.5\text{ cm}^3}$$

45. The relationship between pressure change and volume change is given by Eq. 9–7.

$$\Delta V = -V_0\frac{\Delta P}{B} \rightarrow \Delta P = -\frac{\Delta V}{V_0}B = -(0.10\times10^{-2})(90\times10^9\text{ N/m}^2) = \boxed{9.0\times10^7\text{ N/m}^2}$$

$$\frac{\Delta P}{P_{\text{atm}}} = \frac{9.0\times10^7\text{ N/m}^2}{1.0\times10^5\text{ N/m}^2} = \boxed{9.0\times10^2}$$, or 900 atmospheres

46. The Young's modulus is the stress divided by the strain.

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell_0} = \frac{(13.4\text{ N})/\left[\pi\left(\frac{1}{2}\times 8.5\times 10^{-3}\text{m}\right)^2\right]}{(3.7\times 10^{-3}\text{ m})/(15\times 10^{-2}\text{ m})} = \boxed{9.6\times 10^6\text{ N/m}^2}$$

47. The mass can be calculated from the equation for the relationship between stress and strain. The force causing the strain is the weight of the mass suspended from the wire. Use Eq. 9–4.

$$\frac{\Delta\ell}{\ell_0} = \frac{1}{E}\frac{F}{A} = \frac{mg}{EA} \quad\to\quad m = \frac{EA}{g}\frac{\Delta\ell}{\ell_0} = (200\times 10^9\text{ N/m}^2)\frac{\pi(1.15\times 10^{-3}\text{ m})^2}{(9.80\text{ m/s}^2)}\frac{0.030}{100} = \boxed{25\text{ kg}}$$

48. The percentage change in volume is found by multiplying the relative change in volume by 100. The change in pressure is 199 times atmospheric pressure, since it increases from atmospheric pressure to 200 times atmospheric pressure. Use Eq. 9–7.

$$100\frac{\Delta V}{V_0} = -100\frac{\Delta P}{B} = -100\frac{199(1.0\times 10^5\text{ N/m}^2)}{90\times 10^9\text{ N/m}^2} = \boxed{-2\times 10^{-2}\%}$$

The negative sign indicates that the interior space got smaller.

49. Elastic potential energy is given by $\text{PE}_{\text{elastic}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}F\Delta x.$ The force is found from Eq. 9–4, using $\Delta\ell$ as $\Delta x.$

$$\text{PE}_{\text{elastic}} = \frac{1}{2}F\Delta x = \frac{1}{2}\left(\frac{EA}{\ell_0}\Delta\ell\right)\Delta\ell = \frac{1}{2}\frac{(2.0\times 10^6\text{ N/m}^2)(0.50\times 10^{-4}\text{ m}^2)}{(3.0\times 10^{-3}\text{ m})}(1.0\times 10^{-3}\text{ m})^2$$

$$= \boxed{1.7\times 10^{-2}\text{ J}}$$

50. Set the compressive strength of the bone equal to the stress of the bone.

$$\text{compressive strength} = \frac{F_{\max}}{A} \quad\to\quad F_{\max} = (170\times 10^6\text{ N/m}^2)(3.0\times 10^{-4}\text{ m}^2) = \boxed{5.1\times 10^4\text{ N}}$$

51. (*a*) The maximum tension can be found from the ultimate tensile strength of the material.

$$\text{tensile strength} = \frac{F_{\max}}{A} \quad\to$$

$$F_{\max} = (\text{tensile strength})A = (500\times 10^6\text{ N/m}^2)\pi(5.00\times 10^{-4}\text{ m})^2 = \boxed{393\text{ N}}$$

(*b*) To prevent breakage, $\boxed{\text{thicker strings}}$ should be used, which will increase the cross-sectional area of the strings and thus increase the maximum force. Breakage occurs because when the strings are hit by the ball, they stretch, increasing the tension. The strings are reasonably tight in the normal racket configuration, so when the tension is increased by a particularly hard hit, the tension may exceed the maximum force.

52. (*a*) Compare the stress on the bone with the compressive strength to see whether the bone breaks.

$$\text{Stress} = \frac{F}{A} = \frac{3.3\times 10^4\text{ N}}{3.6\times 10^{-4}\text{ m}^2}$$

$$= 9.167\times 10^7\text{ N/m}^2 < 1.7\times 10^8\text{ N/m}^2\,\text{(compressive strength of bone)}$$

$\boxed{\text{The bone will not break.}}$

(*b*) The change in length is calculated from Eq. 9–4.

$$\Delta\ell = \frac{\ell_0}{E}\frac{F}{A} = \left(\frac{0.22\text{ m}}{15\times10^9\text{ N/m}^2}\right)(9.167\times10^7\text{ N/m}^2) = \boxed{1.3\times10^{-3}\text{ m}}$$

53. (*a*) The area can be found from the ultimate tensile strength of the material.

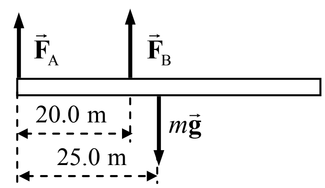
$$\frac{\text{tensile strength}}{\text{safety factor}} = \frac{F}{A} \rightarrow A = F\left(\frac{\text{safety factor}}{\text{tensile strength}}\right) \rightarrow$$

$$A = (270\text{ kg})(9.80\text{ m/s}^2)\frac{7.0}{500\times10^6\text{ N/m}^2} = 3.704\times10^{-5}\text{ m}^2 \approx \boxed{3.7\times10^{-5}\text{ m}^2}$$

(*b*) The change in length can be found from the stress-strain relationship, Eq. 9–4.

$$\frac{F}{A} = E\frac{\Delta\ell}{\ell_0} \rightarrow \Delta\ell = \frac{\ell_0 F}{AE} = \frac{(7.5\text{ m})(320\text{ kg})(9.80\text{ m/s}^2)}{(3.704\times10^{-5}\text{ m}^2)(200\times10^9\text{ N/m}^2)} = \boxed{2.7\times10^{-3}\text{ m}}$$

54. For each support, to find the minimum cross-sectional area with a safety factor means that $\frac{F}{A} = \frac{\text{strength}}{\text{safety factor}}$, where either the tensile or compressive strength is used, as appropriate for each force. To find the force on each support, use the conditions of equilibrium for the beam. Take torques about the left end of the beam, calling counterclockwise torques positive, and also sum the vertical forces, taking upward forces as positive.



$$\sum\tau = F_B(20.0\text{ m}) - mg(25.0\text{ m}) = 0 \rightarrow F_B = \frac{25.0}{20.0}mg = 1.25mg$$

$$\sum F_y = F_A + F_B - mg = 0 \rightarrow F_A = mg - F_B = mg - 1.25mg = -0.25mg$$

Notice that the forces on the supports are the opposite of $\vec{F}_A$ and $\vec{F}_B$. So the force on support A is directed upward, which means that support A is in tension. The force on support B is directed downward, so support B is in compression.

$$\frac{F_A}{A_A} = \frac{\text{tensile strength}}{9.0} \rightarrow$$

$$A_A = 9.0\frac{(0.25mg)}{\text{tensile strength}} = 9.0\frac{(0.25)(2.9\times10^3\text{ kg})(9.80\text{ m/s}^2)}{40\times10^6\text{ N/m}^2} = \boxed{1.6\times10^{-3}\text{ m}^2}$$

$$\frac{F_B}{A_B} = \frac{\text{compressive strength}}{9.0} \rightarrow$$

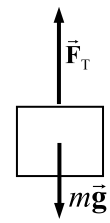
$$A_B = 9.0\frac{(1.25mg)}{\text{compressive strength}} = 9.0\frac{(1.25)(2.9\times10^3\text{ kg})(9.80\text{ m/s}^2)}{35\times10^6\text{ N/m}^2} = \boxed{9.1\times10^{-3}\text{ m}^2}$$

55. The maximum shear stress is to be 1/7th of the shear strength for iron. The maximum stress will occur for the minimum area and thus the minimum diameter.

$$\text{stress}_{\max} = \frac{F}{A_{\min}} = \frac{\text{shear strength}}{7.0} \rightarrow A_1 = \pi\left(\tfrac{1}{2}d\right)^2 = \frac{7.0\,F}{\text{shear strength}} \rightarrow$$

$$d = \sqrt{\frac{4(7.0)F}{\pi(\text{shear strength})}} = \sqrt{\frac{28(3300\text{ N})}{\pi(170\times10^6\text{ N/m}^2)}} = 1.3\times10^{-2}\text{ m} = \boxed{1.3\text{ cm}}$$

56. From the free-body diagram, write Newton’s second law for the vertical direction. Solve for the maximum tension required in the cable, which will occur for an upward acceleration.



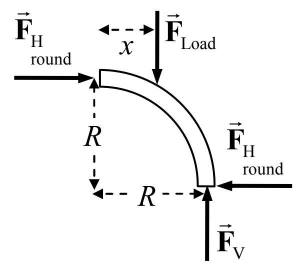
$$\sum F_y = F_{\mathrm{T}} - mg = ma \quad \rightarrow \quad F_{\mathrm{T}} = m(g+a)$$

The maximum stress is to be 1/8th of the tensile strength for steel. The maximum stress will occur for the minimum area and thus the minimum diameter.

$$\text{stress}_{\text{max}} = \frac{F_{\mathrm{T}}}{A_{\text{min}}} = \frac{\text{tensile strength}}{8.0} \quad \rightarrow \quad A_1 = \pi\left(\tfrac{1}{2}d\right)^2 = \frac{8.0F_{\mathrm{T}}}{\text{tensile strength}} \quad \rightarrow$$

$$d = \sqrt{\frac{4(8.0)m(g+a)}{\pi(\text{tensile strength})}} = \sqrt{\frac{32(3100\text{ kg})(11.6\text{ m/s}^2)}{\pi(500\times10^6\text{ N/m}^2)}} = 2.71\times10^{-2}\text{ m} \approx \boxed{2.7\text{ cm}}$$

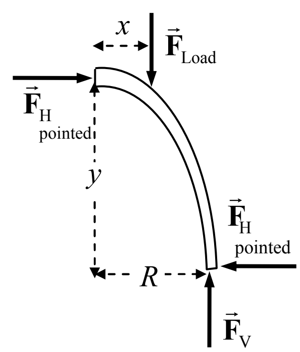
57. Draw free-body diagrams similar to Figs. 9–31a and 9–31b for the forces on the right half of a round arch and a pointed arch. The load force is placed at the same horizontal position on each arch. For each half-arch, take torques about the lower right-hand corner, with counterclockwise as positive.



For the round arch:

$$\sum \tau = F_{\text{Load}}(R-x) - F_{\substack{\text{H}\\\text{round}}} R = 0 \quad \rightarrow \quad F_{\substack{\text{H}\\\text{round}}} = F_{\text{Load}}\frac{R-x}{R}$$

For the pointed arch:



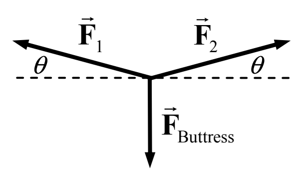
$$\sum \tau = F_{\text{Load}}(R-x) - F_{\substack{\text{H}\\\text{pointed}}} y = 0 \quad \rightarrow \quad F_{\substack{\text{H}\\\text{pointed}}} = F_{\text{Load}}\frac{R-x}{y}$$

Solve for y, given that $F_{\substack{\text{H}\\\text{pointed}}} = \frac{1}{3}F_{\substack{\text{H}\\\text{round}}}$.

$$F_{\substack{\text{H}\\\text{pointed}}} = \tfrac{1}{3}F_{\substack{\text{H}\\\text{round}}} \quad \rightarrow \quad F_{\text{Load}}\frac{R-x}{y} = \tfrac{1}{3}F_{\text{Load}}\frac{R-x}{R} \quad \rightarrow$$

$$y = 3R = 3\left(\tfrac{1}{2}8.0\text{ m}\right) = \boxed{12\text{ m}}$$

58. Write Newton’s second law for the horizontal direction.

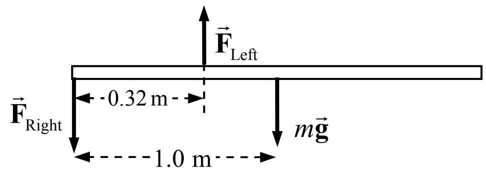


$$\sum F_x = F_2\cos\theta - F_1\cos\theta = 0 \quad \rightarrow \quad F_2 = F_1$$

Thus the two forces are the same size. Now write Newton’s second law for the vertical direction.

$$\sum F_y = F_1\sin\theta + F_1\sin\theta - F_{\text{Buttress}} = 0 \quad \rightarrow \quad F_1 = \frac{F_{\text{Buttress}}}{2\sin\theta} = \frac{4.2\times10^5\text{ N}}{2\,(\sin 5^\circ)} = \boxed{2.4\times10^6\text{ N}}$$

59. (*a*) The pole will exert a downward force and a clockwise torque about the woman's right hand. Thus there must be an upward force exerted by the left hand to cause a counterclockwise torque for the pole to have a net torque of zero about the right hand. The force exerted by the right hand is then of such a magnitude and direction for the net vertical force on the pole to be zero.



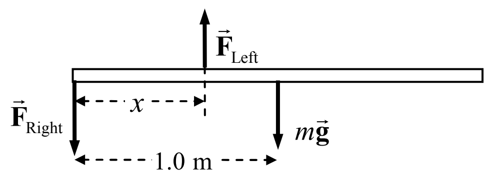
$$\sum \tau = F_{\text{Left}}(0.32\text{ m}) - mg(1.0\text{ m}) = 0 \quad \rightarrow$$

$$F_{\text{Left}} = mg\left(\frac{1.0\text{ m}}{0.32\text{ m}}\right) = \frac{(10.0\text{ kg})(9.80\text{ m/s}^2)}{0.32} = 306.25\text{ N} \approx \boxed{310\text{ N, upward}}$$

$$\sum F_y = F_{\text{Left}} - F_{\text{Right}} - mg = 0 \quad \rightarrow$$

$$F_{\text{Right}} = F_{\text{Left}} - mg = 306.25\text{ N} - (10.0\text{ kg})(9.80\text{ m/s}^2) = 208.25\text{ N} \approx \boxed{210\text{ N, downward}}$$

(*b*) We see that the force due to the left hand is larger than the force due to the right hand, since both the right hand and gravity are downward. Set the left hand force equal to 150 N and calculate the location of the left hand by setting the net torque equal to zero.



$$\sum \tau = F_{\text{Left}}x - mg(1.0\text{ m}) = 0 \quad \rightarrow \quad x = \frac{mg}{F_{\text{Left}}}(1.0\text{ m}) = \frac{98.0\text{ N}}{150\text{ N}}(1.0\text{ m}) = \boxed{0.65\text{ m}}$$

As a check, calculate the force due to the right hand.

$$F_{\text{Right}} = F_{\text{Left}} - mg = 150\text{ N} - 98.0\text{ N} = 52\text{ N} \quad \text{OK}$$

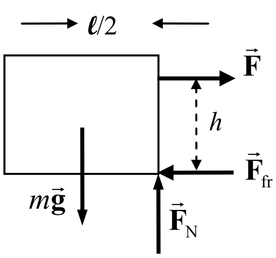
(*c*) Follow the same procedure, setting the left-hand force equal to 85 N:

$$\sum \tau = F_{\text{Left}}x - mg(1.0\text{ m}) = 0 \quad \rightarrow \quad x = \frac{mg}{F_{\text{Left}}}(1.0\text{ m}) = \frac{98.0\text{ N}}{85\text{ N}}(1.0\text{ m}) = 1.153\text{ m} \approx \boxed{1.2\text{ m}}$$

$$F_{\text{Right}} = F_{\text{Left}} - mg = 85\text{ N} - 98.0\text{ N} = -13\text{ N} \quad \text{OK}$$

Note that now the force due to the right hand must be pulling upward, because the left hand is on the opposite side of the center of the pole.

60. If the block is on the verge of tipping, the normal force will be acting at the lower right-hand corner of the block, as shown in the free-body diagram. The block will begin to rotate when the torque caused by the pulling force is larger than the torque caused by gravity. For the block to be able to slide, the pulling force must be as large as the maximum static frictional force. Write the equations of equilibrium for forces in the x and y directions and for torque with the conditions as stated above.



$$\sum F_y = F_N - mg = 0 \quad \rightarrow \quad F_N = mg$$

$$\sum F_x = F - F_{\text{fr}} = 0 \quad \rightarrow \quad F = F_{\text{fr}} = \mu_s F_N = \mu_s mg$$

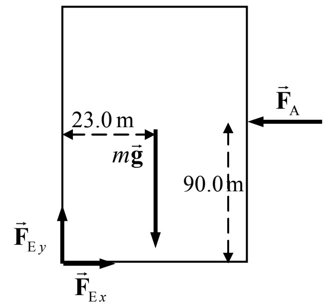
$$\sum \tau = mg\frac{\ell}{2} - Fh = 0 \quad \rightarrow \quad \frac{mg\ell}{2} = Fh = \mu_s mgh$$

Solve for the coefficient of friction in this limiting case, to find $\mu_s = \dfrac{\ell}{2h}$.

(*a*) If $\boxed{\mu_s < \ell/2h}$, then sliding will happen before tipping.

(*b*) If $\boxed{\mu_s > \ell/2h}$, then tipping will happen before sliding.

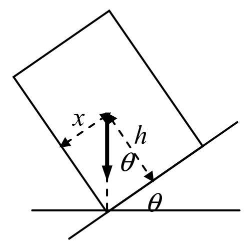
61. Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left-hand corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.



$$\begin{aligned}\sum\tau &= F_A(90.0\text{ m}) - mg(23.0\text{ m})\\ &= [(950\text{ N/m}^2)(180.0\text{ m})(76.0\text{ m})]\,(90.0\text{ m})\\ &\quad -(1.8\times10^7\text{ kg})(9.80\text{ m/s}^2)(23.0\text{ m}) = \boxed{-2.9\times10^9\text{ m}\cdot\text{N}}\end{aligned}$$

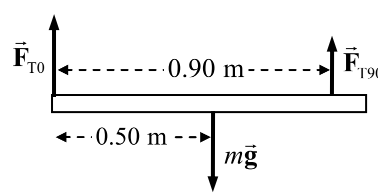
Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus $\boxed{\text{the building will not topple}}$.

62. The truck will not tip as long as a vertical line down from the CG is between the wheels. When that vertical line is at the wheel, it is in unstable equilibrium and will tip if the road is inclined any more. See the diagram for the truck at the tipping angle, showing the truck's weight vector.



$$\tan\theta = \frac{x}{h} \quad\rightarrow\quad \theta = \tan^{-1}\frac{x}{h} = \tan^{-1}\frac{1.2\text{ m}}{2.2\text{ m}} = \boxed{29^\circ}$$

63. (*a*) The meter stick is in equilibrium, so both the net torque and the net force are zero. From the force diagram, write an expression for the net torque about the 90-cm mark, with counterclockwise torques as positive.



$$\sum\tau = mg(0.40\text{ m}) - F_{T0}(0.90\text{ m}) = 0 \quad\rightarrow$$

$$F_{T0} = mg\frac{0.40}{0.90} = (0.180\text{ kg})(9.80\text{ m/s}^2)\frac{0.40}{0.90} = \boxed{0.78\text{ N}}$$

(*b*) Write Newton's second law for the vertical direction with a net force of 0 to find the other tension.

$$\sum F_y = F_{T0} + F_{T90} - mg = 0 \quad\rightarrow$$

$$F_{T90} = mg - F_{T0} = (0.180\text{ kg})(9.80\text{ m/s}^2) - 0.78\text{ N} = \boxed{0.98\text{ N}}$$

64. The maximum compressive force in a column will occur at the bottom. The bottom layer supports the entire weight of the column, so the compressive force on that layer is mg. For the column to be on the verge of buckling, the weight divided by the area of the column will be the compressive strength of the material. The mass of the column is its volume $(\text{area}\times\text{height})$ times its density.

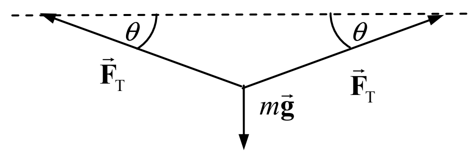
$$\frac{mg}{A} = \text{compressive strength} = \frac{hA\rho g}{A} \quad\rightarrow\quad h = \frac{\text{compressive strength}}{\rho g}$$

Note that the area of the column cancels out of the expression, so the height does not depend on the cross-sectional area of the column.

(*a*) $$h_{\text{steel}} = \frac{\text{compressive strength}}{\rho g} = \frac{500\times10^{6}\ \text{N/m}^2}{(7.8\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{6500\ \text{m}}$$

(*b*) $$h_{\text{granite}} = \frac{\text{compressive strength}}{\rho g} = \frac{170\times10^{6}\ \text{N/m}^2}{(2.7\times10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{6400\ \text{m}}$$

65. The radius of the wire can be determined from the relationship between stress and strain, expressed by Eq. 9–5.

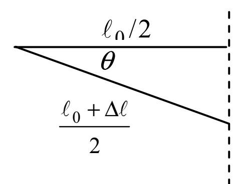


$$\frac{F}{A} = E\frac{\Delta\ell}{\ell_0} \quad\to\quad A = \frac{F\ell_0}{E\Delta\ell} = \pi r^2 \quad\to\quad r = \sqrt{\frac{1}{\pi}\frac{F}{E}\frac{\ell_0}{\Delta\ell}}$$

Use the free-body diagram for the attachment point of the mass and wire to get the wire's tension.

$$\sum F_y = 2F_{\text{T}}\sin\theta - mg = 0 \quad\to\quad F_{\text{T}} = \frac{mg}{2\sin\theta} = \frac{(25\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin 12^\circ} = 589.2\ \text{N}$$

The fractional change in the length of the wire can be found from the geometry of the problem, as seen in the second diagram.

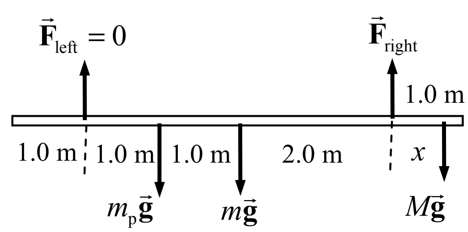


$$\cos\theta = \frac{\ell_0/2}{\frac{\ell_0+\Delta\ell}{2}} \quad\to\quad \frac{\Delta\ell}{\ell_0} = \frac{1}{\cos\theta} - 1 = \frac{1}{\cos 12^\circ} - 1 = 2.234\times10^{-2}$$

Thus, the radius is

$$r = \sqrt{\frac{1}{\pi}\frac{F_{\text{T}}}{E}\frac{\ell_0}{\Delta\ell}} = \sqrt{\frac{1}{\pi}\left(\frac{589.2\ \text{N}}{70\times10^{9}\ \text{N/m}^2}\right)\frac{1}{(2.234\times10^{-2})}} = \boxed{3.5\times10^{-4}\ \text{m}}$$

66. The limiting condition for the painter's safety is the tension in the ropes. The ropes can exert only an upward tension on the scaffold. The tension will be least in the rope that is farther from the painter. The mass of the pail is m_{p}, the mass of the scaffold is m, and the mass of the painter is M.



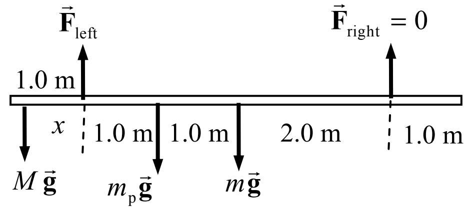
Find the distance to the right that the painter can walk before the tension in the left rope becomes zero. Take torques about the point where the right-side rope is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

$$\sum\tau = mg(2.0\ \text{m}) + m_{\text{p}}g(3.0\ \text{m}) - Mgx = 0 \quad\to$$

$$x = \frac{m(2.0\ \text{m}) + m_{\text{p}}(3.0\ \text{m})}{M} = \frac{(25\ \text{kg})(2.0\ \text{m}) + (4.0\ \text{kg})(3.0\ \text{m})}{65.0\ \text{kg}} = 0.9538\ \text{m} \approx 0.95\ \text{m}$$

The painter can walk to $\boxed{\text{within 5 cm of the right edge}}$ of the scaffold.

Now find the distance to the left that the painter can walk before the tension in the right rope becomes zero. Take torques about the point where the left-side tension is attached to the scaffold, so that its value need not be known. Take counterclockwise torques as positive.

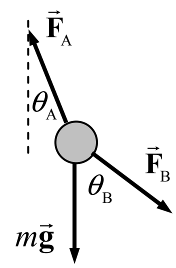


$$\sum \tau = Mgx - m_p g(1.0\text{ m}) - mg(2.0\text{ m}) = 0 \quad \rightarrow$$

$$x = \frac{m(2.0\text{ m}) + m_p(1.0\text{ m})}{M} = \frac{(25\text{ kg})(2.0\text{ m}) + (4.0\text{ kg})(1.0\text{ m})}{65.0\text{ kg}} = 0.8308\text{ m} \approx 0.83\text{ m}$$

The painter can walk to $\boxed{\text{within 17 cm of the left edge}}$ of the scaffold. Both ends are dangerous.

67. See the free-body diagram. The ball is at rest, so it is in equilibrium. Write Newton's second law for the horizontal and vertical directions, and solve for the forces.



$$\sum F_{\text{horiz}} = F_B \sin\theta_B - F_A \sin\theta_A = 0 \quad \rightarrow \quad F_B = F_A \frac{\sin\theta_A}{\sin\theta_B}$$

$$\sum F_{\text{vert}} = F_A \cos\theta_A - F_B \cos\theta_B - mg = 0 \quad \rightarrow \quad F_A \cos\theta_A = F_B \cos\theta_B + mg \quad \rightarrow$$

$$F_A \cos\theta_A = F_A \frac{\sin\theta_A}{\sin\theta_B}\cos\theta_B + mg \quad \rightarrow \quad F_A\left(\cos\theta_A - \frac{\sin\theta_A}{\sin\theta_B}\cos\theta_B\right) = mg \quad \rightarrow$$

$$F_A = mg\frac{\sin\theta_B}{(\cos\theta_A \sin\theta_B - \sin\theta_A \cos\theta_B)} = mg\frac{\sin\theta_B}{\sin(\theta_B - \theta_A)} = (15.0\text{ kg})(9.80\text{ m/s}^2)\frac{\sin 53^\circ}{\sin 31^\circ}$$
$$= 228\text{ N} \approx \boxed{230\text{ N}}$$

$$F_B = F_A\frac{\sin\theta_A}{\sin\theta_B} = (228\text{ N})\frac{\sin 22^\circ}{\sin 53^\circ} = 107\text{ N} \approx \boxed{110\text{ N}}$$

68. The number of supports can be found from the compressive strength of the wood. Since the wood will be oriented longitudinally, the stress will be parallel to the grain.

$$\frac{\text{compressive strength}}{\text{safety factor}} = \frac{\text{load force on supports}}{\text{area of supports}} = \frac{\text{weight of roof}}{(\text{number of supports})(\text{area per support})}$$

$$(\text{number of supports}) = \frac{\text{weight of roof}}{(\text{area per support})}\frac{\text{safety factor}}{\text{compressive strength}}$$
$$= \frac{(1.36\times10^4\text{ kg})(9.80\text{ m/s}^2)}{(0.040\text{ m})(0.090\text{ m})}\frac{12}{(35\times10^6\text{ N/m}^2)} = 12.69\text{ supports}$$

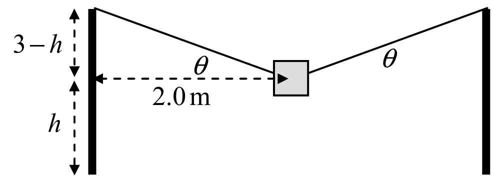
Since there are to be more than 12 supports, and to have the same number of supports on each side, there will be 14 supports, or $\boxed{\text{7 supports on each side}}$. That means there will be 6 support-to-support spans, each of which would be given by $\text{spacing} = \frac{10.0\text{ m}}{6\text{ gaps}} = \boxed{1.66\text{ m/gap}}$.

69. The tension in the string when it breaks is found from the ultimate strength of nylon under tension, from Table 9–2.

$$\frac{F_T}{A} = \text{Tensile strength} \quad \rightarrow$$

$$F_T = A(\text{Tensile strength}) = \pi\left[\tfrac{1}{2}(1.15\times10^{-3}\text{ m})\right]^2 (500\times10^6\text{ N/m}^2) = 519.3\text{ N}$$

From the force diagram for the box, we calculate the angle of the rope relative to the horizontal from Newton's second law in the vertical direction. Note that since the tension is the same throughout the string, the angles must be the same so that the object does not accelerate horizontally.



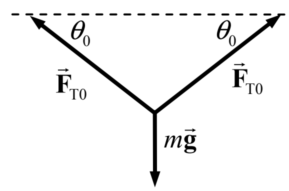
$$\sum F_y = 2F_T \sin\theta - mg = 0 \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{mg}{2F_T} = \sin^{-1}\frac{(25\text{ kg})(9.80\text{ m/s}^2)}{2(519.3\text{ N})} = 13.64^\circ$$

To find the height above the ground, consider the second diagram.

$$\tan\theta = \frac{3.00\text{ m} - h}{2.00\text{ m}} \quad \rightarrow \quad h = 3.00\text{ m} - 2.00\text{ m}(\tan\theta) = 3.00\text{ m} - 2.00\text{ m}(\tan 13.64^\circ) = \boxed{2.51\text{ m}}$$

70. Since the backpack is midway between the two trees, the angles in the free-body diagram are equal. Write Newton's second law for the vertical direction for the point at which the backpack is attached to the cord, with the weight of the backpack being the original downward vertical force.



$$\sum F_y = 2F_{T0}\sin\theta_0 - mg = 0 \quad \rightarrow \quad F_{T0} = \frac{mg}{2\sin\theta_0}$$

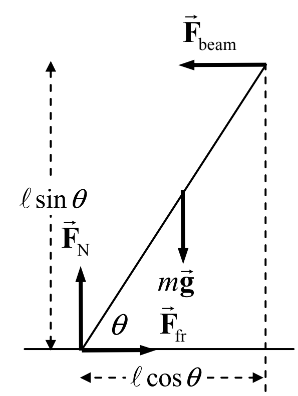
Now assume the bear pulls down with an additional force, F_{bear}. The force equation would be modified as follows:

$$\sum F_y = 2F_{\text{T final}}\sin\theta_{\text{final}} - mg - F_{\text{bear}} = 0 \quad \rightarrow$$

$$F_{\text{bear}} = 2F_{\text{T final}}\sin\theta_{\text{final}} - mg = 2(2F_{T0})\sin\theta_{\text{final}} - mg = 4\left(\frac{mg}{2\sin\theta_0}\right)\sin\theta_{\text{final}} - mg$$

$$= mg\left(\frac{2\sin\theta_{\text{final}}}{\sin\theta_0} - 1\right) = (23.0\text{ kg})(9.80\text{ m/s}^2)\left(\frac{2\sin 27^\circ}{\sin 15^\circ} - 1\right) = 565.3\text{ N} \approx \boxed{570\text{ N}}$$

71. Draw a free-body diagram for one of the beams. By Newton's third law, if the right beam pushes down on the left beam, then the left beam pushes up on the right beam. But the geometry is symmetric for the two beams, so the beam contact force must be horizontal. For the beam to be in equilibrium, $F_N = mg$, and $F_{\text{fr}} = \mu_s F_N = \mu mg$ is the maximum friction force. Take torques about the top of the beam, so that $\vec{\mathbf{F}}_{\text{beam}}$ exerts no torque. Let clockwise torques be positive.



$$\sum\tau = F_N\,\ell\cos\theta - mg\left(\tfrac{1}{2}\ell\right)\cos\theta - F_{\text{fr}}\,\ell\sin\theta = 0 \quad \rightarrow$$

$$\theta = \tan^{-1}\frac{1}{2\mu_s} = \tan^{-1}\frac{1}{2(0.5)} = \boxed{45^\circ}$$

72. (*a*) The fractional decrease in the rod's length is the strain. Use Eq. 9–4. The force applied is the weight of the man.

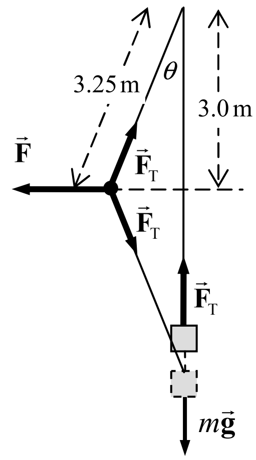
$$\frac{\Delta\ell}{\ell_0} = \frac{F}{AE} = \frac{mg}{\pi r^2 E} = \frac{(65\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.15)^2(200\times10^9\text{ N/m}^2)} = 4.506\times10^{-8} = \boxed{(4.5\times10^{-6})\%}$$

(*b*) The fractional change is the same for the atoms as for the macroscopic material. Let *d* represent the interatomic spacing.

$$\frac{\Delta d}{d_0}=\frac{\Delta \ell}{\ell_0}=4.506\times 10^{-8} \rightarrow$$

$$\Delta d=(4.506\times 10^{-8})d_0=(4.506\times 10^{-8})(2.0\times 10^{-10}\text{ m})=\boxed{9.0\times 10^{-18}\text{ m}}$$

73. (*a*) See the free-body diagram for the system, showing forces on the engine and the forces at the point on the rope where the mechanic is pulling (the point of analysis). Let *m* represent the mass of the engine. The fact that the engine was raised a half-meter means that the part of the rope from the tree branch to the mechanic is 3.25 m, as well as the part from the mechanic to the bumper. From the free-body diagram for the engine, we know that the tension in the rope is equal to the weight of the engine. Use this, along with the equations of equilibrium at the point where the mechanic is pulling, to find the pulling force by the mechanic.



Angle: $\theta=\cos^{-1}\frac{3.0\text{ m}}{3.25\text{ m}}=22.62^\circ$

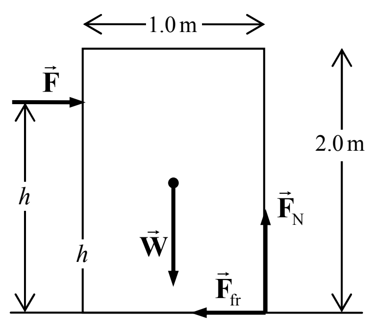
Engine: $\sum F_y=F_{\mathrm{T}}-mg=0 \rightarrow F_{\mathrm{T}}=mg$

Point: $\sum F_x=F-2F_{\mathrm{T}}\sin\theta=0 \rightarrow$

$$F=2mg\sin\theta=2(280\text{ kg})(9.80\text{ m/s}^2)\sin 22.62^\circ=2111\text{ N}\approx\boxed{2100\text{ N}}$$

(*b*) $$\text{mechanical advantage}=\frac{\text{load force}}{\text{applied force}}=\frac{mg}{F}=\frac{(280\text{ kg})(9.80\text{ m/s}^2)}{2111\text{ N}}=\boxed{1.3}$$

74. Consider the free-body diagram for the box. The box is assumed to be in equilibrium, but just on the verge of both sliding and tipping. Since it is on the verge of sliding, the static frictional force is at its maximum value. Use the equations of equilibrium. Take torques about the lower right-hand corner where the box touches the floor, and take clockwise torques as positive. We assume that the box is just barely tipped up on its corner, so that the forces are still parallel and perpendicular to the edges of the box.

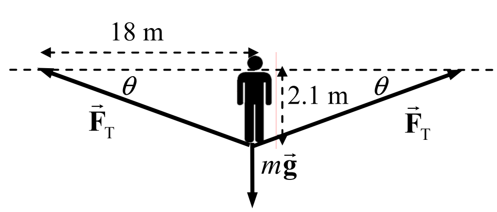


$$\sum F_y=F_{\mathrm{N}}-W=0 \rightarrow F_{\mathrm{N}}=W$$

$$\sum F_x=F-F_{\mathrm{fr}}=0 \rightarrow F=F_{\mathrm{fr}}=\mu W=(0.60)(250\text{ N})=\boxed{150\text{ N}}$$

$$\sum\tau=Fh-W(0.5\text{ m})=0 \rightarrow h=(0.5\text{ m})\frac{W}{F}=(0.5\text{ m})\frac{250\text{ N}}{150\text{ N}}=\boxed{0.83\text{ m}}$$

75. From the free-body diagram (not to scale), write the force equilibrium condition for the vertical direction.



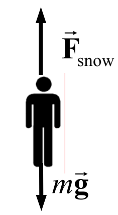
$$\sum F_y=2F_{\mathrm{T}}\sin\theta-mg=0$$

$$F_{\mathrm{T}}=\frac{mg}{2\sin\theta}\approx\frac{mg}{2\tan\theta}=\frac{(60.0\text{ kg})(9.80\text{ m/s}^2)}{2\left(\frac{2.1\text{ m}}{18\text{ m}}\right)}$$

$$=\boxed{2500\text{ N}}$$

Note that the angle is small enough (about 7°) that we have made the substitution $\sin\theta \approx \tan\theta$. $\boxed{\text{It is not possible to increase the tension so that there is no sag.}}$ There must always be a vertical component of the tension to balance the gravity force. The larger the tension gets, the smaller the sag angle will be, however.

76. Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2–11c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area with the strength of body tissue. From the free-body diagram, we have $F_{\text{snow}} - mg = ma \rightarrow F_{\text{snow}} = m(a+g)$.



$$v^2 = v_0^2 - 2a(x-x_0) \quad\rightarrow\quad a = \frac{v^2 - v_0^2}{2(x-x_0)} = \frac{0-(55\text{ m/s})^2}{2(-1.0\text{ m})} = 1513\text{ m/s}^2$$

$$\frac{F_{\text{snow}}}{A} = \frac{m(a+h)}{A} = \frac{(75\text{ kg})(1513\text{ m/s}^2 + 9.80\text{ m/s}^2)}{0.30\text{ m}^2} = 3.81\times10^5\text{ N/m}^2$$

$$\frac{F_{\text{snow}}}{A} < \text{tissue strength} = 5\times10^5\text{ N/m}^2$$

Since $\boxed{\text{the average force on the person is less than the strength of body tissue}}$, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and still sustain injury.

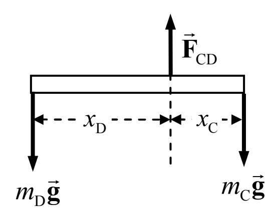
77. The force in the left vertical support column is 44,100 N, in compression. We want a steel column that can handle three times that, or 132,300 N. Steel has a compressive strength of $500\times10^6\text{ N/m}^2$. Use this to find the area.

$$\frac{F}{A} = \frac{132{,}300\text{ N}}{A} = 500\times10^6\text{ N/m}^2 \quad\rightarrow\quad A = \frac{132{,}300\text{ N}}{500\times10^6\text{ N/m}^2} = 2.646\times10^{-4}\text{ m}^2 \approx \boxed{2.6\times10^{-4}\text{ m}^2}$$

If the column were square, each side would be 1.6 cm. If the column were cylindrical, the radius would be 9.2 mm.

78. Each crossbar in the mobile is in equilibrium, so the net torque about the suspension point for each crossbar must be 0. Counterclockwise torques will be taken as positive. The suspension point is used so that the tension in the suspension string need not be known initially. The net vertical force must also be 0.

The bottom bar:

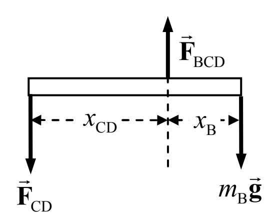


$$\sum\tau = m_{\text{D}}gx_{\text{D}} - m_{\text{C}}gx_{\text{C}} = 0 \quad\rightarrow$$

$$m_{\text{C}} = m_{\text{D}}\frac{x_{\text{D}}}{x_{\text{C}}} = m_{\text{D}}\frac{17.50\text{ cm}}{5.00\text{ cm}} = 3.50m_{\text{D}}$$

$$\sum F_y = F_{\text{CD}} - m_{\text{C}}g - m_{\text{D}}g = 0 \quad\rightarrow\quad F_{\text{CD}} = (m_{\text{C}} + m_{\text{D}})g = 4.50m_{\text{D}}g$$

The middle bar:



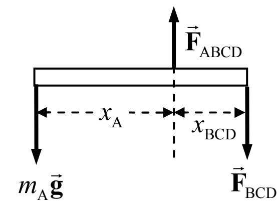
$$\sum\tau = F_{\text{CD}}x_{\text{CD}} - m_{\text{B}}gx_{\text{B}} = 0 \;\rightarrow\; F_{\text{CD}} = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}} \;\rightarrow\; 4.50m_{\text{D}}g = m_{\text{B}}g\frac{x_{\text{B}}}{x_{\text{CD}}}$$

$$m_{\text{D}} = \frac{m_{\text{B}}}{4.50}\frac{x_{\text{B}}}{x_{\text{CD}}} = \frac{(0.748\text{ kg})(5.00\text{ cm})}{(4.50)(15.00\text{ cm})} = 0.05541 \approx \boxed{5.54\times10^{-2}\text{ kg}}$$

$$m_{\text{C}} = 3.50m_{\text{D}} = (3.50)(0.05541\text{ kg}) = \boxed{0.194\text{ kg}}$$

$$\sum F_y = F_{\text{BCD}} - F_{\text{CD}}c - m_{\text{B}}g = 0 \quad\rightarrow\quad F_{\text{BCD}} = F_{\text{CD}} + m_{\text{B}}g = (4.50m_{\text{D}} + m_{\text{B}})g$$

The top bar:

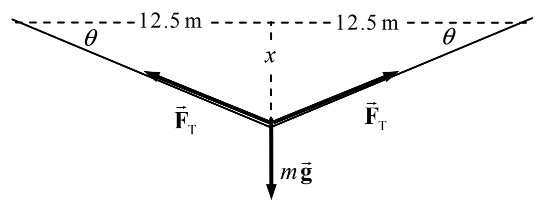


$$\sum \tau = m_A g x_A - F_{BCD} x_{BCD} = 0 \quad \rightarrow$$

$$m_A = \frac{(4.50 m_D + m_B) g x_{BCD}}{g x_A} = (4.50 m_D + m_B)\frac{x_{BCD}}{x_A}$$

$$= [(4.50)(0.05541\text{ kg}) + 0.748\text{ kg}]\frac{7.50\text{ cm}}{30.00\text{ cm}} = \boxed{0.249\text{ kg}}$$

79. (*a*) See the free-body diagram for the Tyrolean traverse technique. We analyze the point on the rope that is at the bottom of the "sag." To include the safety factor, the tension must be no more than 2900 N.



$$\sum F_{\text{vert}} = 2F_T \sin\theta - mg = 0 \quad \rightarrow$$

$$\theta_{\min} = \sin^{-1}\frac{mg}{2F_{T_{\max}}} = \sin^{-1}\frac{(75\text{ kg})(9.80\text{ m/s}^2)}{2(2900\text{ N})} = 7.280°$$

$$\tan\theta_{\min} = \frac{x_{\min}}{12.5\text{ m}} \quad \rightarrow \quad x_{\min} = (12.5\text{ m})\tan(7.280°) = 1.597\text{ m} \approx \boxed{1.6\text{ m}}$$

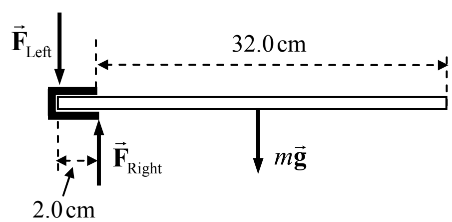
(*b*) Now the sag amount is $x = \frac{1}{4}x_{\min} = \frac{1}{4}(1.597\text{ m}) = 0.3992\text{ m}$. Use that distance to find the tension in the rope.

$$\theta = \tan^{-1}\frac{x}{12.5\text{ m}} = \tan^{-1}\frac{0.3992\text{ m}}{12.5\text{ m}} = 1.829°$$

$$F_T = \frac{mg}{2\sin\theta} = \frac{(75\text{ kg})(9.80\text{ m/s}^2)}{2\sin 1.829°} = 11{,}512\text{ N} \approx \boxed{12{,}000\text{ N}}$$

The rope will not break, but the safety factor will be only about 4 instead of 10.

80. (*a*) The weight of the shelf exerts a downward force and a clockwise torque about the point where the shelf touches the wall. Thus, there must be an upward force and a counterclockwise torque exerted by the slot for the shelf to be in equilibrium. Since any force exerted by the slot will have a short lever arm relative to the point where the shelf touches the wall, the upward force must be larger than the gravity force. Accordingly, there then must be a downward force exerted by the slot at its left edge, exerting no torque, but balancing the vertical forces.



(*b*) Calculate the values of the three forces by first taking torques about the left end of the shelf, with the net torque being zero, and then sum the vertical forces, with the sum being zero.

$$\sum \tau = F_{\text{right}}(0.020\text{ m}) - mg(0.170\text{ m}) = 0 \quad \rightarrow$$

$$F_{\text{right}} = (6.6\text{ kg})(9.80\text{ m/s}^2)\left(\frac{0.170\text{ m}}{0.020\text{ m}}\right) = 549.8\text{ N} \approx \boxed{550\text{ N}}$$

$$\sum F_y = F_{\text{right}} - F_{\text{left}} - mg \quad \rightarrow$$

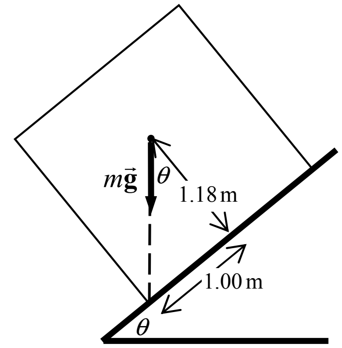
$$F_{\text{left}} = F_{\text{right}} - mg = 549.8\text{ N} - (6.6\text{ kg})(9.80\text{ m/s}^2) = \boxed{490\text{ N}}$$

$$mg = (6.6\text{ kg})(9.80\text{ m/s}^2) = \boxed{65\text{ N}}$$

(*c*) The torque exerted by the support about the left end of the rod is

$$\tau = F_{\text{right}}(2.0\times10^{-2}\text{ m}) = (549.8\text{ N})(2.0\times10^{-2}\text{ m}) = \boxed{11\text{ m}\cdot\text{N}}$$

81. See the free-body diagram for the crate on the verge of tipping. From Fig. 9–16 and the associated discussion, if a vertical line projected downward from the center of gravity falls outside the base of support, then the object will topple. So the limiting case is for the vertical line to intersect the edge of the base of support. Any more tilting and the gravity force would cause the block to tip over, with the axis of rotation through the lower corner of the crate.



$$\tan\theta = \frac{1.00}{1.18} \quad\rightarrow\quad \theta = \tan^{-1}\frac{1.00}{1.18} = \boxed{40^\circ}\text{ (2 significant figures)}$$

The other forces on the block, the normal force and the frictional force, would act at the lower corner. They would cause no torque about the lower corner. The gravity force causes the tipping.

Solutions to Search and Learn Problems

1. For you to remain balanced, your center of mass must be above your base of support on the floor. When you are flat-footed on the floor, your center of mass is above your feet. When you go up onto your tiptoes, your center of mass attempts to move forward so that it will be above your toes. However, due to your finite width and the fact that you cannot move your body inside the wall, your center of mass cannot move forward to be above your toes. You cannot balance on your tiptoes next to the wall.

   If you turn around and place your heels several inches away from the wall, you could lean back and push your back against the wall. In this case your center of mass would be above a point between your feet and the wall. Your feet would create a torque that would rotate your back toward the wall. To prevent from falling over, you would need the normal force of the wall pushing against your back. When your heels are placed against the wall, it is not possible for your center of mass to be between your feet and the wall. Your back cannot therefore push against the wall in this position.

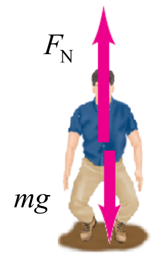
2. As the brick falls, its potential energy is converted into kinetic energy. When the brick hits the floor, work is done on the brick to decelerate it to rest. The amount of work needed to decelerate the brick is equal to the initial potential energy (*mgh*) and is also equal to the product of the average stopping force (*F*) and the brick's compression distance $(\Delta\ell)$. Use Eq. 9–4 to write the compression distance in terms of the force.

$$mgh = F\Delta\ell = F\left(\frac{1}{E}\frac{F}{A}\ell_0\right)$$

   By replacing the strain (F/A) with the ultimate strength of brick, the resulting equation can be solved for the minimum height (*h*) necessary to break the brick when dropped.

$$h = \left(\frac{F}{A}\right)^2\frac{\ell_0 A}{mgE} = \frac{(35\times10^6\text{ N/m}^2)^2(0.040\text{ m})(0.150\text{ m})(0.060\text{ m})}{(1.2\text{ kg})(9.80\text{ m/s}^2)(14\times10^9\text{ N/m}^2)} = \boxed{2.7\text{ m}}$$

3. (*a*) Use conservation of energy to determine the speed when the person reaches the ground. Set the potential energy of the ground as zero $(y = 0)$.



$$\mathrm{KE}_1 + \mathrm{PE}_1 = \mathrm{KE}_2 + \mathrm{PE}_2 \quad \rightarrow \quad 0 + mgy_1 = \tfrac{1}{2}mv^2 + 0$$

$$v = \sqrt{2gy_1} = \sqrt{2(9.8\ \mathrm{m/s^2})(3.0\ \mathrm{m})} = 7.668\ \mathrm{m/s} \approx \boxed{7.7\ \mathrm{m/s}}$$

(*b*) When the person reaches the ground, two forces will act on him: the force of gravity pulling down and the normal force of the ground pushing up. The sum of these two forces provides the net decelerating force. The net work done during deceleration is equal to the change in kinetic energy.

$$\sum Fd = \Delta\mathrm{KE} \quad \rightarrow \quad (mg - F_\mathrm{N})(d) = 0 - \tfrac{1}{2}mv^2$$

$$F_\mathrm{N} = mg + \frac{mv^2}{2d} = (65\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) + \frac{(65\ \mathrm{kg})(7.668\ \mathrm{m/s})^2}{2(0.50\ \mathrm{m})} = 4459\ \mathrm{N} \approx \boxed{4500\ \mathrm{N}}$$

(*c*) Repeat the previous calculation for a stopping distance of $d = 0.010$ m.

$$F_\mathrm{N} = mg + \frac{mv^2}{2d} = (65\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) + \frac{(65\ \mathrm{kg})(7.668\ \mathrm{m/s})^2}{2(0.010\ \mathrm{m})} = 1.917\times10^5\ \mathrm{N} \approx \boxed{1.9\times10^5\ \mathrm{N}}$$

(*d*) The force is evenly spread between each leg, so divide half of the force by the area of the tibia to determine the stress. Then compare this stress to the compressive strength of the tibia given in Table 9–2.

$$\frac{F}{A} = \frac{\tfrac{1}{2}(4459\ \mathrm{N})}{3.0\times10^{-4}\ \mathrm{m^2}} = \boxed{7.4\times10^6\ \mathrm{N/m^2}} < 170\times10^6\ \mathrm{N/m^2}$$

The stress is much less than the compressive strength, so it is unlikely that the tibia will break.

(*e*) Repeating the calculation for the distance of 0.010 m:

$$\frac{F}{A} = \frac{\tfrac{1}{2}(1.917\times10^5\ \mathrm{N})}{3.0\times10^{-4}\ \mathrm{m^2}} = \boxed{3.2\times10^8\ \mathrm{N/m^2}} > 170\times10^6\ \mathrm{N/m^2}$$

The stress is greater than the compressive strength, so the tibia will likely break.

4. (*a*) There are only two forces that produce torques when the axis is chosen about the point where the cable is attached to the beam: the weight of the beam and the vertical component of the force due to the hinge. Since the weight of the beam is downward, it produces a counterclockwise torque. For the net torque to be zero, the vertical component of the force due to the hinge must produce a clockwise torque and therefore must point upward.

(*b*) Choosing the axis of rotation at the wall eliminates the hinge forces from the torque equation, enabling you to solve the torque equation for the tension in the cable directly.

(*c*) First, as in part (*b*), set the sum of the torques around the pivot equal to zero and solve for the tension in the cable.

$$\sum \tau = 0 = -mg(1.10\ \mathrm{m}) - Mg(2.20\ \mathrm{m}) + F_\mathrm{T}\sin\theta\,(2.20\ \mathrm{m}) \quad \rightarrow$$

$$F_\mathrm{T} = \frac{\tfrac{1}{2}mg + Mg}{\sin\theta} = \frac{(12.5\ \mathrm{kg} + 28.0\ \mathrm{kg})}{\sin 30^\circ}(9.8\ \mathrm{m/s^2}) = \boxed{794\ \mathrm{N}}$$

Next, set the sum of the vertical forces and the sum of the horizontal forces equal to zero to determine the components of the force on the hinge.

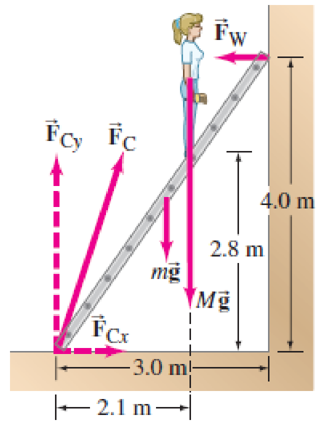
$$\sum F_x = 0 = F_{Hx} - F_{Tx} \quad \rightarrow \quad F_{Hx} = F_{Tx} = F_T \cos\theta = (794\text{ N})\cos 30^\circ = \boxed{687\text{ N}}$$

$$\sum F_y = 0 = F_{Hy} - mg - Mg + F_{Ty} \quad \rightarrow$$

$$F_{Hy} = (m+M)g - F_T \sin\theta = (25\text{ kg} + 28\text{ kg})(9.8\text{ m/s}^2) - (794\text{ N})\sin 30^\circ = \boxed{123\text{ N}}$$

(*d*) You choose the axis of rotation to eliminate one or more unknown values from the torque equation, enabling you to solve for one of the other unknowns.

5. The ladder is in equilibrium, so both the net force and net torque must be zero. Because the ladder is on the verge of slipping, the static frictional force at the ground, F_{Cx} is at its maximum value. Thus, $F_{Cx} = \mu_s F_{Cy}$. Torques are taken about the point of contact of the ladder with the ground, and counterclockwise torques are taken as positive. The three conditions of equilibrium are as follows:



$$\sum F_x = F_{Cx} - F_W = 0 \quad \rightarrow \quad F_{Cx} = F_W$$

$$\sum F_y = F_{Cy} - Mg - mg = 0 \quad \rightarrow$$

$$F_{Cy} = (M+m)g = (67.0\text{ kg})(9.80\text{ m/s}^2) = 656.6\text{ N}$$

$$\sum \tau = F_W(4.0\text{ m}) - mg\left(\tfrac{1}{2}\right)(3.0\text{ m}) - Mg(2.1\text{ m}) = 0$$

Solve the torque equation for F_W.

$$F_W = \left[\frac{\frac{1}{2}(12.0\text{ kg})(3.0\text{ m}) + (55.0\text{ kg})(2.1\text{ m})}{4.0\text{ m}}\right](9.80\text{ m/s}^2) = 327.1\text{ N}$$

The coefficient of friction then is then found from the components of F_C.

$$\mu_s = \frac{F_{Cx}}{F_{Cy}} = \frac{F_W}{F_{Cy}} = \frac{327.1\text{ N}}{656.6\text{ N}} = \boxed{0.50}$$

10

FLUIDS

Responses to Questions

1. Density is the ratio of mass to volume. A high density may mean that lighter molecules are packed more closely together and thus a given amount of mass is occupying a smaller volume, making a higher density. An atom of gold weighs less than an atom of lead, because gold has a lower atomic mass, but the density of gold is higher than that of lead.

2. The sharp end of the pin (with a smaller area) will pierce the skin when pushed with a certain minimum force, but the same force applied in pushing the blunt end of the pen (with a larger area) into the skin does not pierce the skin. Thus, it is pressure (force per unit area) that determines whether the skin is pierced.

3. As the water boils, steam displaces some of the air in the can. With the lid off, the pressure inside is the same as the outside pressure. When the lid is put on, and the water and the can cool, the steam that is trapped in the can condenses back into liquid water. This reduces the pressure in the can to less than atmospheric pressure, and the greater force from the outside air pressure crushes the can.

4. Since the ice floats, the density of ice must be less than that of the water. The mass of the ice displaces a volume of water with the same weight as the ice, whether it is solid or liquid. Thus as the ice melts, the level in the glass stays the same. The ice displaces its melted volume.

5. The density of ice $(917\ \text{kg/m}^3)$ is greater than that of alcohol $(790\ \text{kg/m}^3)$, so the ice cube will not float in a glass of alcohol. The ice cube will sink in the alcohol.

6. Both products have gas dissolved in them (carbonation), making their density less than that of water. The difference is in the sweetener in each product. The Coke® has a significant amount of sugar (or some other sweetener, like high-fructose corn syrup) dissolved in it, increasing its density so that it is greater than that of water. The Diet Coke® has a different, low-calorie sweetener that evidently has a lower density than the Coke sweetener. The density of the Diet Coke (including the can) remains less than that of water. Thus, the Coke sinks, and the Diet Coke floats.

7. Iron ships are not solid iron. If they were, then they would sink. But the ships have quite a bit of open space in their volume (the volume between the deck and the hull, for instance), making their overall density less than that of water. The total mass of iron divided by the total volume of the ship is less than the density of water, so ships made of iron float.

8. Sand must be added to the barge. If sand is removed, then the barge will not need to displace as much water since its weight will be less, and it will rise up in the water, making it even less likely to fit under the bridge. If sand is added, then the barge will sink lower into the water, making it more likely to fit under the bridge. You would have to be careful to not pile the sand up so high that you lose the advantage of adding more sand.

9. As the balloon rises, the air pressure outside the balloon will decrease, becoming lower than the pressure inside the balloon. The excess inside air pressure will cause the balloon to expand, lowering the pressure inside but stretching the balloon in the process. If, at launch, the material of the balloon were already stretched to the limit, then the expansion of the balloon due to the decreasing outside air pressure would cause the balloon to burst. Thus, the balloon is only filled to a fraction of its maximum volume.

10. No. If the balloon is inflated, then the pressure inside the balloon is slightly greater than atmospheric pressure. Thus the air inside the balloon is more dense than the air outside the balloon. Because of the higher density, the weight of the air inside the balloon is greater than the weight of the outside air that has been displaced. This is the same as saying that the buoyant force on the balloon is less than the weight of the air inside the balloon. Therefore, the apparent weight of the filled balloon will be slightly greater than that of the empty balloon.

11. In order to float, you must displace an amount of water equal to your own weight. Salt water is more dense than fresh water, so the volume of salt water you must displace is less than the volume of fresh water. You will float higher in the salt water because you are displacing less water. Less of your body needs to be submerged in the water.

12. As the water falls, its vertical speed is larger when away from the faucet than when close to it, due to gravity. Since the water is essentially incompressible, Eq. 10–4b applies, which says that a faster flow has a smaller cross-sectional area. Thus, the faster moving water has a narrower stream.

13. It is possible. Due to viscosity, some of the air near the train will be pulled along at a speed approximately that of the train. By Bernoulli's principle, that air will be at a lower pressure than air farther from the train. That difference in pressure results in a force toward the train, which could push a lightweight child toward the train. The child would be pushed, not "sucked," but the effect would be the same—a net force toward the train.

14. Water will not flow from the holes when the cup and water are in free fall. The acceleration due to gravity is the same for all falling objects (ignoring friction), so the cup and water would fall together. For the water to flow out of the holes while falling, the water would have to have an acceleration larger than the acceleration due to gravity. Another way to consider the situation is that there will no longer be a pressure difference between the top and bottom of the cup of water, since the lower water molecules don't need to hold up the upper water molecules.

15. The lift generated by wind depends on the speed of the air relative to the wing. For example, an airplane model in a wind tunnel will have lift forces on it even though the model isn't moving relative to the ground. By taking off into the wind, the speed of the air relative to the wing is the sum of the plane's speed and the wind speed. This allows the plane to take off at a lower ground speed, requiring a shorter runway.

16. As the ships move, they drag water with them. The moving water has a lower pressure than stationary water, as shown by Bernoulli's principle. If the ships are moving in parallel paths fairly close together, then the water between them will have a lower pressure than the water on the outside of either one, since it is being dragged by both ships. The ships are in danger of colliding because the higher pressure of the water on the outsides will tend to push them toward each other.

$\boxed{17.}$ The papers move toward each other. Bernoulli's principle says that as the speed of a gas increases, the pressure decreases (when there is no appreciable change in height). As the air passes between the papers, the air pressure between the papers is lowered. The air pressure on the outside of the papers is then greater than that between the papers, so the papers are pushed together.

18. As the car drives through the air, the air inside the car is stationary with respect to the top, but the outside air is moving with respect to the top. There is no appreciable change in height between the two sides of the canvas top. By Bernoulli's principle, the outside air pressure near the canvas top will be less than the inside air pressure. That difference in pressure results in a force that makes the top bulge outward.

19. The roofs are pushed off from the inside. By Bernoulli's principle, the fast moving winds of the storm causes the air pressure above the roof to be quite low, but the pressure inside the house is still near normal levels. There is no appreciable change in height between the two sides of the roof. This pressure difference, combined with the large surface area of the roof, gives a very large force that can push the roof off the house. That is why it is advised to open some windows if a tornado is imminent, so that the pressure inside the house can somewhat equalize with the outside pressure.

20. See the diagram in the textbook. The pressure at the surface of both containers is atmospheric pressure. The pressure in each tube would thus also be atmospheric pressure, at the level of the surface of the liquid in each container. The pressure in each tube will decrease with height by an amount $\rho gh.$ Since the portion of the tube going into the lower container is longer than the portion of the tube going into the higher container, the pressure at the highest point on the right side is lower than the pressure at the highest point on the left side. This pressure difference causes liquid to flow from the left side of the tube to the right side of the tube. And as noted in the question, the tube must be filled with liquid before this can occur.

21. "Blood pressure" should measure the pressure of the blood coming out of the heart. If the cuff is below the level of the heart, then the measured pressure will be the pressure from the pumping of the heart plus the pressure due to the height of blood above the cuff. This reading will be too high. Likewise, if the cuff is above heart level, then the reported pressure measurement will be too low.

Responses to MisConceptual Questions

1. (*c*) Students frequently think that since the wood floats, it experiences a greater buoyancy force. However, both objects experience the same buoyant force since they have the same volume and are in the same fluid. The wood floats because its weight is less than the buoyant force, and the iron sinks because its weight is greater than the buoyant force.

2. (*d*) A common misconception is that container B will have the greater force on the bottom since it holds the greatest weight of water. The force of the water on the bottom of the container is equal to the product of the area of the bottom and the pressure at the bottom. Since all three containers have the same base areas and the same depth of water, the forces on the bottom of each will be equal. The additional weight of the water in B is supported by the diagonal walls. The smaller weight of C is countered by the additional pressure exerted by the diagonal walls on the water.

3. (*c*) Students may think that since part of the wood floats above the water, beaker B will weigh more. Since the wood is in equilibrium, the weight of the water displaced by the wood is equal to the weight of the wood. Therefore, the beakers will weigh the same.

4. (*d*) A common misconception is to consider the ocean liner as a solid object. The ocean liner has an outer shell of steel, which keeps the water out of the interior of the ship. However, most of the volume occupied by the ocean liner is filled with air. This makes the average density of the ocean liner less than the density of the seawater. If a hole were introduced into the outer shell, then the interior would fill with water, increasing the density of the ship until it was greater than that of seawater and then the ship would sink, as happened to the Titanic.

5. i (*b*) and ii (*b*) When the rowboat is floating in the water, the boat will displace a volume of water whose weight is equal to the weight of the boat. When the boat (or the anchor) is removed from the swimming pool, the water is no longer being displaced, so the water level will drop back to its initial level.

6. (*c*) Students may think that since part of the ice floats above the water, the water will overflow as the ice melts. The ice, however, displaces a mass of water equal to the mass of the ice. As the ice melts, its volume decreases until it occupies the same volume as the water that the ice initially displaced.

7. (*a*) A common misconception is that the hot air causes the balloon to rise, so it would rise on the Moon just as it would on Earth. What actually happens on the Earth is the denser cold air around the balloon on Earth is heavier than the hot air in the balloon. This denser air then moves downward, displacing the hot air balloon upward. On the Moon there is no atmosphere to move downward around the balloon, so the balloon would not rise.

8. (*c*) Students may reason that since water is denser than oil, the object would experience a greater buoyant force in water. This would be true if the object was completely submerged in either fluid. Since the object is floating at the surface of the liquid, the buoyant force on the object will be equal to its weight and will therefore be the same in both fluids. The object floats with less volume submerged in the water than is submerged when it floats in the oil.

9. (*d*) The velocity of the water in the pipe depends upon its diameter, as shown by the continuity equation. The pressure depends upon both the change in elevation and the velocity of the water. Therefore, information about how the diameter changes is required to determine how the pressure changes.

10. (*a*) A common misconception is that the pressure would be higher in the faster moving water. The continuity equation requires that the water in the wider pipe travel slower than in the narrow pipe. Bernoulli’s equation shows that the pressure will be higher in the region where the water travels slower (the wider pipe). As water travels from a region of low pressure to one of high pressure, it experiences a retarding force, which decreases the velocity of the water.

11. (*b*) The ball accelerates to the right because the pressure on the left side of the ball is greater than the pressure on the right side. From Bernoulli’s equation, the air on the right side must then be traveling faster than the air on the left side. This difference in air speed is produced by spinning the ball when it is thrown.

12. (*a*) Students may believe that wind above the chimney will blow the smoke back down the chimney. Actually, the wind blowing across the top of the chimney causes the air pressure above the chimney to be lower than the air pressure inside. The greater inside pressure pushes the smoke up the chimney.

Solutions to Problems

1. The mass is found from the density of granite (found in Table 10–1) and the volume of granite.

$$m=\rho V=(2.7\times10^3\ \text{kg/m}^3)(10^8\ \text{m}^3)=2.7\times10^{11}\ \text{kg}\approx\boxed{3\times10^{11}\ \text{kg}}$$

2. The mass is found from the density of air (found in Table 10–1) and the volume of air.

$$m=\rho V=(1.29\ \text{kg/m}^3)(5.6\ \text{m})(3.6\ \text{m})(2.4\ \text{m})=\boxed{62\ \text{kg}}$$

3. The mass is found from the density of gold (found in Table 10–1) and the volume of gold.

$$m=\rho V=(19.3\times10^3\ \text{kg/m}^3)(0.54\ \text{m})(0.31\ \text{m})(0.22\ \text{m})=\boxed{710\ \text{kg}}\quad(\approx 1600\ \text{lb})$$

4. Assume that your density is that of water, and that your mass is 75 kg.

$$V=\frac{m}{\rho}=\frac{75\ \text{kg}}{1.00\times10^3\ \text{kg/m}^3}=\boxed{7.5\times10^{-2}\ \text{m}^3}=75\ \text{L}$$

5. To find the specific gravity of the fluid, take the ratio of the density of the fluid to that of water, noting that the same volume is used for both liquids.

$$\text{SG}_{\text{fluid}}=\frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}=\frac{(m/V)_{\text{fluid}}}{(m/V)_{\text{water}}}=\frac{m_{\text{fluid}}}{m_{\text{water}}}=\frac{89.22\ \text{g}-35.00\ \text{g}}{98.44\ \text{g}-35.00\ \text{g}}=\boxed{0.8547}$$

6. The specific gravity of the mixture is the ratio of the density of the mixture to that of water. To find the density of the mixture, the mass of antifreeze and the mass of water must be known.

$$m_{\text{antifreeze}}=\rho_{\text{antifreeze}}V_{\text{antifreeze}}=\text{SG}_{\text{antifreeze}}\rho_{\text{water}}V_{\text{antifreeze}}\qquad m_{\text{water}}=\rho_{\text{water}}V_{\text{water}}$$

$$\text{SG}_{\text{mixture}}=\frac{\rho_{\text{mixture}}}{\rho_{\text{water}}}=\frac{m_{\text{mixture}}/V_{\text{mixture}}}{\rho_{\text{water}}}=\frac{m_{\text{antifreeze}}+m_{\text{water}}}{\rho_{\text{water}}V_{\text{mixture}}}=\frac{\text{SG}_{\text{antifreeze}}\rho_{\text{water}}V_{\text{antifreeze}}+\rho_{\text{water}}V_{\text{water}}}{\rho_{\text{water}}V_{\text{mixture}}}$$

$$=\frac{\text{SG}_{\text{antifreeze}}V_{\text{antifreeze}}+V_{\text{water}}}{V_{\text{mixture}}}=\frac{(0.80)(4.0\ \text{L})+5.0\ \text{L}}{9.0\ \text{L}}=\boxed{0.91}$$

7. (*a*) The density from the three-part model is found from the total mass divided by the total volume. Let subscript 1 represent the inner core, subscript 2 represent the outer core, and subscript 3 represent the mantle. The radii are then the outer boundaries of the labeled region.

$$\rho_{\substack{\text{three}\\\text{layers}}}=\frac{m_1+m_2+m_3}{V_1+V_2+V_3}=\frac{\rho_1m_1+\rho_2m_2+\rho_3m_3}{V_1+V_2+V_3}=\frac{\rho_1\frac{4}{3}\pi r_1^3+\rho_2\frac{4}{3}\pi(r_2^3-r_1^3)+\rho_3\frac{4}{3}\pi(r_3^3-r_2^3)}{\frac{4}{3}\pi r_1^3+\frac{4}{3}\pi(r_2^3-r_1^3)+\frac{4}{3}\pi(r_3^3-r_2^3)}$$

$$=\frac{\rho_1r_1^3+\rho_2(r_2^3-r_1^3)+\rho_3(r_3^3-r_2^3)}{r_3^3}=\frac{r_1^3(\rho_1-\rho_2)+r_2^3(\rho_2-\rho_3)+r_3^3\rho_3}{r_3^3}$$

$$=\frac{(1220\ \text{km})^3(1900\ \text{kg/m}^3)+(3480\ \text{km})^3(6700\ \text{kg/m}^3)+(6380\ \text{km})^3(4400\ \text{kg/m}^3)}{(6380\ \text{km})^3}$$

$$=5500.6\ \text{kg/m}^3\approx\boxed{5500\ \text{kg/m}^3}$$

(*b*) $$\rho_{\substack{\text{one}\\\text{density}}}=\frac{M}{V}=\frac{M}{\frac{4}{3}\pi R^3}=\frac{5.98\times10^{24}\ \text{kg}}{\frac{4}{3}\pi(6380\times10^3\ \text{m})^3}=5497.3\ \text{kg/m}^3\approx\boxed{5.50\times10^3\ \text{kg/m}^3}$$

$$\%\,\text{difference}=100\left(\frac{\rho_{\substack{\text{one}\\\text{density}}}-\rho_{\substack{\text{three}\\\text{layers}}}}{\rho_{\substack{\text{three}\\\text{layers}}}}\right)=100\left(\frac{5497\ \text{kg/m}^3-5501\ \text{kg/m}^3}{5501\ \text{kg/m}^3}\right)=-0.073\approx\boxed{-0.07\%}$$

8. The pressure is given by Eq. 10–3a.

$$P=\rho gh=(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(46\ \text{m})=\boxed{4.5\times10^5\ \text{N/m}^2}$$

9. The pressure exerted by the heel is caused by the heel pushing down on the floor. That downward push is the reaction to the normal force of the floor on the shoe heel. The normal force on one heel is assumed to be half the weight of the person.

(*a*) $$P_{\text{pointed}}=\frac{\frac{1}{2}W_{\text{person}}}{A_{\text{pointed}}}=\frac{\frac{1}{2}(56\ \text{kg})(9.80\ \text{m/s}^2)}{(0.45\ \text{cm}^2)(0.01\ \text{m/cm})^2}=\boxed{6.1\times10^6\ \text{N/m}^2}$$

(*b*) $$P_{\text{wide}}=\frac{\frac{1}{2}W_{\text{person}}}{A_{\text{wide}}}=\frac{\frac{1}{2}(56\ \text{kg})(9.80\ \text{m/s}^2)}{(16\ \text{cm}^2)(0.01\ \text{m/cm})^2}=\boxed{1.7\times10^5\ \text{N/m}^2}$$

10. Use Eq. 10–3b to find the pressure difference. The density is found in Table 10–1.

$$\Delta P=\rho g\Delta h=(1.05\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.75\ \text{m})$$
$$=1.801\times10^4\ \text{N/m}^2\left(\frac{1\ \text{mm-Hg}}{133\ \text{N/m}^2}\right)=\boxed{135\ \text{mm-Hg}}$$

11. (*a*) The total force of the atmosphere on the table will be the air pressure times the area of the table.

$$F=PA=(1.013\times10^5\ \text{N/m}^2)(1.7\ \text{m})(2.6\ \text{m})=\boxed{4.5\times10^5\ \text{N}}$$

(*b*) Since the atmospheric pressure is the same on the underside of the table (the height difference is minimal), the upward force of air pressure is the same as the downward force of air on the top of the table, $\boxed{4.5\times10^5\ \text{N}}$.

12. The height is found from Eq. 10–3a, using normal atmospheric pressure. The density is found in Table 10–1.

$$P=\rho gh\quad\rightarrow\quad h=\frac{P}{\rho g}=\frac{1.013\times10^5\ \text{N/m}^2}{(0.79\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}=\boxed{13\ \text{m}}$$

That is so tall as to be impractical in many cases.

13. The pressure difference on the lungs is the pressure change from the depth of water. The pressure unit conversion comes from Table 10–2.

$$\Delta P=\rho g\Delta h\quad\rightarrow\quad\Delta h=\frac{\Delta P}{\rho g}=\frac{(-85\ \text{mm-Hg})\left(\frac{133\ \text{N/m}^2}{1\ \text{mm-Hg}}\right)}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}=-1.154\ \text{m}\approx\boxed{-1.2\ \text{m}}$$

He could have been 1.2 m below the surface.

14. The force exerted by the gauge pressure will be equal to the weight of the vehicle.

$$mg = PA = P(\pi r^2) \quad \rightarrow$$
$$m = \frac{P\pi r^2}{g} = \frac{(17.0\text{ atm})\left(\dfrac{1.013\times10^5\text{ N/m}^2}{1\text{ atm}}\right)\pi[\tfrac{1}{2}(0.255\text{ m})]^2}{(9.80\text{ m/s}^2)} = \boxed{8970\text{ kg}}$$

15. The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$mg = 4PA \quad \rightarrow \quad m = \frac{4PA}{g} = \frac{4(2.40\times10^5\text{ N/m}^2)(190\text{ cm}^2)\left(\dfrac{1\text{ m}^2}{10^4\text{ cm}^2}\right)}{(9.80\text{ m/s}^2)} = 1861\text{ kg} \approx \boxed{1900\text{ kg}}$$

$\boxed{16.}$ (*a*) The absolute pressure can be found from Eq. 10–3c, and the total force is the absolute pressure times the area of the bottom of the pool.

$$P = P_0 + \rho gh = 1.013\times10^5\text{ N/m}^2 + (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(1.8\text{ m})$$
$$= 1.189\times10^5\text{ N/m}^2 \approx \boxed{1.19\times10^5\text{ N/m}^2}$$
$$F = PA = (1.189\times10^5\text{ N/m}^2)(28.0\text{ m})(8.5\text{ m}) = \boxed{2.8\times10^7\text{ N}}$$

(*b*) The pressure against the side of the pool, near the bottom, will be the same as the pressure at the bottom. Pressure is not directional. $P = \boxed{1.19\times10^5\text{ N/m}^2}$.

17. (*a*) The gauge pressure is given by Eq. 10–3a. The height is the height from the bottom of the hill to the top of the water tank.

$$P_G = \rho gh = (1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)[6.0\text{ m} + (75\text{ m})\sin 61^\circ] = \boxed{7.0\times10^5\text{ N/m}^2}$$

(*b*) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$h = 6.0\text{ m} + (75\text{ m})\sin 61^\circ = \boxed{72\text{ m}}$$

18. The pressures at points a and b are equal since they are at the same height in the same fluid. If the pressures were unequal, then the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_a = P_b \quad \rightarrow \quad P_0 + \rho_{oil} g h_{oil} = P_0 + \rho_{water} g h_{water} \quad \rightarrow \quad \rho_{oil} h_{oil} = \rho_{water} h_{water} \quad \rightarrow$$
$$\rho_{oil} = \frac{\rho_{water} h_{water}}{h_{oil}} = \frac{(1.00\times10^3\text{ kg/m}^3)(0.272\text{ m} - 0.0862\text{ m})}{(0.272\text{ m})} = \boxed{683\text{ kg/m}^3}$$

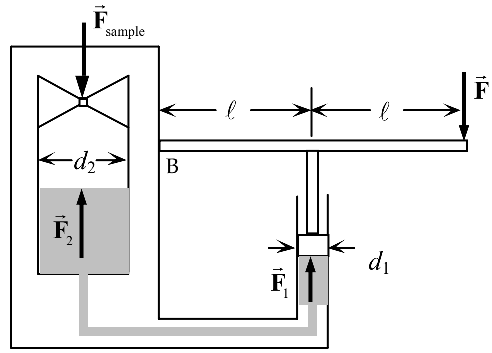
19. If the atmosphere were of uniform density, then the pressure at any height *h* would be $P = P_0 - \rho gh$. At the top of the uniform atmosphere, the pressure would be 0. Thus solve for the height at which the pressure becomes 0, using a density of half of the atmospheric density at sea level.

$$P = P_0 - \rho gh = 0 \quad \rightarrow \quad h = \frac{P_0}{\rho g} = \frac{(1.013\times10^5\text{ N/m}^2)}{\frac{1}{2}(1.29\text{ kg/m}^3)(9.80\text{ m/s}^2)} = \boxed{1.60\times10^4\text{ m}}$$

20. The minimum gauge pressure would cause the water to come out of the faucet with very little speed. This means that the gauge pressure needed must be enough to hold the water at this elevation. Use Eq. 10–3a.

$$P_G = \rho gh = (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(44 \text{ m}) = \boxed{4.3\times10^5 \text{ N/m}^2}$$

21. Consider the lever (handle) of the press. The net torque on that handle is 0. Use that to find the force exerted by the hydraulic fluid upward on the small cylinder (and the lever). Then Pascal's principle can be used to find the upward force on the large cylinder, which is the same as the force on the sample.



$$\sum\tau = F(2\ell) - F_1\ell = 0 \rightarrow F_1 = 2F$$

$$P_1 = P_2 \quad\rightarrow\quad \frac{F_1}{\pi\left(\frac{1}{2}d_1\right)^2} = \frac{F_2}{\pi\left(\frac{1}{2}d_2\right)^2} \quad\rightarrow$$

$$F_2 = F_1(d_2/d_1)^2 = 2F(d_2/d_1)^2 = F_{sample} \quad\rightarrow$$

$$P_{sample} = \frac{F_{sample}}{A_{sample}} = \frac{2F(d_2/d_1)^2}{A_{sample}} = \frac{2(320 \text{ N})(5)^2}{4.0\times10^{-4} \text{ m}^2} = \boxed{4.0\times10^7 \text{ N/m}^2}$$

22. The pressure in the tank is atmospheric pressure plus the pressure difference due to the column of mercury, as given in Eq. 10–3c.

(*a*) $P = P_0 + \rho gh = 1.04 \text{ bar} + \rho_{Hg}gh$

$$= (1.04 \text{ bar})\left(\frac{1.00\times10^5 \text{ N/m}^2}{1 \text{ bar}}\right) + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.185 \text{ m}) = \boxed{1.29\times10^5 \text{ N/m}^2}$$

(*b*) $P = (1.04 \text{ bar})\left(\frac{1.00\times10^5 \text{ N/m}^2}{1 \text{ bar}}\right) + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(-0.056 \text{ m}) = \boxed{9.7\times10^4 \text{ N/m}^2}$

23. If the iron is floating, then the net force on it is zero. The buoyant force on the iron must be equal to its weight. The buoyant force is equal to the weight of the mercury displaced by the submerged iron.

$$F_{buoyant} = m_{Fe}g \quad\rightarrow\quad \rho_{Hg}gV_{submerged} = \rho_{Fe}gV_{total} \quad\rightarrow$$

$$\frac{V_{submerged}}{V_{total}} = \frac{\rho_{Fe}}{\rho_{Hg}} = \frac{7.8\times10^3 \text{ kg/m}^3}{13.6\times10^3 \text{ kg/m}^3} = \boxed{0.57} \approx 57\%$$

24. The difference between the actual mass and the apparent mass is the mass of the water displaced by the rock. The mass of the water displaced is the volume of the rock times the density of water, and the volume of the rock is the mass of the rock divided by its density. Combining these relationships yields an expression for the density of the rock.

$$m_{actual} - m_{apparent} = \Delta m = \rho_{water}V_{rock} = \rho_{water}\frac{m_{rock}}{\rho_{rock}} \quad\rightarrow$$

$$\rho_{rock} = \rho_{water}\frac{m_{rock}}{\Delta m} = (1.00\times10^3 \text{ kg/m}^3)\frac{9.28 \text{ kg}}{9.28 \text{ kg} - 6.18 \text{ kg}} = \boxed{2990 \text{ kg/m}^3}$$

25. (*a*) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, so their sum is equal to the weight of the hull. The buoyant force is the weight of the water displaced.

$$T + F_{\text{buoyant}} = mg \quad \rightarrow$$

$$T = mg - F_{\text{buoyant}} = m_{\text{hull}}g - \rho_{\text{water}}V_{\text{sub}}g = m_{\text{hull}}g - \rho_{\text{water}}\frac{m_{\text{hull}}}{\rho_{\text{hull}}}g = m_{\text{hull}}g\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{hull}}}\right)$$

$$= (1.8\times10^4\ \text{kg})(9.80\ \text{m/s}^2)\left(1 - \frac{1.00\times10^3\ \text{kg/m}^3}{7.8\times10^3\ \text{kg/m}^3}\right) = 1.538\times10^5\ \text{N} \approx \boxed{1.5\times10^5\ \text{N}}$$

(*b*) When the hull is completely out of the water, the tension in the crane's cable must be equal to the weight of the hull.

$$T = mg = (1.8\times10^4\ \text{kg})(9.80\ \text{m/s}^2) = 1.764\times10^5\ \text{N} \approx \boxed{1.8\times10^5\ \text{N}}$$

26. The buoyant force of the balloon must equal the weight of the balloon plus the weight of the helium in the balloon plus the weight of the load. For calculating the weight of the helium, we assume it is at 0°C and 1 atm pressure. The buoyant force is the weight of the air displaced by the volume of the balloon.

$$F_{\text{buoyant}} = \rho_{\text{air}}V_{\text{balloon}}g = m_{\text{He}}g + m_{\text{balloon}}g + m_{\text{cargo}}g \quad \rightarrow$$

$$m_{\text{cargo}} = \rho_{\text{air}}V_{\text{balloon}} - m_{\text{He}} - m_{\text{balloon}} = \rho_{\text{air}}V_{\text{balloon}} - \rho_{\text{He}}V_{\text{balloon}} - m_{\text{balloon}} = (\rho_{\text{air}} - \rho_{\text{He}})V_{\text{balloon}} - m_{\text{balloon}}$$

$$= (1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3)\tfrac{4}{3}\pi(7.15\ \text{m})^3 - 930\ \text{kg} = \boxed{770\ \text{kg}} = 7600\ \text{N}$$

$\boxed{27.}$ The apparent weight is the actual weight minus the buoyant force, as shown in Example 10–8.The buoyant force is the weight of a mass of water occupying the volume of the metal sample.

$$m_{\text{apparent}}g = m_{\text{metal}}g - F_{\text{B}} = m_{\text{metal}}g - V_{\text{metal}}\rho_{\text{H}_2\text{O}}g = m_{\text{metal}}g - \frac{m_{\text{metal}}}{\rho_{\text{metal}}}\rho_{\text{H}_2\text{O}}g \quad \rightarrow$$

$$m_{\text{apparent}} = m_{\text{metal}} - \frac{m_{\text{metal}}}{\rho_{\text{metal}}}\rho_{\text{H}_2\text{O}} \quad \rightarrow$$

$$\rho_{\text{metal}} = \frac{m_{\text{metal}}}{(m_{\text{metal}} - m_{\text{apparent}})}\rho_{\text{H}_2\text{O}} = \frac{63.5\ \text{g}}{(63.5\ \text{g} - 55.4\ \text{g})}(1000\ \text{kg/m}^3) = 7840\ \text{kg/m}^3$$

Based on the density, the metal is probably $\boxed{\text{iron or steel}}$.

28. The difference between the actual mass and the apparent mass of the aluminum is the mass of the air displaced by the aluminum. The mass of the air displaced is the volume of the aluminum times the density of air, and the volume of the aluminum is the actual mass of the aluminum divided by the density of aluminum. Combining these relationships yields an expression for the actual mass.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{air}}V_{\text{Al}} = \rho_{\text{air}}\frac{m_{\text{actual}}}{\rho_{\text{Al}}} \quad \rightarrow$$

$$m_{\text{actual}} = \frac{m_{\text{apparent}}}{1 - \dfrac{\rho_{\text{air}}}{\rho_{\text{Al}}}} = \frac{4.0000\ \text{kg}}{1 - \dfrac{1.29\ \text{kg/m}^3}{2.70\times10^3\ \text{kg/m}^3}} = \boxed{4.0019\ \text{kg}}$$

29. The buoyant force on the drum must be equal to the weight of the steel plus the weight of the gasoline. The weight of each component is its respective volume times the density. The buoyant force is the weight of total volume of displaced water. We assume that the drum just barely floats—in other words, the volume of water displaced is equal to the total volume of gasoline and steel.

$$F_B = W_{steel} + W_{gasoline} \rightarrow (V_{gasoline} + V_{steel})\rho_{water} g = V_{steel}\rho_{steel} g + V_{gasoline}\rho_{gasoline} g \rightarrow$$

$$V_{gasoline}\rho_{water} + V_{steel}\rho_{water} = V_{steel}\rho_{steel} + V_{gasoline}\rho_{gasoline} \rightarrow$$

$$V_{steel} = V_{gasoline}\left(\frac{\rho_{water} - \rho_{gasoline}}{\rho_{steel} - \rho_{water}}\right) = (210\text{ L})\left(\frac{1000\text{ kg/m}^3 - 680\text{ kg/m}^3}{7800\text{ kg/m}^3 - 1000\text{ kg/m}^3}\right) = 9.882\text{ L} \approx \boxed{9.9\times10^{-3}\text{ m}^3}$$

30. (*a*) The buoyant force is the weight of the water displaced, using the density of seawater.

$$F_{buoyant} = m_{\substack{water\\displaced}}\, g = \rho_{water} V_{displaced}\, g$$

$$= (1.025\times10^3\text{ kg/m}^3)(69.6\text{ L})\left(\frac{1\times10^{-3}\text{ m}^3}{1\text{ L}}\right)(9.80\text{ m/s}^2) = \boxed{699\text{ N}}$$

(*b*) The weight of the diver is $m_{diver} g = (72.8\text{ kg})(9.80\text{ m/s}^2) = 713\text{ N}.$ Since the buoyant force is not as large as her weight, $\boxed{\text{she will sink}}$, although she will do so very gradually since the two forces are almost the same.

$\boxed{31.}$ The buoyant force on the ice is equal to the weight of the ice, since it floats.

$$F_{buoyant} = W_{ice} \rightarrow m_{\substack{seawater\\submerged}}\, g = m_{ice} g \rightarrow m_{\substack{seawater\\submerged}} = m_{ice} \rightarrow \rho_{seawater} V_{seawater} = \rho_{ice} V_{ice} \rightarrow$$

$$(\text{SG})_{seawater}\,\rho_{water} V_{\substack{submerged\\ice}} = (\text{SG})_{ice}\,\rho_{water} V_{ice} \rightarrow (\text{SG})_{seawater} V_{\substack{submerged\\ice}} = (\text{SG})_{ice} V_{ice} \rightarrow$$

$$V_{\substack{submerged\\ice}} = \frac{(\text{SG})_{ice}}{(\text{SG})_{seawater}} V_{ice} = \frac{0.917}{1.025} V_{ice} = 0.895\, V_{ice}$$

Thus, the fraction above the water is $V_{above} = V_{ice} - V_{submerged} = 0.105\, V_{ice}$ or $\boxed{10.5\%}$.

32. (*a*) The difference between the actual mass and the apparent mass of the aluminum ball is the mass of the liquid displaced by the ball. The mass of the liquid displaced is the volume of the ball times the density of the liquid, and the volume of the ball is the mass of the ball divided by its density. Combining these relationships yields an expression for the density of the liquid.

$$m_{actual} - m_{apparent} = \Delta m = \rho_{liquid} V_{ball} = \rho_{liquid}\frac{m_{ball}}{\rho_{Al}} \rightarrow$$

$$\rho_{liquid} = \frac{\Delta m}{m_{ball}}\rho_{Al} = \frac{(3.80\text{ kg} - 2.10\text{ kg})}{3.80\text{ kg}}(2.70\times10^3\text{ kg/m}^3) = \boxed{1210\text{ kg/m}^3}$$

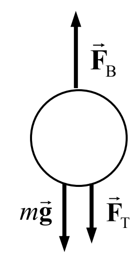
(*b*) Generalizing the relation from above, we have $\boxed{\rho_{liquid} = \left(\frac{m_{object} - m_{apparent}}{m_{object}}\right)\rho_{object}}$.

33. The buoyancy force due to the submerged empty soda bottles must equal the weight of the child. To find the minimum number of bottles (*N*), we assume that each bottle is completely submerged, so displaces 1.0 L of water.

$$F_{\text{buoyant}} = NV_{\text{bottle}}\rho_{\text{water}}g = m_{\text{child}}g \quad \rightarrow$$

$$N = \frac{m_{\text{child}}}{V_{\text{bottle}}\rho_{\text{water}}} = \frac{32\text{ kg}}{(1.0\text{ L})\left(\dfrac{10^{-3}\text{ m}^3}{1\text{ L}}\right)(1000\text{ kg/m}^3)} = \boxed{32\text{ bottles}}$$

34. There are three forces on the chamber: the weight of the chamber, the tension in the cable, and the buoyant force. See the free-body diagram.



(*a*) The buoyant force is the weight of water displaced by the chamber.

$$F_{\text{buoyant}} = \rho_{\text{water}}V_{\text{chamber}}g = \rho_{\text{water}}\tfrac{4}{3}\pi R_{\text{chamber}}^3 g$$
$$= (1.025\times10^3\text{ kg/m}^3)\tfrac{4}{3}\pi(2.60\text{ m})^3(9.80\text{ m/s}^2)$$
$$= 7.3953\times10^5\text{ N} \approx \boxed{7.40\times10^5\text{ N}}$$

(*b*) To find the tension, use Newton's second law for the stationary chamber.

$$F_{\text{buoyant}} = mg + F_{\text{T}} \quad \rightarrow$$

$$F_{\text{T}} = F_{\text{buoyant}} - mg = 7.3953\times10^5\text{ N} - (7.44\times10^4\text{ kg})(9.80\text{ m/s}^2) = \boxed{1.0\times10^4\text{ N}}$$

35. The difference between the actual mass and the apparent mass is the mass of the alcohol displaced by the wood. The mass of the alcohol displaced is the volume of the wood times the density of the alcohol, the volume of the wood is the mass of the wood divided by the density of the wood, and the density of the alcohol is its specific gravity times the density of water.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{alc}}V_{\text{wood}} = \rho_{\text{alc}}\frac{m_{\text{actual}}}{\rho_{\text{wood}}} = \text{SG}_{\text{alc}}\rho_{\text{H}_2\text{O}}\frac{m_{\text{actual}}}{\rho_{\text{wood}}} \quad \rightarrow$$

$$\frac{\rho_{\text{wood}}}{\rho_{\text{H}_2\text{O}}} = \text{SG}_{\text{wood}} = \text{SG}_{\text{alc}}\frac{m_{\text{actual}}}{(m_{\text{actual}} - m_{\text{apparent}})} = (0.79)\frac{0.48\text{ kg}}{(0.48\text{ kg} - 0.047\text{ kg})} = \boxed{0.88}$$

36. Use the definition of density and specific gravity, and then solve for the fat fraction, *f*.

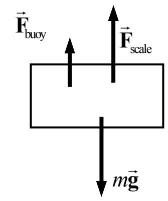
$$m_{\text{fat}} = mf = V_{\text{fat}}\rho_{\text{fat}}; \quad m_{\substack{\text{fat}\\\text{free}}} = m(1-f) = V_{\substack{\text{fat}\\\text{free}}}\rho_{\substack{\text{fat}\\\text{free}}}$$

$$\rho_{\text{body}} = X\rho_{\text{water}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{fat}} + m_{\substack{\text{fat}\\\text{free}}}}{V_{\text{fat}} + V_{\substack{\text{fat}\\\text{free}}}} = \frac{m}{\dfrac{mf}{\rho_{\text{fat}}} + \dfrac{m(1-f)}{\rho_{\substack{\text{fat}\\\text{free}}}}} = \frac{1}{\dfrac{f}{\rho_{\text{fat}}} + \dfrac{(1-f)}{\rho_{\substack{\text{fat}\\\text{free}}}}} \quad \rightarrow$$

$$f = \frac{\rho_{\text{fat}}\rho_{\substack{\text{fat}\\\text{free}}}}{X\rho_{\text{water}}(\rho_{\substack{\text{fat}\\\text{free}}} - \rho_{\text{fat}})} - \frac{\rho_{\text{fat}}}{(\rho_{\substack{\text{fat}\\\text{free}}} - \rho_{\text{fat}})} = \frac{(0.90\text{ g/cm}^3)(1.10\text{ g/cm}^3)}{X(1.0\text{ g/cm}^3)(0.20\text{ g/cm}^3)} - \frac{(0.90\text{ g/cm}^3)}{(0.20\text{ g/cm}^3)}$$

$$= \frac{4.95}{X} - 4.5 \quad \rightarrow \quad \%\text{ Body fat} = 100f = 100\left(\frac{4.95}{X} - 4.5\right) = \boxed{\frac{495}{X} - 450}$$

37. (*a*) The free-body diagram for the athlete shows three forces—the athlete's weight, the buoyancy force, and the upward force of the scale. Those forces must add to give 0, and that can be used to find the volume of the athlete.



$$F_{\text{buoyant}} + F_{\text{scale}} - F_{\text{weight}} = \rho_{\text{water}} V g + m_{\text{apparant}} g - m_{\text{actual}} g = 0 \quad \rightarrow$$

$$V = \frac{m_{\text{actual}} - m_{\text{apparant}}}{\rho_{\text{water}}} = \frac{70.2\text{ kg} - 3.4\text{ kg}}{1000\text{ kg/m}^3} = \boxed{6.68 \times 10^{-2}\text{ m}^3}$$

(*b*) The specific gravity is the athlete's density divided by the density of water.

$$\text{SG} = \frac{\rho_{\text{athlete}}}{\rho_{\text{water}}} = \frac{m/(V - V_{\text{R}})}{\rho_{\text{water}}} = \frac{(70.2\text{ kg})/(6.68 \times 10^{-2}\text{ m}^3 - 1.3 \times 10^{-3}\text{ m}^3)}{1000\text{ kg/m}^3} = 1.072 \approx \boxed{1.07}$$

(*c*) We use the formula given with the problem.

$$\%\text{ Body fat} = \frac{495}{\text{SG}} - 450 = \frac{495}{1.072} - 450 = \boxed{12\%}$$

38. The buoyant force must be equal to the combined weight of the helium balloons and the person. We ignore the buoyant force due to the volume of the person, and we ignore the mass of the balloon material.

$$F_{\text{B}} = (m_{\text{person}} + m_{\text{He}})g \;\rightarrow\; \rho_{\text{air}} V_{\text{He}} g = (m_{\text{person}} + \rho_{\text{He}} V_{\text{He}})g \;\rightarrow\; V_{\text{He}} = N\tfrac{4}{3}\pi r^3 = \frac{m_{\text{person}}}{(\rho_{\text{air}} - \rho_{\text{He}})} \;\rightarrow$$

$$N = \frac{3m_{\text{person}}}{4\pi r^3(\rho_{\text{air}} - \rho_{\text{He}})} = \frac{3(72\text{ kg})}{4\pi(0.165\text{ m})^3(1.29\text{ kg/m}^3 - 0.179\text{ kg/m}^3)} = 3444 \approx \boxed{3400\text{ balloons}}$$

39. There will be a downward gravity force and an upward buoyant force on the fully submerged tank. The buoyant force is constant, but the gravity force will decrease as the air is removed. Take upward to be positive.

$$F_{\text{full}} = F_{\text{B}} - m_{\text{total}} g = \rho_{\text{water}} V_{\text{tank}} g - (m_{\text{tank}} + m_{\text{air}})g$$
$$= [(1025\text{ kg/m}^3)(0.0157\text{ m}^3) - 17.0\text{ kg}]\,(9.80\text{ m/s}^2) = -8.89\text{ N} \approx \boxed{9\text{ N downward}}$$
$$F_{\text{empty}} = F_{\text{B}} - m_{\text{total}} g = \rho_{\text{water}} V_{\text{tank}} g - (m_{\text{tank}} + m_{\text{air}})g$$
$$= [(1025\text{ kg/m}^3)(0.0157\text{ m}^3) - 14.0\text{ kg}]\,(9.80\text{ m/s}^2) = 20.51\text{ N} \approx \boxed{21\text{ N upward}}$$

40. For the combination to just barely sink, the total weight of the wood and lead must be equal to the total buoyant force on the wood and the lead.

$$F_{\text{weight}} = F_{\text{buoyant}} \quad \rightarrow \quad m_{\text{wood}} g + m_{\text{Pb}} g = V_{\text{wood}} \rho_{\text{water}} g + V_{\text{Pb}} \rho_{\text{water}} g \quad \rightarrow$$

$$m_{\text{wood}} + m_{\text{Pb}} = \frac{m_{\text{wood}}}{\rho_{\text{wood}}}\rho_{\text{water}} + \frac{m_{\text{Pb}}}{\rho_{\text{Pb}}}\rho_{\text{water}} \quad \rightarrow \quad m_{\text{Pb}}\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{Pb}}}\right) = m_{\text{wood}}\left(\frac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right) \quad \rightarrow$$

$$m_{\text{Pb}} = m_{\text{wood}} \frac{\left(\dfrac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right)}{\left(1 - \dfrac{\rho_{\text{water}}}{\rho_{\text{Pb}}}\right)} = m_{\text{wood}} \frac{\left(\dfrac{1}{\text{SG}_{\text{wood}}} - 1\right)}{\left(1 - \dfrac{1}{\text{SG}_{\text{Pb}}}\right)} = (3.65\text{ kg}) \frac{\left(\dfrac{1}{0.50} - 1\right)}{\left(1 - \dfrac{1}{11.3}\right)} = \boxed{4.00\text{ kg}}$$

41. We apply the equation of continuity at constant density, Eq. 10–4b. The flow rate out of the duct must be equal to the flow rate into the room.

$$A_{\text{duct}}v_{\text{duct}} = \pi r^2 v_{\text{duct}} = \frac{V_{\text{room}}}{t_{\substack{\text{to fill}\\\text{room}}}} \rightarrow v_{\text{duct}} = \frac{V_{\text{room}}}{\pi r^2 t_{\substack{\text{to fill}\\\text{room}}}} = \frac{(8.2\text{ m})(5.0\text{ m})(3.5\text{ m})}{\pi(0.12\text{ m})^2(12\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right)} = \boxed{4.4\text{ m/s}}$$

42. Use Eq. 10–4b, the equation of continuity for an incompressible fluid, to compare blood flow in the aorta and in the major arteries.

$$(Av)_{\text{aorta}} = (Av)_{\text{arteries}} \rightarrow$$

$$v_{\text{arteries}} = \frac{A_{\text{aorta}}}{A_{\text{arteries}}} v_{\text{aorta}} = \frac{\pi(1.2\text{ cm})^2}{2.0\text{ cm}^2}(40\text{ cm/s}) = 90.5\text{ cm/s} \approx \boxed{0.9\text{ m/s}}$$

43. We may apply Torricelli's theorem, Eq. 10–6.

$$v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2(9.80\text{ m/s}^2)(4.7\text{ m})} = \boxed{9.6\text{ m/s}}$$

44. Bernoulli's equation is evaluated with $v_1 = v_2 = 0.$ Let point 1 be the initial point and point 2 be the final point.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow P_1 + \rho g y_1 = P_2 + \rho g y_2 \rightarrow$$

$$P_2 - P_1 = \rho g(y_1 - y_2) \rightarrow \Delta P = -\rho g \Delta y$$

But a change in the *y* coordinate is the opposite of the change in depth, which is what is represented in Eq. 10–3b. So our final result is $\Delta P = \rho_0 g \Delta h,$ Eq. 10–3b.

45. The pressure head can be interpreted as an initial height for the water, with a speed of 0 and at atmospheric pressure. Apply Bernoulli's equation to the faucet location and the pressure head location to find the speed of the water at the faucet, and then calculate the volume flow rate. Since the faucet is open, the pressure there will be atmospheric as well.

$$P_{\text{faucet}} + \tfrac{1}{2}\rho v_{\text{faucet}}^2 + \rho g y_{\text{faucet}} = P_{\text{head}} + \tfrac{1}{2}\rho v_{\text{head}}^2 + \rho g y_{\text{head}} \rightarrow$$

$$v_{\text{faucet}}^2 = \frac{2}{\rho}(P_{\text{head}} - P_{\text{faucet}}) + v_{\text{head}}^2 + 2g(y_{\text{head}} - y_{\text{faucet}}) = 2gy_{\text{head}} \rightarrow$$

$$v_{\text{faucet}} = \sqrt{2gy_{\text{head}}}$$

$$\text{Volume flow rate} = Av = \pi r^2\sqrt{2gy_{\text{head}}} = \pi\left[\tfrac{1}{2}(1.85\times10^{-2}\text{ m})\right]^2\sqrt{2(9.80\text{ m/s}^2)(12.0\text{ m})}$$

$$= \boxed{4.12\times10^{-3}\text{ m}^3/\text{s}}$$

46. The flow speed is the speed of the water in the input tube. The entire volume of the water in the tank is to be processed in 4.0 h. The volume of water passing through the input tube per unit time is the volume rate of flow, as expressed in the text in the paragraph following Eq. 10–4b.

$$\frac{V}{\Delta t} = Av \rightarrow v = \frac{V}{A\Delta t} = \frac{\ell wh}{\pi r^2 \Delta t} = \frac{(0.36\text{ m})(1.0\text{ m})(0.60\text{ m})}{\pi(0.015\text{ m})^2(3.0\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)} = 0.02829\text{ m/s} \approx \boxed{2.8\text{ cm/s}}$$

47. Apply Bernoulli's equation with point 1 being the water main and point 2 being the top of the spray. The velocity of the water will be zero at both points. The pressure at point 2 will be atmospheric pressure. Measure heights from the level of point 1.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow$$

$$P_1 - P_{\text{atm}} = \rho g y_2 = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(16\ \text{m}) = \boxed{1.6\times10^5\ \text{N/m}^2}$$

48. The volume flow rate of water from the hose, multiplied by the time of filling, must equal the volume of the pool. The volume flow rate is given in the text immediately following Eq. 10–4b.

$$(Av)_{\text{hose}} = \frac{V_{\text{pool}}}{t} \quad\rightarrow\quad t = \frac{V_{\text{pool}}}{A_{\text{hose}}v_{\text{hose}}} = \frac{\pi\left[\tfrac{1}{2}(6.1\ \text{m})\right]^2(1.4\ \text{m})}{\pi\left[\tfrac{1}{2}\left(\tfrac{5}{8}\text{in.}\right)\left(\dfrac{1\ \text{m}}{39.37\ \text{in.}}\right)\right]^2(0.40\ \text{m/s})} = 5.168\times10^5\ \text{s}$$

$$5.168\times10^5\ \text{s}\left(\frac{1\ \text{day}}{60\times60\times24\ \text{s}}\right) = \boxed{6.0\ \text{days}}$$

49. We assume that there is no appreciable height difference between the two sides of the roof. Then the net force on the roof due to the air is the difference in pressure on the two sides of the roof times the area of the roof. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \quad\rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 A_{\text{roof}} = \tfrac{1}{2}(1.29\ \text{kg/m}^3)\left[(180\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2(6.2\ \text{m})(12.4\ \text{m})$$

$$= \boxed{1.2\times10^5\ \text{N}}$$

50. Use the equation of continuity (Eq. 10–4b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 10–5) to relate the pressure conditions at the two locations. The two locations are at the same height. Express the pressures as atmospheric pressure plus gauge pressure. We use subscript 1 for the larger diameter and subscript 2 for the smaller diameter.

$$A_1 v_1 = A_2 v_2 \quad\rightarrow\quad v_2 = v_1\frac{A_1}{A_2} = v_1\frac{\pi r_1^2}{\pi r_2^2} = v_1\frac{r_1^2}{r_2^2}$$

$$P_0 + P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad\rightarrow$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 = P_2 + \tfrac{1}{2}\rho v_1^2\frac{r_1^4}{r_2^4} \quad\rightarrow\quad v_1 = \sqrt{\frac{2(P_1-P_2)}{\rho\left(\dfrac{r_1^4}{r_2^4}-1\right)}} \quad\rightarrow$$

$$A_1 v_1 = \pi r_1^2\sqrt{\frac{2(P_1-P_2)}{\rho\left(\dfrac{r_1^4}{r_2^4}-1\right)}} = \pi(3.0\times10^{-2}\ \text{m})^2\sqrt{\frac{2(33.5\times10^3\ \text{Pa} - 22.6\times10^3\ \text{Pa})}{(1.00\times10^3\ \text{kg/m}^3)\left(\dfrac{(3.0\times10^{-2}\ \text{m})^4}{(2.25\times10^{-2}\ \text{m})^4}-1\right)}}$$

$$= \boxed{9.0\times10^{-3}\ \text{m}^3/\text{s}}$$

51. The air pressure inside the hurricane can be estimated by using Bernoulli's equation, Eq. 10–5. Assume that the pressure outside the hurricane is atmospheric pressure, the speed of the wind outside the hurricane is 0, and the two pressure measurements are made at the same height.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \rightarrow$$

$$P_{\text{inside}} = P_{\text{outside}} - \tfrac{1}{2}\rho_{\text{air}} v_{\text{inside}}^2$$

$$= 1.013\times10^5 \text{ Pa} - \tfrac{1}{2}\left(1.29 \text{ kg/m}^3\right)\left[(300 \text{ km/h})\left(\frac{1000 \text{ m}}{\text{km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right]^2$$

$$= \boxed{9.7\times10^4 \text{ Pa}} \approx 0.96 \text{ atm}$$

52. The lift force would be the difference in pressure between the two wing surfaces times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation, Eq. 10–5. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1 and the top surface point 2.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2)$$

$$F_{\text{lift}} = (P_1 - P_2)(\text{Area of wing}) = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$$

$$= \tfrac{1}{2}(1.29 \text{ kg/m}^3)[(280 \text{ m/s})^2 - (150 \text{ m/s})^2](88 \text{ m}^2) = \boxed{3.2\times10^6 \text{ N}}$$

53. Use the equation of continuity (Eq. 10–4b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 10–5) to relate the conditions at the street to those at the top floor. Express the pressure as atmospheric pressure plus gauge pressure.

$$A_{\text{street}} v_{\text{street}} = A_{\text{top}} v_{\text{top}} \quad \rightarrow$$

$$v_{\text{top}} = v_{\text{street}}\frac{A_{\text{street}}}{A_{\text{top}}} = (0.78 \text{ m/s})\frac{\pi\left[\tfrac{1}{2}(5.0\times10^{-2} \text{ m})\right]^2}{\pi\left[\tfrac{1}{2}(2.8\times10^{-2} \text{ m})\right]^2} = 2.487 \text{ m/s} \approx \boxed{2.5 \text{ m/s}}$$

$$P_0 + P_{\substack{\text{gauge}\\ \text{street}}} + \tfrac{1}{2}\rho v_{\text{street}}^2 + \rho g y_{\text{street}} = P_0 + P_{\substack{\text{gauge}\\ \text{top}}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} \quad \rightarrow$$

$$P_{\substack{\text{gauge}\\ \text{top}}} = P_{\substack{\text{gauge}\\ \text{street}}} + \tfrac{1}{2}\rho\left(v_{\text{street}}^2 - v_{\text{top}}^2\right) + \rho g(y_{\text{street}} - y_{\text{top}})$$

$$= (3.8 \text{ atm})\left(\frac{1.013\times10^5 \text{ Pa}}{\text{atm}}\right) + \tfrac{1}{2}(1.00\times10^3 \text{ kg/m}^3)\left[(0.78 \text{ m/s})^2 - (2.487 \text{ m/s})^2\right]^2$$

$$+ (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(-16 \text{ m})$$

$$= 2.250\times10^5 \text{ Pa}\left(\frac{1 \text{ atm}}{1.013\times10^5 \text{ Pa}}\right) \approx \boxed{2.2 \text{ atm}}$$

54. Consider the volume of fluid in the pipe. At each end of the pipe there is a force toward the contained fluid, given by $F = PA.$ Since the area of the pipe is constant, we have $F_{\text{net}} = (P_1 - P_2)A.$ Then, since the power required is the force on the fluid times its velocity, and $AV = Q =$ volume rate of flow, we have power $= F_{\text{net}}v = (P_1 - P_2)Av = \boxed{(P_1 - P_2)Q}.$

55. Apply both Bernoulli's equation and the equation of continuity between the two openings of the tank. Note that the pressure at each opening will be atmospheric pressure.

$$A_2 v_2 = A_1 v_1 \quad \rightarrow \quad v_2 = v_1 \frac{A_1}{A_2}$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad v_1^2 - v_2^2 = 2g(y_2 - y_1) = 2gh$$

$$v_1^2 - \left(v_1 \frac{A_1}{A_2}\right) = 2gh \quad \rightarrow \quad v_1^2\left(1 - \frac{A_1^2}{A_2^2}\right) = 2gh \quad \rightarrow \quad \boxed{v_1 = \sqrt{\frac{2gh}{(1 - A_1^2/A_2^2)}}}$$

56. (*a*) Apply the equation of continuity and Bernoulli's equation at the same height to the wide and narrow portions of the tube.

$$A_2 v_2 = A_1 v_1 \quad \rightarrow \quad v_2 = v_1 \frac{A_1}{A_2}; \quad P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \quad \rightarrow \quad \frac{2(P_1 - P_2)}{\rho} = v_2^2 - v_1^2 \quad \rightarrow$$

$$\left(v_1 \frac{A_1}{A_2}\right)^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} \quad \rightarrow \quad v_1^2\left(\frac{A_1^2}{A_2^2} - \frac{A_2^2}{A_2^2}\right) = \frac{2(P_1 - P_2)}{\rho} \quad \rightarrow$$

$$v_1^2 = \frac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)} \quad \rightarrow \quad \boxed{v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}}$$

(*b*) $$v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$= \pi\left[\tfrac{1}{2}(0.010\text{ m})\right]^2 \sqrt{\frac{2(18\text{ mm-Hg})\left(\dfrac{133\text{ N/m}^2}{\text{mm-Hg}}\right)}{(1000\text{ kg/m}^3)\left(\pi^2\left[\tfrac{1}{2}(0.035\text{ m})\right]^4 - \pi^2\left[\tfrac{1}{2}(0.010\text{ m})\right]^4\right)}} = \boxed{0.18\text{ m/s}}$$

57. There is a forward force on the exiting water, so by Newton's third law there is an equal force pushing backward on the hose. To keep the hose stationary, you push forward on the hose, so the hose pushes backward on you. So the force on the exiting water is the same magnitude as the force on the person holding the hose. Use Newton's second law and the equation of continuity to find the force. Note that the 450 L/min flow rate is the volume of water being accelerated per unit time. Also, the flow rate is the product of the cross-sectional area of the moving fluid and the speed of the fluid, so $V/t = A_1 v_1 = A_2 v_2$.

$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_2 - v_1}{t} = \rho\left(\frac{V}{t}\right)(v_2 - v_1) = \rho\left(\frac{V}{t}\right)\left(\frac{A_2 v_2}{A_2} - \frac{A_1 v_1}{A_1}\right) = \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$$= \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{\pi r_2^2} - \frac{1}{\pi r_1^2}\right)$$

$$= \left(1.00\times10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{420\text{ L}}{\text{min}}\times\frac{1\text{ min}}{60\text{ s}}\times\frac{1\text{ m}^3}{1000\text{ L}}\right)^2 \frac{1}{\pi}\left(\frac{1}{\left[\tfrac{1}{2}(0.75\times10^{-2}\text{ m})\right]^2} - \frac{1}{\left[\tfrac{1}{2}(7.0\times10^{-2}\text{ m})\right]^2}\right)$$

$$= 1103\text{ N} \approx \boxed{1100\text{ N}}$$

58. Apply Eq. 10–8 for the viscosity force. Use the average radius to calculate the plate area.

$$F=\eta A\frac{\upsilon}{\ell} \quad\rightarrow\quad \eta=\frac{F\ell}{A\upsilon}=\frac{\left(\dfrac{\tau}{r_{\text{inner}}}\right)(r_{\text{outer}}-r_{\text{inner}})}{(2\pi r_{\text{avg}}h)(\omega r_{\text{inner}})}$$

$$=\frac{\left(\dfrac{0.024\text{ m}\cdot\text{N}}{0.0510\text{ m}}\right)(0.20\times10^{-2}\text{ m})}{2\pi(0.0520\text{ m})(0.120\text{ m})\left(57\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\text{ rad}}{\text{rev}}\times\dfrac{1\text{ min}}{60\text{ s}}\right)(0.0510\text{ m})}=\boxed{7.9\times10^{-2}\text{ Pa}\cdot\text{s}}$$

59. Use Poiseuille's equation (Eq. 10–9) to find the pressure difference.

$$Q=\frac{\pi R^4(P_2-P_1)}{8\eta L} \quad\rightarrow (P_2-P_1)=\frac{8Q\eta L}{\pi R^4}$$

$$(P_2-P_1)=\frac{8\left[6.2\times10^{-3}\dfrac{\text{L}}{\text{min}}\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{10^{-3}\text{ m}^3}{\text{L}}\right)\right](0.2\text{ Pa}\cdot\text{s})(0.102\text{ m})}{\pi\left[\frac{1}{2}(1.8\times10^{-3}\text{ m})\right]^4}=\boxed{8200\text{ Pa}}$$

60. From Poiseuille's equation, Eq. 10–9, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus $\dfrac{Q}{R^4}=\dfrac{V}{t}\dfrac{1}{R^4}$ is constant. If the volume of water used to water the garden is to be same in both cases, then tR^4 is constant.

$$t_1R_1^4=t_2R_2^4 \quad\rightarrow\quad t_2=t_1\left(\frac{R_1}{R_2}\right)^4=t_1\left(\frac{3/8}{5/8}\right)^4=0.13t_1$$

Thus the $\boxed{\text{time has been cut by 87\%}}$.

61. Use Poiseuille's equation, Eq. 10–9, to find the radius, and then double the radius to find the diameter.

$$Q=\frac{\pi R^4(P_2-P_1)}{8\eta\ell} \quad\rightarrow\quad d=2R=2\left[\frac{8\eta\ell Q}{\pi(P_2-P_1)}\right]^{1/4} \rightarrow$$

$$d=2\left[\frac{8(1.8\times10^{-5}\text{ Pa}\cdot\text{s})(15.5\text{ m})\left(\dfrac{(8.0\text{ m})(14.0\text{ m})(4.0\text{ m})}{900\text{ s}}\right)}{\pi(0.710\times10^{-3}\text{ atm})(1.013\times10^5\text{ Pa/atm})}\right]^{1/4}=\boxed{0.094\text{ m}}$$

62. Use Poiseuille's equation, Eq. 10–9, to find the pressure difference.

$$Q=\frac{\pi R^4(P_2-P_1)}{8\eta\ell} \quad\rightarrow$$

$$(P_2-P_1)=\frac{8Q\eta\ell}{\pi R^4}=\frac{8(650\text{ cm}^3/\text{s})(10^{-6}\text{ m}^3/\text{cm}^3)(0.20\text{ Pa}\cdot\text{s})(1600\text{ m})}{\pi(0.145\text{ m})^4}$$

$$=1198\text{ Pa}\approx\boxed{1200\text{ Pa}}$$

63. (*a*) We calculate the Reynolds number with the given formula.

$$Re = \frac{2\bar{v}r\rho}{\eta} = \frac{2(0.35\ \text{m/s})(0.80\times10^{-2}\ \text{m})(1.05\times10^{3}\ \text{kg/m}^3)}{4\times10^{-3}\ \text{Pa}\cdot\text{s}} = 1470$$

The flow is $\boxed{\text{laminar}}$ at this speed.

(*b*) Doubling the velocity doubles the Reynolds number, to 2940. The flow is now $\boxed{\text{turbulent}}$.

64. From Poiseuille's equation, Eq. 10–9, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus, Q/R^4 is constant.

$$\frac{Q_{\text{final}}}{R_{\text{final}}^4} = \frac{Q_{\text{initial}}}{R_{\text{initial}}^4} \rightarrow R_{\text{final}} = \left(\frac{Q_{\text{final}}}{Q_{\text{initial}}}\right)^{1/4} R_{\text{initial}} = (0.35)^{1/4} R_{\text{initial}} = 0.769 R_{\text{initial}}$$

The radius has been reduced by about $\boxed{23\%}$.

65. The pressure drop per cm can be found from Poiseuille's equation, Eq. 10–9, using a length of 1 cm. The volume flow rate is the area of the aorta times the speed of the moving blood.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta\ell} \rightarrow$$

$$\frac{(P_2 - P_1)}{\ell} = \frac{8\eta Q}{\pi R^4} = \frac{8\eta\pi R^2 v}{\pi R^4} = \frac{8\eta v}{R^2} = \frac{8(4\times10^{-3}\ \text{Pa}\cdot\text{s})(0.4\ \text{m/s})}{(1.2\times10^{-2}\ \text{m})^2} = 88.9\ \text{Pa/m} = \boxed{0.89\ \text{Pa/cm}}$$

66. The fluid pressure must be 78 torr higher than air pressure as it exits the needle so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 78 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 10–3c to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta_{\text{blood}}\ell} \rightarrow P_2 = P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4} = \rho_{\text{blood}} g \Delta h \rightarrow \Delta h = \frac{1}{\rho_{\text{blood}} g}\left(P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4}\right)$$

$$\Delta h = \frac{1}{\left(1050\ \frac{\text{kg}}{\text{m}^3}\right)(9.80\ \text{m/s}^2)}\left((78\ \text{mm-Hg})\left(\frac{133\ \text{N/m}^2}{1\ \text{mm-Hg}}\right) + \frac{8(4\times10^{-3}\ \text{Pa}\cdot\text{s})(0.025\ \text{m})\left(\frac{2.0\times10^{-6}\ \text{m}^3}{60\ \text{s}}\right)}{\pi(0.4\times10^{-3}\ \text{m})^4}\right) = 1.04\ \text{m} \approx \boxed{1.0\ \text{m}}$$

67. In Fig. 10–34, we have $\gamma = F/2\ell$. Use this to calculate the surface tension.

$$\gamma = \frac{F}{2\ell} = \frac{3.4\times10^{-3}\ \text{N}}{2(0.070\ \text{m})} = \boxed{2.4\times10^{-2}\ \text{N/m}}$$

68. As in Fig. 10–34, there are two surfaces being increased, so $\gamma = F/2\ell$. Use this to calculate the force.

$$\gamma = F/2\ell \quad \rightarrow \quad F = 2\gamma\ell = 2(0.025\text{ N/m})(0.215\text{ m}) = \boxed{1.1\times10^{-2}\text{ N}}$$

69. (*a*) We assume that the weight of the platinum ring is negligible. Then the surface tension is the force to lift the ring divided by the length of surface that is being pulled. Surface tension will act at both edges of the ring, as in Fig. 10–36 (b). Thus, $\boxed{\gamma = \dfrac{F}{2(2\pi r)} = \dfrac{F}{4\pi r}}$.

(*b*) $$\gamma = \frac{F}{4\pi r} = \frac{6.20\times10^{-3}\text{ N}}{4\pi(2.9\times10^{-2}\text{ m})} = \boxed{1.7\times10^{-2}\text{ N/m}}$$

70. From Example 10–15, we have $2\pi r\gamma\cos\theta = \frac{1}{6}mg$. The maximum mass will occur at $\theta = 0°$.

$$2\pi r\gamma\cos\theta = \tfrac{1}{6}mg \quad \rightarrow \quad m_{\text{max}} = \frac{12\pi r\gamma}{g} = \frac{12\pi(3.0\times10^{-5}\text{ m})(0.072\text{ N/m})}{9.80\text{ m/s}^2} = 8.3\times10^{-6}\text{ kg}$$

This is much less than the insect's mass, so $\boxed{\text{the insect will not remain on top of the water}}$.

71. As an estimate, we assume that the surface tension force acts vertically. We assume that the free-body diagram for the cylinder is similar to Fig. 10–36a. The weight must equal the total surface tension force. The needle is of length ℓ.

$$mg = 2F_{\text{T}} \quad \rightarrow \quad \rho_{\text{needle}}\pi\left(\tfrac{1}{2}d_{\text{needle}}\right)^2\ell g = 2\gamma\ell \quad \rightarrow$$

$$d_{\text{needle}} = \sqrt{\frac{8\gamma}{\rho_{\text{needle}}\pi g}} = \sqrt{\frac{8(0.072\text{ N/m})}{(7800\text{ kg/m}^3)\pi(9.80\text{ m/s}^2)}} = 1.55\times10^{-3}\text{ m} \approx \boxed{1.5\text{ mm}}$$

72. The difference in pressure from the heart to the calf is given by Eq. 10–3b.

$$\Delta P = \rho g\Delta h = (1.05\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(1\text{ m}) = 1.029\times10^4\text{ Pa} \approx \boxed{1\times10^4\text{ Pa}}$$

73. (*a*) The fluid in the needle is confined, so Pascal's principle may be applied.

$$P_{\text{plunger}} = P_{\text{needle}} \rightarrow \frac{F_{\text{plunger}}}{A_{\text{plunger}}} = \frac{F_{\text{needle}}}{A_{\text{needle}}} \rightarrow F_{\text{needle}} = F_{\text{plunger}}\frac{A_{\text{needle}}}{A_{\text{plunger}}} = F_{\text{plunger}}\frac{\pi r_{\text{needle}}^2}{\pi r_{\text{plunger}}^2}$$

$$= F_{\text{plunger}}\frac{r_{\text{needle}}^2}{r_{\text{plunger}}^2} = (3.2\text{ N})\frac{(0.10\times10^{-3}\text{ m})^2}{(0.65\times10^{-2}\text{ m})^2} = \boxed{7.6\times10^{-4}\text{ N}}$$

(*b*) $$F_{\text{plunger}} = P_{\text{plunger}}A_{\text{plunger}} = (75\text{ mm-Hg})\left(\frac{133\text{ N/m}^2}{1\text{ mm-Hg}}\right)\pi(0.65\times10^{-2}\text{ m})^2 = \boxed{1.3\text{ N}}$$

74. The pressures for parts (*a*) and (*b*) are gauge pressures, relative to atmospheric pressure. The pressure change due to depth in a fluid is given by Eq. 10–3b, $\Delta P = \rho g \Delta h$.

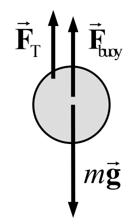
$$(a)\quad \Delta h = \frac{\Delta P}{\rho g} = \frac{(52\text{ mm-Hg})\left(\dfrac{133\text{ N/m}^2}{1\text{ mm-Hg}}\right)}{\left(1.00\dfrac{\text{g}}{\text{cm}^3}\times\dfrac{1\text{ kg}}{1000\text{ g}}\times\dfrac{10^6\text{ cm}^3}{1\text{ m}^3}\right)(9.80\text{ m/s}^2)} = \boxed{0.71\text{ m}}$$

$$(b)\quad \Delta h = \frac{\Delta P}{\rho g} = \frac{(680\text{ mm-H}_2\text{O})\left(\dfrac{9.81\text{ N/m}^2}{1\text{ mm-H}_2\text{O}}\right)}{\left(1.00\dfrac{\text{g}}{\text{cm}^3}\times\dfrac{1\text{ kg}}{1000\text{ g}}\times\dfrac{10^6\text{ cm}^3}{1\text{ m}^3}\right)(9.80\text{ m/s}^2)} = \boxed{0.68\text{ m}}$$

(*c*) For the fluid to just barely enter the vein, the fluid pressure must be the same as the blood pressure.

$$\Delta h = \frac{\Delta P}{\rho g} = \frac{(75\text{ mm-Hg})\left(\dfrac{133\text{ N/m}^2}{1\text{ mm-Hg}}\right)}{\left(1.00\dfrac{\text{g}}{\text{cm}^3}\times\dfrac{1\text{ kg}}{1000\text{ g}}\times\dfrac{10^6\text{ cm}^3}{1\text{ m}^3}\right)(9.80\text{ m/s}^2)} = \boxed{1.0\text{ m}}$$

75. The ball has three vertical forces on it: string tension, buoyant force, and gravity. See the free-body diagram for the ball. The net force must be 0.



$$F_\text{net} = F_\text{T} + F_\text{buoy} - mg = 0 \quad\rightarrow$$
$$F_\text{T} = mg - F_\text{buoy} = \tfrac{4}{3}\pi r^3\rho_\text{Cu}g - \tfrac{4}{3}\pi r^3\rho_\text{water}g = \tfrac{4}{3}\pi r^3 g(\rho_\text{Cu} - \rho_\text{water})$$
$$= \tfrac{4}{3}\pi(0.013\text{ m})^3(9.80\text{ m/s}^2)(8900\text{ kg/m}^3 - 1000\text{ kg/m}^3) = 0.7125\text{ N} \approx \boxed{0.71\text{ N}}$$

Since the water pushes up on the ball via the buoyant force, there is a downward force on the water due to the ball, equal in magnitude to the buoyant force. That mass equivalent of that force (indicated by $m_\text{B} = F_\text{B}/g$) will show up as an increase in the balance reading.

$$F_\text{B} = \tfrac{4}{3}\pi r^3\rho_\text{water}g \quad\rightarrow$$
$$m_\text{B} = \frac{F_\text{B}}{g} = \tfrac{4}{3}\pi r^3\rho_\text{water} = \tfrac{4}{3}\pi(0.013\text{ m})^3(1000\text{ kg/m}^3) = 9.203\times10^{-3}\text{ kg} = 9.203\text{ g}$$

$$\text{Balance reading} = 975.0\text{ g} + 9.2\text{ g} = \boxed{984.2\text{ g}}$$

76. The change in pressure with height is given by Eq. 10–3b.

$$\Delta P = \rho g\Delta h \quad\rightarrow\quad \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{(1.29\text{ kg/m}^3)(9.80\text{ m/s}^2)(380\text{ m})}{1.013\times10^5\text{ Pa}} = 0.047 \quad\rightarrow$$

$$\boxed{\Delta P = 0.047\text{ atm}}$$

77. (*a*) The input pressure is equal to the output pressure.

$$P_{\text{input}} = P_{\text{output}} \quad \rightarrow \quad F_{\text{input}}/A_{\text{input}} = F_{\text{output}}/A_{\text{output}} \quad \rightarrow$$

$$A_{\text{input}} = A_{\text{output}} \frac{F_{\text{input}}}{F_{\text{output}}} = \pi(9.0\times10^{-2}\text{ m})^2 \frac{380\text{ N}}{(960\text{ kg})(9.80\text{ m/s}^2)} = 1.028\times10^{-3}\text{ m}^2$$

$$\approx \boxed{1.0\times10^{-3}\text{ m}^2}$$

(*b*) The work is the force needed to lift the car (its weight) times the vertical distance lifted.

$$W = mgh = (960\text{ kg})(9.80\text{ m/s}^2)(0.42\text{ m}) = 3951\text{ J} \approx \boxed{4.0\times10^3\text{ J}}$$

(*c*) The work done by the input piston is equal to the work done in lifting the car.

$$W_{\text{input}} = W_{\text{output}} \quad \rightarrow \quad F_{\text{input}}d_{\text{input}} = F_{\text{output}}d_{\text{ouptut}} = mgh \quad \rightarrow$$

$$h = \frac{F_{\text{input}}d_{\text{input}}}{mg} = \frac{(380\text{ N})(0.13\text{ m})}{(960\text{ kg})(9.80\text{ m/s}^2)} = 5.251\times10^{-3}\text{ m} \approx \boxed{5.3\times10^{-3}\text{ m}}$$

(*d*) The number of strokes is the full distance divided by the distance per stroke.

$$h_{\text{full}} = Nh_{\text{stroke}} \quad \rightarrow \quad N = \frac{h_{\text{full}}}{h_{\text{stroke}}} = \frac{0.42\text{ m}}{5.251\times10^{-3}\text{ m}} = \boxed{80\text{ strokes}}$$

(*e*) The work input is the input force times the total distance moved by the input piston.

$$W_{\text{input}} = NF_{\text{input}}d_{\text{input}} \quad \rightarrow \quad 80(380\text{ N})(0.13\text{ m}) = 3952\text{ J} \approx \boxed{4.0\times10^3\text{ J}}$$

Since the work input is equal to the work output, energy is conserved.

78. The pressure change due to a change in height is given by Eq. 10–3b. That pressure is the excess force on the eardrum divided by the area of the eardrum.

$$\Delta P = \rho g \Delta h = \frac{F}{A} \quad \rightarrow$$

$$F = \rho g \Delta h A = (1.29\text{ kg/m}^3)(9.80\text{ m/s}^2)(1250\text{ m})(0.20\times10^{-4}\text{ m}^2) = 0.3161\text{ N} \approx \boxed{0.32\text{ N}}$$

79. The change in pressure with height is given by Eq. 10–3b.

$$\Delta P = \rho g \Delta h \quad \rightarrow \quad \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{(1.05\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(6\text{ m})}{1.013\times10^5\text{ Pa}} = 0.609 \quad \rightarrow$$

$$\boxed{\Delta P = 0.6\text{ atm}}$$

80. The pressure head can be interpreted as an initial height for the water, with a speed of 0 and at atmospheric pressure. Apply Bernoulli's equation to the faucet location and the pressure head location to find the speed of the water at the faucet. Since the faucet is open, the pressure there will be atmospheric as well.

$$P_{\text{faucet}} + \tfrac{1}{2}\rho v_{\text{faucet}}^2 + \rho g y_{\text{faucet}} = P_{\text{head}} + \tfrac{1}{2}\rho v_{\text{head}}^2 + \rho g y_{\text{head}} \quad \rightarrow$$

$$y_{\text{head}} = \frac{v_{\text{faucet}}^2}{2g} = \frac{(9.2\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{4.3\text{ m}}$$

81. The pressure difference due to the lungs is the pressure change in the column of water.

$$\Delta P = \rho g \Delta h \quad \rightarrow \quad \Delta h = \frac{\Delta P}{\rho g} = \frac{(75\text{ mm-Hg})\left(\dfrac{133\text{ N/m}^2}{1\text{ mm-Hg}}\right)}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 1.018\text{ m} \approx \boxed{1.0\text{ m}}$$

82. The force can be found by multiplying the pressure and the area of the pump cylinder.

$$F_\text{i} = P_\text{i}A = (2.10\times10^5\text{ N/m}^2)\pi(0.0125\text{ m})^2 = 1.0\times10^2\text{ N}$$
$$F_\text{f} = P_\text{f}A = (3.10\times10^5\text{ N/m}^2)\pi(0.0125\text{ m})^2 = 1.5\times10^2\text{ N}$$

The range of forces is $\boxed{100\text{ N} \le F \le 150\text{ N}}$.

83. The pressure would be the weight of the ice divided by the area covered by the ice. The volume of the ice is represented by V and its thickness by d. The volume is also the mass of the ice divided by the density of the ice.

$$P = \frac{F}{A} = \frac{mg}{V/d} = \frac{mgd}{V} = \frac{mgd}{m/\rho} = gd\rho = (9.80\text{ m/s}^2)(2000\text{ m})(917\text{ kg/m}^3) = 1.80\times10^7\text{ Pa}$$
$$\approx \boxed{2\times10^7\text{ Pa}}$$

84. The buoyant force, equal to the weight of mantle displaced, must be equal to the weight of the continent. Let h represent the full height of the continent and y represent the height of the continent above the surrounding rock.

$$W_\text{continent} = W_{\substack{\text{displaced}\\\text{mantle}}} \quad \rightarrow \quad Ah\rho_\text{continent}g = A(h-y)\rho_\text{mantle}g \quad \rightarrow$$
$$y = h\left(1 - \frac{\rho_\text{continent}}{\rho_\text{mantle}}\right) = (35\text{ km})\left(1 - \frac{2800\text{ kg/m}^3}{3300\text{ kg/m}^3}\right) = \boxed{5.3\text{ km}}$$

85. The "extra" buoyant force on the ship, due to the loaded fresh water, is the weight of "extra" displaced seawater, as indicated by the ship floating lower in the sea. This buoyant force is given by $F_\text{buoyant} = V_\text{displaced}\rho_{\substack{\text{sea}\\\text{water}}}g$. But this extra buoyant force is what holds up the fresh water, so that force must also be equal to the weight of the fresh water.

$$F_\text{buoyant} = V_\text{displaced}\rho_{\substack{\text{sea}\\\text{water}}}g = m_\text{fresh}g \;\rightarrow\; m_\text{fresh} = (2240\text{ m}^2)(8.25\text{ m})(1025\text{ kg/m}^3) = \boxed{1.89\times10^7\text{ kg}}$$

This can also be expressed as a volume.

$$V_\text{fresh} = \frac{m_\text{fresh}}{\rho_\text{fresh}} = \frac{1.89\times10^7\text{ kg}}{1.00\times10^3\text{ kg/m}^3} = \boxed{1.89\times10^4\text{ m}^3} = \boxed{1.89\times10^7\text{ L}}$$

86. The buoyant force must be equal to the weight of the water displaced by the full volume of the logs and must also be equal to the full weight of the raft plus the passengers. Let N represent the number of passengers.

Weight of water displaced by logs = Weight of people + Weight of logs

$$12(V_{\log}\rho_{\text{water}}g) = Nm_{\text{person}}g + 12(V_{\log}\rho_{\log}g) \quad \rightarrow$$

$$N = \frac{12V_{\log}(\rho_{\text{water}} - \rho_{\log})}{m_{\text{person}}} = \frac{12\pi r_{\log}^2 l_{\log}(\rho_{\text{water}} - \text{SG}_{\log}\rho_{\text{water}})}{m_{\text{person}}} = \frac{12\pi r_{\log}^2 l_{\log}\rho_{\text{water}}(1 - \text{SG}_{\log})}{m_{\text{person}}}$$

$$= \frac{12\pi(0.225\text{ m})^2(6.5\text{ m})(1000\text{ kg/m}^3)(1-0.60)}{68\text{ kg}} = 72.97$$

Thus $\boxed{72}$ people can stand on the raft without getting wet. When the 73rd person gets on, the raft will sink under the surface.

87. We assume that the air pressure is due to the weight of the atmosphere, with the area equal to the surface area of the Earth.

$$P = \frac{F}{A} \quad \rightarrow \quad F = PA = mg \quad \rightarrow$$

$$m = \frac{PA}{g} = \frac{4\pi R_{\text{Earth}}^2 P}{g} = \frac{4\pi(6.38\times10^6\text{ m})^2(1.013\times10^5\text{ N/m}^2)}{9.80\text{ m/s}^2} = \boxed{5.29\times10^{18}\text{ kg}}$$

88. The work done during each heartbeat is the force on the fluid times the distance that the fluid moves in the direction of the force. That can be converted to pressure times volume.

$$W = F\Delta\ell = PA\Delta\ell = PV \quad \rightarrow$$

$$\text{Power} = \frac{W}{t} = \frac{PV}{t} = \frac{(105\text{ mm-Hg})\left(\frac{133\text{ N/m}^2}{1\text{ mm-Hg}}\right)(70\times10^{-6}\text{ m}^3)}{\left(\frac{1}{70}\text{min}\right)\left(\frac{60\text{ s}}{\text{min}}\right)} = 1.14\text{ W} \approx \boxed{1\text{ W}}$$

89. (*a*) We assume that the water is launched at ground level. Since it also lands at ground level, the level range formula from Chapter 3 may be used.

$$R = \frac{v_0^2\sin 2\theta}{g} \rightarrow v_0 = \sqrt{\frac{Rg}{\sin 2\theta}} = \sqrt{\frac{(6.0\text{ m})(9.80\text{ m/s}^2)}{\sin 70^\circ}} = 7.910\text{ m/s} \approx \boxed{7.9\text{ m/s}}$$

(*b*) The volume rate of flow is the area of the flow times the speed of the flow. Multiply by 4 for the four sprinkler heads.

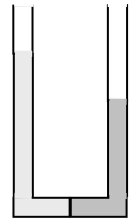
$$\text{Volume flow rate} = Av = 4\pi r^2 v = 4\pi(1.5\times10^{-3}\text{ m})^2(7.910\text{ m/s})$$

$$= 2.236\times10^{-4}\text{ m}^3\text{/s}\left(\frac{1\text{ L}}{1.0\times10^{-3}\text{ m}^3}\right) \approx \boxed{0.22\text{ L/s}}$$

(*c*) Use the equation of continuity to calculate the flow rate in the supply pipe.

$$(Av)_{\text{supply}} = (Av)_{\text{heads}} \rightarrow v_{\text{supply}} = \frac{(Av)_{\text{heads}}}{A_{\text{supply}}} = \frac{2.236\times10^{-4}\text{ m}^3\text{/s}}{\pi(0.95\times10^{-2}\text{ m})^2} = \boxed{0.79\text{ m/s}}$$

90. The pressure at the top of each liquid will be atmospheric pressure, and the pressure at the place where the two fluids meet must be the same if the fluid is to be stationary. In the diagram, the darker color represents the water and the lighter color represents the alcohol. Write the expression for the pressure at a depth for both liquids, starting at the top of each liquid with atmospheric pressure.



$$P_{\text{alcohol}} = P_0 + \rho_{\text{alcohol}} g \Delta h_{\text{alcohol}} = P_{\text{water}} = P_0 + \rho_{\text{water}} g \Delta h_{\text{water}} \quad \rightarrow$$
$$\rho_{\text{alcohol}} \Delta h_{\text{alcohol}} = \rho_{\text{water}} \Delta h_{\text{water}} \quad \rightarrow$$
$$\Delta h_{\text{water}} = \Delta h_{\text{alcohol}} \frac{\rho_{\text{alcohol}}}{\rho_{\text{water}}} = 16.0 \text{ cm}(0.790) = \boxed{12.6 \text{ cm}}$$

91. The force is the pressure times the surface area.

$$F = PA = (120 \text{ mm-Hg})\left(\frac{133 \text{ N/m}^2}{1 \text{ mm-Hg}}\right)(82\times10^{-4} \text{ m}^2) = 130.9 \text{ N} \approx \boxed{130 \text{ N}}$$

92. The upward force due to air pressure on the bottom of the wing must be equal to the weight of the airplane plus the downward force due to air pressure on the top of the wing. Bernoulli's equation can be used to relate the forces due to air pressure. We assume that there is no appreciable height difference between the top and the bottom of the wing.

$$P_{\text{top}} A + mg = P_{\text{bottom}} A \quad \rightarrow \quad (P_{\text{bottom}} - P_{\text{top}}) = \frac{mg}{A}$$
$$P_0 + P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} = P_0 + P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}}$$
$$v_{\text{top}}^2 = \frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2 \quad \rightarrow \quad v_{\text{top}} = \sqrt{\frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2} = \sqrt{\frac{2mg}{\rho A} + v_{\text{bottom}}^2}$$
$$v_{\text{top}} = \sqrt{\frac{2(1.7\times10^6 \text{ kg})(9.80 \text{ m/s}^2)}{(1.29 \text{ kg/m}^3)(1200 \text{ m}^2)} + (95 \text{ m/s})^2} = 174.8 \text{ m/s} \approx \boxed{170 \text{ m/s}}$$

93. Since we are ignoring viscosity, this problem can be considered using Bernoulli's equation. First, we find the speed of the water as it exits the nozzle, using conservation of mechanical energy.

$$\text{PE}_{\substack{\text{at}\\\text{top}}} = \text{KE}_{\substack{\text{at}\\\text{nozzle}}} \quad \rightarrow \quad mgh_{\substack{\text{nozzle}\\\text{to top}}} = \tfrac{1}{2}mv_{\text{nozzle}}^2 \quad \rightarrow$$
$$v_{\text{nozzle}} = \sqrt{2gh_{\substack{\text{nozzle}\\\text{to top}}}} = \sqrt{2(9.80 \text{ m/s}^2)(0.12 \text{ m})} = 1.534 \text{ m/s}$$

Now use the equation of continuity, Eq. 10–4b, to find the speed of the water at the pump.

$$A_{\text{nozzle}} v_{\text{nozzle}} = A_{\substack{\text{pump}\\\text{pipe}}} v_{\text{pump}} \quad \rightarrow \quad v_{\text{pump}} = v_{\text{nozzle}} \frac{A_{\text{nozzle}}}{A_{\substack{\text{pump}\\\text{pipe}}}} = v_{\text{nozzle}} \frac{\pi\left(\tfrac{1}{2} d_{\text{nozzle}}\right)^2}{\pi\left(\tfrac{1}{2} d_{\substack{\text{pump}\\\text{pipe}}}\right)^2} = v_{\text{nozzle}} \left(\frac{d_{\text{nozzle}}}{d_{\substack{\text{pump}\\\text{pipe}}}}\right)^2$$
$$= (1.534 \text{ m/s})\left(\frac{0.60 \text{ cm}}{1.2 \text{ cm}}\right) = 0.3835 \text{ m/s}$$

Finally, use Bernoulli's equation (Eq. 10–5) to relate the pressure at the nozzle (atmospheric pressure) to the pressure at the pump. Note that the "gauge" pressure at the nozzle is 0.

$$P_0 + P_{\substack{\text{gauge}\\\text{nozzle}}} + \tfrac{1}{2}\rho v_{\text{nozzle}}^2 + \rho g y_{\text{nozzle}} = P_0 + P_{\substack{\text{gauge}\\\text{pump}}} + \tfrac{1}{2}\rho v_{\text{pump}}^2 + \rho g y_{\text{pump}} \quad \rightarrow$$

$$P_{\substack{\text{gauge}\\\text{pump}}} = \tfrac{1}{2}\rho(v_{\text{nozzle}}^2 - v_{\text{pump}}^2) + \rho g(y_{\text{nozzle}} - y_{\text{pump}}) = \rho\left[\tfrac{1}{2}(v_{\text{nozzle}}^2 - v_{\text{pump}}^2) + g(y_{\text{nozzle}} - y_{\text{pump}})\right]$$

$$= (1.00\times10^3\ \text{kg/m}^3)\left\{\tfrac{1}{2}\left[(1.534\ \text{m/s})^2 - (0.3835\ \text{m/s})^2\right] + (9.80\ \text{m/s}^2)(1.1\ \text{m})\right\}$$

$$= 1.188\times10^4\ \text{N/m}^2 \approx \boxed{1.2\times10^4\ \text{N/m}^2}$$

94. We assume that there is no appreciable height difference to be considered between the two sides of the window. Then the net force on the window due to the air is the difference in pressure on the two sides of the window times the area of the window. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \quad \rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \quad \rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 A_{\text{roof}} = \tfrac{1}{2}(1.29\ \text{kg/m}^3)\left[(180\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2 (6.0\ \text{m}^2) = \boxed{9700\ \text{N}}$$

95. From Poiseuille's equation, the viscosity can be found from the volume flow rate, the geometry of the tube, and the pressure difference. The pressure difference over the length of the tube is the same as the pressure difference due to the height of the reservoir, assuming that the open end of the needle is at atmospheric pressure.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta\ell};\quad P_2 - P_1 = \rho_{\text{blood}} g h \rightarrow \eta = \frac{\pi R^4 (P_2 - P_1)}{8Q\ell} = \frac{\pi R^4 \rho_{\text{blood}} g h}{8Q\ell}$$

$$\eta = \frac{\pi(0.20\times10^{-3}\ \text{m})^4 (1.05\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.40\ \text{m})}{8\left[4.1\ \dfrac{\text{cm}^3}{\text{min}} \times \dfrac{1\ \text{min}}{60\ \text{s}} \times \dfrac{10^{-6}\ \text{m}^3}{\text{cm}^3}\right](3.8\times10^{-2}\ \text{m})} = \boxed{3.5\times10^{-3}\ \text{Pa}\cdot\text{s}}$$

96. We assume that the water is launched from the same level at which it lands. Then the level range formula, derived in Example 3–10, applies. That formula is $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$. If the range has increased by a factor of 4, then the initial speed has increased by a factor of 2. The equation of continuity is then applied to determine the change in the hose opening. The water will have the same volume rate of flow whether the opening is large or small.

$$(Av)_{\substack{\text{fully}\\\text{open}}} = (Av)_{\substack{\text{partly}\\\text{open}}} \quad \rightarrow \quad A_{\substack{\text{partly}\\\text{open}}} = A_{\substack{\text{fully}\\\text{open}}} \frac{v_{\substack{\text{fully}\\\text{open}}}}{v_{\substack{\text{partly}\\\text{open}}}} = A_{\substack{\text{fully}\\\text{open}}}\left(\frac{1}{2}\right)$$

Thus, $\boxed{1/2}$ of the hose opening was blocked.

97. The buoyant force on the wood must be equal to the combined weight of the wood and copper.

$$(m_{\text{wood}} + m_{\text{Cu}})g = V_{\text{wood}}\rho_{\text{water}}g = \frac{m_{\text{wood}}}{\rho_{\text{wood}}}\rho_{\text{water}}g \quad \rightarrow \quad m_{\text{wood}} + m_{\text{Cu}} = \frac{m_{\text{wood}}}{\rho_{\text{wood}}}\rho_{\text{water}} \quad \rightarrow$$

$$m_{\text{Cu}} = m_{\text{wood}}\left(\frac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1\right) = (0.40\text{ kg})\left(\frac{1000\text{ kg/m}^3}{600\text{ kg/m}^3} - 1\right) = \boxed{0.27\text{ kg}}$$

98. (*a*) We assume that the tube in the pail is about 4.0 cm below the surface of the liquid in the pail so that the pressure at that two tube ends is approximately the same. Apply Bernoulli's equation to the two ends of the tube.

$$P_{\text{sink}} + \tfrac{1}{2}\rho v_{\text{sink}}^2 + \rho g y_{\text{sink}} = P_{\text{pail}} + \tfrac{1}{2}\rho v_{\text{pail}}^2 + \rho g y_{\text{pail}} \quad \rightarrow$$

$$v_{\text{pail}} = \sqrt{2g(y_{\text{sink}} - y_{\text{pail}})} = \sqrt{2(9.80\text{ m/s}^2)(0.40\text{ m})} = \boxed{2.8\text{ m/s}}$$

(*b*) The volume flow rate (at the pail end of the tube) times the time must equal the volume of water in the sink.

$$(Av)_{\text{pail}}t = V_{\text{sink}} \quad \rightarrow \quad t = \frac{V_{\text{sink}}}{(Av)_{\text{pail}}} = \frac{(0.38\text{ m}^2)(4.0\times10^{-2}\text{ m})}{\pi(1.15\times10^{-2}\text{ m})^2(2.8\text{ m/s})} = \boxed{13\text{ s}}$$

99. From Poiseuille's equation, the volume flow rate Q is proportional to R^4 if all other factors are the same. Thus, Q/R^4 is constant. Also, if the diameter is reduced by 25%, then so is the radius.

$$\frac{Q_{\text{final}}}{R_{\text{final}}^4} = \frac{Q_{\text{initial}}}{R_{\text{initial}}^4} \quad \rightarrow \quad \frac{Q_{\text{final}}}{Q_{\text{initial}}} = \frac{R_{\text{final}}^4}{R_{\text{initial}}^4} = (0.75)^4 = 0.32$$

$\boxed{\text{The flow rate is 32\% of the original value.}}$

Solutions to Search and Learn Problems

1. When the block is submerged in the water, the water exerts an upward buoyant force on the block equal to the weight of the water displaced. By Newton's third law, the block then exerts an equal force down on the water. Since the two objects are placed symmetrically about the pivot, they will be balanced when the forces on the two sides of the board are equal.

$$(5.0\text{ kg})g = (4.0\text{ kg})g + (0.5\text{ kg})g + \rho_{\text{water}}V_{\text{displaced}}g$$

$$V_{\text{displaced}} = \frac{(5.0\text{ kg})g - (4.0\text{ kg})g - (0.5\text{ kg})g}{\rho_{\text{water}}g} = \frac{0.5\text{ kg}}{1000\text{ kg/m}^3} = 5\times10^{-4}\text{ m}^3\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)$$

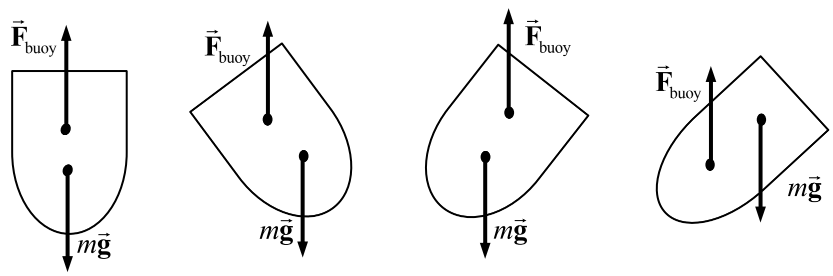
$$= 500\text{ cm}^3$$

The block has a volume $V = (10\text{ cm})^3 = 1000\text{ cm}^3$, so $\boxed{\text{half of the cube}}$ will be submerged. The density of the cube does not affect the solution. Therefore, the aluminum and lead cube would be submerged by the same amount.

2. (*a*) The buoyant force on the object is equal to the weight of the fluid displaced. The force of gravity of the fluid can be considered to act at the center of gravity of the fluid (see Section 7–8). If the object were removed from the fluid and that space re-filled with an equal volume of fluid, then

that fluid would be in equilibrium. Since there are only two forces on that volume of fluid, gravity and the buoyant force, they must be equal in magnitude and act at the same point. Otherwise, they would be a "couple" (see Fig. 9–5), exert a nonzero torque, and cause rotation of the fluid. Since the fluid does not rotate, we may conclude that the buoyant force acts at the center of gravity.

(*b*) From the diagrams below, if the center of buoyancy (the point where the buoyancy force acts) is above the center of gravity (the point where gravity acts) of the entire ship, then when the ship tilts, the net torque about the center of mass will tend to reduce the tilt. If the center of buoyancy is below the center of gravity of the entire ship, then when the ship tilts, the net torque about the center of mass will tend to increase the tilt. Stability is achieved when the center of buoyancy is $\boxed{\text{above the center of gravity}}$.



No torque (Stable) | CW torque (Stabilizing) | CCW torque (Stabilizing) | CW torque (Unstabilizing)

3. (*a*) The mass of water in the tube is the density of the water times the volume of the tube.

$$m = \rho V = \rho\pi r^2 h = (1.00\times10^3\ \text{kg/m}^3)\pi(0.30\times10^{-2}\ \text{m})^2(12\ \text{m}) = 0.3393\ \text{kg} \approx \boxed{0.34\ \text{kg}}$$

(*b*) The net force exerted on the lid is the gauge pressure of the water times the area of the lid. The gauge pressure is found from Eq. 10–3a.

$$F = P_{\text{gauge}}A = (\rho g h)(\pi R^2) = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(12\ \text{m})\pi(0.21\ \text{m})^2 = \boxed{1.6\times10^4\ \text{N}}$$

4. (*a*) Let object 1 be the object with the greater volume, so $V_1 > V_2$. The apparent weight of each object is the difference between its actual weight and the buoyant force. Since both objects are fully submerged, the buoyant force is equal to the product of the density of the water, their volumes, and the acceleration of gravity. We set the apparent weights equal and solve for the actual weight of object 1. We use the symbol W for the actual weight.

$$W_1 - \rho_{\text{water}}V_1 g = W_2 - \rho_{\text{water}}V_2 g \quad \rightarrow \quad W_1 = W_2 + \rho_{\text{water}}(V_1 - V_2)g$$

$\boxed{\text{The larger object has the greater weight.}}$

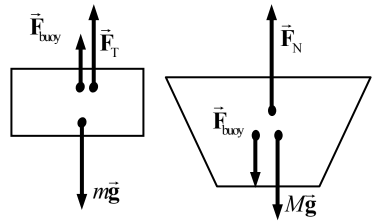
(*b*) Since $V_1 > V_2$ and their apparent weights are equal, the ratio of the apparent weight of object 1 to its volume will be less than the ratio of the apparent weight of object 2 to its volume.

$$\frac{W_1 - \rho_{\text{water}}V_1 g}{V_1} < \frac{W_2 - \rho_{\text{water}}V_2 g}{V_2} \rightarrow \frac{m_1 g}{V_1} - \rho_{\text{water}}g < \frac{m_2 g}{V_2} - \rho_{\text{water}}g \rightarrow$$

$$\rho_1 - \rho_{\text{water}} < \rho_2 - \rho_{\text{water}} \rightarrow \boxed{\rho_1 < \rho_2}$$

$\boxed{\text{The smaller object has the greater density.}}$

5. Draw free-body diagrams for the brick (mass m) and for the tub of water (mass M). Note that the water exerts an upward buoyant force on the brick, so by Newton's third law the brick exerts an equal downward buoyant force on the water. The normal force on the container of water is the scale reading. Then using Newton's second law in equilibrium for the water, solve for the normal force on the water. The densities are found in Table 10–1.



$$F_{\text{B}} = \rho_{\text{water}} V_{\text{brick}} g = \rho_{\text{water}} \left(\frac{m}{\rho_{\text{brick}}} \right) g = \left(\frac{\rho_{\text{water}}}{\rho_{\text{brick}}} \right) mg$$

$$\sum \vec{\mathbf{F}} = 0 = F_{\text{N}} - F_{\text{B}} - Mg \rightarrow F_{\text{N}} = F_{\text{B}} + Mg = \left(\frac{\rho_{\text{water}}}{\rho_{\text{brick}}} \right) mg + Mg$$

$$F_{\text{N}} = \left(\frac{1.000 \times 10^3 \text{ kg/m}^3}{2.3 \times 10^3 \text{ kg/m}^3} \right) 50 \text{ N} + 100 \text{ N} = 121.7 \text{ N} \approx \boxed{1.2 \times 10^2 \text{ N}}$$

6.

Assumptions in Bernoulli's equation	Modifications if assumptions were not made
Steady flow	If the flow rate can vary with time, then each of the terms in Bernoulli's equation could also vary with time. Additional terms would be needed to account for the energy needed to change the flow rates.
Laminar flow	Without laminar flow, turbulence and eddy currents could exist. These would create energy losses due to heating that would need to be accounted for in the equation.
Incompressible fluid	Work is done on the fluid as it compresses, and the fluid does work as it expands. This energy would need to be accounted for in the equation.
Nonviscous fluid	Greater pressure differences would be needed to overcome energy lost to viscous forces. Pressure and velocity would also depend upon distance from pipe walls.

7. From Section 9–5, Eq. 9–7 gives the change in volume due to pressure change as $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta P}{B}$, where B is the bulk modulus of the water, given in Table 9–1. The pressure increase with depth for a fluid of constant density is given by $\Delta P = \rho g \Delta h$, where Δh is the depth of descent. If the density change is small, then we can use the initial value of the density to calculate the pressure change, so $\Delta P \approx \rho_0 g \Delta h$. Finally, consider a constant mass of water. That constant mass will relate the volume and density at the two locations by $M = \rho V = \rho_0 V_0$. Combine these relationships and solve for the density deep in the sea, ρ.

$$\rho V = \rho_0 V_0 \quad \rightarrow \quad \rho = \frac{\rho_0 V_0}{V} = \frac{\rho_0 V_0}{V_0 + \Delta V} = \frac{\rho_0 V_0}{V_0 + \left(-V_0 \dfrac{\Delta P}{B} \right)} = \frac{\rho_0}{1 - \dfrac{\rho_0 g h}{B}}$$

$$\rho = \frac{1025 \text{ kg/m}^3}{1 - \dfrac{(1025 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.4 \times 10^3 \text{ m})}{2.0 \times 10^9 \text{ N/m}^2}} = 1054 \text{ kg/m}^3 \approx \boxed{1.05 \times 10^3 \text{ kg/m}^3}$$

$$\frac{\rho}{\rho_0} = \frac{1054}{1025} = 1.028$$

The density at the 5.4-km depth is about $\boxed{\text{3\% larger}}$ than the density at the surface.

11

Oscillations and Waves

Responses to Questions

$\boxed{1.}$ The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.

2. Since the real spring has mass, the mass that is moving is greater than the mass at the end of the spring. Since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, a larger mass means a smaller frequency. Thus the true frequency will be smaller than the massless spring approximation. And since the true frequency is smaller, the true period will be larger than the massless spring approximation. About one-third of the mass of the spring contributes to the total mass value.

3. The maximum speed is given by $\upsilon_{\max} = A\sqrt{k/m}$. Various combinations of changing A, k, and/or m can result in a doubling of the maximum speed. For example, if k and m are kept constant, then doubling the amplitude will double the maximum speed. Or if A and k are kept constant, then reducing the mass to one-fourth of its original value will double the maximum speed. Note that changing either k or m will also change the frequency of the oscillator, since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. So doubling the frequency (no matter how it is done) will also double the maximum speed.

4. The period of a pendulum clock is inversely proportional to the square root of g, by Eq. 11–11a, $T = 2\pi\sqrt{\ell/g}$. When taken to high altitude, g will decrease (by a small amount), which means that the period will increase. If the period is too long, then the clock is running slow, so will lose time.

$\boxed{5.}$ The tire swing is a good approximation of a simple pendulum. Pull the tire back a short distance and release it so that it oscillates as a pendulum in simple harmonic motion with a small amplitude. Measure the period of the oscillations and calculate the length of the pendulum from the expression $T = 2\pi\sqrt{\ell/g} \;\rightarrow\; \ell = \frac{gT^2}{4\pi^2}$. The length ℓ is the distance from the center of the tire to the branch. The height of the branch is ℓ plus the height of the center of the tire above the ground.

6. The displacement and velocity vectors are in the same direction while the oscillator is moving away from its equilibrium position. The displacement and acceleration vectors are never in the same direction.

7. The two masses reach the equilibrium point simultaneously. The period of oscillation is independent of amplitude and will be the same for both systems.

8. When walking at a normal pace, the period of a walking step is about 1 second. The faster you walk, the shorter the period. The shorter your legs, the shorter the period. If you approximate your leg as a pendulum of length 1 m, then the period would be $T = 2\pi\sqrt{\ell/g} = 2$ seconds.

9. When you rise to a standing position, you raise your center of mass and effectively shorten the length of the swing. The period of the swing will decrease, and the frequency will increase.

10. To make the water slosh, you must shake the water (and the pan) at the natural frequency for water waves in the pan. The water then is in resonance, or in a standing wave pattern, and the amplitude of oscillation gets large. That natural frequency is determined in part by the size of the pan—smaller pans will slosh at higher frequencies, corresponding to shorter wavelengths for the standing waves. The period of the shaking must be the same as the time it takes a water wave to make a "round trip" in the pan.

11. The frequency of a simple periodic wave is equal to the frequency of its source. The wave is created by the source moving the wave medium that is in contact with the source. If you have one end of a taut string in your hand, and you move your hand with a frequency of 2 Hz, then the end of the string in your hand will be moving at 2 Hz, because it is in contact with your hand. Then those parts of the medium that you are moving exert forces on adjacent parts of the medium and cause them to oscillate. Since those two portions of the medium stay in contact with each other, they also must be moving with the same frequency. That can be repeated all along the medium, so the entire wave throughout the medium has the same frequency as the source.

12. The speed of the transverse wave is the speed at which the wave disturbance moves along the cord. For a uniform cord, that speed is constant and depends on the tension in the cord and the mass density of the cord. The speed of a tiny piece of the cord is how fast the piece of cord moves perpendicularly to the cord as the disturbance passes by. That speed is not constant—if a sinusoidal wave is traveling on the cord, the speed of each piece of the cord will be given by the speed relationship of a simple harmonic oscillator (Eq. 11–9), which depends on the amplitude of the wave, the frequency of the wave, and the specific time of observation.

13. (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
 (*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

14. From Eq. 11–14b, the speed of waves in a gas is given by $v = \sqrt{B/\rho}$. A decrease in the density due to a temperature increase therefore leads to a higher speed of sound. We expect the speed of sound to increase as temperature increases.

15. For a rope with a fixed end, the reflected pulse is inverted relative to the incoming pulse. For a rope with a free end, the reflected pulse is not inverted. See Fig. 11–33 for an illustration. For the fixed end, the rope puts a force on the connecting point as the pulse reaches the connecting point. The connecting point puts an equal and opposite force on the rope. This force is what generates the inverted reflected

pulse. The point of connection is a node—a point of no motion. For a free end, the incoming pulse "whips" the end of the rope in a direction transverse to the wave motion, stretching it upward. As that whipped end is pulled back toward the equilibrium position, this whipping motion generates a wave much in the same way that the pulse was originally created and thus creates a wave that is not reflected.

16. Although both longitudinal and transverse waves can travel through solids, only longitudinal waves can travel through liquids. Since longitudinal waves, but no transverse waves, are detected on the Earth diametrically opposite the location of an earthquake, there must be some liquid as part of the Earth's interior.

17. The speed of a longitudinal wave is given in general by $\upsilon = \sqrt{\text{elastic force factor/inertia factor}}$. Even though the density of solids is 1000 to 10,000 times greater than that of air, the elastic force factor (bulk modulus) of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids and accounts for the speed of sound in most solids being greater than in air.

18. (*a*) Similar to the discussion in Section 11–9 for spherical waves, as a circular wave expands, the circumference of the wave increases. For the energy in the wave to be conserved, as the circumference increases, the intensity has to decrease. The intensity of the wave is proportional to the square of the amplitude.
    (*b*) The water waves will decrease in amplitude due to dissipation of energy from viscosity in the water (dissipative or frictional energy loss).

$\boxed{19.}$ Assuming the two waves are in the same medium, then they will both have the same speed. Since $\upsilon = f\lambda$, the wave with the smaller wavelength will have twice the frequency of the other wave. From Eq. 11–18, the intensity of a wave is proportional to the square of the frequency of the wave. Thus, the wave with the shorter wavelength will transmit four times as much energy as the other wave.

20. The frequency must stay the same because the media is continuous—the end of one section of cord is physically tied to the other section of cord. If the end of the first section of cord is vibrating up and down with a given frequency, then since it is attached to the other section of cord, the other section must vibrate at the same frequency. If the two pieces of cord did not move at the same frequency, then they would not stay connected, and the waves would not pass from one section to another.

21. Assuming that there are no dissipative processes, then yes, the energy is conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity and therefore kinetic energy.

$\boxed{22.}$ Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

23. From Eq. 11–13, the speed of waves on the string is $\upsilon = \sqrt{F_T/\mu}$. Equation 11–9b can be used to find the fundamental frequency of oscillation for a string with both ends fixed, $f_1 = \dfrac{\upsilon}{2\ell}$. Combining these two relationships gives $f_1 = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$. By wrapping the string with wire, the mass per unit length (μ) of the string can be greatly increased without changing the length or the tension of the string. This gives the string a low fundamental frequency.

24. The energy of a wave is not localized at one point, because the wave is not localized at one point, so referring to the energy "at a node" being zero is not a meaningful statement. Due to the interference of the waves, the total energy of the medium particles at the nodes is zero, but the energy of the medium is not zero at points of the medium that are not nodes. In fact, the antinodes have more energy than they would have if only one of the two waves were present.

25. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large-amplitude oscillations, even when the generating oscillations from the hand are small.

26. This description of waves works well initially for both descriptions, but the waves continue after the initial motion. When the center is struck, a wave does move from the center to the rim, but then it reflects from the rim back to the center. Likewise, when the rim is struck, a wave does move from the rim to the center, but the wave does not "stop" at the center. Once reaching the center, it then spreads out again to the rim. The amplitude of the waves also changes as the waves travel. As the radius increases, the amplitude decreases, and as the radius decreases, the amplitude increases.

27. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave compared with the size of the obstacle. A hill is much larger than the wavelength of FM waves, so there will be a "shadow" region behind the hill. However, the hill is not large compared with the wavelength of AM signals, so the AM radio waves will bend around the hill.

Responses to MisConceptual Questions

1. (*e*) At $x = \pm A$, the velocity is zero, but at these points the acceleration is a maximum. At $x = 0$, the acceleration is zero, but the velocity is a maximum. For all other points, both the velocity and acceleration are nonzero. Thus there are no points where the acceleration and velocity are simultaneously zero.

2. (*a*, *c*, *d*) At the turning points in the oscillation $(x = \pm A)$, the velocity is zero and the acceleration is a maximum, so (*a*) is true. At the center of the oscillation $(x = 0)$, the acceleration is zero and the velocity is a maximum, so (*c*) is true. Since the velocity is only zero at the turning points where the acceleration is a maximum, there is no point where both the velocity and acceleration are zero, so (*b*) is not true. At all other points besides $x = \pm A$ and $x = 0$, both the acceleration and velocity are nonzero values, so (*d*) is also true.

3. (*c*) Students may believe that the period is proportional to the mass and therefore think that doubling the mass will double the period. However, the period is proportional to the square root of the mass, as seen in Eq. 11–6a. Therefore, the mass must be quadrupled (to 4*M*) for the period to double.

4. (*b*) A common misconception is that the amplitude of oscillation affects the frequency. Eq. 11–6b shows that the frequency can be increased by increasing *k* or decreasing *m*. The frequency does not depend upon the amplitude.

5. (*a*) The small angle approximation is valid only in units of radians because the angle in radians is equal to the ratio of the arc length to the radius. At small angles the arc length can be approximated as a straight line, being the opposite leg of a right triangle with hypotenuse equal

to the radius of the circle. This ratio is equal to the sine of the angle, so for small angles the angle in radians is equal to the sine of the angle.

6. (*e*) A common misconception is that the starting angle, or amplitude of oscillation, affects the period of a pendulum. Eq. 11–11a shows that the period of a small-amplitude pendulum is determined by the length of the string and the acceleration of gravity, not the amplitude. Both oscillations will then have the same period.

7. (*c*) Students may erroneously believe that the mass of the child will affect the period of oscillation. However, Eq. 11–11a shows that the period is determined by the length of the swing cords and the acceleration of gravity. It does not depend upon the weight of the child. Since the swings are identical, they should oscillate with the same period.

8. (*a*) To speed up the pendulum, the period of the oscillation must be decreased. Equation 11–11a shows that the period is proportional to the square root of the length, so shortening the string will decrease the period. The period does not depend upon the mass of the bob, so changing the mass will not affect the period.

9. (*e*) Students frequently have trouble distinguishing between the motion of a point on a cord and the motion of a wave on the cord. As a wave travels down the cord, a point on the cord will move vertically between the lowest point of the wave and the highest point on the wave. The wave and the point have the same amplitude. The point on the cord completes one up and down oscillation as each wavelength passes that point. Therefore, the motion of the point on the cord has the same frequency as the wave. The speed of the wave on the string is determined by the wavelength and frequency. It is constant in time. The point on the cord moves perpendicular to the wave with a speed that varies with time. The maximum speed of the point is proportional to the wave amplitude and the wave frequency. Changing the amplitude will change the maximum speed of the point on the cord, but it does not change the wave speed. The wave speed and string speed therefore are not equal.

10. (*a*) A common misconception is that the waves are objects that can collide. Waves obey the superposition principle such that at any point on the rope the total displacement is the sum of the displacements from each wave. The waves pass through each other unaffected.

11. (*d*) Equation 11–13 shows that the wave speed on a cord is related to the tension in the cord and the mass per unit length of the cord. The wave speed does not depend upon the amplitude, frequency, or wavelength. Stretching the cord increases the tension and decreases the mass per unit length, both of which increase the speed of the wave on the cord.

12. (*d*) The point on the string does not move horizontally, so answers (*a*) and (*b*) cannot be correct. The string has zero velocity only at the turning points (top and bottom), so (*e*) cannot be correct. Examining the graph shows that as the wave moves to the right the crest is approaching point B, so the string at B is traveling upward at this instant.

13. (*d*) Students frequently confuse the medium (an object) with the wave motion and answer that the waves will collide and bounce off of each other. The waves obey the superposition principle such that at any point in the lake the amplitude of the wave is the sum of the individual amplitudes of each wave. This produces the various patterns when they overlap. The waves, however, will pass through each other and continue in their same pattern after they pass.

14. (*c*) The speed of the wave along the Slinky depends upon the mass of the Slinky and the tension caused by stretching it. Since this has not changed, the wave speed remains constant. The wave

speed can also be written as the product of the wavelength and frequency. Therefore, as the frequency is increased, the wavelength must decrease.

15. (*a*) A common misconception is that a wave transports matter as well as energy. However, as shown by a transverse wave on a horizontal string, the wave transports the disturbance down the string, but each part of the string stays at its initial horizontal position.

Solutions to Problems

1. The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A.$ So the total distance $= 4A = 4(0.21\text{ m}) = \boxed{0.84\text{ m}}$.

2. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(66\text{ kg})(9.80\text{ m/s}^2)}{5.0\times10^{-3}\text{ m}} = 1.294\times10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.294\times10^5\text{ N/m}}{1766\text{ kg}}} = 1.362\text{ Hz} \approx \boxed{1.4\text{ Hz}}$$

3. The spring constant is the ratio of external applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{210\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.61\text{ m}} = \frac{135\text{ N}}{0.24\text{ m}} = 562.5\text{ N/m} \approx \boxed{560\text{ N/m}}$$

4. The period is 2.0 seconds, and the mass is 32 kg. The spring constant is calculated from Eq. 11–6a.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad\rightarrow\quad T^2 = 4\pi^2\frac{m}{k} \quad\rightarrow\quad k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{32\text{ kg}}{(2.0\text{ s})^2} = 315.8\text{ N/m} \approx \boxed{320\text{ N/m}}$$

5. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(2.4\text{ kg})(9.80\text{ m/s}^2)}{0.036\text{ m}} = 653\text{ N/m} \approx \boxed{650\text{ N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A = \boxed{2.1\text{ cm}}$.

The frequency of oscillation is found from the oscillating mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{653\text{ N/m}}{2.4\text{ kg}}} = 2.625\text{ Hz} \approx \boxed{2.6\text{ Hz}}$$

$\boxed{6.}$ The relationship between frequency, mass, and spring constant is Eq. 11–6b, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad\rightarrow\quad k = 4\pi^2 f^2 m = 4\pi^2(4.0\text{ Hz})^2(2.2\times10^{-4}\text{ kg}) = 0.1390\text{ N/m} \approx \boxed{0.14\text{ N/m}}$$

(*b*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.1390\text{ N/m}}{4.4\times10^{-4}\text{ kg}}} = 2.828\text{ Hz} \approx \boxed{2.8\text{ Hz}}$$

7. The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \to \quad \frac{k}{4\pi^2} = mf^2 = \text{constant} \quad \to \quad m_1 f_1^2 = m_2 f_1^2 \quad \to$$

$$(m\ \text{kg})(0.83\ \text{Hz})^2 = (m\ \text{kg} + 0.78\ \text{kg})(0.60\ \text{Hz})^2 \quad \to \quad m = \frac{(0.78\ \text{kg})(0.60\ \text{Hz})^2}{(0.83\ \text{Hz})^2 - (0.60\ \text{Hz})^2} = \boxed{0.85\ \text{kg}}$$

8. We assume that downward is the positive direction of motion. For this motion, we have $k = 305$ N/m, $A = 0.280$ m, $m = 0.235$ kg, and $\omega = \sqrt{k/m} = \sqrt{(305\ \text{N/m})/0.235\ \text{kg}} = 36.026$ rad/s.

(*a*) Since the mass has a zero displacement and a positive velocity at $t = 0$, the equation is a sine function.

$$\boxed{y(t) = (0.280\ \text{m})\sin[(36.0\ \text{rad/s})t]}$$

(*b*) The period of oscillation is given by $T = \frac{2\pi}{\omega} = \frac{2\pi}{36.026\ \text{rad/s}} = 0.17441$ s. The spring will have its maximum extension at times given by the following:

$$t_{\max} = \frac{T}{4} + nT = \boxed{4.36\times10^{-2}\ \text{s} + n(0.174\ \text{s}),\ n = 0, 1, 2, \cdots}$$

The spring will have its minimum extension at times given by the following:

$$t_{\min} = \frac{3T}{4} + nT = \boxed{1.31\times10^{-1}\ \text{s} + n(0.174\ \text{s}),\ n = 0, 1, 2, \cdots}$$

9. (*a*) For A, the amplitude is $A_{\text{A}} = \boxed{2.5\ \text{m}}$. For B, the amplitude is $A_{\text{B}} = \boxed{3.5\ \text{m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_{\text{A}} = \boxed{0.25\ \text{Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_{\text{B}} = \boxed{0.50\ \text{Hz}}$.

(*c*) For A, the period is $T_{\text{A}} = \boxed{4.0\ \text{s}}$. For B, the period is $T_{\text{B}} = \boxed{2.0\ \text{s}}$.

10. (*a*) We find the effective spring constant from the mass and the frequency of oscillation, using Eq. 11–6b.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \to$$

$$k = 4\pi^2 mf^2 = 4\pi^2(0.052\ \text{kg})(3.0\ \text{Hz})^2 = 18.476\ \text{N/m} \approx \boxed{18\ \text{N/m}}$$

(*b*) Since the objects are the same size and shape, we anticipate that the spring constant is the same.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{18.476\ \text{N/m}}{0.28\ \text{kg}}} = 1.293\ \text{Hz} \approx \boxed{1.3\ \text{Hz}}$$

11. If the energy of the SHO is half potential and half kinetic, then the potential energy is half the total energy. The total energy is the potential energy when the displacement has the value of the amplitude.

$$E_{\text{pot}} = \tfrac{1}{2}E_{\text{tot}} \quad \to \quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad \to \quad \boxed{x = \pm\frac{1}{\sqrt{2}}A \approx \pm 0.707A}$$

12. When the object is at rest, the magnitude of the spring force is equal to the force of gravity. This determines the spring constant. The period can then be found.

$$\sum F_{\text{vertical}} = kx_0 - mg \quad \rightarrow \quad k = mg/x_0$$
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/x_0)}} = 2\pi\sqrt{\frac{x_0}{g}} = 2\pi\sqrt{\frac{0.14\text{ m}}{9.80\text{ m/s}^2}} = \boxed{0.75\text{ s}}$$

13. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(1.65\text{ kg})(9.80\text{ m/s}^2)}{0.215\text{ m}} = 75.209\text{ N/m}$$

After being stretched farther and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position, independent of the amplitude.

$$\tfrac{1}{4}T = \tfrac{1}{4}2\pi\sqrt{m/k} = \frac{\pi}{2}\sqrt{\frac{1.65\text{ kg}}{75.209\text{ N/m}}} = \boxed{0.233\text{ s}}$$

14. The general form of the motion is $x = A\cos\omega t = 0.650\cos 8.40t$.

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.650\text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 8.40\text{ s}^{-1} \quad \rightarrow \quad f = \dfrac{8.40\text{ s}^{-1}}{2\pi} = 1.337\text{ Hz} \approx \boxed{1.34\text{ Hz}}$

(*c*) The total energy is given by the kinetic energy at the maximum speed.

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(1.15\text{ kg})[(8.40\text{ s}^{-1})(0.650\text{ m})]^2 = 17.142\text{ J} \approx \boxed{17.1\text{ J}}$$

(*d*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(1.15\text{ kg})(8.40\text{ s}^{-1})^2(0.360\text{ m})^2 = 5.258\text{ J} \approx \boxed{5.26\text{ J}}$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 17.142\text{ J} - 5.258\text{ J} = 11.884\text{ J} \approx \boxed{11.9\text{ J}}$$

15. (*a*) At equilibrium, the velocity is its maximum, as given in Eq. 11–7.

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A = \omega A = 2\pi fA = 2\pi(2.2\text{ Hz})(0.15\text{ m}) = 2.073\text{ m/s} \approx \boxed{2.1\text{ m/s}}$$

(*b*) From Eq. 11–5b, we find the speed at any position.

$$v = v_{\text{max}}\sqrt{1 - \frac{x^2}{A^2}} = (2.073\text{ m/s})\sqrt{1 - \frac{(0.10\text{ m})^2}{(0.15\text{ m})^2}} = 1.545\text{ m/s} \approx \boxed{1.5\text{ m/s}}$$

(*c*) $E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}(0.25\text{ kg})(2.073\text{ m/s})^2 = 0.5372\text{ J} \approx \boxed{0.54\text{ J}}$

(*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function.

$$x = (0.15\text{ m})\cos(2\pi(2.2\text{ Hz})t) \quad \rightarrow \quad \boxed{x = (0.15\text{ m})\cos(4.4\pi t)}$$

16. The spring constant is found from the ratio of applied force to displacement.

$$k=\frac{F}{x}=\frac{91.0\text{ N}}{0.175\text{ m}}=520\text{ N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_{\text{i}}=E_{\text{f}}\quad\to\quad\tfrac{1}{2}kx_{\max}^2=\tfrac{1}{2}mv_{\max}^2\quad\to\quad v_{\max}=x_{\max}\sqrt{\frac{k}{m}}=(0.175\text{ m})\sqrt{\frac{520\text{ N/m}}{0.160\text{ kg}}}=\boxed{9.98\text{ m/s}}$$

17. To compare the total energies, we can compare the maximum potential energies. Since the frequencies and the masses are the same, the spring constants are the same.

$$\frac{E_{\substack{\text{high}\\\text{energy}}}}{E_{\substack{\text{low}\\\text{energy}}}}=\frac{\tfrac{1}{2}kA_{\substack{\text{high}\\\text{energy}}}^2}{\tfrac{1}{2}kA_{\substack{\text{low}\\\text{energy}}}^2}=\frac{A_{\substack{\text{high}\\\text{energy}}}^2}{A_{\substack{\text{low}\\\text{energy}}}^2}=3\quad\to\quad\boxed{\frac{A_{\substack{\text{high}\\\text{energy}}}}{A_{\substack{\text{low}\\\text{energy}}}}=\sqrt{3}}$$

18. (*a*) The spring constant can be found from the mass and the frequency of oscillation.

$$\omega=\sqrt{\frac{k}{m}}=2\pi f\quad\to\quad k=4\pi^2f^2m=4\pi^2(2.5\text{ Hz})^2(0.24\text{ kg})=59.22\text{ N/m}\approx\boxed{59\text{ N/m}}$$

(*b*) The energy can be found from the maximum potential energy.

$$E=\tfrac{1}{2}kA^2=\tfrac{1}{2}(59.22\text{ N/m})(0.045\text{ m})^2=5.996\times10^{-2}\text{ J}\approx\boxed{0.060\text{ J}}$$

19. (*a*) The work done to compress a spring is stored as potential energy.

$$W=\tfrac{1}{2}kx^2\quad\to\quad k=\frac{2W}{x^2}=\frac{2(3.6\text{ J})}{(0.13\text{ m})^2}=426.0\text{ N/m}\approx\boxed{430\text{ N/m}}$$

(*b*) The distance that the spring was compressed becomes the amplitude of its motion. The maximum acceleration occurs at the maximum displacement and is given by $a_{\max}=\frac{k}{m}A.$ Solve this for the mass.

$$a_{\max}=\frac{k}{m}A\quad\to\quad m=\frac{k}{a_{\max}}A=\left(\frac{426\text{ N/m}}{12\text{ m/s}^2}\right)(0.13\text{ m})=4.615\text{ kg}\approx\boxed{4.6\text{ kg}}$$

$\boxed{20.}$ (*a*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}}=\tfrac{1}{2}mv^2+\tfrac{1}{2}kx^2=\tfrac{1}{2}kA^2\quad\to$$

$$A=\sqrt{\frac{m}{k}v^2+x^2}=\sqrt{\frac{2.7\text{ kg}}{310\text{ N/m}}(0.55\text{ m/s})^2+(0.020\text{ m})^2}=5.509\times10^{-2}\text{ m}\approx\boxed{5.5\times10^{-2}\text{ m}}$$

(*b*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}}=\tfrac{1}{2}mv^2+\tfrac{1}{2}kx^2=\tfrac{1}{2}kA^2=\tfrac{1}{2}mv_{\max}^2\quad\to$$

$$v_{\max}=A\sqrt{\frac{k}{m}}=(5.509\times10^{-2}\text{ m})\sqrt{\frac{310\text{ N/m}}{2.7\text{ kg}}}=0.5903\text{ m/s}\approx\boxed{0.59\text{ m/s}}$$

21. (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.885\text{ kg}/(184\text{ N/m})} = 0.4358\text{ s} \approx \boxed{0.436\text{ s}}$$

$$f = 1/T = 1/(0.4358\text{ s}) = \boxed{2.29\text{ Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$\upsilon_{\max} = A\sqrt{k/m} \quad \rightarrow$$

$$A = \upsilon_{\max}\sqrt{m/k} = (2.26\text{ m/s})\sqrt{0.885\text{ kg}/(184\text{ N/m})} = 0.1567\text{ m} \approx \boxed{0.157\text{ m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude.

$$a_{\max} = Ak/m = (0.1567\text{ m})(184\text{ N/m})/(0.885\text{ kg}) = \boxed{32.6\text{ m/s}^2}$$

(*d*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}m\upsilon_{\max}^2 = \tfrac{1}{2}(0.885\text{ kg})(2.26\text{ m/s})^2 = 2.2601\text{ J} \approx \boxed{2.26\text{ J}}$$

(*e*) Use conservation of mechanical energy for the oscillator, noting that we found total energy in part (*d*) and that another expression for the total energy is $E_{\text{total}} = \tfrac{1}{2}kA^2$.

$$E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}m\upsilon^2 = \tfrac{1}{2}kA^2 \quad \rightarrow \quad \tfrac{1}{2}k(0.40\ A)^2 + \text{KE} = E_{\text{total}} = \tfrac{1}{2}kA^2 \quad \rightarrow$$

$$\text{KE} = \tfrac{1}{2}kA^2 - \tfrac{1}{2}k(0.40A)^2 = \tfrac{1}{2}kA^2(1-0.40^2) = E_{\text{total}}(0.84) = (2.2601\text{ J})(0.84) = \boxed{1.90\text{ J}}$$

$\boxed{22.}$ We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\text{before}} = p_{\text{after}} \quad \rightarrow \quad m\upsilon_0 = (m+M)\upsilon_{\max} \quad \rightarrow \quad \upsilon_{\max} = \frac{m}{m+M}\upsilon_0$$

$$\tfrac{1}{2}(m+M)\upsilon_{\max}^2 = \tfrac{1}{2}kA^2 \quad \rightarrow \quad \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}\upsilon_0\right)^2 = \tfrac{1}{2}kA^2 \quad \rightarrow$$

$$\upsilon_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\text{ m})}{(7.870\times10^{-3}\text{ kg})}\sqrt{(162.7\text{ N/m})(7.870\times10^{-3}\text{ kg}+4.148\text{ kg})}$$

$$= \boxed{312.6\text{ m/s}}$$

23. The period of the jumper's motion is $T = \dfrac{43.0\text{ s}}{7\text{ cycles}} = 6.143\text{ s}.$ The spring constant can then be found from the period and the jumper's mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(65.0\text{ kg})}{(6.143\text{ s})^2} = 68.004\text{ N/m} \approx \boxed{68.0\text{ N/m}}$$

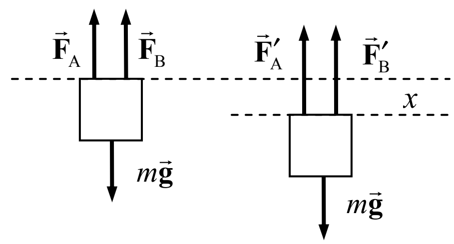
The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{68.004\text{ N/m}} = 9.37\text{ m}$$

Thus the unstretched bungee cord must be $25.0\text{ m} - 9.37\text{ m} = \boxed{15.6\text{ m}}$.

24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives the following:

$$\sum F_y = F_A + F_B - mg = 0 \quad \rightarrow \quad F_A + F_B = mg$$



Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's second law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + (F_A + F_B - mg) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{effective}}}{m}} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

25. (*a*) The object starts at the maximum displacement in the positive direction so will be represented by a cosine function. The mass, period, and amplitude are given.

$$A = 0.16 \text{ m}; \; \omega = \frac{2\pi}{T} = \frac{2\pi}{0.45 \text{ s}} = 13.96 \text{ rad/s} \quad \rightarrow \quad \boxed{y = (0.16 \text{ m}) \cos (14t)}$$

(*b*) The time to reach the equilibrium is one-quarter of a period, so $\frac{1}{4}(0.45 \text{ s}) = \boxed{0.11 \text{ s}}$.

(*c*) The maximum speed is given by Eq. 11–7a.

$$v_{\max} = 2\pi fA = \omega A = (13.96 \text{ rad/s})(0.16 \text{ m}) = \boxed{2.2 \text{ m/s}}$$

(*d*) The maximum acceleration is found from Eq. 11–10.

$$a_{\max} = \frac{kA}{m} = \omega^2 A = (13.96 \text{ rad/s})^2 (0.16 \text{ m}) = \boxed{31 \text{ m/s}^2}$$

The maximum acceleration occurs at the endpoints of the motion and is first attained at the $\boxed{\text{release point}}$.

26. Each object will pass through the origin at the times when the argument of its sine function is a multiple of π.

A: $4.0t_A = n_A\pi \quad \rightarrow \quad t_A = \frac{1}{4}n_A\pi, n_A = 1,2,3,\ldots$ so $t_A = \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, \pi, \frac{5}{4}\pi, \frac{3}{2}\pi, \frac{7}{4}\pi, 2\pi, \frac{9}{4}\pi, \ldots$

B: $3.0t_B = n_B\pi \quad \rightarrow \quad t_B = \frac{1}{3}n_B\pi, n_B = 1,2,3,\ldots$ so $t_B = \frac{1}{3}\pi, \frac{2}{3}\pi, \pi, \frac{4}{3}\pi, \frac{5}{3}\pi, 2\pi, \frac{7}{3}\pi, \frac{8}{3}\pi, 3\pi, \ldots$

Thus we see the first three times are $\boxed{\pi \text{ s}, 2\pi \text{ s, and } 3\pi \text{ s}}$ or $\boxed{3.1 \text{ s}, 6.3 \text{ s, and } 9.4 \text{ s}}$.

27. The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{\ell/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{\ell/g_{\text{Mars}}}}{2\pi\sqrt{\ell/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (1.85 \text{ s})\sqrt{\frac{1}{0.37}} = \boxed{3.0 \text{ s}}$$

28. The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$. Solve for the length using a period of 2.0 seconds.

$$T = 2\pi\sqrt{\ell/g} \quad \rightarrow \quad \ell = \frac{T^2 g}{4\pi^2} = \frac{(2.0\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2} = \boxed{0.99\ \text{m}}$$

29. (*a*) The period is given by $T = 50\ \text{s}/28\ \text{cycles} = \boxed{1.8\ \text{s}}$.

(*b*) The frequency is given by $f = 28\ \text{cycles}/50\ \text{s} = \boxed{0.56\ \text{Hz}}$.

30. The period of a pendulum is given by Eq. 11–11a, $T = 2\pi\sqrt{\ell/g}$.

(*a*) $T = 2\pi\sqrt{\ell/g} = 2\pi\sqrt{\dfrac{0.47\ \text{m}}{9.80\ \text{m/s}^2}} = \boxed{1.4\ \text{s}}$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob and no restoring force to cause oscillations. Thus there will be no period—the pendulum will not oscillate, so no period can be defined.

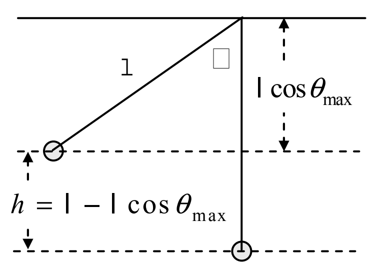
31. There are $(24\ \text{h})(60\ \text{min/h})(60\ \text{s/min}) = 86{,}400\ \text{s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), so the clock should make 43,200 cycles per day. After one day, the clock in question is 21 seconds slow, which means that it has made 10.5 fewer cycles than are required for precise timekeeping. Thus the clock is only making 43,189.5 cycles in a day. Accordingly, the period of the clock must be decreased by a factor of $\dfrac{43{,}189.5}{43{,}200}$.

$$T_{\text{new}} = \frac{43{,}189.5}{43{,}200} T_{\text{old}} \quad \rightarrow \quad 2\pi\sqrt{\ell_{\text{new}}/g} = \left(\frac{43{,}189.5}{43{,}200}\right) 2\pi\sqrt{\ell_{\text{old}}/g} \quad \rightarrow$$

$$\ell_{\text{new}} = \left(\frac{43{,}189.5}{43{,}200}\right)^2 \ell_{\text{old}} = \left(\frac{43{,}189.5}{43{,}200}\right)^2 (0.9930\ \text{m}) = 0.9925\ \text{m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.5 mm}}$.

32. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad \text{KE}_{\text{top}} + \text{PE}_{\text{top}} = \text{KE}_{\text{bottom}} + \text{PE}_{\text{bottom}} \quad \rightarrow$$

$$0 + mgh = \tfrac{1}{2} m v_{\text{max}}^2 \quad \rightarrow \quad v_{\text{max}} = \sqrt{2gh} = \boxed{\sqrt{2g\ell(1-\cos\theta_{\text{max}})}}$$

33. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta = \theta_0 \cos\omega t = \theta_0 \cos\dfrac{2\pi t}{T}$. Solve for the time for the position to decrease to half of the amplitude.

$$\theta_{1/2} = \tfrac{1}{2}\theta_0 = \theta_0 \cos\frac{2\pi t_{1/2}}{T} \quad \rightarrow \quad \frac{2\pi t_{1/2}}{T} = \cos^{-1}\tfrac{1}{2} = \frac{\pi}{3} \quad \rightarrow \quad t_{1/2} = \tfrac{1}{6}T$$

It takes $\frac{1}{6}T$ for the position to change from $+10°$ to $+5°$. It takes $\frac{1}{4}T$ for the position to change from $+10°$ to 0º. Thus it takes $\frac{1}{4}T-\frac{1}{6}T=\frac{1}{12}T$ for the position to change from $+5°$ to 0º. Due to the symmetric nature of the cosine function, it will also take $\frac{1}{12}T$ for the position to change from 0º to $-5°$, so going from $+5°$ to $-5°$ takes $\frac{1}{6}T$. The second half of the cycle will be identical to the first, so the total time spent between $+5°$ and $-5°$ is $\frac{1}{3}T$. So the pendulum spends $\boxed{\text{one-third}}$ of its time between $+5°$ and $-5°$.

34. The equation of motion for an object in SHM that has the maximum displacement at $t=0$ is given by $x=A\cos(2\pi ft)$. We let x be the position of the pendulum bob in terms of arc length measured relative to the lowest point of the pendulum. For a pendulum of length ℓ, the arc length is given by $x=\ell\theta$, so $x_{\max}=A=\ell\theta_{\max}$, where θ must be measured in radians. The equation for the pendulum's angular displacement is then found from the equation for the arc length.

$$\ell\theta=\ell\theta_{\max}\cos(2\pi ft) \quad\rightarrow\quad \theta=\theta_{\max}\cos(2\pi ft)$$

If both sides of the equation are multiplied by $180°/\pi$ rad, then the angles can be measured in degrees. Thus the angular displacement of the pendulum can be written as below. Note that the argument of the cosine function is still in radians.

$$\theta=\theta_{\max}\cos(2\pi ft)=12°\cos(5.0\pi t)=(\pi/15)\cos(5.0\pi t)$$

(*a*) $\theta(t=0.25\text{ s})=(\pi/15)\cos(5.0\pi(0.25))=\boxed{-0.15\text{ rad}}$

(*b*) $\theta(t=1.60\text{ s})=(\pi/15)\cos(5.0\pi(1.60))=\boxed{\pi/15\text{ rad}}$ (The time is exactly 4 periods.)

(*c*) $\theta(t=500\text{ s})=(\pi/15)\cos(5.0\pi(500))=\boxed{\pi/15\text{ rad}}$ (The time is exactly 1250 periods.)

35. The wave speed is given by $\upsilon=\lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$\upsilon=\lambda f=\lambda/T=(7.0\text{ m})/(3.0\text{ s})=\boxed{2.3\text{ m/s}}$

36. The distance between wave crests is the wavelength of the wave.

$\lambda=\upsilon/f=(343\text{ m/s})/282\text{ Hz}=\boxed{1.22\text{ m}}$

37. The elastic and bulk moduli are taken from Table 9–1 in Chapter 9. The densities are taken from Table 10–1 in Chapter 10.

(*a*) For water: $\upsilon=\sqrt{B/\rho}=\sqrt{\dfrac{2.0\times10^9\text{ N/m}^2}{1.00\times10^3\text{ kg/m}^3}}=\boxed{1400\text{ m/s}}$

(*b*) For granite: $\upsilon=\sqrt{E/\rho}=\sqrt{\dfrac{45\times10^9\text{ N/m}^2}{2.7\times10^3\text{ kg/m}^3}}=\boxed{4100\text{ m/s}}$

(*c*) For steel: $\upsilon=\sqrt{E/\rho}=\sqrt{\dfrac{200\times10^9\text{ N/m}^2}{7.8\times10^3\text{ kg/m}^3}}=\boxed{5100\text{ m/s}}$

38. To find the wavelength, use $\lambda = \upsilon/f$.

AM:

$$\lambda_1 = \frac{\upsilon}{f_1} = \frac{3.00\times10^8\ \text{m/s}}{550\times10^3\ \text{Hz}} = 545\ \text{m} \quad \lambda_2 = \frac{\upsilon}{f_2} = \frac{3.00\times10^8\ \text{m/s}}{1600\times10^3\ \text{Hz}} = 188\ \text{m} \quad \boxed{\text{AM: 190 m to 550 m}}$$

FM: $$\lambda_1 = \frac{\upsilon}{f_1} = \frac{3.00\times10^8\ \text{m/s}}{88\times10^6\ \text{Hz}} = 3.41\ \text{m} \quad \lambda_2 = \frac{\upsilon}{f_2} = \frac{3.00\times10^8\ \text{m/s}}{108\times10^6\ \text{Hz}} = 2.78\ \text{m} \quad \boxed{\text{FM: 2.8 m to 3.4 m}}$$

39. (*a*) Both waves travel the same distance, so $\Delta x = \upsilon_1 t_1 = \upsilon_2 t_2$. We let the smaller speed be υ_1 and the larger speed be υ_2. The slower wave will take longer to arrive, so t_1 is greater than t_2.

$$t_1 = t_2 + 1.5\ \text{min} = t_2 + 90\ \text{s} \quad \rightarrow \quad \upsilon_1(t_2 + 102\ \text{s}) = \upsilon_2 t_2 \quad \rightarrow$$

$$t_2 = \frac{\upsilon_1}{\upsilon_2 - \upsilon_1}(90\ \text{s}) = \frac{5.5\ \text{km/s}}{8.5\ \text{km/s} - 5.5\ \text{km/s}}(90\ \text{s}) = 165\ \text{s}$$

$$\Delta x = \upsilon_2 t_2 = (8.5\ \text{km/s})(165\ \text{s}) = \boxed{1400\ \text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius 1400 km from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

40. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, Eq. 11–13 gives $\upsilon = \sqrt{F_T/\mu}$.

$$\upsilon = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T}{m/L}} \quad \rightarrow \quad \Delta t = \frac{\Delta x}{\sqrt{\frac{F_T}{m/L}}} = \frac{8.0\ \text{m}}{\sqrt{\frac{120\ \text{N}}{(0.65\ \text{kg})/(8.0\ \text{m})}}} = \boxed{0.21\ \text{s}}$$

41. For a cord under tension, we have from Eq. 11–13 that $\upsilon = \sqrt{F_T/\mu}$. The speed is also the distance traveled by the wave divided by the elapsed time, $\upsilon = \frac{\Delta x}{\Delta t}$. The distance traveled is the length of the cord.

$$\upsilon = \sqrt{\frac{F_T}{\mu}} = \frac{\Delta x}{\Delta t} \quad \rightarrow \quad F_T = \mu\upsilon^2 = \mu\frac{\ell^2}{(\Delta t)^2} = \frac{m}{\ell}\frac{\ell^2}{(\Delta t)^2} = \frac{m\ell}{(\Delta t)^2} = \frac{(0.40\ \text{kg})(8.7\ \text{m})}{(0.85\ \text{s})^2} = \boxed{4.8\ \text{N}}$$

$\boxed{42.}$ The speed of the water wave is given by Eq. 11–14b, $\upsilon = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 9–1, and ρ is the density of seawater, from Table 10–1. The wave travels twice the depth of the ocean during the elapsed time.

$$\upsilon = \frac{2\ell}{t} \quad \rightarrow \quad \ell = \frac{\upsilon t}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{2.4\ \text{s}}{2}\sqrt{\frac{2.0\times10^9\ \text{N/m}^2}{1.025\times10^3\ \text{kg/m}^3}} = \boxed{1700\ \text{m}}$$

43. The speed of the waves on the cord can be found from Eq. 11–13, $\upsilon = \sqrt{F_T/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x = \upsilon\Delta t = \Delta t\sqrt{\frac{F_T}{\mu}} = \Delta t\sqrt{\frac{F_T}{m/\Delta x}} \quad \rightarrow \quad \Delta x = (\Delta t)^2\frac{F_T}{m} = (0.55\ \text{s})^2\frac{35\ \text{N}}{0.50\ \text{kg}} = \boxed{21\ \text{m}}$$

44. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. 11–16b applies, which states that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\text{ km}}/I_{15\text{ km}} = (15\text{ km})^2/(45\text{ km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\text{ km}}/A_{15\text{ km}} = 15\text{ km}/45\text{ km} = \boxed{0.33}$$

45. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source, as given by Eq. 11–16b. Thus Ir^2 will be constant.

$$I_{\text{near}}r_{\text{near}}^2 = I_{\text{far}}r_{\text{far}}^2 \quad \rightarrow$$

$$I_{\text{near}} = I_{\text{far}}\frac{r_{\text{far}}^2}{r_{\text{near}}^2} = (3.0\times10^6\text{ W/m}^2)\frac{(54\text{ km})^2}{(1.0\text{ km})^2} = 8.748\times10^9\text{ W/m}^2 \approx \boxed{8.7\times10^9\text{ W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = (8.748\times10^9\text{ W/m}^2)(2.0\text{ m}^2) = \boxed{1.7\times10^{10}\text{ W}}$$

46. The bug moves in SHM as the wave passes. The maximum KE of a particle in SHM is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$. Compare the two KE maxima.

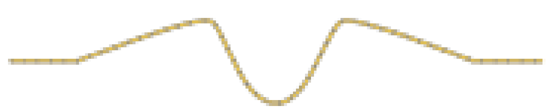
$$\frac{\text{KE}_2}{\text{KE}_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2.25\text{ cm}}{3.5\text{ cm}}\right)^2 = \boxed{0.41}$$

47. From Eq. 11–18, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

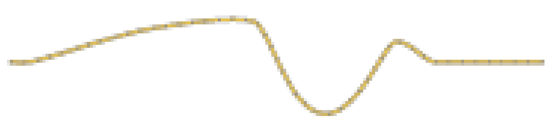
$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 5.0 \quad \rightarrow \quad A_2/A_1 = \sqrt{5.0} = \boxed{2.2}$$

The more energetic wave has the larger amplitude.

48. (*a*)



(*b*)



(*c*) The energy is $\boxed{\text{all kinetic energy}}$ at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

49. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, so the first four harmonics are $\boxed{f_1 = 440\text{ Hz, } f_2 = 880\text{ Hz, } f_3 = 1320\text{ Hz, and } f_4 = 1760\text{ Hz}}$.

50. The fundamental frequency of the full string is given by Eq. 11–19b, $f_{\text{unfingered}} = \dfrac{v}{2\ell} = 294\text{ Hz}$. If the length is reduced to two-thirds of its current value, and the velocity of waves on the string is not changed, then the new frequency will be

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}\ell\right)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right) f_{\text{unfingered}} = \left(\frac{3}{2}\right) 294\text{ Hz} = \boxed{441\text{ Hz}}$$

51. Four loops is the standing wave pattern for the fourth harmonic, with a frequency given by $f_4 = 4f_1 = 240\text{ Hz}$. Thus, $\boxed{f_1 = 60\text{ Hz, } f_2 = 120\text{ Hz, } f_3 = 180\text{ Hz, and } f_5 = 300\text{ Hz}}$ are all other resonant frequencies, where f_1 is the fundamental or first harmonic, f_2 is the first overtone or second harmonic, f_3 is the second overtone or third harmonic, and f_5 is the fourth overtone or fifth harmonic.

52. Adjacent nodes are separated by a half-wavelength, as examination of Fig. 11–41b will show.

$$\lambda = \frac{v}{f} \quad \to \quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{97\text{ m/s}}{2(475\text{ Hz})} = 0.10211\text{ m} \approx \boxed{0.10\text{ m}}$$

53. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)\, f_1 - nf_1 = f_1 = 350\text{ Hz} - 280\text{ Hz} = \boxed{70\text{ Hz}}$$

54. The speed of waves on the string is given by Eq. 11–13, $v = \sqrt{F_T/\mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 11–19b, $f_n = \dfrac{nv}{2\ell_{\text{vib}}}$, where ℓ_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{m/\ell}} \qquad f_1 = \frac{1}{2(0.62\text{ m})}\sqrt{\frac{520\text{ N}}{(3.4\times10^{-3}\text{ kg})/(0.92\text{ m})}} = 302.51\text{ Hz}$$

$$f_2 = 2f_1 = 605.01\text{ Hz} \qquad f_3 = 3f_1 = 907.52\text{ Hz}$$

So the three frequencies are $\boxed{300\text{ Hz, } 610\text{ Hz, and } 910\text{ Hz}}$, to 2 significant figures.

$\boxed{55.}$ The string must oscillate in a standing wave pattern to have a certain number of loops. The frequencies of the standing waves will all be 60.0 Hz, the same as the oscillator. That frequency is also expressed by Eq. 11–19b, $f_n = \dfrac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 11–13, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $f_n = \dfrac{n\upsilon}{2\ell} = \dfrac{n}{2\ell}\sqrt{\dfrac{F_{\mathrm{T}}}{\mu}} = \dfrac{n}{2\ell}\sqrt{\dfrac{mg}{\mu}} \rightarrow m = \dfrac{4\ell^2 f_n^2 \mu}{n^2 g}$

$$m_1 = \frac{4(1.50\text{ m})^2(60.0\text{ Hz})^2(3.5\times10^{-4}\text{ kg/m})}{1^2(9.80\text{ m/s}^2)} = 1.157\text{ kg} \approx \boxed{1.2\text{ kg}}$$

(*b*) $m_2 = \dfrac{m_1}{2^2} = \dfrac{1.157\text{ kg}}{4} = \boxed{0.29\text{ kg}}$

(*c*) $m_5 = \dfrac{m_1}{5^2} = \dfrac{1.157\text{ kg}}{25} = \boxed{4.6\times10^{-2}\text{ kg}}$

56. The tension in the string is the weight of the hanging mass, $F_{\mathrm{T}} = mg$. The speed of waves on the string can be found by $\upsilon = \sqrt{\dfrac{F_{\mathrm{T}}}{\mu}} = \sqrt{\dfrac{mg}{\mu}}$, and the frequency is given as $f = 60$ Hz. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{\upsilon}{f} = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{1}{60.0\text{ Hz}}\sqrt{\frac{(0.080\text{ kg})(9.80\text{ m/s}^2)}{(3.5\times10^{-4}\text{ kg/m})}} = 0.7888\text{ m}$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus, $\ell = \lambda/2, \lambda, 3\lambda/2, \cdots$. This gives $\ell = 0.39\text{ m}, 0.79\text{ m}, 1.18\text{ m}, 1.58\text{ m}, \cdots$ as the possible lengths, so there are $\boxed{\text{three}}$ standing wave patterns that may be achieved.

57. From the description of the water's behavior, there is an antinode at each end of the tub and a node in the middle. Thus, one wavelength is twice the tub length.

$$\upsilon = \lambda f = (2\ell_{\text{tub}})f = 2(0.75\text{ m})(0.85\text{ Hz}) = 1.275\text{ m/s} \approx \boxed{1.3\text{ m/s}}$$

58. The speed in the second medium can be found from the law of refraction, Eq. 11–20.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} \rightarrow \upsilon_2 = \upsilon_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\text{ km/s})\left(\frac{\sin 33^\circ}{\sin 44^\circ}\right) = \boxed{6.3\text{ km/s}}$$

59. The angle of refraction can be found from the law of refraction, Eq. 11–20. The relative velocities can be found from the relationship given in the problem.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} = \frac{331+0.60T_2}{331+0.60T_1} \rightarrow \sin\theta_2 = \sin 25^\circ\left(\frac{331+0.60(-15)}{331+0.60(15)}\right) = \sin 25^\circ\left(\frac{322}{340}\right) = 0.4002$$

$$\theta_2 = \sin^{-1} 0.4002 = 23.59^\circ \approx \boxed{24^\circ}$$

60. The angle of refraction can be found from the law of refraction, Eq. 11–20. The relative velocities can be found from Eq. 11–14a.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} = \frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\mathrm{SG}_1\rho_{\text{water}}}{\mathrm{SG}_2\rho_{\text{water}}}} = \sqrt{\frac{\mathrm{SG}_1}{\mathrm{SG}_2}}$$

$$\sin\theta_2 = \sin\theta_1\sqrt{\frac{\mathrm{SG}_1}{\mathrm{SG}_2}} = \sin 38^\circ\sqrt{\frac{3.6}{2.5}} = 0.7388 \rightarrow \theta_2 = \sin^{-1} 0.7388 = \boxed{48^\circ}$$

61. The wavelength is to be 1.0 m. Use Eq. 11–12.

$$v = f\lambda \quad \rightarrow \quad f = \frac{v}{\lambda} = \frac{344\ \text{m/s}}{0.75\ \text{m}} = 458.7\ \text{Hz} \approx \boxed{460\ \text{Hz}}$$

There will be significant diffraction only for wavelengths larger than the width of the window, so waves with frequencies $\boxed{\text{lower than 460 Hz}}$ would diffract when passing through this window.

62. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. We can measure both the amount of stretch of the fire net and the vertical displacement for gravitational potential energy by the variable y, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height $(y_{\text{top}} = 20.0\ \text{m})$ and the lowest location of the person $(y_{\text{bottom}} = -1.4\ \text{m})$. The person has no kinetic energy at either location.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}ky_{\text{bottom}}^2$$

$$k = 2mg\frac{(y_{\text{top}} - y_{\text{bottom}})}{y_{\text{bottom}}^2} = 2(62\ \text{kg})(9.80\ \text{m/s}^2)\frac{[20.0\ \text{m} - (-1.4\ \text{m})]}{(-1.4\ \text{m})^2} = 1.3268\times10^4\ \text{N/m}$$

(*a*) If the person were to lie on the fire net, the person would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$F_{\text{ext}} = k|y| = mg \quad \rightarrow \quad |y| = \frac{mg}{k} = \frac{(62\ \text{kg})(9.80\ \text{m/s}^2)}{1.3268\times10^4\ \text{N/m}} = \boxed{4.6\times10^{-2}\ \text{m}}$$

(*b*) To find the amount of stretch given a starting height of 38 m, again use conservation of energy. Note that there is no kinetic energy at the top or bottom positions.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}ky_{\text{bottom}}^2 \quad \rightarrow \quad y_{\text{bottom}}^2 + \frac{2mg}{k}y_{\text{bottom}} - \frac{2mg}{k}y_{\text{top}} = 0$$

$$y_{\text{bottom}}^2 + 2\frac{(62\ \text{kg})(9.80\ \text{m/s}^2)}{1.3268\times10^4\ \text{N/m}}y_{\text{bottom}} - 2\frac{(62\ \text{kg})(9.80\ \text{m/s}^2)}{1.3268\times10^4\ \text{N/m}}(38\ \text{m}) = 0 \quad \rightarrow$$

$$y_{\text{bottom}}^2 + 0.091589\,y_{\text{bottom}} - 3.4804 = 0 \quad \rightarrow \quad y_{\text{bottom}} = -1.9119\ \text{m},\ 1.8204\ \text{m}$$

This is a quadratic equation. The solution is the negative root, since the net must be below the unstretched position. The result is that it stretches $\boxed{1.9\ \text{m}}$ down if the person jumps from 38 m.

63. Apply conservation of mechanical energy to the car, calling condition 1 to be before the collision and condition 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_1 = 0$ and $v_2 = 0$.

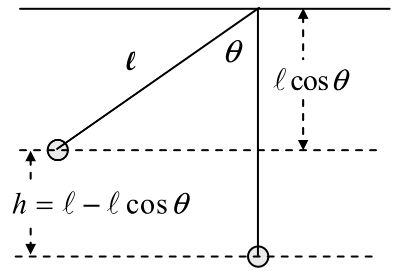
$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$x_2 = \sqrt{\frac{m}{k}}\,v_1 = \sqrt{\frac{1300\ \text{kg}}{410\times10^3\ \text{N/m}}}(2.0\ \text{m/s}) = 0.1126\ \text{m} \approx \boxed{0.11\ \text{m}}$$

64. (*a*) The frequency can be found from the length of the pendulum and the acceleration due to gravity.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{0.72\ \text{m}}} = 0.5872\ \text{Hz} \approx \boxed{0.59\ \text{Hz}}$$

(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.



$$E_{top} = E_{bottom} \quad \rightarrow \quad KE_{top} + PE_{top} = KE_{bottom} + PE_{bottom}$$

$$0 + mg(\ell - \ell\cos\theta) = \tfrac{1}{2}mv_{bottom}^2 + 0$$

$$v_{bottom} = \sqrt{2g\ell(1-\cos\theta)}$$
$$= \sqrt{2(9.80\text{ m/s}^2)(0.72\text{ m})(1-\cos 12^\circ)} = 0.5553\text{ m/s} \approx \boxed{0.56\text{ m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{total} = \tfrac{1}{2}mv_{bottom}^2 = \tfrac{1}{2}(0.295\text{ kg})(0.5553\text{ m/s})^2 = \boxed{4.5\times10^{-2}\text{ J}}$$

65. For the penny to stay on the block at all times means that there will be a normal force on the penny from the block, exerted upward. If down is taken to be the positive direction, then the net force on the penny is $F_{net} = mg - F_N = ma$. Solving for the magnitude of the normal force gives $F_N = mg - ma$. This expression is always positive if the acceleration is upward $(a < 0)$, so there is no possibility of the penny losing contact while accelerating upward. But if a downward acceleration were to be larger than *g*, then the normal force would go to zero, since the normal force cannot switch directions $(F_N > 0)$. Thus the limiting condition is $a_{down} = g$. This is the maximum value for the acceleration. For SHM, we also know that $a_{max} = \omega^2 A = \frac{k}{M+m}A \approx \frac{k}{M}A$. Equate these two values for the acceleration.

$$a_{max} = \frac{k}{M}A = g \quad \rightarrow \quad \boxed{A = \frac{Mg}{k}}$$

66. Block *m* stays on top of block *M* (executing SHM relative to the ground) without slipping due to static friction. The maximum static frictional force on *m* is $F_{fr_{max}} = \mu_s mg$. This frictional force causes block *m* to accelerate, so $ma_{max} = \mu_s mg \quad \rightarrow \quad a_{max} = \mu_s g$. Thus, for the blocks to stay in contact without slipping, the maximum acceleration of block *M* is also $a_{max} = \mu_s g$. But an object in SHM has a maximum acceleration given by $a_{max} = \omega^2 A = \frac{k}{M_{total}}A$. Equate these two expressions for the maximum acceleration.

$$a_{max} = \frac{k}{M_{total}}A = \mu_s g \quad \rightarrow \quad A = \frac{\mu_s g}{k}(M+m) = \frac{(0.30)(9.80\text{ m/s}^2)}{130\text{ N/m}}(7.25\text{ kg}) = \boxed{0.16\text{ m}}$$

67. The frequency of a simple pendulum is given by Eq. 11–11b, $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$. The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.

(*a*) $$f_{new} = \frac{1}{2\pi}\sqrt{\frac{g+a}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{g+0.35\,g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{1.35\,g}{\ell}} = \sqrt{1.35}\frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \sqrt{1.35}\,f = \boxed{1.16\,f}$$

(*b*) $f_{\text{new}} = \dfrac{1}{2\pi}\sqrt{\dfrac{g+a}{\ell}} = \dfrac{1}{2\pi}\sqrt{\dfrac{g-0.35\,g}{\ell}} = \dfrac{1}{2\pi}\sqrt{\dfrac{0.65\,g}{\ell}} = \sqrt{0.65}\dfrac{1}{2\pi}\sqrt{\dfrac{g}{\ell}} = \sqrt{0.65}\;f = \boxed{0.81\,f}$

68. The equation of motion is $x = 0.25\sin\;(4.70\,t) = A\sin\omega t.$

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.25\text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 4.70\text{ rad/s} \quad\rightarrow\quad f = \dfrac{4.70\text{ s}^{-1}}{2\pi} = \boxed{0.748\text{ Hz}}$.

(*c*) The period is the reciprocal of the frequency. $T = 1/f = \dfrac{2\pi}{4.70\text{ s}^{-1}} = \boxed{1.34\text{ s}}$.

(*d*) The total energy is given by the following:

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.650\text{ kg})[(4.70\text{ s}^{-1})(0.25\text{ m})]^2 = 0.4487\text{ J} \approx \boxed{0.45\text{ J}}$$

(*e*) The potential energy is given by

$$\text{PE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.650\text{ kg})(4.70\text{ s}^{-1})^2(0.15\text{ m})^2 = 0.1615\text{ J} \approx \boxed{0.16\text{ J}}$$

The kinetic energy is given by

$$\text{KE} = E_{\text{total}} - \text{PE} = 0.4487\text{ J} - 0.1615\text{ J} = 0.2872\text{ J} \approx \boxed{0.29\text{ J}}$$

69. The spring constant does not change, but the mass does, so the frequency will change. Use Eq. 11–6b to relate the spring constant, the mass, and the frequency for oxygen (O) and sulfur (S).

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad\rightarrow\quad \frac{k}{4\pi^2} = f^2m = \text{constant} \quad\rightarrow\quad f_{\text{O}}^2m_{\text{O}} = f_{\text{S}}^2m_{\text{S}} \quad\rightarrow$$

$$f_{\text{S}} = f_{\text{O}}\sqrt{\frac{m_{\text{O}}}{m_{\text{S}}}} = (3.7\times10^{13}\text{ Hz})\sqrt{\frac{16.0}{32.0}} = \boxed{2.6\times10^{13}\text{ Hz}}$$

70. Assume the block has a cross-sectional area of A. In the equilibrium position, the net force on the block is zero, so $F_{\text{buoy}} = mg.$ When the block is pushed into the water (downward) an additional distance $\Delta x,$ there is an increase in the buoyancy force (F_{extra}) equal to the weight of the additional water displaced. The weight of the extra water displaced is the density of water times the volume displaced.

$$F_{\text{extra}} = m_{\substack{\text{extra}\\ \text{water}}}\,g = \rho_{\text{water}}V_{\substack{\text{extra}\\ \text{water}}}\,g = \rho_{\text{water}}gA\Delta x = (\rho_{\text{water}}gA)\Delta x$$

This is the net force on the displaced block. Note that if the block is pushed down, the additional force is upward. And if the block were to be displaced upward by a distance $\Delta x,$ the buoyancy force would actually be less than the weight of the block by the amount $F_{\text{extra}},$ so there would be a net force downward of magnitude $F_{\text{extra}}.$ In both upward and downward displacement, there is a net force of magnitude $(\rho_{\text{water}}gA)\Delta x$ but opposite to the direction of displacement. So we can write $F_{\text{net}} = -(\rho_{\text{water}}gA)\Delta x,$ indicating that the direction of the force is opposite to the direction of the displacement. This is the equation of simple harmonic motion, with a "spring constant" of $\boxed{k = \rho_{\text{water}}gA}$.

71. The force of the man's weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of

the force put on the raft by the man. This is analogous to pulling down on a mass–spring system that is in equilibrium by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft–water system acts like a spring, with a spring constant found as follows:

$$k = \frac{F_{\text{ext}}}{x} = \frac{(68\text{ kg})(9.80\text{ m/s}^2)}{3.5\times10^{-2}\text{ m}} = 1.904\times10^4\text{ N/m}$$

(*a*) The frequency of vibration is determined by the "spring constant" and the mass of the raft.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.904\times10^4\text{ N/m}}{320\text{ kg}}} = 1.228\text{ Hz} \approx \boxed{1.2\text{ Hz}}$$

(*b*) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus all elastic potential energy.

$$E_{\text{total}} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(1.904\times10^4\text{ N/m})(3.5\times10^{-2}\text{ m})^2 = 11.66\text{ J} \approx \boxed{12\text{ J}}$$

72. The pebble losing contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be g downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus, if the board's downward acceleration exceeds g, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\max} = \frac{kA}{m} = \omega^2 A = 4\pi^2 f^2 A.$

$$a_{\max} = 4\pi^2 f^2 A \le g \quad \rightarrow \quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.80\text{ m/s}^2}{4\pi^2(2.8\text{ Hz})^2} \le \boxed{3.2\times10^{-2}\text{ m}}$$

73. (*a*) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$k = m\frac{v_1^2}{x_2^2} = (950\text{ kg})\frac{(25\text{ m/s})^2}{(4.0\text{ m})^2} = 3.711\times10^4\text{ N/m} \approx \boxed{3.7\times10^4\text{ N/m}}$$

(*b*) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$\tfrac{1}{2}T = \tfrac{1}{2}2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{950\text{ kg}}{3.711\times10^4\text{ N/m}}} = \boxed{0.50\text{ s}}$$

74. (*a*) The relationship between the velocity and the position of a SHO is given by Eq. 11–5b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$v = \pm v_{\max}\sqrt{1 - x^2/A^2} = \tfrac{1}{2}v_{\max} \quad \rightarrow \quad \pm\sqrt{1 - x^2/A^2} = \tfrac{1}{2} \quad \rightarrow \quad 1 - x^2/A^2 = \tfrac{1}{4} \quad \rightarrow$$

$$x^2/A^2 = \tfrac{3}{4} \quad \rightarrow \quad \boxed{x = \pm\sqrt{3}A/2 \approx \pm\, 0.866A}$$

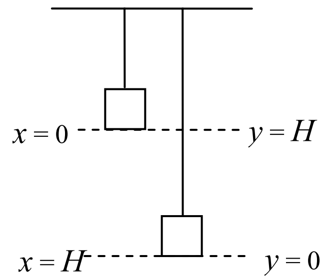
(*b*) Since $F = -kx = ma$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a = -xk/m.$ Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\boxed{\pm\tfrac{1}{2}x_0}$.

75. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 11–6b.

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k=4\pi^2 f^2 m=4\pi^2(2.83\times10^{13}\ \text{Hz})(16.00\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)=\boxed{8.40\times10^2\ \text{N/m}}$$

76. Consider energy conservation for the mass over the range of motion from letting go (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no KE at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable x represents the amount that the spring is stretched from the equilibrium position.



$$E_{\text{top}}=E_{\text{bottom}} \quad\rightarrow\quad \tfrac{1}{2}mv_{\text{top}}^2+mgy_{\text{top}}+\tfrac{1}{2}kx_{\text{top}}^2=\tfrac{1}{2}mv_{\text{bottom}}^2+mgy_{\text{bottom}}+\tfrac{1}{2}kx_{\text{bottom}}^2$$

$$0+mgH+0=0+0+\tfrac{1}{2}kH^2 \quad\rightarrow\quad \frac{k}{m}=\frac{2g}{H}=\omega^2 \quad\rightarrow\quad \omega=\sqrt{\frac{2g}{H}}$$

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{2g}{H}}=\frac{1}{2\pi}\sqrt{\frac{2(9.80\ \text{m/s}^2)}{0.270\ \text{m}}}=\boxed{1.36\ \text{Hz}}$$

77. The maximum velocity is given by Eq. 11–7.

$$v_{\max}=\frac{2\pi A}{T}=\frac{2\pi(0.15\ \text{m})}{7.0\ \text{s}}=\boxed{0.13\ \text{m/s}}$$

The maximum acceleration is given right after Eq. 11–10.

$$a_{\max}=\frac{kA}{m}=\omega^2A=\frac{4\pi^2A}{T^2}=\frac{4\pi^2(0.15\ \text{m})}{(7.0\ \text{s})^2}=0.1209\ \text{m/s}^2\approx\boxed{0.12\ \text{m/s}^2}$$

$$\frac{a_{\max}}{g}=\frac{0.1209\ \text{m/s}^2}{9.80\ \text{m/s}^2}=1.2\times10^{-2}=\boxed{1.2\%}$$

78. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda=16$ cm. The wave speed can be calculated from the frequency and the wavelength.

$$v=\lambda f=(16\ \text{cm})(1\ \text{Hz})=16\ \text{cm/s}\approx\boxed{20\ \text{cm/s}}$$

79. (*a*) The amplitude is half the peak-to-peak distance, so $\boxed{0.06\ \text{m}}$.

(*b*) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{\text{total}}=\tfrac{1}{2}kA^2$. Compare the two kinetic energy maxima.

$$\frac{\text{KE}_{2\max}}{\text{KE}_{1\max}}=\frac{\tfrac{1}{2}kA_2^2}{\tfrac{1}{2}kA_1^2}=\left(\frac{A_2}{A_1}\right)^2=\left(\frac{0.16\ \text{m}}{0.06\ \text{m}}\right)^2=\boxed{7.1}$$

80. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it—the normal force upward from the ground and the weight downward due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of g downward. Thus, the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than g. Any larger downward acceleration, and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as given after Eq. 11–10.

$$a_{\max} = \frac{kA}{m} = \omega^2 A > g \quad \rightarrow \quad A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.80\ \text{m/s}^2}{4\pi^2 (0.60\ \text{Hz})^2} = \boxed{0.69\ \text{m}}$$

81. (*a*) The overtones are given by $f_n = nf_1, n = 2,3,4,\ldots$.

G: $f_2 = 2(392\ \text{Hz}) = \boxed{784\ \text{Hz}}$ $\quad f_3 = 3(392\ \text{Hz}) = 1176\ \text{Hz} \approx \boxed{1180\ \text{Hz}}$

B: $f_2 = 2(494\ \text{Hz}) = \boxed{988\ \text{Hz}}$ $\quad f_3 = 3(440\ \text{Hz}) = 1482\ \text{Hz} \approx \boxed{1480\ \text{Hz}}$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings. The mass enters the problem through the mass per unit length.

$$\frac{f_G}{f_A} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\frac{F_T}{\mu_G}}}{\sqrt{\frac{F_T}{\mu_A}}} = \frac{\sqrt{\frac{F_T}{m_G/\ell}}}{\sqrt{\frac{F_T}{m_A/\ell}}} = \sqrt{\frac{m_A}{m_G}} \quad \rightarrow \quad \frac{m_G}{m_A} = \left(\frac{f_A}{f_G}\right)^2 = \left(\frac{494}{392}\right)^2 = \boxed{1.59}$$

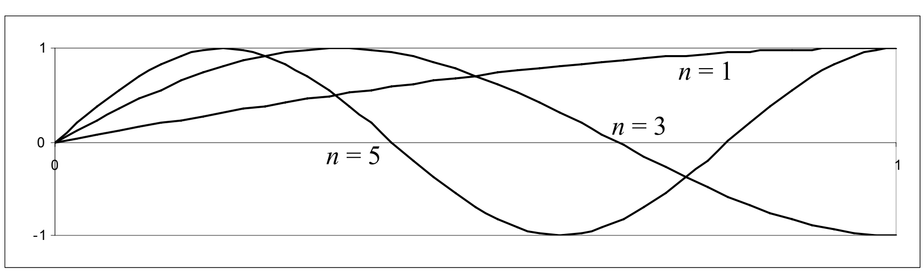
(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_B} = \frac{v/\lambda_G}{v/\lambda_B} = \frac{\lambda_B}{\lambda_G} = \frac{2\ell_B}{2\ell_G} \quad \rightarrow \quad \frac{\ell_G}{\ell_B} = \frac{f_B}{f_G} = \frac{494}{392} = \boxed{1.26}$$

(*d*) If the two strings have the same length, they have the same wavelength. They also have the same mass per unit length. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_B}{f_A} = \frac{v_B/\lambda}{v_A/\lambda} = \frac{v_B}{v_A} = \frac{\sqrt{\frac{F_{TB}}{\mu}}}{\sqrt{\frac{F_{TA}}{\mu}}} = \sqrt{\frac{F_{TB}}{F_{TA}}} \quad \rightarrow \quad \frac{F_{TB}}{F_{TA}} = \left(\frac{f_B}{f_A}\right)^2 = \left(\frac{392}{494}\right)^2 = \boxed{0.630}$$

82.



For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at one end and an antinode at the other end include $3\lambda/4, 5\lambda/4, \cdots$. See the diagrams. The general relationship is $\ell = (2n-1)\lambda/4,\ n = 1,2,3,\cdots$. Solving for the wavelength gives the following:

$$\boxed{\lambda = \frac{4\ell}{2n-1},\ n = 1, 2, 3, \cdots}$$

83. Relative to the fixed needle position, the ripples are moving with a linear velocity given by $v = \omega r$.

$$v = \left(33\tfrac{1}{3}\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)(0.102\text{ m}) = 0.3560\text{ m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f = \frac{v}{\lambda} = \frac{0.3560\text{ m/s}}{1.55\times10^{-3}\text{ m}} = \boxed{230\text{ Hz}}\ \text{(3 significant figures)}$$

84. The wave speed is given by Eq. 11–12, $v = \lambda f$, while the maximum speed of particles on the cord is given by Eq. 11–7, $v_{\text{max}} = 2\pi Af$. We equate the two expressions.

$$\lambda f = 2\pi Af \quad\rightarrow\quad A = \frac{\lambda}{2\pi} = \frac{10.0\text{ cm}}{2\pi} = \boxed{1.59\text{ cm}}$$

85. From the given data, $A = 0.50\text{ m}$ and $v = 2.5\text{ m}/4.0\text{ s} = 0.625\text{ m/s}$. We use Eq. 11–17b for the average power, with the density of seawater from Table 10–1. We estimate the area of the chest as $(0.30\text{ m})^2$. Answers may vary according to the approximation used for the area of the chest.

$$\overline{P} = 2\pi^2\rho S v f^2 A^2 = 2\pi^2(1025\text{ kg/m}^3)(0.30\text{ m})^2(0.625\text{ m/s})(0.25\text{ Hz})^2(0.50\text{ m})^2$$
$$= \boxed{18\text{ W}}$$

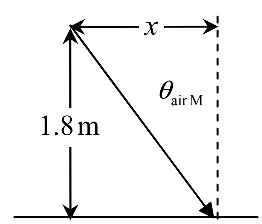
86. The unusual decrease of water corresponds to a trough in Fig. 11–24. The crest or peak of the wave is then one-half wavelength distant, traveling at 550 km/h.

$$\Delta x = vt = \tfrac{1}{2}\lambda \quad\rightarrow\quad t = \frac{\lambda}{2v} = \frac{235\text{ km}}{2(550\text{ km/h})}\left(\frac{60\text{ min}}{1\text{ h}}\right) = \boxed{13\text{ min}}$$

87. (*a*) Equation 11–20 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text{water}} = 90°$ or $\sin\theta_{\text{water}} = 1$. The air is the "incident" media. Thus the incident angle is given by the following:

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}} = \frac{v_{\text{air}}}{v_{\text{water}}};\ \theta_{\text{air}} = \theta_{\text{i}} = \sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right] \quad\rightarrow\quad \boxed{\theta_{\text{iM}} = \sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right] = \sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.



$$\theta_{\text{air M}} = \sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}} = \sin^{-1}\frac{343\text{ m/s}}{1440\text{ m/s}} = 13.8°$$

$$\tan\theta_{\text{air M}} = \frac{x}{1.8\text{ m}} \quad\rightarrow\quad x = (1.8\text{ m})\tan 13.8° = \boxed{0.44\text{ m}}$$

Solutions to Search and Learn Problems

1. For an underdamped oscillation, the period is approximately equal to the period of an undamped oscillation. The period is related to the mass and spring constant as in Eq. 11–6a. To measure the period of oscillation, you can push down on the bumpers of the car and time the oscillation period. Then solving Eq. 11–6a gives the effective spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \to \quad k = m\left(\frac{2\pi}{T}\right)^2$$

This spring constant is for the four springs acting together. To determine the spring constant of each individual spring, you would divide the mass of the car by four.

$$k_1 = \frac{m}{4}\left(\frac{2\pi}{T}\right)^2 = \frac{m\pi^2}{T^2}$$

2. The resonant angular frequency of the spring is equal to the angular frequency of the tires when traveling at 90.0 km/h.

$$\omega = \frac{v}{r} = \frac{90.0 \text{ km/h}}{\frac{1}{2}(0.58 \text{ m})}\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 86.21 \text{ rad/s}$$

The spring constant can be calculated from this angular frequency and the mass of the tire/wheel combination.

$$\omega^2 = \frac{k}{m} \quad \to \quad k = \omega^2 m = (86.21 \text{ rad/s})^2(17.0 \text{ kg}) = 1.263\times10^5 \text{ N/m}$$

Hooke's law (Eq. 6–8) can then be used to determine the distance the spring is compressed when the mass is added. We use the absolute value of the force and the distance, so that there is no negative sign in the equation.

$$F = kx \quad \to \quad x = \frac{F}{k} = \frac{mg}{k} = \frac{\frac{1}{4}(280 \text{ kg})(9.80 \text{ m/s}^2)}{1.263\times10^5 \text{ N/m}} = 5.432\times10^{-3} \text{ m} \approx \boxed{5.4\times10^{-3} \text{ m}}$$

3. Forced resonance and underdamping are factors in the pronounced rattle and vibration. Forced resonance becomes a factor because hitting the bumps in the road is "forcing" action on the car. Assuming that the bumps are regularly spaced and the car has a constant speed, then that forcing will have a particular frequency. The forcing becomes "forced resonance" when the frequency of hitting the road bumps matches a resonant frequency of some part in the car. For example, if the frequency at which the car hits the bumps in the road is in resonance with the natural frequency of the springs, the car will have a large amplitude of oscillation and will shake strongly. If those springs are underdamped, then the oscillation resulting from hitting one bump is still occurring when you hit the next bump, so the amplitude increases with each bump. A significant oscillation amplitude can be built up. If the springs were critically damped or overdamped, the oscillation would not build up—it would dampen before the next bump was encountered.

4. One wavelength, or one full oscillation, corresponds to 360°—a full cycle of the sinusoidal oscillation that is creating the wave disturbance. Therefore, a half of a wavelength ($\lambda/2$) corresponds to a half oscillation, or 180°.

5. We must make several assumptions. Consider a static displacement of the trampoline, by someone sitting on the trampoline mat. The upward elastic force of the trampoline must equal the downward

force of gravity. We estimate that a 75-kg person will depress the trampoline about 25 cm at its midpoint. Hooke's law (Eq. 6–8) can then be solved for the spring constant.

$$kx = mg \quad \rightarrow \quad k = \frac{mg}{x} = \frac{(75\text{ kg})(9.80\text{ m/s}^2)}{0.25\text{ m}} = 2940\text{ N/m} \approx \boxed{3000\text{ N/m}}$$

Answers will vary based on the assumptions made.

6. The addition of the support will force the bridge to have an oscillation node at the center of the span. This makes the new fundamental frequency equal to the first overtone of the original fundamental frequency. If the wave speed in the bridge material doesn't change, then the resonant frequency will double, to $\boxed{6.0\text{ Hz}}$. Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.

12

SOUND

Responses to Questions

1. Sound exhibits several phenomena that give evidence that it is a wave. Interference is a wave phenomenon, and sound produces interference (such as beats). Diffraction is a wave phenomenon, and sound can be diffracted (such as sound being heard around corners). Refraction is a wave phenomenon, and sound exhibits refraction when passing obliquely from one medium to another. Sound also requires a medium, a characteristic of mechanical waves.

2. Evidence that sound is a form of energy is found in the fact that sound can do work. A sound wave created in one location can cause the mechanical vibration of an object at a different location. For example, sound can set eardrums in motion, make windows rattle, or even shatter a glass. See Fig. 11–19 for a photograph of a goblet shattering from the sound of a trumpet.

3. The child speaking into a cup creates sound waves that cause the bottom of the cup to vibrate. Since the string is tightly attached to the bottom of the cup, the vibrations of the cup are transmitted to longitudinal waves in the string. These waves travel down the string and cause the bottom of the receiver's cup to vibrate back and forth. This relatively large vibrating surface moves the adjacent air and generates sound waves from the bottom of the cup that travel up into the cup. These waves are incident on the receiver's ear, and the receiver hears the sound from the speaker.

4. If the frequency were to change, the two media could not stay in contact with each other. If the two media vibrated with different frequencies, then particles from the two media initially in contact could not stay in contact with each other. But particles must be in contact in order for the wave to be transmitted from one medium to the other, so the frequency does not change. Since the wave speed changes in passing from air into water and the frequency does not change, we expect the wavelength to change. Sound waves travel about four times faster in water than in air, so we expect the wavelength in water to be about four times longer than it is in air.

5. If the speed of sound in air depended significantly on frequency, then the sounds that we hear would be separated in time according to frequency. For example, if a chord were played by an orchestra, then we would hear the high notes at one time, the middle notes at another, and the lower notes at still another. This effect is not heard for a large range of distances, indicating that the speed of sound in air does not depend significantly on frequency.

6. The sound-producing anatomy of a person includes various resonating cavities, such as the throat. The relatively fixed geometry of these cavities determines the relatively fixed wavelengths of sound that a person can produce. Those wavelengths have associated frequencies given by $f = \upsilon/\lambda.$ The speed of sound is determined by the gas that is filling the resonant cavities. If the person has inhaled helium, then the speed of sound will be much higher than normal, since the speed of sound waves in helium is about 3 times that in air. Thus, the person's frequencies will go up by about a factor of 3. This is about a 1.5-octave shift, so the person's voice sounds very high pitched.

7. The basic equation determining the pitch of the organ pipe is either $f_{\text{closed}} = \dfrac{n\upsilon}{4\ell},\ n = \text{odd integer},$ for a closed pipe, or $f_{\text{open}} = \dfrac{n\upsilon}{2\ell},\ n = \text{integer},$ for an open pipe. In each case, the frequency is proportional to the speed of sound in air. Since the speed is a function of temperature, and the length of any particular pipe is very nearly constant over the relatively small range of temperatures in a room, the frequency is also a function of temperature. Thus, when the temperature changes, the resonant frequencies of the organ pipes change. Since the speed of sound increases with temperature, as the temperature increases, the pitch of the pipes increases as well.

8. A tube of a given length will resonate (permit standing waves) at certain frequencies. When a mix of frequencies is input to the tube, only those frequencies close to resonant frequencies will produce sound that persists, because standing waves are created for those frequencies. Frequencies far from resonant frequencies will not persist very long at all—they will "die out" quickly. If, for example, two adjacent resonances of a tube are at 100 Hz and 200 Hz, then sound input near one of those frequencies will persist and sound relatively loud. A sound input near 150 Hz would fade out quickly and thus have a reduced amplitude as compared to the resonant frequencies. The length of the tube can therefore be chosen to "filter" certain frequencies, if those filtered frequencies are not close to resonant frequencies.

9. For a string with fixed ends, the fundamental frequency is given by $f = \dfrac{\upsilon}{2\ell},$ so the length of string for a given frequency is $\ell = \dfrac{\upsilon}{2f}.$ For a string, if the tension is not changed while fretting, the speed of sound waves will be constant. Thus, for two frequencies $f_1 < f_2,$ the spacing between the frets corresponding to those frequencies is given as follows:

$$\ell_1 - \ell_2 = \frac{\upsilon}{2f_1} - \frac{\upsilon}{2f_2} = \frac{\upsilon}{2}\left(\frac{1}{f_1} - \frac{1}{f_2}\right)$$

Now see Table 12–3. Each note there corresponds to one fret on the guitar neck. Notice that as the adjacent frequencies increase, the interfrequency spacing also increases. The change from C to $\text{C}^{\#}$ is 15 Hz, while the change from G to $\text{G}^{\#}$ is 23 Hz. Thus, their reciprocals get closer together, so from the above formula, the length spacing gets closer together. Consider a numerical example.

$$\ell_{\text{C}} - \ell_{\text{C}^{\#}} = \frac{\upsilon}{2}\left(\frac{1}{262} - \frac{1}{277}\right) = \frac{\upsilon}{2}(2.07\times10^{-4}) \qquad \ell_{\text{G}} - \ell_{\text{G}^{\#}} = \frac{\upsilon}{2}\left(\frac{1}{392} - \frac{1}{415}\right) = \frac{\upsilon}{2}(1.41\times10^{-4})$$

$$\frac{\ell_{\text{G}} - \ell_{\text{G}^{\#}}}{\ell_{\text{C}} - \ell_{\text{C}^{\#}}} = 0.68$$

The G to $\text{G}^{\#}$ spacing is only about 68% of the C to $\text{C}^{\#}$ spacing.

10. When you first hear the truck, you cannot see it. There is no straight-line path from the truck to you. The sound waves that you are hearing are therefore arriving at your location due to diffraction. Long wavelengths are diffracted more than short wavelengths, so you are initially only hearing sound with long wavelengths, which are low-frequency sounds. After you can see the truck, you are able to receive all frequencies being emitted by the truck, not just the lower frequencies. Thus, the sound "brightens" due to your hearing more high-frequency components.

11. The wave pattern created by standing waves does not "travel" from one place to another. The node locations are fixed in space. Any one point in the medium has the same amplitude at all times. Thus, the interference can be described as "interference in space"—moving the observation point from one location to another changes the interference from constructive (antinode) to destructive (node). To experience the full range from node to antinode, the position of observation must change, but all observations could be made at the same time by a group of observers.

    The wave pattern created by beats does travel from one place to another. Any one point in the medium will at one time have a 0 amplitude (node) and half a beat period later, have a maximum amplitude (antinode). Thus, the interference can be described as "interference in time." To experience the full range from constructive interference to destructive interference, the time of observation must change, but all observations could be made at the same position.

12. If the frequency of the speakers is lowered, then the wavelength will be increased. Each circle in the diagram will be larger, so the points C and D will move farther apart.

13. *Active noise reduction* devices work on the principle of destructive interference. If the electronics are fast enough to detect the noise, invert it, and create the opposite wave (180° out of phase with the original) in significantly less time than one period of the components of the noise, then the original noise and the created noise will be approximately in a destructive interference relationship. The person wearing the headphones will then hear a net sound signal that is very low in intensity.

14. For the two waves shown, the frequency of beating is higher in wave (a)—the beats occur more frequently. The beat frequency is the difference between the two component frequencies, so since (a) has a higher beat frequency, the component frequencies are farther apart in (a).

15. There is no Doppler shift if the source and observer move in the same direction, with the same velocity. Doppler shift is caused by relative motion between source and observer, and if both source and observer move in the same direction with the same velocity, there is no relative motion.

16. If the wind is blowing but the listener is at rest with respect to the source, the listener will not hear a Doppler effect. We analyze the case of the wind blowing from the source toward the listener. The moving air (wind) has the same effect as if the speed of sound had been increased by an amount equal to the wind speed. The wavelength of the sound waves (distance that a wave travels during one period of time) will be increased by the same percentage that the wind speed is relative to the still-air speed of sound. Since the frequency is the speed divided by the wavelength, the frequency does not change, so there is no Doppler effect to hear. Alternatively, the wind has the same effect as if the air were not moving but the source and listener were moving at the same speed in the same direction. See Question 15 for a discussion of that situation. Finally, since there is no relative motion between the source and the listener, there is no Doppler shift.

17. The highest frequency of sound will be heard at position C, while the child is swinging forward. Assuming the child is moving with SHM, the highest speed is at the equilibrium point, point C. And to have an increased pitch, the relative motion of the source and detector must be toward each other. The child would also hear the lowest frequency of sound at point C, while swinging backward.

Responses to MisConceptual Questions

1. (*a*) Students may answer that the speed of sound is the same, if they do not understand that the speed of sound is not constant, but depends upon the temperature of the air. When it is hotter, the speed of sound is greater, so it takes less time for the echo to return.

2. (*d*) Sound waves are longitudinal waves, so (*a*) is incorrect. The sound waves can be characterized either by the longitudinal displacement of the air molecules or by the pressure differences that cause the displacements.

3. (*b*) A common misconception is to treat the sound intensity level as a linear scale instead of a logarithmic scale. If the sound intensity doubles, the intensity level increases by about 3 dB, so the correct answer is 73 dB.

4. (*e*) Students often think that the sound intensity is the same as loudness and therefore mistakenly answer that doubling the intensity will double the loudness. However, the ear interprets loudness on a logarithmic scale. For something to sound twice as loud, it must have an intensity that is 10 times as great.

5. (*b*) The octave is a measure of musical frequency, not loudness. Raising a note by one octave requires doubling the frequency. Therefore, raising a note by two octaves is doubling the frequency twice, which is the same as quadrupling the frequency.

6. (*e*) In a string or open tube the lowest vibration mode is equal to half of a wavelength. In a tube closed at one end the lowest vibration mode is equal to a quarter of a wavelength. Therefore, none of the listed objects have a lowest vibration mode equal to a wavelength.

7. (*e*) A common misconception is that the frequency of a sound changes as it passes from air to water. The frequency is the number of wave crests that pass a certain point per unit time. If this value were to change as it entered the water, then wave crests would build up or be depleted over time. This would make the interface an energy source or sink, which it is not. The speed of sound in water is greater than in air, so the speed of the wave changes. Since the frequency cannot change, the increase in speed results in an increase in wavelength.

8. (*e*) As the string oscillates, it causes the air to vibrate at the same frequency. Therefore, the sound wave will have the same frequency as the guitar string, so answers (*b*) and (*c*) are incorrect. The speed of sound in air at 20°C is 343 m/s. The speed of sound in the string is the product of the wavelength and frequency, 462 m/s, so the sound waves in air have a shorter wavelength than the waves on the string.

9. (*c*) Pushing the string straight down onto a fret does not affect the tension a significant amount, due to the fret being so close to the string. The amplitude of the wave is determined by how hard the string is plucked, not by pushing the string onto the frets. When the string is pushed down, its effective length is shortened, which shortens the wavelength and thus increases the oscillation frequency. (The wave speed on the string doesn't change due to fretting the string.)

10. (*a*) The fundamental wavelength of an open-ended organ pipe is twice the length of the pipe. If one end is closed, then the fundamental wavelength is four times the length of the pipe. Since the wavelength doubles when one end of the pipe is closed off, and the speed of sound remains constant, the fundamental frequency is cut in half.

11. (*c*) The two speakers create sound waves that interact as described by the principle of superposition. When the waves overlap, the frequency remains the same; it does not double. If the speakers occupied the same location so that each point in the room were equidistant from the speakers, then the intensity would double everywhere. However, the speakers are separated by a distance of 10 m. Since the path lengths from each speaker to different locations around the room are not the same, at some points in the room the path difference will be an odd integer number of half wavelengths, so the sounds will destructively interfere. At other locations in the room the speakers will be equidistant, or the path difference will be an integral number of whole wavelengths, and the sounds will constructively interfere. This results in dead spots and loud spots in the room.

12. (*c*) A common misconception is that since the cars are moving there must be a Doppler shift. In this situation, however, there is no relative motion between the two vehicles. The two vehicles travel in the same direction at the same speed. Since the distance between them does not change, you and your sister will hear the horn sound at the same frequency.

13. (*e*) As the string vibrates, each part of the string (other than the nodes) oscillates at the same frequency, so answer (*a*) is true. This oscillation excites the air to vibrate at that frequency, so answer (*c*) is true. The wave relationships in answers (*b*) and (*d*) are true for any wave, so both are true in this case as well. However, the speed of the wave on the string is determined by the tension and mass of the string, and the speed of sound in the air is determined by the temperature, pressure, and density of the air. The two speeds are not necessarily the same. Since the sound wave and wave on the string have the same frequencies, but not necessarily the same wave speeds, they do not necessarily have the same wavelengths. Thus, (*e*) is not true.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted frequencies as correct to the number of digits shown, especially where other values might indicate that. For example, in Problem 49, values of 350 Hz and 355 Hz are used. We took both of those values to have 3 significant figures.

1. The round-trip time for sound is 2.5 seconds, so the time for sound to travel the length of the lake is 1.25 seconds. Use the time and the speed of sound to determine the length of the lake.

$$d = \upsilon t = (343\text{ m/s})(1.25\text{ s}) = 429\text{ m} \approx \boxed{430\text{ m}}$$

2. The round-trip time for sound is 2.0 seconds, so the time for sound to travel the length of the lake is 1.0 seconds. Use the time and the speed of sound in water to determine the depth of the lake.

$$d = \upsilon t = (1560\text{ m/s})(1.0\text{ s}) = 1560\text{ m} = \boxed{1600\text{ m}}$$

3. (*a*) $\lambda_{20\text{ Hz}} = \dfrac{\upsilon}{f} = \dfrac{343\text{ m/s}}{20\text{ Hz}} = \boxed{17\text{ m}}$ $\quad \lambda_{20\text{ kHz}} = \dfrac{\upsilon}{f} = \dfrac{343\text{ m/s}}{2.0\times10^{4}\text{ Hz}} = \boxed{1.7\times10^{-2}\text{ m}}$

The range is from 1.7 cm to 17 m.

(*b*) $\lambda = \dfrac{\upsilon}{f} = \dfrac{343\text{ m/s}}{18\times10^{6}\text{ Hz}} = \boxed{1.9\times10^{-5}\text{ m}}$

4. The distance that the sound travels is the same on both days and is equal to the speed of sound times the elapsed time. Use the temperature-dependent relationship for the speed of sound in air.

$$d = v_1 t_1 = v_2 t_2 \quad \rightarrow \quad [(331 + 0.6(31))\text{ m/s}](4.80\text{ s}) = [(331 + 0.6(T_2))\text{ m/s}](5.20\text{ s}) \quad \rightarrow$$

$$T_2 = \boxed{-14^\circ\text{C}}$$

5. (*a*) For the fish, the speed of sound in sea water must be used.

$$d = vt \quad \rightarrow \quad t = \frac{d}{v} = \frac{1550\text{ m}}{1560\text{ m/s}} = \boxed{0.994\text{ s}}$$

(*b*) For the fishermen, the speed of sound in air must be used.

$$d = vt \quad \rightarrow \quad t = \frac{d}{v} = \frac{1550\text{ m}}{343\text{ m/s}} = \boxed{4.52\text{ s}}$$

$\boxed{6.}$ The two sound waves travel the same distance. The sound will travel faster in the concrete and thus take a shorter time.

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{concrete}} t_{\text{concrete}} = v_{\text{concrete}}(t_{\text{air}} - 0.80\text{ s}) \quad \rightarrow \quad t_{\text{air}} = \frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}}(0.80\text{ s})$$

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{air}}\left(\frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}}\right)(0.80\text{ s})$$

The speed of sound in concrete is obtained from Eq. 11–14a, Table 9–1, and Table 10–1.

$$v_{\text{concrete}} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{20\times10^9\text{ N/m}^2}{2.3\times10^3\text{ kg/m}^3}} = 2949\text{ m/s}$$

$$d = v_{\text{air}} t_{\text{air}} = (343\text{ m/s})\left(\frac{2949\text{ m/s}}{2949\text{ m/s} - 343\text{ m/s}}\right)(0.80\text{ s}) = 310.5\text{ m} \approx \boxed{310\text{ m}}$$

7. The total time T is the time for the stone to fall (t_{down}) plus the time for the sound to come back to the top of the cliff (t_{up}): $T = t_{\text{up}} + t_{\text{down}}$. Use constant-acceleration relationships for an object dropped from rest that falls a distance h in order to find t_{down}, with down as the positive direction. Use the constant speed of sound to find t_{up} for the sound to travel a distance h.

$$\text{down: } y = y_0 + v_0 t_{\text{down}} + \tfrac{1}{2} a t_{\text{down}}^2 \quad \rightarrow \quad h = \tfrac{1}{2} g t_{\text{down}}^2 \qquad \text{up: } h = v_{\text{snd}} t_{\text{up}} \quad \rightarrow \quad t_{\text{up}} = \frac{h}{v_{\text{snd}}}$$

$$h = \tfrac{1}{2} g t_{\text{down}}^2 = \tfrac{1}{2} g (T - t_{\text{up}})^2 = \tfrac{1}{2} g \left(T - \frac{h}{v_{\text{snd}}}\right)^2 \quad \rightarrow \quad h^2 - 2v_{\text{snd}}\left(\frac{v_{\text{snd}}}{g} + T\right)h + T^2 v_{\text{snd}}^2 = 0$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$h^2 - 2(343\text{ m/s})\left(\frac{343\text{ m/s}}{9.80\text{ m/s}^2} + 2.7\text{ s}\right)h + (2.7\text{ s})^2(343\text{ m/s})^2 = 0 \quad \rightarrow$$

$$h^2 - (25862\text{ m})h + 8.5766\times10^5\text{ m}^2 = 0 \quad \rightarrow \quad h = \frac{25862 \pm 25796}{2} = 25{,}829\text{ m}, \ \boxed{33\text{ m}}$$

The larger root is impossible since it takes more than 2.7 s for the rock to fall that distance, so $\boxed{h = 33\text{ m}}$.

8. $120\text{ dB} = 10\ \log\frac{I_{120}}{I_0} \quad\rightarrow\quad I_{120} = 10^{12}\ I_0 = 10^{12}(1.0\times10^{-12}\text{ W/m}^2) = \boxed{1.0\text{ W/m}^2}$

$20\text{ dB} = 10\ \log\frac{I_{20}}{I_0} \quad\rightarrow\quad I_{20} = 10^{2}\ I_0 = 10^{2}(1.0\times10^{-12}\text{ W/m}^2) = \boxed{1.0\times10^{-10}\text{ W/m}^2}$

The pain level is 10^{10} times more intense than the whisper.

9. $\beta = 10\log\frac{I}{I_0} = 10\log\frac{1.5\times10^{-6}\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2} = 61.76\text{ dB} \approx \boxed{62\text{ dB}}$

10. Compare the two power output ratings using the definition of decibels.

$\beta = 10\log\frac{P_{150}}{P_{100}} = 10\log\frac{120\text{ W}}{75\text{ W}} = \boxed{2.0\text{ dB}}$

This would barely be perceptible.

11. From Example 12–4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, the sound level for one firecracker will be $85\text{ dB} - 3\text{ dB} = \boxed{82\text{ dB}}$.

12. From Example 12–4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, if two engines are shut down, the intensity will be cut in half, and the sound level will be 137 dB. Then, if one more engine is shut down, the intensity will be cut in half again, and the sound level will drop by 3 more dB, to a final value of $\boxed{134\text{ dB}}$.

13. For the 82-dB device: $82\text{ dB} = 10\log(I_{\text{signal}}/I_{\text{noise}})_{\text{tape}} \quad\rightarrow\quad (I_{\text{signal}}/I_{\text{noise}})_{\text{tape}} = 10^{8.2} = \boxed{1.6\times10^{8}}$.

For the 98-dB device: $98\text{ dB} = 10\log(I_{\text{signal}}/I_{\text{noise}})_{\text{tape}} \quad\rightarrow\quad (I_{\text{signal}}/I_{\text{noise}})_{\text{tape}} = 10^{9.8} = \boxed{6.3\times10^{9}}$.

14. (*a*) The energy absorbed per second is the power of the wave, which is the intensity times the area.

$$55\text{ dB} = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{5.5}\ I_0 = 10^{5.5}(1.0\times10^{-12}\text{ W/m}^2) = 3.162\times10^{-7}\text{ W/m}^2$$

$$P = IA = (3.162\times10^{-7}\text{ W/m}^2)(5.0\times10^{-5}\text{ m}^2) = 1.581\times10^{-11}\text{ W} \approx \boxed{1.6\times10^{-11}\text{ J/s}}$$

(*b*) $1.0\text{ J}\left(\frac{1\text{ s}}{1.581\times10^{-11}\text{ J}}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = 2001\text{ yr} \approx \boxed{2.0\times10^{3}\text{ yr}}$

$\boxed{15.}$ (*a*) Find the intensity from the 130-dB value, and then find the power output corresponding to that intensity at that distance from the speaker.

$$\beta = 130\text{ dB} = 10\log\frac{I_{2.8\text{ m}}}{I_0} \quad\rightarrow\quad I_{2.8\text{ m}} = 10^{13}\ I_0 = 10^{13}(1.0\times10^{-12}\text{ W/m}^2) = 10\text{ W/m}^2$$

$$P = IA = 4\pi r^2 I = 4\pi(2.5\text{ m})^2(10\text{ W/m}^2) = 785.4\text{ W} \approx \boxed{790\text{ W}}$$

(*b*) Find the intensity from the 85-dB value, and then from the power output, find the distance corresponding to that intensity.

$$\beta = 85\text{ dB} = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{8.5}\, I_0 = 10^{8.5}(1.0\times10^{-12}\text{ W/m}^2) = 3.162\times10^{-4}\text{ W/m}^2$$

$$P = 4\pi r^2 I \quad\rightarrow\quad r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{785.4\text{ W}}{4\pi(3.162\times10^{-4}\text{ W/m}^2)}} = 444.6\text{ m} \approx \boxed{440\text{ m}}$$

16. The first person is a distance of $r_1 = 100\,\text{m}$ from the explosion, while the second person is a distance $r_2 = \sqrt{5}(100\text{ m})$ from the explosion. The intensity detected away from the explosion is inversely proportional to the square of the distance from the explosion.

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left[\frac{\sqrt{5}(100\text{ m})}{100\text{ m}}\right]^2 = 5;\quad \beta = 10\log\frac{I_1}{I_2} = 10\log 5 = 6.99\text{ dB} \approx \boxed{7\text{ dB}}$$

17. (*a*) The intensity is proportional to the square of the amplitude, so if the amplitude is 3.5 times greater, the intensity will $\boxed{\text{increase by a factor of } 3.5^2 = 12.25 \approx 12}$.

(*b*) $\beta = 10\log I/I_0 = 10\log 12.25 = 10.88\text{ dB} \approx \boxed{11\text{ dB}}$

18. The intensity is given by Eq. 11–18, $I = 2\rho v\pi^2 f^2 A^2$. If the only difference in two sound waves is their frequencies, then the ratio of the intensities is the ratio of the square of the frequencies.

$$\frac{I_{2f}}{I_f} = \frac{(2.2f)^2}{f^2} = \boxed{4.8}$$

$\boxed{19.}$ The intensity is given by Eq. 11–18, $I = 2\pi^2 v\rho f^2 A^2$, using the density of air (from Table 10–1) and the speed of sound in air.

$$I = 2\rho v\pi^2 f^2 A^2 = 2(1.29\text{ kg/m}^3)(343\text{ m/s})\pi^2(440\text{ Hz})^2(1.3\times10^{-4}\text{ m})^2 = 28.576\text{ W/m}^2$$

$$\beta = 10\log\frac{I}{I_0} = 10\log\frac{28.576\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2} = 134.56\text{ dB} \approx \boxed{130\text{ dB}}$$

Note that according to Fig. 12–6, this is above the threshold of pain at that frequency.

20. (*a*) According to Table 12–2, the intensity of normal conversation, at a distance of about 50 cm from the speaker, is about $3\times10^{-6}\text{ W/m}^2$. The intensity is the power output per unit area, so the power output can be found. The area to use is the surface area of a sphere.

$$I = \frac{P}{A} \quad\rightarrow\quad P = IA = I(4\pi r^2) = (3\times10^{-6}\text{ W/m}^2)4\pi(0.50\text{ m})^2 = 9.425\times10^{-6}\text{ W} \approx \boxed{9.4\times10^{-6}\text{ W}}$$

(*b*) $60\text{ W}\left(\dfrac{1\text{ person}}{9.425\times10^{-6}\text{ W}}\right) = 6.37\times10^6 \approx \boxed{6\text{ million people}}$

21. The intensity of the sound is defined to be the power per unit area. We assume that the sound spreads out spherically from the loudspeaker.

(*a*) $I_{220} = \dfrac{220\text{ W}}{4\pi(3.5\text{ m})^2} = 1.429\text{ W/m}^2 \quad I_{45} = \dfrac{45\text{ W}}{4\pi(3.5\text{ m})^2} = 0.292\text{ W/m}^2$

$$\beta_{220} = 10\log\frac{I_{220}}{I_0} = 10\log\frac{1.429\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2} = 121.55\text{ dB} \approx \boxed{122\text{ dB}}$$

$$\beta_{45} = 10\log\frac{I_{45}}{I_0} = 10\log\frac{0.292\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2} = 114.66\text{ dB} \approx \boxed{115\text{ dB}}$$

(*b*) According to the textbook, for a sound to be perceived as twice as loud as another, the intensities need to differ by a factor of 10, or differ by 10 dB. They differ by only about 7 dB. $\boxed{\text{The expensive amp will not sound twice as loud as the cheaper one.}}$

22. From Fig. 12–6, a 100-Hz tone at 50 dB has a loudness of about 20 phons. At 5000 Hz, 20 phons corresponds to about $\boxed{20\text{ dB}}$. Answers may vary due to estimation in the reading of the graph.

23. From Fig. 12–6, at 40 dB the low-frequency threshold of hearing is about $\boxed{70\text{–}80\text{ Hz}}$. There is no intersection of the threshold of hearing with the 40-dB level on the high-frequency side of the chart, so we assume that a 40-dB signal can be heard all the way up to the highest frequency that a human can hear, $\boxed{20{,}000\text{ Hz}}$. Answers may vary due to estimation in the reading of the graph.

24. (*a*) From Fig. 12–6, at 100 Hz, the threshold of hearing (the lowest detectable intensity by the ear) is approximately $5\times10^{-9}\text{ W/m}^2$. The threshold of pain is about 5 W/m^2. The ratio of highest to lowest intensity is thus $\dfrac{5\text{ W/m}^2}{5\times10^{-9}\text{ W/m}^2} = \boxed{10^9}$.

(*b*) At 5000 Hz, the threshold of hearing is about 10^{-13} W/m^2, and the threshold of pain is about 10^{-1} W/m^2. The ratio of highest to lowest intensity is $\dfrac{10^{-1}\text{ W/m}^2}{10^{-13}\text{ W/m}^2} = \boxed{10^{12}}$.

Answers may vary due to estimation in the reading of the graph.

25. Each octave is a doubling of frequency. The number of octaves, *n*, can be found from the following:

$$20{,}000\text{ Hz} = 2^n(20\text{ Hz}) \rightarrow 1000 = 2^n \rightarrow \log 1000 = n\log 2 \rightarrow$$
$$n = \frac{\log 1000}{\log 2} = 9.97 \approx \boxed{10\text{ octaves}}$$

26. For a closed tube, Fig. 12–12 indicates that $f_1 = \dfrac{v}{4\ell}$. We assume the bass clarinet is at room temperature.

$$f_1 = \frac{v}{4\ell} \rightarrow \ell = \frac{v}{4f_1} = \frac{343\text{ m/s}}{4(69\text{ Hz})} = \boxed{1.2\text{ m}}$$

27. For a vibrating string, the frequency of the fundamental mode is given by Eq. 11–19b combined with Eq. 11–13.

$$f=\frac{v}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{F_{\mathrm{T}}}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{F_{\mathrm{T}}}{m/\ell}} \quad\rightarrow\quad F_{\mathrm{T}}=4\ell f^2 m=4(0.32\text{ m})(440\text{ Hz})^2(3.5\times10^{-4}\text{ kg})=\boxed{87\text{ N}}$$

$\boxed{28.}$ (*a*) If the pipe is closed at one end, only the odd harmonic frequencies are present.

$$f_n=\frac{n\ell}{4\ell}=nf_1,\ n=1,3,5,\cdots \quad\rightarrow\quad f_1=\frac{v}{4\ell}=\frac{343\text{ m/s}}{4(1.16\text{ m})}=\boxed{73.9\text{ Hz}}$$

$$f_3=3f_1=\boxed{222\text{ Hz}} \quad f_5=5f_1=\boxed{370\text{ Hz}} \quad f_7=7f_1=\boxed{517\text{ Hz}}$$

(*b*) If the pipe is open at both ends, then all the harmonic frequencies are present.

$$f_n=\frac{nv}{2\ell}=nf_1,\ n=1,3,5,\cdots \quad\rightarrow\quad f_1=\frac{v}{2\ell}=\frac{343\text{ m/s}}{2(1.16\text{ m})}=\boxed{148\text{ Hz}}$$

$$f_2=2f_1=\boxed{296\text{ Hz}} \quad f_3=3f_1=\boxed{444\text{ Hz}} \quad f_4=4f_1=\boxed{591\text{ Hz}}$$

29. (*a*) The length of the tube is one-fourth of a wavelength for this (one end closed) tube, so the wavelength is four times the length of the tube.

$$f=\frac{v}{\lambda}=\frac{343\text{ m/s}}{4(0.24\text{ m})}=\boxed{360\text{ Hz}}$$

(*b*) If the bottle is one-third full, then the effective length of the air column is reduced to 12 cm.

$$f=\frac{v}{\lambda}=\frac{343\text{ m/s}}{4(0.16\text{ m})}=\boxed{540\text{ Hz}}$$

30. For a pipe open at both ends, the fundamental frequency is given by $f_1=\frac{v}{2\ell}$, so the length for a given fundamental frequency is $\ell=\frac{v}{2f_1}$.

$$\ell_{20\text{ Hz}}=\frac{343\text{ m/s}}{2(20\text{ Hz})}=\boxed{8.6\text{ m}} \qquad \ell_{20\text{ kHz}}=\frac{343\text{ m/s}}{2(20{,}000\text{ Hz})}=\boxed{8.6\times10^{-3}\text{ m}}$$

31. For a fixed string, the frequency of the *n*th harmonic is given by $f_n=nf_1$. Thus, the fundamental for this string is $f_1=f_3/3=540\text{ Hz}/3=180\text{ Hz}$. When the string is fingered, it has a new length of 70% of the original length. The fundamental frequency of the vibrating string is also given by $f_1=\frac{v}{2\ell}$, and v is constant for the string, assuming its tension is not changed.

$$f_{1\,\text{fingered}}=\frac{v}{2\ell_{\text{fingered}}}=\frac{v}{2(0.70)\ell}=\frac{1}{0.70}f_1=\frac{180\text{ Hz}}{0.70}=\boxed{260\text{ Hz}}$$

32. We approximate the shell as a closed tube of length 15 cm and calculate the fundamental frequency.

$$f=\frac{v}{4\ell}=\frac{343\text{ m/s}}{4(0.15\text{ m})}=572\text{ Hz}\approx\boxed{570\text{ Hz}}$$

33. (*a*) We assume that the speed of waves on the guitar string does not change when the string is fretted. The fundamental frequency is given by $f = \frac{v}{2\ell}$, so the frequency is inversely proportional to the length.

$$f \propto \frac{1}{\ell} \quad \to \quad f\ell = \text{constant}$$

$$f_\text{E}\ell_\text{E} = f_\text{A}\ell_\text{A} \quad \to \quad \ell_\text{A} = \ell_\text{E}\frac{f_\text{E}}{f_\text{A}} = (0.68\text{ m})\left(\frac{330\text{ Hz}}{440\text{ Hz}}\right) = 0.51\text{ m}$$

The string should be fretted a distance $0.68\text{ m} - 0.51\text{ m} = \boxed{0.17\text{ m}}$ from the tuning nut of the guitar: (at the right-hand node in Fig. 12–8a of the textbook).

(*b*) The string is fixed at both ends and is vibrating in its fundamental mode. Thus, the wavelength is twice the length of the string (see Fig. 12–7).

$$\lambda = 2\ell = 2(0.51\text{ m}) = \boxed{1.02\text{ m}}$$

(*c*) The frequency of the sound will be the same as that of the string, $\boxed{440\text{ Hz}}$. The wavelength is given by the following:

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440\text{ Hz}} = \boxed{0.78\text{ m}}$$

34. (*a*) At $T = 18°\text{C}$, the speed of sound is given by $v = (331 + 0.60(18))\text{ m/s} = 341.8\text{ m/s}$. For an open pipe, the fundamental frequency is given by $f = \frac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \quad \to \quad \ell = \frac{v}{2f} = \frac{341.8\text{ m/s}}{2(262\text{ Hz})} = \boxed{0.652\text{ m}}$$

(*b*) The frequency of the standing wave in the tube is $\boxed{262\text{ Hz}}$. The wavelength is twice the length of the pipe, $\boxed{1.30\text{ m}}$.

(*c*) The wavelength and frequency are the same in the air, because it is air that is resonating in the organ pipe. The frequency is $\boxed{262\text{ Hz}}$ and the wavelength is $\boxed{1.30\text{ m}}$.

35. The speed of sound will change as the temperature changes, and that will change the frequency of the organ. Assume that the length of the pipe (and thus the resonant wavelength) does not change.

$$f_{22.0} = \frac{v_{22.0}}{\lambda} \quad f_{11} = \frac{v_{11}}{\lambda} \quad \Delta f = f_{11} - f_{22.0} = \frac{v_{11} - v_{22.0}}{\lambda}$$

$$\frac{\Delta f}{f} = \frac{\dfrac{v_{11} - v_{22.0}}{\lambda}}{\dfrac{v_{22.0}}{\lambda}} = \frac{v_{11}}{v_{22.0}} - 1 = \frac{331 + 0.60(11)}{331 + 0.60(22.0)} - 1 = -1.92\times10^{-2} = \boxed{-1.9\%}$$

36. A flute is a tube that is open at both ends, so the fundamental frequency is given by $f = \frac{v}{2\ell}$, where ℓ is the distance from the mouthpiece (antinode) to the first open side hole in the flute tube (antinode).

$$f = \frac{v}{2\ell} \quad \to \quad \ell = \frac{v}{2f} = \frac{343\text{ m/s}}{2(349\text{ Hz})} = \boxed{0.491\text{ m}}$$

37. (*a*) At $T = 22°\text{C}$, the speed of sound is $v = (331 + 0.60(22))\text{ m/s} = 344.2\text{ m/s}$. For an open pipe, the fundamental frequency is given by $f = \dfrac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \quad \rightarrow \quad \ell = \frac{v}{2f} = \frac{344.2\text{ m/s}}{2(294\text{ Hz})} = 0.58537\text{ m} \approx \boxed{0.585\text{ m}}$$

(*b*) The speed of sound in helium is given in Table 12–1 as 1005 m/s. Use this and the pipe's length to find the pipe's fundamental frequency.

$$f = \frac{v}{2\ell} = \frac{1005\text{ m/s}}{2(0.58537\text{ m})} = 858.43\text{ Hz} \approx \boxed{858\text{ Hz}}$$

38. (*a*) The difference between successive overtones for this pipe is 176 Hz. The difference between successive overtones for an open pipe is the fundamental frequency, and each overtone is an integer multiple of the fundamental. Since 264 Hz is not a multiple of 176 Hz, 176 Hz cannot be the fundamental, so the pipe cannot be open. Thus, it must be a $\boxed{\text{closed}}$ pipe.

(*b*) For a closed pipe, the successive overtones differ by twice the fundamental frequency. Thus, 176 Hz must be twice the fundamental, so the fundamental is $\boxed{88\text{ Hz}}$. This is verified since 264 Hz is three times the fundamental, 440 Hz is five times the fundamental, and 616 Hz is seven times the fundamental.

39. (*a*) The difference between successive overtones for an open pipe is the fundamental frequency.

$$f_1 = 330\text{ Hz} - 275\text{ Hz} = \boxed{55\text{ Hz}}$$

(*b*) The fundamental frequency is given by $f_1 = \dfrac{v}{2\ell}$. Solve this for the speed of sound.

$$v = 2\ell f_1 = 2(1.70\text{ m})(55\text{ Hz}) = 187\text{ m/s} \approx \boxed{190\text{ m/s}}$$

40. The difference in frequency for two successive harmonics is 40 Hz. For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz, with 280 Hz being the 7th harmonic and 320 Hz being the 8th harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz. But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz. So the pipe must be an $\boxed{\text{open pipe}}$.

$$f = \frac{v}{2\ell} \quad \rightarrow \quad \ell = \frac{v}{2f} = \frac{[331 + 0.60(23.0)]\text{m/s}}{2(40\text{ Hz})} = \boxed{4.3\text{ m}}$$

41. (*a*) The harmonics for the open pipe are $f_n = \dfrac{nv}{2\ell}$. To be audible, they must be below 20 kHz.

$$\frac{nv}{2\ell} < 2\times 10^4\text{ Hz} \quad \rightarrow \quad n < \frac{2(2.18\text{ m})(2\times 10^4\text{ Hz})}{343\text{ m/s}} = 254.2$$

Since there are 254 harmonics, there are $\boxed{253\text{ overtones}}$.

(*b*) The harmonics for the closed pipe are $f_n = \dfrac{nv}{4\ell}$, n odd. Again, they must be below 20 kHz.

$$\frac{nv}{4\ell} < 2\times 10^4\text{ Hz} \quad \rightarrow \quad n < \frac{4(2.18\text{ m})(2\times 10^4\text{ Hz})}{343\text{ m/s}} = 508.5$$

The values of n must be odd, so $n = 1, 3, 5, \cdots, 507$. There are 254 harmonics, so there are $\boxed{253\text{ overtones}}$.

42. A tube closed at both ends will have standing waves with displacement nodes at each end, so it has the same harmonic structure as a string that is fastened at both ends. Thus, the wavelength of the fundamental frequency is twice the length of the hallway, $\lambda_1 = 2\ell = 18\text{ m}$.

$$f_1 = \frac{v}{\lambda_1} = \frac{343\text{ m/s}}{18\text{ m}} = 19.056\text{ Hz} \approx \boxed{19\text{ Hz}};\ f_2 = 2f_1 = \boxed{38\text{ Hz}}$$

43. The tension and mass density of the string do not change, so the wave speed is constant. The frequency ratio for two adjacent notes is to be $2^{1/12}$. The frequency is given by $f = \dfrac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \quad\to\quad \frac{f_{\substack{\text{1st}\\\text{fret}}}}{f_{\text{unfingered}}} = \frac{\dfrac{v}{2\ell_{\substack{\text{1st}\\\text{fret}}}}}{\dfrac{v}{2\ell_{\text{unfingered}}}} = 2^{1/12} \quad\to\quad \ell_{\substack{\text{1st}\\\text{fret}}} = \frac{\ell_{\text{unfingered}}}{2^{1/12}} = \frac{75.0\text{ cm}}{2^{1/12}} = 70.79\text{ cm}$$

$$\ell_{\substack{\text{2nd}\\\text{fret}}} = \frac{\ell_{\substack{\text{1st}\\\text{fret}}}}{2^{1/12}} = \frac{\ell_{\text{unfingered}}}{2^{2/12}} \quad\to\quad \ell_{\substack{n\text{th}\\\text{fret}}} = \frac{\ell_{\text{unfingered}}}{2^{n/12}};\quad x_{\substack{n\text{th}\\\text{fret}}} = \ell_{\text{unfingerd}} - \ell_{\substack{n\text{th}\\\text{fret}}} = \ell_{\text{unfingered}}(1-2^{-n/12})$$

$$x_1 = (75.0\text{ cm})(1-2^{-1/12}) = \boxed{4.2\text{ cm}};\quad x_2 = (75.0\text{ cm})(1-2^{-2/12}) = \boxed{8.2\text{ cm}}$$

$$x_3 = (75.0\text{ cm})(1-2^{-3/12}) = \boxed{11.9\text{ cm}};\quad x_4 = (75.0\text{ cm})(1-2^{-4/12}) = \boxed{15.5\text{ cm}}$$

$$x_5 = (75.0\text{ cm})(1-2^{-5/12}) = \boxed{18.8\text{ cm}};\quad x_6 = (75.0\text{ cm})(1-2^{-6/12}) = \boxed{22.0\text{ cm}}$$

44. The ear canal can be modeled as a closed pipe of length 2.5 cm. The resonant frequencies are given by $f_n = \dfrac{nv}{4\ell}$, n odd. The first several frequencies are calculated here.

$$f_n = \frac{nv}{4\ell} = \frac{n(343\text{ m/s})}{4(2.5\times10^{-2}\text{ m})} = n(3430\text{ Hz}),\ n\text{ odd}$$

$$\boxed{f_1 = 3430\text{ Hz}\quad f_3 = 10{,}300\text{ Hz}\quad f_5 = 17{,}200\text{ Hz}}$$

In the graph, the most sensitive frequency is between 3000 and 4000 Hz. This corresponds to the fundamental resonant frequency of the ear canal. The sensitivity decrease above 4000 Hz, but is seen to “flatten out” around 10,000 Hz again, indicating higher sensitivity near 10,000 Hz than at surrounding frequencies. This 10,000-Hz relatively sensitive region corresponds to the first overtone resonant frequency of the ear canal.

45. From Eq. 11–18, the intensity is proportional to the square of the amplitude and the square of the frequency. From Fig. 12–15, the relative amplitudes are $\dfrac{A_2}{A_1} \approx 0.4$ and $\dfrac{A_3}{A_1} \approx 0.15$.

$$I = 2\pi^2 v\rho f^2 A^2 \quad\to\quad \frac{I_2}{I_1} = \frac{2\pi^2 v\rho f_2^2 A_2^2}{2\pi^2 v\rho f_1^2 A_1^2} = \frac{f_2^2 A_2^2}{f_1^2 A_1^2} = \left(\frac{f_2}{f_1}\right)^2 \left(\frac{A_2}{A_1}\right)^2 = 2^2(0.4)^2 = \boxed{0.64}$$

$$\frac{I_3}{I_1} = \left(\frac{f_3}{f_1}\right)^2 \left(\frac{A_3}{A_1}\right)^2 = 3^2(0.15)^2 = \boxed{0.20}$$

$$\beta_{2-1} = 10\log\frac{I_2}{I_1} = 10\log 0.64 = \boxed{-2\text{ dB}};\quad \beta_{3-1} = 10\log\frac{I_3}{I_1} = 10\log 0.24 = \boxed{-7\text{ dB}}$$

Answers may vary due to the reading of the figure.

46. The beat period is 2.0 seconds, so the beat frequency is the reciprocal of that, 0.50 Hz. Thus, the other string is off in frequency by $\boxed{\pm 0.50\ \text{Hz}}$. The beating does not tell the tuner whether the second string is too high or too low.

47. The 5000-Hz shrill whine is the beat frequency generated by the combination of the two sounds. This means that the brand X whistle is either 5000 Hz higher or 5000 Hz lower than the known-frequency whistle. If it were 5000 Hz lower, then it would just barely be in the audible range for humans. Since humans cannot hear it, the brand X whistle must be 5000 Hz higher than the known frequency whistle. Thus, the brand X frequency is $23.5\ \text{kHz} + 5\ \text{kHz} = \boxed{28.5\ \text{kHz}}$. Since the original frequencies are good to 0.1 KHz, we assume that the 5000-Hz value is 5.0 kHz.

48. The beat frequency is the difference in the two frequencies, or $277\ \text{Hz} - 262\ \text{Hz} = \boxed{15\ \text{Hz}}$. If both frequencies are reduced by a factor of 4, then the difference between the two frequencies will also be reduced by a factor of 4, so the beat frequency will be $\frac{1}{4}(15\ \text{Hz}) = 3.75\ \text{Hz} \approx \boxed{3.8\ \text{Hz}}$.

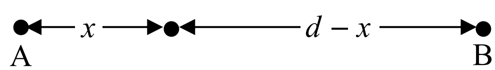
49. Since there are 3 beats/s when sounded with the 350-Hz tuning fork, the guitar string must have a frequency of either 347 Hz or 353 Hz. Since there are 8 beats/s when sounded with the 355-Hz tuning fork, the guitar string must have a frequency of either 347 Hz or 363 Hz. The common value is $\boxed{347\ \text{Hz}}$.

50. The fundamental frequency of the violin string is given by $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu}} = 294\ \text{Hz}$. Change the tension to find the new frequency and then subtract the two frequencies to find the beat frequency.

$$f' = \frac{1}{2\ell}\sqrt{\frac{(0.975)F_\text{T}}{\mu}} = \sqrt{0.975}\,\frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu}} = \sqrt{0.975}\,f$$

$$\Delta f = f - f' = f(1-\sqrt{0.975}) = (294\ \text{Hz})(1-\sqrt{0.975}) = \boxed{3.7\ \text{Hz}}$$

51. (*a*) Since the sounds are initially 180° out of phase, another 180° of phase must be added by a path length difference. Thus, the difference of the distances from the speakers to the point of constructive interference must be half of a wavelength. See the diagram.



$$(d-x) - x = \tfrac{1}{2}\lambda \quad\rightarrow\quad d = 2x + \tfrac{1}{2}\lambda \quad\rightarrow\quad d_\text{min} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{343\ \text{m/s}}{2(305\ \text{Hz})} = \boxed{0.562\ \text{m}}$$

This minimum distance occurs when the observer is right at one of the speakers. If the speakers are separated by more than 0.583 m, the location of constructive interference will be moved away from the speakers, along the line between the speakers.

(*b*) Since the sounds are already 180° out of phase, as long as the listener is equidistant from the speakers, there will be completely destructive interference. So even if the speakers have a tiny separation, the point midway between them will be a point of completely destructive interference. The minimum separation between the speakers is $\boxed{0}$.

52. The beat frequency is 3 beats per 2.5 seconds, or 1.2 Hz. We assume the strings are the same length and the same mass density.

(*a*) The other string is either $220.0\ \text{Hz} - 1.2\ \text{Hz} = \boxed{218.8\ \text{Hz}}$ or $220.0\ \text{Hz} + 1.2\ \text{Hz} = \boxed{221.2\ \text{Hz}}$.

(*b*) Since $f = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$, we have $f \propto \sqrt{F_T} \rightarrow \dfrac{f}{\sqrt{F_T}} = \dfrac{f'}{\sqrt{F'_T}} \rightarrow F'_T = F_T\left(\dfrac{f'}{f}\right)^2$.

To change 218.8 Hz to 220.0 Hz: $F' = F_T\left(\dfrac{220.0}{218.8}\right)^2 = 1.011F_T$, $\boxed{\text{1.1\% increase}}$.

To change 221.2 Hz to 220.0 Hz: $F' = F_T\left(\dfrac{220.0}{221.2}\right)^2 = 0.9892F_T$, $\boxed{\text{1.1\% decrease}}$.

53. (a) For destructive interference, the smallest path difference must be one-half wavelength. Thus, the wavelength in this situation must be twice the path difference, or 1.00 m.

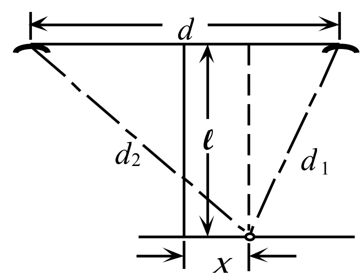
$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{1.00\text{ m}} = \boxed{343\text{ Hz}}$$

(*b*) There will also be destructive interference if the path difference is 1.5 wavelengths, 2.5 wavelengths, etc.

$$\Delta\ell = 1.5\lambda \rightarrow \lambda = \frac{0.50\text{ m}}{1.5} = 0.333\text{ m} \rightarrow f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.33\text{ m}} = 1029\text{ Hz} \approx \boxed{1000\text{ Hz}}$$

$$\Delta\ell = 2.5\lambda \rightarrow \lambda = \frac{0.50\text{ m}}{2.5} = 0.20\text{ m} \rightarrow f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.20\text{ m}} = 1715\text{ Hz} \approx \boxed{1700\text{ Hz}}$$

54. (*a*) The microphone must be moved to the right until the difference in distances from the two sources is half a wavelength. See the diagram. We square the expression, collect terms, isolate the remaining square root, and square again.



$$d_2 - d_1 = \tfrac{1}{2}\lambda \rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}d + x\right)^2 + \ell^2} - \sqrt{\left(\tfrac{1}{2}d - x\right)^2 + \ell^2} = \tfrac{1}{2}\lambda \rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}d + x\right)^2 + \ell^2} = \tfrac{1}{2}\lambda + \sqrt{\left(\tfrac{1}{2}d - x\right)^2 + \ell^2} \rightarrow$$

$$\left(\tfrac{1}{2}d + x\right)^2 + \ell^2 = \tfrac{1}{4}\lambda^2 + 2\left(\tfrac{1}{2}\lambda\right)\sqrt{\left(\tfrac{1}{2}d - x\right)^2 + \ell^2} + \left(\tfrac{1}{2}d - x\right)^2 + \ell^2 \rightarrow$$

$$2dx - \tfrac{1}{4}\lambda^2 = \lambda\sqrt{\left(\tfrac{1}{2}d - x\right)^2 + \ell^2} \rightarrow 4d^2x^2 - 2(2dx)\tfrac{1}{4}\lambda^2 + \tfrac{1}{16}\lambda^4 = \lambda^2\left[\left(\tfrac{1}{2}d - x\right)^2 + \ell^2\right]$$

$$4d^2x^2 - dx\lambda^2 + \tfrac{1}{16}\lambda^4 = \tfrac{1}{4}d^2\lambda^2 - dx\lambda^2 + x^2\lambda^2 + \lambda^2\ell^2 \rightarrow x = \lambda\sqrt{\frac{\left(\tfrac{1}{4}d^2 + \ell^2 - \tfrac{1}{16}\lambda^2\right)}{(4d^2 - \lambda^2)}}$$

The values are $d = 3.00$ m, $\ell = 3.20$ m, and $\lambda = v/f = (343\text{ m/s})/(474\text{ Hz}) = 0.7236$ m.

$$x = (0.7236\text{ m})\sqrt{\frac{\tfrac{1}{4}(3.00\text{ m})^2 + (3.20\text{ m})^2 - \tfrac{1}{16}(0.7236\text{ m})^2}{4(3.00\text{ m})^2 - (0.7236\text{ m})^2}} = \boxed{0.429\text{ m}}$$

(*b*) When the speakers are exactly out of phase, the maxima and minima will be interchanged. The intensity $\boxed{\text{maxima are 0.429 m to the left or right of the midpoint}}$, and the intensity $\boxed{\text{minimum is at the midpoint}}$.

55. (*a*) To find the beat frequency, calculate the frequency of each sound and then subtract the two frequencies.

$$f_{\text{beat}} = |f_1 - f_2| = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right| = (343\text{ m/s})\left|\frac{1}{2.54\text{ m}} - \frac{1}{2.72\text{ m}}\right| = 8.936\text{ Hz} \approx \boxed{8.9\text{ Hz}}$$

(*b*) The speed of sound is 343 m/s, and the beat frequency is 8.936 Hz. The regions of maximum intensity are one "beat wavelength" apart.

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{8.936\text{ Hz}} = 38.38\text{ m} \approx \boxed{38\text{ m}}$$

56. (*a*) Observer moving toward stationary source:

$$f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 + \frac{30.0\text{ m/s}}{343\text{ m/s}}\right)(1650\text{ Hz}) = \boxed{1790\text{ Hz}}$$

(*b*) Observer moving away from stationary source:

$$f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 - \frac{30.0\text{ m/s}}{343\text{ m/s}}\right)(1650\text{ Hz}) = \boxed{1510\text{ Hz}}$$

57. The moving object can be treated as a moving "observer" for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$f'_{\text{object}} = f_{\text{bat}}\left(1 - \frac{v_{\text{object}}}{v_{\text{snd}}}\right)$$

Then the object can be treated as a moving source emitting the frequency f'_{object} and the bat as a stationary observer.

$$f''_{\text{bat}} = \frac{f'_{\text{object}}}{\left(1 + \frac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{\left(1 - \frac{v_{\text{object}}}{v_{\text{snd}}}\right)}{\left(1 + \frac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{(v_{\text{snd}} - v_{\text{object}})}{(v_{\text{snd}} + v_{\text{object}})}$$

$$= (5.00\times10^4\text{ Hz})\left(\frac{343\text{ m/s} - 27.0\text{ m/s}}{343\text{ m/s} + 27.0\text{ m/s}}\right) = \boxed{4.27\times10^4\text{ Hz}}$$

58. The frequency received by the stationary car is higher than the frequency emitted by the stationary car, by $\Delta f = 4.5$ Hz.

$$f_{\text{obs}} = f_{\text{source}} + \Delta f = \frac{f_{\text{source}}}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)} \quad \rightarrow$$

$$f_{\text{source}} = \Delta f\left(\frac{v_{\text{snd}}}{v_{\text{source}}} - 1\right) = (4.5\text{ Hz})\left(\frac{343\text{ m/s}}{18\text{ m/s}} - 1\right) = 81.25\text{ Hz} \approx \boxed{81\text{ Hz}}$$

59. The wall can be treated as a stationary "observer" for calculating the frequency it receives. The bat is flying toward the wall.

$$f'_{\text{wall}} = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}$$

Then the wall can be treated as a stationary source emitting the frequency f'_{wall} and the bat as a moving observer, flying toward the wall.

$$f''_{\text{bat}} = f'_{\text{wall}}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{(v_{\text{snd}} + v_{\text{bat}})}{(v_{\text{snd}} - v_{\text{bat}})}$$

$$= (3.00\times10^4 \text{ Hz})\frac{343 \text{ m/s} + 6.0 \text{ m/s}}{343 \text{ m/s} - 6.0 \text{ m/s}} = \boxed{3.11\times10^4 \text{ Hz}}$$

$\boxed{60.}$ We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$f_{\text{obs}} = \frac{f_{\text{source}}}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{75 \text{ Hz}}{\left(1 - \frac{14.0 \text{ m/s}}{343 \text{ m/s}}\right)} = 78.19 \text{ Hz} \qquad f_{\text{beat}} = 78.19 \text{ Hz} - 75 \text{ Hz} = 3.19 \text{ Hz} \approx \boxed{3 \text{ Hz}}$$

61. The ocean wave has $\lambda = 44$ m and $v = 18$ m/s relative to the ocean floor. The frequency of the ocean wave is then $f = \frac{v}{\lambda} = \frac{18 \text{ m/s}}{44 \text{ m}} = 0.4091$ Hz.

(*a*) For the boat traveling west, the boat will encounter a Doppler shifted frequency, for an observer moving toward a stationary source. The speed $v = 18$ m/s represents the speed of the waves in the stationary medium, so it corresponds to the speed of sound in the Doppler formula. The time between encountering waves is the period of the Doppler shifted frequency.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) f = \left(1 + \frac{14 \text{ m/s}}{18 \text{ m/s}}\right)(0.4091 \text{ Hz}) = 0.7273 \text{ Hz} \quad \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.7273 \text{ Hz}} = 1.375 \text{ s} \approx \boxed{1.4 \text{ s}}$$

(*b*) For the boat traveling east, the boat will encounter a Doppler shifted frequency, for an observer moving away from a stationary source.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) f = \left(1 - \frac{14 \text{ m/s}}{18 \text{ m/s}}\right)(0.4091 \text{ Hz}) = 0.09091 \text{ Hz} \quad \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.09091 \text{ Hz}} = \boxed{11 \text{ s}}$$

62. (*a*) The observer is stationary, and the source is moving. First the source is approaching, and then the source is receding.

$$120.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=33.33\text{ m/s}$$

$$f'_{\substack{\text{source}\\\text{moving}\\\text{toward}}}=f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}=(1580\text{ Hz})\frac{1}{\left(1-\dfrac{33.33\text{ m/s}}{343\text{ m/s}}\right)}=\boxed{1750\text{ Hz}}$$

$$f'_{\substack{\text{source}\\\text{moving}\\\text{away}}}=f\frac{1}{\left(1+\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}=(1580\text{ Hz})\frac{1}{\left(1+\dfrac{33.33\text{ m/s}}{343\text{ m/s}}\right)}=\boxed{1440\text{ Hz}}$$

(*b*) Both the observer and the source are moving, so use Eq. 12–4.

$$90.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=25\text{ m/s}$$

$$f'_{\text{approaching}}=f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}+25\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})}=\boxed{1880\text{ Hz}}$$

$$f'_{\text{receding}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}-25\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})}=\boxed{1340\text{ Hz}}$$

(*c*) Both the observer and the source are moving, so again use Eq. 12–4.

$$80.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)=22.22\text{ m/s}$$

$$f'_{\substack{\text{police}\\\text{car}\\\text{approaching}}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}-22.22\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})}=\boxed{1640\text{ Hz}}$$

$$f'_{\substack{\text{police}\\\text{car}\\\text{receding}}}=f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})}=(1580\text{ Hz})\frac{(343\text{ m/s}+22.22\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})}=\boxed{1530\text{ Hz}}$$

63. The maximum Doppler shift occurs when the heart has its maximum velocity. Assume that the heart is moving away from the original source of sound. The beats arise from the combining of the original 2.25-MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts—one for the heart receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{heart}}=f_{\text{original}}\left(1-\frac{v_{\text{heart}}}{v_{\text{snd}}}\right);\quad f''_{\text{detector}}=\frac{f'_{\text{heart}}}{\left(1+\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)}=f_{\text{original}}\frac{\left(1-\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)}{\left(1+\dfrac{v_{\text{heart}}}{v_{\text{snd}}}\right)}=f_{\text{original}}\frac{(v_{\text{snd}}-v_{\text{heart}})}{(v_{\text{snd}}+v_{\text{heart}})}$$

$$\Delta f=f_{\text{original}}-f''_{\text{detector}}=f_{\text{original}}-f_{\text{original}}\frac{(v_{\text{snd}}-v_{\text{blood}})}{(v_{\text{snd}}+v_{\text{blood}})}=f_{\text{original}}\frac{2v_{\text{blood}}}{(v_{\text{snd}}+v_{\text{blood}})}\quad\rightarrow$$

$$v_{\text{blood}}=v_{\text{snd}}\frac{\Delta f}{2f_{\text{original}}-\Delta f}=(1.54\times10^{3}\text{ m/s})\frac{240\text{ Hz}}{2(2.25\times10^{6}\text{ Hz})-240\text{ Hz}}=\boxed{0.0821\text{ m/s}}$$

If instead we had assumed that the heart was moving toward the original source of sound, we would get $v_{\text{blood}} = v_{\text{snd}} \dfrac{\Delta f}{2 f_{\text{original}} + \Delta f}$. Since the beat frequency is much smaller than the original frequency, the Δf term in the denominator does not significantly affect the answer.

64. We represent the Mach number by the symbol M.

(*a*) $M = \dfrac{v_{\text{obj}}}{v_{\text{snd}}} \rightarrow v_{\text{obj}} = M v_{\text{snd}} = (0.33)[(331\text{ m/s}) + 0.60(24)] = 113.98\text{ m/s} \approx \boxed{110\text{ m/s}}$

(*b*) $M = \dfrac{v_{\text{obj}}}{v_{\text{snd}}} \rightarrow v_{\text{snd}} = \dfrac{v_{\text{obj}}}{M} = \dfrac{3000\text{ km/h}}{3.1} = 967.7\text{ km/h}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = \boxed{270\text{ m/s}}$

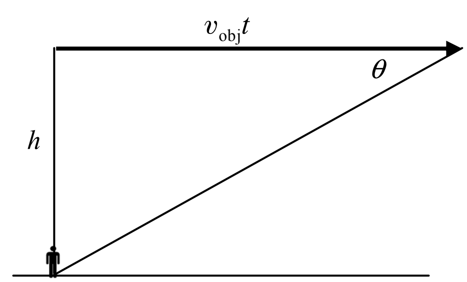
65. The speed is found from Eq. 12–5.

$$\sin\theta = \frac{v_{\text{wave}}}{v_{\text{obj}}} \rightarrow v_{\text{obj}} = \frac{v_{\text{wave}}}{\sin\theta} = \frac{2.2\text{ km/h}}{\sin 12^\circ} = 10.58\text{ km/h} \approx \boxed{11\text{ km/h}}$$

66. (*a*) The angle of the shock wave front relative to the direction of motion is given by Eq. 12–5.

$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}} = \frac{v_{\text{snd}}}{2.1 v_{\text{snd}}} = \frac{1}{2.1} \rightarrow \theta = \sin^{-1}\frac{1}{2.1} = 28.44^\circ = \boxed{28^\circ}$$

(*b*) The displacement of the plane $(v_{\text{obj}}t)$ from the time it passes overhead to the time the shock wave reaches the observer is shown, along with the shock wave front. From the displacement and height of the plane, the time is found.



$$\tan\theta = \frac{h}{v_{\text{obj}}t} \rightarrow$$

$$t = \frac{h}{v_{\text{obj}}\tan\theta} = \frac{6500\text{ m}}{(2.1)(310\text{ m/s})\tan 28.44^\circ} = 18\ 44\text{ s} \approx \boxed{18\text{ s}}$$

67. (*a*) The Mach number is the ratio of the object's speed to the speed of sound.

$$M = \frac{v_{\text{obs}}}{v_{\text{sound}}} = \frac{(1.5\times10^4\text{ km/h})\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{42\text{ m/s}} = 99.21 \approx \boxed{99}$$

(*b*) Use Eq. 12–5 to find the angle.

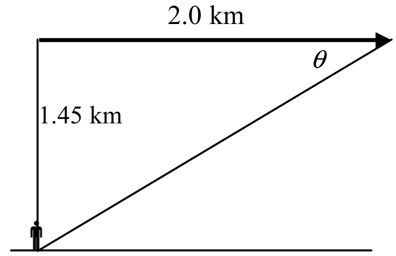
$$\theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{1}{M} = \sin^{-1}\frac{1}{99.21} = \boxed{0.58^\circ}$$

68. From Eq. 12–7, $\sin\theta = \dfrac{v_{\text{snd}}}{v_{\text{obj}}}$. The speed of sound in the ocean is taken from Table 12–1.

(*a*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{343\text{ m/s}}{9200\text{ m/s}} = \boxed{2.1^\circ}$

(*b*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{1560\text{ m/s}}{9200\text{ m/s}} = \boxed{9.8^\circ}$

69. (*a*) The displacement of the plane from the time it passes overhead to the time the shock wave reaches the listener is shown, along with the shock wave front. From the displacement and height of the plane, the angle of the shock wave front relative to the direction of motion can be found.



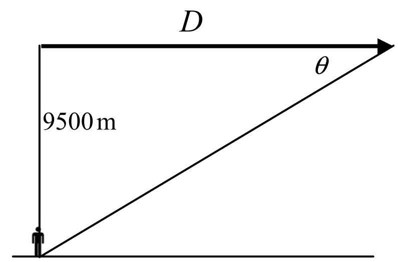
$$\tan\theta = \frac{1.45\text{ km}}{2.0\text{ km}} \rightarrow \theta = \tan^{-1}\frac{1.45}{2.0} = 35.94° \approx \boxed{36°}$$

(*b*) The speed and Mach number are found from Eq. 12–5.

$$\upsilon_{\text{obj}} = \frac{\upsilon_{\text{snd}}}{\sin\theta} = \frac{330\text{ m/s}}{\sin 35.94°} = 562.2\text{ m/s} \approx \boxed{560\text{ m/s}}$$

$$M = \frac{\upsilon_{\text{obj}}}{\upsilon_{\text{snd}}} = \frac{1}{\sin\theta} = \frac{1}{\sin 35.94°} = \boxed{1.7}$$

70. Find the angle of the shock wave, then the distance the plane has traveled when the shock wave reaches the observer. Use Eq. 12–5.



$$\theta = \sin^{-1}\frac{\upsilon_{\text{snd}}}{\upsilon_{\text{obj}}} = \sin^{-1}\frac{\upsilon_{\text{snd}}}{2.0\upsilon_{\text{snd}}} = \sin^{-1}\frac{1}{2.0} = 30°$$

$$\tan\theta = \frac{9500\text{ m}}{D} \rightarrow D = \frac{9500\text{ m}}{\tan 30°} = 16{,}454\text{ m} \approx \boxed{16\text{ km}}$$

71. The minimum time between pulses would be the time for a pulse to travel from the boat to the maximum distance and back again. The total distance traveled by the pulse will be 170 m at the speed of sound in fresh water, 1440 m/s.

$$d = \upsilon t \rightarrow t = \frac{d}{\upsilon} = \frac{170\text{ m}}{1440\text{ m/s}} = \boxed{0.12\text{ s}}$$

72. The single mosquito creates a sound intensity of $I_0 = 1\times10^{-12}\text{ W/m}^2$. Thus, 200 mosquitoes will create a sound intensity of 200 times that of a single mosquito.

$$I = 100I_0 \quad \beta = 10\log\frac{200I_0}{I_0} = 10\log 200 = \boxed{23\text{ dB}}$$

73. The two sound level values must be converted to intensities; then the intensities are added and converted back to sound level.

$$I_{82}:\ 81\text{ dB} = 10\log\frac{I_{81}}{I_0} \rightarrow I_{81} = 10^{8.1}I_0 = 1.259\times10^8 I_0$$

$$I_{87}:\ 87\text{ dB} = 10\log\frac{I_{87}}{I_0} \rightarrow I_{87} = 10^{8.7}I_0 = 5.012\times10^8 I_0$$

$$I_{\text{total}} = I_{82} + I_{87} = (6.271\times10^8)I_0 \rightarrow$$

$$\beta_{\text{total}} = 10\log\frac{6.271\times10^8 I_0}{I_0} = 10\log 6.597\times10^8 = 87.97 \approx \boxed{88\text{ dB}}$$

74. The power output is found from the intensity, which is the power radiated per unit area.

$$115\text{ dB} = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{11.5}I_0 = 10^{11.5}(1.0\times10^{-12}\text{ W/m}^2) = 3.162\times10^{-1}\text{ W/m}^2$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} \quad\rightarrow\quad P = 4\pi r^2 I = 4\pi(8.25\text{ m})^2(3.162\times10^{-1}\text{ W/m}^2) = 270.45\text{W} \approx \boxed{270\text{ W}}$$

The answer has 3 significant figures.

75. Relative to the 1000-Hz output, the 15-kHz output is –12 dB.

$$-12\text{ dB} = 10\log\frac{P_{15\text{ kHz}}}{225\text{ W}} \quad\rightarrow\quad -1.2 = \log\frac{P_{15\text{ kHz}}}{225\text{ W}} \quad\rightarrow\quad 10^{-1.2} = \frac{P_{15\text{ kHz}}}{225\text{ W}} \quad\rightarrow$$

$$P_{15\text{ kHz}} = (225\text{ W})(10^{-1.2}) = \boxed{14\text{ W}}$$

76. The 130-dB level is used to find the intensity, and the intensity is used to find the power. It is assumed that the jet airplane engine radiates equally in all directions.

$$\beta = 130\text{ dB} = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = 10^{13}I_0 = 10^{13}(1.0\times10^{-12}\text{ W/m}^2) = 1.0\times10^{1}\text{ W/m}^2$$

$$P = IA = I\pi r^2 = (1.0\times10^{1}\text{ W/m}^2)\pi(2.0\times10^{-2})^2 = \boxed{0.013\text{ W}}$$

77. (*a*) The gain is given by $\beta = 10\log\frac{P_{\text{out}}}{P_{\text{in}}} = 10\log\frac{135\text{ W}}{1.0\times10^{-3}\text{ W}} = \boxed{51\text{ dB}}$.

(*b*) We solve the gain equation for the noise power level.

$$\beta = 10\log\frac{P_{\text{signal}}}{P_{\text{noise}}} \quad\rightarrow\quad P_{\text{noise}} = \frac{P_{\text{signal}}}{10^{\beta/10}} = \frac{10\text{ W}}{10^{93/10}} = \boxed{5\times10^{-9}\text{ W}}$$

78. The strings are both tuned to the same frequency, and they have the same length. The mass per unit length is the density times the cross-sectional area. The frequency is related to the tension by Eqs. 11–13 and 11–19b.

$$f = \frac{v}{2\ell};\ v = \sqrt{\frac{F_{\text{T}}}{\mu}} \quad\rightarrow\quad f = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\rho\pi r^2}} \quad\rightarrow\quad F_{\text{T}} = 4\ell^2\rho f^2\pi r^2 \quad\rightarrow$$

$$\frac{F_{\text{T high}}}{F_{\text{T low}}} = \frac{4\ell^2\rho f^2\pi r_{\text{high}}^2}{4\ell^2\rho f^2\pi r_{\text{low}}^2} = \left(\frac{r_{\text{high}}}{r_{\text{low}}}\right)^2 = \left(\frac{\frac{1}{2}d_{\text{high}}}{\frac{1}{2}d_{\text{low}}}\right)^2 = \left(\frac{0.724\text{ mm}}{0.699\text{ mm}}\right)^2 = \boxed{1.07}$$

$\boxed{79.}$ The apparatus is a closed tube. The water level is the closed end, so it is a node of air displacement. As the water level lowers, the distance from one resonance level to the next corresponds to the distance between adjacent nodes, which is one-half wavelength.

$$\Delta\ell = \tfrac{1}{2}\lambda \quad\rightarrow\quad \lambda = 2\Delta\ell = 2(0.395\text{ m} - 0.125\text{ m}) = 0.540\text{ m}$$

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.540\text{ m}} = \boxed{635\text{ Hz}}$$

80. The fundamental frequency of a tube closed at one end is given by $f_1 = \frac{v}{4\ell}$. The change in air temperature will change the speed of sound, resulting in two different frequencies.

$$\frac{f_{31.0^\circ\text{C}}}{f_{25.0^\circ\text{C}}} = \frac{\dfrac{v_{31.0^\circ\text{C}}}{4\ell}}{\dfrac{v_{31.0^\circ\text{C}}}{4\ell}} = \frac{v_{31.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}} \quad\rightarrow\quad f_{31.0^\circ\text{C}} = f_{25.0^\circ\text{C}}\left(\frac{v_{31.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}}\right)$$

$$\Delta f = f_{31.0^\circ\text{C}} - f_{25.0^\circ\text{C}} = f_{25.0^\circ\text{C}}\left(\frac{v_{31.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}} - 1\right) = (349\text{ Hz})\left(\frac{331+0.60(31.0)}{331+0.60(25.0)} - 1\right) = 3.63\text{ Hz} \approx \boxed{4\text{ Hz}}$$

81. Call the frequencies of four strings of the violin f_A, f_B, f_C, and f_D, with f_A the lowest pitch. The mass per unit length will be named μ. All strings are the same length and have the same tension. For a string with both ends fixed, the fundamental frequency is given by $f_1 = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu}}$.

$$f_B = 1.5 f_A \quad\rightarrow\quad \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_B}} = 1.5\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \quad\rightarrow\quad \mu_B = \frac{\mu_A}{(1.5)^2} = \boxed{0.44\mu_A}$$

$$f_C = 1.5 f_B = (1.5)^2 f_A \quad\rightarrow\quad \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_C}} = (1.5)^2\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \quad\rightarrow\quad \mu_C = \frac{\mu_A}{(1.5)^4} = \boxed{0.20\mu_A}$$

$$f_D = 1.5 f_C = (1.5)^3 f_A \quad\rightarrow\quad \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_D}} = (1.5)^3\frac{1}{2\ell}\sqrt{\frac{F_T}{\mu_A}} \quad\rightarrow\quad \mu_D = \frac{\mu_A}{(1.5)^6} = \boxed{0.088\mu_A}$$

82. We combine the expression for the frequency in a closed tube with the Doppler shift for a source moving away from a stationary observer, Eq. 12–2b.

$$f_1 = \frac{v_{\text{snd}}}{4\ell}$$

$$f' = \frac{f_1}{\left(1 + \dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{v_{\text{snd}}}{4\ell\left(1 + \dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{343\text{ m/s}}{4(7.10\times10^{-2}\text{ m})\left(1 + \dfrac{25\text{ m/s}}{343\text{ m/s}}\right)} = 1127\text{ Hz} \approx \boxed{1130\text{ Hz}}$$

83. The effective length of the tube is $\ell_{\text{eff}} = \ell + \frac{1}{3}D = 0.55\text{ m} + \frac{1}{3}(0.030\text{ m}) = 0.56\text{ m}$.

Uncorrected frequencies: $f_n = \frac{(2n-1)v}{4\ell},\ n = 1, 2, 3, \cdots \quad\rightarrow$

$$f_{1-4} = (2n-1)\frac{343\text{ m/s}}{4(0.55\text{ m})} = 156\text{ Hz, } 468\text{ Hz, } 770\text{ Hz, } 1090\text{ Hz}$$

Corrected frequencies: $f_n = \frac{(2n-1)v}{4\ell_{\text{eff}}},\ n = 1, 2, 3, \cdots \quad\rightarrow$

$$f_{1-4} = (2n-1)\frac{343\text{ m/s}}{4(0.56\text{ m})} = 153\text{ Hz, } 459\text{ Hz, } 766\text{ Hz, } 1070\text{ Hz}$$

$$\approx \boxed{150\text{ Hz, } 460\text{ Hz, } 770\text{ Hz, } 1100\text{ Hz}}$$

84. As the train approaches, the observed frequency is given by $f'_{\text{approach}} = \dfrac{f}{\left(1 - \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)}$. As the train recedes, the observed frequency is given by $f'_{\text{recede}} = \dfrac{f}{\left(1 + \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)}$. Solve each expression for f, equate them, and then solve for v_{train}.

$$f'_{\text{approach}}\left(1 - \frac{v_{\text{train}}}{v_{\text{snd}}}\right) = f'_{\text{recede}}\left(1 + \frac{v_{\text{train}}}{v_{\text{snd}}}\right) \rightarrow$$

$$v_{\text{train}} = v_{\text{snd}} \frac{(f'_{\text{approach}} - f'_{\text{recede}})}{(f'_{\text{approach}} + f'_{\text{recede}})} = (343 \text{ m/s}) \frac{(565 \text{ Hz} - 486 \text{ Hz})}{(565 \text{ Hz} + 486 \text{ Hz})} = \boxed{26 \text{ m/s}}$$

85. The Doppler shift is 3.5 Hz, and the emitted frequency from both trains is 508 Hz. Thus, the frequency received by the conductor on the stationary train is 511.5 Hz. Use this to find the moving train's speed.

$$f' = f \frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{source}})} \rightarrow v_{\text{source}} = \left(1 - \frac{f}{f'}\right) v_{\text{snd}} = \left(1 - \frac{508 \text{ Hz}}{511.5 \text{ Hz}}\right)(343 \text{ m/s}) = \boxed{2.35 \text{ m/s}}$$

86. (*a*) Since both speakers are moving toward the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears $\boxed{\text{no beats}}$.

(*b*) The observer will detect an increased frequency from the speaker moving toward him and a decreased frequency from the speaker moving away. The difference in those two frequencies will be the beat frequency that is heard.

$$f'_{\text{towards}} = f \frac{1}{\left(1 - \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)} \qquad f'_{\text{away}} = f \frac{1}{\left(1 + \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)}$$

$$f'_{\text{towards}} - f'_{\text{away}} = f \frac{1}{\left(1 - \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)} - f \frac{1}{\left(1 + \dfrac{v_{\text{train}}}{v_{\text{snd}}}\right)} = f\left[\frac{v_{\text{snd}}}{(v_{\text{snd}} - v_{\text{train}})} - \frac{v_{\text{snd}}}{(v_{\text{snd}} + v_{\text{train}})}\right]$$

$$(348 \text{ Hz})\left[\frac{343 \text{ m/s}}{343 \text{ m/s} - 12.0 \text{ m/s}} - \frac{343 \text{ m/s}}{(343 \text{ m/s} + 12.0 \text{ m/s})}\right] = 24.38 \text{ Hz} \approx \boxed{24 \text{ Hz}}$$

(*c*) Since both speakers are moving away from the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears $\boxed{\text{no beats}}$.

87. For each pipe, the fundamental frequency is given by $f = \dfrac{v}{2\ell}$. Find the frequency of the shortest pipe.

$$f = \frac{v}{2\ell} = \frac{343 \text{ m/s}}{2\ (2.40 \text{ m})} = 71.46 \text{ Hz}$$

The longer pipe has a lower frequency. Since the beat frequency is 6.0 Hz, the frequency of the longer pipe must be 65.46 Hz. Use that frequency to find the length of the longer pipe.

$$f = \frac{v}{2\ell} \rightarrow \ell = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(65.46 \text{ Hz})} = \boxed{2.62 \text{ m}}$$

88. Use Eq. 12–4, which applies when both source and observer are in motion. There will be two Doppler shifts in this problem—first for the emitted sound with the bat as the source and the moth as the observer and then for the reflected sound with the moth as the source and the bat as the observer.

$$f'_{\text{moth}} = f_{\text{bat}}\frac{(v_{\text{snd}}+v_{\text{moth}})}{(v_{\text{snd}}-v_{\text{bat}})} \quad f''_{\text{bat}} = f'_{\text{moth}}\frac{(v_{\text{snd}}+v_{\text{bat}})}{(v_{\text{snd}}-v_{\text{moth}})} = f_{\text{bat}}\frac{(v_{\text{snd}}+v_{\text{moth}})}{(v_{\text{snd}}-v_{\text{bat}})}\frac{(v_{\text{snd}}+v_{\text{bat}})}{(v_{\text{snd}}-v_{\text{moth}})}$$

$$= (51.35\text{ kHz})\frac{(343+5.0)}{(343-7.8)}\frac{(343+7.8)}{(343-5.0)} = \boxed{55.3\text{ kHz}}$$

89. It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus, the time for the pulse to travel to the moth and back again is 67.0 ms. The distance to the moth is half the distance that the sound can travel in 67.0 ms, since the sound must reach the moth and return during the 67.0 ms.

$$d = v_{\text{snd}}t = (343\text{ m/s})\tfrac{1}{2}(67.0\times10^{-3}\text{ s}) = \boxed{11.5\text{ m}}$$

90. The person will hear a frequency $f'_{\text{toward}} = f\left(1+\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk toward. The person will hear a frequency $f'_{\text{away}} = f\left(1-\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk away from. The beat frequency is the difference in those two frequencies.

$$f'_{\text{towards}} - f'_{\text{away}} = f\left(1+\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) - f\left(1-\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) = 2f\frac{v_{\text{walk}}}{v_{\text{snd}}} = 2(282\text{ Hz})\frac{1.6\text{ m/s}}{343\text{ m/s}} = \boxed{2.6\text{ Hz}}$$

91. The ratio of sound intensity passing through the door to the original sound intensity is a 30-dB decrease.

$$\beta = 10\log I/I_0 = -30 \quad\rightarrow\quad \log I/I_0 = -3 \quad\rightarrow\quad I = 10^{-3}I_0$$

Only $\boxed{1/1000}$ of the sound intensity passes through the door.

92. The alpenhorn can be modeled as an open tube, so the fundamental frequency is $f = \dfrac{v}{2\ell}$, and the overtones are given by $f_n = \dfrac{nv}{2\ell}$, $n = 1, 2, 3, \ldots$.

$$f_1 = \frac{v}{2\ell} = \frac{343\text{ m/s}}{2(3.4\text{ m})} = 50.44\text{ Hz} \approx \boxed{50\text{ Hz}}$$

$$f_n = nf_1 = f_{\text{F\#}} \quad\rightarrow\quad n(50.44\text{ Hz}) = 370\text{ Hz} \quad\rightarrow\quad n = \frac{370}{50.44} = 7.34$$

Thus, the 7th harmonic, which is the $\boxed{\text{6th overtone}}$, is close to $\text{F}^{\#}$.

93. The walls of the room must be air displacement nodes, so the dimensions of the room between two parallel boundaries correspond to a half wavelength of sound. Fundamental frequencies are then given by $f = \dfrac{v}{2\ell}$.

$$\text{Length: } f = \frac{v}{2\ell} = \frac{343\text{ m/s}}{2(4.7\text{ m})} = \boxed{36\text{ Hz}} \qquad \text{Width: } f = \frac{v}{2\ell} = \frac{343\text{ m/s}}{2(3.6\text{ m})} = \boxed{48\text{ Hz}}$$

$$\text{Height: } f = \frac{v}{2\ell} = \frac{343\text{ m/s}}{2(2.8\text{ m})} = \boxed{61\text{ Hz}}$$

94. (*a*) The "singing" rod is manifesting standing waves. When holding the rod at its midpoint, it has a node at its midpoint and antinodes at its ends. Thus, the length of the rod is a half wavelength. The speed of sound in aluminum is found in Table 12–1.

$$f = \frac{v}{\lambda} = \frac{v}{2\ell} = \frac{5100\text{ m/s}}{1.6\text{ m}} = 3187.5\text{ Hz} \approx \boxed{3200\text{ Hz}}$$

(*b*) The wavelength of sound in the rod is twice the length of the rod, $\boxed{1.6\text{ m}}$.

(*c*) The wavelength of the sound in air is determined by the frequency and the speed of sound in air.

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{3187.5\text{ Hz}} = 0.1076\text{ m} \approx \boxed{0.11\text{ m}}$$

95. Equation 11–18 gives the relationship between intensity and the displacement amplitude: $I = 2\pi^2 v\rho f^2 A^2$, where A is the displacement amplitude. Thus, $I \propto A^2$, or $A \propto \sqrt{I}$. Since the intensity increased by a factor of 10^{12}, the amplitude would increase by a factor of the square root of the intensity increase, or $\boxed{10^6}$.

96. The beats arise from the combining of the original 3.5-MHz frequency with the reflected signal, which has been Doppler shifted. There are two Doppler shifts—one for the blood cells receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{blood}} = f_{\text{original}}\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right) \quad f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{(v_{\text{snd}} - v_{\text{blood}})}{(v_{\text{snd}} + v_{\text{blood}})}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{(v_{\text{snd}} - v_{\text{blood}})}{(v_{\text{snd}} + v_{\text{blood}})} = f_{\text{original}}\frac{2v_{\text{blood}}}{(v_{\text{snd}} + v_{\text{blood}})}$$

$$= (3.5\times10^6\text{ Hz})\frac{2(3.0\times10^{-2})}{(1.54\times10^3\text{ m/s} + 3.0\times10^{-2})} = 136.36\text{ Hz} \approx \boxed{140\text{ Hz}}$$

Solutions to Search and Learn Problems

1. The intensity can be found from the sound level in decibels.

$$\beta = 10\log\frac{I}{I_0} \quad \rightarrow \quad I = 10^{\beta/10} I_0 = 10^{12}(10^{-12}\text{ W/m}^2) = 1.0\text{ W/m}^2$$

Consider a square perpendicular to the direction of travel of the sound wave. The intensity is the energy transported by the wave across a unit area perpendicular to the direction of travel, per unit time. So $I = \frac{\Delta E}{S\Delta t}$, where S is the area of the square. Since the energy is "moving" with the wave, the "speed" of the energy is v, the wave speed. In a time Δt, a volume equal to $\Delta V = Sv\Delta t$ would contain all of the energy that had been transported across the area S. Combine these relationships to find the energy in the volume.

$$I = \frac{\Delta E}{S\Delta t} \quad \rightarrow \quad \Delta E = IS\Delta t = \frac{I\Delta V}{v} = \frac{(1.0\text{ W/m}^2)(0.010\text{ m})^3}{343\text{ m/s}} = \boxed{2.9\times10^{-9}\text{ J}}$$

2. As the car approaches, the frequency of the sound from the engine is Doppler shifted up, as given by Eq. 12–2a. As the car moves away, the frequency of the engine sound is Doppler shifted down, as given by Eq. 12–2b. Since the frequency shift is exactly an octave, we know that $f'_{\text{toward}} = 2f'_{\text{away}}$. We then solve for the car's speed.

$$f'_{\text{toward}} = \frac{f}{\left(1-\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)};\quad f'_{\text{away}} = \frac{f}{\left(1+\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)}$$

$$f'_{\text{toward}} = 2f'_{\text{away}} \rightarrow \frac{f}{\left(1-\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} = 2\frac{f}{\left(1+\dfrac{v_{\text{source}}}{v_{\text{snd}}}\right)} \rightarrow \left(1+\frac{v_{\text{source}}}{v_{\text{snd}}}\right) = 2\left(1-\frac{v_{\text{source}}}{v_{\text{snd}}}\right) \rightarrow$$

$$v_{\text{source}} = \tfrac{1}{3}v_{\text{snd}} = \tfrac{1}{3}(343\text{ m/s}) = \boxed{114\text{ m/s}}$$

3. Since the sound is loudest at points equidistant from the two sources, the two sources must be in phase. The difference in distance from the two sources must be an odd number of half wavelengths for destructive interference. See Fig. 12–17 and the accompanying discussion for a derivation of that relationship.

$$0.25\text{ m} = \lambda/2 \rightarrow \lambda = 0.50\text{ m}$$

$$f = v/\lambda = 343\text{ m/s}/0.50\text{ m} = 686\text{ Hz}$$

$$0.25\text{ m} = 3\lambda/2 \rightarrow \lambda = 0.1667\text{ m}$$

$$f = v/\lambda = 343\text{ m/s}/0.1667\text{ m} = 2060\text{ Hz (out of range)}$$

Thus, we see that the frequency must be about $\boxed{690\text{ Hz}}$.

4. The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other, so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, as if the speed of sound were different, but the frequency of the waves doesn't change. We do a detailed analysis of this claim in part (*a*).

(*a*) The wind velocity is a movement of the medium, so it adds or subtracts from the speed of sound in the medium. Because the wind is blowing away from the observer, the effective speed of sound is $v_{\text{snd}} - v_{\text{wind}}$. The wavelength of the waves traveling toward the observer is $\lambda_a = (v_{\text{snd}} - v_{\text{wind}})/f_0$, where f_0 is the frequency of the sound emitted by the factory whistle. This wavelength approaches the observer at a relative speed of $v_{\text{snd}} - v_{\text{wind}}$. Thus, the observer hears the frequency calculated here.

$$f_a = \frac{v_{\text{snd}} - v_{\text{wind}}}{\lambda_a} = \frac{v_{\text{snd}} - v_{\text{wind}}}{\left(\dfrac{v_{\text{snd}} - v_{\text{wind}}}{f_0}\right)} = f_0 = \boxed{770\text{ Hz}}$$

(*b*) Because the wind is blowing toward the observer, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The same kind of analysis as applied in part (*a*) gives $f_b = \boxed{770\text{ Hz}}$.

(*c*) Because the wind is blowing perpendicular to the line toward the observer, the effective speed of sound along that line is v_{snd}. Since there is no relative motion of the whistle and observer, there will be no change in frequency, so $f_c = \boxed{770\text{ Hz}}$.

(*d*) This is just like part (*c*), so $f_d = \boxed{770\text{ Hz}}$.

(*e*) Because the wind is blowing toward the cyclist, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The wavelength traveling toward the cyclist is $\lambda_e = (v_{\text{snd}} + v_{\text{wind}})/f_0$ and approaches the cyclist at a relative speed of $v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}}$. The cyclist will hear the following frequency:

$$f_e = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{\lambda_e} = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{(v_{\text{snd}} + v_{\text{wind}})} f_0 = \frac{(343 + 15.0 + 12.0)\text{m/s}}{(343 + 15.0)}(770\text{ Hz})$$
$$= \boxed{796\text{ Hz}}$$

(*f*) Since the wind is not changing the speed of the sound waves moving toward the cyclist, the speed of sound is 343 m/s. The observer is moving toward a stationary source at 12.0 m/s.

$$f_f = f\left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) = (770\text{ Hz})\left(1 + \frac{12.0\text{ m/s}}{343\text{ m/s}}\right) = \boxed{797\text{ Hz}}$$

5. For a tube open at both ends, all integer harmonics are allowed, with $f_n = nf_1$. Thus, consecutive harmonics differ by the fundamental frequency. The four consecutive harmonics give the following values for the fundamental frequency:

$$f_1 = 523\text{ Hz} - 392\text{ Hz} = 131\text{ Hz}, \quad 659\text{ Hz} - 523\text{ Hz} = 136\text{ Hz}, \quad 784\text{ Hz} - 659\text{ Hz} = 125\text{ Hz}$$

The average of these is $f_1 = \frac{1}{3}(131\text{ Hz} + 136\text{ Hz} + 125\text{ Hz}) \approx 131\text{ Hz}$. We use that for the fundamental frequency.

(*a*) $f_1 = \dfrac{v}{2\ell} \quad \to \quad \ell = \dfrac{v}{2f_1} = \dfrac{343\text{ m/s}}{2(131\text{ Hz})} = \boxed{1.31\text{ m}}$

Note that the bugle is coiled like a trumpet, so the full length fits in a smaller distance.

(*b*) $f_n = nf_1 \quad \to \quad n_{\text{G4}} = \dfrac{f_{\text{G4}}}{f_1} = \dfrac{392\text{ Hz}}{131\text{ Hz}} = 2.99; \quad n_{\text{C5}} = \dfrac{f_{\text{C5}}}{f_1} = \dfrac{523\text{ Hz}}{131\text{ Hz}} = 3.99;$

$$n_{\text{E5}} = \frac{f_{\text{E5}}}{f_1} = \frac{659\text{ Hz}}{131\text{ Hz}} = 5.03; \quad n_{\text{G5}} = \frac{f_{\text{G5}}}{f_1} = \frac{784\text{ Hz}}{131\text{ Hz}} = 5.98$$

The harmonics are $\boxed{3,\ 4,\ 5,\text{ and }6}$.

13

TEMPERATURE AND KINETIC THEORY

Responses to Questions

1. Because the atomic mass of copper is smaller than that of lead, an atom of copper has less mass than an atom of lead. Thus, 1 kg of copper will have more atoms than 1 kg of lead.

2. Properties of materials that can be exploited for the making of a thermometer include:
   (i) Volume of a liquid (mercury or alcohol thermometer)
   (ii) Electrical resistance (covered in a later chapter)
   (iii) Color (frequency) of emitted light from a heated object (covered in a later chapter)
   (iv) Volume of a gas
   (v) Expansion of a metal (bimetallic strip)

3. 1 C° is larger than 1 F°. There are 100 C° between the freezing and boiling temperatures of water, while there are 180 F° between the same two temperatures.

4. To be precise, ℓ_0 is to be the initial length of the object. In practice, however, since the value of the coefficient of expansion is so small, there will be little difference in the calculation of $\Delta\ell$ caused by using either the initial or final length, unless the temperature change is quite large.

5. When heated, the aluminum expands more than the iron, because the expansion coefficient of aluminum is larger than that of iron. Thus the aluminum will be on the outside of the curve.

6. The steam pipe can have a large temperature change as the steam enters or leaves the pipe. If the pipe is fixed at both ends and the temperature changes significantly, then there will be large thermal stresses that might break joints. The "U" in the pipe allows for expansion and contraction, which is not possible at the fixed ends. This is similar to the joints placed in concrete roadway surfaces to allow expansion and contraction.

7. The bimetallic strip is made of two types of metal joined together. The metal of the outside strip has a higher coefficient of linear expansion than that of the inside strip, so it will expand and contract more dramatically. As the temperature cools, the strip will coil more tightly. That will tilt the liquid mercury vessel so that the mercury goes over the two contacts, making a complete circuit and turning on the heater. As the temperature rises, the strip will "uncoil," moving the liquid mercury off of the contacts and shutting off the heater. Moving the temperature setting lever changes the initial position of the

glass vessel and thus changes the temperature at which the coil has expanded or shrunk enough to move the liquid mercury.

8. If one part is heated or cooled more than another part, there will be more expansion or contraction of one part of the glass compared to an adjacent part. This causes internal stress forces that may exceed the maximum strength of the glass.

9. If water is added quickly to an overheated engine, it comes into contact with the very hot metal parts of the engine. Some areas of the metal parts will cool off very rapidly; others will not. Some of the water will quickly turn to steam and will expand rapidly. The net result can be a cracked engine block or radiator, due to the thermal stress and/or the emission of high-temperature steam from the radiator. Water should always be added slowly, with the engine running. The water will mix with the hotter water already in the system and will circulate through the engine, gradually cooling all parts at about the same rate.

10. The coefficient of expansion is derived from a ratio of lengths: $\alpha = \frac{\Delta\ell}{\ell_0}\frac{1}{\Delta T}$. The length units cancel, so the coefficient does not depend on the specific length unit used in its determination, as long as the same units are used for both $\Delta\ell$ and ℓ_0.

11. The glass is the first to warm due to the hot water, so the glass will initially expand a small amount. As the glass initially expands, the alcohol level will decrease. As thermal equilibrium is reached, the alcohol will expand more than the glass expands, since alcohol has a larger coefficient of expansion than water, and the alcohol level will rise to indicate the higher temperature.

12. Since Pyrex glass has a smaller coefficient of linear expansion than ordinary glass, it will expand less than ordinary glass when heated, making it less likely to crack from internal stresses. Pyrex glass is therefore more suitable for applications involving heating and cooling. An ordinary glass mug may expand to the point of cracking if boiling water is poured in it, whereas a Pyrex mug will not.

13. On a hot day, the pendulum will be slightly longer than at 20°C, due to thermal expansion of the brass rod. Since the period of a pendulum is proportional to the square root of its length, the period will be slightly longer on the hot day, meaning that the pendulum takes more time for one oscillation. Thus, the clock will run slow.

14. The soda is mostly water. As water cools below 4°C it expands. There is more expansion of the soda as it cools below 4°C and freezes than there is available room in the can (the can has actually shrunk a small amount, making the mismatch more pronounced), so the freezing soda pushes against the can surfaces hard enough to push them outward. Evidently the top and bottom of the can are the weakest parts.

15. The buoyant force on the aluminum sphere is the weight of the water displaced by the sphere, which is the volume of the sphere times the density of water times *g*. As the substances are heated, the volume of the sphere increases and the density of the water decreases (because of its increased volume). Since the volume expansion coefficient of the water is almost three times larger than that of the aluminum, the fractional decrease in the water density is larger than the fractional increase in the aluminum volume. Thus, the product of the volume of the sphere and the density of water decreases, and the buoyant force gets smaller.

16. For an absolute vacuum, no. But for most "vacuums," there are still a few molecules in the containers, and the temperature can be determined from the (very low) pressure.

$\boxed{17.}$ (*a*) Because the escape velocity for the Moon is 1/5 that of the Earth, heavy molecules with lower speeds will be able to escape. The Moon may have started with an atmosphere, but over time almost all of the molecules of gas have escaped.

(*b*) Hydrogen is the lightest gas. For a given kinetic energy (temperature) it has the highest speed and will be most likely to escape.

18. Boiling occurs when the saturated vapor pressure equals the external pressure. When we say the oxygen "boils" at –183°C, we mean that the saturated vapor pressure for oxygen will be 1 atm (the same as atmospheric pressure) at a temperature of –183°C. At this temperature and pressure, liquid oxygen will vaporize.

19. The freezing point of water decreases slightly with higher pressure. The wire exerts a large pressure on the ice (due to the weights hung at each end). The ice under the wire will melt, allowing the wire to move lower into the block. Once the wire has passed a given position, the water now above the wire will have only atmospheric pressure on it and will refreeze. This process allows the wire to pass all the way through the block and yet leave a solid block of ice behind.

20. (*a*) A pressure cooker is sealed, so as the temperature of its contents increases, the number of particles inside and the volume are kept constant. Thus, the pressure increases according to the ideal gas law. Assuming there is water inside the pressure cooker, an increased pressure yields a higher boiling point for the water. The water in which the food is prepared will boil at a higher temperature than normal, thereby cooking the food faster.

(*b*) At high altitudes, the atmospheric pressure is less than it is at sea level. If atmospheric pressure decreases, the boiling point of water will decrease, so boiling occurs at a lower temperature than at sea level. Food being cooked in an open pot (including pasta and rice) will need to cook longer at this lower temperature to be properly prepared.

(*c*) It is actually easier to boil water at higher altitude, because the water boils at a lower temperature. Thus, it will take less time to add enough heat to the water to bring it to the boiling temperature.

$\boxed{21.}$ Liquids boil when their saturated vapor pressure equals the external pressure. For water, from Table 13–3, the saturated vapor pressure of water at 20°C is about 0.023 atm. So if the external pressure is lowered to that level (about 2.3% of normal air pressure), the water will boil at that low temperature.

22. Exhaled air contains a large amount of water vapor and is initially at a temperature equal to body temperature. When the exhaled air comes into contact with the external air on a cold day, it cools rapidly and reaches the dew point. At the dew point temperature, the air can no longer hold all the water vapor and water condenses into little droplets, forming a cloud. The white cloud seen is due to the condensed water vapor.

24. The water in the radiator of an overheated automobile engine is under pressure. Similar to a pressure cooker, that high pressure keeps the water in the liquid state even though the water is quite hot—hotter than 100°C. When the cap is opened, the pressure is suddenly lowered, and the superheated water boils quickly and violently. That hot steam can cause severe burns if it contacts the skin. Also, the violent bursting forth of steam propels some of the overheated water out of the radiator as well, which can spray onto the person opening the cap and again cause serious burns.

Responses to MisConceptual Questions

1. (*c*) Students may confuse thermal expansion with elasticity and surmise that the narrower rod would expand more for the same temperature change. However, in thermal expansion the change in length is independent of the rod's diameter or cross-sectional area.

2. (*d*) Equation 13–1a for thermal expansion shows that the change in length depends upon the initial length, the change in temperature, and the coefficient of thermal expansion (which depends upon the type of material).

3. (*c*) Many students have the misconception that as the plate expands, the hole will get smaller. To understand what actually happens, imagine that the hole is filled with a steel disk. As the plate and the steel disk are heated, both will expand. After the plate and disk are heated, the disk is removed from the plate. Since the disk expanded, the hole that is left must also have expanded. As the steel ring is heated its circumference will expand, causing its interior to also expand.

4. (*b*) A common error is to treat the temperature as doubling. If the temperature doubled, the pressure would also double. However, the temperature is given in degrees Celsius, not kelvins. When the temperature is converted to kelvins, it is easy to see that the temperature only increases by about 25%, from 373 K to 437 K. The pressure then also increases by about 25%. (The actual amount is 26.8%.)

5. (*b*) The absolute (Kelvin) scale must be used in the ideal gas law. If the Celsius scale is used, it appears that the temperature has doubled. However, 0°C is an arbitrarily chosen point on the temperature scale and cannot be used to determine temperature ratios. When the temperatures are converted to kelvins (293 K and 313 K) it can be seen that their ratio is about 1.07.

6. (*c*) By the ideal gas law, when the temperature is held constant, the pressure is proportional to the number of moles and inversely proportional to the volume. If the second bottle has twice the volume with only half the number of moles, it would only experience one-fourth the pressure.

7. (*e*) A common misconception is that if the temperature increases, both the pressure and volume will increase. However, by the ideal gas law, when the temperature increases, the product of the pressure and volume must increase. This increase can occur by increasing the pressure, increasing the volume, or both.

8. (*c*) The temperature of the gas is a measure of the average kinetic energy of the gas molecules.

9. (*e*) Some students might erroneously relate the temperature of the gas to the velocity of the gas molecules and surmise that the rms speeds would be equal. However, when the two gases are at the same temperature, the molecules will have the same average kinetic energy. The kinetic energy is proportional to the mass of the molecule and the square of the rms speed. Since mass B is half the mass of A, the speed of molecules of mass B must be $\sqrt{2} \approx 1.4\times$ greater than the speed of molecules of mass A.

10. (*a*) In the mixture, the oxygen molecules and helium atoms will be at the same temperature, which means that their average molecular kinetic energies will be the same. Since a helium atom has less mass than an oxygen molecule, the helium atoms will be moving faster than the oxygen molecules on average.

11. (*d*) The temperature of an ideal gas is a measure of the average kinetic energy of the gas, so increasing the temperature will increase the average kinetic energy—(*a*) is true. Even though the temperature is proportional to the average kinetic energy of the molecules, each molecule can

have a random kinetic energy (according to the Maxwell distribution of speeds). The speeds of individual molecules will vary about this average, so (*d*) is false. Per the ideal gas law, the product of the pressure and volume is proportional to the temperature. Therefore, if pressure or volume is held constant as the temperature increases, the other parameter (volume or pressure) must increase, so (*b*) and (*c*) are true. For a gas to be ideal, it is assumed that the space occupied by the gas is mostly empty. For this to be true, the molecules are assumed to be far apart compared with their size. Thus, (*e*) is also true.

12. (*a*) Students frequently do not understand that gauge pressure and temperature in Celsius are a comparison of pressure and temperature to an arbitrary zero point. For the ideal gas law to hold, the temperature, pressure, and volume must be measured relative to the true zero points: absolute zero pressure, absolute zero temperature, and zero volume. The volume units are not critical, since changing them only affects the value of the ideal gas constant. Gauge pressure and Celsius temperature have additive terms that change the functional form of the ideal gas law.

13. (*c*) The rms speed of a gas is an average molecular speed found by taking the square root of the average of the square of the molecule speeds. Consider a gas in which 2/3 of the molecules are at rest and 1/3 move at a constant speed. The most probable speed is 0, but the rms speed is greater than zero. Therefore, the most probable speed is not necessarily the rms speed, so (*a*) is false. The rms speed is an average speed; therefore, it will always be equal to or smaller than the maximum speed, so (*b*) is false. The temperature of a gas is determined by the average molecular kinetic energy. As the temperature increases, the molecular kinetic energy increases; therefore, the rms speed must also increase, so (*c*) is true.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic masses of gold and silver from the periodic table.

$$\frac{N_{\text{Au}}}{N_{\text{Ag}}}=\frac{\dfrac{2.75\times10^{-2}\text{ kg}}{(196.96655\text{ u/atom})(1.66\times10^{-27}\text{ kg/u})}}{\dfrac{2.75\times10^{-2}\text{ kg}}{(107.8682\text{ u/atom})(1.66\times10^{-27}\text{ kg/u})}}=\frac{107.8682}{196.96655}=0.548 \quad\rightarrow\quad \boxed{N_{\text{Au}}=0.548N_{\text{Ag}}}$$

Because a gold atom is heavier than a silver atom, there are fewer gold atoms in the given mass.

2. The number of atoms is found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of copper from the periodic table.

$$N_{\text{Cu}}=\frac{3.4\times10^{-3}\text{ kg}}{(63.546\text{ u/atom})(1.66\times10^{-27}\text{ kg/u})}=\boxed{3.2\times10^{22}\text{ atoms of Cu}}$$

3. (*a*) $T(°\text{C})=\frac{5}{9}[T(°\text{F})-32]=\frac{5}{9}[68-32]=\boxed{20°\text{C}}$

(*b*) $T(°\text{F})=\frac{9}{5}T(°\text{C})+32=\frac{9}{5}(1900)+32=3452°\text{F}\approx\boxed{3500°\text{F}}$

4. High: $T(°\mathrm{C}) = \frac{5}{9}[T(°\mathrm{F}) - 32] = \frac{5}{9}[136 - 32] = \boxed{57.8°\mathrm{C}}$

   Low: $T(°\mathrm{C}) = \frac{5}{9}[T(°\mathrm{F}) - 32] = \frac{5}{9}[-129 - 32] = \boxed{-89.4°\mathrm{C}}$

5. $T(°\mathrm{F}) = \frac{9}{5}T(°\mathrm{C}) + 32 = \frac{9}{5}(38.9°\mathrm{C}) + 32 = \boxed{102.0°\mathrm{F}}$

6. (*a*) $T(°\mathrm{F}) = \frac{9}{5}T(°\mathrm{C}) + 32 = \frac{9}{5}(-18) + 32 = -0.4°\mathrm{F} \approx \boxed{0°\mathrm{F}}$

   (*b*) $T(^{\mathrm{o}}\mathrm{C}) = \frac{5}{9}[T(^{\mathrm{o}}\mathrm{F}) - 32] = \frac{5}{9}[-18 - 32] = -27.78°\mathrm{C} \approx \boxed{-28°\mathrm{C}}$

7. The two temperatures, as given by the conversions, are to be the same.

$$T(°\mathrm{F}) = \tfrac{9}{5}T(°\mathrm{C}) + 32 \quad \rightarrow \quad x = \tfrac{9}{5}x + 32 \quad \rightarrow \quad -32 = \tfrac{4}{5}x \quad \rightarrow \quad x = (-32)\tfrac{5}{4} = -40$$

   Thus, $\boxed{-40°\mathrm{C} = -40°\mathrm{F}}$.

8. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows:

$$\frac{\Delta T}{\Delta \ell} = \frac{100.0°\mathrm{C} - 0.0°\mathrm{C}}{22.79\ \mathrm{cm} - 12.61\ \mathrm{cm}} = 9.823\mathrm{C}°/\mathrm{cm}$$

   Then the temperature corresponding to length ℓ is $T(\ell) = 0.0°\mathrm{C} + (\ell - 12.61\ \mathrm{cm})(9.823\mathrm{C}°/\mathrm{cm})$.

   (*a*) $T(18.70\ \mathrm{cm}) = 0.0°\mathrm{C} + (18.70\ \mathrm{cm} - 12.61\ \mathrm{cm})(9.823\mathrm{C}°/\mathrm{cm}) = \boxed{59.8°\mathrm{C}}$

   (*b*) $T(14.60\ \mathrm{cm}) = 0.0°\mathrm{C} + (14.60\ \mathrm{cm} - 12.61\ \mathrm{cm})(9.823\mathrm{C}°/\mathrm{cm}) = \boxed{19.5°\mathrm{C}}$

9. Take the 300-m height to be the height in January. Then the increase in the height of the tower as the temperature rises is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T = (12\times10^{-6}/\mathrm{C}°)(300\ \mathrm{m})(25°\mathrm{C} - 2°\mathrm{C}) = \boxed{0.08\ \mathrm{m}}$$

10. When the concrete cools in the winter, it will contract, and there will be no danger of buckling. Thus, the low temperature in the winter is not a factor in the design of the highway. But when the concrete warms in the summer, it will expand. A crack must be left between the slabs equal to the increase in length of the concrete as it heats from 15°C to 50°C.

$$\Delta \ell = \alpha \ell_0 \Delta T = (12\times10^{-6}/\mathrm{C}°)(12\ \mathrm{m})(50°\mathrm{C} - 15°\mathrm{C}) = \boxed{5.0\times10^{-3}\ \mathrm{m}}$$

11. The increase in length of the table is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T = (0.20\times10^{-6}/\mathrm{C}°)(1.8\ \mathrm{m})(6.0\ \mathrm{C}°) = \boxed{2.2\times10^{-6}\ \mathrm{m}}$$

   For steel, $\Delta \ell = \alpha \ell_0 \Delta T = (12\times10^{-6}/\mathrm{C}°)(1.8\ \mathrm{m})(8.0\ \mathrm{C}°) = \boxed{1.7\times10^{-4}\ \mathrm{m}}$.

   The change for Super Invar is only $\frac{1}{60}$ of the change for steel.

12. The increase in length of the rod is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T \quad \rightarrow \quad \Delta T = \frac{\Delta \ell}{\alpha \ell_0} \quad \rightarrow \quad T = T_0 + \frac{\Delta \ell}{\alpha \ell_0} = 25°\mathrm{C} + \frac{0.015}{19\times10^{-6}/\mathrm{C}°} = 814.5°\mathrm{C} \approx \boxed{810°\mathrm{C}}$$

13. The rivet must be cooled so that its diameter becomes the same as the diameter of the hole.

$$\Delta\ell = \alpha\ell_0\Delta T \quad\rightarrow\quad \ell-\ell_0=\alpha\ell_0(T-T_0)$$

$$T=T_0+\frac{\ell-\ell_0}{\alpha\ell_0}=22^\circ\text{C}+\frac{1.870\text{ cm}-1.872\text{ cm}}{(12\times10^{-6}/\text{C}^\circ)(1.872\text{ cm})}=-67^\circ\text{C}\approx\boxed{-70^\circ\text{C}}$$

The temperature of "dry ice" is about -80°C, so this process will be successful.

14. The amount of water that can be added to the container is the final volume of the container minus the final volume of the water. Also note that the original volumes of the water and the container are the same. We assume that the density of water is constant over the temperature change involved.

$$V_{\text{added}}=(V_0+\Delta V)_{\text{container}}-(V_0+\Delta V)_{\text{H}_2\text{O}}=\Delta V_{\text{container}}-\Delta V_{\text{H}_2\text{O}}=(\beta_{\text{container}}-\beta_{\text{H}_2\text{O}})V_0\Delta T$$
$$=(27\times10^{-6}/\text{C}^\circ-210\times10^{-6}/\text{C}^\circ)(450.0\text{ mL})(-80.0\text{ C}^\circ)=\boxed{6.59\text{ mL}}$$

15. The change in volume of the aluminum is given by the volume expansion formula, Eq. 13–2. The percent change is found by taking the change, dividing by the original volume, and then multiplying by 100.

$$\frac{\Delta V}{V_0}(100)=\frac{\beta V_0\Delta T}{V_0}(100)=\beta\Delta T(100)=(75\times10^{-6}/\text{C}^\circ)(160^\circ\text{C}-30^\circ\text{C})(100)=0.975=\boxed{0.98\%}$$

$\boxed{16.}$ (*a*) The amount of water lost is the final volume of the water minus the final volume of the container. Also note that the original volumes of the water and the container are the same.

$$V_{\text{lost}}=(V_0+\Delta V)_{\text{H}_2\text{O}}-(V_0+\Delta V)_{\text{container}}=\Delta V_{\text{H}_2\text{O}}-\Delta V_{\text{container}}=\beta_{\text{H}_2\text{O}}V_0\Delta T-\beta_{\text{container}}V_0\Delta T$$

$$\beta_{\text{container}}=\beta_{\text{H}_2\text{O}}-\frac{V_{\text{lost}}}{V_0\Delta T}=210\times10^{-6}/\text{C}^\circ-\frac{(0.35\text{ g})\left(\dfrac{1\text{ mL}}{0.98324\text{ g}}\right)}{(55.50\text{ mL})(60^\circ\text{C}-20^\circ\text{C})}=\boxed{5.0\times10^{-5}/\text{C}^\circ}$$

(*b*) From Table 13–1, the most likely material is $\boxed{\text{copper}}$.

$\boxed{17.}$ The sum of the original diameter plus the expansion must be the same for both the plug and the ring.

$$(\ell_0+\Delta\ell)_{\text{iron}}=(\ell_0+\Delta\ell)_{\text{brass}}\quad\rightarrow\quad\ell_{\text{iron}}+\alpha_{\text{iron}}\ell_{\text{iron}}\Delta T=\ell_{\text{brass}}+\alpha_{\text{brass}}\ell_{\text{brass}}\Delta T$$

$$\Delta T=\frac{\ell_{\text{brass}}-\ell_{\text{iron}}}{\alpha_{\text{iron}}\ell_{\text{iron}}-\alpha_{\text{brass}}\ell_{\text{brass}}}=\frac{8.755\text{ cm}-8.741\text{ cm}}{(12\times10^{-6}/\text{C}^\circ)(8.741\text{ cm})-(19\times10^{-6}/\text{C}^\circ)(8.755\text{ cm})}$$
$$=-228\text{C}^\circ=T_{\text{final}}-T_{\text{initial}}=T_{\text{final}}-15^\circ\text{C}\quad\rightarrow\quad T_{\text{final}}=-213^\circ\text{C}\approx\boxed{-210^\circ\text{C}}$$

18. Since the coefficient of volume expansion is much larger for the coolant than for the aluminum and the steel, the coolant will expand more than the aluminum and steel, so coolant will overflow the cooling system. Use Eq. 13–2.

$$\Delta V=\Delta V_{\text{coolant}}-\Delta V_{\text{aluminum}}-\Delta V_{\text{steel}}=\beta_{\text{coolant}}V_{\text{coolant}}\Delta T-\beta_{\text{aluminum}}V_{\text{aluminum}}\Delta T-\beta_{\text{steel}}V_{\text{steel}}\Delta T$$
$$=(\beta_{\text{coolant}}V_{\text{coolant}}-\beta_{\text{aluminum}}V_{\text{aluminum}}-\beta_{\text{steel}}V_{\text{steel}})\Delta T$$
$$=[(410\times10^{-6}/\text{C}^\circ)(14.0\text{ L})-(75\times10^{-6}/\text{C}^\circ)(3.5\text{ L})-(35\times10^{-6}/\text{C}^\circ)(10.5\text{ L})](12\text{ C}^\circ)$$
$$=0.06132\text{ L}\approx\boxed{61.3\text{ mL}}$$

19. The thermal stress must compensate for the thermal expansion. *E* is Young's modulus for the aluminum.

$$\text{Stress}=F/A=\alpha E\Delta T=(25\times10^{-6}/\text{C}^\circ)(70\times10^9\text{ N/m}^2)(35^\circ\text{C}-12^\circ\text{C})=\boxed{4.0\times10^7\text{ N/m}^2}$$

20. Notation: we will use T for the period, and "Temp" for the temperature. The pendulum has a period of $T_0 = 2\pi\sqrt{\ell_0/g}$ at 17°C and a period of $T = 2\pi\sqrt{\ell/g}$ at 28°C. Notice that $T > T_0$ since $\ell > \ell_0$. With every swing of the clock, the heated clock will indicate that a time T_0 has passed, but the actual amount of time that has passed is T. Thus, the heated clock is "losing time" by an amount of $\Delta T = T - T_0$ every swing. The fractional loss is given by $\frac{\Delta T}{T_0}$ and is found as follows:

$$\frac{\Delta T}{T_0} = \frac{T - T_0}{T_0} = \frac{2\pi\sqrt{\ell/g} - 2\pi\sqrt{\ell_0/g}}{2\pi\sqrt{\ell_0/g}} = \frac{\sqrt{\ell} - \sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0 + \Delta\ell} - \sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0 + \alpha\ell_0(\Delta\text{Temp})} - \sqrt{\ell_0}}{\sqrt{\ell_0}}$$

$$= \sqrt{1 + \alpha(\Delta\text{Temp})} - 1 = \sqrt{1 + (19\times10^{-6}/\text{C}°)(12\ \text{C}°)} - 1 = 1.1399\times10^{-4}$$

Thus, the amount of time lost in any time period T_0 is $\Delta T = (1.1399\times10^{-4})T_0$. For one year, we have the following:

$$\Delta T = (1.1399\times10^{-4})(3.156\times10^{7}\ \text{s}) = 3598\ \text{s} \approx \boxed{60\ \text{min}}$$

21. Use the relationship that $T(\text{K}) = \frac{5}{9}[T(°\text{F}) - 32] + 273.15$.

$$T(\text{K}) = \tfrac{5}{9}[T(°\text{F}) - 32] + 273.15 \quad \rightarrow$$

$$T(°\text{F}) = \tfrac{9}{5}[T(\text{K}) - 273.15] + 32 = \tfrac{9}{5}[0\ \text{K} - 273.15] + 32 = \boxed{-459.67°\text{F}}$$

22. Use the relationship that $T(\text{K}) = T(°\text{C}) + 273.15$.

(*a*) $T(\text{K}) = T(°\text{C}) + 273.15 = 4270\ \text{K} \approx \boxed{4300\ \text{K}}$; $T(\text{K}) = T(°\text{C}) + 273.15 = \boxed{15\times10^{6}\ \text{K}}$

(*b*) $\%\ \text{error} = \frac{\Delta T}{T(\text{K})}\times100 = \frac{273.15}{T(\text{K})}\times100$

4000°C: $\frac{273.15}{4270}\times100 \approx \boxed{6.4\%}$ $\quad$ 15×10^{6} °C: $\frac{273.15}{15\times10^{6}}\times100 \approx \boxed{1.8\times10^{-3}\%}$

23. Assume the gas is ideal. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1} = (3.50\ \text{m}^3)\left(\frac{1.00\ \text{atm}}{3.20\ \text{atm}}\right)\frac{(273 + 38.0)\ \text{K}}{273\ \text{K}} = \boxed{1.25\ \text{m}^3}$$

24. Assume the air is an ideal gas. Since the amount of air is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (293\ \text{K})\left(\frac{40\ \text{atm}}{1\ \text{atm}}\right)\left(\frac{1}{9}\right) = 1302\ \text{K} = 1029°\text{C} \approx \boxed{1000°\text{C}}$$

25. (*a*) Assume that the helium is an ideal gas and then use the ideal gas law to calculate the volume. Absolute pressure must be used, even though gauge pressure is given.

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(16.00\ \text{mol})(8.314\text{J/mol}\cdot\text{K})(283.15\ \text{K})}{(1.350\ \text{atm})(1.013\times10^{5}\ \text{Pa/atm})} = \boxed{0.2754\ \text{m}^3}$$

(*b*) Since the amount of gas is not changed, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad T_2 = T_1\frac{P_2}{P_1}\frac{V_2}{V_1} = (283.15\ \text{K})\left(\frac{2.00\ \text{atm}}{1.350\ \text{atm}}\right)\left(\frac{1}{2}\right) = 210\ \text{K} = \boxed{-63°\text{C}}$$

26. Assume that the nitrogen and carbon dioxide are ideal gases and that the volume and temperature are constant for the two gases. From the ideal gas law, the value of $\frac{P}{n} = \frac{RT}{V}$ is constant. Also note that concerning the ideal gas law, the identity of the gas is unimportant, as long as the number of moles is considered.

$$\frac{P_1}{n_1} = \frac{P_2}{n_2} \quad \rightarrow \quad P_2 = P_1 \frac{n_2}{n_1} = (3.45\text{ atm})\left(\frac{\dfrac{21.6\text{ kg CO}_2}{44.01\times10^{-3}\text{ kg CO}_2/\text{mol}}}{\dfrac{21.6\text{ kg N}_2}{28.01\times10^{-3}\text{ kg N}_2/\text{mol}}} \right) = \boxed{2.20\text{ atm}}$$

27. (*a*) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n = (28.5\text{ kg})\frac{1\text{ mole N}_2}{28.01\times10^{-3}\text{ kg}} = 1017\text{ mol}$$

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(1017\text{ mol})(8.314\text{J/mol}\cdot\text{K})(273\text{ K})}{1.013\times10^{5}\text{ Pa}} = 22.79\text{ m}^3 \approx \boxed{22.8\text{ m}^3}$$

(*b*) Hold the volume and temperature constant and again use the ideal gas law.

$$n = (28.5\text{ kg} + 32.2\text{ kg})\frac{1\text{ mole N}_2}{28.01\times10^{-3}\text{ kg}} = 2167\text{ mol}$$

$$PV = nRT \quad \rightarrow$$

$$P = \frac{nRT}{V} = \frac{(2167\text{ mol})(8.314\text{J/mol}\cdot\text{K})(273\text{ K})}{22.79\text{ m}^3} = \boxed{2.16\times10^{5}\text{ Pa} = 2.13\text{ atm}}$$

28. We assume that the mass of air is unchanged and the volume of air is unchanged (since the tank is rigid). Use the ideal gas law.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \rightarrow \quad T_2 = T_1\frac{P_2}{P_1} = [(273+29)\text{K}]\left(\frac{191\text{ atm}}{204\text{ atm}}\right) = 283\text{ K} = \boxed{10°\text{C}}$$

The answer has 2 significant figures.

$\boxed{29.}$ Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n = (105.0\text{ kg})\frac{1\text{ mole Ar}}{39.95\times10^{-3}\text{ kg}} = 2628\text{ mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{(2628\text{ mol})(8.314\text{J/mol}\cdot\text{K})(273.15\text{ K} + 21.6\text{ K})}{(38.0\text{ L})(1.00\times10^{-3}\text{ m}^3/\text{L})} = \boxed{1.69\times10^{8}\text{ Pa}}$$

This is 1670 atm.

30. We assume that the gas is ideal, that the amount of gas is constant, and that the volume of the gas is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \rightarrow \quad T_2 = T_1\frac{P_2}{P_1} = [(273.15+20.0)\text{K}]\left(\frac{2.00\text{ atm}}{1.00\text{ atm}}\right) = 586.3\text{ K} = 313.15°\text{C} \approx \boxed{313°\text{C}}$$

31. Assume that the air is an ideal gas and that the volume of the tire remains constant. We consider two states: the original state 1, with temperature T_1, pressure P_1, and amount of gas n_1; and the final state 2, with temperature T_2, pressure P_1, and amount of gas n_2. The ideal gas law is used.

$$P_1V = n_1RT_1;\; P_1V = n_2RT_2 \quad \rightarrow \quad \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{(273+15)\text{K}}{(273+38)\text{K}} = 0.926$$

Thus, $1-0.926 = 0.074 = \boxed{7.4\%}$ must be removed.

32. Assume the oxygen is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad P_2 = P_1\frac{V_1}{V_2}\frac{T_2}{T_1} = (2.45\text{ atm})\left(\frac{61.5\text{ L}}{38.8\text{ L}}\right)\frac{(273.15+56.0)\text{K}}{(273.15+18.0)\text{K}} = \boxed{4.39\text{ atm}}$$

33. The pressure inside the bag will change to the surrounding air pressure as the volume of the bag changes. We assume the amount of gas and temperature of the gas are constant. Use the ideal gas law.

$$P_1V_1 = P_2V_2 \quad \rightarrow \quad V_2 = V_1\frac{P_1}{P_2} = V_1\left(\frac{1.0\text{ atm}}{0.75\text{ atm}}\right) = 1.33\,V_1$$

Thus, the bag has expanded by $\boxed{33\%}$.

34. Assume the helium is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease by 30%.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}\frac{T_2}{T_1} = \left(\frac{1.0}{0.70}\right)\frac{(273+5.0)\text{K}}{(273+20.0)\text{K}} = 1.3954 \approx \boxed{1.4\text{ times the original volume}}$$

35. We calculate the density of water vapor, with a molecular mass of 18.0 grams per mole, from the ideal gas law.

$$PV = nRT \quad \rightarrow \quad \frac{n}{V} = \frac{P}{RT} \quad \rightarrow$$

$$\rho = \frac{m}{V} = \frac{Mn}{V} = \frac{MP}{RT} = \frac{(0.0180\text{ kg/mol})(1.013\times10^5\text{ Pa})}{(8.314\text{J/mol}\cdot\text{K})(373\text{ K})} = \boxed{0.588\text{ kg/m}^3}$$

The density from Table 10–1 is 0.598 kg/m^3. Because this gas is very "near" a phase change state (water can also exist as a liquid at this temperature and pressure), we would not expect it to act like an ideal gas. It is reasonable to expect that the molecules will have other interactions besides purely elastic collisions. That is evidenced by the fact that steam can form droplets, indicating an attractive force between the molecules.

36. We ignore the weight of the stopper. Initially there is a net force (due to air pressure) on the stopper of 0, because the pressure is the same both above and below the stopper. With the increase in temperature, the pressure inside the tube will increase, so there will be a net upward force given by $F_{\text{net}} = (P_{\text{in}} - P_{\text{out}})A$, where A is the cross-sectional area of the stopper. The inside pressure can be expressed in terms of the inside temperature by means of the ideal gas law for a constant volume and constant mass of gas.

$$\frac{P_{\text{in}}V_{\text{tube}}}{T_{\text{in}}} = \frac{P_0 V_{\text{tube}}}{T_0} \quad \rightarrow \quad P_{\text{in}} = P_0 \frac{T_{\text{in}}}{T_0}$$

$$F_{\text{net}} = (P_{\text{in}} - P_{\text{out}})A = \left(P_0 \frac{T_{\text{in}}}{T_0} - P_0 \right) A = P_0 \left(\frac{T_{\text{in}}}{T_0} - 1 \right) A \quad \rightarrow$$

$$T_{\text{in}} = \left(\frac{F_{\text{net}}}{P_0 A} + 1 \right) T_0 = \left[\frac{(10.0\ \text{N})}{(1.013\times10^5\ \text{Pa})\pi(0.0075\ \text{m})^2} + 1 \right] (273\ \text{K} + 18\ \text{K}) = 454\ \text{K} = \boxed{181^\circ\text{C}}$$

37. The ideal gas law can be used to relate the volume at the surface to the submerged volume of the bubble. We assume the amount of gas in the bubble doesn't change as it rises. The pressure at the submerged location is found from Eq. 10–3c.

$$PV = nRT \quad \rightarrow \quad \frac{PV}{T} = nR = \text{constant} \quad \rightarrow \quad \frac{P_{\text{surface}}V_{\text{surface}}}{T_{\text{surface}}} = \frac{P_{\text{submerged}}V_{\text{submerged}}}{T_{\text{submerged}}} \quad \rightarrow$$

$$V_{\text{surface}} = V_{\text{submerged}} \frac{P_{\text{submerged}}}{P_{\text{surface}}} \frac{T_{\text{surface}}}{T_{\text{submerged}}} = V_{\text{submerged}} \frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}} \frac{T_{\text{surface}}}{T_{\text{submerged}}}$$

$$= (1.00\ \text{cm}^3) \frac{[1.013\times10^5 + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(41.0\ \text{m})]}{(1.013\times10^5)\text{Pa}} \frac{(273.15 + 18.5)\text{K}}{(273.15 + 5.5)\text{K}}$$

$$= 5.198\ \text{cm}^3 = \tfrac{4}{3}\pi r^3 \quad \rightarrow \quad r = \left[\frac{3(5.198\ \text{cm}^3)}{4\pi} \right]^{1/3} = \boxed{1.07\ \text{cm}}$$

38. At STP, 1 mole of ideal gas occupies 22.4 L.

$$\frac{1\ \text{mole}}{22.4\ \text{L}} \left(\frac{6.02\times10^{23}\ \text{molecules}}{\text{mole}} \right) \left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^3} \right) = \boxed{2.69\times10^{25}\ \text{molecules/m}^3}$$

39. We assume that the water is at 4°C, so its density is $1000\ \text{kg/m}^3$.

$$1.000\ \text{L} \left(\frac{10^{-3}\ \text{m}^3}{1\ \text{L}} \right) \left(\frac{1000\ \text{kg}}{1\ \text{m}^3} \right) \left(\frac{1\ \text{mol}}{(15.9994 + 2\times1.00794)\times10^{-3}\ \text{kg}} \right) = \boxed{55.51\ \text{mol}}$$

$$55.51\ \text{mol} \left(\frac{6.022\times10^{23}\ \text{molecules}}{1\ \text{mol}} \right) = \boxed{3.343\times10^{25}\ \text{molecules}}$$

40. (*a*) Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\text{Earth}}^2$ and a thickness Δy. Then use the density of sea water to find the mass and the molecular weight of water to find the number of moles.

$$\text{Volume} = 0.75(4\pi R_{\text{Earth}}^2)\Delta y = 0.75(4\pi)(6.38\times10^6\ \text{m})^2(3\times10^3\ \text{m}) = 1.15\times10^{18}\ \text{m}^3$$

$$1.15\times10^{18}\ \text{m}^3 \left(\frac{1025\ \text{kg}}{\text{m}^3} \right) \left(\frac{1\ \text{mol}}{18\times10^{-3}\ \text{kg}} \right) = 6.55\times10^{22}\ \text{moles} \approx \boxed{7\times10^{22}\ \text{moles}}$$

(*b*) $6.55\times10^{22}\ \text{moles}(6.02\times10^{23}\ \text{molecules/1 mol}) \approx \boxed{4\times10^{46}\ \text{molecules}}$

41. Assume the gas is ideal at those low pressures and use the ideal gas law.

$$PV = NkT \quad \rightarrow \quad \frac{N}{V} = \frac{P}{kT} = \frac{1\times10^{-12}\ \text{N/m}^2}{(1.38\times10^{-23}\ \text{J/K})(273\ \text{K})} = \left(3\times10^{8}\ \frac{\text{molecules}}{\text{m}^3}\right)\left(\frac{10^{-6}\ \text{m}^3}{1\ \text{cm}^3}\right)$$
$$= \boxed{300\ \text{molecules/cm}^3}$$

42. We assume an ideal gas at STP. Example 13–9 shows that the molar volume of this gas is 22.4 L. We calculate the actual volume of one mole of gas particles, assuming a volume of ℓ_0^3, and then find the ratio of the actual volume of the particles to the volume of the gas.

$$\frac{V_{\text{molecules}}}{V_{\text{gas}}} = \frac{(6.02\times10^{23}\ \text{molecules})((3.0\times10^{-10}\ \text{m})^3/\text{molecule})}{(22.4\ \text{L})(1\times10^{-3}\ \text{m}^3/1\ \text{L})} = \boxed{7.3\times10^{-4}}$$

The molecules take up less than 0.1% of the volume of the gas.

43. (*a*) The average translational kinetic energy of a gas molecule is $\frac{3}{2}kT$.

$$\text{KE}_{\text{avg}} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})(273\ \text{K}) = \boxed{5.65\times10^{-21}\ \text{J}}$$

(*b*) The total translational kinetic energy is the average kinetic energy per molecule times the number of molecules.

$$\text{KE}_{\text{total}} = N(\text{KE}_{\text{avg}}) = (1.0\ \text{mol})\left(\frac{6.02\times10^{23}\ \text{molecules}}{1\ \text{mol}}\right)\tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})(298\ \text{K})$$
$$= \boxed{3700\ \text{J}}$$

44. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. Helium has an atomic mass of 4.0.

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(6000\ \text{K})}{4.0(1.66\times10^{-27}\ \text{kg})}} = 6116\ \text{m/s} \approx \boxed{6\times10^{3}\ \text{m/s}}$$

45. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature must be in kelvins.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273\ \text{K}+160\ \text{K}}{273\ \text{K}+20\ \text{K}}} = \boxed{1.22}$$

46. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. Since the rms speed is proportional to the square root of the absolute temperature, to triple the rms speed without changing the mass, the absolute temperature must be multiplied by a factor of 9.

$$T_{\text{fast}} = 4T_{\text{slow}} = 9(273+20)\text{K} = 2637\ \text{K} = 2364^\circ\text{C} \approx \boxed{2360^\circ\text{C}}$$

47. The average kinetic molecular energy is $\frac{3}{2}kT$. Set this equal to the kinetic energy of the paper clip.

$$\tfrac{1}{2}mv^2 = \tfrac{3}{2}kT \quad \rightarrow \quad v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(273+22)\text{K}}{1.0\times10^{-3}\ \text{kg}}} = \boxed{3.5\times10^{-9}\ \text{m/s}}$$

48. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = 1.040 = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} \quad \rightarrow$$

$$T_2 = T_1(1.040)^2 = (293.15\ \text{K})(1.040)^2 = 317.07\ \text{K} = \boxed{43.9°\text{C}}$$

49. From the ideal gas law, $PV = nRT$, if the volume and amount of gas are held constant, the temperature is proportional to the pressure, $PV = nRT \quad \rightarrow \quad P = \frac{nR}{V}T = (\text{constant})T$. Thus, the temperature will be tripled. Since the rms speed is proportional to the square root of the temperature, $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$, v_{rms} will be multiplied by a factor of $\boxed{\sqrt{3}} \approx 1.73$.

50. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \quad \rightarrow \quad kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = \frac{M}{V}$. Combining these relationships gives the following:

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

51. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$, where m is the mass of one particle.

$$\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_1 N_A}{m_2 N_A}} = \sqrt{\frac{M_1}{M_2}} \quad \rightarrow \quad \boxed{\frac{(v_{\text{rms}})_2}{(v_{\text{rms}})_1} = \sqrt{\frac{M_1}{M_2}}}$$

52. The temperature of the nitrogen gas is found from the ideal gas law, and then the rms speed is found from the temperature.

$$PV = nRT \quad \rightarrow \quad T = \frac{PV}{nR}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3k}{m}\frac{PV}{nR}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})}{28(1.66\times10^{-27}\ \text{kg})}\frac{(2.9\ \text{atm})(1.013\times10^5\ \text{Pa/atm})(8.5\ \text{m}^3)}{(2100\ \text{mol})(8.314\text{J/mol}\cdot\text{K})}}$$

$$= 356.9\ \text{m/s} \approx \boxed{360\ \text{m/s}}$$

53. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$\frac{(v_{\text{rms}})_{(^{235}\text{UF}_6)}}{(v_{\text{rms}})_{(^{238}\text{UF}_6)}} = \frac{\sqrt{3kT/m_{(^{235}\text{UF}_6)}}}{\sqrt{3kT/m_{(^{238}\text{UF}_6)}}} = \sqrt{\frac{m_{(^{238}\text{UF}_6)}}{m_{(^{235}\text{UF}_6)}}} = \sqrt{\frac{238.050788 + 6(18.998403)}{235.043930 + 6(18.998403)}} = 1.004298 \approx \boxed{1.004}$$

54. (*a*) The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(273\ \text{K})}{2(15.9994)32(1.66\times10^{-27}\ \text{kg})}} = \boxed{461\ \text{m/s}}$$

(*b*) Assuming that the particle has no preferred direction, we have the following:

$$v_{\text{rms}}^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2 \quad \rightarrow \quad v_x = v_{\text{rms}}/\sqrt{3}$$

The time for one crossing of the room is then given by $t = d/v_x = \sqrt{3}d/v_{\text{rms}}$, so the time for a round trip is $2\sqrt{3}d/v_{\text{rms}}$. Thus, the number of back and forth round trips per second is the reciprocal of this time, $\dfrac{v_{\text{rms}}}{2\sqrt{3}d}$.

$$\text{\# round trips per second} = \frac{v_{\text{rms}}}{2\sqrt{3}d} = \frac{461\text{ m/s}}{2\sqrt{3}(5.0\text{ m})} = 26.62 \approx \boxed{27\text{ round trips per second}}$$

55. From Fig. 13–23, we see that CO_2 is a $\boxed{\text{vapor}}$ at 35 atm and 35°C.

56. (*a*) From Fig. 13–23, at atmospheric pressure, CO_2 can exist as $\boxed{\text{solid or vapor}}$.

(*b*) From Fig. 13–23, for CO_2 to exist as a liquid, $\boxed{5.11\text{ atm} \le P \le 73\text{ atm}}$ and $\boxed{-56.6°\text{C} \le T \le 31°\text{C}}$.

57. (*a*) From Fig. 13–22, water is $\boxed{\text{vapor}}$ when the pressure is 0.01 atm and the temperature is 90°C.

(*b*) From Fig. 13–22, water is $\boxed{\text{solid}}$ when the pressure is 0.01 atm and the temperature is −20°C.

58. (*a*) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a vapor at some pressure between 1.0 atm and 0.006 atm. It would still be a vapor at 0.004 atm.

(*b*) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a solid at a pressure of 1.0 atm, and then becomes a vapor at some pressure lower than 0.006 atm. It would be a vapor at 0.004 atm.

59. From Table 13–3, the saturated vapor pressure at 30°C is 4240 Pa. Since the relative humidity is 75%, the partial pressure of water is as follows:

$$P_{\text{water}} = 0.75P_{\text{saturated}} = 0.75(4240\text{ Pa}) = 3180\text{ Pa} \approx \boxed{3200\text{ Pa}}$$

60. At the boiling temperature, the external air pressure equals the saturated vapor pressure. Thus, from Table 13–3, for 80°C the saturated air pressure is $\boxed{355\text{ torr}}$ or $\boxed{4.73\times10^4\text{ Pa}}$ or $\boxed{0.467\text{ atm}}$.

61. From Table 13–3, if the temperature is 25°C, the saturated vapor pressure is 23.8 torr. If the relative humidity is 65%, then the partial pressure of water is 65% of the saturated vapor pressure, or 15.47 torr. The dew point is the temperature at which the saturated vapor pressure is 15.47 torr, and from Table 13–3 that is between 15°C and 20°C. Since there is no entry for 15.47 torr, the temperature can be estimated by a linear interpolation. Between 15°C and 20°C, the temperature change per torr is as follows:

$$\frac{(20-15)\text{ C}°}{(17.5-12.8)\text{ torr}} = 1.064\text{ C}°/\text{torr}$$

Thus, the temperature corresponding to 15.47 torr is as follows:

$$15°\text{C} + [(15.47-12.8)\text{ torr}](1.064\text{ C}°/\text{torr}) = 17.8°\text{C} \approx \boxed{18°\text{C}}$$

62. At the boiling temperature, the air pressure equals the saturated vapor pressure. The pressure of 0.80 atm is equal to 8.10×10^4 Pa. From Table 13–3, the temperature is between 90°C and 100°C. Since there is no entry for 8.10×10^4 Pa, the temperature can be estimated by a linear interpolation. Between 90°C and 100°C, the temperature change per Pa is as follows:

$$\frac{(100-90)\text{ C}^\circ}{(10.1-7.01)\times10^4\text{ Pa}} = 3.236\times10^{-4}\text{ C}^\circ/\text{Pa}$$

Thus, the temperature corresponding to 7.27×10^4 Pa is

$$90^\circ\text{C} + [(8.10-7.01)\times10^4\text{ Pa}](3.236\times10^{-4}\text{ C}^\circ/\text{Pa}) = 93.53^\circ\text{C} \approx \boxed{94^\circ\text{C}}.$$

63. The volume, temperature, and pressure of the water vapor are known. We use the ideal gas law to calculate the mass. The pressure is found from Table 13–3 to be 3170 Pa.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{(0.55)(3170\text{ Pa})(5.0\text{ m})(6.0\text{ m})(2.4\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(273.15\text{ K}+25\text{ K})} = 50.48\text{ mol}$$

$$m_{H_2O} = (50.48\text{ mol})(0.018\text{ kg/mol}) = \boxed{0.91\text{ kg}}$$

64. For boiling to occur at 120°C, the pressure inside the cooker must be the saturated vapor pressure of water at that temperature. That value can be found in Table 13–3. For the mass to stay in place and contain the steam inside the cooker, the weight of the mass must be greater than the force exerted by the gauge pressure from the gas inside the cooker. The limiting case, to hold the temperature right at 120°C, would be with the mass equal to that force.

$$mg = F_{\substack{\text{gauge}\\ \text{pressure}}} = (P_{\text{inside}} - P_{\text{atm}})A = (P_{\text{inside}} - P_{\text{atm}})\pi r^2 \quad \rightarrow$$

$$m = \frac{(P_{\text{inside}} - P_{\text{atm}})\pi r^2}{g} = \frac{(1.99\times10^5\text{ Pa} - 1.01\times10^5\text{ Pa})\pi(1.5\times10^{-3}\text{ m})^2}{9.80\text{ m/s}^2} = 0.07068\text{ kg} \approx \boxed{71\text{ g}}$$

65. From Table 13–3, the saturated vapor pressure at 20°C is 2330 Pa. The total amount of water vapor that can be in the air can be found from the saturated vapor pressure, using the ideal gas law.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{(2330\text{ Pa})(420\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(273+20)\text{ K}} = 401.7\text{ moles}$$

Since the relative humidity is 65%, only 65% of the total possible water is in the air. Thus, 35% of the total possible water can still evaporate into the air.

$$m_{\text{evaporate}} = 0.35(401.7\text{ moles})\left(\frac{18\times10^{-3}\text{ kg}}{1\text{ mole}}\right) = 2.531\text{ kg} \approx \boxed{2.5\text{ kg}}$$

66. The outside air is saturated at 5°C, so the vapor pressure of water is 872 Pa, as read from Table 13–3. The ideal gas law gives the following result for the change in volume of the given mass of air:

$$PV = nRT \quad \rightarrow \quad \frac{P}{nR} = \frac{T}{V} = \text{constant} \quad \rightarrow \quad \frac{T_1}{V_1} = \frac{T_2}{V_2}$$

Thus the vapor pressure of a given mass of air that moves from outside to inside is as follows:

$$P_{\text{in}} = \frac{nRT_{\text{in}}}{V_{\text{in}}} = \frac{nRT_{\text{out}}}{V_{\text{out}}} = P_{\text{out}} = 872\text{ Pa}$$

The saturated vapor pressure at 22°C is estimated to be 2666 Pa by using linear interpolation.

$$\frac{P_{25°C}-P_{20°C}}{25°C-20°C}=\frac{P_{22°C}-P_{20°C}}{22°C-20°C} \quad \to \quad \frac{3170\text{ Pa}-2330\text{ Pa}}{5\text{ C°}}=\frac{P_{22°C}-2330\text{ Pa}}{2\text{C°}} \quad \to \quad P_{22°C}=2666\text{ Pa}$$

Thus the the relative humidity is as follows:

$$\text{relative humidity}=\frac{872\text{ Pa}}{2666\text{ Pa}}=0.327\approx\boxed{33\%}$$

67. (*a*) The true atmospheric pressure will be $\boxed{\text{greater}}$ than the reading from the barometer. In Fig. 10–8, if there is a vapor pressure at the top of the tube, then $P_{\text{atm}}-\rho gh=P_{\text{vapor}}$. The reading from the barometer will be $\rho gh=P_{\text{atm}}-P_{\text{vapor}}<P_{\text{atm}}$.

(*b*) The percent error is found from the atmospheric pressure and the vapor pressure.

$$\%\text{ error}=\left(\frac{\rho gh-P_{\text{atm}}}{P_{\text{atm}}}\right)\times100=\left(-\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right)\times100=\left(-\frac{0.0015\text{ mm}-\text{Hg}}{760\text{ mm}-\text{Hg}}\right)\times100$$

$$=\boxed{\left(-2.0\times10^{-4}\right)\%}$$

(*c*) From Table 13–3, the saturated water vapor pressure at STP is 611 Pa.

$$\%\text{ error}=\left(\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right)\times100=\left(\frac{611\text{ Pa}}{1.013\times10^{5}\text{ Pa}}\right)\times100=\boxed{0.603\ \%}$$

68. From Example 13–19, we have an expression for the time to diffuse a given distance. Divide the distance by the time to get the average speed.

$$t=\frac{\bar{C}}{\Delta C}\frac{(\Delta x)^2}{D}=\frac{\frac{1}{2}(1.00+0.50)\text{ mol/m}^3}{(1.00-0.50)\text{ mol/m}^3}\frac{(25\times10^{-6}\text{ m})^2}{(95\times10^{-11}\text{ m}^2/\text{s})}=0.9868\text{ s}\approx\boxed{0.99\text{ s}}$$

$$v_{\text{diffuse}}=\frac{\Delta x}{t}=\frac{25\times10^{-6}\text{ m}}{0.9868\text{ s}}=\boxed{2.5\times10^{-5}\text{ m/s}}$$

The rms thermal speed is given by Eq. 13–9, $v_{\text{rms}}=\sqrt{3kT/m}$.

$$v_{\text{rms}}=\sqrt{3kT/m}=\sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(293\text{ K})}{75(1.66\times10^{-27}\text{ kg})}}=312.1\text{ m/s}\approx\boxed{310\text{ m/s}}$$

$$\frac{v_{\text{diffuse}}}{v_{\text{rms}}}=\frac{2.5\times10^{-5}\text{ m/s}}{312.1\text{ m/s}}=8.0\times10^{-8}\quad\text{(about 7 orders of magnitude difference)}$$

69. (*a*) Use the ideal gas law to find the concentration of the oxygen. We assume that the air pressure is 1.00 atm, so the pressure caused by the oxygen is 0.21 atm.

$$PV=nRT\quad\to$$

$$\frac{n}{V}=\frac{P}{RT}=\frac{(0.21\text{ atm})(1.013\times10^{5}\text{ Pa/atm})}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})}=8.732\text{ mol/m}^3\approx\boxed{8.7\text{ mol/m}^3}$$

Half of this is 4.366 mol/m^3.

(*b*) Use Eq. 13–10 to calculate the diffusion rate.

$$J = DA\frac{C_1 - C_2}{\Delta x} = (1\times 10^{-5}\ \text{m}^2/\text{s})(2\times 10^{-9}\ \text{m}^2)\left(\frac{8.732\ \text{mol/m}^3 - 4.366\ \text{mol/m}^3}{2\times 10^{-3}\ \text{m}}\right)$$

$$= 4.366\times 10^{-11}\ \text{mol/s} \approx \boxed{4\times 10^{-11}\ \text{mol/s}}$$

(*c*) From Example 13–19, we have an expression for the time to diffuse a given distance.

$$t = \frac{\bar{C}}{\Delta C}\frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(8.732\ \text{mol/m}^3 + 4.366\ \text{mol/m}^3)}{(8.732\ \text{mol/m}^3 - 4.366\ \text{mol/m}^3)}\frac{(2\times 10^{-3}\ \text{m})^2}{1\times 10^{-5}\ \text{m}^2/\text{s}} = \boxed{0.6\ \text{s}}$$

70. Since the glass does not expand, the measuring cup will contain 375 mL of hot water. Find the volume of water after it cools.

$$\Delta V = V_0\beta\Delta T = (375\ \text{mL})(210\times 10^{-6}/\text{C}°)(20°\text{C} - 95°\text{C}) = -5.91\ \text{mL} \approx \boxed{-5.9\ \text{mL}}$$

The volume of cool water is 5.9 mL less than the desired volume of 375 mL.

71. (*a*) At 37°C, the tape will expand from its calibration, so it will $\boxed{\text{read low}}$.

(*b*) $\dfrac{\Delta\ell}{\ell_0} = \alpha\Delta T = (12\times 10^{-6}/°\text{C})(37°\text{C} - 14°\text{C}) = 2.76\times 10^{-4} \approx \boxed{(2.8\times 10^{-2})\%}$

72. The net force on each side of the box will be the pressure difference between the inside and outside of the box times the area of a side of the box. The outside pressure is 1 atmosphere. The ideal gas law is used to find the pressure inside the box, assuming that the mass of gas and the volume are constant.

$$\frac{P}{T} = \frac{nR}{V} = \text{constant} \quad\to\quad \frac{P_2}{T_2} = \frac{P_1}{T_1} \quad\to\quad P_2 = P_1\frac{T_2}{T_1} = (1.00\ \text{atm})\frac{(273+165)\ \text{K}}{(273+15)\ \text{K}} = 1.521\ \text{atm}$$

The area of a side of the box is given by the following.

$$\text{Area} = \ell^2 = [(\text{Volume of box})^{1/3}]^2 = (6.15\times 10^{-2}\ \text{m}^2)^{2/3} = 1.5581\times 10^{-1}\ \text{m}^2$$

The net force on a side of the box is the pressure difference times the area.

$$F = (\Delta\ \text{Pressure})(\text{Area}) = (0.521\ \text{atm})(1.013\times 10^5\ \text{Pa})(1.5581\times 10^{-1}\ \text{m}^2) = 8221\ \text{N} \approx \boxed{8200\ \text{N}}$$

73. Assume the helium is an ideal gas. The volume of the cylinder is constant, and we assume that the temperature of the gas is also constant in the cylinder. From the ideal gas law, $PV = nRT$, under these conditions the amount of gas is proportional to the absolute pressure.

$$PV = nRT \quad\to\quad \frac{P}{n} = \frac{RT}{V} = \text{constant} \quad\to\quad \frac{P_1}{n_1} = \frac{P_2}{n_2} \quad\to\quad \frac{n_2}{n_1} = \frac{P_2}{P_1} = \frac{5\ \text{atm} + 1\ \text{atm}}{32\ \text{atm} + 1\ \text{atm}} = \frac{6}{33}$$

Thus, $\boxed{6/33 = 0.182 \approx 18\%}$ of the original gas remains in the cylinder.

$\boxed{74.}$ Assume that the air in the lungs is an ideal gas, that the amount of gas is constant, and that the temperature is constant. The ideal gas law then says that the value of PV is constant. The pressure a distance *h* below the surface of the water is discussed in Chapter 10 and is given by $P = P_0 + \rho gh$, where P_0 is atmospheric pressure and ρ is the density of the water.

$$(PV)_{\text{surface}} = (PV)_{\text{submerged}} \quad \rightarrow \quad V_{\text{surface}} = V_{\text{submerged}} \frac{P_{\text{submerged}}}{P_{\text{surface}}} = V_{\text{submerged}} \frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}}$$

$$= (5.5\text{ L})\frac{1.013\times10^5\text{ Pa} + (1.0\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(9\text{ m})}{1.013\times10^5\text{ Pa}} = 10.29\text{ L} \approx \boxed{10\text{ L}}$$

It is obviously very dangerous to have the lungs attempt to inflate to almost twice their volume. Thus, it is $\boxed{\text{not advisable}}$ to quickly rise to the surface.

75. To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N_2 (molecular weight 28) and 30% O_2 (molecular weight 32), then the average molecular weight is $0.70(28) + 0.30(32) = 29.$

(*a*) Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{ Pa})(1200\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(288\text{ K})} = 5.076\times10^4\text{ moles}$$

$$m = (5.076\times10^4\text{ moles})(29\times10^{-3}\text{ kg/mol}) = 1472\text{ kg} \approx \boxed{1500\text{ kg}}$$

(*b*) Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{ Pa})(1200\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(258\text{ K})} = 5.666\times10^4\text{ moles}$$

$$m = (5.666\times10^4\text{ moles})(29\times10^{-3}\text{ kg/mol}) = 1643\text{ kg}$$

The mass entering the house is $1643\text{ kg} - 1472\text{ kg} = 171\text{ kg} \approx \boxed{200\text{ kg}}$.

76. Assume the air is an ideal gas, and that the pressure is 1.0 atm.

$$PV = NkT \quad \rightarrow$$

$$N = \frac{PV}{kT} = \frac{(1.013\times10^5\text{ Pa})(6.0\times3.0\times2.5)\text{ m}^3}{(1.38\times10^{-23}\text{ J/K})(273+22)\text{ K}} = 1.1197\times10^{27}\text{ molecules} \approx \boxed{1.1\times10^{27}\text{ molecules}}$$

$$1.1197\times10^{27}\text{ molecules}\left(\frac{1\text{ mole}}{6.02\times10^{23}\text{ molecules}}\right) = 1860\text{ moles} \approx \boxed{1900\text{ moles}}$$

77. (*a*) The iron floats in the mercury because $\rho_{\text{Hg}} > \rho_{\text{Fe}}$, as seen in Table 10–1. As the substances are heated, the density of both substances will decrease due to volume expansion. The density of the mercury decreases more upon heating than the density of the iron, because $\beta_{\text{Hg}} > \beta_{\text{Fe}}$, as seen in Table 13–1. The net effect is that the densities get closer together, so relatively more mercury will have to be displaced to hold up the iron, and the iron will float $\boxed{\text{lower}}$ in the mercury.

(*b*) The fraction of the volume submerged is $V_{\underset{\text{displaced}}{\text{Hg}}}/V_{\text{Fe}}$. Both volumes expand as heated. The subscript "displaced" is dropped for convenience.

$$\text{fractional change} = \frac{V_{\text{Hg}}/V_{\text{Fe}} - V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}} = \frac{\dfrac{V_{0\,\text{Hg}}(1+\beta_{\text{Hg}}\Delta T)}{V_{0\,\text{Fe}}(1+\beta_{\text{Fe}}\Delta T)} - V_{0\,\text{Hg}}/V_{0\,\text{Fe}}}{V_{0\,\text{Hg}}/V_{0\,\text{Fe}}} = \frac{(1+\beta_{\text{Hg}}\Delta T)}{(1+\beta_{\text{Fe}}\Delta T)} - 1$$

$$= \frac{1+(180\times10^{-6}/\text{C}°)(25\text{C}°)}{1+(35\times10^{-6})(25\text{C}°)} - 1 = \frac{1.0045}{1.000875} - 1 = 3.6\times10^{-3}$$

$$\%\text{ change} = (3.6\times10^{-3})(100) = \boxed{0.36\%}$$

78. We find the number of moles of helium in the balloon from the ideal gas law.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{(1.08)(1.013\times10^5\ \text{Pa})\frac{4}{3}\pi(0.240\ \text{m})^3}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 2.6006\ \text{mol} \approx \boxed{2.60\ \text{mol}}$$

$$2.6006\ \text{mol}\left(\frac{4.00\ \text{g}}{1\ \text{mol}}\right) = \boxed{10.4\ \text{g}}$$

79. We assume the temperature is constant. As the oxygen pressure drops to atmospheric pressure, we can find the volume that it occupies at atmospheric pressure. We assume the final pressure inside the cylinder is atmospheric pressure. The gas would quit flowing at that pressure.

$$P_1V_1 = P_2V_2 \quad \rightarrow \quad V_2 = V_1\frac{P_1}{P_2} = (14\ \text{L})\frac{(1.38\times10^7\ \text{Pa} + 1.013\times10^5\ \text{Pa})}{1.013\times10^5\ \text{Pa}} = 1921\ \text{L}$$

14 L of that gas is not available—it is left in the container. So there is a total of 1907 L available.

$$(1907\ \text{L})\frac{1\ \text{min}}{2.1\ \text{L}} = 908.1\ \text{min} \approx \boxed{910\ \text{min}} \approx \boxed{15\ \text{h}}$$

80. The gap will be the radius of the lid minus the radius of the jar. Also note that the original radii of the lid and the jar are the same.

$$r_{\text{gap}} = (r_0 + \Delta r)_{\text{lid}} - (r_0 + \Delta r)_{\text{jar}} = \Delta r_{\text{lid}} - \Delta r_{\text{jar}} = (\alpha_{\text{brass}} - \alpha_{\text{glass}})r_0\Delta T$$

$$= (19\times10^{-6}/\text{C}° - 9\times10^{-6}/\text{C}°)(4.0\ \text{cm})(40\ \text{C}°) = \boxed{1.6\times10^{-3}\ \text{cm}}$$

81. (*a*) Assume that a mass M of gasoline with volume V_0 at 0°C is under consideration, so its density is $\rho_0 = M/V_0$. At a temperature of 33°C, the same mass has a volume $V = V_0(1+\beta\Delta T)$.

$$\rho = \frac{M}{V} = \frac{M}{V_0(1+\beta\Delta T)} = \frac{\rho_0}{1+\beta\Delta T} = \frac{0.68\times10^3\ \text{kg/m}^3}{1+(950\times10^{-6}/\text{C}°)(33\ \text{C}°)} = 0.6593\times10^3\ \text{kg/m}^3$$

$$\approx \boxed{0.66\times10^3\ \text{kg/m}^3}$$

(*b*) Calculate the percentage change in the density.

$$\%\text{ change} = \frac{(0.6593 - 0.68)\times10^3\ \text{kg/m}^3}{0.68\times10^3\ \text{kg/m}^3}\times100 = \boxed{-3.0\%}$$

82. The change in length is to be restricted to $\Delta\ell < 1.0\times10^{-6}$ m.

$$\Delta\ell = \alpha\ell_0\Delta T \le 1.0\times10^{-6}\ \text{m} \quad \rightarrow \quad \Delta T \le \frac{1.0\times10^{-6}\ \text{m}}{(9\times10^{-6}/\text{C}°)(1.0\ \text{m})} \le 0.11\ \text{C}°$$

Thus, the temperature would have to be controlled to within $\boxed{\pm0.11\ \text{C}°}$

83. The original length of the steel band is $\ell_0 = 2\pi R_{\text{Earth}}$. At the higher temperature, the length of the band is $\ell = \ell_0 + \Delta\ell = 2\pi R = 2\pi(R_{\text{Earth}} + \Delta R)$. The change in radius, ΔR, would be the height above the Earth.

$$\Delta\ell = \alpha\ell_0\Delta T = 2\pi\Delta R \quad \rightarrow$$

$$\Delta R = \frac{\alpha\ell_0\Delta T}{2\pi} = \alpha R_{\text{Earth}}\Delta T = (12\times10^{-6}/\text{C}^\circ)(6.38\times10^{6}\ \text{m})(55^\circ\text{C} - 25^\circ\text{C}) = 2297\ \text{m} \approx \boxed{2300\ \text{m}}$$

84. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. Hydrogen atoms have a mass of 1 atomic mass unit.

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(2.7\ \text{K})}{1(1.66\times10^{-27}\ \text{kg})}} = \boxed{260\ \text{m/s}}$$

The pressure is found from the ideal gas law, $PV = NkT$.

$$PV = NkT \quad \rightarrow \quad P = \frac{NkT}{V} = \frac{(1)(1.38\times10^{-23}\ \text{J/K})(2.7\ \text{K})}{1\ \text{cm}^3\left(\dfrac{1\times10^{-6}\ \text{m}^3}{1\ \text{cm}^3}\right)} = 3.726\times10^{-17}\ \text{Pa}\left(\frac{1\ \text{atm}}{1.01\times10^{5}\ \text{Pa}}\right)$$

$$= 3.689\times10^{-22}\ \text{atm} \approx \boxed{3.7\times10^{-22}\ \text{atm}}$$

85. It is stated in the text that the relationship $v_{\text{rms}} = \sqrt{3kT/m}$ is applicable to molecules within living cells at body temperature (37°C). The rms speed is given by Eq. 13–9.

(*a*) $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}{(89\ \text{u})(1.66\times10^{-27}\ \text{kg/u})}} = 294.7\ \text{m/s} \approx \boxed{290\ \text{m/s}}$$

(*b*) $$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(310\ \text{K})}{(8.5\times10^{4}\ \text{u})(1.66\times10^{-27}\ \text{kg/u})}} = 9.537\ \text{m/s} \approx \boxed{9.5\ \text{m/s}}$$

86. The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. Set the escape velocity to v_{rms} solve for *T*.

(*a*) For oxygen molecules:

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{2(15.9994)(1.66\times10^{-27}\ \text{kg})(1.12\times10^{4}\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})} = \boxed{1.61\times10^{5}\ \text{K}}$$

(*b*) For helium atoms:

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{(4.002602)(1.66\times10^{-27}\ \text{kg})(1.12\times10^{4}\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})} = \boxed{2.01\times10^{4}\ \text{K}}$$

(*c*) Because the "escape temperature" is so high for oxygen, very few oxygen molecules ever escape the atmosphere. But helium, with one-eighth the mass, can escape at a much lower temperature. While the temperature of the Earth is not close to 2.0×10^{4} K today, during the Earth's formation its temperature was possibly much hotter—presumably hot enough that helium was able to escape the atmosphere.

87. The gravitational potential energy is given by $\text{PE} = mgh$, and the average kinetic energy is $\text{KE} = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$. We find the ratio of potential energy to kinetic energy. The molecular mass of oxygen molecules is 32.0 u.

$$\frac{\text{PE}}{\text{KE}} = \frac{mgh}{\frac{3}{2}kT} = \frac{(32.0)(1.66\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)(1.00\ \text{m})}{\frac{3}{2}(1.38\times10^{-23}\ \text{J/K})(296\ \text{K})} = \boxed{8.50\times10^{-5}}$$

$\boxed{\text{Yes}}$, it is reasonable to neglect the gravitational potential energy.

88. The temperature can be found from the rms speed by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. The molecular mass of nitrogen molecules is 28.

$$v_{\text{rms}} = \sqrt{3kT/m} \quad \rightarrow$$

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{(28)(1.66\times10^{-27}\ \text{kg})\left[(4.2\times10^4\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{3(1.38\times10^{-23}\ \text{J/K})} = \boxed{1.5\times10^5\ \text{K}}$$

89. Assume that the water is an ideal gas and that the temperature is constant. From Table 13–3, saturated vapor pressure at 85°C is midway between the given saturated vapor pressures for 80°C and 90°C, which is 5.87×10^4 Pa. To have a relative humidity of 10%, the vapor pressure will be 5.87×10^3 Pa. Use the ideal gas law to calculate the amount of water.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} = \frac{(5.87\times10^3\ \text{Pa})(8.5\ \text{m}^3)}{(8.315\ \text{J/mol}\cdot\text{K})(273+85)\ \text{K}} = 16.76\ \text{moles}$$

$$16.76\ \text{moles}\left(\frac{18\times10^{-3}\ \text{kg}}{1\ \text{mole}}\right) = 0.3017\ \text{kg} \approx \boxed{0.30\ \text{kg}}$$

90. Following the development of the kinetic molecular theory in Section 13–10 of the textbook, the tennis balls hitting the trash can lid are similar to the particles colliding with the walls of a container causing pressure. Quoting from the text, "the average force averaged over many collisions will be equal to the force exerted during one collision divided by the time between collisions." That average force must be the weight of the trash can lid in order to suspend it. The fact that the collisions are elastic means that the change in momentum of one of the balls is twice its original momentum.

$$F_{\text{avg}} = M_{\text{lid}}g;\ F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m_{\text{ball}}v_{\text{ball}} - (-m_{\text{ball}}v_{\text{ball}})}{\Delta t} = \frac{2m_{\text{ball}}v_{\text{ball}}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{2m_{\text{ball}}v_{\text{ball}}}{M_{\text{lid}}g}$$

The above expression is "seconds per ball," so its reciprocal will be "balls per second."

$$\text{balls/s} = \frac{1}{\Delta t} = \frac{M_{\text{lid}}g}{2m_{\text{ball}}v_{\text{ball}}} = \frac{(0.50\ \text{kg})(9.80\ \text{m/s}^2)}{2(0.060\ \text{kg})(15\ \text{m/s})} = \boxed{2.7\ \text{balls/s}}$$

91. Assume that the water vapor behaves like an ideal gas. At 20°C, the saturated vapor pressure is 2.33×10^3 Pa. Using the ideal gas law, find the number of moles of water in the air at both 95% and 40%. Subtract those mole amounts to find the amount of water that must be removed.

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT} \quad \rightarrow$$

$$n_1 - n_2 = \frac{V}{RT}(P_1 - P_2) = \frac{(105\text{ m}^2)(2.4\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}(2.33\times10^3\text{ Pa})(0.95-0.40) = 134.8\text{ mol}$$

$$134.8\text{ mol}\left(\frac{18\times10^{-3}\text{ kg}}{1\text{ mol}}\right) = 2.426\text{ kg} \approx \boxed{2.4\text{ kg}}$$

Solutions to Search and Learn Problems

1. The first method for calculating the thermal expansion is given in Eq. 13–2, with the coefficient of expansion given in Table 13–1. Using this method, we calculate the volume change for each temperature difference as follows:

$$\Delta V_A = \beta V_0 \Delta T_A = (3400\times10^{-6}/\text{C}^\circ)(1000\text{ L})(0^\circ\text{C} - -100^\circ\text{C}) = \boxed{340\text{ L}}$$

$$\Delta V_B = \beta V_0 \Delta T_B = (3400\times10^{-6}/\text{C}^\circ)(1000\text{ L})(100^\circ\text{C} - 0^\circ\text{C}) = \boxed{340\text{ L}}$$

Note that the textbook states that this method is accurate only if the change in volume is small compared to the actual volume. This is obviously not the case here, so the values above are not particularly accurate.

The second method is to use the ideal gas law of Eq. 13–3.

$$pV = nRT \quad\to\quad \frac{pV_1}{pV_2} = \frac{nRT_1}{nRT_2} \quad\to\quad V_2 = V_1\frac{T_2}{T_1} \quad\to\quad \Delta V = V_2 - V_1 = V_1\left(\frac{T_2}{T_1} - 1\right)$$

The temperature must be converted to kelvins for this method.

$$\Delta V_A = V_1\left(\frac{T_{2A}}{T_{1A}} - 1\right) = (1000\text{ L})\left(\frac{273\text{ K}}{173\text{ K}} - 1\right) = \boxed{580\text{ L}}$$

$$\Delta V_B = V_1\left(\frac{T_{2B}}{T_{1B}} - 1\right) = (1000\text{ L})\left(\frac{373\text{ K}}{273\text{ K}} - 1\right) = \boxed{370\text{ L}}$$

The answers are different because the first method assumed small temperature changes, such that the relationship between the temperature and volume could be assumed linear. The second method accounts for a nonlinear relationship over large changes in volume and temperature.

2. (*a*) We treat the air as an ideal gas. Since the amount of air and temperature of the air are the same in both cases, the ideal gas law says $PV = nRT$ is a constant.

$$P_2V_2 = P_1V_1 \quad\to\quad V_2 = V_1\frac{P_1}{P_2} = (11.3\text{ L})\frac{180\text{ atm}}{1.00\text{ atm}} = 2034\text{ L} \approx \boxed{2030\text{ L}}$$

(*b*) Before entering the water, the air coming out of the tank will be at 1.00 atm pressure, so the person will be able to breathe 2034 L of air.

$$t = 2034\text{ L}\left(\frac{1\text{ breath}}{2.0\text{ L}}\right)\left(\frac{1\text{ min}}{12\text{ breaths}}\right) = 84.75\text{ min} \approx \boxed{85\text{ min}}$$

(*c*) When the person is underwater, the temperature and pressure will be different. Use the ideal gas law to relate the original tank conditions to the underwater breathing conditions. The amount of gas will be constant, so $PV/T = nR$ will be constant. The pressure a distance *h* below the surface of the water is given in Eq. 10–3c, $P = P_0 + \rho gh,$ where P_0 is atmospheric pressure and ρ is the density of the sea water.

$$\frac{P_2V_2}{T_2}=\frac{P_1V_1}{T_1} \rightarrow V_2=V_1\frac{P_1}{P_2}\frac{T_2}{T_1}$$

$$V_2=(11.3\text{ L})\left[\frac{(180\text{ atm})(1.013\times10^5\text{ Pa/atm})}{1.013\times10^5\text{ Pa}+(1.025\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(23.0\text{ m})}\right]\left(\frac{283\text{ K}}{291\text{ K}}\right)$$

$$=602.95\text{ L} \qquad t=(602.95\text{ L})\left(\frac{1\text{ breath}}{2.0\text{ L}}\right)\left(\frac{1\text{ min}}{12\text{ breaths}}\right)=25.12\text{ min}\approx\boxed{25\text{ min}}$$

3. The change in radius with heating does not cause a torque on the rotating wheel, so the wheel's angular momentum does not change. Also recall that for a cylindrical wheel rotating about its axis, the moment of inertia is $I=\frac{1}{2}mr^2$.

$$L_0=L_{\text{final}} \rightarrow I_0\omega_0=I_{\text{final}}\omega_{\text{final}} \rightarrow \omega_{\text{final}}=\frac{I_0\omega_0}{I_{\text{final}}}=\frac{\frac{1}{2}mr_0^2\omega_0}{\frac{1}{2}mr^2}=\frac{r_0^2\omega_0}{r^2}$$

$$\frac{\Delta\omega}{\omega}=\frac{\omega_{\text{final}}-\omega_0}{\omega_0}=\frac{\frac{r_0^2\omega_0}{r^2}-\omega_0}{\omega_0}=\frac{r_0^2}{r^2}-1=\frac{r_0^2}{(r_0+\Delta r)^2}-1=\frac{r_0^2}{(r_0+\alpha r_0\Delta T)^2}-1=\frac{1}{(1+\alpha\Delta T)^2}-1$$

$$=\frac{1-(1+2\alpha\Delta T+(\alpha\Delta T)^2)}{(1+\alpha\Delta t)^2}=\frac{-2\alpha\Delta T-(\alpha\Delta T)^2}{(1+\alpha\Delta T)^2}=-\alpha\Delta T\frac{2+\alpha\Delta T}{(1+\alpha\Delta T)^2}$$

Now assume that $\alpha\Delta T\ll 1$, so $\frac{\Delta\omega}{\omega}=-\alpha\Delta T\frac{2+\alpha\Delta T}{(1+\alpha\Delta T)^2}\approx-2\alpha\Delta T$. Evaluate at the given values.

$$-2\alpha\Delta T=-2(25\times10^{-6}/\text{C}^\circ)(80.0\text{ C}^\circ)=\boxed{-4.0\times10^{-3}}$$

The negative sign means that the frequency has decreased. This is reasonable, because the rotational inertia increased due to the thermal expansion.

4. There are three forces to consider: the buoyant force upward (which is the weight of the cold air displaced by the volume of the balloon), the downward weight of the hot air inside the balloon, and the downward weight of the passengers and equipment. For the balloon to rise at constant speed, the buoyant force must equal the sum of the two weights.

$$F_{\text{buoyant}}=m_{\text{hot}}g+3300\text{ N} \rightarrow V\rho_{\text{cold}}g=V\rho_{\text{hot}}g+3300\text{ N}$$

The ideal gas law can be written in terms of the gas density ρ and the molecular mass M as follows:

$$PV=nRT=\frac{m}{M}RT \rightarrow \frac{PM}{R}=\frac{m}{V}T=\rho T$$

The gas inside and outside the balloon is air, so M is the same for inside and outside. Also, since the balloon is open to the atmosphere, the pressure inside the balloon is the same as the pressure outside the balloon. Thus, the ideal gas law reduces to $\rho T=\text{constant}=(\rho T)_{\text{cold}}=(\rho T)_{\text{hot}}$. This equation can be used to eliminate the density of the hot gas from the force equation, and the resulting equation can be solved for the temperature of the gas inside the balloon.

$$V\rho_{\text{cold}}g=V\rho_{\text{hot}}g+3300\text{ N}=V\rho_{\text{cold}}\frac{T_{\text{cold}}}{T_{\text{hot}}}g+3300\text{ N} \rightarrow$$

$$T_{\text{hot}}=\frac{V\rho_{\text{cold}}T_{\text{cold}}g}{(V\rho_{\text{cold}}g-3300\text{ N})}=\frac{(1800\text{ m}^3)(1.29\text{ kg/m}^3)(273\text{ K})(9.80\text{ m/s}^2)}{[(1800\text{ m}^3)(1.29\text{ kg/m}^3)(9.80\text{ m/s}^2)-3300\text{ N}]}$$
$$=319.3\text{ K}\quad(2\text{ significant figures})\quad\rightarrow\quad 319.3\text{ K}-273.15\text{ K}=46.15°\text{C}\approx\boxed{50°\text{C}}$$

One factor limiting the maximum altitude would be that as the balloon rises, the density of the air decreases, and thus the temperature required gets higher. Eventually the air would be too hot and the balloon fabric might be damaged.

5. We assume that the last breath Galileo took has been spread uniformly throughout the atmosphere since his death and that each molecule remains in the atmosphere. Calculate the number of molecules in Galileo's last breath and divide it by the volume of the atmosphere to get "Galileo molecules/m^3." Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breaths.

$$PV=NkT\quad\rightarrow\quad N=\frac{PV}{kT}=\frac{(1.01\times10^5\text{ Pa})(2.0\times10^{-3}\text{ m}^3)}{(1.38\times10^{-23}\text{ J/K})(300\text{ K})}=4.9\times10^{22}\text{ molecules}$$

$$\text{Atmospheric volume}=4\pi R_{\text{Earth}}^2h=4\pi(6.38\times10^6\text{ m})^2(1.0\times10^4\text{ m})=5.1\times10^{18}\text{ m}^3$$

$$\frac{\text{Galileo molecules}}{\text{m}^3}=\frac{4.9\times10^{22}\text{ molecules}}{5.8\times10^{18}\text{ m}^3}=9.6\times10^3\text{ molecules/m}^3$$

$$\frac{\text{Galileo molecules}}{\text{breath}}=9.6\times10^3\frac{\text{molecules}}{\text{m}^3}\left(\frac{2.0\times10^{-3}\text{ m}^3}{1\text{ breath}}\right)=19\frac{\text{molecules}}{\text{breath}}\approx\boxed{20\frac{\text{molecules}}{\text{breath}}}$$

6. (*a*) We calculate the volume per molecule from the ideal gas law and assume the molecular volume is spherical.

$$PV=NkT\quad\rightarrow\quad\frac{V}{N}=\frac{kT}{P}=\tfrac{4}{3}\pi r^3\quad\rightarrow$$

$$r_{\substack{\text{inter-}\\\text{molecular}}}=\left(\frac{3kT}{4\pi P}\right)^{1/3}=\left(\frac{3(1.38\times10^{-23}\text{ J/K})(273\text{ K})}{4\pi(1.01\times10^5\text{ Pa})}\right)^{1/3}=\boxed{2.07\times10^{-9}\text{ m}}$$

The intermolecular distance would be twice this "radius," so about 4×10^{-9} m. This is about 14 times larger than the molecular diameter.

$$\frac{d_{\substack{\text{inter-}\\\text{molecular}}}}{d_{\substack{\text{actual}\\\text{molecule}}}}\approx\frac{4.14\times10^{-9}\text{ m}}{3\times10^{-10}\text{ m}}=13.8\quad\rightarrow\quad\boxed{\frac{d_{\substack{\text{inter-}\\\text{molecular}}}}{d_{\substack{\text{actual}\\\text{molecule}}}}\approx14}$$

(*b*) Now we scale the molecular diameter up to 4 cm. The intermolecular distance would be 13.8 times that, which is about $\boxed{55\text{ cm}}$.

(*c*) We replace the pressure in part (*a*) with 3 atm.

$$r_{\substack{\text{inter-}\\\text{molecular}}}=\left(\frac{3kT}{4\pi P}\right)^{1/3}=\left(\frac{3(1.38\times10^{-23}\text{ J/K})(273\text{ K})}{4\pi(3\text{ atm})(1.01\times10^5\text{ Pa/atm})}\right)^{1/3}=\boxed{1.44\times10^{-9}\text{ m}}$$

$$\frac{d_{\substack{\text{inter-}\\\text{molecular}}}}{d_{\substack{\text{actual}\\\text{molecule}}}}\approx\frac{2(1.44)\times10^{-9}\text{ m}}{3\times10^{-10}\text{ m}}=9.58\quad\rightarrow\quad\boxed{\frac{d_{\substack{\text{inter-}\\\text{molecular}}}}{d_{\substack{\text{actual}\\\text{molecule}}}}\approx9.6}$$

We see that the intermolecular distance has decreased with the increased pressure, from about 14 times larger to only about 10 times larger.

(*d*) We are to calculate the volume occupied by the molecules themselves, compared to the total volume of the gas. The total volume is the volume of the molecules plus the volume of the intermolecular distance. Since we don't know the actual number of gas particles, we use the volume of one gas particle and the volume associated with the intermolecular distance.

$$\frac{V_{\substack{\text{actual}\\\text{molecule}}}}{V_{\substack{\text{inter-}\\\text{molecular}}}+V_{\substack{\text{actual}\\\text{molecule}}}}=\frac{\frac{4}{3}\pi\left(\frac{1}{2}d_{\substack{\text{actual}\\\text{molecule}}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}d_{\substack{\text{inter-}\\\text{molecule}}}\right)^3+\frac{4}{3}\pi\left(\frac{1}{2}d_{\substack{\text{actual}\\\text{molecule}}}\right)^3}=\frac{\left(d_{\substack{\text{actual}\\\text{molecule}}}\right)^3}{\left(d_{\substack{\text{inter-}\\\text{molecule}}}\right)^3+\left(d_{\substack{\text{actual}\\\text{molecule}}}\right)^3}$$

$$=\frac{1}{\left(d_{\substack{\text{inter-}\\\text{molecule}}}\Big/d_{\substack{\text{actual}\\\text{molecule}}}\right)^3+1}$$

$$P=1\text{ atm:}\quad \%=\left(\frac{1}{13.8^3+1}\right)100=\boxed{(3.8\times10^{-2})\%}$$

$$P=3\text{ atm:}\quad \%=\left(\frac{1}{9.58^3+1}\right)100=\boxed{0.11\%}$$

14

HEAT

Responses to Questions

1. The work goes primarily into increasing the temperature of the orange juice, by increasing the average kinetic energy of the molecules comprising the orange juice.

2. When a hot object warms a cooler object, energy is transferred from the hot object to the cold object. Temperature does NOT flow. The temperature changes of the two objects are not necessarily equal in magnitude. Under certain circumstances, they can be equal in magnitude, however. In an ideal case (where no heat energy is lost to the surroundings), the amount of heat lost by the warmer object is the same as the amount of heat gained by the cooler object.

3. (*a*) Internal energy depends on both the number of molecules of the substance and the temperature of the substance. Heat will flow naturally from the object with the higher temperature to the object with the lower temperature. The object with the high temperature may or may not be the object with the higher internal energy.
   (*b*) The two objects may consist of one with a higher temperature and smaller number of molecules, and the other with a lower temperature and a larger number of molecules. In that case it is possible for the objects to have the same internal energy, but heat will still flow from the object with the higher temperature to the one with the lower temperature.

4. The water will coat the plants, so the water, not the plant, is in contact with the cold air. Thus, as the air cools, the water cools before the plant does—the water insulates the plant. As the water cools, it releases energy, and raises the temperature of its surroundings, which includes the plant. Particularly if the water freezes, relatively large amounts of heat are released due to the relatively large heat of fusion for water.

5. Because the specific heat of water is quite large, it can contain a relatively large amount of thermal energy per unit mass with a relatively small increase in temperature. Since the water is a liquid, it is relatively easy to transport from one location to another, so large quantities of energy can be moved from one place to another with relative simplicity by water. And the water will give off a large amount of energy as it cools.

6. The water on the cloth jacket will evaporate. Evaporation is a cooling process since energy is required to change the liquid water to vapor. As water evaporates from the moist cloth, the canteen surface is cooled.

$\boxed{7.}$ Steam at 100°C contains more thermal energy than water at 100°C. The difference is due to the latent heat of vaporization, which for water is quite high. As the steam touches the skin and condenses, a large amount of energy is released, causing more severe burns. And the condensed water is still at 100°C, so more burning can occur as that water cools.

8. Evaporation involves water molecules escaping the intermolecular bonds that hold the water together in the liquid state. It takes energy for the molecules to break those bonds (to overcome the bonding forces). This energy is the latent heat of vaporization. The most energetic molecules (those having the highest speed) are the ones that have the most energy (from their kinetic energy) to be able to overcome the bonding forces. The slower moving molecules remain, lowering the average kinetic energy and thus lowering the internal energy and temperature of the liquid.

9. The pasta will not cook faster if the water is boiling faster. The rate at which the pasta cooks depends on the temperature at which it is cooking, and the boiling water is the same temperature whether it is boiling fast or slow.

10. Even though the temperature is high in the upper atmosphere, which means that the gas particles are moving very fast, the density of gas particles is very low. There would be relatively very few collisions of high-temperature gas particles with the animal, so very little warming of the animal would occur. Instead, the animal would radiate heat to the rarified atmosphere. The emissivity of the animal is much greater than that of the rarified atmosphere, so the animal will lose much more energy by radiation than it can gain from the atmosphere.

$\boxed{11.}$ Snow consists of crystals with tiny air pockets in between the flakes. Air is a good insulator, so when the Arctic explorers covered themselves with snow they were using its low thermal conductivity to keep heat from leaving their bodies. (In a similar fashion, down comforters keep you warm because of all the air trapped in between the feathers.) Snow would also protect the explorers from the very cold wind and prevent heat loss by convection.

12. We assume that the wet sand has been wetted fairly recently with water that is cooler than the sand's initial temperature. Water has a higher heat capacity than sand, so for equal masses of sand and water, the sand will cool more than the water warms as their temperatures move toward equilibrium. Thus, the wet sand may actually be cooler than the dry sand. Also, if both the wet and dry sand are at a lower temperature than your feet, the sand with the water in it is a better thermal conductor, so heat will flow more rapidly from you into the wet sand than into the dry sand, giving more of a sensation of having touched something cold.

13. An object with "high heat content" does not have to have a high temperature. If a given amount of heat energy is transferred into equal-mass samples of two substances initially at the same temperature, the substance with the lower specific heat will have the higher final temperature. But both substances would have the same "heat content" relative to their original state. So an object with "high heat content" might be made of material with a very high specific heat and therefore not necessarily be at a high temperature.

14. A hot-air furnace heats primarily by air convection. A return path (often called a "cold air return") is necessary for the convective currents to be able to completely circulate. If the flow of air is blocked, then the convective currents and the heating process will be interrupted. Heating will be less efficient and less uniform if the convective currents are prevented from circulating.

15. A ceiling fan makes more of a breeze when it is set to blow the air down (usually called the "forward" direction by fan manufacturers). This is the setting for the summer, when the breeze will feel cooling since it accelerates evaporation from the skin. In the winter, the fan should be set to pull air up. This

forces the warmer air at the top of the room to move out toward the walls and down. The relocation of warmer air keeps the room feeling warmer, and there is less "breeze" effect on the occupants of the room.

16. When the garment is fluffed up, it will have the most air trapped in its structure. The air has a low thermal conductivity, and the more the garment can be "fluffed," the more air it will trap, making it a better insulator. The "loft" value is similar to the *R*-value of insulation, since the thicker the insulation, the higher the *R*-value. The rate of thermal conduction is inversely proportional to the thickness of the conductor, so a thick conductor (high loft value) means a lower thermal conduction rate, so a lower rate of losing body heat.

17. For all mechanisms of cooling, the rate of heat transfer from the hot object to the cold one is dependent on surface area. The heat sink with fins provides much more surface area than just a solid piece of metal, so there is more cooling of the microprocessor chip. A major mechanism for cooling the heat sink is that of convection. More air is in contact with the finned heat sink than would be in contact with a solid piece of metal. There is often a fan circulating air around that heat sink as well, so that heated air can continually be replaced with cool air to promote more cooling.

18. When there is a temperature difference in air, convection currents arise. Since the temperature of the land rises more rapidly than that of the water, due to the large specific heat of water, the air above the land will be warmer than the air above the water. The warm air above the land will rise, and that rising warm air will be replaced by cooler air from over the body of water. The result is a breeze from the water toward the land.

19. We assume that the temperature in the house is higher than that under the house. Thus, heat will flow through the floor out of the house. If the house sits directly on the ground or on concrete, the heat flow will warm the ground or concrete. Dirt and concrete are relatively poor conductors of heat, so the thermal energy that goes into them will stay for a relatively long time, allowing their temperature to rise and thus reducing the heat loss through the floor. If the floor is over a crawlspace, then the thermal energy from the floor will be heating air instead of dirt or concrete. If that warmed air gets moved away by wind currents or by convection and replaced with colder air, then the temperature difference between the inside and outside will stay large, and more energy will leave through the floor, making the inside of the house cooler.

20. Air is a poorer conductor of heat than water by roughly a factor of 20, so the rate of heat loss from your body to the air is roughly 20 times less than the rate of heat loss from your body to the water. Thus, you lose heat quickly in the water and feel cold. Another contributing factor is that water has a high heat capacity, so as heat leaves your body and enters the water, the temperature rise for the water close to your body is small. Air has a smaller heat capacity, so the temperature rise for the air close to your body is larger. This reduces the temperature difference between your body and the air, which reduces the rate of heat loss to the air as well.

21. A thermometer in the direct sunlight would gain thermal energy (and thus show a higher temperature) due to receiving radiation directly from the Sun. The emissivity of air is small, so it does not gain as much energy from the Sun as the mercury and glass do. The thermometer is to reach its equilibrium temperature by heat transfer with the air, in order to measure the air temperature.

22. Premature babies have underdeveloped skin, and they can lose a lot of moisture through their skin by evaporation. For a baby in a very warm environment, like an incubator at 37°C, there will be a large evaporative effect. A significant increase in evaporation occurs at incubator temperatures, and that evaporation of moisture from the baby will cool the baby dramatically. Thus, an incubator must have not only a high temperature but also a high humidity. Other factors might include radiative

energy loss, blood vessels being close to the skin surface so there is less insulation than in a more mature baby, and low food consumption to replace lost energy. Also, the smaller the child, the larger the surface-area-to-volume ratio, which leads to relatively more heat being lost through the body surface area than for an older, larger baby.

23. An ordinary fan does not cool the air directly. It actually warms the air slightly, because the motor used to power the fan will exhaust some heat into the air, and the increase in average kinetic energy of the air molecules caused by the fan blades pushing them means the air temperature increases slightly. The reason for using the fan is that it keeps air moving. The human body warms the air immediately around it, assuming the air is initially cooler than the body. If that warmed air stays in contact with the body, then the body will lose little further heat after the air is warmed. The fan, by circulating the air, removes the heated air from close to the body and replaces it with cooler air. Likewise, the body is also cooled by evaporation of water from the skin. As the relative humidity of the air close to the body increases, less water can be evaporated, and cooling by evaporation is decreased. The fan, by circulating the air, removes the hot, humid air from close to the body and replaces it with cooler, less humid air, so that evaporation can continue.

24. (*a*) (1) Ventilation around the edges is cooling by convection.
(2) Cooling through the frame is cooling by conduction.
(3) Cooling through the glass panes is cooling by conduction and radiation.

(*b*) Heavy curtains can reduce all three heat losses. The curtains will prevent air circulation around the edges of the windows, thus reducing the convection cooling. The curtains are more opaque than the glass, preventing the electromagnetic waves responsible for radiation heat transfer from reaching the glass. And the curtains provide another layer of insulation between the outdoors and the warm interior of the room, lowering the rate of conduction.

25. The thermal conductivity of the wood is about 2000 times less than that of the aluminum. Thus, it takes a long time for energy from the wood to flow into your hand. Your skin temperature rises very slowly due to contact with the wood compared to contact with the aluminum, so the sensation of heating is much less.

26. The Earth cools primarily by radiation. The clouds act as insulation in that they absorb energy from the radiating Earth and reradiate some of it back to the Earth, reducing the net amount of radiant energy loss.

$\boxed{27.}$ Shiny surfaces have low values of ε, the emissivity. Thus, the net rate of heat flow from the person to the surroundings (outside the blanket) will be low, since most of the heat is reflected by the blanket back to the person, and the person will stay warmer. The blanket will also prevent energy loss due to wind (convection) and will insulate the person, reducing heat transfer by conduction.

28. Cities situated on the ocean have less extreme temperatures because the oceans are a heat reservoir. Due to ocean currents, the temperature of the ocean in a locale will be fairly constant during a season. In the winter, the ocean temperature remains above freezing. Thus, if the air and land near the ocean get colder than the oceans, the oceans will release thermal energy, moderating the temperature of the nearby region. Likewise, in the warm seasons, the ocean temperatures will be cooler than the surrounding land mass, which heats up more easily than the water. Then the oceans will absorb thermal energy from the surrounding areas, again moderating the temperature.

29. The specific heat of water is large, so water is able to absorb a lot of heat energy with just a small temperature change. This helps to keep the temperature of the cup lower, preventing it from burning. Without the water, the cup's temperature would increase quickly, and then it would burn.

30. The air just outside the window will be somewhat warmed by conduction of heat energy through the window to the air. On a windy day, convection removes that warmer air from near the outside surface of the window. This increases the rate of heat conduction through the window, thus making the window feel colder than on a day with no wind.

Responses to MisConceptual Questions

1. (*c*) A common misconception is that "cold" flows from the ice into the tea. When the ice is placed in the tea, the ice has less kinetic energy per molecule than the tea, so in molecular collisions between the tea and ice, energy transfers from the tea into the ice. This energy transfer cools the tea as it melts the ice and then heats up the ice. The transfer of energy from the warmer tea to the colder ice is called "heat."

2. (*c*) Students frequently interpret having more ice with being colder. However, whenever ice and water are mixed together and are in thermal equilibrium they will be at the melting/freezing point of the water. Therefore, the two containers will be at the same temperature.

3. (*a*) When two objects are in thermal equilibrium, heat does not transfer between them. This occurs when the two objects are at the same temperature. Internal energy is an extrinsic property that depends upon the amount of the substance present. Therefore, two gases in thermal equilibrium with each other would have different internal energies if one consisted of one mole of gas and the other consisted of two moles of gas. Heat is a transfer of energy between objects that are not in thermal equilibrium. Heat is not a property of an object.

4. (*c*) Phase changes occur at specific temperatures (melting point, boiling point, or sublimation point) while heat is being added to or removed from the material. During the phase change the temperature remains constant. The thermal energy at this temperature is equal to the intermolecular binding energy. These energies are much lower than the energy necessary to break the molecules apart into their atoms or to change the chemical composition of the molecules.

5. (*a*) A common misconception is that as heat is added to water the temperature will always rise. However, Fig. 14–5 shows that heat is added to water at its melting and boiling points without the temperature changing. At these temperatures the water is undergoing a phase change.

6. (*d*) Heat is able to transfer through a vacuum by radiation, but heat requires a medium to transfer by conduction and by convection. Therefore, the vacuum in a thermos prevents heat loss by conduction and convection.

7. (*c*) Students often think that heat is a substance or a property of a material. When two materials are at different temperatures (that is, they have different average kinetic energies per molecule), energy can transfer from the hot object to the cold object. This transfer of energy is called heat.

8. (*d*) Radiation is emitted by all objects not at absolute zero. Very hot objects, such as the Sun, emit radiation in the visible spectrum, so they appear to be glowing. If the temperature of an object is less than that of its surroundings, it has a net gain in energy as it absorbs more radiation than it emits. However, it is still emitting radiation. The amount of radiation emitted is independent of the object's specific heat.

9. (*c*) The problem does not specify the initial temperatures of the ice and water. If the ice and water are both initially at 0°C, then none of the ice will melt, since heat will not transfer between them. Alternatively, if the ice is at 0°C and the water is at 100°C, then the water can provide $Q = mc\Delta T = (0.010\text{ kg})(4186\text{ J/kg}\cdot\text{C}°)(100\,\text{C}°) = 4186\text{ J}$ of heat as it cools to 0°C. The ice only needs $Q = mL = (0.010\text{ kg})(333\text{ kJ/kg}) = 3330\text{ J}$ to melt completely, so in this case all the ice would melt. Since the water could provide more heat than needed, you need to know the initial temperatures of the ice and water to determine just how much of the ice would melt.

10. (*d*) As the two objects are in thermal contact, the heat given off by the hot object will equal the heat absorbed by the cold object. The objects have the same specific heat, so the heat transfer is proportional to the product of the mass of each object and its change in temperature. The object with the smaller mass will then have the larger temperature change.

11. (*d*) The specific heat of an object is a measure of how much heat is required to change its temperature. Water has a high specific heat (much higher than air), so its temperature remains fairly constant even though the surrounding air may experience large temperature fluctuations.

12. (*a*) The two objects absorb the same amount of heat from the stove. From Eq. 14–2, given that the masses are the same, the object with the higher specific heat will experience the smaller temperature increase and will therefore be cooler.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. The kilocalorie is the heat needed to raise 1 kg of water by 1 C°. Use this relation to find the change in the temperature.

$$(8200\text{ J})\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\frac{(1\text{ kg})(1\text{C}°)}{1\text{ kcal}}\left(\frac{1}{3.0\text{ kg}}\right) = 0.653\,\text{C}°$$

Thus, the final temperature is $10.0°\text{C} + 0.653°\text{C} \approx \boxed{10.7°\text{C}}$.

2. The kilocalorie is the heat needed to raise 1 kg of water by 1 C°. Use the definition to find the heat needed.

$$(34.0\text{ kg})(95°\text{C} - 15°\text{C})\frac{1\text{ kcal}}{(1\text{ kg})(1\text{ C}°)}\left(\frac{4186\text{ J}}{1\text{ kcal}}\right) = 1.139\times10^{7}\text{ J} \approx \boxed{1.1\times10^{7}\text{ J}}$$

3. Find the mass of warmed water from the volume of water and its density of 1025 kg/m^3. Then use the fact that 1 kcal of energy raises 1 kg of water by $1\,\text{C}°$ and that the water warms by $25\,\text{C}°$.

$$V = At = \frac{m}{\rho} \quad\rightarrow\quad m = \rho At = (1025\text{ kg/m}^3)(1.0\text{ m}^2)(0.5\times10^{-3}\text{ m}) = 0.5125\text{ kg}$$

$$(0.5125\text{ kg})(25\text{ C}°)\frac{(1\text{ kcal})}{(1\text{ kg})(1\text{ C}°)} = 12.8\text{ kcal};\;\; 12.8\text{ kcal}\left(\frac{1\text{ bar}}{300\text{ kcal}}\right) = 0.043\text{ bars} \approx \boxed{0.04\text{ bars}}$$

4. (*a*) $$2500\ \text{Cal}\left(\frac{4.186\times10^{3}\ \text{J}}{1\ \text{Cal}}\right)=\boxed{1.0\times10^{7}\ \text{J}}$$

(*b*) $$2500\ \text{Cal}\left(\frac{1\ \text{kWh}}{860\ \text{Cal}}\right)=\boxed{2.9\ \text{kWh}}$$

(*c*) At 10 cents per day, the food energy costs $\boxed{\$0.29\text{ per day}}$. It would be impossible to feed yourself in the United States on this amount of money.

5. On page 79 of the textbook, the conversion $1\ \text{lb}=4.44822\ \text{N}$ is given. We use that value.

$$1\,\text{Btu}=(1\ \text{lb})(1\ \text{F}^\circ)\left(\frac{4.44822\ \text{N}}{1\ \text{lb}}\right)\left(\frac{1\ \text{kg}}{9.80\text{m/s}^2}\right)\left(\frac{5/9\ \text{C}^\circ}{1\ \text{F}^\circ}\right)\frac{1\ \text{kcal}}{(1\ \text{kg})(1\ \text{C}^\circ)}=0.2522\ \text{kcal}\approx\boxed{0.252\ \text{kcal}}$$

$$0.2522\ \text{kcal}\left(\frac{4186\ \text{J}}{1\ \text{kcal}}\right)=\boxed{1056\ \text{J}}$$

6. The energy generated by using the brakes must equal the car's initial kinetic energy, since its final kinetic energy is 0.

$$Q=\tfrac{1}{2}mv_0^2=\tfrac{1}{2}(1300\ \text{kg})\left[(95\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2=4.526\times10^{4}\ \text{J}\approx\boxed{4.5\times10^{4}\ \text{J}}$$

$$4.526\times10^{4}\ \text{J}\left(\frac{1\ \text{kcal}}{4186\ \text{J}}\right)=108.1\ \text{kcal}\approx\boxed{110\ \text{kcal}}$$

7. The energy input is causing a certain rise in temperature, which can be expressed as a number of joules per hour per C°. Convert that to mass using the definition of kcal, relates mass to heat energy.

$$\left(\frac{3.2\times10^{7}\ \text{J/h}}{30\ \text{C}^\circ}\right)\left(\frac{1\ \text{kcal}}{4186\ \text{J}}\right)\frac{(1\ \text{kg})(1\ \text{C}^\circ)}{1\ \text{kcal}}=254.8\ \text{kg/h}\approx\boxed{250\ \text{kg/h}}$$

8. The wattage rating is 375 joules per second. Note that 1 L of water has a mass of 1 kg.

$$\left((2.5\times10^{-1}\ \text{L})\left(\frac{1\ \text{kg}}{1\ \text{L}}\right)(60\ \text{C}^\circ)\right)\frac{1\ \text{kcal}}{(1\ \text{kg})(1\ \text{C}^\circ)}\left(\frac{4186\ \text{J}}{\text{kcal}}\right)\left(\frac{1\ \text{s}}{375\ \text{J}}\right)=167\ \text{s}\approx\boxed{170\ \text{s}=2.8\ \text{min}}$$

9. The heat absorbed can be calculated from Eq. 14–2. Note that 1 L of water has a mass of 1 kg.

$$Q=mc\Delta T=\left[(18\ \text{L})\left(\frac{1\times10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{1.0\times10^{3}\ \text{kg}}{1\ \text{m}^3}\right)\right](4186\ \text{J/kg}\cdot\text{C}^\circ)(95^\circ\text{C}-15^\circ\text{C})=\boxed{6.0\times10^{6}\ \text{J}}$$

10. The specific heat can be calculated from Eq. 14–2.

$$Q=mc\Delta T\quad\rightarrow\quad c=\frac{Q}{m\Delta T}=\frac{1.35\times10^{5}\ \text{J}}{(4.1\ \text{kg})(37.2^\circ\text{C}-18.0^\circ\text{C})}=1715\ \text{J/kg}\cdot\text{C}^\circ\approx\boxed{1700\ \text{J/kg}\cdot\text{C}^\circ}$$

11. (*a*) The heat absorbed can be calculated from Eq. 14–2. Note that 1 L of water has a mass of 1 kg.

$$Q = mc\Delta T = \left[(1.0\text{ L})\left(\frac{1\times10^{-3}\text{ m}^3}{1\text{ L}}\right)\left(\frac{1.0\times10^3\text{ kg}}{1\text{ m}^3}\right)\right](4186\text{ J/kg}\cdot\text{C}^\circ)(100^\circ\text{C}-20^\circ\text{C})$$
$$= 3.349\times10^5\text{ J} \approx \boxed{3.3\times10^5\text{ J}}$$

(*b*) Power is the rate of energy usage.

$$P = \frac{\Delta E}{\Delta t} = \frac{Q}{\Delta t} \quad\rightarrow\quad \Delta t = \frac{Q}{P} = \frac{3.349\times10^5\text{ J}}{60\text{ W}} = 5582\text{ s} \approx \boxed{5600\text{ s}} \approx 93\text{ min}$$

12. The heat absorbed by all three substances is given by Eq. 14–2, $Q = mc\Delta T$. Thus, the amount of mass can be found as $m = \frac{Q}{c\Delta T}$. The heat and temperature change are the same for all three substances.

$$m_{\text{Cu}} : m_{\text{Al}} : m_{\text{H}_2\text{O}} = \frac{Q}{c_{\text{Cu}}\Delta T} : \frac{Q}{c_{\text{Al}}\Delta T} : \frac{Q}{c_{\text{H}_2\text{O}}\Delta T} = \frac{1}{c_{\text{Cu}}} : \frac{1}{c_{\text{Al}}} : \frac{1}{c_{\text{H}_2\text{O}}} = \frac{1}{390} : \frac{1}{900} : \frac{1}{4186}$$
$$= \frac{4186}{390} : \frac{4186}{900} : \frac{4186}{4186} = \boxed{10.7 : 4.65 : 1}$$

$\boxed{13.}$ The heat must warm both the water and the pot to 100°C. The heat is also the power times the time. The temperature change is 89 C°.

$$Q = Pt = (m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T_{\text{H}_2\text{O}} \quad\rightarrow$$
$$t = \frac{(m_{\text{Al}}c_{\text{Al}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T_{\text{H}_2\text{O}}}{P} = \frac{[(0.28\text{ kg})(900\text{ J/kg}\cdot\text{C}^\circ) + (0.75\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ)](89\text{ C}^\circ)}{750\text{ W}}$$
$$= 402\text{ s} \approx \boxed{4.0\times10^2\text{ s} = 6.7\text{ min}}$$

14. The heat lost by the copper must be equal to the heat gained by the aluminum and the water. The aluminum and water have the same temperature change.

$$m_{\text{Cu}}c_{\text{Cu}}(T_{\text{i Cu}} - T_{\text{eq}}) = m_{\text{Al}}c_{\text{Al}}(T_{\text{eq}} - T_{\text{i Al}}) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{i H}_2\text{O}})$$
$$(0.265\text{ kg})(390\text{ J/kg}\cdot\text{C}^\circ)(245^\circ\text{C} - T_{\text{eq}}) = \left[\begin{array}{l}(0.145\text{ kg})(900\text{ J/kg}\cdot\text{C}^\circ)\\ +(0.825\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ)\end{array}\right](T_{\text{eq}} - 12.0^\circ\text{C})$$
$$T_{\text{eq}} = 18.532^\circ\text{C} \approx \boxed{18.5^\circ\text{C}}$$

15. The heat gained by the glass thermometer must be equal to the heat lost by the water.

$$m_{\text{glass}}c_{\text{glass}}(T_{\text{eq}} - T_{\text{glass}}) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{H}_2\text{O}} - T_{\text{eq}})$$
$$(31.5\text{ g})(0.20\text{ cal/g}\cdot\text{C}^\circ)(41.8^\circ\text{C} - 23.6^\circ\text{C}) = (135\text{ g})(1.00\text{ cal/g}\cdot\text{C}^\circ)(T_{\text{H}_2\text{O}} - 41.8^\circ\text{C})$$
$$T_{\text{H}_2\text{O}} = \boxed{42.6^\circ\text{C}}$$

16. The heat lost by the horseshoe must be equal to the heat gained by the iron pot and the water. Note that 1 L of water has a mass of 1 kg.

$$m_{\text{shoe}}c_{\text{Fe}}(T_{\text{i shoe}} - T_{\text{eq}}) = m_{\text{pot}}c_{\text{Fe}}(T_{\text{eq}} - T_{\text{i pot}}) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{i H}_2\text{O}})$$

$$(0.40\text{ kg})(450\text{ J/kg}\cdot\text{C}^\circ)(T_{\text{i shoe}} - 25.0^\circ\text{C}) = (0.30\text{ kg})(450\text{ J/kg}\cdot\text{C}^\circ)(25.0\text{ C}^\circ - 20.0\text{ C}^\circ) + (1.25\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ)(25.0\text{ C}^\circ - 20.0\text{ C}^\circ)$$

$$T_{\text{i shoe}} = 174^\circ\text{C} \approx \boxed{170^\circ\text{C}}$$

17. The heat lost by the iron must be the heat gained by the aluminum and the glycerin.

$$m_{\text{Fe}}c_{\text{Fe}}(T_{\text{i Fe}} - T_{\text{eq}}) = m_{\text{Al}}c_{\text{Al}}(T_{\text{eq}} - T_{\text{i Al}}) + m_{\text{gly}}c_{\text{gly}}(T_{\text{eq}} - T_{\text{i gly}})$$

$$(0.290\text{ kg})(450\text{ J/kg}\cdot\text{C}^\circ)(142\text{ C}^\circ) = (0.095\text{ kg})(900\text{ J/kg}\cdot\text{C}^\circ)(28\text{ C}^\circ) + (0.250\text{ kg})c_{\text{gly}}(28\text{ C}^\circ)$$

$$c_{\text{gly}} = 2305\text{ J/kg}\cdot\text{C}^\circ \approx \boxed{2300\text{ J/kg}\cdot\text{C}^\circ}$$

18. (*a*) Since $Q = mc\Delta T$ and $Q = C\Delta T$, equate these two expressions for Q and solve for C.

$$Q = mc\Delta T = C\Delta T \quad \rightarrow \quad \boxed{C = mc}$$

(*b*) For 1.0 kg of water: $C = mc = (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ) = \boxed{4.2\times10^3\text{ J/C}^\circ}$

(*c*) For 45 kg of water: $C = mc = (45\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ) = \boxed{1.9\times10^5\text{ J/C}^\circ}$

19. We assume that all of the kinetic energy of the hammer goes into heating the nail.

$$\text{KE} = Q \quad \rightarrow \quad 8\left(\tfrac{1}{2}m_{\text{hammer}}\,v_{\text{hammer}}^2\right) = m_{\text{nail}}c_{\text{Fe}}\Delta T \quad \rightarrow$$

$$\Delta T = \frac{8\left(\tfrac{1}{2}m_{\text{hammer}}\,v_{\text{hammer}}^2\right)}{m_{\text{nail}}c_{\text{Fe}}} = \frac{4(1.20\text{ kg})(7.5\text{m/s})^2}{(0.014\text{ kg})(450\text{ J/kg}\cdot\text{C}^\circ)} = 42.86\text{ C}^\circ \approx \boxed{43\text{ C}^\circ}$$

20. The heat lost by the substance must be equal to the heat gained by the aluminum, water, and glass.

$$m_{\text{x}}c_{\text{x}}(T_{\text{i x}} - T_{\text{eq}}) = m_{\text{Al}}c_{\text{Al}}(T_{\text{eq}} - T_{\text{i Al}}) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{i H}_2\text{O}}) + m_{\text{glass}}c_{\text{glass}}(T_{\text{eq}} - T_{\text{i glass}})$$

$$c_{\text{x}} = \frac{m_{\text{Al}}c_{\text{Al}}(T_{\text{eq}} - T_{\text{i Al}}) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{i H}_2\text{O}}) + m_{\text{glass}}c_{\text{glass}}(T_{\text{eq}} - T_{\text{i glass}})}{m_{\text{x}}(T_{\text{ix}} - T_{\text{eq}})}$$

$$= \frac{\begin{bmatrix}(0.105\text{ kg})(900\text{ J/kg}\cdot\text{C}^\circ) + (0.185\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ) \\ +(0.017\text{ kg})(840\text{ J/kg}\cdot\text{C}^\circ)\end{bmatrix}(35.0^\circ\text{C} - 10.5^\circ\text{C})}{(0.215\text{ kg})(330^\circ\text{C} - 35.0^\circ\text{C})}$$

$$= 341.16\text{ J/kg}\cdot\text{C}^\circ \approx \boxed{341\text{ J/kg}\cdot\text{C}^\circ}$$

21. 65% of the original potential energy of the aluminum goes to heating the aluminum.

$$0.65\,\text{PE} = Q \quad \rightarrow \quad 0.65 m_{\text{Al}}gh = m_{\text{Al}}c_{\text{Al}}\Delta T \quad \rightarrow$$

$$\Delta T = \frac{0.65\,gh}{c_{\text{Al}}} = \frac{0.65(9.80\text{ m/s}^2)(55\text{ m})}{(900\text{ J/kg}\cdot\text{C}^\circ)} = \boxed{0.39\text{ C}^\circ}$$

22. The heat released by the 15 grams of candy in the burning is equal to the heat absorbed by the aluminum and water.

$$\begin{aligned} Q_{\substack{15\,\mathrm{g}\\ \text{candy}}} &= (m_{\mathrm{Al}}c_{\mathrm{Al}} + m_{\mathrm{H_2O}}c_{\mathrm{H_2O}})\Delta T \\ &= [(0.325\text{ kg} + 0.624\text{ kg})(0.22\text{ kcal/kg}\cdot\mathrm{C}^\circ) + (1.75\text{ kg})(1.00\text{ kcal/kg}\cdot\mathrm{C}^\circ)](53.5^\circ\mathrm{C} - 15.0^\circ\mathrm{C}) \\ &= 75.41\text{ kcal} \end{aligned}$$

The heat released by 65 grams of the candy would be 65/15 times that released by the 15 grams.

$$Q_{\substack{65\,\mathrm{g}\\ \text{candy}}} = \tfrac{65}{15} Q_{\substack{15\,\mathrm{g}\\ \text{candy}}} = \tfrac{65}{15}(75.41\text{ kcal}) = 326.8\text{ kcal} \approx \boxed{330\text{ Cal}}$$

23. The heat released by the 10 grams of fudge cookies in the burning process is equal to the heat absorbed by the aluminum and water.

$$\begin{aligned} Q_{\substack{100\,\mathrm{g}\\ \text{cookies}}} &= (m_{\mathrm{Al}}c_{\mathrm{Al}} + m_{\mathrm{H_2O}}c_{\mathrm{H_2O}})\Delta T \\ &= [(0.615\text{ kg} + 0.524\text{ kg})(0.22\text{ kcal/kg}\cdot\mathrm{C}^\circ) + (2.00\text{ kg})(1.00\text{ kcal/kg}\cdot\mathrm{C}^\circ)](36.0^\circ\mathrm{C} - 15.0^\circ\mathrm{C}) \\ &= 47.26\text{ kcal} \approx 47.3\text{ kcal} \end{aligned}$$

Since this is the energy content of 10 grams, the content of 100 grams would be 10 times this, or $\boxed{473\text{ kcal}}$.

24. The oxygen is all at the boiling point, so any heat added will cause oxygen to evaporate (as opposed to raising its temperature). We assume that all the heat goes to the oxygen and none to the flask.

$$Q = mL_{\mathrm{V}} \quad\rightarrow\quad m = \frac{Q}{L_{\mathrm{V}}} = \frac{3.40\times10^5\text{ J}}{2.1\times10^5\text{ J/kg}} = \boxed{1.6\text{ kg}}$$

25. The silver must be heated to the melting temperature and then melted.

$$\begin{aligned} Q &= Q_{\text{heat}} + Q_{\text{melt}} = mc\Delta T + mL_{\mathrm{F}} \\ &= (23.50\text{ kg})(230\text{ J/kg}\cdot\mathrm{C}^\circ)(961^\circ\mathrm{C} - 25^\circ\mathrm{C}) + (23.50\text{ kg})(0.88\times10^5\text{ J/kg}) = \boxed{7.1\times10^6\text{ J}} \end{aligned}$$

26. Assume that the heat from the person is only used to evaporate the water. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the person's temperature is closer to room temperature than 100°C.

$$Q = mL_{\mathrm{V}} \quad\rightarrow\quad m = \frac{Q}{L_{\mathrm{V}}} = \frac{185\text{ kcal}}{585\text{ kcal/kg}} = \boxed{0.316\text{ kg}} = 316\text{ mL}$$

27. The heat lost by the steam condensing and then cooling to 30°C must be equal to the heat gained by the ice melting and then warming to 30°C.

$$m_{\text{steam}}[L_{\mathrm{V}} + c_{\mathrm{H_2O}}(T_{\text{isteam}} - T_{\text{eq}})] = m_{\text{ice}}[L_{\mathrm{F}} + c_{\mathrm{H_2O}}(T_{\text{eq}} - T_{\text{iice}})]$$

$$\begin{aligned} m_{\text{steam}} &= m_{\text{ice}}\frac{[L_{\mathrm{F}} + c_{\mathrm{H_2O}}(T_{\text{eq}} - T_{\text{iice}})]}{[L_{\mathrm{V}} + c_{\mathrm{H_2O}}(T_{\text{isteam}} - T_{\text{eq}})]} = (1.00\text{ kg})\frac{[3.33\times10^5\text{ J/kg} + (4186\text{ J/kg}\cdot\mathrm{C}^\circ)(30^\circ\mathrm{C})]}{[22.6\times10^5\text{ J/kg} + (4186\text{ J/kg}\cdot\mathrm{C}^\circ)(30^\circ\mathrm{C})]} \\ &= \boxed{0.18\text{ kg}} \end{aligned}$$

28. Assume that all of the heat lost by the ice cube in cooling to the temperature of the liquid nitrogen is used to boil the nitrogen, so none is used to raise the temperature of the nitrogen. The boiling point of the nitrogen is $77\text{ K} = -196°\text{C}$.

$$m_{\text{ice}}c_{\text{ice}}\left(T_{\substack{\text{ice}\\\text{initial}}} - T_{\substack{\text{ice}\\\text{final}}}\right) = m_{\text{nitrogen}}L_{\text{V}} \quad \rightarrow$$

$$m_{\text{nitrogen}} = \frac{m_{\text{ice}}c_{\text{ice}}\left(T_{\substack{\text{ice}\\\text{initial}}} - T_{\substack{\text{ice}\\\text{final}}}\right)}{L_{\text{V}}} = \frac{(2.8\times10^{-2}\text{ kg})(2100\text{ J/kg}\cdot\text{C°})(0°\text{C} - -196°\text{C})}{200\times10^{3}\text{ J/kg}} = \boxed{5.8\times10^{-2}\text{ kg}}$$

29. (*a*) The energy absorbed from the body must warm the snow to the melting temperature, melt the snow, and then warm the melted snow to the final temperature.

$$Q_a = Q_{\substack{\text{warm}\\\text{snow}}} + Q_{\text{melt}} + Q_{\substack{\text{warm}\\\text{liquid}}} = mc_{\text{snow}}\Delta T_1 + mL_{\text{F}} + mc_{\text{liquid}}\Delta T_2 = m[c_{\text{snow}}\Delta T_1 + L_{\text{F}} + c_{\text{liquid}}\Delta T_2]$$

$$= (1.0\text{ kg})[(2100\text{ J/kg}\cdot\text{C°})(15\text{ C°}) + (3.33\times10^{5}\text{ J/kg}) + (4186\text{ J/kg}\cdot\text{C°})(37\text{ C°})]$$

$$= \boxed{5.2\times10^{5}\text{ J}}$$

(*b*) The energy absorbed from the body only has to warm the melted snow to the final temperature.

$$Q_b = Q_{\substack{\text{heat}\\\text{liquid}}} = mc_{\text{liquid}}\Delta T_2 = (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(35\text{ C°}) = \boxed{1.5\times10^{5}\text{ J}}$$

30. (*a*) The heater must heat both the boiler and the water at the same time.

$$Q_1 = Pt_1 = (m_{\text{Fe}}c_{\text{Fe}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T \quad \rightarrow$$

$$t_1 = \frac{(m_{\text{Fe}}c_{\text{Fe}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T}{P} = \frac{[(180\text{ kg})(450\text{ J/kg}\cdot\text{C°}) + (730\text{ kg})(4186\text{ J/kg}\cdot\text{C°})](82\text{ C°})}{5.8\times10^{7}\text{ J/h}}$$

$$= 4.435\text{ h} \approx \boxed{4.4\text{ h}}$$

(*b*) Assume that after the water starts to boil, all the heat energy goes into boiling the water, and none goes to raising the temperature of the iron or the steam.

$$Q_2 = Pt_2 = m_{\text{H}_2\text{O}}L_{\text{V}} \quad \rightarrow \quad t_2 = \frac{m_{\text{H}_2\text{O}}L_{\text{V}}}{P} = \frac{(730\text{ kg})(22.6\times10^{5}\text{ J/kg})}{5.8\times10^{7}\text{ J/h}} = 28.445\text{ h}$$

Thus, the total time is $t_1 + t_2 = 4.435\text{ h} + 28.445\text{ h} = 32.88\text{ h} \approx \boxed{33\text{ h}}$.

31. The heat lost by the aluminum and the water must equal the heat needed to melt the mercury and to warm the mercury to the equilibrium temperature.

$$m_{\text{Al}}c_{\text{Al}}(T_{\text{Al}} - T_{\text{eq}}) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{H}_2\text{O}} - T_{\text{eq}}) = m_{\text{Hg}}[L_{\text{F}} + c_{\text{Hg}}(T_{\text{eq}} - T_{\text{melt}})]$$

$$L_{\text{F}} = \frac{m_{\text{Al}}c_{\text{Al}}(T_{\text{Al}} - T_{\text{eq}}) + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{H}_2\text{O}} - T_{\text{eq}})}{m_{\text{Hg}}} = c_{\text{Hg}}(T_{\text{eq}} - T_{\text{melt}})$$

$$= \frac{[(0.620\text{ kg})\times(900\text{ J/kg}\cdot\text{C°}) + (0.400\text{ kg})\times(4186\text{ J/kg}\cdot\text{C°})](12.80°\text{C} - 5.06°\text{C})]}{1.00\text{ kg}}$$

$$- (138\text{ J/kg}\cdot\text{C°})[5.06°\text{C} - (-39.0°\text{C})]$$

$$= \boxed{1.12\times10^{4}\text{ J/kg}}$$

32. The kinetic energy of the bullet is assumed to warm the bullet and melt it.

$$\tfrac{1}{2}mv^2 = Q = mc_{\text{Pb}}(T_{\text{melt}} - T_{\text{initial}}) + mL_{\text{F}} \quad \rightarrow$$

$$v = \sqrt{2[c_{\text{Pb}}(T_{\text{melt}} - T_{\text{initial}}) + L_{\text{F}}]} = \sqrt{2[(130\ \text{J/kg}\cdot\text{C}^\circ)(327^\circ\text{C} - 20^\circ\text{C}) + (0.25\times10^5\ \text{J/kg})]}$$
$$= \boxed{360\ \text{m/s}}$$

$\boxed{33.}$ Assume that all of the melted ice stays at 0°C, so that all the heat is used in melting ice and none in warming water. The available heat is half of the original kinetic energy.

$$\tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{skater}}v^2\right) = Q = m_{\text{ice}}L_{\text{F}} \quad \rightarrow$$

$$m_{\text{ice}} = \frac{\frac{1}{4}m_{\text{skater}}v^2}{L_{\text{F}}} = \frac{\frac{1}{4}(64\ \text{kg})(7.5\ \text{m/s})^2}{3.33\times10^5\ \text{J/kg}} = \boxed{2.7\times10^{-3}\ \text{kg}} = 2.7\ \text{g}$$

34. The heat lost by the aluminum and 310 g of liquid water must be equal to the heat gained by the ice in warming in the solid state, melting, and warming in the liquid state.

$$m_{\text{Al}}c_{\text{Al}}(T_{\text{iAl}} - T_{\text{eq}}) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{iH}_2\text{O}} - T_{\text{eq}}) = m_{\text{ice}}[c_{\text{ice}}(T_{\text{melt}} - T_{\text{i ice}}) + L_{\text{F}} + c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{melt}})]$$

$$m_{\text{ice}} = \frac{[(0.085\ \text{kg})(900\ \text{J/kg}\cdot\text{C}^\circ)(3.0\ \text{C}^\circ) + (0.31\ \text{kg})(4186\ \text{J/kg}\cdot\text{C}^\circ)(3.0\ \text{C}^\circ)]}{[(2100\ \text{J/kg}\cdot\text{C}^\circ)(8.5\ \text{C}^\circ) + 3.3\times10^5\ \text{J/kg} + (4186\ \text{J/kg}\cdot\text{C}^\circ)(17\ \text{C}^\circ)]} = \boxed{9.8\times10^{-3}\ \text{kg}}$$

35. Assume that the kinetic energy of the bullet was all converted into heat, which melted the ice.

$$\tfrac{1}{2}m_{\text{bullet}}v^2 = Q = m_{\text{ice}}L_{\text{F}} \quad \rightarrow$$

$$m_{\text{ice}} = \frac{\frac{1}{2}m_{\text{bullet}}v^2}{L_{\text{F}}} = \frac{\frac{1}{2}(5.5\times10^{-2}\ \text{kg})(250\ \text{m/s})^2}{3.33\times10^5\ \text{J/kg}} = \boxed{5.2\times10^{-3}\ \text{kg}} = 5.2\ \text{g}$$

36. The heat conduction rate is given by Eq. 14–5.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (0.84\ \text{J/s}\cdot\text{m}\cdot\text{C}^\circ)(3.0\ \text{m}^2)\frac{[15.0^\circ\text{C} - (-5^\circ\text{C})]}{3.2\times10^{-3}\ \text{m}} = \boxed{1.6\times10^4\ \text{W}}$$

37. The heat conduction rate is given by Eq. 14–5.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (380\ \text{J/s}\cdot\text{m}\cdot\text{C}^\circ)\pi(1.0\times10^{-2}\ \text{m})^2\frac{(460^\circ\text{C} - 22^\circ\text{C})}{0.56\ \text{m}} = \boxed{93\ \text{W}}$$

38. (*a*) The power radiated is given by Eq. 14–6. The temperature of the tungsten is $273\ \text{K} + 25\ \text{K} = 298\ \text{K}$.

$$\frac{\Delta Q}{\Delta t} = \varepsilon\sigma AT^4 = (0.35)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)4\pi(0.19\ \text{m})^2(298\ \text{K})^4 = \boxed{71\ \text{W}}$$

(*b*) The net flow rate of energy is given by Eq. 14–7. The temperature of the surroundings is 268 K.

$$\frac{\Delta Q}{\Delta t} = \varepsilon\sigma A\left(T_1^4 - T_1^4\right) = (0.35)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)4\pi(0.19\ \text{m})^2\left[(298\ \text{K})^4 - (268\ \text{K})^4\right]$$
$$= \boxed{25\ \text{W}}$$

39. Eq. 14–8 gives the heat absorption rate for an object facing the Sun. The heat required to melt the ice is the mass of the ice times the latent heat of fusion for the ice. The mass is found by multiplying the volume of ice by its density.

$$\Delta Q = mL_F = \rho V L_F = \rho A(\Delta x)L_F \qquad \frac{\Delta Q}{\Delta t} = (1000\,\text{W/m}^2)\varepsilon A\cos\theta \quad \rightarrow$$

$$\Delta t = \frac{\rho A(\Delta x)L_F}{(1000\text{ W/m}^2)\varepsilon A\cos\theta} = \frac{\rho(\Delta x)L_F}{(1000\text{ W/m}^2)\varepsilon\cos\theta}$$

$$= \frac{(9.17\times10^2\text{ kg/m}^3)(1.0\times10^{-2}\text{ m})(3.33\times10^5\text{ J/kg})}{(1000\text{ W/m}^2)(0.050)\cos 35^\circ} = \boxed{7.5\times10^4\text{ s}} \approx 21\text{ h}$$

40. The distance can be calculated from the heat conduction rate, given by Eq. 14–5. The rate is given as a power (150 W = 150 J/s).

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} \quad \rightarrow \quad \ell = kA\frac{T_1 - T_2}{P} = (0.2\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(1.5\text{ m}^2)\frac{0.50\text{ C}^\circ}{150\text{ W}} = \boxed{1.0\times10^{-3}\text{ m}}$$

$\boxed{41.}$ This is a heat transfer by conduction, so Eq. 14–5 is applicable.

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} = (0.84\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(16\text{ m}^2)\frac{30^\circ\text{C} - 10^\circ\text{C}}{0.14\text{ m}} = 1920\text{ W}$$

It would take 19.2 lightbulbs, so $\boxed{\text{20 bulbs}}$ are needed to maintain the temperature difference.

42. This is an example of heat conduction. The temperature difference can be calculated by Eq. 14–5.

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} \quad \rightarrow \quad \Delta T = \frac{P\ell}{kA} = \frac{(95\text{ W})(5.0\times10^{-4}\text{ m})}{(0.84\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)4\pi(3.0\times10^{-2}\text{ m})^2} = \boxed{5.0\text{ C}^\circ}$$

43. This is an example of heat conduction. The heat conducted is the heat released by the melting ice, $Q = m_{\text{ice}}L_F$. The area through which the heat is conducted is the total area of the six surfaces of the box, and the length of the conducting material is the thickness of the Styrofoam. We assume that all of the heat conducted into the box goes into melting the ice and none into raising the temperature inside the box. The time can then be calculated by Eq. 14–5.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} \quad \rightarrow \quad t = \frac{m_{\text{ice}}L_F\ell}{kA\Delta T}$$

$$= \frac{(8.2\text{ kg})(3.33\times10^5\text{ J/kg})(1.5\times10^{-2}\text{ m})}{2(0.023\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)[2(0.25\text{ m})(0.35\text{ m}) + 2(0.25\text{ m})(0.55\text{ m}) + 2(0.35\text{ m})(0.55\text{ m})](34\text{ C}^\circ)}$$

$$= 3.136\times10^4\text{ s} \approx \boxed{3.1\times10^4\text{ s}} \approx 8.7\text{ h}$$

44. For the temperature at the joint to remain constant, the heat flow in the rods must be the same. Note that the cross-sectional areas and lengths are the same. Use Eq. 14–5 for heat conduction.

$$\left(\frac{Q}{t}\right)_{\text{Cu}} = \left(\frac{Q}{t}\right)_{\text{Al}} \quad \rightarrow \quad k_{\text{Cu}}A\frac{T_{\text{hot}} - T_{\text{middle}}}{\ell} = k_{\text{Al}}A\frac{T_{\text{middle}} - T_{\text{cool}}}{\ell} \quad \rightarrow$$

$$T_{\text{middle}} = \frac{k_{\text{Cu}}T_{\text{hot}} + k_{\text{Al}}T_{\text{cool}}}{k_{\text{Cu}} + k_{\text{Al}}} = \frac{(380\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(205^\circ\text{C}) + (200\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(0.0^\circ\text{C})}{380\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ + 200\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ} = 134.3^\circ\text{C} \approx \boxed{130\ ^\circ\text{C}}$$

45. The conduction rates through the two materials must be equal. If they were not, the temperatures in the materials would be changing. Call the temperature at the boundary between the materials T_x.

$$\frac{Q}{t} = k_1 A \frac{T_1 - T_x}{\ell_1} = k_2 A \frac{T_x - T_2}{\ell_2} \quad \rightarrow \quad \frac{Q}{t}\frac{\ell_1}{k_1 A} = T_1 - T_x ; \frac{Q}{t}\frac{\ell_2}{k_2 A} = T_x - T_2$$

Add these two equations together, and solve for the heat conduction rate.

$$\frac{Q}{t}\frac{\ell_1}{k_1 A} + \frac{Q}{t}\frac{\ell_2}{k_2 A} = T_1 - T_x + T_x - T_2 \quad \rightarrow \quad \frac{Q}{t}\left(\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2}\right)\frac{1}{A} = T_1 - T_2 \quad \rightarrow$$

$$\frac{Q}{t} = A\frac{(T_1 - T_2)}{\left(\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2}\right)} = A\frac{(T_1 - T_2)}{(R_1 + R_2)}$$

The *R*-value for the brick needs to be calculated, using the definition of *R* given on page 402 of the textbook.

$$R = \frac{\ell}{k} = \frac{4\text{ in.}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)}{0.84\frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\left(\frac{1\text{ Btu}}{1055\text{ J}}\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)\left(\frac{5\text{ C}^\circ}{9\text{ F}^\circ}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)} = 0.69\text{ft}^2\cdot\text{h}\cdot\text{F}^\circ/\text{Btu}$$

$$\frac{Q}{t} = A\frac{(T_1 - T_2)}{(R_1 + R_2)} = (195\text{ ft}^2)\frac{(35\text{ F}^\circ)}{(0.69 + 19)\text{ft}^2\cdot\text{h}\cdot\text{F}^\circ/\text{Btu}} = 347\text{ Btu/h} \approx \boxed{350\text{ Btu/h}}$$

This is about 100 watts.

46. The heat needed to warm the liquid can be calculated by Eq. 14–2.

$$Q = mc\Delta T = (0.35\text{ kg})(1.00\text{ kcal/kg}\cdot\text{C}^\circ)(37^\circ\text{C} - 5^\circ\text{C}) = 11.2\text{ kcal} \approx \boxed{11\text{ Cal}}$$

47. (*a*) Use Eq. 14–6 for total power radiated.

$$\frac{Q}{t} = \varepsilon\sigma A T^4 = \varepsilon\sigma 4\pi R_{\text{Sun}}^2 T^4 = (1.0)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)4\pi(7.0\times10^8\text{ m})^2(5500\text{ K})^4$$

$$= 3.195\times10^{26}\text{ W} \approx \boxed{3.2\times10^{26}\text{ W}}$$

(*b*) Assume that the energy from the Sun is distributed symmetrically over a spherical surface with the Sun at the center.

$$\frac{P}{A} = \frac{Q/t}{4\pi R_{\text{Sun-Earth}}^2} = \frac{3.195\times10^{26}\text{ W}}{4\pi(1.5\times10^{11}\text{ m})^2} = 1.130\times10^3\text{ W/m}^2 \approx \boxed{1100\text{ W/m}^2}$$

48. The heat released can be calculated by Eq. 14–2. To find the mass of the water, use the density (of pure water).

$$Q = mc\Delta T = \rho V c\Delta T = (1.0\times10^3\text{ kg/m}^3)(1.0\times10^3\text{ m})^3(4186\text{ J/kg}\cdot\text{C}^\circ)(1\text{ C}^\circ) = \boxed{4\times10^{15}\text{ J}}$$

49. The heat gained by the ice (to melt it and warm it) must be equal to the heat lost by the steam (in condensing and cooling).

$$mL_F + mc_{H_2O}(T_{eq} - 0) = mL_V + mc_{H_2O}(100°C - T_{eq})$$

$$T_{eq} = \frac{L_V - L_F}{2c_{H_2O}} + 50°C = \frac{2260 \text{ kJ/kg} - 333 \text{ kJ/kg}}{2(4.186 \text{ kJ/kg} \cdot C°)} + 50°C = 280°C$$

This answer is not possible. Because this answer is too high, the steam must not all condense, and none of it must cool below 100°C. Calculate the energy needed to melt a kilogram of ice and warm it to 100°C.

$$Q = mL_F + mc_{H_2O}(T_{eq} - 0) = (1 \text{ kg})[333 \text{ kJ/kg} + (4.186 \text{ kJ/kg} \cdot C°)(100 \text{ C°})] = 751.6 \text{ kJ}$$

Calculate the mass of steam that needs to condense in order to provide this much energy.

$$Q = mL_V \quad \rightarrow \quad m = \frac{Q}{L_V} = \frac{751.6 \text{ kJ}}{2260 \text{ kJ/kg}} = 0.333 \text{ kg}$$

Thus, one-third of the original steam mass must condense to liquid at 100°C in order to melt the ice and warm the melted ice to 100°C. $\boxed{\text{The final mixture will be at 100°C, with 1/3 of the total mass as steam, and 2/3 of the total mass as water.}}$

$\boxed{50.}$ Use the heat conduction rate equation, Eq. 14–5.

(*a*) $$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (0.025 \text{ J/s} \cdot \text{m} \cdot C°)(0.95 \text{ m}^2)\frac{[34°C - (-18°C)]}{3.5\times10^{-2} \text{ m}} = 35.3 \text{ W} \approx \boxed{35 \text{ W}}$$

(*b*) $$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (0.56 \text{ J/s} \cdot \text{m} \cdot C°)(0.95 \text{ m}^2)\frac{[34°C - (-18°C)]}{5.0\times10^{-3} \text{ m}} = 5533 \text{ W} \approx \boxed{5500 \text{ W}}$$

51. The temperature rise can be calculated from Eq. 14–2.

$$Q = mc\Delta T \quad \rightarrow \quad \Delta T = \frac{Q}{mc} = \frac{(0.80)(200 \text{ kcal/h})(0.75 \text{ h})}{(70 \text{ kg})(0.83 \text{ kcal/kg} \cdot C°)} = 2.065°C \approx \boxed{2 \text{ C°}}$$

52. For an estimate of the heat conduction rate, use Eq. 14–5.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} = (0.2 \text{ J/s} \cdot \text{m} \cdot C°)(1.5 \text{ m}^2)\frac{(37°C - 34°C)}{4.0\times10^{-2} \text{ m}} = 22.5 \text{ W} \approx \boxed{20 \text{ W}}$$

This is only about 10% of the cooling capacity that is needed for the body. Thus, convection cooling is clearly necessary.

53. We are told that 80% of the cyclist's energy goes toward evaporation. The energy needed for evaporation is equal to the mass of the water times the latent heat of vaporization for water. Note that 1 L of water has a mass of 1 kg. Also, we use the heat of vaporization at room temperature (585 kcal/kg), since the cyclist's temperature is closer to room temperature than 100°C.

$$0.80Q_{\text{rider}} = m_{H_2O}L_V \rightarrow Q_{\text{rider}} = \frac{m_{H_2O}L_V}{0.80} = \frac{(9.0 \text{ kg})(585 \text{ kcal/kg})}{0.8} = 6581 \text{ kcal} \approx \boxed{6.6\times10^3 \text{ kcal}}$$

54. Since 30% of the heat generated is lost up the chimney, the heat required to heat the house is 70% of the heat provided by the coal.

$$2.0\times10^5 \text{ MJ} = 0.70(30\times10^6 \text{ MJ/kg})(m \text{ kg}) \quad \rightarrow \quad m = \frac{2.0\times10^5 \text{ MJ}}{0.70(30 \text{ MJ/kg})} = \boxed{9500 \text{ kg}}$$

55. We assume that the initial kinetic energy of the bullet all goes into heating the wood and the bullet.

$$\tfrac{1}{2}m_{\text{bullet}}v_{\text{i}}^2 = Q = m_{\text{bullet}}c_{\text{lead}}\Delta T_{\text{lead}} + m_{\text{wood}}c_{\text{wood}}\Delta T_{\text{wood}} \quad \rightarrow$$

$$v_{\text{i}} = \sqrt{\frac{(m_{\text{bullet}}c_{\text{lead}} + m_{\text{wood}}c_{\text{wood}})\Delta T}{\tfrac{1}{2}m_{\text{bullet}}}}$$

$$= \sqrt{\frac{[(0.015\text{ kg})(130\text{ J/kg}\cdot\text{C}°) + (35\text{ kg})(1700\text{ J/kg}\cdot\text{C}°)](0.020\text{ C}°)}{\tfrac{1}{2}(0.015\text{ kg})}} = 398.3\text{m/s}$$

$$\approx \boxed{4.0\times10^2\text{ m/s}}$$

56. We assume that the starting speed of the boulder is zero, and that 50% of the original potential energy of the boulder goes to heating the boulder.

$$\tfrac{1}{2}\text{PE} = Q \quad \rightarrow \quad \tfrac{1}{2}(mgh) = mc_{\text{marble}}\Delta T \quad \rightarrow \quad \Delta T = \frac{\tfrac{1}{2}gh}{c_{\text{marble}}} = \frac{0.50(9.80\text{ m/s}^2)(120\text{ m})}{860\text{ J/kg}\cdot\text{C}°} = \boxed{0.68\text{ C}°}$$

57. The heat lost by the lead must be equal to the heat gained by the water. Note that 1 L of water has a mass of 1 kg.

$$m_{\text{Pb}}c_{\text{Pb}}(T_{\text{i Pb}} - T_{\text{eq}}) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{i H}_2\text{O}})$$

$$(2.3\text{ kg})(130\text{ J/kg}\cdot\text{C}°)(T_{\text{i Pb}} - 32.0°\text{C}) = (2.5\text{ kg})(4186\text{ J/kg}\cdot\text{C}°)(12.0\text{ C}°) \quad \rightarrow$$

$$T_{\text{i Pb}} = 452°\text{C} \approx \boxed{450°\text{C}}$$

58. (*a*) The energy required to raise the temperature of the water is given by Eq. 14–2.

$$Q = mc\Delta T \quad \rightarrow$$

$$\frac{Q}{\Delta t} = mc\frac{\Delta T}{\Delta t} = (0.250\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right)\left(\frac{80\text{ C}°}{105\text{ s}}\right) = 797\text{ W} \approx \boxed{800\text{ W (2 significant figures)}}$$

(*b*) After 105 s, the water is at 100°C. So for the remaining 15 s the energy input will boil the water. Use the heat of vaporization.

$$Q = mL_{\text{V}} \quad \rightarrow \quad m = \frac{Q}{L_{\text{V}}} = \frac{(\text{Power})\Delta t}{L_{\text{V}}} = \frac{(797\text{ W})(15\text{ s})}{2260\text{ J/g}} = \boxed{5.3\text{ g}}$$

59. Assume that the loss of kinetic energy is all turned into heat, which changes the temperature of the squash ball.

$$\text{KE}_{\text{lost}} = Q \quad \rightarrow \quad \tfrac{1}{2}m(v_{\text{i}}^2 - v_{\text{f}}^2) = mc\Delta T \quad \rightarrow \quad \Delta T = \frac{v_{\text{i}}^2 - v_{\text{f}}^2}{2c} = \frac{(22\text{ m/s})^2 - (12\text{ m/s})^2}{2(1200\text{ J/kg}\cdot\text{C}°)} = \boxed{0.14\text{ C}°}$$

60. (*a*) We consider just the 30 m of crust immediately below the surface of the Earth, assuming that all the heat from the interior gets transferred to the surface, so it all passes through this 30-m layer. This is a heat conduction problem, so Eq. 14–5 is appropriate. The radius of the Earth is about 6.38×10^6 m.

$$\frac{Q}{t} = kA\frac{T_1 - T_2}{\ell} \quad \rightarrow \quad Q_{\text{interior}} = kA\frac{T_1 - T_2}{\ell}t = (0.80\text{ J/s}\cdot\text{m}\cdot\text{C}°)4\pi R_{\text{Earth}}^2\frac{1.0\text{ C}°}{30\text{ m}}\left[1\text{ h}\left(\frac{3600\text{ s}}{\text{h}}\right)\right]$$

$$= 4.910\times10^{16}\text{ J} \approx \boxed{4.9\times10^{16}\text{ J}}$$

(*b*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, so it has an area of πR_{Earth}^2. Multiply this area by the solar constant of 1350 W/m^2 to get the amount of energy incident on the Earth from the Sun per second, and then convert to energy per day.

$$Q_{\text{Sun}} = \pi R_{\text{Earth}}^2 (1350\ \text{W/m}^2)\left[1\ \text{h}\left(\frac{3600\ \text{s}}{\text{h}}\right)\right] = 6.215\times10^{20}\ \text{J}$$

$$\frac{Q_{\text{Sun}}}{Q_{\text{interior}}} = \frac{\pi R_{\text{Earth}}^2 (1350\ \text{W/m}^2)\left[1\ \text{h}\left(\frac{3600\ \text{s}}{\text{day}}\right)\right]}{(0.80\ \text{J/s}\cdot\text{m}\cdot\text{C}^\circ)4\pi R_{\text{Earth}}^2 \frac{1.0\ \text{C}^\circ}{30\ \text{m}}\left[1\ \text{h}\left(\frac{3600\ \text{s}}{\text{h}}\right)\right]} = \frac{(1350\ \text{W/m}^2)}{(0.80\ \text{J/s}\cdot\text{m}\cdot\text{C}^\circ)4\left(\frac{1.0\ \text{C}^\circ}{30\ \text{m}}\right)}$$

$$= 1.266\times10^4$$

So $\boxed{Q_{\text{Sun}} \approx 1.3\times10^4\ Q_{\text{interior}}}$.

61. Assume that the final speed of the meteorite, as it completely melts, is 0, and that all of its initial kinetic energy was used in heating the iron to the melting point and then melting the iron.

$$\tfrac{1}{2}mv_{\text{i}}^2 = mc_{\text{Fe}}(T_{\text{melt}} - T_{\text{i}}) + mL_{\text{F}} \quad \rightarrow$$

$$v_{\text{i}} = \sqrt{2[c_{\text{Fe}}(T_{\text{melt}} - T_{\text{i}}) + L_{\text{F}}]} = \sqrt{2[(450\ \text{J/kg}\cdot\text{C}^\circ)(1538^\circ\text{C} - (-105^\circ\text{C})) + 2.89\times10^5\ \text{J/kg}]}$$

$$= \boxed{1430\ \text{m/s}}$$

62. We assume that the lightbulb emits energy by radiation, so Eq. 14–7 applies. Use the data for the 75-W bulb to calculate the product $\varepsilon\sigma A$ for the bulb, and then calculate the temperature of the 150-W bulb.

$$(Q/t)_{75\ \text{W}} = \sigma A\left(T_{75\ \text{W}}^4 - T_{\text{room}}^4\right) \quad \rightarrow$$

$$\varepsilon\sigma A = \frac{(Q/t)_{75\ \text{W}}}{\left(T_{75\ \text{W}}^4 - T_{\text{room}}^4\right)} = \frac{(0.90)(75\ \text{W})}{[(273+75)\text{K}]^4 - [(273+18)\text{K}]^4} = 9.006\times10^{-9}\ \text{W/K}^4$$

$$(Q/t)_{150\ \text{W}} = \varepsilon\sigma A\left(T_{150\ \text{W}}^4 - T_{\text{room}}^4\right) \quad \rightarrow$$

$$T_{150\ \text{W}} = \left[\frac{(Q/t)_{150\ \text{W}}}{\varepsilon\sigma A} + T_{\text{room}}^4\right]^{1/4} = \left[\frac{(0.90)(150\ \text{W})}{(9.006\times10^{-9}\ \text{W/K}^4)} + (291\ \text{K})^4\right]^{1/4}$$

$$= 386\ \text{K} = 113^\circ\text{C} \approx \boxed{110^\circ\text{C}}$$

63. The body's metabolism (blood circulation in particular) provides cooling by convection. If the metabolism has stopped, then heat loss will be by conduction and radiation, at a rate of 200 W, as given. The change in temperature is related to the body's heat loss by Eq. 14–2, $Q = mc\Delta T$.

$$\frac{Q}{t} = P = \frac{mc\Delta T}{t} \quad \rightarrow$$

$$t = \frac{mc\Delta T}{P} = \frac{(65\ \text{kg})(3470\ \text{J/kg}\cdot\text{C}^\circ)(36.6^\circ\text{C} - 35.6^\circ\text{C})}{200\ \text{W}} = 1128\ \text{s} \approx \boxed{1100\ \text{s}} = 19\ \text{min}$$

64. (*a*) The bullet will gain an amount of heat equal to 50% of its loss of kinetic energy. Initially assume that the phase of the bullet does not change, so that all of the heat causes a temperature increase.

$$\tfrac{1}{2}\left[\tfrac{1}{2}m\left(v_{\text{i}}^2 - v_{\text{f}}^2\right)\right] = Q = mc_{\text{Pb}}\Delta T \quad \rightarrow \quad \Delta T = \frac{\tfrac{1}{4}\left(v_{\text{i}}^2 - v_{\text{f}}^2\right)}{c_{\text{Pb}}} = \frac{(220\ \text{m/s})^2 - (160\ \text{m/s})^2}{4(130\ \text{J/kg}\cdot\text{C}^\circ)} = \boxed{44\ \text{C}^\circ}$$

(*b*) The final temperature of the bullet would be about $64°\text{C}$, which is not above the melting temperature of lead, which is $327°\text{C}$. $\boxed{\text{Thus, none of the bullet will melt.}}$

65. (*a*) The rate of absorbing heat for an object facing the Sun is given by Eq. 14–8.

$$\frac{\Delta Q}{\Delta t}=(1000\ \text{W/m}^2)\varepsilon A\cos\theta=(1000\ \text{W/m}^2)(0.85)(40\ \text{cm}^2)\left(\frac{1\ \text{m}^2}{1\times10^4\ \text{cm}^2}\right)(1)=\boxed{3.4\ \text{W}}$$

(*b*) The rise in temperature is related to the absorbed heat by Eq. 14–2. We assume that all absorbed heat raises the temperature of the leaf.

$$\Delta Q=mc\Delta T \quad\rightarrow\quad \Delta T=\frac{\Delta Q}{mc}\quad\rightarrow$$

$$\frac{\Delta T}{\Delta t}=\frac{1}{mc}\frac{\Delta Q}{\Delta t}=\frac{3.4\text{W}}{(4.5\times10^{-4}\ \text{kg})(0.80\ \text{kcal/kg}\cdot\text{C}°)}\left(\frac{1\ \text{kcal}}{4186\ \text{J}}\right)=2.256\text{C}°/\text{s}\approx\boxed{2.3\text{C}°/\text{s}}$$

(*c*) This temperature rise cannot continue for hours. In one hour, the leaf's temperature would increase to over $8000°\text{C}$, and the leaf would combust long before that temperature was reached.

(*d*) We assume that the rate of heat loss by radiation must equal the rate of heat absorption of solar energy. Note that the area of the leaf that radiates is twice the area that absorbs heat energy.

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\substack{\text{solar}\\ \text{heating}}}=\left(\frac{\Delta Q}{\Delta t}\right)_{\text{radiation}}\quad\rightarrow\quad(1000\ \text{W/m}^2)\varepsilon A_{\text{absorb}}\cos\theta=\varepsilon\sigma A_{\text{radiate}}(T_1^4-T_2^4)\quad\rightarrow$$

$$T_1=\left[\frac{(1000\ \text{W/m}^2)\cos\theta}{2\sigma}+T_2^4\right]^{1/4}=\left[\frac{(1000\ \text{W/m}^2)(1)}{2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}+(297\ \text{K})^4\right]^{1/4}$$

$$=359\ \text{K}=\boxed{86°\text{C}}$$

This is very hot, which indicates that the leaf must lose energy by other means than just radiation, in order to cool to a more reasonable temperature.

(*e*) The leaf can also lose heat by $\boxed{\text{conduction}}$ to the cooler air around it; by $\boxed{\text{convection}}$, as the wind continually moves cooler air over the surface of the leaf; and by $\boxed{\text{evaporation}}$ of water.

66. The rate of energy absorption from the Sun must be equal to the rate of losing energy by radiation plus the rate of losing energy by evaporation if the leaf is to maintain a steady temperature. The latent heat of evaporation is taken to be the value at 20°C, which is 2450 kJ/kg. Also note that the area of the leaf that radiates is twice the area that absorbs heat energy.

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\substack{\text{solar}\\ \text{heating}}}=\left(\frac{\Delta Q}{\Delta t}\right)_{\text{radiation}}+\left(\frac{\Delta Q}{\Delta t}\right)_{\text{evaporation}}\quad\rightarrow$$

$$\left(1000\ \text{W/m}^2\right)\varepsilon A_{\text{absorb}}\cos\theta=\varepsilon\sigma A_{\text{radiate}}\left(T_1^4-T_2^4\right)+\frac{m_{H_2O}L_{\text{evaporation}}}{\Delta t}\quad\rightarrow$$

$$\frac{m_{H_2O}}{\Delta t}=\varepsilon A_{\text{absorb}}\frac{\left(1000\ \text{W/m}^2\right)\cos\theta-2\sigma\left(T_1^4-T_2^4\right)}{L_{\text{evaporation}}}$$

$$=(0.85)\left(40\times10^{-4}\ \text{m}^2\right)\frac{\left(1000\ \text{W/m}^2\right)(1)-2\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(308\ \text{K})^4-(297\ \text{K})^4\right]}{\left(2.45\times10^6\ \text{J/kg}\right)}$$

$$=1.196\times10^{-6}\ \text{kg/s}\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=\boxed{4.3\ \text{g/h}}$$

67. (*a*) The amount of heat energy required is given by Eq. 14–2. One liter of water has a mass of 1 kg.

$$Q = mc\Delta T = (245\text{ kg})(4186\text{ J/kg}\cdot\text{C}°)(45°\text{C} - 10°\text{C}) = 3.589\times10^{7}\text{ J} \approx \boxed{3.6\times10^{7}\text{ J}}$$

(*b*) The heat energy is the power input times the time.

$$Q = Pt \quad\rightarrow\quad t = \frac{Q}{P} = \frac{3.589\times10^{7}\text{ J}}{9.5\times10^{3}\text{ W}} = 3778\text{ s} \approx \boxed{3800\text{ s}} \approx 63\text{ min}$$

68. We model the heat loss as conductive, so that, using Eq. 14–5, $\frac{Q}{t} = \frac{kA}{\ell}\Delta T \quad\rightarrow\quad Q = \alpha t\Delta T$, where α describes the average heat conductivity properties of the house, such as insulation materials and surface area of the conducting surfaces. It could have units of $\text{J/h}\cdot°\text{C}$. We see that the heat loss is proportional to the product of elapsed time and the temperature difference. We assume that the proportionality constant α does not vary during the day, so that, for example, heating by direct sunlight through windows is not considered. We also assume that α is independent of temperature, so it is the same during both the day and the night.

$$Q_{\substack{\text{turning}\\\text{down}}} = (\alpha\text{ J/h}\cdot\text{C}°)(15\text{ h})(22°\text{C} - 8°\text{C}) + (\alpha\text{ J/h}\cdot\text{C}°)(9\text{ h})(16°\text{C} - 0°\text{C}) = 354\alpha\text{ J}$$

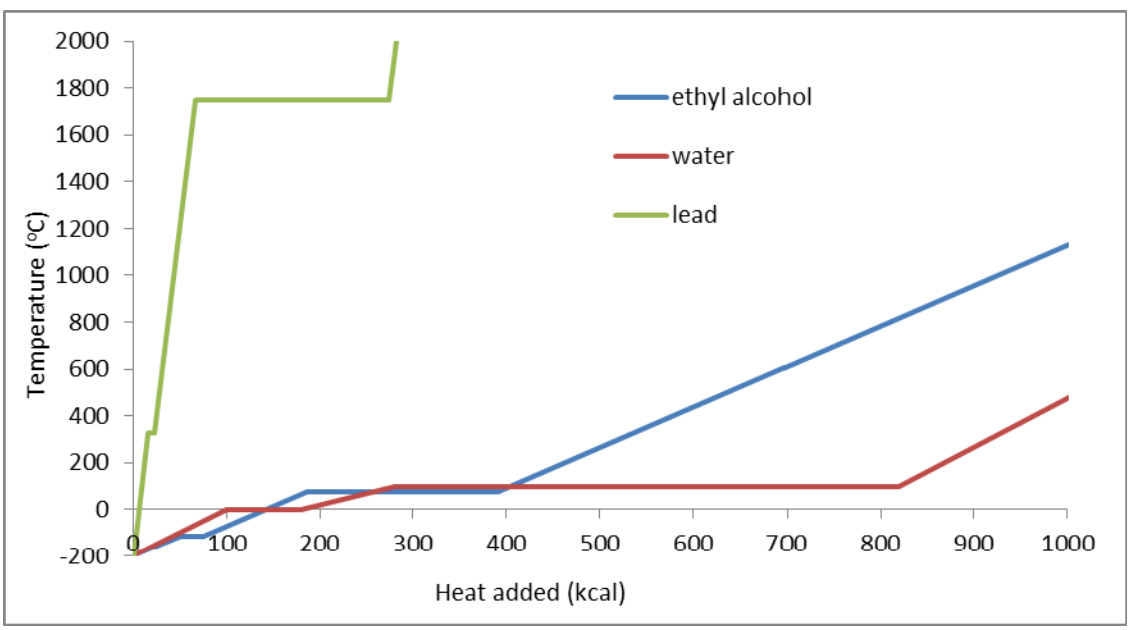
$$Q_{\substack{\text{not turning}\\\text{down}}} = (\alpha\text{ J/h}\cdot\text{C}°)(15\text{ h})(22°\text{C} - 8°\text{C}) + (\alpha\text{ J/h}\cdot\text{C}°)(9\text{ h})(22°\text{C} - 0°\text{C}) = 408\alpha\text{ J}$$

$$\frac{\Delta Q}{Q_{\substack{\text{turning}\\\text{down}}}} = \frac{408\alpha\text{ J} - 354\alpha\text{ J}}{354\alpha\text{ J}} = 0.1525 \approx \boxed{15\%}$$

Keeping the thermostat "up" in this model requires about 15% more heat than turning it down.

Solutions to Search and Learn Problems

1.



We chose a starting temperate of $-200°C$, so that all three substances would be in the solid state. Lead has small specific and latent heats, so the temperature of lead increases rapidly as a function of input heat. Water has the largest specific and latent heats and therefore has the smallest temperature increase. There is no temperature range for which water, lead, and ethyl alcohol are all liquid. All three are solids at temperatures below $-114°C$. All three are gases for temperatures above $1750°C$.

2. (*a*) The cross-sectional area of the Earth, perpendicular to the Sun, is a circle of radius R_{Earth}, so it has an area of πR_{Earth}^2. Multiply this area by the solar constant to get the rate at which the Earth is receiving solar energy.

$$\frac{Q}{t} = \pi R_{\text{Earth}}^2 (\text{solar constant}) = \pi(6.38\times10^6 \text{ m})^2(1350 \text{ W/m}^2) = \boxed{1.73\times10^{17} \text{ W}}$$

(*b*) Use Eq. 14–7 to calculate the rate of heat output by radiation, and assume that the temperature of space is 0 K. The whole sphere is radiating heat back into space, so we use the full surface area of the Earth, $4\pi R_{\text{Earth}}^2$.

$$\frac{Q}{t} = \varepsilon\sigma A T^4 \quad \rightarrow \quad T = \left(\frac{Q}{t}\frac{1}{\varepsilon\sigma A}\right)^{1/4}$$

$$= \left[(1.73\times10^{17} \text{ J/s})\frac{1}{(1.0)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)4\pi(6.38\times10^6 \text{ m})^2}\right]^{1/4} = \boxed{278 \text{ K} = 5°\text{C}}$$

3. We take the 1000 J as an exact number.

(*a*) Solve Eq. 14–2 for the final temperature of the ice. Use the specific heat of ice.

$$Q = mc(T - T_0) \quad \rightarrow \quad T = T_0 + \frac{Q}{mc} = -20°\text{C} + \frac{1000 \text{ J}}{(0.100 \text{ kg})(2100 \text{ J/kg°C})} = \boxed{-15°\text{C}}$$

(*b*) The heat will melt part of the ice. Solve Eq. 14–4 for the mass of ice that will melt, with *L* being the latent heat of fusion of water. The temperature will remain constant at 0°C.

$$Q = mL \quad \rightarrow \quad m = \frac{Q}{L} = \frac{1000 \text{ J}}{333 \text{ kJ/kg}} = \boxed{3.00 \text{ g of ice will melt}}$$

(*c*) Solve Eq. 14–2 for the final temperature of the water. Use the specific heat of water.

$$Q = mc(T - T_0) \quad \rightarrow \quad T = T_0 + \frac{Q}{mc} = 10°\text{C} + \frac{1000 \text{ J}}{(0.100 \text{ kg})(4186 \text{ J/kg°C})} = \boxed{12°\text{C}}$$

(*d*) The heat will convert some of the water into steam. Solve Eq. 14–4 for the mass of the water that will be converted to steam, with *L* being the latent heat of vaporization of water. The temperature will remain constant at 100°C.

$$Q = mL \quad \rightarrow \quad m = \frac{Q}{L} = \frac{1000 \text{ J}}{2260 \text{ kJ/kg}} = \boxed{0.442 \text{ g of water will be converted to steam}}$$

(*e*) Solve Eq. 14–2 for the final temperature of the steam. Use the specific heat of steam.

$$Q = mc(T - T_0) \quad \rightarrow \quad T = T_0 + \frac{Q}{mc} = 110°\text{C} + \frac{1000 \text{ J}}{(0.100 \text{ kg})(2010 \text{ J/kg°C})} = \boxed{115°\text{C}}$$

4. (*a*) To calculate heat transfer by conduction, use Eq. 14–5 for all three areas—walls, roof, and windows. Each area has the same temperature difference.

$$\frac{Q_{\text{conduction}}}{t}=\left[\left(\frac{kA}{\ell}\right)_{\text{walls}}+\left(\frac{kA}{\ell}\right)_{\text{roof}}+\left(\frac{kA}{\ell}\right)_{\text{windows}}\right](T_1-T_2)$$

$$=\left[\begin{array}{l}\dfrac{(0.023\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(410\text{ m}^2)}{0.195\text{ m}}+\dfrac{(0.1\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(250\text{ m}^2)}{0.055\text{ m}}\\ +\dfrac{(0.84\text{ J/s}\cdot\text{m}\cdot\text{C}^\circ)(33\text{ m}^2)}{6.5\times10^{-3}\text{ m}}\end{array}\right](38\text{ C}^\circ)$$

$$=1.812\times10^5\text{ W}\approx\boxed{1.8\times10^5\text{ W}}$$

(*b*) The energy being added must both heat the air and replace the energy lost by conduction, as considered above. The heat required to raise the temperature is given by Eq. 14–2, $Q_{\substack{\text{raise}\\ \text{temp}}}=m_{\text{air}}c_{\text{air}}(\Delta T)_{\text{warming}}$. The mass of the air can be found from the density of the air times its volume. The conduction heat loss is proportional to the temperature difference between the inside and outside, which, if we assume the outside temperature is still $-15°\text{C}$, varies from 30 C° to 38 C°. We will estimate the average temperature difference as 34C° and scale the answer from part (*a*) accordingly.

$$Q_{\text{added}}=Q_{\substack{\text{raise}\\ \text{temp}}}+Q_{\text{conduction}}=\rho_{\text{air}}Vc_{\text{air}}(\Delta T)_{\text{warming}}+\left(\frac{Q_{\text{conduction}}}{t}\right)(1800\text{ s})$$

$$=\left(1.29\frac{\text{kg}}{\text{m}^3}\right)(750\text{m}^3)\left(0.24\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}\right)\left(\frac{4186\text{ J}}{\text{kcal}}\right)(8°\text{C})$$

$$+\left(1.812\times10^5\frac{\text{J}}{\text{s}}\right)\left(\frac{34°\text{C}}{38°\text{C}}\right)(1800\text{ s})=2.996\times10^8\text{ J}\approx\boxed{3.0\times10^8\text{ J}}$$

(*c*) We assume a month is 30 days.

$$0.9Q_{\text{gas}}=\left(\frac{Q}{t}\right)_{\text{conduction}}t_{\text{month}}\quad\rightarrow$$

$$Q_{\text{gas}}=\frac{1}{0.9}\left(\frac{Q}{t}\right)_{\text{conduction}}t_{\text{month}}=\frac{1}{0.9}(1.812\times10^5\text{ J/s})(30\text{ days})\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)=5.219\times10^{11}\text{ J}$$

$$5.276\times10^{11}\text{ J}\left(\frac{1\text{ kg}}{5.4\times10^7\text{ J}}\right)\left(\frac{\$0.080}{\text{kg}}\right)=\$773.12\approx\boxed{\$770}$$

As an extra comment, almost 90% of the heat loss is through the windows. Investing in insulated window drapes and double-paned windows in order to reduce the thermal conductivity of the windows could mean major savings for this homeowner.

15

THE LAWS OF THERMODYNAMICS

Responses to Questions

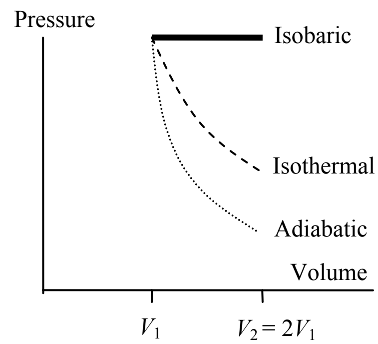
1. Since the process is isothermal, there is no change in the internal energy of the gas. Thus $\Delta U = Q - W = 0 \rightarrow Q = W$, so the heat absorbed by the gas is equal to the work done by the gas. Thus 3700 J of heat was added to the gas.

2. Mechanical energy can be transformed completely into heat. When a moving object slides across a rough level floor and eventually stops, the mechanical energy of the moving object has been transformed completely into heat. Also, if a moving object were to be used to compress a frictionless piston containing an insulated gas, the kinetic energy of the object would become internal energy of the gas. A gas that expands adiabatically (without heat transfer) transforms internal energy into mechanical energy, by doing work on its surroundings at the expense of its internal energy. Of course, that is an ideal (reversible) process. In any nonideal process, only a fraction of the internal energy can be changed into mechanical energy. Some of the internal energy might also be changed into heat.

3. It is possible for temperature (and thus internal energy) to remain constant in a system even though there is heat flow into or out of the system. By the first law of thermodynamics, there must be an equal amount of work done on or by the system, so that $\Delta U = Q - W = 0 \rightarrow Q = W$. The isothermal expansion or compression of a gas is an example of this situation. A change of state (melting, freezing, boiling, condensing, evaporating) is another example of heat transfer without a corresponding temperature change.

4. If the gas is compressed adiabatically, then no heat enters or leaves from the gas. The compression means that work was done ON the gas. By the first law of thermodynamics, $\Delta U = Q - W$, since $Q = 0$, then $\Delta U = -W$. The change in internal energy is equal to the opposite of the work done by the gas or is equal to the work done on the gas. Since positive work was done on the gas, the internal energy of the gas increased, and that corresponds to an increase in temperature. This is conservation of energy—the work done on the gas becomes internal energy of the gas particles, and the temperature increases accordingly.

5. ΔU is proportional to the change in temperature. The change in the internal energy is zero for the isothermal process, greatest for the isobaric process, and least (negative) for the adiabatic process. The work done, W, is the area under the curve and is greatest for the isobaric process and least for the adiabatic process. From the first law of thermodynamics, Q is the sum of ΔU and W and is zero for the adiabatic process and a maximum for the isobaric process.



6. (*a*) When the lid is removed, the chlorine gas mixes with the air in the room around the bottle so that eventually both the room and the bottle contain a mixture of air and chlorine.
   (*b*) The reverse process, in which the individual chlorine particles reorganize so that they are all in the bottle, violates the second law of thermodynamics and does not occur naturally. It would require a spontaneous decrease in entropy.
   (*c*) Adding a drop of food coloring to a glass of water is another example of an irreversible process; the food coloring will eventually disperse throughout the water but will not ever gather into a drop again. The toppling of buildings during an earthquake is another example. The toppled building will not ever become "reconstructed" by another earthquake.

7. No. The definition of heat engine efficiency as $e = W/Q_L$ does not account for Q_H, the energy needed to produce the work. Efficiency should relate the input energy and the output work. This definition of efficiency is also not useful because if the exhaust heat Q_L is less than the work done W (which is possible), the "efficiency" would exceed unity.

8. (*a*) In an internal combustion engine, the high-temperature reservoir is the ignited gas–air mixture in the cylinder. The low-temperature reservoir is the "outside" air. The burned gases leave through the exhaust pipe.
   (*b*) In the steam engine, the high-temperature reservoir is the heated, high-pressure steam from the boiler. The low-temperature reservoir is the condensed water in the condenser.

   In the cases of both these engines, these areas are not technically heat "reservoirs," because each one is not at a constant temperature.

9. To utilize thermal energy from the ocean, a heat engine would need to be developed that operated between two different temperatures. If surface temperature water was to be both the source and the exhaust, then no work could be extracted. If the temperature difference between surface and deep ocean waters were to be used, then there would be considerable engineering obstacles, high expense, and potential environmental difficulties involved in having a heat engine that connected surface water and deep ocean water. Likewise, if the difference in temperature between tropical water and arctic water were to be used, then major difficulties would be involved because of the large distances involved.

$\boxed{10.}$ It is possible to warm the kitchen in the winter by having the oven door open. The oven heating elements radiate heat energy into the oven cavity, and if the oven door is open, then the oven is just heating a bigger volume than usual. There is no thermodynamic cycle involved here. However, you cannot cool the kitchen by having the refrigerator door open. The refrigerator exhausts more heat than it removes from the refrigerated volume, so the room actually gets warmer with the refrigerator door open because of the work done by the refrigerator compressor. If you could have the refrigerator exhaust into some other room, then the refrigerator would be similar to an air conditioner, and it could cool the kitchen, while heating up some other space. Or you could unplug the refrigerator and open the door. That would cool the room somewhat, but would heat up the contents of the refrigerator, which is probably not a desired outcome!

11. For a refrigerator, $\text{COP} = Q_L/W$. That definition makes sense because we are interested in removing heat from the low-temperature reservoir (the interior of the refrigerator). The more heat that can be removed per amount of input work, the better (more efficient) the refrigerator is.

    For a heat pump, $\text{COP} = Q_H/W$. The objective of the heat pump is to heat (deliver Q_H) rather than cool (remove Q_L). It is the heat delivered to the house that is important now. The more heat that can be delivered to the house per amount of input work, the better the heat pump is.

12. Any air conditioner-type heat engine will remove heat from the room (Q_L—the low-temperature input). Work (W) is input to the device to enable it to remove heat from the low-temperature region. By the second law of thermodynamics (conservation of energy), there must be a high-temperature exhaust heat Q_H which is larger than Q_L. Perhaps the inventor has come up with some clever method of having that exhaust heat move into a well-insulated heat "sink," like a container of water. But eventually the addition of that heat to the device will cause the device to become warmer than the room itself, and then heat will be transferred to the room. One very simple device that could do what is described in the question would be a fan blowing over a large block of ice. Heat from the room will enter the ice; cool air from near the surface of the ice will be blown by the fan. But after the ice melts, the fan motor would again heat the air.

13. Some processes that would obey the first law of thermodynamics but not the second, if they actually occurred, include:
    - a cup of tea warms itself by gaining thermal energy from the cooler air molecules around it;
    - a ball sitting on a soccer field gathers energy from its surroundings and begins to roll;
    - a bowl of popcorn placed in the refrigerator "un-pops" as it cools;
    - an empty perfume bottle is placed in a room containing perfume molecules, and all of the perfume molecules move into the bottle from various directions at the same time;
    - water on the sidewalk coalesces into droplets that are propelled upward and rise into the air;
    - a house gets warmer in the winter while the outdoors gets colder, due to heat moving from the outdoors to inside the house.

14. While the state of the papers has changed from disorder to order, they did not do so spontaneously. An outside source (you) caused the increase in order. You had to provide energy to do this (through your metabolic processes), and in doing so, your entropy increased more than the entropy of the papers decreased. The overall effect is that the entropy of the universe increased, satisfying the second law of thermodynamics.

15. The first statement, "You can't get something for nothing," is a whimsical way of saying that energy is conserved. For instance, one way to write the first law is $W = Q - \Delta U$. This says that work done by a system must have a source—either heat is input to the system or the internal energy of the system is lowered. It "costs" energy—either heat energy or internal energy—to get work done. Another way to say this is that no heat engine can be built which puts out more energy in the form of work than it extracts in the form of heat or internal energy.

The second statement, "You can't even break even," reflects the fact that a consequence of the second law is that there is no heat engine that is 100% efficient. Even though the first law is satisfied by an engine that takes in 100 J of heat and outputs 100 J of work, the second law says that that is impossible. If 100 J of heat were taken in, then less than 100 J of work will be output from the heat engine, even if it is an ideal heat engine. Some energy will be "lost" as exhaust energy.

16. (*a*) If a gas expands adiabatically, then $Q = 0$, so $\Delta S = 0$ by Eq. 15–8, $\Delta S = Q/T$.

    (*b*) If a gas expands isothermally, then there is no change in its internal energy, and the gas does work on its surroundings. Thus by the first law of thermodynamics, there must be heat flow into the gas, so $\Delta S > 0$—the entropy of the gas increases.

17. One kilogram of liquid iron will have greater entropy, since it is less ordered than solid iron and its molecules have more thermal motion. In addition, heat must be added to solid iron to melt it; the addition of heat will increase the entropy of the iron.

18. (*a*) The erosion of soil due to water flow over the ground.

    (*b*) The oxidation of various metals (copper, zinc, iron, etc.) when left exposed to the air.

    (*c*) Fallen leaves decaying in the woods.

    (*d*) A pile of compost decomposing.

    (*e*) A landslide.

    The reverse of these processes is not observed.

19. In an action movie, you might see a building or car changing from an exploded state to an unexploded state, or a bullet that was fired going backward into the gun and the gunpowder "unexploding." In a movie with vehicle crashes, you might observe two collided vehicles separating from each other, becoming unwrecked as they separate, or someone "unwrite" something on a piece of paper—moving a pen over paper, taking away written marks as the pen moves.

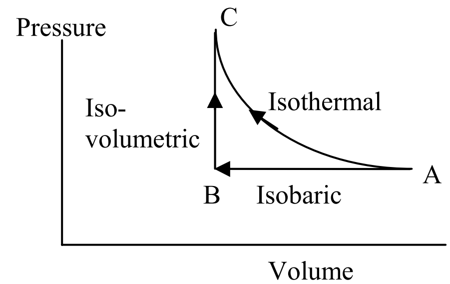
20. The synthesis of complex molecules from simple molecules does involve a decrease in entropy of the constituent molecules, since they have become more "structured" or "ordered." However, the molecules are not a closed system. This process does not occur spontaneously or in isolation. The living organism in which the synthesis process occurs is part of the environment that must be considered for the overall change in entropy. The "living organism and environment" combination will have an increase in entropy that is larger than the decrease in entropy of the molecules, so overall, the second law is still satisfied, and the entropy of the entire system will increase.

Responses to MisConceptual Questions

1. (*d*) An isobaric process is one in which the pressure is kept constant. In a compression the volume of the gas decreases. By Eq. 15–3, the work done by the gas is negative, so an external force had to do work on the gas. In isobaric processes heat is allowed to flow into or out of the system and the internal energy changes.

2. (*c*) According to the second law of thermodynamics, it is impossible for heat to be entirely converted into work in a cycle or a heat engine. However, the question does not specify that we must consider a complete cycle. In an isothermal process $Q = W$. In an isothermal compression work is entirely converted into heat, and in an isothermal expansion heat is entirely converted into work.

3. (*c*) A common misconception is that the work done in moving an object between two states is independent of the path followed. In the graph shown, the work done in going from point A to B to C by the isobaric and isovolumetric processes is equal to the area under the AB line. The work done by the isothermal process is the area under the curved line. Since the AC line includes all of the area under the AB line as well as the area between the AB and AC lines, more work is done on the gas in the isothermal process.



4. (*d*) In an isothermal process the internal energy remains constant $(\Delta U = 0)$. In an expansion the gas does work on the surroundings $(W = 0)$. Since the internal energy is constant and the work is positive, the first law of thermodynamics requires the heat absorbed also be positive $(Q = 0)$.

5. (*d*) Students may misunderstand the difference between isothermal (temperature remains constant so $\Delta U = 0$) and adiabatic $(Q = 0)$. In an isothermal process heat can be absorbed, as long as an equal amount of work is done, so statement (i) is not true. For an ideal gas the temperature is proportional to the internal energy of the gas, statements (ii) and (iii) are equivalent, and both are true.

6. (*b*) As the gas expands, its volume increases and it does work on the surroundings. Since no heat is absorbed while the gas does this work, the first law of thermodynamics says that the internal energy and temperature of the gas must decrease. For the volume to increase as the temperature decreases, the ideal gas law requires that the pressure also decrease.

7. (*d*) A frequent misconception made in calculating the efficiency of an engine is to leave the temperatures in degrees Celsius, which would imply an efficiency of 50%. However, when the temperatures are properly converted to kelvins, Eq. 15–5 gives the efficiency as only about 34%.

8. (*d*) A common misconception in this situation is not realizing that a heat cycle running in reverse, like a refrigerator, must have a high-temperature exhaust. Furthermore, that high-temperature exhaust is the sum of the heat removed from the inside of the refrigerator and the work done by the refrigerator's compressor.

9. (*a, c*) The maximum efficiency of an engine is given by Eq. 15–5, which can be written in the form $e = (T_H - T_C)/T_H$. Increasing the temperature difference, as in (*a*), results in a higher efficiency. In (*b*) the temperature difference remains the same, while the hot temperature increases, which results in a lower efficiency. In (*c*) the efficiency increases as the temperature difference remains the same, but T_H decreases. In (*d*) the temperature difference decreases, which lowers the efficiency.

10. (*a*) The text states that "real engines that are well designed reach 60 to 80% of the Carnot efficiency." The cooling system of the engine keeps the high temperature at about 120°C (400 K) and the exhaust is about room temperature (300 K). The maximum efficiency would then be around 25%. Eighty percent of this maximum would be closest to 20% efficient. Any of the other choices for this question are not reasonable.

11. (*b*) Heat must be added to the ice cube to melt it. The change in entropy is the ratio of the heat added to the temperature of the ice cube. Since heat is absorbed in the process, the entropy increases.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. Use the first law of thermodynamics, Eq. 15–1, and the definition of internal energy, Eq. 14–1. Since the work is done by the gas, it is positive.

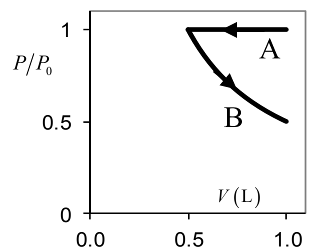
   (*a*) The temperature does not change, so $\boxed{\Delta U = 0}$.

   (b) $\Delta U = Q - W \rightarrow Q = \Delta U + W = 0 + 4.30\times10^3\ \text{J} = \boxed{4.30\times10^3\ \text{J}}$

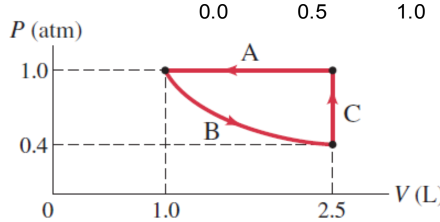
2. For the drawing of the graph, the pressure is given relative to the starting pressure, which is taken to be P_0.

   Segment A is the cooling at constant pressure.

   Segment B is the isothermal expansion.



3. Segment A is the compression at constant pressure. Since the process is at a constant pressure, the path on the diagram is horizontal from 2.5 L to 1.0 L.



   Segment B is the isothermal expansion. Since the temperature is constant, the ideal gas law says that the product PV is constant. Since the volume is increased by a factor of 2.5, the pressure must be divided by 2.5, so the final point on this segment is at a pressure of $1\ \text{atm}/2.5 = 0.4\ \text{atm}$. The path is a piece of a hyperbola.

   Segment C is the pressure increase at constant volume. Since the process is at a constant volume, the path on the diagram is vertical from 0.4 atm to 1.0 atm.

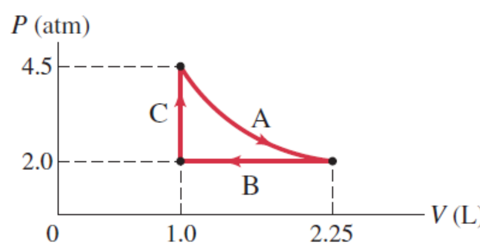
4. (*a*) The work done by a gas at constant pressure is found from Eq. 15–3.

$$W = P\Delta V = (1\ \text{atm})\left(\frac{1.01\times10^5\ \text{Pa}}{1\ \text{atm}}\right)(16.2\ \text{m}^3 - 12.0\ \text{m}^3) = 4.242\times10^5\ \text{J} \approx \boxed{4.2\times10^5\ \text{J}}$$

   (*b*) The change in internal energy is calculated from the first law of thermodynamics.

$$\Delta U = Q - W = (254\ \text{kcal})\left(\frac{4186\ \text{J}}{1\ \text{kcal}}\right) - 4.242\times10^5\ \text{J} = \boxed{6.4\times10^5\ \text{J}}$$

5. The pressure must be converted to absolute pressure in order to use the ideal gas equation, so the initial pressure is 4.5 atm absolute pressure, and the lower pressure is 2.0 atm absolute pressure. Segment A is the isothermal expansion. The temperature and the amount of gas are constant, so $PV = nRT$ is constant. Since the pressure is reduced by a factor of 2.25, the volume increases by a factor of 2.25, to a final volume of 2.25 L. Segment B is the compression at constant pressure, and segment C is the pressure increase at constant volume.



6. (*a*) Since the container has rigid walls, there is no change in volume.

$$W = P\Delta V = \boxed{0}$$

(*b*) Use the first law of thermodynamics to find the change in internal energy.

$$\Delta U = Q - W = (-465\text{ kJ}) - 0 = \boxed{-465\text{ kJ}}$$

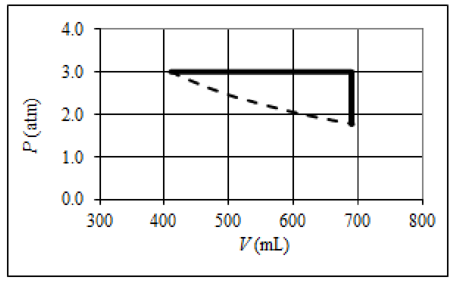
$\boxed{7.}$ (*a*) Since the process is adiabatic, $Q = \boxed{0}$.

(*b*) Use the first law of thermodynamics to find the change in internal energy.

$$\Delta U = Q - W = 0 - (-2630\text{ J}) = \boxed{2630\text{ J}}$$

(*c*) Since the internal energy is proportional to the temperature, a rise in internal energy means a $\boxed{\text{rise}}$ in temperature.

8. A graph of the process is shown. The expansion process is the horizontal line, and the constant-volume process is the vertical line. The dashed line is an isotherm starting from the original state.



(*a*) Work is only done in the expansion at constant pressure, since there must be a volume change in order for there to be work done.

$$W = P\Delta V$$

$$= (3.0\text{ atm})\left(\frac{1.01\times10^5\text{ Pa}}{1\text{ atm}}\right)(0.28\text{ L})\frac{1\times10^{-3}\text{ m}^3}{1\text{ L}} = \boxed{85\text{ J}}$$

(*b*) Use the first law of thermodynamics to find the heat flow. Notice that the temperature change over the entire process is 0, so there is no change in internal energy.

$$\Delta U = Q - W = 0 \quad\to\quad Q = W = \boxed{85\text{ J}}$$

9. Since the expansion is adiabatic, there is no heat flow into or out of the gas. Use the first law of thermodynamics to calculate the temperature change.

$$\Delta U = Q - W \quad\to\quad \tfrac{3}{2}nR\Delta T = 0 - W \quad\to$$

$$\Delta T = -\tfrac{2}{3}\frac{W}{nR} = -\frac{2(8300\text{ J})}{3(8.5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = -78.3\text{ K} \approx \boxed{-78\text{ K}}$$

10. (*a*) No work is done during the first step, since the volume is constant. The work in the second step is given by $W = P\Delta V$.

$$W = P\Delta V = (1.4\text{ atm})\left(\frac{1.01\times10^5\text{ Pa}}{1\text{ atm}}\right)(9.3\text{ L} - 5.9\text{ L})\left(\frac{1\times10^{-3}\text{ m}^3}{1\text{ L}}\right) = \boxed{480\text{ J}}$$

(*b*) Since there is no overall change in temperature, $\Delta U = \boxed{0}$.

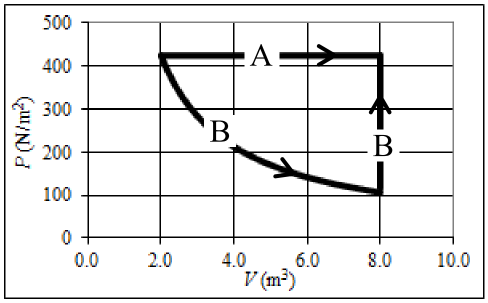
(*c*) The heat flow can be found from the first law of thermodynamics.

$$\Delta U = Q - W \to Q = \Delta U + W = 0 + 480\text{ J} = \boxed{480\text{ J (into the gas)}}$$

11. (*a*) Since the gas is well insulated, no heat can flow into or out of the gas. When the gas is compressed, work is done on that gas. Thus the gas gains energy. That energy manifests as an increase in the average kinetic energy of the gas particles, so the temperature of the gas increases.

(*b*) When the gas expands, the opposite effect occurs. The gas does work on the piston during the expansion. To accomplish that work, the energy of the gas decreases. Since the gas is well insulated, no heat can flow into the gas to compensate for that lost work, so the average kinetic energy of the gas particles decreases, and thus there is a decrease in temperature.

12. (a) See the diagram. The isobaric expansion is just a horizontal line on the graph.



(*b*) The work done is found from Eq. 15–3.

$$W = P\Delta V$$
$$= (425\ \text{N/m}^2)(8.00\ \text{m}^3 - 2.00\ \text{m}^3) = \boxed{2550\ \text{J}}$$

The change in internal energy depends on the temperature change, which can be related to the ideal gas law, $PV = nRT$.

$$\Delta U = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(nRT_2 - nRT_1)$$
$$= \tfrac{3}{2}[(PV)_2 - (PV)_1] = \tfrac{3}{2}P\Delta V = \tfrac{3}{2}W = \tfrac{3}{2}(2550\ \text{J}) = \boxed{3830\ \text{J}}$$

(*c*) For the isothermal expansion, since the volume expands by a factor of 4, the pressure drops by a factor of 4 to $106\ \text{N/m}^2$. See the diagram.

(*d*) The change in internal energy only depends on the initial and final temperatures. Since those temperatures are the same for process B as they are for process A, the internal energy change is the same for process B as for process A, $\boxed{3830\ \text{J}}$.

13. For the path ac, use the first law of thermodynamics to find the change in internal energy.

$$\Delta U_{\text{ac}} = Q_{\text{ac}} - W_{\text{ac}} = -63\ \text{J} - (-35\ \text{J}) = -28\ \text{J}$$

Since internal energy only depends on the initial and final temperatures, this $\overrightarrow{\Delta U}$ applies to any path that starts at a and ends at c. And for any path that starts at c and ends at a, $\Delta U_{\text{ca}} = -\Delta U_{\text{ac}} = 28\ \text{J}$.

(*a*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta U_{\text{abc}} = Q_{\text{abc}} - W_{\text{abc}} \quad \rightarrow \quad Q_{\text{abc}} = \Delta U_{\text{abc}} + W_{\text{abc}} = -28\ \text{J} + (-54\ \text{J}) = \boxed{-82\ \text{J}}$$

(*b*) Since the work along path bc is 0, $W_{\text{abc}} = W_{\text{ab}} = P_{\text{b}}\Delta V_{\text{ab}} = P_{\text{b}}(V_{\text{b}} - V_{\text{a}})$. Also note that the work along path da is 0.

$$W_{\text{cda}} = W_{\text{cd}} = P_{\text{c}}\Delta V_{\text{cd}} = P_{\text{c}}(V_{\text{d}} - V_{\text{c}}) = \tfrac{1}{2}P_{\text{b}}(V_{\text{a}} - V_{\text{b}}) = -\tfrac{1}{2}W_{\text{abc}} = -\tfrac{1}{2}(-54\ \text{J}) = \boxed{27\ \text{J}}$$

(*c*) Use the first law of thermodynamics to find Q_{abc}.

$$\Delta U_{\text{cda}} = Q_{\text{cda}} - W_{\text{cda}} \quad \rightarrow \quad Q_{\text{cda}} = \Delta U_{\text{cda}} + W_{\text{cda}} = 28\ \text{J} + 27\ \text{J} = \boxed{55\ \text{J}}$$

(*d*) As found above, $U_{\text{c}} - U_{\text{a}} = \Delta U_{\text{ca}} = -\Delta U_{\text{ac}} = \boxed{28\ \text{J}}$.

(*e*) $U_{\rm d} - U_{\rm c} = 12\ \text{J} \quad \rightarrow \quad U_{\rm d} = U_{\rm c} + 12\ \text{J} \quad \rightarrow$

$$\Delta U_{\rm da} = U_{\rm a} - U_{\rm d} = U_{\rm a} - U_{\rm c} - 12\ \text{J} = \Delta U_{\rm ca} - 12\ \text{J} = 28\ \text{J} - 12\ \text{J} = 16\ \text{J}$$

Use the first law of thermodynamics to find $Q_{\rm da}$.

$$\Delta U_{\rm da} = Q_{\rm da} - W_{\rm da} \rightarrow Q_{\rm da} = \Delta U_{\rm da} + W_{\rm da} = 16\ \text{J} + 0 = \boxed{16\ \text{J}}$$

14. In Example 15–7, the total energy transformed was 1.15×10^{7} J. We will subtract the energy for 1 hour of desk work and add the energy for 1 hour of running.

$$\text{Energy} = 1.15\times10^{7}\ \text{J} + [-115\ \text{J/s} + 1150\ \text{J/s}](3600\ \text{s/h}) = \boxed{1.52\times10^{7}\ \text{J}} \approx 3640\ \text{Cal}$$

15. Follow the pattern set in Example 15–7. Find the average rate by dividing the total energy for the day by 24 hours.

$$\text{Avg. energy} = \left[\begin{array}{l}(8.0\ \text{h})(70\ \text{J/s}) + (6.0\ \text{h})(115\ \text{J/s}) + (6.0\ \text{h})(230\ \text{J/s}) \\ +(2.0\ \text{h})(115\ \text{J/s}) + (1.5\ \text{h})(460\ \text{J/s}) + (0.5\ \text{h})(1150\ \text{J/s})\end{array}\right] \Big/ 24\ \text{h} = 172\ \text{W} \approx \boxed{170\ \text{W}}$$

16. From Table 15–2, the change in metabolic rate if one hour of sleeping is exchanged for light activity is an addition of $230\ \text{watts} - 70\ \text{watts} = 160\ \text{watts}$. Note that this increased rate is only applicable for one hour per day.

$$\left(160\frac{\text{J}}{\text{s}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)\left(\frac{1\ \text{h}}{\text{day}}\right)\left(\frac{365\ \text{day}}{1\ \text{y}}\right)\left(\frac{1\ \text{kg fat}}{4\times10^{7}\ \text{J}}\right) = 5.256\ \text{kg} \approx \boxed{5.3\ \text{kg}}\left(\frac{2.20\ \text{lb}}{1\ \text{kg}}\right) = \boxed{12\ \text{lb}}$$

17. (*a*) The person runs seven times per week, 30 minutes each time. We use Table 15–2.

$$\left(1150\frac{\text{J}}{\text{s}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{30\ \text{min}}{\text{run}}\right)\left(\frac{7\ \text{runs}}{1\ \text{week}}\right) = 1.449\times10^{7}\ \text{J/week} \approx \boxed{1.4\times10^{7}\ \text{J}}\ \text{in one week}$$

(*b*) Convert the energy used to run from joules to Calories.

$$1.449\times10^{7}\ \text{J}\left(\frac{1\ \text{Cal}}{4.186\times10^{3}\ \text{J}}\right) = 3462\ \text{Cal} \approx \boxed{3500\ \text{Cal}}$$

18. The efficiency of a heat engine is given by Eq. 15–4a.

$$e = \frac{W}{Q_{\rm H}} = \frac{W}{W + Q_{\rm L}} = \frac{2600\ \text{J}}{2600\ \text{J} + 8200\ \text{J}} = 0.24 = \boxed{24\%}$$

19. The maximum (or Carnot) efficiency is given by Eq. 15–5, with temperatures in kelvins.

$$e = 1 - \frac{T_{\rm L}}{T_{\rm H}} = 1 - \frac{(345+273)\ \text{K}}{(560+273)\ \text{K}} = 0.258 = \boxed{25.8\%}$$

We assume that both temperatures are measured to the same precision—the nearest degree.

$\boxed{20.}$ The Carnot efficiency is given by Eq. 15–5, with temperatures in kelvins.

$$e = 1 - \frac{T_{\rm L}}{T_{\rm H}} \quad \rightarrow \quad T_{\rm H} = \frac{T_{\rm L}}{1-e} = \frac{(230+273)\ \text{K}}{1-0.34} = 762\ \text{K} = 489^\circ\text{C} \approx \boxed{490^\circ\text{C}}$$

21. The efficiency of a heat engine is given by Eq. 15–4a.

$$e=\frac{W}{Q_{\mathrm{H}}}=\frac{9200\ \mathrm{J}}{(25.0\ \mathrm{kcal})(4186\ \mathrm{J/kcal})}=0.0879\approx\boxed{8.8\ \%}$$

22. Calculate the Carnot efficiency for the given temperatures, using Eq. 15–5.

$$e_{\mathrm{ideal}}=1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}=1-\frac{77\ \mathrm{K}}{293\ \mathrm{K}}=0.7372\approx\boxed{74\%}$$

23. A 10°C decrease in the low-temperature reservoir will give a greater improvement in the efficiency of a Carnot engine. By definition, T_{L} is less than T_{H}, so a 10°C change will be a larger percentage change in T_{L} than in T_{H}, yielding a greater improvement in efficiency. As an example, we use the values from Problem 22 above.

$$e_{\substack{\mathrm{lower}\\T_{\mathrm{L}}}}=1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}=1-\frac{67\ \mathrm{K}}{293\ \mathrm{K}}=0.7713\ \text{compared to}\ 0.7372$$

$$e_{\substack{\mathrm{higher}\\T_{\mathrm{H}}}}=1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}=1-\frac{77\ \mathrm{K}}{303\ \mathrm{K}}=0.7458\ \text{compared to}\ 0.7372$$

We see that the decrease in the lower temperature was more effective. Here is a more rigorous proof. Note that we never multiply by a negative value, so the original ordering of $e_{\substack{\mathrm{lower}\\T_{\mathrm{L}}}}$ on the left of the comparison and $e_{\substack{\mathrm{higher}\\T_{\mathrm{H}}}}$ on the right of the comparison is preserved. We use the sign $<=>$ to mean "compare to."

$$e_{\substack{\mathrm{lower}\\T_{\mathrm{L}}}}=1-\frac{T_{\mathrm{L}}-T_0}{T_{\mathrm{H}}};\quad e_{\substack{\mathrm{higher}\\T_{\mathrm{H}}}}=1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}+T_0}$$

$$1-\frac{T_{\mathrm{L}}-T_0}{T_{\mathrm{H}}}<=>1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}+T_0};\quad -\frac{T_{\mathrm{L}}-T_0}{T_{\mathrm{H}}}<=>-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}+T_0};\quad \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}+T_0}<=>\frac{T_{\mathrm{L}}-T_0}{T_{\mathrm{H}}};$$

$$T_{\mathrm{L}}T_{\mathrm{H}}<=>(T_{\mathrm{L}}-T_0)(T_{\mathrm{H}}+T_0);\quad T_0T_{\mathrm{H}}+T_0T_0<=>T_{\mathrm{L}}T_0;\quad T_{\mathrm{H}}+T_0<=>T_{\mathrm{L}}$$

Since the left-hand side of this last expression is larger than the right-hand side, $e_{\substack{\mathrm{lower}\\T_{\mathrm{L}}}}>e_{\substack{\mathrm{higher}\\T_{\mathrm{H}}}}$. Thus in general, $\boxed{\text{a change in the low-temperature reservoir has a larger effect}}$ on the efficiency than the same change in the high-temperature reservoir.

24. The efficiency of a heat engine is given by Eq. 15–4a.

$$e=\frac{W}{Q_{\mathrm{H}}}=\frac{W}{W+Q_{\mathrm{L}}}\quad\rightarrow\quad Q_{\mathrm{L}}=W(1/e-1)\quad\rightarrow$$

$$Q_{\mathrm{L}}/t=W/t(1/e-1)=(580\ \mathrm{MW})(1/0.32-1)=1232\ \mathrm{MW}\approx\boxed{1200\ \mathrm{MW}}$$

25. The maximum (or Carnot) efficiency is given by Eq. 15–5, with temperatures in kelvins.

$$e=1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}=1-\frac{(330+273)\ \mathrm{K}}{(660+273)\ \mathrm{K}}=0.3537$$

Thus the total power generated can be found as follows.

$$\text{Actual power}=(\text{Total power})(\text{max. eff.})(\text{operating eff.})\quad\rightarrow$$

$$\text{Total power} = \frac{\text{Actual power}}{(\text{max. eff.})(\text{operating eff.})} = \frac{1.4\text{ GW}}{(0.3537)(0.65)} = 6.089\text{ GW}$$

$$\text{Exhaust power} = \text{Total power} - \text{Actual power} = 6.089\text{ GW} - 1.4\text{ GW} = 4.689\text{ GW}$$

$$= (4.689\times10^{9}\text{ J/s})(3600\text{ s/h}) = 1.688\times10^{13}\text{ J/h} \approx \boxed{1.7\times10^{13}\text{ J/h}}$$

26. Find the intake temperature from the original Carnot efficiency, and then recalculate the exhaust temperature for the new Carnot efficiency, using the same intake temperature.

$$e_1 = 1 - \frac{T_{L1}}{T_H} \rightarrow T_H = \frac{T_{L1}}{1-e_1} = \frac{(340+273)\text{ K}}{1-0.36} = 958\text{ K}$$

$$e_2 = 1 - \frac{T_{L2}}{T_H} \rightarrow T_{L2} = T_H(1-e_2) = (958\text{ K})(1-0.42) = 556\text{ K} = 283^\circ\text{C} \approx \boxed{280^\circ\text{C}}$$

27. This is a perfect Carnot engine, so its efficiency is given by Eqs. 15–4a and 15–5. Use these two expressions to solve for the rate of heat output.

$$e = 1 - \frac{T_L}{T_H} = 1 - \frac{(45+273)\text{K}}{(210+273)\text{K}} = 0.3416 \quad e = \frac{W}{Q_H} = \frac{W}{W+Q_L} \rightarrow Q_L = W(1/e-1)$$

$$Q_L/t = W/t(1/e-1) = (910\text{ W})(1/0.3416-1) = 1754\text{ W} \approx \boxed{1800\text{ W}}$$

28. (*a*) The work done per second is found from the engine specifications.

$$\frac{W}{t} = \left(180\frac{\text{J}}{\text{cycle}\cdot\text{cylinder}}\right)(4\text{ cylinders})\left(25\frac{\text{cycles}}{\text{s}}\right) = \boxed{1.8\times10^{4}\text{ J/s}}$$

(*b*) The efficiency is given by Eq. 15–4a.

$$e = \frac{W}{Q_H} = \rightarrow Q_H = \frac{W}{e} \rightarrow Q_H/t = \frac{W/t}{e} = \frac{1.8\times10^{4}\text{ J/s}}{0.22} = 8.182\times10^{4}\text{ J/s} \approx \boxed{8.2\times10^{4}\text{ J/s}}$$

(*c*) Divide the energy in a gallon of gasoline by the rate at which that energy gets used.

$$\frac{130\times10^{6}\text{ J/gal}}{8.182\times10^{4}\text{ J/s}} = 1589\text{ s} \approx \boxed{26\text{ min}}$$

29. This is a perfect Carnot engine, so its efficiency is given by Eqs. 15–4a and 15–5. Equate these two expressions for the efficiency.

$$e = 1 - T_L/T_H = W/Q_H \rightarrow$$

$$T_L = T_H\left(1-\frac{W}{Q_H}\right) = T_H\left(1-\frac{W/t}{Q_H/t}\right) = [(520+273)\text{ K}]\left(1-\frac{5.2\times10^{5}\text{ J/s}}{(950\text{ kcal/s})(4186\text{ J/kcal})}\right)$$

$$= 689\text{ K} = 416^\circ\text{C} \approx \boxed{420^\circ\text{C}}$$

30. Find the exhaust temperature from the original Carnot efficiency, and then recalculate the intake temperature for the new Carnot efficiency, using the same exhaust temperature. Use Eq. 15–5.

$$e_1 = 1 - T_L/T_{H1} \rightarrow T_L = T_{H1}(1-e) = [(580+273)\text{ K}](1-0.22) = 665.3\text{ K}$$

$$e_2 = 1 - T_L/T_{H2} \rightarrow T_{H2} = \frac{T_L}{1-e_2} = \frac{665.3\text{ K}}{1-0.42} = 1147\text{ K} = 874^\circ\text{C} \approx \boxed{870^\circ\text{C}}$$

31. We calculate both the energy per second (power) delivered by the gasoline and the energy per second (power) needed to overcome the drag forces. The ratio of these is the efficiency, as given by Eq. 15–4a.

$$\frac{W}{t}=P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}}=Fv=(350\text{ N})\left(55\frac{\text{mi}}{\text{h}}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)=8604\text{ W}$$

$$\frac{Q_{\text{H}}}{t}=P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}}=\left(3.2\times10^{7}\ \frac{\text{J}}{\text{L}}\right)\left(\frac{3.8\text{ L}}{1\text{ gal}}\right)\left(\frac{1\text{ gal}}{32\text{ mi}}\right)\left(55\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)=58056\text{ W}$$

$$e=\frac{W}{Q_{\text{H}}}=\frac{P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}}}{P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}}}=\frac{8604\text{ W}}{58056\text{ W}}=0.148\approx\boxed{0.15}$$

32. The ideal coefficient of performance is given by Eq. 15–6c.

$$\text{COP}_{\text{ideal}}=\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}=\frac{(273+2.5)\text{ K}}{(22-2.5)\text{ K}}=14.13\approx\boxed{14}$$

33. The coefficient of performance for a refrigerator is given by Eq. 15–6c, using absolute temperatures.

$$\text{COP}=\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}=\frac{(-8+273)\text{ K}}{(33+273)\text{ K}-(-8+273)\text{ K}}=6.463\approx\boxed{6.5}$$

34. The coefficient of performance for a refrigerator is given by Eq. 15–6c, using absolute temperatures.

$$\text{COP}=\frac{T_{\text{L}}}{T_{\text{H}}-T_{\text{L}}}\quad\rightarrow\quad T_{\text{L}}=T_{\text{H}}\left(\frac{\text{COP}}{1+\text{COP}}\right)=[(22+273)\text{ K}]\left(\frac{7.0}{8.0}\right)=258.1\text{ K}=-14.9^{\circ}\text{C}\approx\boxed{-15^{\circ}\text{C}}$$

35. We initially assume a COP of 3.0. For a heat pump the COP is given by Eq. 15–7.

(*a*) $$\text{COP}=\frac{Q_{\text{H}}}{W}\quad\rightarrow\quad W=\frac{Q_{\text{H}}}{\text{COP}}=\frac{3100\text{ J}}{3.0}=1033\text{ J}\approx\boxed{1.0\times10^{3}\text{ J}}$$

(*b*) The calculation doesn't depend on the outdoor temperature, so $W=\boxed{1.0\times10^{3}\text{ J}}$.

(*c*) The efficiency of a perfect Carnot engine is given by Eqs. 15–4a and 15–5. Equate these two expressions to solve for the work required.

$$e=1-\frac{T_{\text{L}}}{T_{\text{H}}};\quad e=\frac{W}{Q_{\text{H}}}\quad\rightarrow\quad 1-\frac{T_{\text{L}}}{T_{\text{H}}}=\frac{W}{Q_{\text{H}}}\quad\rightarrow\quad W=Q_{\text{H}}\left(1-\frac{T_{\text{L}}}{T_{\text{H}}}\right)$$

$$W=Q_{\text{H}}\left(1-\frac{T_{\text{L}}}{T_{\text{H}}}\right)=3100\text{ J}\left(1-\frac{0+273}{22+273}\right)=\boxed{230\text{ J}}$$

$$W=Q_{\text{H}}\left(1-\frac{T_{\text{L}}}{T_{\text{H}}}\right)=3100\text{ J}\left(1-\frac{-15+273}{22+273}\right)=\boxed{390\text{ J}}$$

36. The COP for an ideal heat pump is given by Eq. 15–7.

(*a*) $$\text{COP}=\frac{Q_{\text{H}}}{W}=\frac{Q_{\text{H}}}{Q_{\text{H}}-Q_{\text{L}}}=\frac{T_{\text{H}}}{T_{\text{H}}-T_{\text{L}}}=\frac{(24+273)\text{ K}}{18\text{ K}}=16.5\approx\boxed{17}$$

(*b*) $\text{COP} = \dfrac{Q_\text{H}}{W} \quad \rightarrow \quad Q_\text{H} = (W/t)(t)(\text{COP}) = (1200\text{ W})(3600\text{ s})(16.5) = 7.128\times10^7\text{ J} \approx \boxed{7.1\times10^7\text{ J}}$

37. The coefficient of performance for a refrigerator is given by Eq. 15–6a and is the heat removed from the low-temperature area divided by the work done to remove the heat. In this case, the heat removed is the latent heat released by the freezing ice, and the work done is 1.2 kW times the elapsed time. The mass of water frozen is its density times its volume.

$$\text{COP} = \frac{Q_\text{L}}{W} = \frac{mL_\text{F}}{W} = \frac{\rho V L_\text{F}}{Pt} \quad \rightarrow$$

$$V = \frac{(\text{COP})Pt}{\rho L_\text{F}} = \frac{(6.0)(1200\text{ W})(3600\text{ s})}{(1.0\times10^3\text{ kg/m}^3)(3.33\times10^5\text{ J/kg})} = 0.0778\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) \approx \boxed{78\text{ L}}$$

38. The \$2000 worth of heat provided by the electric heater is the same amount of heat that the heat pump would need to provide, so this Q_H costs \$2,000. The amount of energy required to run the heat pump to deliver that same amount of heat is found from the coefficient of performance.

$$\text{COP} = \frac{Q_\text{H}}{W} \quad \rightarrow \quad W = \frac{Q_\text{H}}{\text{COP}} = \frac{Q_\text{H}}{2.9}$$

So if the cost for Q_H is divided by 2.9, we get the cost of running the heat pump to deliver the needed heat. Subtract that from the total cost to get the savings.

$$\text{Savings} = \$2{,}000 - \frac{\$2{,}000}{2.9} \approx \boxed{\$1310}$$

Divide the cost of the heat pump by the annual savings to find the break-even time.

$$\frac{\$15{,}000}{\$1310/\text{year}} = 11.45\text{ years} \approx \boxed{11\text{ years}}$$

The total savings over 20 years is the savings in heating costs minus the price of the heat pump.

$$\text{Total savings} = (\$1310/\text{year})(20\text{ years}) - \$15{,}000 = \$11{,}200 \approx \boxed{\$11{,}000}$$

39. Heat energy is taken away from the water, so the change in entropy will be negative. The heat transfer is the mass of the steam times the latent heat of vaporization.

$$\Delta S = \frac{Q}{T} = -\frac{mL_\text{V}}{T} = -\frac{(0.320\text{ kg})(22.6\times10^5\text{ J/kg})}{(273+100)\text{ K}} = -1939\text{ J/K} \approx \boxed{-1900\text{ J/K}}$$

40. The heat added to the water is found from Eq. 14–2, $\Delta Q = mc\Delta T$. Use the average temperature of 50°C in Eq. 15–8 for entropy.

$$\Delta S = \frac{Q}{T} = \frac{mc\Delta T}{T} = \frac{(1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(100\text{ C°})}{(273+50)\text{ K}} = 1296\text{ J/K} \approx \boxed{1300\text{ J/K}}$$

$\boxed{41.}$ Heat energy is taken away from the water, so the change in entropy will be negative. The heat taken away from the water is found from $\Delta Q = mL_\text{F}$. Note that 1.00 m^3 of water has a mass of $1.00\times10^3\text{ kg}$. Use Eq. 15–8 to calculate the entropy change.

$$\Delta S = \frac{Q}{T} = -\frac{mL_\text{fusion}}{T} = -\frac{(1.00\times10^3\text{ kg})(3.33\times10^5\text{ J/kg})}{273\text{ K}} = \boxed{-1.22\times10^6\text{ J/K}}$$

42. Energy has been made "unavailable" in the frictional stopping of the sliding box. We take that "lost" kinetic energy as the heat term of the entropy calculation, Eq. 15–8.

$$\Delta S = \frac{Q}{T} = \frac{\frac{1}{2}mv_0^2}{T} = \frac{\frac{1}{2}(5.8\text{ kg})(4.0\text{ m/s})^2}{293\text{ K}} = 0.1584\text{ J/K} \approx \boxed{0.16\text{ J/K}}$$

Since this is a decrease in "availability," the entropy of the universe has increased.

43. There are three terms of entropy to consider. First, there is a loss of entropy from the water for the freezing process, ΔS_1. Second, there is a loss of entropy from that newly formed ice as it cools to $-8.0°\text{C}$, ΔS_2. That process has an "average" temperature of $-4.0°\text{C}$. Finally, there is a gain of entropy by the "great deal of ice," ΔS_3, as the heat lost from the original mass of water in steps 1 and 2 goes into that great deal of ice. Since it is a large quantity of ice, we assume that its temperature does not change during the processes. The density of water is 1000 kg per cubic meter.

$$\Delta S_1 = \frac{Q_1}{T_1} = -\frac{mL_\text{F}}{T_1} = -\frac{(1.00\times10^3\text{ kg})(3.33\times10^5\text{ J/kg})}{273\text{ K}} = -1.2198\times10^6\text{ J/K}$$

$$\Delta S_2 = \frac{Q_2}{T_2} = -\frac{mc_\text{ice}\Delta T_2}{T_2} = -\frac{(1.00\times10^3\text{ kg})(2100\text{ J/kg}\cdot\text{C°})(8.0\text{ C°})}{(-4+273)\text{ K}} = -6.2453\times10^4\text{ J/K}$$

$$\Delta S_3 = \frac{Q_3}{T_3} = \frac{-Q_1-Q_2}{T_3} = \frac{mL_\text{F}+mc_\text{ice}\Delta T_2}{T_3}$$

$$= \frac{(1.00\times10^3\text{ kg})[(3.33\times10^5\text{ J/kg})+(2100\text{ J/kg}\cdot\text{C°})(8\text{ C°})]}{(-8+273)\text{ K}} = 1.32\times10^6\text{ J/K}$$

44. The same amount of heat that leaves the high-temperature heat source enters the low-temperature body of water.

$$\Delta S = \Delta S_1 + \Delta S_2 = -\frac{Q}{T_\text{high}} + \frac{Q}{T_\text{low}} = Q\left(\frac{1}{T_\text{low}} - \frac{1}{T_\text{high}}\right) \rightarrow$$

$$\frac{\Delta S}{t} = \frac{Q}{t}\left(\frac{1}{T_\text{low}} - \frac{1}{T_\text{high}}\right) = (8.40\text{ cal/s})\left(\frac{4.186\text{ J}}{1\text{ cal}}\right)\left(\frac{1}{(22+273)\text{ K}} - \frac{1}{(225+273)\text{ K}}\right)$$

$$= \boxed{4.86\times10^{-2}\ \frac{\text{J/K}}{\text{s}}}$$

45. The equilibrium temperature is found using calorimetry, from Chapter 14. The heat lost by the aluminum is equal to the heat gained by the water. We assume that the Styrofoam insulates the mixture.

$$m_\text{Al}c_\text{Al}(T_\text{iAl} - T_\text{f}) = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_\text{f} - T_{\text{iH}_2\text{O}}) \rightarrow$$

$$T_\text{f} = \frac{m_\text{Al}c_\text{Al}T_\text{iAl} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}T_{\text{iH}_2\text{O}}}{m_\text{Al}c_\text{Al} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}}$$

$$= \frac{(2.8\text{ kg})(900\text{ J/kg}\cdot\text{C°})(28.5°\text{C}) + (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(20.0°\text{C})}{(2.8\text{ kg})(900\text{ J/kg}\cdot\text{C°}) + (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°})} = 23.19°\text{C}$$

The amount of heat lost by the aluminum, and gained by the water, is

$$Q = m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T_\text{f} - T_{\text{iH}_2\text{O}}) = (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(23.19°\text{C} - 20.0°\text{C}) = 1.335\times10^4\text{ J}$$

In calculating the entropy change, we will need to use estimates for the temperatures of the water and the aluminum since their temperatures are not constant. We will use their average temperatures.

$$T_{\substack{\mathrm{H_2O}\\ \mathrm{avg}}} = (20°\mathrm{C} + 23.19°\mathrm{C})/2 = 21.60°\mathrm{C} \quad T_{\substack{\mathrm{Al}\\ \mathrm{avg}}} = (28.5°\mathrm{C} + 23.19°\mathrm{C})/2 = 25.85°\mathrm{C}$$

$$\Delta S = \Delta S_{\mathrm{Al}} + \Delta S_{\mathrm{H_2O}} = -\frac{Q}{T_{\substack{\mathrm{Al}\\ \mathrm{avg}}}} + \frac{Q}{T_{\substack{\mathrm{H_2O}\\ \mathrm{avg}}}} = (1.335\times10^4\ \mathrm{J})\left(\frac{1}{(21.60+273)\ \mathrm{K}} - \frac{1}{(25.85+273)\ \mathrm{K}}\right)$$

$$= 0.6444\ \mathrm{J/K} = \boxed{0.64\ \mathrm{J/K}}$$

46. Take the energy transfer to use as the initial kinetic energy of the rock, because this energy becomes "unusable" after the collision—it is transferred to the environment. We assume that the rock and the environment are both at temperature T_0.

$$\Delta S = Q/T \quad \rightarrow \quad \boxed{\Delta S = \mathrm{KE}/T_0}$$

47. The same amount of heat that leaves the high-temperature water will enter the low-temperature water. Since the two masses of water are the same, the equilibrium temperature will be the midpoint between the two initial temperatures, $40°\mathrm{C}$. The average temperature of the cool water is $(35°\mathrm{C} + 40°\mathrm{C})/2 = 37.5°\mathrm{C}$, and the average temperature of the warm water is $(45°\mathrm{C} + 40°\mathrm{C})/2 = 42.5°\mathrm{C}$.

$$\Delta S = \Delta S_1 + \Delta S_2 = -\frac{Q}{T_{\mathrm{high}}} + \frac{Q}{T_{\mathrm{low}}} = mc\Delta T\left(\frac{1}{T_{\mathrm{low}}} - \frac{1}{T_{\mathrm{high}}}\right)$$

$$= (1.0\ \mathrm{kg})(4186\ \mathrm{J/kg\cdot C°})(5\ \mathrm{C°})\left(\frac{1}{(37.5+273)\ \mathrm{K}} - \frac{1}{(42.5+273)\ \mathrm{K}}\right) = 1.068\ \mathrm{J/K} \approx \boxed{1.1\ \mathrm{J/K}}$$

48. (*a*) $e_{\mathrm{actual}} = W/Q_{\mathrm{H}} = 550\ \mathrm{J}/2500\ \mathrm{J} = 0.22;\quad e_{\mathrm{ideal}} = 1 - T_{\mathrm{L}}/T_{\mathrm{H}} = 1 - 650\ \mathrm{K}/970\ \mathrm{K} = 0.330$

Thus $e_{\mathrm{actual}}/e_{\mathrm{ideal}} = 0.220/0.330 = 0.667 \approx \boxed{67\ \%\ \text{of ideal}}$.

(*b*) The heat reservoirs do not change temperature during the operation of the engine. There is an entropy loss from the input reservoir, because it loses heat, and an entropy gain for the output reservoir, because it gains heat. Note that $Q_{\mathrm{L}} = Q_{\mathrm{H}} - W = 2500\ \mathrm{J} - 550\ \mathrm{J} = 1950\ \mathrm{J}$.

$$\Delta S = \Delta S_{\mathrm{input}} + \Delta S_{\mathrm{output}} = -\frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{L}}}{T_{\mathrm{L}}} = -\frac{2500\ \mathrm{J}}{970\ \mathrm{K}} + \frac{1950\ \mathrm{J}}{650\ \mathrm{K}} = \boxed{0.42\ \mathrm{J/K}}$$

(*c*) For the Carnot engine, the exhaust energy will be $Q_{\mathrm{L}} = Q_{\mathrm{H}}(1 - e_{\mathrm{Carnot}}) = Q_{\mathrm{H}}T_{\mathrm{L}}/T_{\mathrm{H}}$.

$$\Delta S = \Delta S_{\mathrm{input}} + \Delta S_{\mathrm{output}} = -\frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{L}}}{T_{\mathrm{L}}} = -\frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{H}}T_{\mathrm{L}}/T_{\mathrm{H}}}{T_{\mathrm{L}}} = -\frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} + \frac{Q_{\mathrm{H}}}{T_{\mathrm{H}}} = \boxed{0}$$

A numeric calculation might give a very small number due to not keeping all digits in the calculation.

49. When throwing two dice, there are 36 possible microstates.

(*a*) The possible microstates that give a total of 4 are (1)(3), (2)(2), and (3)(1). Thus the probability of getting a 5 is $3/36 = \boxed{1/12}$.

(*b*) The possible microstates that give a total of 10 are (4)(6), (5)(5), and (6)(4). Thus the probability of getting a 10 is $3/36 = \boxed{1/12}$.

50. From the table below, we see that there are a total of $2^6 = 64$ microstates.

Macrostate	Possible Micostates (H = heads, T = tails)						Number of Microstates
6 heads, 0 tails	H H H H H H						1
5 heads, 1 tails	H H H H H T	H H H H T H	H H H T H H	H H T H H H	H T H H H H	T H H H H H	6
4 heads, 2 tails	H H H H T T	H H H T H T	H H T H H T	H T H H H T	T H H H H T		15
	H H H T T H	H H T H T H	H T H H T H	T H H H T H	HHTTH H		
	H T H T H H	T H H T H H	H T T H H H	T H T H H H	T T H H H H		
3 heads, 3 tails	H H H T T T	H H T H T T	H T H H T T	T H H H T T	H H T T H T		20
	H T H T H T	T H H T H T	H T T H H T	T H T H H T	T T H H H T		
	T T T H T H	T T H T H H	T H T T H H	H T T T H H	T T H H T H		
	T H T H T H	H T T H T H	T H H T T H	H T H T T H	H H T T TH		
2 heads, 4 tails	T T T T H H	T T T H T H	T T H T T H	T H T T T H	H T T T T H		15
	T T T H H T	T T H T H T	T H T T H T	H T T T H T	T T H H T T		
	T H T H T T	H T T H T T	T H H T T T	H T H T T T	H H T T T T		
1 heads, 5 tails	T T T T T H	T T T T H T	T T T H T T	T T H T T T	T H T T T T	H T T T T T	6
0 heads, 6 tails	T T T T T T						1

(*a*) The probability of obtaining three heads and three tails is $\boxed{20/64}$ or $\boxed{5/16}$.

(*b*) The probability of obtaining six heads is $\boxed{1/64}$.

51. (*a*) From the table below, we see that there are 10 macrostates and a total of 27 microstates.

Macrostate	Macrostate (r = red, o = orange, g = green)			Number of Microstates
3 red, 0 orange, 10 0 green	r r r			1
2 red, 1 orange, 10 0 green	r r o	r o r	o r r	3
2 red, 0 orange, 10 1 green	r r g	r g r	g r r	3
1 red, 2 orange, 10 0 green	r o o	o r o	o o r	3
1 red, 0 orange, 10 2 green	r g g	g r g	g g r	3
1 red, 1 orange, 10 1 green	r o g o g r	r g o g r o	o r g g o r	6
0 red, 3 orange, 10 0 green	o o o			1
0 red, 2 orange, 10 1 green	g o o	o g o	o o g	3
0 red, 1 orange, 10 2 green	o g g	g o g	g g o	3
0 red, 0 orange, 10 3 green	g g g			1

(*b*) The probability of obtaining all 3 beans red is $\boxed{1/27}$.

(*c*) The probability of obtaining 2 greens and 1 orange is $\boxed{3/27}$ or $\boxed{1/19}$.

52. A macrostate is a set of five cards from the deck, as given in the problem. For example, four aces and a king is a macrostate. Two jacks, two queens, and an ace is a macrostate. A microstate is a specific set of cards that meets the criterion of a certain macrostate. For example, the set (ace of spades, ace of clubs, ace of hearts, ace of diamonds, king of spades) is a microstate of the macrostate of four aces and a king. The problem then is asking for the relative number of microstates for the four given macrostates.

(*a*) There are only four microstates for this macrostate, corresponding to the particular suit to which the king belongs.

(*b*) Since every card is specified, there is only one microstate for this macrostate.

(*c*) There are six possible jack pairs (spade/club, spade/heart, spade/diamond, club/heart, club/diamond, and heart/diamond), six possible queen pairs, and four possible aces, so there are $6\times6\times4=144$ card combinations, or 144 microstates for this macrostate.

(*d*) There are 52 possibilities for the first card, 48 possibilities for the second card, and so on. It is apparent that there are many more microstates for this macrostate than for any of the other listed macrostates.

Thus in order of increasing probability, we have $\boxed{(b), (a), (c), (d)}$.

53. The required area is $\left(24\dfrac{10^3\ \text{W}\cdot\text{h}}{\text{day}}\right)\left(\dfrac{1\ \text{day}}{9\ \text{h Sun}}\right)\left(\dfrac{1\ \text{m}^2}{40\ \text{W}}\right)=66.7\ \text{m}^2\approx\boxed{70\ \text{m}^2}$. A small house with 1000 ft^2 of floor space and a roof tilted at 30° would have a roof area of $(1000\ \text{ft}^2)\left(\dfrac{1}{\cos 30°}\right)\left(\dfrac{1\ \text{m}}{3.28\ \text{ft}}\right)^2=110\ \text{m}^2$, which is about 50% larger than the area needed, so $\boxed{\text{the cells would fit on the house}}$. But not all parts of the roof would have 9 hours of sunlight, so more than the minimum number of cells would be needed.

54. (*a*) Assume that there are no dissipative forces present, so the energy required to pump the water to the lake is just the gravitational potential energy of the water.

$$\text{PE}_{\text{grav}}=mgh=(1.00\times10^5\ \text{kg/s})(10.0\ \text{h})(9.80\ \text{m/s}^2)(115\ \text{m})=1.127\times10^9\ \text{W}\cdot\text{h}$$
$$\approx\boxed{1.13\times10^6\ \text{kW}\cdot\text{h}}$$

(*b*) $$\frac{(1.127\times10^6\ \text{kW}\cdot\text{h})(0.75)}{14\ \text{h}}=\boxed{6.0\times10^4\ \text{kW}}=60\ \text{MW}$$

55. We assume that the electrical energy comes from the 100% effective conversion of the gravitational potential energy of the water.

$$W=mgh\quad\rightarrow$$
$$P=\frac{W}{t}=\frac{m}{t}gh=\rho\frac{V}{t}gh=(1.00\times10^3\ \text{kg/m}^3)(32\ \text{m}^3/\text{s})(9.80\ \text{m/s}^2)(48\ \text{m})$$
$$=\boxed{1.5\times10^7\ \text{W}}=15\ \text{MW}$$

56. According to the heat figures provided by the inventor, the engine is 67% efficient:

$$e=\frac{W}{Q_{\text{H}}}=\frac{W/t}{Q_{\text{H}}/t}=\frac{2.00\ \text{MW}}{3.00\ \text{MW}}=0.667$$

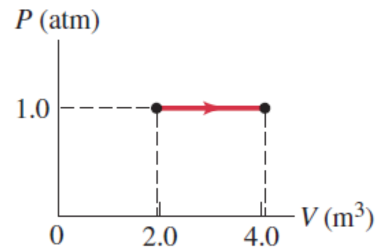
The ideal engine efficiency at the operating temperatures is given by Eq. 15–5.

$$e_{\text{ideal}}=1-\frac{T_{\text{L}}}{T_{\text{H}}}=1-\frac{215\ \text{K}}{425\ \text{K}}=0.494$$

Thus his engine is not possible, even if it were ideal. So $\boxed{\text{yes}}$, there is something fishy about his claim. His engine is even better than ideal, which means that the quoted values cannot be true.

57. (*a*) The work done at constant pressure is Eq. 15–3, $W = P\Delta V$.

$$W = P\Delta V$$
$$= (1.00\text{ atm})(1.01\times10^5\text{ Pa/atm})(4.1\text{ m}^3 - 1.9\text{ m}^3)$$
$$= 2.22\times10^5\text{ J} \approx \boxed{2.2\times10^5\text{ J}}$$



(*b*) Use the first law of thermodynamics, Eq. 15–1.

$$\Delta U = Q - W = 5.80\times10^5\text{ J} - 2.22\times10^5\text{ J} = \boxed{3.6\times10^5\text{ J}}$$

(*c*) See the adjacent graph.

$\boxed{58.}$ The coefficient of performance for an ideal refrigerator is given by Eq. 15–6c, with temperatures in kelvins. Use that expression to find the temperature inside the refrigerator.

$$\text{COP} = \frac{T_L}{T_H - T_L} \quad\rightarrow\quad T_L = T_H\frac{\text{COP}}{1+\text{COP}} = [(32+273)\text{ K}]\frac{4.6}{5.6} = 251\text{ K} = \boxed{-22°\text{C}}$$

59. The minimum value for T_H would occur if the engine were a Carnot engine. We calculate the efficiency of the engine from the given data and use this as a Carnot efficiency to calculate T_H.

$$\frac{W}{t} = P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}} = 7000\text{ W};\quad \frac{Q_H}{t} = P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}} = \left(3.2\times10^7\,\frac{\text{J}}{\text{L}}\right)\left(\frac{1\text{ L}}{17{,}000\text{ m}}\right)\left(\frac{21.8\text{ m}}{1\text{ s}}\right) = 41{,}035\text{ W}$$

$$e = \frac{W}{Q_H} = \frac{P_{\substack{\text{output}\\ \text{(to move}\\ \text{car)}}}}{P_{\substack{\text{input}\\ \text{(from}\\ \text{gasoline)}}}} = \frac{7000\text{ W}}{41{,}035\text{ W}} = 1 - \frac{T_L}{T_H} \quad\rightarrow\quad T_H = \frac{T_L}{(1-e)} = \frac{(273+25)\text{ K}}{\left(1 - \dfrac{7000\text{ W}}{41{,}035\text{ W}}\right)} = 359\text{ K} = \boxed{86°\text{C}}$$

60. (*a*) The heat that must be removed from the water (Q_L) is found in three parts—that from cooling the liquid water to the freezing point, freezing the liquid water, and then cooling the ice to the final temperatures.

$$Q_L = m(c_{\text{liquid}}\Delta T_{\text{liquid}} + L_F + c_{\text{ice}}\Delta T_{\text{ice}})$$
$$= (0.65\text{ kg})\left[\begin{array}{l}(4186\text{ J/kg}\cdot\text{C°})(25\text{ C°}) + (3.33\times10^5\text{ J/kg})\\ +(2100\text{ J/kg}\cdot\text{C°})(17\text{ C°})\end{array}\right] = 3.077\times10^5\text{ J}$$

The Carnot efficiency can be used to find the work done by the refrigerator.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W+Q_L} \quad\rightarrow$$

$$W = Q_L\left(\frac{T_H}{T_L} - 1\right) = (3.077\times10^5\text{ J})\left(\frac{(25+273)\text{ K}}{(-17+273)\text{ K}} - 1\right) = 5.048\times10^4\text{ J} \approx \boxed{5.0\times10^4\text{ J}}$$

(*b*) Use the compressor wattage to calculate the time. The compressor power can be expressed as one-fourth of the nominal power, since it only runs 25% of the time.

$$P = W/t \quad\rightarrow\quad t = W/P = 5.048\times10^4\text{ J}/[(105\text{ W})\,0.25] = 1923\text{ s} \approx \boxed{32\text{ min}}$$

61. (*a*) Calculate the Carnot efficiency for an engine operated between the given temperatures.

$$e_{\text{ideal}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(273+4)\text{ K}}{(273+27)\text{ K}} = 0.077 = \boxed{7.7\%}$$

(*b*) Such an engine might be feasible in spite of the low efficiency because of the large volume of "fuel" (ocean water) available. Ocean water would appear to be an almost inexhaustible source of heat energy.

(*c*) The pumping of water between radically different depths would probably move smaller sea-dwelling creatures from their natural location, perhaps killing them in the transport process. This might affect the food chain of other local sea-dwelling creatures. Mixing the water at different temperatures will also disturb the environment of sea-dwelling creatures. There is a significant dynamic of energy exchange between the ocean and the atmosphere, so any changing of surface temperature water might affect at least the local climate, and perhaps also cause larger-scale climate changes.

62. Heat will enter the freezer due to conductivity, at a rate given by Eq. 14–5. This is the heat that must be removed from the freezer to keep it at a constant temperature, so is the value of Q_L in the equation for the COP, Eq. 15–6a. The work in the COP is the work provided by the cooling motor. The motor must remove the heat in 15% of the time that it takes for the heat to enter the freezer, so that it only runs 15% of the time. To find the minimum power requirement, we assume the freezer is ideal in its operation.

$$\frac{Q_L}{t} = kA\frac{T_H - T_L}{\ell}; \quad \text{COP} = \frac{Q_L}{W} = \frac{Q_L/t}{W/(0.15t)} = \frac{T_L}{T_H - T_L} \quad \rightarrow$$

$$W/t = \frac{Q_L/t}{(0.15)}\left(\frac{T_H - T_L}{T_L}\right) = \frac{kA\frac{T_H - T_L}{\ell}}{(0.15)}\left(\frac{T_H - T_L}{T_L}\right) = \frac{(0.050\ \text{W/m}\cdot\text{K})(8.0\ \text{m}^2)\frac{37\ \text{K}}{0.12\ \text{m}}}{(0.15)}\left(\frac{37\ \text{K}}{258\ \text{K}}\right)$$

$$= 117.9\ \text{W} \approx \boxed{120\ \text{W}} \approx 0.16\,\text{hp}$$

63. We start with Eq. 15–6a for the COP of a refrigerator. The heat involved is the latent heat of fusion for water.

$$\text{COP} = \frac{Q_L}{W} \quad \rightarrow \quad W = \frac{Q_L}{\text{COP}} \quad \rightarrow$$

$$W/t = \frac{Q_L/t}{\text{COP}} = \frac{5\ \text{tons}}{0.18\ \text{COP}_{\text{ideal}}} = \frac{5(909\ \text{kg/d})(3.33\times10^5\ \text{J/kg})}{0.18\left(\frac{273\ \text{K} + 22\ \text{K}}{13\ \text{K}}\right)} = 3.705\times10^8\ \text{J/d}$$

$$\text{cost/h} = (3.705\times10^8\ \text{J/d})\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{kWh}}{3.600\times10^6\ \text{J}}\right)\left(\frac{\$0.10}{\text{kWh}}\right) = \boxed{\$0.43/\text{h}}$$

64. Take the energy transfer to use as the initial kinetic energy of the cars, because this energy becomes "unusable" after the collision—it is transferred to the environment.

$$\Delta S = \frac{Q}{T} = \frac{2(\frac{1}{2}mv_0^2)}{T} = \frac{(1100\ \text{kg})\left[(85\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(20 + 273)\ \text{K}} = 2093\ \text{J/K} \approx \boxed{2100\ \text{J/K}}$$

65. (*a*) The equilibrium temperature is found using calorimetry, from Chapter 14. The heat lost by the water is equal to the heat gained by the aluminum.

$$m_{H_2O}c_{H_2O}(T_{iH_2O} - T_f) = m_{Al}c_{Al}(T_f - T_{iAl}) \quad \rightarrow$$

$$T_f = \frac{m_{Al}c_{Al}T_{iAl} + m_{H_2O}c_{H_2O}T_{iH_2O}}{m_{Al}c_{Al} + m_{H_2O}c_{H_2O}}$$
$$= \frac{(0.11\text{ kg})(900\text{ J/kg}\cdot\text{C°})(35\text{°C}) + (0.15\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(45\text{°C})}{(0.11\text{ kg})(900\text{ J/kg}\cdot\text{C°}) + (0.15\text{ kg})(4186\text{ J/kg}\cdot\text{C°})} = 43.64\text{°C} \approx \boxed{44\text{°C}}$$

(*b*) The amount of heat lost by the aluminum, and gained by the water, is

$$Q = m_{H_2O}c_{H_2O}(T_{i\,H_2O} - T_f) = (0.15\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(45\text{°C} - 43.64\text{°C}) = 853.9\text{ J}$$

In calculating the entropy change, we need to use estimates for the temperatures of the water and the aluminum since their temperatures are not constant. We will use their average temperatures.

$$T_{\underset{\text{avg}}{H_2O}} = (45\text{°C} + 43.64\text{°C})/2 = 44.32\text{°C}; \quad T_{\underset{\text{avg}}{Al}} = (35\text{°C} + 43.64\text{°C})/2 = 39.32\text{°C}$$

$$\Delta S = \Delta S_{Al} + \Delta S_{H_2O} = -\frac{Q}{T_{\underset{\text{avg}}{H_2O}}} + \frac{Q}{T_{\underset{\text{avg}}{Al}}} = (853.9\text{ J})\left(\frac{1}{(39.32+273)\text{ K}} - \frac{1}{(44.32+273)\text{ K}}\right)$$
$$= 0.0431\text{ J/K} \approx \boxed{0.043\text{ J/K}}$$

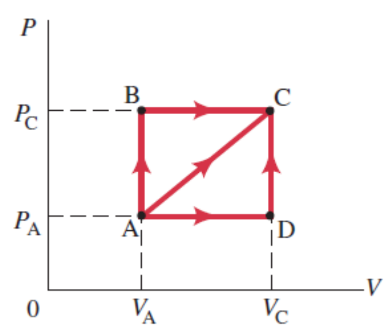
66. The efficiency is given by Eq. 15–4a, $e = W/Q_H = \frac{W/t}{Q_H/t}$, so the input power (Q_H/t) and the useful power (W/t) are needed.

$$W/t = (25\text{ hp})(746\text{ W/hp}) = 1.865\times10^4\text{ J/s}$$
$$Q_H/t = \left(\frac{3.0\times10^4\text{ kcal}}{1\text{ gal}}\right)\left(\frac{1\text{ gal}}{41\text{ km}}\right)\left(\frac{110\text{ km}}{1\text{ h}}\right)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 9.359\times10^4\text{ J/s}$$
$$e = \frac{W/t}{Q_H/t} = \frac{1.865\times10^4\text{ J/s}}{9.359\times10^4\text{ J/s}} = 0.199 \approx \boxed{20\%}$$

67. Find the original intake temperature T_{H1} from the original Carnot efficiency and then recalculate the intake temperature for the new Carnot efficiency, T_{H2}, using the same exhaust temperature. Use Eq. 15–5 for the Carnot efficiency.

$$e_1 = 1 - \frac{T_L}{T_{H1}} \rightarrow T_{H1} = \frac{T_L}{1-e_1} \qquad e_2 = 1 - \frac{T_L}{T_{H2}} \rightarrow T_{H2} = \frac{T_L}{1-e_2}$$
$$T_{H2} - T_{H1} = T_L\left(\frac{1}{1-e_2} - \frac{1}{1-e_1}\right) = (273\text{ K} + 20\text{ K})\left(\frac{1}{1-0.35} - \frac{1}{1-0.25}\right) = 60.10\text{ K} \approx \boxed{60\text{ K}}$$

68. Note that there is NO work done as the gas goes from state A to state B or state D to state C, because there is no volume change. In general, the work done can be found from the area under the *PV* curve representing the process under consideration.



(*a*) $W_{ADC} = \boxed{P_A(V_C - V_A)}$

(*b*) $W_{ABC} = \boxed{P_C(V_C - V_A)}$

(*c*) $W_{AC} = \boxed{\tfrac{1}{2}(P_C + P_A)(V_C - V_A)}$

(Use the formula for the area of a trapezoid.)

69. (*a*) The exhaust heating rate is found from the delivered power and the efficiency. Use the output energy with Eq. 14–2, $Q = mc\Delta T = \rho Vc\Delta T$, to calculate the volume of air that is heated. The efficiency is given by Eq. 15–4a.

$$e = W/Q_H = W/(Q_L + W) \quad \rightarrow \quad Q_L = W(1/e - 1) \quad \rightarrow$$

$$Q_L/t = (W/t)(1/e - 1) = (8.5\times10^8\ \text{W})(1/0.38 - 1) = 1.387\times10^9\ \text{W}$$

$$Q_L = mc\Delta T \quad \rightarrow \quad Q_L/t = \frac{mc\Delta T}{t} = \frac{\rho Vc\Delta T}{t} \quad \rightarrow \quad V/t = \frac{(Q_L/t)}{\rho c\Delta T}$$

The change in air temperature is $7.0\ \text{C}°$. The heated air is at a constant pressure of 1 atm.

$$V/t = \frac{(Q_L/t)t}{\rho c\Delta T} = \frac{(1.387\times10^9\ \text{W})(8.64\times10^4\ \text{s/day})}{(1.3\ \text{kg/m}^3)(1.0\times10^3\ \text{J/kg}\cdot\text{C}°)(7.0\ \text{C}°)}$$

$$= 1.317\times10^{10}\ \text{m}^3/\text{day}\left(\frac{10^{-9}\ \text{km}^3}{1\ \text{m}^3}\right) = 13.17\ \text{km}^3/\text{day} \approx \boxed{13\ \text{km}^3/\text{day}}$$

This could affect the local climate around the power plant.

(*b*) If the air is 180 m thick, find the area by dividing the volume by the thickness.

$$A = \frac{\text{Volume}}{\text{thickness}} = \frac{13.17\ \text{km}^3}{0.18\ \text{km}} = \boxed{73\ \text{km}^2}$$

This would be a square of approximately 8.5 km or 5.3 miles to a side. Thus the local climate for a few miles around the power plant might be heated significantly.

70. (*a*) The exhaust heating rate can be found from the delivered power P and the Carnot efficiency. Then use Eq. 14–2, $Q = mc\Delta T$, to calculate the temperature change of the cooling water. Eqs. 15–4 and 15–5 for efficiency are also used.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{Q_L + W} \quad \rightarrow \quad Q_L = W\frac{T_L}{T_H - T_L} \quad \rightarrow \quad Q_L/t = W/t\frac{T_L}{T_H - T_L} = P\frac{T_L}{T_H - T_L}$$

$$Q_L = mc\Delta T \quad \rightarrow \quad Q_L/t = \frac{m}{t}c\Delta T = \rho\frac{V}{t}c\Delta T$$

Equate the two expressions for Q_L/t, and solve for ΔT.

$$P\frac{T_L}{T_H - T_L} = \rho\frac{V}{t}c\Delta T \quad \rightarrow \quad \Delta T = \frac{P}{\rho\frac{V}{t}c}\left(\frac{T_L}{T_H - T_L}\right)$$

$$= \frac{8.8\times10^8\ \text{W}}{(1.0\times10^3\ \text{kg/m}^3)(37\ \text{m}^3/\text{s})(4186\ \text{J/kg}\cdot\text{C}°)}\frac{285\ \text{K}}{(625\ \text{K} - 285\ \text{K})} = 4.763\ \text{K} = \boxed{4.8\ \text{C}°}$$

(*b*) The addition of heat per kilogram for the downstream water is $Q_L/t = c\Delta T$. We use the "average" temperature of the river water for the calculation: $T = T_0 + \frac{1}{2}\Delta T$. Now the entropy increase can be calculated using Eq. 15–8.

$$\Delta S = \frac{Q}{T} = \frac{c\Delta T}{T_0 + \frac{1}{2}\Delta T} = \frac{(4186\ \text{J/kg}\cdot\text{C}°)(4.763\ \text{K})}{[285 + \frac{1}{2}(4.763)]\ \text{K}} = 69.38\ \text{J/kg}\cdot\text{K} \approx \boxed{69\ \text{J/kg}\cdot\text{K}}$$

71. (*a*) Calculate the Carnot efficiency by Eq. 15–5 and compare it to the 15% actual efficiency.

$$e_{\text{Carnot}} = 1 - T_L/T_H = 1 - (85 + 273)\ \text{K}/(495 + 273)\ \text{K} = 0.534 = 53.4\%$$

Thus the engine's relative efficiency is $e_{\text{actual}}/e_{\text{Carnot}} = 0.15/0.534 = 0.281 = \boxed{28\%}$.

(*b*) Take the stated 135 hp as the useful power obtained from the engine. Use the efficiency to calculate the exhaust heat.

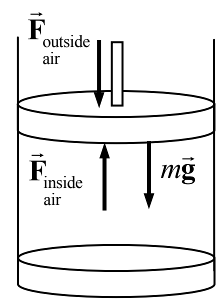
$$P = \frac{W}{t} = (135\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = \boxed{1.01\times10^5\text{ W}}$$

$$e = \frac{W}{Q_H} = \frac{W}{Q_L + W} \quad \rightarrow$$

$$Q_L = W\left(\frac{1}{e} - 1\right) = Pt\left(\frac{1}{e} - 1\right) = (1.01\times10^5\text{ J/s})(1\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1}{0.15} - 1\right)$$

$$= 2.054\times10^9\text{ J} \approx \boxed{2.1\times10^9\text{ J}} = (2.054\times10^9\text{ J})\left(\frac{1\text{ kcal}}{4186\text{ J}}\right) = \boxed{4.9\times10^5\text{ kcal}}$$

72. The net force on the piston must be 0, so the weight of the piston must be equal to the net force exerted by the gas pressures on both sides of the piston. See the free-body diagram.



$$\sum F = F_{\substack{\text{inside}\\\text{air}}} - F_{\substack{\text{outside}\\\text{air}}} - mg = 0 = P_{\text{inside}}A - P_{\text{outside}}A - mg = 0$$

$$P_{\text{inside}} = P_{\text{outside}} + \frac{mg}{A} = (1.0\text{ atm})\left(1.01\times10^5\ \frac{\text{Pa}}{\text{atm}}\right) + \frac{(0.15\text{ kg})(9.8\text{ m/s}^2)}{0.080\text{ m}^2}$$

$$= 1.0102\times10^5\text{ Pa} \approx 1\text{ atm}$$

We see that the weight of the piston is negligible compared to the pressure forces.

When the gas is heated, we assume that the inside pressure does not change. Since the weight of the piston does not change, and the outside air pressure does not change, the inside air pressure cannot change. Thus the expansion is at a constant pressure, so the work done can be calculated. Use this with the first law of thermodynamics to find the heat required for the process.

$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}PV \quad \rightarrow \quad \Delta U = \tfrac{3}{2}P\Delta V = Q - W$$

$$Q = \Delta U + W = \tfrac{3}{2}P\Delta V + P\Delta V = \tfrac{5}{2}P\Delta V = \tfrac{5}{2}PA\Delta y = 2.5(1.01\times10^5\text{ Pa})(0.080\text{ m}^2)(1.0\times10^{-2}\text{ m})$$

$$= 202\text{ J} \approx \boxed{2.0\times10^2\text{ J}}$$

73. (*a*) Multiply the power, the time, and the mass per joule relationship for the fat.

$$(95\text{ J/s})(3600\text{ s/h})(24\text{ h/d})(1.0\text{ kg fat}/3.7\times10^7\text{ J}) = 0.2218\text{ kg/d} \approx \boxed{0.22\text{ kg/d}}$$

(*b*) $1.0\text{ kg}(1\text{ d}/0.2218\text{ kg}) = \boxed{4.5\text{ days}}$

74. (*a*) For each engine, the efficiency is given by $e = 0.65e_{\text{Carnot}}$. Thus

$$e_1 = 0.65e_{C-1} = 0.65\left(1 - \frac{T_{L1}}{T_{H1}}\right) = 0.65\left[1 - \frac{(440+273)\text{ K}}{(750+273)\text{ K}}\right] = 0.197$$

$$e_2 = 0.65e_{C-2} = 0.65\left(1 - \frac{T_{L2}}{T_{H2}}\right) = 0.65\left[1 - \frac{(270+273)\text{ K}}{(415+273)\text{ K}}\right] = 0.137$$

For the first engine, the input heat is from the coal.

$$W_1 = e_1 Q_{H1} = e_1 Q_{coal} \quad \text{and} \quad Q_{L1} = Q_{H1} - W_1 = (1 - e_1) Q_{coal}$$

For the second engine, the input heat is the output heat from the first engine.

$$W_2 = e_2 Q_{H2} = e_2 Q_{L1} = e_2 (1 - e_1) Q_{coal}$$

Add the two work expressions together and solve for Q_{coal}.

$$W_1 + W_2 = e_1 Q_{coal} + e_2 (1 - e_1) Q_{coal} = (e_1 + e_2 - e_1 e_2) Q_{coal}$$

$$Q_{coal} = \frac{W_1 + W_2}{e_1 + e_2 - e_1 e_2} \quad \to \quad Q_{coal}/t = \frac{(W_1 + W_2)/t}{e_1 + e_2 - e_1 e_2}$$

Calculate the rate of coal use from the required rate of input energy, Q_{coal}/t.

$$Q_{coal}/t = \frac{950 \times 10^6 \text{ W}}{0.197 + 0.137 - (0.197)(0.137)} = 3.094 \times 10^9 \text{ J/s}$$

$$(3.094 \times 10^9 \text{ J/s})\left(\frac{1 \text{ kg}}{2.8 \times 10^7 \text{ J}}\right) = 110.5 \text{ kg/s} \approx \boxed{110 \text{ kg/s}}$$

(*b*) The heat exhausted into the water will make the water temperature rise according to Eq. 14–2. The heat exhausted into water is the heat from the coal minus the useful work.

$$Q_{exhaust} = Q_{coal} - W; \quad Q_{exhaust} = m_{H_2O} c_{H_2O} \Delta T_{H_2O} \quad \to \quad m_{H_2O} = \frac{Q_{exhaust}}{c_{H_2O} \Delta T_{H_2O}} = \frac{Q_{coal} - W}{c_{H_2O} \Delta T_{H_2O}}$$

$$\frac{m_{H_2O}}{t} = \frac{(Q_{coal}/t) - (W/t)}{c_{H_2O} \Delta T_{H_2O}} = \frac{(3.094 \times 10^9 \text{ J/s}) - (9.50 \times 10^8) \text{ J/s}}{(4186 \text{ J/kg} \cdot \text{C}°)(4.5 \text{ C}°)} = 1.13 \times 10^5 \text{ kg/s}$$

$$= \left(1.138 \times 10^5 \frac{\text{kg}}{\text{s}}\right)\left(3600 \frac{\text{s}}{\text{h}}\right)\left(\frac{1 \text{ m}^3}{1000 \text{ kg}}\right)\left(\frac{1 \text{ L}}{10^{-3} \text{ m}^3}\right)\left(\frac{1 \text{ gal}}{3.785 \text{ L}}\right) = \boxed{1.1 \times 10^8 \text{ gal/h}}$$

75. According to Table 15–2, riding a bicycle at a racing pace requires an input of 1270 watts. That value is used to calculate the work input to the heat pump. The coefficient of performance equation, Eq. 15–7, is then used to calculate the heat delivered by the heat pump.

$$\text{COP} = \frac{Q_H}{W} \quad \to \quad Q_H = W(\text{COP}) = \left(1270 \frac{\text{J}}{\text{s}}\right)(1800 \text{ s})(2.0) = 4.572 \times 10^6 \text{ J} \approx \boxed{4.6 \times 10^6 \text{ J}}$$

76. The radiant energy that enters the room is the heat to be removed at the low temperature. It can be related to the work necessary to remove it through the ideal efficiency, Eq. 15–5. We then subtract the two rates of doing work to find the savings.

$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W + Q_L} \quad \to \quad W = Q_L\left(\frac{T_H}{T_L} - 1\right) \quad \to \quad W/t = Q_L/t\left(\frac{T_H}{T_L} - 1\right)$$

$$(W/t)_{4800} = (4800 \text{ W})\left(\frac{T_H}{T_L} - 1\right) \quad (W/t)_{500} = (500 \text{ W})\left(\frac{T_H}{T_L} - 1\right)$$

$$(W/t)_{savings} = (W/t)_{4800} - (W/t)_{500} = (4800 \text{ W} - 500 \text{ W})\left(\frac{(273 + 32) \text{ K}}{(273 + 21) \text{ K}} - 1\right) = 160.9 \text{ W} \approx \boxed{160 \text{ W}}$$

77. (*a*) The total rate of adding heat to the house by the heat pump must equal the rate of heat loss by conduction.

$$\frac{Q_L + W}{\Delta t} = (650\ \text{W/C°})(T_{\text{in}} - T_{\text{out}})$$

Since the heat pump is ideal, we have the following from the efficiency.

$$1 - \frac{T_{\text{outside}}}{T_{\text{inside}}} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{Q_L}{Q_L + W} = \frac{W}{Q_L + W} \rightarrow Q_L + W = W\frac{T_{\text{inside}}}{T_{\text{inside}} - T_{\text{outside}}}$$

Combine these two expressions and solve for T_{out}.

$$\frac{Q_L + W}{\Delta t} = (650\ \text{W/C°})(T_{\text{inside}} - T_{\text{outside}}) = \frac{W}{\Delta t}\frac{T_{\text{inside}}}{(T_{\text{inside}} - T_{\text{outside}})} \rightarrow$$

$$(T_{\text{inside}} - T_{\text{outside}})^2 = \frac{W}{\Delta t}\frac{T_{\text{inside}}}{(650\ \text{W/C°})} \rightarrow$$

$$T_{\text{outside}} = T_{\text{inside}} - \sqrt{\frac{W}{\Delta t}\frac{T_{\text{inside}}}{(650\ \text{W/C°})}} = 295\ \text{K} - \sqrt{(1500\ \text{W})\frac{295\ \text{K}}{(650\ \text{W/C°})}} = 269\ \text{K} = \boxed{-4°\text{C}}$$

(*b*) If the outside temperature is $8°\text{C}$, then the rate of heat loss by conduction is found to be $(650\ \text{W/C°})(14\ \text{C°}) = 9100\ \text{W}$. The heat pump must provide this much power to the house in order for the house to stay at a constant temperature. That total power is $(Q_L + W)/\Delta t$. Use this to solve for the rate at which the pump must do work.

$$(Q_L + W)/\Delta t = \frac{W}{\Delta t}\left(\frac{T_{\text{inside}}}{T_{\text{inside}} - T_{\text{outside}}}\right) = 9100\ \text{W} \rightarrow$$

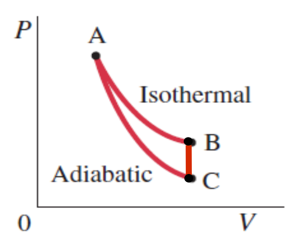
$$\frac{W}{\Delta t} = 9100\ \text{W}\left(\frac{T_{\text{inside}} - T_{\text{outside}}}{T_{\text{inside}}}\right) = 9100\ \text{W}\left(\frac{14\ \text{K}}{295\ \text{K}}\right) = 432\ \text{W}$$

Since the maximum power the pump can provide is 1500 W, the pump must work $\frac{432\ \text{W}}{1500\ \text{W}} = 0.29$ or 29% of the time.

Solutions to Search and Learn Problems

1. If water vapor condenses on the outside of a cold glass of water, the internal energy of the water vapor has decreased, by an amount equal to the heat of vaporization of the water vapor times the mass of water that has condensed. Heat energy left the water vapor, causing it to condense, and heat energy entered the glass of water and the air, causing them to get slightly warmer. No work is done, but heat is exchanged.

2. The first step is an isothermal expansion—the volume increases and the pressure decreases as the temperature stays constant. It is represented by the line from A to B on the diagram. The second step must be at a constant volume since no work is done, so is a vertical line. It is represented by the line from B to C on the diagram. The third step is adiabatic and must be a compression since the work done is negative. It is represented by the line from C to A on the diagram.



3. A Carnot engine is a four-step cycle consisting of, in order, an isothermal expansion, an adiabatic expansion, an isothermal compression, and an adiabatic compression. Each step of the cycle is a reversible process, so the net entropy change for the cycle is zero. Since the processes are reversible, the Carnot engine is the most efficient engine possible. A true Carnot engine is not practical to run and cannot be constructed, but it provides a theoretical limit to the maximum efficiency.

4. (*a*) For sale: Portable air conditioner. Place this air conditioner anywhere in your house and it will remove 500 J of heat per second from the air while using only 100 W of electrical power. No external exhaust vents needed.

   (*b*) For sale: Automobile with 100% efficient engine. All of the gasoline energy goes directly into making the car move. No radiator (cooling system) or exhaust pipe needed.

5. The total entropy change would consist of two parts:

   (i) the energy taken away from the body at the higher body temperature divided by the body temperature; and

   (ii) the energy delivered to the environment at the lower environmental temperature divided by the environmental temperature.

   The person spends 7.0 hours sleeping, 1.0 hour running, 11.0 hours sitting, and 5.0 hours in light activity. An average body temperature is about 310 K, and an average environmental temperature is about 295 K.

$$Q = [(70\text{ W})(7\text{ h}) + (1150\text{ W})(1\text{ h}) + (115\text{ W})(11\text{ h}) + (230\text{ W})(5\text{ h})]\left(\frac{3600\text{ s}}{\text{h}}\right)$$

$$= 1.46\times10^{7}\text{ J}$$

$$\Delta S = \left(\frac{-Q}{T_{\text{body}}} + \frac{Q}{T_{\text{env}}}\right) = (1.46\times10^{7}\text{ J})\left(-\frac{1}{310\text{ K}} + \frac{1}{295\text{ K}}\right) = 2394\text{J/K} \approx \boxed{2400\text{ J/K}}$$

6. The energy necessary to heat the water can be obtained using Eq. 14–2. The specific heat of the water is $4186\text{ J/kg}\cdot\text{C}°$.

$$Q = mc\Delta T = (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C}°)(95°\text{C} - 25°\text{C}) = 2.9302\times10^{5}\text{ J}$$

   The intensity of sunlight at the Earth's surface is $\sim 1000\text{ W/m}^2$. The photovoltaic panel can therefore produce energy at this rate.

$$\frac{Q}{t} = (1000\text{ W/m}^2)(1.5\text{ m}^2)(0.20) = 300\text{ J/s}$$

   Dividing the energy needed to heat the water by the rate at which energy is available will give the time required to heat the water using the photovoltaic cell.

$$t = \frac{Q}{Q/t} = \frac{2.9302\times10^{5}\text{ J}}{300\text{ J/s}} = \boxed{977\text{ s} \approx 16\text{ minutes}}$$

   Using the curved mirror allows all of the energy in the sunlight $[(1000\text{ W/m}^2)(1.5\text{ m}^2) = 1500\text{ J/s}]$ to go into heating the water.

$$t = \frac{Q}{Q/t} = \frac{2.9302\times10^{5}\text{ J}}{1500\text{ J/s}} = \boxed{195\text{ s} \approx 3\text{ minutes}}$$

7. To find the mass of water removed, find the energy that is removed from the low-temperature reservoir from the work input and the Carnot efficiency. Then use the latent heat of vaporization to determine the mass of water from the energy required for the condensation. Note that the heat of vaporization used is that given in Chapter 14 for evaporation at 20°C.

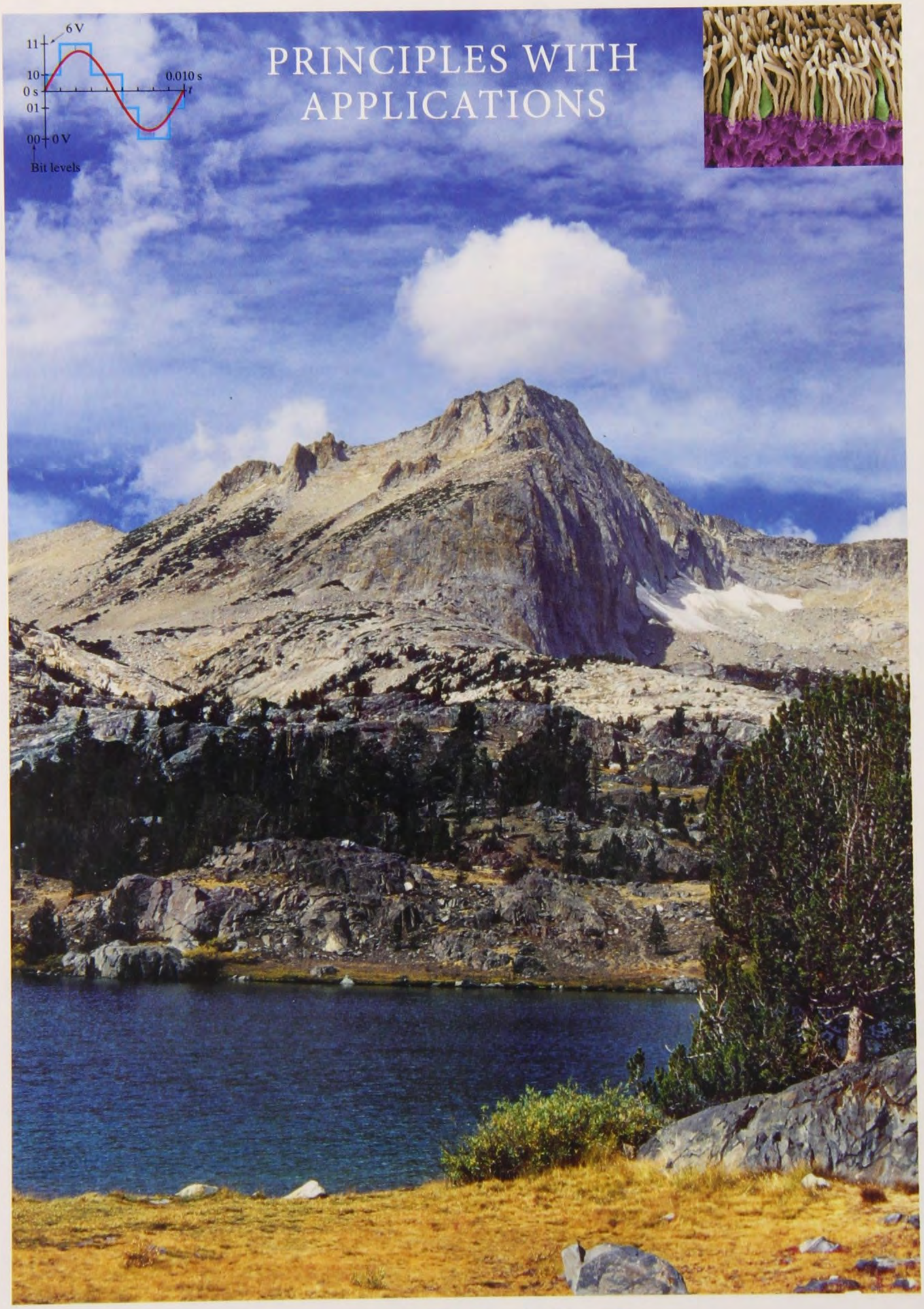
$$e = 1 - \frac{T_L}{T_H} = \frac{W}{Q_H} = \frac{W}{W + Q_L} \quad \rightarrow \quad Q_L = W\frac{T_L}{(T_H - T_L)} = mL_V$$

$$m = \frac{W}{L_V}\frac{T_L}{(T_H - T_L)} = \frac{(600\text{ W})(3600\text{ s})}{(2.45 \times 10^6\text{ J/kg})}\frac{(273 + 8)\text{ K}}{17\text{ K}} = 14.6\text{ kg} \approx \boxed{15\text{ kg}}$$

SEVENTH EDITION

VOLUME II

PHYSICS



PRINCIPLES WITH APPLICATIONS

DOUGLAS C.
GIANCOLI

16

Electric Charge and Electric Field

Responses to Questions

1. A plastic ruler is suspended by a thread and then rubbed with a cloth. As shown in Fig. 16–2, the ruler is negatively charged. Now bring the charged comb close to the ruler. If the ruler is repelled by the comb, then the comb is negatively charged. If the ruler is attracted by the comb, then the comb is positively charged.

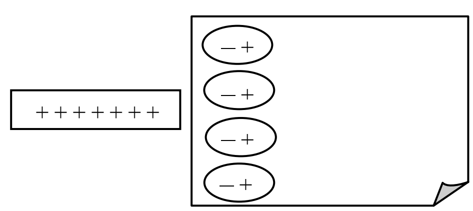
2. The clothing becomes charged as a result of being tossed about in the dryer and rubbing against the dryer sides and other clothes. When you put on the charged object (shirt), it causes charge separation (polarization) within the molecules of your skin (see Fig. 16–9), which results in attraction between the shirt and your skin.

3. Fog or rain droplets tend to form around ions because water is a polar molecule (Fig. 16–4), with a positive region and a negative region. The charge centers on the water molecule will be attracted to the ions or electrons in the air.

4. A plastic ruler that has been rubbed with a cloth is charged. When brought near small pieces of paper, it will cause separation of charge (polarization) in the bits of paper, which will cause the paper to be attracted to the ruler. A small amount of charge is able to create enough electric force to be stronger than gravity. Thus the paper can be lifted.

   On a humid day this is more difficult because the water molecules in the air are polar. Those polar water molecules will be attracted to the ruler and to the separated charge on the bits of paper, neutralizing the charges and thus reducing the attraction.

5. See Fig. 16–9 in the text. The part of the paper near the charged rod becomes polarized—the negatively charged electrons in the paper are attracted to the positively charged rod and move toward it within their molecules. The attraction occurs because the negative charges in the paper are closer to the positive rod than are the positive charges in the paper; therefore, the attraction between the unlike charges is greater than the repulsion between the like charges.



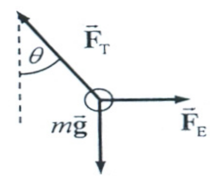
6. The *net charge* on a conductor is the sum of all of the positive and negative charges in the conductor. If a neutral conductor has extra electrons added to it, then the net charge is negative. If a neutral

conductor has electrons removed from it, then the net charge is positive. If a neutral conductor has the same amount of positive and negative charge, then the net charge is zero.

The "free charges" in a conductor are electrons that can move about freely within the material because they are only loosely bound to their atoms. The "free electrons" are also referred to as "conduction electrons." A conductor may have a zero net charge but still have substantial free charges.

7. For each atom in a conductor, only a small number of its electrons are free to move. For example, every atom of copper has 29 electrons, but only 1 or 2 from each atom are free to move easily. Also, not all of the free electrons move. As electrons move toward a region, causing an excess of negative charge, that region then exerts a large repulsive force on other electrons, preventing them from all moving to that same region.

8. The electroscope leaves are connected together at the top. The horizontal component of this tension force balances the electric force of repulsion. The vertical component of the tension force balances the weight of the leaves.



9. The balloon has been charged. The excess charge on the balloon is able to polarize the water molecules in the stream of water, similar to Fig. 16–9. This polarization results in a net attraction of the water toward the balloon, so the water stream curves toward the balloon.

10. Coulomb's law and Newton's law are very similar in form. When expressed in SI units, the magnitude of the constant in Newton's law is very small, while the magnitude of the constant in Coulomb's law is quite large. Newton's law says the gravitational force is proportional to the product of the two masses, while Coulomb's law says the electrical force is proportional to the product of the two charges. Newton's law produces only attractive forces, since there is only one kind of gravitational mass. Coulomb's law produces both attractive and repulsive forces, since there are two kinds of electrical charge.

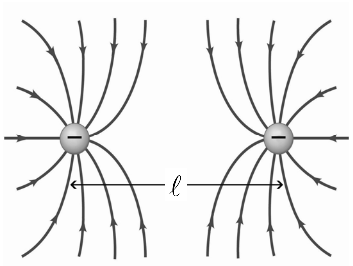
11. Assume that the charged plastic ruler has a negative charge residing on its surface. That charge polarizes the charge in the neutral paper, producing a net attractive force. When the piece of paper then touches the ruler, the paper can get charged by contact with the ruler, gaining a net negative charge. Then, since like charges repel, the paper is repelled by the comb.

12. For the gravitational force, we don't notice it because the force is very weak, due to the very small value of G, the gravitational constant, and the relatively small value of ordinary masses. For the electric force, we don't notice it because ordinary objects are electrically neutral to a very high degree. We notice our weight (the force of gravity) due to the huge mass of the Earth, making a significant gravity force. We notice the electric force when objects have a net static charge (like static cling from the clothes dryer), creating a detectable electric force.

13. The test charge creates its own electric field, The measured electric field is the sum of the original electric field plus the field of the test charge. If the test charge is small, then the field that it causes is small. Therefore, the actual measured electric field is not much different than the original field.

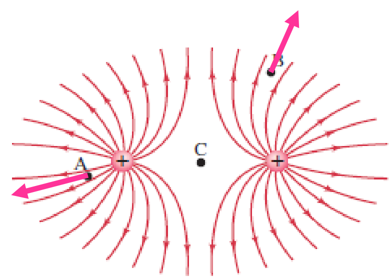
14. A negative test charge could be used. For the purposes of defining directions, the electric field might then be defined as the OPPOSITE of the force on the test charge, divided by the test charge. Equation 16–3 might be changed to $\vec{\mathbf{E}} = -\vec{\mathbf{F}}/q,\ q < 0$.

15.



16. The electric field is strongest to the right of the positive charge (on the line connecting the two charges), because the individual fields from the positive charge and negative charge both are in the same direction (to the right) at that point, so they add to make a stronger field. The electric field is weakest to the left of the positive charge, because the individual fields from the positive charge and negative charge are in opposite directions at that point, so they partially cancel each other. Another indication is the spacing of the field lines. The field lines are closer to each other to the right of the positive charge and farther apart to the left of the positive charge.

17. At point A, the direction of the net force on a positive test charge would be down and to the left, parallel to the nearby electric field lines. At point B, the direction of the net force on a positive test charge would be up and to the right, parallel to the nearby electric field lines. At point C, the net force on a positive test charge would be 0. In order of decreasing field strength, the points would be ordered A, B, C.



$\boxed{18.}$ Electric field lines show the direction of the force on a test charge placed at a given location. The electric force has a unique direction at each point. If two field lines cross, it would indicate that the electric force is pointing in two directions at once, which is not possible.

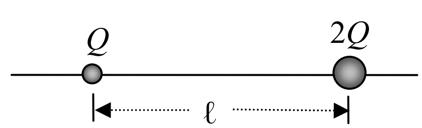
19. From rule 1: A test charge would be either attracted directly toward or repelled directly away from a point charge, depending on the sign of the point charge. So the field lines must be directed either radially toward or radially away from the point charge.

From rule 2: The magnitude of the field due to the point charge only depends on the distance from the point charge. Thus the density of the field lines must be the same at any location around the point charge for a given distance from the point charge.

From rule 3: If the point charge is positive, then the field lines will originate from the location of the point charge. If the point charge is negative, then the field lines will end at the location of the point charge.

Based on rules 1 and 2, the lines are radial and their density is constant for a given distance. This is equivalent to saying that the lines must be symmetrically spaced around the point charge.

20. The two charges are located as shown in the diagram.



(*a*) If the signs of the charges are opposite, then the point on the line where $E = 0$ will lie to the left of Q. In that region the electric fields from the two charges will point in opposite directions, and the point will be closer to the smaller charge.

(*b*) If the two charges have the same sign, then the point on the line where $E = 0$ will lie between the two charges, closer to the smaller charge. In this region, the electric fields from the two charges will point in opposite directions.

21. We assume that there are no other forces (like gravity) acting on the test charge. The direction of the electric field line shows the direction of the force on the test charge. The acceleration is always parallel to the force by Newton's second law, so the acceleration lies along the field line. If the particle is at rest initially and then is released, the initial velocity will also point along the field line, and the particle will start to move along the field line. However, once the particle has a velocity, it will not follow the field line unless the line is straight. The field line gives the direction of the acceleration, or the direction of the change in velocity.

22. The electric flux depends only on the charge enclosed by the gaussian surface (Eq. 16–9), not on the shape of the surface. Φ_E will be the same for the cube as for the sphere.

Responses to MisConceptual Questions

1. (*a*) The two charges have opposite signs, so the force is attractive. Since Q_2 is located on the positive *x* axis relative to Q_1 at the origin, the force on Q_1 will be in the positive *x* direction.

2. (*d*) The signs of the charges are still opposite, so the force remains attractive. However, since Q_2 is now located at the origin with Q_1 on the positive *x* axis, the force on Q_1 will now be toward the origin, or in the negative *x* direction.

3. (*e*) A common misconception is that the object with the greater charge and smaller mass has a greater force of attraction. However, Newton's third law applies here. The force of attraction is the same for both lightning bugs.

4. (*d*) Students sometimes believe that electric fields only exist on electric field lines. This is incorrect. The field lines represent the direction of the electric field in the region of the lines. The magnitude of the field is proportional to the density of the field lines. At point 1 the field lines are closer together than they are at point two. Therefore, the field at point 1 is larger than the field at point 2.

5. (*d*) A common misconception students have is recognizing which object is creating the electric field and which object is interacting with the field. In this question the positive point charge is creating the field. The field from a positive charge always points away from the charge. When a negative charge interacts with an electric field, the force on the negative charge is in the opposite direction from the field. The negative charge experiences a force toward the positive charge.

6. (*a*) An object acquires a positive charge when electrons are removed from the object. Since electrons have mass, as they are removed the mass of the object decreases.

7. (*a*) Students frequently think of the plates as acting like point charges with the electric field increasing as the plates are brought closer together. However, unlike point charges, the electric field lines from each plate are parallel with uniform density. Bringing the plates closer together does not affect the electric field between them.

8. (*b*) The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor, and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.

9. (*d*) Students may equate the lack of the electric force between everyday objects as a sign that the electric force is weaker than other forces in nature. Actually, the electric force between charged particles is much greater than the gravitational force between them. The apparent lack of electric force between everyday objects is because most objects are electrically neutral. That is, most objects have the same number of positive and negative charges in them.

10. (*b*) In a lightning storm, charged particles in the clouds and ground create large electric fields that ionize the air, creating lightning bolts. People are good conductors of electric charge and when located in the electric field can serve as conduits of the lightning bolt. The inside of a metal car acts like a cavity in a conductor, shielding the occupants from the external electric fields. In each of the other options the person remains in the storm's electric field and therefore may be struck by the lightning. If the car is struck by lightning, then the electricity will pass along the exterior of the car, leaving the people inside unharmed.

11. (*a*, *c*, *e*) Lightning will generally travel from clouds to the tallest conductors in the area. If the person is in the middle of a grassy field, near the tallest tree, or on a metal observation town, then he or she is likely to be struck by lightning. A person inside a wooden building or inside a car is somewhat shielded from the lightning.

12. (*d*) The electric field at the fourth corner is the vector sum of the electric fields from the charges at the other three corners. The two positive charges create equal-magnitude electric fields pointing in the positive x direction and in the negative y direction. These add to produce an electric field in the direction of (*d*) with magnitude $\sqrt{2}$ times the magnitude of the electric field from one of the charges. The electric field from the negative charge points in the direction of (*b*). However, since the charge is $\sqrt{2}$ times farther away than the positive charges, the magnitude of the electric field will be smaller than the field from the positive charges. The resulting field will then be in the direction of (*d*).

13. (*e*) A common misconception is that the metal ball must be negatively charged. While a negatively charged ball will be attracted to the positive rod, a neutral conductor will become polarized and also be attracted to the positive rod.

Solutions to Problems

1. Use Coulomb's law (Eq. 16–1) to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.602\times10^{-19}\ \mathrm{C})(26\times1.602\times10^{-19}\ \mathrm{C})}{(1.5\times10^{-12}\ \mathrm{m})^2} = \boxed{2.7\times10^{-3}\ \mathrm{N}}$$

2. Use the charge per electron to find the number of electrons.

$$(-48.0\times10^{-6}\ \mathrm{C})\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\ \mathrm{C}}\right) = 2.996\times10^{14}\ \text{electrons} \approx \boxed{3.00\times10^{14}\ \text{electrons}}$$

3. Use Coulomb's law (Eq. 16–1) to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(25\times10^{-6}\ \mathrm{C})(2.5\times10^{-3}\ \mathrm{C})}{(0.16\ \mathrm{m})^2} = 2.194\times10^4\ \mathrm{N} \approx \boxed{2.2\times10^4\ \mathrm{N}}$$

4. Use Coulomb's law (Eq. 16–1) to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\ \text{C})^2}{(4.0\times10^{-15}\ \text{m})^2} = 14.42\ \text{N} \approx \boxed{14\ \text{N}}$$

5. The charge on the plastic comb is negative, so the comb has gained electrons and has a negative charge.

$$\frac{\Delta m}{m} = \frac{(-3.0\times10^{-6}\ \text{C})\left(\dfrac{1\,e^-}{-1.602\times10^{-19}\ \text{C}}\right)\left(\dfrac{9.109\times10^{-31}\ \text{kg}}{1\,e^-}\right)}{0.0090\ \text{kg}} = 1.90\times10^{-15} \approx \boxed{(1.9\times10^{-13})\%}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \dfrac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, then the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64(4.2\times10^{-2}\ \text{N}) = 2.688\ \text{N} \approx \boxed{2.7\ \text{N}}$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \dfrac{1}{r^2}$, if the force is tripled, then the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{6.52\ \text{cm}}{\sqrt{3}} = \boxed{3.76\ \text{cm}}$$

8. Use the charge per electron and the mass per electron.

$$(-28\times10^{-6}\ \text{C})\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\ \text{C}}\right) = 1.748\times10^{14} \approx \boxed{1.7\times10^{14}\ \text{electrons}}$$

$$(1.748\times10^{14}\ e^-)\left(\frac{9.109\times10^{-31}\ \text{kg}}{1\,e^-}\right) = 1.592\times10^{-16}\ \text{kg} \approx \boxed{1.6\times10^{-16}\ \text{kg}}$$

$\boxed{9.}$ To find the number of electrons, convert the mass to moles and the moles to atoms and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$12\ \text{kg Au} = (12\ \text{kg Au})\left(\frac{1\ \text{mole Au}}{0.197\ \text{kg}}\right)\left(\frac{6.022\times10^{23}\ \text{atoms}}{1\ \text{mole}}\right)\left(\frac{79\ \text{electrons}}{1\ \text{atom}}\right)\left(\frac{-1.602\times10^{-19}\ \text{C}}{\text{electron}}\right)$$

$$= -4.642\times10^8\ \text{C} \approx \boxed{-4.6\times10^8\ \text{C}}$$

The net charge of the bar is $\boxed{0}$, since there are equal numbers of protons and electrons.

10. Take the ratio of the electric force divided by the gravitational force. Note that the distance is not needed for the calculation.

$$\frac{F_\text{E}}{F_\text{G}} = \frac{k\dfrac{Q_1Q_2}{r^2}}{G\dfrac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\ \text{kg})(1.67\times10^{-27}\ \text{kg})} = \boxed{2.27\times10^{39}}$$

The electric force is about 2.27×10^{39} times stronger than the gravitational force for the given scenario.

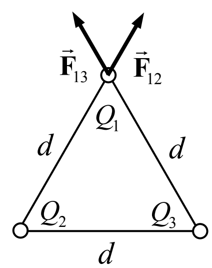
11. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$.

$$F_{+65} = -k\frac{(65\ \mu\text{C})(48\ \mu\text{C})}{(0.35\ \text{m})^2} + k\frac{(65\ \mu\text{C})(95\ \mu\text{C})}{(0.70\ \text{m})^2} = -115.65\ \text{N} \approx \boxed{-120\ \text{N}}\ \text{(to the left)}$$

$$F_{+48} = k\frac{(48\ \mu\text{C})(65\ \mu\text{C})}{(0.35\ \text{m})^2} + k\frac{(48\ \mu\text{C})(95\ \mu\text{C})}{(0.35\ \text{m})^2} = 563.49\ \text{N} \approx \boxed{560\ \text{N}}\ \text{(to the right)}$$

$$F_{-95} = -k\frac{(95\ \mu\text{C})(65\ \mu\text{C})}{(0.70\ \text{m})^2} - k\frac{(95\ \mu\text{C})(48\ \mu\text{C})}{(0.35\ \text{m})^2} = -447.84\ \text{N} \approx \boxed{-450\ \text{N}}\ \text{(to the left)}$$

12. The forces on each charge lie along a line connecting the charges. Let d represent the length of a side of the triangle, and let Q represent the charge at each corner. Since the triangle is equilateral, each angle is 60°.



$$F_{12} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{12x} = k\frac{Q^2}{d^2}\cos 60°,\ F_{12y} = k\frac{Q^2}{d^2}\sin 60°$$

$$F_{13} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{13x} = -k\frac{Q^2}{d^2}\cos 60°,\ F_{13y} = k\frac{Q^2}{d^2}\sin 60°$$

$$F_{1x} = F_{12x} + F_{13x} = 0 \qquad F_{1y} = F_{12y} + F_{13y} = 2k\frac{Q^2}{d^2}\sin 60° = \sqrt{3}k\frac{Q^2}{d^2}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{3}k\frac{Q^2}{d^2} = \sqrt{3}(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(17.0\times10^{-6}\ \text{C})^2}{(0.150\ \text{m})^2} = 199.96\ \text{N}$$

$$\approx \boxed{2.00\times10^{2}\ \text{N}}$$

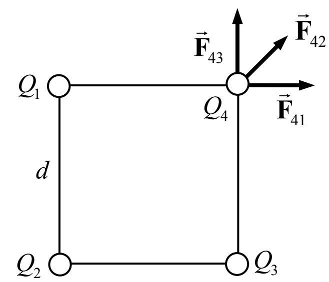
The direction of $\vec{\mathbf{F}}_1$ is in the $\boxed{y \text{ direction}}$. Also notice that it lies along the bisector of the opposite side of the triangle. Thus the force on the lower left charge is of magnitude $\boxed{2.00\times10^{2}\ \text{N}}$ and will point $\boxed{30° \text{ below the } -x \text{ axis}}$. Finally, the force on the lower right charge is of magnitude $\boxed{2.00\times10^{2}\ \text{N}}$ and will point $\boxed{30° \text{ below the } +x \text{ axis}}$.

13. The spheres can be treated as point charges since they are spherical. Thus Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude Q of charge, since that amount was removed from one (initially neutral) sphere and added to the other.

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q^2}{r^2} \quad\rightarrow\quad Q = r\sqrt{\frac{F}{k}} = (0.24\ \text{m})\sqrt{\frac{1.7\times0^{-2}\ \text{N}}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$= 3.3007\times10^{-7}\ \text{C}\left(\frac{1\ \text{electron}}{1.602\times10^{-19}\ \text{C}}\right) = 2.06\times10^{12} \approx \boxed{2.1\times10^{12}\ \text{electrons}}$$

14. Determine the force on the upper right charge and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100-m length of a side of the square, and let the variable Q represent the 6.15-mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{41x} = k\frac{Q^2}{d^2},\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \quad\rightarrow\quad F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \quad\rightarrow\quad F_{43x} = 0,\ F_{43y} = k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(1+\frac{\sqrt{2}}{4}\right) = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(1+\frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.15\times10^{-3}\ \text{C})^2}{(0.100\ \text{m})^2}\left(\sqrt{2}+\frac{1}{2}\right) = 6.507\times10^7\ \text{N} \approx \boxed{6.51\times10^7\ \text{N}}$$

$$\theta = \tan^{-1}\frac{F_{4y}}{F_{4x}} = \boxed{45^\circ} \text{ above the } x \text{ direction}$$

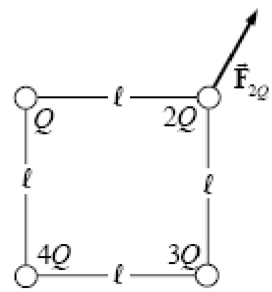
For each charge, the net force will be the magnitude determined above and will lie along the line from the center of the square out toward the charge.

15. Take the lower left-hand corner of the square to be the origin of coordinates. The $2Q$ will have a rightward force on it due to Q, an upward force on it due to $3Q$, and a diagonal force on it due to $4Q$. Find the components of each force, add the components, find the magnitude of the net force, and find the direction of the net force. At the conclusion of the problem there is a diagram showing the net force on the charge $2Q$.

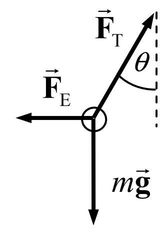
$$2Q:\quad F_{2Qx} = k\frac{(2Q)Q}{\ell^2} + k\frac{(2Q)(4Q)}{2\ell^2}\cos 45^\circ = k\frac{Q^2}{\ell^2}\left(2+2\sqrt{2}\right) = 4.8284\frac{kQ^2}{\ell^2}$$

$$F_{2Qy} = k\frac{(2Q)(3Q)}{\ell^2} + k\frac{(2Q)(4Q)}{2\ell^2}\sin 45^\circ = k\frac{Q^2}{\ell^2}\left(6+2\sqrt{2}\right) = 8.8284\frac{kQ^2}{\ell^2}$$

$$F_{2Q} = \sqrt{F_{2Qx}^2 + F_{2Qy}^2} = \boxed{10.1\frac{kQ^2}{\ell^2}} \qquad \theta_{2Q} = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{8.8284}{4.8284} = \boxed{61^\circ}$$



16. The wires form two sides of an isosceles triangle, so the two charges are separated by a distance $\ell = 2(78\text{ cm})\sin 26° = 68.4\text{ cm}$ and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force as given by Coulomb's law and equate the two expressions for the electric force to find the charge.



$$\sum F_y = F_T\cos\theta - mg = 0 \quad\to\quad F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_T\sin\theta - F_E = 0 \quad\to\quad F_E = F_T\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$F_E = k\frac{(Q/2)^2}{\ell^2} = mg\tan\theta \quad\to\quad Q = 2\ell\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2(0.684\text{ m})\sqrt{\frac{(21\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 26°}{(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}} = 4.572\times10^{-6}\text{C} \approx \boxed{4.6\times10^{-6}\text{C}}$$

17. (*a*) If the force is repulsive, then both charges must be positive since the total charge is positive. Call the total charge Q.

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \quad\to\quad Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \tfrac{1}{2}\left[(90.0\times10^{-6}\text{ C}) \pm \sqrt{(90.0\times10^{-6}\text{ C})^2 - 4\frac{(12.0\text{ N})(0.280\text{ m})^2}{(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}\right]$$

$$= \boxed{88.8\times10^{-6}\text{ C},\ 1.2\times10^{-6}\text{ C}}$$

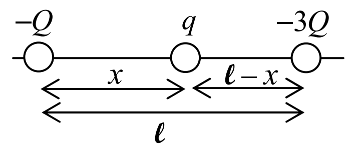
(*b*) If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (*a*).

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \frac{1}{2}\left[(90.0\times10^{-6}\text{ C}) \pm \sqrt{(90.0\times10^{-6}\text{ C})^2 - 4\frac{(-12.0\text{ N})(0.280\text{ m})^2}{(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}\right]$$

$$= \boxed{91.1\times10^{-6}\text{ C}, -1.1\times10^{-6}\text{ C}}$$

18. The negative charges will repel each other, so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables. For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < \ell$.



$$\text{Left: } k\frac{Qq}{x^2} = k\frac{3Q^2}{\ell^2} \qquad \text{Right: } k\frac{3Qq}{(\ell - x)^2} = k\frac{3Q^2}{\ell^2} \quad \rightarrow$$

$$k\frac{Qq}{x^2} = k\frac{3Qq}{(\ell - x)^2} \quad \rightarrow \quad x = \frac{\ell}{\sqrt{3}+1} = 0.366\ell$$

$$k\frac{Qq}{x^2} = k\frac{3Q^2}{\ell^2} \quad \rightarrow \quad q = 3Q\frac{x^2}{\ell^2} = Q\frac{3}{\left(\sqrt{3}+1\right)^2} = 0.402Q$$

Thus the charge should be of magnitude $\boxed{0.40\,Q}$, and a distance $\boxed{0.37\ell \text{ from } -Q \text{ toward } -3Q}$.

19. Use Eq. 16–3 to calculate the force.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \quad \rightarrow \quad \vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-1.602\times10^{-19}\text{ C})(2460\text{ N/C east}) = \boxed{3.94\times10^{-16}\text{ N west}}$$

20. Use Eq. 16–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{1.86\times10^{-14}\text{ N south}}{1.602\times10^{-19}\text{ C}} = \boxed{1.16\times10^5\text{ N/C south}}$$

21. Use Eq. 16–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{33.0\times10^{-6}\text{ C}}{(0.217\text{ m})^2} = \boxed{6.30\times10^6\text{ N/C upward}}$$

Note that the electric field points away from the positive charge.

22. Use Eq. 16–3 to calculate the electric field.

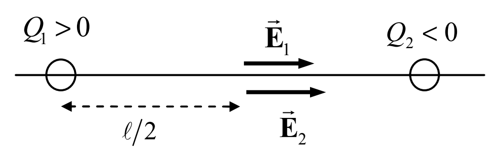
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{6.4\text{ N down}}{-7.3\times10^{-6}\text{ C}} = \boxed{8.8\times10^5\text{ N/C up}}$$

23. Assuming the electric force is the only force on the electron, then Newton's second law may be used with Eq. 16–5 to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \to \quad a = \frac{|q|}{m}E = \frac{(1.602\times10^{-19}\text{ C})}{(9.11\times10^{-31}\text{ kg})}756\text{ N/C} = \boxed{1.33\times10^{14}\text{ m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

24. The electric field due to the positive charge (Q_1) will point away from it, and the electric field due to the negative charge (Q_2) will point toward it. Thus both fields point in the same direction, toward the negative charge, and the magnitudes can be added.

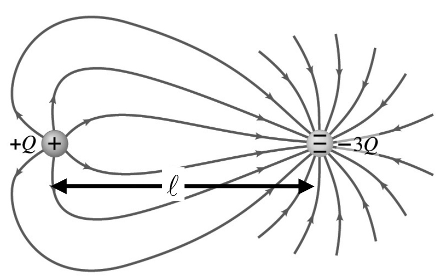


$$E = |E_1| + |E_2| = k\frac{|Q_1|}{r_1^2} + k\frac{|Q_2|}{r_2^2} = k\frac{|Q_1|}{(\ell/2)^2} + k\frac{|Q_2|}{(\ell/2)^2} = \frac{4k}{\ell^2}(|Q_1| + |Q_2|)$$

$$= \frac{4(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}{(0.060\text{ m})^2}(8.0\times10^{-6}\text{ C} + 5.8\times10^{-6}\text{ C}) = 1.3782\times10^8\text{ N/C} \approx \boxed{1.4\times10^8\text{ N/C}}$$

The direction is $\boxed{\text{toward the negative charge}}$.

25.



26. Assuming the electric force is the only force on the electron, Newton's second law may be used to find the electric field strength.

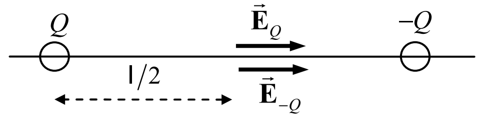
$$F_{\text{net}} = ma = qE \quad \to$$

$$E = \frac{ma}{q} = \frac{(1.673\times10^{-27}\text{ kg})(2.4\times10^6)(9.80\text{ m/s}^2)}{(1.602\times10^{-19}\text{ C})} = 0.2456\text{ N/C} \approx \boxed{0.25\text{ N/C}}$$

27. Since the electron accelerates from rest toward the north, the net force on it must be to the north. Assuming the electric force is the only force on the electron, Newton's second law and Eq. 16–5 may be used to find the electric field.

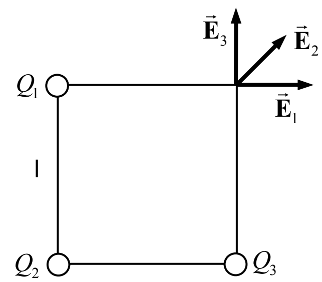
$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \quad \to \quad \vec{\mathbf{E}} = \frac{m}{q}\vec{\mathbf{a}} = \frac{(9.11\times10^{-31}\text{ kg})}{(-1.602\times10^{-19}\text{ C})}(105\text{ m/s}^2\text{ north}) = \boxed{5.97\times10^{-10}\text{ N/C south}}$$

28. The field due to the negative charge will point toward the negative charge, and the field due to the positive charge will point toward the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.



$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(\ell/2)^2} = \frac{8kQ}{\ell^2} \quad \rightarrow \quad Q = \frac{E\ell^2}{8k} = \frac{(386\text{ N/C})(0.160\text{ m})^2}{8(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)} = \boxed{1.37\times10^{-10}\text{ C}}$$

29. The field at the upper right-hand corner of the square is the vector sum of the fields due to the other three charges. Let ℓ represent the 1.22-m length of a side of the square, and let Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{\ell^2} \quad \rightarrow \quad E_{1x} = k\frac{Q}{\ell^2},\ E_{1y} = 0$$

$$E_2 = k\frac{Q}{2\ell^2} \quad \rightarrow \quad E_{2x} = k\frac{Q}{2\ell^2}\cos 45^\circ = k\frac{\sqrt{2}Q}{4\ell^2},\ E_{2y} = k\frac{\sqrt{2}Q}{4\ell^2}$$

$$E_3 = k\frac{Q}{\ell^2} \quad \rightarrow \quad E_{3x} = 0,\ E_{1y} = k\frac{Q}{\ell^2}$$

Add the x and y components together to find the total electric field, noting that $E_x = E_y$.

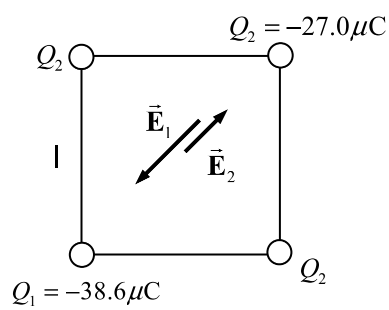
$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{\ell^2} + k\frac{\sqrt{2}Q}{4\ell^2} + 0 = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{\ell^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(3.25\times10^{-6}\text{ C})}{(1.22\text{ m})^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{3.76\times10^4\text{ N/C}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \boxed{45.0^\circ} \text{ from the } x \text{ direction}$$

30. The fields at the center due to the two $-27.0\ \mu\text{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, so only the other two charges need to be considered. The field due to each of the other charges will point directly toward its source charge. Accordingly, the two fields are in opposite directions and can be combined algebraically. The distance from each charge to the center is $\ell/\sqrt{2}$.



$$E = E_1 - E_2 = k\frac{|Q_1|}{\ell^2/2} - k\frac{|Q_2|}{\ell^2/2} = k\frac{|Q_1|-|Q_2|}{\ell^2/2} \quad = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(38.6-27.0)\times10^{-6}\ \text{C}}{(0.425\ \text{m})^2/2}$$

$$= \boxed{1.15\times10^6\ \text{N/C, toward the} -38.6\ \mu\text{C charge}}$$

31. Choose the rightward direction to be positive. Then the field due to $+Q$ will be positive, and the field due to $-Q$ will be negative.

$$E = k\frac{Q}{(x+a)^2} - k\frac{Q}{(x-a)^2} = kQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right) = \boxed{\frac{-4kQxa}{(x^2-a^2)^2}}$$

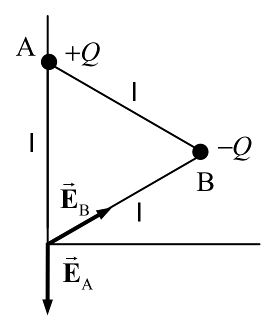
The negative sign means the field points to the $\boxed{\text{left}}$.

$\boxed{32.}$ For the net field to be zero at point P, the magnitudes of the fields created by Q_1 and Q_2 must be equal. Also, the distance x will be taken as positive to the left of Q_1. That is the only region where the total field due to the two charges can be zero. Let the variable ℓ represent the 12-cm distance.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \quad\rightarrow\quad k\frac{|Q_1|}{x^2} = k\frac{Q_2}{(x+\ell)^2} \quad\rightarrow$$

$$x = \ell\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2}-\sqrt{|Q_1|}\right)} = (12\ \text{cm})\frac{\sqrt{32\mu\text{C}}}{\left(\sqrt{45\mu\text{C}}-\sqrt{32\mu\text{C}}\right)} = 64.57\ \text{cm} \approx \boxed{65\ \text{cm}}$$

33. The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from the origin to point B, 30° above the x axis. We first solve the problem symbolically and then substitute in the values.



$$E_A = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Ax} = 0,\ E_{Ay} = -k\frac{Q}{\ell^2}$$

$$E_B = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Bx} = k\frac{Q}{\ell^2}\cos30^\circ = k\frac{\sqrt{3}Q}{2\ell^2},$$

$$E_{By} = k\frac{Q}{\ell^2}\sin30^\circ = k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2\ell^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{k^2Q^2}{4\ell^4}} = \sqrt{\frac{4k^2Q^2}{4\ell^4}} = \frac{kQ}{\ell^2}$$

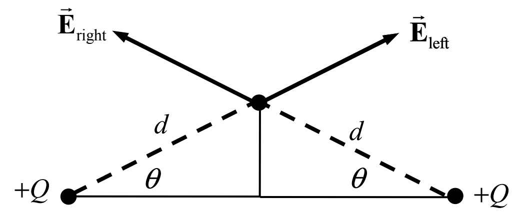
$$= \frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(26\times10^{-6}\ \text{C})}{(0.080\ \text{m})^2} = 3.651\times10^7\ \text{N/C} \approx \boxed{3.7\times10^7\ \text{N/C}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{Q}{2\ell^2}}{k\frac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{-1}{\sqrt{3}} = \boxed{330^\circ}$$

34. The charges must be of the same sign so that the electric fields created by the two charges oppose each other and add to zero. The magnitudes of the two electric fields must be equal.

$$E_1 = E_2 \quad\rightarrow\quad k\frac{Q_1}{(\ell/3)^2} = k\frac{Q_2}{(2\ell/3)^2} \quad\rightarrow\quad 9Q_1 = \frac{9Q_2}{4} \quad\rightarrow\quad \frac{Q_1}{Q_2} = \boxed{\frac{1}{4}}$$

35. In each case, find the vector sum of the field caused by the charge on the left $(\vec{\mathbf{E}}_{\text{left}})$ and the field caused by the charge on the right $(\vec{\mathbf{E}}_{\text{right}})$.



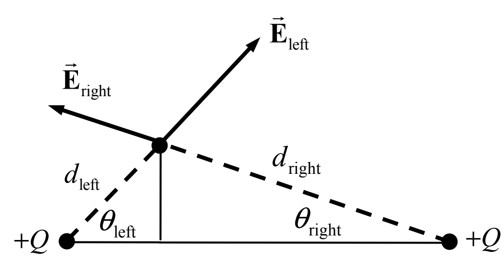
Point A: From the symmetry of the geometry, in calculating the electric field at point A, only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta = \tan^{-1}\frac{5.0}{10.0} = 26.6^\circ$$

$$d = \sqrt{(0.050\text{ m})^2 + (0.100\text{ m})^2} = 0.1118\text{ m}$$

$$E_{\text{A}} = 2\frac{kQ}{d^2}\sin\theta = 2(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.7\times10^{-6}\,\text{C}}{(0.1118\text{ m})^2}\sin 26.6^\circ = \boxed{3.0\times10^6\ \text{N/C}} \qquad \theta_{\text{A}} = \boxed{90^\circ}$$

Point B: Now the point is not symmetrically placed. The horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.



$$\theta_{\text{left}} = \tan^{-1}\frac{5.0}{5.0} = 45^\circ \qquad \theta_{\text{left}} = \tan^{-1}\frac{5.0}{15.0} = 18.4^\circ$$

$$d_{\text{left}} = \sqrt{(0.050\text{ m})^2 + (0.050\text{ m})^2} = 0.0707\text{ m}$$

$$d_{\text{right}} = \sqrt{(0.050\text{ m})^2 + (0.150\text{ m})^2} = 0.1581\text{ m}$$

$$E_x = (\vec{\mathbf{E}}_{\text{left}})_x + (\vec{\mathbf{E}}_{\text{right}})_x = k\frac{Q}{d_{\text{left}}^2}\cos\theta_{\text{left}} - k\frac{Q}{d_{\text{right}}^2}\cos\theta_{\text{right}}$$

$$= (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.7\times10^{-6}\ \text{C})\left[\frac{\cos 45^\circ}{(0.0707\text{ m})^2} - \frac{\cos 18.4^\circ}{(0.1581\text{ m})^2}\right] = 4.372\times10^6\ \text{N/C}$$

$$E_y = (\vec{\mathbf{E}}_{\text{left}})_y + (\vec{\mathbf{E}}_{\text{right}})_y = k\frac{Q}{d_{\text{left}}^2}\sin\theta_{\text{left}} + k\frac{Q}{d_{\text{right}}^2}\sin\theta_{\text{right}}$$

$$= (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.7\times10^{-6}\ \text{C})\left[\frac{\sin 45^\circ}{(0.0707\text{ m})^2} + \frac{\sin 18.4^\circ}{(0.1581\text{ m})^2}\right] = 6.509\times10^6\ \text{N/C}$$

$$E_{\text{B}} = \sqrt{E_x^2 + E_y^2} = 7.841\times10^6\ \text{N/C} \approx \boxed{7.8\times10^6\ \text{N/C}} \qquad \theta_B = \tan^{-1}\frac{E_y}{E_x} = 56.11^\circ \approx \boxed{56^\circ}$$

The results are consistent with Fig. 16–32b. In the figure, the field at point A points straight up, matching the calculations. The field at point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at point B than at point A, indicating that the field is stronger there, again matching the calculations.

36. We assume that gravity can be ignored, which is proven in part (*b*).

(*a*) The electron will accelerate to the right. The magnitude of the acceleration can be found from setting the net force equal to the electric force on the electron. The acceleration is constant, so constant-acceleration relationships can be used.

$$F_{\text{net}} = ma = |q|E \quad \to \quad a = \frac{|q|E}{m}$$

$$v^2 = v_0^2 + 2a\Delta x \quad \to \quad v = \sqrt{2a\Delta x} = \sqrt{2\frac{|q|E}{m}\Delta x}$$

$$= \sqrt{2\frac{(1.602\times10^{-19}\ \text{C})(1.45\times10^4\ \text{N/C})}{(9.11\times10^{-31}\ \text{kg})}(1.60\times10^{-2}\ \text{m})} = \boxed{9.03\times10^6\ \text{m/s}}$$

(*b*) The value of the acceleration caused by the electric field is compared to *g*.

$$a = \frac{|q|E}{m} = \frac{(1.602\times10^{-19}\ \text{C})(1.45\times10^4\ \text{N/C})}{(9.11\times10^{-31}\ \text{kg})} = 2.55\times10^{15}\ \text{m/s}^2$$

$$a/g = (2.55\times10^{15}\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = 2.60\times10^{14}$$

The acceleration due to gravity can be ignored compared to the acceleration caused by the electric field.

37. (*a*) The net force between the thymine and adenine is due to the following forces.

O–H attraction: $$F_{\text{OH}} = k\frac{(0.4e)(0.2e)}{(1.80\times10^{-10}\ \text{m})^2} = \frac{0.08ke^2}{(1.80\times10^{-10}\ \text{m})^2}$$

O–N repulsion: $$F_{\text{ON}} = k\frac{(0.4e)(0.2e)}{(2.80\times10^{-10}\ \text{m})^2} = \frac{0.08ke^2}{(2.80\times10^{-10}\ \text{m})^2}$$

N–N repulsion: $$F_{\text{NN}} = k\frac{(0.2e)(0.2e)}{(3.00\times10^{-10}\ \text{m})^2} = \frac{0.04ke^2}{(3.00\times10^{-10}\ \text{m})^2}$$

H–N attraction: $$F_{\text{HN}} = k\frac{(0.2e)(0.2e)}{(2.00\times10^{-10}\ \text{m})^2} = \frac{0.04ke^2}{(2.00\times10^{-10}\ \text{m})^2}$$

$$F_{\text{A-T}} = F_{\text{OH}} - F_{\text{ON}} - F_{\text{NN}} + F_{\text{HN}} = \left(\frac{0.08}{1.80^2} - \frac{0.08}{2.80^2} - \frac{0.04}{3.00^2} + \frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times10^{-10}\ \text{m}}\right)^2\frac{ke^2}{d^2}$$

$$= (0.02004)\frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{(1.0\times10^{-10}\ \text{m})^2} = 4.623\times10^{-10}\ \text{N} \approx \boxed{5\times10^{-10}\ \text{N}}$$

(*b*) The net force between the cytosine and guanine is due to the following forces.

O–H attraction: $F_{\text{OH}} = k\dfrac{(0.4e)(0.2e)}{(1.90\times10^{-10}\ \text{m})^2} = \dfrac{0.08ke^2}{(1.90\times10^{-10}\ \text{m})^2}$ (two of these)

O–N repulsion: $F_{\text{ON}} = k\dfrac{(0.4e)(0.2e)}{(2.90\times10^{-10}\ \text{m})^2} = \dfrac{0.08ke^2}{(2.90\times10^{-10}\ \text{m})^2}$ (two of these)

H–N attraction: $F_{\text{HN}} = k\dfrac{(0.2e)(0.2e)}{(2.00\times10^{-10}\ \text{m})^2} = \dfrac{0.04ke^2}{(2.00\times10^{-10}\ \text{m})^2}$

N–N repulsion: $F_{\text{NN}} = k\dfrac{(0.2e)(0.2e)}{(3.00\times10^{-10}\ \text{m})^2} = \dfrac{0.04ke^2}{(3.00\times10^{-10}\ \text{m})^2}$

$$F_{\text{C–G}} = 2F_{\text{OH}} - 2F_{\text{ON}} - F_{\text{NN}} + F_{\text{HN}} = \left(2\frac{0.08}{1.90^2} - 2\frac{0.08}{2.90^2} - \frac{0.04}{3.00^2} + \frac{0.04}{2.00^2}\right)\left(\frac{1}{1.0\times10^{-10}\ \text{m}}\right)^2 \frac{ke^2}{d^2}$$
$$= (0.03085)\frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{(1.0\times10^{-10}\ \text{m})^2} = 7.116\times10^{-10}\ \text{N} \approx \boxed{7\times10^{-10}\ \text{N}}$$

(*c*) For the 10^5 pairs of molecules, we assume that half are A–T pairs and half are C–G pairs. We average the above results and multiply by 10^5.

$$F_{\text{net}} = \tfrac{1}{2}10^5(F_{\text{A–T}} + F_{\text{C–G}}) = 10^5(4.623\times10^{-10}\ \text{N} + 7.116\times10^{-10}\ \text{N})$$
$$= 5.850\times10^{-5}\ \text{N} \approx \boxed{6\times10^{-5}\ \text{N}}$$

38. Use Gauss's law to determine the enclosed charge. Note that the size of the box does not enter into the calculation.

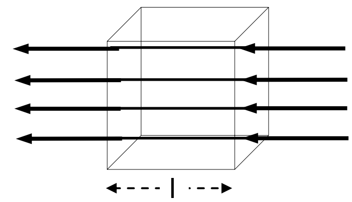
$$\Phi_E = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow Q_{\text{encl}} = \Phi_E\varepsilon_0 = (1850\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = \boxed{1.64\times10^{-8}\ \text{C}}$$

39. (*a*) Use Gauss's law to determine the electric flux.

$$\Phi_E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{-1.0\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{-1.1\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

(*b*) Since there is no charge enclosed by surface A_2, $\Phi_E = \boxed{0}$.

40. (*a*) Assuming that there is no charge contained within the cube, the net flux through the cube is $\boxed{0}$. All of the field lines that enter the cube also leave the cube.



(*b*) Four faces have no flux through them, because no field lines pass through them. In the diagram, the left face has a positive flux and the right face has the opposite amount of negative flux.

$$\Phi_{\text{left}} = EA = E\ell^2 = (7.50\times10^3\ \text{N/C})(8.50\times10^{-2}\ \text{m})^2$$

$$\boxed{\Phi_{\text{left}} = 54.2\ \text{N}\cdot\text{m}^2/\text{C}};\ \boxed{\Phi_{\text{right}} = -54.2\ \text{N}\cdot\text{m}^2/\text{C}};\ \boxed{\Phi_{\text{other}} = 0}$$

41. Equation 16–10 applies. Note that the separation distance does not enter into the calculation.

$$E = \frac{Q/A}{\varepsilon_0} \rightarrow Q = \varepsilon_0 EA = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(130\ \text{N/C})(0.85\ \text{m})^2 = \boxed{8.3\times10^{-10}\ \text{C}}$$

42. The electric field can be calculated by Eq. 16–4a, and that can be solved for the charge.

$$E = k\frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{(3.75\times10^2\ \text{N/C})(3.50\times10^{-2}\ \text{m})^2}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{5.11\times10^{-11}\ \text{C}}$$

This is about 3×10^8 electrons. Since the field points toward the ball, the charge must be $\boxed{\text{negative}}$.

43. See Fig. 16–34 in the text for additional insight into this problem.

(*a*) Inside the inner radius of the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.

(*b*) There is no field inside the conducting material: $\boxed{E = 0}$.

(*c*) Outside the shell, the field is that of the point charge, $\boxed{E = k\frac{Q}{r^2}}$.

(*d*) The shell does not affect the field due to Q alone, except in the shell material, where the field is 0. The charge Q does affect the shell—it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.

44. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\text{electric}} = F_{\text{gravitational}} \rightarrow k\frac{e^2}{r^2} = mg \rightarrow$$

$$r = e\sqrt{\frac{k}{mg}} = (1.602\times10^{-19}\ \text{C})\sqrt{\frac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}} = \boxed{5.08\ \text{m}}$$

45. Water has an atomic mass of 18, so 1 mole of water molecules has a mass of 18 grams. Each water molecule contains 10 protons.

$$75\ \text{kg}\left(\frac{6.02\times10^{23}\ \text{H}_2\text{O molecules}}{0.018\ \text{kg}}\right)\left(\frac{10\ \text{protons}}{1\ \text{molecule}}\right)\left(\frac{1.60\times10^{-19}\ \text{C}}{\text{proton}}\right) = \boxed{4.0\times10^9\ \text{C}}$$

46. Calculate the total charge on all electrons in 3.0 g of copper and compare $32\mu\text{C}$ to that value. The molecular weight of copper is 63.5 g, and its atomic number is 29.

$$\text{Total electron charge} = 3.0\text{ g}\left(\frac{1\text{ mole}}{63.5\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{\text{mole}}\right)\left(\frac{29\,e}{\text{atoms}}\right)\left(\frac{1.602\times10^{-19}\text{ C}}{1\,e}\right) = 1.32\times10^{5}\text{ C}$$

$$\text{Fraction lost} = \frac{32\times10^{-6}\text{ C}}{1.32\times10^{5}\text{ C}} = \boxed{2.4\times10^{-10}}$$

47. Use Eq. 16–4a to calculate the magnitude of the electric charge on the Earth. We use the radius of the Earth as the distance.

$$E = k\frac{Q}{r^2} \quad\rightarrow\quad Q = \frac{Er^2}{k} = \frac{(150\text{ N/C})(6.38\times10^{6}\text{ m})^2}{8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\text{ C}}$$

Since the electric field is pointing toward the Earth's center, the charge must be $\boxed{\text{negative}}$.

48. (*a*) From Problem 47, we know that the electric field is pointed toward the Earth's center. Thus an electron in such a field would experience an upward force of magnitude $F_{\text{E}} = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \quad\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\text{ C})(150\text{ N/C})}{(9.11\times10^{-31}\text{ kg})} = 2.638\times10^{13}\text{ m/s}^2 \approx \boxed{2.6\times10^{13}\text{ m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downward force of magnitude $F_{\text{E}} = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_{\text{E}} = eE = ma \quad\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\text{ C})(150\text{ N/C})}{(1.67\times10^{-27}\text{ kg})} = 1.439\times10^{10}\text{ m/s}^2 \approx \boxed{1.4\times10^{10}\text{ m/s}^2\text{, down}}$$

(*c*) For the electron: $\frac{a}{g} = \frac{2.638\times10^{13}\text{ m/s}^2}{9.80\text{ m/s}^2} \approx \boxed{2.7\times10^{12}}$

For the proton: $\frac{a}{g} = \frac{1.439\times10^{10}\text{ m/s}^2}{9.80\text{ m/s}^2} \approx \boxed{1.5\times10^{9}}$

$\boxed{49}$. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_{\text{E}} = |q|E = mg \quad\rightarrow\quad neE = \tfrac{4}{3}\pi r^3\rho g \quad\rightarrow$$

$$n = \frac{4\pi r^3\rho g}{3eE} = \frac{4\pi(1.8\times10^{-5}\text{ m})^3(1.00\times10^{3}\text{ kg/m}^3)(9.80\text{ m/s}^2)}{3(1.602\times10^{-19}\text{ C})(150\text{ N/C})} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\text{ electrons}}$$

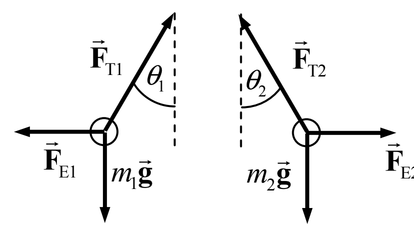
50. There are four forces to calculate. Call the rightward direction the positive direction. The value of k is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of e is 1.602×10^{-19} C.

$$F_{\text{net}} = F_{\text{CH}} + F_{\text{CN}} + F_{\text{OH}} + F_{\text{ON}} = \frac{k(0.40e)(0.20e)}{(1\times10^{-9}\ \text{m})^2}\left[-\frac{1}{(0.30)^2}+\frac{1}{(0.40)^2}+\frac{1}{(0.18)^2}-\frac{1}{(0.28)^2}\right]$$
$$= 2.445\times10^{-10}\ \text{N} \approx \boxed{2.4\times10^{-10}\ \text{N}}$$

51. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\text{E}} = F_{\text{radial}} \rightarrow k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \rightarrow$$
$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\ \text{C})^2}{(9.109\times10^{-31}\ \text{kg})(2.2\times10^{6}\ \text{m/s})^2} = \boxed{5.2\times10^{-11}\ \text{m}}$$

52. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and the electric force of repulsion is always horizontal. Likewise, the small-angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force on each charge. Take the right to be the positive horizontal direction and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.



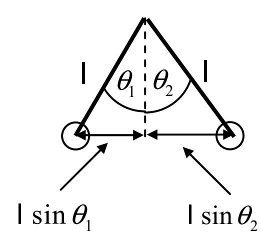
(*a*) $\sum F_{1x} = F_{\text{T1}}\sin\theta_1 - F_{\text{E1}} = 0 \rightarrow F_{\text{E1}} = F_{\text{T1}}\sin\theta_1$

$$\sum F_{1y} = F_{\text{T1}}\cos\theta_1 - m_1 g \rightarrow F_{\text{T1}} = \frac{m_1 g}{\cos\theta_1} \rightarrow F_{\text{E1}} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

A completely parallel analysis would give $F_{\text{E2}} = m_2 g\theta_2$. Since the electric forces are a Newton's third law pair, they can be set equal to each other in magnitude. Note that the explicit form of Coulomb's law was not necessary for this analysis.

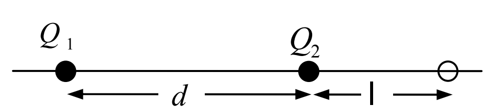
$$F_{\text{E1}} = F_{\text{E2}} \rightarrow m_1 g\theta_1 = m_2 g\theta_2 \rightarrow \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

(*b*) The horizontal distance from one sphere to the other, represented by d, is found by the small-angle approximation. See the diagram. Use the relationship derived above that $F_{\text{E}} = mg\theta$ to solve for the distance.



$$d = \ell(\theta_1+\theta_2) = 2\ell\theta_1 \rightarrow \theta_1 = \frac{d}{2\ell}$$
$$m_1 g\theta_1 = F_{\text{E1}} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2\ell} \rightarrow \boxed{d = \left(\frac{4\ell kQ^2}{mg}\right)^{1/3}}$$

53. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (Q_2). Also, in between the two charges, the fields due to the two charges are in the same direction and cannot cancel. Thus the only



place where the field can be zero is closer to the weaker charge, but not between them. In the diagram, this means that ℓ must be positive. Evaluate the net field at that location.

$$E = -k\frac{|Q_2|}{\ell^2} + k\frac{Q_1}{(\ell+d)^2} = 0 \quad \rightarrow \quad |Q_2|(\ell+d)^2 = Q_1\ell^2 \quad \rightarrow$$

$$\ell = \frac{\sqrt{|Q_2|}}{\sqrt{Q_1} - \sqrt{|Q_2|}}d = \frac{\sqrt{5.0\times10^{-6}\text{ C}}}{\sqrt{2.5\times10^{-5}\text{ C}} - \sqrt{5.0\times10^{-6}\text{ C}}}(2.4\text{ m}) = \boxed{\begin{array}{l}1.9\text{ m from } Q_2,\\ 4.3\text{ m from } Q_1\end{array}}$$

54. The electric field at the surface of the pea is given by Eq. 16–4a. Solve that equation for the charge.

$$E = k\frac{Q}{r^2} \quad \rightarrow \quad Q = \frac{Er^2}{k} = \frac{(3\times10^6\text{ N/C})(3.75\times10^{-3}\text{ m})^2}{8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{5\times10^{-9}\text{ C}} \ (\approx 3\times10^9 \text{ electrons})$$

55. There will be a rightward force on Q_1 due to Q_2, given by Coulomb's law. There will be a leftward force on Q_1 due to the electric field created by the parallel plates. Let right be the positive direction.

$$\sum F = k\frac{|Q_1Q_2|}{x^2} - |Q_1|E$$

$$= (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(6.7\times10^{-6}\text{ C})(1.8\times10^{-6}\text{ C})}{(0.47\text{ m})^2} - (6.7\times10^{-6}\text{ C})(5.3\times10^4\text{ N/C})$$

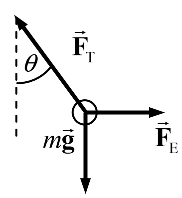
$$= \boxed{0.14\text{ N, right}}$$

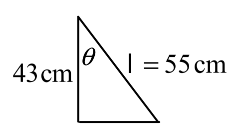
56. The weight of the sphere is the density times the volume. The electric force is given by Eq. 16–1, with both spheres having the same charge, and the separation distance equal to their diameter.

$$mg = k\frac{Q^2}{(d)^2} \quad \rightarrow \quad \rho\tfrac{4}{3}\pi r^3 g = \frac{kQ^2}{(2r)^2} \quad \rightarrow$$

$$Q = \sqrt{\frac{16\rho\pi g r^5}{3k}} = \sqrt{\frac{16(35\text{ kg/m}^3)\pi(9.80\text{ m/s}^2)(1.0\times10^{-2}\text{ m})^5}{3(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}} = \boxed{8.0\times10^{-9}\text{ C}}$$

$\boxed{57}$. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is $\boxed{\text{positive}}$. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions and use those equations to find the magnitude of the charge.





$$\theta = \cos^{-1}\frac{43}{55} = 38.6°$$

$$\sum F_x = F_\text{E} - F_\text{T}\sin\theta = 0 \quad \rightarrow \quad F_\text{E} = F_\text{T}\sin\theta = QE$$

$$\sum F_y = F_\text{T}\cos\theta - mg = 0 \quad \rightarrow \quad F_\text{T} = \frac{mg}{\cos\theta} \quad \rightarrow \quad QE = mg\tan\theta$$

$$Q = \frac{mg\tan\theta}{E} = \frac{(1.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 38.6°}{(9500\text{ N/C})} = \boxed{8.2\times10^{-7}\text{ C}}$$

58. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_{\mathrm{AB}} = \boxed{\frac{kQ^2}{R^2}, \text{ away from B}}$$

(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_{\mathrm{AB}} = k\frac{QQ/2}{R^2} = \boxed{\frac{kQ^2}{2R^2}, \text{ away from B}}$$

(*c*) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

$$F_{\mathrm{AB}} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}, \text{ away from B}}$$

59. (*a*) Outside of a sphere, the electric field of a uniformly charged sphere is the same as if all the charge were at the center of the sphere. Thus we can use Eq. 16–4a. According to Example 16–6, each toner particle has the same charge as 20 electrons. Let *N* represent the number of toner particles.

$$E = k\frac{Q}{r^2} = k\frac{NQ_0}{r^2} \quad \rightarrow$$

$$N = \frac{Er^2}{kQ_0} = \frac{(5000\,\mathrm{N/C})(0.50\ \mathrm{m})^2}{(8.988\times10^9\ \mathrm{N\cdot m^2/C^2})(20)(1.6\times10^{-19}\ \mathrm{C})} = 4.346\times10^{10} \approx \boxed{4\times10^{10}\,\text{particles}}$$

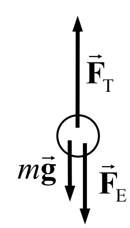
(*b*) According to Example 16–6, each toner particle has a mass of 9.0×10^{-16} kg.

$$m = Nm_0 = (4.346\times10^{10})(9.0\times10^{-16}\ \mathrm{kg}) = 3.911\times10^{-5} \approx \boxed{4\times10^{-5}\ \mathrm{kg}}$$

60. Since the gravity force is downward, the electric force must be upward. Since the charge is positive, the electric field must also be upward. Equate the magnitudes of the two forces and solve for the electric field.

$$F_{\text{electric}} = F_{\text{gravitational}} \quad \rightarrow \quad qE = mg \quad \rightarrow \quad E = \frac{mg}{q} = \frac{(1.67\times10^{-27}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(1.602\times10^{-19}\ \mathrm{C})} = \boxed{1.02\times10^{-7}\ \mathrm{N/C,\ up}}$$

61. The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed $\boxed{\text{down}}$. Use the free-body diagram to write an expression for the magnitude of the electric field.



$$\sum F = F_{\mathrm{T}} - mg - F_{\mathrm{E}} = 0 \quad \rightarrow \quad F_{\mathrm{E}} = QE = F_{\mathrm{T}} - mg \quad \rightarrow$$

$$E = \frac{F_{\mathrm{T}} - mg}{Q} = \frac{5.18\ \mathrm{N} - (0.185\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{3.40\times10^{-7}\ \mathrm{C}} = \boxed{9.90\times10^6\ \mathrm{N/C}}$$

62. (*a*) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use Eq. 2–11c with a final velocity of 0.

$$F = ma = qE = -eE \quad \rightarrow \quad a = -\frac{eE}{m};\ v^2 = v_0^2 + 2a\Delta x = 0 \quad \rightarrow$$

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\frac{eE}{m}\right)} = \frac{mv_0^2}{2eE} = \frac{(9.11\times10^{-31}\ \text{kg})(5.32\times10^{6}\ \text{m/s})^2}{2(1.60\times10^{-19}\ \text{C})(9.45\times10^{3}\ \text{N/C})} = \boxed{8.53\times10^{-3}\ \text{m}}$$

(*b*) Find the elapsed time from constant-acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity. Use Eq. 2–11a.

$$v = v_0 + at \quad \rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-2v_0}{\left(-\frac{eE}{m}\right)} = \frac{2mv_0}{eE} = \frac{2(9.11\times10^{-31}\ \text{kg})(5.32\times10^{6}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(9.45\times10^{3}\ \text{N/C})} = \boxed{6.41\times10^{-9}\ \text{s}}$$

63. On the x axis, the electric field can only be zero at a location closer to the smaller magnitude charge. The field will never be zero to the left of the midpoint between the two charges. In between the two charges, the field due to both charges will point to the left. Thus the total field cannot be zero there. So the only place on the x axis where the field can be zero is to the right of the negative charge, and x must be positive. Calculate the field at point P and set it equal to zero.

$$E = k\frac{(-Q/2)}{x^2} + k\frac{Q}{(x+d)^2} = 0 \quad \rightarrow \quad 2x^2 = (x+d)^2 \quad \rightarrow \quad x = \boxed{\frac{d}{\sqrt{2}-1} = d\left(\sqrt{2}+1\right) \approx 2.41\,d}$$

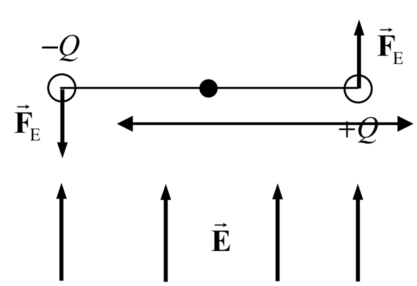
The field cannot be zero at any points off the x axis. For any point off the x axis, the electric fields due to the two charges will not be along the same line, so they can never combine to give 0.

64. To find the number of electrons, convert the mass to moles and the moles to atoms and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

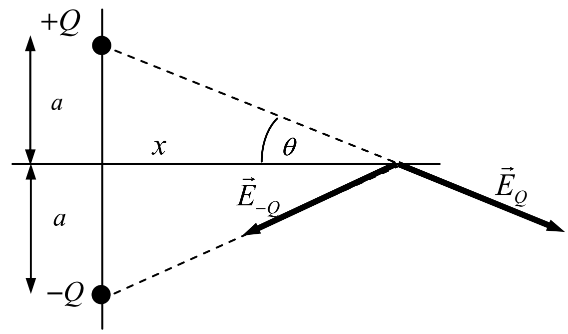
$$25\ \text{kg Al} = (25\ \text{kg Al})\left(\frac{1\ \text{mole Al}}{2.7\times10^{-2}\ \text{kg}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{1\ \text{mole}}\right)\left(\frac{13\ \text{electrons}}{1\ \text{molecule}}\right)\left(\frac{-1.602\times10^{-19}\ \text{C}}{\text{electron}}\right)$$

$$= \boxed{-1.2\times10^{9}\ \text{C}}$$

65. The electric field will put a force of magnitude $F_E = QE$ on each charge. The distance of each charge from the pivot point is $\ell/2$, so the torque caused by each force is $\tau = F_E r_\perp = QE\left(\frac{1}{2}\ell\right)$. Both torques will tend to make the rod rotate counterclockwise in the diagram. The net torque is $\tau_{\text{net}} = 2\left(\frac{QE\ell}{2}\right) = \boxed{QE\ell}$, and it is $\boxed{\text{counterclockwise}}$. However, if the charges were switched, the torque would be clockwise.



66. From the diagram, we see that the x components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative y direction and will be twice the y component of either electric field vector.



$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$
$$= \frac{2kQ}{x^2+a^2}\frac{a}{(x^2+a^2)^{1/2}}$$
$$= \boxed{\frac{2kQa}{(x^2+a^2)^{3/2}} \text{ in the negative } y \text{ direction}}$$

67. One sphere will have a positive charge, and the other sphere will have the same amount of negative charge. First solve for that charge by equating the electric force to the gravitational force. Then compare that charge to the total charge. N represents the number of electrons moved from one sphere to the other.

$$F_{\text{electric}} = F_{\text{gravitational}} \quad \rightarrow \quad k\frac{(Ne)^2}{r^2} = G\frac{m^2}{r^2} \quad \rightarrow$$

$$N = \frac{m}{e}\sqrt{\frac{G}{k}} = \frac{(0.12\,\text{kg})}{(1.602\times10^{-19}\,\text{C})}\sqrt{\frac{6.67\times10^{-11}}{8.988\times10^{9}}} = 6.453\times10^{6}\,\text{electrons}$$

$$\text{fraction} = \frac{6.453\times10^{6}\,\text{electrons}}{7.22\times10^{24}\,\text{electrons}} = \boxed{8.94\times10^{-19}}$$

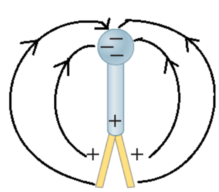
Solutions to Search and Learn Problems

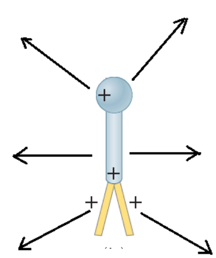
1. (*a*) When the leaves are charged by induction, no additional charge is added to the leaves. As the charged rod is near the top of the electroscope, it repels the charge onto the leaves, causing them to separate. When the rod is removed, the charge returns to its initial equilibrium position and the leaves come back together.

   (*b*) When the leaves are charged by conduction, positive charge is placed onto the electroscope from the rod, causing the leaves to separate. When the rod is removed, the charge remains on the electroscope and the leaves remain separated.

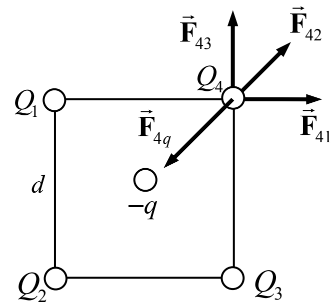
   (*c*) Yes. The electroscope has a negative charge on the top sphere and on the leaves. Therefore, the electroscope has a total net negative charge, so it must have been charged by conduction.

   (*d*) In Fig. 16–11a the electroscope is charged by induction, so the net charge is zero. Electric field lines leaving the positive end of the electroscope end on the negative end. In Fig. 16–11b the electroscope has a net positive charge. Electric field lines originate on the electroscope and point away from it.





2. A negative charge must be placed at the center of the square. Let $Q = 6.4\ \mu\text{C}$ be the charge at each corner, let $-q$ be the magnitude of negative charge in the center, and let $d = 9.2\text{ cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, then the net force on each other corner charge will also be zero. The force on the charge in the center is also zero, from the symmetry of the problem.



$$F_{41} = k\frac{Q^2}{d^2} \quad \rightarrow \quad F_{41x} = k\frac{Q^2}{d^2},\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \quad \rightarrow \quad F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \quad \rightarrow \quad F_{43x} = 0,\ F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \quad \rightarrow \quad F_{4qx} = -k\frac{2qQ}{d^2}\cos 45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \quad \rightarrow$$

$$q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = (6.4\times10^{-6}\text{ C})\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 6.1\times10^{-6}\text{ C}$$

So the charge to be placed is $-q = \boxed{-6.1\times10^{-6}\text{ C}}$. Note that the length of the sides of the square does not enter into the calculation.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say toward the right, then it would be closer to the right charges than the left and would be attracted more to the right. Likewise, the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape; rather, it would have a tendency to move away from it if disturbed.

3. This is a constant-acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be ℓ, the vertical gap between the plates be h, and the initial velocity be v_0. Notice that the vertical motion has a maximum displacement of $h/2$. Let upward be the positive vertical direction. We calculate the vertical acceleration produced by the electric field as the electron crosses the region.

$$F_y = ma_y = qE = -eE \quad \rightarrow \quad a_y = -\frac{eE}{m}$$

Since the electron has constant horizontal velocity in the region, we use the horizontal component of the velocity and the length of the region to solve for the time of flight across the region.

$$\ell = v_0\cos\theta_0(t) \quad \rightarrow \quad t = \frac{\ell}{v_0\cos\theta_0}$$

In half this time, the electron will travel vertically from the middle of the region to the highest point. At the highest point the vertical velocity must be zero. Using Eq. 2–11a with the known acceleration and time to reach the highest point, we can solve for the square of the initial speed of the electron.

$$\upsilon_{y\,\text{top}} = \upsilon_{0y} + a_y t_{\text{top}} \quad \rightarrow \quad 0 = \upsilon_0 \sin\theta_0 + \left(-\frac{eE}{m}\right)\left(\frac{1}{2}\frac{\ell}{\upsilon_0\cos\theta_0}\right) \quad \rightarrow \quad \upsilon_0^2 = \frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)$$

Finally, using Eq. 2–11a we solve for the initial angle.

$$y_{\text{top}} = y_0 + \upsilon_{0y}t_{\text{top}} + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}h = \upsilon_0 \sin\theta_0\left(\frac{1}{2}\frac{\ell}{\upsilon_0\cos\theta_0}\right) - \frac{1}{2}\frac{eE}{m}\left(\frac{1}{2}\frac{\ell}{\upsilon_0\cos\theta_0}\right)^2 \quad \rightarrow$$

$$h = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\upsilon_0^2} = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\dfrac{eE}{2m}\left(\dfrac{\ell}{\sin\theta_0\cos\theta_0}\right)} = \ell\tan\theta_0 - \tfrac{1}{2}\ell\tan\theta_0$$

$$h = \tfrac{1}{2}\ell\tan\theta_0 \quad \rightarrow \quad \theta_0 = \tan^{-1}\frac{2h}{\ell} = \tan^{-1}\frac{2(1.0\text{ cm})}{7.2\text{ cm}} = 15.52^\circ \approx \boxed{16^\circ}$$

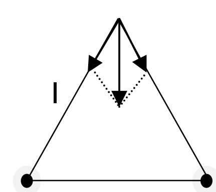
4. Coulomb observed experimentally that the force between two charged objects is directly proportional to the charge on each one. For example, if the charge on either object is tripled, then the force is tripled. This is not in agreement with a force that is proportional to the *sum* of the charges instead of to the *product* of the charges. Also, a charged object is not attracted to or repelled from a neutral insulating object. But if the numerator in Coulomb's law were proportional to the sum of the charges, then there would be a force between a neutral object and a charged object, because their sum would not be 0.

5. Two forces act on the mass: gravity and the electric force. Since the acceleration is smaller than the acceleration of gravity, the electric force must be upward, opposite the direction of the electric field. The charge on the object is therefore negative. Using Newton's second law, with down as the positive direction, we can calculate the magnitude of the charge on the object.

$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} \quad \rightarrow \quad mg + qE = ma \quad \rightarrow$$

$$q = \frac{m(a-g)}{E} = \frac{(1.0\text{ kg})(8.0\text{ m/s}^2 - 9.8\text{ m/s}^2)}{150\text{ N/C}} = \boxed{-1.2\times10^{-2}\text{ C}}$$

6. The magnitude of the electric field at the third vertex is the vector sum of the electric fields from each of the two charges. If we place the two charges at the lower two vertices, then the electric field at the top vertex will be vertically downward. By symmetry, the horizontal components of the electric field will cancel, leaving only the vertical components for the net field.



$$E = E_y = \frac{ke}{\ell^2}\sin 30^\circ + \frac{ke}{\ell^2}\sin 30^\circ = \boxed{\frac{ke}{\ell^2}}$$

The electric force on a third negative charge would be upward (opposite the direction of the electric field) and equal to the product of the electric field magnitude and the magnitude of the charge.

$$F = eE = \boxed{\frac{ke^2}{\ell^2}\text{, upward}}$$

7. Set the Coulomb electrical force equal to the Newtonian gravitational force on the Moon.

$$F_{\mathrm{E}} = F_{\mathrm{G}} \quad \rightarrow \quad k\frac{Q^2}{r_{\mathrm{orbit}}^2} = G\frac{M_{\mathrm{Moon}}M_{\mathrm{Earth}}}{r_{\mathrm{orbit}}^2} \quad \rightarrow$$

$$Q = \sqrt{\frac{GM_{\mathrm{Moon}}M_{\mathrm{Earth}}}{k}} = \sqrt{\frac{(6.67\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(7.35\times 10^{22}\ \mathrm{kg})(5.98\times 10^{24}\ \mathrm{kg})}{(8.988\times 10^{9}\ \mathrm{N\cdot m^2/C^2})}} = \boxed{5.71\times 10^{13}\ \mathrm{C}}$$

17

ELECTRIC POTENTIAL

Responses to Questions

1. If two points are at the same potential, then no NET work is done in moving a test charge from one point to the other. Along some segments of the path, some positive work might be done, but along other segments of the path, negative work would be done. And if the object is moved strictly along an equipotential line, then no work would be done along any segment of the path.

   Along any segment of the path where positive or negative work is done, a force would have to be exerted. If the object is moved along an equipotential line, then no force would be exerted along that segment of the path.

   This is analogous to climbing up and then back down a flight of stairs to get from one point to another point on the same floor of a building. Gravitational potential increased while going up the stairs, and decreased while going down the stairs. A force was required both to go up the stairs and to go down the stairs. If instead you walked on the level from one point to another, then the gravitational potential was constant, and no force was needed to change gravitational potential.

2. A negative charge will move toward a region of higher potential. A positive charge will move toward a region of lower potential. The potential energy of both charges decreases as they move, because they gain kinetic energy.

3. (*a*) Electric potential, a scalar, is the electric potential energy per unit charge at a point in space. Electric field, a vector, is the electric force per unit charge at a point in space.
   (*b*) Electric potential energy is the work done against the electric force in moving a charge from a specified location of zero potential energy to some other location. Electric potential is the electric potential energy per unit charge.

4. The potential energy of the electron is proportional to the voltage used to accelerate it. Thus, if the voltage is multiplied by a factor of 4, then the potential energy is increased by a factor of 4 also. Then, by energy conservation, we assume that all of the potential energy is converted to kinetic energy during the acceleration process. Thus the kinetic energy has increased by a factor of 4 also. Finally, since the speed is proportional to the square root of kinetic energy, the speed must increase by a factor of 2.

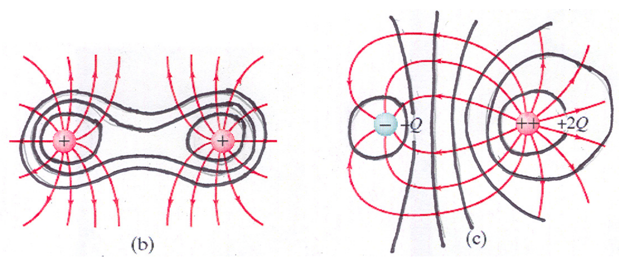
5. The electric field is zero at the midpoint of the line segment joining the two equal positive charges. The electric field due to each charge is of the same magnitude at that location, because the location is equidistant from both charges, but the two fields are in the opposite direction. Thus the net electric field is zero there. The electric potential is never zero along that line, except at infinity. The electric potential due to each charge is positive, so the total potential, which is the algebraic sum of the two potentials, is always positive.

6. A negative particle will have its electric potential energy decrease if it moves from a region of low electric potential to one of high potential. By Eq. 17–3, if the charge is negative and the potential difference is positive, then the change in potential energy will be negative and therefore decrease.

7. There is no general relationship between the value of V and the value of $\vec{\mathbf{E}}$. Instead, the magnitude of $\vec{\mathbf{E}}$ is equal to the rate at which V decreases over a short distance. Consider the point midway between two positive charges. $\vec{\mathbf{E}}$ is 0 there, but V is high. Or consider the point midway between two negative charges. $\vec{\mathbf{E}}$ is also 0 there, but V is low, because it is negative. Finally, consider the point midway between positive and negative charges of equal magnitude. There $\vec{\mathbf{E}}$ is not 0, because it points toward the negative charge, but V is zero.

8. Two equipotential lines cannot cross. That would indicate that a region in space had two different values for the potential. For example, if a 40-V line and a 50-V line crossed, then the potential at the point of crossing would be both 40 V and 50 V, which is impossible. As an analogy, imagine contour lines on a topographic map. They also never cross because one point on the surface of the Earth cannot have two different values for elevation above sea level. Likewise, the electric field is perpendicular to the equipotential lines. If two lines crossed, then the electric field at that point would point in two different directions simultaneously, which is not possible.

9. The equipotential lines (in black) are perpendicular to the electric field lines (in red).



10. Any imbalance of charge that exists would quickly be resolved. Suppose the positive plate, connected to the positive terminal of the battery, had more charge than the negative plate. Then negative charges from the negative battery terminal would be attracted to the negative plate by the more charged positive plate. This would continue only until the negative plate had the same magnitude of charge as the positive plate. If the negative plate became "overcharged," then the opposite transfer of charge would take place, again until equilibrium was reached. Another way to explain the balance of charge is that neither the battery nor the capacitor can create or destroy charge. Since they were neutral before they were connected, they must be neutral after they are connected. The charge removed from one plate appears as excess charge on the other plate. This is true regardless of the conductor size or shape.

11. (*a*) Once the two spheres are placed in contact with each other, they effectively become one larger conductor. They will have the same potential because the potential everywhere on a conducting surface is constant.

(*b*) Because the spheres are identical in size, an amount of charge $Q/2$ will flow from the initially charged sphere to the initially neutral sphere so that they will have equal charges.

12. A force is required to increase the separation of the plates of an isolated capacitor because you are pulling a positive plate away from a negative plate. Since unlike charges attract, it takes a force to move the oppositely charged plates apart. The work done in increasing the separation goes into increasing the electric potential energy stored between the plates. The capacitance decreases, and the potential between the plates increases since the charge has to remain the same.

13. If the electric field in a region of space is uniform, then you can infer that the electric potential is increasing or decreasing uniformly in that region. For example, if the electric field is 10 V/m in a region of space, then you can infer that the potential difference between two points 1 meter apart (measured parallel to the direction of the field) is 10 V.

    If the electric potential in a region of space is uniform, then you can infer that the electric field there is zero.

14. The electric potential energy of two unlike charges is negative if we take the 0 location for potential energy to be when the charges are infinitely far apart. The electric potential energy of two like charges is positive. In the case of unlike charges, work must be done to separate the charges. In the case of like charges, work must be done to move the charges together.

15. (*c*) If the voltage across a capacitor is doubled, then the amount of energy it can store is quadrupled: $\text{PE} = \frac{1}{2}CV^2$.

16. (*a*) If the capacitor is isolated, then Q remains constant, and $\text{PE} = \frac{1}{2}\frac{Q^2}{C}$ becomes $\text{PE}' = \frac{1}{2}\frac{Q^2}{KC}$ and the stored energy decreases.

    (*b*) If the capacitor remains connected to a battery so V does not change, then $\text{PE} = \frac{1}{2}CV^2$ becomes $\text{PE}' = \frac{1}{2}KCV^2$ and the stored energy increases.

17. (*a*) When the dielectric is removed, the capacitance decreases by a factor of K.

    (*b*) The charge decreases since $Q = CV$ and the capacitance decreases while the potential difference remains constant. The "lost" charge returns to the battery.

    (*c*) The potential difference stays the same because it is equal to the battery voltage.

    (*d*) If the potential difference remains the same and the capacitance decreases, then the energy stored in the capacitor must also decrease, since $\text{PE} = \frac{1}{2}CV^2$.

    (*e*) The electric field between the plates will stay the same because the potential difference across the plates and the distance between the plates remain constant.

18. We meant that the capacitance did not depend on the amount of charge stored or on the potential difference between the capacitor plates. Changing the amount of charge stored or the potential difference will not change the capacitance.

Responses to MisConceptual Questions

1. (*b*) The two different concepts, electric potential and electric potential energy, are often confused, since their names are so similar. The electric potential is determined by the electric field and is independent of the charge placed in the field. Therefore, doubling the charge will not affect the electric potential. The electric potential energy is the product of the electric potential and the electric charge. Doubling the charge will double the electric potential energy.

2. (*a*) It is very important to realize that the electric field is a vector, while the electric potential is a scalar. The electric field at the point halfway between the two charges is the sum of the electric fields from each charge. The magnitudes of the fields will be the same, but they will point in opposite directions. Their sum is therefore zero. The electric potential from each charge is positive. Since the potential is a scalar, the net potential at the midpoint is the sum of the two potentials, which will also be positive.

3. (*b*) The net electric field at the center is the vector sum of the electric fields due to each charge. The fields will have equal magnitudes at the center, but the fields from the charges at opposite corners point in opposite directions, so the net field will be zero. The electric potential from each charge is a nonzero scalar. At the center the magnitudes of the four potentials are equal and sum to a nonzero value.

4. (*d*) A common misconception is that the electric field and electric potential are proportional to each other. However, the electric field is proportional to the change in the electric potential. If the electric potential is constant (no change), then the electric field must be zero. The electric potential midway between equal but opposite charges is zero, but the electric field is not zero at that point, so (*a*) is incorrect. The electric field midway between two equal positive charges is zero, but the electric potential is not zero at that point, so (*b*) is incorrect. The electric field between two oppositely charged parallel plates is constant, but the electric potential decreases as one moves from the positive plate toward the negative plate, so (*c*) is incorrect.

5. (*b*) It is commonly thought that it only takes $2W$ to bring together the three charges. However, it takes W to bring two charges together. When the third charge is brought in, it is repelled by both of the other charges. It therefore takes an additional $2W$ to bring in the third charge. Adding the initial work to bring the first two charges together gives a total work of $3W$.

6. (*e*) A common misconception is that the electron feels the greater force because it experiences the greater acceleration. However, the magnitude of the force is the product of the electric field and the charge. Since both objects have the same magnitude of charge and are in the same electric field, they will experience the same magnitude force (but in opposite directions). Since the electron is lighter, it will experience a greater acceleration.

7. (*c*) It is often thought that the electron, with its greater acceleration and greater final speed, will have the greater final kinetic energy. However, the increase in kinetic energy is equal to the decrease in potential energy, which is the product of the object's charge and the change in potential. However, in this situation, the increase in kinetic energy is equal to the decrease in electric potential energy, which is the product of the object's charge and the change in electric potential through which it passes. The change in potential for each object has the same magnitude, but they have opposite signs since they move in opposite directions in the field. The magnitude of the charge of each object is the same, but they have opposite signs; thus they experience the same change in electric potential energy and therefore they have the same final kinetic energy.

8. (*d*), (*e*) The capacitance is determined by the shape of the capacitor (area of plates and separation distance) and the material between the plates (dielectric). The capacitance is the constant ratio of the charge on the plates to the potential difference between them. Changing the charge will change the potential difference but not the capacitance, so (*d*) is one correct answer. Changing the energy stored in a capacitor will change the charge on the plates and the potential difference between them, but will not change the capacitance, so (*e*) is also correct.

9. (*b*) When the plates were connected to the battery, a charge was established on the plates. As the battery is disconnected, this charge remains constant on the plates. The capacitance decreases as the plates are pulled apart, since the capacitance is inversely proportional to the separation distance. For the charge to remain constant with smaller capacitance, the voltage between the plates increases.

10. (*c*) A vector has both direction and magnitude. Electric potential, electric potential energy, and capacitance all have magnitudes but not direction. Electric field lines have a direction at every point in space, and the density of the electric field lines gives us information about the magnitude of the electric field in that region of space.

11. (*b*) The similarity in the names of these concepts can lead to confusion. The electric potential is determined by the electric field and is independent of the charge placed in the field. Therefore, changing the charge will not affect the electric potential. The electric potential energy is the product of the electric potential and the electric charge. Changing the sign of the charge will change the sign of the electric potential energy.

Solutions to Problems

1. The work done by the electric field can be found from Eq. 17–2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -(-7.7\times10^{-6}\text{ C})(+65\text{ V}) = \boxed{5.0\times10^{-4}\text{ J}}$$

2. The work done by the electric field can be found from Eq. 17–2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -(1.60\times10^{-19}\text{ C})(-45\text{ V} - 125\text{ V}) = \boxed{2.72\times10^{-17}\text{ J}}$$

$$= -(1\,e)(-170\text{ V}) = \boxed{1.70\times10^{2}\text{ eV}}$$

3. Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.

$$\Delta\text{PE} = q\Delta V = -\Delta\text{KE} \rightarrow$$

$$\Delta V = \frac{-\Delta\text{KE}}{q} = \frac{\text{KE}_{\text{initial}} - \text{KE}_{\text{final}}}{q} = \frac{mv^2}{2q} = \frac{(9.11\times10^{-31}\text{ kg})(6.0\times10^{5}\text{ m/s})^2}{2(-1.60\times10^{-19}\text{ C})} = -1.025\text{ V} \approx \boxed{-1.0\text{ V}}$$

The final potential should be lower than the initial potential in order to stop the electron.

4. The kinetic energy gained is equal to the work done on the electron by the electric field. The potential difference must be positive for the electron to gain potential energy. Use Eq. 17–2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \rightarrow W_{\text{ba}} = -qV_{\text{ba}} = -(-1.60\times10^{-19}\text{ C})(1.85\times10^{4}\text{ V}) = \boxed{2.96\times10^{-15}\text{ J}}$$

$$= -(-1\,e)(1.85\times10^{4}\text{ V}) = \boxed{1.85\times10^{4}\text{ eV}}$$

5. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 17–2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{6.45\times10^{-16}\text{ J}}{(-1.60\times10^{-19}\text{ C})} = \boxed{4030\text{ V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

6. We assume that the electric field is uniform. Use Eq. 17–4b, using the magnitude of the electric field.

$$E = \frac{V_{ba}}{d} = \frac{220\text{ V}}{6.8\times 10^{-3}\text{ m}} = \boxed{3.2\times 10^{4}\text{ V/m}}$$

7. The magnitude of the voltage can be found from Eq. 17–4b, using the magnitude of the electric field.

$$E = \frac{V_{ba}}{d} \rightarrow V_{ba} = Ed = (525\text{ V/m})(11.0\times 10^{-3}\text{ m}) = \boxed{5.78\text{ V}}$$

8. The distance between the plates is found from Eq. 17–4b, using the magnitude of the electric field.

$$E = \frac{V_{ba}}{d} \rightarrow d = \frac{V_{ba}}{E} = \frac{45\text{ V}}{1900\text{ V/m}} = \boxed{2.4\times 10^{-2}\text{ m}}$$

9. The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge. The helium nucleus has a charge of $2e$.

$$\Delta V = \frac{\Delta\text{PE}}{q} = -\frac{\Delta\text{KE}}{q} = -\frac{85.0\times 10^{3}\text{ eV}}{2e} = \boxed{-4.25\times 10^{4}\text{ V}}$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.

10. Find the distance corresponding to the maximum electric field, using Eq. 17–4b for the magnitude of the electric field.

$$E = \frac{V_{ba}}{d} \rightarrow d = \frac{V_{ba}}{E} = \frac{45\text{ V}}{3\times 10^{6}\text{ V/m}} = 1.5\times 10^{-5}\text{ m} \approx \boxed{2\times 10^{-5}\text{ m}}$$

11. By the work-energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 17–2b.

$$W_{\text{external}} + W_{\text{electric}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \rightarrow W_{\text{external}} - q(V_B - V_A) = \text{KE}_{\text{final}} \rightarrow$$

$$(V_B - V_A) = \frac{W_{\text{external}} - \text{KE}_{\text{final}}}{q} = \frac{15.0\times 10^{-4}\text{ J} - 4.82\times 10^{-4}\text{ J}}{-6.50\times 10^{-6}\text{ C}} = \boxed{-157\text{ V}}$$

Since the potential difference is negative, we see that $V_a > V_b$.

12. The kinetic energy of the electron is given in each case. Use the kinetic energy to find the speed.

(*a*) $$\tfrac{1}{2}mv^2 = \text{KE} \rightarrow v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(850\text{ eV})(1.60\times 10^{-19}\text{ J/eV})}{9.11\times 10^{-31}\text{ kg}}} = \boxed{1.7\times 10^{7}\text{ m/s}}$$

(*b*) $$\tfrac{1}{2}mv^2 = \text{KE} \rightarrow v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(0.50\times 10^{3}\text{ eV})(1.60\times 10^{-19}\text{ J/eV})}{9.11\times 10^{-31}\text{ kg}}} = \boxed{1.3\times 10^{7}\text{ m/s}}$$

13. The kinetic energy of the proton is given. Use the kinetic energy to find the speed.

$$\tfrac{1}{2}mv^2 = \text{KE} \rightarrow v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(4.2\times 10^{3}\text{ eV})(1.60\times 10^{-19}\text{ J/eV})}{1.67\times 10^{-27}\text{ kg}}} = \boxed{9.0\times 10^{5}\text{ m/s}}$$

$\boxed{14.}$ The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$\tfrac{1}{2}mv^2 = \text{KE} \quad \rightarrow \quad v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(5.53\times10^6 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{6.64\times10^{-27} \text{ kg}}} = \boxed{1.63\times10^7 \text{ m/s}}$$

15. Use Eq. 17–4b without the negative sign in order to find the magnitude of the voltage difference.

$$E = \frac{\Delta V}{\Delta x} \quad \rightarrow \quad \Delta V = E\Delta x = (3\times10^6 \text{ V/m})(1\times10^{-3} \text{ m}) = \boxed{3000 \text{ V}}$$

No harm is done because very little charge is transferred between you and the doorknob.

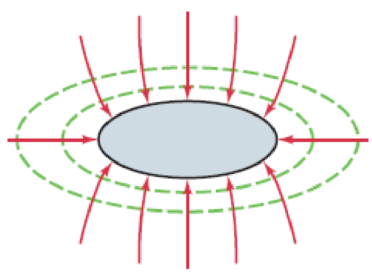
16. (*a*) The electron was accelerated through a potential difference of 4.8 kV (moving from low potential to high potential) in gaining 4.8 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron and has the exact opposite charge. Thus the proton gains the same kinetic energy, $\boxed{4.8 \text{ keV}}$.

(*b*) Both the proton and the electron have the same kinetic energy. Use that fact to find the ratio of the speeds.

$$\tfrac{1}{2}m_{\text{p}}v_{\text{p}}^2 = \tfrac{1}{2}m_{\text{e}}v_{\text{e}}^2 \quad \rightarrow \quad \frac{v_{\text{e}}}{v_{\text{p}}} = \sqrt{\frac{m_{\text{p}}}{m_{\text{e}}}} = \sqrt{\frac{1.67\times10^{-27} \text{ kg}}{9.11\times10^{-31} \text{ kg}}} = \boxed{42.8}$$

The electron will be moving 42.8 times as fast as the proton.

17.



18. Use Eq. 17–5 to find the potential.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{3.00\times10^{-6} \text{ C}}{1.50\times10^{-1} \text{ m}} = \boxed{1.80\times10^5 \text{ V}}$$

19. Use Eq. 17–5 to find the charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \quad \rightarrow \quad Q = (4\pi\varepsilon_0)rV = \left(\frac{1}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}\right)(0.15 \text{ m})(165 \text{ V}) = \boxed{2.8\times10^{-9} \text{ C}}$$

$\boxed{20.}$ The work required is the difference in potential energy between the two locations. The test charge has potential energy due to each of the other charges, given in Conceptual Example 17–7 as $\text{PE} = k\dfrac{Q_1Q_2}{r}$. So to find the work, calculate the difference in potential energy between the two locations. Let Q represent the $35\ \mu\text{C}$ charge, let q represent the $0.50\ \mu\text{C}$ test charge, and let d represent the 46-cm distance.

$$\text{PE}_{\text{initial}} = \frac{kQq}{d/2} + \frac{kQq}{d/2} \qquad \text{PE}_{\text{final}} = \frac{kQq}{[d/2 - 0.12 \text{ m}]} + \frac{kQq}{[d/2 + 0.12 \text{ m}]}$$

$$\text{Work} = \text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = \frac{kQq}{[d/2 - 0.12\text{ m}]} + \frac{kQq}{[d/2 + 0.12\text{ m}]} - 2\left(\frac{kQq}{d/2}\right)$$

$$= kQq\left[\frac{1}{[0.23\text{ m} - 0.12\text{ m}]} + \frac{1}{[0.23\text{ m} + 0.12\text{ m}]} - \frac{2}{0.23\text{ m}}\right]$$

$$= (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(35\times10^{-6}\text{ C})(0.50\times10^{-6}\text{ C})(3.25\text{ m}^{-1}) = 0.5117\text{ J} \approx \boxed{0.51\text{ J}}$$

An external force needs to do positive work to move the charge.

21. (*a*) The electric potential is given by Eq. 17–5.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{1.60\times10^{-19}\text{ C}}{2.5\times10^{-15}\text{ m}} = 5.754\times10^{5}\text{ V} \approx \boxed{5.8\times10^{5}\text{ V}}$$

(*b*) The potential energy of a pair of charges is derived in Conceptual Example 17.7.

$$\text{PE} = k\frac{Q_1Q_2}{r} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\text{ C})^2}{2.5\times10^{-15}\text{ m}} = \boxed{9.2\times10^{-14}\text{ J}} = 0.58\text{ MeV}$$

22. The potential at the corner is the sum of the potentials due to each of the charges, found using Eq. 17–5.

$$V = \frac{k(3Q)}{\ell} + \frac{kQ}{\sqrt{2}\ell} + \frac{k(-2Q)}{\ell} = \frac{kQ}{\ell}\left(1 + \frac{1}{\sqrt{2}}\right) = \boxed{\frac{kQ}{2\ell}(2+\sqrt{2})}$$

23. By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed and has no kinetic energy. When the electron is far away, there is no potential energy.

$$E_{\text{initial}} = E_{\text{final}} \quad\to\quad \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad\to\quad \frac{k(-e)(Q)}{r} = \tfrac{1}{2}mv^2 \quad\to$$

$$v = \sqrt{\frac{2k(-e)(Q)}{mr}} = \sqrt{\frac{2(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})(-6.50\times10^{-9}\text{ C})}{(9.11\times10^{-31}\text{ kg})(0.245\text{ m})}}$$

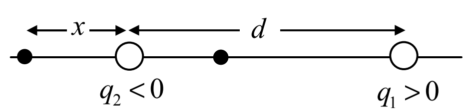
$$= \boxed{9.15\times10^{6}\text{ m/s}}$$

24. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$E_{\text{initial}} = E_{\text{final}} \quad\to\quad \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad\to\quad \frac{kQ^2}{r} = 2(\tfrac{1}{2}mv^2) \quad\to$$

$$v = \sqrt{\frac{kQ^2}{mr}} = \sqrt{\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(9.5\times10^{-6}\text{ C})^2}{(1.0\times10^{-6}\text{ kg})(0.053\text{ m})}} = \boxed{3.9\times10^{3}\text{ m/s}}$$

25. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges

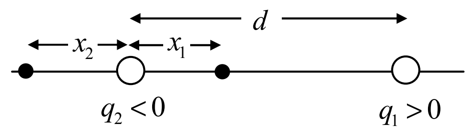


are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, they are labeled x. Take the positive direction to the right.

$$E = k\frac{|q_2|}{x^2} - k\frac{q_1}{(d+x)^2} = 0 \quad\rightarrow\quad |q_2|(d+x)^2 = q_1x^2 \quad\rightarrow$$

$$x = \frac{\sqrt{|q_2|}}{\sqrt{q_1}-\sqrt{|q_2|}}d = \frac{\sqrt{2.0\times10^{-6}\text{ C}}}{\sqrt{3.0\times10^{-6}\text{ C}}-\sqrt{2.0\times10^{-6}\text{ C}}}(4.0\text{ cm}) = \boxed{18\text{ cm left of }q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.

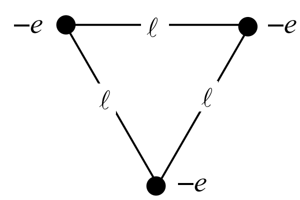


$$V_{\text{location 1}} = \frac{kq_1}{(d-x_1)} + \frac{kq_2}{x_1} = 0 \quad\rightarrow\quad x_1 = \frac{q_2d}{(q_2-q_1)} = \frac{(-2.0\times10^{-6}\text{ C})(4.0\text{ cm})}{(-5.0\times10^{-6}\text{ C})}$$

$$V_{\text{location 2}} = \frac{kq_1}{(d+x_2)} + \frac{kq_2}{x_2} = 0 \quad\rightarrow\quad x_2 = -\frac{q_2d}{(q_1+q_2)} = -\frac{(-2.0\times10^{-6}\text{ C})(4.0\text{ cm})}{(1.0\times10^{-6}\text{ C})} = 8.0\text{ cm}$$

So the two locations where the potential is zero are $\boxed{\text{1.6 cm from the negative charge toward the positive charge and 8.0 cm from the negative charge away from the positive charge.}}$

26. Let the side length of the equilateral triangle be $\boxed{\ell.}$ Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0.$ The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.



Thus $W_2 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}.$ The work done in bringing the third electron to its final location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell} - \frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell}.$ The total work done is the sum $W_1 + W_2 + W_3.$

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\ell} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{\ell} = \frac{3(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(1.0\times10^{-10}\text{ m})}$$

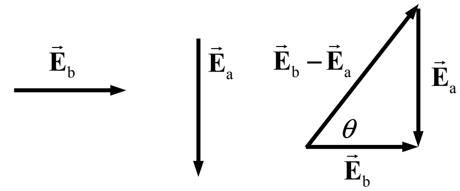
$$= \boxed{6.9\times10^{-18}\text{ J}} = 6.9\times10^{-18}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{43\text{ eV}}$$

27. (*a*) The potential due to a point charge is given by Eq. 17–5.

$$V_{\text{ba}} = V_{\text{b}} - V_{\text{a}} = \frac{kq}{r_{\text{b}}} - \frac{kq}{r_{\text{a}}} = kq\left(\frac{1}{r_{\text{b}}} - \frac{1}{r_{\text{a}}}\right)$$

$$= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-3.8\times10^{-6}\text{ C})\left(\frac{1}{0.88\text{ m}} - \frac{1}{0.62\text{ m}}\right) = \boxed{1.6\times10^4\text{ V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 16–4a. The direction of the electric field due to a negative charge is toward the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.



$$\vec{\mathbf{E}}_\mathrm{b} = \frac{k|q|}{r_b^{\,2}}\text{ right} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.8\times10^{-6}\ \mathrm{C})}{(0.88\ \mathrm{m})^2}\text{ right} = 4.4114\times10^4\ \mathrm{V/m,\ right}$$

$$\vec{\mathbf{E}}_\mathrm{a} = \frac{k|q|}{r_b^{\,2}}\text{ down} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.8\times10^{-6}\ \mathrm{C})}{(0.62\ \mathrm{m})^2}\text{ down} = 8.8871\times10^4\ \mathrm{V/m,\ down}$$

$$\left|\vec{\mathbf{E}}_\mathrm{b} - \vec{\mathbf{E}}_\mathrm{a}\right| = \sqrt{E_\mathrm{a}^2 + E_\mathrm{b}^2} = \sqrt{(4.4114\times10^4\ \mathrm{V/M})^2 (8.8871\times10^4\ \mathrm{V/m})^2} = \boxed{9.9\times10^4\ \mathrm{V/m}}$$

$$\theta = \tan^{-1}\frac{E_\mathrm{a}}{E_\mathrm{b}} = \tan^{-1}\frac{88871}{44114} = \boxed{64^\circ}$$

28. The potential energy of the two-charge configuration (assuming they are both point charges) is given in Conceptual Example 17–7. Note that the charges have equal but opposite charge.

$$\mathrm{PE} = k\frac{Q_1Q_2}{r} = -k\frac{e^2}{r}$$

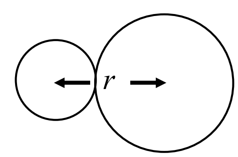
$$\Delta\mathrm{PE} = \mathrm{PE}_\mathrm{final} - \mathrm{PE}_\mathrm{initial} = ke^2\left(\frac{1}{r_\mathrm{initial}} - \frac{1}{r_\mathrm{final}}\right)$$

$$= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2\left(\frac{1}{0.110\times10^{-9}\ \mathrm{m}} - \frac{1}{0.100\times10^{-9}\ \mathrm{m}}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)$$

$$= -1.31\ \mathrm{eV}$$

Thus $\boxed{1.3\ \mathrm{eV}}$ of potential energy was lost.

29. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.



$$\mathrm{PE}_\mathrm{initial} = \mathrm{PE}_\mathrm{final} \quad\rightarrow\quad eV_\mathrm{initial} = k\frac{e(14e)}{r} \quad\rightarrow$$

$$V_\mathrm{initial} = k\frac{14e}{r} = (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(14)(1.60\times10^{-19}\ \mathrm{C})}{(1.2+3.6)\times10^{-15}\ \mathrm{m}} = \boxed{4.2\times10^6\ \mathrm{V}}$$

30. Use Eq. 17–5.

$$V_\mathrm{BA} = V_\mathrm{B} - V_\mathrm{A} = \left(\frac{kq}{d-b} + \frac{k(-q)}{b}\right) - \left(\frac{kq}{b} + \frac{k(-q)}{d-b}\right) = kq\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right)$$

$$= 2kq\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2kq(2b-d)}{b(d-b)}}$$

31. (*a*) The electric potential is found from Eq. 17–5.

$$V_\mathrm{initial} = k\frac{q_\mathrm{p}}{r_\mathrm{atom}} = (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.60\times10^{-19}\ \mathrm{C})}{0.53\times10^{-10}\ \mathrm{m}} \approx \boxed{27\ \mathrm{V}}$$

(*b*) The kinetic energy can be found from the fact that the magnitude of the net force on the electron, which is the attraction by the proton, is causing circular motion.

$$|F_{\text{net}}| = \frac{m_e v^2}{r_{\text{atom}}} = k\frac{q_p|q_e|}{r_{\text{atom}}^2} \rightarrow m_e v^2 = k\frac{e^2}{r_{\text{atom}}} \rightarrow \text{KE} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}k\frac{e^2}{r_{\text{aton}}}$$

$$= \tfrac{1}{2}(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.53\times10^{-10}\ \text{m})} = 2.171\times10^{-18}\ \text{J} \approx \boxed{2.2\times10^{-18}\ \text{J}}$$

$$= 2.171\times10^{-18}\ \text{J}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}13.57\ \text{eV} \approx \boxed{14\ \text{eV}}$$

(*c*) The total energy of the electron is the sum of its KE and PE. The PE is found from Eq. 17–2a and is negative since the electron's charge is negative.

$$E_{\text{total}} = \text{PE}+\text{KE} = -eV + \tfrac{1}{2}m_e v^2 = -k\frac{e^2}{r_{\text{atom}}} + \tfrac{1}{2}k\frac{e^2}{r_{\text{atom}}} = -\tfrac{1}{2}k\frac{e^2}{r_{\text{atom}}}$$

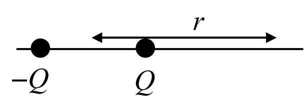
$$= -2.171\times10^{-18}\ \text{J} \approx \boxed{-2.2\times10^{-18}\ \text{J}} \approx \boxed{-14\ \text{eV}}$$

(*d*) If the electron is taken to infinity at rest, then both its PE and KE would be 0. The amount of energy needed by the electron to have a total energy of 0 is just the opposite of the answer to part (*c*), $\boxed{2.2\times10^{-18}\ \text{J}}$ or $\boxed{14\ \text{eV}}$.

32. The dipole moment is the product of the magnitude of one of the charges and the separation distance.

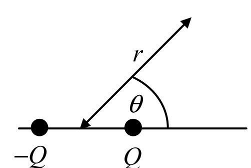
$$p = Q\ell = (1.60\times10^{-19}\ \text{C})(0.53\times10^{-10}\ \text{m}) = \boxed{8.5\times10^{-30}\ \text{C}\cdot\text{m}}$$

33. The potential due to the dipole is given by Eq. 17–6b.



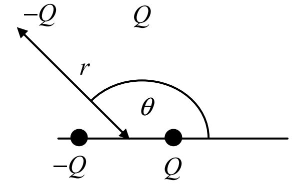
(*a*) $$V = \frac{kp\cos\theta}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.2\times10^{-30}\ \text{C}\cdot\text{m})\cos 0}{(2.4\times10^{-9}\ \text{m})^2}$$

$$= \boxed{6.6\times10^{-3}\ \text{V}}$$



(*b*) $$V = \frac{kp\cos\theta}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.2\times10^{-30}\ \text{C}\cdot\text{m})\cos 45^\circ}{(2.4\times10^{-9}\ \text{m})^2}$$

$$= \boxed{4.6\times10^{-3}\ \text{V}}$$



(*c*) $$V = \frac{kp\cos\theta}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.2\times10^{-30}\ \text{C}\cdot\text{m})\cos 135^\circ}{(2.4\times10^{-9}\ \text{m})^2}$$

$$= \boxed{-4.6\times10^{-3}\ \text{V}}$$

34. We assume that $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ are equal in magnitude and that each makes a 52° angle with $\vec{\mathbf{p}}$. The magnitude of $\vec{\mathbf{p}}_1$ is also given by $p_1 = qd$, where q is the charge on the hydrogen atom and d is the distance between the H and the O.

$$p = 2p_1\cos 52^\circ \rightarrow p_1 = \frac{p}{2\cos 52^\circ} = qd \rightarrow$$

$$q = \frac{p}{2d\cos 52^\circ} = \frac{6.1\times10^{-30}\ \text{C}\cdot\text{m}}{2(0.96\times10^{-10}\ \text{m})\cos 52^\circ} = \boxed{5.2\times10^{-20}\ \text{C}},\ \text{about } 3.2\times \text{electron's charge}$$

35. The capacitance is found from Eq. 17–7.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{2.5\times10^{-3}\ \text{C}}{960\ \text{V}} = \boxed{2.6\times10^{-6}\ \text{F}}$$

36. The voltage is found from Eq. 17–7.

$$Q = CV \quad \rightarrow \quad V = \frac{Q}{C} = \frac{16.5\times10^{-8}\ \text{C}}{8.5\times10^{-9}\ \text{F}} = \boxed{19\ \text{V}}$$

37. We assume the capacitor is fully charged, according to Eq. 17–7.

$$Q = CV = (5.00\times10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{6.00\times10^{-5}\ \text{C}}$$

38. The area can be found from Eq. 17–8.

$$C = \frac{\varepsilon_0 A}{d} \quad \rightarrow \quad A = \frac{Cd}{\varepsilon_0} = \frac{(0.20\ \text{F})(3.2\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{7.2\times10^{7}\ \text{m}^2}$$

Note that this is not very realistic—if it were square, each side would be about 8500 m.

39. Let Q_1 and V_1 be the initial charge and voltage on the capacitor, and let Q_2 and V_2 be the final charge and voltage on the capacitor. Use Eq. 17–7 to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \quad Q_2 = CV_2 \quad Q_2 - Q_1 = CV_2 - CV_1 = C(V_2 - V_1) \quad \rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{15\times10^{-6}\ \text{C}}{24\ \text{V}} = \boxed{6.3\times10^{-7}\ \text{F}}$$

40. The desired electric field is the value of V/d for the capacitor. Combine Eq. 17–7 and Eq. 17–8 to find the charge.

$$Q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(45.0\times10^{-4}\ \text{m}^2)(8.50\times10^{5}\ \text{V/m})$$

$$= \boxed{3.39\times10^{-8}\ \text{C}}$$

41. Combine Eq. 17–7 and Eq. 17–8 to find the area.

$$Q = CV = \frac{\varepsilon_0 AV}{d} = \varepsilon_0 AE \quad \rightarrow \quad A = \frac{Q}{\varepsilon_0 E} = \frac{(4.2\times10^{-6}\ \text{C})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\left(\dfrac{2000\ \text{V}}{10^{-3}\ \text{m}}\right)} = \boxed{0.24\ \text{m}^2}$$

42. The work to move the charge between the capacitor plates is $W = qV$, where V is the voltage difference between the plates, assuming that $q << Q$ so that the charge on the capacitor does not change appreciably. The charge is then found from Eq. 17–7.

$$W = qV = q\left(\frac{Q}{C}\right) \quad \rightarrow \quad Q = \frac{CW}{q} = \frac{(15\times10^{-6}\ \text{F})(18\ \text{J})}{0.30\times10^{-3}\ \text{C}} = \boxed{0.90\ \text{C}}$$

The assumption that $q << Q$ is justified.

43. Use Eq. 17–8 for the capacitance.

$$C=\frac{\varepsilon_0 A}{d} \quad\rightarrow\quad d=\frac{\varepsilon_0 A}{C}=\frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.0\times10^{-4}\ \mathrm{m^2})}{(1\ \mathrm{F})}=\boxed{9\times10^{-16}\ \mathrm{m}}$$

$\boxed{\text{No}}$, this is not practically achievable. The gap would have to be smaller than the radius of a proton.

$\boxed{44.}$ From Eq. 17–4a, the voltage across the capacitor is the magnitude of the electric field times the separation distance of the plates. Use that with Eq. 17–7.

$$Q=CV=CEd \quad\rightarrow\quad E=\frac{Q}{Cd}=\frac{(62\times10^{-6}\ \mathrm{C})}{(0.80\times10^{-6}\ \mathrm{F})(2.0\times10^{-3}\ \mathrm{m})}=\boxed{3.9\times10^{4}\ \mathrm{V/m}}$$

45. The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge, and there is no neutralization of charge by combining positive and negative charges. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_{1\,\text{initial}}=C_1V_{1\,\text{initial}} \quad Q_{2\,\text{initial}}=C_2V_{2\,\text{initial}} \quad Q_{1\,\text{final}}=C_1V_{\text{final}} \quad Q_{2\,\text{final}}=C_2V_{\text{final}}$$

$$Q_{\text{Total}}=Q_{1\,\text{initial}}+Q_{2\,\text{initial}}=Q_{1\,\text{final}}+Q_{2\,\text{final}}=C_1V_{1\,\text{initial}}+C_2V_{2\,\text{initial}}=C_1V_{\text{final}}+C_2V_{\text{final}} \quad\rightarrow$$

$$V_{\text{final}}=\frac{C_1V_{1\,\text{initial}}+C_2V_{2\,\text{initial}}}{C_1+C_2}=\frac{(2.50\times10^{-6}\ \mathrm{F})(746\ \mathrm{V})+(6.80\times10^{-6}\ \mathrm{F})(562\ \mathrm{V})}{(9.30\times10^{-6}\ \mathrm{F})}$$

$$=611.46\ \mathrm{V}\approx\boxed{611\ \mathrm{V}}=V_1=V_2$$

$$Q_{1\,\text{final}}=C_1V_{\text{final}}=(2.50\times10^{-6}\ \mathrm{F})(611.46)=\boxed{1.53\times10^{-3}\ \mathrm{C}}$$

$$Q_2=C_2V_{\text{final}}=(6.80\times10^{-6}\ \mathrm{F})(611.46)=\boxed{4.16\times10^{-3}\ \mathrm{C}}$$

46. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}}=C_1V_{1\,\text{initial}} \quad Q_{1\,\text{final}}=C_1V_{\text{final}} \quad Q_{2\,\text{final}}=C_2V_{\text{final}}$$

$$Q_{\text{Total}}=Q_{1\,\text{final}}+Q_{2\,\text{final}}=(C_1+C_2)V_{\text{final}} \quad\rightarrow\quad C_1V_{1\,\text{initial}}=(C_1+C_2)V_{\text{final}} \quad\rightarrow$$

$$C2=C1\left(\frac{V_{1\,\text{initial}}}{V_{\text{final}}}-1\right)=(7.7\times10^{-6}\ \mathrm{F})\left(\frac{165\ \mathrm{V}}{15\ \mathrm{V}}-1\right)=\boxed{7.7\times10^{-5}\ \mathrm{F}}$$

47. Use Eq. 17–9 to calculate the capacitance with a dielectric.

$$C=K\varepsilon_0\frac{A}{d}=(2.2)(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})\frac{(6.6\times10^{-2}\ \mathrm{m})^2}{(1.8\times10^{-3}\ \mathrm{m})}=\boxed{4.7\times10^{-11}\ \mathrm{F}}$$

48. Use Eq. 17–9 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0 \frac{A}{d} = (7)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\frac{\pi(5.0\times10^{-2}\ \text{m})^2}{(2.8\times10^{-3}\ \text{m})} = \boxed{1.7\times10^{-10}\ \text{F}}$$

49. Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of K, the dielectric constant.

$$Q = C_{\text{initial}}V_{\text{initial}} = C_{\text{final}}V_{\text{final}} \rightarrow V_{\text{final}} = V_{\text{initial}}\frac{C_{\text{initial}}}{C_{\text{final}}} = V_{\text{initial}}\frac{C_{\text{initial}}}{KC_{\text{initial}}} = (21.0\ \text{V})\frac{1}{2.2} = \boxed{9.5\ \text{V}}$$

50. The initial charge on the capacitor is $Q_{\text{initial}} = C_{\text{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\text{final}} = KC_{\text{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text{final}} = C_{\text{final}}V$.

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}} = C_{\text{final}}V - C_{\text{initial}}V = (K-1)C_{\text{initial}}V = (7-1)(3.5\times10^{-9}\ \text{F})(32\ \text{V})$$
$$= \boxed{6.7\times10^{-7}\ \text{C}}$$

51. The capacitance is found from Eq. 17–7, with the voltage given by Eq. 17–4 (ignoring the sign).

$$Q = CV = C(Ed) \rightarrow C = \frac{Q}{Ed} = \frac{0.675\times10^{-6}\ \text{C}}{(8.24\times10^4\ \text{V/m})(1.95\times10^{-3}\ \text{m})} = \boxed{4.20\times10^{-9}\ \text{F}}$$

The plate area is found from Eq. 17–9.

$$C = K\varepsilon_0\frac{A}{d} \rightarrow A = \frac{Cd}{K\varepsilon_0} = \frac{(4.20\times10^{-9}\ \text{F})(1.95\times10^{-3}\ \text{m})}{(3.75)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{0.247\ \text{m}^2}$$

52. The stored energy is given by Eq. 17–10.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(2.8\times10^{-9}\ \text{F})(650\ \text{V})^2 = \boxed{5.9\times10^{-4}\ \text{J}}$$

53. The capacitance can be found from the stored energy using Eq. 17–10.

$$\text{PE} = \tfrac{1}{2}CV^2 \rightarrow C = \frac{2(\text{PE})}{V^2} = \frac{2(1200\ \text{J})}{(5.0\times10^3\ \text{V})^2} = \boxed{9.6\times10^{-5}\ \text{F}}$$

54. The two charged plates form a capacitor. Use Eq. 17–8 to calculate the capacitance and Eq. 17–10 for the energy stored in the capacitor.

$$C = \frac{\varepsilon_0 A}{d} \quad \text{PE} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 d}{\varepsilon_0 A} = \frac{1}{2}\frac{(3.7\times10^{-4}\ \text{C})^2(1.5\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})^2} = \boxed{1.8\times10^3\ \text{J}}$$

55. (*a*) Use Eq. 17–8 to estimate the capacitance.

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(4.5\ \text{in.}\times0.0254\ \text{m/in.})^2}{(4\times10^{-2}\ \text{m})} = 9.081\times10^{-12}\ \text{F} \approx \boxed{9\times10^{-12}\ \text{F}}$$

(*b*) Use Eq. 17–7 to estimate the charge on each plate.

$$Q = CV = (9.081\times10^{-12}\ \text{F})(9\ \text{V}) = 8.173\times10^{-11}\ \text{C} \approx \boxed{8\times10^{-11}\ \text{C}}$$

(*c*) Use Eq. 17–4b to estimate the (assumed uniform) electric field between the plates. The actual location of the field measurement is not critical in this approximation.

$$E = \frac{V}{d} = \frac{9\ \text{V}}{4\times10^{-2}\ \text{m}} = 225\ \text{V/m} \approx \boxed{200\ \text{V/m}}$$

(*d*) By energy conservation, the work done by the battery to charge the plates is the potential energy stored in the capacitor, given by Eq. 17–10.

$$\text{PE} = \tfrac{1}{2}QV = \tfrac{1}{2}(8.173\times10^{-11}\ \text{C})(9\ \text{V}) = 3.678\times10^{-10}\ \text{J} \approx \boxed{4\times10^{-10}\ \text{J}}$$

(*e*) If a dielectric is inserted, then the $\boxed{\text{capacitance}}$ changes. Thus the $\boxed{\text{charge}}$ on the capacitor and the $\boxed{\text{work done}}$ by the battery also change. The electric field does not change because it only depends on the battery voltage and the plate separation.

56\. (*a*) From Eq. 17–8 for the capacitance, $C = \varepsilon_0 \frac{A}{d}$, since the separation of the plates is halved, the capacitance is doubled. Then from Eq. 17–10 for the energy, $\text{PE} = \frac{1}{2}\frac{Q^2}{C}$, since the charge is constant and the capacitance is doubled, the energy is halved. So the energy stored changes by a factor of $\boxed{\tfrac{1}{2}}$.

(*b*) The work done to move the plates together will be negative work, since the plates will naturally attract each other. The work is the change in the stored energy.

$$\text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = \tfrac{1}{2}\text{PE}_{\text{initial}} - \text{PE}_{\text{initial}} = -\tfrac{1}{2}\text{PE}_{\text{initial}} = -\tfrac{1}{2}\left(\tfrac{1}{2}\frac{Q^2}{C}\right) = -\tfrac{1}{4}\frac{Q^2}{\varepsilon_0 \frac{A}{d}} = \boxed{-\frac{Q^2 d}{4\varepsilon_0 A}}$$

57\. The energy density is given by Eq. 17–11.

$$\text{Energy density} = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(150\ \text{V/m})^2 = \boxed{1.0\times10^{-7}\ \text{J/m}^3}$$

58\. (*a*) Before the two capacitors are connected, all of the stored energy is in the first capacitor. Use Eq. 17–10.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(3.70\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = 2.664\times10^{-4}\ \text{J} \approx \boxed{2.66\times10^{-4}\ \text{J}}$$

(*b*) After the first capacitor is disconnected from the battery, the total charge must remain constant, and the voltage across each capacitor must be the same, since each capacitor plate is connected to a corresponding plate on the other capacitor by a connecting wire which always has a constant potential. Use the total charge and the fact of equal potentials to find the charge on each capacitor and then calculate the stored energy.

$$Q_{\text{total}} = C_1 V_{\text{initial}} = (3.70\times10^{-6}\ \text{F})(12.0\ \text{V}) = 4.44\times10^{-5}\ \text{C}$$

$$Q_1 = C_1 V \quad Q_2 = C_2 V \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{\text{total}} - Q_1}{C_2} \quad \rightarrow$$

$$Q_1 = Q_{\text{total}}\frac{C_1}{C_1 + C_2} = (4.44\times10^{-5}\ \text{C})\frac{(3.70\times10^{-6}\ \text{F})}{(8.70\times10^{-6}\ \text{F})} = 1.888\times10^{-5}\ \text{C}$$

$$Q_2 = Q_{\text{total}} - Q_1 = 4.44\times10^{-5}\ \text{C} - 1.888\times10^{-5}\ \text{C} = 2.552\times10^{-5}\ \text{C}$$

$$\text{PE}_{\text{total}} = \text{PE}_1 + \text{PE}_2 = \tfrac{1}{2}\frac{Q_1^2}{C_1} + \tfrac{1}{2}\frac{Q_2^2}{C_2} = \tfrac{1}{2}\left[\frac{(1.888\times10^{-5}\ \text{C})^2}{(3.70\times10^{-6}\ \text{F})} + \frac{(2.552\times10^{-5}\ \text{C})^2}{(5.00\times10^{-6}\ \text{F})}\right]$$

$$= 1.133\times10^{-4}\ \text{J} \approx \boxed{1.13\times10^{-4}\ \text{J}}$$

(*c*) Subtract the two energies to find the change.

$$\Delta\text{PE} = \text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = 1.133\times10^{-4}\ \text{J} - 2.664\times10^{-4}\ \text{J} = \boxed{-1.53\times10^{-4}\ \text{J}}$$

59. To turn decimal into binary, we need to use the largest power of 2 that is smaller than the decimal number with which we start. That would be $2^6 = 64,$ so there is a "1" in the 64's place. Subtracting 64 from 116 leaves 52. The next power of 2 is $2^5 = 32,$ so there is a "1" in the 32's place. Subtracting 32 from 52 leaves 20. The next power of 2 is $2^4 = 16,$ so there is a "1" in the 16's place. Subtracting 16 from 20 leaves 4, so there is a "0" in the 8's place, a "1" in the 4's place, a "0" in the 2's place, and a "0" in the 1's place. $64+32+16+0+4+0+0=116,$ so 116 decimal = $\boxed{\text{1110100 binary.}}$

60. The given binary number has a "1" in the 1's place, the 4's place, the 16's place, and the 64's place, so we have $1+4+16+64=85,$ and 01010101 binary = $\boxed{\text{85 decimal}}$. Another way to calculate it is that $01010101 = 2^0 + 2^2 + 2^4 + 2^6 = 1+4+16+64 = 85.$

61. Each "1" in the given binary contributes a certain power of 2 to the total value.

$$1010101010101010 = 2^1 + 2^3 + 2^5 + 2^7 + 2^9 + 2^{11} + 2^{13} + 2^{15}$$
$$= 2+8+32+128+512+2048+8192+32768 = \boxed{\text{43,690 decimal}}$$

62. (*a*) A 4-bit number can represent 16 different values, from 0 to 15. Thus if 5.0 volts is to be the maximum value, then each binary number would represent $5.0\ \text{V}/15 = 1/3\ \text{V}.$ Since 1011 binary $= 1+2+8 = 11$ decimal, 1011 binary $= 11(\tfrac{1}{3}\text{V}) = \boxed{3\tfrac{2}{3}\text{V}}$.

(*b*) 2.0 volts is 6 units of 1/3 volt each, so the binary representation is $\boxed{0110}$, which is equal to 6 in the decimal system.

63. (*a*) A 16-bit sampling would be able to represent $2^{16} = \boxed{65{,}536}$ different voltages.

(*b*) A 24-bit sampling would be able to represent $2^{24} = \boxed{16{,}777{,}216}$ different voltages.

(*c*) For 3 subpixels each with an 8-bit value, 24 bits are needed. This is the same as part (*b*), so $2^{24} = \boxed{16{,}777{,}216}$ different colors are possible.

64. We see that they have a base 4 counting system, because the decimal number 5 is represented as 11, which would be a "1" in the 1's place and a "1" in the 4's place. Making an analogy to humans, we have 5 digits on each hand and use base 10. Since the extraterrestrials use base 4, they must have 2 fingers on each hand, for a $\boxed{\text{total of 4 fingers}}$.

65. (*a*) The monitor screen is redrawn at regular intervals. Even though the mouse cursor moves in a continuous motion, its location is only drawn at those regular intervals. It is somewhat like a strobe light effect.

(*b*) Let the interval between images be "T" and construct a table.

Position #1 Time = 0

Position #2 Time = T

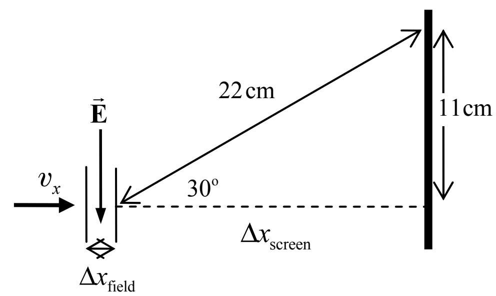
Position #3 Time = 2T

Position #4 Time = 3T

Position #15 Time = 14T = 0.25 seconds

Thus there are 15 images in 0.25 seconds. The refresh time would be $T = 0.25/14 = 0.018$ second. The refresh rate is the reciprocal of this, which is $\boxed{56\text{ Hz}}$.

66. Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved. The horizontal speed of the electron v_x can be found from the accelerating potential V. Next, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron an upward velocity, v_y. We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen, moving at an angle of approximately 30°.



Acceleration:

$$\text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad \rightarrow \quad eV = \tfrac{1}{2}mv_x^2 \quad \rightarrow \quad v_x = \sqrt{2eV/m}$$

Deflection:

$$\text{Time in field:} \quad \Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_{y0} + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} mv_x^2}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m\dfrac{2eV}{m}}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2(9.0\times10^3\text{ V})(0.11\text{ m})}{[(0.22\text{ m})\cos 30°](0.028\text{ m})}$$

$$= 3.712\times10^5\text{ V/m} \approx \boxed{3.7\times10^5\text{ V/m}}$$

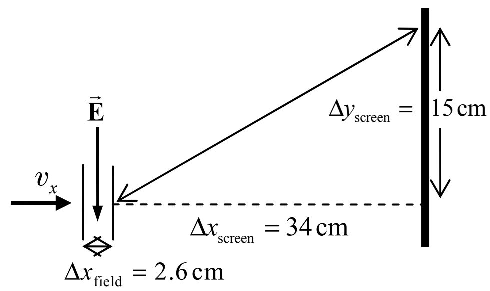
As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_{y0}t_{\text{field}} + \tfrac{1}{2}a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE(\Delta x_{\text{field}})^2}{2m\left(\dfrac{2eV}{m}\right)} = \frac{E(\Delta x_{\text{field}})^2}{4V}$$

$$= \frac{(3.712\times10^5 \text{ V/m})(2.8\times10^{-2} \text{ m})^2}{4(9000 \text{ V})} = 8.1\times10^{-3} \text{ m}$$

This is about 7% of the total 11-cm vertical deflection. For an estimation, our approximation is acceptable.

67. If there were no deflecting field, then the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons toward one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the electron's motion and see the diagram, which is a top view. First, during the horizontal acceleration phase, energy will be conserved. The horizontal speed of the electron v_x can be found from the accelerating potential V. Next, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, v_y. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.



Acceleration:

$$\text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad \rightarrow \quad eV = \tfrac{1}{2}mv_x^2 \quad \rightarrow \quad v_x = \sqrt{2eV/m}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_{y0} + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \dfrac{2eV}{m}}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2(6.0\times10^3 \text{ V})(0.15 \text{ m})}{(0.34 \text{ m})(0.026 \text{ m})}$$

$$= 2.04\times10^5 \text{ V/m} \approx 2.0\times10^5 \text{ V/m}$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE(\Delta x_{\text{field}})^2}{2m\left(\frac{2eV}{m}\right)} = \frac{E(\Delta x_{\text{field}})^2}{4V}$$

$$= \frac{(2.04\times10^5\ \text{V/m})(2.6\times10^{-2}\ \text{m})^2}{4(6000\ \text{V})} = 6\times10^{-3}\ \text{m}$$

This is about 4% of the total 15-cm vertical deflection. Thus for an estimation, our approximation is acceptable. The field must vary from $\boxed{+2.0\times10^5\ \text{V/m to} -2.0\times10^5\ \text{V/m}}$.

68. (*a*) The energy is related to the charge and the potential difference by Eq. 17–3.

$$\Delta\text{PE} = q\Delta V \quad \to \quad \Delta V = \frac{\Delta\text{PE}}{q} = \frac{5.2\times10^6\ \text{J}}{4.0\ \text{C}} = \boxed{1.3\times10^6\ \text{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C. Use Eq. 14–2 and Eq. 14–4.

$$Q = mc\Delta T + mL_{\text{F}} \quad \to$$

$$m = \frac{Q}{c\Delta T + L_{\text{F}}} = \frac{5.2\times10^6\ \text{J}}{\left(4186\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(80\ \text{C}^\circ) + \left(22.6\times10^5\ \frac{\text{J}}{\text{kg}}\right)} = \boxed{2.0\ \text{kg}}$$

69. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

$$F_{\text{E}} = mg;\ F_{\text{E}} = |q|E = eV/d \quad \to \quad eV/d = mg \quad \to$$

$$V = \frac{mgd}{e} = \frac{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)(0.024\ \text{m})}{1.60\times10^{-19}\ \text{C}} = \boxed{1.3\times10^{-12}\ \text{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

70. The energy stored in the capacitor is given by Eq. 17–10, $\text{PE} = \frac{1}{2}CV^2$. As the plates are separated, the voltage is held constant by the battery. Since the separation distance has doubled, the capacitance has been multiplied by ½, using Eq. 17–8. Thus the stored energy has been $\boxed{\text{multiplied by ½}}$.

71. The capacitor is charged and isolated, so the amount of charge on the capacitor cannot change.

(*a*) The stored energy is given by Eq. 17–10, $\text{PE} = \frac{1}{2}QV$. The charge does not change, and the potential difference is doubled, so the stored energy is $\boxed{\text{multiplied by 2}}$.

(*b*) The stored energy can also be given by $\text{PE} = \frac{1}{2}\frac{Q^2}{C}$. The charge does not change. The capacitance would be reduced by a factor of 2 if the plate separation were doubled, according to Eq. 17–8, so the stored energy is $\boxed{\text{multiplied by 2}}$.

72. The energy in the capacitor, given by Eq. 17–10, is the heat energy absorbed by the water, given by Eq. 14–2.

$$\text{PE} = Q_{\text{heat}} \quad \rightarrow \quad \tfrac{1}{2}CV^2 = mc\Delta T \quad \rightarrow$$

$$V = \sqrt{\frac{2mc\Delta T}{C}} = \sqrt{\frac{2(2.8\text{ kg})\left(4186\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(95^\circ\text{C} - 21^\circ\text{C})}{4.0\text{ F}}} = 658\text{ V} \approx \boxed{660\text{ V}}$$

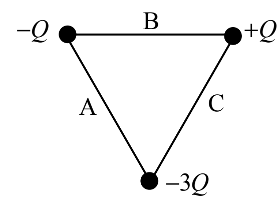
73. The proton would gain half the kinetic energy as compared to the alpha particle. The alpha particle has twice the charge of the proton, so it will have twice the potential energy for the same voltage. Thus $\boxed{\text{the alpha will have twice the kinetic energy of the proton}}$ after acceleration.

74. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$. Combine Eq. 17–7 with Eq. 17–8 and Eq. 17–4.

$$Q = CV = \frac{\varepsilon_0 A}{d}V = \varepsilon_0 A\frac{V}{d} = \varepsilon_0 AE = (8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(65\times10^{-4}\text{ m}^2)(3.0\times10^6\text{ V/m})$$

$$= \boxed{1.7\times10^{-7}\text{ C}}$$

75. Use Eq. 17–5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3}\ell/2$.



$$V_{\text{A}} = \frac{k(-Q)}{\ell/2} + \frac{k(-3Q)}{\ell/2} + \frac{k(Q)}{\sqrt{3}\ell/2} = \frac{2kQ}{\ell}\left(-4 + \frac{1}{\sqrt{3}}\right) = \boxed{-6.85\frac{kQ}{\ell}}$$

$$V_{\text{B}} = \frac{k(-Q)}{\ell/2} + \frac{k(Q)}{\ell/2} + \frac{k(-3Q)}{\sqrt{3}\ell/2} = -2\sqrt{3}\frac{kQ}{\ell} = \boxed{-3.46\frac{kQ}{\ell}}$$

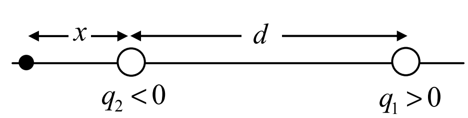
$$V_{\text{C}} = \frac{k(Q)}{\ell/2} + \frac{k(-3Q)}{\ell/2} + \frac{k(-Q)}{\sqrt{3}\ell/2} = -\frac{2kQ}{\ell}\left(2 + \frac{1}{\sqrt{3}}\right) = \boxed{-5.15\frac{kQ}{\ell}}$$

76. The energy is given by Eq. 17–10. Calculate the energy difference for the two different amounts of charge and then solve for the difference.

$$\text{PE} = \tfrac{1}{2}\frac{Q^2}{C} \quad \rightarrow \quad \Delta\text{PE} = \tfrac{1}{2}\frac{(Q+\Delta Q)^2}{C} - \tfrac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}[(Q+\Delta Q)^2 - Q^2] = \frac{\Delta Q}{2C}[2Q + \Delta Q] \quad \rightarrow$$

$$Q = \frac{C(\Delta\text{PE})}{\Delta Q} - \tfrac{1}{2}\Delta Q = \frac{(17.0\times10^{-6}\text{ F})(15.2\text{ J})}{(13.0\times10^{-3}\text{ C})} - \tfrac{1}{2}(13.0\times10^{-3}\text{ C}) = \boxed{13.4\times10^{-2}\text{ C}}$$

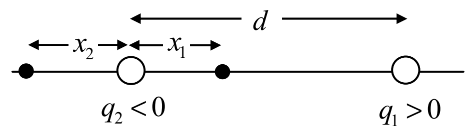
77. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, they are labeled x.



$$E = k\frac{|q_2|}{x^2} - k\frac{q_1}{(d+x)^2} = 0 \quad \rightarrow \quad |q_2|(d+x)^2 = q_1 x^2 \quad \rightarrow$$

$$x = \frac{\sqrt{|q_2|}}{\sqrt{q_1} - \sqrt{|q_2|}} d = \frac{\sqrt{2.6\times10^{-6}\text{ C}}}{\sqrt{3.4\times10^{-6}\text{ C}} - \sqrt{2.6\times10^{-6}\text{ C}}}(2.5\text{ cm}) = 17.42\text{ cm} \approx \boxed{17\text{ cm left of } q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. Consider locations between the charges (x_1) and to the left of the negative charge (x_2) as shown in the diagram.



$$V_{\text{location 1}} = \frac{kq_1}{(d - x_1)} + \frac{kq_2}{x_1} = 0 \quad \rightarrow \quad x_1 = \frac{q_2 d}{(q_2 - q_1)} = \frac{(-2.6\times10^{-6}\text{ C})(2.5\text{ cm})}{(-6.0\times10^{-6}\text{ C})} = 1.1\text{ cm}$$

$$V_{\text{location 2}} = \frac{kq_1}{(d + x_2)} + \frac{kq_2}{x_2} = 0 \quad \rightarrow \quad x_2 = -\frac{q_2 d}{(q_1 - q_2)} = \frac{(-2.6\times10^{-6}\text{ C})(2.5\text{ cm})}{(0.8\times10^{-6}\text{ C})} = 8.1\text{ cm}$$

So the two locations where the potential is zero are $\boxed{\text{1.1 cm from the negative charge toward the positive charge and 8.1 cm from the negative charge away from the positive charge}}$.

78. Since the electric field points downward, the surface of the Earth is at a lower potential than above the surface. Call the potential of the surface of the Earth 0. Then a height of 2.00 m has a potential of 300 V. We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0) and at ground level (where their electrical and gravitational potential energies are 0).

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad mgh + qV = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{2\left(gh + \frac{qV}{m}\right)}$$

$$v_+ = \sqrt{2\left[(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{(6.5\times10^{-4}\text{ C})(300\text{ V})}{(0.670\text{ kg})}\right]} = 6.3073\text{ m/s}$$

$$v_- = \sqrt{2\left[(9.80\text{ m/s}^2)(2.00\text{ m}) + \frac{(-6.5\times10^{-4}\text{ C})(300\text{ V})}{(0.670\text{ kg})}\right]} = 6.2143\text{ m/s}$$

$$v_+ - v_- = 6.3073\text{ m/s} - 6.2143\text{ m/s} = 0.093\text{ m/s} \approx \boxed{0.09\text{ m/s}}$$

79. (*a*) Use Eq. 17–10 to calculate the stored energy.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(5.0\times10^{-8}\text{ F})(3.5\times10^4\text{ V})^2 = 30.625\text{ J} \approx \boxed{31\text{ J}}$$

(*b*) The power is the energy converted per unit time.

$$P = \frac{\text{Energy}}{\text{time}} = \frac{0.12(30.625\text{ J})}{6.2\times10^{-6}\text{ s}} = \boxed{5.9\times10^5\text{ W}}$$

80. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\text{KE}_{\text{initial}} = \text{PE}_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 = qV \quad \rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(-1.60\times10^{-19}\text{ C})(-3.02\text{ V})}{9.11\times10^{-31}\text{ kg}}} = \boxed{1.03\times10^6\text{ m/s}}$$

81. Let d_1 represent the distance from the left charge to point B, and let d_2 represent the distance from the right charge to point B. Let Q represent the positive charges, and let q represent the negative charge that moves. The change in potential energy is given by Eq. 17–3.

$$\mathrm{PE_B} - \mathrm{PE_A} = qV_{\mathrm{BA}} = q(V_{\mathrm{B}} - V_{\mathrm{A}})$$

$$d_1 = \sqrt{12^2 + 14^2}\,\mathrm{cm} = 18.44\ \mathrm{cm} \quad d_2 = \sqrt{14^2 + 24^2}\,\mathrm{cm} = 27.78\ \mathrm{cm}$$

$$\mathrm{PE_B} - \mathrm{PE_A} = q(V_{\mathrm{B}} - V_{\mathrm{A}}) = q\left[\left(\frac{kQ}{0.1844\ \mathrm{m}} + \frac{kQ}{0.2778\ \mathrm{m}}\right) - \left(\frac{kQ}{0.12\ \mathrm{m}} + \frac{kQ}{0.24\ \mathrm{m}}\right)\right]$$

$$= kQq\left[\left(\frac{1}{0.1844\ \mathrm{m}} + \frac{1}{0.2778\ \mathrm{m}}\right) - \left(\frac{1}{0.12\ \mathrm{m}} + \frac{1}{0.24\ \mathrm{m}}\right)\right]$$

$$= (8.99\times10^{9})(-1.5\times10^{-6}\ \mathrm{C})(38\times10^{-6}\ \mathrm{C})(-3.477\ \mathrm{m}^{-1}) = 1.782\ \mathrm{J} \approx \boxed{1.8\ \mathrm{J}}$$

82. (*a*) The initial capacitance is obtained directly from Eq. 17–9.

$$C_0 = \frac{K\varepsilon_0 A}{d} = \frac{3.7(8.85\times10^{-12}\ \mathrm{F/m})(0.21\ \mathrm{m})(0.14\ \mathrm{m})}{0.11\times10^{-3}\ \mathrm{m}} = 8.752\times10^{-9}\ \mathrm{F} \approx \boxed{8.8\ \mathrm{nF}}$$

(*b*) Maximum charge will occur when the electric field between the plates is equal to the dielectric strength. The charge will be equal to the capacitance multiplied by the maximum voltage, where the maximum voltage is the electric field times the separation distance of the plates.

$$Q_{\max} = C_0 V = C_0 E d = (8.752\times10^{-9}\ \mathrm{F})(15\times10^{6}\ \mathrm{V/m})(0.11\times10^{-3}\ \mathrm{m})$$

$$= 1.444\times10^{-5}\ \mathrm{C} \approx \boxed{14\ \mu\mathrm{C}}$$

83. Use Eq. 17–7 with Eq. 17–9 to find the charge.

$$Q = CV = K\varepsilon_0 \frac{A}{d} V = (3.7)(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})\frac{\pi(0.55\times10^{-2}\ \mathrm{m})^2}{(0.10\times10^{-3}\ \mathrm{m})}(12\ \mathrm{V}) = \boxed{3.7\times10^{-10}\ \mathrm{C}}$$

84. (*a*) Use Eq. 17–5 to calculate the potential due to the charges. Let the distance between the charges be *d*.

$$V_{\mathrm{mid}} = \frac{kQ_1}{(d/2)} + \frac{kQ_2}{(d/2)} = \frac{2k}{d}(Q_1 + Q_2) = \frac{2(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})}{(0.23\ \mathrm{m})}(3.5\times10^{-6}\ \mathrm{C} - 7.2\times10^{-6}\ \mathrm{C})$$

$$= \boxed{-2.9\times10^{5}\ \mathrm{V}}$$

(*b*) Use Eq. 16–4a to calculate the electric field. Note that the field due to each of the two charges will point to the left, away from the positive charge and toward the negative charge. Find the magnitude of the field using the absolute value of the charges.

$$E_{\mathrm{mid}} = \frac{kQ_1}{(d/2)^2} + \frac{k|Q_2|}{(d/2)^2} = \frac{4k}{d^2}(Q_1 + |Q_2|)$$

$$= \frac{4(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})}{(0.23\ \mathrm{m})^2}(3.5\times10^{-6}\ \mathrm{C} + 7.2\times10^{-6}\ \mathrm{C}) = \boxed{7.3\times10^{6}\ \mathrm{V/m,\ left}}$$

85. (*a*) Use Eq. 17–7 and Eq. 17–8 to calculate the charge.

$$Q = CV = \frac{\varepsilon_0 A}{d} V = \frac{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(3.0\times10^{-4}\ \mathrm{m^2})}{(5.0\times10^{-4}\ \mathrm{m})}(12\ \mathrm{V}) = 6.372\times10^{-11}\ \mathrm{C}$$

$$\approx \boxed{6.4\times10^{-11}\ \mathrm{C}}$$

(*b*) The charge does not change. Since the capacitor is not connected to a battery, no charge can flow to it or from it. Thus $Q = 6.372\times10^{-11}\ \text{C} \approx \boxed{6.4\times10^{-11}\ \text{C}}$.

(*c*) The separation distance was multiplied by a factor of 1.5. The capacitance is inversely proportional to the separation distance, so the capacitance was divided by a factor of 1.5. Since $Q = CV$, if the charge does not change and the capacitance is divided by a factor of 1.5, then the voltage must have increased by a factor of 1.5.

$$V_{\text{final}} = 1.5V_{\text{initial}} = 1.5(12\ \text{V}) = \boxed{18\ \text{V}}$$

(*d*) The work done is the change in energy stored in the capacitor.

$$W = \text{PE}_{\text{final}} - \text{PE}_{\text{initial}} = \tfrac{1}{2}QV_{\text{final}} - \tfrac{1}{2}QV_{\text{initial}} = \tfrac{1}{2}Q(V_{\text{final}} - V_{\text{initial}})$$
$$= \tfrac{1}{2}(6.372\times10^{-11}\ \text{C})(18\ \text{V} - 12\ \text{V}) = 1.912\times10^{-10}\ \text{J} \approx \boxed{2\times10^{-10}\ \text{J}}$$

86. The energy stored in the capacitor is given by Eq. 17–10. The final energy is half the initial energy. Find the final charge and then subtract the final charge from the initial charge to find the charge lost.

$$E_{\text{final}} = \tfrac{1}{2}E_{\text{initial}} \rightarrow \tfrac{1}{2}\frac{Q_{\text{final}}^2}{C} = \tfrac{1}{2}\tfrac{1}{2}\frac{Q_{\text{initial}}^2}{C} \rightarrow Q_{\text{final}} = \frac{1}{\sqrt{2}}Q_{\text{initial}}$$

$$Q_{\text{lost}} = Q_{\text{initial}} - Q_{\text{final}} = Q_{\text{initial}}\left(1 - \frac{1}{\sqrt{2}}\right) = CV\left(1 - \frac{1}{\sqrt{2}}\right) = (2.1\times10^{-6}\ \text{F})(6.0\ \text{V})(0.2929)$$
$$= \boxed{3.7\times10^{-6}\ \text{C}}$$

87. (*a*) We assume that Q_2 is held at rest. The energy of the system will then be conserved, with the initial potential energy of the system all being changed to kinetic energy after a very long time.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \rightarrow \frac{kQ_1Q_2}{r} = \tfrac{1}{2}m_1\upsilon_1^2 \rightarrow$$

$$\upsilon_1 = \sqrt{\frac{2kQ_1Q_2}{m_1 r}} = \sqrt{\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.5\times10^{-6}\ \text{C})^2}{(1.5\times10^{-6}\ \text{kg})(4.0\times10^{-2}\ \text{m})}} = \boxed{3.6\times10^3\ \text{m/s}}$$

(*b*) In this case, both the energy and the momentum of the system will be conserved. Since the initial momentum is zero, the magnitudes of the momenta of the two charges will be equal.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_1\upsilon_1 + m_2\upsilon_2 \rightarrow \upsilon_2 = -\upsilon_1\frac{m_1}{m_2}$$

$$E_{\text{initial}} = E_{\text{final}} \rightarrow \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \rightarrow$$

$$\frac{kQ_1Q_2}{r} = \tfrac{1}{2}m_1\upsilon_1^2 + \tfrac{1}{2}m_2\upsilon_2^2 = \tfrac{1}{2}\left[m_1\upsilon_1^2 + m_2\left(-\upsilon_1\frac{m_1}{m_2}\right)\right] = \tfrac{1}{2}\frac{m_1}{m_2}(m_1 + m_2)\upsilon_1^2$$

$$\upsilon_1 = \sqrt{\frac{2kQ_1Q_2m_2}{m_1(m_1 + m_2)r}} = \sqrt{\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.5\times10^{-6}\ \text{C})^2(2.5\times10^{-6}\ \text{kg})}{(1.5\times10^{-6}\ \text{kg})(4.0\times10^{-6}\ \text{m})(4.0\times10^{-2}\ \text{m})}}$$
$$= \boxed{2.8\times10^3\ \text{m/s}}$$

88. Calculate $V_{AB} = V_A - V_B$. Represent the 0.10-m distance by the variable *d*.

$$V_{AB} = V_A - V_B = \left(\frac{kq_1}{d} + \frac{kq_2}{\sqrt{2}d}\right) - \left(\frac{kq_2}{d} + \frac{kq_1}{\sqrt{2}d}\right) = \frac{k}{d}(q_1 - q_2)\left(1 - \frac{1}{\sqrt{2}}\right)$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{0.1\ \text{m}}(4.5\times10^{-6}\ \text{C})\left(1-\frac{1}{\sqrt{2}}\right) = \boxed{1.2\times10^5\ \text{V}}$$

89. The potential of the Earth will increase because the "neutral" Earth will now be charged by the removing of the electrons. The excess charge will be the elementary charge times the number of electrons removed. We approximate this change in potential by using a spherical Earth with all the excess charge at the surface.

$$Q = \left(\frac{1.602\times10^{-19}\ \text{C}}{e^-}\right)\left(\frac{10\ e^-}{\text{H}_2\text{O molecule}}\right)\left(\frac{6.02\times10^{23}\ \text{molecules}}{0.018\ \text{kg}}\right)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right)\tfrac{4}{3}\pi(0.00175\ \text{m})^3$$
$$= 1203\ \text{C}$$
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_{\text{Earth}}} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1203\ \text{C}}{6.38\times10^6\ \text{m}} = \boxed{1.7\times10^6\ \text{V}}$$

90. As an estimate, the length of the bolt would be the voltage difference of the bolt divided by the breakdown electric field of air.

$$\frac{5\times10^7\ \text{V}}{3\times10^6\ \text{V/m}} = 16.7\ \text{m} \approx \boxed{20\ \text{m}}$$

If the lightning strikes from a height of 750 m (roughly a half-mile), then 40 or more steps might be involved.

91. If we assume that the electric field is uniform, then we can use Eq. 17–4a to estimate the magnitude of the electric field. From Eq. 16–10 we have an expression for the electric field due to a pair of oppositely charged planes. We approximate the area of a shoe as $30\ \text{cm}\times8\ \text{cm}$. Answers will vary with the shoe size approximation.

$$E = \frac{V}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad\rightarrow$$
$$Q = \frac{\varepsilon_0 A V}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.024\ \text{m}^2)(6.0\times10^3\ \text{V})}{1.0\times10^{-3}\ \text{m}} = \boxed{1.3\times10^{-6}\ \text{C}}$$

92. Use Eq. 17–7.

$$\Delta Q = C\Delta V;\ \Delta t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{C\Delta V}{\Delta Q/\Delta t} = \frac{(1200\ \text{F})(6.0\ \text{V})}{1.0\times10^{-3}\ \text{C/s}} = 7.2\times10^6\ \text{s}\left(\frac{1\ \text{d}}{86{,}400\ \text{s}}\right) = \boxed{83\ \text{d}}$$

93. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates and the time the electron spends between the plates.

Horizontal:

$$\text{PE}_{\text{inital}} = \text{KE}_{\text{final}} \quad\rightarrow\quad qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Vertical:

$$F_E = qE_y = ma = m\frac{(v_y - v_{0y})}{t} \quad \rightarrow \quad {}_y = \frac{qE_y t}{m} = \frac{qE_y \Delta x}{mv_x}$$

Combined:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\frac{250\text{ V}}{0.013\text{ m}}\right)(0.065\text{ m})}{2(2200\text{ V})} = 0.284$$

$$\theta = \tan^{-1}0.284 = \boxed{16^\circ}$$

94. (*a*) The absolute value of the charge on each plate is given by Eq. 17–7. The plate with electrons has a net negative charge.

$$Q = CV \quad \rightarrow \quad N(-e) = -CV \quad \rightarrow$$

$$N = \frac{CV}{e} = \frac{(35\times10^{-15}\text{ F})(1.5\text{ V})}{1.60\times10^{-19}\text{ C}} = 3.281\times10^5 \approx \boxed{3.3\times10^5 \text{ electrons}}$$

(*b*) Since the charge is directly proportional to the potential difference, a 2.0% decrease in potential difference corresponds to a 2.0% decrease in charge.

$$\Delta Q = 0.020Q$$

$$\Delta t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{0.020Q}{\Delta Q/\Delta t} = \frac{0.020CV}{\Delta Q/\Delta t} = \frac{0.020(35\times10^{-15}\text{ F})(1.5\text{ V})}{0.30\times10^{-15}\text{ C/s}} = \boxed{3.5\text{ s}}$$

95. (*a*) From Problem 94, we have $C = 35\times10^{-15}$ F. Use Eq. 17–9 to calculate the area.

$$C = K\varepsilon_0\frac{A}{d} \quad \rightarrow \quad A = \frac{Cd}{K\varepsilon_0} = \frac{(35\times10^{-15}\text{ F})(2.0\times10^{-9}\text{ m})}{(25)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} = 3.164\times10^{-13}\text{ m}^2\left(\frac{10^6\ \mu\text{m}}{1\text{ m}}\right)^2$$

$$= 0.3164\ \mu\text{m}^2 \approx \boxed{0.32\ \mu\text{m}^2}$$

(*b*) Half of the area of the cell is used for capacitance, so 1.5 cm^2 is available for capacitance. Each capacitor is one bit.

$$1.5\text{ cm}^2\left(\frac{10^6\ \mu\text{m}}{10^2\text{ cm}}\right)^2\left(\frac{1\text{ bit}}{0.32\mu\text{m}^2}\right)\left(\frac{1\text{ byte}}{8\text{ bits}}\right) = 5.86\times10^7\text{ bytes} \approx \boxed{59\text{ Mbytes}}$$

96. (*a*) Use Eq. 17–8 to calculate the capacitance.

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.0\text{ m}^2)}{(3.0\times10^{-3}\text{ m})} = \boxed{5.9\times10^{-9}\text{ F}}$$

Use Eq. 17–7 to calculate the charge.

$$Q_0 = C_0V_0 = (5.9\times10^{-9}\text{ F})(35\text{ V}) = 2.065\times10^{-7}\text{ C} \approx \boxed{2.1\times10^{-7}\text{ C}}$$

The electric field is the potential difference divided by the plate separation.

$$E_0 = \frac{V_0}{d} = \frac{35\text{ V}}{3.0\times10^{-3}\text{ m}} = 11667\text{ V/m} \approx \boxed{1.2\times10^4\text{ V/m}}$$

Use Eq. 17–10 to calculate the energy stored.

$$\text{PE}_0 = \tfrac{1}{2}C_0V_0^2 = \tfrac{1}{2}(5.9\times10^{-9}\text{ F})(35\text{ V})^2 = 3.614\times10^{-6}\text{ J} \approx \boxed{3.6\times10^{-6}\text{ J}}$$

(*b*) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$C = KC_0 = 3.2(5.9\times10^{-9}\text{ F}) = 1.888\times10^{-8}\text{ F} \approx \boxed{1.9\times10^{-8}\text{ F}}$$

The voltage doesn't change. Use Eq. 17–7 to calculate the charge.

$$Q = CV = KC_0V = 3.2(5.9\times10^{-9}\text{ F})(35\text{ V}) = 6.608\times10^{-7}\text{ C} \approx \boxed{6.6\times10^{-7}\text{ C}}$$

Since the battery is still connected, the voltage is the same as before. Thus the electric field doesn't change.

$$E = E_0 = \boxed{1.2\times10^{4}\text{ V/m}}$$

Use Eq. 17–10 to calculate the energy stored.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}KC_0V^2 = \tfrac{1}{2}(3.2)(5.9\times10^{-9}\text{ F})(35\text{ V})^2 = \boxed{1.2\times10^{-5}\text{ J}}$$

Solutions to Search and Learn Problems

1. (*a*) An equipotential surface is one in which all points are at the same potential.
   (*b*) An equipotential surface must be perpendicular to the electric field at any point.
   (*c*) Equipotential lines and surfaces are always continuous.
   (*d*) Equipotential lines and surfaces never end.
   (*e*) The entire volume (including the surface) of a conductor is an equipotential.
   (*f*) No work is required to move a charged particle from one point on an equipotential surface to another point on that surface.
   (*g*) Equipotential lines and surfaces cannot cross.
   (*h*) Electric field lines point from higher equipotential surfaces to lower equipotential surfaces.
   (*i*) Closer spacing of equipotentials indicates a larger electric field.

2. (*a*) The contour values on the map are in feet (the mountains are about 2 miles above sea level). There are six contour lines between Iceberg Lake and Cecile Lake. Each line is to be interpreted as an 80-V difference, so the voltage difference is 480 V. The two lakes appear to be about 0.5 km apart. The magnitude of the electric field is equal to the change in voltage divided by the separation distance, Eq. 17–4a.

$$E = \frac{V_{\text{ba}}}{d} = \frac{480\text{ V}}{500\text{ m}} = 0.96\text{ V/m} \approx \boxed{1\text{ V/m}}$$

Iceberg Lake is lower than Cecile Lake, so the direction of the electric field is basically $\boxed{\text{in the } +y \text{ direction}}$. Answers may vary due to interpreting the figure.

   (*b*) At the Minaret Mine, the higher elevations are upward and leftward. The elevation rises about 400 ft (five contour lines) in about 500 m as you move in that direction. Thus the electric field points downward and to the right. Calculate the magnitude by Eq. 17–4a.

$$E = \frac{V_{\text{ab}}}{d} = \frac{400\text{ V}}{500\text{ m}} = \boxed{0.8\text{ V/m, }45^\circ\text{ below the } +x \text{ axis}}$$

Again, answers may vary due to estimations in interpreting the figure.

$\boxed{3.}$ (*a*) Use Eq. 17–8.

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(110\times10^6\ \text{m}^2)}{(1500\ \text{m})} = 6.49\times10^{-7}\ \text{F} \approx \boxed{6.5\times10^{-7}\ \text{F}} = 65\mu\text{F}$$

(*b*) Use Eq. 17–7.

$$Q = CV = (6.49\times10^{-7}\ \text{F})(3.5\times10^7\ \text{V}) = 22.715\ \text{C} \approx \boxed{23\ \text{C}}$$

(*c*) Use Eq. 17–10.

$$\text{PE} = \tfrac{1}{2}QV = \tfrac{1}{2}(22.715\ \text{C})(3.5\times10^7\ \text{V}) = \boxed{4.0\times10^8\ \text{J}}$$

4. (*a*) Both of these equations for the potential energy can be obtained from Eq. 17–10 by using Eq. 17–7 first to eliminate the charge from the equation, and second to eliminate the voltage.

$$\text{PE} = \tfrac{1}{2}QV = \tfrac{1}{2}(CV)V = \tfrac{1}{2}CV^2$$

$$= \tfrac{1}{2}QV = \tfrac{1}{2}Q\left(\frac{Q}{C}\right) = \tfrac{1}{2}\frac{Q^2}{C}$$

(*b*) The first equation $(\text{PE} = \tfrac{1}{2}CV^2)$ is used when the capacitance and voltage difference across the capacitor is known, as when the capacitor is connected to a battery. The second equation $\left(\text{PE} = \tfrac{1}{2}\frac{Q^2}{C}\right)$ is used when the capacitance and charge are known, as when a specific charge is placed on an isolated capacitor.

(*c*) Since the voltage is constant, we use the first equation for the potential energy, with the capacitance equal to the product of the initial capacitance and the dielectric constant of the paper.

$$\text{PE} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(KC_0)V^2 = K(\tfrac{1}{2}C_0V^2) = K(\text{PE}_0)$$

The potential energy in the capacitor will increase by a factor equal to the dielectric constant. For a paper dielectric, the $\boxed{\text{potential increases by a factor of 3.7}}$.

(*d*) In this case the charge is constant, so we use the second equation to determine the final energy.

$$\text{PE} = \frac{Q^2}{2C} = \frac{Q^2}{2KC_0} = \frac{1}{K}\left(\frac{Q^2}{2C_0}\right) = \frac{1}{K}(\text{PE}_0)$$

The potential energy in the capacitor decreases by a factor equal to the dielectric constant. For quartz, the potential energy decreases to $1/4.3 = 0.23$, or $\boxed{\text{23\% of the initial value}}$.

5. (*a*) A horsepower is equal to 746 watts.

$$P = (75\times10^3\ \text{W})\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 100.5\ \text{hp} \approx \boxed{1.0\times10^2\ \text{hp}}$$

(*b*) The energy is the product of the power and time elapsed.

$$E = Pt = (75\times10^3\ \text{W})(5.0\ \text{h})(3600\ \text{s/h}) = 1.35\times10^9\ \text{J} \approx \boxed{1.4\times10^9\ \text{J}}$$

(*c*) Use Eq. 17–10, in terms of the capacitance and voltage, to solve for the capacitance.

$$\text{PE} = \tfrac{1}{2}CV^2 \quad \rightarrow \quad C = \frac{2\text{PE}}{V^2} = \frac{2(1.35\times10^9\ \text{J})}{(850\ \text{V})^2} = 3.737\times10^3\ \text{F} \approx \boxed{3.7\times10^3\ \text{F}}$$

(*d*) According to the text, the capacitance of 0.1 g of activated carbon is about 1 F. We again use Eq. 17–10 to find the capacitance and then convert to a mass of carbon.

$$C=\frac{2\text{PE}}{V^2}=\frac{2(1.35\times10^9\ \text{J})}{(10\ \text{V})^2}=2.7\times10^7\ \text{F}\,;\ 2.7\times10^7\ \text{F}\left(\frac{0.1\ \text{g}}{1\ \text{F}}\right)=\boxed{2.7\times10^6\ \text{g}}=2.7\times10^3\ \text{kg}$$

(*e*) Cars have a mass on the order of 1000–2000 kg, so this capacitor would more than double the mass of the car. $\boxed{\text{This is not practical.}}$

6. (*a*) If N electrons flow onto the plate, then the charge on the top plate is $-Ne,$ and the positive charge associated with the capacitor is $Q=Ne.$ Since $Q=CV,$ we have

$Ne=CV \quad\rightarrow\quad \boxed{V=Ne/C},$ showing that V is proportional to N.

(*b*) Given $\Delta V=1\ \text{mV}$ and we want $\Delta N=1,$ solve for the capacitance.

$$V=\frac{Ne}{C}\quad\rightarrow\quad\Delta V=\frac{e\Delta N}{C}\quad\rightarrow$$

$$C=e\frac{\Delta N}{\Delta V}=(1.60\times10^{-19}\ \text{C})\frac{1}{1\times10^{-3}\ \text{V}}=1.60\times10^{-16}\ \text{F}\approx\boxed{2\times10^{-16}\ \text{F}}$$

(*c*) Use Eq. 17–9 to solve for the side length of the capacitor.

$$C=\varepsilon_0K\frac{A}{d}=\varepsilon_0K\frac{\ell^2}{d}\quad\rightarrow$$

$$\ell=\sqrt{\frac{Cd}{\varepsilon_0K}}=\sqrt{\frac{(1.60\times10^{-16}\ \text{F})(100\times10^{-9}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3)}}=7.76\times10^{-7}\ \text{m}\left(\frac{10^6\ \mu\text{m}}{1\ \text{m}}\right)=\boxed{0.8\ \mu\text{m}}$$

18

ELECTRIC CURRENTS

Responses to Questions

1. In the circuit (not in the battery), electrons flow from high potential energy (at the negative terminal) to low potential energy (at the positive terminal). Inside the battery, the chemical reaction does work on the electrons to take them from low potential energy to high potential energy (to the negative terminal). A chemical description could say that the chemical reaction taking place at the negative electrode leaves electrons behind on the terminal, and the positive ions created at the negative electrode pull electrons off the positive electrode.

2. Battery energy is what is being "used up." As charges leave the battery terminal, they have a relatively high potential energy. Then as the charges move through the flashlight bulb, they lose potential energy. The battery uses a chemical reaction to replace the potential energy of the charges, by lowering the battery's chemical potential energy. When a battery is "used up," it is unable to give potential energy to charges.

$\boxed{3.}$ Ampere-hours measures charge. The ampere is a charge per unit time, and the hour is a time, so the product is charge. One ampere-hour of charge is 3600 coulombs of charge.

4. Resistance is given by the relationship $R = \rho L/A.$ If the ratio of resistivity to area is the same for the copper wire and the aluminum wire, then the resistances will be the same. Thus if $\rho_{Cu}/A_{Cu} = \rho_{Al}/A_{Al}$ or $A_{Al}/A_{Cu} = \rho_{Al}/\rho_{Cu},$ then the resistances will be the same.

   Also, resistance changes with temperature. By having the two wires at different temperatures, it might be possible to have their resistance be the same.

5. The terminal of the battery (usually the negative one) is connected to the metal chassis, frame, and engine block of the car. This means that all voltages used for electrical devices in the car are measured with respect to the car's frame. Also, since the frame is a large mass of metal, it can supply charges for current without significantly changing its electric potential.

6. To say that $P = V^2/R$ indicates a decrease in power as resistance increases implies that the voltage is constant. To say that $P = I^2R$ indicates an increase in power as resistance increases implies that the current is constant. Only one of those can be true for any given situation in which the resistance is changing. If the resistance changes and the voltage is constant, then the current must also change. Likewise, if the resistance changes and the current is constant, then the voltage must also change.

7. When a lightbulb burns out, its filament burns to the point of breakage. Once the filament (part of the conducting path for the electricity flowing through the bulb) is broken, current can no longer flow through the bulb. It no longer gives off any light.

8. We assume that the voltage is the same in both cases. If the resistance increases, then the power delivered to the heater will decrease according to $P = V^2/R$. If the power decreases, then the heating process will slow down.

9. Resistance is given by the relationship $R = \rho L/A$. Thus, to minimize the resistance, you should have a small length and a large cross-sectional area. Likewise, to maximize the resistance, you should have a large length and a small cross-sectional area.
   (*a*) For the least resistance, connect the wires to the faces that have dimensions of 2*a* by 3*a*, which maximizes the area $(6a^2)$ and minimizes the length (*a*).
   (*b*) For the greatest resistance, connect the wires to the faces that have dimensions of *a* by 2*a*, which minimizes the area $(2a^2)$ and maximizes the length (3*a*).

10. When a lightbulb is first turned on, it will be cool and the filament will have a lower resistance than when it is hot. This lower resistance means that there will be more current through the bulb while it is cool. This momentary high current will make the filament quite hot. If the temperature is too high, then the filament will vaporize, and the current will no longer be able to flow in the bulb. After the light has been on for some time, the filament is at a constant high temperature, with a higher resistance and a lower current. Since the temperature is constant, there is less thermal stress on the filament than when the light is first turned on.

11. Assuming that both lightbulbs have the same voltage, since $P = IV$, the higher-power bulb will draw the most current. Likewise, assuming that both lightbulbs have the same voltage, since $P = V^2/R$, the higher-power bulb will have the lower resistance. So the 100-W bulb will draw the most current, and the 75-W bulb will have the higher resistance.

12. Transmission lines have resistance and therefore will change some electrical energy to thermal energy (heat) as the electrical energy is transmitted. We assume that the resistance of the transmission lines is constant. Then the "lost" power is given by $P_{\text{lost}} = I^2R$, where *I* is the current carried by the transmission lines. The transmitted power is given by $P_{\text{trans}} = IV$, where *V* is the voltage across the transmission lines. For a given value of $P_{\text{trans}} = IV$, the higher the voltage is, the lower the current has to be. As the current is decreased, $P_{\text{lost}} = I^2R$ is also decreased, so there is a lower amount of power lost.

13. The 15-A fuse is blowing because the circuit is carrying more than 15 A of current. The circuit is probably designed to only carry 15 A, so there might be a "short" or some other malfunction causing the current to exceed 15 A. Replacing the 15-A fuse with a 25-A fuse will allow more current to flow and thus make the wires carrying the current get hotter. A fire or damage to certain kinds of electrical equipment might result. The blown fuse is a warning that something is wrong with the circuit.

14. At only 10 Hz, the metal filament in the wire will go on and off 20 times per second. (It has a maximum magnitude of current at the maximum current in each direction.) The metal filament has time to cool down and get dim during the low current parts of the cycle, and your eye can detect this. At 50 or 60 Hz, the filament never cools enough to dim significantly. Also, the human eye and brain cannot distinguish the on-off cycle of lights when they are operated at the normal 60-Hz frequency. At

much lower frequencies, such as 5 Hz, the eye and brain are able to process the on-off cycle of the lights, and they will appear to flicker.

15. There are several factors which can be considered. As the voltage reverses with each cycle of AC, the potential energy of the electrons is raised again. Thus with each "pass" through the light, the electrons lose their potential energy and then get it back again. Secondly, the heating of the filament (which causes the light) does not depend on the direction of the current, but only on the fact that a current exists, so the light occurs as the electrons move in both directions. Also, at 60 Hz, the current peaks 120 times per second. The small amount of time while the magnitude of the current is small is not long enough for the hot metal filament to cool down, so it stays lit the entire cycle. Finally, the human eye sees anything more rapid than about 20 Hz as continuous, even if it is not. So even if the light was to go dim during part of the cycle, our eyes would not detect it.

16. When the toaster is first turned on, the Nichrome wire is at room temperature. The wire starts to heat up almost immediately. Since the resistance increases with temperature, the resistance will be increasing as the wire heats. Assuming the voltage supplied is constant, the current will be decreasing as the resistance increases.

17. Current is NOT used up in a resistor. The current that flows into the resistor is the same as the current that flows out of the resistor. If that were not the case, then there would be either an increase or decrease in the charge of the resistor, but the resistor actually stays neutral, indicating equal charge flow both in and out. What does get "used up" is potential energy. The charges that leave a resistor have lower potential energy than the charges that enter a resistor. The amount of energy decrease per unit time is given by I^2R.

18. If you turn on an electric appliance when you are outside with bare feet, and the appliance shorts out through you, the current has a direct path to ground through your feet, and you will receive a severe shock. If you are inside wearing socks and shoes with thick soles, and the appliance shorts out, the current will not have an easy path to ground through you and will most likely find an alternate path. You might receive a mild shock, but not a severe one.

19. In the two wires described, the drift velocities of the electrons will be about the same, but the current density, and therefore the current, in the wire with twice as many free electrons per atom will be twice as large as in the other wire.

20. (*a*) If the length of the wire doubles, then its resistance also doubles, so the current in the wire will be reduced by a factor of 2. Drift velocity is proportional to current, so the drift velocity will be halved.
    (*b*) If the wire's radius is doubled, then the drift velocity remains the same. (Although, since there are more charge carriers, the current will quadruple.)
    (*c*) If the potential difference doubles while the resistance remains constant, then the drift velocity and current will also double.

Responses to MisConceptual Questions

1. (*c*) It may be thought that the orientation of the battery is important for the bulb to work properly. But the lightbulb and it will glow equally bright regardless of the direction in which current flows through it.

2. (*c*) A common misconception is that the lightbulb "uses up" the current, causing more current to flow in one portion of a loop than in another. For a single loop, the current is the same at every

point in the loop. Therefore, the amount of current that flows through the lightbulb is the same as the amount that flows through the battery.

3. (*a*) Ohm's law is an empirical law showing that for some materials the current through the material is proportional to the voltage across the material. For other materials, such as diodes, fluorescent lightbulbs, and superconductors, Ohm's law is not valid. Since it is not valid for all objects, it cannot be a fundamental law of physics.

4. (*e*) A common misconception is that the electrons are "used up" by the lightbulb. A good analogy would be water flowing across a water wheel in a flour mill. The water flows onto the wheel at the top (high potential) and causes the wheel to rotate as the water descends along the wheel. The amount of water that leaves the bottom of the wheel is the same as the amount that entered at the top, but it does so at a lower point (low potential). The change in potential energy goes into work in the wheel. In a lightbulb, electrons at higher potential energy enter the lightbulb and give off that energy as they pass through the bulb. As with the water on the wheel, the number of electrons exiting and entering the bulb is the same.

5. (*e*) A misconception commonly found when dealing with electric circuits is that electrons are "used up" by the lightbulb. The current is a measure of the rate that electrons pass a given point. If the current were different at two points in the circuit, then electrons would be building up (or being depleted) between those two points. The buildup of electrons would cause the circuit to be time dependent and not a steady state system. The flow of electrons (current) must be the same at all points in a loop.

6. (*b*) Ohm's law requires that the ratio of voltage to current be constant. Since it is not constant in this case, the material does not obey Ohm's law.

7. (*d*) A common misconception is that the charge or current is "used up" in a resistor. The resistor removes energy from the system, such that the electrons exiting the resistor have less potential energy than the electrons entering, but the number of electrons (charge carriers) entering and exiting the resistor is the same. The rate of electrons entering and exiting is equal to the constant current in the circuit.

8. (*c*) Since the unit of kilowatt-hour contains the word "watt," it is often incorrectly thought to be a unit of power. However, the kilowatt-hour is the product of the unit of power (kilowatt) and a unit of time (hour), resulting in a unit of energy.

9. (*b*) Each device added to the circuit is added in parallel. The voltage across the circuit does not change as the devices are added. Each new device, however, creates a path for additional current to flow, causing the current in the circuit breaker to increase. When the current becomes too high, the circuit breaker will open.

10. (*b*) For current to flow through an object it must complete a circuit. When a bird lands on a wire, the bird creates a loop in parallel with the segment of wire between its feet. The voltage drop across the bird is equal to the voltage drop across the wire between the bird's feet. Since the wire has a small resistance, there will be very little voltage drop across the wire between the bird's feet and very little voltage drop across the bird. The bird is a good conductor (similar to a human), but since there is little voltage drop, it will experience little current flow. When a ladder is placed between the ground and the wire, it creates a path for the current to flow from the high-voltage wire to ground (zero voltage). This large potential difference will enable a large current to flow through the ladder.

11. (*b*) A common misconception is that the electrons must travel from the switch to the lightbulb for the lightbulb to turn on. This is incorrect because there are electrons throughout the circuit, not just at the switch. When the switch is turned on, the electric potential across the circuit creates an electric field in the wire that causes all of the conduction electrons in the wire to move.

Solutions to Problems

1. Use the definition of current, Eq. 18–1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad 1.60\text{ A} = \frac{1.60\text{ C}}{\text{s}} \times \frac{1\text{ electron}}{1.60 \times 10^{-19}\text{ C}} = \boxed{1.00 \times 10^{19}\text{ electrons/s}}$$

2. Use the definition of current, Eq. 18–1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad \Delta Q = I\Delta t = (6.7\text{ A})(5.0\text{ h})(3600\text{ s/h}) = \boxed{1.2 \times 10^{5}\text{ C}}$$

3. Use the definition of current, Eq. 18–1.

$$I = \frac{\Delta Q}{\Delta t} = \frac{(1200\text{ ions})(1.60 \times 10^{-19}\text{ C/ion})}{3.1 \times 10^{-6}\text{ s}} = \boxed{6.2 \times 10^{-11}\text{ A}}$$

4. Use Eq. 18–2 for resistance.

$$R = \frac{V}{I} = \frac{120\text{ V}}{4.6\text{ A}} = \boxed{26\ \Omega}$$

5. Use Eq. 18–2 for the voltage.

$$V = IR = (0.25\text{ A})(4800\ \Omega) = \boxed{1200\text{ V}}$$

6. The ampere-hour is a unit of charge.

$$(75\text{A} \cdot \text{h})\left(\frac{1\text{ C/s}}{1\text{ A}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{2.7 \times 10^{5}\text{ C}}$$

7. (*a*) Use Eq. 18–2 to find the current.

$$V = IR \quad \rightarrow \quad I = \frac{V}{R} = \frac{240\text{ V}}{8.6\ \Omega} = 27.91\text{ A} \approx \boxed{28\text{ A}}$$

(*b*) Use the definition of current, Eq. 18–1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad \Delta Q = I\Delta t = (27.91\text{ A})(50\text{ min})(60\text{ s/min}) = \boxed{8.4 \times 10^{4}\text{ C}}$$

8. Find the potential difference from the resistance and the current, using Eq. 18–2.

$$R = (2.5 \times 10^{-5}\ \Omega/\text{m})(4.0 \times 10^{-2}\text{ m}) = 1.0 \times 10^{-6}\ \Omega$$
$$V = IR = (4100\text{ A})(1.0 \times 10^{-6}\ \Omega) = \boxed{4.1 \times 10^{-3}\text{ V}}$$

9. (*a*) Use Eq. 18–2 for resistance.

$$R = \frac{V}{I} = \frac{120\text{ V}}{13.5\text{ A}} = 8.889\ \Omega \approx \boxed{8.9\ \Omega}$$

(*b*) Use the definition of current, Eq. 18–1.

$$I = \frac{\Delta Q}{\Delta t} \quad \rightarrow \quad \Delta Q = I\Delta t = (13.5\text{ A})(15\text{ min})(60\text{ s/min}) = 1.215\times 10^4\text{ C} \approx \boxed{1.2\times 10^4\text{ C}}$$

10. Find the current from the voltage and resistance and then find the number of electrons from the current.

$$V = IR \quad \rightarrow \quad I = \frac{V}{R} = \frac{4.5\text{ V}}{1.3\ \Omega} = 3.462\text{ A}$$

$$3.462\text{ A} = 3.462\frac{\text{C}}{\text{s}} \times \frac{1\text{ electron}}{1.60\times 10^{-19}\text{ C}} \times \frac{60\text{ s}}{1\text{ min}} = \boxed{1.3\times 10^{21}\text{ electrons/min}}$$

11. (*a*) If the voltage drops by 15%, and the resistance stays the same, then by Eq. 18–2, $V = IR$, the current will also drop by 15%.

$$I_{\text{final}} = 0.85 I_{\text{initial}} = 0.85(5.60\text{ A}) = 4.76\text{ A} \approx \boxed{4.8\text{ A}}$$

(*b*) If the resistance drops by 15% (the same as being multiplied by 0.85), and the voltage stays the same, then by Eq. 18–2, the current must be divided by 0.85.

$$I_{\text{final}} = \frac{I_{\text{initial}}}{0.85} = \frac{5.60\text{ A}}{0.85} = 6.588\text{ A} \approx \boxed{6.6\text{ A}}$$

12. Use Eq. 18–3 to find the diameter, with the area $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} \quad \rightarrow \quad d = \sqrt{\frac{4\ell\rho}{\pi R}} = \sqrt{\frac{4(1.00\text{ m})(5.6\times 10^{-8}\ \Omega\cdot\text{m})}{\pi(0.32\ \Omega)}} = \boxed{4.7\times 10^{-4}\text{ m}}$$

13. Use Eq. 18–3 to calculate the resistance, with the area $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = (1.68\times 10^{-8}\ \Omega\cdot\text{m})\frac{4(5.4\text{ m})}{\pi(1.5\times 10^{-3}\text{ m})^2} = \boxed{5.1\times 10^{-2}\ \Omega}$$

14. Use Eq. 18–3 to calculate the resistances, with the area $A = \pi r^2 = \pi d^2/4$, so $R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2}$.

$$\frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{\rho_{\text{Al}}\frac{4\ell_{\text{Al}}}{\pi d_{\text{Al}}^2}}{\rho_{\text{Cu}}\frac{4\ell_{\text{Cu}}}{\pi d_{\text{Cu}}^2}} = \frac{\rho_{\text{Al}}\ell_{\text{Al}}d_{\text{Cu}}^2}{\rho_{\text{Cu}}\ell_{\text{Cu}}d_{\text{Al}}^2} = \frac{(2.65\times 10^{-8}\ \Omega\cdot\text{m})(10.0\text{ m})(1.8\text{ mm})^2}{(1.68\times 10^{-8}\ \Omega\cdot\text{m})(24.0\text{ m})(2.2\text{ mm})^2} = \boxed{0.44}$$

15. Use Eq. 18–3 to express the resistances, with the area $A = \pi r^2 = \pi d^2/4$, so $R = \rho \frac{\ell}{A} = \rho \frac{4\ell}{\pi d^2}$.

$$R_W = R_{Cu} \rightarrow \rho_W \frac{4\ell}{\pi d_W^2} = \rho_{Cu} \frac{4\ell}{\pi d_{Cu}^2} \rightarrow$$

$$d_W = d_{Cu}\sqrt{\frac{\rho_W}{\rho_{Cu}}} = (2.2\text{ mm})\sqrt{\frac{5.6\times10^{-8}\ \Omega\cdot\text{m}}{1.68\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{4.0\text{ mm}}$$

The diameter of the tungsten should be 4.0 mm.

16. Since the resistance is directly proportional to the length, the length of the long piece must be 4.0 times the length of the short piece.

$$\ell = \ell_{\text{short}} + \ell_{\text{long}} = \ell_{\text{short}} + 4.0\ell_{\text{short}} = 5.0\ell_{\text{short}} \rightarrow \ell_{\text{short}} = 0.20\ell,\ \ell_{\text{long}} = 0.80\ell$$

Make the cut at $\boxed{\text{20\% of the length of the wire}}$.

$$\ell_{\text{short}} = 0.20\ell,\ \ell_{\text{long}} = 0.80\ell \rightarrow R_{\text{short}} = 0.2\,R = 0.2(15\ \Omega) = \boxed{3.0\ \Omega},\ R_{\text{long}} = 0.8\,R = \boxed{12.0\ \Omega}$$

17. Calculate the voltage drop by combining Ohm's law (Eq. 18–2) with the expression for resistance, Eq. 18–3.

$$V = IR = I\frac{\rho\ell}{A} = I\frac{4\rho\ell}{\pi d^2} = (12\text{ A})\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(21\text{ m})}{\pi(1.628\times10^{-3}\text{ m})^2} = \boxed{2.0\text{ V}}$$

18. The wires have the same resistance and the same resistivity.

$$R_{\text{long}} = R_{\text{short}} \rightarrow \frac{\rho\ell_{\text{long}}}{A_1} = \frac{\rho\ell_{\text{short}}}{A_2} \rightarrow \frac{(4)2\ell_{\text{short}}}{\pi d_{\text{long}}^2} = \frac{4\ell_{\text{short}}}{\pi d_{\text{short}}^2} \rightarrow \boxed{\frac{d_{\text{long}}}{d_{\text{short}}} = \sqrt{2}}$$

19. In each case calculate the resistance by using Eq. 18–3 for resistance.

(*a*) $$R_x = \frac{\rho\ell_x}{A_{yz}} = \frac{(3.0\times10^{-5}\ \Omega\cdot\text{m})(1.0\times10^{-2}\text{ m})}{(2.0\times10^{-2}\text{ m})(4.0\times10^{-2}\text{ m})} = 3.75\times10^{-4}\ \Omega \approx \boxed{3.8\times10^{-4}\ \Omega}$$

(*b*) $$R_y = \frac{\rho\ell_y}{A_{xz}} = \frac{(3.0\times10^{-5}\ \Omega\cdot\text{m})(2.0\times10^{-2}\text{ m})}{(1.0\times10^{-2}\text{ m})(4.0\times10^{-2}\text{ m})} = \boxed{1.5\times10^{-3}\ \Omega}$$

(*c*) $$R_z = \frac{\rho\ell_z}{A_{xy}} = \frac{(3.0\times10^{-5}\ \Omega\cdot\text{m})(4.0\times10^{-2}\text{ m})}{(1.0\times10^{-2}\text{ m})(2.0\times10^{-2}\text{ m})} = \boxed{6.0\times10^{-3}\ \Omega}$$

20. The length of the new wire is half the length of the original wire, and the cross-sectional area of the new wire is twice that of the original wire. Use Eq. 18–3.

$$R_0 = \rho\frac{\ell_0}{A_0} \quad \ell = \tfrac{1}{2}\ell_0 \quad A = 2A_0 \quad R = \rho\frac{\ell}{A} = \rho\frac{\tfrac{1}{2}\ell_0}{2A_0} = \tfrac{1}{4}\rho\frac{\ell_0}{A_0} = \tfrac{1}{4}R_0$$

The new resistance is $\boxed{\text{one-fourth}}$ of the original resistance.

21. Use Eq. 18–4 multiplied by ℓ/A so that it expresses resistance instead of resistivity.

$$R = R_0[1+\alpha(T-T_0)] = 1.12R_0 \quad \rightarrow \quad 1+\alpha(T-T_0) = 1.12 \quad \rightarrow$$
$$T - T_0 = \frac{0.12}{\alpha} = \frac{0.12}{0.0068\,(\mathrm{C}^\circ)^{-1}} = 17.6\ \mathrm{C}^\circ \approx \boxed{18\ \mathrm{C}^\circ}$$

So raise the temperature by $22\,\mathrm{C}^\circ$ to a final temperature of $42^\circ\mathrm{C}$.

22. Use Eq. 18–4 for the resistivity.

$$\rho_{\mathrm{TAl}} = \rho_{0\mathrm{Al}}[1+\alpha_{\mathrm{Al}}(T-T_0)] = \rho_{0\mathrm{W}} \quad \rightarrow$$
$$T = T_0 + \frac{1}{\alpha_{\mathrm{Al}}}\left(\frac{\rho_{0\mathrm{W}}}{\rho_{0\mathrm{Al}}} - 1\right) = 20^\circ\mathrm{C} + \frac{1}{0.00429(\mathrm{C}^\circ)^{-1}}\left(\frac{5.6\times10^{-8}\ \Omega\cdot\mathrm{m}}{2.65\times10^{-8}\ \Omega\cdot\mathrm{m}} - 1\right) = 279.49^\circ\mathrm{C} \approx \boxed{280^\circ\mathrm{C}}$$

23. Use Eq. 18–4 multiplied by ℓ/A so that it expresses resistances instead of resistivity.

$$R = R_0[1+\alpha(T-T_0)] \quad \rightarrow$$
$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20^\circ\mathrm{C} + \frac{1}{0.0045(\mathrm{C}^\circ)^{-1}}\left(\frac{140\ \Omega}{12\ \Omega} - 1\right) = 2390^\circ\mathrm{C} \approx \boxed{2400^\circ\mathrm{C}}$$

24. The original resistance is $R_0 = V/I_0$, and the high-temperature resistance is $R = V/I$, where the two voltages are the same. The two resistances are related by Eq. 18–4, multiplied by ℓ/A so that it expresses resistance instead of resistivity.

$$R = R_0[1+\alpha(T-T_0)] \quad \rightarrow \quad T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = T_0 + \frac{1}{\alpha}\left(\frac{V/I}{V/I_0} - 1\right) = T_0 + \frac{1}{\alpha}\left(\frac{I_0}{I} - 1\right)$$
$$= 23.5^\circ\mathrm{C} + \frac{1}{0.00429(\mathrm{C}^\circ)^{-1}}\left(\frac{0.4212\ \mathrm{A}}{0.3818\ \mathrm{A}} - 1\right) = \boxed{47.6^\circ\mathrm{C}}$$

25. The total resistance is to be 3200 ohms (R_{total}) at all temperatures. Write each resistance in terms of Eq. 18–4 (with $T_0 = 0^\circ\mathrm{C}$), multiplied by ℓ/A to express resistance instead of resistivity.

$$\begin{aligned} R_{\mathrm{total}} &= R_{0\mathrm{C}}[1+\alpha_{\mathrm{C}}T] + R_{0\mathrm{N}}[1+\alpha_{\mathrm{N}}T] = R_{0\mathrm{C}} + R_{0\mathrm{C}}\alpha_{\mathrm{C}}T + R_{0\mathrm{N}} + R_{0\mathrm{N}}\alpha_{\mathrm{N}}T \\ &= R_{0\mathrm{C}} + R_{0\mathrm{N}} + (R_{0\mathrm{C}}\alpha_{\mathrm{C}} + R_{0\mathrm{N}}\alpha_{\mathrm{N}})T \end{aligned}$$

For the above to be true, the terms with a temperature dependence must cancel, and the terms without a temperature dependence must add to R_{total}. Thus we have two equations in two unknowns.

$$0 = (R_{0\mathrm{C}}\alpha_{\mathrm{C}} + R_{0\mathrm{N}}\alpha_{\mathrm{N}})T \quad \rightarrow \quad R_{0\mathrm{N}} = -\frac{R_{0\mathrm{C}}\alpha_{\mathrm{C}}}{\alpha_{\mathrm{N}}}$$
$$R_{\mathrm{total}} = R_{0\mathrm{C}} + R_{0\mathrm{N}} = R_{0\mathrm{C}} - \frac{R_{0\mathrm{C}}\alpha_{\mathrm{C}}}{\alpha_{\mathrm{N}}} = \frac{R_{0\mathrm{C}}(\alpha_{\mathrm{N}} - \alpha_{\mathrm{C}})}{\alpha_{\mathrm{N}}} \quad \rightarrow$$
$$R_{0\mathrm{C}} = R_{\mathrm{total}}\frac{\alpha_{\mathrm{N}}}{(\alpha_{\mathrm{N}} - \alpha_{\mathrm{C}})} = (3.20\ \mathrm{k}\Omega)\frac{0.0004(\mathrm{C}^\circ)^{-1}}{0.0004(\mathrm{C}^\circ)^{-1} + 0.0005(\mathrm{C}^\circ)^{-1}} = 1.422\ \mathrm{k}\Omega \approx \boxed{1.42\ \mathrm{k}\Omega}$$
$$R_{0\mathrm{N}} = R_{\mathrm{total}} - R_{0\mathrm{C}} = \boxed{1.78\ \mathrm{k}\Omega}$$

26. (*a*) Calculate each resistance separately using Eq. 18–3 and then add the resistances together to find the total resistance.

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{4\rho_{\text{Cu}}\ell}{\pi d^2} = \frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(5.0\ \text{m})}{\pi(1.4\times10^{-3}\ \text{m})^2} = 0.05457\ \Omega$$

$$R_{\text{Al}} = \frac{\rho_{\text{Al}}\ell}{A} = \frac{4\rho_{\text{Al}}\ell}{\pi d^2} = \frac{4(2.65\times10^{-8}\ \Omega\cdot\text{m})(5.0\ \text{m})}{\pi(1.4\times10^{-3}\ \text{m})^2} = 0.08607\ \Omega$$

$$R_{\text{total}} = R_{\text{Cu}} + R_{\text{Al}} = 0.05457\ \Omega + 0.08607\ \Omega = 0.14064\ \Omega \approx \boxed{0.141\ \Omega}$$

(*b*) The current through the wire is the voltage divided by the total resistance.

$$I = \frac{V}{R_{\text{total}}} = \frac{95\times10^{-3}\ \text{V}}{0.14064\ \Omega} = 0.6755\ \text{A} \approx \boxed{0.68\ \text{A}}$$

(*c*) For each segment of wire, Ohm's law is true. Both wires have the current found in (*b*) above.

$$V_{\text{Cu}} = IR_{\text{Cu}} = (0.6755\ \text{A})(0.05457\ \Omega) \approx \boxed{0.037\ \text{V}}$$

$$V_{\text{Al}} = IR_{\text{Al}} = (0.6755\ \text{A})(0.08607\ \Omega) \approx \boxed{0.058\ \text{V}}$$

Notice that the total voltage is 95 mV.

27. Use Eq. 18–5 to find the power from the voltage and the current.

$$P = IV = (0.24\ \text{A})(3.0\ \text{V}) = \boxed{0.72\ \text{W}}$$

28. Use Eq. 18–6b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{3300\ \text{W}} = \boxed{17\ \Omega}$$

29. Use Eq. 18–6b to find the voltage from the power and the resistance.

$$P = \frac{V^2}{R} \quad\rightarrow\quad V = \sqrt{PR} = \sqrt{(0.25\ \text{W})(3900\ \Omega)} = \boxed{31\ \text{V}}$$

30. Use Eq. 18–6b to find the resistance and Eq. 18–5 to find the current.

(*a*) $$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{(110\ \text{V})^2}{75\ \text{W}} = 161.3\ \Omega \approx \boxed{160\ \Omega}$$

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} = \frac{75\ \text{W}}{110\ \text{V}} = 0.6818\ \text{A} \approx \boxed{0.68\ \text{A}}$$

(*b*) $$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{(110\ \text{V})^2}{250\ \text{W}} = 48.4\ \Omega \approx \boxed{48\ \Omega}$$

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} = \frac{250\ \text{W}}{110\ \text{V}} = 2.273\ \text{A} \approx \boxed{2.3\ \text{A}}$$

31. The battery's potential energy is equal to the charge that it can deliver times its voltage.

$$\text{PE} = QV \quad \rightarrow \quad Q = \frac{\text{PE}}{V} = \frac{16\ \text{kW}\cdot\text{h}\left(\dfrac{1000\ \text{W}}{1\ \text{kW}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right)}{340\ \text{V}} = \boxed{1.7\times10^5\ \text{C}}$$

32. The power needed is the voltage times the current, Eq. 18–5.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} = \frac{45{,}000\ \text{W}}{340\ \text{V}} = 132.4\ \text{A} \approx \boxed{130\ \text{A}}$$

33. (*a*) Since $P = \dfrac{V^2}{R} \quad \rightarrow \quad R = \dfrac{V^2}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the $\boxed{\text{950-W}}$ setting has the higher resistance.

(*b*) $$R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{950\ \text{W}} = 15.16\ \Omega \approx \boxed{15\ \Omega}$$

(*c*) $$R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{1450\ \text{W}} = 9.93\ \Omega \approx \boxed{9.9\ \Omega}$$

34. (*a*) Use Eq. 18–2 to find the resistance.

$$R = \frac{V}{I} = \frac{12\ \text{V}}{0.60\ \text{A}} = \boxed{20\ \Omega}\ \text{(2 significant figures)}$$

(*b*) An amount of charge ΔQ loses a potential energy of $(\Delta Q)V$ as it passes through the resistor. The amount of charge is found from Eq. 18–1.

$$\Delta\text{PE} = (\Delta Q)V = (I\Delta t)V = (0.60\ \text{A})(60\ \text{s})(12\ \text{V}) = \boxed{430\ \text{J}}$$

35. (*a*) Use Eq. 18–5 to find the current.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} = \frac{130\ \text{W}}{120\ \text{V}} = 1.08\ \text{A} \approx \boxed{1.1\ \text{A}}$$

(*b*) Use Eq. 18–6b to find the resistance.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{130\ \text{W}} \approx 110.8\ \Omega \approx \boxed{110\ \Omega}$$

36. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. 18–6a, $P = \dfrac{V^2}{R}$. Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120 V, the power will be reduced by a factor of 4. Thus in the United States the bulb will appear only about $\boxed{\text{1/4 as bright}}$ as it does in Europe.

$\boxed{37.}$ To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$\text{Energy} = P(\text{in kW})t(\text{in h}) = (550\ \text{W})\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right)(5.0\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)$$
$$= 0.04583\ \text{kWh} \approx \boxed{0.046\ \text{kWh}}$$

To find the cost of the energy used in a month, multiply by 4 days per week of usage, 4 weeks per month, and the cost per kWh.

$$\text{Cost} = \left(0.04583\ \frac{\text{kWh}}{\text{d}}\right)\left(\frac{4\ \text{d}}{1\ \text{week}}\right)\left(\frac{4\ \text{weeks}}{1\ \text{month}}\right)\left(\frac{9.0\ \text{cents}}{\text{kWh}}\right) \approx \boxed{6.6\ \text{cents/month}}$$

38. To find the cost of the energy, multiply the kilowatts of power consumption by the number of hours in operation by the cost per kWh.

$$\text{Cost} = (25\ \text{W})\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right)(365\ \text{d})\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\$0.095}{\text{kWh}}\right) \approx \boxed{\$\,21}$$

39. The $\text{A}\cdot\text{h}$ rating is the amount of charge that the battery can deliver. The potential energy of the charge is the charge times the voltage.

$$\text{PE} = QV = (65\ \text{A}\cdot\text{h})\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)(12\ \text{V}) = \boxed{2.8\times10^{6}\ \text{J}} \times \frac{1\ \text{W}\cdot\text{s}}{1\ \text{J}} \times \frac{1\ \text{kW}}{1000\ \text{W}} \times \frac{1\ \text{h}}{3600\ \text{s}} = 0.78\ \text{kWh}$$

40. (*a*) Calculate the resistance from Eq. 18–2 and the power from Eq. 18–5.

$$R = \frac{V}{I} = \frac{3.0\ \text{V}}{0.38\ \text{A}} = 7.895\ \Omega \approx \boxed{7.9\ \Omega} \qquad P = IV = (0.38\ \text{A})(3.0\ \text{V}) = 1.14\ \text{W} \approx \boxed{1.1\ \text{W}}$$

(*b*) If four D-cells are used, then the voltage will double, to 6.0 V. Assuming that the resistance of the bulb stays the same (by ignoring heating effects in the filament), the power that the bulb would need to dissipate is given by Eq. 18–6b, $P = \frac{V^2}{R}$. A doubling of the voltage means the power is increased by a factor of $\boxed{4}$. This should not be tried because the bulb is probably not rated for such a high wattage. The filament in the bulb would probably burn out, and the glass bulb might even explode if the filament burns violently.

41. Each bulb will draw an amount of current found from Eq. 18–5.

$$P = IV \quad \rightarrow \quad I_{\text{bulb}} = \frac{P}{V}$$

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

$$I_{\text{total}} = nI_{\text{bulb}} = n\frac{P}{V} \quad \rightarrow \quad n = \frac{VI_{\text{total}}}{P} = \frac{(120\ \text{V})(15\ \text{A})}{75\ \text{W}} = \boxed{24\ \text{bulbs}}$$

42. Find the power dissipated in the cord by Eq. 18–6a, using Eq. 18–3 for the resistance.

$$P = I^2R = I^2\rho\frac{2L}{A} = (18.0\ \text{A})^2(1.68\times10^{-8}\ \Omega\cdot\text{m})\frac{2(2.7\ \text{m})}{\pi\frac{1}{4}(0.129\times10^{-2}\ \text{m})^2} = 22.49\ \text{W} \approx \boxed{22\ \text{W}}$$

43. Use Eqs. 18–3 and 18–6b to calculate the length of the wire.

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \frac{4\rho\ell}{\pi d^2};\; P = \frac{V^2}{R} = \frac{V^2}{\dfrac{4\rho\ell}{\pi d^2}} \quad\rightarrow$$

$$\ell = \frac{V^2\pi d^2}{4\rho P} = \frac{(1.5\text{ V})^2\pi(5.0\times10^{-4}\text{ m})^2}{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(18\text{ W})} = 1.461\text{ m} \approx \boxed{1.5\text{ m}}$$

If the voltage increases by a factor of 6 without the resistance changing, then the power will increase by a factor of 36. The blanket would theoretically be able to deliver 540 W of power, which might make the material catch on fire or burn the occupant.

44. Find the current used to deliver the power in each case and then find the power dissipated in the resistance at the given current.

$$P_{\text{delivered}} = IV \quad\rightarrow\quad I = \frac{P_{\text{delivered}}}{V} \qquad P_{\text{dissipated}} = I^2R = \frac{P_{\text{delivered}}^2}{V^2}R$$

$$P_{\substack{\text{dissipated}\\ 12{,}000\text{ V}}} = \frac{(7.5\times10^5\text{ W})^2}{(1.2\times10^4\text{ V})^2}(3.0\ \Omega) = 11{,}719\text{ W}$$

$$P_{\substack{\text{dissipated}\\ 50{,}000\text{ V}}} = \frac{(7.5\times10^5\text{ W})^2}{(5\times10^4\text{ V})^2}(3.0\ \Omega) = 675\text{ W} \qquad \text{difference} = 11{,}719\text{ W} - 675\text{ W} = \boxed{1.1\times10^4\text{ W}}$$

45. (*a*) By conservation of energy and the efficiency claim, 85% of the electrical power emitted by the heater must be the rate at which energy is absorbed by the water. The power emitted by the heater is given by Eq. 18–5.

$$0.85P_{\substack{\text{emitted by}\\ \text{heater}}} = P_{\substack{\text{absorbed}\\ \text{by water}}} \quad\rightarrow\quad 0.85(IV) = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \quad\rightarrow$$

$$I = \frac{mc\Delta T}{0.85\,Vt} = \frac{(0.120\,\text{kg})(4186\text{ J/kg})(95°\text{C} - 25°\text{C})}{(0.85)(12\text{ V})(480\text{ s})} = 7.182\text{ A} \approx \boxed{7.2\text{ A}}$$

(*b*) Use Ohm's law to find the resistance of the heater.

$$V = IR \quad\rightarrow\quad R = \frac{V}{I} = \frac{12\text{ V}}{7.182\text{ A}} = \boxed{1.7\ \Omega}$$

46. The water temperature rises by absorbing the heat energy that the electromagnet dissipates. Express both energies in terms of power, which is energy per unit time. We assume that 240 V has 3 significant figures. The power emitted by powering the electromagnet is given by Eq. 18–5.

$$P_{\text{electric}} = P_{\substack{\text{to heat}\\ \text{water}}} \quad\rightarrow\quad IV = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \quad\rightarrow$$

$$\frac{m}{t} = \frac{IV}{c\Delta T} = \frac{(21.5\text{ A})(240\text{ V})}{(4186\text{ J/kg}\cdot°\text{C})(6.50\text{ C}°)} = 0.1896\text{ kg/s} \approx \boxed{0.190\text{ kg/s}}$$

This is 190 mL per second.

47. Use Ohm's law, Eq. 18–2, and the relationship between peak and rms values, Eq. 18–8a.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{R} = \sqrt{2}\frac{220\text{ V}}{2700\ \Omega} = \boxed{0.12\text{ A}}$$

48. Find the peak current from Ohm's law and then find the rms current from the relationship between peak and rms values.

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{180\text{ V}}{310\ \Omega} = 0.58065\text{ A} = \boxed{0.58\text{ A}} \qquad I_{\text{rms}} = I_{\text{peak}}/\sqrt{2} = (0.58065\text{ A})/\sqrt{2} = \boxed{0.41\text{ A}}$$

49. (*a*) When everything electrical is turned off, no current will be flowing into the house, even though a voltage is being supplied. Since for a given voltage, the more resistance, the lower the current, a zero current corresponds to an $\boxed{\text{infinite resistance}}$.

(*b*) Assuming that the voltage is 120 V, use Eq. 18–6ba to calculate the resistance.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{2(75\text{ W})} = \boxed{96\ \Omega}$$

50. The power and current can be used to find the peak voltage and then the rms voltage can be found from the peak voltage.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}} \quad \rightarrow \quad V_{\text{rms}} = \frac{\sqrt{2}\overline{P}}{I_{\text{peak}}} = \frac{\sqrt{2}(1500\text{ W})}{6.4\text{ A}} = \boxed{330\text{ V}}$$

$\boxed{51.}$ Use the average power and rms voltage to calculate the peak voltage and peak current.

(*a*) $V_{\text{peak}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(660\text{ V}) = 933.4\text{ V} \approx \boxed{930\text{ V}}$

(*b*) $\overline{P} = I_{\text{rms}}V_{\text{rms}} = \dfrac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}} \quad \rightarrow \quad I_{\text{peak}} = \dfrac{\sqrt{2}\overline{P}}{V_{\text{rms}}} = \dfrac{\sqrt{2}(1800\text{ W})}{660\text{ V}} = \boxed{3.9\text{ A}}$

52. (*a*) We use Eqs. 18–9c and 18–9a.

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} \quad \rightarrow \quad V_{\text{rms}} = \sqrt{\overline{P}R} = \sqrt{(100\text{ W})(8\ \Omega)} = 28.3\text{ V} \approx \boxed{30\text{ V}}$$

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{100\text{ W}}{28.3\text{ V}} = 3.54\text{ A} \approx \boxed{4\text{ A}}$$

(*b*) We repeat the process using the lower power.

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} \quad \rightarrow \quad V_{\text{rms}} = \sqrt{\overline{P}R} = \sqrt{(1.0\text{ W})(8\ \Omega)} = 2.83\text{ V} \approx \boxed{3\text{ V}}$$

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{1.0\text{ W}}{2.83\text{ V}} = 0.354\text{ A} \approx \boxed{0.4\text{ A}}$$

53. (*a*) We assume that the 2.2 hp is the average power, so the maximum power is twice that, or 4.4 hp, as seen in Fig. 18–22.

$$4.4\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right)=3282\text{ W}\approx\boxed{3300\text{ W}}$$

(*b*) Use the average power and the rms voltage to find the peak current.

$$\overline{P}=I_{\text{rms}}V_{\text{rms}}=\frac{I_{\text{peak}}}{\sqrt{2}}V_{\text{rms}}\quad\rightarrow\quad I_{\text{peak}}=\frac{\sqrt{2}\overline{P}}{V_{\text{rms}}}=\frac{\sqrt{2}\left[\frac{1}{2}(3282\text{ W})\right]}{240\text{ V}}=\boxed{9.7\text{ A}}$$

54. (*a*) The average power used can be found from the resistance and the rms voltage by Eq. 18–9c.

$$\overline{P}=\frac{V_{\text{rms}}^2}{R}=\frac{(240\text{ V})^2}{38\ \Omega}=1516\text{ W}\approx\boxed{1500\text{ W}}$$

(*b*) The maximum power is twice the average power, and the minimum power is 0.

$$P_{\text{max}}=2\overline{P}=2(1516\text{ W})\approx\boxed{3.0\times10^3\text{ W}}\quad P_{\text{min}}=\boxed{0}$$

55. We follow exactly the derivation in Example 18–14, which results in an expression for the drift velocity.

$$v_{\text{d}}=\frac{I}{neA}=\frac{I}{\dfrac{N(1\text{ mole})}{m(1\text{ mole})}\rho_{\text{D}}e\pi\left(\frac{1}{2}d\right)^2}=\frac{4Im}{N\rho_{\text{D}}e\pi d^2}$$

$$=\frac{4(2.7\times10^{-6}\text{ A})(63.5\times10^{-3}\text{ kg})}{(6.02\times10^{23})(8.9\times10^3\text{ kg/m}^3)(1.60\times10^{-19}\text{ C})\pi(0.65\times10^{-3}\text{ m})^2}=\boxed{6.0\times10^{-10}\text{ m/s}}$$

56. (*a*) Use Ohm's law to find the resistance.

$$V=IR\quad\rightarrow\quad R=\frac{V}{I}=\frac{22.0\times10^{-3}\text{ V}}{0.75\text{ A}}=0.02933\ \Omega\approx\boxed{0.029\ \Omega}$$

(*b*) Find the resistivity from Eq. 18–3.

$$R=\frac{\rho L}{A}\quad\rightarrow\quad\rho=\frac{RA}{L}=\frac{R\pi r^2}{L}=\frac{(0.02933\ \Omega)\pi(1.0\times10^{-3}\text{ m})^2}{(4.80\text{ m})}=\boxed{1.9\times10^{-8}\ \Omega\cdot\text{m}}$$

(*c*) Find the number of electrons per unit volume from Eq. 18–10.

$$I=neAv_{\text{d}}\quad\rightarrow$$

$$n=\frac{I}{eAv_{\text{d}}}=\frac{I}{e\pi r^2v_{\text{d}}}=\frac{(0.75\text{ A})}{(1.60\times10^{-19}\text{ C})\pi(1.0\times10^{-3}\text{ m})^2(1.7\times10^{-5}\text{ m/s})}=\boxed{8.8\times10^{28}\text{ electrons/m}^3}$$

57. We are given a net charge, a concentration, and a speed (like the drift speed) for both types of ions. From that we can use Eq. 18–10 to determine the current per unit area. Both currents are in the same direction in terms of conventional current—positive charge moving north has the same effect as negative charge moving south—so they can be added.

$$\begin{aligned}
&I = neAv_{\text{d}} \quad \rightarrow \\
&\frac{I}{A} = (nev_{\text{d}})_{\text{He}} + (nev_{\text{d}})_{\text{O}} \\
&\quad = (2.4\times10^{12}\ \text{ions/m}^3)(2\ \text{charges/ion})(1.60\times10^{-19}\ \text{C/charge})(2.0\times10^{6}\ \text{m/s})] + \\
&\qquad\qquad [(7.0\times10^{11}\ \text{ions/m}^3)(-1\ \text{charge/ion})(-1.60\times10^{-19}\ \text{C/charge})(6.2\times10^{6}\ \text{m/s})] \\
&\quad = \boxed{2.2\ \text{A/m}^2\text{, north}}
\end{aligned}$$

58. The magnitude of the electric field is the voltage change per unit meter.

$$|E| = \frac{\Delta V}{\Delta x} = \frac{70\times10^{-3}\ \text{V}}{1.0\times10^{-8}\ \text{m}} = \boxed{7\times10^{6}\ \text{V/m}}$$

59. The speed is the change in position per unit time.

$$v = \frac{\Delta x}{\Delta t} = \frac{7.20\times10^{-2}\ \text{m} - 3.70\times10^{-2}\ \text{m}}{0.0063\ \text{s} - 0.0052\ \text{s}} = \boxed{32\ \text{m/s}}$$

Two measurements are needed because there may be a time delay from the stimulation of the nerve to the generation of the action potential.

60. The power is the work done per unit time. The work done to move a charge through a potential difference is the charge times the potential difference. The charge density must be multiplied by the surface area of the cell (the surface area of an open tube, length times circumference) to find the actual charge moved.

$$\begin{aligned}
P &= \frac{W}{t} = \frac{QV}{t} = \frac{Q}{t}V \\
&= \left(3\times10^{-7}\ \frac{\text{mol}}{\text{m}^2\cdot\text{s}}\right)\left(6.02\times10^{23}\ \frac{\text{ions}}{\text{mol}}\right)\left(1.6\times10^{-19}\ \frac{\text{C}}{\text{ion}}\right)(0.10\ \text{m})\pi(20\times10^{-6}\ \text{m})(0.030\ \text{V}) \\
&= \boxed{5\times10^{-9}\ \text{W}}
\end{aligned}$$

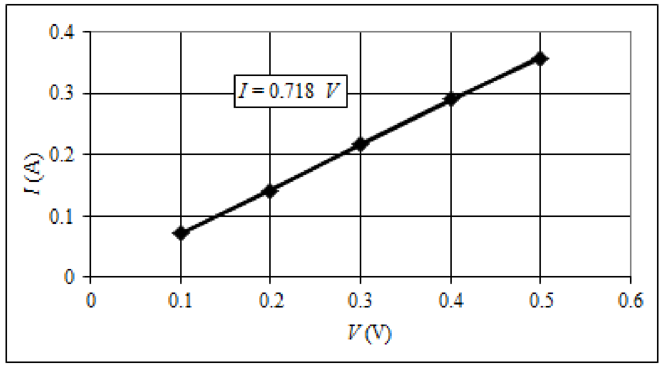
61. The energy supplied by the battery is the energy consumed by the lights.

$$E_{\text{supplied}} = E_{\text{consumed}} \quad \rightarrow \quad Q\Delta V = Pt \quad \rightarrow$$

$$t = \frac{Q\Delta V}{P} = \frac{(75\ \text{A}\cdot\text{h})(3600\ \text{s/h})(12\ \text{V})}{92\ \text{W}} = 35{,}217\ \text{s}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 9.783\ \text{h} \approx \boxed{9.8\ \text{h}}$$

62. (*a*) If the wire obeys Ohm's law, then $V = IR$ or $I = V/R$, showing a linear relationship between I and V. A graph of I vs. V should give a straight line with a slope of $1/R$ and a y intercept of 0.

(*b*) From the graph and the calculated linear fit, we see that the wire obeys Ohm's law.



$$\text{slope} = \frac{1}{R} \quad \rightarrow$$

$$R = \frac{1}{0.718}\,\text{A/V} = \boxed{1.39\ \Omega}$$

(*c*) Use Eq. 18–3 to find the resistivity.

$$R = \rho\frac{\ell}{A} \quad \rightarrow \quad \rho = \frac{AR}{\ell} = \frac{\pi d^2 R}{4\ell} = \frac{\pi(3.2\times10^{-4}\ \text{m})^2(1.39\ \Omega)}{4(0.11\ \text{m})} = \boxed{1.0\times10^{-6}\ \Omega\cdot\text{m}}$$

From Table 18–1, the material is $\boxed{\text{Nichrome}}$.

63. Use Eq. 18–5 to calculate the current. We assume that 120 V has 3 significant figures.

$$P = IV \quad \rightarrow \quad I = \frac{P}{V} = \frac{746\ \text{W}}{120\ \text{V}} = \boxed{6.22\ \text{A}}$$

64. From Eq. 18–2, if $R = V/I$, then $G = I/V$.

$$G = \frac{I}{V} = \frac{0.44\ \text{A}}{3.0\ \text{V}} = 0.1467\ \text{S} \approx \boxed{0.15\ \text{S}}$$

65. Use Eq. 18–6b to express the resistance in terms of the power and Eq. 18–3 to express the resistance in terms of the wire geometry.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} \qquad R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2} = 4\rho\frac{L}{\pi d^2}$$

$$4\rho\frac{L}{\pi d^2} = \frac{V^2}{P} \quad \rightarrow \quad d = \sqrt{\frac{4\rho LP}{\pi V^2}} = \sqrt{\frac{4(9.71\times10^{-8}\ \Omega\cdot\text{m})(3.8\ \text{m})(1500\ \text{W})}{\pi(110\ \text{V})^2}} = \boxed{2.4\times10^{-4}\ \text{m}}$$

66. (*a*) Calculate the total kWh used per day and then multiply by the number of days and the cost per kWh.

$$(2.2\ \text{kW})(2.0\ \text{h/d}) + 4(0.1\ \text{kW})(6.0\ \text{h/d}) + (3.0\ \text{kW})(1.0\ \text{h/d}) + (2.0\ \text{kWh/d})$$
$$= 11.8\ \text{kWh/d}$$

$$\text{Cost} = (11.8\ \text{kWh/d})(30\ \text{d})\left(\frac{\$0.115}{\text{kWh}}\right) = \$40.71 \approx \boxed{\$40.70\ \text{per month}}$$

(*b*) The energy required by the household is 35% of the energy that needs to be supplied by the power plant.

Household energy = 0.35(Coal mass)(Coal energy per mass) $\rightarrow$

$$\text{Coal mass} = \frac{\text{Household energy}}{(0.35)(\text{Coal energy per mass})} = \frac{(11.8\text{ kWh/d})(365\text{ d})\left(\frac{1000\text{ W}}{\text{kW}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)}{(0.35)\left(7500\frac{\text{kcal}}{\text{kg}}\right)\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)}$$

$$= 1411\text{ kg} \approx \boxed{1400\text{ kg of coal}}$$

67. To deliver 15 MW of power at 120 V, a current of $I = \frac{P}{V} = \frac{1.5\times10^7\text{ W}}{120\text{ V}} = 125{,}000\text{ A}$ is required.

Calculate the power dissipated in the resistors using the current and the resistance. There are two wires, so we consider 2.0 m of wire.

$$P = I^2R = I^2\rho\frac{L}{A} = I^2\rho\frac{L}{\pi r^2} = 4I^2\rho\frac{L}{\pi d^2} = 4(1.25\times10^5\text{ A})^2(1.68\times10^{-8}\ \Omega\cdot\text{m})\frac{2(1.0\text{ m})}{\pi(5.0\times10^{-3}\text{ m})^2}$$

$$= 2.674\times10^7\text{ W}$$

$$\text{Cost} = (\text{Power})(\text{Time})(\text{Rate per kWh}) = (2.674\times10^7\text{W})\left(\frac{1\text{ kW}}{1000\text{ W}}\right)(1\text{ h})\left(\frac{\$0.12}{\text{kWh}}\right)$$

$$= \$3209 \approx \boxed{\$3200\text{ per hour per meter}}$$

68. (*a*) Use Eq. 18–6b to relate the power to the voltage for a constant resistance.

$$P = \frac{V^2}{R} \quad\rightarrow\quad \frac{P_{105}}{P_{117}} = \frac{(105\text{ V})^2/R}{(117\text{ V})^2/R} = \frac{(105\text{ V})^2}{(117\text{ V})^2} = 0.805\text{ or a }\boxed{19.5\%\text{ decrease}}$$

(*b*) The lower power output means that the resistor is generating less heat, so the resistor's temperature would be lower. The lower temperature results in a lower value of the resistance, which would increase the power output at the lower voltages. Thus the $\boxed{\text{decrease would be smaller}}$ than the value given in the first part of the problem.

69. Assume that we have a meter of wire, carrying 35 A of current, and dissipating 1.5 W of heat. The power dissipated is $P_R = I^2R,$ and the resistance is $R = \frac{\rho\ell}{A}.$

$$P_R = I^2R = I^2\frac{\rho\ell}{A} = I^2\frac{\rho\ell}{\pi r^2} = I^2\frac{4\rho\ell}{\pi d^2} \quad\rightarrow$$

$$d = \sqrt{I^2\frac{4\rho\ell}{P_R\pi}} = 2I\sqrt{\frac{\rho\ell}{P_R\pi}} = 2(35\text{ A})\sqrt{\frac{(1.68\times10^{-8}\ \Omega\cdot\text{m})(1.0\text{ m})}{(1.5\text{ W})\pi}} = \boxed{4.2\times10^{-3}\text{ m}}$$

70. (*a*) The resistance at the operating temperature can be calculated directly from Eq. 18–6b.

$$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{75\text{ W}} = \boxed{190\ \Omega}$$

(*b*) The resistance at room temperature is found by converting Eq. 18–4 into an equation for resistances and solving for R_0.

$$R = R_0[1 + a(T - T_0)]$$

$$R_0 = \frac{R}{[1+\alpha(T-T_0)]} = \frac{192\ \Omega}{\left[1+(0.0045\text{ K}^{-1})(2800\text{ K}-293\text{ K})\right]} = 15.63\ \Omega \approx \boxed{16\ \Omega}$$

71. (*a*) The angular frequency is $\omega = 210$ rad/s.

$$f = \frac{\omega}{2\pi} = \frac{210 \text{ rad/s}}{2\pi} = \boxed{33 \text{ Hz}}$$

(*b*) The maximum current is 1.40 A.

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{1.40 \text{ A}}{\sqrt{2}} = \boxed{0.990 \text{ A}}$$

(*c*) For a resistor, $V = IR$.

$$V = IR = (1.40 \text{ A})(\sin 210\, t)(24.0\ \Omega) = \boxed{(33.6 \sin 210t)\,\text{V}}$$

72. (*a*) The power delivered to the interior is 65% of the power drawn from the source.

$$P_{\text{interior}} = 0.65\, P_{\text{source}} \quad \rightarrow \quad P_{\text{source}} = \frac{P_{\text{interior}}}{0.65} = \frac{950 \text{ W}}{0.65} = 1462 \text{ W} \approx \boxed{1500 \text{ W}}$$

(*b*) The current drawn is current from the source. The source power is used to calculate the current.

$$P_{\text{source}} = IV_{\text{source}} \quad \rightarrow \quad I = \frac{P_{\text{source}}}{V_{\text{source}}} = \frac{1462 \text{ W}}{120 \text{ V}} = 12.18 \text{ A} \approx \boxed{12 \text{ A}}$$

73. The volume of wire is unchanged by the stretching. The volume is equal to the length of the wire times its cross-sectional area. Since the length was increased by a factor of 1.5, the area was decreased by a factor of 1.5. Use Eq. 18–3.

$$R_0 = \rho\frac{\ell_0}{A_0} \quad \ell = 1.5\ell_0 \quad A = \tfrac{1}{1.5}A_0 \quad R = \rho\frac{\ell}{A} = \rho\frac{1.5\ell_0}{\frac{1}{1.5}A_0} = 2.25\rho\frac{\ell_0}{A_0} = 2.25R_0 = \boxed{2.25\ \Omega}$$

74. The long, thick conductor is labeled as conductor number 1, and the short, thin conductor is labeled as number 2. The power transformed by a resistor is given by Eq. 18–6, $P = V^2/R$, and both have the same voltage applied. Use Eq. 18–3 for resistance.

$$R_1 = \rho\frac{\ell_1}{A_1} \quad R_2 = \rho\frac{\ell_2}{A_2} \quad \ell_1 = 2\ell_2 \quad A_1 = 4A_2 \ (\text{diameter}_1 = 2\text{diameter}_2)$$

$$\frac{P_1}{P_2} = \frac{V_1^2/R_1}{V_2^2/R_2} = \frac{R_2}{R_1} = \frac{\rho\dfrac{\ell_2}{A_2}}{\rho\dfrac{\ell_1}{A_1}} = \frac{\ell_2}{\ell_1}\frac{A_1}{A_2} = \frac{1}{2}\times 4 = 2 \quad \boxed{P_1 : P_2 = 2:1}$$

75. (*a*) From Eq. 18–5, if power P is delivered to the transmission line at voltage V, then there must be a current $I = P/V$. As this current is carried by the transmission line, there will be power losses of I^2R due to the resistance of the wire. This power loss can be expressed as $\Delta P = I^2R = \boxed{P^2R/V^2}$. Equivalently, there is a voltage drop across the transmission lines of $V' = IR$. Thus the voltage available to the users is $V - V'$, and the power available to the users is $P' = (V - V')I = VI - V'I = VI - I^2R = P - I^2R$. The power loss is $\Delta P = P - P' = P - (P - I^2R) = I^2R = \boxed{P^2R/V^2}$.

(*b*) Since $\Delta P \propto \dfrac{1}{V^2}$, V should be $\boxed{\text{as large as possible}}$ to minimize ΔP.

76. (*a*) Use Eq. 18–6b.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(240\text{ V})^2}{2800\text{ W}} = 20.57\ \Omega \approx \boxed{21\ \Omega}$$

(*b*) Only 75% of the heat from the oven is used to heat the water.

$$0.75(P_{\text{oven}})t = \text{Heat absorbed by water} = mc\Delta T \quad \rightarrow$$

$$t = \frac{mc\Delta T}{0.75(P_{\text{oven}})} = \frac{(0.120\text{ L})\left(\dfrac{1\text{ kg}}{1\text{ L}}\right)(4186\text{ J/kg}\cdot\text{C}^\circ)(85\text{ C}^\circ)}{0.65(2800\text{ W})} = 23.46\text{ s} \approx \boxed{23\text{ s}}$$

(*c*) $$\frac{11\text{ cents}}{\text{kWh}}(2.8\text{ kW})(23.46\text{ s})\frac{1\text{ h}}{3600\text{ s}} = \boxed{0.20\text{ cents}} \ \ (1/5\text{ of a cent})$$

$\boxed{77.}$ (*a*) The horsepower required is the power dissipated by the frictional force, since we are neglecting the energy used for acceleration.

$$P = F\upsilon = (440\text{ N})(45\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 5500\text{ W}$$

$$P = 5500\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 7.373\text{ hp} \approx \boxed{7.4\text{ hp}}$$

(*b*) The charge available by each battery is $Q = 95\text{A}\cdot\text{h} = 95\text{ C/s}\cdot 3600\text{ s} = 3.42\times10^5\text{ C}$, so the total charge available is 24 times that. The potential energy of that charge is the charge times the voltage. That energy must be delivered (batteries discharged) in a certain amount of time to produce the 3000 W necessary. The speed of the car times the discharge time is the range of the car between recharges.

$$P = \frac{\text{PE}}{t} = \frac{QV}{t} \quad \rightarrow \quad t = \frac{QV}{P} = \frac{d}{\upsilon} \quad \rightarrow$$

$$d = \upsilon t = \upsilon\frac{QV}{P} = \upsilon\frac{QV}{F\upsilon} = \frac{QV}{F} = \frac{24(3.42\times10^5\text{ C})(12\text{ V})}{440\text{ N}} = 2.24\times10^5\text{ m} \approx \boxed{220\text{ km}}$$

78. The mass of the wire is the density of copper times the volume of the wire, and the resistance of the wire is given by Eq. 18–3. We represent the mass density by ρ_{m} and the resistivity by ρ.

$$R = \rho\frac{\ell}{A} \quad \rightarrow \quad A = \frac{\rho\ell}{R} \qquad m = \rho_{\text{m}}\ell A = \rho_{\text{m}}L\frac{\rho\ell}{R} \quad \rightarrow$$

$$\ell = \sqrt{\frac{mR}{\rho_{\text{m}}\rho}} = \sqrt{\frac{(0.0155\text{ kg})(15.2\ \Omega)}{(8.9\times10^3\ \text{kg/m}^3)(1.68\times10^{-8}\ \Omega\cdot\text{m})}} = 39.70\text{ m} \approx \boxed{39.7\text{ m}}$$

$$A = \frac{\rho\ell}{R} = \pi\left(\tfrac{1}{2}d\right)^2 \quad \rightarrow \quad d = \sqrt{\frac{4\rho\ell}{\pi R}} = \sqrt{\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(39.70\text{ m})}{\pi(15.2\ \Omega)}} = \boxed{2.36\times10^{-4}\ \text{m}}$$

79. The resistance can be calculated from the power and voltage, and then the diameter of the wire can be calculated from the resistance.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} \qquad R = \frac{\rho L}{A} = \frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2} \quad \rightarrow \quad \frac{V^2}{P} = \frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2} \quad \rightarrow$$

$$d = \sqrt{\frac{4\rho L P}{\pi V^2}} = \sqrt{\frac{4(100\times10^{-8}\ \Omega\cdot\text{m})(3.5\ \text{m})(95\ \text{W})}{\pi(120\ \text{V})^2}} = \boxed{1.7\times10^{-4}\ \text{m}}$$

80. Use Eq. 18–6b.

(*a*) $P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{12\ \Omega} = \boxed{1200\ \text{W}}$

(*b*) $P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{140\ \Omega} = 103\ \text{W} \approx \boxed{100\ \text{W}}$ (2 significant figures)

81. Use Eq. 18–6b for the power in each case, assuming the resistance is constant.

$$\frac{P_{13.8\ \text{V}}}{P_{12.0\ \text{V}}} = \frac{(V^2/R)_{13.8\ \text{V}}}{(V^2/R)_{12.0\ \text{V}}} = \frac{13.8^2}{12.0^2} = 1.3225 = \boxed{32\%\ \text{increase}}$$

$\boxed{82.}$ The resistance of the filament when the flashlight is on is $R = \frac{V}{I} = \frac{3.0\ \text{V}}{0.20\ \text{A}} = 15\ \Omega$. That can be used with a combination of Eqs. 18–3 and 18–4 to find the temperature.

$$R = R_0[1+\alpha(T-T_0)] \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20^\circ\text{C} + \frac{1}{0.0045(\text{C}^\circ)^{-1}}\left(\frac{15\ \Omega}{1.5\ \Omega} - 1\right) = 2020^\circ\text{C} \approx \boxed{2000^\circ\text{C}}$$

83. The solution assumes that the 40-W values are precise to 2 significant figures.

(*a*) The current can be found from Eq. 18–5.

$$I = P/V \quad I_A = P_A/V_A = 40\text{W}/120\ \text{V} = \boxed{0.33\ \text{A}} \quad I_B = P_B/V_B = 40\text{W}/12\ \text{V} = \boxed{3.3\ \text{A}}$$

(*b*) The resistance can be found from Eq. 18–6b.

$$R = \frac{V^2}{P} \quad R_A = \frac{V_A^2}{P_A} = \frac{(120\ \text{V})^2}{40\ \text{W}} = \boxed{360\ \Omega} \qquad R_B = \frac{V_B^2}{P_B} = \frac{(12\ \text{V})^2}{40\ \text{W}} = \boxed{3.6\ \Omega}$$

(*c*) The charge is the current times the time.

$$Q = It \quad Q_A = I_A t = (0.33\ \text{A})(3600\ \text{s}) = \boxed{1200\ \text{C}}$$

$$Q_B = I_B t = (3.3\ \text{A})(3600\ \text{s}) = \boxed{1.2\times10^4\ \text{C}}$$

(*d*) The energy is the power times the time, and the power is the same for both bulbs.

$$E = Pt \quad E_A = E_B = (40\ \text{W})(3600\ \text{s}) = \boxed{1.4\times10^5\ \text{J}}$$

(*e*) $\boxed{\text{Bulb B}}$ requires a larger current and should have larger diameter connecting wires to avoid overheating the connecting wires.

84. (*a*) The power is given by Eq. 18–5.

$$P = IV = (18\text{ A})(220\text{ V}) = 3960\text{W} \approx \boxed{4.0\times10^3\text{ W}}$$

(*b*) The power dissipated by the resistor is calculated by Eq. 18–6b, and the resistance is calculated by Eq. 18–3.

$$P_R = I^2R = I^2\frac{\rho\ell}{A} = I^2\frac{\rho\ell}{\pi r^2} = I^2\frac{4\rho\ell}{\pi d^2} = (18\text{ A})^2\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})2(3.5\text{ m})}{\pi(1.628\times10^{-3}\text{ m})^2} = 18.304\text{ W}$$

$$\approx \boxed{18\text{ W}}$$

(*c*) Recalculate using the alternate wire.

$$P_R = I^2\frac{4\rho\ell}{\pi d^2} = (18\text{ A})^2\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})2(3.5\text{ m})}{\pi(2.053\times10^{-3}\text{ m})^2} = 11.510\text{ W} \approx \boxed{12\text{ W}}$$

(*d*) The savings is due to the power difference.

$$\text{Savings} = (18.304\text{ W} - 11.510\text{ W})\left(\frac{1\text{ kW}}{1000\text{ W}}\right)(30\text{ d})\left(\frac{12\text{ h}}{1\text{ d}}\right)\left(\frac{\$0.12}{1\text{ kWh}}\right)$$

$$= \$0.2935/\text{month} \approx \boxed{\text{29 cents per month}}$$

85. (*a*) The energy stored is electrical potential energy, found by multiplying the available charge by the potential.

$$\text{PE} = QV = \left[(100\text{A}\cdot\text{h})\left(\frac{1\text{ C/s}}{1\text{ A}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\right](12\text{ V}) = 4.32\times10^6\text{ J} \approx \boxed{4\times10^6\text{ J}}$$

(*b*) The work done by the potential energy applies a force to overcome the retarding force and thus does the same amount of work as the retarding force.

$$W = \Delta\text{PE} = Fd \quad\to\quad d = \frac{\Delta\text{PE}}{F} = \frac{4.32\times10^6\text{ J}}{210\text{ N}} = 2.057\times10^4\text{ m} \approx \boxed{2\times10^4\text{ m}}$$

86. The volume of wire $(\text{volume} = \text{length}\times\text{cross-sectional area})$ remains constant as the wire is stretched from the original length of ℓ_0 to the final length of $2\ell_0$. Thus the cross-sectional area changes from A_0 to $\frac{1}{2}A_0$. Use Eq. 18–3 for the resistance and Eq. 18–6b for the power dissipated.

$$R_0 = \frac{\rho L_0}{A_0};\quad R = \frac{\rho L}{A} = \frac{\rho 2L_0}{\frac{1}{2}A_0} = 4\frac{\rho L_0}{A_0} = 4R_0;\quad P_0 = \frac{V^2}{R_0};\quad P = \frac{V^2}{R} = \frac{V^2}{4R_0} = \frac{1}{4}\frac{V^2}{R_0} = \frac{1}{4}P_0$$

The $\boxed{\text{power is reduced by a factor of 4}}$.

87. The wasted power is due to losses in the wire. The current in the wire can be found from Eq. 18–5.

(*a*)
$$P_{\text{dissipated}} = I^2R = \frac{P_{\text{delivered}}^2}{V^2}R = \frac{P_{\text{delivered}}^2}{V^2}\frac{\rho L}{A} = \frac{P_{\text{delivered}}^2}{V^2}\frac{\rho L}{\pi r^2} = \frac{P_{\text{delivered}}^2}{V^2}\frac{4\rho L}{\pi d^2}$$

$$= \frac{(1450\text{ W})^2}{(120\text{ V})^2}\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(25.0\text{ m})}{\pi(2.59\times10^{-3}\text{ m})^2} = 11.639\text{ W} \approx \boxed{12\text{ W}}$$

(*b*)
$$P_{\text{dissipated}} = \frac{P_{\text{delivered}}^2}{V^2}\frac{4\rho L}{\pi d^2} = \frac{(1450\text{ W})^2}{(120\text{ V})^2}\frac{4(1.68\times10^{-8}\ \Omega\cdot\text{m})(25.0\text{ m})}{\pi(4.12\times10^{-3}\text{ m})^2} = \boxed{4.6\text{ W}}$$

88. (*a*) The D-cell provides 25 mA at 1.5 V for 820 h, at a cost of \$1.70.

$$\text{Energy} = Pt = VIt = (1.5\text{ V})(0.025\text{ A})(820\text{ h})\left(\frac{1\text{ kW}}{1000\text{ W}}\right) = 0.03075\text{ kWh}$$

$$\text{Cost/kWh} = \frac{\$1.70}{0.03075\text{ kWh}} = \$55.28/\text{kWh} \approx \boxed{\$55/\text{kWh}}$$

(*b*) The AA-cell provides 25 mA at 1.5 V for 120 h, at a cost of \$1.25.

$$\text{Energy} = Pt = VIt = (1.5\text{ V})(0.025\text{ A})(120\text{ h})\left(\frac{1\text{ kW}}{1000\text{ W}}\right) = 0.0045\text{ kWh}$$

$$\text{Cost/kWh} = \frac{\$1.25}{0.0045\text{ kWh}} = \$277.78/\text{kWh} \approx \boxed{\$280/\text{kWh}}$$

(*c*) Compare the battery costs with a normal commercial electric rate.

$$\text{D-cell: } \frac{\$55.28/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{550\times \text{as costly}}$$

$$\text{AA-cell: } \frac{\$277.78/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{2800\times \text{as costly}}$$

89. Eq. 18–3 can be used. The area to be used is the cross-sectional area of the pipe.

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(r_{\text{outside}}^2 - r_{\text{inside}}^2)} = \frac{(1.68\times10^{-8}\ \Omega\cdot\text{m})(10.0\text{ m})}{\pi\left[(2.50\times10^{-2}\text{ m})^2 - (1.50\times10^{-2}\text{ m})^2\right]} = \boxed{1.34\times10^{-4}\ \Omega}$$

90. Model the protons as moving in a continuous beam of cross-sectional area A. Then by Eq. 18–10, $I = neAv_d$. The variable n is the number of protons per unit volume, so $n = \frac{N}{A\ell}$, where N is the number of protons in the beam and ℓ is the circumference of the ring. The "drift" velocity in this case is the speed of light.

$$I = neAv_d = \frac{N}{A\ell}eAv_d = \frac{N}{\ell}ev_d \quad \to$$

$$N = \frac{I\ell}{ev_d} = \frac{(11\times10^{-3}\text{ A})(6300\text{ m})}{(1.60\times10^{-19}\text{ C})(3.00\times10^{8}\text{ m/s})} = \boxed{1.4\times10^{12}\text{ protons}}$$

91. When the tank is empty, the entire length of the wire is in a nonsuperconducting state and has a nonzero resistivity, which we call ρ. Then the resistance of the wire when the tank is empty is given by $R_0 = \rho\frac{\ell}{A} = \frac{V_0}{I}$. When a length x of the wire is superconducting, that portion of the wire has 0 resistance. Then the resistance of the wire is only due to the length $\ell - x$, and $R = \rho\frac{\ell - x}{A} = \rho\frac{\ell}{A}\frac{\ell - x}{\ell} = R_0\frac{\ell - x}{\ell}$. This resistance, combined with the constant current, gives $V = IR$.

$$V = IR = \left(\frac{V_0}{R_0}\right)R_0\frac{\ell - x}{\ell} = V_0\left(1 - \frac{x}{\ell}\right) = V_0(1 - f) \quad \to \quad \boxed{f = 1 - \frac{V}{V_0}}$$

Thus a measurement of the voltage can give the fraction of the tank that is filled with liquid helium.

Solutions to Search and Learn Problems

1. Ohm's law is an empirical law that applies only to certain materials. Newton's laws are general laws that apply to all objects.

2. (*a*) The efficiency of the bulb is proportional to the power radiated from the bulb. The power radiated is given by Eq. 14–6. The subscript "H" is used for the halogen bulb and "T" for the traditional incandescent bulb.

$$\frac{(Q/t)_\mathrm{H}-(Q/t)_\mathrm{T}}{(Q/t)_\mathrm{T}}=\frac{T_\mathrm{H}^4-T_\mathrm{T}^4}{T_\mathrm{T}^4}=\frac{(2900\ \mathrm{K})^4-(2700\ \mathrm{K})^4}{(2900\ \mathrm{K})^4}=0.331\approx\boxed{33\%}$$

The halogen bulb is about 33% more efficient than the traditional incandescent bulb.

(*b*) The radiated power is proportional to the power consumed by the lightbulb. Since the halogen bulb is 33% more efficient than the incandescent, the power input can be decreased by a factor of 1.33 to provide the same output intensity.

$$P_\mathrm{H}=P_\mathrm{T}=\frac{100\ \mathrm{W}}{1.33}=\boxed{75\ \mathrm{W}}$$

3. The resistor code in Fig. 18–12 shows that the resistance is $24\times10^1=240\ \Omega.$ Solve for the resistivity by inserting this resistance into Eq. 18–3 along with the length and diameter of the resistor.

$$\rho=R\frac{A}{\ell}=R\frac{\pi d^2}{4\ell}=(240\ \Omega)\frac{\pi(2.15\times10^{-3}\ \mathrm{m})^2}{4(9.00\times10^{-3}\ \mathrm{m})}=9.68\times10^{-2}\ \Omega\cdot\mathrm{m}\approx\boxed{97\times10^{-3}\ \Omega\cdot\mathrm{m}}$$

Using Table 18–1, the only material that has a resistivity in this range is $\boxed{\text{germanium}}$.

4. For the cylindrical wire, its (constant) volume is given by $V=\ell_0A_0=\ell A,$ so $A=\dfrac{V}{\ell}.$ Combine this relationship with Eq. 18–3. We assume that $\Delta\ell\ll\ell_0,$ and use that fact in the third line where we drop the term $\rho\dfrac{(\Delta\ell)^2}{V_0}.$

$$R_0=\rho\frac{\ell_0}{A_0}=\rho\frac{\ell_0^2}{V_0}$$

$$R=\rho\frac{\ell}{A}=\rho\frac{\ell^2}{V_0}=\rho\frac{(\ell_0+\Delta\ell)^2}{V_0}=\rho\frac{\left(\ell_0^2+2\ell\Delta\ell+(\Delta\ell)^2\right)}{V_0}=\rho\frac{\ell_0^2}{V_0}+2\rho\frac{\ell_0\Delta\ell}{V_0}+\rho\frac{(\Delta\ell)^2}{V_0}=R_0+\Delta R$$

$$R-R_0=\Delta R=2\rho\frac{\ell_0\Delta\ell}{V_0}+\rho\frac{(\Delta\ell)^2}{V_0}\approx2\rho\frac{\ell_0\Delta\ell}{V_0}=2\frac{R_0}{\ell_0}\Delta\ell\quad\rightarrow\quad\frac{\Delta R}{2R_0}=\frac{\Delta\ell}{\ell_0}$$

This is true for any initial conditions, so $\boxed{\dfrac{\Delta\ell}{\ell_0}=\tfrac{1}{2}\dfrac{\Delta R}{R_0}}$

5. The heater must heat $130\ \mathrm{m}^3$ of air per hour from $5^\circ\mathrm{C}$ to $22^\circ\mathrm{C}$ and also replace the heat being lost at a rate of 850 kcal/h. Use Eq. 14–2 to calculate the energy needed to heat the air. The density of air is found in Table 10–1.

$$Q = mc\Delta T \quad \rightarrow \quad \frac{Q}{t} = \frac{m}{t}c\Delta T = \left(130\frac{\text{m}^3}{\text{h}}\right)\left(1.29\frac{\text{kg}}{\text{m}^3}\right)\left(0.17\frac{\text{kcal}}{\text{kg}\cdot\text{C}°}\right)(17\text{C}°) = 485\frac{\text{kcal}}{\text{h}}$$

$$\text{Power required} = 485\frac{\text{kcal}}{\text{h}} + 850\frac{\text{kcal}}{\text{h}} = 1335\frac{\text{kcal}}{\text{h}}\left(\frac{4186\ \text{J}}{\text{kcal}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 1552\ \text{W} \approx \boxed{1550\ \text{W}}$$

6. (*a*) We use Eq. 18–3.

$$\frac{R_{\text{Cu}}}{R_{\text{Al}}} = \frac{\rho_{\text{Cu}}\ell/A}{\rho_{\text{Al}}\ell/A} = \frac{\rho_{\text{Cu}}}{\rho_{\text{Al}}} = \frac{1.68\times10^{-8}\ \Omega\cdot\text{m}}{2.65\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{0.634}$$

(*b*) Use Eq. 18–3 with the resistivity of copper to find the resistance.

$$R_{\text{Cu}} = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = (1.68\times10^{-8}\ \Omega\cdot\text{m})\frac{4(125\ \text{m})}{\pi(1.63\times10^{-3}\ \text{m})^2} = \boxed{1.01\ \Omega}$$

(*c*) Use Eq. 18–3 with the resistivity of aluminum.

$$R_{\text{Al}} = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = (2.65\times10^{-8}\ \Omega\cdot\text{m})\frac{4(125\ \text{m})}{\pi(1.63\times10^{-3}\ \text{m})^2} = \boxed{1.59\ \Omega}$$

(*d*) Use Eq. 18–6a to determine the power dissipated in each wire.

$$P_{\text{Cu}} = I^2R_{\text{Cu}} = (18\ \text{A})^2(1.01\ \Omega) = 327\ \text{W} \approx \boxed{330\ \text{W}}$$

$$P_{\text{Al}} = I^2R_{\text{Al}} = (18\ \text{A})^2(1.59\ \Omega) = 515\ \text{W} \approx \boxed{520\ \text{W}}$$

(*e*) Set the resistances in Eq. 18–3 equal and solve for the ratio of the wire diameters.

$$R_{\text{Al}} = R_{\text{Cu}} \quad \rightarrow \quad \rho_{\text{Al}}\frac{4\ell}{\pi d_{\text{Al}}^2} = \rho_{\text{Cu}}\frac{4\ell}{\pi d_{\text{Cu}}^2} \quad \rightarrow$$

$$d_{\text{Al}} = d_{\text{Cu}}\sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = 1.63\ \text{mm}\sqrt{\frac{2.65\times10^{-8}\ \Omega\cdot\text{m}}{1.68\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{2.05\ \text{mm}}$$

(*f*) Use the density of copper to find the mass of the copper wire in part (*b*):

$$m_{\text{Cu}} = \rho_{\text{Cu}}V = \frac{\rho_{\text{Cu}}\pi d^2\ell}{4} = \frac{(8.9\times10^3\ \text{kg/m}^3)\pi(1.63\times10^{-3}\ \text{m})^2(125\ \text{m})}{4} = 2.32\ \text{kg}$$

Set the mass of aluminum equal to the mass of the copper and then calculate the diameter of the aluminum wire.

$$m_{\text{Al}} = \frac{\rho_{\text{Al}}\pi d^2\ell}{4} \quad \rightarrow \quad d = \sqrt{\frac{4m_{\text{Al}}}{\rho_M\pi\ell}} = \sqrt{\frac{4(2.32\ \text{kg})}{(2.70\times10^3\ \text{kg/m}^3)\pi(125\ \text{m})}} = 2.96\times10^{-3}\ \text{m}$$

Using the length, diameter, and resistivity of aluminum, calculate the resistance of the aluminum wire.

$$R_{\text{Al}} = \frac{4\rho_{\text{Al}}\ell}{\pi d^2} = \frac{4(2.65\times10^{-8}\ \Omega\cdot\text{m})(125\ \text{m})}{\pi(2.96\times10^{-3}\ \text{m})^2} = 0.481\ \Omega$$

In part (*b*) above, the copper has a resistance of $1.01\ \Omega$. In Section 18–4 it states that an aluminum wire of the same mass and length as copper wire would have a smaller resistance. We see that this is true, with the resistance of the aluminum being about 48% of the resistance of copper.

7. The electrons are assumed to be moving with simple harmonic motion. During one cycle, an object in simple harmonic motion will move a distance equal to twice its amplitude—it will move from its minimum position to its maximum position. From Eq. 11–7, we know that $v_{\max} = A\omega,$ where ω is the angular frequency of oscillation and A is the amplitude. From Eq. 18–10, we see that $I_{\max} = ne(\text{Area})v_{\max}.$ Finally, the maximum current can be related to the power by Eqs. 18–8 and 18–9. The charge carrier density, n, is calculated in Example 18–14.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}} = \frac{1}{\sqrt{2}}I_{\max}V_{\text{rms}}$$

$$A = \frac{v_{\max}}{\omega} = \frac{I_{\max}}{\omega ne(\text{Area})} = \frac{\sqrt{2}\,\overline{P}}{\omega ne\left(\frac{1}{4}\pi d^2\right)V_{\text{rms}}}$$

$$= \frac{4\sqrt{2}\,(650\text{ W})}{2\pi(60\text{ Hz})(8.4\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})\pi(1.7\times10^{-3}\text{ m})^2(120\text{ V})} = 6.66\times10^{-7}\text{ m}$$

The electron will move this distance in both directions from its equilibrium point, so its maximum displacement is from one extreme to the other, which is twice the amplitude $\approx \boxed{1.3\times10^{-6}\text{ m}}$.

8. (*a*) We model the axon as a cylindrically shaped parallel plate capacitor, with opposite charges on each side. The area of the capacitor plates is the area of the walls of the cylindrical surface area.

$$C = \frac{K\varepsilon_0 A}{d} = \frac{K\varepsilon_0 2\pi r\ell}{d} = \frac{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)2\pi(10\times10^{-6}\text{ m})(0.1\text{ m})}{10^{-8}\text{ m}}$$

$$= 1.67\times10^{-8}\text{ F} \approx \boxed{10^{-8}\text{ F}}$$

(*b*) In one action, the voltage changes from –70 mV to +30 mV, or a difference of 100 mV. From the capacitance and change in voltage we calculate the change in charge during the action.

$$Q = CV = (1.67\times10^{-8}\text{ F})(0.1\text{ V}) = 1.67\times10^{-9}\text{ C}$$

Each sodium ion is singly ionized and carries a charge of $1.6\times10^{-19}\text{C}.$ Use this to calculate the number of ions that have moved.

$$n = \frac{Q}{q} = \frac{1.67\times10^{-9}\text{ C}}{1.6\times10^{-19}\text{ C/ion}} = 1.04\times10^{10}\text{ ions}$$

Table 18–2 gives the concentration of sodium ions inside the axon as $15\text{ mol/m}^3.$ Multiplying this concentration by the volume of the axon and Avogadro's number gives the number of sodium ions in the axon. We divide the change in ions by the total number to determine the relative change during the action.

$$\frac{n}{N} = \frac{1.04\times10^{10}\text{ ions}}{(15\text{ mol/m}^3)(6.02\times10^{23}\text{ ions/mol})\pi(10\times10^{-6}\text{ m})^2(0.1\text{ m})} 3.67\times10^{-5} \approx 4\times10^{-5}$$

The number of ions changes by $\boxed{\text{about 1 per 40,000}}$. This tiny change is not measureable.

19

DC CIRCUITS

Responses to Questions

$\boxed{1.}$ Even though the bird's feet are at high potential with respect to the ground, there is very little potential difference between them, because they are close together on the wire. The resistance of the bird is much greater than the resistance of the wire between the bird's feet. These two resistances are in parallel, so very little current will pass through the bird as it perches on the wire. However, when you put a metal ladder up against a power line, you provide a direct connection between the high potential line and ground. The ladder will have a large potential difference between its top and bottom. A person standing on the ladder will also have a large potential difference between his or her hands and feet. Even if the person's resistance is large, the potential difference will be great enough to produce a current through the person's body large enough to cause substantial damage or death.

2. Series: The main disadvantage of Christmas tree lights connected in series is that when one bulb burns out, a gap is created in the circuit and none of the bulbs remains lit. Finding the burned-out bulb requires replacing each individual bulb one at a time until the string of bulbs comes back on. As an advantage, the bulbs are slightly easier to wire in series. Only one wire needs to be attached from bulb to bulb. Also, a "blinker bulb" can make the entire string flash on and off by cutting off the current.

   Parallel: The main advantage of connecting the bulbs in parallel is that one burned-out bulb does not affect the rest of the strand and is easy to identify and replace. As a disadvantage, wiring the bulbs in parallel is slightly more difficult. Two wires must be attached from bulb to bulb.

3. If 20 of the 6-V lamps were connected in series and then connected to the 120-V line, there would be a voltage drop of 6 V for each of the lamps, and they would not burn out due to too much voltage. Being in series, if one of the bulbs went out for any reason, then they would all turn off.

4. If the lightbulbs are in series, then each will have the same current. The power dissipated by the bulb as heat and light is given by $P = I^2R.$ Thus the bulb with the higher resistance (R_2) will be brighter. If the bulbs are in parallel, then each will have the same voltage. The power dissipated by the bulb as heat and light is given by $P = V^2/R.$ Thus the bulb with the lower resistance (R_1) will be brighter.

$\boxed{5.}$ The outlets are connected in parallel to each other, because you can use one outlet without using the other. If they were in series, then both outlets would have to be used at the same time to have a completed circuit. Also, both outlets supply the same voltage to whatever device is plugged in to the outlet, which indicates that they are wired in parallel to the voltage source.

6. The power output from a resistor is given by $P = V^2/R$. To maximize this value, the voltage needs to be as large as possible and the resistance as small as possible. That can be accomplished by putting the two batteries in series and then connecting the two resistors in parallel to each other, across the full two-battery voltage.

7. The battery has to supply less power when the two resistors are connected in series than it has to supply when only one resistor is connected. $P = V^2/R$, so if V is constant and R increases, then the power decreases.

$\boxed{8.}$ We assume that the bulbs are in parallel with each other. The overall resistance decreases and more current is drawn from the source. A bulb rated at 60 W and 120 V has a resistance of 240 Ω. A bulb rated at 100 W and 120 V has a resistance of 144 Ω. When only the 60-W bulb is on, the total resistance is $240\ \Omega$. When both bulbs are lit, the total resistance is the combination of the two resistances in parallel, which is only 90 Ω.

9. The energy stored in a capacitor network can be calculated by $\mathrm{PE} = \frac{1}{2}CV^2$. Since the voltage for the capacitor network is the same in this problem for both configurations, the configuration with the highest equivalent capacitance will store the most energy. The parallel combination has the highest equivalent capacitance, so it stores the most energy. Another way to consider this is that the total stored energy is the sum of the quantity $\mathrm{PE} = \frac{1}{2}CV^2$ for each capacitor. Each capacitor has the same capacitance, but in the parallel circuit, each capacitor has a larger voltage than in the series circuit. Thus the parallel circuit stores more energy.

10. No, the sign of the battery's emf does not depend on the direction of the current through the battery. The sign of the battery's emf depends on the direction you go through the battery in applying the loop rule. If you go from negative pole to positive pole, then the emf is added. If you go from positive pole to negative pole, then the emf is subtracted.

    But the terminal voltage does depend on the direction of the current through the battery. If current is flowing through the battery in the normal orientation (leaving the positive terminal, flowing through the circuit, and arriving at the negative terminal), then there is a voltage drop across the internal resistance, and the terminal voltage is less than the emf. If the current flows in the opposite sense (as in charging the battery), then there is a voltage rise across the terminal resistance, and the terminal voltage is higher than the emf.

11. (*a*) With the batteries in series, a greater voltage is delivered to the lamp, and the lamp will burn brighter.

    (*b*) With the batteries in parallel, the voltage across the lamp is the same as for either battery alone. Each battery supplies only half of the current going through the lamp, so the batteries will last longer (and the bulb stay lit longer) as compared to just having one battery.

12. When batteries are connected in series, their emfs add together, producing a larger potential. For instance, if there are two 1.5-V batteries in series in a flashlight, then the potential across the bulb will be 3.0 V, and there will be more current in the bulb. The batteries do not need to be identical in this case. When batteries are connected in parallel, the currents they can generate add together, producing a larger current over a longer time period. Batteries in this case need to be nearly identical, or the battery with the larger emf will end up charging the battery with the smaller emf.

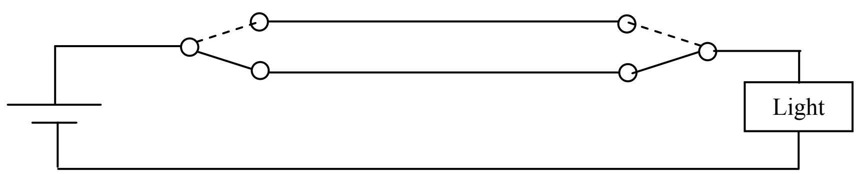
13. Yes. When a battery is being charged, current is forced through it "backward" and $V_{\text{terminal}} = \mathscr{E} + Ir$, so $V_{\text{terminal}} > \mathscr{E}$.

14. Refer to Fig. 19–2. Connect the battery to a known resistance R and measure the terminal voltage V_{ab}. The current in the circuit is given by Ohm's law to be $I = \dfrac{V_{\text{ab}}}{R}$. It is also true that $V_{\text{ab}} = \mathscr{E} - Ir$, so the internal resistance can be calculated by $r = \dfrac{\mathscr{E} - V_{\text{ab}}}{I} = R\dfrac{\mathscr{E} - V_{\text{ab}}}{V_{\text{ab}}}$.

15. No. As current passes through the resistor in the RC circuit, energy is dissipated in the resistor. Therefore, the total energy supplied by the battery during the charging is the combination of the energy dissipated in the resistor and the energy stored in the capacitor.

16. (*a*) stays the same; (*b*) increases; (*c*) decreases; (*d*) increases; (*e*) increases; (*f*) decreases; (*g*) decreases; (*h*) increases; (*i*) stays the same.

17. The following is one way to accomplish the desired task.



In the current configuration, the light would be on. If either switch is moved, then the light will go out. But if either switch were moved again, then the light would come back on.

$\boxed{18.}$ The soles of your shoes are made of a material which has a relatively high resistance, and there is relatively high resistance flooring material between your shoes and the literal ground (the Earth). With that high resistance, a malfunctioning appliance would not be able to cause a large current flow through your body. The resistance of bare feet is much less than that of shoes, and the feet are in direct contact with the ground, so the total resistance is much lower and a larger current would flow through your body.

19. In an analog ammeter, the internal resistor, or shunt resistor, has a small value and is in parallel with the galvanometer, so that the overall resistance of the ammeter is very small. In an analog voltmeter, the internal resistor has a large value and is in series with the galvanometer, and the overall resistance of the voltmeter is very large.

20. If you mistakenly use an ammeter where you intend to use a voltmeter, then you are inserting a short in parallel with some resistance. That means that the resistance of the entire circuit has been lowered, and almost all of the current will flow through the low-resistance ammeter. Ammeters usually have a fairly small current limit, so the ammeter will very likely be damaged in such a scenario. Also, if the ammeter is inserted across a voltage source, then the source will provide a large current, and again the meter will almost certainly be damaged or at least disabled by burning out a fuse.

21. An ammeter is placed in series with a given circuit element in order to measure the current through that element. If the ammeter did not have very low (ideally, zero) resistance, then its presence in the circuit would change the current it is attempting to measure by adding more resistance in series. An ideal ammeter has zero resistance and thus does not change the current it is measuring.

A voltmeter is placed in parallel with a circuit element in order to measure the voltage difference across that element. If the voltmeter does not have a very high resistance, then its presence in parallel will lower the overall resistance and affect the circuit. An ideal voltmeter has infinite resistance so that when placed in parallel with circuit elements it will not change the value of the voltage it is reading.

22. When the voltmeter is connected to the circuit, it reduces the resistance of that part of the circuit. That will make the resistor + voltmeter combination a smaller fraction of the total resistance of the circuit than the resistor was alone, which means that it will have a smaller fraction of the total voltage drop across it.

23. A voltmeter has a very high resistance. When it is connected to the battery, very little current will flow. A small current results in a small voltage drop due to the internal resistance of the battery, and the emf and terminal voltage (measured by the voltmeter) will be very close to the same value. However, when the battery is connected to the lower-resistance flashlight bulb, the current will be higher and the voltage drop due to the internal resistance of the battery will also be higher. As a battery is used, its internal resistance increases. Therefore, the terminal voltage will be significantly lower than the emf: $V_{\text{terminal}} = \mathscr{E} - Ir.$ A lower terminal voltage will result in a dimmer bulb and usually indicates a "used-up" battery.

Responses to MisConceptual Questions

1. (*a*, *c*) Resistors are connected in series when there are no junctions between the resistors. With no junctions between the resistors, any current flowing through one of the resistors must also flow through the other resistor.

2. (*d*) It might be easy to think that all three resistors are in parallel because they are on three branches of the circuit. However, following a path from the positive terminal of the battery, all of the current from the battery passes through R_2 before reaching the junction at the end of that branch. The current then splits and part of the current passes through R_1 while the remainder of the current passes through R_3 before meeting at the right junction.

3. (*c*) A common misconception is that the smaller resistor would have the larger current. However, the two resistors are in series, so they must have the same current flowing through them. It does not matter which resistor is first.

4. (*a*) It might seem that current must flow through each resistor and lightbulb. However, current will only flow through the lightbulb if there is a potential difference across the bulb. If we consider the bottom branch of the circuit to be at ground, then the left end of the lightbulb will be at a potential of 10 V. The top branch will be at 20 V. Since the two resistors are identical, the voltage drop across each will be half the voltage from the top to bottom, or 10 V each. This makes the right side of the lightbulb also at 10 V. Since both sides of the lightbulb are at the same potential, no current will flow through the lightbulb.

5. (*d*) Resistors R_1 and R_2 are in parallel, so each has half of the current from the battery. R_3 and R_4 are in series and add to produce twice as much resistance as R_5. Since they are in parallel with R_5, one-third of the current from the battery goes through them, while two-thirds goes through R_5. The greatest current therefore goes through R_5.

6. (*d*) Current takes the path of least resistance. Since bulb A is in parallel with the short circuit, all of the current will pass through the short circuit, causing bulb A to go out.

7. (*b*) A common misconception is that adding another parallel branch would have no effect on the voltage across R_4. However, when the switch is closed, the additional parallel resistor makes the effective resistance of the parallel resistors smaller, and therefore the resistance of the entire circuit gets smaller. With a smaller effective resistance, a greater current flows through the battery and through R_1, resulting in a greater voltage drop across R_1. Since the voltage from the battery is equal to the sum of the voltages across R_1 and R_4, increasing the voltage across R_1 results in a decrease in voltage across R_4.

8. (*a*) As explained in the answer to Question 7, when the switch is closed, it adds an additional resistor in parallel to R_3 and R_4, making the effective resistance of the entire circuit smaller. With less effective resistance, a greater current flows through the circuit, increasing the potential difference across R_1.

9. (*b*) As the capacitor charges, the voltage drop across the capacitor increases, thereby diminishing the voltage drop across the resistor. As the voltage drop across the resistor decreases, the current decreases.

10. (*c*) Even though steady current cannot flow through a capacitor, charge can build up on the capacitor, allowing current to initially flow in the circuit. As the charge builds on the capacitor, the voltage drop across the capacitor increases, and the current decreases. The rate of charging is determined by the time constant, which is the product of *R* and *C*.

11. (*b*) It might seem that a capacitor would discharge linearly in time, losing one-fourth of the charge every second. This is incorrect, as the capacitor discharges exponentially. That is, every 2.0 seconds, half of the remaining charge on the capacitor will discharge. After 2.0 seconds, half of the charge remains. After 4.0 seconds, half of the half, or one-fourth, of the charge remains. After 6.0 seconds, one-eighth of the charge remains.

12. (*a*) To double the heart rate, the time of discharging must be shorter, so the discharge rate must be faster. The resistance limits the rate at which the current can flow through the circuit. Decreasing the resistance will increase the current flow, causing the capacitor to discharge faster.

13. (*c*) In a two-prong cord, one prong is at high voltage and the other is grounded. Electricity flows through the appliance between these two wires. However, if there is a short between the high-voltage wire and the casing, then the casing can become charged and electrocute a person touching the case. The third prong connects the external case to ground, so that the case cannot become charged.

14. (*c*) Connecting capacitors in series effectively increases the plate separation distance, decreasing the net capacitance. Connecting capacitors in parallel effectively increases the plate area, increasing the net capacitance. Therefore, when capacitors are connected in series, the effective capacitance will be less than the capacitance of the smallest capacitor, and when connected in parallel, the effective capacitance will be greater than the capacitance of the largest capacitance.

15. (*b*) The ammeter is placed in series with the circuit and therefore should have a small resistance, so there is minimal voltage drop across the ammeter. The voltmeter is placed in parallel with the circuit. It should have a large resistance so that minimal current from the circuit passes through the voltmeter instead of passing through the circuit.

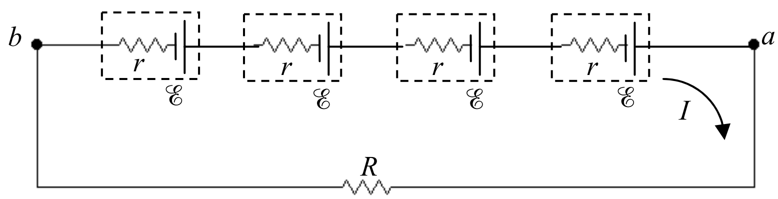
Solutions to Problems

1. See Fig. 19–2 for a circuit diagram for this problem. Using the same analysis as in Example 19–1, the current in the circuit is $I = \dfrac{\mathscr{E}}{R+r}$. Use Eq. 19–1 to calculate the terminal voltage.

(*a*) $$V_{\text{ab}} = \mathscr{E} - Ir = \mathscr{E} - \left(\frac{\mathscr{E}}{R+r}\right)r = \frac{\mathscr{E}(R+r) - \mathscr{E}r}{R+r} = \mathscr{E}\frac{R}{R+r} = (6.00\text{ V})\frac{71.0\ \Omega}{(71.0+0.900)\Omega} = \boxed{5.92\text{ V}}$$

(*b*) $$V_{\text{ab}} = \mathscr{E}\frac{R}{R+r} = (6.00\text{ V})\frac{710\ \Omega}{(710+0.900)\Omega} = \boxed{5.99\text{ V}}$$

2. See the circuit diagram. The current in the circuit is *I*. The voltage V_{ab} is given by Ohm's law to be $V_{\text{ab}} = IR$. That same voltage is the terminal voltage of the series emf.



$$V_{\text{ab}} = (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) = 4(\mathscr{E} - Ir) \quad \text{and} \quad V_{\text{ab}} = IR$$

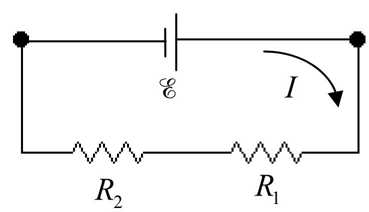
$$4(\mathscr{E} - Ir) = IR \quad \rightarrow \quad r = \frac{\mathscr{E} - \frac{1}{4}IR}{I} = \frac{(1.50\text{ V}) - \frac{1}{4}(0.45\text{ A})(12.0\ \Omega)}{0.45\text{ A}} = 0.333\ \Omega \approx \boxed{0.33\ \Omega}$$

$\boxed{3.}$ See Fig. 19–2 for a circuit diagram for this problem. Use Eq. 19–1.

$$V_{\text{ab}} = \mathscr{E} - Ir \quad \rightarrow \quad r = \frac{\mathscr{E} - V_{\text{ab}}}{I} = \frac{12.0\text{ V} - 8.8\text{ V}}{95\text{ A}} = \boxed{0.034\ \Omega}$$

$$V_{\text{ab}} = IR \quad \rightarrow \quad R = \frac{V_{\text{ab}}}{I} = \frac{8.8\text{ V}}{95\text{ A}} = \boxed{0.093\ \Omega}$$

4. The equivalent resistance is the sum of the two resistances: $R_{\text{eq}} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \dfrac{\mathscr{E}}{R_{\text{eq}}} = \dfrac{\mathscr{E}}{R_1 + R_2}$. The voltage across the 1800-Ω resistor is given by Ohm's law.



$$V_{2200} = IR_2 = \frac{\mathscr{E}}{R_1 + R_2}R_2 = \mathscr{E}\frac{R_2}{R_1 + R_2} = (12\text{ V})\frac{1800\ \Omega}{650\ \Omega + 1800\ \Omega} = \boxed{8.8\text{ V}}$$

5. (*a*) For the resistors in series, use Eq. 19–3, which says the resistances add linearly.

$$R_{\text{eq}} = 3(45\ \Omega) + 3(65\ \Omega) = \boxed{330\ \Omega}$$

(*b*) For the resistors in parallel, use Eq. 19–4, which says the resistances add reciprocally.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{45\ \Omega} + \frac{1}{45\ \Omega} + \frac{1}{45\ \Omega} + \frac{1}{65\ \Omega} + \frac{1}{65\ \Omega} + \frac{1}{65\ \Omega} = \frac{3}{45\ \Omega} + \frac{3}{65\ \Omega} = \frac{3(65\ \Omega) + 3(45\ \Omega)}{(65\ \Omega)(45\ \Omega)} \quad \rightarrow$$

$$R_{\text{eq}} = \frac{(65\ \Omega)(45\ \Omega)}{3(65\ \Omega) + 3(45\ \Omega)} = \boxed{8.9\ \Omega}$$

$\boxed{6.}$ (*a*) The maximum resistance is made by combining the resistors in series.

$$R_{\text{eq}} = R_1 + R_2 + R_3 = 580\ \Omega + 790\ \Omega + 1200\ \Omega = \boxed{2.57\ \text{k}\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

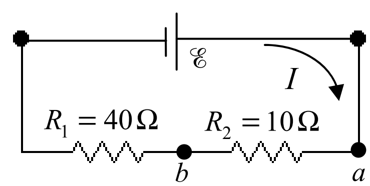
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \rightarrow$$

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{580\ \Omega} + \frac{1}{790\ \Omega} + \frac{1}{1200\ \Omega}\right)^{-1} = \boxed{260\ \Omega}$$

7. The equivalent resistance of five 100-Ω resistors in parallel is found, and then that resistance is divided by $10\ \Omega$ to find the number of 10-Ω resistors needed.

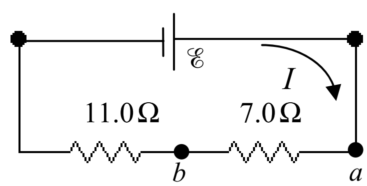
$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{5}{100\ \Omega}\right)^{-1} = 20\ \Omega = n(10\ \Omega) \quad \rightarrow \quad n = \frac{20\ \Omega}{10\ \Omega} = \boxed{2}$$

8. The ratio R_1/R_2 must be 4/1. Since both resistors have the same current, the voltage drop across R_1 (4/5 of the battery voltage) will be 4 times that across R_2 (1/5 of the battery voltage). So if $R_1 = 40\ \Omega$ and $R_2 = 10\ \Omega$, then the current would be $\mathscr{E}/50$. The voltage across R_1 would be found from Ohm's law to be



$V_1 = IR_1 = \dfrac{\mathscr{E}}{50}(40) = 0.8\mathscr{E}$, and the voltage across R_2 would be $V_2 = IR_2 = \dfrac{\mathscr{E}}{50}(10) = 0.2\mathscr{E}$.

9. Connecting 18 of the resistors in series will enable you to make a voltage divider with a 3.5-V output. To get the desired output, measure the voltage across 7 consecutive series resistors. In the diagram that would be the voltage between points a and b.



$$R_{\text{eq}} = 18(1.0\ \Omega) \qquad I = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{\mathscr{E}}{18.0\ \Omega}$$

$$V_{\text{ab}} = (7.0\ \Omega)I = (7.0\ \Omega)\frac{\mathscr{E}}{18.0\ \Omega} = (7.0\ \Omega)\frac{9.0\ \text{V}}{18.0\ \Omega} = \boxed{3.5\ \text{V}}$$

10. The resistors can all be connected in series.

$$R_{\text{eq}} = R + R + R = 3(1.70\ \text{k}\Omega) = \boxed{5.10\ \text{k}\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \quad \rightarrow \quad R_{\text{eq}} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{1.70\ \text{k}\Omega}{3} = \boxed{567\ \Omega}$$

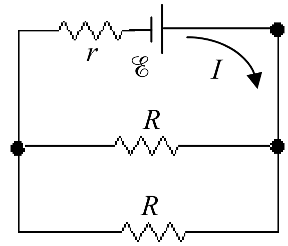
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \quad \rightarrow \quad R_{\text{eq}} = \frac{2R}{3} = \frac{2(1.70\ \text{k}\Omega)}{3} = \boxed{1.13\ \text{k}\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(1.70\ \text{k}\Omega) = \boxed{2.55\ \text{k}\Omega}$$

11. The resistance of each bulb can be found from its power rating.



$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(12.0\ \text{V})^2}{4.0\ \text{W}} = 36\ \Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \rightarrow \quad R_{\text{eq}} = \frac{R}{2} = \frac{36\ \Omega}{2} = 18\ \Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{\text{ab}} = IR_{\text{eq}} \quad \rightarrow \quad I = \frac{V_{\text{ab}}}{R_{\text{eq}}} = \frac{V_{\text{ab}}}{R/2} = \frac{2V_{\text{ab}}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 19–1.

$$V_{\text{ab}} = \mathcal{E} - Ir \quad \rightarrow \quad r = \frac{\mathcal{E} - V_{\text{ab}}}{I} = \frac{\mathcal{E} - V_{\text{ab}}}{\left(\frac{2V_{\text{ab}}}{R}\right)} = R\frac{\mathcal{E} - V_{\text{ab}}}{2V_{\text{ab}}} = (36\ \Omega)\frac{12.0\ \text{V} - 11.8\ \text{V}}{2(11.8\ \text{V})} = 0.305\ \Omega \approx \boxed{0.3\ \Omega}$$

12. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text{eq}} = 8R$. The current flowing through the bulbs is then $V_{\text{tot}} = IR_{\text{eq}} \quad \rightarrow \quad I = \frac{V_{\text{tot}}}{R_{\text{eq}}} = \frac{V_{\text{tot}}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V = IR = \frac{V_{\text{tot}}}{8R}R = \frac{V_{\text{tot}}}{8} = \frac{120\ \text{V}}{8} = \boxed{15\ \text{V}}$$

(*b*) $$I = \frac{V_{\text{tot}}}{8R} \quad \rightarrow \quad R = \frac{V_{\text{tot}}}{8I} = \frac{120\ \text{V}}{8(0.45\ \text{A})} = 33.33\ \Omega \approx \boxed{33\ \Omega}$$

$$P = I^2R = (0.45\ \text{A})^2(33.33\ \Omega) = 6.75\ \text{W} \approx \boxed{6.8\ \text{W}}$$

13. We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, so $I = 8I_R$. The voltage drop across the long leads is $V_{\text{leads}} = Ir = 8I_{\text{R}}r = 8(0.21\ \text{A})(1.4\ \Omega) = 2.352\ \text{V}$. Thus the voltage across each of the parallel resistors is $V_{\text{R}} = V_{\text{tot}} - V_{\text{leads}} = 120\ \text{V} - 2.352\ \text{V} = 117.6\ \text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_{\text{R}} = I_{\text{R}}R \quad \rightarrow \quad R = \frac{V_{\text{R}}}{I_{\text{R}}} = \frac{117.6\ \text{V}}{0.21\ \text{A}} = \boxed{560\ \Omega}$$

The total power delivered is $P = V_{\text{tot}}I$, and the "wasted" power is I^2r. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted} = \frac{I^2 r}{IV_{\text{tot}}} = \frac{Ir}{V_{\text{tot}}} = \frac{8(0.21\text{ A})(1.4\ \Omega)}{120\text{ V}} = 0.0196 \approx \boxed{0.020}$$

So about 2% of the power is wasted.

14. To fix this circuit, connect another resistor in parallel with the $480\text{-}\Omega$ resistor so that the equivalent resistance is the desired $350\ \Omega$.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \to \quad R_2 = \left(\frac{1}{R_{\text{eq}}} - \frac{1}{R_1}\right)^{-1} = \left(\frac{1}{350\ \Omega} - \frac{1}{480\ \Omega}\right)^{-1} = 1292\ \Omega \approx \boxed{1300\ \Omega}$$

So solder a $1300\text{-}\Omega$ resistor in parallel with the $480\text{-}\Omega$ resistor.

15. Each bulb will get one-eighth of the total voltage, so $V_{\text{bulb}} = \frac{V_{\text{tot}}}{8}$. That could be proven using a similar argument to that used in Problem 12. Use that voltage and the power dissipated by each bulb to calculate the resistance of a bulb.

$$P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R} \quad \to \quad R = \frac{V_{\text{bulb}}^2}{P} = \frac{V_{\text{tot}}^2}{64P} = \frac{(120\text{ V})^2}{64(7.0\text{ W})} = \boxed{32\ \Omega}$$

16. (*a*) The equivalent resistance is found by combining the $750\text{-}\Omega$ and $680\text{-}\Omega$ resistors in parallel and then adding the $990\text{-}\Omega$ resistor in series with that parallel combination.

$$R_{\text{eq}} = \left(\frac{1}{750\ \Omega} + \frac{1}{680\ \Omega}\right)^{-1} + 990\ \Omega = 357\ \Omega + 990\ \Omega = 1347\ \Omega \approx \boxed{1350\ \Omega}$$

(*b*) The current delivered by the battery is $I = \frac{V}{R_{\text{eq}}} = \frac{12.0\text{ V}}{1347\ \Omega} = 8.909\times10^{-3}\text{ A}$. This is the current in the $990\text{-}\Omega$ resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{990} = IR = (8.909\times10^{-3}\text{ A})(990\ \Omega) = 8.820\text{ V} \approx \boxed{8.8\text{ V}}$$

Thus the voltage across the parallel combination must be $12.0\text{ V} - 8.8\text{ V} = \boxed{3.2\text{ V}}$. This is the voltage across both the $750\text{-}\Omega$ and $680\text{-}\Omega$ resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$V_{\text{parallel}} = IR_{\text{parallel}} = (8.909\times10^{-3}\text{ A})(357\ \Omega) = 3.181\text{ V} \approx 3.2\text{ V}$$

(*c*) The current in the $990\text{-}\Omega$ resistor was already seen to be $\boxed{8.91\text{ mA}}$. The other currents can be found from Ohm's law.

$$I_{750} = \frac{V_{750}}{R_{750}} = \frac{3.18\text{ V}}{470\ \Omega} \approx \boxed{4.2\text{ mA}}; \quad I_{680} = \frac{V_{750}}{R_{680}} = \frac{3.18\text{ V}}{680\ \Omega} \approx \boxed{4.7\text{ mA}}$$

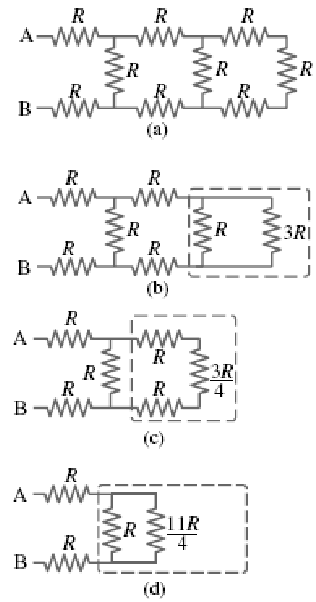
Note that these last two currents add to be the current in the $990\text{-}\Omega$ resistor.

17. The resistance of each bulb can be found by using Eq. 18–6, $P = V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P = V^2/R \quad \to \quad \frac{1}{R} = \frac{P}{V^2}$$

$$R_{\text{eq}} = \left(\frac{1}{R_{75}} + \frac{1}{R_{25}}\right)^{-1} = \left(\frac{75\text{ W}}{(120\text{ V})^2} + \frac{25\text{ W}}{(120\text{ V})^2}\right)^{-1} = 144\ \Omega \approx \boxed{140\ \Omega}$$

18. (*a*) The three resistors on the far right are in series, so their equivalent resistance is 3R. That combination is in parallel with the next resistor to the left, as shown in the dashed box in figure (b). The equivalent resistance of the dashed box is found as follows.



$$R_{\text{eq1}} = \left(\frac{1}{R} + \frac{1}{3R}\right)^{-1} = \tfrac{3}{4}R$$

This equivalent resistance of $\frac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in figure (c). The equivalent resistance of that dashed box is $R_{\text{eq2}} = 2R + \frac{3}{4}R = \frac{11}{4}R$. This $\frac{11}{4}R$ is in parallel with the next resistor to the left, as shown in figure (d). The equivalent resistance of that dashed box is found as follows.

$$R_{\text{eq2}} = \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1} = \tfrac{11}{15}R$$

This is in series with the last two resistors, the ones connected directly to A and B. The final equivalent resistance is then

$$R_{\text{eq}} = 2R + \tfrac{11}{15}R = \tfrac{41}{15}R = \tfrac{41}{15}(175\ \Omega) = 478.33\ \Omega \approx \boxed{478\ \Omega}$$

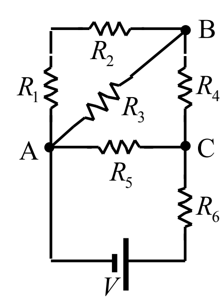
(*b*) The current flowing from the battery is found from Ohm's law.

$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{50.0\text{ V}}{478.33\ \Omega} = 0.10453\text{ A} \approx \boxed{0.105\text{ A}}$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\frac{11}{4}R$, as shown in figure (d). The voltage across R and across $\frac{11}{4}R$ must be the same, since they are in parallel. Use this to find the desired current.

$$V_R = V_{\frac{11}{4}R} \quad\rightarrow\quad I_R R = I_{\frac{11}{4}R}\left(\tfrac{11}{4}R\right) = (I_{\text{total}} - I_R)\left(\tfrac{11}{4}R\right) \quad\rightarrow$$
$$I_R = \tfrac{11}{15}I_{\text{total}} = \tfrac{11}{15}(0.10453\text{ A})I_{\text{total}} = \boxed{0.0767\text{ A}}$$

19. The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \frac{2}{3}R + R = \frac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \frac{5}{8}R$.

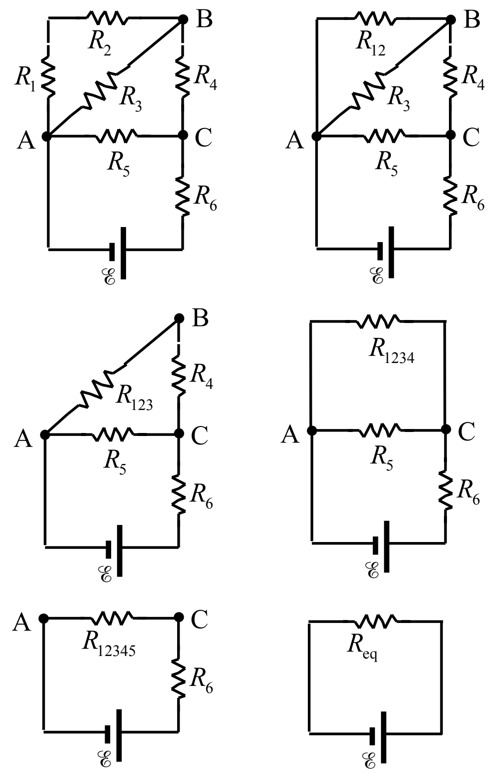


Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.

$$R_{\text{eq}} = \tfrac{5}{8}R + R = \boxed{\tfrac{13}{8}R}$$

A more detailed solution with more circuit diagrams is given in Problem 20.

20. We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams. The initial diagram has been rotated by 90 degrees.



R_1 and R_2 are in series.

$$R_{12} = R_1 + R_2 = R + R = 2R$$

R_{12} and R_3 are in parallel.

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \tfrac{2}{3}R$$

R_{123} and R_4 are in series.

$$R_{1234} = R_{123} + R_4 = \tfrac{2}{3}R + R = \tfrac{5}{3}R$$

R_{1234} and R_5 are in parallel.

$$R_{12345} = \left(\frac{1}{R_{1234}} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{\tfrac{5}{3}R} + \frac{1}{R}\right)^{-1} = \tfrac{5}{8}R$$

R_{12345} and R_6 are in series, producing the equivalent resistance.

$$R_{\text{eq}} = R_{12345} + R_6 = \tfrac{5}{8}R + R = \tfrac{13}{8}R$$

Now work "backward" from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the same voltage as their equivalent resistance. To avoid rounding errors, we do not use numeric values until the end of the problem.

$$I_{\text{eq}} = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{\tfrac{13}{8}R} = \boxed{\frac{8\mathcal{E}}{13R} = I_6} = I_{12345}$$

$$V_5 = V_{1234} = V_{12345} = I_{12345}R_{12345} = \left(\frac{8\mathcal{E}}{13R}\right)\left(\tfrac{5}{8}R\right) = \tfrac{5}{13}\mathcal{E}; \quad I_5 = \frac{V_5}{R_5} = \frac{\tfrac{5}{13}\mathcal{E}}{R} = \boxed{\frac{5\mathcal{E}}{13R} = I_5}$$

$$I_{1234} = \frac{V_{1234}}{R_{1234}} = \frac{\tfrac{5}{13}\mathcal{E}}{\tfrac{5}{3}R} = \boxed{\frac{3\mathcal{E}}{13R} = I_4} = I_{123}; \quad V_{123} = I_{123}R_{123} = \left(\frac{3\mathcal{E}}{13R}\right)\left(\tfrac{2}{3}R\right) = \tfrac{2}{13}\mathcal{E} = V_{12} = V_3$$

$$I_3 = \frac{V_3}{R_3} = \boxed{\frac{2\mathcal{E}}{13R} = I_3}; \quad I_{12} = \frac{V_{12}}{R_{12}} = \frac{\tfrac{2}{13}\mathcal{E}}{2R} = \boxed{\frac{\mathcal{E}}{13R} = I_1 = I_2}$$

Now substitute in numerical values.

$$I_1 = I_2 = \frac{\mathscr{E}}{13R} = \frac{12.0\text{ V}}{13(3250\ \Omega)} = \boxed{0.284\text{ mA}};\quad I_3 = \frac{2\mathscr{E}}{13R} = \boxed{0.568\text{ mA}};\quad I_4 = \frac{3\mathscr{E}}{13R} = \boxed{0.852\text{ mA}};$$

$$I_5 = \frac{5\mathscr{E}}{13R} = \boxed{1.42\text{ mA}};\quad I_6 = \frac{8\mathscr{E}}{13R} = \boxed{2.27\text{ mA}};\quad V_{\text{AB}} = V_3 = \tfrac{2}{13}\mathscr{E} = \boxed{1.85\text{ V}}$$

21. It is given that the power used when the resistors are in series is one-fourth the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 18–6b, along with the definitions of series and parallel equivalent resistance.

$$P_{\text{series}} = \tfrac{1}{4}P_{\text{parallel}} \;\rightarrow\; \frac{V^2}{R_{\text{series}}} = \frac{1}{4}\frac{V^2}{R_{\text{parallel}}} \;\rightarrow\; R_{\text{series}} = 4R_{\text{parallel}} \;\rightarrow\; (R_1+R_2) = 4\frac{R_1R_2}{(R_1+R_2)} \;\rightarrow$$

$$(R_1+R_2)^2 = 4R_1R_2 \;\rightarrow\; R_1^2 + 2R_1R_2 + R_2^2 - 4R_1R_2 = 0 = (R_1-R_2)^2 \;\rightarrow\; R_1 = R_2$$

Thus the two resistors must be the same, so the "other" resistor is $\boxed{4.8\text{ k}\Omega}$.

22. We label identical resistors from left to right as R_{left}, R_{middle}, and R_{right}. When the switch is opened, the equivalent resistance of the circuit increases from $\tfrac{3}{2}R + r$ to $2R + r$. Thus the current delivered by the battery decreases, from $\dfrac{\mathscr{E}}{\tfrac{3}{2}R+r}$ to $\dfrac{\mathscr{E}}{2R+r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle}.

$$\boxed{V_{\text{left}}\text{ decreases; }V_{\text{middle}}\text{ increases; }V_{\text{right}}\text{ goes to 0}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$$\boxed{I_{\text{left}}\text{ decreases; }I_{\text{middle}}\text{ increases; }I_{\text{right}}\text{ goes to 0}}$$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the $\boxed{\text{terminal voltage increases}}$.

(*d*) With the switch closed, the equivalent resistance is $\tfrac{3}{2}R + r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathscr{E}}{\tfrac{3}{2}R+r}$, and the terminal voltage is given by Eq. 19–1.

$$V_{\substack{\text{terminal}\\\text{closed}}} = \mathscr{E} - I_{\text{closed}}r = \mathscr{E} - \frac{\mathscr{E}}{\tfrac{3}{2}R+r}r = \mathscr{E}\left(1 - \frac{r}{\tfrac{3}{2}R+r}\right) = (9.0\text{ V})\left(1 - \frac{0.50\ \Omega}{\tfrac{3}{2}(5.50\ \Omega)+0.50\ \Omega}\right)$$

$$= 8.486\text{ V} \approx \boxed{8.5\text{ V}}$$

(*e*) With the switch open, the equivalent resistance is $2R + r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathscr{E}}{2R+r}$, and again the terminal voltage is given by Eq. 19–1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathscr{E} - I_{\text{closed}}r = \mathscr{E} - \frac{\mathscr{E}}{2R+r}r = \mathscr{E}\left(1 - \frac{r}{2R+r}\right) = (9.0\text{ V})\left(1 - \frac{0.50\ \Omega}{2(5.50\ \Omega)+0.50\ \Omega}\right)$$

$$= 8.609\text{ V} \approx \boxed{8.6\text{ V}}$$

23. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2R = \frac{V^2}{R} \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}, \quad V = \sqrt{RP}$$

$$I_{1400} = \sqrt{\frac{0.5\text{ W}}{1.4\times10^3\ \Omega}} = 0.0189\text{ A} \quad V_{1800} = \sqrt{(0.5\text{ W})(1.4\times10^3\ \Omega)} = 26.5\text{ V}$$

$$I_{2500} = \sqrt{\frac{0.5\text{ W}}{2.5\times10^3\ \Omega}} = 0.0141\text{ A} \quad V_{2500} = \sqrt{(0.5\text{ W})(2.8\times10^3\ \Omega)} = 35.4\text{ V}$$

$$I_{3700} = \sqrt{\frac{0.5\text{ W}}{3.7\times10^3\ \Omega}} = 0.0116\text{ A} \quad V_{3700} = \sqrt{(0.5\text{ W})(3.7\times10^3\ \Omega)} = 43.0\text{ V}$$

The parallel resistors have to have the same voltage, so the voltage across that combination is limited to 35.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}} = \frac{V_{\text{parallel}}}{R_{\text{parallel}}} = V_{\text{parallel}}\left(\frac{1}{R_{2500}} + \frac{1}{R_{3700}}\right) = (35.4\text{ V})\left(\frac{1}{2500\ \Omega} + \frac{1}{3700\ \Omega}\right) = 0.0237\text{ A}$$

This is more than the maximum current that can be in R_{1400}. Thus the maximum current that R_{1400} can carry, 0.0189 A, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of R_{2500} and R_{3700} added to R_{1400}.

$$V_{\max} = I_{\max}R_{\text{eq}} = I_{\max}\left[R_{1800}\left(\frac{1}{R_{2800}} + \frac{1}{R_{3700}}\right)^{-1}\right] = (0.0189\text{ A})\left[1400\ \Omega\left(\frac{1}{2500\ \Omega} + \frac{1}{3700\ \Omega}\right)^{-1}\right]$$

$$= 54.66\text{ V} \approx \boxed{55\text{ V}}$$

24. (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with R_3 and R_4, which lowers the net resistance of the parallel portion of the circuit and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since R_1 is in series with the battery, its voltage will increase. Because of that increase, the voltage across R_3 and R_4 must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across R_2 until the switch was closed, its voltage will increase. To summarize:

$$\boxed{V_1 \text{ and } V_2 \text{ increase; } V_3 \text{ and } V_4 \text{ decrease}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$\boxed{I_1 \text{ and } I_2 \text{ increase; } I_3 \text{ and } I_4 \text{ decrease}}$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is R_3 and R_4 in parallel, combined with R_1 in series.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 155\ \Omega + \left(\frac{2}{155\ \Omega}\right)^{-1} = 232.5\ \Omega$$

The current delivered by the battery is the same as the current through R_1.

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\ \text{V}}{232.5\ \Omega} = 0.09462\ \text{A} = I_1$$

The voltage across R_1 is found by Ohm's law.

$$V_1 = IR_1 = (0.09462\ \text{A})(155\ \Omega) = 14.666\ \text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across R_1.

$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\ \text{V} - 14.666\ \text{V} = 7.334\ \text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_{\text{p}}}{R_2} = \frac{7.334\ \text{V}}{155\ \Omega} = 0.04732\ \text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.0946\ \text{A} \quad I_3 = I_4 = 0.0473\ \text{A}}$$

After the switch is closed, the equivalent resistance is R_2, R_3, and R_4 in parallel, combined with R_1 in series. Do a similar analysis.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 155\ \Omega + \left(\frac{3}{155\ \Omega}\right)^{-1} = 206.7\ \Omega$$

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\ \text{V}}{206.7\ \Omega} = 0.1064\ \text{A} = I_1 \quad V_1 = IR_1 = (0.1064\ \text{A})(155\ \Omega) = 16.49\ \text{V}$$

$$V_{\text{p}} = V_{\text{battery}} - V_1 = 22.0\ \text{V} - 16.49\ \text{V} = 5.51\ \text{V} \quad I_2 = \frac{V_{\text{p}}}{R_2} = \frac{5.51\ \text{V}}{155\ \Omega} = 0.0355\ \text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one-third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.106\ \text{A} \quad I_2 = I_3 = I_4 = 0.0355\ \text{A}}$$

$\boxed{\text{Yes}}$, the predictions made in part (*b*) are all confirmed.

25. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I=\frac{\mathcal{E}}{R_{\text{eq}}}=\frac{9.0\text{ V}}{(9.5+14.0+2.0)\Omega}=0.3529\text{ A}\approx\boxed{0.35\text{ A}}$$

$$\sum\text{voltages}=9.0\text{ V}-(9.5\ \Omega)(0.3529\text{ A})-(14.0\ \Omega)(0.3529\text{ A})-(2.0\ \Omega)(0.3529\text{ A})$$
$$=9.0\text{ V}-3.35\text{ V}-4.94\text{ V}-0.71\text{ V}=\boxed{0}$$

26. Apply Kirchhoff's loop rule to the circuit, starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(2.0\ \Omega)+18\text{ V}-I(4.8\ \Omega)-12\text{ V}-I(1.0\ \Omega)=0\quad\rightarrow\quad I=\frac{6\text{ V}}{7.8\ \Omega}=0.769\text{ A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and emf from left to right. Note that for the 12-V battery, there is a voltage gain going across the internal resistance from left to right.

18-V battery: $V_{\text{terminal}}=-I(2.0\ \Omega)+18\text{ V}=-(0.769\text{ A})(2.0\ \Omega)+18\text{ V}=16.46\text{ V}\approx\boxed{16\text{ V}}$

12-V battery: $V_{\text{terminal}}=I(1.0\ \Omega)+12\text{ V}=(0.769\text{ A})(1.0\ \Omega)+12\text{ V}=12.769\text{ V}\approx\boxed{13\text{ V}}$

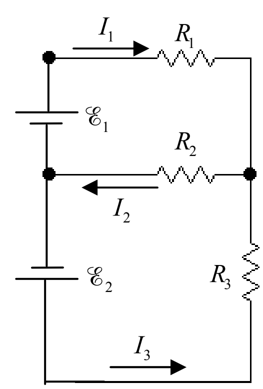
27. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR+\mathcal{E}-IR-IR+\mathcal{E}-IR=0\quad\rightarrow\quad I=\frac{\mathcal{E}}{2R}$$

$$V_{\text{ab}}=V_{\text{a}}-V_{\text{b}}=-IR+\mathcal{E}-IR=\mathcal{E}-2IR=\mathcal{E}-2\frac{\mathcal{E}}{2R}R=\boxed{0}$$

Notice that the actual values for the battery voltages and the resistances were not used.

$\boxed{28.}$ (*a*) There are three currents involved, so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2=I_1+I_3\quad\rightarrow\quad I_1=I_2-I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathcal{E}_1-I_1R_1-I_2R_2=0\quad\rightarrow\quad 9=25I_1+68I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2-I_3R_3-I_2R_2=0\quad\rightarrow\quad 12=35I_3+68I_2$$

Substitute $I_1=I_2-I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9=25I_1+68I_2=25(I_2-I_3)+68I_2=93I_2-25I_3;\ 12=35I_3+68I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 35I_3 + 68I_2 \quad \rightarrow \quad I_2 = \frac{12 - 35I_3}{68}$$

$$9 = 93I_2 - 25I_3 = 93\left(\frac{12 - 35I_3}{68}\right) - 25I_3 \quad \rightarrow \quad 612 = 1116 - 3255I_3 - 1700I_3 \quad \rightarrow$$

$$I_3 = \frac{504}{4955} = 0.1017 \text{ A} \approx \boxed{0.10 \text{ A, up}}; \quad I_2 = \frac{12 - 35I_3}{68} = 0.1241 \text{ A} \approx \boxed{0.12 \text{ A, left}}$$

$$I_1 = I_2 - I_3 = 0.0224 \text{ A} \approx \boxed{0.02 \text{ A, right}}$$

(*b*) We can include the internal resistances simply by adding $1.0\,\Omega$ to R_1 and R_3. So let $R_1 = 26\,\Omega$ and let $R_3 = 36\,\Omega$. Now re-work the problem exactly as in part (*a*).

$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

$$\mathscr{E}_1 - I_1R_1 - I_2R_2 = 0 \quad \rightarrow \quad 9 = 26I_1 + 68I_2$$

$$\mathscr{E}_2 - I_3R_3 - I_2R_2 = 0 \quad \rightarrow \quad 12 = 36I_3 + 68I_2$$

$$9 = 26I_1 + 68I_2 = 26(I_2 - I_3) + 68I_2 = 94I_2 - 26I_3;\ 12 = 36I_3 + 68I_2$$

$$12 = 36I_3 + 68I_2 \quad \rightarrow \quad I_2 = \frac{12 - 36I_3}{68} = \frac{3 - 9I_3}{17}$$

$$9 = 94I_2 - 26I_3 = 94\left(\frac{3 - 9I_3}{17}\right) - 26I_3 \quad \rightarrow \quad 153 = 282 - 846I_3 - 442I_3 \quad \rightarrow$$

$$I_3 = \frac{129}{1288} = 0.1002 \text{ A} \approx \boxed{0.10 \text{ A, up}}; \quad I_2 = \frac{3 - 12I_3}{17} = 0.1234 \text{ A} \approx \boxed{0.12 \text{ A, left}}$$

$$I_1 = I_2 - I_3 = 0.0232 \text{ A} \approx \boxed{0.02 \text{ A, right}}$$

The currents are unchanged to the nearest 0.01 A by the inclusion of the internal resistances.

29. This circuit is identical to Example 19–8 and Fig. 19–13 except for the numeric values. So we may copy the equations developed in that Example but use the current values.

Eq. (i): $I_3 = I_1 + I_2$ Eq. (ii): $-34I_1 + 45 - 48I_3 = 0$

Eq. (iii): $-34I_1 + 19I_2 - 85 = 0$ Eq. (iv): $I_2 = \dfrac{85 + 34I_1}{19} = 4.474 + 1.789I_1$

Eq. (v): $I_3 = \dfrac{45 - 34I_1}{48} = 0.938 - 0.708I_1$

$$I_3 = I_1 + I_2 \quad \rightarrow \quad 0.938 - 0.708I_1 = I_1 + 4.474 + 1.789I_1 \quad \rightarrow \quad I_1 = -1.011 \text{ A}$$

$$I_2 = 4.474 + 1.789I_1 = 2.665 \text{ A}; \quad I_3 = 0.938 - 0.708I_1 = 1.654 \text{ A}$$

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = -I_1(34\,\Omega) = -(-1.011 \text{ A})(34\,\Omega) = 34.37 \text{ V} \approx \boxed{34 \text{ V}}$$

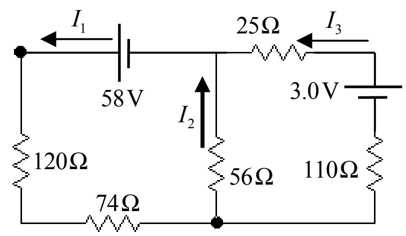
Slight differences might be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = \mathscr{E}_1 - I_2(19\,\Omega) = 85 \text{ V} - (2.665 \text{ A})(19\,\Omega) = 34.365 \text{ V} \approx 34 \text{ V}$$

(*b*) For the 85-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

$$85 = \text{V battery: } V_{\text{terminal}} = \mathscr{E}_1 - I_2 r = 85\text{ V} - (2.665\text{ A})(1.0\ \Omega) = \boxed{82\text{ V}}$$
$$45 = \text{V battery: } V_{\text{terminal}} = \mathscr{E}_2 - I_3 r = 45\text{ V} - (1.654\text{ A})(1.0\ \Omega) = \boxed{43\text{ V}}$$

30. There are three currents involved, so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58\text{ V} - I_1(120\ \Omega) - I_1(74\ \Omega) - I_2(56\ \Omega) = 0 \quad \rightarrow \quad 58 = 194 I_1 + 56 I_2$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\text{ V} - I_3(25\ \Omega) + I_2(56\ \Omega) - I_3(110\ \Omega) = 0 \quad \rightarrow \quad 3 = -56 I_2 + 135 I_3$$

Substitute $I_1 = I_2 + I_3$ into the left loop equation, so that there are two equations with two unknowns.

$$58 = 194(I_2 + I_3) + 56 I_2 = 250 I_2 + 194 I_3$$

Solve the right loop equation for I_2 and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$3 = -56 I_2 + 135 I_3 \quad \rightarrow \quad I_2 = \frac{135 I_3 - 3}{56}; \quad 58 = 250 I_2 + 194 I_3 = 250\left(\frac{135 I_3 - 3}{56}\right) + 194 I_3 \quad \rightarrow$$
$$3998 = 44614 I_3$$
$$I_3 = 0.08961\text{ A}; \quad I_2 = \frac{135 I_3 - 3}{56} = 0.1625\text{ A}; \quad I_1 = I_2 + I_3 = 0.2521\text{ A}$$

The current in each resistor is as follows:

$$\boxed{120\ \Omega\text{: }0.25\text{ A} \quad 74\ \Omega\text{: }0.25\text{ A} \quad 56\ \Omega\text{: }0.16\text{ A} \quad 25\ \Omega\text{: }0.090\text{ A} \quad 110\ \Omega\text{: }0.090\text{ A}}$$

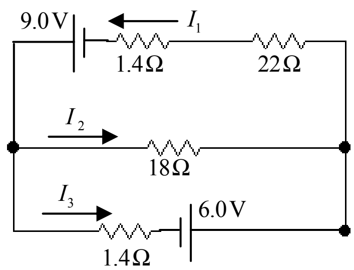
31. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

$$V_3 - I_1 R_1 + V_1 = 0 \quad \rightarrow \quad I_1 = \frac{V_3 + V_1}{R_1} = \frac{6.0\text{ V} + 9.0\text{ V}}{22\ \Omega} = \boxed{0.68\text{ A, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3 - I_2 R_2 = 0 \quad \rightarrow \quad I_2 = \frac{V_3}{R_2} = \frac{6.0\ \text{V}}{18\ \Omega} = \boxed{0.33\ \text{A, left}}$$

32. There are three currents involved, so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner and progressing counterclockwise.

$$-I_3(1.4\ \Omega) + 6.0\ \text{V} - I_1(22\ \Omega) - I_1(1.4\ \Omega) + 9.0\ \text{V} = 0 \quad \rightarrow$$
$$15 = 23.4 I_1 + 1.4 I_3$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the lower left corner and progressing counterclockwise.

$$-I_3(1.4\ \Omega) + 6.0\ \text{V} + I_2(18\ \Omega) = 0 \quad \rightarrow \quad 6 = -18 I_2 + 1.4 I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$15 = 23.4 I_1 + 1.4 I_3 = 23.4(I_2 + I_3) + 1.4 I_3 = 23.4 I_2 + 24.8 I_3; \quad 6 = -18 I_2 + 1.4 I_3$$

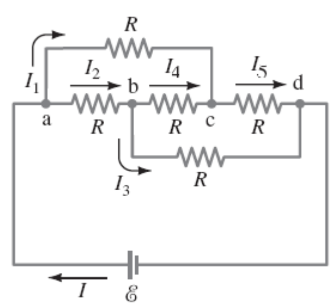
Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = -18 I_2 + 1.4 I_3 \ \rightarrow \ I_2 = \frac{-6 + 1.4 I_3}{18}$$

$$15 = 23.4 I_2 + 24.8 I_3 = 23.4\left(\frac{-6 + 1.4 I_3}{18}\right) + 24.8 I_3 \ \rightarrow \ 270 = -140.4 + 32.76 I_3 + 446.4 I_3 \ \rightarrow$$

$$I_3 = \frac{410.4}{479.16} = 0.8565\ \text{A}; \quad I_2 = \frac{-6 + 1.4 I_3}{18} = \frac{-6 + 1.4(0.8565)}{18} = -0.2667\ \text{A} \approx \boxed{0.27\ \text{A, left}}$$
$$I_1 = I_2 + I_3 = 0.5898\ \text{A} \approx \boxed{0.59\ \text{A, left}}$$

33. (*a*) We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a–c), we write equations relating the entering and exiting currents.



$I = I_1 + I_2$ [1]

$I_2 = I_3 + I_4$ [2]

$I_1 + I_4 = I_5$ [3]

We use Kirchhoff's loop rule to write equations for loops abca, abcda, and bdcb.

$$0 = -I_2R - I_4R + I_1R \qquad [4]$$
$$0 = -I_2R - I_3R + \mathscr{E} \qquad [5]$$
$$0 = -I_3R + I_5R + I_4R \qquad [6]$$

We have six unknown currents and six equations. We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current I_5. Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate I_2.

$$0 = -I_3R + (I_1 + I_4)R + I_4R \;\rightarrow\; 0 = -I_3R + I_1R + 2I_4\mathrm{R} \qquad [6']$$
$$I = I_1 + I_3 + I_4 \qquad [1']$$
$$0 = -(I_3 + I_4)R - I_4R + I_1R \;\rightarrow\; 0 = -I_3R - 2I_4\mathrm{R} + I_1R \qquad [4']$$
$$0 = -(I_3 + I_4)R - I_3R + \mathscr{E} \;\rightarrow\; 0 = -I_4R - 2I_3R + \mathscr{E} \qquad [5']$$

Next we solve Eq. [4′] for I_4 and insert the result into Eqs. [1′], [5′], and [6′].

$$0 = -I_3R - 2I_4R + I_1R \;\rightarrow\; I_4 = \tfrac{1}{2}I_1 - \tfrac{1}{2}I_3$$
$$I = I_1 + I_3 + \tfrac{1}{2}I_1 - \tfrac{1}{2}I_3 \;\rightarrow\; I = \tfrac{3}{2}I_1 + \tfrac{1}{2}I_3 \qquad [1'']$$
$$0 = -I_3R + I_1R + 2(\tfrac{1}{2}I_1 - \tfrac{1}{2}I_3)R = -2I_3R + 2I_1\mathrm{R} \;\rightarrow\; I_1 - I_3 \qquad [6'']$$
$$0 = -(\tfrac{1}{2}I_1 - \tfrac{1}{2}I_3)R - 2I_3R + \mathscr{E} \;\rightarrow\; 0 = -\tfrac{1}{2}I_1R - \tfrac{3}{2}I_3\mathrm{R} + \mathscr{E} \qquad [5'']$$

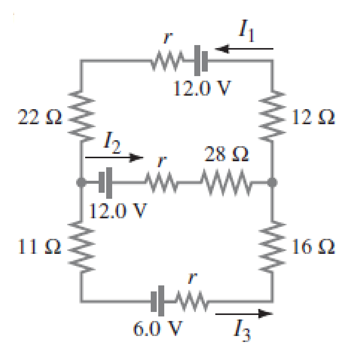
Finally we substitute Eq. [6″] into Eq. [5″] and solve for I_1. We insert this result into Eq. [1″] to write an equation for the current through the battery in terms of the battery emf and resistance.

$$0 = -\tfrac{1}{2}I_1R - \tfrac{3}{2}I_1R + \mathscr{E} \;\rightarrow\; I_1 = \frac{\mathscr{E}}{2R}; \quad I = \tfrac{3}{2}I_1 + \tfrac{1}{2}I_1 = 2I_1 \;\rightarrow \boxed{I = \frac{\mathscr{E}}{R}}$$

(*b*) We divide the battery emf by the current to determine the effective resistance.

$$R_{\mathrm{eq}} = \frac{\mathscr{E}}{I} = \frac{\mathscr{E}}{\mathscr{E}/R} = \boxed{R}$$

34. (*a*) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery and progressing counterclockwise. We add series resistances.

$$12.0\text{ V} - I_2(12\ \Omega) + 12.0\text{ V} - I_1(35\ \Omega) = 0 \;\rightarrow\; 24 = 35I_1 + 12I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery and progressing clockwise.

$$12.0\text{ V} - I_2(12\ \Omega) - 6.0\text{ V} + I_3(28\ \Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 28I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 35I_1 + 12I_2 = 35(I_2 + I_3) + 12I_2 = 47I_2 + 35I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for I_3.

$$6 = 12I_2 - 28I_3 \quad\rightarrow\quad I_2 = \frac{6 + 28I_3}{12} = \frac{3 + 14I_3}{6}; \quad 24 = 47I_2 + 35I_3 = 47\left(\frac{3 + 14I_3}{6}\right) + 35I_3 \quad\rightarrow$$

$$I_3 = \boxed{3.46\text{ mA}}; \quad I_2 = \frac{6 + 28I_3}{12} = \boxed{0.508\text{ A}}; \quad I_1 = I_2 + I_3 = \boxed{0.511\text{ A}}$$

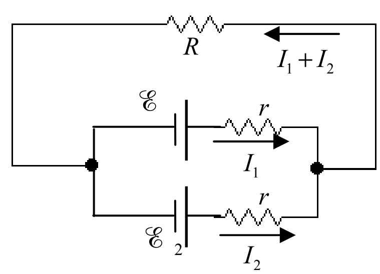
(*b*) The terminal voltage of the 6.0-V battery is $6.0\text{ V} - I_3 r = 6.0\text{ V} - (3.46\times10^{-3}\text{ A})(1.0\ \Omega)$ $= 5.997\text{ V} \approx \boxed{6.0\text{ V}}$.

35. This problem is the same as Problem 34, except the total resistance in the top branch is now $23\ \Omega$ instead of $35\ \Omega$. We simply reproduce the adjusted equations here without the prose.

$$I_1 = I_2 + I_3$$
$$12.0\text{ V} - I_2(12\ \Omega) + 12.0\text{ V} - I_1(23\ \Omega) = 0 \quad\rightarrow\quad 24 = 23I_1 + 12I_2$$
$$12.0\text{ V} - I_2(12\ \Omega) - 6.0\text{ V} + I_3(28\ \Omega) = 0 \quad\rightarrow\quad 6 = 12I_2 - 28I_3$$
$$24 = 23I_1 + 12I_2 = 23(I_2 + I_3) + 12I_2 = 35I_2 + 23I_3$$

$$6 = 12I_2 - 28I_3 \quad\rightarrow\quad I_2 = \frac{6 + 28I_3}{12} = \frac{3 + 14I_3}{6}; \quad 24 = 35I_2 + 23I_3 = 35\left(\frac{3 + 14I_3}{6}\right) + 23I_3 \quad\rightarrow$$
$$39 = 628I_3$$
$$I_3 = 0.0621\text{ A}; \quad I_2 = \frac{6 + 28I_3}{12} = 0.6449\text{ A}; \quad I_1 = I_2 + I_3 = 0.707\text{ A} \approx \boxed{0.71\text{ A}}$$

36. Define I_1 to be the current to the right through the 2.00-V battery ($\mathscr{E}_1$), and I_2 to be the current to the right through the 3.00 V battery ($\mathscr{E}_2$). At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule, first to the upper loop and then to the outer loop, and solve for the currents.



$$\mathscr{E}_1 - I_1 r - (I_1 + I_2)R = 0 \quad\rightarrow\quad \mathscr{E}_1 - (R + r)I_1 - RI_2 = 0$$
$$\mathscr{E}_2 - I_2 r - (I_1 + I_2)R = 0 \quad\rightarrow\quad \mathscr{E}_2 - RI_1 - (R + r)I_2 = 0$$

Solve the first equation for I_2 and substitute into the second equation to solve for I_1.

$$\mathscr{E}_1 - (R + r)I_1 - RI_2 = 0 \quad\rightarrow\quad I_2 = \frac{\mathscr{E}_1 - (R + r)I_1}{R} = \frac{2.00 - 4.350I_1}{4.00} = 0.500 - 1.0875I_1$$
$$\mathscr{E}_2 - RI_1 - (R + r)I_2 = 3.00 - 4.00I_1 - (4.35)(0.500 - 1.0875I_1) = 0 \quad\rightarrow$$
$$0.825 = -0.7306I_1$$
$$I_1 = -1.129\text{ A}; \quad I_2 = 0.500 - 1.0875I_1 = 1.728\text{ A}$$

The voltage across R is its resistance times $I = I_1 + I_2$.

$$V_R = R(I_1 + I_2) = (4.00\ \Omega)(-1.129\ \text{A} + 1.728\ \text{A}) = 2.396\ \text{V} \approx \boxed{2.40\ \text{V}}$$

Note that the top battery is being charged—the current is flowing through it from positive to negative.

37. Take 100 of the batteries and connect them in series, which would give a total voltage of 300 volts. Do that again with the next 100 batteries, and again with the last 100 batteries. That gives three sets, each with a total voltage of 300 volts. Then those three sets can be connected in parallel with each other. The total combination of batteries would have a potential difference of 300 volts.

Another possibility is to connect three batteries in parallel, which would provide a potential difference of 3 volts. Make 100 sets of those three-battery combinations and connect those 100 sets in series. That total combination would also have a potential difference of 300 volts.

38. (*a*) Capacitors in parallel add according to Eq. 19–5.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6(4.8\times10^{-6}\ \text{F}) = \boxed{2.88\times10^{-5}\ \text{F}} = 28.8\ \mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 19–6.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_6}\right)^{-1} = \left(\frac{6}{4.8\times10^{-6}\ \text{F}}\right)^{-1} = \frac{4.8\times10^{-6}\ \text{F}}{6} = \boxed{8.0\times10^{-7}\ \text{F}}$$

39. The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} + C_3 = \left(\frac{1}{3.00\times10^{-6}\ \text{F}} + \frac{1}{4.00\times10^{-6}\ \text{F}}\right)^{-1} + 2.00\times10^{-6}\ \text{F} = \boxed{3.71\times10^{-6}\ \text{F}} = 3.71\ \mu\text{F}$$

40. (*a*) The full voltage is across the $2.00\text{-}\mu\text{F}$ capacitor, so $\boxed{V_3 = 21.0\ \text{V}}$. To find the voltage cross the two capacitors in series, find their equivalent capacitance and the charge stored. That charge will be the same for both of the series capacitors. Finally, use that charge to determine the voltage on each capacitor. The sum of the voltages across the series capacitors is 26.0 V.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{3.00\times10^{-6}\ \text{F}} + \frac{1}{4.00\times10^{-6}\ \text{F}}\right)^{-1} = 1.714\times10^{-6}\ \text{F}$$

$$Q_{\text{eq}} = C_{\text{eq}}V = (1.714\times10^{-6}\ \text{F})(21.0\ \text{V}) = 3.599\times10^{-5}\ \text{C}$$

$$V_1 = \frac{Q_{\text{eq}}}{C} = \frac{3.599\times10^{-5}\ \text{C}}{3.00\times10^{-6}\ \text{F}} = \boxed{12.0\ \text{V}} \quad V_2 = \frac{Q_{\text{eq}}}{C} = \frac{3.599\times10^{-5}\ \text{C}}{4.00\times10^{-6}\ \text{F}} = \boxed{9.0\ \text{V}}$$

(*b*) We have found two of the three charges already. The charge $Q_{\text{eq}} = 3.599\times10^{-5}\ \text{C}$ is the charge on the two capacitors in series. The other charge is found by using the full voltage.

$$Q_3 = C_3V = (2.00\times10^{-6}\ \text{F})(21.0\ \text{V}) = 4.20\times10^{-5}\ \text{C}$$

$$\boxed{Q_1 = Q_2 = 3.60\times10^{-5}\ \text{C};\quad Q_3 = 4.20\times10^{-5}\ \text{C}}$$

41. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2} \quad\rightarrow\quad \frac{1}{C_2}=\frac{1}{C_{\text{eq}}}-\frac{1}{C_1}=\frac{C_1-C_{\text{eq}}}{C_1C_{\text{eq}}} \quad\rightarrow$$

$$C_2=\frac{C_1C_{\text{eq}}}{C_1-C_{\text{eq}}}=\frac{(2.9\times10^{-9}\text{ F})(1.2\times10^{-9}\text{ F})}{(2.9\times10^{-9}\text{ F})-(1.2\times10^{-9}\text{ F})}=2.047\times10^{-9}\text{ F}\approx\boxed{2.0\text{ nF}}$$

$\boxed{\text{Yes}}$, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

42. Capacitors in parallel add linearly, so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\text{-}\mu\text{F}$ capacitor.

$$7.0\ \mu\text{F}+C=16\ \mu\text{F} \quad\rightarrow\quad C=\boxed{9\ \mu\text{F connected in parallel}}$$

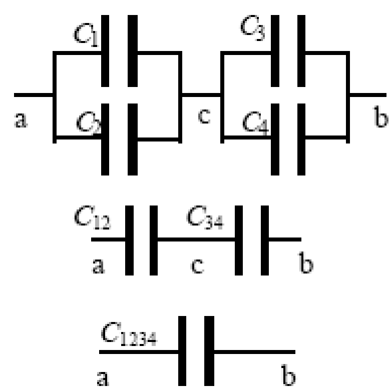
43. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max}=C_1+C_2+C_3=3.2\times10^{-9}\text{ F}+5.8\times10^{-9}\text{ F}+1.00\times10^{-8}\text{ F}=\boxed{1.90\times10^{-8}\text{ F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}=\left(\frac{1}{3.2\times10^{-9}\text{ F}}+\frac{1}{5.8\times10^{-9}\text{ F}}+\frac{1}{1.00\times10^{-8}\text{ F}}\right)^{-1}=\boxed{1.7\times10^{-9}\text{ F in series}}$$

44. From the diagram, we see that C_1 and C_2 are in parallel and C_3 and C_4 are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.



$$C_{12}=C_1+C_2;\ C_{34}=C_3+C_4;$$

$$\frac{1}{C_{1234}}=\frac{1}{C_{12}}+\frac{1}{C_{34}}=\frac{1}{C_1+C_2}+\frac{1}{C_3+C_4} \quad\rightarrow$$

$$C_{1234}=\boxed{\frac{(C_1+C_2)(C_3+C_4)}{(C_1+C_2+C_3+C_4)}}$$

45. The voltage across C_1 is the full 10 V from the battery. Since the other two capacitors are identical, they will each have 5 V across them so that the total voltage across the two of them is 10 V. Thus $V_1/V_2=10\text{ V}/5\text{ V}=\boxed{2/1}$.

46. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $$Q_1=Q_2=Q_{\text{eq}}=C_{\text{eq}}V=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}V=\left(\frac{1}{0.50\times10^{-6}\text{ F}}+\frac{1}{1.40\times10^{-6}\text{ F}}\right)^{-1}(9.0\text{ V})$$

$$=3.316\times10^{-6}\text{ C}$$

$$V_1=\frac{Q_1}{C_1}=\frac{3.316\times10^{-6}\text{ C}}{0.50\times10^{-6}\text{ F}}=6.632\text{ V}\approx\boxed{6.6\text{ V}} \qquad V_2=\frac{Q_2}{C_2}=\frac{3.316\times10^{-6}\text{ C}}{1.4\times10^{-6}\text{ F}}=2.369\text{ V}\approx\boxed{2.4\text{ V}}$$

(*b*) $Q_1 = Q_2 = Q_{eq} = 3.316\times10^{-6}\ \text{C} \approx \boxed{3.3\times10^{-6}\ \text{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\ \text{V}}$ $V_2 = \boxed{9.0\ \text{V}}$ $Q_1 = C_1V_1 = (0.50\times10^{-6}\ \text{F})(9.0\ \text{V}) = \boxed{4.5\times10^{-6}\ \text{C}}$

$Q_2 = C_2V_2 = (1.4\times10^{-6}\ \text{F})(9.0\ \text{V}) = \boxed{1.3\times10^{-5}\ \text{C}}$

47. The energy stored by a capacitor is given by Eq. 17–10, $\text{PE} = \frac{1}{2}CV^2$.

$$\text{PE}_{\text{final}} = 4\text{PE}_{\text{initial}} \quad \rightarrow \quad \tfrac{1}{2}C_{\text{final}}V_{\text{final}}^2 = 4\tfrac{1}{2}C_{\text{initial}}V_{\text{initial}}^2$$

One simple way to accomplish this is to have $C_{\text{final}} = 4C_{\text{initial}}$ and $V_{\text{final}} = V_{\text{initial}}$. In order to keep the voltage the same for both configurations, any additional capacitors must be connected in $\boxed{\text{parallel}}$ to the original capacitor. In order to multiply the capacitance by a factor of 4, we recognize that capacitors added in parallel add linearly. Thus if a capacitor of value $3C = \boxed{750\ \text{pF}}$ were connected in parallel to the original capacitor, then the final capacitance would be 4 times the original capacitance with the same voltage, so the potential energy would increase by a factor of 4.

48. The capacitors are in parallel, so the potential is the same for each capacitor, and the total charge on the capacitors is the sum of the individual charges. We use Eqs. 17–7 and 17–8.

$$Q_1 = C_1V = \varepsilon_0\frac{A_1}{d_1}V;\ Q_2 = C_2V = \varepsilon_0\frac{A_2}{d_2}V;\ Q_3 = C_3V = \varepsilon_0\frac{A_3}{d_3}V$$

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 = \varepsilon_0\frac{A_1}{d_1}V + \varepsilon_0\frac{A_2}{d_2}V + \varepsilon_0\frac{A_3}{d_3}V = \left(\varepsilon_0\frac{A_1}{d_1} + \varepsilon_0\frac{A_2}{d_2} + \varepsilon_0\frac{A_3}{d_3}\right)V$$

$$C_{\text{net}} = \frac{Q_{\text{total}}}{V} = \frac{\left(\varepsilon_0\frac{A_1}{d_1} + \varepsilon_0\frac{A_2}{d_2} + \varepsilon_0\frac{A_3}{d_3}\right)V}{V} = \left(\varepsilon_0\frac{A_1}{d_1} + \varepsilon_0\frac{A_2}{d_2} + \varepsilon_0\frac{A_3}{d_3}\right) = C_1 + C_2 + C_3$$

49. We have $C_{\text{P}} = C_1 + C_2 = 35.0\ \mu\text{F}$ and $\frac{1}{C_{\text{S}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.8\ \mu\text{F}}$. Solve for C_1 and C_2 in terms of C_{P} and C_{S}.

$$\frac{1}{C_{\text{S}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{C_{\text{P}} - C_1} = \frac{(C_{\text{P}} - C_1) + C_1}{C_1(C_{\text{P}} - C_1)} = \frac{C_{\text{P}}}{C_1(C_{\text{P}} - C_1)} \quad \rightarrow$$

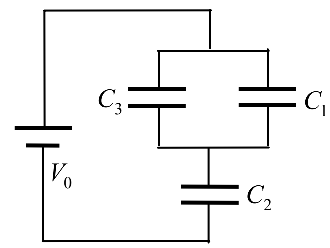
$$\frac{1}{C_{\text{S}}} = \frac{C_{\text{P}}}{C_1(C_{\text{P}} - C_1)} \quad \rightarrow \quad C_1^2 - C_{\text{P}}C_1 + C_{\text{P}}C_{\text{S}} = 0 \quad \rightarrow$$

$$C_1 = \frac{C_{\text{P}} \pm \sqrt{C_{\text{P}}^2 - 4C_{\text{P}}C_{\text{S}}}}{2} = \frac{35.0\ \mu\text{F} \pm \sqrt{(35.0\ \mu\text{F})^2 - 4(35.0\ \mu\text{F})(4.8\ \mu\text{F})}}{2}$$
$$= 29.3\ \mu\text{F},\ 5.7\ \mu\text{F}$$

$$C_2 = C_{\text{P}} - C_1 = 35.0\ \mu\text{F} - 29.3\ \mu\text{F} = 5.7\ \mu\text{F} \text{ or } 35.0\ \mu\text{F} - 5.7\ \mu\text{F} = 29.3\ \mu\text{F}$$

So the two values are $\boxed{29.3\ \mu\text{F and } 5.7\ \mu\text{F}}$.

50. We want a small voltage drop across C_1. Since $V = Q/C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put C_1 and C_3 in parallel with each other, and then put that combination in series with C_2. See the diagram. To calculate the voltage across C_1, find the equivalent capacitance and the net charge. That charge is used to find the voltage drop across C_2, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_1 + C_3} = \frac{C_1 + C_2 + C_3}{C_2(C_1 + C_3)} \rightarrow C_{\text{eq}} = \frac{C_2(C_1 + C_3)}{C_1 + C_2 + C_3}; \quad Q_{\text{eq}} = C_{\text{eq}} V_0; \quad V_2 = \frac{Q_2}{C_2} = \frac{Q_{\text{eq}}}{C_2};$$

$$V_1 = V_0 - V_2 = V_0 - \frac{Q_{\text{eq}}}{C_2} = V_0 - \frac{C_{\text{eq}} V_0}{C_2} = V_0 - \frac{\frac{C_2(C_1 + C_3)}{C_1 + C_2 + C_3} V_0}{C_2} = \frac{C_2}{C_1 + C_2 + C_3} V_0 = \frac{1.5\ \mu\text{F}}{6.5\ \mu\text{F}}(12\ \text{V})$$

$$= \boxed{2.8\ \text{V}}$$

51. (*a*) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3, and then that combination is in series with C_4. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \rightarrow C_{12} = \tfrac{1}{2}C; \quad C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \rightarrow C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V = \frac{3}{5}CV$. This is the charge on both of the series components of C_{1234}.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \rightarrow V_{123} = \tfrac{2}{5}V$$

$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \rightarrow V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\frac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12};\ Q_{12} = C_{12}V_{12} = \left(\tfrac{1}{2}C\right)\left(\tfrac{2}{5}V\right) = \tfrac{1}{5}CV$$
$$V_{123} = \tfrac{2}{5}V = V_3;\ Q_3 = C_3V_3 = C\left(\tfrac{2}{5}V\right) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12}.

$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \quad \rightarrow \quad V_1 = \tfrac{1}{5}V;\quad Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \quad \rightarrow \quad V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV;\quad Q_3 = \tfrac{2}{5}CV;\quad Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V;\quad V_3 = \tfrac{2}{5}V;\quad V_4 = \tfrac{3}{5}V \end{array}}$$

52. We take each of the given times as the time constant of the *RC* combination.

$$\tau = RC \quad \rightarrow \quad R = \frac{\tau}{C}$$
$$R_{1\text{s}} = \frac{\tau}{C} = \frac{1\text{ s}}{1\times10^{-6}\text{ F}} = \boxed{1\times10^{6}\ \Omega};\quad R_{2\text{s}} = \frac{\tau}{C} = \frac{2\text{ s}}{1\times10^{-6}\text{ F}} = \boxed{2\times10^{6}\ \Omega};$$
$$R_{4\text{s}} = \frac{\tau}{C} = \frac{4\text{ s}}{1\times10^{-6}\text{ F}} = \boxed{4\times10^{6}\ \Omega};\quad R_{2\text{s}} = \frac{\tau}{C} = \frac{8\text{ s}}{1\times10^{-6}\text{ F}} = \boxed{8\times10^{6}\ \Omega};$$
$$R_{15\text{s}} = \frac{\tau}{C} = \frac{15\text{ s}}{1\times10^{-6}\text{ F}} = \boxed{15\times10^{6}\ \Omega}$$

So we estimate the range of resistance to be $\boxed{1\text{ M}\Omega - 15\text{ M}\Omega}$.

53. From Eq. 19–7, the product RC is equal to the time constant.

$$\tau = RC \quad \rightarrow \quad R = \frac{\tau}{C} = \frac{3.0\text{ s}}{3.0\times10^{-6}\text{ F}} = \boxed{1.0\times10^{6}\ \Omega}$$

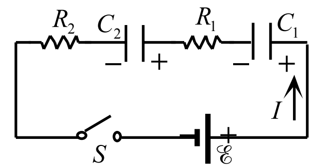
54. (*a*) From Eq. 19–7, the product *RC* is equal to the time constant.

$$\tau = RC \quad \rightarrow \quad C = \frac{\tau}{R} = \frac{18.0\times10^{-6}\text{ s}}{15.0\times10^{3}\ \Omega} = \boxed{1.20\times10^{-9}\text{ F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, then the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathscr{E}(1 - e^{-t/\tau}) \quad \rightarrow \quad e^{-t/\tau} = \left(1 - \frac{V_C}{\mathscr{E}}\right) \quad \rightarrow \quad -\frac{t}{\tau} = \ln\left(1 - \frac{V_C}{\mathscr{E}}\right) \quad \rightarrow$$
$$t = -\tau\ln\left(1 - \frac{V_C}{\mathscr{E}}\right) = -(18.0\times10^{-6}\text{ s})\ln\left(1 - \frac{16.0\text{ V}}{24.0\text{ V}}\right) = \boxed{1.98\times10^{-5}\text{ s}}$$

55. The current for a capacitor-charging circuit is given by Eq. 19–7d, with R the equivalent series resistance and C the equivalent series capacitance.



$$I=\frac{\mathscr{E}}{R_{\text{eq}}}e^{-(t/R_{\text{eq}}C_{\text{eq}})}\quad\rightarrow$$

$$t=-R_{\text{eq}}C_{\text{eq}}\ln\left(\frac{IR_{\text{eq}}}{\mathscr{E}}\right)=-(R_1+R_2)\left(\frac{C_1C_2}{C_1+C_2}\right)\ln\left[\frac{I(R_1+R_2)}{\mathscr{E}}\right]$$

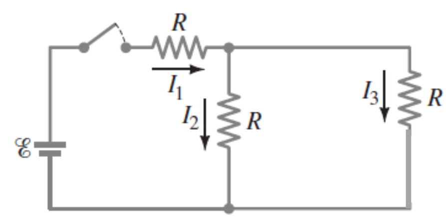
$$=-(4400\ \Omega)\left[\frac{(3.8\times10^{-6}\ \text{F})^2}{7.6\times10^{-6}\ \text{F}}\right]\ln\left[\frac{(1.50\times10^{-3}\ \text{A})(4400\ \Omega)}{(16.0\text{V})}\right]=\boxed{7.4\times10^{-3}\ \text{s}}$$

56. The voltage of the discharging capacitor is given by $V_C=V_0e^{-t/RC}$. The capacitor voltage is to be $0.0025\,V_0$.

$$V_C=V_0e^{-t/RC}\quad\rightarrow\quad 0.0010\,V_0=V_0e^{-t/RC}\quad\rightarrow\quad 0.0010=e^{-t/RC}\quad\rightarrow\quad \ln(0.010)=-\frac{t}{RC}\quad\rightarrow$$

$$t=-RC\ln(0.010)=-(8.7\times10^3\ \Omega)(3.0\times10^{-6}\ \text{F})\ln(0.0025)=\boxed{0.16\ \text{s}}$$

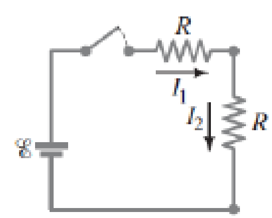
57. (*a*) At $t=0$, the capacitor is uncharged, so there is no voltage difference across it. The capacitor is a "short," so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the equal, parallel resistors.



$$R_{\text{eq}}=R+\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}=\tfrac{3}{2}R\quad\rightarrow$$

$$I_1=\frac{\mathscr{E}}{R_{\text{eq}}}=\frac{\mathscr{E}}{\frac{3}{2}R}=\boxed{\frac{2\mathscr{E}}{3R}};\quad I_2=I_3=\tfrac{1}{2}I_1=\boxed{\frac{\mathscr{E}}{3R}}$$

(*b*) At $t=\infty$, the capacitor will be fully charged and there will be no current in the branch containing the capacitor, so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.



$$R_{\text{eq}}=R+R=2R\quad\rightarrow\quad I_1=I_2=\frac{\mathscr{E}}{R_{\text{eq}}}=\boxed{\frac{\mathscr{E}}{2R}};\quad I_3=\boxed{0}$$

(*c*) At $t=\infty$, since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore, the voltage difference across the capacitor equals the voltage difference across the resistor through which I_2 flows.

$$V_C=V_{R_2}=I_2R=\left(\frac{\mathscr{E}}{2R}\right)R=\boxed{\tfrac{1}{2}\mathscr{E}}$$

58. (*a*) With the switch open, the resistors are in series with each other, so have the same current. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \quad \rightarrow \quad I = \frac{V}{R_1 + R_2} = \frac{24\text{ V}}{8.8\,\Omega + 4.4\,\Omega} = 1.818\text{ A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\text{ A})(4.4\,\Omega) = \boxed{8.0\text{ V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow \quad C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.48\,\mu\text{F})(0.36\,\mu\text{F})}{(0.48\,\mu\text{F} + 0.36\,\mu\text{F})} = 0.2057\,\mu\text{F}$$

$$Q_{\text{eq}} = VC_{\text{eq}} = (24.0\text{ V})(0.2057\,\mu\text{F}) = 4.937\,\mu\text{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\text{-}\mu\text{F}$ capacitor.

$$V_{\text{b}} = \frac{Q_2}{C_2} = \frac{4.937\,\mu\text{C}}{0.36\,\mu\text{F}} = 13.7\text{ V} \approx \boxed{14\text{ V}}$$

(*c*) The switch is now closed. After equilibrium has been reached for a long time, there is no current flowing in the capacitors, so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, so point b must be at the same potential as point a, $\boxed{8.0\text{ V}}$. This also means that the voltage across C_2 is 8.0 V, and the voltage across C_1 is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1 C_1 = (16\text{ V})(0.48\,\mu\text{F}) = 7.68\,\mu\text{C}$$

$$Q_2 = V_2 C_2 = (8.0\text{ V})(0.36\,\mu\text{F}) = 2.88\,\mu\text{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of C_1 had the same magnitude as the charge on the positive plate of C_2. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of C_1 and the charge on the positive plate of C_2.

$$Q_{\text{b}} = -Q_1 + Q_2 = -7.68\,\mu\text{C} + 2.88\,\mu\text{C} = -4.80\,\mu\text{C} \approx -4.8\,\mu\text{C}$$

Thus $\boxed{4.8\,\mu\text{C}}$ of charge has passed through the switch, from right to left.

59. (*a*) The full-scale current is the reciprocal of the sensitivity.

$$I_{\substack{\text{full}\\\text{scale}}} = \frac{1}{35{,}000\,\Omega/\text{V}} = \boxed{2.9\times10^{-5}\text{ A}} \text{ or } 29\,\mu\text{A}$$

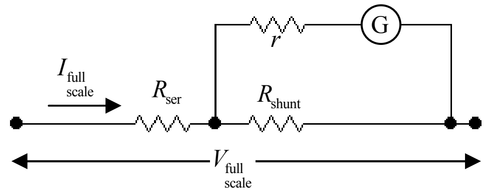
(*b*) The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\substack{\text{full}\\\text{scale}}}(\text{sensitivity}) = (250\text{ V})(35{,}000\,\Omega/\text{V}) = 8.75\times10^{6}\,\Omega \approx \boxed{8.8\times10^{6}\,\Omega}$$

60. The total resistance with the ammeter present is $R_{eq} = 720\ \Omega + 480\ \Omega + 53\ \Omega = 1253\ \Omega$. The voltage supplied by the battery is found from Ohm's law to be $V_{battery} = IR_{eq} = (5.25\times10^{-3}\ \text{A})(1253\ \Omega)$ $= 6.578\ \text{V}$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{eq} = 1200\ \Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{battery}}{R'_{eq}} = \frac{6.578\ \text{V}}{1200\ \Omega} = 5.48\times10^{-3}\ \text{A} = \boxed{5.48\ \text{mA}}$$

61. To make a voltmeter, a resistor R_{ser} must be placed in series with the existing meter so that the desired full-scale voltage corresponds to the full-scale current of the galvanometer. We know that 35 mA produces full-scale deflection of the galvanometer, so the voltage drop across the total meter must be 25 V when the current through the meter is 35 mA.



$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{eq} = I_{\substack{\text{full}\\\text{scale}}} \left[R_{ser} + \left(\frac{1}{r} + \frac{1}{R_{shunt}} \right)^{-1} \right] \rightarrow$$

$$R_{ser} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left(\frac{1}{r} + \frac{1}{R_{shunt}} \right)^{-1} = \frac{25\ \text{V}}{35\times10^{-3}\ \text{A}} - \left(\frac{1}{33\ \Omega} + \frac{1}{0.20\ \Omega} \right)^{-1} = 714.1\ \Omega \approx \boxed{710\ \Omega}$$

The sensitivity is $\frac{714\ \Omega}{25\ \text{V}} = \boxed{29\ \Omega/\text{V}}$.

$\boxed{62.}$ (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Fig. 19–31 for a circuit diagram.

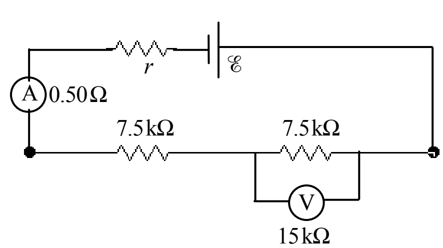
$$V_{shunt} = V_G \quad \rightarrow \quad (I - I_G) R_{shunt} = I_G r \quad \rightarrow$$

$$R_{shunt} = \frac{I_G r}{(I - I_G)} = \frac{(55\times10^{-6}\ \text{A})(32\ \Omega)}{(25\ \text{A} - 55\times10^{-6}\ \text{A})} = \boxed{7.0\times10^{-5}\ \Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full-scale current of the galvanometer. See Fig. 19–32 for a circuit diagram.

$$V_{\text{full scale}} = I_G (R_{ser} + r) \quad \rightarrow \quad R_{ser} = \frac{V_{\text{full scale}}}{I_G} - r = \frac{250\ \text{V}}{55\times10^{-6}\ \text{A}} - 32\ \Omega = \boxed{4.5\times10^{6}\ \Omega}$$

63. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.



$$R_{\text{eq}} = 1.0\ \Omega + 0.50\ \Omega + 7500\ \Omega + \frac{(7500\ \Omega)(15{,}000\ \Omega)}{(7500\ \Omega + 15{,}000\ \Omega)}$$

$$= 12501.5\ \Omega \approx 12500\ \Omega;\ I_{\text{source}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{12.0\text{ V}}{12500\ \Omega} = \boxed{9.60\times10^{-4}\text{ A}}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{\text{meter}} = I_{\text{source}}R_{\text{eq}} = (9.60\times10^{-4}\text{ A})\frac{(7500\ \Omega)(15{,}000\ \Omega)}{(7500\ \Omega + 15{,}000\ \Omega)} = \boxed{4.8\text{ V}}$$

If there were no meters at all, then the equivalent resistance, delivered current, and voltage across the resistor would be as follows.

$$R_{\text{eq}} = 1.0\ \Omega + 7500\ \Omega + 7500\ \Omega = 15{,}001\ \Omega$$

$$I_{\text{source}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{12.0\text{ V}}{15{,}001\ \Omega} = 8.00\times10^{-4}\text{ A}$$

$$V_{7500\Omega} = I_{\text{source}}R = (8.00\times10^{-4}\text{ A})(7500\ \Omega) = 6.00\text{ V}$$

We also calculated the % difference from the readings with realistic meters.

$$(\%\text{ diff})_I = \left(\frac{9.60\times10^{-4}\text{ A} - 8.00\times10^{-4}\text{ A}}{8.00\times10^{-4}\text{ A}}\right)\times100 = \boxed{20\ \%}$$

$$(\%\text{ diff})_V = \left(\frac{4.8\text{ V} - 6.0\text{ V}}{6.0\text{ V}}\right)\times100 = \boxed{-20\ \%}$$

The current reads 20% higher with the meters present, and the voltage reads 20% lower.

64. We know from Example 19–17 that the voltage across the resistor without the voltmeter connected is 4.0 V. Thus the minimum acceptable voltmeter reading is 95% of that: (0.95)(4.0 V) = 3.8 V. The voltage across the other resistor would then be 4.2 V, which is used to find the current in the circuit.

$$I = \frac{V_2}{R_2} = \frac{4.2\text{ V}}{15{,}000\ \Omega} = 2.8\times10^{-4}\text{ A}$$

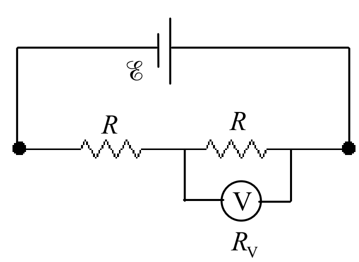
This current is used with the voltmeter reading to find the equivalent resistance of the meter–resistor combination, from which the voltmeter resistance can be found.

$$R_{\text{comb}} = \frac{V_{\text{comb}}}{I} = \frac{3.8\text{ V}}{2.8\times10^{-4}\text{ A}} = 13{,}570\ \Omega$$

$$\frac{1}{R_{\text{comb}}} = \frac{1}{R_1} + \frac{1}{R_{\text{meter}}} \rightarrow \frac{1}{R_{\text{meter}}} = \frac{1}{R_{\text{comb}}} - \frac{1}{R_1} \rightarrow$$

$$R_{\text{meter}} = \frac{R_1R_{\text{comb}}}{R_1 - R_{\text{comb}}} = \frac{(15{,}000\ \Omega)(13{,}570\ \Omega)}{15{,}000\ \Omega - 13{,}570\ \Omega} = 142{,}300\ \Omega \approx \boxed{140\text{ k}\Omega}$$

65. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0-volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.



$$R_{\mathrm{p}} = \left(\frac{1}{R} + \frac{1}{R_{\mathrm{V}}}\right)^{-1} = \frac{R_{\mathrm{V}}R}{R_{\mathrm{V}} + R} = \frac{(3000\ \Omega)(9400\ \Omega)}{3000\ \Omega + 9400\ \Omega} = 2274\ \Omega$$

$$R_{\mathrm{eq}} = R + R_{\mathrm{p}} = 2274\ \Omega + 9400\ \Omega = 11{,}674\ \Omega$$

Using the meter reading of 1.9 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the emf of the battery.

$$I = \frac{V}{R_{\mathrm{p}}} = \frac{1.9\ \mathrm{V}}{2274\ \Omega} = 8.355\times10^{-4}\ \mathrm{A}$$

$$\mathscr{E} = IR_{\mathrm{eq}} = (8.355\times10^{-4}\ \mathrm{A})(11{,}674\ \Omega) = 9.754\ \mathrm{V} \approx \boxed{9.8\ \mathrm{V}}$$

66. Because the voltmeter is called "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

$$\text{Case 1: } V_{\mathrm{meter}} = V_1 = I_1R_1 \quad \mathscr{E} - I_1r - I_1R_1 = 0 \quad \to \quad \mathscr{E} = I_1(r + R_1) = \frac{V_1}{R_1}(r + R_1)$$

$$\text{Case 2: } V_{\mathrm{meter}} = V_2 = I_2R_2 \quad \mathscr{E} - I_2r - I_2R_2 = 0 \quad \to \quad \mathscr{E} = I_2(r + R_2) = \frac{V_2}{R_2}(r + R_2)$$

Solve these two equations for the two unknowns of $\mathscr{E}$ and r.

$$\mathscr{E} = \frac{V_1}{R_1}(r + R_1) = \frac{V_2}{R_2}(r + R_2) \quad \to$$

$$r = R_1R_2\left(\frac{V_2 - V_1}{V_1R_2 - V_2R_1}\right) = (35\ \Omega)(14.0\ \Omega)\left(\frac{8.1\ \mathrm{V} - 9.7\ \mathrm{V}}{(9.7\ \mathrm{V})(14.0\ \Omega) - (8.1\ \mathrm{V})(35\ \Omega)}\right) = 5.308\ \Omega \approx \boxed{5.3\ \Omega}$$

$$\mathscr{E} = \frac{V_1}{R_1}(r + R_1) = \frac{9.7\ \mathrm{V}}{35\ \Omega}(5.308\ \Omega + 35\ \Omega) = 11.17\ \mathrm{V} \approx \boxed{11.2\ \mathrm{V}}$$

67. We can use a voltage divider circuit, as discussed in Example 19–3b and illustrated in Fig. 19–6b. The current to be passing through the body is given by Ohm's law.

$$I = \frac{V_{\mathrm{body}}}{R_{\mathrm{body}}} = \frac{0.25\ \mathrm{V}}{1800\ \Omega} = 1.389\times10^{-4}\ \mathrm{A}$$

This is the current in the entire circuit. Use this to find the resistor to put in series with the body.

$$V_{\mathrm{battery}} = I(R_{\mathrm{body}} + R_{\mathrm{series}}) \quad \to$$

$$R_{\mathrm{series}} = \frac{V_{\mathrm{battery}}}{I} - R_{\mathrm{body}} = \frac{1.5\ \mathrm{V}}{1.389\times10^{-4}\ \mathrm{A}} - 1800\ \Omega = 8999\ \Omega = \boxed{9.0\ \mathrm{k}\Omega}$$

68. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50-W output, use the higher-resistance filament}}$. For the $\boxed{\text{100-W output, use the lower-resistance filament}}$. For the $\boxed{\text{150-W output, use the filaments in parallel}}$.

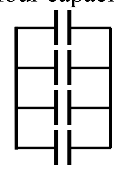
(*b*) $P = V^2/R \;\rightarrow\; R = \dfrac{V^2}{P} \quad R_1 = \dfrac{(120\text{ V})^2}{50\text{ W}} = 288\ \Omega \approx \boxed{290\ \Omega} \quad R_2 = \dfrac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega \approx \boxed{140\ \Omega}$

As a check, the parallel combination of the resistors gives the following.

$$R_{\text{p}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(288\ \Omega)(144\ \Omega)}{288\ \Omega + 144\ \Omega} = 96\ \Omega \quad P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{96\ \Omega} = 150\text{ W}$$

69. There are nine values of effective capacitance that can be obtained from the four capacitors.

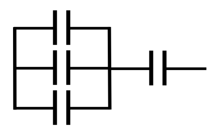
All four in parallel: $C_{\text{eq}} = C + C + C + C = \boxed{4C}$



All four in series: $C_{\text{eq}} = \left(\dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C}\right)^{-1} = \boxed{\tfrac{1}{4}C}$

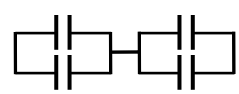
(Three in parallel) in series with one:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C + C + C} + \frac{1}{C} = \frac{1}{3C} + \frac{1}{C} = \frac{4}{3C} \;\rightarrow\; C_{\text{eq}} = \boxed{\tfrac{3}{4}C}$$



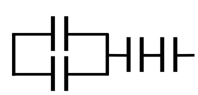
(Two in parallel) in series with (two in parallel):

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C + C} + \frac{1}{C + C} = \frac{1}{2C} + \frac{1}{2C} = \frac{2}{2C} \;\rightarrow\; C_{\text{eq}} = \boxed{C}$$



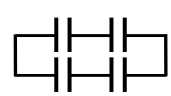
(Two in parallel) in series with (two in series):

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C + C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{2C} + \frac{2}{C} = \frac{5}{2C} \;\rightarrow\; C_{\text{eq}} = \boxed{\tfrac{2}{5}C}$$



(Two in series) in parallel with (two in series):

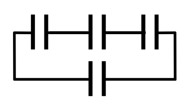
$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \left(\frac{2}{C}\right)^{-1} + \left(\frac{2}{C}\right)^{-1} = \frac{C}{2} + \frac{C}{2} = C$$



(not a new value)

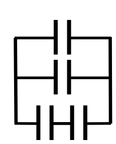
(Three in series) in parallel with one:

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} + C = \left(\frac{3}{C}\right)^{-1} + C = \boxed{\tfrac{4}{3}C}$$

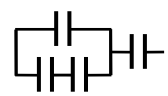


(Two in series) in parallel with (two in parallel):

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} + C + C = \left(\frac{2}{C}\right)^{-1} + 2C = \boxed{\tfrac{5}{2}C}$$



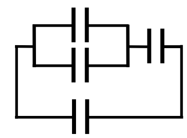
((Two in series) in parallel with one) in series with one:



$$\frac{1}{C_{\text{eq}}}=\left[\left(\frac{1}{C}+\frac{1}{C}\right)^{-1}+C\right]^{-1}+\frac{1}{C}=\left[\frac{C}{2}+C\right]^{-1}+\frac{1}{C}$$

$$=\frac{2}{3C}+\frac{1}{C}=\frac{5}{3C}\quad\rightarrow\quad C_{\text{eq}}=\boxed{\tfrac{3}{5}C}$$

((Two in parallel) in series with one) in parallel with one:



$$C_{\text{eq}}=\left(\frac{1}{C}+\frac{1}{2C}\right)^{-1}+C=C+\frac{2C}{3}=\boxed{\tfrac{5}{3}C}$$

70. The equivalent resistance is the sum of all the resistances. The current is found from Ohm's law.

$$I=\frac{\mathcal{E}}{R_{\text{eq}}}=\frac{240\text{ V}}{3\times10^{4}\ \Omega}=\boxed{8\times10^{-3}\text{ A}}$$

This is 8 milliamps, and 10 milliamps is considered to be a dangerous level in that it can cause sustained muscular contraction. The 8 milliamps could certainly be felt by the patient and could be painful.

$\boxed{71.}$ The capacitor will charge up to 75% of its maximum value and then discharge. The charging time is the time for one heartbeat.

$$t_{\text{beat}}=\frac{1\text{ min}}{72\text{ beats}}\times\frac{60\text{ s}}{1\text{ min}}=0.8333\text{ s}$$

$$V=V_0\left(1-e^{-\frac{t}{RC}}\right)\ \rightarrow\ 0.75\,V_0=V_0\left(1-e^{-\frac{t_{\text{beat}}}{RC}}\right)\ \rightarrow\ e^{-\frac{t_{\text{beat}}}{RC}}=0.25\ \rightarrow\ \left(-\frac{t_{\text{beat}}}{RC}\right)=\ln(0.25)\ \rightarrow$$

$$R=-\frac{t_{\text{beat}}}{C\ln(0.25)}=-\frac{0.8333\text{ s}}{(6.5\times10^{-6}\text{ F})(-1.3863)}=\boxed{9.2\times10^{4}\ \Omega}$$

72. (*a*) Apply Ohm's law to find the current.

$$I=\frac{V_{\text{body}}}{R_{\text{body}}}=\frac{120\text{ V}}{950\ \Omega}=0.126\text{ A}\approx\boxed{0.13\text{ A}}$$

(*b*) The description of "alternative path to ground" is a statement that the 25-Ω path is in parallel with the body. Thus the full 110 V is still applied across the body, so the current is the same: $\boxed{0.13\text{ A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will be divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}}=V_{\text{alternate}}\ \rightarrow\ I_{\text{body}}R_{\text{body}}=I_{\text{alternate}}R_{\text{alternate}}=(I_{\text{total}}-I_{\text{body}})R_{\text{alternate}}\ \rightarrow$$

$$I_{\text{body}}=I_{\text{total}}\frac{R_{\text{alternate}}}{(R_{\text{body}}+R_{\text{alternate}})}=(1.5\text{ A})\frac{25\ \Omega}{950\ \Omega+25\ \Omega}=0.03846\text{ A}\approx\boxed{38\text{ mA}}$$

This is still a very dangerous current.

73. The original resistance of the motor is found from Ohm's law.

$$V = IR \quad \rightarrow \quad R_{\text{original}} = \frac{V}{I} = \frac{12\text{ V}}{5.0\text{ A}} = 2.4\ \Omega$$

(*a*) Now another resistor is put in series to reduce the current to 2.0 A. We assume the battery voltage has not changed.

$$V = IR \quad \rightarrow \quad R_{\text{original}} + R_{\text{series}} = \frac{V}{I} = \frac{12\text{ V}}{2.0\text{ A}} = 6.0\ \Omega \quad \rightarrow \quad R_{\text{series}} = 6.0\ \Omega - 2.4\ \Omega = \boxed{3.6\ \Omega}$$

(*b*) $P = I^2R = (2.0\text{ A})^2(3.6\ \Omega) = 14.4\text{ W} \approx \boxed{14\text{ W}}$

74. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}} = V_{\text{DA}} \quad \rightarrow \quad I_3R_3 = I_1R_1 \quad \rightarrow \quad \frac{R_3}{R_1} = \frac{I_1}{I_3}$$

$$V_{\text{CB}} = V_{\text{CD}} \quad \rightarrow \quad I_3R_x = I_1R_2 \quad \rightarrow \quad R_x = R_2\frac{I_1}{I_3} = \boxed{R_2R_3/R_1}$$

(*b*) $R_x = R_2\dfrac{R_3}{R_1} = (972\ \Omega)\left(\dfrac{78.6\ \Omega}{590\ \Omega}\right) = 129\ \Omega \approx \boxed{130\ \Omega}$

75. Divide the power by the required voltage to determine the current drawn by the hearing aid.

$$I = \frac{P}{V} = \frac{2.5\text{ W}}{4.0\text{ V}} = 0.625\text{ A}$$

Use Eq. 19–1 to calculate the terminal voltage across the series combinations of three batteries for both mercury (Hg) and dry (D) cells.

$$V_{\text{Hg}} = 3(\mathscr{E} - Ir) = 3[1.35\text{ V} - (0.625\text{ A})(0.030\ \Omega)] = 3.99\text{ V}$$
$$V_{\text{D}} = 3(\mathscr{E} - Ir) = 3[1.50\text{ V} - (0.625\text{ A})(0.35\ \Omega)] = 3.84\text{ V}$$

The terminal voltage of the mercury cell batteries is closer to the required 4.0 V than the voltage from the dry cell.

76. One way is to connect *N* resistors in series. If each resistor can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{\text{eq}}}{7} = \frac{3200\ \Omega}{7} = 457\ \Omega \approx 460\ \Omega$$

So connect $\boxed{\text{7 resistors of } 460\ \Omega \text{ each, rated at ½ W, in series}}$. Each resistor would have to have a voltage of about 15 V in order to not exceed the power rating. Thus the total voltage would be limited to about 105 V.

Alternately, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{\text{eq}}} = 7\left(\frac{1}{R}\right) \quad \rightarrow \quad R = 7R_{\text{eq}} = 7(3200\ \Omega) = 22.4\ \text{k}\Omega$$

So connect $\boxed{\text{7 resistors of } 22.4\ \text{k}\Omega \text{ each, rated at ½ W, in parallel}}$. Each resistor could have about 105 V across it in this configuration.

77. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\ \text{V}}{0.80\ \text{V/cell}} = 150 \text{ cells}$ are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add, making a total of $\frac{1.3\ \text{A}}{350\times10^{-3}\ \text{A/bank}} = 3.71 \text{ banks} \approx 4 \text{ banks}.$ So the total number of cells is $\boxed{600 \text{ cells}}$. The panel area is $600 \text{ cells } (9.0\times10^{-4}\ \text{m}^2/\text{cell}) = \boxed{0.54\ \text{m}^2}$. The cells should be wired in $\boxed{\text{4 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.4 A at 120 V.

78. There are two answers because it is not known which direction the given current is flowing through the 4.0-kΩ resistor. Assume the current is to the right. The voltage across the 4.0-kΩ resistor is given by Ohm's law as $V = IR = (3.10\times10^{-3}\ \text{A})(4000\ \Omega) = 12.4\ \text{V}.$ The voltage drop across the $8.0\ \text{k}\Omega$ must be the same, so the current through it is $I = \frac{V}{R} = \frac{12.4\ \text{V}}{8000\ \Omega} = 1.55\times10^{-3}\ \text{A}.$ The total current in the circuit is the sum of the two currents: $I_{\text{tot}} = 4.65\times10^{-3}\ \text{A}.$ That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{\text{ba}} - (3200\ \Omega)I_{\text{tot}} - 12.4\ \text{V} - 12.0\ \text{V} - (1.0\ \Omega)I_{\text{tot}} \quad \rightarrow$$
$$V_{\text{ba}} = 24.4\text{V} + (3201\ \Omega)(4.65\times10^{-3}\ \text{A}) = 39.28\ \text{V} \approx \boxed{39\ \text{V}}$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 12.4 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$V_{\text{ab}} + (3200\ \Omega)I_{\text{tot}} + 12.4\ \text{V} - 12.0\ \text{V} + (1.0\ \Omega)I_{\text{tot}} \quad \rightarrow$$
$$V_{\text{ab}} = -0.4\text{V} - (3201\ \Omega)(4.65\times10^{-3}\ \text{A}) = -15.28\ \text{V} \approx \boxed{-15\ \text{V}}$$

79. (*a*) If the terminal voltage is to be 3.5 V, then the voltage across R_1 will be 8.5 V. This can be used to find the current, which then can be used to find the value of R_2.

$$V_1 = IR_1 \quad \rightarrow \quad I = \frac{V_1}{R_1} \quad V_2 = IR_2 \quad \rightarrow$$
$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (14.5\ \Omega)\frac{3.5\ \text{V}}{8.5\ \text{V}} = 5.971\ \Omega \approx \boxed{6.0\ \Omega}$$

(*b*) If the load has a resistance of $7.0\ \Omega$, then the parallel combination of R_2 and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2 R_{\text{load}}}{R_2 + R_{\text{load}}} = \frac{(5.971\ \Omega)(7.0\ \Omega)}{(5.971\ \Omega + 7.0\ \Omega)} = 3.222\ \Omega \quad R_{\text{eq}} = 3.222\ \Omega + 14.5\ \Omega = 17.722\ \Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12.0\ \text{V}}{17.722\ \Omega} = 0.6771\ \text{A} \quad V_{\text{T}} = IR_{2+\text{load}} = (0.6771\ \text{A})(3.222\ \Omega) = 2.182\ \text{V} \approx \boxed{2.2\ \text{V}}$$

The presence of the load has affected the terminal voltage significantly.

80. The terminal voltage and current are given for two situations. Apply Eq. 19–1 to both of these situations and solve the resulting two equations for the two unknowns.

$$V_1 = \mathscr{E} - I_1 r;\ V_2 = \mathscr{E} - I_2 r \quad \rightarrow \quad \mathscr{E} = V_1 + I_1 r = V_2 + I_2 r \quad \rightarrow$$

$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\ \text{V} - 40.8\ \text{V}}{8.40\ \text{A} - 2.80\ \text{A}} = 1.161\ \Omega \approx \boxed{1.2\ \Omega}$$

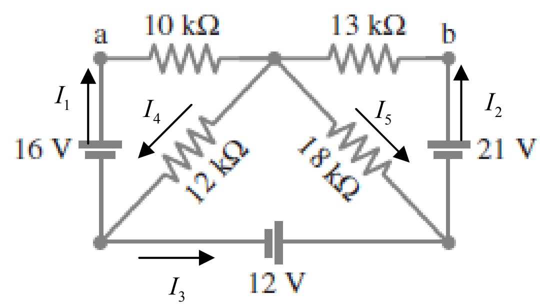
$$\mathscr{E} = V_1 + I_1 r = 40.8\ \text{V} + (8.40\ \text{A})(1.161\ \Omega) = \boxed{50.6\ \text{V}}$$

81. The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2 R \quad \rightarrow \quad I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.80\ \text{W}}{33\ \Omega}} = 0.1557\ \text{A}$$

$$R_{\text{eq}} = 33\ \Omega + \left(\frac{1}{68\ \Omega} + \frac{1}{85\ \Omega}\right)^{-1} = 70.78\ \Omega \quad V = IR_{\text{eq}} = (0.1557\ \text{A})(70.78\ \Omega) = 11.02\ \text{V} \approx \boxed{11\ \text{V}}$$

82. There are three loops, so there are three loop equations necessary to solve the circuit. We identify five currents in the diagram, so we need two junction equations to complete the analysis. We also assume that the $10\text{-k}\Omega$ resistor is known to 2 significant digits, like the other resistors. Finally, note that if we use the resistances in "kilo-ohms," the currents will be in "milli-amps."



Lower left junction: $I_4 = I_1 + I_3$

Lower right junction: $I_2 = I_3 + I_5$

Left loop, clockwise: $16\ \text{V} - 10I_1 - 12I_4 = 0$

Right loop, counterclockwise: $21\ \text{V} - 13I_2 - 18I_5 = 0$

Bottom loop, counterclockwise: $12\ \text{V} + 18I_5 - 12I_4 = 0$

$$16-10I_1-12(I_1+I_3)=0 \;\rightarrow\; 16-22I_1-12I_3=0 \;\rightarrow\; I_1=\frac{16-12I_3}{22}$$

$$21-13(I_3+I_5)-18I_5=0 \;\rightarrow\; 21-13I_3-31I_5=0 \;\rightarrow\; I_5=\frac{21-13I_3}{31}$$

$$12+18I_5-12(I_1+I_3)=0 \;\rightarrow\; 12-12I_1-12I_3+18I_5=0$$

Substitute the results of the first two equations into the third equation.

$$12-12\left[\frac{16-12I_3}{22}\right]-12I_3+18\left[\frac{21-13I_3}{31}\right]=0$$

$$12-\tfrac{12}{22}(16-12I_3)-12I_3+\tfrac{18}{31}(21-13I_3)=0$$

$$12-\tfrac{6}{11}16+\tfrac{18}{31}21=(12+\tfrac{18}{31}13-\tfrac{6}{11}12)I_3$$

$$15.466=13.003I_3 \;\rightarrow\; I_3=\frac{15.466}{13.003}=1.189\text{ mA}$$

(a) $I_1=\dfrac{16-12I_3}{22}=\dfrac{16-12(1.189)}{22}=0.0787\text{ mA}\approx\boxed{7.9\times10^{-5}\text{ A, upward}}$

(b) Reference all voltages to the lower left corner of the circuit diagram. Then $V_a=16\text{ V}$, $V_b=12\text{ V}+21\text{ V}=33\text{ V}$, and $V_a-V_b=16\text{ V}-33\text{ V}=\boxed{-17\text{ V}}$.

83. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\text{ V}-I(50\,\Omega+20\,\Omega+10\,\Omega)=0 \;\rightarrow\; I=6.0\text{ V}/80\,\Omega=0.075\text{ A}$$

If the switches are both closed, then the $20\text{-}\Omega$ resistor is in parallel with *R*. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the $20\text{-}\Omega$ resistor having the current found previously.

$$6.0\text{ V}-I(50\,\Omega)-(0.075\text{ A})(20\,\Omega)=0 \;\rightarrow\; I=\frac{6.0\text{ V}-(0.075\text{ A})(20\,\Omega)}{50\,\Omega}=0.090\text{ A}$$

This is the current in the parallel combination. Since 0.075 A is in the $20\text{-}\Omega$ resistor, 0.015 A must be in *R*. The voltage drops across *R* and the $20\text{-}\Omega$ resistor are the same since they are in parallel.

$$V_{20}=V_R \;\rightarrow\; I_{20}R_{20}=I_RR \;\rightarrow\; R=R_{20}\frac{I_{20}}{I_R}=(20\,\Omega)\frac{0.075\text{ A}}{0.015\text{ A}}=\boxed{100\,\Omega}$$

84. (*a*) The $12\text{-}\Omega$ and the $22\text{-}\Omega$ resistors are in parallel, with a net resistance R_{1-2} as follows.

$$R_{1-2}=\left(\frac{1}{12\,\Omega}+\frac{1}{22\,\Omega}\right)^{-1}=7.765\,\Omega$$

R_{1-2} is in series with the $4.5\text{-}\Omega$ resistor, for a net resistance R_{1-2-3} as follows.

$$R_{1-2-3}=4.5\,\Omega+7.765\,\Omega=12.265\,\Omega$$

That net resistance is in parallel with the $14\text{-}\Omega$ resistor, for a final equivalent resistance as follows.

$$R_{\text{eq}} = \left(\frac{1}{12.265\ \Omega} + \frac{1}{14\ \Omega}\right)^{-1} = 6.538\ \Omega \approx \boxed{6.5\ \Omega}$$

(*b*) Find the current in the $14\text{-}\Omega$ resistor by using Kirchhoff's loop rule for the loop containing the battery and the $14\text{-}\Omega$ resistor.

$$\mathcal{E} - I_{14}R_{14} = 0 \quad \rightarrow \quad I_{18} = \frac{\mathcal{E}}{R_{18}} = \frac{6.0\ \text{V}}{14\ \Omega} = 0.4286\ \text{A} \approx \boxed{0.43\ \text{A}}$$

(*c*) Find the current in R_{1-2} and the $4.5\text{-}\Omega$ resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors R_{1-2} and the $4.5\text{-}\Omega$ resistor. Those resistances are in series.

$$\mathcal{E} - I_{1-2}R_{1-2} - I_{1-2}R_{4.5} = 0 \quad \rightarrow \quad I_{1-2} = \frac{\mathcal{E}}{R_{1-2} + R_{4.5}} = \frac{6.0\ \text{V}}{7.765\ \Omega + 4.5\ \Omega} = 0.4892\ \text{A}$$

This current divides to go through the $12\text{-}\Omega$ and $22\text{-}\Omega$ resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the $12\text{-}\Omega$ resistor.

$$I_{1-2} = I_{12} + I_{25} \quad \rightarrow \quad I_{25} = I_{1-2} - I_{12}$$
$$V_{R_{12}} = V_{R_{25}} \quad \rightarrow \quad I_{12}R_{12} = I_{25}R_{25} = (I_{1-2} - I_{12})R_{25} \quad \rightarrow$$
$$I_{12} = I_{1-2}\frac{R_{25}}{(R_{12} + R_{25})} = (0.4892\ \text{A})\frac{22\ \Omega}{34\ \Omega} = 0.3165\ \text{A} \approx \boxed{0.32\ \text{A}}$$

(*d*) The current in the $4.5\text{-}\Omega$ resistor was found above to be $I_{1-2} = 0.4892\ \text{A}.$ Find the power accordingly.

$$P_{4.5} = I_{1-2}^2 R_{4.5} = (0.4892\ \text{A})^2(4.5\ \Omega) = 1.077\ \text{W} \approx \boxed{1.1\ \text{W}}$$

85. (*a*) We assume that the ammeter is ideal so has 0 resistance but that the voltmeter has resistance R_{V}. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, so it is reading the voltage across the battery.

$$V = IR_{\text{eq}} = I\frac{1}{\frac{1}{R} + \frac{1}{R_{\text{V}}}} \quad \rightarrow \quad V\left(\frac{1}{R} + \frac{1}{R_{\text{V}}}\right) = I \quad \rightarrow \quad \frac{1}{R} + \frac{1}{R_{\text{V}}} = \frac{I}{V} \quad \rightarrow \quad \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_{\text{V}}}}$$

(*b*) We now assume the voltmeter is ideal, so has an infinite resistance but that the ammeter has resistance R_{A}. We also assume that the voltmeter is accurate and is reading the voltage across the battery.

$$V = IR_{\text{eq}} = I(R + R_{\text{A}}) \quad \rightarrow \quad R + R_{\text{A}} = \frac{V}{I} \quad \rightarrow \quad \boxed{R = \frac{V}{I} - R_{\text{A}}}$$

86. (*a*) The light will first flash when the voltage across the capacitor reaches 90.0 V.

$$V = \mathscr{E}_0\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow$$

$$t = -RC\ln\left(1 - \frac{V}{\mathscr{E}_0}\right) = -(2.35\times10^6\ \Omega)(0.150\times10^{-6}\ \text{F})\ln\left(1 - \frac{90}{105}\right) = \boxed{0.686\ \text{s}}$$

(*b*) We see from the equation that $t \propto R$, so if R increases, then the $\boxed{\text{time will increase}}$.

(*c*) The capacitor discharges through a very low resistance (the lamp filled with ionized gas), so the discharge time constant is very short. Thus the flash is very brief.

(*d*) Once the lamp has flashed, the stored energy in the capacitor is gone, and there is no source of charge to maintain the lamp current. The lamp "goes out," the lamp resistance increases, and the capacitor starts to recharge. It charges again for 0.686 second, and the process will repeat.

87. We find the current for the bulb from the bulb's ratings. Also, since the bulb will have 3.0 V across it, the resistor R will have 6.0 V across it.

$$P_{\text{bulb}} = I_{\text{bulb}}V_{\text{bulb}} \rightarrow I_{\text{bulb}} = \frac{P_{\text{bulb}}}{V_{\text{bulb}}} = \frac{2.0\ \text{W}}{3.0\ \text{V}} = 0.667\ \text{A};\ R = \frac{V_R}{I_R} = \frac{6.0\ \text{V}}{0.667\ \text{A}} = \boxed{9.0\ \Omega}$$

88. (*a*) As shown, the equivalent capacitance is found from adding the capacitance of C_1 to the series combination of C_1 and C_1. That net capacitance is used to calculate the stored energy.

$$C_{\text{net}} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = 25.4\ \mu\text{F} + \left(\frac{2}{25.4\ \mu\text{F}}\right)^{-1} = 38.1\ \mu\text{F}$$

$$\text{PE} = \tfrac{1}{2}C_{\text{net}}V^2 = \tfrac{1}{2}(38.1\times10^{-6}\ \text{F})(10.0\ \text{V})^2 = 1.905\times10^{-3}\ \text{J} \approx \boxed{1.91\times10^{-3}\ \text{J}}$$

(*b*) If all the capacitors were in series, then the equivalent capacitance is 1/3 the value of one of the capacitors.

$$C_{\text{net}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{3}{25.4\ \mu\text{F}}\right)^{-1} = 8.467\ \mu\text{F}$$

$$\text{PE} = \tfrac{1}{2}C_{\text{net}}V^2 = \tfrac{1}{2}(8.467\times10^{-6}\ \text{F})(10.0\ \text{V})^2 \approx \boxed{4.24\times10^{-4}\ \text{J}}$$

(*c*) If all the capacitors were in parallel, then the equivalent capacitance is 3 times the value of one of the capacitors.

$$C_{\text{net}} = C_1 + C_2 + C_3 = 3(25.4\ \mu\text{F}) = 76.2\ \mu\text{F}$$

$$\text{PE} = \tfrac{1}{2}C_{\text{net}}V^2 = \tfrac{1}{2}(76.2\times10^{-6}\ \text{F})(10.0\ \text{V})^2 \approx \boxed{3.81\times10^{-4}\ \text{J}}$$

89. After a long time, the only current is in the resistors. The voltage across the $3.3\text{-k}\Omega$ resistor is found using the left loop.

$$12.0\text{V} - I(4600\ \Omega) = 0 \rightarrow I = \frac{12.0\text{V}}{4600\ \Omega} = 2.609\times10^{-3}\ \Omega \rightarrow$$

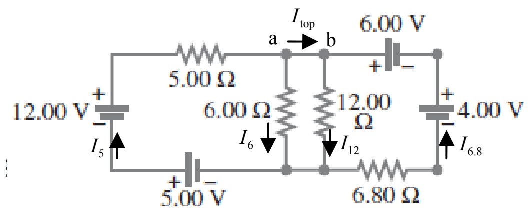
$$V_{3.3\ \text{k}\Omega} = IR = (2.609\times10^{-3}\ \Omega)(3300\ \Omega) = 8.609\ \text{V}$$

The voltage across the $3.3\ \text{k}\Omega$ is the voltage across each capacitor.

$$Q_{12\ \mu\text{F}} = CV = (12\times10^{-6}\ \text{F})(8.609\ \text{V}) = 1.033\times10^{-4}\ \text{C} \approx \boxed{1.0\times10^{-4}\ \text{C}}$$

$$Q_{48\ \mu\text{F}} = CV = (48\times10^{-6}\ \text{F})(8.609\ \text{V}) = 4.132\times10^{-4}\ \text{C} \approx \boxed{4.1\times10^{-4}\ \text{C}}$$

90. We have labeled currents through the resistors with the value of the specific resistance and the emfs with the appropriate voltage value. We apply the junction rule to points a and b and then apply the loop rule to the left loop, the middle loop, and the right loop. This enables us to solve for all of the currents.



$$I_5 = I_6 + I_{top};\quad I_{top} + I_{6.8} = I_{12} \quad\rightarrow\quad I_5 - I_6 = I_{12} - I_{6.8} \quad\rightarrow\quad I_5 + I_{6.8} = I_{12} + I_6 \quad [1]$$

$$17\ \text{V} - I_5(5.00\ \Omega) - I_6(6.00\ \Omega) = 0 \qquad [2]\ \text{(left loop)}$$

$$10\ \text{V} - I_{12}(12.00\ \Omega) - I_{6.8}(6.80\ \Omega) = 0 \qquad [3]\ \text{(right loop)}$$

$$I_{12}(12.00\ \Omega) - I_6(6.00\ \Omega) = 0 \qquad [4]\ \text{(center loop)}$$

Solve Eq. [4] to get $I_6 = 2I_{12}$. Substitute that into the other equations.

$$I_5 + I_{6.8} = 3I_{12} \quad [1]$$

$$17\ \text{V} - I_5(5.00\ \Omega) - 2I_{12}(6.00\ \Omega) = 0 \quad [2]$$

$$10\ \text{V} - I_{12}(12.00\ \Omega) - I_{6.8}(6.80\ \Omega) = 0 \quad [3]$$

Use Eq. [1] to eliminate $I_{6.8}$ by $I_{6.8} = 3I_{12} - I_5$.

$$17\ \text{V} - I_5(5.00\ \Omega) - I_{12}(12.00\ \Omega) = 0 \quad [2]$$

$$10\ \text{V} - I_{12}(12.00\ \Omega) - (3I_{12} - I_5)(6.80\ \Omega) = 0 \quad\rightarrow\quad 10\ \text{V} - I_{12}(32.40\ \Omega) + I_5(6.80\ \Omega) = 0 \quad [3]$$

Use Eq. [2] to eliminate I_5 by $I_5 = \dfrac{17\ \text{V} - I_{12}(12.00\ \Omega)}{(5.00\ \Omega)}$, and then substitute into Eq. [3] and solve for solve for I_{12}. Then back-substitute to find the other currents.

$$10\ \text{V} - I_{12}(32.40\ \Omega) + \left[\frac{17\ \text{V} - I_{12}(12.00\ \Omega)}{(5.00\ \Omega)}\right](6.80\ \Omega) = 0 \quad\rightarrow$$

$$10\ \text{V}(5.00\ \Omega) - I_{12}(32.40\ \Omega)(5.00\ \Omega) + [17\ \text{V} - I_{12}(12.00\ \Omega)](6.80\ \Omega) = 0 \quad\rightarrow$$

$$I_{12} = \frac{10\ \text{V}(5.00\ \Omega) + 17\ \text{V}(6.80\ \Omega)}{(12.00\ \Omega)(6.80\ \Omega) + (32.40\ \Omega)(5.00\ \Omega)} = 0.6798\ \text{A} \quad\rightarrow\quad I_{12} = \boxed{0.680\ \text{A}}$$

$$I_5 = \frac{17\ \text{V} - I_{12}(12.00\ \Omega)}{(5.00\ \Omega)} = \frac{17\ \text{V} - (0.6798\ \text{A})(12.00\ \Omega)}{(5.00\ \Omega)} = 1.768\ \text{A} \quad\rightarrow\quad I_5 = \boxed{1.77\ \text{A}}$$

$$I_{6.8} = 3I_{12} - I_5 = 3(0.6798\ \text{A}) - 1.768\ \text{A} = 0.2714\ \text{A} \quad\rightarrow\quad I_{6.8} = \boxed{0.271\ \text{A}}$$

$$I_6 = 2I_{12} = 2(0.6798\ \text{A}) = 1.3596\ \text{A} \quad\rightarrow\quad I_6 = \boxed{1.36\ \text{A}}$$

91. (*a*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19–5. The stored energy is given by Eq. 17–10.

$$C_{\text{eq}} = C_1 + C_2 = 0.65\ \mu\text{F};\ \text{PE} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(0.65\times10^{-6}\ \text{F})(24\ \text{V})^2 = \boxed{1.9\times10^{-4}\ \text{J}}$$

(*b*) When the capacitors are connected in parallel, their equivalent capacitance is found from Eq. 19–6. The stored energy is again given by Eq. 17–10.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad\rightarrow\quad C_{\text{eq}} = \frac{C_1C_2}{C_1+C_2} = \frac{(0.45\ \mu\text{F})(0.20\ \mu\text{F})}{0.65\ \mu\text{F}} = 0.1385\ \mu\text{F}$$

$$\text{PE} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(0.1385\times10^{-6}\ \text{F})(24\ \text{V})^2 = \boxed{4.0\times10^{-5}\ \text{J}}$$

(*c*) The charge is found from Eq. 17–7, $Q = CV$.

$$Q_{\text{parallel}} = (C_{\text{eq}})_{\text{parallel}}V = (0.65\ \mu\text{F})(24\ \text{V}) = \boxed{16\ \mu\text{C}}$$

$$Q_{\text{series}} = (C_{\text{eq}})_{\text{series}}V = (0.1385\ \mu\text{F})(45\ \text{V}) = \boxed{3.3\ \mu\text{C}}$$

92. Because $Q = CV$ and the capacitors both have the same voltage applied, it will be the case that $Q_1 > Q_2$. Because of conservation of charge, when the capacitors are hooked up, the total charge that is available is $Q_1 - Q_2$. That amount of charge redistributes over the capacitor plates, and once equilibrium has been reached, the total charge on the plates is given by $q_1 + q_2 = Q_1 - Q_2$. The other constraint on this system is that once equilibrium has been reached, the capacitors must have the same voltage, so $\dfrac{q_1}{C_1} = \dfrac{q_2}{C_2}$.

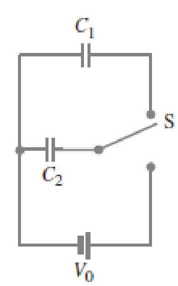
$$Q_1 = C_1V = (2.2\ \mu\text{F})(24\ \text{V}) = 52.8\ \mu\text{F};\ Q_2 = C_2V = (1.2\ \mu\text{F})(24\ \text{V}) = 28.8\ \mu\text{F}$$

$$Q_1 - Q_2 = 52.8\ \mu\text{F} - 28.8\ \mu\text{F} = 24.0\ \mu\text{F} = q_1 + q_2 \quad\rightarrow\quad q_2 = 24.0\ \mu\text{F} - q_1$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{24.0\ \mu\text{F} - q_1}{C_2} \quad\rightarrow\quad q_1 = \frac{24.0\ \mu\text{F}}{\left(1+\dfrac{C_2}{C_1}\right)} = \frac{24.0\ \mu\text{F}}{\left(1+\dfrac{1.2\ \mu\text{F}}{2.2\ \mu\text{F}}\right)} = 15.5\ \mu\text{C}$$

$$\boxed{q_1 = 15.5\ \mu\text{C},\quad q_2 = 8.5\ \mu\text{C}}$$

93. When the switch is down, the initial charge on C_2 is calculated from Eq. 17–7.



$$Q_2 = C_2V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \quad\rightarrow\quad Q_2' = Q_1'\frac{C_2}{C_1}$$

The sum of the charges on the two capacitors is equal to the initial charge on C_2.

$$Q_2 = Q_2' + Q_1' = Q_1'\frac{C_2}{C_1} + Q_1' = Q_1'\left(\frac{C_1 + C_2}{C_1}\right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1'\left(\frac{C_1 + C_2}{C_1}\right) = C_2 V_0 \quad \rightarrow \quad Q_1' = \boxed{\frac{C_1 C_2}{C_1 + C_2}V_0};\ Q_2' = Q_1'\frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_1 + C_2}V_0}$$

94. The power delivered to the starter is equal to the square of the current in the circuit multiplied by the resistance of the starter. Since the resistors in each circuit are in series, we calculate the currents as the battery emf divided by the sum of the resistances.

$$\frac{P}{P_0} = \frac{I^2 R_S}{I_0^2 R_S} = \left(\frac{I}{I_0}\right)^2 = \left(\frac{\mathscr{E}/R_{eq}}{\mathscr{E}/R_{0eq}}\right)^2 = \left(\frac{R_{0eq}}{R_{eq}}\right)^2 = \left(\frac{r + R_S}{r + R_S + R_C}\right)^2$$
$$= \left(\frac{0.02\ \Omega + 0.15\ \Omega}{0.02\ \Omega + 0.15\ \Omega + 0.10\ \Omega}\right)^2 = \boxed{0.40}$$

95. (*a*) From the diagram, we see that one group of four plates is connected together, and the other group of four plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$. Each capacitor would have the same voltage across it.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{eq} = 7C = 7\varepsilon_0\frac{A}{d}$$
$$C_{min} = 7\varepsilon_0\frac{A_{min}}{d} = 7(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\frac{(2.0\times10^{-4}\ \text{m}^2)}{(1.6\times10^{-3}\ \text{m})} = 7.7\times10^{-12}\ \text{F}$$
$$C_{max} = 7\varepsilon_0\frac{A_{max}}{d} = 7(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\frac{(9.0\times10^{-4}\ \text{m}^2)}{(1.6\times10^{-3}\ \text{m})} = 3.5\times10^{-11}\ \text{F}$$

So the range is $\boxed{\text{from 7.7 pF to 35 pF}}$.

96. Capacitors in series each store the same amount of charge, so the capacitance on the unknown capacitor is also 125 pC. We calculate the voltage across each capacitor, and then calculate the unknown capacitance.

$$V_{175} = \frac{Q_{175}}{C_{175}} = \frac{125\times10^{-12}\ \text{C}}{175\times10^{-12}\ \text{F}} = 0.714\ \text{V}$$
$$V_{unknown} = 25.0\ \text{V} - V_{185} = 25.0\ \text{V} - 0.714\ \text{V} = 24.286\ \text{V}$$
$$C_{unknown} = \frac{Q}{V} = \frac{125\times10^{-12}\ \text{C}}{24.286\ \text{V}} = \boxed{5.15\times10^{-12}\ \text{C}}$$

Thus the voltage across the unknown capacitor is $25\text{ V}-V_{185}=25\text{ V}-0.676\text{ V}=24.324\text{ V}$. The capacitance can be calculated from the voltage across and charge on that capacitor.

$$C=\frac{Q}{V}=\frac{125\times10^{-12}\text{ C}}{24.324\text{ V}}=\boxed{5.14\text{ pF}}$$

97. The first capacitor is charged and has charge $Q_0=C_1V_0$ on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, so the final charge is equal to the initial charge. Initially treat capacitors C_2 and C_3 as their equivalent capacitance,

$C_{23}=\dfrac{C_2C_3}{C_2+C_3}=\dfrac{(2.0\ \mu\text{F})(2.4\ \mu\text{F})}{4.4\ \mu\text{F}}=1.091\ \mu\text{F}$. The final voltage across C_1 and C_{23} must be the same. The charge on C_2 and C_3 must be the same. Use Eq. 17–7.

$$Q_0=C_1V_0=Q_1+Q_{23}=C_1V_1+C_{23}V_{23}=C_1V_1+C_{23}V_1 \quad\rightarrow$$
$$V_1=\frac{C_1}{C_1+C_{23}}V_0=\frac{1.0\ \mu\text{F}}{1.0\ \mu\text{F}+1.091\ \mu\text{F}}(24\text{ V})=11.48\text{ V}=V_1=V_{23}$$

$$Q_1=C_1V_1=(1.0\ \mu\text{F})(11.48\text{ V})=11.48\ \mu\text{C}$$
$$Q_{23}=C_{23}V_{23}=(1.091\ \mu\text{F})(11.48\text{ V})=12.52\ \mu\text{C}=Q_2=Q_3$$
$$V_2=\frac{Q_2}{C_2}=\frac{12.52\ \mu\text{C}}{2.0\ \mu\text{F}}=6.26\text{ V};\quad V_3=\frac{Q_3}{C_3}=\frac{12.52\ \mu\text{C}}{2.4\ \mu\text{F}}=5.22\text{ V}$$

To summarize: $\boxed{Q_1=11\ \mu\text{C}, V_1=11\text{ V};\quad Q_2=13\ \mu\text{C}, V_2=6.3\text{ V};\quad Q_3=13\ \mu\text{C}, V_3=5.2\text{ V}}$.

Solutions to Search and Learn Problems

1.

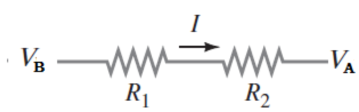
	R_{eq}	C_{eq}
Series	$R_{eq}=R_1+R_2+\cdots$	$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots$
Parallel	$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots$	$C_{eq}=C_1+C_2+\cdots$

When resistors are connected in *series*, the equivalent resistance is the *sum* of the individual resistances, $R_{\underset{\text{series}}{\text{eq}}}=R_1+R_2+\cdots$. The current has to go through each additional resistance if the resistors are in series and therefore the equivalent resistance is greater than any individual resistance. In contrast, when capacitors are in *parallel*, the equivalent capacitance is equal to the sum of the individual capacitors, $C_{\underset{\text{parallel}}{\text{eq}}}=C_1+C_2+\cdots$. Charge drawn from the battery can go down any one of the different branches and land on any one of the capacitors, so the overall capacitance is greater than that of each individual capacitor.

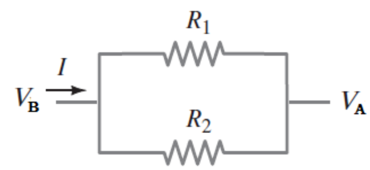
When resistors are connected in *parallel*, the current from the battery or other source divides into the different branches so the equivalent resistance is less than any individual resistor in the circuit. The corresponding expression is $\frac{1}{R_{\text{eq}}}_{\text{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots.$ The formula for the equivalent capacitance of capacitors in *series* follows this same form, $\frac{1}{C_{\text{eq}}}_{\text{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots.$ When capacitors are in series, the overall capacitance is less than the capacitance of any individual capacitor. Charge leaving the first capacitor lands on the second rather than going straight to the battery, so on.

Compare the expressions defining resistance $(R = V/I)$ and capacitance $(C = Q/V)$. Resistance is proportional to voltage, whereas capacitance is inversely proportional to voltage.

2.



Series



Parallel

Property	Resistors in Series	Resistors in Parallel
Equivalent resistance	$R_1 + R_2$	$R_1 R_2/(R_1 + R_2)$
Current through equivalent resistance	$(V_B - V_A)/(R_1 + R_2)$	$(V_B - V_A)(R_1 + R_2)/(R_1 R_2)$
Voltage across equivalent resistance	$V_B - V_A$	$V_B - V_A$
Voltage across the pair of resistors	$V_B - V_A$	$V_B - V_A$
Voltage across each resistor	$V_1 = (V_B - V_A)R_1/(R_1 + R_2)$ $V_2 = (V_B - V_A)R_2/(R_1 + R_2)$	$V_1 = V_B - V_A$ $V_2 = V_B - V_A$
Voltage at a point between the resistors	$(V_A R_2 + V_B R_1)/(R_1 + R_2)$	Not applicable
Current through each resistor	$I_1 = (V_B - V_A)/(R_1 + R_2)$ $I_2 = (V_B - V_A)/(R_1 + R_2)$	$I_1 = (V_B - V_A)/R_1$ $I_2 = (V_B - V_A)/R_2$

3. (*a*) Use Eq. 19–7a to solve for the unknown resistance, where the ratio $V_C/\mathscr{E} = 0.95$.

$$0.95 = 1 - e^{-t/RC} \quad \rightarrow \quad R = -\frac{t}{C\ln(1-0.95)} = -\frac{2.0\text{ s}}{(1.0\times10^{-6}\text{ F})\ln(0.05)} = \boxed{6.7\times10^{5}\ \Omega}$$

(*b*) The defibrillator paddles are usually moistened so that the resistance of the skin is about 1000 Ω. Use Eq. 19–7c to solve for the capacitance.

$$\tau = RC \quad \rightarrow \quad C = \frac{\tau}{R} = \frac{10\times10^{-3}\text{ s}}{1000\ \Omega} = \boxed{10\times10^{-6}\text{ F}}$$

4. When the galvanometer gives a null reading, no current is passing through the galvanometer or the emf that is being measured. All of the current is flowing through the slide wire resistance. Application of the loop rule to the lower loop gives $\mathscr{E} - IR = 0,$ since there is no current through the emf to cause a voltage drop across any internal resistance in the emf or any resistance in the galvanometer. The

current flowing through the slide wire resistor is independent of the emf in the lower loop because no current is flowing through the lower loop. We solve the loop rule equation for the current and set the currents equal. The resulting equation is solved for the unknown emf.

$$\mathscr{E}_x - IR_x = 0; \quad \mathscr{E}_\text{s} - IR_\text{s} = 0 \quad \rightarrow \quad I = \frac{\mathscr{E}_x}{R_x} = \frac{\mathscr{E}_\text{s}}{R_\text{s}} \quad \rightarrow \quad \boxed{\mathscr{E}_x = \left(\frac{R_x}{R_\text{s}}\right)\mathscr{E}_\text{s}}$$

5. Note that, based on the significant figures of the resistors, that the $1.0\text{-}\Omega$ resistor will not change the equivalent resistance of the circuit as determined by the resistors in the switch bank.

Case 1: $n = 0$ switch closed. The effective resistance of the circuit is $16.0\ \text{k}\Omega$. The current in the circuit is $I = \dfrac{16\ \text{V}}{16.0\ \text{k}\Omega} = 1.0\ \text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (1.0\ \text{mA})(1.0\ \Omega) = \boxed{1.0\ \text{mV}}$.

Case 2: $n = 1$ switch closed. The effective resistance of the circuit is $8.0\ \text{k}\Omega$. The current in the circuit is $I = \dfrac{16\ \text{V}}{8.0\ \text{k}\Omega} = 2.0\ \text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (2.0\ \text{mA})(1.0\ \Omega) = \boxed{2.0\ \text{mV}}$.

Case 3: $n = 2$ switch closed. The effective resistance of the circuit is $4.0\ \text{k}\Omega$. The current in the circuit is $I = \dfrac{16\ \text{V}}{4.0\ \text{k}\Omega} = 4.0\ \text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (4.0\ \text{mA})(1.0\ \Omega) = \boxed{4.0\ \text{mV}}$.

Case 4: $n = 3$ and $n = 0$ switches closed. The effective resistance of the circuit is found by the parallel combination of the $2.0\text{-k}\Omega$ and $16.0\text{-k}\Omega$ resistors.

$$R_\text{eq} = \left(\frac{1}{2.0\ \text{k}\Omega} + \frac{1}{16.0\ \text{k}\Omega}\right)^{-1} = 1.778\ \text{k}\Omega$$

The current in the circuit is $I = \dfrac{16\ \text{V}}{1.778\ \text{k}\Omega} = 8.999\ \text{mA} \approx 9.0\ \text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR = (9.0\ \text{mA})(1.0\ \Omega) = \boxed{9.0\ \text{mV}}$.

So in each case, the voltage across the $1.0\text{-}\Omega$ resistor, if taken in mV, is the expected analog value corresponding to the digital number set by the switches.

20

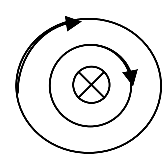
MAGNETISM

Responses to Questions

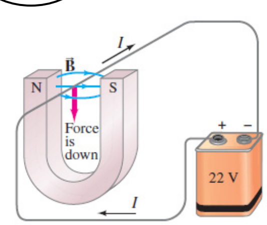
1. The Earth's magnetic field is not always parallel to the surface of the Earth—it may have a component perpendicular to the Earth's surface. The compass will tend to line up with the local direction of the magnetic field, so one end of the compass will dip downward. The angle that the Earth's magnetic field makes with the horizontal is called the dip angle.

2. The pole on a magnetic compass needle that points geographically northward is defined at the north pole of the compass. This north pole is magnetically attracted to the south pole of other magnets, so the Earth's magnetic field must have a south pole at the geographic north pole.

3. The magnetic field lines form clockwise circles centered on the wire.



4. The force is downward. The field lines point from the north pole to the south pole, or left to right. Use the right-hand rule. Your fingers point in the direction of the current (away from you). Curl them in the direction of the field (to the right). Your thumb points in the direction of the force (downward). See Fig. 20–11a, copied here.



(a)

5. A magnet will not attract just any metallic object. For example, while a magnet will attract paper clips and nails, it will not attract coins or pieces of aluminum foil. This is because magnets will only attract other ferromagnetic materials (iron, cobalt, nickel, gadolinium, and some of their oxides and alloys). Iron and its alloys (such as steel) are the only common materials. These ferromagnetic materials contain magnetic domains that can be made to temporarily align when a strong magnet is brought near. The alignment occurs in such a way that the north pole of the domain points toward the south pole of the strong magnet, and vice versa, which creates the attraction.

6. No, they are not both magnets. If they were both magnets, then they would repel one another when they were placed with like poles facing each other. However, if one is a magnet and the other isn't, then they will attract each other no matter which ends are placed together. The magnet will cause an alignment of the domains of the nonmagnet, causing an attraction.

7. Typical current in a house circuit is 60 Hz AC. Due to the mass of the compass needle, its reaction to 60 Hz (changing direction back and forth at 60 complete cycles per second) will probably not be noticeable. A DC current in a single wire could affect a compass, depending on the relative orientation of the wire and the compass, the magnitude of the current, and the distance from the wire to the compass. A DC current being carried by two very close wires in opposite directions would not have much of an effect on the compass needle, since the two currents would cause magnetic fields that tended to cancel each other.

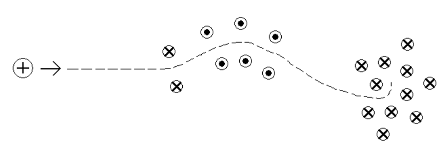
8. The magnetic force will be exactly perpendicular to the velocity, which means that the force is perpendicular to the direction of motion. Since there is no component of force in the direction of motion, the work done by the magnetic force will be zero, and the kinetic energy of the particle will not change. The particle will change direction, but not change speed.

9. Use the right-hand rule to determine the direction of the force on each particle. In the plane of the diagram, the magnetic field is coming out of the page for points above the wire and is going into the page for points below the wire.
   a: force down, toward the wire
   b: force to the left, opposite of the direction of the current
   c: force up, toward the wire
   d: force to the left, opposite of the direction of the current

$\boxed{10.}$ Charge a experiences an initial upward force. By the right-hand rule, charge a must be positive.
Charge b experiences no force, so charge b must be uncharged.
Charge c experiences an initial downward force. By the right-hand rule, charge c must be negative.

11. A magnet can attract an iron bar, so by Newton's third law, an iron bar can attract a magnet. Another consideration is that the iron has domains that can be made slightly magnetic by an external magnetic field, as opposed to a substance like plastic or wood. Thus the iron is also a magnet, at least when it is close to the magnet.

12. The right-hand rule tells us that as the positive particle moves to the right, if it is deflected upward, then the magnetic field must be into the page, and if it is deflected downward, then the magnetic field must be out of the page. Also, the stronger the magnetic field, the more tightly the charged particle will turn. The magnetic field at the first bend must be relatively small and pointed into the page. The magnetic field at the second bend must be medium in size and pointed out of the page. The magnetic field at the third bend must be relatively very large and pointed into the page. In terms of the indicated letters:
   a: no field
   b: relatively weak field, into the page
   c: moderate strength field, out of the page
   d: no field
   e: very strong field, into the page



13. Section 17–11 shows that a CRT television picture is created by shooting a beam of electrons at a fluorescent screen. Wherever the electrons hit the screen, the screen glows momentarily. When you hold a strong magnet too close to the screen, the magnetic field puts a force on the moving electrons $(F = qvB \sin\theta)$, which causes them to bend away from their original destination position on the screen. These errant electrons then cause parts of the screen to light up that aren't supposed to be lit and cause other parts to be slightly darker than they are supposed to be. The picture sometimes goes completely black where the field is strongest because at these points all of the electrons that were supposed to hit that particular spot have been deflected by the strong field.

14. Put one end of one rod close to one end of another rod. The ends will either attract or repel. Continue trying all combinations of rods and ends until two ends repel each other. Then the two rods used in that case are the magnets.

15. No, you cannot set a resting electron into motion with a magnetic field (no matter how big the field is). A magnetic field can only put a force on a *moving* charge. Thus, with no force (which means no acceleration), the velocity of the electron will not change—it will remain at rest. However, you can set a resting electron into motion with an electric field. An electric field will put a force on *any* charged particle, moving or not. Thus, the electric force can cause the electron to accelerate from rest to a higher speed.

16. The particle will move in an elongating helical path in the direction of the electric field (for a positive charge). The radius of the helix will remain constant, but the pitch (see Problem 79) will increase, because the particle will accelerate along the electric field lines.

17. Yes. One possible situation is that the magnetic field is parallel or antiparallel to the velocity of the charged particle. In this case, the magnetic force would be zero, and the particle would continue moving in a straight line. Another possible situation is that there is an electric field with a magnitude and direction (perpendicular to the magnetic field) such that the electric and magnetic forces on the particle cancel each other out. The net force would be zero and the particle would continue moving in a straight line.

18. No. A moving charged particle can be deflected sideways with an electric field that points perpendicular to the direction of the velocity, even when the magnetic field in the region is zero.

19. Consider that the two wires are both horizontal, with the lower one carrying a current pointing east and with the upper one carrying a current pointing north. The magnetic field from the upper wire points downward on the east half of the lower wire and points upward on the west half of the lower wire. Using the right-hand rule on the east half of the lower wire, where the magnetic field is downward, the force points to the north. Using the right-hand rule on the west half of the lower wire, where the magnetic field is upward, the force points to the south. Thus, the lower wire experiences a counterclockwise torque about the vertical direction due to the magnetic forces from the upper wire. This torque is attempting to rotate the two wires so that their currents are parallel. Likewise, the upper wire would experience a clockwise torque tending to align the currents in the wires.

20. (*a*) The current in the lower wire is pointing in the opposite direction of the current in the upper wire. If the current in the lower wire is toward the north, then it creates a magnetic field pointing east at the upper wire. Using the right-hand rule, the current of the upper wire must be pointing toward the south so that the magnetic force created on it is upward.

    (*b*) No, the lower wire is not stable. If the lower wire falls away from the upper wire, then the upward magnetic force on it will weaken, and there will be a net downward force on the wire. It would continue to fall away. Also, if the lower wire were to be moved toward the upper wire, the upward magnetic force on it would increase, and the lower wire would accelerate upward.

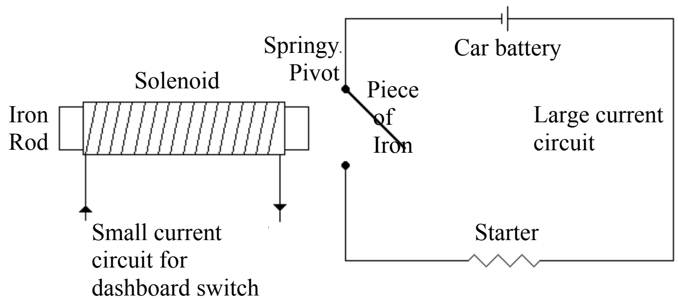
21. The equation for the magnetic field strength inside a solenoid is given by $B = \dfrac{\mu_0 NI}{\ell}$, Eq. 20–8.

    (*a*) The magnetic field strength is not affected if the diameter of the loops doubles. The equation shows that the magnetic field is independent of the diameter of the solenoid.

    (*b*) If the spacing between the loops doubles, then the length of the solenoid would increase by a factor of 2, so the magnetic field strength would also decrease by a factor of 2.

    (*c*) If the solenoid's length is doubled along with the doubling of the total number of loops, then the number of loops per unit length remains the same, and the magnetic field strength is not affected.

22. To design a relay, place the iron rod inside of a solenoid and then point one end of the solenoid/rod combination at the piece of iron on a pivot. A spring normally holds the piece of iron away from a switch, making an open circuit where current cannot flow. When the relay is activated with a small current, a relatively strong magnetic field is created inside the solenoid, which aligns most of the magnetic domains in the iron rod and produces a strong magnetic field at the end of the solenoid/rod combination. This magnetic field attracts the piece of iron on the springy pivot, which causes it to move toward the switch, connecting it and allowing current to flow through the large current circuit.



23. The two ions will come out of the velocity selector portion of the mass spectrometer at the same speed, since $\upsilon = E/B$. Once the charges reach the area of the mass spectrometer where there is only a magnetic field, the difference in the ions' charges will have an effect. Eq. 20–12 for the mass spectrometer says that $m = qBB'r/E$, which can be rearranged as $r = mE/qBB'$. Since every quantity in the equation is constant except q, the doubly ionized ion will hit the film at a half of the radius as the singly ionized ion.

24. The magnetic domains in the unmagnetized piece of iron are initially pointing in random directions, as in Fig. 20–42a (which is why it appears to be unmagnetized). When the south pole of a strong external magnet is brought close to the random magnetic domains of the iron, many of the domains will rotate slightly so that their north poles are closer to the external south pole, which causes the unmagnetized iron to be attracted to the magnet. Similarly, when the north pole of a strong external magnet is brought close to the random magnetic domains of the iron, many of the domains will rotate slightly so that their south poles are closer to the external north pole, which causes the unmagnetized iron to now be attracted to the magnet. Thus, either pole of a magnet will attract an unmagnetized piece of iron.

$\boxed{25.}$ Initially, both the nail and the paper clip have all of their magnetic domains pointing in random directions (which is why they appear to be unmagnetized). Thus, when you bring them close to each other, they are not attracted to or repelled from each other. Once the nail is in contact with a magnet (let's say the north pole), many of the nail's domains will align in such a way that the end of the nail that is touching the magnet becomes a south pole, due to the strong attraction, and the opposite end of the nail then becomes a north pole. Now, when you bring the nail close to the paper clip, there are mainly north poles of nail domains close to the paper clip, which causes some of the domains in the paper clip to align in such a way that the end near the nail becomes a south pole. Since the nail's domains are only partially aligned, it will not be a strong magnet and thus the alignment of the paper clip's domains will be even weaker. The attraction of the paper clip to the nail will be weaker than the attraction of the nail to the magnet.

Responsess to MisConceptual Questions

1. (*a*, *b*, *d*, *e*) A common misconception is that only permanent magnets (such as a magnet and the Earth) create magnetic fields. However, moving charges and electric currents also produce magnetic fields. Stationary charges, ordinary pieces of iron, and other pieces of metal do not create magnetic fields.

2. (*c*) It is common to confuse the directions of the magnetic fields with electric fields and thus indicate that the magnetic field points toward or away from the current. Since the current is flowing into the page, the right-hand rule indicates that the magnetic field is in the clockwise direction around the current. At point A the field points downward.

3. (*b*) The right-hand rule indicates that the magnetic field is clockwise around the current, so at point B the current is to the left.

4. (*a*) The charged particle only experiences a force when it has a component of velocity perpendicular to the magnetic field. When it moves parallel to the field, it follows a straight line at constant speed.

5. (*c*) Electric fields are created by charged objects whether the charges are moving or not. Magnetic fields are created by moving charged objects. Since the proton is charged and moving, it creates both an electric field and a magnetic field.

6. (*a*) A stationary charged particle does not experience a force in a magnetic field. Therefore, the particle must be moving to experience a force. The force is a maximum when the particle is moving perpendicular to the field, not parallel to the field. Since the force is perpendicular to the motion of the particle, it acts as a centripetal force, changing the particle's direction but not its kinetic energy. That is, since the force is perpendicular to the motion, it does no work on the particle. The direction of the force is always perpendicular to the direction of motion and also perpendicular to the magnetic field.

7. (*c*) Section 20–5 shows that the magnetic field from a current is directed circularly around the wire, is proportional to the current flowing in the wire, and is inversely proportional to the distance from the wire. A constant current produces a magnetic field, so the current does not need to be changing.

8. (*e*) It is common to confuse the direction of electric fields (which point toward or away from the charges) with magnetic fields, which always make circles around the current.

9. (*c*) A common misconception is that a force always does work on the object. Since the magnetic force is perpendicular to the velocity of the proton, the force acts as a centripetal force, changing the proton's direction, but not doing any work and thus not changing its kinetic energy.

10. (*a*) A common misconception is that a constant magnetic field can change the magnitude of the particle's velocity. However, the magnetic force is always perpendicular to the velocity, so it can do no work on the particle. The magnetic force only serves as a centripetal force to change the particle's direction.

11. (*e*) Equation 20–3 shows that the magnetic force depends upon the particle's charge, its velocity, and the strength of the external magnetic field. The direction of the force is always perpendicular to the magnetic field and the velocity of the particle. Therefore, all four statements are accurate.

12. (*c*) This question requires a consideration of Newton's third law. The force that one wire exerts on a second must be equal in magnitude, but opposite in direction, to the force that the second exerts on the first.

Solutions to Problems

1. (*a*) Use Eq. 20–1 to calculate the force with an angle of $90°$ and a length of 1 meter.

$$F = I\ell B \sin\theta \quad \rightarrow \quad F/\ell = IB\sin\theta = (6.40\ \text{A})(0.90\ \text{T})\sin 90° = \boxed{5.8\ \text{N/m}}$$

(*b*) Change the angle to $35.0°$.

$$F/\ell = IB\sin\theta = (6.40\ \text{A})(0.90\ \text{T})\sin 35.0° = \boxed{3.3\ \text{N/m}}$$

2. Use Eq. 20–2.

$$F_{\text{max}} = I\ell B \quad \rightarrow \quad I = \frac{F_{\text{max}}}{\ell B} = \frac{0.625\ \text{N}}{(4.80\ \text{m})(8.00\times10^{-2}\ \text{T})} = \boxed{1.63\ \text{A}}$$

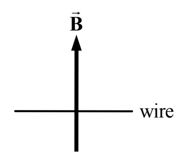
3. Use Eq. 20–1 to calculate the force.

$$F = I\ell B\sin\theta = (120\ \text{A})(240\ \text{m})(5.0\times10^{-5}\ \text{T})\sin 68° = \boxed{1.3\ \text{N}}$$

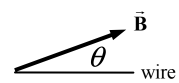
4. The dip angle is the angle between the Earth's magnetic field and the current in the wire. Use Eq. 20–1 to calculate the force.

$$F = I\ell B\sin\theta = (4.5\ \text{A})(2.6\ \text{m})(5.5\times10^{-5}\ \text{T})\sin 41° = \boxed{4.2\times10^{-4}\ \text{N}}$$

5. To have the maximum force, the current must be perpendicular to the magnetic field, as shown in the first diagram. Use $\frac{F}{\ell} = 0.45\frac{F_{\text{max}}}{\ell}$ to find the angle between the wire and the magnetic field, illustrated in the second diagram. Use Eq. 20–1.



$$\frac{F}{\ell} = 0.45\frac{F_{\text{max}}}{\ell} \quad \rightarrow \quad IB\sin\theta = 0.45IB \quad \rightarrow \quad \theta = \sin^{-1} 0.45 = \boxed{27°}$$



6. Use Eq. 20–2. The length of wire in the B field is the same as the diameter of the pole faces.

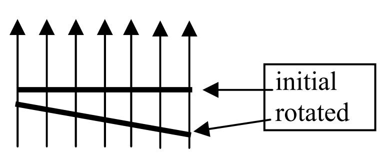
$$F_{\text{max}} = I\ell B \quad \rightarrow \quad B = \frac{F_{\text{max}}}{I\ell} = \frac{1.28\ \text{N}}{(6.45\ \text{A})(0.555\ \text{m})} = \boxed{0.358\ \text{T}}$$

7. (*a*) By the right-hand rule, the magnetic field must be pointing up, so the top pole face must be a south pole.

(*b*) Use Eq. 20–2 to relate the maximum force to the current. The length of wire in the magnetic field is equal to the diameter of the pole faces.

$$F_{\text{max}} = I\ell B \quad \rightarrow \quad I = \frac{F_{\text{max}}}{\ell B} = \frac{(8.50\times10^{-2}\ \text{N})}{(0.100\ \text{m})(0.220\ \text{T})} = 3.864\ \text{A} \approx \boxed{3.86\ \text{A}}$$

(*c*) If the wire is tipped so that it points $10.0°$ downward, then the angle between the wire and the magnetic field is changed to $100.0°$. But the length of wire now in the field has increased from ℓ to $\ell/\cos 10.0°$. The net effect of the changes is that the force has not changed.



$$F_{\text{initial}} = I\ell B; \quad F_{\text{rotated}} = I(\ell/\cos 10.0°)B\sin 100.0° = I\ell B = F_{\text{initial}} = \boxed{8.50\times10^{-2}\ \text{N}}$$

$\boxed{8.}$ The magnetic force must be equal in magnitude to the force of gravity on the wire. The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field. The mass of the wire is the density of copper times the volume of the wire.

$$F_B = mg \quad \to \quad I\ell B = \rho\pi\left(\tfrac{1}{2}d\right)^2 \ell g \quad \to$$

$$I = \frac{\rho\pi d^2 g}{4B} = \frac{(8.9\times10^3\ \text{kg/m}^3)\pi(1.00\times10^{-3}\ \text{m})^2(9.80\ \text{m/s}^2)}{4(5.0\times10^{-5}\ \text{T})} = \boxed{1400\ \text{A}}$$

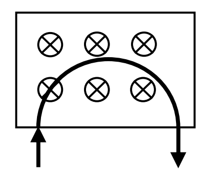
This answer does not seem feasible. The current is very large, and the resistive heating in the thin copper wire would probably melt it.

9. The maximum magnetic force as given in Eq. 20–4 can be used since the velocity is perpendicular to the magnetic field.

$$F_{\text{max}} = qvB = (1.60\times10^{-19}\ \text{C})(7.75\times10^5\ \text{m/s})(0.45\ \text{T}) = \boxed{5.6\times10^{-14}\ \text{N}}$$

By the right-hand rule, the force must be directed to the $\boxed{\text{north}}$.

10. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.



$$F_{\text{max}} = qvB = m\frac{v^2}{r} \quad \to$$

$$r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\ \text{kg})(1.70\times10^6\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.640\ \text{T})} = \boxed{1.51\times10^{-5}\ \text{m}}$$

Assuming the magnetic field existed in a sharply defined region, the electron would make only one-half of a rotation and then exit the field going in the opposite direction from which it came.

11. In this scenario, the magnetic force is causing centripetal motion, so it must have the form of a centripetal force. The magnetic force is perpendicular to the velocity at all times for circular motion.

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \quad \to \quad B = \frac{mv}{qr} = \frac{(6.6\times10^{-27}\ \text{kg})(1.6\times10^6\ \text{m/s})}{2(1.60\times10^{-19}\ \text{C})(0.14\ \text{m})} = \boxed{0.24\ \text{T}}$$

12. Since the charge is negative, the answer is the OPPOSITE of the result given from the right-hand rule applied to the velocity and magnetic field.

(*a*) no force
(*b*) downward
(*c*) upward
(*d*) inward, into the page
(*e*) to the left
(*f*) to the left

13. The right-hand rule applied to the velocity and magnetic field would give the direction of the force. Use this to determine the direction of the magnetic field given the velocity and the force.

(*a*) to the right
(*b*) downward
(*c*) into the page

14. The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force.

$$F_{\mathrm{E}} = F_{\mathrm{B}} \rightarrow qE = q\upsilon B \rightarrow \upsilon = \frac{E}{B} = \frac{7.7\times10^{3}\ \mathrm{V/m}}{7.5\times10^{-3}\ \mathrm{T}} = 1.027\times10^{6}\ \mathrm{m/s} \approx \boxed{1.0\times10^{6}\ \mathrm{m/s}}$$

If the electric field is turned off, then the magnetic force will cause circular motion.

$$F_{\mathrm{B}} = q\upsilon B = m\frac{\upsilon^2}{r} \rightarrow r = \frac{m\upsilon}{qB} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(1.027\times10^{6}\ \mathrm{m/s})}{(1.60\times10^{-19}\ \mathrm{C})(7.5\times10^{-3}\ \mathrm{T})} = \boxed{7.8\times10^{-4}\ \mathrm{m}}$$

15. (*a*) The speed of the ion can be found using energy conservation. The electric potential energy of the ion becomes kinetic energy as it is accelerated.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow qV = \tfrac{1}{2}m\upsilon^2 \rightarrow$$

$$\upsilon = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2[2(1.60\times10^{-19}\ \mathrm{C})](3700\ \mathrm{V})}{(6.6\times10^{-27}\ \mathrm{kg})}} = 5.99\times10^{5}\ \mathrm{m/s} \approx \boxed{6.0\times10^{5}\ \mathrm{m/s}}$$

(*b*) Since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$F_{\max} = q\upsilon B = m\frac{\upsilon^2}{r} \rightarrow$$

$$r = \frac{m\upsilon}{qB} = \frac{(6.6\times10^{-27}\ \mathrm{kg})(5.99\times10^{5}\ \mathrm{m/s})}{2(1.60\times10^{-19}\ \mathrm{C})(0.340\ \mathrm{T})} = 3.63\times10^{-2}\ \mathrm{m} \approx \boxed{3.6\times10^{-2}\ \mathrm{m}}$$

(*c*) The period can be found from the speed and the radius.

$$T = \frac{2\pi r}{\upsilon} = \frac{2\pi(3.63\times10^{-2}\ \mathrm{m})}{5.99\times10^{5}\ \mathrm{m/s}} = \boxed{3.8\times10^{-7}\ \mathrm{s}}$$

16. (*a*) From Example 20–6, we have $r = \dfrac{m\upsilon}{qB}$, so $\upsilon = \dfrac{rqB}{m}$. The kinetic energy is given by

$$\mathrm{KE} = \tfrac{1}{2}m\upsilon^2 = \tfrac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m},\ \text{so we see that } \boxed{\mathrm{KE} \propto r^2}.$$

(b) The angular momentum of a particle moving in a circular path is $L = m\upsilon r$. From Example 20–6, we have $r = \dfrac{m\upsilon}{qB}$, so $\upsilon = \dfrac{rqB}{m}$. Combining these relationships gives the following.

$$L = m\upsilon r = m\frac{rqB}{m}r = \boxed{qBr^2}$$

$\boxed{17.}$ The kinetic energy of the proton can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined.

$$\mathrm{KE} = \tfrac{1}{2}m\upsilon^2 \rightarrow \upsilon = \sqrt{\frac{2\ \mathrm{KE}}{m}} \qquad q\upsilon B = \frac{m\upsilon^2}{r} \rightarrow$$

$$r = \frac{m\upsilon}{qB} = \frac{m\sqrt{\dfrac{2\ \mathrm{KE}}{m}}}{qB} = \frac{\sqrt{2\ \mathrm{KE}\ m}}{qB} = \frac{\sqrt{2(1.5\times10^{6}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})(1.67\times10^{-27}\ \mathrm{kg})}}{(1.60\times10^{-19}\ \mathrm{C})(0.30\ \mathrm{T})} = \boxed{0.59\ \mathrm{m}}$$

18. The magnetic field can be found from Eq. 20–4, and the direction is found from the right-hand rule. Remember that the charge is negative.

$$F_{\text{max}} = qvB \quad \rightarrow \quad B = \frac{F_{\text{max}}}{qv} = \frac{6.2\times10^{-13}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(2.8\times10^{6}\ \text{m/s})} = \boxed{1.4\ \text{T}}$$

The direction would have to be $\boxed{\text{east}}$ for the right-hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

19. The kinetic energy is used to determine the speed of the particles, and then the speed can be used to determine the radius of the circular path, since the magnetic force is causing centripetal acceleration.

$$\text{KE} = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2\ \text{KE}}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2\ \text{KE}}{m}}}{qB} = \frac{\sqrt{2m\ \text{KE}}}{qB}$$

$$\frac{r_{\text{p}}}{r_{\text{e}}} = \frac{\frac{\sqrt{2m_{\text{p}}\ \text{KE}}}{qB}}{\frac{\sqrt{2m_{\text{e}}\ \text{KE}}}{qB}} = \sqrt{\frac{m_{\text{p}}}{m_{\text{e}}}} = \sqrt{\frac{1.67\times10^{-27}\ \text{kg}}{9.11\times10^{-31}\ \text{kg}}} = \boxed{42.8}$$

20. The velocity of each charged particle can be found using energy conservation. The electrical potential energy of the particle becomes kinetic energy as it is accelerated. Then, since the particle is moving perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path, and the radius can be determined in terms of the mass and charge of the particle.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad qV = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{\text{max}} = qvB = m\frac{v^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_{\text{d}}}{r_{\text{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\text{d}}V}{q_{\text{d}}}}}{\frac{1}{B}\sqrt{\frac{2m_{\text{p}}V}{q_{\text{p}}}}} = \frac{\sqrt{\frac{m_{\text{d}}}{m_{\text{p}}}}}{\sqrt{\frac{q_{\text{d}}}{q_{\text{p}}}}} = \frac{\sqrt{2}}{\sqrt{1}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_{\text{d}} = \sqrt{2}r_{\text{p}}}$$

$$\frac{r_{\alpha}}{r_{\text{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\alpha}V}{q_{\alpha}}}}{\frac{1}{B}\sqrt{\frac{2m_{\text{p}}V}{q_{\text{p}}}}} = \frac{\sqrt{\frac{m_{\alpha}}{m_{\text{p}}}}}{\sqrt{\frac{q_{\alpha}}{q_{\text{p}}}}} = \frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_{\alpha} = \sqrt{2}r_{\text{p}}}$$

21. The magnetic force produces an acceleration perpendicular to the original motion. If that acceleration is small, then it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity v and the distance traveled ℓ. The starting speed in the perpendicular direction will be zero.

$$F_\perp = ma_\perp = qvB \quad\rightarrow\quad a_\perp = \frac{qvB}{m}$$

$$d_\perp = v_{0\perp}t + \tfrac{1}{2}a_\perp t^2 = \tfrac{1}{2}\frac{qvB}{m}\left(\frac{\ell}{v}\right)^2 = \frac{qB\ell^2}{2mv} = \frac{(18.5\times10^{-9}\ \text{C})(5.00\times10^{-5}\ \text{T})(1.50\times10^{3}\ \text{m})^2}{2(3.40\times10^{-3}\ \text{kg})(155\ \text{m/s})}$$

$$= \boxed{1.97\times10^{-6}\ \text{m}}$$

This small distance justifies the assumption of constant acceleration.

22. (*a*) The discussion of the Hall effect in the text states that the Hall emf is proportional to the magnetic field causing the effect. We can use this proportionality to determine the unknown resistance. Since the new magnetic field is oriented 90° to the surface, the full magnetic field will be used to create the Hall potential.

$$\frac{V_{\text{Hall}}'}{V_{\text{Hall}}} = \frac{B_\perp'}{B_\perp} \quad\rightarrow\quad B_\perp' = \frac{V_{\text{Hall}}'}{V_{\text{Hall}}}B_\perp = \frac{63\ \text{mV}}{12\ \text{mV}}(0.10\ \text{T}) = \boxed{0.53\ \text{T}}$$

(*b*) When the field is oriented at 60° to the surface, the magnetic field, $B\sin 60°$, is used to create the Hall potential.

$$B_\perp' \sin 60° = \frac{V_{\text{Hall}}'}{V_{\text{Hall}}}B_\perp \quad\rightarrow\quad B_\perp' = \frac{63\ \text{mV}}{12\ \text{mV}}\frac{(0.10\ \text{T})}{\sin 60°} = \boxed{0.61\ \text{T}}$$

23. (*a*) The sign of the ions will not change the magnitude of the Hall emf but will determine the polarity of the emf.

(*b*) The flow velocity corresponds to the drift velocity in the Hall effect relationship.

$$V_{\text{Hall}} = vBd \quad\rightarrow\quad v = \frac{V_{\text{Hall}}}{Bd} = \frac{(0.13\times10^{-3}\ \text{V})}{(0.070\ \text{T})(0.0033\ \text{m})} = \boxed{0.56\ \text{m/s}}$$

24. (*a*) The drift velocity is found from $V_{\text{Hall}} = v_{\text{d}}Bd$.

$$v_{\text{d}} = \frac{V_{\text{Hall}}}{Bd} = \frac{(1.02\times10^{-6}\ \text{V})}{(1.2\ \text{T})(0.018\ \text{m})} = 4.722\times10^{-5}\ \text{m/s} \approx \boxed{4.7\times10^{-5}\ \text{m/s}}$$

(*b*) We now find the density by using Eq. 18–10.

$$n = \frac{I}{eAv_d} = \frac{15\ \text{A}}{(1.6\times10^{-19}\ \text{C})(1.0\times10^{-3}\ \text{m})(0.018\ \text{m})(4.722\times10^{-5}\ \text{m/s})}$$

$$= \boxed{1.1\times10^{29}\ \text{electrons/m}^3}$$

25. We assume that the jumper cable is a long straight wire and use Eq. 20–6.

$$B_{\text{cable}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(65\ \text{A})}{2\pi(4.5\times10^{-2}\ \text{m})} = 2.889\times10^{-4}\ \text{T} \approx \boxed{2.9\times10^{-4}\ \text{T}}$$

Compare this with the Earth's field of 0.5×10^{-4} T.

$$B_{\text{cable}}/B_{\text{Earth}} = \frac{2.889\times10^{-4}\ \text{T}}{5.0\times10^{-5}\ \text{T}} = 5.77,\ \text{so}\ \boxed{\text{the field of the cable is about 5.8 times that of the Earth.}}$$

26. We assume that the wire is long and straight and use Eq. 20–6.

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} \quad \rightarrow \quad I = \frac{2\pi r B_{\text{wire}}}{\mu_0} = \frac{2\pi(0.12\text{ m})(0.50\times10^{-4}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = \boxed{30\text{ A}} \text{ (2 significant figures)}$$

27. Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 20–7 to calculate the magnitude of the force.

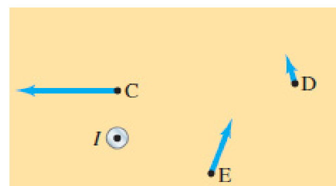
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}{2\pi}\frac{(25\text{ A})^2}{(0.040\text{ m})}(25\text{ m}) = \boxed{7.8\times10^{-2}\text{ N, attractive}}$$

28. Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 20–7 to calculate the magnitude of the second current.

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 \quad \rightarrow$$

$$I_2 = \frac{2\pi}{\mu_0}\frac{F_2}{\ell_2}\frac{d}{I_1} = \frac{2\pi}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}(7.8\times10^{-4}\text{ N/m})\frac{0.090\text{ m}}{28\text{ A}} = 12.54\text{ A} \approx \boxed{13\text{ A upward}}$$

29. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a line perpendicular to the radius, directed so that the perpendicular line would be part of a counterclockwise circle. The relative magnitude is given by the length of the arrow. The farther a point is from the wire, the weaker the field.



$\boxed{30.}$ For the experiment to be accurate to $\pm3.0\%$, the magnetic field due to the current in the cable must be less than or equal to 3.0% of the Earth's magnetic field. Use Eq. 20–6 to calculate the magnetic field due to the current in the cable.

$$B_{\text{cable}} = \frac{\mu_0 I}{2\pi r} \le 0.030 B_{\text{Earth}} \quad \rightarrow \quad I \le \frac{2\pi r(0.030 B_{\text{Earth}})}{\mu_0} = \frac{2\pi(1.00\text{ m})(0.030)(0.50\times10^{-4}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 7.5\text{ A}$$

Thus the maximum allowable current is $\boxed{7.5\text{ A}}$.

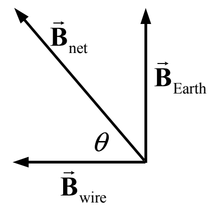
31. The magnetic field at the loop due to the long wire is into the page and can be calculated by Eq. 20–6. The force on the segment of the loop closest to the wire is toward the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is difficult to calculate the total force on either the left or right segments of the loop. Using the right-hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, then the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, so only the parallel segments need to be considered in the calculation. Use Eq. 20–7 to find the force.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} = \frac{\mu_0}{2\pi}I_1 I_2 \ell\left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}}\right)$$

$$= \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}(3.5\text{ A})^2(0.100\text{ m})\left(\frac{1}{0.030\text{ m}} - \frac{1}{0.080\text{ m}}\right) = \boxed{5.1\times10^{-6}\text{ N, toward wire}}$$

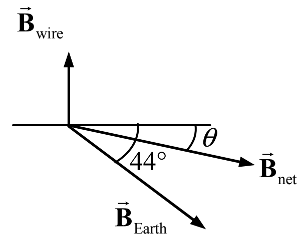
32. At the location of the compass, the magnetic field caused by the wire will point to the west, and the part of the Earth's magnetic field that turns the compass is pointing due north. The compass needle will point in the direction of the NET magnetic field.



$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(48\ \text{A})}{2\pi(0.18\ \text{m})} = 5.33\times10^{-5}\ \text{T}$$

$$\theta = \tan^{-1}\frac{B_{\text{Earth}}}{B_{\text{wire}}} = \tan^{-1}\frac{4.5\times10^{-5}\ \text{T}}{5.33\times10^{-5}\ \text{T}} = \boxed{40^\circ\text{N of W}}\ \text{(2 significant figures)}$$

33. The magnetic field due to the long horizontal wire points straight up at the point in question, and its magnitude is given by Eq. 20–6. The two fields are oriented as shown in the diagram. The net field is the vector sum of the two fields.



$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(24.0\ \text{A})}{2\pi(0.200\ \text{m})} = 2.40\times10^{-5}\ \text{T}$$

$$B_{\text{Earth}} = 5.0\times10^{-5}\ \text{T}$$

$$B_{\text{net}\,x} = B_{\text{Earth}}\cos 44^\circ = 3.60\times10^{-5}\ \text{T} \qquad B_{\text{net},\,y} = B_{\text{wire}} - B_{\text{Earth}}\sin 44^\circ = -1.07\times10^{-5}\ \text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{(3.60\times10^{-5}\ \text{T})^2 + (-1.07\times10^{-5}\ \text{T})^2} = \boxed{3.8\times10^{-5}\ \text{T}}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-1.07\times10^{-5}\ \text{T}}{3.60\times10^{-5}\ \text{T}} = \boxed{17^\circ\ \text{below the horizontal}}$$

34. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate by the charge of a proton. Then use Eq. 20–6 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.5\times10^{9}\ \text{protons/s})(1.60\times10^{-19}\ \text{C/proton})}{2\pi(1.5\ \text{m})} = \boxed{5.3\times10^{-17}\ \text{T}}$$

$\boxed{35.}$ (*a*) If the currents are in the same direction, then the magnetic fields at the midpoint between the two currents will oppose each other, so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(0.010\ \text{m})}(I - 25\ \text{A}) = \boxed{(2.0\times10^{-5}\ \text{T/A})(I - 25\ \text{A})}$$

(*b*) If the currents are in the opposite direction, then the magnetic fields at the midpoint between the two currents will reinforce each other, so their magnitudes should be added.

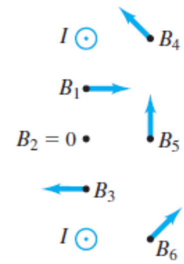
$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(0.010\ \text{m})}(I + 25\ \text{A}) = \boxed{(2.0\times10^{-5}\ \text{T/A})(I + 25\ \text{A})}$$

36. Using the right-hand rule, we see that if the currents flow in the same direction, then the magnetic fields will oppose each other between the wires and therefore can equal zero at a given point. Set the sum of the magnetic fields from the two wires equal to zero at the point 2.2 cm from the first wire and use Eq. 20–6 to solve for the unknown current.

$$B_{\text{net}} = 0 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} \quad \rightarrow$$

$$I_2 = \left(\frac{r_2}{r_1}\right) I_1 = \left(\frac{7.0\ \text{cm} - 2.2\ \text{cm}}{2.2\ \text{cm}}\right)(2.0\ \text{A}) = \boxed{4.4\ \text{A, same direction as other current}}$$

37. Use the right-hand rule to determine the direction of the magnetic field from each wire. Remembering that the magnetic field is inversely proportional to the distance from the wire, qualitatively add the magnetic field vectors. The magnetic field at point 2 is zero.



38. (*a*) We assume that the power line is long and straight and use Eq. 20–6.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(95\ \text{A})}{2\pi(8.5\ \text{m})} = 2.235\times 10^{-6}\ \text{T} \approx \boxed{2.2\times 10^{-6}\ \text{T}}$$

The direction at the ground, from the right-hand rule, is $\boxed{\text{south}}$. Compare this with the Earth's field of $0.5\times 10^{-4}\ \text{T}$, which points approximately north.

$$B_{\text{line}}/B_{\text{Earth}} = \frac{2.235\times 10^{-6}\ \text{T}}{0.5\times 10^{-4}\ \text{T}} = 0.0447$$

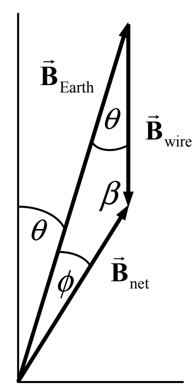
$\boxed{\text{The field of the cable is about 4\% that of the Earth.}}$

(*b*) We solve for the distance where $B_{\text{line}} = B_{\text{Earth}}$.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = B_{\text{Earth}} \quad\rightarrow\quad r = \frac{\mu_0 I}{2\pi B_{\text{Earth}}} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(95\ \text{A})}{2\pi(0.5\times 10^{-4}\ \text{T})} = 0.38\ \text{m} \approx \boxed{0.4\ \text{m}}$$

So about 0.4 m below the wire, the net B field would be 0, assuming that the Earth's field points straight north at this location.

39. The Earth's magnetic field is present at both locations in the problem, and we assume that it is the same at both locations. The field east of a vertical wire must be pointing either due north or due south. The compass shows the direction of the net magnetic field, and it changes from 17° E of N to 32° E of N when taken inside. That is a "southerly" change (rather than a "northerly" change"), so the field due to the wire must be pointing due south. See the diagram, in which $\theta = 17°$, $\theta + \phi = 32°$, and $\beta + \theta + \phi = 180°$. Thus $\phi = 15°$ and $\beta = 148°$. Use the law of sines to find the magnitude of $\vec{\mathbf{B}}_{\text{wire}}$ and then use Eq. 20–6 to find the magnitude of the current.



$$\frac{B_{\text{wire}}}{\sin\phi} = \frac{B_{\text{Earth}}}{\sin\beta} \quad\rightarrow\quad B_{\text{wire}} = B_{\text{Earth}}\frac{\sin\phi}{\sin\beta} = \frac{\mu_0}{2\pi}\frac{I}{r} \quad\rightarrow$$

$$I = B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}\frac{2\pi}{\mu_0}r = (5.0\times 10^{-5}\ \text{T})\frac{\sin 15°}{\sin 148°}\frac{2\pi}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}}(0.120\ \text{m}) = 14.65\ \text{A} \approx \boxed{15\ \text{A}}$$

Since the field due to the wire is due south, the current in the wire must be $\boxed{\text{downward}}$.

40. The fields created by the two wires will oppose each other, so the net field is the difference between the magnitudes of the two fields. The positive direction for the fields is taken to be into the page, so the closer wire creates a field in the positive direction, and the more distant wire creates a field in the negative direction. Let d be the separation distance of the wires.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{\text{closer}}} - \frac{\mu_0 I}{2\pi r_{\text{farther}}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}} - \frac{1}{r_{\text{farther}}}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r - \frac{1}{2}d} - \frac{1}{r + \frac{1}{2}d}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{d}{\left(r-\frac{1}{2}d\right)\left(r+\frac{1}{2}d\right)}\right)$$

$$= \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(24.5\ \mathrm{A})}{2\pi}\left(\frac{0.0028\ \mathrm{m}}{(0.10\ \mathrm{m}-0.0014\ \mathrm{m})(0.10\ \mathrm{m}+0.0014\ \mathrm{m})}\right)$$

$$= 1.372\times10^{-6}\ \mathrm{T} \approx \boxed{1.4\times10^{-6}\ \mathrm{T}}$$

Compare this with the Earth's field of 0.5×10^{-4} T.

$$B_{\mathrm{net}}/B_{\mathrm{Earth}} = \frac{1.372\times10^{-6}\ \mathrm{T}}{0.5\times10^{-4}\ \mathrm{T}} = 0.027$$

$\boxed{\text{The field of the wires is about 3\% that of the Earth.}}$

41. The center of the third wire is 5.6 mm from the left wire and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. 20–7 to calculate the force per unit length.

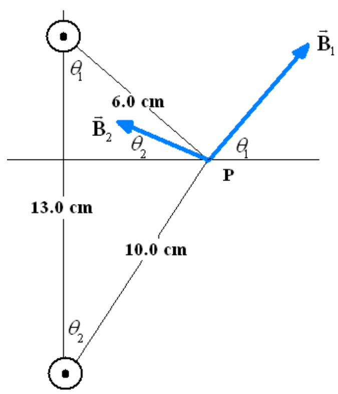
$$F_{\mathrm{near}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{near}}}\ell_{\mathrm{near}} \quad \rightarrow$$

$$\frac{F_{\mathrm{near}}}{\ell_{\mathrm{near}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{near}}} = \frac{4\pi\times10^{-7}\ \mathrm{T\cdot m/A}}{2\pi}\frac{(25.0\ \mathrm{A})(24.5\ \mathrm{A})}{2.8\times10^{-3}\ \mathrm{m}} = \boxed{4.4\times10^{-2}\ \mathrm{N/m,\ attract\ (to\ the\ right)}}$$

$$F_{\mathrm{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{far}}}\ell_{\mathrm{far}} \quad \rightarrow$$

$$\frac{F_{\mathrm{far}}}{\ell_{\mathrm{far}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\mathrm{far}}} = \frac{4\pi\times10^{-7}\ \mathrm{T\cdot m/A}}{2\pi}\frac{(25.0\ \mathrm{A})(24.5\ \mathrm{A})}{5.6\times10^{-3}\ \mathrm{m}} = \boxed{2.2\times10^{-2}\ \mathrm{N/m,\ repel\ (to\ the\ left)}}$$

42. Since the magnetic field from a current-carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define $\vec{\mathbf{B}}_1$ as the field from the top wire and $\vec{\mathbf{B}}_2$ as the field from the bottom wire. We use Eq. 20–6 to calculate the magnitude of each individual field.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(28\ \mathrm{A})}{2\pi(0.060\ \mathrm{m})} = 9.333\times10^{-5}\ \mathrm{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(28\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})} = 5.600\times10^{-5}\ \mathrm{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal.

$$\theta_1 = \cos^{-1}\left(\frac{(0.060\ \mathrm{m})^2+(0.130\ \mathrm{m})^2-(0.100\ \mathrm{m})^2}{2(0.060\ \mathrm{m})(0.130\ \mathrm{m})}\right) = 47.7^\circ$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.100\ \mathrm{m})^2+(0.130\ \mathrm{m})^2-(0.060\ \mathrm{m})^2}{2(0.100\ \mathrm{m})(0.130\ \mathrm{m})}\right) = 26.3^\circ$$

Using the magnitudes and angles of each magnetic field, we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net }x} = B_1\cos(\theta_1) - B_2\cos\theta_2 = (9.333\times10^{-5}\ \text{T})\cos 47.7° - (5.600\times10^{-5}\ \text{T})\cos 26.3° = 1.261\times10^{-5}\ \text{T}$$

$$B_{\text{net }y} = B_1\sin(\theta_1) + B_2\sin\theta_1 = (9.333\times10^{-4}\ \text{T})\sin 47.7° + (5.600\times10^{-5}\ \text{T})\sin 26.3° = 9.384\times10^{-5}\ \text{T}$$

$$B = \sqrt{B_{\text{net, }x}^2 + B_{\text{net, }y}^2} = \sqrt{(1.261\times10^{-5}\ \text{T})^2 + (9.384\times10^{-5}\ \text{T})^2} = 9.468\times10^{-5}\ \text{T}$$

$$\theta = \tan^{-1}\frac{B_{\text{net, }y}}{B_{\text{net, }y}} = \tan^{-1}\frac{8.379\times10^{-5}\ \text{T}}{1.126\times10^{-5}\ \text{T}} = 82.3°$$

$$\vec{\mathbf{B}} = 9.468\times10^{-5}\ \text{T at } 82.3° \approx \boxed{9.5\times10^{-5}\ \text{T at } 82°}$$

43. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r}\sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi(0.100\ \text{m})}\sqrt{(20.0\ \text{A})^2 + (12.0\ \text{A})^2}$$
$$= \boxed{4.66\times10^{-5}\ \text{T}}$$

44. Use Eq. 20–8 for the field inside a solenoid.

$$B = \frac{\mu_0 NI}{\ell} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(460)(2.0\ \text{A})}{0.12\ \text{m}} = \boxed{9.6\times10^{-3}\ \text{T}}$$

45. Use Eq. 20–8 for the field inside a solenoid.

$$B = \frac{\mu_0 NI}{\ell} \quad\rightarrow\quad I = \frac{B\ell}{\mu_0 N} = \frac{(4.65\times10^{-3}\ \text{T})(0.300\ \text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(935)} = \boxed{1.19\ \text{A}}$$

46. The field inside a solenoid is given by Eq. 20–8.

$$B = \frac{\mu_0 NI}{\ell} \quad\rightarrow\quad N = \frac{B\ell}{\mu_0 I} = \frac{(0.030\ \text{T})(0.42\ \text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.5\ \text{A})} = \boxed{2.2\times10^3\ \text{turns}}$$

47. The field due to the solenoid is given by Eq. 20–8. Since the field due to the solenoid is perpendicular to the current in the wire, Eq. 20–2 can be used to find the force on the wire segment.

$$F = I_{\text{wire}}\ell_{\text{wire}}B_{\text{solenoid}} = I_{\text{wire}}\ell_{\text{wire}}\frac{\mu_0 NI_{\text{solenoid}}}{\ell_{\text{solenoid}}} = (22\ \text{A})(0.030\ \text{m})\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(38\ \text{A})(550)}{(0.15\ \text{m})}$$
$$= 0.1156\ \text{N} \approx \boxed{0.12\ \text{N to the south}}$$

48. Since the mass of copper is fixed and the density is fixed, the volume of copper is fixed, and we designate it as $V_{\text{Cu}} = m_{\text{Cu}}\rho_{\text{Cu}} = \ell_{\text{Cu}}A_{\text{Cu}}$. We call the fixed voltage V_0. The magnetic field in the solenoid is given by Eq. 20–8. For the resistance, we used the resistivity-based definition from Eq. 18–3, with resistivity represented by $\rho_{\text{R Cu}}$.

$$B = \frac{\mu_0 NI}{\ell_{\text{sol}}} = \mu_0 \frac{N}{\ell_{\text{sol}}}\frac{V_0}{R_{\text{Cu}}} = \mu_0 \frac{N}{\ell_{\text{sol}}}\frac{V_0}{\rho_{\text{R Cu}}\frac{\ell_{\text{Cu}}}{A_{\text{Cu}}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}}\frac{A_{\text{Cu}}}{\ell_{\text{Cu}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}}\frac{m_{\text{Cu}}\rho_{\text{Cu}}}{\ell_{\text{Cu}}^2}$$

$$= \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2}$$

The number of turns of wire is the length of wire divided by the circumference of the solenoid.

$$N = \frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}} \quad \rightarrow \quad B = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{\frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}}}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{2\pi\rho_{\text{R Cu}}}\frac{1}{\ell_{\text{sol}} r_{\text{sol}}\ell_{\text{Cu}}}$$

The first factor in the expression for B is made of constants, so we have $B \propto \dfrac{1}{\ell_{\text{sol}} r_{\text{sol}}\ell_{\text{Cu}}}$. Thus we want the wire to be $\boxed{\text{short and fat}}$. Also, the radius of the solenoid should be small and the length of the solenoid should be small.

49. (*a*) Each loop of wire produces a field along its axis, similar to Fig. 20–9. For path 1, with all the loops taken together, that symmetry leads to a magnetic field that is the same anywhere along the path and parallel to the path. One side of every turn of the wire is enclosed by path 1, so the enclosed current is NI. Apply Eq. 20–9.

$$\sum B_{\parallel}\Delta\ell = \mu_0 NI \quad \rightarrow \quad B(2\pi R) = \mu_0 NI \quad \rightarrow \quad \boxed{B = \mu_0 NI/2\pi R}$$

(*b*) For path 2, each loop of wire pierces the enclosed area twice—once going up and once going down. Ampere's law takes the direction of the current into account, so the net current through the area enclosed by path 2 is 0.

$$\sum B_{\parallel}\Delta\ell = \mu_0 NI \quad \rightarrow \quad B(2\pi R) = 0 \quad \rightarrow \quad \boxed{B = 0}$$

(*c*) The field inside a toroid is $\boxed{\text{not uniform}}$. As seen in the result to part (*a*), the field varies inversely as the radius of the toroid, so $\boxed{B \propto \dfrac{1}{R}}$. The field is strongest at the inside wall of the toroid, and weakest at the outside wall.

50. (*a*) Ampere's law (Eq. 20–9) says that along a closed path, $\sum B_{\parallel}\Delta\ell = \mu_0 I_{\text{encl}}$. For the path, choose a circle of radius r, centered on the center of the wire, greater than the radius of the inner wire and less than the radius of the outer cylindrical braid. Because the wire is long and straight, the magnetic field is tangent to the chosen path, so $B_{\parallel} = B$. The current enclosed is I.

$$\mu_0 I_{\text{encl}} = \sum B_{\parallel}\Delta\ell = \sum B\Delta\ell = B\sum\Delta\ell = B(2\pi r) \quad \rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

(*b*) We make a similar argument, but now choose the path to be a circle of radius *r*, greater than the radius of the outer cylindrical braid. Because the wire and the braid are long and straight, the magnetic field is tangent to the chosen path, so $B_{\|} = B.$ The current enclosed by the path is zero, since there are two equal but oppositely directed currents.

$$\mu_0 I_{\text{encl}} = \sum B_{\|}\Delta\ell = \sum B\Delta\ell = B\sum\Delta\ell = B(2\pi r) \quad \rightarrow \quad B = \frac{\mu_0 I_{\text{encl}}}{2\pi r} = 0$$

51. If the face of the loop of wire is parallel to the magnetic field, then the angle between the perpendicular to the loop and the magnetic field is $90°$. Use Eq. 20–10 to calculate the magnetic field strength.

$$\tau = NIAB\sin\theta \quad \rightarrow \quad B = \frac{\tau}{NIA\sin\theta} = \frac{0.325\ \text{m}\cdot\text{N}}{(1)(5.70\ \text{A})(0.220\ \text{m})^2\sin 90°} = \boxed{1.18\ \text{T}}$$

52. From Eq. 20–10, we see that the torque is proportional to the current, so if the current drops by 12%, then the output torque will also $\boxed{\text{drop by 12\%}}$. Thus the final torque is 0.88 times the initial torque.

53. In Section 20–10, it is shown that the angular deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.760 times its original value, then the current must be increased by dividing the original value by 0.760 to obtain the same deflection.

$$(IB)_{\text{initial}} = (IB)_{\text{final}} \quad \rightarrow \quad I_{\text{final}} = \frac{I_{\text{initial}}B_{\text{initial}}}{B_{\text{final}}} = \frac{(53.0\ \mu\text{A})B_{\text{initial}}}{0.760B_{\text{initial}}} = \boxed{69.7\ \mu\text{A}}$$

54. (*a*) The torque is given by Eq. 20–10. The angle is the angle between the *B* field and the perpendicular to the coil face.

$$\tau = NIAB\sin\theta = 9(7.20\ \text{A})\left[\pi\left(\frac{0.120\ \text{m}}{2}\right)^2\right](5.50\times10^{-5}\ \text{T})\sin 34.0° = \boxed{2.25\times10^{-5}\ \text{m}\cdot\text{N}}$$

(*b*) In Example 20–13 it is stated that if the coil is free to turn, then it will rotate toward the orientation so that the angle is $0°$. In this case, that means the $\boxed{\text{north}}$ edge of the coil will rise, so that a perpendicular to its face will be parallel with the Earth's magnetic field.

55. The radius and magnetic field values can be used to find the speed of the protons. The electric field is then found from the fact that the magnetic force must be the same magnitude as the electric force for the protons to have straight paths.

$$qvB = mv^2/r \quad \rightarrow \quad v = qBr/m \qquad F_{\text{E}} = F_{\text{B}} \quad \rightarrow \quad qE = qvB \quad \rightarrow$$

$$E = vB = qB^2r/m = \frac{(1.60\times10^{-19}\ \text{C})(0.566\ \text{T})^2(6.10\times10^{-2}\ \text{m})}{1.67\times10^{-27}\ \text{kg}} = \boxed{1.87\times10^{6}\ \text{V/m}}$$

The direction of the electric field must be perpendicular to both the velocity and the magnetic field and must be in the opposite direction to the magnetic force on the protons.

56. The magnetic force on the ions causes them to move in a circular path, so the magnetic force is a centripetal force. This results in the ion mass being proportional to the path's radius of curvature.

$$qvB = mv^2/r \;\rightarrow\; m = qBr/v \;\rightarrow\; m/r = qB/v = \text{constant} = 76\text{ u}/22.8\text{ cm}$$

$$\frac{m_{21.0}}{21.0\text{ cm}} = \frac{76\text{ u}}{22.8\text{ cm}} \;\rightarrow\; m_{21.0} = 70\text{ u} \qquad \frac{m_{21.6}}{21.6\text{ cm}} = \frac{76\text{ u}}{22.8\text{ cm}} \;\rightarrow\; m_{21.6} = 72\text{ u}$$

$$\frac{m_{21.9}}{21.9\text{ cm}} = \frac{76\text{ u}}{22.8\text{ cm}} \;\rightarrow\; m_{21.9} = 73\text{ u} \qquad \frac{m_{22.2}}{22.2\text{ cm}} = \frac{76\text{ u}}{22.8\text{ cm}} \;\rightarrow\; m_{22.2} = 74\text{ u}$$

The other masses are $\boxed{70\text{ u, }72\text{ u, }73\text{ u, and }74\text{ u}}$.

57. The location of each line on the film is twice the radius of curvature of the ion. The radius of curvature can be found from Eq. 20–12.

$$m = \frac{qBB'r}{E} \;\rightarrow\; r = \frac{mE}{qBB'} \;\rightarrow\; 2r = \frac{2mE}{qBB'}$$

$$2r_{12} = \frac{2(12)(1.67\times10^{-27}\text{ kg})(2.88\times10^{4}\text{ V/m})}{(1.60\times10^{-19}\text{ C})(0.68\text{ T})^2} = 1.560\times10^{-2}\text{ m}$$

$$2r_{13} = 1.690\times10^{-2}\text{ m} \qquad 2r_{14} = 1.820\times10^{-2}\text{ m}$$

The distances between the lines are the following.

$$2r_{13} - 2r_{12} = 1.690\times10^{-2}\text{ m} - 1.560\times10^{-2}\text{ m} = 1.11\times10^{-3}\text{ m} \approx \boxed{1.3\times10^{-3}\text{ m}}$$

$$2r_{14} - 2r_{13} = 1.820\times10^{-2}\text{ m} - 1.690\times10^{-2}\text{ m} = 1.12\times10^{-3}\text{ m} \approx \boxed{1.3\times10^{-3}\text{ m}}$$

If the ions are doubly charged, then the value of q in the denominator of the expression would double, so the actual distances on the film would be halved. Thus the distances between the lines would also be halved.

$$2r_{13} - 2r_{12} = 8.451\times10^{-3}\text{ m} - 7.801\times10^{-3}\text{ m} = \boxed{6.5\times10^{-4}\text{ m}}$$

$$2r_{14} - 2r_{13} = 9.101\times10^{-3}\text{ m} - 8.451\times10^{-3}\text{ m} = \boxed{6.5\times10^{-4}\text{ m}}$$

58. The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, since the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ions to move in a circular path.

$$qvB = \frac{mv^2}{R} \;\rightarrow\; v = \frac{qBR}{m} \qquad qV = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m} \;\rightarrow\; m = \frac{qR^2B^2}{2V}$$

$\boxed{59.}$ Since the particle is undeflected in the crossed fields, its speed is given by $v = E/B$, as stated in Section 20–11. Without the electric field, the particle will travel in a circle due to the magnetic force. Using the centripetal acceleration, we can calculate the mass of the particle. Also, the charge must be an integer multiple of the fundamental charge.

$$qvB = \frac{mv^2}{r} \;\rightarrow$$

$$m = \frac{qBr}{v} = \frac{qBr}{(E/B)} = \frac{neB^2r}{E} = \frac{n(1.60\times10^{-19}\text{ C})(0.034\text{ T})^2(0.027\text{ m})}{1.5\times10^{3}\text{ V/m}} = n(3.3\times10^{-27}\text{ kg}) \approx n(2.0\text{ u})$$

The particle has an atomic mass of a multiple of 2.0 u. The simplest two cases are that it could be a hydrogen-2 nucleus (called a deuteron) or a helium-4 nucleus (called an alpha particle): $\boxed{{}^2_1\text{H},\ {}^4_2\text{He}}$.

60. The particles in the mass spectrometer follow a semicircular path as shown in Fig. 20–41. A particle has a displacement of $2r$ from the point of entering the semicircular region to where it strikes the film. So if the separation of the two molecules on the film is 0.50 mm, then the difference in radii of the two molecules is 0.25 mm. The mass to radius ratio is the same for the two molecules.

$$qvB = mv^2/r \quad \to \quad m = qBr/v \quad \to \quad m/r = \text{constant}$$

$$\left(\frac{m}{r}\right)_{\text{CO}} = \left(\frac{m}{r}\right)_{\text{N}_2} \quad \to \quad \frac{28.0106\text{ u}}{r} = \frac{28.0134\text{ u}}{r + 2.5\times10^{-4}\text{ m}} \quad \to \quad r = \boxed{2.5\text{ m}}$$

61. The field inside the solenoid is given by Eq. 20–8 with μ_0 replaced by the permeability of the iron.

$$B = \frac{\mu NI}{\ell} = \frac{3000(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(380)(0.35\text{ A})}{(1.0\text{ m})} = \boxed{0.5\text{ T}}$$

We assume that the factor of 3000 only has 1 significant figure.

62. The field inside the solenoid is given by Eq. 20–8 with μ_0 replaced by the permeability of the iron.

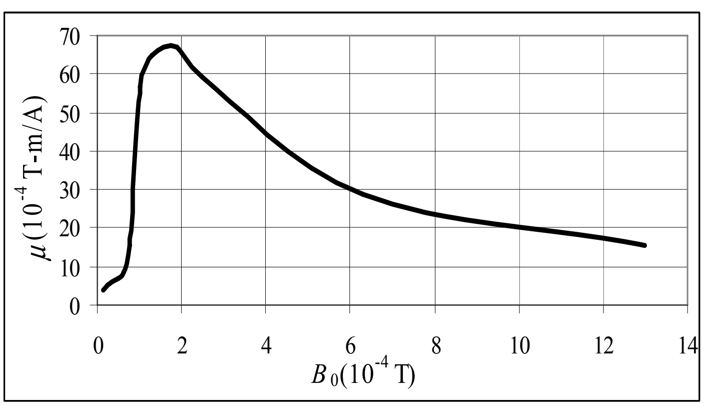
$$B = \frac{\mu NI}{\ell} \quad \to \quad \mu = \frac{B\ell}{NI} = \frac{(2.2\text{ T})(0.38\text{ m})}{(780)(48\text{ A})} = 2.239\times10^{-5}\text{ T}\cdot\text{m/A} \approx \boxed{2.2\times10^{-5}\text{ T}\cdot\text{m/A}} \approx 18\mu_0$$

63. The magnetic permeability is found from the two fields.

$$B_0 = \mu_0 nI; \quad B = \mu nI; \quad \frac{B}{B_0} = \frac{\mu}{\mu_0} \quad \to \quad \mu = \mu_0\frac{B}{B_0}$$

Here is a data table with the given values as well as the calculated values of μ.

For the graph, we have not plotted the last three data points so that the structure for low fields is seen. It would appear from the data that the value of μ is asymptotically approaching 0 for large fields.



$B_0(10^{-4}$ T)	B (T)	μ (10^{-4})(T-m/A)
0	0	
0.13	0.0042	4.06
0.25	0.01	5.03
0.50	0.028	7.04
0.63	0.043	8.58
0.78	0.095	15.3
1.0	0.45	56.5
1.3	0.67	64.8
1.9	1.01	66.8
2.5	1.18	59.3
6.3	1.44	28.7
13.0	1.58	15.3
130	1.72	1.66
1300	2.26	0.218
10000	3.15	0.040

64. (*a*) Use Eq. 20–6 to calculate the field due to a long straight wire.

$$B_{\text{A at B}} = \frac{\mu_0 I_{\text{A}}}{2\pi r_{\text{A to B}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.0\ \text{A})}{2\pi(0.15\ \text{m})} = 2.667\times10^{-6}\ \text{T} \approx \boxed{2.7\times10^{-6}\ \text{T}}$$

(*b*) $$B_{\text{B at A}} = \frac{\mu_0 I_{\text{B}}}{2\pi r_{\text{B to A}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.0\ \text{A})}{2\pi(0.15\ \text{m})} = 5.333\times10^{-6}\ \text{T} \approx \boxed{5.3\times10^{-6}\ \text{T}}$$

(*c*) The two fields are not equal and opposite. Each individual field is due to a single wire and has no dependence on the other wire. The magnitude of current in the second wire has nothing to do with the value of the field caused by the first wire.

(*d*) Use Eq. 20–7 to calculate the force per unit length on one wire due to the other wire. The forces are attractive since the currents are in the same direction.

$$\frac{F_{\text{on A due to B}}}{\ell_{\text{A}}} = \frac{F_{\text{on B due to A}}}{\ell_{\text{B}}} = \frac{\mu_0}{2\pi}\frac{I_{\text{A}} I_{\text{B}}}{d_{\text{A to B}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{(2.0\ \text{A})(4.0\ \text{A})}{(0.15\ \text{m})}$$
$$= 1.067\times10^{-5}\ \text{N/m} \approx \boxed{1.1\times10^{-5}\ \text{N/m}}$$

These two forces per unit length are equal and opposite because they are a Newton's third law pair of forces.

65. The magnetic force produces centripetal acceleration.

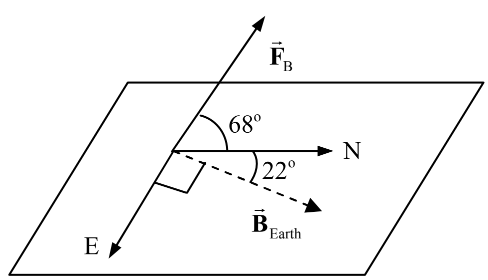
$$qvB = mv^2/r \quad\rightarrow\quad mv = p = qBr \quad\rightarrow\quad B = \frac{p}{qr} = \frac{4.8\times10^{-21}\ \text{kg}\cdot\text{m/s}}{(1.60\times10^{-19}\ \text{C})(1.1\ \text{m})} = \boxed{2.7\times10^{-2}\ \text{T}}$$

The magnetic field must point $\boxed{\text{upward}}$ to cause an inward-pointing (centripetal) force that steers the protons clockwise.

66. There will be no force on either the top or bottom part of the wire, because the current is either parallel to or opposite to the magnetic field. So the only force is on the left branch, which we define to be of length ℓ. Since the current is perpendicular to the magnetic field, use Eq. 20–2. The magnetic field can be calculated by Eq. 20–8 for the magnetic field inside a solenoid. By the right-hand rule, the force on the left branch is up out of the page.

$$F = I_{\text{wire}}\ell_{\text{wire}}B = I_{\text{wire}}\ell_{\text{wire}}\left(\frac{\mu_0 N I_{\text{solenoid}}}{\ell_{\text{solenoid}}}\right) = (5.0\ \text{A})\ell_{\text{wire}}\left[\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(700)(7.0\ \text{A})}{0.15\ \text{m}}\right]$$
$$= 0.2\ell_{\text{wire}}\ \text{N (assuming } \ell_{\text{wire}} \text{ given in m)}$$

67. We assume that the horizontal component of the Earth's magnetic field is pointing due north. The Earth's magnetic field also has the dip angle of 22°. The angle between the magnetic field and the eastward current is 90°. Use Eq. 20–1 to calculate the magnitude of the force.



$$F = I\ell B\sin\theta = (330\ \text{A})(18\ \text{m})(5.0\times10^{-5}\ \text{T})\sin 90°$$
$$= 0.297\ \text{N} \approx \boxed{0.30\ \text{N}}$$

Using the right-hand rule with the eastward current and the Earth's magnetic field, the force on the wire is $\boxed{\text{northerly and 68° above the horizontal}}$.

68. From Example 20–6, we have $r=\dfrac{mv}{qB}$. The quantity mv is the momentum, p, so $r=\dfrac{p}{qB}$. Thus $\boxed{p=qBr}$.

69. The airplane is a charge moving in a magnetic field. Since it is flying perpendicular to the magnetic field, Eq. 20–4 applies.

$$F_{\text{max}}=qvB=(1280\times10^{-6}\ \text{C})(120\ \text{m/s})(5.0\times10^{-5}\ \text{T})=\boxed{7.7\times10^{-6}\ \text{N}}$$

70. The field inside a solenoid is given by Eq. 20–8.

$$B=\frac{\mu_0 NI}{\ell}\ \rightarrow\ N=\frac{B\ell}{\mu_0 I}=\frac{(0.050\ \text{T})(0.32\ \text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(6.4\ \text{A})}=1989\approx\boxed{2.0\times10^{3}\ \text{turns}}$$

71. The magnetic force must be equal in magnitude to the weight of the electron.

$$mg=qvB\ \rightarrow\ v=\frac{mg}{qB}=\frac{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}{(1.60\times10^{-19}\ \text{C})(0.50\times10^{-4}\ \text{T})}=\boxed{1.1\times10^{-6}\ \text{m/s}}$$

The magnetic force must point upward, so by the right-hand rule and the negative charge of the electron, the electron must be moving $\boxed{\text{west}}$.

72. (*a*) The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, assuming the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

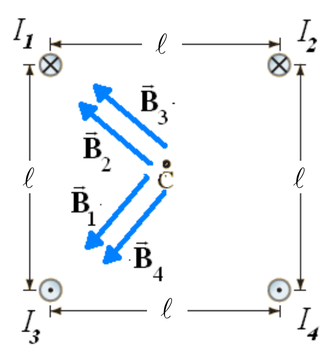
$$\text{PE}_{\text{initial}}=\text{KE}_{\text{final}}\ \rightarrow\ qV=\tfrac{1}{2}mv^2\ \rightarrow\ v=\sqrt{\frac{2qV}{m}}\qquad qvB=\frac{mv^2}{r}\ \rightarrow$$

$$r=\frac{mv}{qB}=\frac{m\sqrt{\dfrac{2qV}{m}}}{qB}=\sqrt{\frac{2mV}{qB^2}}=\sqrt{\frac{2(6.6\times10^{-27}\ \text{kg})(3200\ \text{V})}{2(1.60\times10^{-19}\ \text{C})(0.240\ \text{T})^2}}=4.787\times10^{-2}\ \text{m}\approx\boxed{4.8\times10^{-2}\ \text{m}}$$

(*b*) The period is the circumference of the circular path divided by the speed.

$$T=\frac{2\pi r}{v}=\frac{2\pi\sqrt{\dfrac{2mV}{qB^2}}}{\sqrt{\dfrac{2qV}{m}}}=\frac{2\pi m}{qB}=\frac{2\pi(6.6\times10^{-27}\ \text{kg})}{2(1.60\times10^{-19}\ \text{C})(0.240\ \text{T})}=\boxed{5.5\times10^{-7}\ \text{s}}$$

73. The magnetic field at the center of the square is the vector sum of the magnetic field created by each current. Since the magnitudes of the currents are equal and the distance from each corner to the center is the same, the magnitude of the magnetic field from each wire is the same and is given by Eq. 20–6. The direction of the magnetic field is directed by the right-hand rule and is shown in the diagram. By symmetry, we see that the vertical components of the magnetic field cancel and the horizontal components add.



$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45° \text{ to the left}$$

$$= 4\left(\frac{\mu_0 I}{2\pi \frac{\sqrt{2}}{2}\ell}\right)\frac{\sqrt{2}}{2} \text{ to the left} = \boxed{\frac{2\mu_0 I}{\pi \ell} \text{ to the left}}$$

74. (*a*) For the beam of electrons to be undeflected, the magnitude of the magnetic force must equal the magnitude of the electric force. We assume that the magnetic field will be perpendicular to the velocity of the electrons so that the maximum magnetic force is obtained.

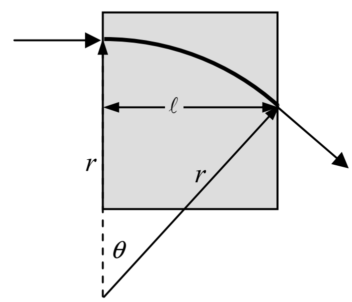
$$F_B = F_E \rightarrow qvB = qE \rightarrow B = \frac{E}{v} = \frac{12{,}000 \text{ V/m}}{4.8\times10^6 \text{ m/s}} = \boxed{2.5\times10^{-3} \text{ T}}$$

(*b*) Since the electric field is pointing south, the electric force is to the north. Thus the magnetic force must be to the south. Using the right-hand rule with the negative electrons, the magnetic field must be vertically upward.

(*c*) If the electric field is turned off, then the magnetic field will cause a centripetal force, moving the electrons in a circular path. The frequency is the cyclotron frequency, Eq. 20–5.

$$f = \frac{qB}{2\pi m} = \frac{qE}{2\pi m v} = \frac{(1.60\times10^{-19} \text{ C})(12{,}000 \text{ V/m})}{2\pi(9.11\times10^{-31} \text{ kg})(4.8\times10^6 \text{ m/s})} = \boxed{7.0\times10^7 \text{ Hz}}$$

75. The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 20–6 as $r = \frac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large, so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small, so they will exit below the second tube. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.



$$\sin\theta = \frac{\ell}{r} \rightarrow$$

$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell q B}{m v} = \sin^{-1}\frac{(5.0\times10^{-2} \text{ m})(1.60\times10^{-19} \text{ C})(0.41 \text{ T})}{(1.67\times10^{-27} \text{ kg})(2.5\times10^6 \text{ m/s})} = \sin^{-1} 0.7856 = \boxed{52°}$$

76. With one turn of wire, we have $B = \frac{\mu_0 I}{2r}$. Use the radius of the Earth for r.

$$B = \frac{\mu_0 I}{2r} \rightarrow I = \frac{2rB}{\mu_0} = \frac{2(6.38\times10^6 \text{ m})(1\times10^{-4} \text{ T})}{4\pi\times10^{-7} \text{ T}\cdot\text{m/A}} = 1.015\times10^9 \text{ A} \approx \boxed{1\times10^9 \text{ A}}$$

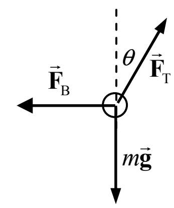
77. The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB = \frac{mv^2}{r} \quad \rightarrow \quad v = \frac{qBr}{m} \qquad \mathrm{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$

$$\Delta\mathrm{KE} = \frac{q^2B^2}{2m}(r_2^2 - r_1^2) = \frac{(1.60\times10^{-19}\ \mathrm{C})^2(0.010\ \mathrm{T})^2}{2(1.67\times10^{-27}\ \mathrm{kg})}[(8.5\times10^{-3}\ \mathrm{m})^2 - (10.0\times10^{-3}\ \mathrm{m})^2]$$

$$= \boxed{-2.1\times10^{-20}\ \mathrm{J}} \text{ or } -0.13\ \mathrm{eV}$$

78. There are three forces on each wire—its weight, the magnetic force of repulsion from the other wire, and the tension in the attached string. See the diagram. The magnetic force is given by Eq. 20–7. The mass of the wire is its density times its volume. The length of the current-carrying wires is ℓ. The net force in both the vertical and horizontal directions is zero.



$$F_{\mathrm{B}} = F_{\mathrm{T}}\sin\theta; \quad F_{\mathrm{T}}\cos\theta = mg \quad \rightarrow \quad F_{\mathrm{T}} = \frac{mg}{\cos\theta} \quad \rightarrow$$

$$F_{\mathrm{B}} = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = \rho_{\mathrm{Al}}\pi r_{\mathrm{Al}}^2\ell g\tan\theta$$

$$F_{\mathrm{B}} = \frac{\mu_0}{2\pi}\frac{I^2}{S}\ell \quad \rightarrow \quad \frac{\mu_0}{2\pi}\frac{I^2}{S}\ell = \rho_{\mathrm{Al}}\pi r_{\mathrm{Al}}^2\ell g\tan\theta \quad \rightarrow$$

$$I = \sqrt{\frac{2\pi S\rho_{\mathrm{Al}}\pi r_{\mathrm{Al}}^2 g\tan\theta}{\mu_0}} = \sqrt{\frac{2\pi[2(0.50\ \mathrm{m})\sin3^\circ](2700\ \mathrm{kg/m^3})\pi(2.1\times10^{-4}\ \mathrm{m})^2(9.8\ \mathrm{m/s^2})\tan3^\circ}{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})}}$$

$$= \boxed{7.1\ \mathrm{A}}$$

79. The centripetal force is caused by the magnetic field and is given by Eq. 20–3.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad \rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(3.0\times10^6\ \mathrm{m/s})\sin45^\circ}{(1.60\times10^{-19}\ \mathrm{C})(0.23\ \mathrm{T})} = 5.251\times10^{-5}\ \mathrm{m} \approx \boxed{5.3\times10^{-5}\ \mathrm{m}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, so the pitch is that velocity component times the period of the circular motion.

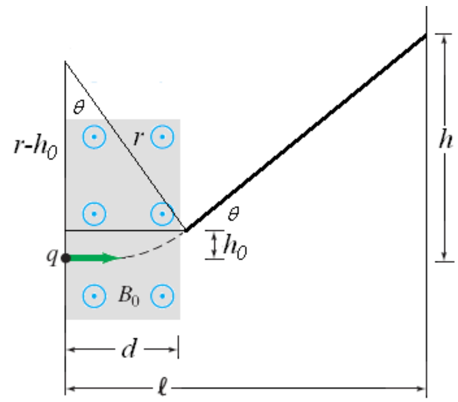
$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\dfrac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_\parallel T = v\cos45^\circ\left(\frac{2\pi m}{qB}\right) = (3.0\times10^6\ \mathrm{m/s})\cos45^\circ\frac{2\pi(9.11\times10^{-31}\ \mathrm{kg})}{(1.60\times10^{-19}\ \mathrm{C})(0.23\ \mathrm{T})} = \boxed{3.3\times10^{-4}\ \mathrm{m}}$$

80. The maximum torque is found using Eq. 20–10 with $\sin\theta = 1$. Set the current equal to the voltage divided by the resistance and the area as the square of the side length.

$$\tau = NIAB = N\left(\frac{V}{R}\right)\ell^2 B = 20\left(\frac{9.0\ \mathrm{V}}{28\ \Omega}\right)(0.050\ \mathrm{m})^2(0.020\ \mathrm{T}) = \boxed{3.2\times10^{-4}\ \mathrm{m\cdot N}}$$

81. We find the speed of the electron using conservation of energy. The accelerating potential energy becomes the kinetic energy of the electron.



$$eV = \tfrac{1}{2}mv^2 \quad \rightarrow$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(2200\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(9.11\times10^{-31}\text{ kg})}}$$

$$= 2.78\times10^{7}\text{ m/s}$$

Upon entering the magnetic field, the electron is traveling horizontally. The magnetic field will cause the path of the electron to be an arc of a circle of radius r and deflect an angle θ from the horizontal. While in the field, the electron will travel a horizontal distance d and a vertical distance h_0. Our approximation will be to ignore the distance h_0. We have the following relationships.

$$\tan\theta = \frac{h}{\ell};\ \sin\theta = \frac{d}{r};\ r = \frac{mv}{eB} \quad \rightarrow$$

$$\theta = \tan^{-1}\frac{h}{\ell} = \tan^{-1}\frac{11\text{ cm}}{22\text{ cm}} = 26.6^\circ;\ r = \frac{d}{\sin\theta} = \frac{0.038\text{ m}}{\sin 26.6^\circ} = 0.0849\text{ m}$$

$$B = \frac{mv}{er} = \frac{(9.11\times10^{-31}\text{ kg})(2.78\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0849\text{ m})} = 1.86\times10^{-3}\text{ T} \approx \boxed{1.9\times10^{-3}\text{ T}}$$

82. The magnetic field from the wire at the location of the plane is perpendicular to the velocity of the plane since the plane is flying parallel to the wire. We calculate the force on the plane, and thus the acceleration, using Eq. 20–4, with the magnetic field of the wire given by Eq. 20–6.

$$F = qvB = qv\frac{\mu_0 I}{2\pi r}$$

$$a = \frac{F}{m} = \frac{qv}{m}\frac{\mu_0 I}{2\pi r} = \frac{(18.0\times10^{-3}\text{ C})(3.4\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(25\text{ A})}{2\pi(0.175\text{ kg})(0.086\text{ m})}$$

$$= 2.033\times10^{-5}\text{ m/s}^2\left(\frac{1\text{ g}}{9.80\text{ m/s}^2}\right) = \boxed{2.1\times10^{-6}\text{ g's}}$$

83. Since the magnetic and gravitational force along the entire rod is uniform, we consider the two forces acting at the center of mass of the rod. To be balanced, the net torque about the fulcrum must be zero. Using the usual sign convention for torques and Eq. 8–10a, we solve for the magnetic force on the rod.

$$\sum\tau = 0 = Mg\left(\tfrac{1}{4}\ell\right) - mg\left(\tfrac{1}{4}\ell\right) - F_M\left(\tfrac{1}{4}\ell\right) \quad \rightarrow \quad F_M = (M-m)g$$

We solve for the current using Eq. 20–2.

$$I = \frac{F}{\ell B} = \frac{(M-m)g}{\ell B} = \frac{(6.0m-m)g}{\ell B} = \boxed{\frac{5.0mg}{\ell B}}$$

The right-hand rule indicates that the current must flow $\boxed{\text{toward the left}}$ since the magnetic field is into the page and the magnetic force is downward.

84. The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is 6.38×10^6 km, and the altitude is added to that.

$$F_B = qvB = m\frac{v^2}{r} \quad \to \quad v = \frac{qrB}{m} = \frac{(1.60\times10^{-19}\ \text{C})(6.385\times10^6\ \text{m})(0.50\times10^{-4}\ \text{T})}{238(1.66\times10^{-27}\ \text{kg})} = \boxed{1.3\times10^8\ \text{m/s}}$$

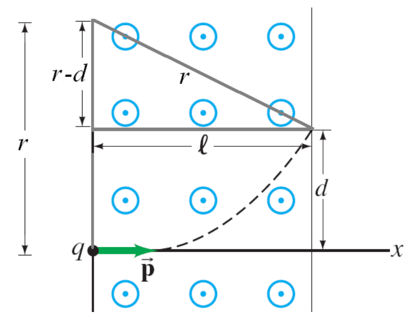
Compare the size of the magnetic force to the force of gravity on the ion.

$$\frac{F_B}{F_g} = \frac{qvB}{mg} = \frac{(1.60\times10^{-19}\ \text{C})(1.3\times10^8\ \text{m/s})(0.50\times10^{-4}\ \text{T})}{238(1.66\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{2.7\times10^8}$$

It is fine to ignore gravity—the magnetic force is almost 300 million times larger than gravity.

85. (*a*) For the particle to move upward, the magnetic force must point upward; by the right-hand rule we see that the force on a positively charged particle would be downward. Therefore, the charge on the particle must be $\boxed{\text{negative}}$.

(*b*) In the figure we have created a right triangle to relate the horizontal distance ℓ, the displacement d, and the radius of curvature, r. Using the Pythagorean theorem we can write an expression for the radius in terms of the other two distances.



$$r^2 = (r-d)^2 + \ell^2 \quad \to \quad r = \frac{d^2+\ell^2}{2d}$$

Since the momentum is perpendicular to the magnetic field, we can solve for the momentum by relating the maximum force (Eq. 20–4) to the centripetal force on the particle.

$$F_{\text{max}} = qvB_0 = \frac{mv^2}{r} \quad \to \quad p = mv = qB_0 r = \boxed{\frac{qB_0(d^2+\ell^2)}{2d}}$$

86. Example 18–10 estimates the current in a lightning bolt at 100 A. Use Eq. 20–2, with the Earth's magnetic field. We estimate the flag pole as being 65 m tall.

$$F = I\ell B = (100\ \text{A})(6\ \text{m})(0.5\times10^{-4}\ \text{T}) = \boxed{0.03\ \text{N}}$$

87. The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, then the magnetic force will be constant, so constant-acceleration kinematics (Eq. 2–11c) can be used.

$$v^2 = v_0^2 + 2a\Delta x \quad \to \quad a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{\text{net}} = ma = I\ell B \quad \to \quad I = \frac{ma}{\ell B} = \frac{m\left(\dfrac{v^2}{2\Delta x}\right)}{\ell B} = \frac{mv^2}{2\ell B\Delta x} = \frac{(1.5\times10^{-3}\ \text{kg})(28\ \text{m/s})^2}{2(0.28\ \text{m})(1.7\ \text{T})(1.0\ \text{m})} = \boxed{1.2\ \text{A}}$$

Using the right-hand rule, for the force on the bar to be in the direction of the acceleration shown in Fig. 20–71, the magnetic field must be $\boxed{\text{down}}$.

88. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 20–6 as $r = \dfrac{mv}{qB}$. For an object moving in circular motion, the period is given by $T = \dfrac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T = \frac{2\pi r}{v} \quad \rightarrow \quad f = \frac{\ell}{T} = \frac{v}{2\pi \dfrac{mv}{qB}} = \boxed{\frac{Bq}{2\pi m}}$$

In particular, we note that this frequency is independent of the radius, so the same frequency can be used throughout the acceleration. This frequency is also given as Eq. 20–5.

(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, then the particles receive an energy increase of $\text{KE} = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of two gaps, so the total KE increase is $\boxed{2qV_0}$.

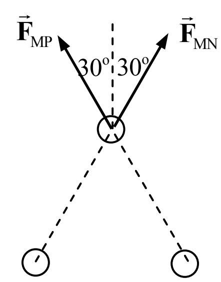
(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$v = \frac{rqB}{m}$$

$$\text{KE}_{\text{max}} = \tfrac{1}{2} m v_{\text{max}}^2 = \tfrac{1}{2} m \left(\frac{r_{\text{max}} qB}{m} \right)^2 = \tfrac{1}{2} \frac{r_{\text{max}}^2 q^2 B^2}{m} = \tfrac{1}{2} \frac{(2.0\text{ m})^2 (1.60\times10^{-19}\text{ C})^2 (0.50\text{ T})^2}{1.67\times10^{-27}\text{ kg}}$$

$$= 7.66\times10^{-12}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}} \right)\left(\frac{1\text{ MeV}}{10^6\text{ eV}} \right) = \boxed{48\text{ MeV}}$$

89. (*a*) The forces on wire M due to the other wires are repelling forces, away from the other wires. Use Eq. 20–7 to calculate the force per unit length on wire M due to each of the other wires and then add the force vectors together. The horizontal parts of the forces cancel, and the sum is vertical.

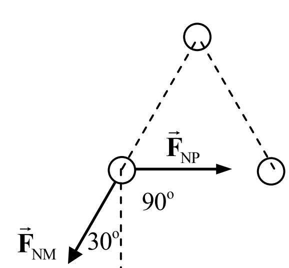


$$\frac{F_{\text{M net }y}}{\ell_{\text{M}}} = \frac{F_{\text{MN}}}{\ell_{\text{M}}} \cos 30^\circ + \frac{F_{\text{MP}}}{\ell_{\text{M}}} \cos 30^\circ$$

$$= \frac{\mu_0}{2\pi} \frac{I_{\text{M}} I_{\text{N}}}{d_{\text{MN}}} \cos 30^\circ + \frac{\mu_0}{2\pi} \frac{I_{\text{M}} I_{\text{P}}}{d_{\text{MP}}} \cos 30^\circ$$

$$= 2 \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}{2\pi} \frac{(8.00\text{ A})^2}{(0.038\text{ m})} \cos 30^\circ = \boxed{5.8\times10^{-4}\text{ N/m, upward}}$$

The forces on wire N due to the other wires are an attractive force toward wire P and a repelling force away from wire M. Use Eq. 20–7 to calculate the force per unit length on wire N due to each of the other wires and then add the force vectors together. From symmetry, we expect the net force to lie exactly between the two individual force vectors, which is 60° below the horizontal.



$$F_{\text{N net }y} = -F_{\text{NM}}\cos 30° = -\frac{\mu_0}{2\pi}\frac{I_{\text{M}}I_{\text{N}}}{d_{\text{MN}}}\cos 30°$$

$$= -\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{(8.00\ \text{A})^2}{(0.038\ \text{m})}\cos 30° = -2.917\times10^{-4}\ \text{N/m}$$

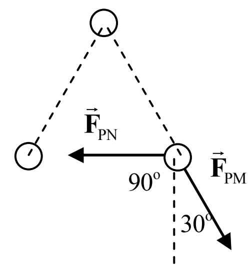
$$F_{\text{N net }x} = -F_{\text{NM}}\sin 30° + F_{\text{NP}}$$

$$= -\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{(8.00\ \text{A})^2}{(0.038\ \text{m})}\sin 30° + \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{(8.00\ \text{A})^2}{(0.038\ \text{m})}$$

$$= 1.684\times10^{-5}\ \text{N/m}$$

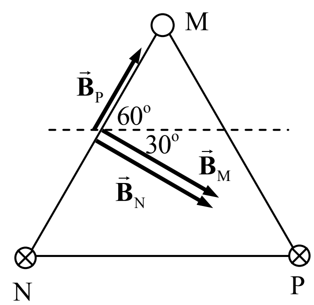
$$F_{\text{N net}} = \sqrt{F_{\text{N net }x}^2 + F_{\text{N net }y}^2} = 3.4\times10^{-4}\ \text{N/m} \quad \theta = \tan^{-1}\frac{F_{\text{N net }y}}{F_{\text{N net }x}} = \tan^{-1}\frac{-2.917}{1.684} = \boxed{300°}$$

The forces on wire P due to the other wires are an attractive force toward wire N and a repelling force away from wire M. Use Eq. 20–7 to calculate the force per unit length on wire P due to each of the other wires and then add the force vectors together. From symmetry, this is just a mirror image of the previous solution, so the net force is as follows.



$$F_{\text{P net}} = \boxed{3.4\times10^{-4}\ \text{N/m}} \quad \theta = \boxed{240°}$$

(*b*) There will be three magnetic fields to sum—one from each wire. Each field will point perpendicularly to the line connecting the wire to the midpoint. The two fields due to M and N are drawn slightly separated from each other, but should be collinear. The magnitude of each field is given by Eq. 20–6.



$$\vec{\mathbf{B}}_{\text{net}} = \vec{\mathbf{B}}_{\text{M}} + \vec{\mathbf{B}}_{\text{N}} + \vec{\mathbf{B}}_{\text{P}}$$

$$B_{\text{M}} = B_{\text{N}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{M}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{8.00\ \text{A}}{0.019\ \text{m}} = 8.421\times10^{-5}\ \text{T}$$

$$B_{\text{P}} = \frac{\mu_0}{2\pi}\frac{I}{r_{\text{P}}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\frac{8.00\ \text{A}}{\sqrt{3}(0.019\ \text{m})} = 4.862\times10^{-5}\ \text{T}$$

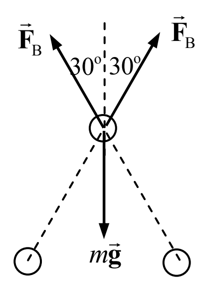
$$B_{\text{net }x} = 2B_{\text{M}}\cos 30° + B_{\text{P}}\cos 60° = 2(8.421\times10^{-5}\ \text{T})\cos 30°$$
$$+ (4.862\times10^{-5}\ \text{T})\cos 60° = 1.702\times10^{-4}\ \text{T}$$

$$B_{\text{net }y} = -2B_{\text{M}}\sin 30° + B_{\text{P}}\sin 60° = -2(8.421\times10^{-5}\ \text{T})\sin 30°$$
$$+ (4.862\times10^{-5}\ \text{T})\sin 60° = -4.210\times10^{-5}\ \text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net }x}^2 + B_{\text{net }y}^2} = \sqrt{(1.702\times10^{-4}\ \text{T})^2 + (-4.210\times10^{-5}\ \text{T})} = \boxed{1.75\times10^{-4}\ \text{T}}$$

$$\theta_{\text{net}} = \tan^{-1}\frac{B_{\text{net }y}}{B_{\text{net }x}} = \tan^{-1}\frac{-4.210\times10^{-5}\ \text{T}}{1.702\times10^{-4}\ \text{T}} = \boxed{-14°}$$

The net field points slightly below the horizontal direction.



90. Each of the bottom wires will repel the top wire since each bottom current is opposite to the top current. The repelling forces will be along the line directly from the bottom wires to the top wires. Only the vertical components of those forces will be counteracting gravity. Use Eq. 20–7 to calculate the magnetic forces. The mass of the wire is its density times its volume. The length of the wire is represented by ℓ.

$$mg = 2F_B \cos 30° \rightarrow \rho_{Cu} \ell \pi \left(\tfrac{1}{2} d_{Cu}\right)^2 g = 2\frac{\mu_0}{2\pi} \frac{I_{bottom} I_{Cu}}{d} \ell \cos 30° \rightarrow$$

$$I_{Cu} = \frac{\rho_{Cu} \pi^2 \left(\tfrac{1}{2} d_{Cu}\right)^2 dg}{\mu_0 I_{bottom} \cos 30°} = \frac{(8900\ \text{kg/m}^3)\pi^2 (5.0\times10^{-4}\ \text{m})^2 (0.038\ \text{m})(9.80\ \text{m/s}^2)}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(75\ \text{A})(\cos 30°)}$$
$$= 100.2\ \text{A} \approx \boxed{1.0\times10^2\ \text{A}}$$

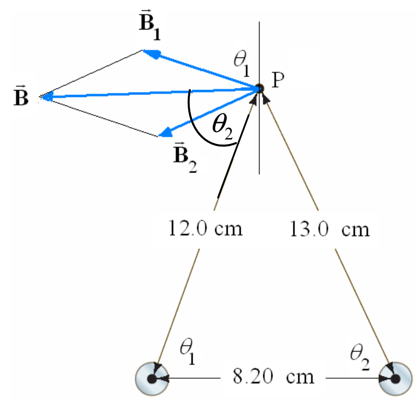
91. We approximate the magnetic field by using Eq. 20–6. The current is found from Eq. 18–5.

$$I = \frac{P}{V} \rightarrow B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r}\frac{P}{V} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(13\ \text{m})}\frac{(46\times10^6\ \text{W})}{(2.4\times10^5\ \text{V})} = 2.949\times10^{-6}\ \text{T} \approx \boxed{2.9\times10^{-6}\ \text{T}}$$

$$\frac{B_{wire}}{B_{Earth}} = \frac{2.949\times10^{-6}\ \text{T}}{5\times10^{-5}\ \text{T}} = 5.90\times10^{-2} \approx \boxed{6\%}$$

The power line is almost certainly AC, so the voltage, power, current, and the magnitude of the magnetic field are most likely rms values.

92. The net magnetic field is the vector sum of the magnetic fields produced by each current-carrying wire. Since the individual magnetic fields encircle the wire producing it, the field is perpendicular to the radial line from the wire to point P. We let $\vec{\mathbf{B}}_1$ be the field from the left wire and $\vec{\mathbf{B}}_2$ designate the field from the right wire. The magnitude of the magnetic field vectors is calculated from Eq. 20–6.



$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(19.2\ \text{A})}{2\pi(0.12\ \text{m})} = 3.2000\times10^{-5}\ \text{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(19.2\ \text{A})}{2\pi(0.13\ \text{m})} = 2.9538\times10^{-5}\ \text{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the horizontal. Since the magnetic fields are perpendicular to the radial lines, these angles are the same as the angles the magnetic fields make with the vertical.

$$\theta_1 = \cos^{-1}\left(\frac{(0.12\ \text{m})^2 + (0.082\ \text{m})^2 - (0.13\ \text{m})^2}{2(0.12\ \text{m})(0.082\ \text{m})}\right) = 77.606°$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.13\ \text{m})^2 + (0.082\ \text{m})^2 - (0.12\ \text{m})^2}{2(0.13\ \text{m})(0.082\ \text{m})}\right) = 64.364°$$

Using the magnitudes and angles of each magnetic field, we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{net\,x} = -B_1 \sin(\theta_1) - B_2 \sin\theta_2 = -(3.2000\times10^{-5}\ \text{T})\sin 77.606° - (2.9538\times10^{-5}\ \text{T})\sin 64.364°$$
$$= -57.88\times10^{-6}\ \text{T}$$

$$B_{net\,y} = B_1 \cos(\theta_1) - B_2 \cos\theta_1 = (3.2000\times10^{-5}\ \text{T})\cos 77.606° - (2.9538\times10^{-5}\ \text{T})\cos 64.364°$$
$$= -5.911\times10^{-6}\ \text{T}$$

$$B = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{(-57.88\times10^{-6}\ \text{T})^2 + (-5.911\times10^{-6}\ \text{T})^2} = 5.82\times10^{-5}\ \text{T}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-5.911\times10^{-6}\ \text{T}}{-57.88\times10^{-6}\ \text{T}} = 5.83^\circ$$

$$\boxed{\vec{\mathbf{B}} = 5.82\times10^{-5}\ \text{T at } 5.83^\circ \text{ below the negative } x \text{ axis}}$$

Solutions to Search and Learn Problems

1. There are three magnetic force equations given in this chapter.

   (i) $F = I\ell B\sin\theta$ This is the force on a current-carrying wire in a magnetic field. The current is measured in amperes, the length in meters, and the magnetic field in teslas. A tesla is equivalent to a newton per ampere per meter.

   $$(\text{A})(\text{m})(\text{T}) = (\text{A})(\text{m})\left(\frac{\text{N}}{\text{A}\cdot\text{m}}\right) = (\text{N})$$

   (ii) $F = qvB\sin\theta$ This is the force on a moving charged particle in a magnetic field. The charge is measured in coulombs, the velocity in meters per second, and the magnetic field in teslas.

   $$(\text{C})\left(\frac{\text{m}}{\text{s}}\right)(\text{T}) = \left(\frac{\text{C}}{\text{s}}\right)(\text{m})(\text{T}) = (\text{A})(\text{m})\left(\frac{\text{N}}{\text{A}\cdot\text{m}}\right) = (\text{N})$$

   (iii) $F = \dfrac{\mu_0 I_1 I_2}{2\pi d}\ell_2$ This is the force between two parallel currents—specifically, the force on a length ℓ_2 of the wire carrying current I_2, due to another wire carrying current I_1. The permeability of free space has units of $\text{T}\cdot\text{m/A}$.

   $$\left(\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{(\text{A})(\text{A})}{(\text{m})}(\text{m}) = (\text{T}\cdot\text{m})(\text{A}) = \left(\frac{\text{N}}{\text{A}\cdot\text{m}}\cdot\text{m}\right)(\text{A}) = (\text{N})$$

2. (*a*) The electron, a negative particle, accelerates opposite the direction of the electric field. Therefore, for the electron to accelerate east, the electric field should point to the west.

   (*b*) By the right-hand rule, the magnetic field must point upward for the magnetic force on a north-moving electron to point westward.

   (*c*) For the electron to not accelerate, the magnetic force must be equal in magnitude but opposite in direction to the electric force. Set the magnitudes of the forces from Eq. 16–5 and Eq. 20–4 equal and solve for the magnetic field. The direction of the field is upward.

   $$qE = qvB \quad\rightarrow\quad B = \frac{E}{v} = \frac{330\ \text{V/m}}{3.0\times10^4\ \text{m/s}} = \boxed{0.011\ \text{T}}$$

   (*d*) If the electron is moving faster, then the magnetic force will be greater than the electric force and the electron will accelerate westward. If it is moving slower, then the magnetic force will be smaller than the electric force and the electron will accelerate eastward.

   (*e*) The two forces must be equal and opposite for the particle to travel undeflected. The ratio of the electric to magnetic field can be calculated by setting the forces equal.

   $$qE = qvB \quad\rightarrow\quad \frac{E}{B} = v = \boxed{5.5\times10^4\ \text{m/s}}$$

   The value of B cannot be determined unless the value of E is known.

3. (*a*) In Example 20–6, the radius of curvature for an electron in a uniform magnetic field was shown to be $r = mv/qB$. If the kinetic energy (and therefore the speed) of the particle remains constant as the magnetic field doubles, then the radius of curvature is cut in half. The angular momentum of a particle is given by $L = mvr$, so if the speed remains constant as the radius of curvature decreases, $\boxed{\text{the angular momentum will be cut in half}}$.

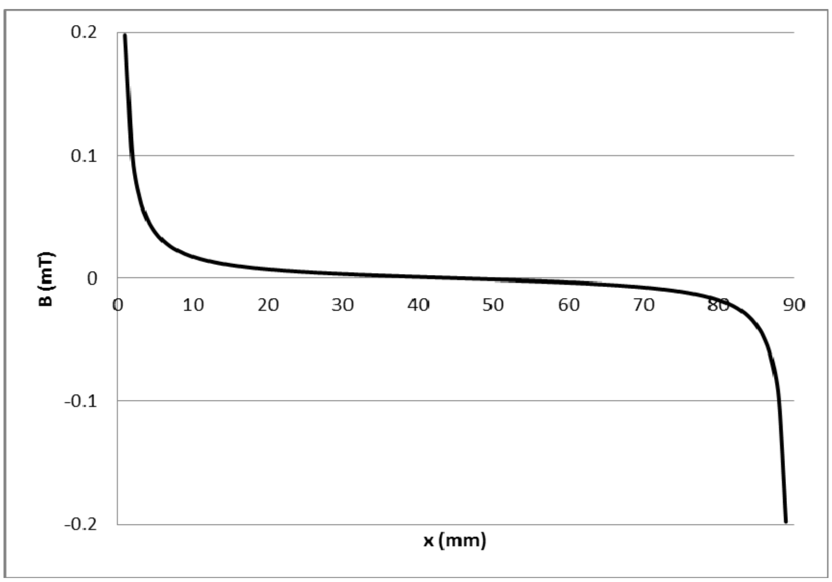
(*b*) The magnetic dipole moment is defined in Eq. 20–11 as $M = NIA$. The number of turns, *N*, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume that the electron is moving in a circular orbit of radius *r*, then the area is $A = \pi r^2$. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mvr$. Combining these relationships gives the magnetic moment in terms of the angular momentum.

$$M = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emvr}{2m} = \frac{e}{2m}mvr = \frac{e}{2m}L$$

4. (*a*) The left current creates a field on the *x* axis that points in the positive *y* direction, and the right current creates a field on the *x* axis pointing in the negative *y* direction. If the distance from the left wire to a point on the positive *x* axis is *x*, then the distance from the right wire to that same point is $d - x$. We can write the net magnetic field, taking upward as positive, using Eq. 20–6.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(d-x)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right) = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x(d-x)}\right)}$$

(*b*)



5. Using the configuration shown in Fig. 20–76, the downward electric field puts a force on the charged particles. Positive charges are accelerated downward, and negative particles are accelerated upward. As the ions move vertically, perpendicularly to the magnetic field, the magnetic field now puts a force on them. Using "right-hand rule 1," we see that both charges of ions experience a force out of the page, which is parallel to the axis of the blood vessel.

21

ELECTROMAGNETIC INDUCTION AND FARADAY'S LAW

Responses to Questions

1. The advantage of using many turns (N = large number) in Faraday's experiments is that the emf and induced current are proportional to N, which makes it easier to experimentally measure those quantities. Many turns in the primary coil will make a larger magnetic flux, and many turns in the output coil will produce a larger output voltage and current.

$\boxed{2.}$ Magnetic flux is proportional to the total number of magnetic field lines passing through an enclosed loop area: $\Phi_B = BA\cos\theta$, so the flux is proportional to the magnitude of the magnetic field. The magnetic flux depends not only on the field itself, but also on the area and on the angle between the field and the area. Thus, they also have different units (magnetic field = tesla = T; magnetic flux = $\mathrm{T\cdot m^2}$ = Wb). Another difference is that magnetic field is a vector, but magnetic flux is a scalar.

3. (*a*) A current is induced in the ring when you move the south pole toward the ring. An emf and current are induced in the ring due to the changing magnetic flux. As the magnet gets closer to the ring, more magnetic field lines are going through the ring. Using Lenz's law and the right-hand rule, the direction of the induced current when you bring the south pole toward the ring is clockwise. In this case, the number of magnetic field lines coming through the loop and pointing toward you is increasing (remember that magnetic field lines point toward the south pole of the magnet). The induced current in the loop will oppose this change in flux and will attempt to create magnetic field lines through the loop that point away from you. A clockwise induced current will provide this opposing magnetic field.

   (*b*) A current is not induced in the ring when the magnet is held steady within the ring. An emf and current are not induced in the ring since the magnetic flux through the ring is not changing while the magnet is held steady.

   (*c*) A current is induced in the ring when you withdraw the magnet. An emf and current are induced in the ring due to the changing magnetic flux. As you pull the magnet out of the ring toward you, fewer magnetic field lines are going through the ring. Using Lenz's law and the right-hand rule again, the direction of the induced current when you withdraw the south pole from the ring is counterclockwise. In this case, the number of magnetic field lines coming through the loop and pointing toward you is decreasing. The induced current in the loop will oppose this change in flux and will create more magnetic field lines through the loop that point toward you. A counterclockwise induced current will provide this opposing magnetic field.

4. (*a*) The magnetic field through the loop due to the current-carrying wire will be into the page. As the wire loop is pulled away, the flux will decrease since the magnetic field is inversely proportional to the distance from the wire. Current will be induced to increase the inward magnetic field, which means that the induced current will be clockwise.

   (*b*) If the wire loop is stationary but the current in the wire is decreased, then the inward magnetic field through the loop will again be decreasing, so again the induced current will be clockwise.

5. (*a*) There will be no magnetic field lines "piercing" the loop from top to bottom or vice versa. All of the field lines are parallel to the face of the loop. Thus there will be no magnetic flux in the loop and no change of flux in the loop, so there will be no induced current.

   (*b*) Now, since the magnet is much thicker than the loop, there will be some field lines that pierce the loop from top to bottom. The flux will increase as the magnet gets closer, so a current will be induced that makes upward-pointing field lines through the loop. The current will be counterclockwise.

6. (*a*) Yes. As the battery is connected to the front loop and current starts to flow, it will create an increasing magnetic field that points away from you and down through the two loops. Because the magnetic flux will be increasing in the second loop, an emf and current will be induced in the second loop.

   (*b*) The induced current in the second loop starts to flow as soon as the current in the front loop starts to increase and create a magnetic field (basically, immediately upon the connection of the battery to the front loop).

   (*c*) The current in the second loop stops flowing as soon as the current in the front loop becomes steady. Once the battery has increased the current in the front loop from zero to its steady-state value, the magnetic field it creates is also steady. Since the magnetic flux through the second loop is no longer changing, the induced current goes to zero.

   (*d*) The induced current in the second loop is counterclockwise. Since the increasing clockwise current in the front loop is causing an increase in the number of magnetic field lines down through the second loop, Lenz's law states that the second loop will attempt to oppose this change in flux. To oppose this change, the right-hand rule indicates that a counterclockwise current will be induced in the second loop.

   (*e*) Yes. Since both loops carry currents and create magnetic fields while the current in the front loop is increasing from the battery, each current will "feel" the magnetic field caused by the other loop.

   (*f*) The force between the two loops will repel each other. The front loop is creating a magnetic field pointed toward the second loop. This changing magnetic field induces a current in the second loop to oppose the increasing magnetic field, and this induced current creates a magnetic field pointing toward the front loop. These two magnetic fields will act like two north poles pointing at each other and repel. We can also explain it by saying that the two currents are in opposite directions, and opposing currents exert a repelling force on each other.

7. Yes, a current will be induced in the second coil. It will start when the battery is disconnected from the first coil and stop when the current falls to zero in the first coil. The current in the second loop will be clockwise. All of the reasoning is similar to that given for Question 6, except now the current is decreasing instead of increasing.

8. (*a*) The induced current in R_{A} is to the right as coil B is moved toward coil A. As B approaches A, the magnetic flux through coil A increases (there are now more magnetic field lines in coil A pointing to the left). The induced emf in coil A creates a current to produce a magnetic field that opposes this increase in flux, with the field pointing to the right through the center of the coil. A current through R_{A} to the right will produce this opposing field.

(*b*) The induced current in R_A is to the left as coil B is moved away from coil A. As B recedes from A, the magnetic flux through coil A decreases (there are now fewer magnetic field lines in coil A pointing to the left). The induced emf in coil A creates a current to produce a magnetic field that opposes this decrease in flux, pointing to the left through the center of the coil. A current through R_A to the left will produce this opposing field.

(*c*) The induced current in R_A is to the left as R_B in coil B is increased. As R_B increases, the current in coil B decreases, which also decreases the magnetic field coil B produces. As the magnetic field from coil B decreases, the magnetic flux through coil A decreases (there are now fewer magnetic field lines in coil A pointing to the left). The induced emf in coil A creates a current to produce a magnetic field pointing to the left through the center of the coil, opposing the decrease in flux. A current through R_A to the left will produce this opposing field.

9. The net current in the wire and shield combination is 0. Thus to the "outside," there is no magnetic field created by the combination. If there is an external magnetic field, it will not be influenced by the signal current, and then by Newton's third law, the signal current will not be influenced by an external magnetic field either.

10. One advantage of placing the two insulated wires carrying ac close together is that the magnetic field created by the changing current moving one way in one wire is approximately cancelled out by the magnetic field created by the current moving in the opposite direction in the second wire. Also, since large loops of wire in a circuit can generate a large self-induced back emf, by placing the two wires close to each other, or even twisting them about each other, the effective area of the current loop is decreased and the induced current is minimized.

11. When the motor first starts up, there is only a small back emf in the circuit (back emf is proportional to the rotation speed of the motor). This allows a large current to flow to the refrigerator. The power source for the house can be treated as an emf with an internal resistance. This large current to the refrigerator motor from the power source reduces the voltage across the power source because of its internal resistance. Since the power source voltage has decreased, other items (like lights) will have a lower voltage across them and receive less current, so may "dim." As the motor speeds up to its normal operational speed, the back emf increases to its normal level and the current delivered to the motor is now limited to its usual amount. This current is no longer enough to significantly reduce the output voltage of the power source, so the other devices then get their normal voltage. Thus, the lights flicker just when the refrigerator motor first starts up. A heater, on the other hand, draws a large amount of current (it is a very low-resistance device) at all times. (The heat-producing element is not a motor, so has very little induction associated with it.) The source is then continually delivering a large current, which continually reduces the output voltage of the power source. In an ideal situation, the source could provide any amount of current to the whole circuit in either situation. In reality, though, the higher current in the wires causes bigger losses of energy along the way to the devices, so the lights dim.

12. Figure 21–17 shows that the induced current in the upper armature segment points into the page. This can be shown using the right-hand rule. The charges in the top metal armature segment are moving in the direction of the velocity shown with the green arrow (up and to the right), and these moving charges are in a magnetic field shown with the blue arrows (to the right). The right-hand rule says that the charges experience a force into the page producing the induced current. This induced current is also in the same magnetic field. Using another right-hand rule, a current-carrying wire, with the current going into the page (as in the upper armature segment) in a magnetic field pointed to the right, will experience a force in a downward direction. This downward force exerts a counterclockwise torque on the armature while it is rotation in a clockwise direction. The back emf is opposing the motion of the armature during its operation.

13. Eddy current brakes will work on metallic wheels, such as copper and aluminum. Eddy current brakes do not need to act on ferromagnetic wheels. The external magnetic field of the eddy brake just needs to interact with the "free" conduction electrons in the metal wheels in order to have the braking effect. First, the magnetic field creates eddy currents in the moving metal wheel using the free conduction electrons (the right-hand rule says moving charges in a magnetic field will experience a magnetic force, making them move and creating an eddy current). This eddy current is also in the braking magnetic field. The right-hand rule says these currents will experience a force opposing the original motion of the piece of metal and the eddy current brake will begin to slow the wheel. Good conductors, such as copper and aluminum, have many free conduction electrons and will allow large eddy currents to be created, which in turn will provide good braking results.

$\boxed{14.}$ As a magnet falls through a metal tube, an increase in the magnetic flux is created in the areas ahead of it in the tube. This flux change induces a current to flow around the tube walls to create an opposing magnetic field in the tube (Lenz's law). This induced magnetic field pushes against the falling magnet and reduces its acceleration. The speed of the falling magnet increases until the magnetic force on the magnet is the same size as the gravity force. The opposing magnetic field cannot cause the magnet to actually come to a stop, since then the flux would become a constant and the induced current would disappear, as would the opposing magnetic field. Thus, the magnet reaches a state of equilibrium and falls at a constant terminal velocity. The weight of the magnet is balanced by the upward force from the eddy currents.

15. The nonferrous materials are not magnetic, but they are conducting. As they pass by the permanent magnets, eddy currents will be induced in them. The eddy currents provide a "braking" mechanism which will cause the metallic materials to slide more slowly down the incline than the nonmetallic materials. The nonmetallic materials will reach the bottom with larger speeds. The nonmetallic materials can therefore be separated from the metallic, nonferrous materials by placing bins at different distances from the bottom of the incline. The closest bin will catch the metallic materials, since their projectile velocities off the end of the incline will be small. The bin for the nonmetallic materials should be placed farther away to catch the higher-velocity projectiles.

16. The slots in the metal bar prevent the formation of large eddy currents, which would slow the bar's fall through the region of magnetic field. The smaller eddy currents then experience a smaller opposing force to the motion of the metal bar. Thus, the slotted bar falls more quickly through the magnetic field.

17. This is similar to the situation accompanying Fig. 21–20. As the aluminum sheet is moved through the magnetic field, eddy currents are created in the sheet. The magnetic force on these induced currents opposes the motion. Thus it requires some force to pull the sheet out.

18. The speed of the magnet in case (b) will be larger than that in case (a). As the bar magnet falls through the loop, it sets up an induced current in the loop, which opposes the change in flux. This current acts like a magnet that is opposing the physics magnet, repelling it and so reducing its speed. Then, after the midpoint of the magnet passes through the loop, the induced current will reverse its direction and attract the falling magnet, again reducing its speed.

19. As the metal bar enters (or leaves) the magnetic field during the swinging motion, areas of the metal bar experience a change in magnetic flux. This changing flux induces eddy currents with the "free" conduction electrons in the metal bar. These eddy currents are then acted on by the magnetic field, and the resulting force opposes the motion of the swinging metal bar. This opposing force acts on the bar no matter which direction it is swinging through the magnetic field, thus damping the motion during both directions of the swing.

20. To determine the ratio of turns on the two coils of a transformer without taking it apart, apply a known ac input source voltage to one pair of leads and carefully measure the output voltage across the other two leads. Then, $V_{\text{source}}/V_{\text{output}} = N_{\text{source}}/N_{\text{output}}$, which provides us with the ratio of turns on the two coils. To determine which leads are paired with which, you could use an ohmmeter, since the two source wires are in no way electrically connected to the two output wires. If the resistance between two wires is very small, then those two wires are a pair. If the resistance between two wires is infinite, then those two wires are not a pair.

21. Higher voltages, such as 600 V or 1200 V, would be dangerous if they were used in household wires. Having a larger voltage than the typical 120 V would mean that any accidental contact with a "live" wire would send more current through a person's body. Such a large potential difference between household wires and anything that is grounded (other wires, people, etc.) would more easily cause electrical breakdown of the air, and then much more sparking would occur. Basically, this would supply each of the charges in the household wires with much more energy than the lower voltages, which would allow them to arc to other conductors. This would increase the possibility of more short circuits and accidental electrocutions.

22. When 120 V dc is applied to the transformer, there is no induced back emf that would usually occur with 120 V ac. This means that the 120 V dc encounters much less resistance than the 120 V ac, resulting in too much current in the primary coils. This large amount of current could overheat the coils, which are usually wound with many loops of very fine, low-resistance wire, and could melt the insulation and burn out or short out the transformer.

23. (*a*) To create the largest amount of mutual inductance with two flat circular coils of wire, you would place them face-to-face and very close to each other. This way, almost all of the magnetic flux from one coil also goes through the other coil.

(*b*) To create the least amount of mutual inductance with two flat circular coils, you would place them with their faces at right angles. This way, almost none of the magnetic flux from one coil goes through the other coil.

24. (*a*) No. Although the current through an *LR* circuit is described by $I = \dfrac{V_0}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$, we can substitute $\dfrac{V_0}{R} = I_{\max}$. Thus, a given fraction of a maximum possible current, $I/I_{\max}$, is equal to $1 - e^{-\frac{t}{L/R}}$ and is independent of the battery emf.

(*b*) Yes. Since $I = \dfrac{V_0}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$, if a given value of the current is desired, then it is dependent on the value of the battery emf.

25. (*a*) Yes.

(*b*) Yes.

The rms voltages across either an inductor or a capacitor of an *LRC* circuit can be greater than the rms source voltage because the different voltages are out of phase with each other. At any given instant, the voltage across either the inductor or the capacitor could be negative, for example, thus allowing for a very large positive voltage on the other device. (The rms voltages, however, are always positive by definition.)

26. (*a*) The frequency of the source emf does not affect the impedance of a pure resistance. The impedance of a pure resistance is independent of the source emf frequency.

(*b*) The impedance of a pure capacitance varies inversely with the frequency of the source emf according to $X_C = 1/2\pi fC$. As the source frequency gets very small, the impedance of the capacitor gets very large, and as the source frequency gets very large, the impedance of the capacitor gets very small.

(*c*) The impedance of a pure inductance varies directly with the frequency of the source emf according to $X_L = 2\pi fL$. Thus, as the source frequency gets very small, the impedance of the inductor gets very small, and as the source frequency gets very large, the impedance of the inductor gets very large.

(*d*) The impedance of an *LRC* circuit with a small *R* near resonance is very sensitive to the frequency of the source emf. If the frequency is set at resonance exactly, where $X_L = X_C$, then the *LRC* circuit's impedance is very small and equal to *R*. The impedance increases rapidly as the source frequency is either increased or decreased a small amount from resonance.

(*e*) The impedance of an *LRC* circuit with a small *R* very far from resonance depends on whether the source frequency is much higher or much lower than the resonance frequency. If the source frequency is much higher than resonance, then the impedance is directly proportional to the frequency of the source emf. Basically, at extremely high frequencies, the circuit impedance is equal to $2\pi fL$. If the source frequency is much lower than resonance (nearly zero), then the impedance is inversely proportional to the frequency of the source emf. Basically, at extremely low frequencies, the circuit impedance is equal to $1/2\pi fC$.

27. To make the impedance of an *LRC* circuit a minimum, make the resistance very small and make the reactance of the capacitor equal to the reactance of the inductor: $X_L = X_C$, or

$$2\pi fL = \frac{1}{2\pi fC} \quad \rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}}.$$

28. In an *LRC* circuit, the current and the voltage in the circuit both oscillate. The energy stored in the circuit also oscillates and is alternately stored in the magnetic field of the inductor and the electric field of the capacitor.

29. Yes. The instantaneous voltages across the different elements in the circuit will be different, but the current through each element in the series circuit is the same.

Responses to MisConceptual Questions

1. (*b*, *d*) The right-hand rule shows that a clockwise current will create a flux in the loop that points into the page. Since the initial magnetic flux is into the page, the current will be induced when the magnetic flux decreases. This happens when the size of the coil decreases or the magnetic field becomes tilted. Increasing the magnetic field, as in answer (*a*), will create a counterclockwise current. Moving the coil sideways, as in answer (*c*), does not change the flux, so no current would be induced.

2. (*c*) A common misconception is that a moving loop would experience a change in flux. However, if the loop is moving through a constant field without rotation, then the flux through the loop will remain constant and no current will be induced.

3. (*d*) A current is induced in the loop when the flux through the loop is changing. As the loop passes through line J it enters a region with a magnetic field, so the flux through the loop increases and a current will be induced. When the loop passes line K, the flux remains constant, as there is no change in field, so no current is induced. As the loop passes line L, the magnetic field flux through the loop decreases and a current is again induced.

4. (*c, a*) The magnetic field near a long straight wire is inversely proportional to the distance from the wire. For C, the loop remains at the same distance from the wire, so the magnetic flux through the wire remains constant and no current is induced in the loop. For D, the magnetic field from the long wire points into the page in the region of the loop. As the loop moves away from the wire, the magnetic flux decreases, so a clockwise current is induced in the loop to oppose the change in flux.

5. (*c*) The induced current creates a magnetic field that opposes the motion of the magnet. The result is that the magnet has less kinetic energy than it would have had if the loop were not present. The change in gravitational potential energy provides the energy for the current.

6. (*c*) Since the flux through the loop is increasing, an emf will be produced. However, since plastic is not a conductor, no current will be induced. The emf is produced regardless of whether there is a conducting path for current or not.

7. (*b*) A current will be induced in the loop whenever the magnetic flux through the loop is changed. Increasing the current in the wire will increase the magnetic field produced by the wire and therefore the magnetic flux in the loop. Rotating the loop changes the angle between the loop and wire, which will change the flux. Since the magnetic field strength decreases with distance from the wire, moving the loop away from the wire, either with or without rotation, will decrease the flux in the wire. If the loop is moved parallel to the wire, then the flux through the loop will not change, so no current will be created in the loop.

8. (*c*) When a steady current flows in the first coil, it creates a constant magnetic field and therefore a constant magnetic flux through the second coil. Since the flux is not changing, a current will not be induced in the second coil. If the current in the first coil changes, then the flux through the second will also change, inducing a current in the second coil.

9. (*b*) A generator converts mechanical energy into electric energy. The generator's magnetic field remains unchanged as the generator operates, so energy is not being pulled from the magnetic field. Resistance in the coils removes electric energy from the system in the form of heat—it does not provide the energy. In order for the generator to work, an external force is necessary to rotate the generator's axle. This external force does work, which is converted to electric energy.

10. (*c*) Increasing the rotation frequency increases the rate at which the flux changes; therefore, answer (*a*) will increase the output voltage. Answers (*b*), (*d*), and (*e*) all increase the maximum flux in the coil. Since the generator's voltage output is proportional to the rate of change of the flux through the coil, increasing the maximum flux results in a greater voltage output. When the magnetic field is parallel to the generator's axis, little to no flux passes through the coil. With less flux passing through the coil, there will be a lower output voltage.

11. (*d*) A common misconception is that a transformer only changes voltage. However, power is conserved across a transformer, where power is the product of the voltage and current. When a transformer increases the voltage, it must also proportionately decrease the current.

12. (*d*) If the charger unit had a battery, then it could run the laptop without being plugged in. A motor converts electrical energy into mechanical energy, but the laptop charger output is electrical energy, not mechanical energy. A generator converts mechanical energy into electric energy, but the input to the laptop is electrical, not mechanical. The cables to and from the charger unit can be considered transmission lines, but they are not the important component inside the charger unit. A transformer can convert a high-voltage input into a low-voltage output without power loss. This is a significant function of the charger unit.

13. (*a*) In a transformer with no lost flux, the power across the transformer (product of current and voltage) is constant across the transformer. Therefore, if the voltage increases, then the current must decrease across the transformer. If the current increases, then the voltage must decrease across the transformer. A transformer works due to the induced voltage created by the changing flux. A dc circuit does not have a changing flux, so a transformer does not work with dc current.

14. (*e*) It may appear that (*b*) is the correct answer if the problem is interpreted as an ac step-up transformer with twice as many loops in the secondary coil as in the primary. It is true that an ac current would double the voltage and cut the current in half; however, this is a dc current. A dc current does not produce a changing flux, so no current or voltage will be induced in the secondary coil.

15. (*b*) Many people may not realize that generators (rotating coils in a magnetic field) are the heart of most electric power plants that produce the alternating current in wall outlets.

16. (*b*) A common misconception by users of credit card readers is that they are swiping their cards too quickly, so they slow down the swipe. The credit card has a magnetic stripe with information encoded in the magnetic field. Swiping the card more rapidly increases the induced emf, due to the greater rate of change of magnetic flux.

17. (*b*) In a series circuit (whether ac or dc), the current is the same at every point in the circuit. In an ac circuit, the current is out of phase with the voltage across an inductor and the voltage across a capacitor, so (*a*) is incorrect. In nonresonant ac circuits, there is a phase difference between the current and the voltage source, so (*c*) is not correct. The current and voltage across a resistor are always in phase, so the resistor does not change the phase in a circuit, so (*d*) is incorrect.

Solutions to Problems

$\boxed{1.}$ The magnitude of the average induced emf is given by Eq. 21–2b.

$$|\mathscr{E}| = N\frac{\Delta\Phi_B}{\Delta t} = 2\frac{38\text{ Wb} - (-58\text{ Wb})}{0.34\text{ s}} = 564.7\text{ V} \approx \boxed{560\text{ V}}$$

A direction for the emf cannot be specified without more information.

2. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

$\boxed{3.}$ As the coil is pushed into the field, the magnetic flux through the coil increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is $\boxed{\text{counterclockwise}}$.

4. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is $\boxed{\text{counterclockwise}}$ as viewed from the right end of the solenoid.

5. The flux changes because the loop rotates. The angle between the field and the normal to the loop changes from 0° to 90°. The magnitude of the average induced emf is given by Eq. 21–2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{AB\Delta\cos\theta}{\Delta t} = -\frac{\pi(0.0925\text{ m})^2 1.5\text{ T}(\cos 90° - \cos 0°)}{0.20\text{ s}} = \boxed{0.20\text{ V}}$$

$\boxed{6.}$ We choose up as the positive direction. The average induced emf is given by Eq. 21–2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.054\text{ m})^2(-0.25\text{ T} - 0.48\text{ T})}{0.16\text{ s}} = \boxed{4.2\times10^{-2}\text{ V}}$$

7. (*a*) When the plane of the loop is perpendicular to the field lines, the flux is given by the maximum of Eq. 21–1.

$$\Phi_B = BA = B\pi r^2 = (0.50\text{ T})\pi(0.080\text{ m})^2 = 1.005\times10^{-2}\text{ Wb} \approx \boxed{1.0\times10^{-2}\text{ Wb}}$$

(*b*) The angle is $\theta = 90° - 42° = \boxed{48°}$.

(*c*) Use Eq. 21–1.

$$\Phi_B = BA\cos\theta = B\pi r^2 = (0.50\text{ T})\pi(0.080\text{ m})^2\cos 48° = \boxed{6.7\times10^{-3}\text{ Wb}}$$

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, then the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

9. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 21–3.

$$\mathscr{E} = B\ell v = (0.800\text{ T})(0.120\text{ m})(0.180\text{ m/s}) = \boxed{1.73\times10^{-2}\text{ V}}$$

(*b*) Because the upward flux is increasing, the induced flux will be into the page, so the induced current is clockwise. Thus the induced emf in the rod is down, which means that the electric field will be down. The electric field is the induced voltage per unit length.

$$E = \frac{\mathscr{E}}{\ell} = \frac{1.73\times10^{-2}\text{ V}}{0.120\text{ m}} = \boxed{0.144\text{ V/m, down}}$$

10. (*a*) The magnetic flux through the loop is into the page and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The average induced emf is given by Eq. 21–2a.

$$\left|\mathscr{E}_{\text{avg}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B|\Delta A|}{\Delta t} = \frac{(0.65\text{ T})\pi[(0.100\text{ m})^2 - (0.030\text{ m})^2]}{0.50\text{ s}}$$

$$= 3.717\times10^{-2}\text{ V} \approx \boxed{3.7\times10^{-2}\text{ V}}$$

(*c*) We find the average induced current from Ohm's law.

$$I = \frac{\mathcal{E}}{R} = \frac{3.717\times10^{-2}\ \text{V}}{2.5\ \Omega} = \boxed{1.5\times10^{-2}\ \text{A}}$$

11. (*a*) Because the current is constant, there will be no change in flux, so the induced current will be $\boxed{\text{zero}}$.

(*b*) The decreasing current in the wire will cause a decreasing field into the page through the loop. To oppose this decrease, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*c*) The decreasing current in the wire will cause a decreasing field out of the page through the loop. To oppose this decrease, the induced current in the loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*d*) The increasing current in the wire will cause an increasing field out of the page through the loop. To oppose this increase, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

12. The emf induced in the short coil is given by Eq. 21–2b, where *N* is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the *B* field created by the solenoid. The field in a solenoid is given by Eq. 20–8, $B = \mu_0 I N_{\text{solenoid}}/\ell_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathcal{E}| = \frac{N_{\text{short}} A_{\text{short}} \Delta B}{\Delta t} = \frac{N_{\text{short}} A_{\text{short}} \Delta\left(\dfrac{\mu_0 I N_{\text{solenoid}}}{\ell_{\text{solenoid}}}\right)}{\Delta t} = \frac{\mu_0 N_{\text{short}} N_{\text{solenoid}} A_{\text{short}}}{\ell_{\text{solenoid}}}\frac{\Delta I}{\Delta t}$$

$$= \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(14)(600)\pi(0.0125\ \text{m})^2}{(0.25\ \text{m})}\frac{(5.0\ \text{A})}{(0.60\ \text{s})} = \boxed{1.7\times10^{-4}\ \text{V}}$$

The induced emf will oppose the emf that is being used to create the 5.0-A current.

13. Since the antenna is vertical, the maximum emf will occur when the car is traveling perpendicular to the horizontal component of the Earth's magnetic field. This occurs when the car is traveling in the $\boxed{\text{east or west direction}}$. We calculate the magnitude of the emf using Eq. 21–3, where *B* is the horizontal component of the Earth's magnetic field.

$$\mathcal{E} = B_x \ell v = [(5.0\times10^{-5}\ \text{T})\cos 38°](0.55\ \text{m})(30.0\ \text{m/s}) = 6.501\times10^{-4}\ \text{V} = \boxed{0.65\ \text{mV}}$$

14. As the loop is pulled from the field, the flux through the loop decreases, causing an induced emf whose magnitude is given by Eq. 21–3, $\mathcal{E} = B\ell v$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I = \mathcal{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F = I\ell B$.

$$F = I\ell B = \frac{\mathcal{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2}{R} = \frac{(0.550\ \text{T})^2(0.350\ \text{m})^2(3.10\ \text{m/s})}{0.230\ \Omega} = \boxed{0.499\ \text{N}}$$

15. (*a*) There is a conventional current flowing in the rod, from top to bottom, equal to the induced emf divided by the resistance of the bar. That current is in a magnetic field and, by the right-hand rule, will have a force on it (and the bar) to the left in Fig. 21–11a. The external force must equal that magnetic force in order for the bar to move with a constant speed.

$$F_{\text{external}} = F_{\text{magentic}} = I\ell B = \frac{\mathcal{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \boxed{\frac{B^2\ell^2 v}{R}}$$

(*b*) The power required is the external force times the speed of the rod.

$$P = Fv = \boxed{\frac{B^2\ell^2v^2}{R}}$$

16. From the derivation in Problem 15a, we have an expression relating the external force to the magnetic field.

$$F_{\text{external}} = \frac{B^2\ell^2 v}{R} \quad \to \quad B = \sqrt{\frac{F_{\text{external}}R}{\ell^2 v}} = \sqrt{\frac{(0.350\text{ N})(0.25\ \Omega)}{(0.200\text{ m})^2(1.50\text{ m/s})}} = 1.208\text{ T} \approx \boxed{1.2\text{ T}}$$

17. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 21–3.

$$\mathscr{E} = B\ell v = (0.35\text{ T})(0.300\text{ m})(1.6\text{ m/s}) = 0.168\text{ V} \approx \boxed{0.17\text{ V}}$$

(*b*) Find the induced current from Ohm's law.

$$I = \frac{\mathscr{E}}{R} = \frac{0.168\text{ V}}{21.0\ \Omega + 2.5\ \Omega} = 7.149\times10^{-3}\text{ A} \approx \boxed{7.1\times10^{-3}\text{ A}}$$

(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by Eq. 20–2.

$$F = I\ell B = (7.149\times10^{-3}\text{ A})(0.300\text{ m})(0.35\text{ T}) = 7.506\times10^{-4}\text{ N} \approx \boxed{7.5\times10^{-4}\text{ N}}$$

18. (*a*) There is an emf induced in the coil since the flux through the coil changes. The current in the coil is the induced emf divided by the resistance of the coil. The resistance of the coil is found from Eq. 18–3.

$$|\mathscr{E}| = NA_{\text{coil}}\frac{\Delta B}{\Delta t} \qquad R = \frac{\rho\ell}{A_{\text{wire}}}$$

$$I = \frac{\mathscr{E}}{R} = \frac{NA_{\text{coil}}\dfrac{\Delta B}{\Delta t}}{\dfrac{\rho\ell}{A_{\text{wire}}}} = \frac{NA_{\text{coil}}A_{\text{wire}}}{\rho\ell}\frac{\Delta B}{\Delta t}$$

$$= \frac{30[\pi(0.110\text{ m})^2][\pi(1.3\times10^{-3}\text{ m})](8.65\times10^{-3}\text{ T/s})}{(1.68\times10^{-8}\,\Omega\cdot\text{m})30(2\pi)(0.110\text{ m})} = 0.1504\text{ A} \approx \boxed{0.15\text{ A}}$$

(*b*) The rate at which thermal energy is produced in the wire is the power dissipated in the wire.

$$P = I^2R = I^2\frac{\rho\ell}{A_{\text{wire}}} = (0.1504\text{ A})^2\frac{(1.68\times10^{-8}\ \Omega\cdot\text{m})30(2\pi)(0.110\text{ m})}{\pi(1.3\times10^{-3}\text{ m})^2}$$

$$= 1.484\times10^{-3}\text{ W} \approx \boxed{1.5\times10^{-3}\text{ W}}$$

19. The charge that passes a given point is the current times the elapsed time, $Q = I\Delta t.$ The current will be the emf divided by the resistance, $I = \frac{\mathscr{E}}{R}.$ The resistance is given by Eq. 18–3, $R = \frac{\rho\ell}{A_{\text{wire}}},$ and the emf is given by Eq. 21–2b. Combine these equations to find the charge during the operation.

$$|\mathscr{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}|\Delta B|}{\Delta t}; \quad R = \frac{\rho\ell}{A_{\text{wire}}}; \quad I = \frac{\mathscr{E}}{R} = \frac{\dfrac{A_{\text{loop}}|\Delta B|}{\Delta t}}{\dfrac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}} A_{\text{wire}} |\Delta B|}{\rho \ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2 |\Delta B|}{\rho (2\pi) r_{\text{loop}}} = \frac{r_{\text{loop}} \pi r_{\text{wire}}^2 |\Delta B|}{2\rho}$$

$$= \frac{(0.066 \text{ m})\pi(1.125\times10^{-3} \text{ m})^2 (0.670 \text{ T})}{2(1.68\times10^{-8} \ \Omega\cdot\text{m})} = \boxed{5.23 \text{ C}}$$

20. From Eq. 21–5, the induced voltage is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathscr{E}_1}{\omega_1} = \frac{\mathscr{E}_2}{\omega_2} \quad \rightarrow \quad \mathscr{E}_2 = \mathscr{E}_1 \frac{\omega_2}{\omega_1} = (12.7 \text{ V}) \frac{2500 \text{ rpm}}{1100 \text{ rpm}} = 28.86 \text{ V} \approx \boxed{29 \text{ V}}$$

21. (*a*) The rms voltage is found from the peak induced emf. The peak induced emf is calculated from Eq. 21–5.

$$\mathscr{E}_{\text{peak}} = NB\omega A \quad \rightarrow$$

$$V_{\text{rms}} = \frac{\mathscr{E}_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(550)(0.55 \text{ T})(2\pi \text{ rad/rev})(120 \text{ rev/s})\pi(0.040 \text{ m})^2}{\sqrt{2}}$$

$$= 810.7 \text{ V} \approx \boxed{810 \text{ V}}$$

(*b*) To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s.

$\boxed{22.}$ (*a*) The peak current is found from the rms current.

$$I_{\text{peak}} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(70.0 \text{ A}) = \boxed{99.0 \text{ A}}$$

(*b*) The area can be found from Eq. 21–5.

$$\mathscr{E}_{\text{peak}} = NB\omega A = \sqrt{2}\, V_{\text{rms}} \quad \rightarrow$$

$$A = \frac{\sqrt{2}\, V_{\text{rms}}}{NB\omega} = \frac{\sqrt{2}\,(150 \text{ V})}{(950)(0.030 \text{ T})(85 \text{ rev/s})(2\,\pi \text{ rad/rev})} = 0.01394 \text{ m}^2 \approx \boxed{1.4\times10^{-2} \text{ m}^2}$$

23. We assume that the voltage given is an rms value and that the power is an average power, as in Eq. 18–9a. We calculate the resistance of the wire on the armature using Eq. 18–3.

$$R_{\text{wire}} = \frac{\rho\ell}{A} = \frac{(1.68\times10^{-8} \ \Omega\cdot\text{m})[85(4)(0.060 \text{ m})]}{\pi[\frac{1}{2}(5.9\times10^{-4} \text{ m})]^2} = 1.254 \ \Omega$$

$$I_{\text{rms}} = \frac{\overline{P}}{V_{\text{rms}}} = \frac{25.0 \text{ W}}{12.0 \text{ V}} = 2.083 \text{ A}$$

$$V_{\text{circuit}} = V_{\text{bulb}} + V_{\text{wire}} = V_{\text{bulb}} + IR_{\text{wire}} = 12 \text{ V} + (2.083 \text{ A})(1.254 \ \Omega) = 14.61 \text{ V}$$

So the generator's rms emf must be 14.61 volts. We find the frequency of the generator from Eq. 21–5.

$$\frac{\mathscr{E}}{\sqrt{2}} = \frac{2\pi fNBA}{\sqrt{2}} = 14.61 \text{ V} \quad \rightarrow$$

$$f = \frac{(14.61 \text{ V})\sqrt{2}}{2\pi NBA} = \frac{(14.61 \text{ V})\sqrt{2}}{2\pi(85)(0.65 \text{ T})(0.060 \text{ m})^2} = 16.54 \text{ Hz} \approx \boxed{17 \text{ Hz}} = 17 \text{ cycles/s}$$

24. When the motor is running at full speed, the back emf opposes the applied emf, to give the net voltage across the motor.

$$\mathscr{E}_{\text{applied}} - \mathscr{E}_{\text{back}} = IR \rightarrow \mathscr{E}_{\text{back}} = \mathscr{E}_{\text{applied}} - IR = 120 \text{ V} - (8.20 \text{ A})(3.65 \ \Omega) = \boxed{90 \text{ V}}$$

(2 significant figures)

25. From Eq. 21–5, the induced voltage (back emf) is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathscr{E}_1}{\omega_1}=\frac{\mathscr{E}_2}{\omega_2} \rightarrow \mathscr{E}_2=\mathscr{E}_1\frac{\omega_2}{\omega_1}=\mathscr{E}_1\frac{2\pi f_2}{2\pi f_1}=\mathscr{E}_1\frac{f_2}{f_1}=(72\text{ V})\frac{2300\text{ rpm}}{1800\text{ rpm}}=\boxed{92\text{ V}}$$

26. The back emf is proportional to the rotation speed (Eq. 21–5). Thus if the motor is running at half speed, then the back emf is half the original value, or 54 V. Find the new current from writing a loop equation for the motor circuit, from Fig. 21–19.

$$\mathscr{E}-\mathscr{E}_{\text{back}}-IR=0 \rightarrow I=\frac{\mathscr{E}-\mathscr{E}_{\text{back}}}{R}=\frac{120\text{ V}-54\text{ V}}{5.0\ \Omega}=\boxed{13\text{ A}}$$

27. We find the number of turns in the secondary from Eq. 21–6.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}} \rightarrow N_{\text{S}}=N_{\text{P}}\frac{V_{\text{S}}}{V_{\text{P}}}=(148\text{ turns})\frac{13{,}500\text{ V}}{117\text{ V}}\approx\boxed{17{,}100\text{ turns}}$$

28. Because $N_{\text{S}}<N_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer. Use Eq. 21–6 to find the voltage ratio and Eq. 21–7 to find the current ratio.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}}=\frac{120\text{ turns}}{360\text{ turns}}=\boxed{\frac{1}{3}\text{ or }0.33} \qquad \frac{I_{\text{S}}}{I_{\text{P}}}=\frac{N_{\text{P}}}{N_{\text{S}}}=\frac{360\text{ turns}}{120\text{ turns}}=\boxed{3.0}$$

29. Use Eqs. 21–6 and 21–7 to relate the voltage and current ratios.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}};\ \frac{I_{\text{S}}}{I_{\text{P}}}=\frac{N_{\text{P}}}{N_{\text{S}}} \rightarrow \frac{V_{\text{S}}}{V_{\text{P}}}=\frac{I_{\text{P}}}{I_{\text{S}}} \rightarrow \frac{I_{\text{S}}}{I_{\text{P}}}=\frac{V_{\text{P}}}{V_{\text{S}}}=\frac{25\text{ V}}{120\text{ V}}=\boxed{0.21}$$

30. We find the ratio of the number of turns from Eq. 21–6.

$$\frac{N_{\text{S}}}{N_{\text{P}}}=\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{12{,}000\text{ V}}{240\text{ V}}=\boxed{50}\text{ (2 significant figures)}$$

If the transformer is connected backward, then the role of the turns will be reversed:

$$\frac{N_{\text{S}}}{N_{\text{P}}}=\frac{V_{\text{S}}}{V_{\text{P}}} \rightarrow \frac{1}{50}=\frac{V_{\text{S}}}{240\text{ V}} \rightarrow V_{\text{S}}=\frac{1}{50}(240\text{ V})=\boxed{4.8\text{ V}}$$

$\boxed{31.}$ (*a*) Use Eqs. 21–6 and 21–7 to relate the voltage and current ratios.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}};\ \frac{I_{\text{S}}}{I_{\text{P}}}=\frac{N_{\text{P}}}{N_{\text{S}}} \rightarrow \frac{V_{\text{S}}}{V_{\text{P}}}=\frac{I_{\text{P}}}{I_{\text{S}}} \rightarrow V_{\text{S}}=V_{\text{P}}\frac{I_{\text{P}}}{I_{\text{S}}}=(120\text{ V})\frac{0.35\text{ A}}{6.8\text{ A}}=\boxed{6.2\text{ V}}$$

(*b*) Because $V_{\text{S}}<V_{\text{P}}$, this is a $\boxed{\text{step-down}}$ transformer.

32. (*a*) We assume 100% efficiency and find the input voltage from $P=IV$.

$$P=I_{\text{P}}V_{\text{P}} \rightarrow V_{\text{P}}=\frac{P}{I_{\text{P}}}=\frac{95\text{ W}}{25\text{ A}}=3.8\text{ V}$$

Since $V_{\text{P}}<V_{\text{S}}$, this is a $\boxed{\text{step-up}}$ transformer.

(*b*) $$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{12\text{ V}}{3.8\text{ V}}=\boxed{3.2}$$

33. Use Eqs. 21–6 and 21–7 to relate the voltage and current ratios.

$$\frac{V_{\text{S}}}{V_{\text{P}}}=\frac{N_{\text{S}}}{N_{\text{P}}} \rightarrow V_{\text{S}}=V_{\text{P}}\frac{N_{\text{S}}}{N_{\text{P}}}=(120\text{ V})\frac{1240\text{ turns}}{330\text{ turns}}=450.9\text{ V}\approx\boxed{450\text{ V}}$$

$$\frac{I_S}{I_P}=\frac{N_P}{N_S} \quad\rightarrow\quad I_P=I_S\frac{N_S}{N_P}=(15.0\text{ A})\frac{1240\text{ turns}}{330\text{ turns}}=\boxed{56\text{ A}}$$

34. (*a*) The current in the transmission lines can be found from Eq. 18–9a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}}=V_{\text{rms}}I_{\text{rms}} \quad\rightarrow\quad I_{\text{rms}}=\frac{P_{\text{town}}}{V_{\text{rms}}}=\frac{35\times10^6\text{ W}}{45\times10^3\text{ V}}=778\text{ A}$$

$$\mathcal{E}-IR+V_{\text{output}}=0 \quad\rightarrow$$

$$\mathcal{E}=IR+V_{\text{output}}=\frac{P_{\text{town}}}{V_{\text{rms}}}R+V_{\text{rms}}=\frac{35\times10^6\text{ W}}{45\times10^3\text{ V}}(4.6\ \Omega)+45\times10^3\text{ V}=48,578\text{ V}\approx\boxed{49\text{ kV(rms)}}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}}=I_{\text{rms}}^2R$.

$$\text{Fraction wasted}=\frac{P_{\text{loss}}}{P_{\text{total}}}=\frac{P_{\text{loss}}}{P_{\text{town}}+P_{\text{loss}}}=\frac{I_{\text{rms}}^2R}{P_{\text{town}}+I_{\text{rms}}^2R}=\frac{(778\text{ A})^2(4.6\ \Omega)}{35\times10^6\text{ W}+(778\text{ A})^2(4.6\ \Omega)}$$

$$=\boxed{0.074}=7.4\%$$

35. At the power plant, 100 kW of power is delivered at 12,000 V. We first find the initial current from Eq. 18–9a.

$$P=VI \quad\rightarrow\quad I=\frac{P}{V}=\frac{100\times10^3\text{ W}}{12\times10^3\text{ V}}=8.333\text{ A}$$

Then we have a step-up transformer, which steps up the voltage by a factor of 20, so the current is stepped down by a factor of 20.

$$\frac{I_S}{I_P}=\frac{N_P}{N_S} \quad\rightarrow\quad I_S=I_P\frac{N_P}{N_S}=(8.333\text{ A})\frac{1}{20}=0.4167\text{ A}$$

The power lost is given by Eq. 18–6a.

$$P_{\text{lost}}=I^2R= \quad\rightarrow\quad 50\times10^3\text{ W}=(0.4167\text{ A})^2(5\times10^{-5}\ \Omega/\text{m})(x) \quad\rightarrow$$

$$x=\frac{50\times10^3\text{ W}}{(0.4167\text{ A})^2(5\times10^{-5}\ \Omega/\text{m})}=5.76\times10^9\text{ m}\approx\boxed{6\times10^9\text{ m}}$$

Note that we are not taking into account any power losses that occur after the step-down transformers.

36. (*a*) At the power plant, the voltage is changed from 12,000 V to 240,000 V.

$$\frac{N_S}{N_P}=\frac{V_S}{V_P}=\frac{240{,}000\text{ V}}{12{,}000\text{ V}}=\boxed{20}\ \text{(2 significant figures)}$$

(*b*) At the house, the voltage is changed from 7200 V to 240 V.

$$\frac{N_S}{N_P}=\frac{V_S}{V_P}=\frac{240\text{ V}}{7200\text{ V}}=\boxed{3.3\times10^{-2}}$$

37. Without the transformers, we find the delivered current, which is the current in the transmission lines, from the delivered power and the power lost in the transmission lines.

$$P_{\text{out}}=V_{\text{out}}I_{\text{line}} \quad\rightarrow\quad I_{\text{line}}=\frac{P_{\text{out}}}{V_{\text{out}}}=\frac{2.0\times10^6\text{ W}}{120\text{ V}}=1.667\times10^4\text{ A}$$

$$P_{\text{lost}}=I_{\text{line}}^2R_{\text{line}}=(1.667\times10^4\text{ A})^2\,2(0.100\ \Omega)=5.556\times10^7\text{ W}$$

Thus there must be $2.0\times10^6\text{ W}+5.556\times10^7\text{ W}\approx5.756\times10^7\text{ W}$ of power generated at the start of the process.

With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer must still be 1.667×10^4 A. Using the step-down transformer efficiency, we calculate the current in the transmission lines and the loss in the transmission lines.

$$P_{\text{out}}=0.99P_{\substack{\text{line}\\\text{end}}}\quad\rightarrow\quad V_{\text{out}}I_{\text{out}}=0.99V_{\text{line}}I_{\text{line}}\quad\rightarrow$$

$$I_{\text{line}}=\frac{V_{\text{out}}I_{\text{out}}}{0.99V_{\text{line}}}=\frac{(120\text{ V})(1.66\times10^4\text{ A})}{(0.99)(1200\text{ V})}=1.684\times10^3\text{ A}$$

$$P_{\text{lost}}=I_{\text{line}}^2R_{\text{line}}=(1.684\times10^3\text{ A})^2\,2(0.100\,\Omega)=5.671\times10^5\text{ W}=0.5671\text{ MW}$$

The power to be delivered is 2.0 MW. The power that must be delivered to the step-down transformer is $\dfrac{2.0\text{ MW}}{0.99}=2.0202\text{ MW}.$ The power that must be present at the start of the transmission must be $2.0202\text{ MW}+0.5671\text{M}=2.5873\text{MW}$ to compensate for the transmission line loss. The power that must enter the transmission lines from the 99% efficient step-up transformer is $\dfrac{2.5873\text{ MW}}{0.99}=2.6134\text{ MW}.$ So the power saved is as follows.

$$5.756\times10^7\text{ W}-2.6134\times10^6\text{ W}=5.495\times10^7\text{ W}\approx\boxed{55\text{ MW}}$$

38. We set the power loss equal to 2.5% of the total power. Then, using Eq. 18–6a, we write the power loss in terms of the current (equal to the power divided by the voltage drop) and the resistance. Then, using Eq. 18–3, we calculate the cross-sectional area of each wire and the minimum wire diameter. We assume there are two lines to have a complete circuit.

$$P_{\text{loss}}=0.025P=I^2R=\left(\frac{P}{V}\right)^2\left(\frac{\rho\ell}{A}\right)\quad\rightarrow\quad A=\frac{P\rho\ell}{0.025\,V^2}=\frac{\pi d^2}{4}\quad\rightarrow$$

$$d=\sqrt{\frac{4}{0.025\,\pi}\frac{P\rho\ell}{V^2}}=\sqrt{\frac{4(925\times10^6\text{ W})(2.65\times10^{-8}\,\Omega\cdot\text{m})2(185\times10^3\text{ m})}{0.025\,\pi(660\times10^3\text{ V})^2}}=0.03256\text{ m}\approx\boxed{3.3\text{ cm}}$$

The transmission lines must have a diameter greater than or equal to 3.3 cm.

39. Find the induced emf from Eq. 21–9.

$$\mathscr{E}=-L\frac{\Delta I}{\Delta t}=-(0.16\text{ H})\frac{(10.0\text{ A}-25.0\text{ A})}{0.35\text{ s}}=\boxed{6.9\text{ V}}$$

$\boxed{40.}$ Because the current is increasing, the emf is negative. We find the self-inductance from Eq. 21–9.

$$\mathscr{E}=-L\frac{\Delta I}{\Delta t}\quad\rightarrow\quad L=-\mathscr{E}\frac{\Delta t}{\Delta I}=-(-2.50\text{ V})\frac{0.0140\text{ s}}{[0.0310\text{ A}-(-0.0280\text{ A})]}=\boxed{0.593\text{ H}}$$

41. Use the relationship for the inductance of a solenoid, as given in Example 21–13.

$$L=\frac{\mu_0N^2A}{\ell}=\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(8500)^2\pi(1.45\times10^{-2}\text{ m})^2}{0.60\text{ m}}=\boxed{0.10\text{ H}}$$

42. Use the relationship for the inductance of a solenoid, as given in Example 21–13.

$$L=\frac{\mu_0N^2A}{\ell}\quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0A}}=\sqrt{\frac{(0.13\text{ H})(0.300\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\pi(0.029\text{ m})^2}}\approx\boxed{3400\text{ turns}}$$

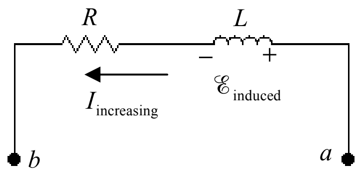
43. (*a*) Use the relationship for the inductance of a solenoid, as given in Example 21–13.

$$L=\frac{\mu_0 N^2 A}{\ell}=\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(2600)^2\pi(0.0125\ \mathrm{m})^2}{(0.282\ \mathrm{m})}=0.01479\ \mathrm{H}\approx\boxed{0.015\ \mathrm{H}}$$

(*b*) Apply the same equation again, solving for the number of turns but using the permeability of iron.

$$L=\frac{\mu N^2 A}{\ell}\ \rightarrow\ N=\sqrt{\frac{L\ell}{\mu A}}=\sqrt{\frac{(0.01479\ \mathrm{H})(0.282\ \mathrm{m})}{(1200)(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})\pi(0.0125\ \mathrm{m})^2}}\approx\boxed{75\ \mathrm{turns}}$$

44. We draw the coil as two elements in series, a pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 21–9. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.



$$V_{\mathrm{ab}}=IR+L\frac{\Delta I}{\Delta t}=(3.00\ \mathrm{A})(2.25\ \Omega)+(0.112\ \mathrm{H})(3.80\ \mathrm{A/s})=\boxed{7.18\ \mathrm{V}}$$

45. The self-inductance of an air-filled solenoid was determined in Example 21–13. We solve this equation for the length of the tube, using the diameter of the wire as the length per turn.

$$L=\frac{\mu_0 N^2 A}{\ell}=\mu_0 n^2 A\ell=\frac{\mu_0 A\ell}{d^2}$$

$$\ell=\frac{Ld^2}{\mu_0\pi r^2}=\frac{(1.0\ \mathrm{H})(0.81\times10^{-3}\ \mathrm{m})^2}{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})\pi(0.060\ \mathrm{m})^2}=46.16\ \mathrm{m}\approx\boxed{46\ \mathrm{m}}$$

The length of the wire needed (L) is equal to the number of turns (the length of the solenoid divided by the diameter of the wire) multiplied by the circumference of the turn.

$$L=\frac{\ell}{d}\pi D=\frac{46.16\ \mathrm{m}}{0.81\times10^{-3}\ \mathrm{m}}\pi(0.12\ \mathrm{m})=21{,}490\ \mathrm{m}\approx\boxed{21\ \mathrm{km}}$$

The resistance is calculated from the resistivity, area, and length of the wire.

$$R=\frac{\rho\ell}{A}=\frac{(1.68\times10^{-8}\ \Omega\cdot\mathrm{m})(21{,}490\ \mathrm{m})}{\pi(0.405\times10^{-3}\ \mathrm{m})^2}=\boxed{0.70\ \mathrm{k\Omega}}$$

46. We assume that the solenoid and the coil have the same cross-sectional area. The magnetic field of the solenoid (which passes through the coil) is $B_1=\mu_0\dfrac{N_1 I_1}{\ell}$. When the current in the solenoid changes, the magnetic field of the solenoid changes, and thus the flux through the coil changes, inducing an emf in the coil.

$$\mathscr{E}=-\frac{N_2 A\Delta B}{\Delta t}=-\frac{N_2 A\Delta\left(\mu_0\dfrac{N_1 I_1}{\ell}\right)}{\Delta t}=-\mu_0\frac{N_1 N_2 A}{\ell}\frac{\Delta(I_1)}{\Delta t}$$

As in Eq. 21–8a, the mutual inductance is the proportionality constant in the above relationship.

$$\mathscr{E}=-\mu_0\frac{N_1 N_2 A}{\ell}\frac{\Delta(I_1)}{\Delta t}\ \rightarrow\ \boxed{M=\mu_0\frac{N_1 N_2 A}{\ell}}$$

47. The magnetic energy in the field is derived from Eq. 21–10.

$$u = \frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{B^2}{\mu_0} \quad \rightarrow$$

$$\text{Energy} = \frac{1}{2}\frac{B^2}{\mu_0}(\text{Volume}) = \frac{1}{2}\frac{B^2}{\mu_0}\pi r^2 L = \frac{1}{2}\frac{(0.72\ \text{T})^2}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}\pi(0.010\ \text{m})^2(0.36\ \text{m}) = \boxed{23\ \text{J}}$$

48. The initial energy stored in the inductor is found from an equation in Section 21–11.

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(45.0\times10^{-3}\ \text{H})(50.0\times10^{-3}\ \text{A})^2 = \boxed{5.63\times10^{-5}\ \text{J}}$$

The final current is found from the final energy, which is 5 times the initial energy.

$$U_\text{f} = 5U_0 \quad \rightarrow \quad \tfrac{1}{2}LI_\text{f}^2 = 5(\tfrac{1}{2}LI_0^2) \quad \rightarrow \quad I_\text{f} = \sqrt{5}I_0$$

The current increases at a constant rate.

$$\frac{\Delta I}{\Delta t} = \frac{I_\text{f} - I_0}{\Delta t} = 0.115\ \text{A/s} \quad \rightarrow$$

$$\Delta t = \frac{I_\text{f} - I_0}{0.115\ \text{A/s}} = \frac{\sqrt{5}I_0 - I_0}{0.115\ \text{A/s}} = \frac{I_0}{0.115\ \text{A/s}}(\sqrt{5}-1) = \frac{(50.0\times10^{-3}\ \text{A})}{0.115\ \text{A/s}}(\sqrt{5}-1) = \boxed{0.537\ \text{s}}$$

49. The magnetic energy in the field is derived from Eq. 21–10. The volume of a relatively thin spherical shell, like the first 10 km above the Earth's surface, is the surface area of the sphere times its thickness.

$$u = \frac{1}{2}\frac{B^2}{\mu_0} \quad \rightarrow$$

$$E = u(\text{volume}) = \frac{1}{2}\frac{B^2}{\mu_0}4\pi R_\text{Earth}^2 h = \frac{1}{2}\frac{(0.50\times10^{-4}\ \text{T})^2}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}4\pi(6.38\times10^6\ \text{m})^2(10^4\ \text{m}) \approx \boxed{5\times10^{15}\ \text{J}}$$

50. (*a*) We set *I* equal to 75% of the maximum value in the equation $I = I_0(1-e^{-t/\tau})$ and solve for the time constant.

$$I = 0.75I_0 = I_0(1-e^{-t/\tau}) \quad \rightarrow \quad \tau = -\frac{t}{\ln(0.25)} = -\frac{(2.56\ \text{ms})}{\ln(0.25\ \text{ms})} = 1.847\ \text{ms} \approx \boxed{1.8\ \text{ms}}$$

(*b*) The resistance can be calculated from the time constant, $\tau = L/R$.

$$R = \frac{L}{\tau} = \frac{31.0\ \text{mH}}{1.847\ \text{ms}} = \boxed{17\ \Omega}$$

51. The potential difference across the resistor is proportional to the current, so we set the current in the equation $I = I_\text{max}e^{-t/\tau}$ equal to $0.025I_0$ and solve for the time.

$$I = 0.025I_0 = I_0 e^{-t/\tau} \quad \rightarrow \quad t = -\tau\ln(0.025) \approx \boxed{3.7\tau}$$

52. When the switch is initially closed, the inductor prevents current from flowing, so the initial current is 0, as shown in Fig. 21–37. If the current is 0, then there is no voltage drop across the resistor (since $V_\text{R} = IR$), so the entire battery voltage appears across the inductor. Apply Eq. 21–9 to find the initial rate of change of the current.

$$V = V_L = L\frac{\Delta I}{\Delta t} \quad \rightarrow \quad \frac{\Delta I}{\Delta t} = \boxed{\frac{V}{L}}$$

The maximum value of the current is reached after a long time, when there is no voltage across the inductor, so the entire battery voltage appears across the resistor. Apply Ohm's law.

$$V = I_{\max} R \;\rightarrow\; I_{\max} = \frac{V}{R}$$

Find the time to reach the maximum current if the rate of current change remained at $\frac{\Delta I}{\Delta t} = \frac{V}{L}.$

$$I_{\max} = I_0 + \frac{\Delta I}{\Delta t}(\text{elapsed time}) \;\rightarrow\; \text{elapsed time} = (I_{\max} - I_0)\frac{\Delta t}{\Delta I} = \left(\frac{V}{R} - 0\right)\frac{L}{V} = \boxed{\frac{L}{R}}$$

53. For an LR circuit, we have $I = I_{\max}(1 - e^{-t/\tau}).$ Solve for t.

$$I = I_{\max}(1 - e^{-t/\tau}) \;\rightarrow\; e^{-t/\tau} = 1 - \frac{I}{I_{\max}} \;\rightarrow\; t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right)$$

(*a*) $I = 0.90 I_{\max} \;\rightarrow\; t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.9) = \boxed{2.3\,\tau}$

(*b*) $I = 0.990 I_{\max} \;\rightarrow\; t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.99) = \boxed{4.6\,\tau}$

(*c*) $I = 0.999 I_{\max} \;\rightarrow\; t = -\tau \ln\left(1 - \frac{I}{I_{\max}}\right) = -\tau \ln(1 - 0.999) = \boxed{6.9\,\tau}$

54. The reactance of a capacitor is given by Eq. 21–12b, $X_C = \frac{1}{2\pi f C}.$

(*a*) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})(6.20\times10^{-6}\text{ F})} = \boxed{428\ \Omega}$

(*b*) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1.00\times10^{6}\text{ Hz})(6.20\times10^{-6}\text{ F})} = \boxed{2.57\times10^{-2}\ \Omega}$

55. We find the frequency from Eq. 21–11b for the reactance of an inductor.

$$X_L = 2\pi f L \;\rightarrow\; f = \frac{X_L}{2\pi L} = \frac{660\ \Omega}{2\pi(0.0320\text{ H})} = 3283\text{ Hz} \approx \boxed{3300\text{ Hz}}$$

56. We find the frequency from Eq. 21–12b for the reactance of a capacitor.

$$X_C = \frac{1}{2\pi f C} \;\rightarrow\; f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(6.10\times10^{3}\ \Omega)(2.40\times10^{-6}\text{ F})} = \boxed{10.9\text{ Hz}}$$

57. We find the reactance from Eq. 21–11b and the current from Ohm's law.

$$X_L = 2\pi f L = 2\pi(10.0\times10^{3}\text{ Hz})(0.26\text{ H}) = 16{,}336\ \Omega \approx \boxed{16\text{ k}\Omega}$$

$$V = I X_L \;\rightarrow\; I = \frac{V}{X_L} = \frac{240\text{ V}}{16{,}336\ \Omega} = \boxed{1.47\times10^{-2}\text{ A}}$$

58. We find the reactance from Ohm's law and the inductance by Eq. 21–11b.

$$V = I X_L \;\rightarrow\; X_L = \frac{V}{I}$$

$$X_L = 2\pi f L \;\rightarrow\; L = \frac{X_L}{2\pi f} = \frac{V}{2\pi f I} = \frac{240\text{ V}}{2\pi(60.0\text{ Hz})12.2\text{ A}} = \boxed{5.22\times10^{-2}\text{ H}}$$

59. (*a*) We find the reactance from Eq. 21–12b.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(720\text{ Hz})(3.0\times10^{-8}\text{ F})} = 7368\ \Omega \approx \boxed{7400\ \Omega}$$

(*b*) We find the peak value of the current from Ohm's law.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{X_C} = \sqrt{2}\frac{2.0\times10^3\text{ V}}{7368\ \Omega} = \boxed{0.38\text{ A}}$$

60. We find the impedance from Eq. 21–14.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\text{ V}}{70\times10^{-3}\text{ A}} = 1700\ \Omega \approx \boxed{2000\ \Omega}$$

61. The impedance of the circuit is given by Eq. 21–15 without a capacitive reactance. The reactance of the inductor is given by Eq. 21–11b.

(*a*) $$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{(36\times10^3\ \Omega)^2 + 4\pi^2(50\text{ Hz})^2(55\times10^{-3}\text{ H})^2} = \boxed{3.6\times10^4\ \Omega}$$

The inductor has essentially no effect on the impedance at this relatively low frequency.

(*b*) $$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2} = \sqrt{(36\times10^3\ \Omega)^2 + 4\pi^2(3.0\times10^4\text{ Hz})^2(55\times10^{-3}\text{ H})^2}$$

$$= 37{,}463\ \Omega \approx \boxed{3.7\times10^4\ \Omega}$$

62. The impedance of the circuit is given by Eq. 21–15 without an inductive reactance. The reactance of the capacitor is given by Eq. 21–12b.

(*a*) $$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}} = \sqrt{(3.5\times10^3\Omega)^2 + \frac{1}{4\pi^2(60\text{ Hz})^2(3.0\times10^{-6}\text{ F})^2}}$$

$$= 3609\ \Omega \approx \boxed{3600\ \Omega}$$

(*b*) $$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}} = \sqrt{(3.5\times10^3\ \Omega)^2 + \frac{1}{4\pi^2(60{,}000\text{ Hz})^2(3.0\times10^{-6}\text{ F})^2}}$$

$$= \boxed{3500\ \Omega}$$

63. Use Eq. 21–15, with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} \quad\to\quad R = \sqrt{Z^2 - X_L^2} = \sqrt{(235\ \Omega)^2 - (115\ \Omega)^2} = \boxed{205\ \Omega}$$

64. The total impedance is given by Eq. 21–15.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(8.70\times10^3\ \Omega)^2 + \left[2\pi(1.00\times10^4\text{ Hz})(2.80\times10^{-2}\text{ H}) - \frac{1}{2\pi(1.00\times10^4\text{ Hz})(6.25\times10^{-9}\text{ F})}\right]^2}$$

$$= 8735.5\ \Omega \approx \boxed{8.74\text{ k}\Omega}$$

The phase angle is given by Eq. 21–16a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \dfrac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi(1.00\times10^4\text{ Hz})(2.80\times10^{-2}\text{ H}) - \dfrac{1}{2\pi(1.00\times10^4\text{ Hz})(6.25\times10^{-9}\text{ F})}}{R}$$

$$= \tan^{-1}\frac{-787\ \Omega}{8700\ \Omega} = \boxed{-5.17^\circ}$$

The voltage is lagging the current, or the current is leading the voltage.

The rms current is given by Eq. 21–14.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{725\text{ V}}{8735.5\ \Omega} = \boxed{8.30\times10^{-2}\text{ A}}$$

65. We use the rms voltage across the resistor to determine the rms current through the circuit. Then, using the rms current and the rms voltage across the capacitor in Eq. 21–13b, we determine the frequency.

$$I_{\text{rms}} = \frac{V_{R,\text{ rms}}}{R} \qquad V_{C,\text{ rms}} = \frac{I_{\text{rms}}}{2\pi fC}$$

$$f = \frac{I_{\text{rms}}}{2\pi CV_{C,\text{ rms}}} = \frac{V_{R,\text{rms}}}{2\pi CRV_{C,\text{ rms}}} = \frac{(3.0\text{ V})}{2\pi(1.0\times10^{-6}\text{ C})(650\ \Omega)(2.7\text{ V})} = 272.1\text{ Hz} \approx \boxed{270\text{ Hz}}$$

Since the voltages in the resistor and capacitor are not in phase, the rms voltage across the power source will not be the sum of the rms voltages across the resistor and capacitor.

66. (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 21–15 with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} = \frac{120\text{ V}}{\sqrt{(2.80\times10^3\ \Omega)^2 + 4\pi^2(60.0\text{ Hz})^2(0.35\text{ H})^2}}$$

$$= \frac{120\text{ V}}{2803\ \Omega} = 0.04281\text{ A} \approx \boxed{4.3\times10^{-2}\text{ A}}$$

(*b*) The phase angle is given by Eq. 21–6a with no capacitive reactance.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi(60.0\text{ Hz})(0.35\text{ H})}{2.80\times10^3\ \Omega} = \boxed{2.7^\circ}$$

The current is lagging the source voltage.

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = (4.281\times10^{-2}\text{ A})^2(2.80\times10^3\ \Omega) = \boxed{5.1\text{ W}}$.

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\text{rms}\\R}} = I_{\text{rms}}R = (4.281\times10^{-2}\text{ A})(2.8\times10^3\ \Omega) = 119.87\text{ V} \approx \boxed{120\text{ V}}$$

$$V_{\substack{\text{rms}\\L}} = I_{\text{rms}}X_L = I_{\text{rms}}2\pi fL = (4.281\times10^{-2}\text{ A})2\pi(60.0\text{ Hz})(0.35\text{ H}) = 5.649\text{ V} \approx \boxed{5.6\text{ V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add up to the applied voltage of 120 V.

67. (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 21–15 with no inductive reactance.

$$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{(2\pi fC)^2}}$$

$$I_{\mathrm{rms}}=\frac{V_{\mathrm{rms}}}{Z}=\frac{V_{\mathrm{rms}}}{\sqrt{R^2+\dfrac{1}{4\pi^2f^2L^2}}}=\frac{120\text{ V}}{\sqrt{(6.60\times10^3\ \Omega)^2+\dfrac{1}{4\pi^2(60.0\text{ Hz})^2(1.80\times10^{-6}\text{ F})^2}}}$$

$$=\frac{120\text{ V}}{6763\ \Omega}=1.774\times10^{-2}\text{ A}\approx\boxed{1.77\times10^{-2}\text{ A}}$$

(*b*) The phase angle is given by Eq. 21–16a with no inductive reactance.

$$\phi=\tan^{-1}\frac{-X_C}{R}=\tan^{-1}\frac{-\dfrac{1}{2\pi fC}}{R}=\tan^{-1}\frac{-\dfrac{1}{2\pi(60.0\text{ Hz})(1.80\times10^{-6}\text{ F})}}{6.60\times10^3\ \Omega}=\boxed{-12.6^\circ}$$

The current is leading the source voltage.

(*c*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\mathrm{rms}\\R}}=I_{\mathrm{rms}}R=(1.774\times10^{-2}\text{ A})(6.60\times10^3\,\Omega)=\boxed{117\text{ V}}$$

$$V_{\substack{\mathrm{rms}\\C}}=I_{\mathrm{rms}}X_C=I_{\mathrm{rms}}\frac{1}{2\pi fC}=(1.774\times10^{-2}\text{ A})\frac{1}{2\pi(60.0\text{ Hz})(1.80\times10^{-6}\text{ F})}=\boxed{26.1\text{ V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add up to the applied voltage of 120 V.

68. (*a*) The impedance of the circuit is given by Eq. 21–15. Divide the source voltage by the impedance to determine the magnitude of the current in the circuit. Finally, multiply the current by the resistance to determine the voltage drop across the resistor.

$$V_R=IR=\frac{V_{\mathrm{in}}}{Z}R=\frac{V_{\mathrm{in}}R}{\sqrt{R^2+1/(2\pi fC)^2}}$$

$$=\frac{(130\text{ mV})(520\ \Omega)}{\sqrt{(520\ \Omega)^2+1/[2\pi(60\text{ Hz})(1.2\times10^{-6}\text{ F})]^2}}=29.77\text{ mV}\approx\boxed{30\text{ mV}}$$

(*b*) Repeat the calculation with a frequency of 6.0 kHz.

$$V_R=\frac{(130\text{ mV})(520\ \Omega)}{\sqrt{(520\ \Omega)^2+1/[2\pi(6000\text{ Hz})(1.2\times10^{-6}\text{ F})]^2}}=129.88\text{ mV}\approx\boxed{130\text{ mV}}$$

Thus the capacitor allows the higher frequency to pass but attenuates the lower frequency.

69. The resonant frequency is found from Eq. 21–19. The resistance does not influence the resonant frequency.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\frac{1}{2\pi}\sqrt{\frac{1}{(55.0\times10^{-6}\text{ H})(3500\times10^{-12}\text{ F})}}=3.627\times10^5\text{ Hz}\approx\boxed{3.6\times10^5\text{ Hz}}$$

70. (*a*) The resonant frequency is given by Eq. 21–19.

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad\rightarrow\quad \frac{f_{580\text{ kHz}}}{f_{1600\text{ kHz}}}=\frac{580\text{ kHz}}{1600\text{ kHz}}=\frac{\frac{1}{2\pi}\sqrt{\frac{1}{LC_{580\text{ kHz}}}}}{\frac{1}{2\pi}\sqrt{\frac{1}{LC_{1600\text{ kHz}}}}} \quad\rightarrow$$

$$C_{1600\text{ kHz}}=\left(\frac{580}{1600}\right)^2 C_{580\text{ kHz}}=\left(\frac{580}{1600}\right)^2(2800\text{ pF})=367.9\text{ pF}\approx\boxed{370\text{ pF}}$$

(*b*) The inductance can be found from the resonant frequency and the capacitance.

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad\rightarrow\quad L=\frac{1}{4\pi^2 f^2 C}=\frac{1}{4\pi^2(5.8\times10^5\text{ Hz})^2(2800\times10^{-12}\text{ F})}=\boxed{2.7\times10^{-5}\text{ H}}$$

$\boxed{71}$. (*a*) We find the capacitance from the resonant frequency, Eq. 21–19.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad\rightarrow\quad C=\frac{1}{4\pi^2 L f_0^2}=\frac{1}{4\pi^2(14.8\times10^{-3}\text{ H})(3600\text{ Hz})^2}\approx\boxed{1.3\times10^{-7}\text{ F}}$$

(*b*) At resonance the impedance is the resistance, so the current is given by Ohm's law.

$$I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{150\text{ V}}{4.10\ \Omega}=\boxed{37\text{ A}}$$

72. Since the circuit is in resonance, we use Eq. 21–19 for the resonant frequency to determine the necessary inductance. We set this inductance equal to the solenoid inductance calculated in Example 21–13, with the area equal to the area of a circle of radius r, the number of turns equal to the length of the wire divided by the circumference of a turn, and the length of the solenoid equal to the diameter of the wire multiplied by the number of turns. We solve the resulting equation for the number of turns.

$$f_0=\frac{1}{2\pi\sqrt{LC}} \quad\rightarrow\quad L=\frac{1}{4\pi^2 f_0^2 C}=\frac{\mu_0 N^2 A}{\ell_{\text{solenoid}}}=\frac{\mu_0\left(\frac{\ell_{wire}}{2\pi r}\right)^2\pi r^2}{Nd} \quad\rightarrow$$

$$N=\frac{\pi f_0^2 C\mu_0\ell_{\text{wire}}^2}{d}=\frac{\pi(18.0\times10^3\text{ Hz})^2(2.6\times10^{-7}\text{ F})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ m})^2}{1.1\times10^{-3}\text{ m}}$$

$$=43.45\approx\boxed{44\text{ loops}}$$

73. (*a*) We calculate the inductance from the resonance frequency.

$$f_0=\frac{1}{2\pi\sqrt{LC}} \quad\rightarrow$$

$$L=\frac{1}{4\pi^2 f_0^2 C}=\frac{1}{4\pi^2(19\times10^3\text{ Hz})^2(2.2\times10^{-9}\text{ F})}=0.03189\text{ H}\approx\boxed{0.032\text{ H}}$$

(*b*) We set the initial energy in the electric field equal to the maximum energy in the magnetic field and solve for the maximum current.

$$\tfrac{1}{2}CV_0^2=\tfrac{1}{2}LI_{\text{max}}^2 \quad\rightarrow\quad I_{\text{max}}=\sqrt{\frac{CV_0^2}{L}}=\sqrt{\frac{(2.2\times10^{-9}\text{ F})(120\text{ V})^2}{(0.03189\text{ H})}}=0.03152\text{A}\approx\boxed{0.032\text{ A}}$$

(*c*) The maximum energy in the inductor is equal to the initial energy in the capacitor.

$$U_{L,\text{max}}=\tfrac{1}{2}CV_0^2=\tfrac{1}{2}(2.2\times10^{-9}\text{ F})(120\text{ V})^2=\boxed{16\ \mu\text{J}}$$

74. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field that is directed out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{clockwise}}$.

(*b*) After a long time, the current in the left-hand loop is constant, so there will be $\boxed{\text{no induced current}}$ in the right-hand coil.

(*c*) If the second loop is pulled to the right, then the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be $\boxed{\text{counterclockwise}}$.

(*d*) Since equilibrium has been reached ("a long time"), there is a steady current in the left-hand loop and steady magnetic flux passing through the right-hand loop. There is no emf being induced in the right-hand loop, so closing a switch in the right-hand loop has $\boxed{\text{no effect}}$.

75. The electrical energy is dissipated because there is current flowing in a resistor. The power dissipation by a resistor is given by $P = I^2R,$ so the energy dissipated is $E = P\Delta t = I^2R\Delta t.$ The current is created by the induced emf caused by the changing B field. The emf is calculated by Eq. 21–2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} \qquad I = \frac{\mathscr{E}}{R} = -\frac{A\Delta B}{R\Delta t}$$

$$E = P\Delta t = I^2R\Delta t = \frac{A^2(\Delta B)^2}{R^2(\Delta t)^2}R\Delta t = \frac{A^2(\Delta B)^2}{R(\Delta t)} = \frac{[(0.240\text{ m})^2]^2[(0-0.665\text{ T})]^2}{(6.10\ \Omega)(0.0400\text{ s})}$$

$$= \boxed{6.01\times10^{-3}\text{ J}}$$

76. (*a*) Because $V_S < V_P,$ this is a $\boxed{\text{step-down}}$ transformer.

(*b*) Assuming 100% efficiency, the power in both the primary and secondary is 45 W. Find the current in the secondary from the relationship $P = IV.$

$$P_S = I_SV_S \quad\rightarrow\quad I_S = \frac{P_S}{V_S} = \frac{45\text{ W}}{12\text{ V}} = \boxed{3.8\text{ A}}$$

(*c*) $$P_P = I_PV_P \quad\rightarrow\quad I_P = \frac{P_P}{V_P} = \frac{45\text{ W}}{120\text{ V}} = \boxed{0.38\text{ A}}$$

(*d*) Find the resistance of the bulb from Ohm's law. The bulb is in the secondary circuit.

$$V_S = I_SR \quad\rightarrow\quad R = \frac{V_S}{I_S} = \frac{12\text{ V}}{3.75\text{ A}} = \boxed{3.2\ \Omega}$$

77. The coil should have a diameter about equal to the diameter of a standard flashlight D-cell so that it will be simple to hold and use. This would give the coil a radius of about 1.5 cm. As the magnet passes through the coil, the field changes direction, so the change in flux for each pass is twice the maximum flux. Let us assume that the magnet is shaken with a frequency of about two shakes per second, so the magnet passes through the coil four times per second. We obtain the number of turns in the coil using Eq. 21–2b.

$$N = \frac{\mathscr{E}}{\Delta\Phi/\Delta t} = \frac{\mathscr{E}\Delta t}{\Delta\Phi} = \frac{\mathscr{E}\Delta t}{2B_0A} = \frac{(3\text{ V})(0.25\text{ s})}{2(0.05\text{ T})\pi(0.015\text{ m})^2} = 10{,}610\text{ turns} \approx \boxed{10{,}000\text{ turns}}$$

The answer will vary somewhat based on the approximations used. For example, a flashlight with a smaller diameter, for AA batteries perhaps, would require more turns.

78. (*a*) Use Ohm's law to see that the current is $120\text{ V}/3.0\ \Omega = \boxed{40\text{ A}}$ (2 significant figures).

(*b*) Find the back emf from the normal operating current.

$$\mathscr{E}_{\text{applied}} - \mathscr{E}_{\text{back}} = IR \quad \rightarrow \quad \mathscr{E}_{\text{back}} = \mathscr{E}_{\text{applied}} - IR = 120\text{ V} - (2.0\text{ A})(3.0\ \Omega) = \boxed{114\text{ V}}$$

(*c*) Use Eq. 18–6a.

$$P = I^2R = (2.0\text{ A})^2(3.0\ \Omega) = \boxed{12\text{ W}}$$

(*d*) Use Eq. 18–6a.

$$P = I^2R = (40\text{ A})^2(3.0\ \Omega) = \boxed{4800\text{ W}}$$

79. Because the transformers are assumed to be perfect, the power loss is due to resistive heating in the transmission lines. Since the town requires 55 MW, the power at the generating plant must be $\dfrac{55\text{ MW}}{0.985} = 55.838\text{ MW}$. Thus the power lost in the transmission is 0.838 MW. This can be used to determine the current in the (two) transmission lines.

$$P = I^2R \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.838\times10^6\text{ W}}{2(56\text{ km})0.10\ \Omega/\text{km}}} = 273.6\text{ A}$$

To produce 55.838 MW of power at 273.6 A, the following voltage is required.

$$V = \frac{P}{I} = \frac{55.838\times10^6\text{ W}}{273.6\text{ A}} = 2.041\times10^5\text{ V} \approx \boxed{200\text{ kV}}$$

The voltage has 2 significant figures.

80. (*a*) From the efficiency of the transformer, we have $P_{\text{S}} = 0.88P_{\text{P}}$. Use this to calculate the current in the primary.

$$P_{\text{S}} = 0.88P_{\text{P}} = 0.88I_{\text{P}}V_{\text{P}} \quad \rightarrow \quad I_{\text{P}} = \frac{P_{\text{S}}}{0.88V_{\text{P}}} = \frac{75\text{ W}}{0.88(110\text{ V})} = 0.7748\text{ A} \approx \boxed{0.77\text{ A}}$$

(*b*) The voltage in both the primary and secondary is proportional to the number of turns in the respective coil. The secondary voltage is calculated from the secondary power and resistance since $P = V^2/R$.

$$\frac{N_{\text{P}}}{N_{\text{S}}} = \frac{V_{\text{P}}}{V_{\text{S}}} = \frac{V_{\text{P}}}{\sqrt{P_{\text{S}}R_{\text{S}}}} = \frac{110\text{ V}}{\sqrt{(75\text{ W})(2.4\ \Omega)}} = \boxed{8.2}$$

81. (*a*) The voltage drop across the lines is due to the resistance.

$$V_{\text{out}} = V_{\text{in}} - IR = 42{,}000\text{ V} - (740\text{ A})(2)(0.95\ \Omega) = 40{,}594\text{ V} \approx \boxed{41\text{ kV}}$$

(*b*) The power input is given by $P_{\text{in}} = IV_{\text{in}}$.

$$P_{\text{in}} = IV_{\text{in}} = (740\text{ A})(42{,}000\text{ V}) = 3.108\times10^7\text{ W} \approx \boxed{3.1\times10^7\text{ W}}$$

(*c*) The power loss in the lines is due to the current in the resistive wires.

$$P_{\text{loss}} = I^2R = (740\text{ A})^2 2(0.95\ \Omega) = 1.040\times10^6\text{ W} \approx \boxed{1.0\times10^6\text{ W}}$$

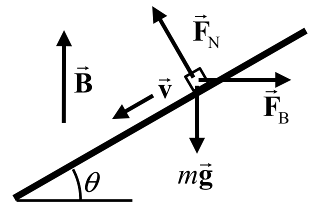
(*d*) The power output is given by $P_{\text{out}} = IV_{\text{out}}$.

$$P_{\text{out}} = IV_{\text{out}} = (740\text{ A})(40{,}594\text{ V}) = 3.004\times10^7\text{ W} \approx \boxed{3.0\times10^7\text{ W}}$$

This could also be found by subtracting the power lost from the input power.

$$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 3.108\times10^7\text{ W} - 1.040\times10^6\text{ W} = 3.004\times10^7\text{ W} \approx \boxed{3.0\times10^7\text{ W}}$$

82. A side view of the rail and bar is shown in the figure. From the discussion in Section 21–3, the emf in the bar is produced by the components of the magnetic field, the length of the bar, and the velocity of the bar, which are all mutually perpendicular. The magnetic field and the length of the bar are already perpendicular. The component of the velocity of the bar that is perpendicular to the magnetic field is $\upsilon\cos\theta$, so the induced emf is given by the following.



$$\mathscr{E} = B\ell\upsilon\cos\theta$$

This produces a current in the wire, which can be found by Ohm's law. That current is pointing into the page on the diagram.

$$I = \frac{\mathscr{E}}{R} = \frac{B\ell\upsilon\cos\theta}{R}$$

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field can be calculated from Eq. 20–2 and will be horizontal, as shown in the diagram.

$$F_B = I\ell B = \frac{B\ell\upsilon\cos\theta}{R}\ell B = \frac{B^2\ell^2\upsilon\cos\theta}{R}$$

For the wire to slide down at a steady speed, the net force along the rail must be zero. Write Newton's second law for forces along the rail, with up the rail being positive.

$$F_{\text{net}} = F_B\cos\theta - mg\sin\theta = 0 \quad\rightarrow\quad \frac{B^2\ell^2\upsilon\cos^2\theta}{R} = mg\sin\theta \quad\rightarrow$$

$$\upsilon = \frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta} = \frac{(0.60\ \Omega)(0.040\ \text{kg})(9.80\ \text{m/s}^2)\sin 6.0^\circ}{(0.45\ \text{T})^2(0.32\ \text{m})^2\cos^2 6.0^\circ} = \boxed{1.199\ \text{m/s} \approx 1.2\ \text{m/s}}$$

83. We find the current in the transmission lines from the power transmitted to the user and then find the power loss in the lines.

$$P_{\mathrm{T}} = I_{\mathrm{L}}V \quad\rightarrow\quad I_{\mathrm{L}} = \frac{P_{\mathrm{T}}}{V} \qquad P_{\mathrm{L}} = I_{\mathrm{L}}^2R_{\mathrm{L}} = \left(\frac{P_{\mathrm{T}}}{V}\right)^2 R_{\mathrm{L}} = \boxed{\frac{P_{\mathrm{T}}^2R_{\mathrm{L}}}{V^2}}$$

84. The induced current in the coil is the induced emf divided by the resistance. The induced emf is found from the changing flux by Eq. 21–2b. The magnetic field of the solenoid, which causes the flux, is given by Eq. 20–8. For the area used in Eq. 21–2b, the cross-sectional area of the solenoid (not the coil) must be used, because all of the magnetic flux is inside the solenoid.

$$I = \frac{\mathscr{E}_{\text{ind}}}{R} \qquad \left|\mathscr{E}_{\text{ind}}\right| = N_{\text{coil}}\frac{\Delta\Phi}{\Delta t} = N_{\text{coil}}A_{\text{sol}}\frac{\Delta B_{\text{sol}}}{\Delta t} \qquad B_{\text{sol}} = \mu_0\frac{N_{\text{sol}}I_{\text{sol}}}{\ell_{\text{sol}}}$$

$$I = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0\dfrac{N_{\text{sol}}}{\ell_{\text{sol}}}\dfrac{\Delta I_{\text{sol}}}{\Delta t}}{R} = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0}{R}\frac{N_{\text{sol}}}{\ell_{\text{sol}}}\frac{\Delta I_{\text{sol}}}{\Delta t}$$

$$= \frac{(190\ \text{turns})\pi(0.045\ \text{m})^2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{12\ \Omega}\frac{(230\ \text{turns})}{(0.01\ \text{m})}\frac{2.0\ \text{A}}{0.10\ \text{s}} = 5.823\times10^{-2}\ \text{A}$$

$$\approx \boxed{5.8\times10^{-2}\ \text{A}}$$

As the current in the solenoid increases, a magnetic field from right to left is created in the solenoid and the loop. The induced current will flow in such a direction as to oppose that field, so must flow from $\boxed{\text{left to right}}$ through the resistor.

85. Putting an inductor in series with the device will protect it from sudden surges in current. The growth of current in an *LR* circuit is given in Section 21–12.

$$I = \frac{V}{R}\left(1 - e^{-tR/L}\right) = I_{\text{max}}\left(1 - e^{-tR/L}\right)$$

The maximum current is 55 mA, and the current is to have a value of 7.5 mA after a time of 120 microseconds. Use this data to solve for the inductance.

$$I = I_{\text{max}}\left(1 - e^{-tR/L}\right) \rightarrow e^{-tR/L} = 1 - \frac{I}{I_{\text{max}}} \rightarrow$$

$$L = -\frac{tR}{\ln\left(1 - \frac{I}{I_{\text{max}}}\right)} = -\frac{(1.2\times10^{-4}\text{ s})(120\ \Omega)}{\ln\left(1 - \frac{7.5\text{ mA}}{55\text{ mA}}\right)} = 0.0982\text{ H} \approx 98\text{ mH in series}$$

$\boxed{\text{Put an inductor of value 98 mH in series with the device.}}$

86. The emf is related to the flux change by Eq. 21–2b. The flux change is caused by the changing magnetic field.

$$|\mathscr{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{A\Delta B}{\Delta t} \rightarrow \frac{\Delta B}{\Delta t} = \frac{|\mathscr{E}|}{NA} = \frac{120\text{ V}}{(35)\pi(6.25\times10^{-2}\text{ m})^2} = \boxed{280\text{ T/s}}$$

87. We find the peak emf from Eq. 21–5.

$$\mathscr{E}_{\text{peak}} = NB\omega A = (125)(0.200\text{ T})(2\pi\text{ rad/rev})(120\text{ rev/s})(6.60\times10^{-2}\text{ m})^2 = \boxed{82\text{ V}}$$

88. (*a*) The electric field energy density is given by Eq. 17–11, and the magnetic field energy density is given by Eq. 21–10.

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.0\times10^4\text{ V/m})$$

$$= 4.425\times10^{-4}\text{ J/m}^3 \approx \boxed{4.4\times10^{-4}\text{ J/m}^3}$$

$$u_B = \tfrac{1}{2}\frac{B^2}{\mu_0} = \tfrac{1}{2}\frac{(2.0\text{ T})^2}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 1.592\times10^6\text{ J/m}^3 \approx \boxed{1.6\times10^6\text{ J/m}^3}$$

$$\frac{u_B}{u_E} = \frac{1.592\times10^6\text{ J/m}^3}{4.425\times10^{-4}\text{ J/m}^3} = \boxed{3.6\times10^9}$$

The energy density in the magnetic field is 3.6 billion times greater than the energy density in the electric field.

(*b*) Set the two densities equal and solve for the magnitude of the electric field.

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = u_B = \tfrac{1}{2}\frac{B^2}{\mu_0} \rightarrow$$

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{2.0\text{ T}}{\sqrt{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}} = \boxed{6.0\times10^8\text{ V/m}}$$

89. If there is no current in the secondary, then there will be no induced emf from the mutual inductance. Therefore, we set the ratio of the voltage to current equal to the inductive reactance (Eq. 21–11b) and solve for the inductance.

$$\frac{V_{\text{rms}}}{I_{\text{rms}}} = X_L = 2\pi fL \rightarrow L = \frac{V_{\text{rms}}}{2\pi f\, I_{\text{rms}}} = \frac{220\text{ V}}{2\pi(60\text{ Hz})(6.3\text{ A})} = \boxed{93\text{ mH}}$$

90. For the current and voltage to be in phase, the net reactance of the capacitor and inductor must be zero, which means that the circuit is at resonance. Thus Eq. 21–19 applies.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \rightarrow \quad C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2 (0.13\text{ H})(1360\text{ Hz})^2} = \boxed{1.1\times10^{-7}\text{ F}}$$

91. The inductance of the solenoid is given by $L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2}{\ell}\frac{\pi d^2}{4}$. The (constant) length of the wire is given by $\ell_{\text{wire}} = N\pi d_{\text{sol}}$, so since $d_{\text{sol }2} = 2d_{\text{sol }1}$, we also know that $N_1 = 2N_2$. The fact that the wire is tightly wound gives $\ell_{\text{sol}} = Nd_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1} = \frac{\frac{\mu_0\pi}{4}\frac{N_2^2}{\ell_{\text{sol }2}}d_{\text{sol }2}^2}{\frac{\mu_0\pi}{4}\frac{N_1^2}{\ell_{\text{sol }1}}d_{\text{sol }1}^2} = \frac{\frac{N_2^2}{\ell_{\text{sol }2}}d_{\text{sol }2}^2}{\frac{N_1^2}{\ell_{\text{sol }1}}d_{\text{sol }1}^2} = \frac{\frac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol }2}}}{\frac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol }1}}} = \frac{\ell_{\text{sol }1}}{\ell_{\text{sol }2}} = \frac{N_1 d_{\text{wire}}}{N_2 d_{\text{wire}}} = \frac{N_1}{N_2} = \frac{2N_2}{N_2} = \boxed{2}$$

92. (*a*) From the text of the problem, the Q factor is the ratio of the voltage across the capacitor or inductor to the voltage across the resistor, at resonance. The resonant frequency is given by Eq. 21–19.

$$Q = \frac{V_L}{V_R} = \frac{I_{\text{res}}X_L}{I_{\text{res}}R} = \frac{2\pi f_{\text{res}}L}{R} = \frac{2\pi\frac{1}{2\pi}\sqrt{\frac{1}{LC}}L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

(*b*) Find the inductance from the resonant frequency and the resistance from the Q factor.

$$f_{\text{res}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \rightarrow$$

$$L = \frac{1}{4\pi^2 C f_{\text{res}}^2} = \frac{1}{4\pi^2(1.0\times10^{-8}\text{ F})(1.0\times10^6\text{ Hz})^2} = 2.533\times10^{-6}\text{ H} \approx \boxed{2.5\times10^{-6}\text{ H}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \rightarrow \quad R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{650}\sqrt{\frac{2.533\times10^{-6}\text{ H}}{1.0\times10^{-8}\text{ F}}} = \boxed{2.4\times10^{-2}\ \Omega}$$

Solutions to Search and Learn Problems

1. (*a*) Kitchens, bathrooms, outdoor areas, and near swimming pools are especially dangerous for touching ground because of the abundance of water in those areas. As shown in Section 19–7, high current through the body can stop the heart. Wet skin has a much smaller resistance than dry skin, so for a given voltage it is more dangerous to ground an object with wet skin than with dry skin.

(*b*) The GFCI works using Ampere's law. Both wires from the power source pass through the center of an iron ring. If the ac current in the circuit passes through both wires (one going in and one going out), then the net current through the ring is exactly zero and there is no magnetic field induced in the ring. If even a small amount of the electric current is shorted, so the exact same current no longer flows through both of the wires, then an ac-induced magnetic field is created in the ring. The changing flux induces a current in the small wire wrapped around the ring, which shuts off the power to the GFCI. This is similar to fuses and circuit breakers, which shut off the power if too much current is flowing through the wires. GFCIs are much more sensitive and they act more quickly than fuses and circuit breakers. This is why electrical codes now require them in kitchens and bathrooms, even though homes already have fuses or circuit breakers. As with a circuit breaker, a GFCI can be reset and used again after it has been triggered.

2. (*a*) Use Eq. 21–2a to calculate the emf induced in the ring, where the flux is the magnetic field multiplied by the area of the ring. Then using Eq. 18–6b, calculate the average power dissipated in the ring as it is moved away. The thermal energy is the average power times the time.

$$\mathscr{E}=-\frac{\Delta\Phi_B}{\Delta t}=-\frac{(\Delta B)A}{\Delta t}=-\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}$$

$$Q=P\Delta t=\left(\frac{\mathscr{E}^2}{R}\right)\Delta t=\left(\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}\right)^2\left(\frac{\Delta t}{R}\right)=\frac{(\Delta B)^2\pi^2 d^4}{16R\Delta t}$$

$$=\frac{(0.68\text{ T})^2\pi^2(0.015\text{ m})^4}{16(55\times10^{-6}\ \Omega)(45\times10^{-3}\text{ s})}=5.834\times10^{-3}\text{ J}\approx\boxed{5.8\text{ mJ}}$$

(*b*) The temperature change is calculated from the thermal energy using Eq. 14–2.

$$\Delta T=\frac{Q}{mc}=\frac{5.834\times10^{-3}\text{ J}}{(15\times10^{-3}\text{ kg})(129\text{ J/kg}\cdot{}^\circ\text{C})}=\boxed{3.0\times10^{-3}\text{C}^\circ}$$

3. (*a*) Since the coils are directly connected to the wheels, the torque provided by the motor (Eq. 20–10) balances the torque caused by the frictional force.

$$\tau=NIAB=Fr\rightarrow I=\frac{Fr}{NAB}=\frac{(250\text{ N})(0.29\text{ m})}{290(0.12\text{ m})(0.15\text{ m})(0.65\text{ T})}=21.37\text{ A}\approx\boxed{21\text{ A}}$$

(*b*) To maintain this speed, the power loss due to the friction must equal the net power provided by the coils. The power provided by the coils is the current through the coils multiplied by the back emf.

$$P=Fv=I\mathscr{E}_{\text{back}}\rightarrow\mathscr{E}_{\text{back}}=\frac{F}{I}=\frac{(250\text{ N})(35\text{ km/h})}{21.37\text{ A}}\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right)=113.7\text{ V}\approx\boxed{110\text{ V}}$$

(*c*) The power dissipated in the coils is the difference between the power produced by the coils and the net power provided to the wheels.

$$P_{\text{loss}}=P-P_{\text{net}}=I\mathscr{E}-I\mathscr{E}_{\text{back}}=(21.37\text{ A})(120\text{ V}-113.7\text{ V})=134.6\text{ W}\approx\boxed{130\text{ W}}$$

(*d*) We divide the net power by the total power to determine the percent used to drive the car.

$$\frac{P_{\text{net}}}{P}=\frac{I\mathscr{E}_{\text{back}}}{I\mathscr{E}}=\frac{113.7\text{ V}}{120\text{ V}}=0.9475\approx\boxed{95\%}$$

4. The power dissipated (wasted) in a transmission line is proportional to the square of the current passing through the lines, or $P=I^2R.$ To minimize the ratio of the power dissipated to the power delivered, it is desirable to have the smallest current possible, which requires a large transmission voltage. However, it is also desirable, for safety reasons, for the voltage at the consumer end to be low. The use of transformers in ac circuits makes it possible to have high voltage in the transmission lines while having low voltage for the consumers.

5. The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, with an amplitude given by Eq. 20–6. Using Eq. 21–1, approximate the flux by using the average distance from the power line to the coil. Take the change in flux to be the maximum flux through the coil, as suggested in the problem hint. Set the maximum emf equal to 170 V and note that the flux changes from the maximum to zero in a quarter cycle. Set the area of the coil equal to the product of the length and the height of the rectangle, and solve for the length.

$$|\mathscr{E}| = N\frac{\Delta\Phi_B}{\Delta t} = \frac{N(\Delta B)A}{\Delta t} = \frac{N\left(\dfrac{\mu_0 I_0}{2\pi r_{\text{avg}}}\right)w\ell}{\Delta t} \quad \rightarrow$$

$$\ell = \frac{|\mathscr{E}|\Delta t}{w}\frac{2\pi r_{\text{avg}}}{N\mu_0 I_0} = \frac{(170\text{ V})\left(\frac{1}{240}\text{ s}\right)}{2.0\text{ m}}\frac{2\pi(6.0\text{ m})}{2000(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(155\text{ A})} = 34.27\text{ m} \approx \boxed{34\text{ m}}$$

This is stealing and unethical because the current in the rectangle creates a back emf in the initial wire, which results in a power loss to the electric company just as if the wire had been directly connected to the line.

6. The average induced emf is given by Eq. 21–2b. Because the coil orientation changes by 180°, the change in flux is the opposite of twice the initial flux. The average current is the induced emf divided by the resistance, and the charge that flows in a given time is the current times the elapsed time.

$$\mathscr{E}_{\text{avg}} = -N\frac{\Delta\Phi_B}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{[(-B)-(+B)]}{\Delta t} = \frac{2NAB}{\Delta t}$$

$$Q = I\Delta t = \frac{\mathscr{E}_{\text{avg}}}{R}\Delta t = \frac{\left(\dfrac{2NAB}{\Delta t}\right)}{R}\Delta t = \frac{2NAB}{R} \quad \rightarrow \quad \boxed{B = \frac{QR}{2NA}}$$

22

ELECTROMAGNETIC WAVES

Responses to Questions

1. If the direction of travel for the EM wave is north and the electric field oscillates in an east–west plane, then the magnetic field must oscillate up and down. For an EM wave, the direction of travel, the electric field, and the magnetic field must all be perpendicular to each other.

2. No, sound is not an electromagnetic wave. Sound is a longitudinal mechanical (pressure) wave, which requires a medium in which to travel. The medium can be a gas, a liquid, or a solid. EM waves do not need a medium in which to travel.

3. EM waves can travel through a perfect vacuum. The energy is carried in the oscillating electric and magnetic fields, and no medium is required to travel. Sound waves cannot travel through a perfect vacuum. A medium is needed to carry the energy of a mechanical wave such as sound, and there is no medium in a perfect vacuum.

4. No. Electromagnetic waves travel at a very large but finite speed. When you flip on a light switch, it takes a very small amount of time for the electrical signal to travel along the wires from the switch to the lightbulb.

5. The wavelengths of radio and TV signals are much longer than those of visible light. Radio waves are on the order of 3 m–30,000 m. TV waves are on the order of 0.3 m–3 m. Visible waves are on the order of 10^{-7} m.

6. It is not necessary to make the lead-in wires to your speakers the exact same length. Since electrical signals in the wires travels at nearly the speed of light, the difference in time between the signals getting to the different speakers will be too small for your ears to detect.

7. Wavelength of 10^3 km: sub-radio waves (or very long radio waves; for example, ELF waves for submarine communication fall into this category). Wavelength of 1 km: radio waves. Wavelength of 1 m: TV signals and microwaves. Wavelength of 1 cm: microwaves and satellite TV signals. Wavelength of 1 mm: microwaves and infrared waves. Wavelength of 1 μm: infrared waves.

$\boxed{8.}$ Yes, radio waves can have the same frequencies as sound waves. These 20- to 20,000-Hz EM waves would have extremely long wavelengths when compared to the sound waves, because of their high speed. A 5000-Hz sound wave has a wavelength of about 70 mm, while a 5000-Hz EM wave has a wavelength of about 60 km.

$\boxed{9.}$ The receiver antenna should also be vertical for obtaining the best reception. The oscillating carrier electric field is up and down, so a vertical antenna would "pick up" that signal better. That is because the electrons in the metal antenna would be forced to oscillate up and down along the entire length of the vertical antenna, creating a stronger signal.

10. Diffraction effects (the bending of waves around the edge of an object) are evident only when the size of the wavelength of the wave is on the order of the size of the object (or larger). AM waves have wavelengths that are on the order of 300 m long, while FM waves have wavelengths on the order of 3 m long. Buildings and hills are much larger than FM waves, so FM waves will not diffract around the buildings and hills. Thus the FM signal will not be received behind the hills or buildings. On the other hand, these objects are smaller than AM waves, so the AM waves will diffract around them easily. The AM signal can be received behind the objects.

11. Cordless phones utilize EM waves when sending information back and forth between the handset (the part you hold up to your ear/mouth) and its base (which is sitting in your house, physically connected to the wire phone lines that lead outside to the phone company's network). These EM waves are usually very weak—you can't walk very far away from the base before you lose the signal. Cell phones utilize EM waves when sending information back and forth between the phone and the nearest tower in your geographical area (which could be miles away from your location). These EM waves need to be much stronger than cordless phone waves (or the cell phone electronics need to be more sensitive) because of the larger distances involved.

12. Transmitting Morse code by flashing a flashlight on and off creates an AM wave. The amplitude of the carrier wave is increasing/decreasing every time you turn the flashlight on and off. The frequency of the carrier wave is visible light, which is on the order of $10^{14}–10^{15}$ Hz.

Responses to MisConceptual Questions

1. (*a*, *b*) All electromagnetic waves have the same velocity in a vacuum. The velocity is the product of the wavelength and frequency. Since X-rays and radio waves have different wavelengths but the same speed, they will also have different frequencies.

2. (*c*) Visible light has a wavelength on the order of 10^{-7} to 10^{-8} m. This is over a thousand times larger than the size of an atom.

3. (*a*, *b*, *c*, *e*, *f*, *g*, *h*) All electromagnetic waves travel at the speed of light. The only listed wave that is not an electromagnetic wave is (*d*) ultrasonic wave, which is a sound wave and would travel at the speed of sound.

4. (*e*) A common misconception is that electromagnetic waves travel at different speeds. They do not, as they all travel at the speed of light.

5. (*c*) In empty space, X-ray radiation and a radio wave both travel at the speed of light. X-ray radiation has a much smaller wavelength than a radio wave. Since wave speed is equal to the product of the wavelength and the frequency, and both waves travel at the same speed, the X-ray

radiation has a higher frequency than a radio wave. The amplitude depends upon the intensity of the wave, so either the X-ray radiation or the radio wave could have the greater amplitude.

6. (*b*) Frequency is the measure of the number of oscillations made per second. The rate at which the electrons oscillate will be equal to the oscillation frequency of the radiation emitted.

7. (*d*) A common misconception is to think that the radiation decreases linearly with distance, such that the intensity would decrease by a factor of two. However, the radiation intensity decreases as the square of the distance. Doubling the distance will decrease the radiation intensity to one-fourth the initial intensity.

8. (*a*) The direction of the magnetic field is perpendicular to the direction the wave is traveling and perpendicular to the electric field, so only north and south are possible answers. The right-hand rule can be used to determine which direction is correct by pointing fingers in the direction of the electric field (west) and bending them in the direction of the magnetic field (north), resulting in the thumb pointing in the direction of the wave (down).

9. (*c*) A misconception that can arise is thinking that the wave intensity is proportional to just one of the field amplitudes. However, the intensity is proportional to the product of the amplitudes, with the field amplitudes proportional to each other. If the intensity doubles, then each field (electric and magnetic) must increase by the same factor, $\sqrt{2}$.

10. (*d*) A common misconception is that every color surface will experience the same radiation pressure. However, the pressure is related to the change in momentum of the light. With a black object, all of the light is absorbed, so the radiation pressure is proportional to the incident momentum. With a colored object, some of the light is reflected, thus increasing the change in momentum of the light and therefore increasing the radiation pressure as compared to the black object. For a white object, the maximum amount of light is reflected off the surface, creating the greatest change in momentum of the light and the greatest radiation pressure.

11. (*c*) A common misconception is that the digital broadcast uses a different frequency than the analog broadcast. This is incorrect—the carrier wave frequency is the same. Since the antenna is designed for operation at the broadcast frequency, the antenna will work just as well with a digital signal.

Solutions to Problems

1. The electric field between the plates is given by Eq. 17–4a, $E = V/d$, where d is the distance between the plates. We use that to find the rate of change of the electric field.

$$E = \frac{V}{d} \quad \rightarrow \quad \frac{\Delta E}{\Delta t} = \frac{1}{d}\frac{\Delta V}{\Delta t} = \left(\frac{1}{0.0011\text{ m}}\right)(120\text{ V/s}) = \boxed{1.1\times 10^{5}\,\frac{\text{V/m}}{\text{s}}}$$

2. The displacement current is shown in Section 22–1 to be $I_{\text{D}} = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$, and the flux is given by $\Phi_E = EA$, where A is the area of the capacitor plates.

$$I_{\text{D}} = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t} = \varepsilon_0 A \frac{\Delta E}{\Delta t} = (8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.058\text{ m})^2\left(1.6\times 10^{6}\,\frac{\text{V}}{\text{m}\cdot\text{s}}\right) = \boxed{4.8\times 10^{-8}\,\text{A}}$$

3. The current in the wires must also be the displacement current in the capacitor. Use the displacement current to find the rate at which the electric field is changing.

$$I_D = \varepsilon_0 \frac{\Delta\Phi_E}{\Delta t} = \varepsilon_0 A \frac{\Delta E}{\Delta t} \quad \rightarrow \quad \frac{\Delta E}{\Delta t} = \frac{I_\text{D}}{\varepsilon_0 A} = \frac{(3.8\text{ A})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.0160\text{ m})^2} = \boxed{1.7\times10^{15}\,\frac{\text{V}}{\text{m}\cdot\text{s}}}$$

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire, Eq. 20–6.

$$B = \frac{\mu_0 I}{2\pi R} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R} = \frac{(10^{-7}\,\text{T}\cdot\text{m/A})\,2\,(32.0\times10^{-3}\,\text{A})}{(0.100\text{ m})} = \boxed{6.40\times10^{-8}\text{ T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero}}$.

5. Use Eq. 22–2.

$$\frac{E_0}{B_0} = c \quad \rightarrow \quad B_0 = \frac{E_0}{c} = \frac{0.72\times10^{-4}\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = \boxed{2.4\times10^{-13}\text{ T}}$$

$\boxed{6.}$ Use Eq. 22–2.

$$\frac{E_0}{B_0} = c \quad \rightarrow \quad E_0 = B_0 c = (10.5\times10^{-9}\text{ T})(3.00\times10^{8}\text{ m/s}) = \boxed{3.15\text{ V/m}}$$

7. The frequency of the two fields must be the same: $\boxed{90.0\text{ kHz}}$.

The rms strength of the electric field is found from Eq. 22–2.

$$E_\text{rms} = cB_\text{rms} = (3.00\times10^{8}\text{ m/s})(7.75\times10^{-9}\text{ T}) = \boxed{2.33\text{ V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north–south line}}$.

8. Use the relationship $d = \upsilon t$ to find the time.

$$d = \upsilon t \quad \rightarrow \quad t = \frac{d}{\upsilon} = \frac{(1.50\times10^{11}\text{ m})}{(3.00\times10^{8}\text{ m/s})} = 5.00\times10^{2}\text{ s} = \boxed{8.33\text{ min}}$$

9. We assume that the only significant time is the transit time from the Earth to the Moon, at the speed of light. We use the Earth–Moon distance but subtract the Earth and Moon radii since the astronaut and the receiver are on the surfaces of the bodies.

$$d = \upsilon t \quad \rightarrow \quad t = \frac{d}{\upsilon} = \frac{d_{\substack{\text{Earth-}\\\text{Moon}}} - r_\text{Earth} - r_\text{Moon}}{c} = \frac{(384\times10^{6}\text{ m} - 6.38\times10^{6}\text{ m} - 1.74\times10^{6}\text{ m})}{(3.00\times10^{8}\text{ m/s})} = \boxed{1.25\text{ s}}$$

10. The frequency of the wave is given by Eq. 22–4.

$$f = \frac{c}{\lambda} = \frac{(3.00\times10^{8}\ \text{m/s})}{(720\times10^{-9}\ \text{m})} = \boxed{4.2\times10^{14}\ \text{Hz}}$$

This frequency is just inside the red end of the visible region, so it is $\boxed{\text{red visible light}}$.

11. The wavelength of the wave is calculated from Eq. 22–4.

$$\lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(7.14\times10^{14}\ \text{Hz})} = \boxed{4.20\times10^{-7}\ \text{m}}$$

This wavelength is $\boxed{\text{violet visible light}}$.

12. The radio frequency is found from Eq. 22–4.

$$f = \frac{c}{\lambda} = \frac{(3.0\times10^{8}\ \text{m/s})}{(49\ \text{m})} = \boxed{6.1\times10^{6}\ \text{Hz}}$$

13. Use Eq. 22–4 to find the frequency of the microwave.

$$c = \lambda f \quad \to \quad f = \frac{c}{\lambda} = \frac{(3.00\times10^{8}\ \text{m/s})}{(1.50\times10^{-2}\ \text{m})} = \boxed{2.00\times10^{10}\ \text{Hz}}$$

14. (*a*) Use Eq. 22–4 to find the wavelength.

$$c = \lambda f \quad \to \quad \lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(1.00\times10^{3}\ \text{Hz})} = \boxed{3.00\times10^{5}\ \text{m}}$$

(*b*) Again use Eq. 22–4, with the speed of sound in place of the speed of light.

$$v = \lambda f \quad \to \quad \lambda = \frac{v}{f} = \frac{(341\ \text{m/s})}{(1.00\times10^{3}\ \text{Hz})} = \boxed{0.341\ \text{m}}$$

(*c*) $\boxed{\text{No}}$, you cannot hear a 1000-Hz EM wave. It takes a pressure wave to excite the auditory system. However, if you applied the 1000-Hz EM wave to a speaker, you could hear the 1000-Hz pressure wave produced by the speaker.

$\boxed{15.}$ Use Eq. 22–4 to find the desired wavelength and frequency.

(*a*) $$c = \lambda f \quad \to \quad \lambda = \frac{c}{f} = \frac{(3.000\times10^{8}\ \text{m/s})}{(22.75\times10^{9}\ \text{Hz})} = \boxed{1.319\times10^{-2}\ \text{m}}$$

(*b*) $$c = \lambda f \quad \to \quad f = \frac{c}{\lambda} = \frac{(3.00\times10^{8}\ \text{m/s})}{(0.12\times10^{-9}\ \text{m})} = \boxed{2.5\times10^{18}\ \text{Hz}}$$

$\boxed{16.}$ (*a*) The radio waves travel at the speed of light, so $\Delta d = v\Delta t.$ The distance is found from the radii of the orbits. For times when Mars is nearest the Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{(230\times10^{9}\ \text{m} - 149.6\times10^{9}\ \text{m})}{(3.000\times10^{8}\ \text{m/s})} = 268\ \text{s} \approx \boxed{300\ \text{s}} \approx 4\ \text{min}$$

(*b*) For times when Mars is farthest from Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{(230\times10^9 \text{ m} + 149.6\times10^9 \text{ m})}{(3.000\times10^8 \text{ m/s})} = 1265 \text{ s} \approx \boxed{1300 \text{ s}} \approx 21 \text{ min}$$

17. We convert the units from light-years to meters.

$$d = (4.2 \text{ ly})(3.00\times10^8 \text{ m/s})(3.16\times10^7 \text{ s/yr}) = \boxed{4.0\times10^{16} \text{ m}}$$

18. The distance is the rate (speed of light) times the time (1 year), with the appropriate units.

$$d = (3.00\times10^8 \text{ m/s})(3.156\times10^7 \text{ s/yr}) = \boxed{9.47\times10^{15} \text{ m}}$$

19. The length of the pulse is $\Delta d = c\Delta t,$ so the number of wavelengths in this length is found by dividing this length by the wavelength.

$$N = \frac{(c\Delta t)}{\lambda} = \frac{(3.00\times10^8 \text{ m/s})(34\times10^{-12} \text{ s})}{(1062\times10^{-9} \text{ m})} = \boxed{9600 \text{ wavelengths}}$$

The time for the length of the pulse to be one wavelength is the wavelength divided by the speed of light.

$$\Delta t' = \frac{\lambda}{c} = \frac{(1062\times10^{-9} \text{ m})}{(3.00\times10^8 \text{ m/s})} = \boxed{3.54\times10^{-15} \text{ s}}$$

20. The eight-sided mirror would have to rotate 1/8 of a revolution for the succeeding mirror to be in position to reflect the light in the proper direction. During this time the light must travel to the opposite mirror and back.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{1}{8}(2\pi \text{ rad})}{(2\Delta x/c)} = \frac{(\pi \text{ rad})c}{8\Delta x} = \frac{(\pi \text{ rad})(3.00\times10^8 \text{ m/s})}{8(35\times10^3 \text{ m})} = \boxed{3400 \text{ rad/s}} \ (3.2\times10^4 \text{ rev/min})$$

$\boxed{21.}$ The mirror must rotate at least 1/8 of a revolution in the time it takes the light beam to travel twice the length of the room.

$$t = \frac{(2)(12 \text{ m})}{(3.0\times10^8 \text{ m/s})} = 8.0\times10^{-8} \text{ s}; \quad \omega = \frac{(\frac{1}{8} \text{ revolution})}{(8.0\times10^{-8} \text{ s})} = \boxed{1.6\times10^6 \text{ revolutions/s}}$$

22. The average energy transferred across unit area per unit time is the average intensity and is given by Eq. 22–8.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}(8.85\times10^{-12} \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})(0.0225 \text{ V/m})^2 = \boxed{6.72\times10^{-7} \text{ W/m}^2}$$

23. The energy per unit area per unit time is the same as the magnitude of the average intensity, as in Eq. 22–8. Use the given area and energy with the intensity to solve for the time.

$$\bar{I} = \frac{1}{2}\frac{cB_0^2}{\mu_0} = \frac{1}{2}\frac{c(\sqrt{2}B_{\text{rms}})^2}{\mu_0} = \frac{cB_{\text{rms}}^2}{\mu_0} = \frac{\Delta U}{A\Delta t}$$

$$\Delta t = \frac{\mu_0 \Delta U}{cB_{\text{rms}}^2 A} = \frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(365 \text{ J})}{(3.00\times10^8 \text{ m/s})(22.5\times10^{-9} \text{ T})^2(1.00\times10^{-4} \text{ m}^2)} = \boxed{3.02\times10^7 \text{ s}} \approx \boxed{350 \text{ days}}$$

24. The energy per unit area per unit time is the same as the magnitude of the average intensity, as in Eq. 22–8. Use the given area and time with the intensity to solve for the energy.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\rm rms})^2 = \varepsilon_0 c E_{\rm rms}^2 = \frac{\Delta U}{A\Delta t} \quad \rightarrow$$

$$\frac{\Delta U}{\Delta t} = c\varepsilon_0 E_{\rm rms}^2 A$$
$$= (3.00\times10^8 \text{ m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.0308 \text{ V/m})^2(1.00\times10^{-4}\,\text{m}^2)(3600 \text{ s/h})$$
$$= \boxed{9.07\times10^{-7} \text{ J/h}}$$

25. The intensity is the power per unit area. The area is the surface area of a sphere, since the wave is spreading spherically. Use Eq. 22–8 to find the rms value of the electric field.

$$\bar{I} = \frac{P}{A} = \frac{(1800 \text{ W})}{[4\pi(5.0 \text{ m})^2]} = 5.730 \text{ W/m}^2 \approx \boxed{5.7 \text{ W/m}^2}$$

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\rm rms})^2 = \varepsilon_0 c E_{\rm rms}^2 \quad \rightarrow$$

$$E_{\rm rms} = \sqrt{\frac{\bar{I}}{c\varepsilon_0}} = \sqrt{\frac{5.730 \text{ W/m}^2}{(3.00\times10^8 \text{ m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}} = \boxed{46 \text{ V/m}}$$

26. (*a*) We find E_0 using Eq. 22–2 with $v = c$.

$$E_0 = cB_0 = (3.00\times10^8 \text{ m/s})(2.2\times10^{-7} \text{ T}) = \boxed{66 \text{ V/m}}$$

(*b*) The average power per unit area is the intensity, given by Eq. 22–8.

$$\bar{I} = \frac{E_0 B_0}{(2\mu_0)} = \frac{(66 \text{ V/m})(2.2\times10^{-7} \text{ T})}{[2(4\pi\times10^{-7}\,\text{N}\cdot\text{s}^2/\text{C}^2)]} = \boxed{5.8 \text{ W/m}^2}$$

27. Use Eq. 22–6a (the instantaneous value of energy density) with Eq. 22–7.

$$u = \varepsilon_0 E^2 \quad \rightarrow \quad \bar{u} = \tfrac{1}{2}\varepsilon_0 E_0^2 = \frac{\bar{I}}{c}; \quad \bar{U} = \bar{u}V = \frac{\bar{I}}{c}V = \frac{1350 \text{ W/m}^2}{3.00\times10^8 \text{ m/s}}(1.00 \text{ m}^3) = \boxed{4.50\times10^{-6} \text{ J}}$$

$\boxed{28.}$ The power output per unit area is the intensity. Use Eq. 22–8 with rms values.

$$\bar{I} = \frac{\bar{P}}{A} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\rm rms})^2 = \varepsilon_0 c E_{\rm rms}^2 \quad \rightarrow$$

$$E_{\rm rms} = \sqrt{\frac{\bar{P}}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158 \text{ W}}{\pi(1.20\times10^{-3} \text{ m})^2(3.00\times10^8 \text{ m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}}$$
$$= 1146.9 \text{ V/m} \approx \boxed{1150 \text{ V/m}}$$

$$B_{\rm rms} = \frac{E_{\rm rms}}{c} = \frac{1146.9 \text{ V/m}}{3.00\times10^8 \text{ m/s}} = \boxed{3.82\times10^{-6} \text{ T}}$$

29. The Sun is assumed to radiate uniformly, so use the intensity at the Earth to calculate the total power output of the sun. Use a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The 1350 W/m^2 is the average intensity.

$$I = \frac{P}{A} \quad \rightarrow \quad P = IA = 4\pi R^2 I = 4\pi(1.496\times10^{11} \text{ m})^2(1350 \text{ W/m}^2) = \boxed{3.80\times10^{26} \text{ W}}$$

30. (*a*) The energy emitted in each pulse is the laser's power output times the duration of the pulse.

$$P=\frac{\Delta W}{\Delta t} \quad\rightarrow\quad \Delta W=P\Delta t=(1.5\times 10^{11}\ \text{W})(1.0\times 10^{-9}\ \text{s})=\boxed{150\ \text{J}}$$

(*b*) We find the rms electric field from the intensity, which is the power per unit area. Use Eq. 22–8a with rms values. The area to use is the cross-sectional area of the cylindrical beam.

$$\bar{I}=\frac{\bar{P}}{A}=\tfrac{1}{2}\varepsilon_0 cE_0^2=\tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\text{rms}})^2=\varepsilon_0 cE_{\text{rms}}^2 \quad\rightarrow$$

$$E_{\text{rms}}=\sqrt{\frac{\bar{P}}{Ac\varepsilon_0}}=\sqrt{\frac{(1.5\times 10^{11}\ \text{W})}{\pi(2.2\times 10^{-3}\ \text{m})^2(3.00\times 10^{8}\ \text{m/s})(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}}$$

$$=\boxed{1.9\times 10^{9}\ \text{V/m}}$$

31. We assume the light energy is all absorbed, so use Eq. 22–10a. The intensity is the power radiated over a spherical surface area.

$$\text{Pressure}=P=\frac{\bar{I}}{c}=\frac{\dfrac{25\ \text{W}}{4\pi(9.5\times 10^{-2}\ \text{m})^2}}{(3.00\times 10^{8}\ \text{m/s})}=7.348\times 10^{-7}\ \text{N/m}^2\approx\boxed{7.3\times 10^{-7}\ \text{N/m}^2}$$

The force on the fingertip is the pressure times the area of the fingertip. We approximate the area of a fingertip to be $1.0\ \text{cm}^2$.

$$F=PA=(7.348\times 10^{-7}\ \text{N/m}^2)(1.0\times 10^{-4}\ \text{m}^2)=\boxed{7.3\times 10^{-11}\ \text{N}}$$

32. The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Earth}}}{I_{\text{Jupiter}}}=\frac{r^2_{\substack{\text{Sun-}\\\text{Jupiter}}}}{r^2_{\substack{\text{Sun-}\\\text{Earth}}}}=\frac{(7.78\times 10^{11}\ \text{m})^2}{(1.496\times 10^{11}\ \text{m})^2}=27.0$$

So it would take an area of $\boxed{27\ \text{m}^2}$ at Jupiter to collect the same radiation as a 1.0-m^2 solar panel at the Earth.

33. We convert the horsepower rating to watts and then use the intensity to relate the wattage to the needed area.

$$I=\frac{P}{A} \quad\rightarrow\quad A=\frac{P}{I}=\frac{100\ \text{hp}\times\dfrac{746\ \text{W}}{1\ \text{hp}}}{200\ \text{W/m}^2}=373\ \text{m}^2\approx\boxed{400\ \text{m}^2}$$

34. Use Eq. 22–4. Note that the higher frequencies have the shorter wavelengths.

(*a*) For FM radio we have the following.

$$\lambda=\frac{c}{f}=\frac{(3.00\times 10^{8}\ \text{m/s})}{(1.08\times 10^{8}\ \text{Hz})}=\boxed{2.78\ \text{m}}\ \text{to}\ \lambda=\frac{c}{f}=\frac{(3.00\times 10^{8}\ \text{m/s})}{(8.8\times 10^{7}\ \text{Hz})}=\boxed{3.41\ \text{m}}$$

(*b*) For AM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(1.7\times10^{6}\ \text{Hz})} = \boxed{180\ \text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(5.35\times10^{5}\ \text{Hz})} = \boxed{561\ \text{m}}$$

35. Use Eq. 22–4.

$$\lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(1.9\times10^{9}\ \text{Hz})} = \boxed{0.16\ \text{m}}$$

36. The frequencies are 980 kHz on the AM dial and 98.1 MHz on the FM dial. From Eq. 22–4, $c = f\lambda$, we see that the lower frequency will have the longer wavelength: $\boxed{\text{the AM station}}$.

When we form the ratio of wavelengths, we get

$$\frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2} = \frac{(98.1\times10^{6}\ \text{Hz})}{(980\times10^{3}\ \text{Hz})} = \boxed{100}$$

The factor is good to 2 significant figures.

37. Each wavelength is found by dividing the speed of light by the frequency, as in Eq. 22–4.

Channel 2: $\lambda_2 = \dfrac{c}{f_2} = \dfrac{(3.00\times10^{8}\ \text{m/s})}{(54.0\times10^{6}\ \text{Hz})} = \boxed{5.56\ \text{m}}$

Channel 51: $\lambda_{51} = \dfrac{c}{f_{51}} = \dfrac{(3.00\times10^{8}\ \text{m/s})}{(692\times10^{6}\ \text{Hz})} = \boxed{0.434\ \text{m}}$

38. The resonant frequency of an *LC* circuit is given by Eq. 21–19. We assume that the inductance is constant and form the ratio of the two frequencies.

$$\frac{f_1}{f_2} = \frac{\dfrac{1}{2\pi\sqrt{LC_1}}}{\dfrac{1}{2\pi\sqrt{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \quad\rightarrow\quad C_2 = \left(\frac{f_1}{f_2}\right)^2 C_1 = \left(\frac{550\ \text{kHz}}{1610\ \text{kHz}}\right)^2 (2500\ \text{pF}) = \boxed{290\ \text{pF}}$$

39. The resonant frequency of an *LC* circuit is given by Eq. 21–19. Solve for the capacitance.

$$f = \frac{1}{2\pi\sqrt{LC}} \quad\rightarrow\quad C = \frac{1}{4\pi^2 f^2 F} = \frac{1}{4\pi^2(98.3\times10^{6}\ \text{Hz})^2(1.8\times10^{-6}\ \text{H})} = \boxed{1.5\times10^{-12}\ \text{F}}$$

$\boxed{40.}$ The resonant frequency of an *LC* circuit is given by Eq. 21–19. Solve for the inductance.

$$f = \frac{1}{2\pi\sqrt{LC}} \quad\rightarrow\quad L = \frac{1}{4\pi^2 f^2 C}$$

$$L_1 = \frac{1}{4\pi^2 f_1^2 C} = \frac{1}{4\pi^2(88\times10^{6}\ \text{Hz})^2(810\times10^{-12}\ \text{F})} = 4.0\times10^{-9}\ \text{H}$$

$$L_2 = \frac{1}{4\pi^2 f_2^2 C} = \frac{1}{4\pi^2(108\times10^{6}\text{Hz})^2(810\times10^{-12}\ \text{F})} = 2.7\times10^{-9}\ \text{H}$$

The range of inductances is $\boxed{2.7\times10^{-9}\ \text{H} \le L \le 4.0\times10^{-9}\ \text{H}}$.

41. (*a*) The minimum value of C corresponds with the higher frequency, according to Eq. 21–19.

$$f=\frac{1}{2\pi\sqrt{LC}} \rightarrow$$

$$L=\frac{1}{4\pi^2 f^2 C}=\frac{1}{4\pi^2(15.0\times10^6\text{ Hz})^2(86\times10^{-12}\text{ F})}=1.31\times10^{-6}\text{ H}\approx\boxed{1.3\times10^{-6}\text{ H}}$$

(*b*) The maximum value of C corresponds with the lower frequency.

$$f=\frac{1}{2\pi\sqrt{LC}} \rightarrow C=\frac{1}{4\pi^2 f^2 F}=\frac{1}{4\pi^2(14.0\times10^6\text{ Hz})^2(1.31\times10^{-6}\text{ H})}=\boxed{9.9\times10^{-11}\text{ F}}$$

42. The rms electric field strength of the beam can be found from the average intensity, which is the power per unit area. Use Eq. 22–8.

$$\bar{I}=\frac{\bar{P}}{A}=\tfrac{1}{2}\varepsilon_0 cE_0^2=\tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\text{rms}})^2=\varepsilon_0 cE_{\text{rms}}^2 \rightarrow$$

$$E_{\text{rms}}=\sqrt{\frac{\bar{P}}{Ac\varepsilon_0}}=\sqrt{\frac{1.3\times10^4\text{ W}}{\pi(750\text{ m})^2(3.00\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}}=1.665\text{ V/m}\approx\boxed{1.7\text{ V/m}}$$

43. The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the intensity.

$$E_{\text{rms}}=\frac{V_{\text{rms}}}{d}=\frac{1.00\times10^{-3}\text{ V}}{1.60\text{ m}}=\boxed{6.25\times10^{-4}\text{ V/m}}$$

$$\bar{I}=\frac{\bar{P}}{A}=\tfrac{1}{2}\varepsilon_0 cE_0^2=\tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\text{rms}})^2=\varepsilon_0 cE_{\text{rms}}^2=\varepsilon_0 c\frac{V_{\text{rms}}^2}{d^2}$$

$$=(3.00\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)\frac{(1.00\times10^{-3}\text{ V})^2}{(1.60\text{ m})^2}=\boxed{1.04\times10^{-9}\text{ W/m}^2}$$

44. We ignore the time for the sound to travel to the microphone. Find the difference between the time for sound to travel to the balcony and for a radio wave to travel 1200 km.

$$\Delta t=t_{\text{radio}}-t_{\text{sound}}=\left(\frac{d_{\text{radio}}}{c}\right)-\left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right)=\left(\frac{1.2\times10^6\text{ m}}{3.00\times10^8\text{ m/s}}\right)-\left(\frac{50.0\text{ m}}{343\text{ m/s}}\right)=-0.142\text{ s},$$

so the $\boxed{\text{person at the radio hears the voice 0.142 s sooner.}}$

45. The ratio of distance to time must be the speed of light.

$$\frac{\Delta x}{\Delta t}=c \rightarrow \Delta t=\frac{\Delta x}{c}=\frac{3\text{ m}}{3.0\times10^8\text{ m/s}}=1\times10^{-8}\text{s}\left(\frac{10^9\text{ ns}}{1\text{ s}}\right)=\boxed{10\text{ ns}}$$

46. The length is found from the speed of light and the duration of the burst.

$$\Delta x=c\Delta t=(3.00\times10^8\text{ m/s})(10^{-8}\text{ s})=\boxed{3\text{ m}}$$

$\boxed{47.}$ The time consists of the time for the radio signal to travel to Earth and the time for the sound to travel from the loudspeaker. We use 343 m/s for the speed of sound.

$$t = t_{\text{radio}} + t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) + \left(\frac{d_{\text{sound}}}{\upsilon_{\text{sound}}}\right) = \left(\frac{3.84\times10^8 \text{ m}}{3.00\times10^8 \text{ m/s}}\right) + \left(\frac{28 \text{ m}}{343 \text{ m/s}}\right) = \boxed{1.36 \text{ s}}$$

Note that about 5% of the time is for the sound wave.

The microphone was inside the helmet. The Moon has essentially no atmosphere, so there would not be sound waves in free space on the Moon. The helmet is filled with breathable air, so the microphone has to be inside the helmet so that the sound waves will travel in the air around the astronaut's head, inside the helmet.

48. The time travel delay is the distance divided by the speed of radio waves (which is the speed of light).

$$t = \frac{d}{c} = \frac{3\times10^6 \text{ m}}{3.00\times10^8 \text{ m/s}} = \boxed{0.01 \text{ s}}$$

49. After the change occurred, we would find out when the change in radiation reached the Earth, traveling at the speed of light.

$$\Delta t = \frac{d}{c} = \frac{(1.50\times10^{14} \text{ m})}{(3.00\times10^8 \text{ m/s})} = 5.00\times10^2 \text{ s} = \boxed{8.33 \text{ min}}$$

50. (*a*) The instantaneous energy density is given by Eq. 22–6a. We convert it to average values and use the rms value of the associated electric field.

$$u = \varepsilon_0 E^2 \quad \rightarrow \quad \overline{u} = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(\sqrt{2}E_{\text{rms}})^2 = \varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{\overline{u}}{\varepsilon_0}} = \sqrt{\frac{4\times10^{-14} \text{ J/m}^3}{8.85\times10^{-12} \text{C}^2/\text{N}\cdot\text{m}^2}} = 0.0672 \text{ V/m} \approx \boxed{0.07 \text{ V/m}}$$

(*b*) The distance to receive a comparable value can be found using the average intensity.

$$\overline{I} = \frac{\overline{P}}{4\pi r^2} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\text{rms}})^2 = \varepsilon_0 c E_{\text{rms}}^2 \quad \rightarrow$$

$$r = \frac{1}{E_{\text{rms}}}\sqrt{\frac{\overline{P}}{4\pi\varepsilon_0 c}} = \frac{1}{0.0672 \text{ V/m}}\sqrt{\frac{7500 \text{ W}}{4\pi(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})}}$$

$$= 7055 \text{ m} \approx \boxed{7 \text{ km}}$$

$\boxed{51.}$ The light has the same intensity in all directions, so use a spherical geometry centered on the source to find the value of the intensity, Then use Eq. 22–8 to find the magnitude of the electric field and Eq. 22–2 with $\upsilon = c$ to find the magnitude of the magnetic field.

$$\overline{I} = \frac{\overline{P}}{A} = \frac{\overline{P}}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{\overline{P}}{2\pi r^2 c\varepsilon_0}} = \sqrt{\frac{(18 \text{ W})}{2\pi(2.50 \text{ m})^2(3.00\times10^8 \text{ m/s})(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)}} = 13.14 \text{ V/m} \approx \boxed{13 \text{ V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{(13.14 \text{ V/m})}{(3.00\times10^8 \text{ m/s})} = \boxed{4.4\times10^{-8} \text{ T}}$$

52. The radiation from the Sun has the same intensity in all directions, so the rate at which energy passes through a sphere centered at the Sun is $P = I(4\pi R^2)$. This rate must be the same at any distance from the Sun. Use this fact to calculate the magnitude of the intensity at Mars, and then use the intensity vector to calculate the rms magnitude of the electric field at Mars.

$$\bar{I}_{\text{Mars}}(4\pi R_{\text{Mars}}^2) = \bar{I}_{\text{Earth}}(4\pi R_{\text{Earth}}^2) \quad \rightarrow$$

$$\bar{I}_{\text{Mars}} = \bar{I}_{\text{Earth}}\left(\frac{R_{\text{Earth}}^2}{R_{\text{Mars}}^2}\right) = \tfrac{1}{2}\varepsilon_0 c E_{0\,\text{Mars}}^2 = \tfrac{1}{2}\varepsilon_0 c\left(\sqrt{2}E_{\text{rms Mars}}\right)^2 = \varepsilon_0 c E_{\text{rms Mars}}^2$$

$$E_{\text{rms Mars}} = \sqrt{\frac{\bar{I}_{\text{Earth}}}{c\varepsilon_0}}\left(\frac{R_{\text{Earth}}}{R_{\text{Mars}}}\right) = \sqrt{\frac{1350\ \text{W/m}^2}{(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}}\left(\frac{1}{1.52}\right) = \boxed{469\ \text{V/m}}$$

53. (*a*) Since intensity is energy per unit time per unit area, the energy is found by multiplying the intensity by the area of the antenna and the elapsed time.

$$\Delta U = IA\Delta t = (1.0\times10^{-13}\ \text{W/m}^2)\pi\left(\frac{0.33\ \text{m}}{2}\right)^2(4.0\ \text{h})(3600\ \text{s/h}) = \boxed{1.2\times10^{-10}\ \text{J}}$$

(*b*) The electric field amplitude can be found from the intensity, using Eq. 22–8. The magnitude of the magnetic field is then found from Eq. 22–2 with $v = c$.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{2\bar{I}}{\varepsilon_0 c}} = \sqrt{\frac{2(1.0\times10^{-13}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 8.679\times10^{-6}\ \text{V/m}$$

$$\approx \boxed{8.7\times10^{-6}\ \text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{8.679\times10^{-6}\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = \boxed{2.9\times10^{-14}\ \text{T}}$$

54. According to Fig. 22–8, satellite TV waves have a wavelength of approximately 0.03 m. Example 22–2 says that the wavelength is 4 times the antenna's length. So the antenna might be $\boxed{0.075\ \text{m}}$ in length.

$\boxed{55.}$ Use the relationship between average intensity (the magnitude of the Poynting vector) and electric field strength, as given by Eq. 22–8. Also use the fact that intensity is power per unit area. We assume that the power is spherically symmetric about the source.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0{}^2 = \frac{\bar{P}}{A} = \frac{\bar{P}}{4\pi r^2} \quad \rightarrow$$

$$r = \sqrt{\frac{\bar{P}}{2\pi\varepsilon_0 c E_0{}^2}} = \sqrt{\frac{25{,}000\ \text{W}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\,(3.00\times10^8\ \text{m/s})(0.020\ \text{V/m})^2}} = 61{,}200\ \text{m}$$

$$\approx \boxed{61\ \text{km}}$$

Thus, to receive the transmission one should be within 61 km of the station.

56. (*a*) The radiation pressure when there is total reflection is given in Eq. 22–10b.

$$P = \frac{2I_{\text{avg}}}{c} = \frac{2(1350\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}} = \boxed{9.00\times10^{-6}\ \text{N/m}^2}$$

(*b*) The force on the sail is the pressure times the area; then use Newton's second law to calculate the acceleration.

$$P=\frac{F}{A}=\frac{ma}{A}\quad\rightarrow\quad a=\frac{PA}{m}=\frac{P}{m/A}=\frac{9.00\times10^{-6}\ \text{N/m}^2}{1\times10^{-3}\ \text{kg/m}^2}=\boxed{9\times10^{-3}\ \text{m/s}^2}$$

This answer is not an accurate description of the full situation. There is also a gravitational attraction due to the Sun's mass, and it might be big enough to change the acceleration calculated here.

(*c*) The required force is the total mass (the payload plus the sail) and is provided by the radiation pressure times the area of the sail.

$$P=\frac{F}{A}=\frac{ma}{A}\quad\rightarrow\quad A=\frac{ma}{P}=\frac{(m_{\text{payload}}+m_{\text{sail}})a}{P}=\frac{(m_{\text{payload}}+(0.001\ \text{kg/m}^2)A)a}{P}\quad\rightarrow$$

$$A=\frac{m_{\text{payload}}a}{[P-(0.001\ \text{kg/m}^2)a]}=\frac{(100\ \text{kg})(1\times10^{-3}\ \text{m/s}^2)}{[(9.00\times10^{-6}\ \text{N/m}^2)-(0.001\ \text{kg/m}^2)(1\times10^{-3}\ \text{m/s}^2)]}$$

$$=1.25\times10^4\,\text{m}^2\approx\boxed{1\times10^4\,\text{m}^2}$$

Again, the Sun's gravitational attraction should also be figured into this before we claim to have a full understanding of the situation.

57. (*a*) The radio waves have the same intensity in all directions. The power crossing a given area is the intensity times the area. The intensity is the total power through the area of a sphere centered at the source.

$$\overline{P}=\overline{I}A=\left(\frac{P_0}{A_{\text{total}}}\right)A=\frac{35{,}000\ \text{W}}{4\pi(1.0\times10^3\ \text{m})^2}(1.0\ \text{m}^2)=2.785\times10^{-3}\ \text{W}\approx\boxed{2.8\ \text{mW}}$$

(*b*) We find the rms value of the electric field from the intensity, using Eq. 22–8.

$$\overline{I}=\frac{\overline{P}}{4\pi r^2}=\tfrac{1}{2}\varepsilon_0 cE_0^2=\tfrac{1}{2}\varepsilon_0 c(\sqrt{2}E_{\text{rms}})^2=\varepsilon_0 cE_{\text{rms}}^2$$

$$E_{\text{rms}}=\sqrt{\frac{\overline{P}}{4\pi r^2 c\varepsilon_0}}=\sqrt{\frac{35{,}000\ \text{W}}{4\pi(1.0\times10^3\ \text{m})^2(3.00\times10^8\ \text{m/s})\,(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}}$$

$$=1.024\ \text{V/m}\approx\boxed{1.0\ \text{V/m}}$$

(*c*) The voltage over the length of the antenna is the electric field times the length of the antenna.

$$V_{\text{rms}}=E_{\text{rms}}d=(1.024\ \text{V/m})(1.0\ \text{m})=\boxed{1.0\ \text{V}}$$

(*d*) We calculate the electric field at the new distance, and then calculate the voltage.

$$E_{\text{rms}}=\sqrt{\frac{\overline{P}}{4\pi r^2 c\varepsilon_0}}=\sqrt{\frac{35{,}000\ \text{W}}{4\pi(5.0\times10^4\ \text{m})^2(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}}$$

$$=2.048\times10^{-2}\ \text{V/m};\ \ V_{\text{rms}}=E_{\text{rms}}d=(2.048\times10^{-2}\ \text{V/m})(1.0\ \text{m})=\boxed{2.0\times10^{-2}\ \text{V}}$$

58. The light has the same intensity in all directions. Use a spherical geometry centered at the source with the definition of the average intensity, Eq. 22–8. We also use Eq. 22–3.

$$\overline{I}=\frac{P_0}{A}=\frac{P_0}{4\pi r^2}=\tfrac{1}{2}c\varepsilon_0 E_0{}^2=\tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0\quad\rightarrow\quad\tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0=\frac{P_0}{4\pi r^2}\quad\rightarrow\quad\boxed{E_0=\sqrt{\frac{\mu_0 cP_0}{2\pi r^2}}}$$

59. The power output of the antenna would be the intensity at a given distance from the antenna times the area of a sphere surrounding the antenna. The intensity is given by Eq. 22–8.

$$\bar{I} = \frac{\bar{P}}{A} = \tfrac{1}{2}c\varepsilon_0 E_0^2$$
$$\bar{P} = 4\pi r^2 \bar{I} = 2\pi r^2 c\varepsilon_0 E_0^2 = 2\pi(0.65\text{ m})^2(3.00\times10^8\text{ m/s})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3\times10^6\text{ V/m})^2$$
$$\approx \boxed{6\times10^{10}\text{ W}}$$

This is many orders of magnitude higher than the power output of commercial radio stations, which are no higher than tens of kilowatts.

60. The wavelength of the AM radio signal is given by Eq. 22–4, $\lambda = \dfrac{c}{f} = \dfrac{(3.0\times10^8\text{ m/s})}{(10^6\text{ Hz})} = 300\text{ m}.$

(*a*) $\tfrac{1}{2}\lambda = \tfrac{1}{2}(300\text{ m}) = \boxed{150\text{ m}}$

(*b*) $\tfrac{1}{4}\lambda = \tfrac{1}{4}(300\text{ m}) = \boxed{75\text{ m}}$

So there must be another way to pick up this signal—we don't see antennas this long for AM radio.

61. We find the average intensity at the 12-km distance from Eq. 22–8. Then the power can be found from the intensity, assuming that the signal is spherically symmetric, using the surface area of a sphere of radius 12 km.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0{}^2 = \tfrac{1}{2}(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(3\times10^8\text{ m/s})(0.12\text{ V/m})^2 = 1.9116\times10^{-5}\text{ W/m}^2$$
$$P = \bar{I}A = (1.9116\times10^{-5}\text{ W/m}^2)4\pi(12\times10^3\text{ m})^2 = \boxed{35\text{ kW}}$$

Solutions to Search and Learn Problems

1. In each case, the required area is the power requirement of the device divided by 20% of the intensity of the sunlight.

(*a*) $A = \dfrac{P}{I} = \dfrac{50\times10^{-3}\text{ W}}{200\text{ W/m}^2} = 2.5\times10^{-4}\text{ m}^2 \approx \boxed{3\text{ cm}^2}$

(*b*) $A = \dfrac{P}{I} = \dfrac{1500\text{ W}}{200\text{ W/m}^2} = 7.5\text{ m}^2 \approx \boxed{8\text{ m}^2}$ (to 1 significant figure)

(*c*) $A = \dfrac{P}{I} = \dfrac{40\text{ hp}(746\text{ W/hp})}{200\text{ W/m}^2} = 149\text{ m}^2 \approx \boxed{100\text{ m}^2}$

(*d*) Calculator: A typical graphing calculator is about $18\text{ cm}\times8\text{ cm},$ which is about $140\text{ cm}^2,$ so $\boxed{\text{yes}}$, the solar panel can be mounted directly on the calculator. Hair dryer: A house of floor area 1000 ft^2 would have a roof area on the order of $100\text{ m}^2,$ so $\boxed{\text{yes}}$, a solar panel on the roof should be able to power the hair dryer. Car: This would require a square panel of side length about 12 m, which is much larger than the car. So $\boxed{\text{no}}$, this panel could not be mounted on a car and used for real-time power.

2. (*a*) From Eq. 22–10b, the radiation pressure is $P = \frac{2\bar{I}}{c}$, where the intensity is the power per unit area delivered to the suit. The force is the pressure times the area.

$$F_{\text{laser}} = PA = \frac{2}{c}\bar{I}A = \frac{2}{c}\left(\frac{3.0\text{ W}}{\pi r^2}\right)(\pi r^2) = \frac{2(3.0\text{ W})}{3.00\times10^8\text{ m/s}} = \boxed{2.0\times10^{-8}\text{ N}}$$

(*b*) Use Newton's law of universal gravitation, Eq. 6–1, to estimate the gravitational force. We take the 30-m distance as having 2 significant figures.

$$F_{\text{grav}} = G\frac{m_{\text{shuttle}}m_{\text{astronaut}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.03\times10^5\text{ kg})(120\text{ kg})}{(30\text{ m})^2}$$

$$= 9.160\times10^{-7}\text{ N} \approx \boxed{9.2\times10^{-7}\text{ N}}$$

(*c*) $\boxed{\text{The gravity force is larger, by a factor of approximately 50.}}$

3. (*a*) The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Sun}}}{I_{\text{Star}}} = \frac{r^2_{\text{Star-Earth}}}{r^2_{\text{Sun-Earth}}} \rightarrow r_{\text{Star-Earth}} = r_{\text{Sun-Earth}}\sqrt{\frac{I_{\text{Sun}}}{I_{\text{Star}}}} = (1.496\times10^{11}\text{ m})\sqrt{\frac{1350\text{ W/m}^2}{1\times10^{-23}\text{ W/m}^2}}\left(\frac{1\text{ ly}}{9.46\times10^{15}\text{ m}}\right)$$

$$= 1.84\times10^8\text{ ly} \approx \boxed{2\times10^8\text{ ly}}$$

(*b*) Compare this distance to the galactic size.

$$\frac{r_{\text{Star-Earth}}}{\text{galactic size}} = \frac{1.84\times10^8\text{ ly}}{1\times10^5\text{ ly}} = 1840 \approx \boxed{2000}$$

The distance to the star is about 2000 times the size of our galaxy.

4. (*a*) Since the cylinder is absorbing the radiation, the pressure (P) is given by Eq. 22–10a, where the intensity is the power (P_{avg}) divided by the cross-sectional area. The force is the pressure multiplied by the cross-sectional area of the cylinder, or the power absorbed divided by the speed of light.

$$P = \frac{\bar{I}}{c} = \frac{P_{\text{avg}}}{cA} = \frac{1.0\text{ W}}{(3.00\times10^8\text{ m/s})\pi(5\times10^{-7}\text{ m})^2} = 4244\text{ W/m}^2 \approx \boxed{4\times10^3\text{ W/m}^2}$$

$$F = PA = \frac{P_{\text{avg}}}{cA}A = \frac{P_{\text{avg}}}{c} = \frac{1.0\text{ W}}{3.00\times10^8\text{ m/s}} = \boxed{3.3\times10^{-9}\text{ N}}$$

(*b*) The acceleration of the cylindrical particle will be the force on it (due to radiation pressure) divided by its mass. The mass of the particle is its volume times the density of water.

$$a = \frac{F}{m} = \frac{F}{\rho_{H_2O}\pi r^3} = \frac{3.3\times10^{-9}\text{ N}}{(1000\text{ kg/m}^2)\pi(5\times10^{-7}\text{ m})^3} = 8.49\times10^6\text{ m/s}^2 \approx \boxed{8\times10^6\text{ m/s}^2}$$

23

LIGHT: GEOMETRIC OPTICS

Responses to Questions

1. It is probably not reasonable to assume that he burned the whole fleet at once, but it does seem possible (although certainly difficult) to start them on fire one at a time. Several attempts have been made to reenact the event in order to test its feasibility. Two of the successful attempts include a 1975 experiment directed by Greek scientist Dr. Ioannis Sakkas and a 2005 experiment performed by a group of engineering students at MIT. (See www.mit.edu for links to both these and other similar experiments.) In both these cases, several individual mirrors operating together (perhaps like a group of soldiers' shields?) simulated a large spherical mirror and were used to ignite a wooden boat. If in fact the story is true, then Archimedes would have needed good weather and an enemy fleet that cooperated by staying relatively still while the focused sunlight heated the wood.

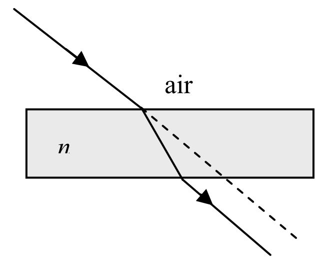
2. The focal length of a plane mirror is infinity. The magnification of a plane mirror is $+1$. As the radius (and focal length) of a spherical mirror increases, the front surface gets more and more flat. The ultimate limit is that as the radius (and focal length) of the spherical mirror goes to infinity, the front surface becomes perfectly flat. For this mirror, the image height and object height are identical (as are the image distance and object distance) and the image is virtual, with a magnification of $m = +1$.

3. The mirror doesn't actually reverse right and left, either, just as it doesn't reverse up and down. Instead it reverses "front" and "back." When you are looking in a flat mirror and move your right hand, it is the image of your right hand that moves in the mirror. When we see our image, though, we imagine it as if it is another person looking back at us. If we saw this other person raise their left hand, then we would see the hand that is on the right side of their body (from our point of view) move.

4. The image is real and inverted, because the magnification is negative. The mirror is concave, because convex mirrors can only form virtual images. The image is on the same side of the mirror as the object; real images are formed by converging light rays, and light rays cannot actually pass through a mirror.

5. Yes, if the mirror is the only piece of optics involved. When a concave mirror produces a real image of a real object, both d_o and d_i are positive. The magnification equation, $m = -\frac{d_i}{d_o}$, results in a negative magnification, which indicates that the image is inverted. If instead a virtual object were involved, then an upright real image would be formed.

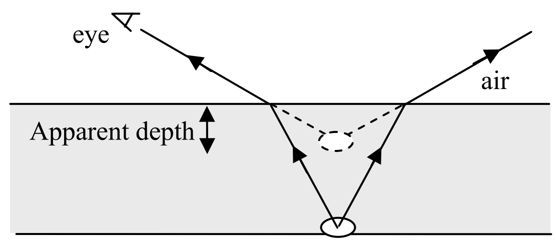
6. A light ray entering the solid rectangular object will exit the other side following a path that is parallel to its original path but displaced slightly from it. The angle of refraction in the glass can be determined geometrically from this displacement and the thickness of the object. The index of refraction can then be determined using Snell's law with this angle of refraction and the original angle of incidence. The speed of light in the material follows from the definition of the index of refraction: $n = c/v.$ Alternatively, if the angles of incidence and refraction at one surface can be measured, then Snell's law can be used to calculate the index of refraction.



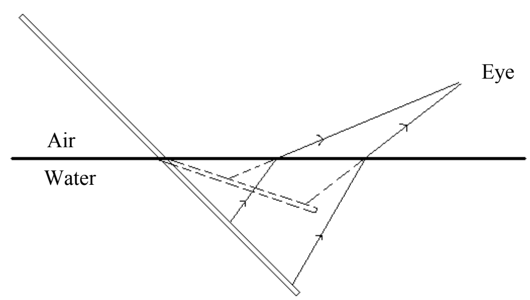
7. This effect is similar to diffuse reflection off of a rough surface. The ripply sea provides many different surface orientations, so the light from the Moon reflects from that surface at many different angles. This makes it look like there is one continuous reflection along the surface of the water between you and the Moon, instead of just one specular reflection spot.

8. When a light ray meets the boundary between two materials perpendicularly, both the angle of incidence and the angle of refraction are 0°. Snell's law shows this to be true for any combination of the indices of refraction of the two materials.

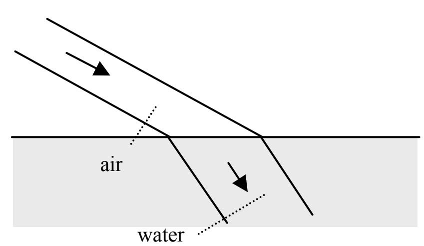
9. You are likely to underestimate the depth. The light rays leaving the bottom of the pool bend away from the normal as they enter the air, so their source appears to be more shallow than it actually is. The greater the viewing angle relative to the normal, the greater the bending of the light and therefore the shallower the apparent depth.



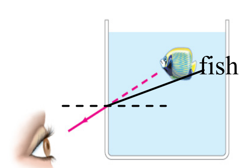
10. Your brain interprets the bending rays as though the part of the stick that is underwater is closer to the surface than it really is, so it looks bent. Each point on the stick that is underwater appears to you to be closer to the surface than it actually is.



11. Because the broad beam hits the surface of the water at an angle, it illuminates an area of the surface that is wider than the beam width. Light from the beam bends toward the normal. The refracted beam is wider than the incident beam because one edge of the beam strikes the surface first, while the other edge travels farther in the air. (See the adjacent diagram.)



12. The light rays from the fish are bent away from the normal as they leave the tank. The fish will appear closer to the side of the tank than it really is.



13. One reason you can see a drop of transparent and colorless water on a surface is refraction. The light leaving the tabletop and going through the drop on the way to your eye will make that portion of the tabletop seem to be raised up, due to depth distortion and/or magnification due to the round drop. A second reason that you can see the drop is reflection. At certain angles, the light sources in the room will reflect off of the drop and give its location away.

14. When the light ray passes from the left material to the middle material, the ray bends toward the normal. This indicates that the index of refraction of the left material is less than that of the middle material. When the light ray passes from the middle material to the right material, the ray bends away from the normal, but not far enough to make the ray parallel to the initial ray, indicating that the index of refraction of the right material is less than that of the middle material but larger than the index of refraction of the left material. The ranking of the indices of refraction is, least to greatest, left, right, and middle. So the middle has the largest index and the left has the smallest.

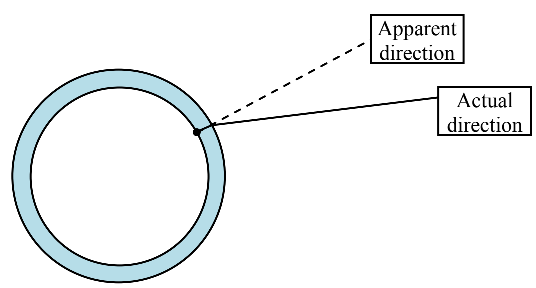
15. As discussed in Question 9, the bottom of the pool and the toy will appear closer to the top of the water than they really are. If the child is not aware of this phenomenon, then she will underestimate the depth of the water.

16. No. Total internal reflection can only occur when light travels from a medium of higher index of refraction to a medium of lower index of refraction.

$\boxed{17.}$ The mirror is concave, and the person is standing inside the focal point so that a virtual, upright image is formed. A convex mirror would also form a virtual, upright image, but the image would be smaller than the object. See Fig. 23–16 and Example 23–4 for similar situations.

18. (*a*) The fact that light rays from stars (including our Sun) bend toward the vertical direction as they pass through Earth's atmosphere makes sense because the index of refraction of the atmosphere is slightly greater than that of the vacuum of space. As the light rays from stars enter the atmosphere, they slow down and bend toward the vertical, according to Snell's law.

(*b*) The apparent positions of stars in the sky are too high when compared to actual positions. Since the light rays bend toward the vertical as they come into the atmosphere and then into our eyes, as we "follow" these rays back into space, which doesn't take the bending into account, the image of the stars appears higher than the actual position. See the simple diagram, which might be considered looking down from the North Pole, with the sun at the left so that it is nighttime for the observer.



$\boxed{19.}$ To make a sharp image of an object that is very far away, the film of a camera must be placed at the focal point of the lens. Objects that are very far away have light rays coming into the camera that are basically parallel; these rays will be bent and focused to create an image at the focal point of the lens, which is where the film should be in order to record a focused image.

20. The lens moves farther away from the film. When the photographer moves closer to his subject, the object distance decreases. The focal length of the lens does not change, so the image distance must increase, by Eq. 23–8, $\frac{1}{f}=\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}$. To get the image and the film at the same place, the lens needs to move away from the film.

21. If the diverging lens is the only piece of optics involved, then it is not possible for it to form a real image. Diverging lenses, by definition, cause light rays to diverge and will not bring rays from a real object to a focal point as required to form a real image. However, if another optical element (for example, a converging lens) forms a virtual object for the diverging lens, then it is possible for the diverging lens to form a real image.

22. Yes, to say that light rays are "reversible" is consistent with the thin lens equation. Since $\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}$, if you interchange the image and object positions, then the equation is unchanged.

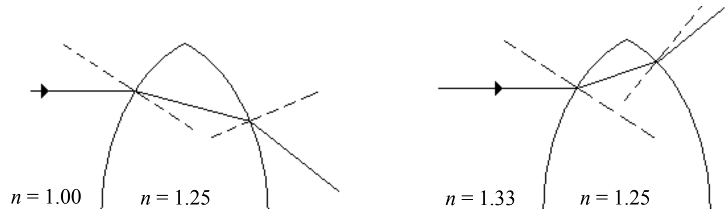
23. Real images can be projected onto a screen. In this situation, real light rays actually focus at the position of the image, and if the screen is placed at the image location, then the image can be seen on the screen. Virtual images cannot be projected onto a screen. In this situation, no real light rays are converging at the position of the virtual image, so they cannot be seen on a screen (recall that a plane mirror's virtual image is actually behind the mirror, where no real light rays travel). Both real images and virtual images can be photographed. Real images can be photographed by putting the film at the image location, so the film takes the place of the screen. Both virtual and real images can be photographed in the same way that you can see both virtual and real images with your eye. As long as the camera (or the eye) is positioned so that diverging rays from the (real or virtual) image enter the camera (or the eye), the converging lens is able to make an image from those rays on the film. There is no difference in the way that the camera records rays that came from a virtual image or rays that came from some object directly. See Fig. 23–37c or Fig. 23–39, and simply replace the eye with a camera. The same image will be recorded in both situations.

24. (*a*) As a thin converging lens is moved closer to a nearby object, the position of the real image changes. As the object distance decreases, the thin lens equation says that the image distance increases in order for the calculation for the focal length to not change. Thus, the position of the image moves farther away from the lens until the object is at the focal point. At that condition, the image is at infinity.
(*b*) As a thin converging lens is moved closer to a nearby object, the size of the real image changes. As the object distance decreases, the thin lens equation says that the image distance increases, which means that the magnification $(m=-d_\text{i}/d_\text{o})$ gets larger. Thus, the size of the image increases until the object reaches the focal point, in which case a real image will no longer be formed.

25. Because there is less refraction when light passes between water and glass than when light passes between air and glass, the light will not be bent as much when the lens is in water, so its focal length will be longer in the water.

26. The forms of the mirror equation and the lens equation are identical. According to the sign conventions, $f>0$ indicates a converging lens or mirror, and $f<0$ indicates a diverging lens or mirror. A value for $d>0$ indicates a real object or image, and $d<0$ indicates a virtual object or image, for both mirrors and lenses. But the positions of the objects and images are different for a mirror and a lens. For a mirror, a real object or image will be in front of the mirror (on the same side as the incoming and outgoing light), and a virtual object or image will be behind the mirror. For a lens, a real image will be on the opposite side of the lens from a real object, and a virtual image will be on the same side of the lens as the real object.

27. No. If a lens with $n = 1.25$ is a converging lens in air, then it will become a diverging lens when placed in water, with $n = 1.33$. The figure on the left shows that as parallel light rays enter the lens when it is in air, at the first surface the ray is bent toward the normal, and at the second surface the ray is bent away from the normal, which is a net converging situation. The figure on the right shows that as parallel light rays enter the lens when it is in water, at the first surface the ray is bent away from the normal, and at the second surface the ray is bent toward the normal, which is a net diverging situation.



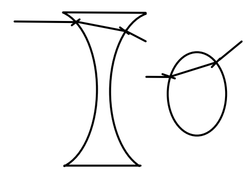
28. (*a*) Yes, the focal length of a lens depends on the surrounding fluid. The relative values of the index of refraction of the fluid and the index of refraction of the lens will determine the refraction of light as it passes from the fluid through the lens and back into the fluid. The amount of refraction of light determines the focal length of the lens, so the focal length will change if the lens is immersed in a fluid.

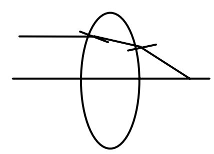
(*b*) No, the focal length of a spherical mirror does not depend on the surrounding fluid. The image formation of the spherical mirror is determined by reflection, not refraction, and is independent of the medium in which the mirror is immersed.

29. The lens material is air and the medium in which the lens is placed is water. Air has a lower index of refraction than water, so the light rays will bend away from the normal when entering the lens and toward the normal when leaving the lens.

(*a*) A converging lens can be made by a shape that is thinner in the middle than it is at the edges.

(*b*) A diverging lens will be thicker in the middle than it is at the edges.





30. A double convex lens causes light rays to converge because the light bends toward the normal as it enters the lens and away from the normal as it exits the lens. The result, due to the curvature of the sides of the lens, is that the light bends toward the principal axis at both surfaces. The more strongly the sides of the lens are curved (which means the thicker the lens), the greater the refraction and the shorter the focal length. This can also be seen from the lensmaker's equation: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. To make a lens that is thick in the middle, the radii of the two convex sides should be small. If R_1 and R_2 decrease in the lensmaker's equation, then the focal length decreases.

31. The image point of an unsymmetrical lens does not change if you turn the lens around. The lensmaker's equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, is symmetrical (even if the lens is not). Turning the lens around inter changes R_1 and R_2, which does not change the value of the focal length or the location of the image point.

32. (*a*) The focal length of a diverging lens cannot be measured directly because the diverging lens cannot form a real image. Therefore, it is very difficult to measure the image distance—some indirect measure (as described in this problem) must be used.

(*b*) For the converging lens to be stronger, it must have a shorter focal length then the absolute value of the focal length of the diverging lens. If the diverging lens is stronger, then instead of the focal point simply being moved back from its original location in Fig. 23–45, the rays would never focus. The diverging lens would diverge the entering rays so much that, again, a virtual image would be formed, and no direct measure would be possible.

Responses to MisConceptual Questions

1. (*a*) It is easy to think that as someone approaches the mirror, they will see less of themselves since their image would be larger or they will see more of themselves as the angle of reflection becomes larger. Actually, they will see the exact same amount, as the two properties (larger image and greater angle) exactly cancel each other. A diagram is included with the solution to Problem 1 showing that a ray entering the eye that has reflected from the bottom of the mirror originates at the waist, regardless of how far the person is from the mirror.

2. (*c*) Since the image from a flat mirror is behind the mirror, the image is virtual. Experience shows that when looking through a flat mirror, the image is upright.

3. (*a*) Parallel rays that reflect off of a mirror will reflect out through the focal point. Therefore, if the source is placed at the focal point, then all of the reflected rays will exit the mirror parallel to the principal axis of the mirror.

4. (*c*) A common misconception is that the fish will appear to be located at its actual depth. However, due to refraction, light from the fish bends away from the normal as it exits the surface. This refraction makes the fish appear closer to the surface than it actually is. See Fig. 23–25 in the text.

5. (*e*) When light travels from a low index to a higher index, it bends toward the normal. When it travels from a high index to a low index, it bends away from the normal. As it passes from region 1 to region 2, it bends toward the normal, so $n_2 > n_1$. As it passes from region 2 to region 3, it bends away from the normal, so $n_2 > n_3$. Since the surfaces are parallel and the angle in region 3 is greater than the initial angle in region 1, $n_1 > n_3$. Combining these three relations gives $n_2 > n_1 > n_3$.

6. (*a*) Students may note that due to refraction the fish is actually lower in the water than it appears and may answer that you should shoot below the image. However, the laser is a light beam that will also refract at the surface, so to hit the fish you would need to aim directly at the fish.

7. (*b*) If all of the light reflected off of the surface, then the water under the lake would be completely dark. However, when you swim underwater on a moonlit night, it is possible to see the Moon. If all of the moonlight entered the water, then you could not see the Moon reflecting off the surface. Therefore, some of the light must enter the water and some must reflect off of the surface.

8. (*c*) A common misconception is that the light travels down the fiber without touching the edges. However, since the index of refraction of the fiber is greater than the index of refraction of the surrounding air, the light inside the fiber can totally internally reflect off of the sides. Since the ends are perpendicular to the sides, the angle of the light relative to the ends is less than the critical angle and the light exits the ends.

9. (*e*) If the object distance is greater than twice the focal length, then the image will be reduced, so (*a*) is incorrect. If the object distance is between the focal length and twice the focal length, then the image will be magnified, so (*b*) is incorrect. If the object is placed between the lens and the focal length, then the virtual image will be upright, so (*c*) is incorrect. If the object is placed at a distance greater than the focal length, then the image will be inverted, so (*d*) is incorrect. Therefore, none of the given statements are true.

10. (*e*) Virtual images are formed by plane mirrors, convex mirrors, concave mirrors (when the object is within the focal distance), diverging lenses, and converging lenses (when the object is within the focal distance). Therefore, virtual images can be formed with plane and curved mirrors and lenses.

11. (*d*) A common misconception is to equate power with magnification. However, power is the term used for the reciprocal of the focal length.

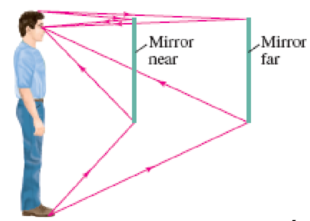
12. (*c*) A common misconception is that each half of the lens produces half of the image. Actually, light from each part of the image passes through each part of the lens. When half of the lens is covered, the image is less intense, but the full image is still present on the screen.

13. (*e*) It might seem reasonable that a lens must be curved on both sides to produce an image. However, images can be produced by plane mirrors, curved mirrors, and lenses curved on one or both sides.

14. (*a*) When the object is outside the focal length, the image is real and inverted. When the object is inside the focal length, the image is virtual and upright. And the closer the object is to the focal length, the greater the magnification, so the image is large in both cases.

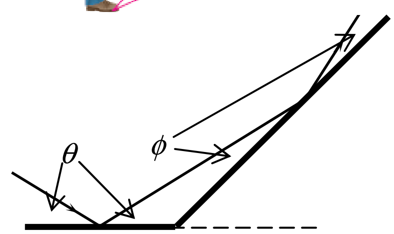
Solutions to Problems

1. The angle of incidence must equal the angle of reflection. We see from the diagram that a ray that reflects from your toes to your eyes must strike the mirror at a point that is half the vertical distance from your toes to your eyes, regardless of your distance from the mirror.



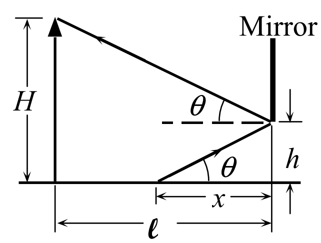
2. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is twice the distance from the object to the mirror, or $\boxed{6.2\text{ m}}$.

3. The law of reflection can be applied twice. At the first reflection, the angle is θ, and at the second reflection, the angle is ϕ. Consider the triangle formed by the mirrors (at angle α) and the first reflected ray.



$$\theta+\alpha+\phi=180° \rightarrow 34°+135°+\phi=180° \rightarrow \boxed{\phi=11°}$$

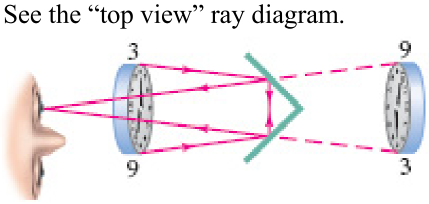
4. The angle of incidence is the angle of reflection. See the diagram for the appropriate lengths.



$$\tan\theta=\frac{(H-h)}{\ell}=\frac{h}{x} \rightarrow$$

$$x=\frac{h\ell}{(H-h)}=\frac{(0.38\text{ m})(2.20\text{ m})}{(1.72\text{ m}-0.38\text{ m})}=\boxed{0.62\text{ m}}$$

5. See the "top view" ray diagram.



6. (*a*) The first image seen will be due to a single reflection off the front (leftmost) mirror. This image will be as far behind the mirror as you are in front of the mirror.

$$D_1=2\times 1.6\text{ m}=\boxed{3.2\text{ m}}$$

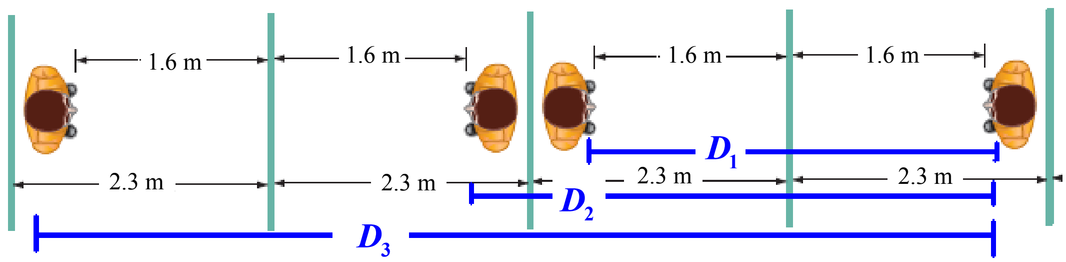
The second image seen will be the image reflected first from the back (rightmost) mirror and then off the front mirror, so it is an image of your back. As seen in the diagram, this image will appear to be twice the distance between the mirrors.

$$D_2 = 1.6\text{ m} + 2.3\text{ m} + (2.3\text{ m} - 1.6\text{ m}) = 2\times 2.3\text{ m} = \boxed{4.6\text{ m}}$$

The third image seen will be the image reflected off the front mirror, the back mirror, and the front mirror again. As seen in the diagram, this image distance will be the sum of twice your distance to the mirror and twice the distance between the mirrors.

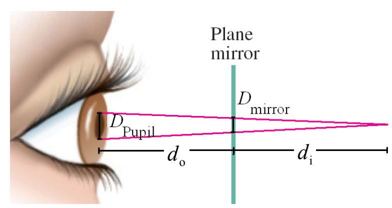
$$D_3 = 1.6\text{ m} + 2.3\text{ m} + 2.3\text{ m} + 1.6\text{ m} = 2\times 1.6\text{ m} + 2\times 2.3\text{ m} = \boxed{7.8\text{ m}}$$

The actual person is to the far right in the diagram.



(*b*) We see from the diagram that the first image is facing $\boxed{\text{toward you}}$; the second image is facing $\boxed{\text{away from you}}$; and the third image is facing $\boxed{\text{toward you}}$.

7. The rays entering your eye are diverging from the virtual image position behind the mirror. Thus the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image.



$$\frac{D_{\text{mirror}}}{d_i} = \frac{D_{\text{pupil}}}{(d_o + d_i)} \rightarrow D_{\text{mirror}} = D_{\text{pupil}}\frac{d_i}{(d_o + d_i)} = \tfrac{1}{2}D_{\text{pupil}}$$

$$A_{\text{mirror}} = \tfrac{1}{4}\pi D_{\text{mirror}}^2 = \tfrac{1}{4}\pi\tfrac{1}{4}D_{\text{pupil}}^2 = \frac{\pi}{16}(4.5\times10^{-3}\text{ m})^2 = \boxed{4.0\times10^{-6}\text{ m}^2}$$

8. The rays from the Sun will be parallel, so the image will be at the focal point, which is half the radius of curvature.

$$r = 2f = 2(18.8\text{ cm}) = \boxed{37.6\text{ cm}}$$

9. To produce an image at infinity, the object must be at the focal point, which is half the radius of curvature.

$$d_o = f = \tfrac{1}{2}r = \tfrac{1}{2}(21.0\text{ cm}) = \boxed{10.5\text{ cm}}$$

10. (*a*) The focal length is half the radius of curvature, so $f = \tfrac{1}{2}r = \tfrac{1}{2}(24\text{ cm}) = \boxed{12\text{ cm}}$.

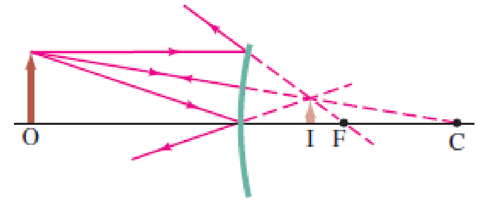
(*b*) Use Eq. 23–2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(38\text{ cm})(12\text{ cm})}{38\text{ cm} - 12\text{ cm}} = 17.54\text{ cm} \approx \boxed{18\text{ cm}}$$

(*c*) The image is $\boxed{\text{inverted}}$, since the magnification is negative.

$$m = -\frac{d_i}{d_o} = -\frac{18\text{ cm}}{38\text{ cm}} = -0.47$$

11. (*a*) From the ray diagram it is seen that the image is virtual. We estimate the image distance as about a third of the radius of curvature, or about $\boxed{-5\text{ cm}}$.



(*b*) Use a focal length of $-8.0\,\text{cm}$ with the object distance of $16.0\,\text{cm}$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(16.0\text{ cm})(-8.0\text{ cm})}{16.0\text{ cm}-(-8.0\text{ cm})}=-5.33\text{ cm}\approx\boxed{-5.3\text{ cm}}$$

(*c*) We find the image size from the magnification.

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o} \rightarrow h_i=h_o\left(\frac{-d_i}{d_o}\right)=(3.0\text{ mm})\left(\frac{5.33\text{ cm}}{16.0\text{ cm}}\right)=\boxed{1.0\text{ mm}}$$

12. The object distance of 2.00 cm and the magnification of $+4.0$ are used to find the image distance. The focal length and radius of curvature can then be found.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_o d_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{4(2.00\text{ cm})}{4-1}=2.677\text{ cm}$$

$$r=2f=2(2.667\text{ cm})=\boxed{5.3\text{ cm}}$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$.

13. The object distance of 3.4 m and the magnification of $+0.5$ are used to find the image distance. The focal length and radius of curvature can then be found.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_o d_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{0.5(3.4\text{ m})}{0.5-1}=-3.4\text{ m}$$

$$r=2f=2(-3.4\text{ m})=\boxed{-6.8\text{ m}}$$

14. Take the object distance to be ∞ and use Eq. 23–2. Note that the image distance is negative since the image is behind the mirror.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{\infty}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=d_i=-19.0\text{ cm} \rightarrow r=2f=\boxed{-38.0\text{ cm}}$$

Because the focal length is negative, the mirror is $\boxed{\text{convex}}$.

15. The image distance can be found from the object distance of 1.9 m and the magnification of $+3$. With the image distance and object distance, the focal length and radius of curvature can be found.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_o d_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}\frac{3(1.9\text{ m})}{3-1}=2.85\text{ m}$$

$$r=2f=2(2.85\text{ m})=\boxed{5.7\text{ m}}$$

16. The object distance changes from 20.0 cm to 30.0 cm. The radius of curvature is still 30.0 cm, so the focal distance is 15.0 cm.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \rightarrow \quad d_i=\frac{d_o f}{d_o-f}=\frac{(30.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm}-15.0\text{ cm}}=30.0\text{ cm}$$

$$m=-\frac{d_i}{d_o}=-\frac{30.0\text{ cm}}{30.0\text{ cm}}=-1.00$$

We see that the image is inverted and the same size as the object. We see that $\boxed{d_o=2f}$ for the "actual-sized image" situation.

17. The ball is a convex mirror with a focal length $f=\frac{1}{2}r=\frac{1}{2}(-4.4\text{ cm})=-2.2\text{ cm}.$ Use Eq. 23–2 to locate the image.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \rightarrow \quad d_i=\frac{d_o f}{d_o-f}=\frac{(25.0\text{ cm})(-2.2\text{ cm})}{25.0\text{ cm}-(-2.2\text{ cm})}=-2.022\text{ cm}\approx -2.0\text{ cm}$$

The image is $\boxed{\text{2.0 cm behind the surface of the ball, virtual, and upright}}$. Note that the magnification is $m=-\frac{d_i}{d_o}=\frac{-(-2.022\text{ cm})}{(25.0\text{ cm})}=+0.081.$

$\boxed{18.}$ The mirror must be $\boxed{\text{convex}}$. Only convex mirrors produce images that are upright and smaller than the object. The object distance of 16.0 m and the magnification of $+0.33$ are used to find the image distance. The focal length and radius of curvature can then be found.

$$m=\frac{-d_i}{d_o} \quad \rightarrow \quad d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \rightarrow \quad f=\frac{d_o d_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{0.33(16.0\text{ m})}{0.33-1}=-7.881\text{ m}$$

$$r=2f=2(-7.881\text{ m})=-15.76\text{ m}\approx \boxed{-16\text{ m}}$$

The negative focal length confirms that the mirror must be convex.

19. The image flips at the focal point, which is half the radius of curvature. Thus the radius is $\boxed{1.0\text{ m}}$.

20. (*a*) We are given that $d_i=d_o$. Use Eq. 23–2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \rightarrow \quad \frac{2}{d_o}=\frac{1}{f} \quad \rightarrow \quad \boxed{d_o=2f=r}$$

The object should be placed at the $\boxed{\text{radius of curvature}}$.

(*b*) Because the image is in front of the mirror, $d_i>0,$ it is $\boxed{\text{real}}$.

(*c*) The magnification is $m=\frac{-d_i}{d_o}=\frac{-d_o}{d_o}=-1.$ Because the magnification is negative, the image is $\boxed{\text{inverted}}$.

(*d*) As found in part (*c*), $m=\boxed{-1.}$

21. (*a*) Producing a larger upright image requires a $\boxed{\text{concave mirror}}$.

(*b*) The image will be $\boxed{\text{upright, virtual, and magnified}}$.

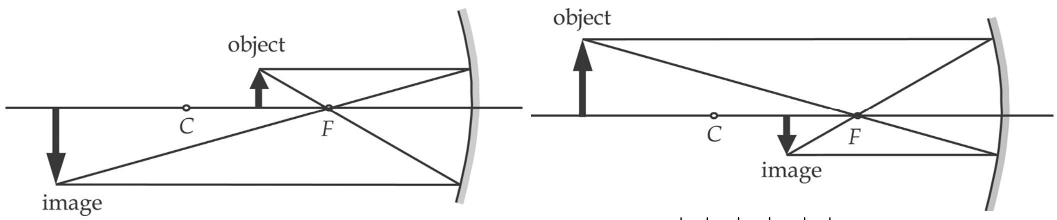
(*c*) We find the image distance from the magnification and use that to find the radius of curvature.

$$m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}} \rightarrow d_{\mathrm{i}}=-md_{\mathrm{o}}$$

$$\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}}=\frac{1}{f} \rightarrow f=\frac{d_{\mathrm{o}}d_{\mathrm{i}}}{d_{\mathrm{o}}+d_{\mathrm{i}}}=\frac{d_{\mathrm{o}}(-md_{\mathrm{o}})}{d_{\mathrm{o}}-md_{\mathrm{o}}}=\frac{md_{\mathrm{o}}}{m-1} \rightarrow$$

$$r=2f=\frac{2md_{\mathrm{o}}}{m-1}=\frac{2(1.40)(20.0\text{ cm})}{1.40-1}=140\text{ cm}=\boxed{1.40\text{ m}}\ \text{(3 significant figures)}$$

22. (*a*)



$|h_{\mathrm{i}}|>|h_{\mathrm{o}}|;\quad |m|>1;\quad d_{\mathrm{o}}<r$ $\qquad$ $|h_{\mathrm{i}}|>|h_{\mathrm{o}}|;\quad |m|>1;\quad d_{\mathrm{o}}<r$

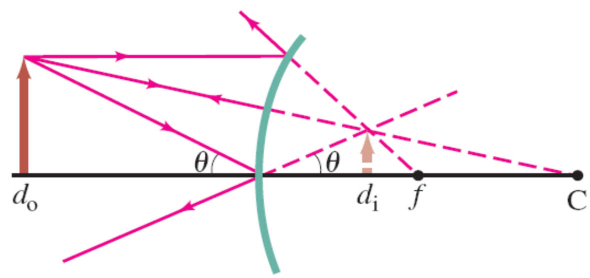
(*b*) Apply Eq. 23–2 and Eq. 23–3.

$$\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}}=\frac{1}{f}=\frac{2}{r} \rightarrow d_{\mathrm{i}}=\frac{rd_{\mathrm{o}}}{(2d_{\mathrm{o}}-r)};\quad m=\frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}=\frac{-r}{(2d_{\mathrm{o}}-r)}$$

If $d_{\mathrm{o}}>r$, then $(2d_{\mathrm{o}}-r)>r$, so $|m|=\dfrac{r}{(2d_{\mathrm{o}}-r)}=\dfrac{r}{(>r)}<1.$

If $\frac{1}{2}r<d_{\mathrm{o}}<r$, then $(2d_{\mathrm{o}}-r)<r$, so $|m|=\dfrac{r}{(2d_{\mathrm{o}}-r)}=\dfrac{r}{(<r)}>1.$

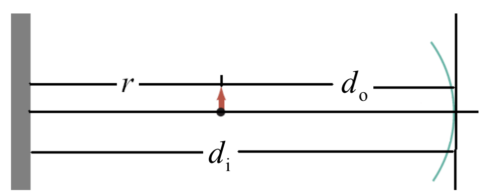
23. Consider the ray that reflects from the center of the mirror, and note that $d_{\mathrm{i}}<0$.



$$\tan\theta=\frac{h_{\mathrm{o}}}{d_{\mathrm{o}}}=\frac{h_{\mathrm{i}}}{-d_{\mathrm{i}}} \rightarrow \frac{-d_{\mathrm{i}}}{d_{\mathrm{o}}}=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}$$

$$m=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}=\boxed{-\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}}}$$

24. The distance between the mirror and the wall is equal to the image distance, which we can calculate using Eq. 23–2. The object is located a distance *r* from the wall, so the object distance will be *r* less than the image distance. The focal length is given by Eq. 23–1. For the object distance to be real, the image distance must be greater than *r*.



$$\frac{1}{f}=\frac{1}{d_{\mathrm{o}}}+\frac{1}{d_{\mathrm{i}}} \rightarrow \frac{2}{r}=\frac{1}{d_{\mathrm{i}}-r}+\frac{1}{d_{\mathrm{i}}} \rightarrow 2d_{\mathrm{i}}^2-4d_{\mathrm{i}}r+r^2=0$$

$$d_{\mathrm{i}}=\frac{4r\pm\sqrt{16r^2-8r^2}}{4}=r\left(1\pm\frac{\sqrt{2}}{2}\right)\approx 0.292r\text{ or }\boxed{1.71r}$$

Use Eq. 23–3 to calculate the magnification: $m=-\dfrac{d_{\mathrm{i}}}{d_{\mathrm{o}}}=\dfrac{1.71r}{1.71r-r}=\boxed{-2.41}$

25. We find the index of refraction from Eq. 23–4.

$$n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.29 \times 10^8 \text{ m/s}} = \boxed{1.31}$$

26. In each case, the speed is found from Eq. 23–4 and the index of refraction.

(*a*) Ethyl alcohol: $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.36} = \boxed{2.21 \times 10^8 \text{ m/s}}$

(*b*) Lucite: $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.51} = \boxed{1.99 \times 10^8 \text{ m/s}}$

(*c*) Crown glass: $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.52} = \boxed{1.97 \times 10^8 \text{ m/s}}$

27. We find the index of refraction from Eq. 23–4.

$$n = \frac{c}{v} = \frac{c}{0.82 v_{\text{water}}} = \frac{c}{0.82\left(\frac{c}{n_{\text{water}}}\right)} = \frac{n_{\text{water}}}{0.82} = \frac{1.33}{0.82} = \boxed{1.62}$$

28. Find the angle of refraction from Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to \quad \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.56}\sin 67^\circ\right) = \boxed{36^\circ}$$

29. Find the angle of refraction from Snell's law. Medium 1 is water, and medium 2 is air.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to \quad \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 35.2^\circ\right) = \boxed{50.1^\circ}$$

30. We find the incident angle in the water from Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to \quad \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 56.0^\circ\right) = \boxed{38.6^\circ}$$

31. We find the incident angle in the air (relative to the normal) from Snell's law.

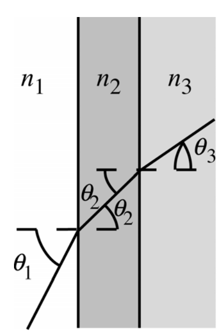
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to \quad \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 36.0^\circ\right) = 51.4^\circ$$

Since this is the angle relative to the vertical, the angle as measured from the horizon is the complementary angle, $90.0^\circ - 51.4^\circ = \boxed{38.6^\circ}$.

32. (*a*) We use Eq. 23–5 to calculate the refracted angle as the light enters the glass $(n = 1.56)$ from the air $(n = 1.00)$.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \to$$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.54}\sin 43.5^\circ\right] = 26.55^\circ \approx \boxed{26.6^\circ}$$



(*b*) We again use Eq. 23–5 using the refracted angle in the glass and the indices of refraction of the glass and water.

$$\theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.54}{1.33}\sin 26.55°\right] = 31.17° \approx \boxed{31.2°}$$

(*c*) We repeat the same calculation as in part (*a*) but using the index of refraction of water.

$$\theta_3 = \sin^{-1}\left[\frac{n_1}{n_3}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33}\sin 43.5°\right] = 31.17° \approx \boxed{31.2°}$$

As expected, the refracted angle in the water is the same whether the light beam first passes through the glass or passes directly into the water.

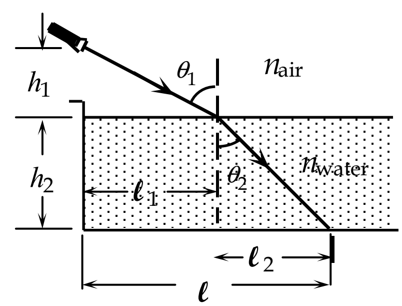
33. The angle of reflection is equal to the angle of incidence and is equal to twice the angle of refraction: $\theta_{\text{ref1}} = \theta_1 = 2\theta_2$. Use Snell's law and a "double-angle" trig identity.

$$n_{\text{air}}\sin\theta_1 = n_{\text{glass}}\sin\theta_2 \quad\to\quad (1.00)\sin 2\theta_2 = (1.51)\sin\theta_2$$

$$\sin 2\theta_2 = 2\sin\theta_2\cos\theta_2 = (1.51)\sin\theta_2 \quad\to\quad \cos\theta_2 = 0.755 \quad\to\quad \theta_2 = 40.97°$$

$$\theta_1 = 2\theta_2 = \boxed{81.9°}$$

34. We find the angle of incidence from the distances.



$$\tan\theta_1 = \frac{\ell_1}{h_1} = \frac{(2.5\text{ m})}{(1.3\text{ m})} = 1.9231 \quad\to\quad \theta_1 = 62.526°$$

Use Snell's law to find the angle in the water.

$$n_{\text{air}}\sin\theta_1 = n_{\text{water}}\sin\theta_2;$$

$$(1.00)\sin 62.526° = (1.33)\sin\theta_2 \quad\to\quad \theta_2 = 41.842°$$

Find the horizontal distance from the edge of the pool.

$$\ell = \ell_1 + \ell_2 = \ell_1 + h_2\tan\theta_2$$

$$= 2.5\text{ m} + (2.1\text{ m})\tan 41.842° = 4.38\text{ m} \approx \boxed{4.4\text{ m}}$$

35. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

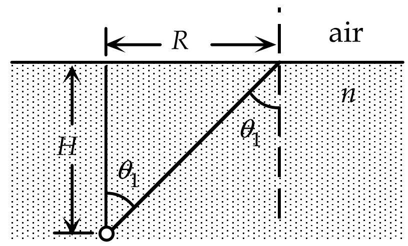
$$n_{\substack{\text{crown}\\\text{glass}}}\sin\theta_1 = n_{\text{water}}\sin\theta_2 \quad\to\quad \theta_1 = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\substack{\text{crown}\\\text{glass}}}}\right) = \sin^{-1}\frac{1.33}{1.52} = \boxed{61.0°}$$

Because crown glass has the higher index, the light must start in $\boxed{\text{crown glass}}$.

36. Use Eq. 23–6.

$$\sin\theta_C = \frac{n_{\text{air}}}{n_{\text{liquid}}} \quad\to\quad n_{\text{liquid}} = \frac{n_{\text{air}}}{\sin\theta_1} = \frac{1.00}{\sin 47.2°} = \boxed{1.36}$$

37. We find the critical angle for light leaving the water, so that the refracted angle is 90°.



$$\sin\theta_C = \frac{n_{air}}{n_{liquid}} \rightarrow \theta_C = \sin^{-1}\left(\frac{1.00}{1.33}\right) = 48.75°$$

If the light is incident at a greater angle than this, then it will totally reflect. Find R from the diagram.

$$R > H\tan\theta_1 = (82.0\text{ cm})\tan 48.75° = \boxed{93.5\text{ cm}}$$

38. Refer to the diagram for the solution to Problem 37. Find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{R}{H} = \frac{(7.6\text{ cm})}{(8.0\text{ cm})} = 0.95 \rightarrow \theta_1 = 43.53°$$

The maximum incident angle for refraction from liquid into air is the critical angle, $\sin\theta_C = \dfrac{1.00}{n_{liquid}}$.

Thus we have the following.

$$\sin\theta_1 \ge \sin\theta_C = \frac{1}{n_{liquid}} \rightarrow \sin 43.53° = 0.6887 \ge \frac{1}{n_{liquid}} \rightarrow n_{liquid} \ge 1.452 \approx \boxed{n_{liquid} \ge 1.5}$$

39. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.00)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \ge 1$, so $n_1 \ge \dfrac{1}{\sin 45°} = \boxed{1.4}$.

(*b*) When there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow (1.58)\sin 45° = (1.33)\sin\theta_2 \rightarrow \sin\theta_2 = 0.84$$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting}}$.

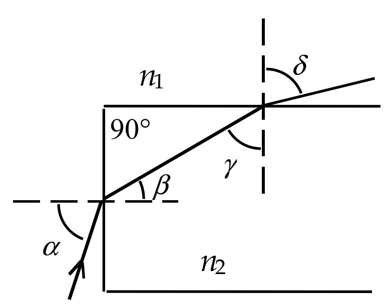
(*c*) For total reflection when there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.33)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \ge 1$.

$$n_1 \ge \frac{1.33}{\sin 45°} = \boxed{1.9}$$

40. (*a*) For the refraction at the side of the rod, we have $n_2\sin\gamma = n_1\sin\delta$. The minimum angle for total reflection γ_{min} occurs when $\delta = 90°$.



$$n_2\sin\gamma_{min} = (1.00)(1) = 1 \rightarrow \sin\gamma_{min} = \frac{1}{n_2}$$

Find the maximum angle of refraction at the end of the rod.

$$\beta_{max} = 90° - \gamma_{min}$$

Because the sine function increases with angle, for the refraction at the end of the rod, we have the following.

$$n_1 \sin \alpha_{\max} = n_2 \sin \beta_{\max} \quad \to \quad (1.00)\sin \alpha_{\max} = n_2 \sin(90^\circ - \gamma_{\min}) = n_2 \cos \gamma_{\min}$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of α is 90°, so $n_2 \cos \gamma_{\min} = 1$ or $\cos \gamma_{\min} = \frac{1}{n_2}$. When we divide this by the result for the refraction at the side, we get $\tan \gamma_{\min} = 1 \quad \to \quad \gamma_{\min} = 45^\circ$. Thus we have the following.

$$\boxed{n_2 \geq \frac{1}{\sin \gamma_{\min}} = \frac{1}{\sin 45^\circ} = 1.414}$$

(*b*) If the fiber were immersed in water, then the value of n_1 would be 1.33. We repeat the analysis with that value.

$$n_2 \sin \gamma_{\min} = (1.33)(1) = 1.33 \quad \to \quad \sin \gamma_{\min} = \frac{1.33}{n_2}; \quad \beta_{\max} = 90^\circ - \gamma_{\min}$$

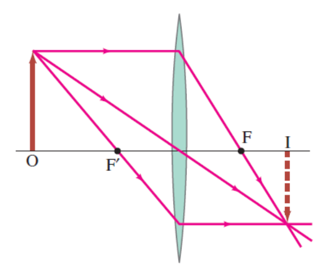
$$n_1 \sin \alpha_{\max} = n_2 \sin \beta_{\max} \quad \to \quad (1.33)\sin \alpha_{\max} = n_2 \sin(90^\circ - \gamma_{\min}) = n_2 \cos \gamma_{\min}$$

$$\alpha_{\max} = 90^\circ \quad \to \quad 1.33 = n_2 \cos \gamma_{\min} \quad \to \quad \cos \gamma_{\min} = \frac{1.33}{n_2}$$

$$\tan \gamma_{\min} = \frac{\sin \gamma_{\min}}{\cos \gamma_{\min}} = \frac{\frac{1.33}{n_2}}{\frac{1.33}{n_2}} = 1 \quad \to \quad \gamma_{\min} = 45^\circ \quad \to \quad \boxed{n_2 \geq \frac{1.33}{\sin \gamma_{\min}} = \frac{1.33}{\sin 45^\circ} = 1.88}$$

The index of refraction of the material must be at least 1.88 to not have losses upon reflection, if immersed in water.

41. (*a*) From the ray diagram, the object distance is about $\boxed{500 \text{ mm}}$.



(*b*) We find the object distance from Eq. 23–8.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \to$$

$$d_o = \frac{f d_i}{d_i - f} = \frac{(215 \text{ mm})(391 \text{ mm})}{391 \text{ mm} - 215 \text{ mm}} = \boxed{478 \text{ mm}}$$

42. (*a*) Forming a real image from parallel rays requires a $\boxed{\text{converging lens}}$.

(*b*) We find the power of the lens from Eqs. 23–7 and 23–8. We treat the object distance as infinite.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P = \frac{1}{\infty} + \frac{1}{0.165 \text{ m}} = \boxed{6.06 \text{ D}}$$

43. (*a*) The power of the lens is given by Eq. 23–7.

$$P = \frac{1}{f} = \frac{1}{0.325 \text{ m}} = \boxed{3.08 \text{ D}}$$

This lens is $\boxed{\text{converging}}$.

(*b*) We find the focal length of the lens from Eq. 23–7.

$$P=\frac{1}{f} \rightarrow f=\frac{1}{D}=-\frac{1}{6.75\text{ D}}=\boxed{-0.148\text{ m}}$$

This lens is diverging.

44. Forming a real image from a real object requires a $\boxed{\text{converging lens}}$. We find the focal length of the lens from Eq. 23–8.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f} \rightarrow f=\frac{d_\text{o}d_\text{i}}{d_\text{o}+d_\text{i}}=\frac{(1.55\text{ m})(0.483\text{ m})}{1.55\text{ m}+0.483\text{ m}}=\boxed{0.368\text{ m}}$$

Because $d_\text{i}>0,$ the image is $\boxed{\text{real}}$.

45. (*a*) We find the image distance from Eq. 23–8.

$$\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f} \rightarrow d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}=\frac{(10.0\text{ m})(0.105\text{ m})}{10.0\text{ m}-0.105\text{ m}}=0.106\text{ m}=\boxed{106\text{ mm}}$$

(*b*) Use the same general calculation.

$$d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}=\frac{(3.0\text{ m})(0.105\text{ m})}{3.0\text{ m}-0.105\text{ m}}=0.109\text{ m}=\boxed{109\text{ mm}}$$

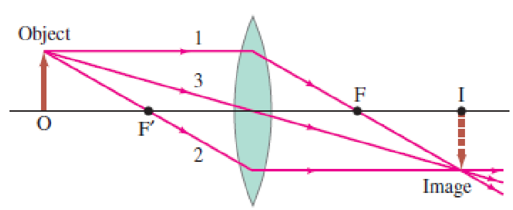
(*c*) Use the same general calculation.

$$d_\text{i}=\frac{d_\text{o}f}{d_\text{o}-f}=\frac{(1.0\text{ m})(0.105\text{ m})}{1.0\text{ m}-0.105\text{ m}}=0.117\text{ m}=\boxed{117\text{ mm}}$$

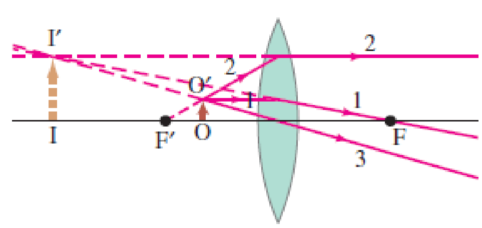
(*d*) We find the smallest object distance from the maximum image distance.

$$\frac{1}{d_{\text{o}_\text{min}}}+\frac{1}{d_{\text{i}_\text{max}}}=\frac{1}{f} \rightarrow d_{\text{o}_\text{min}}=\frac{d_{\text{i}_\text{max}}f}{d_{\text{i}_\text{max}}-f}=\frac{(132\text{ mm})(105\text{ mm})}{132\text{ mm}-105\text{ mm}}=513\text{ mm}=\boxed{0.513\text{ m}}$$

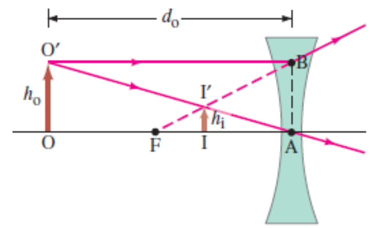
46. A real image is only formed by a converging lens, and only if the object distance is greater than the focal length. The image is on the opposite side of the lens as the object and will always be inverted, as shown in the first diagram. If the object were moved back, then the three rays drawn would never intersect above the axis of symmetry to make an upright image, but only intersect below the axis of symmetry, only making an inverted image.



A virtual image is formed by a converging lens if the object distance is less than the focal length. In the second diagram, the outgoing rays will never intersect since ray 2 is parallel to the symmetry axis and rays 1 and 3 are sloping downward. Also, ray 3 will always slope more than ray 1, so none of those rays ever meet. Thus they can only form a virtual image, and it has to be upright, as seen in the diagram.



A virtual image is formed by a diverging lens, no matter where a real object is placed. Since the rays leaving the lens diverge, they cannot form a real image. And because the ray that hits the lens at point B is sloping upward, it will always (virtually) intersect the other ray to make an upright image.



47. (*a*) We locate the image using Eq. 23–8. Both the focal length and object distance are positive.

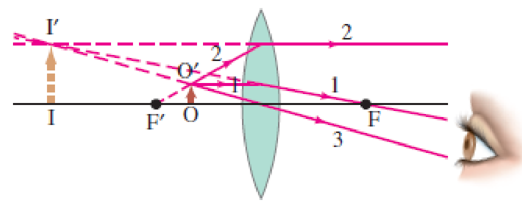
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(16\text{ cm})(28\text{ cm})}{16\text{ cm}-28\text{ cm}}=-37.33\text{ cm}\approx -37\text{ cm}$$

The negative sign means that the image is $\boxed{\text{37 cm behind the lens (virtual)}}$.

(*b*) We find the magnification from Eq. 23–9.

$$m=-\frac{d_i}{d_o}=-\frac{(-37.33\text{ cm})}{(16\text{ cm})}=\boxed{+2.3}$$

48. (*a*) The image should be upright for reading. The image will be $\boxed{\text{virtual, upright, and magnified}}$.



(*b*) Forming a virtual, upright magnified image requires a $\boxed{\text{converging lens}}$.

(*c*) We find the image distance, then the focal length, and then the power of the lens. The object distance is given.

$$m=-\frac{d_i}{d_o} \rightarrow d_i=-md_o$$

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{d_i+d_o}{d_o d_i}=\frac{-md_o+d_o}{d_o(-md_o)}=\frac{m-1}{md_o}=\frac{3.0-1}{(3.0)(0.090\text{ m})}=\boxed{7.4\text{ D}}$$

For reference, the focal length is 13.5 cm.

49. Use Eqs. 23–7 and 23–8 to find the image distance and Eq. 23–9 to find the image height.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_i=\frac{d_o}{Pd_o-1}=\frac{(0.125\text{ m})}{(-7.00\text{ D})(0.125\text{ m})-1}=-0.0667\text{ m}=\boxed{-6.67\text{ cm}}$$

Since the image distance is negative, the image is $\boxed{\text{virtual and behind the lens}}$.

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{(-6.67\text{ cm})}{12.5\text{ cm}}(1.00\text{ mm})=\boxed{0.534\text{ mm (upright)}}$$

50. First, find the original image distance from Eqs. 23–7 and 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_i=\frac{d_o}{Pd_o-1}=\frac{(1.50\text{ m})}{(6.5\text{ D})(1.50\text{ m})-1}=0.1714\text{ m}$$

(*a*) With a new object distance of $d_o=0.60$ m, find the new image distance.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_i=\frac{d_o}{Pd_o-1}=\frac{(0.60\text{ m})}{(6.5\text{ D})(0.60\text{ m})-1}=0.2069\text{ m}$$

Thus the image has moved $0.2069\text{ m}-0.1714\text{ m}=0.0355\text{ m}\approx\boxed{0.04\text{ m}}$ away from the lens.

(*b*) With a new object distance of $d_o = 2.40\text{ m},$ find the new image distance.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \quad\to\quad d_i=\frac{d_o}{Pd_o-1}=\frac{(2.40\text{ m})}{(6.5\text{ D})(2.40\text{ m})-1}=0.1644\text{ m}$$

The image has moved $0.1714\text{ m}-0.1644\text{ m}=\boxed{0.007\text{ m}}$ toward the lens.

51. (*a*) If the image is real, then the focal length must be positive, the image distance must be positive, and the magnification is negative. Thus $d_i = 2.50 d_o$. Use Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{2.50d_o}=\frac{1}{f} \quad\to\quad d_o=\left(\frac{3.50}{2.50}\right)f=\left(\frac{3.50}{2.50}\right)(50.0\text{ mm})=\boxed{70.0\text{ mm}}$$

(*b*) If the image is magnified, then the lens must have a positive focal length, because negative lenses always form reduced images. Since the image is virtual the magnification is positive. Thus $d_i = -2.50 d_o$. Again use Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}-\frac{1}{2.5d_o}=\frac{1}{f} \quad\to\quad d_o=\left(\frac{1.50}{2.50}\right)f=\left(\frac{1.50}{2.50}\right)(50.0\text{ mm})=\boxed{30.0\text{ mm}}$$

52. Since a concave (negative focal length) lens always forms reduced images of a real object, the object in these cases must be virtual (formed by another piece of optics) and have negative object distances in order to form enlarged images. Using Eqs. 23–8 and 23–9, we get this relationship.

$$m=\frac{-d_i}{d_o} \quad\to\quad d_o=\frac{-d_i}{m};\quad \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\to \frac{m}{-d_i}+\frac{1}{d_i}=\frac{1}{f} \quad\to\quad d_i=(1-m)f$$

(*a*) If the image is real, then $d_i > 0$. With $f < 0$, we must have $m > 1$ from the above relationship. Thus $m = +2.50$.

$$d_i=[1-(+2.50)](-50.0\text{ mm})=75.0\text{ mm}$$

$$d_o=\frac{-d_i}{m}=\frac{-(75.0\text{ mm})}{(+2.50)}=\boxed{-30.0\text{ mm}}$$

(*b*) If the image is virtual, then $d_i < 0$. With $f < 0$, we see that $m < 1$. Thus $m = -2.50$.

$$d_i=[1-(-2.50)](-50.0\text{ mm})=-175\text{ mm}$$

$$d_o=-\frac{d_i}{m}=-\frac{(-170\text{ mm})}{(-2.50)}=\boxed{-70.0\text{ mm}}$$

$\boxed{53.}$ From Eq. 23–9, $|h_i|=|h_o|$ when $d_i = d_o$. So find d_o from Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{d_o}=\frac{1}{f} \quad\to\quad d_o=2f=\boxed{64\text{ cm}}$$

54. (*a*) Use Eqs. 23–8 and 23–9.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\to\quad d_i=\frac{d_o f}{d_o-f}=\frac{(1.30\text{ m})(0.135\text{ m})}{1.30\text{ m}-0.135\text{ m}}=0.1506\text{ m}\approx\boxed{151\text{ mm}}$$

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \quad\to\quad h_i=-\frac{d_i}{d_o}h_o=-\frac{0.1506\text{ m}}{1.30\text{ m}}(2.40\text{ cm})=\boxed{-0.278\text{ cm}}$$

The image is behind the lens a distance of 151 mm and is $\boxed{\text{real, inverted, and reduced}}$.

(*b*) Again use Eqs. 23–8 and 23–9.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(1.30\text{ m})(-0.135\text{ m})}{1.30\text{ m}-(-0.135\text{ m})}=-0.1223\text{ m}\approx\boxed{-122\text{ mm}}$$

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{(-0.1223\text{ m})}{1.30\text{ m}}(2.40\text{ cm})=\boxed{0.226\text{ cm}}$$

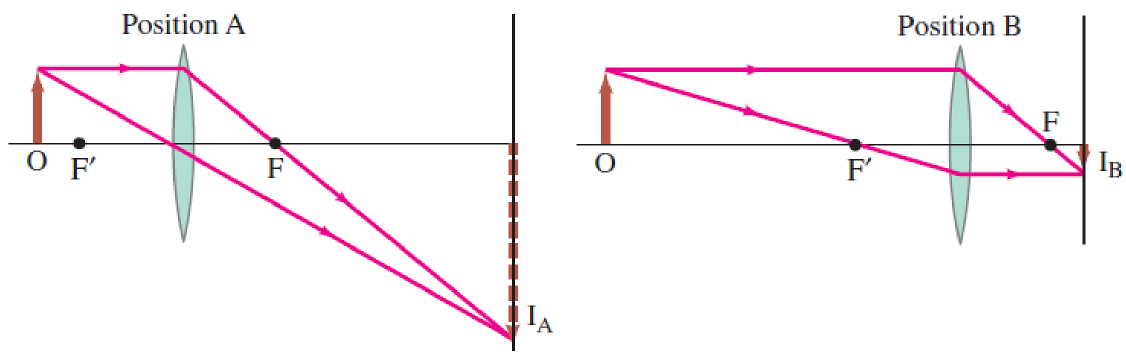
The image is in front of the lens a distance of 122 mm and is $\boxed{\text{virtual, upright, and reduced}}$.

55. The sum of the object and image distances must be the distance between object and screen, which we label as d_T. We solve this relationship for the image distance and use that expression in Eq. 23–8 in order to find the object distance.

$$d_o+d_i=d_T \rightarrow d_i=d_T-d_o;\quad \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(d_T-d_o)}=\frac{1}{f} \rightarrow d_o^2-d_Td_o+fd_T=0 \rightarrow$$

$$d_o=\frac{d_T\pm\sqrt{d_T^2-4fd_T}}{2}=\frac{(86.0\text{ cm})\pm\sqrt{(86.0\text{ cm})^2-4(16.0\text{ cm})(86.0\text{ cm})}}{2}=\boxed{21.3\text{ cm}, 64.7\text{ cm}}$$

Note that to have real values for d_o, we must in general have $d_T^2-4fd_T>0 \rightarrow d_T>4f$.



56. For a real image, both the object and image distances are positive, so the magnification is negative. Use Eqs. 23–8 and 23–9 to find the object and image distances. Since they are on opposite sides of the lens, the distance between them is their sum.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o=md_o=3.25d_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{3.25d_o}=\frac{1}{f} \rightarrow d_o=\left(\frac{4.25}{3.25}\right)f=\left(\frac{4.25}{3.25}\right)(85\text{ cm})=111.2\text{ cm}$$

$$d_i=3.25d_o=3.25(111.2\text{ cm})=361.3\text{ cm}$$

$$d_o+d_i=111.2\text{ cm}+361.3\text{ cm}=472.5\text{ cm}\approx\boxed{470\text{ cm}}$$

57. Find the object distance from Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{fd_i}{d_i-f}=\frac{(0.105\text{ m})(25.5\text{ m})}{25.5\text{ m}-0.105\text{ m}}=\boxed{0.105\text{ m}}$$

Find the size of the image from Eq. 23–9.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \rightarrow \quad |h_i| = \frac{d_i}{d_o} h_o = \frac{25.5\text{ m}}{0.105\text{ m}}(24\text{ mm}) = 5805\text{ mm} \approx \boxed{5.8\text{ m}}$$

58. The first lens is the converging lens. An object at infinity will form an image at the focal point of the converging lens, by Eq. 23–8.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} = \frac{1}{\infty} + \frac{1}{d_{i1}} \quad \rightarrow \quad d_{i1} = f_1 = 20.0\text{ cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, so $d_{o2} = -6.0$ cm. Again use Eq. 23–8.

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \quad \rightarrow \quad d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2} = \frac{(-6.0\text{ cm})(-36.5\text{ cm})}{(-6.0\text{ cm}) - (-36.5\text{ cm})} = 7.2\text{ cm}$$

Thus the final image is real and is $\boxed{\text{7.2 cm beyond the second lens}}$.

59. Find the image formed by the first lens, using Eq. 23–8.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \quad \rightarrow \quad d_{i1} = \frac{d_{o1} f_1}{d_{o1} - f_1} = \frac{(35.0\text{ cm})(25.0\text{ cm})}{(35.0\text{ cm}) - (25.0\text{ cm})} = 87.5\text{ cm}$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance.

$$d_{o2} = 16.5\text{ cm} - 87.5\text{ cm} = -71.0\text{ cm}$$

Find the image formed by the second lens, again using Eq. 23–8.

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \quad \rightarrow \quad d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2} = \frac{(-71.0\text{ cm})(25.0\text{ cm})}{(-71.0\text{ cm}) - (25.0\text{ cm})} = 18.5\text{ cm}$$

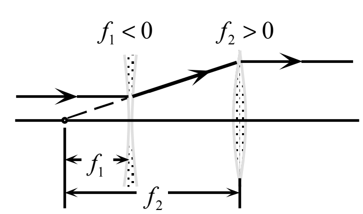
Thus the final image is real and is $\boxed{\text{18.5 cm beyond the second lens}}$.

The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \frac{d_{i1} d_{i2}}{d_{o1} d_{o2}}$$

$$= \frac{(+87.5\text{ cm})(+18.5\text{ cm})}{(+35.0\text{ cm})(-71.0\text{ cm})} = \boxed{-0.651\times\text{ (inverted)}}$$

60. From the ray diagram, the image from the first lens is a virtual image located at the focal point of the first lens. This is a real object for the second lens. Since the light is parallel after leaving the second lens, the object for the second lens must be at its focal point. Let the separation of the lenses be ℓ. Note that the focal length of the diverging lens is negative.



$$|f_1| + \ell = f_2 \quad \rightarrow$$
$$|f_1| = f_2 - \ell = 38.0\text{ cm} - 28.0\text{ cm} = 10.0\text{ cm} \quad \rightarrow \quad f_1 = \boxed{-10.0\text{ cm}}$$

61. (*a*) The first lens is the converging lens. Find the image formed by the first lens.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(60.0\text{ cm})(20.0\text{ cm})}{(60.0\text{ cm})-(20.0\text{ cm})}=30.0\text{ cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, so $d_{o2}=25.0\text{ cm}-30.0\text{ cm}=-5.0\text{ cm}$. Use Eq. 23–8.

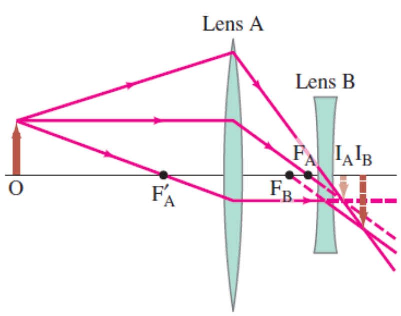
$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-5.0\text{ cm})(-10.0\text{ cm})}{(-5.0\text{ cm})-(-10.0\text{ cm})}=10\text{ cm}$$

Thus the final image is real and is $\boxed{\text{10 cm beyond the second lens}}$. The distance has two significant figures.

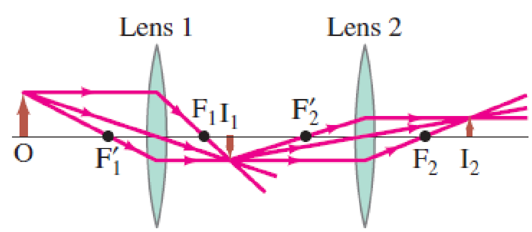
(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}=\frac{(30.0\text{ cm})(10.0\text{ cm})}{(60.0\text{ cm})(-5.0\text{ cm})}=\boxed{-1.0\times}$$

(*c*)



62. (*a*) We see that the image is real and upright. We estimate that it is 30 cm beyond the second lens and that the final image height is half the original object height.



(*b*) Find the image formed by the first lens, using Eq. 23–8.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow$$

$$d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(36\text{ cm})(13\text{ cm})}{(36\text{ cm})-(13\text{ cm})}=20.35\text{ cm}$$

This image is the object for the second lens. Because it is between the lenses, it has a positive object distance.

$$d_{o2}=56\text{ cm}-20.35\text{ cm}=35.65\text{ cm}$$

Find the image formed by the second lens, again using Eq. 23–8.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(35.65\text{ cm})(16\text{ cm})}{(35.65\text{ cm})-(16\text{ cm})}=29.25\text{ cm}$$

Thus the final image is real and is $\boxed{\text{29 cm beyond the second lens}}$.

The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \frac{(20.35\text{ cm})(29.25\text{ cm})}{(36\text{ cm})(35.65\text{ cm})} = \boxed{0.46\times}$$

63. Use Eq. 23–10, the lensmaker's equation.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$
$$f = \frac{1}{(n-1)}\left(\frac{R_1 R_2}{R_1+R_2}\right) = \frac{1}{(1.52-1)}\left(\frac{(-33.4\text{ cm})(-28.8\text{ cm})}{(-33.4\text{ cm})+(-28.8\text{ cm})}\right) = -29.74\text{ cm} \approx \boxed{-29.7\text{ cm}}$$

64. Find the index from Eq. 23–10, the lensmaker's equation.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow \quad n = 1+\frac{1}{f}\left(\frac{R_1 R_2}{R_1+R_2}\right) = 1+\left(\frac{1}{28.9\text{ cm}}\right)\left(\tfrac{1}{2}(34.1\text{ cm})\right) = \boxed{1.590}$$

65. The plane surface has an infinite radius of curvature. Let the plane surface be surface 2, so $R_2 = \infty$.

Use Eq. 23–10.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{\infty}\right) = \frac{(n-1)}{R_1} \rightarrow$$
$$R_1 = (n-1)f = (1.55-1)(16.3\text{ cm}) = 8.965\text{ cm} \approx \boxed{9.0\text{ cm}}$$

66. Find the radius from the Eq. 23–10, the lensmaker's equation, with $R_1 = R_2 = R.$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = (n-1)\left(\frac{2}{R}\right) \rightarrow$$
$$R = 2f(n-1) = 2(22.0\text{ cm})(1.52-1) = 22.88\text{ cm} \approx \boxed{23\text{ cm}}$$

67. Find the radius from the lensmaker's equation, Eq. 23–10.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow \quad P = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$R_2 = \frac{(n-1)R_1}{PR_1-(n-1)} = \frac{(1.56-1)(0.300\text{ m})}{(3.50\text{ D})(0.300\text{ m})-(1.56-1)} = \boxed{0.34\text{ m}}$$

68. First we find the focal length from Eq. 23–10, the lensmaker's equation. Then we use Eq. 23–8 to find the image distance and Eq. 23–9 to find the magnification.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$
$$f = \frac{1}{(n-1)}\left(\frac{R_1 R_2}{R_1+R_2}\right) = \frac{1}{(1.52-1)}\left(\frac{(-22.0\text{ cm})(+18.5\text{ cm})}{(-22.0\text{ cm})+(+18.5\text{ cm})}\right) = 223.6\text{ cm}$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\rightarrow\quad d_i=\frac{d_o f}{d_o-f}=\frac{(96.5\text{ cm})(223.6\text{ m})}{96.5\text{ cm}-223.6\text{ cm}}=-169.77\text{ cm}\approx\boxed{-170\text{ cm}}$$

$$m=-\frac{d_i}{d_o}=-\frac{-169.77\text{ cm}}{96.5\text{ cm}}=1.759\approx\boxed{+1.8}$$

The image is virtual, in front of the lens, and upright.

69. The transit time is the distance traveled divided by the speed of light in a vacuum. We use the Earth–Moon distance, but subtract the Earth and Moon radii since the light travels between the surfaces of the bodies.

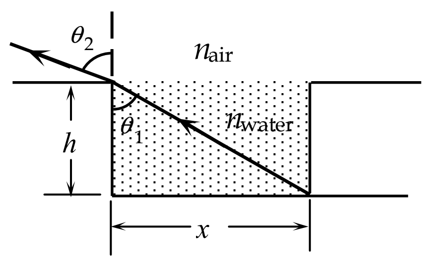
$$d=vt\rightarrow t=\frac{d}{v}=\frac{d_{\substack{\text{Earth}\\\text{Moon}}}-r_{\text{Earth}}-r_{\text{Moon}}}{c}=\frac{(384\times10^6\text{ m}-6.38\times10^6\text{ m}-1.74\times10^6\text{ m})}{(3.00\times10^8\text{ m/s})}=\boxed{1.25\text{ s}}$$

70. The closest image is the image you see in the small mirror. It will be 0.50 m to the right of the small mirror, or $\boxed{\text{1.00 m to your right}}$. You could also consider that light left your face, traveled 0.50 m to the small mirror, and then 0.50 m back to your eyes, for a total of 1.00 m.

The next closest image is one that is behind the full-length mirror. That image would be 1.0 m behind the full length mirror. You can't see that image directly, because your back is facing the full-length mirror. So you would see the "reflection" of that back of your head, by light that traveled from you, to the full-length mirror (1.0 m), back to you (1.0 m), then to the small mirror (0.50 m), and then to your eyes (0.50 m), for a total of $\boxed{\text{3.0 m}}$ from you. You could think about that image as being your reflection in the full-length mirror being reflected by the small mirror.

The next closest image is the one that would be light that left your face, traveled to the small mirror (0.50 m), reflected from the small mirror to the large mirror (1.5 m), reflected from the large mirror to the small mirror (1.5 m), and then reflected into your eyes (0.5 m), for a total distance of $\boxed{\text{4.0 m}}$.

$\boxed{71.}$ Find the angle of incidence for refraction from water into air, using Snell's law.



$$n_{\text{water}}\sin\theta_1=n_{\text{air}}\sin\theta_2\quad\rightarrow$$
$$(1.33)\sin\theta_1=(1.00)\sin(90.0^\circ-13.0^\circ)\quad\rightarrow\quad\theta_1=47.11^\circ$$

$$(1.33)\sin\theta_1=(1.00)\sin(90.0^\circ-13.0^\circ),$$

Find the depth of the pool from $\tan\theta_1=x/h$.

$$\tan 47.11^\circ=(6.50\text{ m})/h\quad\rightarrow\quad h=\boxed{6.04\text{ m}}$$

72. For the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have the following. Use Eq. 23–6.

Plastic to air: $\sin\theta_C=\dfrac{n_{\text{air}}}{n_{\text{plastic}}}\quad\rightarrow\quad n_{\text{plastic}}=\dfrac{1.00}{\sin 37.8^\circ}=1.6316$

Plastic to water: $\sin\theta_C=\dfrac{n_{\text{water}}}{n_{\text{plastic}}}\quad\rightarrow\quad\theta_C=\sin^{-1}\left(\dfrac{1.33}{1.6316}\right)=\boxed{54.6^\circ}$

73. The index of refraction is the speed of light in a vacuum divided by the speed of light in the material, Eq. 23–4. The material can be identified from Table 23–1.

$$n=\frac{c}{v}=\frac{c}{\Delta x/\Delta t}=\frac{3.00\times10^{8}\ \text{m/s}}{(0.500\ \text{m})/(2.63\times10^{-9}\ \text{s})}=\boxed{1.58}$$

The material is $\boxed{\text{light flint glass}}$.

74. We set d_{i} as the original image distance and $d_{\text{i}}+10.0\,\text{cm}$ as the new image distance. Then using Eq. 23–8 for both cases, we eliminate the focal length and solve for the image distance. We insert the real image distance into the initial lens equation and solve for the focal length.

$$\frac{1}{d_{\text{o1}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}=\frac{1}{d_{\text{o2}}}+\frac{1}{d_{\text{i}}+10.0\ \text{cm}}\quad\rightarrow\quad\frac{1}{d_{\text{o1}}}-\frac{1}{d_{\text{o2}}}=\frac{1}{d_{\text{i}}+10.0\ \text{cm}}-\frac{1}{d_{\text{i}}}=\frac{-10.0\ \text{cm}}{d_{\text{i}}(d_{\text{i}}+10.0\ \text{cm})}$$

$$\frac{1}{60.0\ \text{cm}}-\frac{1}{40.0\ \text{cm}}=\frac{-10.0\ \text{cm}}{d_{\text{i}}(d_{\text{i}}+10.0\ \text{cm})}\quad\rightarrow\quad d_{\text{i}}^2+(10.0\ \text{cm})d_{\text{i}}-1200\ \text{cm}^2=0$$

$$d_{\text{i}}=-40.0\ \text{cm or }30.\text{cm}$$

Only the positive image distance will produce the real image.

$$\frac{1}{f}=\frac{1}{d_{\text{o1}}}+\frac{1}{d_{\text{i}}}\Rightarrow f=\frac{d_i d_{o1}}{d_{\text{i}}+d_{o1}}=\frac{(30.0\ \text{cm})(60.0\ \text{cm})}{30.0\ \text{cm}+60.0\ \text{cm}}=\boxed{20.0\ \text{cm}}$$

75. (*a*) Producing a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror}}$.

(*b*) Find the image distance from the magnification, Eq. 23–3.

$$m=\frac{h_{\text{i}}}{h_{\text{o}}}=\frac{-d_{\text{i}}}{d_{\text{o}}}\quad\rightarrow\quad d_{\text{i}}=-\frac{d_{\text{o}}h_{\text{i}}}{h_{\text{o}}}=-\frac{(32\ \text{cm})(3.5\ \text{cm})}{(4.5\ \text{cm})}=-24.9\ \text{cm}\approx\boxed{-25\ \text{cm}}$$

As expected, $d_{\text{i}}<0$. The image is located $\boxed{\text{25 cm behind the surface}}$.

(*c*) Find the focal length from Eq. 23–2.

$$\frac{1}{d_{\text{o}}}+\frac{1}{d_{\text{i}}}=\frac{1}{f}\quad\rightarrow f=\frac{d_{\text{o}}d_{\text{i}}}{d_{\text{o}}+d_{\text{i}}}=\frac{(32\,\text{cm})(-24.9\,\text{cm})}{(32\,\text{cm})+(-24.9\,\text{cm})}=-112.2\,\text{cm}\approx\boxed{-110\,\text{cm}}$$

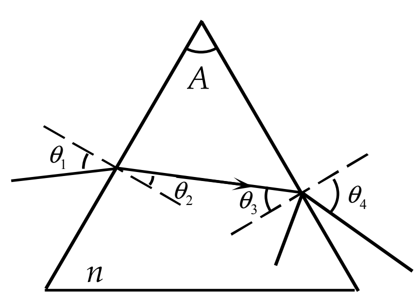
(*d*) The radius of curvature is twice the focal length.

$$r=2f=2(-112.2\,\text{cm})=-224.4\,\text{cm}\approx\boxed{-220\,\text{cm}}$$

76. The length in space of a burst is the speed of light times the elapsed time.

$$d=ct=(3.00\times10^{8}\ \text{m/s})(10^{-8}\ \text{s})=\boxed{3\ \text{m}}$$

77. For refraction at the second surface, use Snell's law.



$$n\sin\theta_3=n_{\text{air}}\sin\theta_4\quad\rightarrow\quad(1.58)\sin\theta_3=(1.00)\sin\theta_4$$

The maximum value of θ_4 before total internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.58)\sin\theta_3\le1.00\quad\rightarrow\quad\sin\theta_3\le0.6329\quad\rightarrow\quad\theta_3\le39.27°$$

We find the refraction angle at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \rightarrow$$
$$\theta_2 = A - \theta_3 = 75° - \theta_3$$

Thus $\theta_2 \geq 75° - 39.27° = 35.73°$.

For the refraction at the first surface, we have the following.

$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2 \quad \rightarrow \quad (1.00)\sin\theta_1 = (1.58)\sin\theta_2 \quad \rightarrow \quad \sin\theta_1 = (1.58)\sin\theta_2$$

Now apply the limiting condition.

$$\sin\theta_1 \geq (1.58)\sin 35.73° = 0.9227 \quad \rightarrow \quad \boxed{\theta_1 \geq 67°}$$

78. (*a*) As the radius of a sphere increases, the surface becomes flatter. The plane mirror can be considered a spherical mirror with an infinite radius, and thus $\boxed{f = \infty}$.

(*b*) When we use the mirror equation, Eq. 23–2, we get the following.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{1}{\infty} = 0 \rightarrow \boxed{d_i = -d_o}$$

(*c*) For the magnification, we have $m = \frac{-d_i}{d_o} = \frac{-(-d_o)}{d_o} = \boxed{+1}$.

(*d*) $\boxed{\text{Yes,}}$ these are consistent with the discussion on plane mirrors.

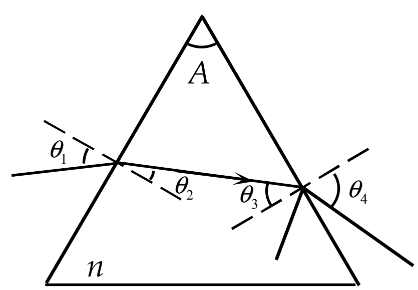
$\boxed{79.}$ A relationship between the image and object distances can be obtained from Eq. 23–3.

$$m = -\frac{1}{2} = -\frac{d_i}{d_o} \quad \rightarrow \quad d_i = \tfrac{1}{2}d_o = \boxed{9\,\text{cm}}$$

Now we find the focal length and the radius of curvature from Eq. 23–2 and Eq. 23–1.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad \frac{1}{18\,\text{cm}} + \frac{1}{9\,\text{cm}} = \frac{1}{f} \quad \rightarrow \quad f = 6\,\text{cm} \quad \rightarrow \quad \boxed{r = 12\,\text{cm}}$$

80. Find the angle θ_2 for the refraction at the first surface.



$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2$$
$$(1.00)\sin 45.0° = (1.54)\sin\theta_2 \quad \rightarrow \quad \theta_2 = 27.33°$$

Find the angle of incidence at the second surface from the triangle formed by the two sides of the prism and the light path.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \rightarrow$$

$$\theta_3 = A - \theta_2 = 60° - 27.33° = 32.67°$$

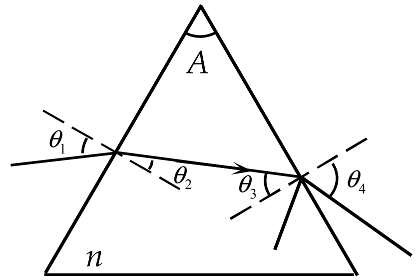
Use refraction at the second surface to find θ_4.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 \quad \rightarrow \quad (1.54)\sin 32.67° = (1.00)\sin\theta_4 \quad \rightarrow \quad \theta_4 = \boxed{56.2° \text{ from the normal}}$$

81. For the refraction at the first surface, we have the following.

$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2 \quad \rightarrow \quad (1.00)\sin\theta_1 = n\sin\theta_2 \quad \rightarrow$$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \rightarrow n = \frac{\sin\theta_1}{\sin\theta_2}$$



Find the angle of incidence at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \rightarrow$$

$$\theta_3 = A - \theta_2$$

For the refraction at the second surface, we have this:

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4 = (1.00)\sin\theta_4$$

The maximum value of θ_4 before internal reflection takes place at the second surface is 90°, which makes the right side of the above equation equal to 1. For internal reflection to occur, we have the following.

$$n \sin\theta_3 = n \sin(A - \theta_2) \geq 1 \quad \rightarrow \quad n(\sin A \cos\theta_2 - \cos A \sin\theta_2) \geq 1$$

Use the result from the first surface to eliminate n.

$$\frac{\sin\theta_1(\sin A \cos\theta_2 - \cos A \sin\theta_2)}{\sin\theta_2} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2} - \cos A\right) \geq 1 \quad \rightarrow$$

$$\tan\theta_2 \leq \frac{\sin A}{\left(\dfrac{1}{\sin\theta_1} + \cos A\right)} = \frac{\sin 65.0°}{\left(\dfrac{1}{\sin 45.0°} + \cos 65.0°\right)} = 0.4934 \quad \rightarrow \quad \theta_2 \leq 26.3°$$

Use the result from the first surface.

$$n_{\min} = \frac{\sin\theta_1}{\sin\theta_{2\max}} = \frac{\sin 45.0°}{\sin 26.3°} = 1.596 \quad \rightarrow \quad \boxed{n \geq 1.60}$$

82. (*a*) For the image to be between the object and the lens, the lens must be $\boxed{\text{diverging}}$ (see Fig. 23–39). Find the focal length from Eq. 23–8. The image distance is negative.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad f = \frac{d_o d_i}{d_o + d_i} = \frac{(37.5\text{ cm})(-8.20\text{ cm})}{37.5\text{ cm} - 8.20\text{ cm}} = \boxed{-10.5\text{ cm}}$$

The image is in front of the lens, so it is $\boxed{\text{virtual}}$.

(*b*) For the image to be farther from the lens than the object but on the same side of the lens as the object, the lens must be converging (see Fig. 23–43). The image distance again is negative.

$$f = \frac{d_o d_i}{d_o + d_i} = \frac{(37.5\text{ cm})(-44.50\text{ cm})}{37.5\text{ cm} - 44.5\text{ cm}} = 238\text{ cm} \approx \boxed{240\text{ cm}}$$

The image is in front of the lens, so again it is virtual.

83. (*a*) Because the Sun is very far away, the image will be at the focal point. We find the size of the image from Eq. 23–9.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} = -\frac{f}{d_o} \quad \rightarrow \quad h_i = -h_o\frac{f}{d_o} = -(1.4\times10^6\text{ km})\frac{35\text{ mm}}{(1.5\times10^8\text{ km})} = \boxed{-0.33\text{ mm}}$$

The negative sign indicates that the image is inverted.

(*b*) Use the same calculation for the 50-mm lens.

$$h_i = -h_o \frac{f}{d_o} = -(1.4\times10^6\ \text{km})\frac{50\ \text{mm}}{(1.5\times10^8\ \text{km})} = \boxed{-0.47\ \text{mm}}$$

(*c*) Use the same calculation for the 105-mm lens.

$$h_i = -h_o \frac{f}{d_o} = -(1.4\times10^6\ \text{km})\frac{105\ \text{mm}}{(1.5\times10^8\ \text{km})} = \boxed{-0.98\ \text{mm}}$$

84. (*a*) The eye must be acting as a convex mirror—otherwise you couldn't see the image.

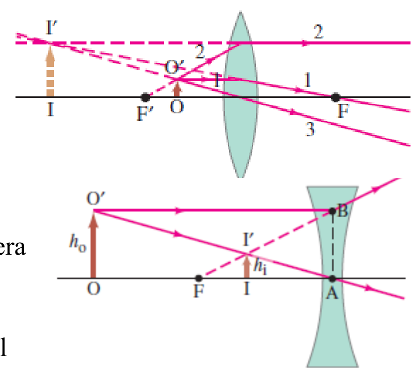
(*b*) The eye is being viewed right side up. Convex mirrors make upright images. Also notice the eyelashes.

(*c*) The eye is convex-shaped. If it were acting as a convex lens, then the object would have to be inside the eye, sending light from inside the eye to outside the eye. That is not possible. So it is acting like a convex mirror, making a virtual, reduced object. This is similar to Fig. 23–19b, in which the "mirror" is the eyeball and the "eye" is the camera taking the photo.

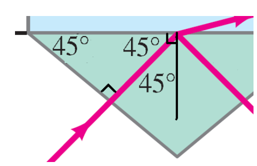
85. The left lens is a converging lens, acting as a magnifying glass, forming a magnified virtual image. The object (the face) is closer to the lens than the focal point. The right lens is a diverging lens, forming a reduced virtual image.

For the left lens: O is the person's face. The observer (the camera taking the picture) is to the right of the lens.

For the right lens: O is the person's face. The observer (the camera taking the picture) is to the right of the lens.



86. For the device to work properly, the light should experience total internal reflection at the top surface of the prism when it is a prism to air interface, but not total internal reflection when the top surface is a prism to water interface. Since the incident ray is perpendicular to the lower surface of the prism, light does not experience refraction at that surface. As shown in the diagram, the incident angle for the upper surface will be 45°. We then use Eq. 23–6 to determine the minimum index of refraction for total internal reflection with an air interface and the maximum index of refraction for a water interface. The usable indices of refraction will lie between these two values.



$$\frac{n_2}{n_1} = \sin\theta_C \quad\rightarrow\quad n_{1,\text{min}} = \frac{n_{\text{air}}}{\sin\theta_C} = \frac{1.00}{\sin 45^\circ} = 1.41 \quad\rightarrow\quad n_{1,\text{max}} = \frac{n_{\text{water}}}{\sin\theta_c} = \frac{1.33}{\sin 45^\circ} = 1.88$$

The index of refraction must fall within the range $\boxed{1.41 < n < 1.88}$. A glass, Lucite, or Plexiglas prism would work.

87. (*a*) When two lenses are placed in contact, the negative of the image of the first lens is the object distance of the second. Using Eq. 23–8, we solve for the image distance of the first lens. By inserting the negative of this image distance into the lens equation for the second lens we obtain a relationship between the initial object distance and final image distance. Since that object/image pair is related by the combination focal length, if we use the lens equation with this relationship, then we obtain the focal length of the lens combination.

$$\frac{1}{f_1}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}}=\frac{1}{f_1}-\frac{1}{d_{o1}}=-\frac{1}{d_{o2}}$$

$$\frac{1}{f_2}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=-\left(\frac{1}{f_1}-\frac{1}{d_{o1}}\right)+\frac{1}{d_{i2}} \Rightarrow \frac{1}{f_2}+\frac{1}{f_1}=\frac{1}{d_{o1}}+\frac{1}{d_{i2}}=\frac{1}{f_T}$$

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2} \rightarrow \boxed{f_T=\frac{f_1 f_2}{f_1+f_2}}$$

(*b*) Setting the power equal to the inverse of the focal length gives the relationship between powers of adjacent lenses.

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \boxed{P_T=P_1+P_2}$$

88. Solve this problem by working backward through the lenses. Use the image distances and focal lengths to calculate the object distances. Let the right lens be lens 2 and the left lens be lens 1. Since the final image from lens 2 is halfway between the lenses, set the image distance of the second lens equal to the negative of half the distance between the lenses. Using Eq. 23–8, solve for the object distance of lens 2. By subtracting this object distance from the distance between the two lenses, find the image distance for lens 1. Then using Eq. 23–8 again, solve for the initial object distance.

$$d_{i2}=-\tfrac{1}{2}\ell=-\tfrac{1}{2}(30.0\text{ cm})=-15.0\text{ cm}$$

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{o2}=\frac{d_{i2}f_2}{d_{i2}-f_2}=\frac{(-15.0\text{ cm})(20.0\text{ cm})}{-15.0\text{ cm}-20.0\text{ cm}}=8.57\text{ cm}$$

$$d_{i1}=\ell-d_{o2}=30.0\text{ cm}-8.57\text{ cm}=21.4\text{ cm}$$

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{o1}=\frac{d_{i1}f_1}{d_{i1}-f_1}=\frac{(21.4\text{ cm})(15.0\text{ cm})}{21.4\text{ cm}-15.0\text{ cm}}=\boxed{50.0\text{ cm}}$$

89. The lens has a focal length of 20 cm. Approach this problem in steps—find the first image formed by the lens, let that image be the object for the mirror, and then use the image from the mirror as the object for the “reversed” light that goes back through the lens. Use Eq. 23–8 for each image formation.

Calculate the location of the first image, using $d_{o1}=30.0$ cm and $f_1=20.0$ cm.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(30.0\text{ cm})(20.0\text{ cm})}{30.0\text{ cm}-20.0\text{ cm}}=60.0\text{ cm}$$

This image is 15 cm from the mirror. Calculate the location of the second image, using $d_{o2}=15.0$ cm and $f_2=25.0$ cm.

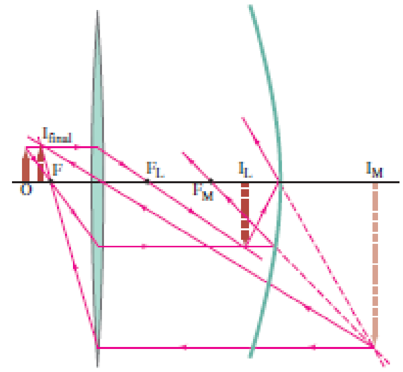
$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(15.0\text{ cm})(25.0\text{ cm})}{15.0\text{ cm}-25.0\text{ cm}}=-37.5\text{ cm}$$

This is a virtual image, 37.5 cm behind the mirror, or 112.5 cm from the lens. The diverging light rays from the mirror will be moving from the mirror toward the lens, so this is a real object for the lens. So calculate the location of the third image, using $d_{o3}=125.0$ cm and $f_3=20.0$ cm.

$$\frac{1}{d_{o3}}+\frac{1}{d_{i3}}=\frac{1}{f_3} \rightarrow d_{i3}=\frac{d_{o3}f_3}{d_{o3}-f_3}=\frac{(112.5\text{ cm})(20.0\text{ cm})}{112.5\text{ cm}-20.0\text{ cm}}=24.3\text{ cm}$$

This would be on the same side of the lens as the original object, so this final image is $\boxed{\text{5.7 cm from the original object, toward the lens}}$.

Here is a ray diagram showing some of the rays used to locate the various images.



90. We use Eq. 23–8 with the final image distance and focal length of the converging lens to determine the location of the object for the second lens. Subtracting this distance from the separation distance between the lenses gives us the image distance from the first lens. Inserting this image distance and object distance into Eq. 23–8, we calculate the focal length of the diverging lens.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \quad \to \quad d_{o2}=\frac{d_{i2}f_2}{d_{i2}-f_2}=\frac{(17.0\text{ cm})(12.0\text{ cm})}{17.0\text{ cm}-12.0\text{ cm}}=40.8\text{ cm}$$

$$d_{i1}=\ell-d_{o2}=30.0\text{ cm}-40.8\text{ cm}=-10.8\text{ cm}$$

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \quad \to \quad f_1=\frac{d_{i1}d_{o1}}{d_{i1}+d_{o1}}=\frac{(-10.8\text{ cm})(25.0\text{ cm})}{-10.8\text{ cm}+25.0\text{ cm}}=\boxed{-19.0\text{ cm}}$$

Solutions to Search and Learn Problems

1. (*a*) Light from a real image actually passes through the image. Light from a virtual image only appears to pass through the image location. A real image can be displayed on a screen. To see a virtual image you must look through the mirror or lens to see the image.

   (*b*) No, your eyes cannot tell the difference. In each case the light entering your eye appears to come from the position of the image.

   (*c*) On a ray diagram, the ray traces pass through the real image. You must extrapolate the rays backward through the mirror/lens to find the location of the virtual image.

   (*d*) You could place a screen at the location of the image. If it is a real image, then it will appear on the screen. If it is a virtual image, then it will not appear on the screen.

   (*e*) Yes, if the photograph is taken through the lens or mirror, then the image would appear in the photograph.

   (*f*) No. The light does not actually pass through the location of the virtual image, so no image would be captured on the film.

   (*g*) Light appears to come from the position of the virtual image, and that light can be captured by the camera. However, the light does not pass through the location of the virtual image, so the film placed at that position would not capture the image.

2. The two students chose different signs for the magnification, that is, one upright and one inverted. The focal length of the lens is $f = 12$ cm. We relate the object and image distances from the magnification.

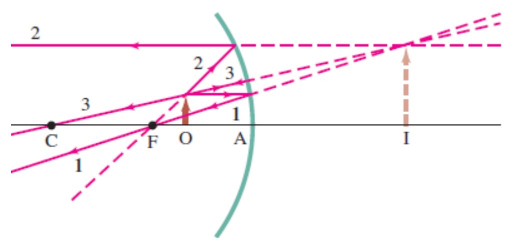
$$m = -\frac{d_i}{d_o} \quad \rightarrow \quad \pm 3 = -\frac{d_i}{d_o} \quad \rightarrow \quad d_i = \mp 3d_o$$

Use this result in the mirror equation.

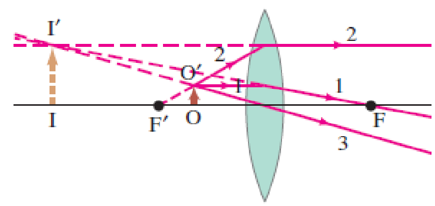
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad \frac{1}{d_o} + \frac{1}{(\mp 3d_o)} = \frac{1}{f} \quad \rightarrow \quad d_o = \frac{2f}{3}, \frac{4f}{3} = 8.0 \text{ cm}, 16 \text{ cm}$$

So the object distances are + 8.0 cm (produces a virtual image) and + 16 cm (produces a real image).

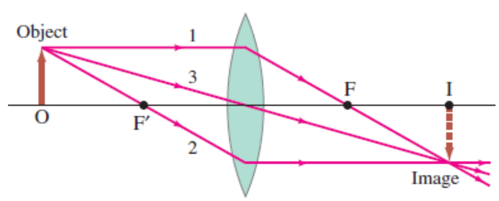
3. (*a*) For a makeup mirror the object O that is to be magnified is also the observer. For a concave mirror, the ray diagram would be as shown here (Fig. 23–17), and the observer can see the magnified virtual image.



For a converging lens, one possibility is that the magnified image would be virtual and behind the observer, as shown in the adjacent diagram (Fig. 23–43, with O as the observer and the object, looking toward the lens). The observer could not see that image, however, so it is not useful.



The final possibility is a converging lens making a real, inverted, and somewhat distant image as shown in the last image (Fig. 23–37c, with O as the observer and the object, looking toward the lens). For the observer (also the object) to see the real image, a screen would need to be placed at the image location. The resulting image would be inverted (making it very inconvenient) and somewhat distant from the observer. Also, for this diagram to be accurate, the object should be closer to the focal point, and the image farther away, probably making it more inconvenient.



(*b*) The objects will have the same magnification. If the object distance is $\frac{1}{2}f$, then the mirror equation and the thin lens equation give the same value for the image distance.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \rightarrow \quad d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} = \left(\frac{1}{f} - \frac{1}{\frac{1}{2}f}\right)^{-1} = -f$$

The magnification equation for mirrors and lenses is the same, which gives the same result for both.

$$m = -\frac{d_i}{d_o} = -\frac{-f}{\frac{1}{2}f} = 2$$

For both the mirror and the lens, the image will be virtual with a magnification of $+2$.

4. (*a*) Yes, the person in the picture shot the image.

   (*b*) The mirror on the left is a concave mirror. Only a concave mirror produces an inverted image. The middle mirror is flat. The mirror on the right is convex.

   (*c*) The middle mirror is flat, so the image in that mirror is the same size as the object. There is no focal point for a flat mirror, so the person could be anywhere relative to the mirror. For the concave mirror (left) the image is inverted and reduced in size, compared to the image in the flat mirror. Therefore, she must be at a distance greater than the center of curvature (so also greater than the focal point). If she were at a distance between the focus point and the radius of curvature, then her image would be enlarged, and the image would be behind the camera and therefore not in focus. And if she were at a distance less than the focus point, then her image would be upright and magnified. In the convex mirror (right), her image is about 1/3 the size of the image in the flat mirror, so the magnification is about 1/3. Solving the mirror equation for $d_\mathrm{i} = -\frac{1}{3}d_o$ gives $d_\mathrm{o} = -2f$. Therefore, she must be at a distance approximately equal in magnitude to the radius of curvature in front of the mirror, or twice the magnitude of the focal length.

5. The statement to justify is this: "Equivalently, the image distance is positive for a real image and negative for a virtual image." The three cases to consider are for a diverging lens, a converging lens with object distance greater than the focal length, and a converging lens with object distance less than the focal length. In each case consider this equation: $\frac{1}{f} = \frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} \rightarrow d_\mathrm{i} = \left(\frac{1}{f} - \frac{1}{d_\mathrm{o}}\right)^{-1}$.

   Case i: Fig. 23–39 shows that the image from a diverging lens is virtual. Since $f < 0$ and $d_\mathrm{o} > 0$, $d_\mathrm{i} = \left(\frac{1}{f} - \frac{1}{d_\mathrm{o}}\right)^{-1} < 0$. Thus the virtual image has a negative image distance.

   Case ii: Fig. 23–40 shows that the image from a converging lens with $d_\mathrm{o} > f$ is a real image. In this case, the image distance must be positive in the lens equation since $f > 0$ and $d_\mathrm{o} > f > 0$. The reciprocal of *f* is greater than the reciprocal of d_o, so $d_\mathrm{i} = \left(\frac{1}{f} - \frac{1}{d_\mathrm{o}}\right)^{-1} > 0$. Therefore, the real image has a positive image distance.

   Case iii: Fig. 23–43 shows that the image from a converging lens with $f > d_o > 0$ is a virtual image. In this case since $f > d_\mathrm{o} > 0$, the reciprocal of *f* is less than the reciprocal of d_o, so $d_\mathrm{i} = \left(\frac{1}{f} - \frac{1}{d_\mathrm{o}}\right)^{-1} < 0$. Therefore, the virtual image has a negative image distance.

6. To create a real image with a single lens, the image distance must be positive. For a single lens the object distance must be positive. If we solve the thin lens equation for the image distance, then we get the following equation.

$$\frac{1}{f} = \frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} \rightarrow d_\mathrm{i} = \left(\frac{1}{f} - \frac{1}{d_\mathrm{o}}\right)^{-1}$$

   Since d_o is positive, the image distance will always be negative if $f < 0$, as it is for a diverging lens. Therefore, a diverging lens cannot produce a real image. If the object is inside the focal length of a converging lens, then d_o will be smaller than f, causing the image distance to be negative and the object to be virtual. Only when the object is outside the focal length will the image distance be positive and the image real. The answer is (c).

7.

<table>
<tr><th rowspan="2"></th><th rowspan="2">Focal Length</th><th colspan="2">Object Distance</th><th colspan="2">Image Distance</th><th>Object Height</th><th>Image Height</th></tr>
<tr><th>In front of</th><th>Behind</th><th>In front of</th><th>Behind</th><td rowspan="5">Positive when right side up

Negative when inverted</td><td rowspan="5">Positive when right side up

Negative when inverted</td></tr>
<tr><td>Concave Mirror</td><td>Positive</td><td>Positive</td><td>Negative</td><td>Positive</td><td>Negative</td></tr>
<tr><td>Convex Mirror</td><td>Negative</td><td>Positive</td><td>Negative</td><td>Positive</td><td>Negative</td></tr>
<tr><td>Converging Lens</td><td>Positive</td><td>Positive</td><td>Negative</td><td>Negative</td><td>Positive</td></tr>
<tr><td>Diverging Lens</td><td>Negative</td><td>Positive</td><td>Negative</td><td>Negative</td><td>Positive</td></tr>
</table>

8. The magnifications in the two figures appear to have the same magnitude with the magnification positive in (a) and negative in (b). In (a) the object distance is 5 cm and in (b) the object distance is 15 cm. Using the magnification equation, we write the image distance in (b) in terms of the image and object distances in (a).

$$m_b = -m_a = -\frac{d_{ib}}{d_{ob}} = \frac{d_{ia}}{d_{oa}} \quad \rightarrow \quad d_{ib} = -\frac{d_{ob}d_{ia}}{d_{oa}} = -\frac{15\text{ cm}}{5\text{ cm}}d_{ia} = -3d_{ia}$$

Then using the thin lens equation for each image, calculate the image distance in figure (a).

$$\frac{1}{f} = \frac{1}{d_{oa}} + \frac{1}{d_{ia}} = \frac{1}{d_{ob}} + \frac{1}{d_{ib}} \quad \rightarrow \quad \frac{1}{d_{oa}} + \frac{1}{d_{ia}} = \frac{1}{d_{ob}} + \frac{1}{-3d_{ia}}$$

$$d_{ia} = \frac{4}{3}\left(\frac{1}{d_{ob}} - \frac{1}{d_{oa}}\right)^{-1} = \frac{4}{3}\left(\frac{1}{15\text{ cm}} - \frac{1}{5\text{ cm}}\right)^{-1} = \boxed{-10\text{ cm}}$$

Now the image distance in figure (b) and the focal length can be determined.

$$d_{ib} = -3d_{ia} = -3(-10\text{ cm}) = \boxed{30\text{ cm}}$$

$$\frac{1}{f} = \frac{1}{d_{oa}} + \frac{1}{d_{ia}} \quad \rightarrow \quad f = \left(\frac{1}{d_{oa}} + \frac{1}{d_{ia}}\right)^{-1} = \left(\frac{1}{5\text{ cm}} + \frac{1}{-10\text{ cm}}\right)^{-1} = \boxed{10\text{ cm}}$$

The focal length is 10 cm. In (a) the object is only half the focal length from the lens and therefore creates a virtual image 10 cm behind the lens, with a magnification of $+2$. In (b) the object is 1.5 times the focal length and therefore creates a real image 30 cm in front of the lens, with a magnification of -2.

24

THE WAVE NATURE OF LIGHT

Responses to Questions

1. Huygens' principle applies to both sound waves and water waves. Huygens' principle applies to all waves that form a wave crest. Sound, water, and light waves all can be represented in this way. The maximum or crest of a water wave is its highest point above the equilibrium level of the water, so a wave front can be formed by connecting all of the local maximum points of a wave.

2. A ray shows the direction of propagation of a wave front. If this information is enough for the situation under discussion, then light can be discussed as rays. This is true in particular for geometric optics. Sometimes, however, the wave nature of light is essential to the discussion. For instance, the double-slit interference pattern depends on the interference of the waves and could not be explained by examining light as only rays.

3. The bending of waves around corners or obstacles is called diffraction. Diffraction is most prominent when the size of the obstacle is on the order of the size of the wavelength. Sound waves have much longer wavelengths than do light waves. As a result, the diffraction of sound waves around a corner of a building or through a doorway is noticeable, and we can hear the sound in the "shadow region," but the diffraction of light waves around a corner is not noticeable because of the very short wavelength of the light.

4. For destructive interference, the path lengths must differ by an odd number of half wavelengths, such as $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc. In general, the path lengths must differ by $\lambda(m + ½)$, where m is an integer. Under these conditions, the wave crests from one ray match up with the wave troughs from the other ray and cancellation occurs (destructive interference).

$\boxed{5.}$ As red light is switched to blue light, the wavelength of the light is decreased. Thus, $d \sin\theta = m\lambda$ says that θ is decreased for a constant m and d. This means that the bright spots on the screen are more closely packed together with blue light than with red light.

6. The wavelength of light in a medium such as water is decreased when compared to the wavelength in air. Thus, $d \sin\theta = m\lambda$ says that θ is decreased for a particular m and d. This means that the bright spots on the screen are more closely packed together in water than in air.

$\boxed{7.}$ The reason you do not get an interference pattern from the two headlights of a distant car is that they are not coherent light sources. The phase relationship between the two headlights is not constant—they have randomly changing phases relative to each other. Thus, you cannot produce zones of destructive and constructive interference where the crests and troughs match up or the crests and crests match up. Also, the headlights are far enough apart that even if they were coherent, the interference pattern would be so tightly packed that it would not be observable with the unaided eye.

8. For a very thin film, there are only a select few distinct visible wavelengths that meet the constructive interference criteria, because the film is only a few wavelengths thick. But for a thick film, there might be many different wavelengths that meet the constructive interference criteria for many different *m* values. Accordingly, many different colors will constructively interfere, and the reflected light will be white (containing all visible wavelengths).

9. As you move farther away from the center of the curved piece of glass on top, the path differences change more rapidly due to the curvature. Thus, you get higher order interference patterns more closely spaced together. An "air wedge," as in Fig. 24–33, has equally spaced interference patterns because as the observation point is moved farther from the contact point of the flat piece of glass on top, the path differences change linearly.

$\boxed{10.}$ These lenses probably are designed to eliminate reflected wavelengths at both the red and the blue ends of the spectrum. The thickness of the coating is designed to cause destructive interference for reflected red and blue light. The reflected light then appears yellow-green.

11. The index of refraction of the oil must be less than the index of refraction of the water. If the oil film appears bright at the edge, then the interference between the light reflected from the top of the oil film and that reflected from the bottom of the oil film at that point must be constructive. The light reflecting from the top surface (the air/oil interface) undergoes a 180° phase shift since the index of refraction of the oil is greater than that of air. The thickness of the oil film at the edge is negligible, so for there to be constructive interference, the light reflecting from the bottom of the oil film (the oil/water interface) must also undergo a 180° phase shift. This will occur only if the index of refraction of the oil is less than that of the water: $1.00 < n < 1.33$.

12. Radio waves have a much longer wavelength than visible light and will diffract around normal-sized objects (like hills). The wavelengths of visible light are very small and will not diffract around normal-sized objects. Thus diffraction allows the radio waves to be "picked up" even if the broadcasting tower is not visible on a line of sight.

13. You see a pattern of dark and bright lines parallel to your fingertips in the narrow opening between your fingers, due to diffraction.

14. (*a*) When you increase the slit width in a single-slit diffraction experiment, the spacing of the fringes decreases. The equation for the location of the minima, $\sin\theta = \dfrac{m\lambda}{D}$, indicates that θ is decreased for a particular *m* and λ when the width *D* increases. This means that the bright spots on the screen are more closely packed together for a wider slit.

(*b*) When you increase the wavelength of light used in a single-slit diffraction experiment, the spacing of the fringes increases. The equation for the location of the minima, $\sin\theta = \dfrac{m\lambda}{D}$, indicates that θ is increased for a particular *m* and *D* when the wavelength increases. This means that the bright spots on the screen are spread farther apart for a longer wavelength.

15. (*a*) A slit width of 60 nm would produce a central maximum so spread out that it would cover the entire width of the screen. No minimum (and therefore no diffraction pattern) will be seen,

because the first minimum would have to satisfy $\sin\theta = \dfrac{\lambda}{D} \approx 10,$ which is not possible. The different wavelengths will all overlap, so the light on the screen will be white. It will also be dim, compared to the source, because it is spread out.

(*b*) For the 60,000-nm slit, the central maximum will be very narrow, about a degree in width for the blue end of the spectrum and about a degree and a half for the red. The diffraction pattern will not be distinct, because most of the intensity will be in the small central maximum and the fringes for the different wavelengths of white light will not coincide.

16. (*a*) If the apparatus is immersed in water, then the wavelength of the light will decrease $\left(\lambda' = \dfrac{\lambda}{n}\right).$ and the diffraction pattern will become more compact.

(*b*) If the apparatus is placed in a vacuum, then the wavelength of the light will increase slightly and the diffraction pattern will spread out very slightly.

17. The interference pattern created by the diffraction grating with 10^4 lines/cm has bright maxima that are more sharply defined and narrower than the interference pattern created by the two slits 10^{-4} cm apart. The spacing of the bright maxima would be the same in both patterns, but for the grating, each maximum would be essentially the same brightness, while for the two-slit pattern, slit width effects would make the maxima for $m > 1$ much less bright than the central maximum.

18. (*a*) The advantage of having many slits in a diffraction grating is that this makes the bright maxima in the interference pattern more sharply defined, brighter, and narrower.

(*b*) The advantage of having closely spaced slits in a diffraction grating is that this spreads out the bright maxima in the interference pattern and makes them easier to resolve.

19. (*a*) Violet light will be at the top of the rainbow created by the diffraction grating. Principal maxima for a diffraction grating are at positions given by $\sin\theta = \dfrac{m\lambda}{d}.$ Violet light has a shorter wavelength than red light so will appear at a smaller angle away from the direction of the horizontal incident beam.

(*b*) Red light will appear at the top of the rainbow created by the prism. The index of refraction for violet light in a given medium is slightly greater than for red light in the same medium, so the violet light will bend more and will appear farther from the direction of the horizontal incident beam.

$\boxed{20.}$ Polarization demonstrates the transverse wave nature of light and cannot be explained if light is considered as a longitudinal wave or as classical particles.

21. Polarized sunglasses completely block horizontally polarized glare at certain reflected angles and also block unpolarized light by 50%. Regular tinted sunglasses reduce the intensity of all incoming light but don’t preferentially reduce the “glare” from smooth reflective surfaces.

$\boxed{22.}$ Take the sunglasses outside and look up at the sky through them. Rotate the sunglasses (about an axis perpendicular to the lens) through at least 180°. If the sky seems to lighten and darken as you rotate the sunglasses, then they are polarizing. You could also look at a liquid crystal display or reflections from a tile floor (with a lot of “glare”) while rotating the glasses and again look for the light to be lighter or darker depending on the rotation angle. Finally, you could put one pair of glasses on top of the other as in Fig. 24–44 and rotate them relative to each other. If the intensity of light that you see through the glasses changes as you rotate them, then the glasses are polarizing.

23. If Earth had no atmosphere, then the color of the sky would be black (and dotted with stars and planets) at all times. This is the condition of the sky that the astronauts found on the Moon, which has no atmosphere. If there were no air molecules to scatter the light from the Sun, then the only light we would see would be from the stars, the planets, the Moon, and direct sunlight. The rest of the sky would be black.

24. If the atmosphere were 50% more dense, then sunlight (after passing through the atmosphere) would be much redder than it is now. As the atmosphere increased in density, more and more of the blue light would be scattered away in all directions, making the light that reaches the ground very red. Think of the color of a deep red sunset, but this might be the color even when the sun was at high elevations.

Responses to MisConceptual Questions

1. (*a*) The width of the fringes is proportional to the wavelength and inversely proportional to the slit spacing. Therefore, since red light has the longer wavelength, red light with small slit spacing will have the largest fringe width.

2. (*a*) At point A the green laser light is at a maximum. The next maximum (B) occurs when the path difference between the point and each of the two slits is equal to one full wavelength, or 530 nm. The minimum (halfway between points A and B) is the point where the difference in distances is equal to a half wavelength (or 265 nm).

3. (*c*) The index of refraction in the water droplets varies slightly with wavelength. Therefore, as sunlight passes through the droplets, the light of different frequencies (colors) refracts at slightly different angles.

4. (*a*) The cause of single-slit diffraction can be difficult to understand. The slit can be divided into a number of individual regions. If the path difference between each pair of regions and a point on the screen is a half wavelength, then destructive interference occurs at that point.

5. (*b*) It might be common to think that the lines are due to the inability of the eye to focus on the point. However, the lines are due to diffraction of the light as it passes through the very small opening between your fingers.

6. (*b*) The spreading out of the light is due to diffraction, with diffraction being significant when the slit size is about the same size as the wavelength of light. For visible light, the wavelength is about 5×10^{-7} m, which is on the order of the slit width. Therefore, the light will spread out wider than the slit. The height opening, which is 4 orders of magnitude larger than the wavelength, will have almost no discernible diffraction, so the light will remain about the same height.

7. (*d*) In Eq. 24–2a the angles of constructive interference are proportional to the wavelength and inversely proportional to the slit width. Therefore, if the slit width and wavelength are changed by the same factor, then the diffraction pattern will not change.

8. (*b*) A common misconception is that the center of the shadow will be the darkest. However, since the center of the disk is equidistant from all points on the edge of the disk, the diffracting light constructively interferes at the center, making a bright point. This bright point is independent of the wavelength of the light.

9. (*c*) It can be difficult to recognize the different scales of length involved with the diffraction of sound versus the diffraction of light. The sound waves of a person's voice have wavelengths about the same size as a doorway and therefore diffract easily around the corner. The wavelength of light is much smaller, approximately 10^{-7} m, so light does not diffract around the corner.

10. (*c*) The CD is made of thinly spaced grooves. As light reflects off of these grooves, the light interferes in the same way as when it passes through a diffraction grating. Note that the diffraction pattern is the same regardless of the data and/or security on the CD. The bit pattern does not determine the diffraction—just the presence of the thinly spaced grooves.

11. (*e*) If the film has regions of lower index of refraction on both sides of the film (or higher on both sides), then a phase shift will occur at one surface but not the other. In this case constructive interference will occur when the thickness is ¼ of a wavelength and destructive interference will occur when the thickness is ½ of a wavelength. If the film has a lower index on one side and a higher index on the other, then either no phase shift will occur at either surface or phase shifts will occur at both surfaces. In this case destructive interference occurs when the film has a thickness of ¼ wavelength and constructive interference occurs when the thickness is ½ of a wavelength. Since the indices of refraction are not specified for the surfaces of the film, none of the answers are always true.

12. (*b*) If two successive polarizers are perpendicular to each other, then light cannot get through. In case 1, the first two polarizers are perpendicular, and in case 2, the last two polarizers are perpendicular. In case 2, no successive polarizers are perpendicular, so some light can pass through.

Solutions to Problems

1. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. Apply this to the fifth order.

$$d \sin\theta = m\lambda \quad \rightarrow \quad \lambda = \frac{d \sin\theta}{m} = \frac{(1.8\times10^{-5}\ \text{m})\sin 8.6°}{5} = \boxed{5.4\times10^{-7}\ \text{m}}$$

$\boxed{2.}$ For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. Apply this to the third order ($m = 3$).

$$d \sin\theta = m\lambda \quad \rightarrow \quad d = \frac{m\lambda}{\sin\theta} = \frac{3(610\times10^{-9}\ \text{m})}{\sin 31°} = \boxed{3.6\times10^{-6}\ \text{m}}$$

$\boxed{3.}$ For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d \sin\theta = m\lambda \quad \rightarrow \quad d\frac{x}{\ell} = m\lambda \quad \rightarrow \quad x = \frac{\lambda m \ell}{d}$$

$$x_1 = \frac{\lambda m_1 \ell}{d};\ x_2 = \frac{\lambda(m+1)\ell}{d} \quad \rightarrow \quad \Delta x = x_2 - x_1 = \frac{\lambda(m+1)\ell}{d} - \frac{\lambda m \ell}{d} = \frac{\lambda \ell}{d}$$

$$\lambda = \frac{d\Delta x}{\ell} = \frac{(4.8\times10^{-5}\ \text{m})(0.085\ \text{m})}{6.50\ \text{m}} = 6.277\times10^{-7}\ \text{m} \approx \boxed{6.3\times10^{-7}\ \text{m}}$$

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{6.277\times10^{-7}\ \text{m}} = \boxed{4.8\times10^{14}\ \text{Hz}}$$

4. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad x = \frac{\lambda m\ell}{d};\quad x_1 = \frac{\lambda_1 m\ell}{d};\quad x_2 = \frac{\lambda_2 m\ell}{d} \quad\rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{(\lambda_2 - \lambda_1)m\ell}{d} = \frac{[(720-660)\times10^{-9}\ m](2)(1.0\ \text{m})}{(6.2\times10^{-4}\ \text{m})} = 1.935\times10^{-4}\ \text{m} \approx \boxed{0.2\ \text{mm}}$$

This justifies using the small angle approximation, since $x \ll \ell$.

5. For destructive interference, the path difference is as follows.

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda \quad\rightarrow\quad \sin\theta = \frac{\left(m+\frac{1}{2}\right)\lambda}{d} = \frac{\left(m+\frac{1}{2}\right)(4.5\ \text{cm})}{(7.5\ \text{cm})} = \left(m+\tfrac{1}{2}\right)(0.60),\quad m = 0, 1, 2, 3, \ldots$$

The angles for the first three regions of complete destructive interference are calculated.

$$\theta_0 = \sin^{-1}\left[\left(0+\tfrac{1}{2}\right)(0.60)\right] = \sin^{-1} 0.30 = 17.46° \approx \boxed{17°}$$
$$\theta_1 = \sin^{-1}\left[\left(1+\tfrac{1}{2}\right)(0.60)\right] = \sin^{-1} 0.90 = 64.16° \approx \boxed{64°}$$
$$\theta_2 = \sin^{-1}\left[\left(2+\tfrac{1}{2}\right)(0.60)\right] = \sin^{-1} 1.50 = \text{impossible}$$

There are only two regions of destructive interference, at $17°$ and $64°$.

6. The slit spacing and the distance from the slits to the screen are the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe ($m = 1$) relative to the central fringe. We indicate the lab laser with subscript 1 and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad x = \frac{\lambda m\ell}{d};\quad x_1 = \frac{\lambda_1\ell}{d};\quad x_2 = \frac{\lambda_2\ell}{d} \quad\rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = (632.8\ \text{nm})\frac{5.14\ \text{mm}}{5.00\ \text{mm}} = 650.52\ \text{nm} \approx \boxed{651\ \text{nm}}$$

7. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad d = \frac{\lambda m\ell}{x} = \frac{(680\times10^{-9}\ \text{m})(3)(2.8\ \text{m})}{38\times10^{-3}\ \text{m}} = \boxed{1.5\times10^{-4}\ \text{m}}$$

8. Using a ruler on Fig. 24–9a, the width of 18 fringes is found to be about 25 mm. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad \lambda = \frac{dx}{m\ell} = \frac{dx}{m\ell} = \frac{(1.7\times10^{-4}\ \text{m})(0.025\ \text{m})}{(18)(0.37\ \text{m})} = \boxed{6.4\times10^{-7}\ \text{m}}$$

$\boxed{9.}$ For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \quad\to\quad d\frac{x}{\ell} = m\lambda \quad\to\quad x = \frac{\lambda m \ell}{d} \quad\to$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{(633\times10^{-9}\text{ m})(3.3\text{ m})}{(6.8\times10^{-5}\text{ m})} = 0.0307\text{ m} = \boxed{3.1\text{ cm}}$$

10. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad\to\quad d\frac{x}{\ell} = m\lambda \quad\to\quad d = \frac{\lambda m\ell}{x} = \frac{(633\times10^{-9}\text{ m})(1)(5.0\text{ m})}{(0.35\text{ m})} = \boxed{9.0\times10^{-6}\text{ m}}$$

11. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{max}} = m\lambda, \quad m = 0, 1, 2, \cdots \quad\to\quad d\sin\theta_{\text{max}} = \left(m - \tfrac{1}{2}\right)\lambda, \quad m = 1, 2, \cdots$$

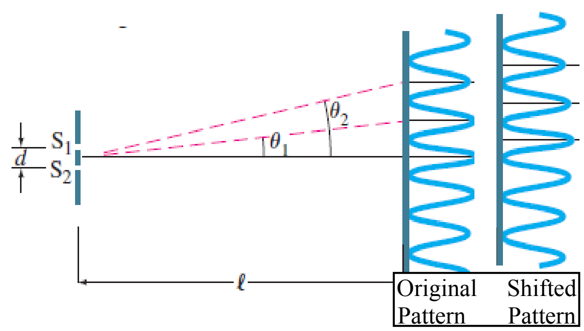
We could express the result as $d\sin\theta_{\text{max}} = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{min}} = \left(m + \tfrac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \cdots \quad\to\quad d\sin\theta_{\text{min}} = m\lambda, \quad m = 0, 1, 2, \cdots$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern}}$. There will be a dark central line instead of a bright central maximum. Every place there was a bright fringe will now have a dark line, and vice versa.

Figure 24–10 is reproduced here, and immediately to the right is the pattern that would appear from the situation described by this problem. Notice that maxima have changed to minima, and vice versa.



12. We equate the expression from Eq. 24–2a for the second-order blue light to Eq. 24–2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$d\sin\theta = m\lambda_{\text{b}} = (2)(480\text{ nm}) = 960\text{ nm}; \quad d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda, \quad m' = 0, 1, 2, \cdots$$

$$\left(m' + \tfrac{1}{2}\right)\lambda = 960\text{ nm} \quad m' = 0 \quad\to\quad \lambda = 1920\text{ nm}; \quad m' = 1 \quad\to\quad \lambda = 640\text{ nm}$$

$$m' = 2 \quad\to\quad \lambda = 384\text{ nm}$$

The only one visible is $\boxed{640\text{ nm}}$. 384 nm is near the low-wavelength limit for visible light.

13. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad x = \frac{\lambda m\ell}{d} \quad\rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{(544\times10^{-9}\text{ m})(4.0\text{ m})}{(1.0\times10^{-3}\text{ m})} = \boxed{2.2\times10^{-3}\text{ m}}$$

14. We have the same setup as in Example 24–3, so the two slits are 0.50 mm apart, and the screen is 2.5 m away. We use Eq. 24–21 with the small-angle approximation as described in that example, and solve for the wavelength.

$$d\sin\theta = d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad \lambda = \frac{dx}{m\ell} = \frac{(5.0\times10^{-4}\text{ m})(2.9\times10^{-3}\text{ m})}{(1)(2.5\text{ m})} = \boxed{5.8\times10^{-7}\text{ m}}$$

From Fig. 24–12, that wavelength is $\boxed{\text{yellow}}$.

15. An expression is derived for the slit separation from the data for the 480-nm light. That expression is then used to find the location of the maxima for the 650-nm light. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda \quad\rightarrow\quad d = \frac{\lambda m\ell}{x} = \frac{\lambda_1 m_1\ell}{x_1} \quad\rightarrow\quad x = \frac{\lambda m\ell}{d} \quad\rightarrow$$

$$x_2 = \frac{\lambda_2 m_2\ell}{\lambda_1 m_1\ell/x_1} = x_1\frac{\lambda_2 m_2}{\lambda_1 m_1} = (16\text{ mm})\frac{(650\text{ nm})(2)}{(480\text{ nm})(3)} = 14.44\text{ mm} \approx \boxed{14\text{ mm}}$$

16. The presence of the water changes the wavelength according to Eq. 24–1, so we must change λ to $\lambda_n = \lambda/n$. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda_n \quad\rightarrow\quad d\frac{x}{\ell} = m\lambda_n \quad\rightarrow\quad x = \frac{\lambda_n m\ell}{d};\quad x_1 = \frac{\lambda m_1\ell}{d};\quad x_2 = \frac{\lambda(m+1)\ell}{d} \quad\rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n\ell}{d} = \frac{\lambda\ell}{nd} = \frac{(470\times10^{-9}\text{ m})(0.400\text{ m})}{(1.33)(6.00\times10^{-5}\text{ m})}$$

$$= 2.356\times10^{-3}\text{ m} \approx \boxed{2.4\times10^{-3}\text{ m}}$$

17. To change the center point from constructive to destructive interference, the phase shift produced by inserting the plastic must be equivalent to half a wavelength. The wavelength of the light is shorter in the plastic than in the air, so the number of wavelengths in the plastic must be ½ greater than the number in the same thickness of air. The number of wavelengths in the distance equal to the thickness of the plate is the thickness of the plate divided by the appropriate wavelength.

$$N_{\text{plastic}} - N_{\text{air}} = \frac{t}{\lambda_{\text{plastic}}} - \frac{t}{\lambda} = \frac{tn_{\text{plastic}}}{\lambda} - \frac{t}{\lambda} = \frac{t}{\lambda}(n_{\text{plastic}} - 1) = \tfrac{1}{2} \quad\rightarrow$$

$$t = \frac{\lambda}{2(n_{\text{plastic}} - 1)} = \frac{680\text{ nm}}{2(1.60 - 1)} = \boxed{570\text{ nm}}$$

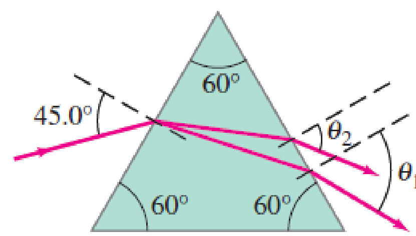
18. We find the speed of light from the index of refraction, $\upsilon = c/n$. We calculate the % difference using the red as the standard. Note that answers will vary due to differences in reading the graph.

$$\frac{(\upsilon_{\text{red}} - \upsilon_{\text{blue}})}{\upsilon_{\text{red}}} = \frac{\left[\left(\dfrac{c}{n_{\text{red}}}\right) - \left(\dfrac{c}{n_{\text{blue}}}\right)\right]}{\left(\dfrac{c}{n_{\text{red}}}\right)} = \frac{(n_{\text{blue}} - n_{\text{red}})}{n_{\text{blue}}} = \frac{(1.615 - 1.640)}{(1.640)} = -0.015 = \boxed{1.5\% \text{ less}}$$

$\boxed{19.}$ We find the angles of refraction in the glass from Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

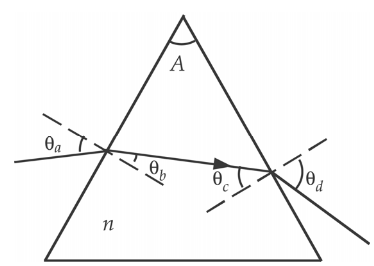
$$(1.00)\sin 65.00° = (1.4831)\sin\theta_{2,450} \rightarrow \theta_{2,450} = 37.67°$$
$$(1.00)\sin 65.00° = (1.4754)\sin\theta_{2,700} \rightarrow \theta_{2,700} = 37.90°$$
$$\theta_{2,700} - \theta_{2,450} = 37.90° - 37.67° = \boxed{0.23°}$$

20. We use Snell's law for the refraction at the first surface, entering the prism. The lower wavelength has the higher index of refraction. The indices of refraction are found from Fig. 24–14.



$$n_{\text{air}} \sin\theta_a = n\sin\theta_b;$$
$$(1.00)\sin 45° = (1.64)\sin\theta_{b,455} \rightarrow \theta_{b,455} = 25.54°$$
$$(1.00)\sin 45° = (1.62)\sin\theta_{b,642} \rightarrow \theta_{b,642} = 25.88°$$

We find the angle of incidence at the second surface, leaving the prism, from the second diagram.



$$(90° - \theta_b) + (90° - \theta_c) + A = 180°$$
$$\theta_{c,455} = A - \theta_{b,455} = 60.00° - 25.54° = 34.46°$$
$$\theta_{c,642} = A - \theta_{b,642} = 60.00° - 25.88° = 34.12°$$

Use Snell's law again for the refraction at the second surface.

$$n\sin\theta_c = n_{\text{air}} \sin\theta_d$$
$$(1.64)\sin 34.46° = (1.00)\sin\theta_{d,455} \rightarrow \theta_{d,455} = \boxed{\theta_1 = 68.1°}$$
$$(1.62)\sin 34.12° = (1.00)\sin\theta_{d,642} \rightarrow \theta_{d,642} = \boxed{\theta_2 = 65.3°}$$

The values may vary slightly from differences in reading the graph.

21. We use Eq. 24–3a to calculate the angular distance from the middle of the central peak to the first minimum. The width of the central peak is twice this angular distance.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{680\times10^{-9}\ \text{m}}{0.0425\times10^{-3}\ \text{m}}\right) = 0.9168°$$
$$\Delta\theta = 2\theta_1 = 2(0.9168) = 1.834° \approx \boxed{1.8°}$$

22. The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using this angle and Eq. 24–3a, we calculate the wavelength used.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(28.0°) = 14.0°$$

$$\sin\theta_1 = \frac{\lambda}{D} \to \lambda = D\sin\theta_1 = (2.60\times10^{-3}\ \text{mm})\sin(14.0°) = 6.29\times10^{-4}\ \text{mm} = \boxed{629\ \text{nm}}$$

23. We set the angle to the first minimum equal to half of the separation angle between the dark bands. We insert this angle into Eq. 24–3a to solve for the slit width.

$$\theta = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(51.0°) = 25.5°$$

$$\sin\theta = \frac{\lambda}{D} \quad\to\quad D = \frac{\lambda}{\sin\theta} = \frac{440\ \text{nm}}{\sin 25.5°} = 1022\ \text{nm} \approx \boxed{1.0\times10^{-6}\ \text{m}}$$

24. We find the angle to the first minimum using Eq. 24–3a. The distance on the screen from the central maximum is found using the distance to the screen and the tangent of the angle. The width of the central maximum is twice the distance from the central maximum to the first minimum.

$$\sin\theta_1 = \frac{\lambda}{D} \quad\to\quad \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{450\times10^{-9}\ \text{m}}{1.0\times10^{-3}\ \text{m}}\right) = 0.02578°$$

$$x_1 = \ell\tan\theta_1 = (6.0\ \text{m})\tan 0.02578° = 2.70\times10^{-3}\ \text{m}$$

$$\Delta x = 2x_1 = 2(2.70\times10^{-3}\ \text{m}) = 5.4\times10^{-3}\ \text{m} = \boxed{0.54\ \text{cm}}$$

25. We find the angle to the first minimum using Eq. 24–3a. The distance on the screen from the central maximum is found using the distance to the screen and the tangent of the angle. The width of the central maximum is twice the distance from the central maximum to the first minimum.

$$\sin\theta_1 = \frac{\lambda}{D} \quad\to\quad \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{558\times10^{-9}\ \text{m}}{3.48\times10^{-5}\ \text{m}}\right) = 0.9187°$$

$$x_1 = \ell\tan\theta_1 = (2.30\ \text{m})\tan 0.9187° = 3.688\times10^{-2}\ \text{m}$$

$$\Delta x = 2x_1 = 2(3.688\times10^{-2}\ \text{m}) = \boxed{7.38\times10^{-2}\ \text{m}}$$

26. (*a*) We use Eq. 24–3b, using m = 1, 2, 3,… to calculate the possible diffraction minima when the wavelength is 0.50 cm.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) = 18.2° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) = 38.7°$$

$$\theta_3 = \sin^{-1}\left(\frac{3\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) = 69.6° \qquad \theta_4 = \sin^{-1}\left(\frac{4\times0.50\ \text{cm}}{1.6\ \text{cm}}\right) \quad\to\quad \text{no solution}$$

There are three diffraction minima: 18°, 39°, and 70°.

(*b*) We repeat the process from part (*a*) using a wavelength of 1.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = 38.7^\circ \qquad \theta_2 = \sin^{-1}\left(\frac{2\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

The only diffraction minimum is at $\boxed{39^\circ}$.

(*c*) We repeat the process from part (*a*) using a wavelength of 3.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 3.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

There are $\boxed{\text{no diffraction minima}}$.

27. (*a*) There will be no diffraction minima if the angle for the first minimum is greater than 90°. We set the angle in Eq. 24–3a equal to 90° and solve for the slit width.

$$\sin\theta = \frac{\lambda}{D} \to D = \frac{\lambda}{\sin 90^\circ} = \boxed{\lambda}$$

(*b*) For no visible light to exhibit a diffraction minimum, the slit width must be equal to the shortest visible wavelength.

$$D = \lambda_{\text{min}} = \boxed{400\text{ nm}}$$

28. The first bright region occurs at $\sin\theta \approx \dfrac{3\lambda}{2D}$. We then find the distance on the screen from the central maximum by multiplying the distance to the screen by the tangent of the angle.

$$\theta_{\text{max}} = \sin^{-1}\left(\frac{3\lambda}{2D}\right) = \sin^{-1}\left[\frac{3(620\times 10^{-9}\text{ m})}{2(3.80\times 10^{-6}\text{ m})}\right] = 14.17^\circ$$

$$x = \ell\tan\theta_1 = (10.0\text{ m})\tan(14.17^\circ) = \boxed{2.52\text{ m}}$$

29. The angle from the central maximum to the first bright maximum is half the angle between the first bright maxima on either side of the central maximum. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 24–3b, setting $m = 3/2$, to calculate the slit width, *D*.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(32^\circ) = 16^\circ$$

$$D\sin\theta_{\text{m}} = m\lambda \to D = \frac{m\lambda}{\sin\theta_1} = \frac{(3/2)(633\text{ nm})}{\sin 16^\circ} = 3445\text{ nm} \approx \boxed{3.4\ \mu\text{m}}$$

30. (*a*) For vertical diffraction we use the height of the slit (1.5 μm) as the slit width in Eq. 24–3a to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is equal to twice this angle.

$$\sin\theta_1 = \frac{\lambda}{D} \to \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times 10^{-9}\text{ m}}{1.5\times 10^{-6}\text{ m}} = 31.3^\circ$$

$$\Delta\theta = 2\theta_1 = 2(31.3^\circ) \approx \boxed{63^\circ}$$

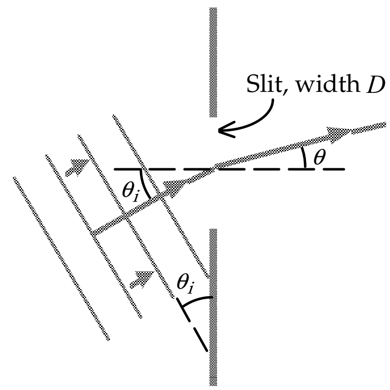
(*b*) To find the horizontal diffraction we use the width of the slit (3.0 μm) in Eq. 24–3a.

$$\sin\theta_1 = \frac{\lambda}{D} \to \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\text{ m}}{3.0\times10^{-6}\text{ m}} = 15.07°$$

$$\Delta\theta = 2\theta_1 = 2(15.07°) \approx \boxed{30°}$$

Note that the $30°$ has 2 significant figures.

31. The path difference between the top and bottom of the slit for the incident wave is $D\sin\theta_i$. The path difference between the top and bottom of the slit for the diffracted wave is $D\sin\theta$. Therefore, the net path difference is $D\sin\theta_i - D\sin\theta$. When $\theta = \theta_i$, the net path difference is 0, and there will be constructive interference. Thus there will be a central maximum at $\theta = 23.0°$. When the net path difference is equal to a multiple of the wavelength, there will be an even number of segments of the wave having a path difference of $\lambda/2$. We set the path difference equal to *m* (an integer) times the wavelength and solve for the angle of the diffraction minimum.



$$D\sin\theta_i - D\sin\theta = m\lambda \quad\to\quad \boxed{\sin\theta = \sin\theta_i - \frac{m\lambda}{D}, \quad m = \pm1, \pm2, \ldots}$$

From this equation we see that when $\theta_i = 28.0°$, the minima will be symmetrically distributed around a central maximum at 28.0°.

32. We use Eq. 24–4 to calculate the angle for the second-order maximum.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2(510\times10^{-9}\text{ m})}{1.35\times10^{-5}\text{ m}}\right) = \boxed{4.3°}$$

33. The slit separation, in cm, is the reciprocal of the number of slits per cm. We use Eq. 24–4 to find the wavelength.

$$d\sin\theta = m\lambda \to \lambda = \frac{d\sin\theta}{m} = \frac{\left(\frac{1}{3800}\text{ cm}\right)\sin 22.0°}{3} = 3.29\times10^{-5}\text{ cm} \approx \boxed{330\text{ nm}}$$

34. Because the angle increases with wavelength, to have a complete order we use the longest wavelength. We set the maximum angle to 90° to determine the largest integer *m* in Eq. 24–4. The slit separation, in cm, is the reciprocal of the number of lines per cm.

$$d\sin\theta = m\lambda \to m = \frac{d\sin\theta}{\lambda N} = \frac{\left(\frac{1}{7400}\text{ cm}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\sin 90°}{(700\times10^{-9}\text{ m})} = 1.93$$

Thus only $\boxed{\text{one full order}}$ (all the way out to red) can be seen on each side of the central white line, although the second order is almost fully visible.

We can also evaluate for the shortest wavelength.

$$d\sin\theta = m\lambda \rightarrow m = \frac{d\sin\theta}{\lambda N} = \frac{\left(\frac{1}{7400}\text{cm}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\sin 90^\circ}{(400\times 10^{-9}\text{ m})} = 3.38$$

Thus part of the third order will be visible—at least the violet part.

35. Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. Solve Eq. 24–4 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

$$d\sin\theta = m\lambda \quad\Rightarrow\quad d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \quad\Rightarrow\quad \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6^\circ}{\sin 53.2^\circ}(632.8\text{ nm}) = \boxed{556\text{ nm}}$$

36. The number of lines per cm is the reciprocal of the slit width in cm. Use Eq. 24–4.

$$d\sin\theta = m\lambda \rightarrow \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 15.0^\circ}{(3)(620\times 10^{-9}\text{ m})\left(\frac{100\text{ cm}}{\text{m}}\right)} = 1392\text{ lines/cm} \approx \boxed{1400\text{ lines/cm}}$$

37. We use Eq. 24–4 to calculate the wavelengths from the given angles. The slit separation, d, is the inverse of the number of slits per cm.

$$d\sin\theta = m\lambda \rightarrow \lambda = \frac{d\sin\theta}{m}$$

$$\lambda_1 = \left(\frac{1}{9800}\text{cm}\right)\sin 28.8^\circ = 4.92\times 10^{-5}\text{ cm} = 492\text{ nm} \approx \boxed{490\text{ nm}}$$

$$\lambda_2 = \left(\frac{1}{9800}\text{cm}\right)\sin 36.7^\circ = 6.10\times 10^{-5}\text{ cm} \approx \boxed{610\text{ nm}}$$

$$\lambda_3 = \left(\frac{1}{9800}\text{cm}\right)\sin 38.6^\circ = 6.37\times 10^{-5}\text{ cm} \approx \boxed{640\text{ nm}}$$

$$\lambda_4 = \left(\frac{1}{9800}\text{cm}\right)\sin 41.2^\circ = 6.72\times 10^{-5}\text{ cm} \approx \boxed{670\text{ nm}}$$

38. We find the first-order angles for the maximum and minimum wavelengths using Eq. 24–4, where the number of lines per centimeter is the reciprocal of the slit separation distance, in cm. Then we set the distance from the central maximum of the maximum and minimum wavelength equal to the distance to the screen multiplied by the tangent of the first-order angle. The width of the spectrum is the difference in these distances.

$$d\sin\theta = m\lambda \rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

$$\theta_1 = \sin^{-1}[(410\times 10^{-7}\text{ cm})(7800\text{ lines/cm})] = 18.65^\circ$$

$$\theta_2 = \sin^{-1}[(750\times 10^{-7}\text{ cm})(7800\text{ lines/cm})] = 35.80^\circ$$

$$\Delta x = x_2 - x_1 = \ell(\tan\theta_2 - \tan\theta_1) = (3.40\text{ m})(\tan 35.80^\circ - \tan 18.65^\circ) = \boxed{1.3\text{ m}}$$

39. We find the second-order angles for the maximum and minimum wavelengths using Eq. 24–4, where the slit separation distance is the inverse of the number of lines per cm. Subtracting these two angles gives the angular width.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

$$\theta_1 = \sin^{-1}[2(4.5\times10^{-7}\text{ m})(6.5\times10^{5}\text{ /m})] = 35.80^\circ$$

$$\theta_2 = \sin^{-1}[2(7.0\times10^{-7}\text{ m})(6.5\times10^{5}\text{ /m})] = 65.50^\circ$$

$$\Delta\theta = \theta_2 - \theta_1 = 65.50^\circ - 35.80^\circ = 29.7^\circ \approx \boxed{30^\circ}$$

Note that the answer has 2 significant figures.

40. We set the diffraction angles as one-half the difference between the angles on opposite sides of the center. Then we solve Eq. 24–4 for the wavelength, with d equal to the inverse of the number of lines per cm.

$$\theta_1 = \frac{\theta_r - \theta_\ell}{2} = \frac{26^\circ38' - (-26^\circ18')}{2} = 26^\circ28' = 26 + 28/60 = 26.47^\circ$$

$$\lambda_1 = d\sin\theta = \left(\frac{1}{9650}\text{cm}\right)\sin 26.47^\circ = 4.618\times10^{-5}\text{ cm} = \boxed{462\text{ nm}}$$

$$\theta_2 = \frac{\theta_{2r} - \theta_{2\ell}}{2} = \frac{41^\circ02' - (-40^\circ27')}{2} = 40^\circ44.5' = 40 + 44.5/60 = 40.74^\circ$$

$$\lambda_2 = \left(\frac{1}{9650}\text{cm}\right)\sin 40.74^\circ = 6.763\times10^{-5}\text{ cm} = \boxed{676\text{ nm}}$$

41. The maximum angle is 90°. The slit separation is the reciprocal of the line spacing.

$$d\sin\theta = m\lambda \to m = \frac{d\sin\theta}{\lambda} = \frac{\left(\frac{1}{6500}\text{cm}\right)\sin 90^\circ}{633\times10^{-7}\text{ cm}} = 2.43$$

Thus $\boxed{\text{the second order}}$ is the highest order that can be seen.

$\boxed{42.}$ We solve Eq. 24–4 for the slit separation width, d, using the given information. Then, setting $m = 3$, we solve for the angle of the third-order maximum.

$$\sin\theta = \frac{m\lambda}{d} \quad\to\quad d = \frac{m\lambda}{\sin\theta} = \frac{1(589\text{ nm})}{\sin 14.5^\circ} = 2352\text{ nm} = \boxed{2.35\ \mu\text{m}}$$

$$\theta_3 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times589\text{ nm}}{2352\text{ nm}}\right) = \boxed{48.7^\circ}$$

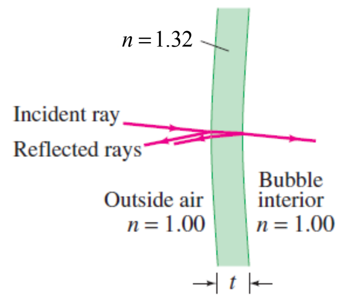
43. Because the angle of the diffracted light increases with wavelength, to have a full order use the largest wavelength, 700 nm. See Fig. 24–26 for an illustration. The maximum angle of diffraction is 90°. Use that angle with the largest wavelength to find the minimum slit separation. The reciprocal of the slit separation gives the number of slits per cm.

$$d\sin\theta = m\lambda \to d = \frac{m\lambda}{\sin\theta} = \frac{2(7.00\times10^{-7}\text{ m})}{\sin 90^\circ} = 1.40\times10^{-6}\text{ m} = 1.40\times10^{-4}\text{ cm}$$

$$\frac{1}{d} = \frac{1}{1.40\times10^{-4}\text{ cm}} = \boxed{7140\text{ slits/cm}}$$

We have assumed that the original wavelength is known to 3 significant figures.

44. From Example 24–11, we see that the thickness is related to the interference maximum wavelength by $t = \lambda/4n$.

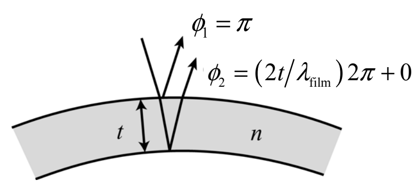


$$t = \lambda/4n \quad \rightarrow \quad \lambda = 4nt = 4(1.32)(120\text{ nm}) = 634\text{ nm} \approx \boxed{630\text{ nm}}$$

45. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals over the length of the plates.

$$\frac{21.5\text{ cm}}{24.5\text{ intervals}} = \boxed{0.878\text{ cm}}$$

46. (*a*) An incident wave that reflects from the outer surface of the bubble has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the inner surface of the bubble has a phase change due to the additional path length, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi$. For destructive interference with a minimum nonzero thickness of bubble, the net phase change must be π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = \pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \frac{\lambda}{2n} = \frac{480\text{ nm}}{2(1.33)} = \boxed{180\text{ nm}}$$

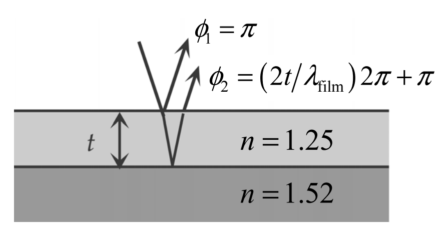
(b) For the next two larger thicknesses, the net phase change would be 3π and 5π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 3\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \frac{480\text{ nm}}{(1.33)} = \boxed{360\text{ nm}}$$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 5\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \frac{3}{2}\frac{480\text{ nm}}{(1.33)} = \boxed{540\text{ nm}}$$

(c) If the thickness were much less than one wavelength, then there would be very little phase change introduced by additional path length, so the two reflected waves would have a phase difference of about $\phi_1 = \pi$. This would produce destructive interference.

$\boxed{47.}$ An incident wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the coating has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference with a minimum nonzero thickness of coating, the net phase change must be 2π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = 2\pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right)$$

The lens reflects the most for $\lambda = 570$ nm. The minimum nonzero thickness occurs for $m = 1$:

$$t_{\text{min}} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\ \text{nm})}{2(1.25)} = \boxed{230\ \text{nm}}$$

Since the middle of the spectrum is being selectively reflected, the transmitted light will be stronger in the red and blue portions of the visible spectrum.

48. (*a*) When illuminated from above at point A, a light ray reflected from the air–oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil–water interface undergoes no phase shift. If the oil thickness at point A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path length traveled by a ray in the oil. Thus the light reflected from the two surfaces will destructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(*b*) From the discussion in part (*a*), the ray reflected from the air–oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray that reflects from the oil–water interface has no phase change due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference, the net phase change must be a multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \quad \to \quad t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for 580 nm, so we use $m = 1$. (The first location that yields constructive interference would be for $m = 0$.)

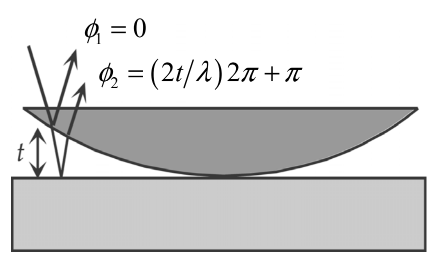
$$t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}} = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\frac{580\ \text{nm}}{1.50} = \boxed{290\ \text{nm}}$$

49. According to Section 24–8, at each surface approximately 4% of the light is reflected. So passing each surface of a lens results in only 96% transmission either into the lens (at the front surface) or on to the next piece of optics (at the second surface). So the light exiting a lens would experience two of these reductions and so would pass only 96% of 96% of the light incident on the lens. This is a factor of $(0.96)^2$ for each lens. Reducing to 50% would involve that factor for each lens. Let n be the number of lenses.

$$0.50 = [(0.96)^2]^n = (0.96)^{2n} \to \ln(0.50) = 2n\ln(0.96) \to n = \frac{\ln(0.50)}{2\ln(0.96)} = 8.49$$

It would take $\boxed{\text{nine lenses}}$ for the light to be reduced to 50% or less.

50. An incident wave that reflects from the convex surface of the lens has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the glass underneath the lens has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark rings), the net phase change must be an odd-integer multiple of π, so



$$\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi, \quad m = 0, 1, 2, \cdots.$$

Because $m = 0$ corresponds to the dark center, m represents the number of the ring.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi, \quad m = 0, 1, 2, \cdots \rightarrow$$

$$t = \tfrac{1}{2}m\lambda_{\text{air}} = \tfrac{1}{2}(35)(560\text{ nm}) = 9800\text{ nm} = \boxed{9.8\ \mu\text{m}}$$

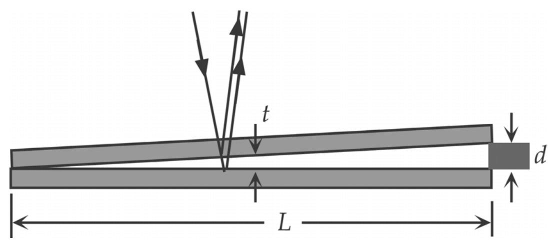
The thickness of the lens is the thickness of the air at the edge of the lens.

51. Since the wedge is now filled with water, we just use the wavelength in the water, as instructed in the problem statement. Since the index of refraction of the water is less than that of the glass, there are no new phase shifts introduced by reflections, so we can use the relationships developed in that example. We "count" the number of wavelengths at the position of the wire.

$$\frac{2t}{\lambda_n} = \frac{2(7.35\times10^{-6}\text{ m})}{6.00\times10^{-7}\text{ m}/1.33} = 32.6 \text{ wavelengths}$$

Thus there will be $\boxed{\text{33 dark bands}}$ across the plates, including the one at the point of contact.

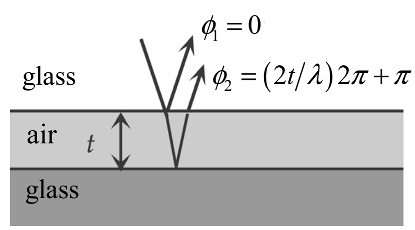
52. An incident wave that reflects from the second surface of the upper piece of glass has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the first surface of the second piece of glass has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark lines), the net phase change must be an odd-integer multiple of π, so $\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi, \quad m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the left edge of the diagram, the 24th dark line corresponds to $m = 23$. The 24th dark line also has a gap thickness of d.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda \quad \rightarrow$$

$$d = \tfrac{1}{2}(23)(670\text{ nm}) = 7705\text{ nm} \approx \boxed{7.7\ \mu\text{m}}$$

53. With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of $\phi_1 = 0$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even nonzero integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = 2m\pi \quad \rightarrow \quad t = \tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda, \quad m = 1, 2, \cdots.$$

The minimum thickness is with $m = 1$.

$$t_{\text{min}} = \tfrac{1}{2}(450\text{ nm})\left(1 - \tfrac{1}{2}\right) = 113\text{ nm} \approx \boxed{110\text{ nm}}$$

For destructive interference, the net phase change must be an odd-integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda, \quad m = 0, 1, 2, \cdots$$

The minimum nonzero thickness is $t_{\text{min}} = \tfrac{1}{2}(450\text{ nm})(1) = 225\text{ nm} \approx \boxed{230\text{ nm}}$

54. When illuminated from above, the light ray reflected from the air–oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil–water interface undergoes no phase shift due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference to occur, the net phase change must be a multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \quad \rightarrow \quad t = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

For $\lambda = 650$ nm, the possible thicknesses are as follows:

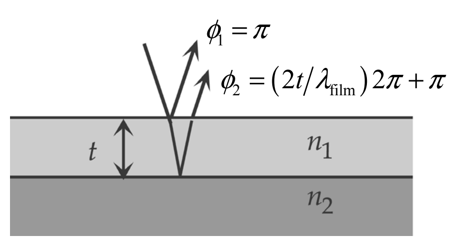
$$t_{650} = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\frac{650\text{ nm}}{1.50} = 108\text{ nm}, 325\text{ nm}, 542\text{ nm}, \cdots$$

For $\lambda = 390$ nm, the possible thicknesses are as follows:

$$t_{390} = \tfrac{1}{2}\left(m + \tfrac{1}{2}\right)\frac{390\text{ nm}}{1.50} = 65\text{ nm}, 195\text{ nm}, 325\text{ nm}, 455\text{ nm}, \cdots$$

The minimum thickness of the oil slick must be $\boxed{325\text{ nm}}$.

55. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to both the additional path length and a phase change of π on reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even nonzero integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{1\text{film}}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{1\text{film}}m_1 = \frac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1, \quad m_1 = 1, 2, 3, \cdots.$$

For destructive interference, the net phase change must be an odd-integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{2\text{film}}}\right)2\pi + \pi\right] - \pi = (2m_2+1)\pi \quad \to \quad t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1), \quad m_2 = 0, 1, 2, \cdots.$$

Set the two expressions for the thickness equal to each other.

$$\tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) \quad \to \quad \frac{2m_2+1}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(655\text{ nm})}{(525\text{ nm})} = 1.2476 \approx 1.25 = \frac{5}{4}$$

Thus we see that $m_1 = m_2 = 2,$ and the thickness of the film is

$$t = \tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{2}\left(\frac{655\text{ nm}}{1.36}\right)(2) = 481.6\text{ nm} \;\text{ or }\; t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) = \tfrac{1}{4}\left(\frac{525\text{ nm}}{1.36}\right)(5) = 482.5\text{ nm}$$

The average thickness, with 3 significant figures, is $\boxed{482\text{ nm}}$.

56. From the discussion in Section 24–9, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda,$ so corresponds to a mirror motion of $\tfrac{1}{2}\lambda.$ Let N be the number of fringe shifts produced by a mirror movement of $\Delta x.$

$$N = \frac{\Delta x}{\tfrac{1}{2}\lambda} \quad \to \quad \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(680)(589\times10^{-9}\text{ m}) = \boxed{2.00\times10^{-4}\text{ m}}$$

$\boxed{57.}$ From the discussion in Section 24–9, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda,$ so it corresponds to a mirror motion of $\tfrac{1}{2}\lambda.$ Let N be the number of fringe shifts produced by a mirror movement of $\Delta x.$

$$N = \frac{\Delta x}{\tfrac{1}{2}\lambda} \quad \to \quad \lambda = \frac{2\Delta x}{N} = \frac{2(1.25\times10^{-4}\text{ m})}{362} = 6.91\times10^{-7}\text{ m} = \boxed{691\text{ nm}}$$

58. From the discussion in Section 24–9, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda,$ so it corresponds to a mirror motion of $\tfrac{1}{2}\lambda.$ Let N be the number of fringe shifts produced by a mirror movement of $\Delta x.$ The thickness of the foil is the distance that the mirror moves during the 296 fringe shifts.

$$N = \frac{\Delta x}{\tfrac{1}{2}\lambda} \quad \to \quad \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(296)(589\times10^{-9}\text{ m}) = \boxed{8.72\times10^{-5}\text{ m}}$$

59. One fringe shift corresponds to an effective change in path length of $\lambda.$ The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a length d, then the number of wavelengths in vacuum is $\dfrac{d}{\lambda},$ and the (greater) number with the gas present is $\dfrac{d}{\lambda_{\text{gas}}} = \dfrac{n_{\text{gas}}d}{\lambda}.$ Because the light passes through the cavity twice, the number of fringe shifts is twice the difference in the number of wavelengths in the two media.

$$N = 2\left(\frac{n_{\text{gas}}d}{\lambda} - \frac{d}{\lambda}\right) = 2\frac{d}{\lambda}(n_{\text{gas}}-1) \quad \to \quad n_{\text{gas}} = \frac{N\lambda}{2d} + 1 = \frac{(158)(632.8\times10^{-9}\text{ m})}{2(1.155\times10^{-2}\text{ m})} + 1 = \boxed{1.004328}$$

The answer is quoted to more significant figures than is justified.

60. Use Eq. 24–5. Since the initial light is unpolarized, the intensity after the first polarizer will be half the initial intensity. Let the initial intensity be I_0.

$$I_1 = \tfrac{1}{2} I_0; \quad I_2 = I_1 \cos^2\theta = \tfrac{1}{2} I_0 \cos^2\theta \quad \rightarrow \quad \frac{I_2}{I_0} = \frac{\cos^2 72^\circ}{2} = \boxed{0.048}, \text{ or } 4.8\%$$

61. We assume that the light is coming from air to glass and use Eq. 24–6b.

$$\tan\theta_{\text{p}} = n_{\text{glass}} = 1.56 \quad \rightarrow \quad \theta_{\text{p}} = \tan^{-1} 1.56 = \boxed{57.3^\circ}$$

62. Let the initial intensity of the unpolarized light be I_0. The intensity after passing through the first Polaroid will be $I_1 = \tfrac{1}{2} I_0$. Then use Eq. 24–5.

$$I_2 = I_1 \cos^2\theta = \tfrac{1}{2} I_0 \cos^2\theta \quad \rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}}$$

(*a*) $\theta = \cos^{-1}\sqrt{\dfrac{2I_2}{I_0}} = \cos^{-1}\sqrt{\dfrac{2}{3}} = 35.3^\circ \approx \boxed{35^\circ}$

(*b*) $\theta = \cos^{-1}\sqrt{\dfrac{2I_2}{I_0}} = \cos^{-1}\sqrt{\dfrac{2}{10}} = 63.4^\circ \approx \boxed{63^\circ}$

63. For the first transmission, the angle between the light and the polarizer is 21.0°. For the second transmission, the angle between the light and the polarizer is 42.0°. Use Eq. 24–5 twice.

$$I_1 = I_0 \cos^2 21.0^\circ; \quad I_2 = I_1 \cos^2 42.0^\circ = I_0(\cos^2 21.0^\circ)(\cos^2 42.0^\circ) = 0.4813 I_0$$

Thus the transmitted intensity is $\boxed{48.1\%}$ of the incoming intensity.

64. For the first transmission, since the incoming light is unpolarized, the transmission is the same as if the angle were 45°. So $I_1 = \tfrac{1}{2} I_0$. The second polarizer is at an angle of 30° relative to the first one, so $I_2 = I_1 \cos^2 30^\circ$. And the third polarizer is at an angle of 60° relative to the second one, so $I_3 = I_2 \cos^2 60^\circ$. Combine these to find the percent that is transmitted out through the third polarizer.

$$I_3 = I_2 \cos^2 60^\circ = I_1 \cos^2 30^\circ \cos^2 60^\circ = \tfrac{1}{2} I_0 \cos^2 30^\circ \cos^2 60^\circ \quad \rightarrow$$
$$\frac{I_3}{I_1} = \tfrac{1}{2}\cos^2 30^\circ \cos^2 60^\circ = \tfrac{1}{2}\left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right) = \frac{3}{32} = 0.09375 \approx \boxed{9.4\%}$$

65. The polarizing angle θ_{p} is found using Eq. 24–6a, $\tan\theta_{\text{p}} = \dfrac{n_2}{n_1}$. For an oil–diamond interface, $\tan\theta_{\text{p}} = \dfrac{2.42}{1.43}$, which gives $\theta_{\text{p}} = 59.4^\circ$. The material $\boxed{\text{does NOT appear to be diamond}}$.

66. If I_0 is the intensity passed by the first Polaroid, then the intensity passed by the second will be I_0 when the two axes are parallel. To calculate a reduction to half intensity, we use Eq. 24–5.

$$I = I_0 \cos^2\theta = \tfrac{1}{2} I_0 \quad \rightarrow \quad \cos^2\theta = \tfrac{1}{2} \quad \rightarrow \quad \theta = \boxed{45^\circ}$$

67. The light is traveling from water to diamond. We use Eq. 24–6a.

$$\tan\theta_{\mathrm{p}}=\frac{n_{\text{diamond}}}{n_{\text{water}}}=\frac{2.42}{1.33}=1.82 \quad\rightarrow\quad \theta_{\mathrm{p}}=\tan^{-1}1.82=\boxed{61.2^\circ}$$

68. The critical angle exists when light passes from a material with a higher index of refraction (n_1) into a material with a lower index of refraction (n_2). Use Eq. 23–6.

$$\frac{n_2}{n_1}=\sin\theta_{\mathrm{C}}=\sin 58^\circ$$

To find Brewster's angle, use Eq. 24–6a. If light is passing from high index to low index, then we have the following:

$$\frac{n_2}{n_1}=\tan\theta_{\mathrm{p}}=\sin 58^\circ \quad\rightarrow\quad \theta_{\mathrm{p}}=\tan^{-1}(\sin 58^\circ)=40.3^\circ\approx\boxed{40^\circ}$$

If light is passing from low index to high index, then we have the following:

$$\frac{n_1}{n_2}=\tan\theta_{\mathrm{p}}=\frac{1}{\sin 58^\circ} \quad\rightarrow\quad \theta_{\mathrm{p}}=\tan^{-1}\left(\frac{1}{\sin 58^\circ}\right)=49.7^\circ\approx\boxed{50^\circ}$$

Note that both answers are given to 2 significant figures.

69. First case: the light is coming from water to air. Use Eq. 24–6a.

$$\tan\theta_{\mathrm{p}}=\frac{n_{\text{air}}}{n_{\text{water}}} \quad\rightarrow\quad \theta_{\mathrm{p}}=\tan^{-1}\frac{n_{\text{air}}}{n_{\text{water}}}=\tan^{-1}\frac{1.00}{1.33}=\boxed{36.9^\circ}$$

Second case: for total internal reflection, the light must also be coming from water into air. Use Eq. 23–6.

$$\sin\theta_{\mathrm{C}}=\frac{n_{\text{air}}}{n_{\text{water}}} \quad\rightarrow\quad \theta_{\mathrm{p}}=\sin^{-1}\frac{n_{\text{air}}}{n_{\text{water}}}=\sin^{-1}\frac{1.00}{1.33}=\boxed{48.8^\circ}$$

Third case: the light is coming from air to water. Use Eq. 24–6b.

$$\tan\theta_{\mathrm{p}}=n_{\text{water}} \quad\rightarrow\quad \theta_{\mathrm{p}}=\tan^{-1}n_{\text{water}}=\tan^{-1}1.33=\boxed{53.1^\circ}$$

Note that the two Brewster's angles add to give 90.0°.

70. When plane-polarized light passes through a sheet oriented at an angle θ, the intensity decreases according to Eq. 24–5, $I=I_0\cos^2\theta$. For the first sheet, with unpolarized light incident, we can treat $\theta=45^\circ$, $\cos^2\theta=\frac{1}{2}$. Then sheets two through six will each reduce the intensity by a factor of $\cos^2 35.0^\circ$. Thus we have the following:

$$I=I_0(\cos^2 45.0^\circ)(\cos^2 35.0)^5=\boxed{0.068I_0}$$

The transmitted intensity is 6.8% of the incident intensity.

71. Let I_0 be the initial intensity. Use Eq. 24–5 for both transmissions of the light.

$$I_1 = I_0 \cos^2 \theta_1; \quad I_2 = I_1 \cos^2 \theta_2 = I_0 \cos^2 \theta_1 \cos^2 \theta_2 = 0.35 I_0 \quad \rightarrow$$
$$\theta_1 = \cos^{-1}\left(\frac{\sqrt{0.35}}{\cos\theta_2}\right) = \cos^{-1}\left(\frac{\sqrt{0.35}}{\cos 48^\circ}\right) = \boxed{28^\circ}$$

72. (*a*) We apply Eq. 24–5 through the successive polarizers. The initial light is unpolarized. Each polarizer is then rotated 30.0° from the previous one.

$$I_1 = \tfrac{1}{2} I_0; \quad I_2 = I_1 \cos^2 \theta_2 = \tfrac{1}{2} I_0 \cos^2 \theta_2; \quad I_3 = I_2 \cos^2 \theta_3 = \tfrac{1}{2} I_0 \cos^2 \theta_2 \cos^2 \theta_3;$$
$$I_4 = I_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 \theta_2 \cos^2 \theta_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 30.0^\circ \cos^2 30.0^\circ \cos^2 30.0^\circ = \boxed{0.211 I_0}$$

(*b*) If the second polarizer is removed, then the angle between polarizers 1 and 3 is now 60.0°.

$$I_1 = \tfrac{1}{2} I_0; \quad I_3 = I_1 \cos^2 \theta_3 = \tfrac{1}{2} I_0 \cos^2 \theta_3;$$
$$I_4 = I_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 \theta_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 60.0^\circ \cos^2 30.0^\circ = 0.0938\, I_0$$

The same value would result by removing the third polarizer, because then the angle between polarizers 2 and 4 would be 60°. The intensity can be decreased by removing either the $\boxed{\text{second or third polarizer}}$.

(*c*) If both the $\boxed{\text{second and third polarizers}}$ are removed, then there are still two polarizers with their axes perpendicular, so no light will be transmitted.

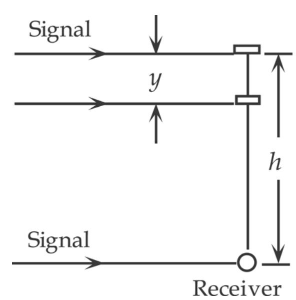
73. (*a*) For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d\frac{x}{\ell} = m\lambda \quad \rightarrow \quad x = \frac{\lambda m \ell}{d} \quad \rightarrow \quad \Delta x = \Delta m \frac{\lambda \ell}{d} \quad \rightarrow$$
$$d = \frac{\lambda \ell \Delta m}{\Delta x} = \frac{(5.0\times10^{-7}\text{ m})(5.0\text{ m})(1)}{(2.0\times10^{-2}\text{ m})} = 1.25\times10^{-4}\text{ m} \approx \boxed{1.3\times10^{-4}\text{ m}}$$

(*b*) For minima, we use Eq. 24–2b. The fourth-order minimum corresponds to $m = 3$, and the fifth-order minimum corresponds to $m = 4$. The slit separation, screen distance, and location on the screen are the same for the two wavelengths.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow \quad d\frac{x}{\ell} = \left(m + \tfrac{1}{2}\right)\lambda \quad \rightarrow \quad \left(m_A + \tfrac{1}{2}\right)\lambda_A = \left(m_B + \tfrac{1}{2}\right)\lambda_B \quad \rightarrow$$
$$\lambda_B = \lambda_A \frac{\left(m_A + \tfrac{1}{2}\right)}{\left(m_B + \tfrac{1}{2}\right)} = (5.0\times10^{-7}\text{ m})\frac{3.5}{4.5} = \boxed{3.9\times10^{-7}\text{ m}}$$

74. The wavelength of the signal is $\lambda = \dfrac{v}{f} = \dfrac{(3.00\times10^8\text{ m/s})}{(75\times10^6\text{ Hz})} = 4.00\text{ m}.$



(a) There is a phase difference between the direct and reflected signals from both the path difference, $\left(\dfrac{h}{\lambda}\right)2\pi,$ and the reflection, π.

The total phase difference is the sum of the two.

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \frac{(122\text{ m})}{(4.00\text{ m})}2\pi + \pi = 62\pi = 31(2\pi)$$

Since the phase difference is an integer multiple of 2π, the interference is $\boxed{\text{constructive}}$.

(*b*) When the plane is 22 m closer to the receiver, the phase difference is as follows.

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi = \left[\frac{(122\text{ m} - 22\text{ m})}{(4.00\text{ m})}\right]2\pi + \pi = 51\pi = \frac{51}{2}(2\pi)$$

Since the phase difference is an odd half-integer multiple of 2π, the interference is $\boxed{\text{destructive}}$.

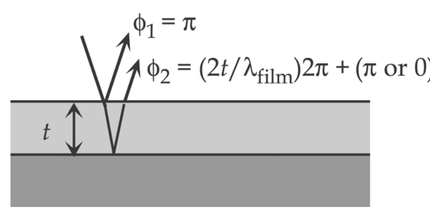
75. We find the angles for the first order from Eq. 24–2a, $d\sin\theta = m\lambda$. We are considering first order, so $m = 1$, and the slit separation (in meters) is the reciprocal of the lines per meter value.

$$\lambda_{\text{H}} = d\sin\theta_{\text{H}} = \frac{\sin\theta_{\text{H}}}{3.60\times10^5\text{ lines/m}} = 6.56\times10^{-7}\text{ m} \quad\rightarrow\quad \theta_{\text{H}} = \sin^{-1}0.2362 = \boxed{13.7^\circ}$$

$$\lambda_{\text{Ne}} = d\sin\theta_{\text{Ne}} = \frac{\sin\theta_{\text{Ne}}}{3.60\times10^5\text{ lines/m}} = 6.50\times10^{-7}\text{ m} \quad\rightarrow\quad \theta_{\text{Ne}} = \sin^{-1}0.2340 = \boxed{13.5^\circ}$$

$$\lambda_{\text{Ar}} = d\sin\theta_{\text{Ar}} = \frac{\sin\theta_{\text{Ar}}}{3.60\times10^5\text{ lines/m}} = 6.97\times10^{-7}\text{ m} \quad\rightarrow\quad \theta_{\text{Ar}} = \sin^{-1}0.2509 = \boxed{14.5^\circ}$$

76. With respect to the incident wave, the wave that reflects from the top surface of the material has a phase change of $\phi_1 = \pi$. If we assume that the film has an index less than that of the glass, then the wave that reflects from the glass has a phase change due to the additional path length and a phase change on reflection, for a total phase change of $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi = \left(\frac{2t}{\lambda/n}\right)2\pi + \pi$.



For destructive interference, the net phase change must be an odd-integer multiple of π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda/n}\right)2\pi + \pi - \pi = (2m-1)\pi \rightarrow n = \tfrac{1}{2}\left(\frac{\lambda}{t}\right)\left(m - \tfrac{1}{2}\right), \quad m = 1, 2, \ldots$$

The minimum index of refraction is found from $m = 1$.

$$n_{\text{min}} = \tfrac{1}{2}\left(\frac{675\text{ nm}}{125\text{ nm}}\right)\left(1 - \tfrac{1}{2}\right) = 1.35$$

This index is smaller than the typical index of glass (about 1.5), so choose a material with $\boxed{n = 1.35}$.

If we now assume that the film has an index greater than glass, then the wave that reflects from the glass has a phase change due to the additional path length and no phase change on reflection, for a total phase change of $\phi_2 = \left(\frac{2t}{\lambda/n}\right)2\pi + 0$.

Repeat the calculation from above for this new phase change.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda/n}\right)2\pi - \pi = (2m-1)\pi \rightarrow n = \frac{m\lambda}{2t}, \quad m = 1, 2, \ldots$$

$$n_{\min} = \frac{\lambda}{2t} = \frac{1}{2}\left(\frac{675 \text{ nm}}{125 \text{ nm}}\right) = 2.70$$

That is a very high index—higher than diamond. It probably is not realistic to look for this material.

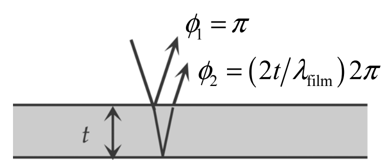
77. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 24–2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 24–7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d}; \quad x_1 = \frac{\lambda_1 m\ell}{d}; \quad x_2 = \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\Delta x = x_1 - x_2 = \frac{\lambda_1 m\ell}{d} - \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\lambda_2 = \lambda_1 - \frac{d\Delta x}{m\ell} = 650\times10^{-9} \text{ m} - \frac{(6.6\times10^{-4} \text{ m})(1.23\times10^{-3} \text{ m})}{2(2.40 \text{ m})} = 4.81\times10^{-7} \text{ m} \approx \boxed{480 \text{ nm}}$$

78. With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the bottom surface of the film has a phase change due to the additional path length and no phase change due to reflection, so



$\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + 0$. For constructive interference, the net phase change must be an integer multiple of 2π.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = 2\pi m \rightarrow t = \tfrac{1}{2}(m+\tfrac{1}{2})\lambda_{\text{film}} = \tfrac{1}{2}(m+\tfrac{1}{2})\frac{\lambda}{n_{\text{film}}}, \quad m = 0, 1, 2, \ldots$$

Evaluate the thickness for the two wavelengths.

$$t = \tfrac{1}{2}(m_1+\tfrac{1}{2})\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}(m_2+\tfrac{1}{2})\frac{\lambda_2}{n_{\text{film}}} \rightarrow$$

$$\frac{(m_2+\frac{1}{2})}{(m_1+\frac{1}{2})} = \frac{(m_1+\frac{3}{2})}{(m_1+\frac{1}{2})} = \frac{\lambda_1}{\lambda_2} = \frac{688.0 \text{ nm}}{491.4 \text{ nm}} = 1.40 = \frac{7}{5} \rightarrow 5(m_1+\tfrac{3}{2}) = 7(m_1+\tfrac{1}{2}) \rightarrow m_1 = 2$$

Thus $m_2 = 3$ and $m_1 = 2$. Evaluate the thickness with either value and the corresponding wavelength.

$$t = \tfrac{1}{2}(m_1+\tfrac{1}{2})\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}(\tfrac{5}{2})\frac{688.0 \text{ nm}}{1.58} = \boxed{544 \text{ nm}}; \quad t = \tfrac{1}{2}(m_2+\tfrac{1}{2})\frac{\lambda_2}{n_{\text{film}}} = \tfrac{1}{2}(\tfrac{7}{2})\frac{491.4 \text{ nm}}{1.58} = \boxed{544 \text{ nm}}$$

79. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order, using Eq. 24–4, by calculating the ratio of the sines for each angle. Since this ratio is greater than one, the maximum angle for the second order is larger than the minimum angle for the first order and the spectra overlap.

$$d\sin\theta = m\lambda \rightarrow \sin\theta = \left(\frac{m\lambda}{d}\right); \quad \frac{\sin\theta_2}{\sin\theta_3} = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} = \frac{2(700 \text{ nm})}{3(400 \text{ nm})} = 1.2$$

To determine which wavelengths overlap, we set this ratio of sines equal to one and solve for the second-order wavelength that overlaps with the shortest wavelength of the third order. We then repeat this process to find the wavelength of the third order that overlaps with the longest wavelength of the second order.

$$\frac{\sin\theta_2}{\sin\theta_3}=1=\frac{2\lambda_2/d}{3\lambda_3/d}=\frac{2\lambda_2}{3\lambda_3} \quad\rightarrow\quad \lambda_3=\frac{2}{3}\lambda_{2,\,\max}=\frac{2}{3}(700\text{ nm})=467\text{ nm}$$

$$\rightarrow\quad \lambda_2=\frac{3}{2}\lambda_{3,\min}=\frac{3}{2}(400\text{ nm})=600\text{ nm}$$

Therefore, the wavelengths $\boxed{\text{600 nm–700 nm of the second order}}$ overlap with the wavelengths $\boxed{\text{400 nm–467 nm of the third order}}$. Note that these wavelengths are independent of the slit spacing.

80. Because the measurements are made far from the antennas, we can use the analysis for the double slit. Use Eq. 24–2a for constructive interference, and Eq. 34–2b for destructive interference. The wavelength of the signal is $\lambda=\dfrac{v}{f}=\dfrac{(3.00\times10^8\text{ m/s})}{(90.3\times10^6\text{ Hz})}=3.322\text{ m}.$

For constructive interference, the path difference is a multiple of the wavelength.

$$d\sin\theta=m\lambda,\quad m=0,1,2,3,....;\quad\rightarrow\quad \theta=\sin^{-1}\frac{m\lambda}{d}$$

$$\theta_{\substack{1\\ \max}}=\sin^{-1}\frac{(1)(3.322\text{ m})}{9.0\text{ m}}=21.7^\circ\approx\boxed{22^\circ};\quad \theta_{\substack{2\\ \max}}=\sin^{-1}\frac{(2)(3.322\text{ m})}{9.0\text{ m}}=47.6^\circ\approx\boxed{48^\circ};$$

$$\theta_{\substack{3\\ \max}}=\sin^{-1}\frac{(3)(3.322\text{ m})}{9.0\text{ m}}=\text{impossible}$$

For destructive interference, the path difference is an odd multiple of half a wavelength.

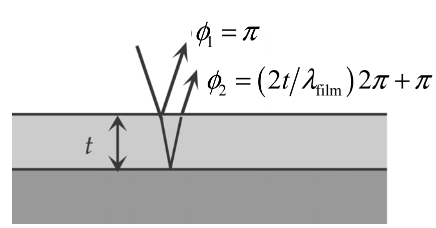
$$d\sin\theta=(m+\tfrac{1}{2})\lambda,\quad m=0,1,2,3,...;\quad\rightarrow\quad \theta=\sin^{-1}\frac{(m+\frac{1}{2})\lambda}{d}$$

$$\theta_{\substack{0\\ \max}}=\sin^{-1}\frac{(\frac{1}{2})(3.322\text{ m})}{9.0\text{ m}}=10.6^\circ\approx\boxed{11^\circ};\quad \theta_{\substack{1\\ \max}}=\sin^{-1}\frac{(\frac{3}{2})(3.39\text{ m})}{9.0\text{ m}}=34.4^\circ\approx\boxed{34^\circ};$$

$$\theta_{\substack{2\\ \max}}=\sin^{-1}\frac{(\frac{5}{2})(3.39\text{ m})}{9.0\text{ m}}=70.3^\circ\approx\boxed{70^\circ};\quad \theta_{\substack{3\\ \max}}=\sin^{-1}\frac{(\frac{7}{2})(3.39\text{ m})}{9.0\text{ m}}=\text{impossible}$$

These angles are applicable both above and below the midline and both to the left and to the right of the antennas.

81. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of $\phi_1=\pi.$ With respect to the incident wave, the wave that reflects from the glass $(n\approx1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and reflection, so $\phi_2=\left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi+\pi.$ For destructive interference, the net phase change must be an odd-integer multiple of $\pi.$



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = (2m+1)\pi \quad \rightarrow$$

$$t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}} = \tfrac{1}{4}(2m+1)\frac{\lambda}{n_{\text{film}}}, \quad m = 0, 1, 2, \ldots$$

The minimum thickness has $m = 0$, so $t_{\text{min}} = \tfrac{1}{4}\dfrac{\lambda}{n_{\text{film}}}$.

(*a*) For the blue light, $t_{\text{min}} = \tfrac{1}{4}\dfrac{(450\text{ nm})}{(1.38)} = 81.52\text{ nm} \approx \boxed{82\text{ nm}}$.

(*b*) For the red light, $t_{\text{min}} = \tfrac{1}{4}\dfrac{(720\text{ nm})}{(1.38)} = 130.4\text{ nm} \approx \boxed{130\text{ nm}}$.

82. The phase difference caused by the path difference back and forth through the coating must correspond to half a wavelength in order to produce destructive interference.

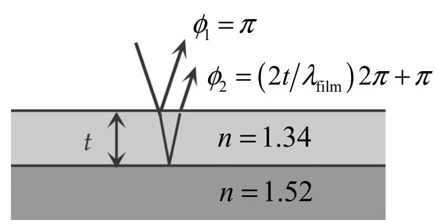
$$2t = \tfrac{1}{2}\lambda \quad \rightarrow \quad t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2\text{ cm}) = \boxed{0.5\text{ cm}}$$

83. We first find the angular half-width for the first order, using Eq. 24–3a, $\sin\theta = \dfrac{\lambda}{D}$. Since this angle is small, we may use the approximation that $\sin\theta \approx \tan\theta$. The width from the central maximum to the first minimum is given by $x = \ell\tan\theta$. That width is then doubled to find the width of the beam, from the first diffraction minimum on one side to the first diffraction minimum on the other side.

$$x = \ell\tan\theta = x\sin\theta$$

$$\Delta x = 2x = 2\ell\sin\theta = 2\ell\frac{\lambda}{D} = \frac{2(3.8\times10^{8}\text{ m})(633\times10^{-9}\text{ m})}{0.010\text{ m}} = \boxed{4.8\times10^{4}\text{ m}} = 48\text{ km}$$

84. With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even nonzero integer multiple of π.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = m2\pi \quad \rightarrow \quad t = \tfrac{1}{2}m\lambda_{\text{film}} = \tfrac{1}{2}m\frac{\lambda}{n_{\text{film}}}, \quad m = 1, 2, 3, \ldots$$

The minimum nonzero thickness occurs for $m = 1$.

$$t_{\text{min}} = \frac{\lambda}{2n_{\text{film}}} = \frac{643\text{ nm}}{2(1.34)} = \boxed{240\text{ nm}}$$

The answer has 3 significant figures.

85. Because the width of the pattern is much smaller than the distance to the screen, the angles from the diffraction pattern for this first order will be small. Thus we may make the approximation that $\sin\theta = \tan\theta$. We find the angle to the first minimum from the distances, using half the width of the full first-order pattern. Then we use Eq. 24–3b to find the slit width.

$$\tan\theta_{1\,\text{min}} = \tfrac{1}{2}\frac{(8.20\text{ cm})}{(315\text{ cm})} = 0.01302 = \sin\theta_{1\,\text{min}}$$

$$D\sin\theta = m\lambda \quad\rightarrow\quad D = \frac{m\lambda}{\sin\theta} = \frac{(1)(415\text{ nm})}{0.01302} = 3.188\times10^{4}\text{ nm} = \boxed{3.19\times10^{-5}\text{ m}}$$

86. If the original intensity is I_0, then the first polarizers will reduce the intensity to $I_1 = \tfrac{1}{2}I_0$. Each subsequent polarizer oriented at an angle θ to the preceding one will reduce the intensity as given by the equation $I_{\text{transmitted}} = I_{\text{incident}}\cos^2\theta$. For n polarizers (including the first one), $I_n = \tfrac{1}{2}I_0(\cos^2\theta)^{n-1}$. Solve for n such that the intensity is $\tfrac{1}{5}I_0$.

$$I_n = \tfrac{1}{5}I_0 = \tfrac{1}{2}I_0(\cos^2 10^\circ)^{n-1} \rightarrow 0.4 = (\cos^2 10^\circ)^{n-1} \rightarrow \ln(0.4) = (n-1)\ln(\cos^2 10^\circ) \rightarrow$$

$$n = 1 + \frac{\ln(0.4)}{\ln(\cos^2 10^\circ)} = 30.93$$

Thus $\boxed{31}$ polarizers are needed for the intensity to drop below $\tfrac{1}{5}$ of its original value.

87. The lines act like a reflection grating, similar to a CD. We assume that we see the first diffractive order, so $m = 1$. Use Eq. 24–4.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d = \frac{m\lambda}{\sin\theta} = \frac{(1)(480\text{ nm})}{\sin 56^\circ} = \boxed{580\text{ nm}}$$

88. We assume that the sound is diffracted when it passes through the doorway and find the angles of the minima from Eq. 24–3b.

$$\lambda = \frac{v}{f};\quad D\sin\theta = m\lambda = \frac{mv}{f} \quad\rightarrow\quad \theta = \sin^{-1}\frac{mv}{Df},\quad m = 1, 2, 3, \ldots$$

$$m = 1:\quad \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(1)(340\text{ m/s})}{(0.88\text{ m})(950\text{ Hz})} = \boxed{24^\circ}$$

$$m = 2:\quad \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(2)(340\text{ m/s})}{(0.88\text{ m})(950\text{ Hz})} = \boxed{54^\circ}$$

$$m = 3:\quad \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(3)(340\text{ m/s})}{(0.88\text{ m})(950\text{ Hz})} = \sin^{-1}1.22 = \text{impossible}$$

Thus the whistle would not be heard clearly at angles of $\boxed{24^\circ\text{ and }54^\circ\text{ on either side of the normal}}$.

89. We find the angles for the first order from Eq. 24–4. The slit spacing is the reciprocal of the number of lines per cm.

$$\theta_1 = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(4.4\times10^{-7}\text{ m})}{\left(\frac{1}{7200}\text{cm}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)} = 18.47^\circ;\quad \theta_2 = \sin^{-1}\frac{(1)(6.8\times10^{-7}\text{ m})}{\left(\frac{1}{7200}\text{cm}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)} = 29.31^\circ$$

The distances from the central white line on the screen are found using the tangent of the angle and the distance to the screen.

$$x_1 = \ell \tan\theta_1 = (2.5\text{ m})\tan 18.47° = 0.835\text{ m};\quad x_2 = \ell\tan\theta_2 = (2.5\text{ m})\tan 29.31° = 1.404\text{ m}$$

Subtracting these two distances gives the linear separation of the two lines.

$$x_2 - x_1 = 1.404\text{ m} - 0.835\text{ m} = 0.569\text{ m} \approx \boxed{0.6\text{ m}}$$

90. Because the angle increases with wavelength, to miss a complete order we use the smallest visible wavelength, 400 nm. The maximum angle is 90°. With these parameters we use Eq. 24–4 to find the slit separation, d. The inverse of the slit separation gives the number of lines per unit length.

$$d\sin\theta = m\lambda \quad\to\quad d = \frac{m\lambda}{\sin\theta} = \frac{2(400\text{ nm})}{\sin 90°} = \boxed{800\text{ nm}}$$

$$\frac{1}{d} = \frac{1}{800\times10^{-7}\text{ cm}} = \boxed{12{,}500\text{ lines/cm}}$$

91. We find the angles for the two first-order peaks from the distance to the screen and the distances along the screen to the maxima from the central peak.

$$\tan\theta_1 = \frac{x_1}{\ell} \quad\to\quad \theta_1 = \tan^{-1}\frac{x_1}{\ell} = \tan^{-1}\frac{(3.32\text{ cm})}{(72.0\text{ cm})} = 2.640°$$

$$\tan\theta_2 = \frac{x_2}{\ell} \quad\to\quad \theta_2 = \tan^{-1}\frac{x_2}{\ell} = \tan^{-1}\frac{(3.71\text{ cm})}{(72.0\text{ cm})} = 2.950°$$

Inserting the wavelength of yellow sodium light and the first-order angle into Eq. 24–4, we calculate the slit separation on the grating. Then, using the slit separation and the second angle, we calculate the wavelength of the second source. Finally, we take the inverse of the slit separation to determine the number of lines per cm on the grating.

$$d\sin\theta_1 = m\lambda_1 \quad\to\quad d = \frac{m\lambda_1}{\sin\theta_1} = \frac{1(589\text{ nm})}{\sin 2.64°} = 12{,}790\text{ nm}$$

$$\lambda_2 = \frac{d\sin\theta_2}{m} = (12{,}790\text{ nm})\sin 2.95° = \boxed{658\text{ nm}}$$

$$\frac{1}{d} = \frac{1\text{ line}}{12{,}790\times10^{-7}\text{ cm}} = \boxed{782\text{ lines/cm}}$$

92. Find the angles for the first order from Eq. 24–4, with $m = 1$. The slit spacing is the inverse of the lines/cm of the grating.

$$d = \frac{1}{7700\text{ lines/cm}}\times\frac{1\text{ cm}}{100\text{ cm}} = \frac{1}{7.7\times10^5}\text{ m};\quad d\sin\theta = m\lambda \quad\to\quad \theta = \sin^{-1}\frac{m\lambda}{d} \quad\to$$

$$\Delta\theta = \sin^{-1}\frac{\lambda_1}{d} - \sin^{-1}\frac{\lambda}{d} = \sin^{-1}\frac{656\times10^{-9}\text{ m}}{\left(\frac{1}{7.7\times10^5}\text{ m}\right)} - \sin^{-1}\frac{410\times10^{-9}\text{ m}}{\left(\frac{1}{7.7\times10^5}\text{ m}\right)} = 11.94° \approx \boxed{12°}$$

93. The $m=1$ brightness maximum for the wavelength of 1200 nm occurs at angle θ. At this same angle $m=2$, $m=3$, etc., brightness maxima may exist for other wavelengths. To find these wavelengths, use Eq. 24–4, keeping $d\sin\theta$ constant, and solve for the wavelengths of higher order.

$$d\sin\theta = m_1\lambda_1 = m\lambda_m \quad \rightarrow \quad \lambda_m = \frac{m_1\lambda_1}{m} = \frac{\lambda_1}{m}$$

$$\lambda_2 = \frac{1200\text{ nm}}{2} = 600\text{ nm} \quad \lambda_3 = \frac{1200\text{ nm}}{3} = 400\text{ nm} \quad \lambda_4 = \frac{1200\text{ nm}}{4} = 300\text{ nm}$$

Higher order maxima will have shorter wavelengths. Therefore, in the range from 360 nm to 2000 nm, the only wavelengths that have maxima at the angle θ are $\boxed{\text{600 nm and 400 nm}}$ besides the 1200 nm.

94. We use the relationships for Brewster's angle, Eq. 24–6b, for light coming from air to water.

$$\tan\theta_\text{p} = n \quad \rightarrow \quad \theta_\text{p} = \tan^{-1} n = \tan^{-1} 1.33 = 53.1^\circ$$

This is the angle from the normal, as seen in Fig. 24–48, so the angle above the horizontal is the complement of Brewster's angle, $90.0^\circ - 53.1^\circ = \boxed{36.9^\circ}$.

95. (*a*) Let the initial unpolarized intensity be I_0. The intensity of the polarized light after passing the first polarizer is $I_1 = \frac{1}{2}I_0$. Apply Eq. 24–5 to find the final intensity.

$$I_2 = I_1\cos^2\theta = I_1\cos^2 90^\circ = \boxed{0}.$$

(*b*) Now the third polarizer is inserted. The angle between the first and second polarizers is 56°, so the angle between the second and third polarizers is 34°. It is still true that $I_1 = \frac{1}{2}I_0$.

$$I_2 = I_1\cos^2 56^\circ = \tfrac{1}{2}I_0\cos^2 56^\circ; \quad I_3 = I_2\cos^2 34^\circ = \tfrac{1}{2}I_0\cos^2 56^\circ\cos^2 34^\circ \quad \rightarrow$$

$$\frac{I_3}{I_0} = \frac{\frac{1}{2}I_0\cos^2 56^\circ\cos^2 34^\circ}{I_0} = 0.1075 \approx \boxed{0.11}$$

(*c*) The two crossed polarizers, which are now numbers 2 and 3, will still not allow any light to pass through them if they are consecutive to each other. Thus, $\frac{I_3}{I_1} = \boxed{0}$.

96. The reduction being investigated is that which occurs when the polarized light passes through the second Polaroid. Let I_1 be the intensity of the light that emerges from the first Polaroid and I_2 be the intensity of the light after it emerges from the second Polaroid. Use Eq. 24–5.

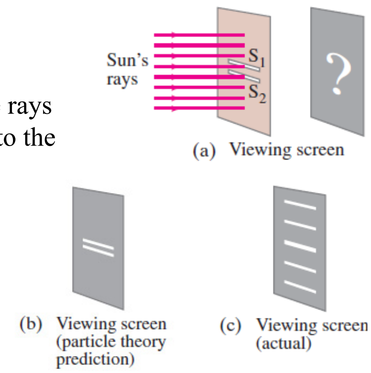
(a) $I_2 = I_1\cos^2\theta = 0.25I_1 \quad \rightarrow \quad \theta = \cos^{-1}\sqrt{0.25} = \boxed{60^\circ}$

(*b*) $I_2 = I_1\cos^2\theta = 0.10I_1 \quad \rightarrow \quad \theta = \cos^{-1}\sqrt{0.10} = \boxed{72^\circ}$

(*c*) $I_2 = I_1\cos^2\theta = 0.010I_1 \quad \rightarrow \quad \theta = \cos^{-1}\sqrt{0.010} = \boxed{84^\circ}$

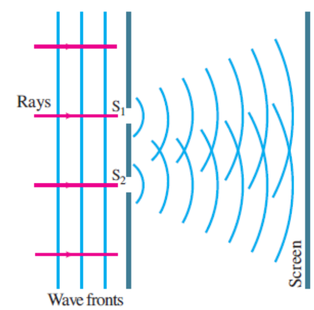
Solutions to Search and Learn Problems

1. Figure 24–5:



   - The light is traveling from left to right.
   - The wave crests are located at regular intervals along the rays of light. A line connecting the wave crests from one ray to the next would run vertically up and down the page.
   - The wave crests are not represented in this figure.
   - The advantage of this figure is to compare the particle theory prediction for what appears on the screen with the actual image.

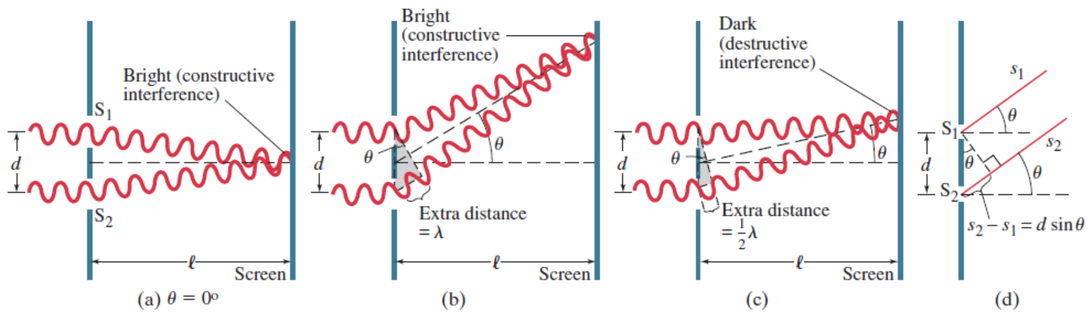
   Figure 24–6:



   - The light is traveling from left to right.
   - The wave crests are in the plane of the page, perpendicular to the direction of motion.
   - The wave crests are represented by the vertical lines to the left of the slits and barrier and by the curved lines to the right of the slits and barrier.
   - An advantage to this figure is that it shows the two slits as point sources due to diffraction and shows the overlap (implying interference) of the waves from the two slit sources when they reach the viewing screen.

   Figure 24–7:

   - The light is traveling from left to right.
   - The wave crests are in the plane of the page, perpendicular to the direction of motion.
   - The wave crests are the maxima of each sinusoidal representation of the two individual rays that are shown in part (a), (b), and (c). They are not shown in part (d).
   - An advantage is that this figure shows two rays in phase [(a) and (b)] or out of phase (c) when they reach the screen, illustrating constructive and destructive interference, respectively.



2. Similarities: Double-slit interference and single-slit diffraction are both caused by the interference of light. In double-slit interference the maxima and minima occur when the path difference between the two slits is equal to integral and half-integral wavelengths. In single-slit diffraction the minima occur when the path difference from different portions of the slit is equal to a half wavelength. The sine of the angle of diffraction in each case is proportional to the wavelength of light and inversely proportional to the slit separation or slit width.

Differences: In double-slit interference the central maximum has the same angular width as each of the other maxima. In single-slit diffraction the central maximum is much wider than any of the other maxima. Double-slit interference has equations for the location of both the maxima and minima, and the maxima are located exactly between the minima. Single-slit diffraction only has a (simple) equation for the location of the minima. The maxima are not located exactly at the midpoint between each minimum.

3. Fig. 24–16: Inside the raindrop the violet light is refracted at a slightly greater angle than the red light, because each wavelength of light has a slightly different index of refraction in water. The two colors then reflect off the back of the raindrop at different positions and exit the front of the raindrop with the violet closer to the horizontal than the red, with the other colors between these two extremes. For an observer to see the violet light, the rain droplets must be lower in the sky than the droplets that refract the red light to the observer. The observer sees the red light from droplets higher in the sky and the violet light from lower droplets.

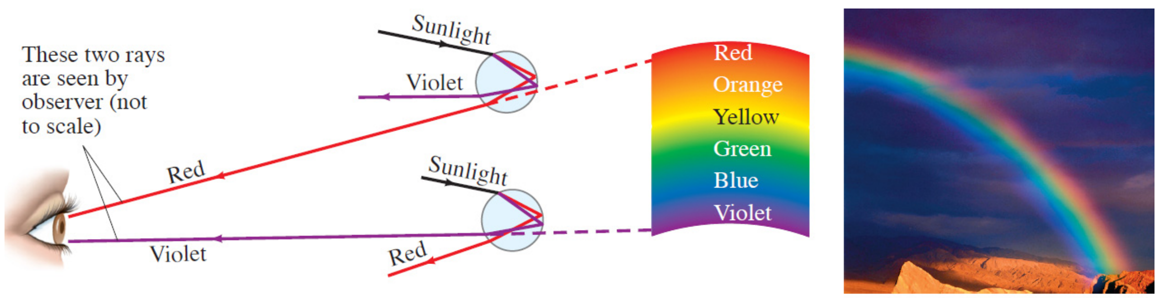
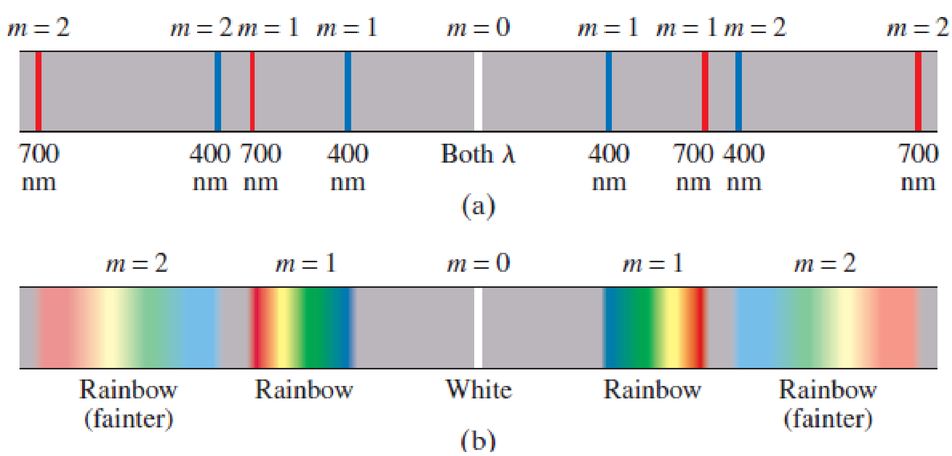
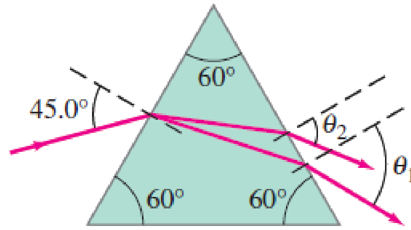


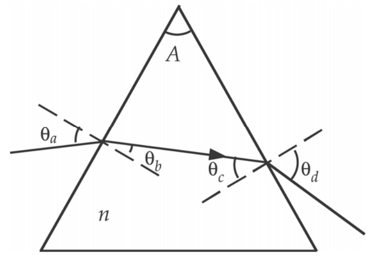
Fig. 24–26: In a diffraction grating the angle of diffraction is directly related to the wavelength. Since red light has a greater wavelength than violet light, it is diffracted more and appears farther from the central maximum than does the violet light.



4. Geometric optics is useful for studying reflection and refraction through lenses and mirrors whose physical dimensions are much larger than the wavelength of light. When dealing with objects (slits and thin films) whose dimensions are roughly the same order of magnitude as the wavelength of light we must use the more complicated wave nature of light. The physical characteristics that determines whether we must use the wave nature are the size of the object interacting with the light and the wavelength of the light.

5. (*a*) For the refraction at the first surface, apply Snell's law (twice) to find the exit angles. Take the indices of refraction from Fig. 24–14. Subscript 1 is the 420-nm light (with the higher index of refraction), and subscript 2 is the 650-nm light. Subscript *a* is the incident angle on the left side, subscript *b* is the angle after the first refraction, subscript *c* is the incident angle on the right side, and subscript *d* is the final refracted angle. For the entrance face:





$$n_{\text{air}} \sin\theta_a = n\sin\theta_b \quad \rightarrow$$
$$(1.00)\sin 45° = (1.585)\sin\theta_{b1} \quad \rightarrow \quad \theta_{b1} = 26.495°$$
$$(1.00)\sin 45° = (1.565)\sin\theta_{b1} \quad \rightarrow \quad \theta_{b1} = 26.861°$$

We find the angle of incidence at the second surface, leaving the prism, from the second diagram.

$$(90° - \theta_b) + (90° - \theta_c) + A = 180°$$
$$\theta_{c1} = A - \theta_{b1} = 60° - 26.495° = 33.505°$$
$$\theta_{c2} = A - \theta_{b2} = 60° - 26.861° = 33.139°$$

Use Snell's law again for the refraction at the second surface.

$$n\sin\theta_c = n_{\text{air}} \sin\theta_d$$
$$(1.585)\sin 33.505° = (1.00)\sin\theta_{d1} \quad \rightarrow \quad \theta_{d1} = \theta_1 = 61.037°$$
$$(1.565)\sin 33.139° = (1.00)\sin\theta_{d2} \quad \rightarrow \quad \theta_{d2} = \theta_2 = 58.820°$$
$$\boxed{\Delta\theta = 2.2°}$$

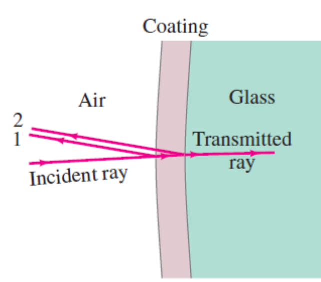
The values may vary due to differences in reading the graph in Fig. 24–14.

(*b*) We use Eq. 24–4 to determine the angles for each wavelength. The slit separation distance is the inverse of the number of slits per centimeter, and $m = 1$.

$$\sin\theta = \frac{m\lambda}{d} \quad \rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda}{d}\right) \qquad d = \frac{1}{5800 \text{ slits/cm}} = 1724 \text{ nm}$$
$$\theta_1 = \sin^{-1}\left(\frac{420 \text{ nm}}{1724 \text{ nm}}\right) = 14.1° \qquad \theta_2 = \sin^{-1}\left(\frac{650 \text{ nm}}{1724 \text{ nm}}\right) = 22.1°$$
$$\Delta\theta = 22.1° - 14.1° = \boxed{8.0°}$$

(*c*) One obvious advantage for using the diffraction grating is that much larger separation angles are possible: 8.0° versus 2.2° in this example. Also with a diffraction grating it is possible to observe second-order diffractions, which give a second, even larger separation angle that can be measured.

6. The incident ray intensity is equal to the sum of the intensities of the transmitted and reflected rays. Therefore, the transmitted ray will be a maximum when the reflected ray is a minimum, and the transmitted ray will be a minimum when the reflected ray is a maximum.

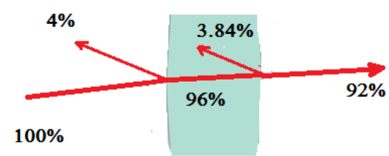


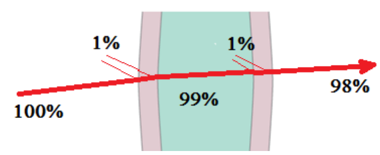
When the index of refraction of the film is less than the index of refraction of the glass $(n_{\text{film}} < n_{\text{glass}})$ then phase changes occur at both film surfaces and the transmission will be a minimum when $2n_{\text{film}}t = m\lambda$ and a maximum when $2n_{\text{film}}t = (m+\frac{1}{2})\lambda,$ where $m = 0, 1, 2, 3, \ldots.$

When the index of refraction of the film is greater than the index of refraction of the glass $(n_{\text{film}} > n_{\text{glass}}),$ a phase change only occurs at the air/film surface. The transmission will be a minimum when $2n_{\text{film}}t = (m+\frac{1}{2})\lambda$ and a maximum when $2n_{\text{film}}t = m\lambda,$ where $m = 0, 1, 2, 3, \ldots.$

In Section 24–8, it states that a single coating of film can reduce the reflection from about 4% to 1% of the incident light. The transmission then would have a minimum value of about 96% if the incident light was a maximum of 99%.

7. About 4% of light is reflected off each surface. Therefore, of 100% incident on the front surface, only 96% reaches the back surface. Four percent of this value or $0.04(0.96) = 3.84\%$ reflects off the back surface, so after one lens only $100\% - 4\% \quad -3.94\% = 92.2\%$ of the light is transmitted. For each lens only 92% of the light incident on the lens would transmit through the lens. After multiple lenses the transmission can become small. If there are six lenses (12 surfaces), then only $(0.96)^{12} = 0.61$ or 61% is transmitted.





To minimize this degradation, it is suggested to coat the lens with a thin film that will cause destructive interference for the reflected light at the center of the visible spectrum. Since the light cannot reflect, a greater percentage of light is transmitted. For the lens in air, this reduces the reflected light from 4% to 1%. For six lenses (12 surfaces), this increases the transmission percentage from 61% to 89%, a significant improvement.

25

Optical Instruments

Responses to Questions

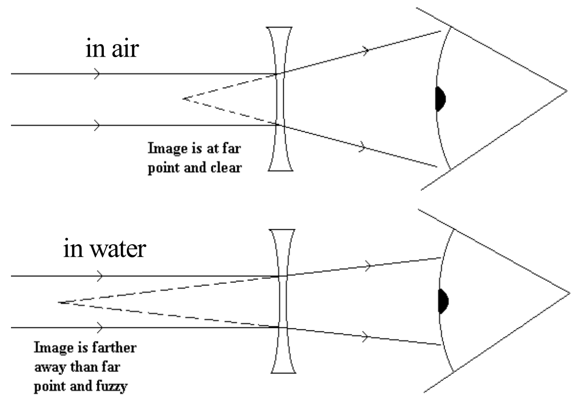
$\boxed{1.}$ The lens equation $\left(\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\right)$ says that as an object gets closer to the lens (d_o decreases) and the focal length of the lens remains constant, the image distance d_i must get larger to create a focused image. Since we cannot move the film or sensor inside the camera, the lens must be moved farther away from the film or sensor. This is unlike the human eye, where the focal length of the lens is changed as the object distance changes.

2. Stopping down a lens to a larger *f*-number means that the lens opening is smaller and only light rays coming through the central part of the lens are accepted. These rays are more nearly parallel than if a smaller *f*-number was used. These rays form smaller circles of confusion, which means that a greater range of object distances will be more sharply focused.

3. If a lens is stopped down too much (too big an *f*-number, so the iris opening is small), then diffraction will occur as light passes through the extremely small lens opening (the light will begin to "bend around" the edges of the stop). This diffraction will blur the image, especially around the edges of the film, and will lead to an image that is less sharp.

$\boxed{4.}$ As people get older, their eyes can no longer accommodate as well. It becomes harder for the muscles to change the shape of the lens, since the lens becomes less flexible with age. In general, people first lose the ability to see far objects, so they need corrective diverging lenses to move their "far point" back toward infinity. Then, as people get older, their near point increases and becomes greater than the ideal value of 25 cm. They may still need the diverging portion of their corrective lenses (kept as the upper part of the corrective lens) so they can have a far point at infinity, but now they also need a converging lens to move the near point back toward 25 cm for seeing close objects. Thus, as people get older, their far point is too close and their near point is too far. Bifocals (with two different focal lengths) can correct both of these problems.

5. No, a nearsighted person will not be able to see clearly if she wears her corrective lenses underwater. A nearsighted person has a far point that is closer than infinity and wears corrective lenses to bring the image of a faraway object to their far point so she can see it clearly. See the pair of diagrams. The object is at infinity. In air, the image is at the (relatively close) far point. If the person's eyes and glasses are underwater, and since the index of refraction of glass is closer to that of water than to that of air, the glasses will not bend the light as much as they did in the air. Therefore, the image of the faraway object will now be at a position that is beyond the person's far point. The image will now be out of focus.



6. This person is nearsighted. Diverging lenses are used to correct nearsightedness, and converging lenses are used to correct farsightedness. If the person's face appears narrower through the glasses, then the image of the face produced by the lenses is smaller than the face, virtual, and upright. Thus, the lenses must be diverging; therefore, the person is nearsighted.

7. To see far objects clearly, you want your eye muscles relaxed, which makes your lens relatively flat (large focal length). When you *f*-stop your eye down by closing your eyelids partially (squinting), you are only using the middle of your lens, where it is the most flat. This creates a smaller circle of confusion for the lens, which helps you see distant objects more clearly.

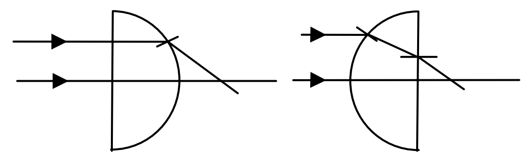
$\boxed{8.}$ The images formed on our retinas are real, so they are inverted. The implication of this is that our brains must flip this inverted image for us so that we can see things upright.

9. All light entering the camera lens while the shutter is open contributes to a single picture. If the camera is moved while the shutter is open, then the position of the image on the film moves. The new image position overlaps the previous image position, causing a blurry final image. With the eye, new images are continuously being formed by the nervous system, so images do not "build up" on the retina and overlap with each other. Your brain "refreshes" the image from the retina about 30 times a second. In other words, your brain and your retina work together in a manner similar to how a motion picture camera and film work together, which is not at all like how a still camera and film work together.

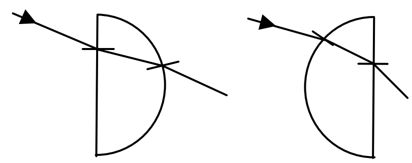
10. Both reading glasses and magnifiers are converging lenses. A magnifier, generally a short focal length lens, is typically used by adjusting the distance between the lens and the object so that the object is exactly at or just inside the focal point. An object exactly at the focal point results in an image that is at infinity and can be viewed with a relaxed eye. If the lens is adjusted so that it focuses the image at the eye's near point, then the magnification is slightly greater. The lenses in reading glasses typically are a fixed distance from the eye. They are used when the near point is too far away to be convenient—such as when reading a book. These lenses cause the rays from a nearby object to converge somewhat before they reach the eye, allowing the eye to focus on an object that is inside the near point. The focal length of the lens needed for reading glasses will depend on the individual eye. For both reading glasses and magnifiers, the lenses allow the eye to focus on an object that is located closer than the near point.

11. The glasses that near-sighted people wear are designed to help the viewing of distant objects by moving the focus point farther back in the eye. Near-sighted eyes in general have good vision for close objects whose focus point already lies on the retina. They can actually see close objects better without the corrective lenses. The near point of near-sighted people can be very close to their eye, so they bring small objects close to their eyes in order to see them most clearly.

12. The curved surface should face the object. If the flat surface faces the object and the rays come in parallel to the optical axis, then no bending will occur at the first surface and all the bending will occur at the second surface. Bending at the two surfaces will clearly not be equal in this case. The bending at the two surfaces may be equal (or at least less different) if the curved surface faces the object.



If the parallel rays from the distant object come in above or below the optical axis with the flat side toward the object, then the first bending is actually away from the axis. In this case also, bending at both surfaces can be equal (or at least less different) if the curved side of the lens faces the object.



13. Chromatic aberrations in lenses occur due to dispersion, which is when different colors (or wavelengths) of light are bent different amounts due to the fact that the index of refraction of most materials varies with wavelength. Thus when light passes through a lens, not all of the different colors come out of the lens at the exact same angle, and they are all focused at different positions. A mirror, in contrast, reflects light off of a smooth metallic surface. This surface reflects all different colors of light at the exact same angle, thus there is no refraction, no dispersion, and no chromatic aberrations. Of course, in most mirrors, there is a piece of glass that covers the metallic reflector. Since the two faces of this piece of glass are parallel to each other, even though refraction and dispersion take place inside the piece of glass, when the light emerges from the parallel face, all of the different colors are once again going in the same direction, and there is no dispersion and no chromatic aberration.

14. A poor-quality, inexpensive lens will not be correctly shaped to fix chromatic aberrations. Thus, the colors you see around the edges of these lenses are from all the different colors of light focusing at different points, instead of all being focused at the same point.

15. Spherical aberrations can be present in a simple lens. To correct this in a simple lens, usually many lenses are used together in combination to minimize the bending at each of the surfaces. Your eye minimizes spherical aberrations by bending the light at many different interfaces as it makes its way through the different parts of the eye (cornea, aqueous humor, lens, vitreous humor, etc.), each with their own n. Also, the cornea is less curved at the edges than it is at the center, and the lens is less dense at the edges than at the center, both of which reduce spherical aberrations in the eye since they cause the rays at the outer edges to be bent less strongly. Curvature of field occurs when the focal plane is not flat. Our curved retina helps with this distortion, whereas a flat piece of film in a camera, for example, wouldn't be able to fix this. Distortion is a result of the variation of the magnification at different distances from the optical axis. This is most common in wide-angle lenses, where it must be corrected for. This is compensated in the human eye because it is a very small lens and our retina is curved. Chromatic aberrations are mostly compensated for in the human eye because the lens absorbs shorter wavelengths and the retina is not very sensitive to most blue and violet wavelengths where most chromatic aberrations occur.

16. The Rayleigh criterion gives us the resolution limit for two objects a distance D apart when using light of wavelength λ: $\theta = \frac{1.22\lambda}{D}$, which can be interpreted as the smaller the angle, the better the resolution. Looking at the two wavelengths given, $\theta_{\text{blue}} = \frac{1.22(450\text{ nm})}{D}$ and $\theta_{\text{red}} = \frac{1.22(700\text{ nm})}{D}$,

we have $\frac{\theta_{\text{blue}}}{\theta_{\text{red}}} = \frac{\frac{1.22(450\text{ nm})}{D}}{\frac{1.22(700\text{ nm})}{D}} = \frac{450}{700} = 0.64.$ This could be expressed as saying that the resolution with blue light is $\frac{1}{0.64} = 1.56$ times the resolution with red light.

17. No. The resolving power of a lens is on the order of the wavelength of the light being used, so it is not possible to resolve details smaller than the wavelength of the light. Atoms have diameters of about 10^{-8} cm, and the wavelength of visible light is on the order of 10^{-5} cm.

18. Violet light would give the best resolution in a microscope, because according to the Rayleigh criterion, a smaller wavelength gives better resolution. Violet light has the shortest wavelength of visible light.

19. For both converging and diverging lenses, the focal point for violet light is closer to the lens than the focal point for red light. The index of refraction for violet light is slightly greater than that for red light for glass, so the violet light bends more, resulting in a focal length of a smaller magnitude.

20. Radiotelescopes are large primarily because the wavelength of radio waves is large—on the order of meters to hundreds of meters. To get better resolution with a large wavelength, the Rayleigh criterion says that a large aperture is needed. Making such large optical telescopes is not done for several reasons: (i) The theoretical resolution obtained by such a large aperture would not be achievable because of atmospheric turbulence that does not affect radiotelescopes; (ii) the much smaller wavelength of light means that the mirror has to be more perfectly shaped than the radiotelescope, and that is very difficult to do with huge mirrors. Some large-aperture optical telescopes (such as the "Very Large Telescope" in Chile) are now being made with multiple small mirrors combined to provide some of the capabilities of a single large mirror.

Responses to MisConceptual Questions

1. (*d*) An object at infinity would produce an image at the focal point. Objects closer than infinity produce images that are farther from the lens than the focal point.

2. (*d*) Mega- is the prefix meaning million. A pixel is a single point on the image. Therefore, a megapixel is a million light-sensitive spots on the detector.

3. (*b*) A nearsighted person cannot see objects clearly beyond her far point. Therefore, glasses create images of distant objects at the observer's far point.

4. (*a*) If the distance from the lens to retina is shorter than normal, then the lens must contract to focus distant objects. It must contract even more to view closer objects. Since it is already partially contracted to view far objects, it will not be able to sufficiently contract to view nearby objects.

5. (*a*) Because people perceive objects as upright, they may think that the image is upright. However, when a converging lens creates a real image, the image is inverted.

6. (*a*) A nearsighted person's retina is farther from the front of the eye than usual. As such, a diverging lens is needed to make the image focus farther from the front of the eye.

7. (*c*) A common misconception is that a magnifying glass makes the object appear to be closer since it appears larger. A magnifying glass creates an enlarged image outside the observer's near point, when the object is very close to the eye (within the near point). The angular magnification of that image is what the observer perceives as larger.

8. (*d*) The limit on magnification using visible light is not due to the lenses, but due to the nature of light. Diffraction limits resolution to the wavelength of the light used. For visible light the wavelength is about 500 nm. Crystal structures are much smaller (1–10 nm) in size and require X-rays to resolve their structure.

9. (*e*) The limit on magnification of visible light is not due to the lenses, but to the nature of light. A magnification of $3000\times$ would magnify objects whose size was on the order of, or smaller than, the wavelength of light. The effects of diffraction prevent clear images of these objects at that great of a magnification.

10. (*a*) The resolving power is inversely proportional to the wavelength of the light. Of the four types of light listed, ultraviolet light has the smallest wavelength and therefore the greatest resolving power.

11. (*a*) Red light has a longer wavelength than blue light and therefore will not have a better resolving power, since resolving power is inversely proportional to the wavelength. The resolving power is related to the angular separation of the objects. Two objects separated by the same distance would have a larger angular separation when they are closer to the observer and therefore a better resolution when they are closer to the observer. Objects that are farther apart from each other are easier to resolve. Therefore, the only correct answer is that the larger the lens diameter, the better the resolving power.

12. (*d*) The *f*-stop value determines the amount of light entering the camera and therefore affects how dark the image is. It does not affect the focus. Since the focal length of the lens is fixed, when the object distance increases, the image distance must decrease. Therefore, the lens must be moved closer to the sensor or film.

13. (*b*, *d*) When the object is placed between the lens and the focal point, the converging lens produces a magnified, upright, virtual image. When the object is just outside the focal length, the lens produces a magnified, inverted, real image. When the object is far from the lens, the lens produces a reduced, inverted, real image. Therefore, the image from a converting lens could be larger or smaller than the object, and the image can be inverted or upright.

Solutions to Problems

1. As implied in Example 25–2, the exposure is proportional to the product of the lens opening area and the exposure time, with the square of the *f*-stop number inversely proportional to the lens opening area. Setting the exposures equal for both exposure times, we solve for the needed *f*-stop number.

$$t_1 \text{ Exposure} = A_1t_1 = A_2t_2 \quad \rightarrow \quad t_1(f\text{-stop}_1)^{-2} = t_2(f\text{-stop}_2)^{-2} \quad \rightarrow$$

$$f\text{-stop}_2 = f\text{-stop}_1\sqrt{\frac{t_2}{t_1}} = 16\sqrt{\frac{1/400\text{ s}}{1/100\text{ s}}} = 8 \text{ or } \boxed{f/8}$$

2. We find the *f*-number from $f\text{-stop} = f/\text{D}$.

$$f\text{-stop} = \frac{f}{D} = \frac{(17\text{ cm})}{(6.0\text{ cm})} = \boxed{\frac{f}{2.8}}$$

3. From the definition of the *f*-stop, we have $f\text{-stop} = \frac{f}{D}$, so $D = \frac{f}{f\text{-stop}}$.

$$D_{1.4} = \frac{65\text{ mm}}{1.4} = 46\text{ mm}; \quad D_{22} = \frac{65\text{ mm}}{22} = 3.0\text{ mm}$$

Thus the range of diameters is $\boxed{3.0\text{ mm} \le D \le 46\text{ mm}}$.

4. As discussed in Example 25–2, to maintain an exposure, the product of the area of the lens opening and the exposure time must be the same. The area is inversely proportional to the *f*-stop value squared. Thus exposure time divided by $(f\text{-stop})^2$ must be constant.

$$\frac{t_1}{(f\text{-stop}_1)^2} = \frac{t_2}{(f\text{-stop}_2)^2} \rightarrow t_2 = t_1 \frac{(f\text{-stop}_2)^2}{(f\text{-stop}_1)^2} = (\tfrac{1}{500}\text{ s})\frac{11^2}{5.6^2} = 7.72\times10^{-3}\text{ s} \approx \boxed{\tfrac{1}{125}\text{ s}}$$

5. Since the object height is equal to the image height, the magnification is –1. We use Eq. 23–9 to obtain the image distance in terms of the object distance. Then we use this relationship with Eq. 23–8 to solve for the object distance.

$$m = -1 = -\frac{d_\text{i}}{d_\text{o}} \rightarrow d_\text{i} = d_\text{o}$$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{o}} = \frac{2}{d_\text{o}} \rightarrow d_o = 2f = 2(55\text{ mm}) = \boxed{110\text{ mm}}$$

The distance between the object and the film is the sum of the object and image distances.

$$d = d_\text{o} + d_\text{i} = d_\text{o} + d_\text{o} = 2d_\text{o} = 2(110\text{ mm}) = \boxed{220\text{ mm}}$$

$\boxed{6.}$ The image distance is found from Eq. 23–9 and then the focal length from Eq. 23–8. The image is inverted.

$$m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}} \rightarrow d_\text{i} = -d_\text{o}\frac{h_\text{i}}{h_\text{o}} = -(65\text{ m})\frac{(-24\text{ mm})}{(34\text{ m})} = 45.9\text{ mm}$$

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f} \rightarrow f = \frac{d_\text{o}d_\text{i}}{d_\text{o}+d_\text{i}} = \frac{(65\text{ m})(0.0459\text{ m})}{65\text{ m} - 0.0459\text{ m}} = 0.0459\text{ m} \approx \boxed{46\text{ mm}}$$

The object is essentially at infinity, so the image distance is equal to the focal length.

7. We calculate the range object distances from Eq. 23–8 using the given focal length and maximum and minimum image distances.

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \rightarrow d_{\text{o, min}} = \frac{fd_{\text{i, max}}}{d_{\text{i,max}} - f} = \frac{(200.0\text{ mm})(208.2\text{ mm})}{208.2\text{ mm} - 200.0\text{ mm}} = 5078\text{ mm} \approx 5.1\text{ m}$$

$$d_{\text{o,max}} = \frac{fd_{\text{i,min}}}{d_{\text{i, min}} - f} = \frac{(200.0\text{ mm})(200.0\text{ mm})}{200.0\text{ mm} - 200.0\text{ mm}} = \infty$$

Thus the range of object distances is $\boxed{5.1\text{ m} \le d_\text{o} < \infty}$.

8. (*a*) Because the Sun is very far away, the image will be at the focal point, or $d_i = f$. We find the magnitude of the size of the image using Eq. 23–9, with the image distance equal to 28 mm.

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o} \quad \rightarrow \quad |h_i| = \frac{h_o d_i}{d_o} = \frac{(1.4\times10^6 \text{ km})(28 \text{ mm})}{1.5\times10^8 \text{ km}} = \boxed{0.26 \text{ mm}}$$

(*b*) We repeat the same calculation with a 50-mm image distance.

$$|h_i| = \frac{(1.4\times10^6 \text{ km})(50 \text{ mm})}{1.5\times10^8 \text{ km}} = \boxed{0.47 \text{ mm}}$$

(*c*) Again, with a 135-mm image distance.

$$|h_i| = \frac{(1.4\times10^6 \text{ km})(135 \text{ mm})}{1.5\times10^8 \text{ km}} = \boxed{1.3 \text{ mm}}$$

(*d*) The equations show that image height is directly proportional to focal length. Therefore, the relative magnifications will be the ratio of focal lengths.

$$\frac{28 \text{ mm}}{50 \text{ mm}} = \boxed{0.56\times} \text{ for the 28-mm lens;} \qquad \frac{135 \text{ mm}}{50 \text{ mm}} = \boxed{2.7\times} \text{ for the 135-mm lens}$$

9. We calculate the maximum and minimum image distances from Eq. 23–8, using the given focal length and maximum and minimum object distances. Subtracting these two distances gives the distance over which the lens must move relative to the plane of the sensor or film.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \rightarrow \quad d_{i,\max} = \frac{f_{\text{do,min}}}{d_{o,\min} - f} = \frac{(135 \text{ mm})(1.30 \text{ m})}{1300 \text{ mm} - 135 \text{ mm}} = 0.151 \text{ m} = 151 \text{ mm}$$

$$d_{i,\min} = \frac{f d_{o,\max}}{d_{o,\max} - f} = \frac{(135 \text{ mm})(\infty)}{\infty - 135 \text{ mm}} = 135 \text{ mm}$$

$$\Delta d = d_{i,\max} - d_{i,\max} = 151 \text{ mm} - 135 \text{ mm} = \boxed{16 \text{ mm}}$$

10. When an object is very far away, the image will be at the focal point. We set the image distance in Eq. 23–9 equal to the focal length to show that the magnification is proportional to the focal length.

$$m = -\frac{d_i}{d_o} = -\frac{f}{d_o} = \left(-\frac{1}{d_o}\right) f = (\text{constant}) f \quad \rightarrow \quad \boxed{m \propto f}$$

11. The length of the eyeball is the image distance for a far object, that is, the focal length of the lens. We find the *f*-number from $f\text{-stop} = f/D$.

$$f\text{-stop} = \frac{f}{D} = \frac{(20 \text{ mm})}{(8.0 \text{ mm})} = \boxed{2.5 \text{ or } \frac{f}{2.5}}$$

12. The actual near point of the person is 52 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 52 cm from the eye, or 50 cm from the lens. We find the power of the lens from Eqs. 23–7 and 23–9.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.23 \text{ m}} + \frac{1}{-0.50 \text{ m}} = \boxed{2.3 \text{ D}}$$

13. The screen placed 55 cm from the eye, or 53.2 cm from the reading glasses lens, is to produce a virtual image 125 cm from the eye, or 123.2 cm from the lens. Find the power of the lens from Eqs. 23–7 and 23–8.

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{0.532\text{ m}}+\frac{1}{-1.232\text{ m}}=\boxed{1.1\text{ D}}$$

14. (*a*) Since the lens power is negative, the lens is diverging, so it produces images closer than the object. Thus the person is $\boxed{\text{nearsighted}}$.

(*b*) We find the far point by finding the image distance for an object at infinity. Since the lens is 2.0 cm in front of the eye, the far point is 2.0 cm farther than the absolute value of the image distance. Use Eqs. 23–7 and 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \quad\rightarrow\quad \frac{1}{\infty}+\frac{1}{d_i}=-5.50\text{ D} \quad\rightarrow\quad d_i=-\frac{1}{5.50\text{ D}}=-0.182\text{ m}=-18.2\text{ cm}$$

$$\text{Far point}=|-18.2\text{ cm}|+2.0\text{ cm}=\boxed{20.2\text{ cm}}\text{ from eye}$$

15. (*a*) The lens should put the image of an object at infinity at the person's far point of 85 cm. Note that the image is still in front of the eye, so the image distance is negative. Use Eqs. 23–7 and 23–8.

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}+\frac{1}{(-0.85\text{ m})}=-1.176\text{ D}\approx\boxed{-1.2\text{ D}}$$

(*b*) To find the near point with the lens in place, we find the object distance to form an image 25 cm in front of the eye.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \quad\rightarrow\quad d_o=\frac{d_i}{Pd_i-1}=\frac{(-0.25\text{ m})}{(-1.176\text{ D})(-0.25\text{ m})-1}=0.354\text{ m}=\boxed{35\text{ cm}}$$

16. The 2.0 cm of a normal eye is the image distance for an object at infinity; thus it is the focal length of the lens of the eye. To find the length of the nearsighted eye, find the image distance (distance from lens to retina) for an object at the far point of the eye. Use Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\rightarrow\quad d_i=\frac{fd_o}{d_o-f}=\frac{(2.0\text{ cm})(17\text{ cm})}{15\text{ cm}}=2.27\text{ cm}$$

$$\left(\frac{1}{17\text{ cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{2.0\text{ cm}}\text{, which gives } d_i=2.27\text{ cm}$$

Thus the difference is $2.27\text{ cm}-2.0\text{ cm}=0.27\text{ cm}\approx\boxed{0.3\text{ cm}}$.

17. The image of an object at infinity is to be formed 14 cm in front of the eye. So for glasses, the image distance is to be $d_i=-12\text{ cm}$, and for contact lenses, the image distance is to be $d_i=-14$ cm.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{1}{\infty}+\frac{1}{d_i} \quad\rightarrow\quad f=d_i \quad\rightarrow\quad P=\frac{1}{f}=\frac{1}{d_i}$$

$$P_{\text{glasses}}=\frac{1}{-0.12\text{ m}}=\boxed{-8.3\text{ D}};\quad P_{\text{contacts}}=\frac{1}{-0.14\text{ m}}=\boxed{-7.1\text{ D}}$$

18. Find the far point of the eye by finding the image distance FROM THE LENS for an object at infinity, using Eq. 23–8.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=f_1=-26.0\text{ cm}$$

Since the image is 26.0 cm in front of the lens, the image is 27.8 cm in front of the eye. The contact lens must put the image of an object at infinity at this same location. Use Eq. 23–8 for the contact lens with an image distance of –27.8 cm and an object distance of infinity.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i2}}=\frac{1}{f_1} \rightarrow f_1=d_{i2}=\boxed{-27.8\text{ cm}}$$

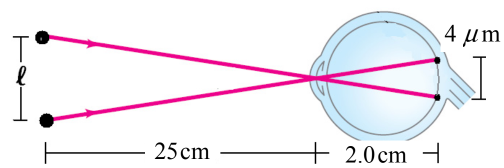
19. (*a*) We find the focal length of the lens for an object at infinity and the image on the retina. The image distance is thus 2.0 cm. Use Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{\infty}+\frac{1}{2.0\text{ cm}}=\frac{1}{f} \rightarrow f=\boxed{2.0\text{ cm}}$$

(*b*) We find the focal length of the lens for an object distance of 38 cm and an image distance of 2.0 cm. Again use Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_o d_i}{d_o+d_i}=\frac{(34\text{ cm})(2.0\text{ cm})}{(34\text{ cm})+(2.0\text{ cm})}=\boxed{1.9\text{ cm}}$$

20. The diagram shows rays from each of the two points as they pass directly through the cornea and onto the lens. These two rays, the distance between the objects (ℓ), and the distance between the images $(4\ \mu\text{m})$ create similar triangles. We set the ratio of the bases and altitudes of these two triangles equal to solve for $\boldsymbol{\ell}$.



$$\frac{\ell}{25\text{ cm}}=\frac{4\ \mu\text{m}}{2.0\text{ cm}} \rightarrow \ell=25\text{ cm}\frac{4\ \mu\text{m}}{2.0\text{ cm}}=\boxed{50\ \mu\text{m}}$$

21. Find the object distance for the contact lens to form an image at the eye's near point, using Eqs. 23–7 and 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_o=\frac{d_i}{Pd_i-1}=\frac{-0.106\text{ m}}{(-4.00\text{ D})(-0.106\text{ m})-1}=0.184\text{ m}=\boxed{18.4\text{ cm}}$$

Likewise, find the object distance for the contact lens to form an image at the eye's far point.

$$d_o=\frac{d_i}{Pd_i-1}=\frac{-0.200\text{ m}}{(-4.0\text{ D})(-0.200\text{ m})-1}=1.00\text{ m}=\boxed{100\text{ cm}} \quad \text{(3 significant figures)}$$

22. We find the focal length from Eq. 25–2a.

$$M=\frac{N}{f} \rightarrow f=\frac{N}{M}=\frac{25\text{ cm}}{3.2}=\boxed{7.8\text{ cm}}$$

23. Find the magnification from Eq. 25–2a.

$$M = \frac{N}{f} = \frac{(25\text{ cm})}{(16\text{ cm})} = \boxed{1.6\times}$$

24. (*a*) We find the focal length with the image at the near point from Eq. 25–2b.

$$M = 1 + \frac{N}{f} \quad \rightarrow \quad f = \frac{N}{M-1} = \frac{25\text{ cm}}{3.5-1} = \boxed{10\text{ cm}} \quad \text{(2 significant figures)}$$

(*b*) If the eye is relaxed, then the image is at infinity, so we use Eq. 25–2a.

$$M = \frac{N}{f} \quad \rightarrow \quad f = \frac{N}{M} = \frac{25\text{ cm}}{3.5} = \boxed{7.1\text{ cm}}$$

25. Maximum magnification is obtained with the image at the near point (which is negative). We find the object distance from Eq. 23–8 and the magnification from Eq. 25–2b.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad d_o = \frac{d_i f}{d_i - f} = \frac{(-25\text{ cm})(8.20\text{ cm})}{(-25\text{ cm}) - (8.20\text{ cm})} = 6.17\text{ cm} \approx \boxed{6.2\text{ cm}}$$

$$M = 1 + \frac{N}{f} = 1 + \frac{25\text{ cm}}{8.20\text{ cm}} = 4.049 \approx \boxed{4.0\times}$$

26. (*a*) We find the image distance from Eq. 23–8.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad d_i = \frac{f d_o}{d_o - f} = \frac{(5.00\text{ cm})(4.85\text{ cm})}{4.85\text{ cm} - 5.00\text{ cm}} = -162\text{ cm} \approx \boxed{-160\text{ cm}}$$

(*b*) The angular magnification is found from the definition given in the text.

$$M = \frac{\theta'}{\theta} = \frac{N}{d_o} = \frac{25\text{ cm}}{4.85\text{ cm}} = 5.155 \approx \boxed{5.2\times}$$

27. (*a*) Use Eq. 25–2b to calculate the angular magnification, since the eye is focused at the near point.

$$M = 1 + \frac{N}{f} = 1 + \frac{(25\text{ cm})}{(9.60\text{ cm})} = \boxed{3.6\times}$$

(*b*) The image is at the near point, so the image distance is –25 cm. Use Eq. 23–8 to find the object distance, and then calculate the image size from Eq. 23–3.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad d_o = \frac{f d_i}{d_i - f} = \frac{(9.60\text{ cm})(-25\text{ cm})}{-25\text{ cm} - 9.60\text{ cm}} = 6.94\text{ cm}$$

$$h_i = -h_o \frac{d_i}{d_o} = -(3.80\text{ mm}) \frac{(-25\text{ cm})}{(6.94\text{ cm})} = \boxed{13.7\text{ mm}}$$

It can be seen from Fig. 25–17 in the text that, for small angles, $M = h_i / h_o$ so $h_i = Mh_o$ $= 3.6(3.80\text{ mm}) = 13.7\text{ mm}$.

(*c*) As found above, $d_o = \boxed{6.9\text{ cm}}$.

28. (*a*) We find the image distance using Eq. 23–8.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{fd_o}{d_o-f}=\frac{(9.2\text{ cm})(8.0\text{ cm})}{8.0\text{ cm}-9.2\text{ cm}}=-61.3\text{ cm}\approx\boxed{-61\text{ cm}}$$

(*b*) The angular magnification is found using Eq. 25–1, with the angles given as defined in Fig. 25–17.

$$M=\frac{\theta'}{\theta}=\frac{(h_o/d_o)}{(h_o/N)}=\frac{N}{d_o}=\frac{25\text{ cm}}{8.0\text{ cm}}=\boxed{3.1\times}$$

29. The focal length is 12 cm. First, find the object distance for an image at infinity. Then find the object distance for an image 25 cm in front of the eye. Use Eq. 23–8.

$$\text{Initial: } \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{\infty}=\frac{1}{f} \rightarrow d_o=f=12\text{ cm}$$

$$\text{Final: } \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{d_i f}{d_i-f}=\frac{(-25\text{ cm})(12\text{ cm})}{(-25\text{ cm})-(12\text{ cm})}=8.1\text{ cm}$$

The lens was moved $12\text{ cm}-8.1\text{ cm}=3.9\text{ cm}\approx\boxed{\text{4 cm toward the fine print}}$.

30. First, find the focal length of the magnifying glass from Eq. 25–2a for a relaxed eye (focused at infinity).

$$M=\frac{N}{f} \rightarrow f=\frac{N}{M}=\frac{25.0\text{ cm}}{3.0}=8.33\text{ cm}$$

(*a*) Again use Eq. 25–2a for a different near point.

$$M_1=\frac{N_1}{f}=\frac{(75\text{ cm})}{(8.33\text{ cm})}=\boxed{9.0\times}$$

(*b*) Again use Eq. 25–2a for a different near point.

$$M_2=\frac{N_2}{f}=\frac{(15\text{ cm})}{(8.33\text{ cm})}=\boxed{1.8\times}$$

Without the lens, the closest an object can be placed is the near point. A farther near point means a smaller angle subtended by the object without the lens, and thus greater magnification.

31. The magnification of the telescope is given by Eq. 25–3.

$$M=-\frac{f_o}{f_e}=-\frac{(82\text{ cm})}{(2.8\text{ cm})}=\boxed{-29\times}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o+f_e=82\text{ cm}+2.8\text{ cm}=84.8\text{ cm}\approx\boxed{85\text{ cm}}$$

32. We find the focal length of the eyepiece from the magnification by Eq. 25–3.

$$M=-\frac{f_o}{f_e} \rightarrow f_e=-\frac{f_o}{M}=-\frac{88\text{ cm}}{-25\times}=3.52\text{ cm}\approx\boxed{3.5\text{ cm}}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o + f_e = 88\text{ cm} + 3.52\text{ cm} = 91.52 \approx \boxed{92\text{ cm}}$$

33. We find the focal length of the objective from Eq. 25–3.

$$M = -f_o/f_e \quad \rightarrow \quad f_o = -Mf_e = (7.0)(3.5\text{ cm}) = 24.5\text{ cm} \approx \boxed{25\text{ cm}}$$

34. The magnification is given by Eq. 25–3.

$$M = -f_o/f_e = -f_oP_e = -(0.75\text{ m})(+25\text{ D}) = -18.75\times \approx \boxed{-19\times}$$

$\boxed{35.}$ For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 25–3 to find the magnification.

$$\ell = f_o + f_e; \quad M = -\frac{f_o}{f_e} = -\frac{f_o}{\ell - f_o} = -\frac{78.5\text{ cm}}{82.0\text{ cm} - 78.5\text{ cm}} = -22.4 \approx \boxed{-22\times}$$

36. For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 25–3 to find the magnification.

$$\ell = f_o + f_e; \quad M = -\frac{f_o}{f_e} = -\frac{f_o}{\ell - f_o} = -\frac{39.0\text{ cm}}{36.8\text{ cm} - 39.0\text{ cm}} = 17.72 \approx \boxed{-18\times}$$

37. The focal length of the objective is just half the radius of curvature. Use Eq. 25–3 for the magnification.

$$M = -\frac{f_o}{f_e} = -\frac{\frac{1}{2}r}{f_e} = -\frac{\frac{1}{2}(6.1\text{ m})}{0.028\text{ m}} = -108.9\times \approx \boxed{-110\times}$$

$\boxed{38.}$ The focal length of the mirror is found from Eq. 25–3. The radius of curvature is twice the focal length.

$$M = -\frac{f_o}{f_e} \quad \rightarrow \quad f_o = -Mf_e = -(150)(0.031\text{ m}) = 4.65\text{ m} \approx \boxed{4.7\text{ m}}; \quad r = 2f = \boxed{9.3\text{ m}}$$

39. The relaxed eye means that the image is at infinity, so the distance between the two lenses is the sum of the focal lengths. Use that relationship with Eq. 25–3 to solve for the focal lengths. Note that the magnification for an astronomical telescope is negative.

$$M = -\frac{f_o}{f_e} \quad \rightarrow \quad f_o = -Mf_e; \quad \ell = f_o + f_e = (1 - M)f_e \quad \rightarrow$$

$$f_e = \frac{\ell}{(1-M)} = \frac{1.10\text{ m}}{1-(-120)} = 9.09\times10^{-3}\text{ m} = \boxed{9.09\text{ mm}}$$

$$f_o = -Mf_e = -(-120)(9.09\times10^{-3}\text{ m}) = \boxed{1.09\text{ m}}$$

40. We use Eq. 25–2a and the magnification of the eyepiece to calculate the focal length of the eyepiece. We set the sum of the focal lengths equal to the length of the telescope to calculate the focal length of the objective. Then, using both focal lengths in Eq. 25–3, we calculate the maximum magnification.

$$f_e = \frac{N}{M} = \frac{25\text{ cm}}{5} = 5\text{ cm}; \quad \ell = f_e + f_o \quad \rightarrow \quad f_o = \ell - f_e = 50\text{ cm} - 5\text{ cm} = 45\text{ cm}$$

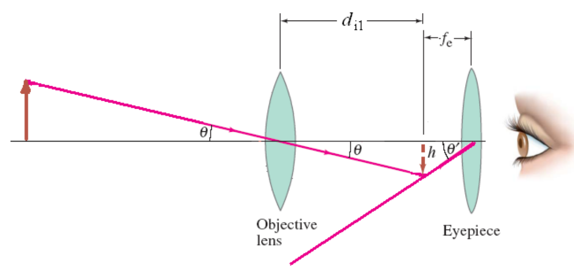
$$M = -\frac{f_o}{f_e} = -\frac{45\text{ cm}}{5\text{ cm}} = \boxed{-9\times}$$

41. Since the star is very far away, the image of the star from the objective mirror will be at the focal length of the objective, which is equal to one-half its radius of curvature (Eq. 23–1). We subtract this distance from the separation distance to determine the object distance for the second mirror. Then, using Eq. 23–8, we calculate the final image distance, which is where the sensor should be placed.

$$d_{i1} = f_o = \frac{R_o}{2} = \frac{3.00\text{ m}}{2} = 1.50\text{ m}; \quad d_{o2} = \ell - d_{i1} = 0.90\text{ m} - 1.50\text{ m} = -0.60\text{ m}$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_e} = \frac{2}{R_e} \rightarrow d_i = \frac{R_e d_{o2}}{2d_{o2} - R_e} = \frac{(-1.50\text{ m})(-0.60\text{ m})}{2(-0.60\text{ m}) - (-1.50\text{ m})} = \boxed{3.0\text{ m}}$$

42. We assume a prism binocular so the magnification is positive, but simplify the diagram by ignoring the prisms. We find the focal length of the eyepiece using Eq. 25–3, with the design magnification.



$$f_e = \frac{f_o}{M} = \frac{26\text{ cm}}{6.5} = 4.0\text{ cm}$$

Using Eq. 23–8 and the objective focal length, we calculate the intermediate image distance. With the final image at infinity (relaxed eye), the secondary object distance is equal to the focal length of the eyepiece. We calculate the angular magnification using Eq. 25–1, with the angles shown in the diagram.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_o} \rightarrow d_{i1} = \frac{f_o d_o}{d_{o1} - f_o} = \frac{(26\text{ cm})(400\text{ cm})}{400\text{ cm} - 26\text{ cm}} = 27.81\text{ cm}$$

$$M = \frac{\theta'}{\theta} = \frac{h/f_e}{h/d_{i1}} = \frac{d_{i1}}{f_e} = \frac{27.81\text{ cm}}{4.0\text{ cm}} = 6.95 \approx \boxed{7.0\times}$$

43. The magnification of the microscope is given by Eq. 25–6b.

$$M = \frac{N\ell}{f_o f_e} = \frac{(25\text{ cm})(17.5\text{ cm})}{(0.65\text{ cm})(1.70\text{ cm})} = 395.9\times \approx \boxed{400\times} \text{ (2 significant figures)}$$

44. We find the focal length of the eyepiece from the magnification of the microscope, using the approximate results of Eq. 25–6b. We already know that $f_o \ll \ell$.

$$M \approx \frac{N\ell}{f_o f_e} \rightarrow f_e = \frac{N\ell}{M f_o} = \frac{(25\text{ cm})(17.5\text{ cm})}{(720)(0.40\text{ cm})} = \boxed{1.5\text{ cm}}$$

Note that this also satisfies the assumption that $f_e \ll \ell$.

45. We use Eq. 25–6b.

$$M = \frac{N\ell}{f_e f_o} = \frac{(25\text{ cm})(17\text{ cm})}{(2.5\text{ cm})(0.33\text{ cm})} = 515.2 \approx \boxed{520\times}$$

46. (*a*) The total magnification is found from Eq. 25–6a.

$$M = M_o M_e = (60.0)(14.0) = \boxed{840\times} \text{ (3 significant figures)}$$

(*b*) With the final image at infinity, we find the focal length of the eyepiece using Eq. 25–5.

$$M_e = \frac{N}{f_e} \quad \to \quad f_e = \frac{N}{M_e} = \frac{25 \text{ cm}}{14.0} = 1.79 \text{ cm} \approx \boxed{1.8 \text{ cm}}$$

Since the image from the objective is at the focal point of the eyepiece, we set the image distance from the objective as the distance between the lenses less the focal length of the eyepiece. Using the image distance and magnification in Eq. 23–9, we calculate the initial object distance. Then using the image and object distance in Eq. 23–8, we calculate the objective focal length.

$$d_i = \ell - f_e = 20.0 \text{ cm} - 1.79 \text{ cm} = 18.21 \text{ cm}$$

$$m = \frac{d_i}{d_o} \to d_o = \frac{d_i}{m} = \frac{18.21 \text{ cm}}{60.0} = 0.3035 \text{ cm}$$

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i} \to f_o = \frac{d_o d_i}{d_o + d_i} = \frac{(0.3035 \text{ cm})(18.21 \text{ cm})}{0.3035 \text{ cm} + 18.21 \text{ cm}} = 0.2985 \text{ cm} \approx \boxed{0.299 \text{ cm}}$$

(*c*) We found the object distance, in part (*b*), $d_o = \boxed{0.304 \text{ cm}}$.

47. (*a*) The total magnification is the product of the magnification of each lens, with the magnification of the eyepiece increased by one, as in Eq. 25–2b.

$$M = M_o(M_e + 1) = (60.0)(14.0 + 1.0) = \boxed{900\times} \text{ (3 significant figures)}$$

(*b*) We find the focal length of the eyepiece using Eq. 25–2b.

$$(M_e + 1) = \frac{N}{f_e} + 1 \quad \to \quad f_e = \frac{N}{M_e} = \frac{25 \text{ cm}}{14.0} = 1.79 \text{ cm} \approx \boxed{1.8 \text{ cm}}$$

Since the image from the eyepiece is at the near point, we use Eq. 23–8 to calculate the location of the object. This object distance is the location of the image from the objective. Subtracting this object distance from the distance between the lenses gives us the image distance from the objective. Using the image distance and magnification in Eq. 23–9, we calculate the initial object distance. Then using the image and object distance in Eq. 23–8 we calculate the objective focal length.

$$\frac{1}{f_e} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \quad \to \quad d_{o2} = \frac{f_e d_{i2}}{d_{i2} - f_e} = \frac{(1.79 \text{ cm})(-25 \text{ cm})}{-25 \text{ cm} - 1.79 \text{ cm}} = 1.67 \text{ cm}$$

$$d_{i1} = \ell - d_{o2} = 20.0 \text{ cm} - 1.67 \text{ cm} = 18.33 \text{ cm}$$

$$m = \frac{d_i}{d_o} \quad \to \quad d_o = \frac{d_i}{m} = \frac{18.33 \text{ cm}}{60.0} = 0.3055 \text{ cm}$$

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i} \quad \to \quad f_o = \frac{d_o d_i}{d_o + d_i} = \frac{(0.3055 \text{ cm})(18.33 \text{ cm})}{0.3055 \text{ cm} + 18.33 \text{ cm}} = \boxed{0.300 \text{ cm}}$$

(*c*) We found the object distance, in part (*b*), $d_o = \boxed{0.306 \text{ cm}}$.

48. (*a*) Since the final image is at infinity (relaxed eye), the image from the objective is at the focal point of the eyepiece. We subtract this distance from the distance between the lenses to calculate the objective image distance. Then using Eq. 23–8, we calculate the object distance.

$$d_{i1} = \ell - f_e = 14.8 \text{ cm} - 1.8 \text{ cm} = 13.0 \text{ cm}$$

$$\frac{1}{f_o} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \quad \to \quad d_{o1} = \frac{f_o d_{i1}}{d_{i1} - f_o} = \frac{(0.80 \text{ cm})(13.0 \text{ cm})}{13.0 \text{ cm} - 0.80 \text{ cm}} = 0.8525 \text{ cm} \approx \boxed{0.85 \text{ cm}}$$

(*b*) With the final image at infinity, the magnification of the eyepiece is given by Eq. 25–6a.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25\text{ cm})}{(1.8\text{ cm})}\left(\frac{14.8\text{ cm}-1.8\text{ cm}}{0.8525\text{ cm}}\right)=212\times\approx\boxed{210\times}$$

49. (*a*) We find the image distance from the objective using Eq. 23–8. For the final image to be at infinity (viewed with a relaxed eye), the objective image distance must be at the focal distance of the eyepiece. We calculate the distance between the lenses as the sum of the objective image distance and the eyepiece focal length.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_o}\quad\rightarrow\quad d_{i1}=\frac{f_o d_{o1}}{d_{o1}-f_o}=\frac{(0.740\text{ cm})(0.790\text{ cm})}{0.790\text{ cm}-0.740\text{ cm}}=11.69\text{ cm}$$

$$\ell=d_{i1}+f_e=11.69\text{ cm}+2.80\text{ cm}=14.49\text{ cm}\approx\boxed{14\text{ cm}}$$

(*b*) We use Eq. 25–6a to calculate the total magnification.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25\text{ cm})}{(2.80\text{ cm})}\left(\frac{14.49\text{ cm}-2.80\text{ cm}}{0.790\text{ cm}}\right)=132\times\approx\boxed{130\times}$$

50. For each objective lens we set the image distance equal to the sum of the focal length and 160 mm. Then using Eq. 23–8 we write a relation for the object distance in terms of the focal length. Using this relation in Eq. 23–9, we write an equation for the magnification in terms of the objective focal length. The total magnification is the product of the magnification of the objective and focal length.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f_o}\quad\rightarrow\quad\frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{d_i}\quad\rightarrow\quad\frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{f_o+160\text{ mm}}\quad\rightarrow\quad d_o=\frac{f_o(f_o+160\text{ mm})}{160\text{ mm}}$$

$$m_o=\frac{d_i}{d_o}=\frac{f_o+160\text{ mm}}{\left[\dfrac{f_o(f_o+160\text{ mm})}{160\text{ mm}}\right]}=\frac{160\text{ mm}}{f_o}$$

Since the objective magnification is inversely proportional to the focal length, the objective with the smallest focal length $(f_o=3.9\text{ mm})$ combined with the largest eyepiece magnification $(M_e=15\times)$ yields the largest overall magnification. The objective with the largest focal length $(f_o=32\text{ mm})$ coupled with the smallest eyepiece magnification $(M_e=5\times)$ yields the smallest overall magnification.

$$M_{\text{largest}}=\frac{160\text{ mm}}{3.9\text{ mm}}(15\times)=615.4\times\approx\boxed{620\times};\qquad M_{\text{smallest}}=\frac{160\text{ mm}}{32\text{ mm}}(5\times)=\boxed{25\times}$$

51. From Chapter 23, Problem 87, we have a relationship between the individual focal lengths and the focal length of the combination.

$$f_T=\frac{f_1f_2}{f_1+f_2}=\frac{(25.3\text{ cm})(-27.8\text{ cm})}{(25.3\text{ cm})+(-27.8\text{ cm})}=281\text{ cm}$$

(*a*) The combination is $\boxed{\text{converging}}$, since f_T is positive. The converging lens is "stronger" than the diverging lens since it has a smaller absolute focal length (or higher absolute power).

(*b*) From above, $f_T=\boxed{281\text{ cm}}$.

52. We use Eq. 23–10 to find the focal length for each color and then Eq. 23–8 to find the image distance. For the planoconvex lens, $R_1 > 0$ and $R_2 = \infty$.

$$\frac{1}{f_{\text{red}}} = (n_{\text{red}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5106 - 1)\left[\left(\frac{1}{14.5\text{ cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{red}} = 28.398\text{ cm}$$

$$\frac{1}{f_{\text{yellow}}} = (n_{\text{yellow}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5226 - 1)\left[\left(\frac{1}{14.5\text{ cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{orange}} = 27.746\text{ cm}$$

We find the image distances from the following:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_{\text{red}}} \rightarrow d_i = \frac{d_o f_{\text{red}}}{d_o - f_{\text{red}}} = \frac{(66.0\text{ cm})(28.398\text{ cm})}{(66.0\text{ cm}) - (28.398\text{ cm})} = 49.845\text{ cm} \approx \boxed{49.8\text{ cm}}$$

$$\frac{1}{d_o} + \frac{1}{d_{i_{\text{yellow}}}} = \frac{1}{f_{\text{yellow}}} \rightarrow d_{i_{\text{yellow}}} = \frac{d_o f_{\text{yellow}}}{d_o - f_{\text{yellow}}} = \frac{(66.0\text{ cm})(27.746\text{ cm})}{(66.0\text{ cm}) - (27.746\text{ cm})} = 47.870\text{ cm} \approx \boxed{47.9\text{ cm}}$$

The images are 1.9 cm apart, an example of chromatic aberration.

53. The angular resolution is given by Eq. 25–7.

$$\theta = \frac{1.22\,\lambda}{D} = \frac{(1.22)(560\times10^{-9}\text{ m})}{(100\text{ in})(0.0254\text{ m/in})} = 2.69\times10^{-7}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right) = \boxed{\left(1.54\times10^{-5}\right)^\circ}$$

54. The resolving power is given by Eq. 25–8.

$$\text{Resolving power} = 1.22\frac{\lambda f}{D} = 1.22\frac{(500\text{ nm})(9\text{ mm})}{(5\text{ mm})} = 1098\text{ nm} \approx \boxed{1000\text{ nm}}$$

55. The angular resolution is given by Eq. 25–7. The distance between the stars (ℓ) is the angular resolution (θ) times the distance to the stars from the Earth (r).

$$\theta = 1.22\frac{\lambda}{D}; \quad \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{(18\text{ ly})\left(\frac{9.46\times10^{15}\text{ m}}{1\text{ ly}}\right)(550\times10^{-9}\text{ m})}{(0.66\text{ m})} = \boxed{1.7\times10^{11}\text{ m}}$$

56. Find the angle subtended by the planet by dividing the orbit radius by the distance of the star to the Earth. Use Eq. 25–7 to calculate the minimum aperture diameter needed to resolve this angle.

$$\theta = \frac{r}{d} = \frac{1.22\,\lambda}{D} \rightarrow$$

$$D = \frac{1.22\,\lambda d}{r} = \frac{1.22(550\times10^{-9}\text{ m})(4\text{ ly})(9.461\times10^{15}\text{ m/ly})}{(1\text{ AU})(1.496\times10^{11}\text{ m/AU})} = 0.17\text{ m} \approx \boxed{20\text{ cm}}$$

57. We find the angular half-width of the flashlight beam using Eq. 25–7 with $D = 5$ cm and $\lambda = 550$ nm. We set the diameter of the beam equal to twice the radius, where the radius of the beam is equal to the angular half-width multiplied by the distance traveled, 3.84×10^8 m.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{0.050\text{ m}} = 1.3\times10^{-5}\text{ rad}$$

$$d = 2(r\theta) = 2(3.84\times10^8\text{ m})(1.3\times10^{-5}\text{ rad}) = \boxed{1.0\times10^4\text{ m}}$$

58. We set the resolving power (RP) as the focal length of the lens multiplied by the angular resolution, as in Eq. 25–8. The resolution is the inverse of the resolving power.

$$\frac{1}{\mathrm{RP}(f/2)}=\left[\frac{1.22\lambda f}{D}\right]^{-1}=\frac{D}{1.22\lambda f}=\frac{25\text{ mm}}{1.22(560\times10^{-6}\text{ mm})(50.0\text{ mm})}=\boxed{730\text{ lines/mm}}$$

$$\frac{1}{\mathrm{RP}(f/16)}=\frac{3.0\text{ mm}}{1.22(560\times10^{-6}\text{ mm})(50.0\text{ mm})}=\boxed{88\text{ lines/mm}}$$

59. To find the focal length of the eyepiece we use Eq. 25–3, where the objective focal length is 2.00 m, θ' is the ratio of the minimum resolved distance and 25 cm, and θ is the ratio of the object on the Moon and the distance to the Moon. We ignore the inversion of the image.

$$\frac{f_o}{f_e}=\frac{\theta'}{\theta}\rightarrow f_e=f_o\frac{\theta}{\theta'}=f_o\frac{(d_o/\ell)}{(d/N)}=(2.0\text{ m})\frac{(6.5\text{ km}/384{,}000\text{ km})}{(0.10\text{ mm}/250\text{ mm})}=0.0846\text{ m}\approx\boxed{8.5\text{ cm}}$$

We use Eq. 35–10 to determine the resolution limit.

$$\theta=1.22\frac{\lambda}{D}=1.22\frac{560\times10^{-9}\text{ m}}{0.11\text{ m}}=\boxed{6.2\times10^{-6}\text{ rad}}$$

This corresponds to a minimum resolution distance of $r=(384{,}000\text{ km})(6.2\times10^{-6}\text{ rad})=\boxed{2.4\text{ km}}$, which is smaller than the 6.5-km object we wish to observe.

$\boxed{60.}$ We use Eq. 25–10 with $m = 1$.

$$m\lambda=2d\sin\phi\quad\rightarrow\quad\phi=\sin^{-1}\frac{m\lambda}{2d}=\sin^{-1}\frac{(1)(0.138\text{ nm})}{2(0.285\text{ nm})}=\boxed{14.0^\circ}$$

61. We use Eq. 25–10 for X-ray diffraction.

(*a*) Apply Eq. 25–10 to both orders of diffraction.

$$m\lambda=2d\sin\phi\quad\rightarrow\quad\frac{m_1}{m_2}=\frac{\sin\phi_1}{\sin\phi_2}\quad\rightarrow\quad\phi_2=\sin^{-1}\left(\frac{m_2}{m_1}\sin\phi_1\right)=\sin^{-1}\left(\frac{2}{1}\sin23.8^\circ\right)=\boxed{53.8^\circ}$$

(b) Use the first-order data.

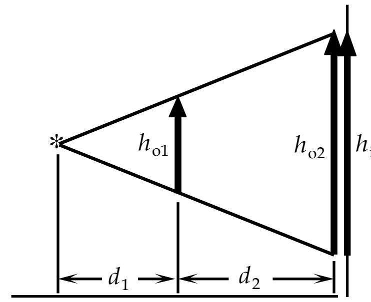
$$m\lambda=2d\sin\phi\quad\rightarrow\quad\lambda=\frac{2d\sin\phi}{m}=\frac{2(0.24\text{ nm})\sin23.8^\circ}{1}=\boxed{0.19\text{ nm}}$$

62. For each diffraction peak, we can measure the angle and count the order. Consider Eq. 25–10.

$$m\lambda=2d\sin\phi\quad\rightarrow\quad\lambda=2d\sin\phi_1;\quad2\lambda=2d\sin\phi_2;\quad3\lambda=2d\sin\phi_3$$

From each equation, all we can find is the ratio $\frac{\lambda}{d}=2\sin\phi=\sin\phi_2=\frac{2}{3}\sin\phi_3$. $\boxed{\text{No}}$, we cannot separately determine the wavelength or the spacing.

63. (a) Because X-ray images are shadows, the image will be the same size as the object, so the magnification is $\boxed{1}$.

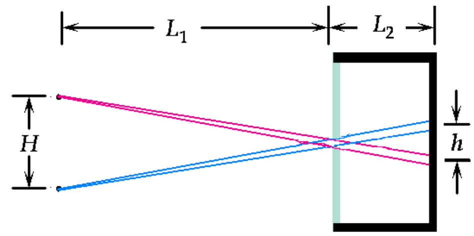


(*b*) The rays from the point source will not refract, so use similar triangles to compare the image size to the object size for the front of the body:

$$m_1 = \frac{h_i}{h_{o1}} = \frac{(d_1 + d_2)}{d_1} = \frac{(15\text{ cm} + 25\text{ cm})}{(15\text{ cm})} = \boxed{2.7}$$

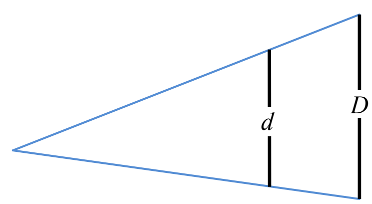
For the back of the body, the image and object have the same size, so the magnification is $\boxed{1}$.

64. We use similar triangles, created from the distances between the centers of the two objects (H) and their ray traces to the hole (L_1) and the distance between the centers of the two images (h) and the distance of the screen to the hole (L_2) to determine h. Note that the diagram is not to scale, since L_1 should be $50\times$ the size of H.



$$\frac{H}{L_1} = \frac{h}{L_2} \to h = H\frac{L_2}{L_1} = (2.0\text{ cm})\frac{7.0\text{ cm}}{100\text{ cm}} = 0.14\text{ cm} = 1.4\text{ mm}$$

Now consider similar triangles from the two rays from just one of the two sources. The base of one triangle is equal to the diameter of the hole (d), and the base of the second triangle is equal to the diameter of the image circle (D). The heights for these two triangles are the distance from object to hole (L_1) and the distance from object to image $(L_1 + L_2)$.



$$\frac{d}{L_1} = \frac{D}{L_1 + L_2} \to D = d\frac{L_1 + L_2}{L_1} = (1.0\text{ mm})\frac{100\text{ cm} + 7.0\text{ cm}}{100\text{ cm}} = 1.07\text{ mm}$$

Since the separation distance of the two images (1.4 mm) is greater than their diameters, the two circles do not overlap.

65. We calculate the effective *f*-number for the pinhole camera by dividing the focal length by the diameter of the pinhole. The focal length is equal to the image distance. Setting the exposures equal for both cameras, where the exposure is proportional to the product of the exposure time and the area of the lens opening (which is inversely proportional to the square of the *f*-stop number), we solve for the exposure time.

$$f\text{-stop}_2 = \frac{f}{D} = \frac{(70\text{ mm})}{(1.0\text{ mm})} = \frac{f}{70}$$

$$\text{Exposure}_1 = \text{Exposure}_2 \quad \to \quad A_1 t_1 = A_2 t_2 \quad \to \quad \frac{\pi}{4}D_1^2 t_1 = \frac{\pi}{4}D_2^2 t_2 \quad \to$$

$$\frac{\pi}{4}\frac{t_1 f^2}{(f\text{-stop}_1)^2} = \frac{\pi}{4}\frac{t_2 f^2}{(f\text{-stop}_2)^2} \quad \to \quad t_2 = t_1\left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2 = \frac{1}{250\text{ s}}\left(\frac{70}{11}\right)^2 = 0.16\text{ s} \approx \boxed{\tfrac{1}{6}\text{ s}}$$

66. We use Eq. 25–3, which relates the magnification to the focal lengths, to write the focal length of the objective lens in terms of the magnification and focal length of the eyepiece. Then setting the sum of the focal lengths equal to the length of the telescope, we solve for the focal length of the eyepiece and the focal length of the objective.

$$M=-\frac{f_o}{f_e} \rightarrow f_o=-Mf_e;\quad \ell=f_e+f_o=f_e(1-M) \rightarrow f_e=\frac{\ell}{1-M}=\frac{28\text{ cm}}{1-(-7.5)}=\boxed{3.3\text{ cm}}$$

$$f_o=\ell-f_e=28\text{ cm}-3.3\text{ cm}=24.7\text{ cm}\approx\boxed{25\text{ cm}}$$

67. (*a*) This is very similar to Example 25–11. We use the same notation as in that Example, and solve for the distance $\boxed{\ell}$.

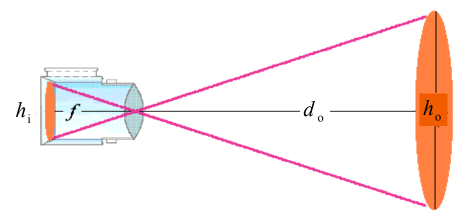
$$s=\ell\theta=\ell\frac{1.22\lambda}{D} \rightarrow \ell=\frac{Ds}{1.22\lambda}=\frac{(6.0\times10^{-3}\text{ m})(2.0\text{ m})}{1.22(560\times10^{-9}\text{ m})}=\boxed{1.8\times10^4\text{ m}}=18\text{ km}\approx11\text{ miles}$$

(*b*) We use the same data for the eye and the wavelength.

$$\theta=\frac{1.22\lambda}{D}=\frac{1.22(560\times10^{-9}\text{ m})}{(6.0\times10^{-3}\text{ m})}=1.139\times10^{-4}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right)\left(\frac{3600''}{1^\circ}\right)=\boxed{23''}$$

This is less than the actual resolution because of atmospheric effects and aberrations in the eye.

68. Since the distance to the sun is much larger than the telescope's focal length, the image distance is about equal to the focal length. Rays from the top and bottom edges of the Sun pass through the lens unrefracted. These rays form similar triangles along with the object and image heights. Calculate the focal length of the telescope by setting the ratio of height to base for each triangle equal.



$$\frac{f}{h_i}=\frac{d_o}{h_o} \rightarrow f=h_i\frac{d_o}{h_o}=(15\text{ mm})\frac{1.5\times10^8\text{ km}}{1.4\times10^6\text{ km}}=1607\text{ mm}\approx\boxed{1.6\text{ m}}$$

69. This is very similar to Example 25–11. We use the same notation as in that Example and solve for the separation s. We use 450 nm for the wavelength of the blue light.

$$s=\ell\theta=\ell\frac{1.22\lambda}{D}=(15\text{ m})\frac{1.22(450\times10^{-9}\text{ m})}{1.0\times10^{-2}\text{ m}}=\boxed{8.2\times10^{-4}\text{ m}}$$

$\boxed{70.}$ We use Eq. 23–9 to write the image distance in terms of the object distance, image height, and object height. Then using Eq. 23–8, we solve for the object distance, which is the distance between the photographer and the subject.

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow \frac{1}{d_i}=-\frac{h_o}{h_i}\frac{1}{d_o}$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\left(-\frac{h_o}{h_i}\frac{1}{d_o}\right)=\left(1-\frac{h_o}{h_i}\right)\frac{1}{d_o} \rightarrow$$

$$d_o=\left(1-\frac{h_o}{h_i}\right)f=\left(1-\frac{1650\text{ mm}}{-8.25\text{ mm}}\right)(220\text{ mm})=44{,}220\text{ mm}\approx\boxed{44\text{ m}}$$

71. The exposure is proportional to the intensity of light, the area of the shutter, and the time. The area of the shutter is proportional to the square of the diameter or inversely proportional to the square of the *f*-stop. Setting the two proportionalities equal, with constant time, we solve for the change in intensity.

$$\frac{I_1 t}{(f\text{-stop}_1)^2} = \frac{I_2 t}{(f\text{-stop}_2)^2} \quad \to \quad \frac{I_2}{I_1} = \left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2 = \left(\frac{16}{5.6}\right)^2 = \boxed{8.2}$$

$\boxed{72.}$ The maximum magnification is achieved with the image at the near point, using Eq. 25–2b.

$$M_1 = 1 + \frac{N_1}{f} = 1 + \frac{(15\text{ cm})}{(9.5\text{ cm})} = \boxed{2.6\times}$$

For an adult, we set the near point equal to 25.0 cm.

$$M_2 = 1 + \frac{N_2}{f} = 1 + \frac{(25\text{ cm})}{(9.5\text{ cm})} = \boxed{3.6\times}$$

The $\boxed{\text{person with the normal eye}}$ (adult) sees more detail.

73. The actual far point of the person is 135 cm. With the lens, an object far away is to produce a virtual image 135 cm from the eye, or 133 cm from the lens. We calculate the power of the upper part of the bifocals using Eq. 23–8 with the power equal to the inverse of the focal length in meters.

$$P_1 = \frac{1}{f_1} = \left(\frac{1}{d_{o1}}\right) + \left(\frac{1}{d_{i1}}\right) = \left(\frac{1}{\infty}\right) + \left(\frac{1}{-1.33\text{ m}}\right) = \boxed{-0.75\text{ D (upper part)}}$$

The actual near point of the person is 45 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens. We again calculate the power using Eq. 23–8.

$$P_2 = \frac{1}{f_2} = \left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{d_{i2}}\right) = \left(\frac{1}{0.23\text{ m}}\right) + \left(\frac{1}{-0.43\text{ m}}\right) = \boxed{+2.0\text{ D (lower part)}}$$

74. The magnification for a relaxed eye is given by Eq. 25–2a.

$$M = N/f = NP = (0.25\text{ m})(+4.0\text{ D}) = \boxed{1.0\times}$$

75. (*a*) The magnification of the telescope is given by Eq. 25–3. The focal lengths are expressed in terms of their powers, from Eq. 23–7.

$$M = -\frac{f_o}{f_e} = -\frac{P_e}{P_o} = -\frac{(5.5\text{ D})}{(2.0\text{ D})} = -2.75\times \approx \boxed{-2.8\times}$$

(*b*) To get a magnification greater than 1, for the eyepiece we use the lens with the smaller focal length, or greater power: $\boxed{5.5\text{ D}}$.

76. Since the microscope is adjusted for a good image with a relaxed eye, the image must be at infinity. Thus the object distance for the camera should be set to $\boxed{\text{infinity}}$.

77. We calculate the man's near point (d_i) using Eqs. 23–7 and 23–8, with the initial object at 0.38 m with a 2.5-D lens. To give him a normal near point, we set the final object distance as 0.25 m and calculate the power necessary to have the image at his actual near point.

$$P_1 = \frac{1}{d_i} + \frac{1}{d_{o1}} \rightarrow \frac{1}{d_i} = P_1 - \frac{1}{d_{o1}} \rightarrow d_i = \frac{d_{o1}}{P_1 d_{o1} - 1} = \frac{0.38\text{ m}}{(2.5\text{ D})(0.38\text{ m}) - 1} = -7.6\text{ m}$$

$$P_2 = \frac{1}{d_i} + \frac{1}{d_{o2}} = \frac{1}{-7.6\text{ m}} + \frac{1}{0.25\text{ m}} = \boxed{+3.9\text{ D}}$$

78. (*a*) The length of the telescope is the sum of the focal lengths. The magnification is the ratio of the focal lengths (Eq. 25–3). For a magnification greater than 1, the lens with the smaller focal length should be the eyepiece. Therefore, the $\boxed{\text{4.0-cm lens should be the eyepiece}}$.

$$\ell = f_o + f_e = 4.0\text{ cm} + 48\text{ cm} = \boxed{52\text{ cm}}$$

$$M = -\frac{f_o}{f_e} = -\frac{(48\text{ cm})}{(4.0\text{ cm})} = \boxed{-12\times}$$

(*b*) We use Eq. 25–6b to solve for the length, ℓ, of the microscope.

$$M = -\frac{N\ell}{f_e f_o} \Rightarrow \ell = \frac{-M f_e f_o}{N} = \frac{-(25)(4.0\text{ cm})(48\text{ cm})}{25\text{ cm}} = 192\text{ cm} = \boxed{1.9\text{ m}}$$

This is far too long to be practical.

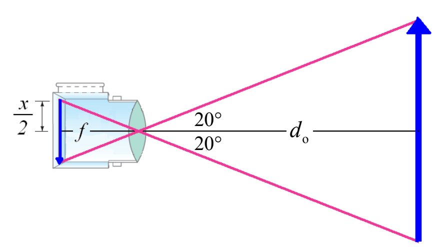
79. We assume that all of the electrical energy input to the tube becomes heat energy. The unit of amps times volts is joules per second, and the kcal units of the specific heat need to be converted to joules.

$$P = IV = \frac{Q}{t} = \frac{mc\Delta T}{t} \rightarrow$$

$$\frac{\Delta T}{t} = \frac{IV}{mc} = \frac{(25\times10^{-3}\text{ A})(95\times10^{3}\text{ V})(60\text{ s/min})}{(0.065\text{ kg})(0.11\text{ kcal/kg}\cdot\text{C}^\circ)(4186\text{ J/kcal})} = \boxed{4.8\times10^{3}\text{ C}^\circ\text{/min}}$$

This is an enormous rate of temperature increase. The cooling water is essential.

80. Consider an object located a distance d_o from a converging lens of focal length f and its real image formed a distance d_i from the lens. If the distance d_o is much greater than the focal length, then the lens equation tells us that the focal length and image distance are equal.



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{f d_o}{d_o - f} \approx \frac{f d_o}{d_o} = f$$

Thus, in a camera, the recording medium of spatial extent x is placed a distance equal to f behind the lens to form a focused image of a distant object. Assume the distant object subtends an angle of 40° at the position of the lens, so that the half-angle subtended is 20°, as shown in the figure. We then use the tangent of this angle to determine the relationship between the focal length and half the image height.

$$\tan 20^\circ = \frac{\frac{1}{2}x}{f} \rightarrow f = \frac{x}{2\tan 20^\circ}$$

(*a*) For a 35-mm camera, we set $x = 36$ mm to calculate the focal length.

$$f = \frac{36 \text{ mm}}{2\tan 20^\circ} = \boxed{49 \text{ mm}}$$

(*b*) For a digital camera, we set $x = 1.60 \text{ cm} = 16.0 \text{ mm}$.

$$f = \frac{16.0 \text{ mm}}{2\tan 20^\circ} = \boxed{22 \text{ mm}}$$

81. The focal length of the eyepiece is found using Eq. 23–7.

$$f_\text{e} = \frac{1}{P_\text{e}} = \frac{1}{19 \text{ D}} = 5.26\times10^{-2} \text{ m} \approx 5.3 \text{ cm}$$

For both object and image far away, find the focal length of the objective from the separation of the lenses.

$$\ell = f_\text{o} + f_\text{e} \rightarrow f_\text{o} = \ell - f_\text{e} = 85 \text{ cm} - 5.3 \text{ cm} = 79.7 \text{ cm}$$

The magnification of the telescope is given by Eq. 25–3.

$$M = -\frac{f_\text{o}}{f_\text{e}} = -\frac{(79.7 \text{ cm})}{(5.3 \text{ cm})} = \boxed{-15\times}$$

82. (*a*) The focal length of the lens is the inverse of the power.

$$f = \frac{1}{P} = \frac{1}{3.50 \text{ D}} = 0.286 \text{ m} = \boxed{28.6 \text{ cm}}$$

(*b*) The lens produces a virtual image at Sam's near point. We set the object distance at 23 cm from the glasses (25 cm from the eyes) and solve for the image distance. We add the 2 cm between the glasses and eyes to determine the near point.

$$P = \frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \rightarrow d_\text{i} = \left(P - \frac{1}{d_\text{o}}\right)^{-1} = \left(3.50 \text{ D} - \frac{1}{0.23 \text{ m}}\right)^{-1} = -1.18 \text{ m}$$

$$N = |d_i| + 0.02 \text{ m} = 1.18 \text{ m} + 0.02 \text{ m} = 1.20 \text{ m} \approx \boxed{1.2 \text{ m}}$$

(*c*) For Pam, find the object distance that has an image at her near point, –0.23 m from the lens.

$$P = \frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \rightarrow d_\text{o} = \left(P - \frac{1}{d_\text{i}}\right)^{-1} = \left(3.50 \text{ D} - \frac{1}{-0.23 \text{ m}}\right)^{-1} = 0.13 \text{ m}$$

Pam's near point with the glasses is 13 cm from the glasses, or $\boxed{15 \text{ cm}}$ from her eyes.

83. For the minimum aperture, the angle subtended at the lens by the smallest feature is the angular resolution, given by Eq. 25–7. We let ℓ represent the spatial separation and r represent the altitude of the camera above the ground.

$$\theta = \frac{1.22\lambda}{D} = \frac{\ell}{r} \rightarrow D = \frac{1.22\lambda r}{\ell} = \frac{1.22(580\times10^{-9} \text{ m})(25{,}000 \text{ m})}{(0.05 \text{ m})} = 0.3538 \text{ m} \approx \boxed{0.4 \text{ m}}$$

84. We find the spacing from Eq. 25–10.

$$m\lambda = 2d\sin\phi \quad \rightarrow \quad d = \frac{m\lambda}{2\sin\phi} = \frac{(2)(9.73\times10^{-11}\text{ m})}{2\sin 21.2^\circ} = \boxed{2.69\times10^{-10}\text{ m}}$$

85. From Eq. 25–7 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between the Earth and Moon to obtain the minimum distance between two objects on the Moon that the Hubble can resolve.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{2.4\text{ m}} = 2.796\times10^{-7}\text{ rad}$$

$$\ell = s\theta = (3.84\times10^{8}\text{ m})(2.796\times10^{-7}\text{ rad}) = 107.4\text{ m} \approx \boxed{110\text{ m}}$$

86. From Eq. 25–7 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between Mars and Earth to obtain the minimum distance between two objects that can be resolved by a person on Mars.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{0.005\text{ m}} = 1.34\times10^{-4}\text{ rad}$$

$$\ell = s\theta = (8\times10^{10}\text{ m})(1.34\times10^{-4}\text{ rad}) = \boxed{1.07\times10^{7}\text{ m}}$$

Since the minimum resolvable distance is much less than the Earth–Moon distance, a person standing on Mars $\boxed{\text{could resolve the Earth and Moon as two separate objects}}$ without a telescope.

87. The required angular resolution from the geometry is $\theta = \frac{s}{\ell}$. This must equal the resolution from the Rayleigh criterion, Eq. 25–7.

$$\theta = \frac{s}{\ell} = \frac{1.22\lambda}{D} \rightarrow D = \frac{1.22\,\lambda\ell}{s} = \frac{1.22(550\times10^{-9}\text{ m})(130\times10^{3}\text{ m})}{(0.05\text{ m})} = 1.74\text{ m} \approx \boxed{2\text{ m}}$$

88. (*a*) For this microscope both the objective and eyepiece have focal lengths of 12 cm. Since the final image is at infinity (relaxed eye), the image from the objective must be at the focal length of the eyepiece. The objective image distance must therefore be equal to the distance between the lenses less the focal length of the objective. We calculate the object distance by inserting the objective focal length and image distance into Eq. 23–8.

$$d_{\text{i1}} = \ell - f_{\text{e}} = 55\text{ cm} - 12\text{ cm} = 43\text{ cm}$$

$$\frac{1}{f_{\text{o}}} = \frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i1}}} \quad \rightarrow \quad d_{\text{o}} = \frac{f_{\text{o}}d_{\text{i1}}}{d_{\text{i1}} - f_{\text{o}}} = \frac{(12\text{ cm})(43\text{ cm})}{43\text{ cm} - 12\text{ cm}} = 16.65\text{ cm} \approx \boxed{17\text{ cm}}$$

(*b*) We calculate the magnification using Eq. 25–6a.

$$M = \frac{N}{f_{\text{e}}}\left(\frac{\ell - f_{\text{e}}}{d_{\text{o}}}\right) = \frac{(25\text{ cm})}{(12\text{ cm})}\left(\frac{55\text{ cm} - 12\text{ cm}}{16.65\text{ cm}}\right) = 5.38\times \approx \boxed{5.4\times}$$

(*c*) We calculate the magnification using Eq. 25–6b and divide the result by the answer to part (*b*) to determine the percent difference.

$$M_{\text{approx}} \approx \frac{N\ell}{f_{\text{e}}f_{\text{o}}} = \frac{(25\text{ cm})(55\text{ cm})}{(12\text{ cm})(12\text{ cm})} = 9.55\times;\ \frac{M_{\text{approx}} - M}{M} = \frac{9.55 - 5.38}{5.38} = 0.775 \approx \boxed{78\%}$$

89. We use Eq. 23–10 with Eq. 23–7 to find the desired radius.

$$\frac{1}{f} = P = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow$$

$$R_2 = \left[\frac{P}{(n-1)} - \frac{1}{R_1}\right]^{-1} = \left[\frac{(-3.5\text{ m}^{-1})}{(1.62-1)} - \frac{1}{0.160\text{ m}}\right]^{-1} = -8.41\times10^{-2}\text{ m} \approx \boxed{-8.41\text{ cm}}$$

Solutions to Search and Learn Problems

1. The optical zoom is preferable because it uses all of the available pixels to maximize the resolution. When a digital zoom is used to take a picture, the size of the pixels is enlarged on the final print, so there is a loss of sharpness.

2. (*a*) False. Contact lenses are placed on your eye, while glasses are typically placed about 2.0 cm in front of the eyes. This extra distance is taken into account when determining the power of the prescription.
   (*b*) True. Farsighted people have a large near point and therefore cannot see close objects clearly.
   (*c*) False. Nearsighted people can see close objects clearly but have a relatively near "far point," preventing them from seeing objects beyond their far point clearly.
   (*d*) False. Astigmatism is corrected by using a nonspherical lens to correct the asymmetric shape of the cornea.

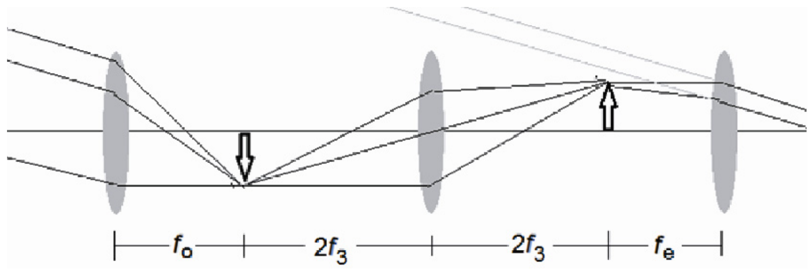
3. Example 25–3 assumes that the sensor has 12 MP. The 6-MP lens will have half as many total pixels, so the number of pixels in each direction would be decreased by the square root of 2, making the pixel dimensions 2828 pixels by 2121 pixels. We calculate the new resolution.

$$\frac{2828\text{ pixels}}{32\text{ mm}} = \boxed{88\text{ pixels/mm}}$$

For the enlargement to 8×10, as in Example 25–4, the size would be magnified by a factor of 8, which reduces the resolution by a factor of 8, giving $\frac{88\text{ pixels/mm}}{8} = \boxed{11\text{ pixels/mm}}$, which is right at the limit of creating a sharp image.

4. (i) A reflecting telescope can be built with a much larger diameter, as the mirror can be made of many small mirrors connected together. The lens, being one single piece of glass, is more difficult to manufacture.
   (ii) A lens must be supported around the outer edge; therefore, due to the large weight of the lens, its size is limited. A mirror can be supported everywhere behind the mirror and is much lighter. Therefore, the mirrors can be much larger in diameter.
   (iii) A lens experiences spherical aberrations, while the shape of a mirror can be adjusted to eliminate spherical aberrations.
   (iv) A lens experiences chromatic aberrations since different colors of light refract in the lens at slightly different angles. The image created by a mirror does not depend upon an index of refraction, since the light does not travel through a different material when reflecting from a telescope mirror.

5. Use the 150-cm lens as the objective and the 1.5-cm lens as the eyepiece. These two lenses produce the magnification of $100\times$, according to Eq. 25–3. The third lens is placed between the objective and the eyepiece to invert the image. The objective lens creates an image of a distance object at its focal point. The image becomes the object of the intermediate lens. The intermediate lens is placed so that this object is twice the lens' focal length away. The lens then creates a real, inverted image located at twice its focal length with no magnification. This image becomes the object of the eyepiece, which is placed at its focal distance away from the image. The image of the eyepiece is virtual and infinitely far away.



Calculate the length of the telescope from the figure.

$$\ell = f_\text{o} + 4f_3 + f_\text{e} = 150\text{ cm} + 4(10\text{ cm}) + 1.5\text{ cm} = 191.5\text{ cm} \approx 190\text{ cm}$$

6. Assume that the pupil is 3.0 mm in diameter and we use light of wavelength 550 nm. The minimum angular separation that we could observe is given near Fig. 25–30.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\text{ m})}{3.0\times10^{-3}\text{ m}} = 2.24\times10^{-4}\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right)\left(\frac{60\text{ arc min}}{1°}\right) = 0.77\text{ arc min}$$

Since this is smaller than the angular separation of Alcor and Mizar, they are resolvable by the naked eye.

26

THE SPECIAL THEORY OF RELATIVITY

Responses to Questions

1. No. Since the windowless car in an exceptionally smooth train moving at a constant velocity is an inertial reference frame and the basic laws of physics are the same in all inertial reference frames, there is no way for you to tell if you are moving or not. The first postulate of the special theory of relativity can be phrased as “No experiment can tell you if an inertial reference frame is at rest or moving uniformly at constant velocity.”

2. The fact that you instinctively think you are moving is consistent with the relativity principle applied to mechanics. Even though you are at rest relative to the ground, when the car next to you creeps forward, you are moving backward relative to that car.

3. The ball will land back in the worker’s hand. Both the ball and the car are already moving forward (relative to the ground), so when the ball is thrown straight up into the air with respect to the car, it will continue to move forward at the same rate as the car and fall back down to land in his hand.

4. Whether you say that the Earth goes around the Sun or the Sun goes around the Earth depends on your reference frame. It is valid to say either one, depending on which frame you choose. The laws of physics, though, won’t be the same in each of these reference frames, since the Earth is accelerating as it goes around the Sun. The Sun is nearly an inertial reference frame, but the Earth is not.

5. The starlight would pass you at a speed of *c*. The speed of light has the same speed in any reference frame in empty space, according to the second postulate of special relativity.

6. The clocks are not at fault and they are functioning properly. Time itself is actually measured to pass more slowly in moving reference frames when compared to a rest frame. Any measurement of time (heartbeats or decay rates, for instance) would be measured as slower than normal when viewed by an observer outside the moving reference frame.

7. Time actually passes more slowly in the moving reference frame, including aging and other life processes. It is not just that it seems this way—time has been measured to pass more slowly in the moving reference frame, as predicted by special relativity.

$\boxed{8.}$ This is an example of the "twin paradox." This situation would be possible if the woman was traveling at high enough speeds during her trip. Time would have passed more slowly for her so she would have aged less than her son, who stayed on Earth. (Note that the situations of the woman and son are not symmetric; she must undergo acceleration during her journey.)

$\boxed{9.}$ You would not notice a change in your own heartbeat, mass, height, or waistline. No matter how fast you are moving relative to Earth, you are at rest in your own reference frame. Thus, you would not notice any changes in your own characteristics. To observers on Earth, you are moving away at 0.6*c*, which gives $\gamma = 1.25.$ If we assume that you are standing up, so that your body is perpendicular to the direction of motion, then to the observers on Earth, it would appear that your heartbeat has slowed by a factor of 1/1.25 = 0.80, that your mass has increased by a factor of 1.25, and that your waistline has decreased by a factor of 0.80 (all due to the relativity equations for time dilation, mass increase, and length contraction), but that your height would be unchanged (since there is no relative motion between you and Earth in that direction). Also see the section of the text on "rest mass and relativistic mass" for comments about mass change and relativity.

10. Yes. However, at a speed of only 90 km/h, υ/c is very small, and therefore γ is very close to 1, so the effects would not be noticeable.

11. Length contraction and time dilation would not occur. If the speed of light were infinite, then υ/c would be zero for all finite values of υ, and therefore γ would always be 1, resulting in $\Delta t = \Delta t_0$ and $\ell = \ell_0$.

12. Both the length contraction and time dilation formulas include the term $\sqrt{1-\upsilon^2/c^2}$. If *c* were not the limiting speed in the universe, then it would be possible to have a situation with $\upsilon > c$. However, this would result in a negative number under the square root, which gives an imaginary number as a result, indicating that *c* must be the limiting speed. Also, assuming that the relativistic formulas were still correct, as υ gets very close to *c*, an outside observer should be able to show that $\ell = \ell_o\sqrt{1-\upsilon^2/c^2}$ is getting smaller and smaller and that the limit as $\upsilon \to c$ is $\ell \to 0$. This would show that *c* is a limiting speed, since nothing can get smaller than having a length of 0. A similar analysis for time dilation shows that $\Delta t = \dfrac{\Delta t_o}{\sqrt{1-\upsilon^2/c^2}}$ is getting longer and longer and that the limit as $\upsilon \to c$ is $\Delta t \to \infty$. This would show that *c* is a limiting speed, since the slowest that time can pass is that it comes to a stop.

13. If the speed of light were 25 m/s, then we would see relativistic effects all the time, something like the Chapter-Opening Figure or Figure 26–12 with Question 14. Everything moving relative to us would look length contracted, and time dilation would have to be taken into account for many events. There would be no "absolute time" on which we would all agree, so it would be more difficult, for instance, to plan to meet friends for lunch at a certain time. Many "twin paradox" kind of events would occur, and the momentum of moving objects would become very large, making it very difficult to change their motion. One of the most unusual changes for today's modern inhabitants of Earth would be that nothing would be able to move faster than 25 m/s, which is only about 56 mi/h.

14. Mr Tompkins appears shrunk in the horizontal direction, since that is the direction of his motion, and normal size in the vertical direction, perpendicular to his direction of motion. This length contraction is a result of the fact that, to the people on the sidewalk, Mr Tompkins is in a moving frame of reference. If the speed of light were only 20 mi/h, then the amount of contraction, which depends on γ, would be enough to be noticeable. Therefore, Mr Tompkins and his bicycle appear very skinny. (Compare to the Chapter-Opening Figure, which is shown from Mr Tompkins' viewpoint. In this case, Mr Tompkins sees himself as "normal," but all the objects moving with respect to him are contracted.)

15. No. The relativistic momentum of the electron is given by $p = \gamma m\upsilon = \dfrac{m\upsilon}{\sqrt{1-\upsilon^2/c^2}}$. At low speeds (compared to c), this reduces to the classical momentum, $p = m\upsilon$. As υ approaches c, γ approaches infinity, so there is no upper limit to the electron's momentum.

16. No. Accelerating a particle with nonzero mass up to the speed of light would require an infinite amount of energy, since $\text{KE} = \dfrac{mc^2}{\sqrt{1-\upsilon^2/c^2}} - mc^2$, so that is not possible.

17. No, $E = mc^2$ does not conflict with the conservation of energy; it actually *completes* it. Since this equation shows us that mass and energy are interconvertible, it says it is now necessary to include mass as a form of energy in the analysis of physical processes.

$\boxed{18.}$ Yes. One way to describe the energy stored in the compressed spring is to say it is a mass increase (although it would be so small that it could not be measured). This mass will convert back to energy when the spring is uncompressed.

$\boxed{19.}$ Matter and energy are interconvertible (matter can be converted into energy and energy can be converted into matter), thus we should say "energy can neither be created nor destroyed."

20. No, our intuitive notion that velocities simply add is not completely wrong. Our intuition is based on our everyday experiences, and at these everyday speeds our intuition regarding how velocities add is correct. Our intuition does break down, though, at very high speeds, where we have to take into account relativistic effects. Relativity does not contradict classical mechanics, but it is a more general theory, whereas classical mechanics is a limiting case.

Responses to MisConceptual Questions

1. (*a*) Proper length is the length measured by a person at rest with the object measured. The ship's captain is at rest with the ship, so that measurement is the proper length.

2. (*c*) A common misconception is that the distance between the objects is important when measuring relativistic effects. The important parameter is the relative velocity. The ship's captain is at rest with the flashlight and therefore measures the proper time. The space-dock personnel measure the dilated time, which is always longer than the proper time. Since the dilated time is 1.00 s, the proper time must be shorter, or 0.87 s.

3. (*c*) The proper time interval is measured at rest with respect to the flashlight, or the 0.87-s measurement.

4. (*b*) The ship's captain will have aged less. While the ship is moving at constant speed, both the captain and the space-dock personnel record that the other's clock is running slowly. However, the ship must accelerate (and therefore change frames of reference) when it turns around. The space-dock personnel do not accelerate; therefore, they are correct in their measurement that they have aged more than the ship's captain.

5. (*f*) A common misconception is that the person on the spacecraft would observe the change in her measurement of time, or that the person on the spacecraft would see the Earth clocks running fast. Actually, the person on the spacecraft would observe her time running normally. Since the spacecraft and Earth are moving relative to each other, observers on the spacecraft and observers on the Earth would both see the others' clocks running slowly.

6. (*c*) It might be assumed that the speed of the ship should be added to the speed of light to obtain a relative speed of 1.5*c*. However, the second postulate of relativity is that the speed of light is 1.0*c*, as measured by any observer.

7. (*d*) A common misconception is that the relativistic formulas are only valid for speeds close to the speed of light. The relativistic formulas are always valid. However, for speeds much smaller than the speed of light, time dilations, length contractions, and changes in mass are insignificant.

8. (*d*, *e*) Due to relativistic effects, the observers will not necessarily agree on the time an event occurs, the distance between events, the time interval between events, or the simultaneity of two events. However, the postulates of relativity state that both observers agree on the validity of the laws of physics and on the speed of light.

9. (*d*) It is common to think that one frame of reference is preferable to another. Due to relativistic effects, observers in different frames of reference may make different time measurements. However, each measurement is correct in that frame of reference.

10. (*e*) Even though the rocket ship is going faster and faster relative to some stationary frame, you remain at rest relative to the rocket ship. Therefore, in your frame of reference, your mass, length, and time remain the same.

11. (*d*) A common misconception is that there is a stationary frame of reference and that motion relative to this frame can be measured. Special relativity demonstrates that a stationary frame does not exist, but all motion is relative. If the spaceship has no means to observe the outside (and make a reference to another object), then there is no way to measure your velocity.

12. (*b*) To an observer on Earth, the pendulum would appear to be moving slowly. Therefore, the period would be longer than 2.0 seconds.

13. (*a*) A common misconception is to add the velocities together to obtain a speed of 1.5*c*. However, due to relativistic effects, the relative speed between two objects must be less than the speed of light. Since the objects are moving away from each other the speed must be greater than the speed of each object, so it must be greater than 0.75*c*. The only option that is greater than 0.75*c* and less than 1.0*c* is 0.96*c*.

Solutions to Problems

1. You measure the contracted length. Find the rest length from Eq. 26–3a.

$$\ell_0=\frac{\ell}{\sqrt{1-v^2/c^2}}=\frac{44.2\text{ m}}{\sqrt{1-(0.850)^2}}=\boxed{83.9\text{ m}}$$

2. We find the lifetime at rest from Eq. 26–1a.

$$\Delta t_0=\Delta t\sqrt{1-v^2/c^2}=(4.76\times10^{-6}\text{ s})\sqrt{1-\left(\frac{2.70\times10^8\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}=\boxed{2.07\times10^{-6}\text{ s}}$$

3. The measured distance is the contracted length. Use Eq. 26–3a.

$$\ell=\ell_0\sqrt{1-v^2/c^2}=(135\text{ ly})\sqrt{1-\left(\frac{2.90\times10^8\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}=34.6\text{ ly}\approx\boxed{35\text{ ly}}$$

4. The speed is determined from the time dilation relationship, Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad \rightarrow$$

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{2.60\times10^{-8}\ \text{s}}{4.40\times10^{-8}\ \text{s}}\right)^2} = \boxed{0.807c} = 2.42\times10^8\ \text{m/s}$$

$\boxed{5.}$ The speed is determined from the length contraction relationship, Eq. 26–3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-\left(\frac{35\ \text{ly}}{49\ \text{ly}}\right)^2} = \boxed{0.70c} = 2.1\times10^8\ \text{m/s}$$

6. The speed is determined from the length contraction relationship, Eq. 26–3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-(0.900)^2} = \boxed{0.436c} = 1.31\times10^8\ \text{m/s}$$

7. (*a*) We use Eq. 26–3a for length contraction with the contracted length 99.0% of the rest length.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-(0.990)^2} = \boxed{0.141c}$$

(*b*) We use Eq. 26–1a for time dilation with the time as measured from a relative moving frame 1.00% greater than the rest time.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{1}{1.0100}\right)^2} = \boxed{0.140c}$$

We see that a speed of $0.14c$ results in about a 1% relativistic effect.

8. The speed is determined from the length contraction relationship, Eq. 26–3a. Then the time is found from the speed and the contracted distance.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow$$

$$v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}; \quad t = \frac{\ell}{v} = \frac{\ell}{c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}} = \frac{25\ \text{ly}}{c\sqrt{1-\left(\frac{25\ \text{ly}}{62\ \text{ly}}\right)^2}} = \frac{(25\ \text{yr})c}{c(0.915)} = \boxed{27\ \text{yr}}$$

That is the time according to you, the traveler. The time according to an observer on Earth would be as follows.

$$t = \frac{\ell_0}{v} = \frac{(62\ \text{yr})c}{c(0.915)} = \boxed{68\ \text{yr}}$$

9. (*a*) The measured length is the contracted length. We find the rest length from Eq. 26–3a.

$$\ell_0 = \frac{\ell}{\sqrt{1-v^2/c^2}} = \frac{4.80\ \text{m}}{\sqrt{1-(0.720)^2}} = \boxed{6.92\ \text{m}}$$

Distances perpendicular to the motion do not change, so the rest height is $\boxed{1.35\,\text{m}}$.

(*b*) The time in the spacecraft is the proper time, found from Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (20.0\text{ s})\sqrt{1-(0.720)^2} = \boxed{13.9\text{ s}}$$

(*c*) To your friend, you moved at the same relative speed: $\boxed{0.720c}$.

(*d*) She would measure the same time dilation: $\boxed{13.9\text{ s}}$.

10. (*a*) To an observer on Earth, 21.6 ly is the rest length, so the time will be the distance divided by the speed.

$$t_{\text{Earth}} = \frac{\ell_0}{v} = \frac{(21.6\text{ ly})}{0.950c} = 22.74\text{ yr} \sim \boxed{22.7\text{ yr}}$$

(*b*) We find the proper time (the time that passes on the spacecraft) from Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (22.74\text{ yr})\sqrt{1-(0.950)^2} = 7.101\text{ yr} \approx \boxed{7.10\text{ yr}}$$

(*c*) To the spacecraft observer, the distance to the star is contracted. Use Eq. 26–3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} = (21.6\text{ ly})\sqrt{1-(0.950)^2} = 6.7446\text{ ly} \approx \boxed{6.74\text{ ly}}$$

(*d*) To the spacecraft observer, the speed of the spacecraft is as follows:

$$v = \frac{\ell}{\Delta t} = \frac{(6.745\text{ ly})}{7.101\text{ yr}} = \boxed{0.95c}\text{, as expected.}$$

$\boxed{11.}$ (*a*) In the Earth frame, the clock on the *Enterprise* will run slower. Use Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = (5.0\text{ yr})\sqrt{1-(0.70)^2} = \boxed{3.6\text{ yr}}$$

(*b*) Now we assume that the 5.0 years is the time as measured on the *Enterprise*. Again use Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad\rightarrow\quad \Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{(5.0\text{ yr})}{\sqrt{1-(0.70)^2}} = \boxed{7.0\text{ yr}}$$

12. The dimension along the direction of motion is contracted, and the other two dimensions are unchanged. Use Eq. 26–3a to find the contracted length.

$$\ell = \ell_0\sqrt{1-v^2/c^2};\quad V = \ell(\ell_0)^2 = (\ell_0)^3\sqrt{1-v^2/c^2} = (2.6\text{ m})^3\sqrt{1-(0.80)^2} = 10.55\text{ m}^3 \approx \boxed{11\text{ m}^3}$$

13. The change in length is determined from the length contraction relationship, Eq. 26–3a. The speed is very small compared to the speed of light, so we use the binomial expansion.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad\rightarrow$$

$$\frac{\ell}{\ell_0} = \sqrt{1-v^2/c^2} = \left(1-\frac{v^2}{c^2}\right)^{1/2} \approx 1-\tfrac{1}{2}\frac{v^2}{c^2} = 1-\tfrac{1}{2}\left(\frac{11.2\times10^3\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2 = 1-6.97\times10^{-10}$$

So the percent decrease is $\boxed{(6.97\times10^{-8})\%}$.

14. We find the speed of the particle in the lab frame and use that to find the rest frame lifetime and distance.

$$v = \frac{\Delta x_{\text{lab}}}{\Delta t_{\text{lab}}} = \frac{1.00\text{ m}}{3.40\times10^{-9}\text{ s}} = 2.941\times10^8\text{ m/s} = 0.9803c$$

(*a*) Find the rest frame lifetime from Eq. 26–1a.

$$\Delta t_0 = \Delta t_{\text{lab}}\sqrt{1-v^2/c^2} = (3.40\times10^{-9}\ \text{s})\sqrt{1-(0.9803)^2} = 6.715\times10^{-10}\ \text{s} \approx \boxed{6.7\times10^{-10}\ \text{s}}$$

(*b*) In its rest frame, the particle will travel the distance given by its speed and the rest lifetime.

$$\Delta x_0 = v\Delta t_0 = (2.941\times10^8\ \text{m/s})(6.715\times10^{-10}\ \text{s}) = \boxed{0.20\ \text{m}}$$

This could also be found from the length contraction relationship: $\Delta x_0 = \dfrac{\Delta x_{\text{lab}}}{\sqrt{1-v^2/c^2}}$.

15. In the Earth frame, the average lifetime of the pion will be dilated according to Eq. 26–1a. The speed of the pion will be the distance moved in the Earth frame times the dilated time.

$$v = \frac{d}{\Delta t} = \frac{d}{\Delta t_0}\sqrt{1-v^2/c^2} \quad\rightarrow$$

$$v = c\sqrt{\frac{1}{1+\left(\dfrac{c\Delta t_0}{d}\right)^2}} = c\sqrt{\frac{1}{1+\left(\dfrac{(3.00\times10^8\ \text{m/s})(2.6\times10^{-8}\ \text{s})}{(32\ \text{m})}\right)^2}} = \boxed{0.9716c}$$

Note that the significant figure addition rule gives 4 significant figures for the value under the radical sign.

$\boxed{16.}$ The momentum of the proton is given by Eq. 26–4.

$$p = \gamma mv = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{(1.67\times10^{-27}\ \text{kg})(0.68)(3.00\times10^8\ \text{m/s})}{\sqrt{1-0.68^2}} = \boxed{4.6\times10^{-19}\ \text{kg}\cdot\text{m/s}}$$

17. (*a*) We compare the classical momentum to the relativistic momentum (Eq. 26–4).

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\left(\dfrac{mv}{\sqrt{1-v^2/c^2}}\right)} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.15)^2} = 0.989$$

The classical momentum is about $\boxed{-1.1\%}$ in error. The negative sign means that the classical momentum is smaller than the relativistic momentum.

(*b*) We again compare the two momenta.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\dfrac{mv}{\sqrt{1-v^2/c^2}}} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.75)^2} = 0.66$$

The classical momentum is $\boxed{-34\%}$ in error.

18. The momentum at the higher speed is to be twice the initial momentum. We designate the initial state with the subscript 0 and the final state with the subscript f.

$$\frac{p_{\text{f}}}{p_0} = \frac{\left(\dfrac{mv_{\text{f}}}{\sqrt{1-v_{\text{f}}^2/c^2}}\right)}{\left(\dfrac{mv_0}{\sqrt{1-v_0^2/c^2}}\right)} = 2 \quad\rightarrow\quad \frac{\left(\dfrac{v_{\text{f}}^2}{1-v_{\text{f}}^2/c^2}\right)}{\left(\dfrac{v_0^2}{1-v_0^2/c^2}\right)} = 4 \quad\rightarrow$$

$$\left(\frac{v_f^2}{1-v_f^2/c^2}\right)=4\left(\frac{v_0^2}{1-v_0^2/c^2}\right)=4\left[\frac{(0.22c)^2}{1-(0.22)^2}\right]=0.203c^2 \rightarrow v_f^2=\left(\frac{0.203}{1.203}\right)c^2 \rightarrow v_f=\boxed{0.41c}$$

19. The two momenta, as measured in the frame in which the particle was initially at rest, will be equal to each other in magnitude. The lighter particle is designated with the subscript 1 and the heavier particle with the subscript 2.

$$p_1=p_2 \rightarrow \frac{m_1 v_1}{\sqrt{1-v_1^2/c^2}}=\frac{m_2 v_2}{\sqrt{1-v_2^2/c^2}} \rightarrow$$

$$\frac{v_1^2}{\left(1-v_1^2/c^2\right)}=\left(\frac{m_2}{m_1}\right)^2\frac{v_2^2}{\left(1-v_2^2/c^2\right)}=\left(\frac{6.68\times10^{-27}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right)^2\left[\frac{(0.60c)^2}{1-(0.60)^2}\right]=9.0c^2 \quad \rightarrow$$

$$v_1=\sqrt{0.90}c=\boxed{0.95c}$$

20. We find the proton's momenta using Eq. 26–4.

$$p_{0.45}=\frac{m_p v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}=\frac{m_p(0.45c)}{\sqrt{1-(0.45)^2}}=0.5039m_pc; \quad p_{0.85}=\frac{m_p v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}=\frac{m_p(0.85c)}{\sqrt{1-(0.85)^2}}=1.6136m_pc$$

$$p_{0.98}=\frac{m_p v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}=\frac{m_p(0.98c)}{\sqrt{1-(0.98)^2}}=4.9247m_pc$$

(*a*) $$\left(\frac{p_2-p_1}{p_1}\right)100=\left(\frac{1.6136m_pc-0.5039m_pc}{0.5039m_pc}\right)100=220.2\approx\boxed{220\%}$$

(*b*) $$\left(\frac{p_2-p_1}{p_1}\right)100=\left(\frac{4.9247m_pc-1.6136m_pc}{1.6136m_pc}\right)100=205.2\approx\boxed{210\%}$$

21. The rest energy of the electron is given by Eq. 26–7.

$$E=mc^2=(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})^2=8.199\times10^{-14}\text{ J}\approx\boxed{8.20\times10^{-14}\text{ J}}$$

$$=\frac{(8.199\times10^{-14}\text{ J})}{(1.60\times10^{-13}\text{ J/MeV})}=0.5124\text{ MeV}\approx\boxed{0.512\text{ MeV}}$$

This does not agree with the value inside the front cover of the book because of significant figures. If more significant digits were used for the given values, then a value of 0.511 MeV would be obtained.

22. We find the loss in mass from Eq. 26–7.

$$m=\frac{E}{c^2}=\frac{(200\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(3.00\times10^8\text{ m/s})^2}=3.56\times10^{-28}\text{ kg}\approx\boxed{4\times10^{-28}\text{ kg}}$$

23. We find the mass conversion from Eq. 26–7.

$$m=\frac{E}{c^2}=\frac{(1\times10^{20}\text{ J})}{(3.00\times10^8\text{ m/s})^2}=1111\text{ kg}\approx\boxed{1000\text{ kg}}$$

24. We calculate the mass from Eq. 26–7.

$$m = \frac{E}{c^2} = \frac{1}{c^2}(mc^2) = \frac{1}{c^2}\frac{(1.67\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})^2}{(1.60\times10^{-13}\text{ J/MeV})} = \boxed{939\text{ MeV}/c^2}$$

To get the same answer as inside the front cover of the book, we would need more significant figures for each constant.

25. This increase in mass is found from Eq. 26–8.

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(4.82\times10^4\text{ J})}{(3.00\times10^8\text{ m/s})^2} = \boxed{5.36\times10^{-13}\text{ kg}}$$

Note that this is so small, most chemical reactions are considered to have mass conserved.

26. We find the speed in terms of c. The kinetic energy is given by Eq. 26–5b and the momentum by Eq. 26–4.

$$\upsilon = \frac{(2.90\times10^8\text{ m/s})}{(3.00\times10^8\text{ m/s})} = 0.9667c$$

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-0.9667^2}}-1\right)(938.3\text{ MeV}) = 2728.2\text{ MeV} \approx \boxed{2.7\text{ GeV}}$$

$$p = \gamma m\upsilon = \frac{1}{\sqrt{1-0.9667^2}}(938.3\text{ MeV}/c^2)(0.9667\,c) = 3544\text{ MeV}/c \approx \boxed{3.5\text{ GeV}/c}$$

27. The total energy of the proton is the kinetic energy plus the mass energy. Use Eq. 26–9 to find the momentum.

$$E = \text{KE} + mc^2;$$

$$(pc)^2 = E^2 - (mc^2)^2 = (\text{KE} + mc^2)^2 - (mc^2)^2 = \text{KE}^2 + 2\text{KE}(mc^2)$$

$$pc = \sqrt{\text{KE}^2 + 2\text{KE}(mc^2)} = \text{KE}\sqrt{1+2\frac{mc^2}{\text{KE}}} = (950\text{ MeV})\sqrt{1+2\frac{938.3\text{ MeV}}{950\text{ MeV}}} = 1638\text{ MeV}$$

$$p = 1638\text{ MeV}/c \approx \boxed{1.6\text{ GeV}/c}$$

28. We use Eq. 26–5b to find the speed from the kinetic energy.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-\upsilon^2/c^2}}-1\right)mc^2 \quad \rightarrow$$

$$\upsilon = c\sqrt{1-\frac{1}{\left(\frac{\text{KE}}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{1.12\text{ MeV}}{0.511\text{ MeV}}+1\right)^2}} = \boxed{0.9497c}$$

29. (*a*) The work is the change in kinetic energy. Use Eq. 26–5b. The initial kinetic energy is 0.

$$W = \Delta\text{KE} = \text{KE}_{\text{final}} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-0.985^2}}-1\right)(938.3\text{ MeV}) = 4.499\times10^3\text{ MeV}$$

$$\approx \boxed{4.5\text{ GeV}}$$

(*b*) The momentum of the proton is given by Eq. 26–4.

$$p=\gamma m v=\frac{1}{\sqrt{1-0.985^2}}(938.3\ \mathrm{MeV}/c^2)(0.985c)=5.36\times10^3\ \mathrm{MeV}/c\approx\boxed{5.4\ \mathrm{GeV}/c}$$

$\boxed{30.}$ The relationship between kinetic energy and mass energy is given in Eq. 26–5b.

$$\mathrm{KE}=(\gamma-1)mc^2=0.33mc^2 \quad\rightarrow\quad \frac{1}{\sqrt{1-v^2/c^2}}-1=0.33 \quad\rightarrow\quad v=c\sqrt{1-\frac{1}{(1.33)^2}}=\boxed{0.659c}$$

$\boxed{31.}$ The kinetic energy is equal to the rest energy. We use Eq. 26–5b.

$$\mathrm{KE}=(\gamma-1)mc^2=mc^2 \quad\rightarrow\quad \frac{1}{\sqrt{1-v^2/c^2}}-1=1 \quad\rightarrow\quad v=c\sqrt{1-\frac{1}{(2)^2}}=\boxed{0.866c}$$

The momentum is found from Eq. 26–6a and Eq. 26–9.

$$E=\mathrm{KE}+mc^2=2mc^2 \quad\rightarrow\quad E^2=4m^2c^4$$

$$E^2=p^2c^2+m^2c^4 \quad\rightarrow\quad p^2c^2=E^2-m^2c^4=3m^2c^4 \quad\rightarrow$$

$$p=\sqrt{3}mc=\sqrt{3}(9.11\times10^{-31}\ \mathrm{kg})(3.00\times10^8\ \mathrm{m/s})=\boxed{4.73\times10^{-22}\ \mathrm{kg\cdot m/s}}$$

$$4.73\times10^{-22}\ \mathrm{kg\cdot m/s}\left(\frac{3.0\times10^8\ \mathrm{m/s}}{c}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)\frac{1\ \mathrm{MeV}}{10^6\ \mathrm{eV}}=0.887\ \mathrm{MeV}/c$$

32. Use Eq. 26–5b to calculate the kinetic energy of the proton. Note that the classical answer would be that a doubling of speed would lead to a fourfold increase in kinetic energy. Subscript 1 represents the lower speed $\left(v=\frac{1}{3}c\right)$ and subscript 2 represents the higher speed $\left(v=\frac{2}{3}c\right)$.

$$\mathrm{KE}=(\gamma-1)mc^2 \quad\rightarrow\quad \frac{\mathrm{KE}_2}{\mathrm{KE}_1}=\frac{(\gamma_2-1)mc^2}{(\gamma_1-1)mc^2}=\frac{\left(1-\left(\frac{2}{3}\right)^2\right)^{-1/2}-1}{\left(1-\left(\frac{1}{3}\right)^2\right)^{-1/2}-1}=\frac{(5/9)^{-1/2}-1}{(8/9)^{-1/2}-1}=\frac{0.3416}{0.0607}=\boxed{5.6}$$

33. We find the energy equivalent of the mass from Eq. 26–7.

$$E=mc^2=(1.0\times10^{-3}\ \mathrm{kg})(3.00\times10^8\ \mathrm{m/s})^2=\boxed{9.0\times10^{13}\ \mathrm{J}}$$

We assume that this energy is used to increase the gravitational potential energy.

$$E=mgh \quad\rightarrow\quad m=\frac{E}{hg}=\frac{9.0\times10^{13}\ \mathrm{J}}{(1.0\times10^3\ \mathrm{m})(9.80\ \mathrm{m/s^2})}=\boxed{9.2\times10^9\ \mathrm{kg}}$$

34. The work is the change in kinetic energy. Use Eq. 26–5b. The initial kinetic energy is 0.

$$W_1=(\gamma_{0.90}-1)mc^2;\quad W_2=\mathrm{KE}_{0.99c}-\mathrm{KE}_{0.90c}=(\gamma_{0.99}-1)mc^2-(\gamma_{0.90}-1)mc^2$$

$$\frac{W_2}{W_1}=\frac{(\gamma_{0.99}-1)mc^2-(\gamma_{0.90}-1)mc^2}{(\gamma_{0.90}-1)mc^2}=\frac{\gamma_{0.99}-\gamma_{0.90}}{\gamma_{0.90}-1}=\frac{\frac{1}{\sqrt{1-0.99^2}}-\frac{1}{\sqrt{1-0.90^2}}}{\frac{1}{\sqrt{1-0.90^2}}-1}=\boxed{3.7}$$

35. Each photon has momentum 0.65 MeV/c. Thus each photon has an energy of 0.65 MeV. Assuming that the photons have opposite initial directions, then the total momentum is 0, so the product mass will not be moving. Thus all of the photon energy can be converted into the mass energy of the particle. Thus the heaviest particle would have a mass of $\boxed{1.30\ \text{MeV}/c^2}$, which is 2.32×10^{-30} kg.

36. Since the proton was accelerated by a potential difference of 165 MV, its potential energy decreased by 165 MeV, so its kinetic energy increased from 0 to 165 MeV. Use Eq. 26–5b to find the speed from the kinetic energy.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 \quad\rightarrow$$

$$v = c\sqrt{1-\frac{1}{\left(\frac{\text{KE}}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{165\ \text{MeV}}{938.3\ \text{MeV}}+1\right)^2}} = \boxed{0.5261c}$$

37. Since the electron was accelerated by a potential difference of 31 kV, its potential energy decreased by 31 keV, so its kinetic energy increased from 0 to 31 keV. Use Eq. 26–5b to find the speed from the kinetic energy.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 \quad\rightarrow$$

$$v = c\sqrt{1-\frac{1}{\left(\frac{\text{KE}}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\frac{0.031\ \text{MeV}}{0.511\ \text{MeV}}+1\right)^2}} = \boxed{0.333c}$$

38. We use Eqs. 26–6a and 26–9 in order to find the mass.

$$E^2 = p^2c^2 + m^2c^4 = (\text{KE}+mc^2)^2 = \text{KE}^2 + 2\text{KE}\ mc^2 + m^2c^4 \quad\rightarrow$$

$$m = \frac{p^2c^2-\text{KE}^2}{2\ \text{KE}\ c^2} = \frac{(121\ \text{MeV}/c)^2c^2-(45\ \text{MeV})^2}{2(45\ \text{MeV})c^2} = \boxed{140\ \text{MeV}/c^2} \approx 2.5\times10^{-28}\ \text{kg}$$

The particle is most likely a π^0 meson.

39. (*a*) Since the kinetic energy is half the total energy and the total energy is the kinetic energy plus the rest energy, the kinetic energy must be equal to the rest energy. We also use Eq. 26–5b.

$$\text{KE} = \tfrac{1}{2}E = \tfrac{1}{2}(\text{KE}+mc^2) \quad\rightarrow\quad \text{KE} = mc^2$$

$$\text{KE} = (\gamma-1)mc^2 = mc^2 \quad\rightarrow\quad \gamma = 2 = \frac{1}{\sqrt{1-v^2/c^2}} \quad\rightarrow\quad v = \sqrt{\tfrac{3}{4}}\,c = \boxed{0.866c}$$

(*b*) In this case, the kinetic energy is half the rest energy.

$$\text{KE} = (\gamma-1)mc^2 = \tfrac{1}{2}mc^2 \quad\rightarrow\quad \gamma = \tfrac{3}{2} = \frac{1}{\sqrt{1-v^2/c^2}} \quad\rightarrow\quad v = \sqrt{\tfrac{5}{9}}\,c = \boxed{0.745c}$$

40. We use Eq. 26–5b for the kinetic energy and Eq. 26–4 for the momentum.

$$\frac{v}{c}=\frac{8.65\times10^{7}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}=0.2883$$

$$\text{KE}=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=\left(\frac{1}{\sqrt{1-(0.2883)^2}}-1\right)(938.3\text{ MeV})=\boxed{41.6\text{ MeV}}$$

$$p=\gamma mv=\frac{mv}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{mc^2(v/c)}{\sqrt{1-v^2/c^2}}=\frac{1}{c}\frac{(938.3\text{ MeV})(0.2883)}{\sqrt{1-(0.2883)^2}}=\boxed{283\text{ MeV}/c}$$

Evaluate with the classical expressions.

$$\text{KE}_\text{c}=\tfrac{1}{2}mv^2=\tfrac{1}{2}mc^2\left(\frac{v}{c}\right)^2=\tfrac{1}{2}(938.3\text{ MeV})(0.2883)^2=39.0\text{ MeV}$$

$$p_\text{c}=mv=\frac{1}{c}mc^2\left(\frac{v}{c}\right)=(938.3\text{ MeV}/c)(0.2883)=271\text{ MeV}/c$$

Calculate the percent error.

$$\text{Error}_\text{KE}=\frac{\text{KE}_\text{c}-\text{KE}}{\text{KE}}\times100=\frac{39.0-41.6}{41.6}\times100=\boxed{-6.3\%}$$

$$\text{Error}_p=\frac{p_\text{c}-p}{p}\times100=\frac{271-283}{283}\times100=\boxed{-4.2\%}$$

41. (*a*) The kinetic energy is found from Eq. 26–5b.

$$\text{KE}=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=\left(\frac{1}{\sqrt{1-0.15^2}}-1\right)(1.7\times10^4\text{ kg})(3.00\times10^8\text{ m/s})^2$$
$$=1.751\times10^{19}\text{ J}\approx\boxed{1.8\times10^{19}\text{ J}}$$

(*b*) Use the classical expression and compare the two results.

$$\text{KE}=\tfrac{1}{2}mv^2=\tfrac{1}{2}(1.7\times10^4\text{ kg})[(0.15)(3.00\times10^8\text{ m/s})]^2=1.721\times10^{19}\text{ J}$$

$$\%\text{ error}=\frac{(1.721\times10^{19}\text{ J})-(1.751\times10^{19}\text{ J})}{(1.751\times10^{19}\text{ J})}\times100=\boxed{-1.7\%}$$

The classical value is 1.7% too low.

42. All of the energy, both rest energy and kinetic energy, becomes electromagnetic energy. We use Eq. 26–6b. The masses are the same.

$$E_\text{total}=E_1+E_2=\gamma_1mc^2+\gamma_2mc^2=(\gamma_1+\gamma_2)mc^2=\left(\frac{1}{\sqrt{1-0.53^2}}+\frac{1}{\sqrt{1-0.65^2}}\right)(105.7\text{ MeV})$$
$$=263.7\text{ MeV}\approx\boxed{260\text{ MeV}}$$

43. We let M represent the mass of the new particle. The initial energy is due to both incoming particles, and the final energy is the rest energy of the new particle. Use Eq. 26–6b for the initial energies.

$$E = 2(\gamma mc^2) = Mc^2 \quad \rightarrow \quad M = 2\gamma m = \boxed{\frac{2m}{\sqrt{1-v^2/c^2}}}$$

We assumed that energy is conserved, so there was $\boxed{\text{no loss of energy}}$ in the collision.

The final kinetic energy is 0, so all of the kinetic energy was lost.

$$\text{KE}_{\text{lost}} = \text{KE}_{\text{initial}} = 2(\gamma - 1)mc^2 = \boxed{\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)2mc^2}$$

44. By conservation of energy, the rest energy of the americium nucleus is equal to the rest energies of the other particles plus the kinetic energy of the alpha particle.

$$m_{\text{Am}}c^2 = (m_{\text{Np}} + m_\alpha)c^2 + \text{KE}_\alpha \quad \rightarrow$$

$$m_{\text{Np}} = m_{\text{Am}} - m_\alpha - \frac{\text{KE}_\alpha}{c^2} = 241.05682\text{ u} - 4.00260\text{ u} - \frac{5.5\text{ MeV}}{c^2}\left(\frac{1\text{ u}}{931.49\text{ MeV}/c^2}\right) = \boxed{237.04832\text{ u}}$$

45. We use Eqs. 26–6a and 26–9.

$$E = \text{KE} + mc^2;\ (pc)^2 = E^2 - (mc^2)^2 = (\text{KE} + mc^2)^2 - (mc^2)^2 = \text{KE}^2 + 2\text{KE}(mc^2) \quad \rightarrow$$

$$\boxed{p = \frac{\sqrt{\text{KE}^2 + 2\text{KE}(mc^2)}}{c}}$$

46. The kinetic energy of 998 GeV is used to find the speed of the protons. Since the energy is 1000 times the mass, we expect the speed to be very close to c. Use Eq. 26–5b.

$$\text{KE} = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 \quad \rightarrow$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{\text{KE}}{mc^2} + 1\right)^2}} = c\sqrt{1 - \frac{1}{\left(\frac{998\text{ GeV}}{0.938\text{ GeV}} + 1\right)^2}} = c \text{ (to 7 significant figures)}$$

$$B = \frac{\gamma m v^2}{rqv} = \frac{\gamma m v}{rq} \approx \frac{\left(\frac{K}{mc^2} - 1\right)mc}{rq} = \frac{\left(\frac{998\text{ GeV}}{0.938\text{ GeV}} - 1\right)(1.673\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})}{(1.0\times10^3\text{ m})(1.60\times10^{-19}\text{ C})} = \boxed{3.3\text{ T}}$$

$\boxed{47.}$ Take the positive direction to be the direction of motion of the person's rocket. Consider the Earth as reference frame S and the person's rocket as reference frame S′. Then the speed of the rocket relative to Earth is $v = 0.40c$, and the speed of the meteor relative to the rocket is $u' = 0.40c$. Calculate the speed of the meteor relative to the rocket using Eq. 26–10.

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{(0.40c + 0.40c)}{[1 + (0.40)(0.40)]} = \boxed{0.69c}$$

48. (*a*) Take the positive direction to be the direction of motion of spaceship 1. Consider spaceship 2 as reference frame S and the Earth as reference frame S'. The velocity of the Earth relative to spaceship 2 is $v = 0.60c$. The velocity of spaceship 1 relative to the Earth is $u' = 0.60c$. Solve for the velocity of spaceship 1 relative to spaceship 2, u, using Eq. 26–10.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(0.60c+0.60c)}{[1+(0.60)(0.60)]} = \boxed{0.882c}$$

(*b*) Now consider spaceship 1 as reference frame S. The velocity of the Earth relative to spaceship 1 is $v = -0.60c$. The velocity of spaceship 2 relative to the Earth is $u' = -0.60c$. Solve for the velocity of spaceship 2 relative to spaceship 1, u, using Eq. 26–10.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(-0.60c-0.60c)}{[1+(-0.60)(-0.60)]} = \boxed{-0.882c}$$

As expected, the two relative velocities are the opposite of each other.

49. (*a*) We take the positive direction in the direction of the first spaceship. We choose reference frame S as the Earth and reference frame S' as the first spaceship. So $v = 0.65c$. The speed of the second spaceship relative to the first spaceship is $u' = 0.82c$. We use Eq. 26–10 to solve for the speed of the second spaceship relative to the Earth, u.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(0.82c+0.65c)}{[1+(0.65)(0.82)]} = \boxed{0.959c}$$

(*b*) The only difference is now that $u' = -0.82c$.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(-0.82c+0.65c)}{[1+(0.65)(-0.82)]} = -0.36c$$

The Problem asks for the speed, which is $\boxed{0.36c}$.

50. We take the positive direction as the direction of the *Enterprise*. Consider the alien vessel as reference frame S and the Earth as reference frame S'. The velocity of the Earth relative to the alien vessel is $v = -0.60c$. The velocity of the *Enterprise* relative to the Earth is $u' = 0.90c$. Solve for the velocity of the *Enterprise* relative to the alien vessel, u, using Eq. 26–10.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{(0.90c-0.60c)}{[1+(-0.60)(0.90)]} = \boxed{0.65c}$$

We could also have made the *Enterprise* reference frame S, with $v = -0.90c$, and the velocity of the alien vessel relative to the Earth as $u' = 0.60c$. The same answer would result.
Choosing the two spacecraft as the two reference frames would also work. Let the alien vessel be reference frame S and the *Enterprise* be reference frame S'. Then we have the velocity of the Earth relative to the alien vessel as $u = -0.60c$ and the velocity of the Earth relative to the *Enterprise* as $u' = -0.90c$. We solve for v, the velocity of the *Enterprise* relative to the alien vessel.

$$u = \frac{(u' + v)}{\left(1 + \frac{vu'}{c^2}\right)} \rightarrow v = \frac{u - u'}{\left(1 - \frac{u'u}{c^2}\right)} = \frac{(-0.60c) - (-0.90c)}{\left(1 - \frac{(-0.90c)(-0.60c)}{c^2}\right)} = \boxed{0.65c}$$

51. We take the positive direction as the direction of the motion of the second pod. Consider the first pod as reference frame S and the spacecraft as reference frame S′. The velocity of the spacecraft relative to the first pod is $v = 0.70c.$ The velocity of the second pod relative to the spacecraft is $u' = 0.80c.$ Solve for the velocity of the second pod relative to the first pod, $u,$ using Eq. 26–10.

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{(0.80c + 0.70c)}{[1 + (0.70)(0.80)]} = \boxed{0.962c}$$

52. We take the positive direction as the direction of motion of rocket A. Consider rocket A as reference frame S and the Earth as reference frame S′. The velocity of the Earth relative to rocket A is $v = -0.65c.$ The velocity of rocket B relative to the Earth is $u' = 0.95c.$ Solve for the velocity of rocket B relative to rocket A, $u,$ using Eq. 26–10.

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{(0.95c - 0.65c)}{[1 + (-0.65)(0.95)]} = \boxed{0.78c}$$

Note that a Galilean analysis would have resulted in $u = 0.30c.$

53. We assume that the given speed of $0.90c$ is relative to the planet that you are approaching. We take the positive direction as the direction that you are traveling. Consider your spaceship as reference frame S and the planet as reference frame S′. The velocity of the planet relative to you is $v = -0.90c.$ The velocity of the probe relative to the planet is $u' = 0.95c.$ Solve for the velocity of the probe relative to your spaceship, $u,$ using Eq. 26–10.

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{(0.95c - 0.90c)}{[1 + (-0.90)(0.95)]} = 0.34c \approx \boxed{0.3c}$$

54. The kinetic energy is given by Eq. 26–5b.

$$\text{KE} = (\gamma - 1)mc^2 = mc^2 \rightarrow \gamma = 2 = \frac{1}{\sqrt{1 - v^2/c^2}} \rightarrow v = \sqrt{\frac{3}{4}}c = \boxed{0.866c}$$

Notice that the mass of the particle does not affect the result.

55. (*a*) To travelers on the spacecraft, the distance to the star is contracted, according to Eq. 26–3a. This contracted distance is to be traveled in 4.9 years. Use that time with the contracted distance to find the speed of the spacecraft.

$$v = \frac{\Delta x_{\text{spacecraft}}}{\Delta t_{\text{spacecraft}}} = \frac{\Delta x_{\text{Earth}}\sqrt{1 - v^2/c^2}}{\Delta t_{\text{spacecraft}}} \rightarrow$$

$$v = c\frac{1}{\sqrt{1 + \left(\frac{c\Delta t_{\text{spacecraft}}}{\Delta x_{\text{Earth}}}\right)^2}} = c\frac{1}{\sqrt{1 + \left(\frac{4.9\text{ ly}}{4.3\text{ ly}}\right)^2}} = 0.6596c \approx \boxed{0.66c}$$

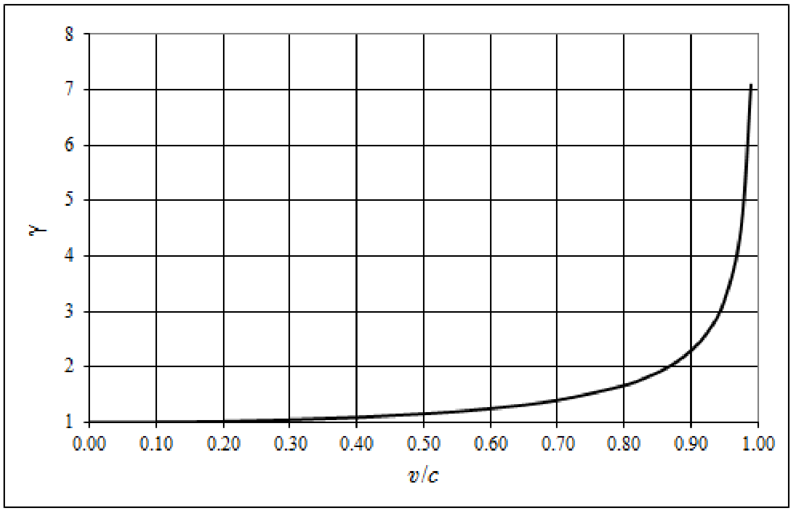
(*b*) Find the elapsed time according to observers on Earth, using Eq. 26–1a.

$$\Delta t_{\text{Earth}} = \frac{\Delta t_{\text{spaceship}}}{\sqrt{1-v^2/c^2}} = \frac{4.9\text{ yr}}{\sqrt{1-0.6596^2}} = \boxed{6.5\text{ yr}}$$

Note that this agrees with the time found from distance and speed.

$$t_{\text{Earth}} = \frac{\Delta x_{\text{Earth}}}{v} = \frac{4.3\text{ ly}}{0.6596c} = 6.5\text{ yr}$$

56. The numerical values and graph were generated in a spreadsheet. The graph is shown also.



v/c	0.00	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99
γ	1.000	1.000	1.001	1.005	1.021	1.048	1.091	1.155	1.250	1.400	1.667	2.294	3.203	7.089

57. Time dilation effects have changed the proper time for one heartbeat, which is given as 1 second, to a dilated time of 2.4 seconds as observed on Earth. Use Eq. 26–1a.

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad \rightarrow \quad = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{1.0\text{ s}}{2.4\text{ s}}\right)^2} = \boxed{0.91c}$$

58. (*a*) We find the speed from Eq. 26–5b.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 = 14{,}000mc^2 \quad \rightarrow$$

$$v = c\sqrt{1-\left(\frac{1}{14{,}001}\right)^2} \approx c - \frac{c}{2}\left(\frac{1}{14{,}001}\right)^2 \quad \rightarrow$$

$$c - v = \frac{c}{2}\left(\frac{1}{14{,}001}\right)^2 = \frac{(3.00\times10^8\text{ m/s})}{2}\left(\frac{1}{14{,}001}\right)^2 = \boxed{0.77\text{ m/s}}$$

(*b*) The tube will be contracted in the rest frame of the electron, according to Eq. 26–3a.

$$\ell_0 = \ell\sqrt{1-v^2/c^2} = (3.0\times10^3\text{ m})\sqrt{1-\left[1-\left(\frac{1}{14{,}001}\right)^2\right]} = \boxed{0.21\text{ m}}$$

59. The minimum energy required would be the energy to produce the pair with no kinetic energy, so the total energy is their rest energy. They both have the same mass. Use Eq. 26–7.

$$E = 2mc^2 = 2(0.511\text{ MeV}) = \boxed{1.022\text{ MeV}(1.637\times10^{-13}\text{ J})}$$

60. The wattage times the time is the energy required. We use Eq. 26–7 to calculate the mass.

$$E = Pt = mc^2 \quad\rightarrow\quad m = \frac{Pt}{c^2} = \frac{(75\text{ W})(3.16\times10^7\text{ s})}{(3.00\times10^8\text{ m/s})^2}\left(\frac{1000\text{ g}}{1\text{ kg}}\right) = \boxed{2.6\times10^{-5}\text{ g}}$$

61. The kinetic energy available comes from the decrease in rest energy.

$$\text{KE} = m_\text{n}c^2 - (m_\text{p}c^2 + m_ec^2 + m_\nu c^2) = 939.57\text{ MeV} - (938.27\text{ MeV} + 0.511\text{ MeV} + 0) = \boxed{0.79\text{ MeV}}$$

62. The increase in kinetic energy comes from the decrease in potential energy.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 \quad\rightarrow$$

$$v = c\left(1 - \frac{1}{\left(\dfrac{\text{KE}}{mc^2}+1\right)^2}\right)^{1/2} = c\left(1 - \frac{1}{\left(\dfrac{6.20\times10^{-14}\text{ J}}{(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})^2}+1\right)^2}\right)^{1/2} = \boxed{0.822c}$$

63. (*a*) We find the rate of mass loss from Eq. 26–7.

$$E = mc^2 \quad\rightarrow\quad \Delta E = (\Delta m)c^2 \quad\rightarrow$$

$$\frac{\Delta m}{\Delta t} = \frac{1}{c^2}\left(\frac{\Delta E}{\Delta t}\right) = \frac{4\times10^{26}\text{ J/s}}{(3.00\times10^8\text{ m/s})^2} = 4.44\times10^9\text{ kg/s} \approx \boxed{4\times10^9\text{ kg/s}}$$

(*b*) Find the time from the mass of the Sun and the rate determined in part (*a*).

$$\Delta t = \frac{m_\text{Earth}}{\Delta m/\Delta t} = \frac{(5.98\times10^{24}\text{ kg})}{(4.44\times10^9\text{ kg/s})(3.156\times10^7\text{ s/yr})} = 4.27\times10^7\text{ yr} \approx \boxed{4\times10^7\text{ yr}}$$

(*c*) We find the time for the Sun to lose all of its mass at this same rate.

$$\Delta t = \frac{m_\text{Sun}}{\Delta m/\Delta t} = \frac{(1.99\times10^{30}\text{ kg})}{(4.44\times10^9\text{ kg/s})(3.156\times10^7\text{ s/yr})} = 1.42\times10^{13}\text{ yr} \approx \boxed{1\times10^{13}\text{ yr}}$$

64. The total binding energy is the energy required to provide the increase in rest energy.

$$E = [(2m_\text{p+e} + 2m_\text{n}) - m_\text{He}]c^2$$

$$= [2(1.00783\text{ u}) + 2(1.00867\text{ u}) - 4.00260\text{ u}]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right) = \boxed{28.32\text{ MeV}}$$

65. The momentum is given by Eq. 26–4, and the energy is given by Eq. 26–6b and Eq. 26–9.

$$p = \gamma mv = \frac{\gamma mc^2 v}{c^2} = \frac{Ev}{c^2} \quad\rightarrow\quad v = \boxed{\frac{pc^2}{E}} = \frac{pc^2}{\sqrt{m^2c^4 + p^2c^2}} = \boxed{\frac{pc}{\sqrt{m^2c^2 + p^2}}}$$

66. (*a*) The magnitudes of the momenta are equal. Use Eq. 26–4.

$$p = \gamma m v = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{1}{c}\frac{mc^2(v/c)}{\sqrt{1-v^2/c^2}} = \frac{1}{c}\frac{(938.3\text{ MeV})(0.990)}{\sqrt{1-0.990^2}} = 6585\text{ MeV}/c$$

$$\approx \boxed{6.59\text{ GeV}/c} = (6.585\text{ GeV}/c)\left(\frac{1\,c}{3.00\times10^8\text{ m/s}}\right)\left(\frac{1.602\times10^{-10}\text{ J/GeV}}{1\text{ GeV}}\right)$$

$$= \boxed{3.52\times10^{-18}\text{ kg}\cdot\text{m/s}}$$

(*b*) Because the protons are moving in opposite directions, the vector sum of the momenta is $\boxed{0}$.

(*c*) In the reference frame of one proton, the laboratory is moving at 0.990*c*. The other proton is moving at $+0.990c$ relative to the laboratory. We find the speed of one proton relative to the other, and then find the momentum of the moving proton in the rest frame of the other proton by using that relative velocity.

$$u = \frac{(v+u')}{\left(1+\frac{vu'}{c^2}\right)} = \frac{[0.990c+(0.990c)]}{[1+(0.990)(0.990)]} = \left(\frac{2(0.990)c}{1+0.990^2}\right) \approx 0.99995c$$

$$p = \gamma m u = \frac{mu}{\sqrt{1-u^2/c^2}} = \frac{1}{c}\frac{mc^2(u/c)}{\sqrt{1-u^2/c^2}} = \frac{1}{c}\frac{(938.3\text{ MeV})\left(\frac{2(0.990)}{1+0.990^2}\right)}{\sqrt{1-\left(\frac{2(0.990)}{1+0.990^2}\right)^2}} = 93358\text{ MeV}/c$$

$$\approx \boxed{93.4\text{ GeV}/c} = (93.358\text{ GeV}/c)\left(\frac{1c}{3.00\times10^8\text{ m/s}}\right)\left(\frac{1.602\times10^{-10}\text{ J/GeV}}{1\text{ GeV}}\right)$$

$$= \boxed{4.99\times10^{-17}\text{ kg}\cdot\text{m/s}}$$

67. We find the loss in mass from Eq. 26–8.

$$\Delta m = \frac{\Delta E}{c^2} = \frac{484\times10^3\text{ J}}{(3.00\times10^8\text{ m/s})^2} = \boxed{5.38\times10^{-12}\text{ kg}}$$

Two moles of water has a mass of 36.0×10^{-3} kg. Find the percentage of mass lost.

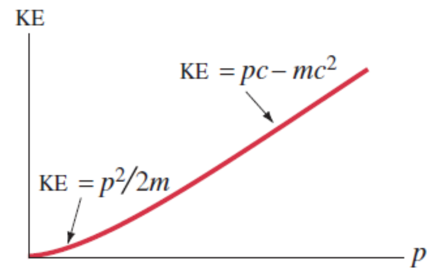
$$\frac{5.38\times10^{-12}\text{ kg}}{36.0\times10^{-3}\text{ kg}} = 1.49\times10^{-10} = \boxed{1.49\times10^{-8}\ \%}$$

68. Use Eq. 26–5b for kinetic energy and Eq. 26–7 for rest energy.

$$\text{KE} = (\gamma-1)m_{Enterprise}c^2 = m_{\text{converted}}c^2 \quad\rightarrow$$

$$m_{\text{converted}} = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)m_{Enterprise} = \left(\frac{1}{\sqrt{1-0.10^2}}-1\right)(6\times10^9\text{ kg}) = \boxed{3\times10^7\text{ kg}}$$

69. (*a*) For a particle of nonzero mass, we derive the following relationship between kinetic energy and momentum.



$$E = \text{KE} + mc^2;$$

$$(pc)^2 = E^2 - (mc^2)^2 = (\text{KE} + mc^2)^2 - (mc^2)^2 \quad \rightarrow$$

$$\text{KE} = \sqrt{(pc)^2 + (mc^2)^2} - mc^2$$

This is the relationship that is graphed. But note the two extreme regions. If the momentum is small, then we have the classical relationship.

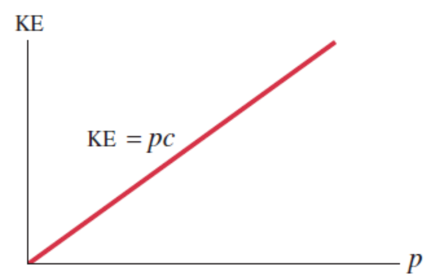
$$\text{KE} = -mc^2 + \sqrt{(mc^2)^2 + (pc)^2} = -mc^2 + mc^2\sqrt{1 + \left(\frac{pc}{mc^2}\right)^2}$$

$$\approx -mc^2 + mc^2\left(1 + \tfrac{1}{2}\left(\frac{pc}{mc^2}\right)^2\right) = \frac{p^2}{2m}$$

If the momentum is large, then we have this relationship, which is linear.

$$\text{KE} = -mc^2 + \sqrt{(mc^2)^2 + (pc)^2} \approx pc - mc^2$$

(*b*) For a particle of zero mass, the relationship is simply $\text{KE} = pc$. See the adjacent graph.



70. (*a*) We set the kinetic energy of the spacecraft equal to the rest energy of an unknown mass, *m*. Use Eqs. 26–5a and 26–7.

$$\text{KE} = (\gamma - 1)m_{\text{ship}}c^2 = mc^2 \quad \rightarrow$$

$$m = (\gamma - 1)m_{\text{ship}} = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)m_{\text{ship}} = \left(\frac{1}{\sqrt{1 - 0.70^2}} - 1\right)(1.6\times10^5\text{ kg}) = \boxed{6.4\times10^4\text{ kg}}$$

(*b*) From the Earth's point of view, the distance is 35 ly and the speed is $0.70c$. Those data are used to calculate the time from the Earth frame, and then Eq. 26–1a is used to calculate the time in the spaceship frame.

$$\Delta t = \frac{d}{v} = \frac{(35\text{ yr})c}{0.70c} = 50\text{ yr}; \quad \Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (50\text{ yr})\sqrt{1 - 0.70^2} = \boxed{36\text{ yr}}$$

71. We assume that one particle is moving in the negative direction in the laboratory frame and the other particle is moving in the positive direction. We consider the particle moving in the negative direction as reference frame S and the laboratory as reference frame S′. The velocity of the laboratory relative to the negative-moving particle is $v = 0.82c$, and the velocity of the positive-moving particle relative to the laboratory frame is $u' = 0.82c$. Solve for the velocity of the positive-moving particle relative to the negative-moving particle, u. Use Eq. 26–10.

$$u = \frac{(v + u')}{\left(1 + \frac{vu'}{c^2}\right)} = \frac{(0.82c + 0.82c)}{[1 + (0.82)(0.82)]} = \boxed{0.981c}$$

72. We consider the motion from the reference frame of the spaceship. The passengers will see the trip distance contracted, as given by Eq. 26–3a. They will measure their speed to be that contracted distance divided by the years of travel time (as measured on the ship). Use that speed to find the work done (the kinetic energy of the ship).

$$\upsilon = \frac{\ell}{\Delta t_0} = \frac{\ell_0\sqrt{1-\upsilon^2/c^2}}{\Delta t_0} \quad\rightarrow\quad \frac{\upsilon}{c} = \frac{1}{\left(\sqrt{1+\left(\frac{c\Delta t_0}{\ell_0}\right)^2}\right)} = \frac{1}{\left(\sqrt{1+\left(\frac{1.0\text{ ly}}{6.6\text{ ly}}\right)^2}\right)} = 0.9887c$$

$$W = \Delta\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-\upsilon^2/c^2}}-1\right)mc^2$$

$$= \left(\frac{1}{\sqrt{1-0.9887^2}}-1\right)(3.6\times10^4\text{ kg})(3.00\times10^8\text{ m/s})^2 = \boxed{1.8\times10^{22}\text{ J}}$$

Note that, according to Problem 73, this is about $100\times$ the annual energy consumption of the United States.

73. The kinetic energy is given by Eq. 26–5b.

$$\text{KE} = (\gamma-1)mc^2 = \left(\frac{1}{\sqrt{1-\upsilon^2/c^2}}-1\right)mc^2 = \left(\frac{1}{\sqrt{1-(0.90)^2}}-1\right)(14{,}500\text{ kg})(3.00\times10^8\text{ m/s})^2$$

$$= \boxed{1.7\times10^{21}\text{ J}}$$

The spaceship's kinetic energy is approximately 20 times as great as the annual U.S. energy use.

74. The pi meson decays at rest, so the momentum of the muon and the neutrino must each have the same magnitude (and opposite directions). The neutrino has no mass, and the total energy must be conserved. We combine these relationships using Eq. 26–9.

$$E_v = (p_v^2c^2 + m_v^2c^4)^{1/2} = p_vc; \quad p_\mu = p_v = p$$

$$E_\pi = E_\mu + E_v \quad\rightarrow\quad m_\pi c^2 = (p_\mu^2c^2 + m_\mu^2c^4)^{1/2} + p_vc = (p^2c^2 + m_\mu^2c^4)^{1/2} + pc \quad\rightarrow$$

$$m_\pi c^2 - pc = (p^2c^2 + m_\mu^2c^4)^{1/2} \quad\rightarrow\quad (m_\pi c^2 - pc)^2 = (p^2c^2 + m_\mu^2c^4)$$

Solve for the momentum.

$$m_\pi^2c^4 - 2m_\pi c^2pc + p^2c^2 = p^2c^2 + m_\mu^2c^4 \quad\rightarrow\quad pc = \frac{m_\pi^2c^2 - m_\mu^2c^2}{2m_\pi}$$

Write the kinetic energy of the muon using Eqs. 26–6b and 26–9.

$$K_\mu = E_\mu - m_\mu c^2; \quad E_\mu = E_\pi - E_v = m_\mu c^2 - pc \quad\rightarrow$$

$$K_\mu = (m_\pi c^2 - pc) - m_\mu c^2 = m_\pi c^2 - m_\mu c^2 - \frac{(m_\pi^2c^2 - m_\mu^2c^2)}{2m_\pi}$$

$$= \frac{2m_\pi(m_\pi c^2 - m_\mu c^2)}{2m_\pi} - \frac{(m_\pi^2c^2 - m_\mu^2c^2)}{2m_\pi}$$

$$= \frac{(2m_\pi^2 - 2m_\mu m_\pi - m_\pi^2 + m_\mu^2)c^2}{2m_\pi} = \frac{(m_\pi^2 - 2m_\mu m_\pi + m_\mu^2)c^2}{2m_\pi} = \boxed{\frac{(m_\pi - m_\mu)^2c^2}{2m_\pi}}$$

$\boxed{75.}$ (*a*) Earth observers see the ship as contracted, as given by Eq. 26–3a.

$$\ell = \ell_0\sqrt{1-v^2/c^2} = (23\text{ m})\sqrt{1-(0.75)^2} = 15.2\text{ m} \approx \boxed{15\text{ m}}$$

(*b*) Earth observers see the launch as dilated (lengthened) in time, as given by Eq. 26–1a.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{28\text{ min}}{\sqrt{1-(0.75)^2}} = 42.3\text{ min} \approx \boxed{42\text{ min}}$$

76. (*a*) The relative speed can be calculated in either frame and will be the same in both frames. The time measured on Earth will be longer than that measured on the spaceship, by Eq. 26–1a.

$$v = \frac{\Delta x_{\text{Earth}}}{\Delta t_{\text{Earth}}};\ \Delta t_{\text{Earth}} = \frac{\Delta t_{\text{spaceship}}}{\sqrt{1-v^2/c^2}} = \frac{\Delta t_{\text{spaceship}}}{\sqrt{1-\left(\dfrac{\Delta x_{\text{Earth}}}{c\Delta t_{\text{Earth}}}\right)^2}} \quad \rightarrow$$

$$(\Delta t_{\text{Earth}})^2 - \left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2 = (\Delta t_{\text{spaceship}})^2 \quad \rightarrow \quad (\Delta t_{\text{Earth}})^2 - \left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2 = (\Delta t_{\text{spaceship}})^2 \quad \rightarrow$$

$$\Delta t_{\text{Earth}} = \sqrt{\left(\frac{\Delta x_{\text{Earth}}}{c}\right)^2 + (\Delta t_{\text{spaceship}})^2} = \sqrt{(6.0\text{ yr})^2 + (3.50\text{ yr})^2} = 6.946\text{ yr} \approx \boxed{6.9\text{ yr}}$$

(*b*) The distance as measured by the spaceship will be contracted.

$$v = \frac{\Delta x_{\text{Earth}}}{\Delta t_{\text{Earth}}} = \frac{\Delta x_{\text{spaceship}}}{\Delta t_{\text{spaceship}}} \quad \rightarrow$$

$$\Delta x_{\text{spaceship}} = \frac{\Delta t_{\text{spaceship}}}{\Delta t_{\text{Earth}}}\Delta x_{\text{Earth}} = \frac{3.50\text{ yr}}{6.946\text{ yr}}(6.0\text{ ly}) = 3.023\text{ ly} \approx \boxed{3.0\text{ ly}}$$

This distance is the same as that found using the length contraction relationship.

77. (*a*) The kinetic energy is 5.00 times greater than the rest energy. Use Eq. 26–5b.

$$\text{KE} = (\gamma-1)mc^2 = 5.00mc^2 \quad \rightarrow \quad \frac{1}{\sqrt{1-v^2/c^2}} = 6.00 \quad \rightarrow \quad v = c\sqrt{1-\frac{1}{(6.00)^2}} = \boxed{0.986c}$$

(*b*) The kinetic energy is 999 times greater than the rest energy. We use the binomial expansion.

$$\text{KE} = (\gamma-1)mc^2 = 999mc^2 \quad \rightarrow \quad \frac{1}{\sqrt{1-v^2/c^2}} = 1000 \quad \rightarrow$$

$$v = c\sqrt{1-\frac{1}{(1000)^2}} \approx c\left(1-\tfrac{1}{2}\left(\frac{1}{1000}\right)^2\right) = \boxed{c(1-5.0\times10^{-7})}$$

78. Every observer will measure the speed of a beam of light to be $\boxed{c}$. Check it with Eq. 26–10.

$$u = \frac{v+u'}{1+vu'} = \frac{(-c)+0.70c}{1+(0.70)(-1)} = -c$$

The beam's speed as a positive value, relative to Earth, is c.

79. From the boy's frame of reference, the pole remains at rest with respect to him. As such, the pole will always remain 13.0 m long. As the boy runs toward the barn, relativity requires that the (relatively moving) barn contract in size, making the barn even shorter than its rest length of 13.0 m. Thus it is impossible, in the boy's frame of reference, for the barn to be longer than the pole. So according to the boy, the pole will never completely fit within the barn.

In the frame of reference at rest with respect to the barn, it is possible for the pole to be shorter than the barn. We use Eq. 26–3a to calculate the speed that the boy would have to run for the contracted length of the pole, ℓ, to equal the length of the barn.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-\ell^2/\ell_0^2} = c\sqrt{1-(10.0\text{ m})^2/(13.0\text{ m})^2} = 0.6390c$$

80. At the North Pole the clock is at rest, while the clock on the Equator travels the circumference of the Earth each day. We divide the circumference of the Earth by the length of the day to determine the speed of the equatorial clock. We set the dilated time equal to 2.0 years and solve for the change in rest times for the two clocks.

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.38\times10^6\text{ m})}{(24\text{ h})(3600\text{ s/h})} = 464\text{ m/s}$$

$$\Delta t = \frac{\Delta t_{0,\text{eq}}}{\sqrt{1-v^2/c^2}} \quad \rightarrow \quad \Delta t_{0,\text{eq}} = \Delta t\sqrt{1-v^2/c^2} \approx \Delta t\left(1+\frac{v^2}{2c^2}\right)$$

$$\Delta t = \frac{\Delta t_{0,\text{pole}}}{\sqrt{1-0}} \quad \rightarrow \quad \Delta t_{0,\text{pole}} = \Delta t$$

$$\left|\Delta t_{0,\text{eq}} - \Delta t_{0,\text{pole}}\right| = \Delta t\left(1+\frac{v^2}{2c^2}\right) - \Delta t$$

$$= \Delta t\frac{v^2}{2c^2} = \frac{(2.0\text{ yr})(464\text{ m/s})^2(3.156\times10^7\text{ s/yr})}{2(3.00\times10^8\text{ m/s})^2} = \boxed{75\ \mu\text{s}}$$

81. We treat the Earth as the stationary frame and the airplane as the moving frame. The elapsed time in the airplane will be dilated to the observers on the Earth. Use Eq. 26–1a.

$$t_{\text{Earth}} = \frac{2\pi r_{\text{Earth}}}{v}; \quad t_{\text{plane}} = t_{\text{Earth}}\sqrt{1-v^2/c^2} = \frac{2\pi r_{\text{Earth}}}{v}\sqrt{1-v^2/c^2}$$

$$\Delta t = t_{\text{Earth}} - t_{\text{plane}} = \frac{2\pi r_{\text{Earth}}}{v}\left(1-\sqrt{1-v^2/c^2}\right) \approx \frac{2\pi r_{\text{Earth}}}{v}\left[1-\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)\right] = \frac{\pi r_{\text{Earth}} v}{c^2}$$

$$= \frac{\pi(6.38\times10^6\text{ m})\left[1300\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]}{(3.00\times10^8\text{ m/s})^2} = \boxed{8.0\times10^{-8}\text{ s}}$$

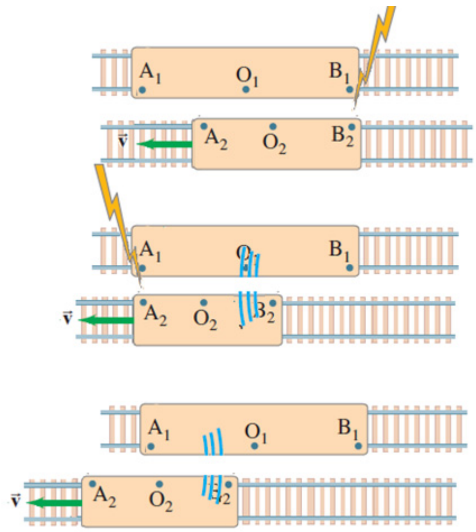
Solutions to Search and Learn Problems

1. To determine Mr Tompkins' speed, examine the clock at Lloyd's Bank. It should be circular, so measure the height of the clock to find the proper length (260 units) and the width of the clock to find the contracted length (120 units). Use these lengths in Eq. 26–3a with $c = 10$ m/s to find his speed.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad \rightarrow \quad v = c\sqrt{1-(\ell/\ell_0)^2} = 10\text{ m/s}\sqrt{1-(120/260)^2} = \boxed{8.9\text{ m/s}}$$

This is not how he would see the world, because there would be additional distortions due to the time it takes light to travel from objects to him. More distant objects would appear rotated.

2. In O_2's frame of reference, O_2 observes that both trains are the same length. But O_2 is observing the contracted length of O_1's train. Thus the proper length of O_1's train is longer than O_2's train. In O_1's frame of reference, O_2 is moving; therefore, O_2's train is length contracted, making it even shorter. When the end of O_2's train passes the end of O_1's train, the first lightning bolt strikes, and both observers record the strike as occurring at the end of their train. A short while later, O_1 observes that O_2's train moves down the track until the front ends of the trains are at the same location. When this occurs, the second lightning bolt strikes, and both observers note that it occurs at the front of their train. Since to observer O_2 the trains are the same length, the strikes occurred at the same time. To observer O_1, the trains are not the same length; therefore, train O_2 had to move between strikes so that the ends were aligned during each strike.



3. The change in length is the difference between the initial length and the contracted length. The binomial expansion is used to simplify the square root expression.

$$\Delta\ell = \ell_0 - \ell = \ell_0 - \ell_0\sqrt{1-v^2/c^2} = \ell_0\left(1-\sqrt{1-v^2/c^2}\right)$$

$$\approx \ell_0\left[1-\left(1-\tfrac{1}{2}v^2/c^2\right)\right] = \tfrac{1}{2}\ell_0 v^2/c^2$$

$$= \tfrac{1}{2}(500\text{ m})(100\text{ km/h})^2\left(\frac{\text{m/s}}{3.6\text{ km/h}}\right)^2 \Big/ (3.00\times10^8\text{ m/s})^2 = \boxed{2.14\times10^{-12}\text{ m}}$$

This length contraction is less than the size of an atom.

4. If the ship travels $0.90c$ at 30° above the horizontal, then we can consider the motion in two stages: horizontal motion of $0.90c\cos30° = 0.779c$ and vertical motion of $0.90c\sin30° = 0.45c.$ The length of the painting will be contracted due to the horizontal motion, and the height will be contracted due to the vertical motion.

$$\ell = \ell_0\sqrt{1-v_x^2/c^2} = 1.50\text{ m}\sqrt{1-0.779^2} = \boxed{0.94\text{ m}}$$

$$h = h_0\sqrt{1-v_y^2/c^2} = 1.00\text{ m}\sqrt{1-0.45^2} = \boxed{0.89\text{ m}}$$

5. Using nonrelativistic mechanics, we would find that the muon could only travel a distance of $(3.00\times10^8\text{ m/s})(2.20\times10^{-6}\text{ s}) = 660\text{ m}$ before decaying. But from an Earth-based reference frame, the muon's "clock" would run slowly, so the time to use is the dilated time. The speed in the Earth's reference frame is then the distance of 10 km divided by the dilated time.

$$v = \frac{\Delta\ell_0}{\Delta t} = \frac{\Delta\ell_0}{\gamma\Delta t_0} = \frac{\Delta\ell_0\sqrt{(1 - v^2/c^2)}}{\Delta t_0} \quad \rightarrow$$

$$\frac{v}{c} = \frac{\Delta\ell_0}{\sqrt{c^2(\Delta t_0)^2 + (\Delta\ell_0)^2}} = \frac{\left(3.0\times10^4\,\text{m}\right)}{\sqrt{(3.00\times10^8\ \text{m/s})^2(2.20\times10^{-6}\ \text{s})^2 + (3.0\times10^4\ \text{m})^2}}$$

$$= 0.9997581c \quad \rightarrow \quad \boxed{v = 0.99976c}$$

Alternatively, we could find the speed for the muon to travel a contracted length during its "normal" lifetime. The contracted length is $\Delta\ell = \Delta\ell_0/\gamma$, so the speed would be $v = \frac{\Delta\ell}{\Delta t_0} = \left(\frac{\Delta\ell_0}{\gamma}\right)\frac{1}{\Delta t_0} = \frac{\Delta\ell_0}{\gamma\Delta t_0}$, which is the same as the earlier expression.

The kinetic energy is found from Eq. 26–5b.

$$\text{KE} = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{(1 - v^2/c^2)}} - 1\right)mc^2 = \left(\frac{1}{\sqrt{(1 - (0.9997581)^2)}} - 1\right)(105.7\ \text{MeV})$$

$$= \boxed{4700\ \text{MeV}}$$

6. The electrostatic force provides the radial acceleration, so those two forces are equated. We solve that relationship for the speed of the electron.

$$\left|F_{\text{electrostatic}}\right| = \left|F_{\text{centripetal}}\right| \quad \rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_{\text{electron}}v^2}{r} \quad \rightarrow$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_{\text{electron}}r}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{9.11\times10^{-31}\ \text{kg}}\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.53\times10^{-10}\ \text{m})}} = 2.18\times10^6\ \text{m/s} = 0.0073c$$

Because this is much less than $0.1c$, the electron is $\boxed{\text{not relativistic}}$.

27

EARLY QUANTUM THEORY AND MODELS OF THE ATOM

Responses to Questions

1. The lightbulb will not produce light as white as the Sun, since the peak of the lightbulb's emitted light is in the infrared. The lightbulb will appear more yellowish than the Sun. The Sun has a spectrum that peaks in the visible range.

2. The difficulty with seeing objects in the dark is that although all objects emit radiation, only a small portion of the electromagnetic spectrum can be detected by our eyes. Usually objects are so cool that they only give off very long wavelengths of light (infrared), which our eyes are unable to detect.

3. Bluish stars are the hottest, whitish-yellow stars are hot, and reddish stars are the coolest. This follows from Wien's law, which says that stars with the shortest wavelength peak in their spectrum have the highest temperatures. Blue is the shortest visible wavelength and red is the longest visible wavelength, so blue is the hottest and red is the coolest.

4. The red bulb used in black-and-white film darkrooms is a very "cool" filament. Thus it does not emit much radiation in the range of visible wavelengths (and the small amount that it does emit in the visible region is not very intense), which means that it will not develop the black-and-white film but will still allow a person to see what's going on. A red bulb will not work very well in a darkroom that is used for color film. It will expose and develop the film during the process, especially at the red end of the spectrum, ruining the film.

5. If the threshold wavelength increases for the second metal, then the second metal has a smaller work function than the first metal. Longer wavelength corresponds to lower energy. It will take less energy for the electron to escape the surface of the second metal.

6. According to the wave theory, light of any frequency can cause electrons to be ejected as long as the light is intense enough. A higher intensity corresponds to a greater electric field magnitude and more energy. Therefore, there should be no frequency below which the photoelectric effect does not occur. According to the particle theory, however, each photon carries an amount of energy which depends upon its frequency. Increasing the intensity of the light increases the number of photons but does not increase the energy of the individual photons. The cutoff frequency is that frequency at which the energy of the photon equals the work function. If the frequency of the incoming light is below the cutoff, then the electrons will not be ejected because no individual photon has enough energy to eject an electron.

7. Individual photons of ultraviolet light are more energetic than photons of visible light and will deliver more energy to the skin, causing burns. UV photons also can penetrate farther into the skin and, once at the deeper level, can deposit a large amount of energy that can cause damage to cells.

8. Cesium will give a higher maximum kinetic energy for the ejected electrons. Since the incident photons bring in a given amount of energy, and in cesium less of this energy goes to releasing the electron from the material (the work function), it will give off electrons with a higher kinetic energy.

9. (*a*) A light source, such as a laser (especially an invisible one), could be directed on a photocell in such a way that as a burglar opened a door or passed through a window, the beam would be blocked from reaching the photocell. A burglar alarm could then be triggered to sound when the current in the ammeter went to 0.

(*b*) A light source could be focused on a photocell in a smoke detector. As the density of smoke particles in the air becomes thicker and thicker, more and more of the light attempting to reach the photocell would be scattered away from the photocell. At some set minimum level the alarm could be triggered to sound.

(*c*) The amount of current in the circuit with the photocell depends on the intensity of the light, as long as the frequency of the light is above the threshold frequency of the photocell material. The ammeter in the light meter's circuit could be calibrated to reflect the light intensity.

10. (*a*) No. The energy of a beam of photons depends not only on the energy of each individual photon but also on the total number of photons in the beam. It is possible that there could be many more photons in the IR beam than in the UV beam. In this instance, even though each UV photon has more energy than each IR photon, the IR beam could have more total energy than the UV beam.

(*b*) Yes. A photon's energy depends on its frequency: $E = hf$. Since infrared light has a lower frequency than ultraviolet light, a single IR photon will always have less energy than a single UV photon.

11. No, fewer electrons are emitted, but each one is emitted with higher kinetic energy, when the 400-nm light strikes the metal surface. The intensity (energy per unit time) of both light beams is the same, but the 400-nm photons each have more energy than the 450-nm photons. Thus there are fewer photons hitting the surface per unit time. This means that fewer electrons will be ejected per unit time from the surface with the 400-nm light. The maximum kinetic energy of the electrons leaving the metal surface will be greater, though, since the incoming photons have shorter wavelengths and more energy per photon, and it still takes the same amount of energy (the work function) to remove each electron. This "extra" energy goes into higher kinetic energy of the ejected electrons.

12. Yes, an X-ray photon that scatters from an electron does have its wavelength changed. The photon gives some of its energy to the electron during the collision and the electron recoils slightly. Thus, the photon has less energy and its wavelength is longer after the collision, since the energy and wavelength are inversely proportional to each other $(E = hf = hc/\lambda)$.

13. In the photoelectric effect, the photons (typically visible frequencies) have only a few eV of energy, whereas in the Compton effect, the energy of the photons (typically X-ray frequencies) is more than 1000 times greater and their wavelength is correspondingly smaller. In the photoelectric effect, the incident photons eject electrons completely out of the target material, while the photons are completely absorbed (no scattered photons). In the Compton effect, the photons are not absorbed but are scattered from the electrons.

14. Light demonstrates characteristics of both waves and particles. Diffraction, interference, and polarization are wave characteristics and are demonstrated, for example, in Young's double-slit experiment. The photoelectric effect and Compton scattering are examples of experiments in which light demonstrates particle characteristics. We can't say that light IS a wave or a particle, but it has properties of each.

15. We say that electrons have wave properties since we see them act like waves when they are diffracted or exhibit two-slit interference. We say that electrons have particle properties since we see them act like particles when they are bent by magnetic fields or accelerated and fired into materials where they scatter other electrons.

16. Both a photon and an electron have properties of waves and properties of particles. They can both be associated with a wavelength and they can both undergo scattering. An electron has a negative charge and has mass, obeys the Pauli exclusion principle, and travels at less than the speed of light. A photon is not charged, has no mass, does not obey the Pauli exclusion principle, and travels at the speed of light.

Property	Photon	Electron
Mass	None	9.11×10^{-31} kg
Charge	None	-1.60×10^{-19} C
Speed	3×10^{8} m/s	$< 3\times10^{8}$ m/s

There are other properties, such as spin, that will be discussed in later chapters.

17. The proton will have the shorter wavelength, since it has a larger mass than the electron and therefore a larger momentum for the same speed $(\lambda = h/p)$.

18. The particles will have the same kinetic energy. But since the proton has a larger mass, it will have a larger momentum than the electron $(\text{KE} = p^2/2m)$. Wavelength is inversely proportional to momentum, so the proton will have the shorter wavelength.

19. In Rutherford's planetary model of the atom, the Coulomb force (electrostatic force) keeps the electrons from flying off into space. Since the protons in the center are positively charged, the negatively charged electrons are attracted to the center by the Coulomb force and orbit the center just like the planets orbiting a sun in a solar system due to the attractive gravitational force.

20. At room temperature, nearly all the atoms in hydrogen gas will be in the ground state. When light passes through the gas, photons are absorbed, causing electrons to make transitions to higher states and creating absorption lines. These lines correspond to the Lyman series since that is the series of transitions involving the ground state or $n = 1$ level. Since there are virtually no atoms in higher energy states, photons corresponding to transitions from $n \geq 2$ to higher states will not be absorbed.

21. To determine whether there is oxygen near the surface of the Sun, you need to collect light coming from the Sun and spread it out using a diffraction grating or prism so you can see the spectrum of wavelengths. If there is oxygen near the surface, then there will be dark (absorption) lines at the wavelengths corresponding to electron transitions in oxygen.

22. (*a*) The Bohr model successfully explains why atoms emit line spectra; it predicts the wavelengths of emitted light for hydrogen; it explains absorption spectra; it ensures the stability of atoms (by decree); and it predicts the ionization energy of hydrogen.

    (*b*) The Bohr model did not give a reason for orbit quantization; it was not successful for multi-electron atoms; it could not explain why some emission lines are brighter than others; and it could not explain the "fine structure" of some very closely spaced spectral lines.

23. The two main difficulties of the Rutherford model of the atom were that (1) it predicted that light of a continuous range of frequencies should be emitted by atoms and (2) it predicted that atoms would be unstable.

24. It is possible for the de Broglie wavelength $(\lambda = h/p)$ of a particle to be bigger than the dimension of the particle. If the particle has a very small mass and a slow speed (like a low-energy electron or proton), then the wavelength may be larger than the dimension of the particle. It is also possible for the de Broglie wavelength of a particle to be smaller than the dimension of the particle if it has a large momentum and a moderate speed (like a baseball). There is no direct connection between the size of a particle and the size of the de Broglie wavelength of a particle. For example, you could also make the wavelength of a proton much smaller than the size of the proton by making it go very fast.

25. Even though hydrogen only has one electron, it still has an infinite number of energy states for that one electron to occupy, and each line in the spectrum represents a transition between two of those possible energy levels. So there are many possible spectral lines. And seeing many lines simultaneously would mean that there would have to be many hydrogen atoms undergoing energy level transitions—a sample of gas containing many H atoms, for example.

26. The closely spaced energy levels in Fig. 27–29 correspond to the different transitions of electrons from one energy state to another—specifically to those that start from closely packed high energy levels, perhaps with $n = 10$ or even higher. When these transitions occur, they emit radiation (photons) that creates the closely spaced spectral lines shown in Fig. 27–24.

27. On average, the electrons of helium are closer to the nucleus than are the electrons of hydrogen. The nucleus of helium contains two protons (positive charges) so attracts each electron more strongly than the single proton in the nucleus of hydrogen. (There is some shielding of the nuclear charge by the "other" electron, but each electron still feels an average attractive force of more than one proton's worth of charge.)

28. The Balmer series spectral lines are in the visible light range and could be seen by early experimenters without special detection equipment. It was only later that the UV (Lyman) and IR (Paschen) regions were explored thoroughly, using detectors other than human sight.

29. When a photon is emitted by a hydrogen atom as the electron makes a transition from one energy state to a lower one, not only does the photon carry away energy and momentum, but to conserve momentum, the atom must also take away some momentum. If the atom carries away some momentum, then it must also carry away some of the available energy, which means that the photon takes away less energy than Eq. 27–10 predicts.

30. (*a*) continuous

    (*b*) line, emission

    (*c*) continuous

    (*d*) line, absorption

    (*e*) continuous with absorption lines (like the Sun)

31. No. At room temperature, virtually all the atoms in a sample of hydrogen gas will be in the ground state. Thus, the absorption spectrum will contain primarily the Lyman lines, as photons corresponding to transitions from the $n = 1$ level to higher levels are absorbed. Hydrogen at very high temperatures will have atoms in excited states. The electrons in the higher energy levels will fall to all lower energy levels, not just the $n = 1$ level. Therefore, emission lines corresponding to transitions to levels higher than $n = 1$ will be present as well as the Lyman lines. In general, you would expect to see only Lyman lines in the absorption spectrum of room temperature hydrogen, but you would find Lyman, Balmer, Paschen, and other lines in the emission spectrum of high-temperature hydrogen.

Responses to MisConceptual Questions

1. (*b*) A common misconception is that the maximum wavelength increases as the temperature increases. However, the temperature and maximum wavelength are inversely proportional. As the temperature increases, the intensity increases and the wavelength decreases.

2. (*a*) A higher work function requires more energy per photon to release the electrons. Blue light has the shortest wavelength and therefore the largest energy per photon.

3. (*b*) The blue light has a shorter wavelength and therefore more energy per photon. Since the beams have the same intensity (energy per unit time per unit area), the red light will have more photons.

4. (*d*) A common misconception is that violet light has more energy than red light. However, the energy is the product of the energy per photon and the number of photons. A single photon of violet light has more energy than a single photon of red light, but if a beam of red light has more photons than the beam of violet light, then the red light could have more energy.

5. (*d*) The energy of the photon E is equal to the sum of the work function of the metal and the kinetic energy of the released photons. If the energy of the photon is more than double the work function, then cutting the photon energy in half will still allow electrons to be emitted. However, if the work function is more than half of the photon energy, then no electrons would be emitted if the photon energy were cut in half.

6. (*b*) The momentum of a photon is inversely proportional to its wavelength. Therefore, doubling the momentum would cut the wavelength in half.

7. (*d*) A commons misconception is that only light behaves as both a particle and a wave. De Broglie postulated, and experiments have confirmed, that in addition to light, electrons and protons (and many other particles) have both wave and particle properties that can be observed or measured.

8. (*d*) A thrown baseball has a momentum on the order of $1\ \text{kg}\cdot\text{m/s}$. The wavelength of the baseball is Planck's constant divided by the momentum. Due to the very small size of Planck's constant, the wavelength of the baseball would be much smaller than the size of a nucleus.

9. (*d*) This Chapter demonstrates the wave-particle duality of light and matter. Both electrons and photons have momentum that is related to their wavelength by $p = h/\lambda$. Young's double-slit experiment demonstrated diffraction with light, and later experiments demonstrated electron diffraction. Therefore, all three statements are correct.

10. (*a*, *c*, *d*) Alpha particles are positive. To produce large angle scattering, the nucleus of the atom would have to repel the alpha particle. Thus the nucleus must be positive. If the nuclear charge was spread out over a large area, then there would be more small-angle scattering but insufficient

force to create large-angle scattering. With a small nucleus, large scattering forces are possible. Since most of the alpha particles pass through the foil undeflected, most of the atom must be empty space. The scattering does not require quantized charge, but it would be possible with a continuous charge distribution. Therefore, answer (*b*) is incorrect.

11. (*d*) A photon is emitted when the electron transitions from a higher state to a lower state. Photons are not emitted when an electron transitions from $2 \rightarrow 5$ or from $5 \rightarrow 8$. The other two transitions are to states that are three states below the original state. The energy levels change more rapidly for lower values of *n*, so the $5 \rightarrow 2$ transition will have a higher energy and therefore a shorter wavelength than the $8 \rightarrow 5$ transition.

12. (*b*) The lowest energy cannot be zero, if zero energy has been defined as when the electron and proton are infinitely far away. As the electron and proton approach each other, their potential energy decreases. The energy levels are quantized and therefore cannot be any value. As shown in the text after Eq. 27–15b, the lowest energy level of the hydrogen atom is –13.6 eV.

13. (*d*) The current model of the atom is the quantum mechanical model. The plum-pudding model was rejected, as it did not explain Rutherford scattering. The Rutherford atom was rejected, as it did not explain the spectral lines emitted from atoms. The Bohr atom did not explain fine structure. Each of these phenomena are explained by the quantum mechanical model.

14. (*a*) Light is a massless particle. Even though it has no mass, it transports energy and therefore has kinetic energy and momentum. Light is also a wave with frequency and wavelength.

Solutions to Problems

In several problems, the value of *hc* is needed. We often use the result of Problem 29, $hc = 1240\ \text{eV}\cdot\text{nm}$.

1. This scenario fits the experiment that results in Eq. 27–1.

$$\frac{e}{m} = \frac{E}{rB^2} = \frac{(640\ \text{V/m})}{(14\times10^{-3}\ \text{m})(0.86\ \text{T})^2} = \boxed{6.2\times10^4\ \text{C/kg}}$$

2. (*a*) The velocity relationship is given right before Eq. 27–1 in the text.

$$\upsilon = \frac{E}{B} = \frac{(1.88\times10^4\ \text{V/m})}{(2.60\times10^{-3}\ \text{T})} = \boxed{7.23\times10^6\ \text{m/s}}$$

(*b*) For the radius of the path in the magnetic field, use an expression derived in Example 20–6.

$$r = \frac{m\upsilon}{qB} = \frac{(9.11\times10^{-31}\ \text{kg})(7.23\times10^6\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(2.90\times10^{-3}\ \text{T})} = 1.58\times10^{-2}\ \text{m} = \boxed{1.58\ \text{cm}}$$

3. The force from the electric field must balance the weight, and the electric field is the potential difference divided by the plate separation.

$$qE = ne\frac{V}{d} = mg \quad \rightarrow \quad n = \frac{dmg}{eV} = \frac{(1.0\times10^{-2}\ \text{m})(2.8\times10^{-15}\ \text{kg})(9.80\ \text{m/s}^2)}{(1.60\times10^{-19}\ \text{C})(340\ \text{V})} = 5.04 \approx \boxed{5\ \text{electrons}}$$

4. Use Wien's law, Eq. 27–2, to find the temperature.

$$T = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{\lambda_P} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(520\times10^{-9}\ \text{m})} = 5577\ \text{K} \approx \boxed{5600\ \text{K}}$$

5. Use Wien's law, Eq. 27–2.

(*a*) $$\lambda_P = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(273\ \text{K})} = 1.06\times10^{-5}\ \text{m} = \boxed{10.6\ \mu\text{m, far infrared}}$$

(*b*) $$\lambda_P = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(3100\ \text{K})} = 9.35410^{-7}\ \text{m} \approx \boxed{940\ \text{nm, near infrared}}$$

(*c*) $$\lambda_P = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(4\ \text{K})} = 7.25\times10^{-4}\ \text{m} \approx \boxed{0.7\ \text{mm, microwave}}$$

6. Use Wein's law, Eq. 27–2.

(*a*) $$T = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{\lambda_P} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(18.0\times10^{-9}\ \text{m})} = \boxed{1.61\times10^{5}\ \text{K}}$$

(*b*) $$\lambda_P = \frac{(2.90\times10^{-3}\ \text{m.K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(2200\ \text{K})} = 1.318\times10^{-6}\ \text{m} \approx \boxed{1.3\ \mu\text{m}}$$

7. Because the energy is quantized according to Eq. 27–3, the difference in energy between adjacent levels is simply $\Delta E = hf$.

$$\Delta E = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(8.1\times10^{13}\ \text{Hz}) = 5.37\times10^{-20}\ \text{J} \approx \boxed{5.4\times10^{-20}\ \text{J}}$$

$$5.37\times10^{-20}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{0.34\ \text{eV}}$$

8. The potential energy is "quantized" in units of *mgh*.

(*a*) $\text{PE}_1 = mgh = (62.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m}) = 121.52\ \text{J} \approx \boxed{122\ \text{J}}$

(*b*) $\text{PE}_2 = mg2h = 2\text{PE}_1 = 2(121.52\ \text{J}) = \boxed{243\ \text{J}}$

(*c*) $\text{PE}_3 = mg3h = 3\text{PE}_1 = 3(121.52\ \text{J}) = \boxed{365\ \text{J}}$

(*d*) $\text{PE}_n = mgnh = n\text{PE}_1 = n(121.52\ \text{J}) = \boxed{(122n)\ \text{J}}$

(*e*) $\Delta E = \text{PE}_2 - \text{PE}_6 = (2-6)(121.52\ \text{J}) = \boxed{-486\ \text{J}}$

9. Use Eq. 27–2 with a temperature of $98°\text{F} = 37°\text{C} = (273+37)\text{K} = 310\ \text{K}$ (3 significant figures).

$$\lambda_P = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(310\ \text{K})} = 9.35\times10^{-6}\ \text{m} = \boxed{9.35\ \mu\text{m}}$$

10. We use Eq. 27–4 to find the energy of the photons.

$$E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(91.7\times10^{6}\ \text{Hz}) = \boxed{6.08\times10^{-26}\ \text{J}}$$

11. We use Eq. 27–4 along with the fact that $f = c/\lambda$ for light. The longest wavelength will have the lowest energy.

$$E_1 = hf_1 = \frac{hc}{\lambda_1} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(400\times10^{-9}\ \text{m})} = 4.97\times10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 3.11\ \text{eV}$$

$$E_2 = hf_2 = \frac{hc}{\lambda_2} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(750\times10^{-9}\ \text{m})} = 2.65\times10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 1.66\ \text{eV}$$

Thus the range of energies is $\boxed{2.7\times10^{-19}\ \text{J} < E < 5.0\times10^{-19}\ \text{J}}$, or $\boxed{1.7\ \text{eV} < E < 3.1\ \text{eV}}$.

12. Use Eq. 27–4 with the fact that $f = c/\lambda$ for light.

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(320\times10^{3}\ \text{eV})} = 3.88\times10^{-12}\ \text{m} \approx \boxed{3.9\times10^{-3}\ \text{nm}}$$

Significant diffraction occurs when the opening is on the order of the wavelength. Thus there would be $\boxed{\text{insignificant diffraction}}$ through the doorway.

13. Use Eq. 27–6.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(5.80\times10^{-7}\ \text{m})} = \boxed{1.14\times10^{-27}\ \text{kg}\cdot\text{m/s}}$$

14. The momentum of the photon is found from Eq. 27–6.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(0.014\times10^{-9}\ \text{m})} = \boxed{4.7\times10^{-23}\ \text{kg}\cdot\text{m/s}}$$

15.

Particle Theory	**Wave Theory**
(1) If the light intensity is increased, the number of electrons ejected should increase.	(1) If the light intensity is increased, the number of electrons ejected should increase.
(2) If the light intensity is increased, the maximum kinetic energy of the electrons ejected should not increase.	(2) If the light intensity is increased, the maximum kinetic energy of the electrons ejected should increase.
(3) If the frequency of the light is increased, the maximum kinetic energy of the electrons should increase.	(3) If the frequency of the light is increased, the maximum kinetic energy of the electrons should not be affected.
(4) There is a "cutoff" frequency, below which no electrons will be ejected, no matter how intense the light.	(4) There should be no lower limit to the frequency—electrons will be ejected for all frequencies.

16. We use Eq. 27–4 with the fact that $f = c/\lambda$ for light.

$$E_{\text{min}} = hf_{\text{min}} \quad \rightarrow \quad f_{\text{min}} = \frac{E_{\text{min}}}{h} = \frac{(0.1\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = 2.41\times10^{13}\ \text{Hz} \approx \boxed{2\times10^{13}\ \text{Hz}}$$

$$\lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{(3.00\times10^8 \text{ m/s})}{(2.41\times10^{13} \text{ Hz})} = 1.24\times10^{-5} \text{ m} \approx \boxed{1\times10^{-5} \text{ m}}$$

17. At the minimum frequency, the kinetic energy of the ejected electrons is 0. Use Eq. 27–5a.

$$\text{KE} = hf_{\text{min}} - W_0 = 0 \quad\rightarrow\quad f_{\text{min}} = \frac{W_0}{h} = \frac{4.8\times10^{-19} \text{ J}}{6.63\times10^{-34} \text{ J}\cdot\text{s}} = \boxed{7.2\times10^{14} \text{ Hz}}$$

18. We divide the minimum energy by the photon energy at 550 nm to find the number of photons.

$$E = nhf = E_{\text{min}} \quad\rightarrow\quad n = \frac{E_{\text{min}}}{hf} = \frac{E_{\text{min}}\lambda}{hc} = \frac{(10^{-18} \text{ J})(550\times10^{-9} \text{ m})}{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})} = 2.77 \approx \boxed{3 \text{ photons}}$$

19. The longest wavelength corresponds to the minimum frequency. That occurs when the kinetic energy of the ejected electrons is 0. Use Eq. 27–5a.

$$\text{KE} = hf_{\text{min}} - W_0 = 0 \quad\rightarrow\quad f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{W_0}{h} \quad\rightarrow$$

$$\lambda_{\text{max}} = \frac{ch}{W_0} = \frac{(3.00\times10^8 \text{ m/s})(6.63\times10^{-34} \text{ J}\cdot\text{s})}{(2.90 \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = \boxed{4.29\times10^{-7} \text{ m}} = 429 \text{ nm}$$

20. The photon of visible light with the maximum energy has the least wavelength. We use 400 nm as the lowest wavelength of visible light and calculate the energy for that wavelength.

$$hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(1.60\times10^{-19} \text{ J/eV})(400\times10^{-9} \text{ m})} = 3.11 \text{ eV}$$

Electrons will not be emitted if this energy is less than the work function.

The metals with work functions greater than 3.11 eV are $\boxed{\text{copper and iron}}$.

21. (*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so the work function is equal to the energy of the photon.

$$W_0 = hf - \text{KE}_{\text{max}} = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{550 \text{ nm}} = 2.255 \text{ eV} \approx \boxed{2.3 \text{ eV}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 27–5b to calculate the maximum kinetic energy.

$$\text{KE}_{\text{max}} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} - 2.255 \text{ eV} = 0.845 \text{ eV}$$

$$V_0 = \frac{\text{KE}_{\text{max}}}{e} = \frac{0.845 \text{ eV}}{e} \approx \boxed{0.85 \text{ V}}$$

22. The photon of visible light with the maximum energy has the minimum wavelength. We use Eq. 27–5b to calculate the maximum kinetic energy.

$$\text{KE}_{\text{max}} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} - 2.48 \text{ eV} = \boxed{0.62 \text{ eV}}$$

$\boxed{23.}$ We use Eq. 27–5b to calculate the maximum kinetic energy. Since the kinetic energy is much less than the rest energy, we use the classical definition of kinetic energy to calculate the speed.

$$\text{KE}_{\text{max}} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240 \text{ eV}\cdot\text{nm}}{365 \text{ nm}} - 2.48 \text{ eV} = \boxed{0.92 \text{ eV}}$$

$$\text{KE}_{\text{max}} = \tfrac{1}{2}mv^2 \quad \to \quad v = \sqrt{\frac{2\text{KE}_{\text{max}}}{m}} = \sqrt{\frac{2(0.92\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = \boxed{5.7\times10^5 \text{ m/s}}$$

This speed is only about 0.2% of the speed of light, so the classical kinetic energy formula is correct.

24. We use Eq. 27–5b to calculate the work function.

$$W_0 = hf - \text{KE}_{\text{max}} = \frac{hc}{\lambda} - \text{KE}_{\text{max}} = \frac{1240 \text{ eV}\cdot\text{nm}}{255 \text{ nm}} - 1.40 \text{ eV} = \boxed{3.46 \text{ eV}}$$

25. Electrons emitted from photons at the threshold wavelength have no kinetic energy. Use Eq. 27–5b with the threshold wavelength to determine the work function.

$$W_0 = \frac{hc}{\lambda} - \text{KE}_{\text{max}} = \frac{hc}{\lambda_{\text{max}}} = \frac{1240 \text{ eV}\cdot\text{nm}}{340 \text{ nm}} = 3.647 \text{ eV}$$

(*a*) Now use Eq. 27–5b again with the work function determined above to calculate the kinetic energy of the photoelectrons emitted by 280-nm light.

$$\text{KE}_{\text{max}} = \frac{hc}{\lambda} - W_0 = \frac{1240 \text{ eV}\cdot\text{nm}}{280 \text{ nm}} - 3.647 \text{ eV} \approx \boxed{0.78 \text{ eV}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons}}$.

26. The energy required for the chemical reaction is provided by the photon. We use Eq. 27–4 for the energy of the photon, where $f = c/\lambda$.

$$E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{630 \text{ nm}} = 1.968 \text{ eV} \approx \boxed{2.0 \text{ eV}}$$

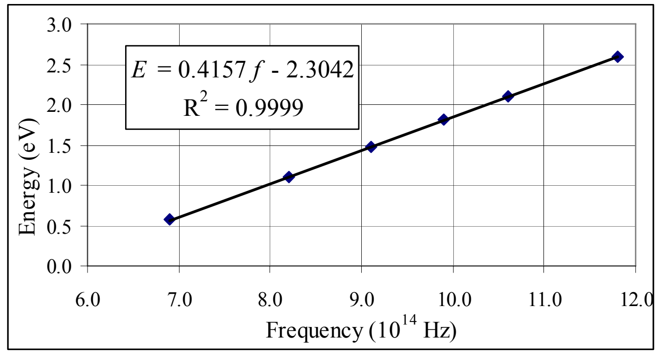
Each reaction takes place in a molecule, so we use the appropriate conversions to convert eV/molecule to kcal/mol.

$$E = \left(\frac{1.968 \text{ eV}}{\text{molecule}}\right)\left(\frac{1.60\times10^{-19} \text{ J}}{\text{eV}}\right)\left(\frac{6.02\times10^{23}\text{molecules}}{\text{mol}}\right)\left(\frac{\text{kcal}}{4186 \text{ J}}\right) = \boxed{45 \text{ kcal/mol}}$$

27. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy of the photoelectrons. We use Eq. 27–5b to calculate the work function, where the maximum kinetic energy is the product of the stopping voltage and electron charge.

$$W_0 = \frac{hc}{\lambda} - \text{KE}_{\text{max}} = \frac{hc}{\lambda} - eV_0 = \frac{1240 \text{ eV}\cdot\text{nm}}{250 \text{ nm}} - (1.64 \text{ V})e = \boxed{3.32 \text{ eV}}$$

28. We plot the maximum (kinetic) energy of the emitted electrons vs. the frequency of the incident radiation. Eq. 27–5b says $\mathrm{KE}_{\max} = hf - W_0$. The best-fit straight line is determined by linear regression in Excel. The slope of the best-fit straight line to the data should give Planck's constant, the x-intercept is the cutoff frequency, and the y-intercept is the opposite of the work function.



(a) $h = (0.4157\ \text{eV}/10^{14}\ \text{Hz})(1.60\times10^{-19}\ \text{J/eV}) = \boxed{6.7\times10^{-34}\ \text{J}\cdot\text{s}}$

(b) $hf_{\text{cutoff}} = W_0 \quad \rightarrow \quad f_{\text{cutoff}} = \dfrac{W_0}{h} = \dfrac{2.3042\ \text{eV}}{(0.4157\ \text{eV}/10^{14}\ \text{Hz})} = \boxed{5.5\times10^{14}\ \text{Hz}}$

(c) $W_0 = \boxed{2.3\ \text{eV}}$

29. Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We insert the values for h and c and convert the resulting units to $\text{eV}\cdot\text{nm}$.

$$E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{\lambda}\frac{(1\ \text{eV}/1.602\times10^{-19}\ \text{J})}{(10^{-9}\ \text{m}/1\ \text{nm})} = \boxed{\frac{1240\ \text{eV}\cdot\text{nm}}{\lambda\ (\text{in nm})}}$$

30. Use Eq. 27–7.

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\phi) \quad \rightarrow \quad \Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) \quad \rightarrow$$

$$\phi = \cos^{-1}\left(1 - \frac{m_e c \Delta\lambda}{h}\right)$$

$$= \cos^{-1}\left(1 - \frac{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(1.7\times10^{-13}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}\right) = 21.58^\circ \approx \boxed{22^\circ}$$

31. The Compton wavelength for a particle of mass m is h/mc.

(a) $\dfrac{h}{m_e c} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = \boxed{2.43\times10^{-12}\ \text{m}}$

(b) $\dfrac{h}{m_p c} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = \boxed{1.32\times10^{-15}\ \text{m}}$

(c) The energy of the photon is given by Eq. 27–4.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{hc}{(h/mc)} = mc^2 = \text{rest energy}$$

32. We find the Compton wavelength shift for a photon scattered from an electron, using Eq. 27–7. The Compton wavelength of a free electron is calculated in Problem 31 above.

$$\lambda' - \lambda = \left(\frac{h}{m_e c}\right)(1-\cos\theta) = \lambda_C(1-\cos\theta) = (2.43\times10^{-3}\ \text{nm})(1-\cos\theta)$$

(*a*) $\lambda'_a - \lambda = (2.43\times10^{-3}\ \text{nm})(1-\cos 45^\circ) = \boxed{7.12\times10^{-4}\ \text{nm}}$

(*b*) $\lambda'_b - \lambda = (2.43\times10^{-3}\ \text{nm})(1-\cos 90^\circ) = \boxed{2.43\times10^{-3}\ \text{nm}}$

(*c*) $\lambda'_c - \lambda = (2.43\times10^{-3}\ \text{nm})(1-\cos 180^\circ) = \boxed{4.86\times10^{-3}\ \text{nm}}$

33. The photon energy must be equal to the kinetic energy of the products plus the mass energy of the products. The mass of the positron is equal to the mass of the electron.

$$E_{\text{photon}} = \text{KE}_{\text{products}} + m_{\text{products}}c^2 \quad \rightarrow$$

$$\text{KE}_{\text{products}} = E_{\text{photon}} - m_{\text{products}}c^2 = E_{\text{photon}} - 2m_{\text{electron}}c^2 = 3.64\ \text{MeV} - 2(0.511\ \text{MeV}) = \boxed{2.62\ \text{MeV}}$$

$\boxed{34.}$ The photon with the longest wavelength has the minimum energy in order to create the masses with no additional kinetic energy. Use Eq. 27–6.

$$\lambda_{\max} = \frac{hc}{E_{\min}} = \frac{hc}{2mc^2} = \frac{h}{2mc} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = \boxed{6.62\times10^{-16}\ \text{m}}$$

This must take place in the presence of some other object in order for momentum to be conserved.

35. The minimum energy necessary is equal to the rest energy of the two muons.

$$E_{\min} = 2mc^2 = 2(207)(0.511\ \text{MeV}) = \boxed{212\ \text{MeV}}$$

The wavelength is given by Eq. 27–6.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(212\times10^{6}\ \text{eV})} = \boxed{5.86\times10^{-15}\ \text{m}}$$

36. Since $\upsilon = 0.001c$, the total energy of the particles is essentially equal to their rest energy. The particles have the same rest energy of 0.511 MeV. Since the total momentum is 0, each photon must have half the available energy and equal momenta.

$$E_{\text{photon}} = m_{\text{electron}}c^2 = \boxed{0.511\ \text{MeV}}; \quad p_{\text{photon}} = \frac{E_{\text{photon}}}{c} = \boxed{0.511\ \text{MeV}/c}$$

$\boxed{37.}$ The energy of the photon is equal to the total energy of the two particles produced. The particles have the same kinetic energy and the same mass.

$$E_{\text{photon}} = 2(\text{KE} + mc^2) = 2(0.285\ \text{MeV} + 0.511\ \text{MeV}) = \boxed{1.592\ \text{MeV}}$$

The wavelength is found from Eq. 27–6.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(1.592\times10^{6}\ \text{eV})} = \boxed{7.81\times10^{-13}\ \text{m}}$$

38. We find the wavelength from Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(0.21\ \text{kg})(0.10\ \text{m/s})} = \boxed{3.2\times10^{-32}\ \text{m}}$$

39. The neutron is not relativistic, so use $p = mv$. Also use Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(8.5\times10^{4}\ \text{m/s})} = \boxed{4.7\times10^{-12}\ \text{m}}$$

40. We assume that the electron is nonrelativistic, and check that with the final answer. We use Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow\quad v = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(0.27\times10^{-9}\ \text{m})} = 2.695\times10^{6}\ \text{m/s} = 0.0089c$$

The use of classical expressions is justified. The kinetic energy is equal to the potential energy change.

$$eV = \text{KE} = \tfrac{1}{2}mv^2 = \frac{\tfrac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.695\times10^{6}\ \text{m/s})^2}{(1.60\times10^{-19}\ \text{J/eV})} = 20.7\ \text{eV}$$

Thus the required potential difference is $\boxed{21\ \text{V}}$.

41. Since the particles are not relativistic, we may use $\text{KE} = p^2/2m.$ We then form the ratio of the kinetic energies, using Eq. 27–8.

$$\text{KE} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2};\quad \frac{\text{KE}_\text{e}}{\text{KE}_\text{p}} = \frac{h^2/2m_\text{e}\lambda^2}{h^2/2m_\text{p}\lambda^2} = \frac{m_\text{p}}{m_\text{e}} = \frac{1.67\times10^{-27}\ \text{kg}}{9.11\times10^{-31}\ \text{kg}} = \boxed{1840}$$

$\boxed{42.}$ (*a*) We find the momentum from Eq. 27–8.

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4.5\times10^{-10}\ \text{m}} = \boxed{1.5\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(*b*) We assume the speed is nonrelativistic.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow\quad v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(4.5\times10^{-10}\ \text{m})} = \boxed{1.6\times10^{6}\ \text{m/s}} \ll c$$

(*c*) We calculate the kinetic energy classically.

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(mc^2)(v/c)^2 = \tfrac{1}{2}(0.511\ \text{MeV})(5.39\times10^{-3})^2 = 7.43\times10^{-6}\ \text{MeV} = 7.43\ \text{eV}$$

This is the energy gained by an electron if accelerated through a potential difference of $\boxed{7.4\ \text{V}}$.

43. Because all of the energies to be considered are much less than the rest energy of an electron, we can use nonrelativistic relationships. We use Eq. 27–8 to calculate the wavelength.

$$\text{KE} = \frac{p^2}{2m} \quad\rightarrow\quad p = \sqrt{2m(\text{KE})};\quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(\text{KE})}}$$

(*a*) $\lambda = \dfrac{h}{\sqrt{2m(\text{KE})}} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(10\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 3.88\times10^{-10}\ \text{m} \approx \boxed{4\times10^{-10}\ \text{m}}$

(*b*) $\lambda = \dfrac{h}{\sqrt{2m(\text{KE})}} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(100\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 1.23\times10^{-10}\ \text{m} \approx \boxed{1\times10^{-10}\ \text{m}}$

(*c*) $\lambda = \dfrac{h}{\sqrt{2m(\text{KE})}} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.0\times10^{3}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = \boxed{3.9\times10^{-11}\ \text{m}}$

44. Since the particles are not relativistic, use $\text{KE} = p^2/2m.$ Form the ratio of the wavelengths, using Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\text{KE}}};\quad \frac{\lambda_{\text{p}}}{\lambda_{\text{e}}} = \frac{\dfrac{h}{\sqrt{2m_{\text{p}}\text{KE}}}}{\dfrac{h}{\sqrt{2m_{\text{e}}\text{KE}}}} = \sqrt{\frac{m_{\text{e}}}{m_{\text{p}}}} < 1$$

Thus $\boxed{\text{the proton has the shorter wavelength}}$, since $m_{\text{e}} < m_{\text{p}}$.

45. The final KE of the electron equals the negative change in its PE as it passes through the potential difference. Compare this energy to the electron's rest energy to determine if it is relativistic.

$$\text{KE} = -q\Delta V = (1\ \text{e})(35\times10^{3}\ \text{V}) = 35\times10^{3}\ \text{eV}$$

Because this is greater than 1% of the electron rest energy, $\boxed{\text{the electron is relativistic}}$. We use Eq. 26–9 to determine the electron momentum and then Eq.27–6 to determine the wavelength.

$$E^2 = [\text{KE} + mc^2]^2 = p^2c^2 + m^2c^4 \quad \Rightarrow \quad p = \frac{\sqrt{\text{KE}^2 + 2(\text{KE})mc^2}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{\text{KE}^2 + 2(\text{KE})mc^2}} = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{(35\times10^{3}\ \text{eV})^2 + 2(35\times10^{3}\ \text{eV})(511\times10^{3}\ \text{eV})}} = \boxed{6.4\times10^{-3}\ \text{nm}}$$

Because $\lambda \ll 5$ cm, $\boxed{\text{diffraction effects are negligible}}$.

46. For diffraction, the wavelength must be on the order of the opening. Find the speed from Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \to \quad v = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1400\ \text{kg})(12\ \text{m})} = \boxed{3.9\times10^{-38}\ \text{m/s}}$$

This is on the order of 10^{-39} times ordinary highway speeds.

47. We relate the kinetic energy to the momentum with a classical relationship, since the electrons are nonrelativistic. We also use Eq. 27–8. We then assume that the kinetic energy was acquired by electrostatic potential energy.

$$\text{KE} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = eV \quad \to$$

$$V = \frac{h^2}{2me\lambda^2} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})(0.26\times10^{-9}\ \text{m})^2} = \boxed{22\ \text{V}}$$

48. The kinetic energy is 2850 eV. That is small enough compared to the rest energy of the electron for the electron to be nonrelativistic. We use Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{(2m\text{KE})^{1/2}} = \frac{hc}{(2mc^2\text{KE})^{1/2}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})[2(0.511\times10^6\text{ eV})(2850\text{ eV})]^{1/2}}$$
$$= 2.30\times10^{-11}\text{ m} = \boxed{23.0\text{ pm}}$$

49. The energy of a level is $E_n = -\frac{(13.6\text{ eV})}{n^2}$, from Eq. 27–15b for $Z = 1$.

(*a*) The transition from $n = 1$ to $n' = 3$ is an $\boxed{\text{absorption}}$, because the $\boxed{\text{final state}}$, $n' = 3$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{3^2}\right) - \left(\frac{1}{1^2}\right)\right] = 12.1\text{ eV}$$

(*b*) The transition from $n = 6$ to $n' = 2$ is an $\boxed{\text{emission}}$, because the $\boxed{\text{initial state}}$, $n' = 2$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = -(E_{n'} - E_n) = (13.6\text{ eV})\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{6^2}\right)\right] = 3.0\text{ eV}$$

(*c*) The transition from $n = 4$ to $n' = 5$ is an $\boxed{\text{absorption}}$, because the $\boxed{\text{final state}}$, $n' = 5$, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{4^2}\right)\right] = 0.31\text{ eV}$$

The photon for the transition from $\boxed{n = 1 \text{ to } n' = 3}$ has the largest energy.

50. Ionizing the atom means removing the electron, or raising it to zero energy.

$$E_{\text{ionization}} = 0 - E_n = 0 - \frac{(-13.6\text{ eV})}{n^2} = \frac{(13.6\text{ eV})}{3^2} = \boxed{1.51\text{ eV}}$$

51. From $\Delta E = \frac{hc}{\lambda}$, we see that the second longest wavelength comes from the transition with the second smallest energy: $\boxed{n = 5 \text{ to } n' = 3}$.

52. Doubly ionized lithium is similar to hydrogen, except that there are three positive charges $(Z = 3)$ in the nucleus. Use Eq. 27–15b with $Z = 3$.

$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{3^2(13.6\text{ eV})}{n^2} = -\frac{(122\text{ eV})}{n^2}$$
$$E_{\text{ionization}} = 0 - E_1 = 0 - \left[-\frac{(122\text{ eV})}{(1)^2}\right] = \boxed{122\text{ eV}}$$

53. We use Eq. 27–10 and Eq. 27–15b. Note that $E_1, E_2,$ and E_3 are calculated in the textbook. We also need $E_4 = \frac{-(13.6\text{ eV})}{4^2} = -0.85\text{ eV}$ and $E_5 = \frac{-(13.6\text{ eV})}{5^2} = -0.54\text{ eV}$.

(*a*) The second Balmer line is the transition from $n = 4$ to $n = 2$.

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{1240\text{ eV}\cdot\text{nm}}{[-0.85\text{ eV} - (-3.40\text{ eV})]} = \boxed{486\text{ nm}}$$

(*b*) The second Lyman line is the transition from $n = 3$ to $n = 1$.

$$\lambda = \frac{hc}{(E_3 - E_1)} = \frac{1240\text{ eV}\cdot\text{nm}}{[-1.51\text{ eV} - (-13.6\text{ eV})]} = \boxed{103\text{ nm}}$$

(*c*) The third Balmer line is the transition from $n = 5$ to $n = 2$.

$$\lambda = \frac{hc}{(E_5 - E_2)} = \frac{1240\text{ eV}\cdot\text{nm}}{[-0.54\text{ eV} - (-3.40\text{ eV})]} = \boxed{434\text{ nm}}$$

54. We evaluate the Rydberg constant using Eq. 27–9 and 27–16. We use hydrogen so $Z = 1$. We also substitute $k = \frac{1}{4\pi\varepsilon_0}$.

$$\frac{1}{\lambda} = R\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) = \frac{2\pi^2 Z^2 e^4 m k^2}{h^3 c}\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) \rightarrow$$

$$R = \frac{2\pi^2 Z^2 e^4 m k^2}{h^3 c} = \frac{2\pi^2 Z^2 e^4 m}{16\pi^2 \varepsilon_0^2 h^3 c}$$

$$= \frac{(1)^2 (1.602176\times10^{-19}\text{ C})^4 (9.109382\times10^{-31}\text{ kg})}{8(8.854188\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)^2 (6.626069\times10^{-34}\text{ J}\cdot\text{s})^3 (2.997925\times10^8\text{ m/s})}$$

$$= 1.097361\times10^7 \frac{\text{C}^4\cdot\text{kg}}{\frac{\text{C}^4}{\text{N}^2\cdot\text{m}^4}\text{J}^3\text{s}^3\text{ m/s}} \approx \boxed{1.0974\times10^7\text{ m}^{-1}}$$

55. The longest wavelength corresponds to the minimum energy, which is the ionization energy.

$$\lambda = \frac{hc}{E_{\text{ion}}} = \frac{1240\text{ eV}\cdot\text{nm}}{13.6\text{ eV}} = \boxed{91.2\text{ nm}}$$

56. The energy of the photon is the sum of the ionization energy of 13.6 eV and the kinetic energy of 11.5 eV. The wavelength is found from Eq. 27–4.

$$hf = \frac{hc}{\lambda} = E_{\text{total}} \rightarrow \lambda = \frac{hc}{E_{\text{total}}} = \frac{1240\text{ eV}\cdot\text{nm}}{25.1\text{ eV}} = \boxed{49.4\text{ nm}}$$

57. Singly ionized helium is like hydrogen, except that there are two positive charges $(Z = 2)$ in the nucleus. Use Eq. 27–15b.

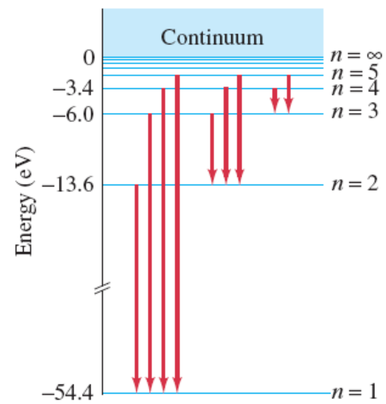
$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}$$

We find the energy of the photon from the $n=5$ to $n=2$ transition in singly ionized helium.

$$\Delta E = E_5 - E_2 = -(54.4\text{ eV})\left[\left(\frac{1}{6^2}\right)-\left(\frac{1}{2^2}\right)\right] = 12.1\text{ eV}$$

Because this is the energy difference for the $n=1$ to $n=3$ transition in hydrogen, the photon can be absorbed by a hydrogen atom which will jump from $\boxed{n=1 \text{ to } n=3}$.

58. Singly ionized helium is like hydrogen, except that there are two positive charges $(Z=2)$ in the nucleus. Use Eq. 27–15b.

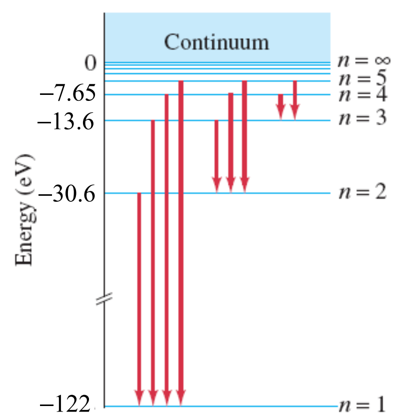


$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}$$

$E_1 = -54.5\text{ eV}$, $E_2 = -13.6\text{ eV}$, $E_3 = -6.0\text{ eV}$, $E_4 = -3.4\text{ eV}$,

$E_5 = -2.2\text{ eV}$, $E_6 = -1.5\text{ eV}$, …

59. Doubly ionized lithium is like hydrogen, except that there are three positive charges $(Z=3)$ in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace e^2 with Ze^2:



$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{3^2(13.6\text{ eV})}{n^2} = -\frac{(122.4\text{ eV})}{n^2}$$

$E_1 = -122\text{ eV}$, $E_2 = -30.6\text{ eV}$, $E_3 = -13.6\text{ eV}$,

$E_4 = -7.65\text{ eV}$, $E_5 = -4.90\text{ eV}$, $E_6 = -3.40\text{ eV}$, …

60. The potential energy for the ground state is given by the charge of the electron times the electric potential caused by the proton.

$$\text{PE} = (-e)V_{\text{proton}} = (-e)\frac{1}{4\pi\varepsilon_0}\frac{e}{r_1} = -\frac{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})^2(1\text{ eV}/1.60\times10^{-19}\text{ J})}{(0.529\times10^{-10}\text{ m})}$$

$$= \boxed{-27.2\text{ eV}}$$

The kinetic energy is the total energy minus the potential energy.

$$\text{KE} = E_1 - \text{PE} = -13.6\text{ eV} - (-27.2\text{ eV}) = \boxed{+13.6\text{ eV}}$$

61. The angular momentum can be used to find the quantum number for the orbit, and then the energy can be found from the quantum number. Use Eqs. 27–11 and 27–15b.

$$L = n\frac{h}{2\pi} \quad\to\quad n = \frac{2\pi L}{h} = \frac{2\pi(5.273\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})} = 5.000 \approx 5$$

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} = -\frac{13.6\text{ eV}}{25} = \boxed{-0.544\text{ eV}}$$

62. The value of n is found from $r_n = n^2 r_1$, and then the energy can be found from Eq. 27–15b.

$$r_n = n^2 r_1 \quad \rightarrow \quad n = \sqrt{\frac{r_n}{r_1}} = \sqrt{\frac{1.00\times10^{-2}\text{ m}}{0.529\times10^{-10}\text{ m}}} = 13,749 \approx \boxed{13,700}$$

$$E = -\frac{(13.6\text{ eV})}{n^2} = -\frac{(13.6\text{ eV})}{(13,749)^2} = \boxed{-7.19\times10^{-8}\text{ eV}}$$

63. The velocity is found from Eq. 27–11 evaluated for $n = 1$.

$$m\upsilon r_n = \frac{nh}{2\pi} \quad \rightarrow$$

$$\upsilon = \frac{h}{2\pi r_1 m_\text{e}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{2\pi(0.529\times10^{-10}\text{ m})(9.11\times10^{-31}\text{ kg})} = 2.190\times10^6\text{ m/s} = \boxed{7.30\times10^{-3}\,c}$$

Since $\upsilon \ll c$, we say $\boxed{\text{yes}}$, nonrelativistic formulas are justified. The relativistic factor is as follows.

$$\sqrt{1-\upsilon^2/c^2} \approx 1 - \tfrac{1}{2}(\upsilon^2/c^2) = 1 - \tfrac{1}{2}\left(\frac{2.190\times10^6\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2 = \boxed{1-2.66\times10^{-5}} \approx 0.99997$$

We see that $\sqrt{1-\upsilon^2/c^2}$ is essentially 1. $\boxed{\text{Yes}}$, nonrelativistic formulas are justified.

64. The electrostatic potential energy is given by Eq. 17–2a. Note that the charge is negative. The kinetic energy is given by the total energy, Eq. 27–15a, minus the potential energy. The Bohr radius is given by Eq. 27–12.

$$\text{PE} = -eV = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\pi m Ze^2}{n^2h^2\varepsilon_0} = -\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}$$

$$\text{KE} = E - \text{PE} = -\frac{Z^2e^4m}{8\varepsilon_0^2h^2n^2} - \left(-\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\right) = \frac{Z^2e^4m}{8n^2h^2\varepsilon_0^2}; \quad \frac{|\text{PE}|}{\text{KE}} = \frac{\dfrac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}}{\dfrac{Z^2e^4m}{8n^2h^2\varepsilon_0^2}} = \frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\,\frac{8n^2h^2\varepsilon_0^2}{Z^2e^4m} = \boxed{2}$$

65. When we compare the gravitational and electric forces, we see that we can use the same expression for the Bohr orbits, Eq. 27–12 and 27–15a, if we replace kZe^2 with $Gm_\text{e}m_\text{p}$.

$$r_1 = \frac{h^2\varepsilon_0}{4\pi^2 m_\text{e} kZe^2} \quad \rightarrow$$

$$r_1 = \frac{h^2}{4\pi^2 G m_\text{e}^2 m_\text{p}} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{4\pi^2(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\text{ kg})^2(1.67\times10^{-27}\text{ kg})}$$

$$= \boxed{1.20\times10^{29}\text{ m}}$$

$$E_1 = -\frac{2\pi^2 Z^2 e^4 m_\text{e} k^2}{h^2} = -(kZe^2)^2\frac{2\pi^2 m_\text{e}}{h^2} \quad \rightarrow \quad E_1 = -\frac{2\pi^2 G^2 m_\text{e}{}^3 m_\text{p}{}^2}{h^2}$$

$$= -\frac{2\pi^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)^2(9.11\times10^{-31}\text{ kg})^3(1.67\times10^{-27}\text{ kg})^2}{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2} = \boxed{-4.22\times10^{-97}\text{ J}}$$

66. Find the peak wavelength from Wien's law, Eq. 27–2.

$$\lambda_\text{P} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{T} = \frac{(2.90\times10^{-3}\ \text{m}\cdot\text{K})}{(2.7\ \text{K})} = 1.1\times10^{-3}\ \text{m} = \boxed{1.1\ \text{mm}}$$

67. To produce a photoelectron, the hydrogen atom must be ionized, so the minimum energy of the photon is 13.6 eV. We find the minimum frequency of the photon from Eq. 27–4.

$$E = hf \rightarrow f = \frac{E}{h} \rightarrow f_\text{min} = \frac{E_\text{min}}{h} = \frac{(13.6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{3.28\times10^{15}\ \text{Hz}}$$

68. From Section 25–11, the spacing between planes, d, for the first-order peaks is given by Eq. 25–10, $\lambda = 2d\sin\theta$. The wavelength of the electrons can be found from their kinetic energy. The electrons are not relativistic at the given energy.

$$\text{KE} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \rightarrow \lambda = \frac{h}{\sqrt{2m\text{KE}}} = 2d\sin\theta \rightarrow$$

$$d = \frac{h}{2\sin\theta\sqrt{2m\text{KE}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2(\sin 38^\circ)\sqrt{2(9.11\times10^{-31}\ \text{kg})(72\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = \boxed{1.2\times10^{-10}\ \text{m}}$$

69. The power rating is the energy produced per second. If this is divided by the energy per photon, then the result is the number of photons produced per second. Use Eq. 27–4.

$$E_\text{photon} = hf = \frac{hc}{\lambda};\quad \frac{P}{E_\text{photon}} = \frac{P\lambda}{hc} = \frac{(720\ \text{W})(12.2\times10^{-2}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})} = \boxed{4.4\times10^{26}\ \text{photons/s}}$$

70. The intensity is the amount of energy per second per unit area reaching the Earth. If that intensity is divided by the energy per photon, then the result will be the photons per second per unit area reaching the Earth. We use Eq. 27–4.

$$E_\text{photon} = hf = \frac{hc}{\lambda}$$

$$I_\text{photons} = \frac{I_\text{sunlight}}{E_\text{photon}} = \frac{I_\text{sunlight}\lambda}{hc} = \frac{(1300\ \text{W/m}^2)(550\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})} = 3.595\times10^{21} \approx \boxed{3.6\times10^{21}\ \frac{\text{photons}}{\text{s}\cdot\text{m}^2}}$$

71. The impulse on the wall is due to the change in momentum of the photons. Each photon is absorbed, so its entire momentum is transferred to the wall.

$$F_\text{on wall}\Delta t = \Delta p_\text{wall} = -\Delta p_\text{photons} = -(0 - np_\text{photon}) = np_\text{photon} = \frac{nh}{\lambda} \rightarrow$$

$$\frac{n}{\Delta t} = \frac{F\lambda}{h} = \frac{(5.8\times10^{-9}\ \text{N})(633\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{5.5\times10^{18}\ \text{photons/s}}$$

72. As light leaves the flashlight, it gains momentum. The momentum change is given by Eq. 22–9a. Dividing the momentum change by the elapsed time gives the force the flashlight must apply to the light to produce the momentum, which is equal to the force that the light applies to the flashlight.

$$\frac{\Delta p}{\Delta t} = \frac{\Delta U}{c\Delta t} = \frac{P}{c} = \frac{2.5\ \text{W}}{3.00\times10^{8}\ \text{m/s}} = \boxed{8.3\times10^{-9}\ \text{N}}$$

73. (*a*) Since $f = c/\lambda$, the photon energy is $E = hc/\lambda$ and the largest wavelength has the smallest energy. In order to eject electrons for all possible incident visible light, the metal's work function must be less than or equal to the energy of a 750-nm photon. Thus the maximum value for the metal's work function W_0 is found by setting the work function equal to the energy of the 750-nm photon.

$$W_0 = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}{(750\times10^{-9}\ \mathrm{m})}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{1.7\ \mathrm{eV}}$$

(*b*) If the photomultiplier is to function only for incident wavelengths less than 410 nm, then we set the work function equal to the energy of the 410-nm photon.

$$W_0 = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}{(410\times10^{-9}\ \mathrm{m})}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{3.0\ \mathrm{eV}}$$

74. (*a*) Calculate the energy from the lightbulb that enters the eye by calculating the intensity of the light at a distance of $\ell = 1.0\ \mathrm{m}$ by dividing the power in the visible spectrum by the surface area of a sphere of radius $\ell = 1.0\ \mathrm{m}$. Multiply the intensity of the light by the area of the pupil (diameter = D) to determine the energy entering the eye per second. Divide this energy by the energy of a photon (Eq. 27–4) to calculate the number of photons entering the eye per second.

$$I = \frac{P}{4\pi\ell^2} \qquad P_\mathrm{e} = I(\pi D^2/4) = \frac{P}{16}\left(\frac{D}{\ell}\right)^2$$

$$n = \frac{P_\mathrm{e}}{hc/\lambda} = \frac{P\lambda}{16hc}\left(\frac{D}{\ell}\right)^2 = \frac{0.030(100\ \mathrm{W})(550\times10^{-9}\ \mathrm{m})}{16(6.626\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}\left(\frac{4.0\times10^{-3}\ \mathrm{m}}{1.0\ \mathrm{m}}\right)^2$$

$$= \boxed{8.3\times10^{12}\ \text{photons/s}}$$

(*b*) Repeat the above calculation for a distance of $\ell = 1.0\ \mathrm{km}$ instead of 1.0 m.

$$I = \frac{P}{4\pi\ell^2} \qquad P_\mathrm{e} = I(\pi D^2/4) = \frac{P}{16}\left(\frac{D}{\ell}\right)^2$$

$$n = \frac{P_\mathrm{e}}{hc/\lambda} = \frac{P\lambda}{16hc}\left(\frac{D}{\ell}\right)^2 = \frac{0.030(100\ \mathrm{W})(550\times10^{-9}\ \mathrm{m})}{16(6.626\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}\left(\frac{4.0\times10^{-3}\ \mathrm{m}}{1.0\times10^{3}\ \mathrm{m}}\right)^2$$

$$= \boxed{8.3\times10^{6}\ \text{photons/s}}$$

75. The total energy of the two photons must equal the total energy (kinetic energy plus mass energy) of the two particles. The total momentum of the photons is 0, so the momentum of the particles must have been equal and opposite. Since both particles have the same mass and had the same initial momentum, they each had the same initial kinetic energy.

$$E_\text{photons} = E_\text{particles} = 2(m_\mathrm{e}c^2 + \mathrm{KE}) \quad \rightarrow$$

$$\mathrm{KE} = \tfrac{1}{2}E_\text{photons} - m_\mathrm{e}c^2 = 0.85\ \mathrm{MeV} - 0.511\ \mathrm{MeV} = \boxed{0.34\ \mathrm{MeV}}$$

76. We calculate the required momentum from de Broglie's relation, Eq. 27–78.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})}{(4.0\times10^{-12}\ \mathrm{m})} = 1.658\times10^{-22}\ \mathrm{kg\cdot m/s}$$

(*a*) For the proton, we use the classical definition of momentum to determine its speed and kinetic energy. We divide the kinetic energy by the charge of the proton to determine the required potential difference.

$$\upsilon = \frac{p}{m} = \frac{1.658\times10^{-22}\ \text{kg}\cdot\text{m/s}}{1.67\times10^{-27}\ \text{kg}} = 9.93\times10^{5}\ \text{m/s} \approx 0.01c$$

$$V = \frac{K}{e} = \frac{m\upsilon^2}{2e} = \frac{(1.67\times10^{-27}\ \text{kg})(9.93\times10^{5}\ \text{m/s})^2}{2(1.60\times10^{-19}\ \text{C})} = \boxed{51\ \text{V}}$$

(*b*) For the electron, if we divide the momentum by the electron mass we obtain a speed that is about 60% of the speed of light. Therefore, we must use Eq. 26–9 to determine the energy of the electron. We then subtract the rest energy from the total energy to determine the kinetic energy of the electron. Finally, we divide the kinetic energy by the electron charge to calculate the potential difference.

$$\begin{aligned} E &= [(pc)^2 + (mc^2)^2]^{\frac{1}{2}} \\ &= [(1.658\times10^{-22}\ \text{kg}\cdot\text{m/s})^2(3.00\times10^{8}\ \text{m/s})^2 + (9.11\times10^{-31}\ \text{kg})^2(3.00\times10^{8}\ \text{m/s})^4]^{\frac{1}{2}} \\ &= 9.950\times10^{-14}\ \text{J} \end{aligned}$$

$$\text{KE} = E - mc^2 = 9.950\times10^{-14}\,\text{J} - (9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 1.391\times10^{-14}\ \text{J}$$

$$V = \frac{\text{KE}}{e} = \frac{1.391\times10^{-14}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = 86{,}925\ \text{V} \approx \boxed{87\ \text{kV}}$$

$\boxed{77.}$ If we ignore the recoil motion, then at the closest approach the kinetic energy of both particles is zero. The potential energy of the two charges must equal the initial kinetic energy of the α particle.

$$\text{KE}_{\alpha} = U = \frac{1}{4\pi\varepsilon_0}\frac{(Z_{\alpha}e)(Z_{\text{Ag}}e)}{r_{\min}} \quad \rightarrow$$

$$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{(Z_{\alpha}e)(Z_{\text{Ag}}e)}{\text{KE}_{\alpha}} = \frac{(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\ \text{C})^2}{(4.8\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})} = 4.74\times10^{-14}\ \text{m}$$

The distance to the "surface" of the gold nucleus is then $4.74\times10^{-14}\,\text{m} - 7.0\times10^{-15}\ \text{m} = \boxed{4.0\times10^{-14}\ \text{m}}$.

78. The decrease in mass occurs because a photon has been emitted. We calculate the fractional change. Since we are told to find the amount of decrease, we use the opposite of the change.

$$\frac{-\Delta m}{m} = \frac{\left(\frac{-\Delta E}{c^2}\right)}{m} = \frac{-\Delta E}{mc^2} = \frac{(13.6\ \text{eV})\left[\left(\frac{1}{1^2}\right)-\left(\frac{1}{3^2}\right)\right]}{(939\times10^{6}\ \text{eV})} = \boxed{1.29\times10^{-8}}$$

79. We calculate the ratio of the forces.

$$\frac{F_{\text{gravitational}}}{F_{\text{electric}}} = \frac{\left(\frac{Gm_{\text{e}}m_{\text{p}}}{r^2}\right)}{\left(\frac{ke^2}{r^2}\right)} = \frac{Gm_{\text{e}}m_{\text{p}}}{ke^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\ \text{kg})(1.67\times10^{-27}\ \text{kg})}{(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}$$

$$= \boxed{4.40\times10^{-40}}$$

$\boxed{\text{Yes}}$, the gravitational force may be safely ignored.

80. The potential difference gives the electrons a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is $-13.6\text{ eV}+12.3\text{ eV}=-1.3\text{ eV}$. From the energy level diagram, Fig. 27–29, we see that this means that the atom could be excited to the $n=3$ state, so the possible transitions when the atom returns to the ground state are $n=3$ to $n=2$, $n=3$ to $n=1$, and $n=2$ to $n=1$. We calculate the wavelengths from the equation above and Eq. 27–16.

$$\lambda_{3\to 2}=\frac{hc}{(E_3-E_2)}=\frac{1240\text{ eV}\cdot\text{nm}}{[-1.5\text{ eV}-(-3.4\text{ eV})]}=\boxed{650\text{ nm}}$$

$$\lambda_{3\to 1}=\frac{hc}{(E_3-E_1)}=\frac{1240\text{ eV}\cdot\text{nm}}{[-1.5\text{ eV}-(-13.6\text{ eV})]}=\boxed{102\text{ nm}}$$

$$\lambda_{2\to 1}=\frac{hc}{(E_2-E_1)}=\frac{1240\text{ eV}\cdot\text{nm}}{[-3.4\text{ eV}-(-13.6\text{ eV})]}=\boxed{122\text{ nm}}$$

81. The stopping potential is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 27–5b to first find the work function and then find the stopping potential for the higher wavelength.

$$\text{KE}_{\text{max}}=eV_0=\frac{hc}{\lambda}-W_0 \quad\rightarrow\quad W_0=\frac{hc}{\lambda_0}-eV_0$$

$$eV_1=\frac{hc}{\lambda_1}-W_0=\frac{hc}{\lambda_1}-\left(\frac{hc}{\lambda_0}-eV_0\right)=hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_0}\right)+eV_0$$

$$=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ J/ev})}\left(\frac{1}{440\times10^{-9}\text{ m}}-\frac{1}{270\times10^{-9}\text{ m}}\right)+2.10\text{ eV}=0.32\text{ eV}$$

The potential difference needed to oppose an electron kinetic energy of 0.32 eV is $\boxed{0.32\text{ V}}$.

82. We assume that the neutron is not relativistic. If the resulting velocity is small, then our assumption will be valid. We use Eq. 27–8.

$$\lambda=\frac{h}{p}=\frac{h}{m\upsilon}\quad\rightarrow\quad \upsilon=\frac{h}{m\lambda}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.67\times10^{-27}\text{ kg})(0.3\times10^{-9}\text{ m})}=1323\text{ m/s}\approx\boxed{1000\text{ m/s}}$$

This is not relativistic, so our assumption was valid.

83. The intensity is the amount of energy hitting the surface area per second. That is found from the number of photons per second hitting the area and the energy per photon, from Eq. 27–4.

$$\bar{I}=\frac{(1.0\times10^{12}\text{ photons})}{(1\text{ m}^2)(1\text{ s})}\frac{hc}{\lambda}=\frac{(1.0\times10^{12})}{(1\text{ m}^2)(1\text{ m}^2)}\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(497\times10^{-9}\text{ m})}=\boxed{4.0\times10^{-7}\text{ W/m}^2}$$

The magnitude of the electric field is found from Eq. 22–8.

$$\bar{I}=\tfrac{1}{2}\varepsilon_0 cE_0^2\quad\rightarrow\quad E_0=\sqrt{\frac{2\bar{I}}{\varepsilon_0 c}}=\sqrt{\frac{2(4.0\times10^{-7}\text{ W/m}^2)}{(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\text{ m/s})}}=\boxed{1.7\times10^{-2}\text{ V/m}}$$

84. The average force on the sail is equal to the impulse (change in momentum) on the sail divided by the time. Since the photons bounce off the mirror, the impulse is equal to twice the incident momentum. We use Eq. 27–6 to write the momentum of the photon in terms of the photon energy. The total photon energy is the intensity of the sunlight multiplied by the area of the sail.

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{2(E/c)}{\Delta t} = \frac{2(E/\Delta t)}{c} = \frac{2IA}{c} = \frac{2(1350\ \text{W/m}^2)(1.0\times10^3\,\text{m})^2}{3.00\times10^8\ \text{m/s}} = \boxed{9.0\ \text{N}}$$

85. The electrons will be nonrelativistic at that low energy. The maximum kinetic energy of the photoelectrons is given by Eq. 27–5b. The kinetic energy determines the momentum, and the momentum determines the wavelength of the emitted electrons. The shortest electron wavelength corresponds to the maximum kinetic energy.

$$\text{KE}_{\text{electron}} = \frac{hc}{\lambda} - W_0 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_{\text{electron}}^2} \rightarrow \lambda_{\text{electron}} = \frac{h}{\sqrt{2m\left(\frac{hc}{\lambda} - W_0\right)}}$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\left(\frac{1240\ \text{eV}\cdot\text{nm}}{280\ \text{nm}} - 2.2\ \text{eV}\right)(1.60\times10^{-19}\ \text{J/eV})}} = \boxed{8.2\times10^{-10}\ \text{m}}$$

86. We first find the work function from the given data. A photon energy of 6.0 eV corresponds to a stopping potential of 3.8 V.

$$eV_0 = hf - W_0 \rightarrow W_0 = hf - eV_0 = 6.0\ \text{eV} - 3.8\ \text{eV} = 2.2\ \text{eV}$$

If the photons' wavelength is doubled, then the energy is halved, from 6.0 eV to 3.0 eV. The maximum kinetic energy would be 0.8 eV. If the photon's wavelength is tripled, then the energy is only 2.0 eV. Since this is less than the work function, $\boxed{\text{no current flows}}$.

87. The theoretical resolution limit is the wavelength of the electron. We find the wavelength from the momentum and find the momentum from the kinetic energy and rest energy. We use the result from Chapter 26, Problem 45. The kinetic energy of the electron is 110 keV.

$$\lambda = \frac{hc}{p} = \frac{hc}{\sqrt{\text{KE}^2 + 2mc^2\text{KE}}}$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})\sqrt{(110\times10^3\text{eV})^2 + 2(0.511\times10^6\text{eV})(110\times10^3\text{eV})}} = \boxed{3.5\times10^{-12}\ \text{m}}$$

88. Hydrogen atoms start in the $n=1$ orbit (ground state). Using Eq. 27–10 and Eq. 27–15b, we determine the state to which the atom is excited when it absorbs a photon of 12.75 eV via collision with an electron. Then, using Eq. 27–16, we calculate all possible wavelengths that can be emitted as the electron cascades back to the ground state.

$$\Delta E = E_{\text{u}} - E_\ell \rightarrow E_{\text{u}} = -\frac{13.6\ \text{eV}}{n^2} = E_\ell + \Delta E \rightarrow n = \sqrt{\frac{-13.6\ \text{eV}}{E_\ell + \Delta E}} = \sqrt{\frac{-13.6\ \text{eV}}{-13.6\ \text{eV} + 12.75\ \text{eV}}} = 4$$

Starting with the electron in the $n=4$ orbit, the following transitions are possible: $n=4$ to $n=3$; $n=4$ to $n=2$; $n=4$ to $n=1$; $n=3$ to $n=2$; $n=3$ to $n=1$; $n=2$ to $n=1$.

$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=5.333\times10^{5}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{1875\ \text{nm}}$$

$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=2.057\times10^{6}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{486.2\ \text{nm}}$$

$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{1^2}-\frac{1}{4^2}\right)=1.028\times10^{7}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{97.23\ \text{nm}}$$

$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.524\times10^{6}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{656.3\ \text{nm}}$$

$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=9.751\times10^{6}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{102.6\ \text{nm}}$$

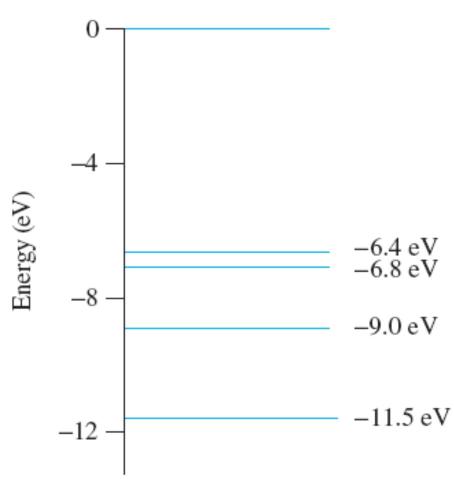
$$\frac{1}{\lambda}=(1.097\times10^{7}\ \text{m}^{-1})\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=8.228\times10^{6}\ \text{m}^{-1} \quad\rightarrow\quad \lambda=\boxed{121.5\ \text{nm}}$$

89. The wavelength is found from Eq. 24–4. The velocity of electrons with the same wavelength (and thus the same diffraction pattern) is found from their momentum, assuming they are not relativistic. We use Eq. 27–8 to relate the wavelength and momentum.

$$d\sin\theta=n\lambda \quad\rightarrow\quad \lambda=\frac{d\sin\theta}{n}=\frac{h}{p}=\frac{h}{mv} \quad\rightarrow$$

$$v=\frac{hn}{md\sin\theta}=\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1)}{(9.11\times10^{-31}\ \text{kg})(0.010\times10^{-3}\ \text{m})(\sin 3.6^\circ)}=1159\ \text{m/s}\approx\boxed{1200\ \text{m/s}}$$

90. (*a*) See the adjacent figure.



(*b*) Absorption of a 5.1-eV photon represents a transition from the $\boxed{\text{ground state}}$ to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state.

$$4\rightarrow3:\quad -6.4\ \text{eV}-(-6.8\ \text{eV})=\boxed{0.4\ \text{eV}}$$

$$4\rightarrow2:\quad -6.4\ \text{eV}-(-9.0\ \text{eV})=\boxed{2.6\ \text{eV}}$$

$$4\rightarrow1:\quad -6.4\ \text{eV}-(-11.5\ \text{eV})=\boxed{5.1\ \text{eV}}$$

$$3\rightarrow2:\quad -6.8\ \text{eV}-(-9.0\ \text{eV})=\boxed{2.2\ \text{eV}}$$

$$3\rightarrow1:\quad -6.8\ \text{eV}-(-11.5\ \text{eV})=\boxed{4.7\ \text{eV}}$$

$$2\rightarrow1:\quad -9.0\ \text{eV}-(-11.5\ \text{eV})=\boxed{2.5\ \text{eV}}$$

91. Find the energy of the photon from Eq. 27–4 and Eq. 27–8.

$$E = hf = h\frac{c}{\lambda} = pc = (3.53\times10^{-28}\ \text{kg}\cdot\text{m/s})(3.00\times10^{8}\ \text{m/s})\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 0.662\ \text{eV}$$

From Fig. 27–29, the Lyman series photons have $13.6\ \text{eV} \geq E \geq 10.2\ \text{eV}$. The Balmer series photons have $3.4\ \text{eV} \geq E \geq 1.9\ \text{eV}$. The Paschen series photons have $1.5\ \text{eV} \geq E \geq 0.65\ \text{eV}$. It appears that this photon belongs to the $\boxed{\text{Paschen series, ejected from energy level 4}}$.

92. (*a*) Use Eq. 27–5b to calculate the maximum kinetic energy of the electron and set this equal to the product of the stopping voltage and the electron charge.

$$\text{KE}_{\text{max}} = hf - W_0 = eV_0 \quad \rightarrow \quad V_0 = \frac{hf - W_0}{e} = \frac{hc/\lambda - W_0}{e}$$

$$V_0 = \frac{(1240\ \text{eV}\cdot\text{nm})/(464\ \text{nm}) - 2.28\ \text{eV}}{e} = 0.3924\ \text{V} \approx \boxed{0.39\ \text{V}}$$

(*b*) Calculate the speed from the nonrelativistic kinetic energy equation and the maximum kinetic energy found in part (*a*).

$$\text{KE}_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 \quad \rightarrow$$

$$v_{\text{max}} = \sqrt{\frac{2\text{KE}_{\text{max}}}{m}} = \sqrt{\frac{2(0.3924\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 3.713\times10^{5}\ \text{m/s} \approx \boxed{3.7\times10^{5}\ \text{m/s}}$$

This is only about $0.001c$, so we are justified in using the classical definition of kinetic energy.

(*c*) We use Eq. 27–8 to calculate the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.713\times10^{5}\ \text{m/s})} = 1.96\times10^{-9}\ \text{m} \approx \boxed{2.0\ \text{nm}}$$

93. The electron acquires a kinetic energy of 96 eV, which is used to find the speed. We use the classical expression.

$$\text{KE} = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(96\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(9.11\times10^{-31}\ \text{kg})}} = 5.807\times10^{6}\ \text{m/s}$$

This is about 2% of the speed of light. The charge-to-mass ratio can be calculated using an expression derived in Example 20–6.

$$r = \frac{mv}{qB} \quad \rightarrow \quad \frac{q}{m} = \frac{v}{Br} = \frac{5.807\times10^{6}\ \text{m/s}}{(3.67\times10^{-4}\ \text{T})(9.0\times10^{-2}\ \text{m})} = \boxed{1.7\times10^{11}\ \text{C/kg}}$$

94. First find the area of a sphere whose radius is the Earth–Sun distance.

$$A = 4\pi r^2 = 4\pi(150\times10^{9}\ \text{m})^2 = 2.83\times10^{23}\ \text{m}^2$$

Multiply this by the given intensity to find the Sun's total power output.

$$(2.83\times10^{23}\text{m}^2)(1350\ \text{W/m}^2) = 3.82\times10^{26}\ \text{W}$$

Multiplying by the number of seconds in a year gives the annual energy output.

$$E = (3.82\times10^{26}\text{ W})(3600\text{ s/h})(24\text{ h/d})(365.25\text{ d/yr}) = 1.20\times10^{34}\text{ J/yr}$$

Finally, we divide by the energy of one photon to find the number of photons per year.

$$\frac{E}{hf} = \frac{E\lambda}{hc} = \frac{(1.20\times10^{34}\text{ J/yr})(550\times10^{-9}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})} = \boxed{3.3\times10^{52}\text{ photons/yr}}$$

95. Use Bohr's analysis of the H atom, replacing the proton mass with Earth's mass, the electron mass with the Moon's mass, and the electrostatic force $F_e = \frac{ke^2}{r^2}$ with the gravitational force, $F_g = \frac{Gm_E m_M}{r^2}$. To account for the change in force, replace ke^2 with $Gm_E m_M$ and set $Z = 1$. With these replacements, write expressions similar to Eq. 27–12 and Eq. 27–15a for the Bohr radius and energy.

$$r_n = \frac{h^2 n^2}{4\pi^2 mke^2} \rightarrow$$

$$r_n = \frac{h^2 n^2}{4\pi^2 Gm_M^2 m_E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{4\pi^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})^2(5.98\times10^{24}\text{ kg})}n^2$$

$$= \boxed{n^2(5.16\times10^{-129}\text{ m})}$$

$$E_n = -\frac{2\pi^2 e^4 mk^2}{n^2h^2} \rightarrow$$

$$E_n = -\frac{2\pi^2 G^2 m_E^2 m_M^3}{n^2h^2} = -\frac{2\pi^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)^2(5.98\times10^{24}\text{ kg})^2(7.35\times10^{22}\text{ kg})^3}{n^2(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}$$

$$= \boxed{-\frac{2.84\times10^{165}\text{ J}}{n^2}}$$

Insert the known masses and Earth–Moon distance into the Bohr radius equation to determine the Bohr state.

$$n = \sqrt{\frac{4\pi^2 Gm_M^2 m_E r_n}{h^2}}$$

$$= \sqrt{\frac{4\pi^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})^2(5.98\times10^{24}\text{ kg})(3.84\times10^{8}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}}$$

$$= 2.73\times10^{68}$$

Since $n \approx 10^{68}$, a value of $\Delta n = 1$ is negligible compared to *n*. Hence the quantization of energy and radius is $\boxed{\text{not apparent}}$.

96. The kinetic energy of the hydrogen gas would have to be the difference between the $n = 1$ and $n = 2$ states of the hydrogen atom, 10.2 eV. Use Eq. 13–8.

$$\text{KE} = \tfrac{3}{2}kT \rightarrow T = \frac{2\text{KE}}{3k} = \frac{2(10.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{7.88\times10^{4}\text{ K}}$$

Solutions to Search and Learn Problems

1. (*a*) J. J. Thomson
   (*b*) Robert A. Millikan
   (*c*) Max Planck
   (*d*) A. H. Compton
   (*e*) Louis de Broglie
   (*f*) C. J. Davisson, L. H. Germer, G. P. Thomson
   (*g*) Ernest Rutherford

2. The principle of complementarity states that to understand an experiment, sometimes you must use the wave theory and sometimes you must use the particle theory. That is, to have a full understanding of various phenomena (like light), you must accept that they have both wave and particle characteristics. For example, to understand Young's double-slit experiment for photons or electrons, you must apply wave theory with interference of the two waves. To understand the photoelectric effect for light or Compton scattering for electrons, you must use particle theory.

3. (*a*) We solve Eq. 24–2a for the slit separation with the wavelength given in terms of the electron momentum from Eq. 27–6. The electron momentum is written in terms of the kinetic energy.

$$m\lambda = d\sin\theta \quad \rightarrow$$

$$d = \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta} = \frac{h}{\sqrt{2meV}\,\sin\theta} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sin 10^\circ\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.602\times10^{-19}\ \text{C})(12\ \text{V})}}$$

$$= \boxed{2.0\times10^{-9}\ \text{m} = 2.0\ \text{nm}}$$

   (*b*) No. The slit separation distance is the same size as the atoms.

4. (*a*) particle
   (*b*) wave
   (*c*) particle
   (*d*) wave

5. (*a*) The minimum energy necessary to initiate the chemical process on the retina is the energy of a single photon of red light. Red light has a wavelength of approximately 750 nm.

$$E_{\text{min}} = hf = h\frac{c}{\lambda} = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{3.00\times10^{8}\ \text{m/s}}{750\times10^{-9}\ \text{m}}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.7\ \text{eV}}$$

   (*b*) The maximum energy is the energy of a photon of violet light of wavelength 400 nm

$$E_{\text{max}} = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{3.00\times10^{8}\ \text{m/s}}{400\times10^{-9}\ \text{m}}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{3.1\ \text{eV}}$$

6. (*a*) Apply conservation of momentum before and after the emission of the photon to determine the recoil speed of the atom, where the momentum of the photon is given by Eq. 27–6. The initial momentum is 0.

$$0 = \frac{h}{\lambda} - m\upsilon \quad \rightarrow \quad \upsilon = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{85(1.66\times10^{-27}\ \text{kg})(780\times10^{-9}\ \text{m})} = \boxed{6.0\times10^{-3}\ \text{m/s}}$$

(*b*) We solve Eq. 18–5 for the lowest achievable temperature, where the recoil speed is the rms speed of the rubidium gas.

$$v=\sqrt{\frac{3kT}{m}} \quad\rightarrow\quad T=\frac{mv^2}{3k}=\frac{85(1.66\times10^{-27}\ \text{kg})(6.0\times10^{-3}\ \text{m/s})^2}{3(1.38\times10^{-23}\ \text{J/K})}=1.2\times10^{-7}\ \text{K}=\boxed{0.12\ \mu\text{K}}$$

7. (*a*) Here are standing wave diagrams for the first three modes of vibration.

(*b*) From the diagram we see that the wavelengths are given by $\lambda_n=\frac{2L}{n},\ \ n=1,\ 2,\ 3,\ \ldots$. The momentum is $p_n=\frac{h}{\lambda}=\frac{nh}{2L}$, so the kinetic energy is $\text{KE}_n=\frac{p_n^2}{2m}=\frac{n^2h^2}{8mL^2}$.

(*c*) Because the potential energy is zero inside the box, the total energy is the kinetic energy. The ground state energy has $n=1$.

$$E_1=\frac{n^2h^2}{8mL^2}=\frac{(1)^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(0.50\times10^{-10}\ \text{m})^2}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{150\ \text{eV}}$$

(*d*) Do the same calculation for the baseball, and then find the speed from the kinetic energy.

$$E_1=\frac{n^2h^2}{8mL^2}=\frac{(1)^2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(0.140\ \text{kg})(0.65\ \text{m})^2}=9.289\times10^{-67}\ \text{J}\approx\boxed{9.3\times10^{-67}\ \text{J}}$$

$$E_1=\tfrac{1}{2}mv^2 \quad\rightarrow\quad v=\sqrt{\frac{2\text{KE}_1}{m}}=\sqrt{\frac{2(9.289\times10^{-67}\ \text{J})}{(0.140\ \text{kg})}}=\boxed{3.6\times10^{-33}\ \text{m/s}}$$

(*e*) Find the width of the box from $E_1=\frac{n^2h^2}{8mL^2}$.

$$E_1=\frac{n^2h^2}{8mL^2} \quad\rightarrow$$

$$L=\frac{nh}{\sqrt{8mE_1}}=\frac{(1)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{8(9.11\times10^{-31}\ \text{kg})(22\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}}=1.3\times10^{-10}\ \text{m}=\boxed{0.13\ \text{nm}}$$

28

QUANTUM MECHANICS OF ATOMS

Responses to Questions

1. (*a*) A matter wave Ψ does not need a medium, but a wave on a string does. The square of the wave function Ψ for a matter wave describes the probability of finding a particle within a certain spatial range. The function for a wave on a string describes the displacement of a piece of the string from its equilibrium position as a function of location and time.

   (*b*) Neither an EM nor a matter wave needs a medium in which to exist. The EM wave describes the amplitudes and directions of the electric and magnetic fields as a function of location and time. An EM wave represents a vector field and can be polarized. A matter wave is a scalar and cannot be polarized.

2. According to Bohr's theory, each electron in an atom travels in a circular orbit and has a precise position and momentum at any point in time. This view is inconsistent with the postulates of quantum mechanics and the uncertainty principle, which does not allow both the position and momentum to be known precisely. According to quantum mechanics, the "orbitals" of electrons do not have precise radii, but describe the probability of finding an electron in a given spatial range.

3. As a particle becomes more massive, the uncertainty of its momentum $(\Delta p = m\Delta v)$ becomes larger, which reduces the uncertainty of its position (due to $\Delta x \geq \hbar/\Delta p$). Thus, with a small uncertainty of its position, we can do a better job of predicting its future position.

4. Because of the very small value of $\hbar$, the uncertainties in both a baseball's momentum and position can be very small compared to typical macroscopic positions and momenta without being close to the limit imposed by the uncertainty principle. For instance, the uncertainty in the baseball's position could be 10^{-10} m, and the uncertainty in the baseball's momentum 10^{-10} kg·m/s, and still easily satisfy the uncertainty principle. Yet we never try to measure the position or momentum of a baseball to that precision. For an electron, however, typical values of uncertainty in position and momentum can be close to the uncertainty principle limit, so the relative uncertainty can be much higher for that small object.

5. No. According to the uncertainty principle, if the needle were balanced, then the position of the center of mass would be known exactly, and there would have to be some uncertainty in its momentum. The center of mass of the needle could not have a zero momentum and therefore would fall over. If the initial momentum of the center of mass of the needle were exactly zero, then there would be uncertainty in its position, and the needle could not be perfectly balanced (with the center of mass over the tip).

6. No. The hot soup will warm the thermometer when they come in contact. Thus the final temperature of the thermometer/soup system will be less than the initial temperature of the soup. Accordingly, the final thermometer reading will be lower than the original temperature of the soup. This is a situation where making a measurement affects the physical situation and causes uncertainty in the measurement.

7. No. However, the greater the precision of the measurement of position, the greater the uncertainty in the measurement of the momentum of the object will be.

8. If you knew the position precisely, then you would know nothing about the momentum.

9. Yes, some of the air escapes the tire (and goes into the tire gauge) in the act of measuring the pressure. It is impossible to avoid this escape. The act of measuring the air pressure in a tire therefore actually changes the pressure, although not by much since very little air escapes compared to the total amount of air in the tire. This is similar to the uncertainty principle, in which one of the two factors limiting the precision of measurement is the interaction between the object being observed, or measured, and the observing instrument. By making the measurement, you have affected the state of the system.

10. Yes, this is consistent with the uncertainty principle for energy $(\Delta E \Delta t \geq \hbar)$. The ground-state energy can be very precisely known because the electrons remain in that state for a very long time. Thus, as $\Delta t \rightarrow \infty$, $\Delta E \rightarrow 0$. However, the electrons do not remain in the excited states for very long, thus the Δt in this case is relatively small, making the ΔE, the energy width, relatively large.

11. The quantum-mechanical model predicts that the electron spends more time near the nucleus. In the Bohr model, the electron in the ground state is in a fixed orbit of definite radius. The electron cannot come any closer to the nucleus than that distance. In the quantum-mechanical model, the most probable location for the electron is the Bohr radius, but it can also be found closer to the nucleus (and farther away). See Fig. 28–6 for a visual representation of this question.

12. As the number of electrons goes up, the number of protons in the nucleus increases, which increases the attraction of the electrons to the center of the atom. Even though the outer electrons are partially screened from the increased nuclear charge by the inner electrons, they are all pulled closer to the more positive nucleus. Also, more states are available in the upper shells to accommodate many more electrons at approximately the same radius.

13. Because the nuclei of hydrogen and helium are different, the energy levels of the atoms are different. The presence of the second electron in helium will also affect its energy levels. If the energy levels are different, then the energy difference between the levels will be different and the spectra will be different.

14. If there were no electron spin, then, according to the Pauli exclusion principle, *s* subshells would be filled with one electron, *p* subshells with three electrons, and *d* subshells with five electrons. The first 20 elements of the Periodic Table would look like the following:

H 1 $1s^1$								
He 2 $2s^1$						Li 3 $2p^1$	Be 4 $2p^2$	B 5 $2p^3$
C 6 $3s^1$						N 7 $3p^1$	O 8 $3p^2$	F 9 $3p^3$
Ne 10 $4s^1$	Na 11 $3d^1$	Mg 12 $3d^2$	Al 13 $3d^3$	Si 14 $3d^4$	P 15 $3d^5$	S 16 $4p^1$	Cl 17 $4p^2$	Ar 18 $4p^3$
K 19 $5s^1$	Ca 20 $4d^1$							

$\boxed{15.}$ (*a*) and (*c*) are allowed for atoms in an excited state. (*b*) is not allowed. Only six electrons are allowed in the 2*p* state.

16. (*a*) Group VII. Outer electrons are $2s^2 2p^5$: fluorine.

(*b*) Group II. Outer electrons are $3s^2$: magnesium.

(*c*) Group VIII. Outer electrons are $3s^2 3p^6$: argon.

(*d*) Group I. Outer electron is $4s^1$: potassium.

17. Both chlorine and iodine are in the same column of the Periodic Table (column VII). Thus, both of their outer electron configurations are p^5 (5 electrons in their outer *p* shell), which makes them both halogens. Halogens lack one electron from a filled shell, so their outer electron orbit shapes are similar and can readily accept an additional electron from another atom. This makes their chemical reaction properties extremely similar.

$\boxed{18.}$ Both sodium and potassium are in the same column of the Periodic Table (column I). Thus, both of their outer electron configurations are s^1 (1 electron in their outer *s* shell), which makes them both alkali metals. Alkali metals have one lone electron in their outermost shell, so their outer electron orbit shapes are similar and they can readily share this electron with another atom (especially since this outermost electron spends a considerable amount of time very far away from the nucleus). This makes their chemical reaction properties extremely similar.

19. Rare-earth elements have similar chemical properties because the electrons in the filled 6*s* or 7*s* suborbitals serve as the valence electrons for all these elements. They all have partially filled inner *f* suborbitals, which are very close together in energy. The different numbers of electrons in the *f* suborbitals have little effect on the chemical properties of these elements.

20. Neon is a "closed shell" atom—a noble gas. All of its shells are completely full and spherically symmetric, so its electrons are not readily ionized. Sodium is a Group I atom (an alkali metal) that has one lone electron in the outermost shell, and this electron spends a considerable amount of time far away from the nucleus and also shielded from the nucleus. The result is that sodium can be ionized much easier than neon, which the two ionization energies demonstrate very well.

21. The difference in energy between $n=1$ and $n=2$ levels is much bigger than between any other combination of energy levels. Thus, electron transitions between the $n=1$ and $n=2$ levels produce photons of higher energy and frequency, which means that these photons have shorter wavelengths (since frequency and wavelength are inversely proportional to each other).

22. The Bohr model does violate the uncertainty principle, specifically by saying that an electron in a given orbit has a precise radius and a precise momentum. The uncertainty principle says that knowing both the location and the momentum precisely is not possible.

23. Noble gases are nonreactive because they have completely filled shells or subshells. In that configuration, other electrons are not attracted, nor are electrons readily lost. Thus they do not react (share or exchange electrons) with other elements. Alkali metals all have a single outer *s* electron. This outer electron is easily removed and can spend much of its time around another atom, forming a molecule. Thus alkali metals are highly reactive.

24. Spontaneous emission occurs when an electron is in an excited state of an atom and it spontaneously (with no external stimulus) drops back down to a lower energy level. To do this, it emits a photon to carry away the excess energy. Stimulated emission occurs when an electron is in an excited state of an atom but a photon strikes the atom and causes or stimulates the electron to make its transition to a lower energy level sooner than it would have done so spontaneously. The stimulating photon has to have the same energy as the difference in energy levels of the transition. The result is two photons of the same frequency—the original one and the one due to the emission.

25. Laser light is coherent (all of the photons are in the same phase) and ordinary light is not coherent (the photons have random phases relative to each other). Laser light is nearly a perfect plane wave, while ordinary light is spherically symmetric (which means that the intensity of laser light remains nearly constant as it moves away from the source, while the intensity of ordinary light drops off as $1/r^2$). Laser light is always monochromatic, while ordinary light can be monochromatic, but it usually is not. However, both travel at *c* and both display the wave–particle duality of photons. Both are created when electrons fall to lower energy levels and emit photons. Both are electromagnetic waves.

26. The 0.0005-W laser beam's intensity is nearly constant as it travels away from its source since it is approximately a plane wave. The street lamp's intensity drops off as $1/r^2$ as it travels away from its source since it is a spherically symmetric wave. So, at a distance, the power from the lamp light actually reaching the camera is much less than 1000 W. Also, the street lamp's intensity is spread out over many wavelengths (many of which are probably not in the visible range), whereas the laser's intensity is all at one (assumed visible) wavelength.

27. No, the intensity of light from a laser beam does not drop off as the inverse square of the distance. Laser light is much closer to being a plane wave than a spherically symmetric wave, so its intensity is nearly constant along the entire beam. It does spread slightly over long distances, so it is not a perfect plane wave.

28. The continuous portion of the X-ray spectrum is due to the "bremsstrahlung" radiation. An incoming electron gives up energy in the collision and emits light. Electrons can give up all or part of their kinetic energy. The maximum amount of energy an electron can give up is its total amount of kinetic energy. In the photon description of light, the maximum electron kinetic energy will correspond to the energy of the shortest-wavelength (highest-energy) photons that can be produced in the collisions. The result is the existence of a "cutoff" wavelength in the X-ray spectrum. An increase in the number of electrons will not change the cutoff wavelength. According to wave theory, an increase in the number of electrons could result in the production of shorter-wavelength photons, which is not observed experimentally.

29. When we use the Bohr theory to calculate the X-ray line wavelengths, we estimate the nuclear charge seen by the transitioning electron as $Z-1,$ assuming that the second electron in the ground state is partially shielding the nuclear charge. This is only an estimate, so we do not expect the calculated wavelengths to agree exactly with the measured values.

30. To figure out which lines in an X-ray spectrum correspond to which transitions, you would use the Bohr model to estimate the differences in the sizes of the energy level jumps that the falling electrons will make. Then, you just need to match the $n=2 \rightarrow n=1$ energy to the K_α line, the $n=3 \rightarrow n=1$ energy to the K_β line, the $n=3 \rightarrow n=2$ jump energy to the L_α line, and so on.

Responses to MisConceptual Questions

1. (*b*) A common error is to add up the coefficients (principal quantum numbers), which would give an answer of 15. However, the exponents represent the number of electrons in each subshell. In this case there are 19 electrons.

2. (*b*) The orbital quantum numbers are represented by the subshell letters: $s = 0$ and $p = 1$. Since only *s* and *p* subshells are present, only the orbital quantum numbers 0 and 1 are in the electron configuration.

3. (*a*) In considering electrons, if only the particle property of electrons is considered, then the electron is viewed as a point particle passing through the slit. The electron also has wave properties and therefore undergoes diffraction as it passes through the slit.

4. (*c*) A common misconception is that all of the electrons have their lowest quantum numbers when in the ground state. However, due to the Pauli exclusion principle, no two electrons can have the same quantum numbers. The ground state of an atom is when each electron occupies the lowest energy state available to it without violating the exclusion principle.

5. (*e*) The Pauli exclusion principle applies to all electrons within the same atom. No two electrons in an atom can have the same set of four quantum numbers. Electrons from different atoms can have the same set of quantum numbers (for example, two separate atoms in the same container or two atoms from different parts of the Periodic Table).

6. (*c*) The Heisenberg uncertainty principle deals with our inability to know the position and momentum of a particle precisely. The more precisely you can determine the position, the less precisely you can determine the momentum, and vice versa.

7. (*a*) In an atom, multiple electrons will be in the same shell and can therefore have the same value of *n.* Electrons in different shells can have the same value of m_ℓ. Electrons in different atoms can also have the same set of four quantum numbers. The Pauli exclusion principle does not allow electrons within the same atom to have the same set of four quantum numbers.

8. (*e*) The Heisenberg uncertainty principle prohibits the exact measurement of position and velocity at the same time; the product of their uncertainties will always be greater than a constant value.

9. (*c*) The uncertainty principle allows for a precise measurement of the position, provided that there is a large uncertainty in the momentum. It also allows for a precise measurement of the momentum, provided that the simultaneous measurement of the position has a large uncertainty. It does not allow for the precise measurement of position and momentum at the same time.

10. (*d*) Laser light is monochromatic (all photons have nearly identical frequencies), is coherent (all photons have the same phase), moves as a beam, and is created by a population inversion. A laser beam usually is intense, but it does not have to be brighter than other sources of light.

Solutions to Problems

1. We find the wavelength of the neutron from Eq. 27–8, using the classical relationship between momentum and kinetic energy for the low energy neutrons. The peaks of the interference pattern are given by Eq. 24–2a and Fig. 24–10. For small angles, we have $\sin\theta = \tan\theta$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_{\text{n}}\text{KE}}}; \quad d \sin\theta = m\lambda, \quad m = 1, 2, \ldots; \quad x = \ell \tan\theta$$

$$\sin\theta = \tan\theta \quad \rightarrow \quad \frac{m\lambda}{d} = \frac{x}{\ell} \quad \rightarrow \quad x = \frac{m\lambda\ell}{d}, \quad m = 1, 2, \ldots \quad \rightarrow$$

$$\Delta x = \frac{\lambda\ell}{d} = \frac{h\ell}{d\sqrt{2m_{\text{n}}\text{KE}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1.0\ \text{m})}{(4.0\times10^{-4}\ \text{m})\sqrt{2(1.67\times10^{-27}\ \text{kg})(0.025\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}}$$

$$= \boxed{4.5\times10^{-7}\ \text{m}}$$

2. We find the wavelength of a pellet from Eq. 27–8. The half-angle for the central circle of the diffraction pattern is given in Section 25–7 as $\sin\theta = \frac{1.22\lambda}{D}$, where D is the diameter of the opening. Assuming the angle is small, the diameter of the spread of the bullet beam is $d = 2\ell \tan\theta = 2\ell \sin\theta$.

$$\lambda = \frac{h}{p} = \frac{h}{mv}; \quad d = 2\ell \tan\theta = 2\ell \sin\theta = 2\ell\frac{1.22\lambda}{D} = 2\ell\frac{1.22h}{Dmv} \quad \rightarrow$$

$$\ell = \frac{Dmvd}{2.44h} = \frac{(3.0\times10^{-3}\ \text{m})(2.0\times10^{-3}\ \text{kg})(120\ \text{m/s})(0.010\ \text{m})}{2.44(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{4.5\times10^{27}\ \text{m}}$$

This is more than 10^{11} light-years.

3. The uncertainty in the velocity is given, so the uncertainty in the momentum can be calculated. Use Eq. 28–1 to find the minimum uncertainty in the position.

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(1200\ \text{m/s})} = \boxed{5.3\times10^{-11}\ \text{m}}$$

4. The uncertainty in position is given. Use Eq. 28–1 to find the uncertainty in the momentum.

$$\Delta p = m\Delta v \geq \frac{\hbar}{\Delta x} \quad \rightarrow \quad \Delta v \geq \frac{\hbar}{m\Delta x} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(2.4\times10^{-8}\ \text{m})} = 4825\ \text{m/s} \approx \boxed{4.8\times10^{3}\ \text{m/s}}$$

$\boxed{5.}$ The minimum uncertainty in the energy is found from Eq. 28–2.

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(1\times10^{-8}\ \text{s})} = 1.055\times10^{-26}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 6.59\times10^{-8}\ \text{eV} \approx \boxed{7\times10^{-8}\ \text{eV}}$$

6. We find the lifetime of the particle from Eq. 28–2.

$$\Delta t \geq \frac{\hbar}{\Delta E} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(2.5\ \text{GeV})(1.60\times10^{-10}\ \text{J/GeV})} = \boxed{2.6\times10^{-25}\ \text{s}}$$

7. We find the uncertainty in the energy of the muon from Eq. 28–2, and then find the uncertainty in the mass.

$$\Delta E \geq \frac{\hbar}{\Delta t}; \Delta E = (\Delta m)c^2 \quad \rightarrow$$

$$\Delta m \geq \frac{\hbar}{c^2\Delta t} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{c^2(2.20\times10^{-6}\ \text{s})} = \left(4.7955\times10^{-29}\ \frac{\text{J}}{c^2}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{3.00\times10^{-10}\ \text{eV}/c^2}$$

8. We find the uncertainty in the energy of the free neutron from Eq. 28–2, and then the mass uncertainty from Eq. 26–7. We assume that the lifetime of the neutron is good to 2 significant figures. The current experimental lifetime of the neutron is between 881 and 882 seconds.

$$\Delta E \geq \frac{\hbar}{\Delta t}; \quad \Delta E = (\Delta m)c^2 \quad \rightarrow \quad \Delta m \geq \frac{\hbar}{c^2 \Delta t} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(3.00\times10^{8}\text{ m/s})^2(880\text{ s})} = \boxed{1.3\times10^{-54}\text{ kg}}$$

9. The uncertainty in the position is found from the uncertainty in the velocity and Eq. 28–1. The uncertainty in velocity is found by multiplying the velocity by its precision.

$$\Delta x_{\text{electron}} \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(120\text{ m/s})(6.5\times10^{-4})} = 1.485\times10^{-3}\text{ m} \approx \boxed{1.5\times10^{-3}\text{ m}}$$

$$\Delta x_{\text{baseball}} \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(0.14\text{ kg})(120\text{ m/s})(6.5\times10^{-4})} = 9.661\times10^{-33}\text{ m} \approx \boxed{9.7\times10^{-33}\text{ m}}$$

$$\frac{\Delta x_{\text{electron}}}{\Delta x_{\text{baseball}}} = \frac{\dfrac{\hbar}{m_{\text{electron}}\Delta v}}{\dfrac{\hbar}{m_{\text{baseball}}\Delta v}} = \frac{m_{\text{baseball}}}{m_{\text{electron}}} = \frac{(0.14\text{ kg})}{(9.11\times10^{-31}\text{ kg})} = 1.5\times10^{29}$$

The uncertainty for the electron is greater by a factor of 1.5×10^{29}.

10. The uncertainty in the energy is found from the lifetime and the uncertainty principle.

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(12\times10^{-6}\text{ s})}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 5.49\times10^{-11}\text{ eV}$$

$$\frac{\Delta E}{E} = \frac{5.49\times10^{-11}\text{ eV}}{5.5\times10^{6}\text{ eV}} = \boxed{1.0\times10^{-17}}$$

11. The uncertainty in position is given. Use Eq. 28–1 to find the uncertainty in the momentum.

$$\Delta p = m\Delta v \geq \frac{\hbar}{\Delta x} \quad \rightarrow \quad \Delta v \geq \frac{\hbar}{m\Delta x} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(15\times10^{-9}\text{ m})} = 7720\text{ m/s} \approx \boxed{7.7\times10^{3}\text{ m/s}}$$

The speed is low enough that the electron may be treated with classical mechanics instead of relativistic mechanics. We calculate the minimum kinetic energy from the speed calculated above.

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.11\times10^{-31}\text{ kg})(7720\text{ m/s})^2 = 2.715\times10^{-23}\text{ J}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right) = 1.694\times10^{-4}\text{ eV}$$

$$= \boxed{2.7\times10^{-23}\text{ J, or }1.7\times10^{-4}\text{ eV}}$$

12. We use the radius as the uncertainty in position for the neutron. We find the uncertainty in the momentum from Eq. 28–1. If we assume that the lowest value for the momentum is the least uncertainty, then we can estimate the lowest possible kinetic energy (nonrelativistic) as the following:

$$E = \frac{(\Delta p)^2}{2m} = \frac{\left(\dfrac{\hbar}{\Delta x}\right)^2}{2m} = \frac{\left(\dfrac{1.055\times10^{-34}\text{ kg}\cdot\text{m/s}}{1.2\times10^{-15}\text{ m}}\right)^2}{2(1.67\times10^{-27}\text{ kg})} = 2.314\times10^{-12}\text{ J}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)$$

$$= 14.46\text{ MeV} \approx \boxed{14\text{ MeV}}$$

13. The momentum of the electron is found from the energy by the classical relationship.

$$p = [2m(\text{KE})]^{\frac{1}{2}} = [2(9.11\times10^{-31}\ \text{kg})(5.00\ \text{keV})(1.60\times10^{-16}\ \text{J/keV})]^{\frac{1}{2}} = 3.817\times10^{-23}\ \text{kg}\cdot\text{m/s}$$

If the uncertainty in the energy is 1.00%, then the uncertainty in momentum can be found using the binomial expansion. Then use Eq. 28–1 to find the uncertainty in position.

$$p' = [2m(1.0100\ \text{KE})]^{\frac{1}{2}} = [2m(\text{KE})]^{\frac{1}{2}}(1+0.0100)^{\frac{1}{2}} \approx p[1+\tfrac{1}{2}(0.0100)]$$

$$\Delta p = p' - p = p\left(\tfrac{1}{2}\right)(0.0100) = (3.8127\times10^{-23}\ \text{kg}\cdot\text{m/s})\left(\tfrac{1}{2}\right)(0.0100) = 1.909\times10^{-25}\ \text{kg}\cdot\text{m/s}$$

$$\Delta x \ge \frac{\hbar}{\Delta p} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(1.909\times10^{-25}\ \text{kg}\cdot\text{m/s})} = \boxed{5.53\times10^{-10}\ \text{m}}$$

14. Use the radius as the uncertainty in position for the electron. We find the uncertainty in the momentum from Eq. 28–1, and then find the energy associated with that momentum from Eq. 26–9.

$$\Delta p \ge \frac{\hbar}{\Delta x} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{(1.0\times10^{-15}\ \text{m})} = 1.055\times10^{-19}\ \text{kg}\cdot\text{m/s}$$

If we assume that the lowest value for the momentum is the least uncertainty, then we can estimate the lowest possible energy.

$$\begin{aligned} E &= (p^2c^2 + m^2c^4)^{1/2} = [(\Delta p)^2c^2 + m^2c^4]^{1/2} \\ &= [(1.055\times10^{-19}\ \text{kg}\cdot\text{m/s})^2(3.00\times10^8\ \text{m/s})^2 + (9.11\times10^{-31}\ \text{kg})^2(3.00\times10^8\ \text{m/s})^4]^{1/2} \\ &= 3.175\times10^{-11}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) \approx \boxed{200\ \text{MeV}} \end{aligned}$$

15. The value of ℓ ranges from 0 to $n-1$. Thus, for $n=6$, $\boxed{\ell = 0, 1, 2, 3, 4, 5}$.

$\boxed{16.}$ The value of m_ℓ can range from $-\ell$ to $+\ell$. Thus, for $\ell = 3$, $\boxed{m_\ell = -3, -2, -1, 0, 1, 2, 3}$.

The possible values of m_s are $\boxed{-\tfrac{1}{2}, +\tfrac{1}{2}}$.

17. The number of electrons in the subshell is determined by the value of ℓ. For each ℓ, the value of m_ℓ can range from $-\ell$ to $+\ell$, which is $2\ell+1$ values. For each m_ℓ value, there are two values of m_s. Thus the total number of states for a given ℓ is $N = 2(2\ell+1)$.

$$N = 2(2\ell+1) = 2[2(3)+1] = \boxed{14\ \text{electrons}}$$

18. The value of ℓ ranges from 0 to $n-1$. Thus for $n=4$, $\ell = 0, 1, 2, 3$. For each ℓ, the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each m_ℓ, there are two values of m_s. Thus the number of states for each ℓ is $2(2\ell+1)$. The number of states is therefore given by the following:

$$N = 2(0+1) + 2(2+1) + 2(4+1) + 2(6+1) = \boxed{32\ \text{states}}$$

We start with $\ell = 0$, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{\begin{array}{l}(4,0,0,-\tfrac{1}{2}),(4,0,0,+\tfrac{1}{2}),(4,1,-1,-\tfrac{1}{2}),(4,1,-1,+\tfrac{1}{2}),(4,1,0,-\tfrac{1}{2}),(4,1,0,+\tfrac{1}{2}),\\(4,1,1,-\tfrac{1}{2}),(4,1,1,+\tfrac{1}{2}),(4,2,-2,-\tfrac{1}{2}),(4,2,-2,+\tfrac{1}{2}),(4,2,-1,-\tfrac{1}{2}),(4,2,-1,+\tfrac{1}{2}),\\(4,2,0,-\tfrac{1}{2}),(4,2,0,+\tfrac{1}{2}),(4,2,1,-\tfrac{1}{2}),(4,2,1,+\tfrac{1}{2}),(4,2,2,-\tfrac{1}{2}),(4,2,2,+\tfrac{1}{2}),\\(4,3,-3,-\tfrac{1}{2}),(4,3,-3,+\tfrac{1}{2}),(4,3,-2,-\tfrac{1}{2}),(4,3,-2,+\tfrac{1}{2}),(4,3,-1,-\tfrac{1}{2}),(4,3,-1,+\tfrac{1}{2}),\\(4,3,0,-\tfrac{1}{2}),(4,3,0,+\tfrac{1}{2}),(4,3,1,-\tfrac{1}{2}),(4,3,1,+\tfrac{1}{2}),(4,3,2,-\tfrac{1}{2}),(4,3,2,+\tfrac{1}{2}),\\(4,3,3,-\tfrac{1}{2}),(4,3,3,+\tfrac{1}{2})\end{array}}$$

19. (*a*) For carbon, $Z = 6$. We start with the $n = 1$ shell, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{(1,0,0,-\tfrac{1}{2}),\ (1,0,0,+\tfrac{1}{2}),\ (2,0,0,-\tfrac{1}{2}),\ (2,0,0,+\tfrac{1}{2}),\ (2,1,-1,-\tfrac{1}{2}),\ (2,1,-1,+\tfrac{1}{2})}$$

Note that, without additional information, there are other possibilities for the last two electrons. The third quantum number for the last two electrons could be –1, 0, or 1.

(*b*) For aluminum, $Z = 13$. We start with the $n = 1$ shell, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{\begin{array}{l}(1,0,0,-\tfrac{1}{2}),\ (1,0,0,+\tfrac{1}{2}),\ (2,0,0,-\tfrac{1}{2}),\ (2,0,0,+\tfrac{1}{2}),\ (2,1,-1,-\tfrac{1}{2}),\ (2,1,-1,+\tfrac{1}{2}),\\(2,1,0,-\tfrac{1}{2}),\ (2,1,0,+\tfrac{1}{2}),\ (2,1,1,-\tfrac{1}{2}),\ (2,1,1,+\tfrac{1}{2}),\ (3,0,0,-\tfrac{1}{2}),\ (3,0,0,+\tfrac{1}{2}),\ (3,1,-1,-\tfrac{1}{2})\end{array}}$$

Note that, without additional information, there are other possibilities for the last electron. The third quantum number could be –2, –1, 0, 1, or 2, and the fourth quantum number could be either $+\frac{1}{2}$ or $-\frac{1}{2}$.

20. For oxygen, $Z = 8$. We start with the $n = 1$ shell, and list the quantum numbers in the order (n, ℓ, m_ℓ, m_s).

$$\boxed{\begin{array}{l}(1,0,0,-\tfrac{1}{2}),\ (1,0,0,+\tfrac{1}{2}),\ (2,0,0,-\tfrac{1}{2}),\ (2,0,0,+\tfrac{1}{2}),\\(2,1,-1,-\tfrac{1}{2}),\ (2,1,-1,+\tfrac{1}{2}),\ (2,1,0,-\tfrac{1}{2}),\ (2,1,0,+\tfrac{1}{2})\end{array}}$$

Note that, without additional information, there are two other possibilities that could substitute for any of the last four electrons. The third quantum number could be changed to 1.

21. The magnitude of the angular momentum depends only on ℓ.

$$L = \sqrt{\ell(\ell+1)}\,\hbar = \boxed{\sqrt{12}\,\hbar} = \sqrt{12}(1.055\times10^{-34}\ \mathrm{J\cdot s}) = \boxed{3.65\times10^{-34}\ \mathrm{J\cdot s}}$$

22. The value of m_ℓ can range from $-\ell$ to $+\ell$, so we have $\boxed{m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4}$.

The value of ℓ can range from 0 to $n-1$. Thus we have $\boxed{n \ge 5}$.

There are two values of m_s: $\boxed{m_s = -\frac{1}{2}, +\frac{1}{2}}$.

23. The value of ℓ can range from 0 to $n-1$. Thus we have $\boxed{n \geq 4}$.

The value of m_ℓ can range from $-\ell$ to $+\ell$, so $\boxed{\ell \geq 3}$, and $\boxed{\ell \leq n-1}$. So $\boxed{3 \leq \ell \leq n-1}$.

There are two values of m_s: $\boxed{m_s = -\frac{1}{2}, +\frac{1}{2}}$.

24. The "g" subshell has $\ell = 4$. The value of m_ℓ can range from $-\ell$ to $+\ell$, so there are 9 values of m_ℓ: $m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4$. Each of the 9 m_ℓ values represents two states, so the "g" subshell can hold $\boxed{\text{18 electrons}}$.

25. The configurations can be written with the subshells in order by energy (4*s* before 3*d*, for example) or with the subshells in principal number order (3*d* before 4*s*, for example, as in Table 28–4). Both are shown here.

(*a*) $Z = 26$ is iron.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2} \text{ or } \boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6}$$

(*b*) $Z = 34$ is selenium.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4} \text{ or } \boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^4}$$

(*c*) $Z = 38$ is strontium.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^2} \text{ or } \boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2}$$

$\boxed{26.}$ The configurations can be written with the subshells in order by energy (4*s* before 3*d*, for example) or with the subshells in principal number order (3*d* before 4*s*, for example, as in Table 28–4). Both are shown here.

(*a*) Silver has $Z = 47$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^1} \text{ or } \boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^1 4d^{10}}$$

(*b*) Gold has $Z = 79$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^1} \text{ or}$$

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 4f^{14} 6s^1 5d^{10}}$$

(*c*) Uranium has $Z = 92$.

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6 5f^3 6d^1 7s^2} \text{ or}$$

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 4f^{14} 6s^2 5d^{10} 6p^6 7s^2 5f^3 6d^1}$$

27. (*a*) The principal quantum number is $n = \boxed{5}$.

(*b*) The energy of the state is found from Eq. 27–15b.

$$E_7 = -\frac{(13.6 \text{ eV})}{n^2} = -\frac{(13.6 \text{ eV})}{5^2} = \boxed{-0.544 \text{ eV}}$$

(*c*) The *d* subshell has $\ell = \boxed{2}$. The magnitude of the angular momentum depends on ℓ only.

$$L = \hbar\sqrt{\ell(\ell+1)} = \boxed{\sqrt{6}\,\hbar}$$

(*d*) For each ℓ the value of m_ℓ can range from $-\ell$ to $+\ell$.

$$\boxed{m_\ell = -2,\ -1,\ 0,\ 1,\ 2}$$

28. The third electron in lithium is in the 2*s* subshell, which is outside the more tightly bound filled 1*s* shell. This makes it appear as if there is a "nucleus" with a net charge of $+1e$. Use the energy of the hydrogen atom, Eq. 27–15b.

$$E_2 = -\frac{(13.6\text{ eV})}{n^2} = -\frac{(13.6\text{ eV})}{2^2} = -3.4\text{ eV}$$

We predict the binding energy to be $\boxed{3.4\text{ eV}}$. Our assumption of complete shielding of the nucleus by the 2*s* electrons is evidently not correct. The partial shielding means the net charge of the "nucleus" is higher than +1*e*, so it holds the outer electron more tightly, requiring more energy to remove it.

29. In a filled subshell, there are an even number of electrons. Those electrons have m_ℓ values of $-\ell, -\ell+\ell, \ldots, -1, 0, 1, \ldots \ell, -1, \ell$. For each of those electrons, $L_z = m_\ell \hbar$. When all of those L_z values are calculated and summed, the total is $-\ell\hbar + (-\ell+1)\hbar + \cdots + (\ell-1)\hbar + \ell\hbar = 0$. Half of the electrons have spin angular momentum of "up," and the other half have spin angular momentum of "down." The sum of those spin angular momenta is also 0. Thus the total angular momentum is 0.

30. (*a*) The $4p \rightarrow 3p$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = 0 \neq \pm 1$.

(*b*) The $3p \rightarrow 1s$ transition is $\boxed{\text{allowed,}}$ because $\Delta\ell = -1$.

(*c*) The $4d \rightarrow 2d$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = 0 \neq \pm 1$.

(*d*) The $5d \rightarrow 3s$ transition is $\boxed{\text{forbidden,}}$ because $\Delta\ell = -2 \neq \pm 1$.

(*e*) The $4s \rightarrow 2p$ transition is $\boxed{\text{allowed,}}$ because $\Delta\ell = +1$.

31. Since the electron is in its lowest energy state, we must have the lowest possible value of *n*. Since $m_\ell = 2$, the smallest possible value of ℓ is $\boxed{\ell = 2}$, and the smallest possible value of *n* is $\boxed{n = 3}$.

32. Photon emission means a jump to a lower state, so $n = 1, 2, 3, 4,$ or 5. For a *d* subshell, $\ell = 2$, and because $\Delta\ell = \pm 1$, the new value of ℓ must be 1 or 3.

(*a*) $\ell = 1$ corresponds to a *p* subshell, and $\ell = 3$ corresponds to an *f* subshell. Keeping in mind that $0 \leq \ell \leq n-1$, we find the following possible destination states: $2p, 3p, 4p, 5p, 4f, 5f$. In the format requested, these states are: $\boxed{(2, 1), (3, 1), (4, 1), (5, 1), (4, 3), (5, 3)}$.

(*b*) In a hydrogen atom, ℓ has no appreciable effect on energy, so for energy purposes there are four possible destination states, corresponding to *n* = 2, 3, 4, and 5. Thus, there are $\boxed{\text{four different photon wavelengths}}$ corresponding to four possible changes in energy.

33. The smallest wavelength X-ray has the most energy, which is the maximum KE of the electron in the tube. A 28.5-kV potential difference means that the electrons have 28.5 keV of KE.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(28.5\times10^{3}\ \text{eV})} = 4.362\times10^{-11}\ \text{m} = \boxed{0.0436\ \text{nm}}$$

The longest wavelength of the continuous spectrum would be at the limit of the X-ray region of the electromagnetic spectrum, generally on the order of $\boxed{1\ \text{nm}}$.

$\boxed{34.}$ The shortest wavelength X-ray has the most energy, which is the maximum kinetic energy of the electron in the tube:

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(0.035\times10^{-9}\ \text{m})} = 3.552\times10^{4}\ \text{eV} \approx 36\ \text{keV}$$

Thus the operating voltage of the tube is $\boxed{36\ \text{kV}}$.

35. The energy of the photon with the shortest wavelength must equal the maximum kinetic energy of an electron. We assume that V is in volts.

$$E = hf_0 = \frac{hc}{\lambda_0} = eV \quad \rightarrow$$

$$\lambda_0 = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})(10^{9}\ \text{nm/m})}{(1.60\times10^{-19}\ \text{C})(V\ \text{V})} = \frac{1243\ \text{nm}}{V} \approx \boxed{\frac{1240\ \text{nm}}{V}}$$

36. The maximum energy is emitted when the shortest wavelength is emitted. That appears to be about 0.025 nm. The minimum energy is emitted when the longest wavelength is emitted. That appears to be about 0.2 nm. Use Eq. 27–4.

$$E_{\text{max}} = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.025\times10^{-9}\ \text{m})}\frac{(1\ \text{eV})}{(1.60\times10^{-19}\ \text{J})} = 49{,}725\ \text{eV}$$
$$\approx \boxed{5.0\times10^{4}\ \text{eV}}$$

$$E_{\text{min}} = hf_{\text{min}} = \frac{hc}{\lambda_{\text{max}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.2\times10^{-9}\ \text{m})}\frac{(1\ \text{eV})}{(1.60\times10^{-19}\ \text{J})} = 6215\ \text{eV} \approx \boxed{6\times10^{3}\ \text{eV}}$$

37. We follow Example 28–7, using the Bohr formula, Eq. 27–16, with Z replaced by $Z-1$.

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 e^4 mk^2}{h^3 c}\right)(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right) = (1.097\times10^{7}\ \text{m}^{-1})(27-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$
$$= 5.562\times10^{9}\,\text{m}^{-1} \quad \rightarrow \quad \lambda = \frac{1}{5.562\times10^{9}\ \text{m}^{-1}} = \boxed{1.798\times10^{-10}\ \text{m}}$$

$\boxed{38.}$ We follow Example 28–7, using the Bohr formula, Eq. 27–16, with Z replaced by $Z-1$.

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 e^4 mk^2}{h^3 c}\right)(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right) = (1.097\times10^{7}\ \text{m}^{-1})(26-1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$
$$= 9.523\times10^{9}\ \text{m}^{-1} \quad \rightarrow \quad \lambda = \frac{1}{9.523\times10^{9}\ \text{m}^{-1}} = \boxed{1.050\times10^{-9}\ \text{m}}$$

39. We assume that there is "shielding" provided by the 1*s* electron that is already at that level. Thus the effective charge "seen" by the transitioning electron is $42-1=41$. We use Eqs. 27–10 and 27–15b.

$$hf=\Delta E=(13.6\text{ eV})(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)$$

$$\lambda=\frac{hc}{\Delta E}=\frac{hc}{(13.6\text{ eV})(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(13.6\text{ eV})(41^2)\left(\frac{1}{1^2}-\frac{1}{3^2}\right)(1.60\times10^{-19}\text{ J/ev})}$$

$$=6.12\times10^{-11}\text{ m}=\boxed{0.0612\text{ nm}}$$

We do not expect perfect agreement because there is some $\boxed{\text{partial shielding provided by the } n=2 \text{ shell,}}$ which was ignored when we replaced *Z* with *Z* – 1. That would make the effective atomic number a little smaller, which would lead to a larger wavelength. The amount of shielding could be estimated by using the actual wavelength and solving for the effective atomic number.

40. The wavelength of the K_α line is calculated for molybdenum in Example 28–7. We use that same procedure. Note that the wavelength is inversely proportional to $(Z-1)^2$.

$$\frac{\lambda_{\text{unknown}}}{\lambda_{\text{Fe}}}=\frac{(Z_{\text{Fe}}-1)^2}{(Z_{\text{unknown}}-1)^2}\quad\rightarrow\quad Z_{\text{unknown}}=\left[(26-1)\sqrt{\frac{194\text{ pm}}{229\text{ pm}}}\right]+1=24$$

The unknown material has *Z* = 24, so it is $\boxed{\text{chromium}}$.

$\boxed{41.}$ The energy of a pulse is the power of the pulse times the duration in time.

$$E=P\Delta t=(0.68\text{ W})(25\times10^{-3}\text{ s})=\boxed{0.017\text{ J}}$$

The number of photons in a pulse is the energy of a pulse divided by the energy of a photon as given in Eq. 27–4.

$$N=\frac{E}{hf}=\frac{E\lambda}{hc}=\frac{(0.017\text{ J})(640\times10^{-9}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}=\boxed{5.5\times10^{16}\text{ photons}}$$

42. Intensity equals power per unit area. The area of the light from the laser is assumed to be in a circular area, while the area intercepted by the light from a lightbulb is the surface area of a sphere.

(*a*) $$I=\frac{P}{\pi r^2}=\frac{0.50\times10^{-3}\text{ W}}{\pi(1.5\times10^{-3}\text{ m})^2}=70.74\text{ W/m}^2\approx\boxed{71\text{ W/m}^2}$$

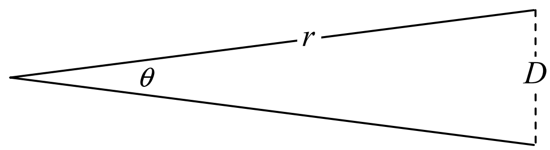
(*b*) $$I=\frac{P}{4\pi r^2}=\frac{100\text{ W}}{4\pi(2.0\text{ m})^2}=1.989\text{ W/m}^2\approx2.0\text{ W/m}^2$$

The laser beam is more intense by a factor of $\frac{70.74\text{ W/m}^2}{1.989\text{ W/m}^2}=35.57\approx\boxed{36}$.

43. Transition from the E_3' state to the E_2' state releases photons with energy 1.96 eV, as shown in Fig. 28–20. The wavelength is determined from the energy.

$$\lambda=\frac{hc}{\Delta E}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(1.96\text{ eV})}=6.34\times10^{-7}\text{ m}=\boxed{634\text{ nm}}$$

44. The angular half-width of the beam is given in Eq. 25–7, $\theta_{1/2} = \dfrac{1.22\lambda}{d}$, where d is the diameter of the diffracting circle. The angular width of the beam is twice this. The linear diameter of the beam is then the angular width times the distance from the source of the light to the observation point, $D = r\theta$, for small angles. See the diagram.



$$\theta_{1/2} = \frac{1.22\lambda}{d} = \frac{1.22(694\times10^{-9}\text{ m})}{(3.0\times10^{-3}\text{ m})} = 2.822\times10^{-4}\text{ rad} \quad\rightarrow\quad \theta = 5.644\times10^{-4}\text{ rad} \approx \boxed{5.6\times10^{-4}\text{ rad}}$$

(*a*) The diameter of the beam when it reaches the satellite is as follows:

$$D = r\theta = (340\times10^{3}\text{ m})(5.644\times10^{-4}\text{ rad}) = \boxed{190\text{ m}}$$

(*b*) The diameter of the beam when it reaches the Moon is as follows:

$$D = r\theta = (384\times10^{6}\text{ m})(5.644\times10^{-4}\text{ rad}) = \boxed{2.2\times10^{5}\text{ m}}$$

45. We use Eq. 28–3 to find $\boldsymbol{\ell}$ and the unnumbered equation following it to find m_ℓ.

$$L = \sqrt{\ell(\ell+1)}\,\hbar \quad\rightarrow\quad \ell(\ell+1) = \frac{L^2}{\hbar^2} = \frac{(6.84\times10^{-34}\text{ J}\cdot\text{s})^2}{(1.055\times10^{-34}\text{ J}\cdot\text{s})^2} = 42$$

$$\ell^2 + \ell - 42 = 0 \quad\rightarrow\quad \ell = \frac{-1\pm\sqrt{1-4(1)(-42)}}{2} = \frac{-1\pm13}{2} = 6, -7 \quad\rightarrow\quad \boxed{\ell = 6}$$

$$L_z = m_\ell\hbar \quad\rightarrow\quad m_\ell = \frac{L_z}{\hbar} = \frac{2.11\times10^{-34}\text{ J}\cdot\text{s}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{2}$$

Since $\ell = 6$, we must have $\boxed{n \ge 7}$.

46. (*a*) The minimum uncertainty in the energy is found from Eq. 28–2.

$$\Delta E \ge \frac{\hbar}{\Delta t} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(1\times10^{-8}\text{ s})} = 1.055\times10^{-26}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 6.59\times10^{-8}\text{ eV} \approx \boxed{10^{-7}\text{ eV}}$$

(*b*) The transition energy can be found from the Bohr hydrogen atom energies, Eq. 27–15b, with $Z = 1$ for hydrogen.

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} \quad\rightarrow\quad E_2 - E_1 = \left[-(13.6\text{ eV})\frac{1^2}{2^2}\right] - \left[-(13.6\text{ eV})\frac{1^2}{1^2}\right] = 10.2\text{ eV}$$

$$\frac{\Delta E}{E_2 - E_1} = \frac{6.59\times10^{-8}\text{ eV}}{10.2\text{ eV}} = 6.46\times10^{-9} \approx \boxed{10^{-8}}$$

The uncertainty in the energy is a very small fraction of the actual transition energy.

(*c*) The wavelength is given by Eq. 27–4.

$$E = hf = \frac{hc}{\lambda} \quad\rightarrow\quad \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(10.2\text{ eV})\left(\dfrac{1.60\times10^{-19}\text{ J}}{\text{eV}}\right)} = 1.22\times10^{-7}\text{ m} = \boxed{122\text{ nm}}$$

Take the width in the line to be the uncertainty in the line's value, based on the uncertainty in the energy. We will use Eq. 27–4 with the binomial expansion.

$$\lambda+\Delta\lambda=\frac{hc}{(E+\Delta E)}=\frac{hc}{E\left(1+\frac{\Delta E}{E}\right)}=\frac{hc}{E}\left(1+\frac{\Delta E}{E}\right)^{-1}\approx\frac{hc}{E}\left(1-\frac{\Delta E}{E}\right)=\lambda\left(1-\frac{\Delta E}{E}\right)\rightarrow$$

$$|\Delta\lambda|=\lambda\frac{\Delta E}{E}=(122\text{ nm})\frac{6.59\times10^{-8}\text{ eV}}{10.2\text{ eV}}=\boxed{8\times10^{-7}\text{ nm}}$$

47. The value of ℓ can range from 0 to $n-1$. Thus for $n=6$, we have $0\le\ell\le5$. The magnitude of the angular momentum is given by Eq. 28–3, $L=\sqrt{\ell(\ell+1)}\,\hbar$.

$$\boxed{L_{\text{min}}=0;\quad L_{\text{max}}=\sqrt{30}\,\hbar}$$

48. (*a*) We find the wavelength from Eq. 27–8.

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(0.012\text{ kg})(150\text{ m/s})}=3.683\times10^{-34}\text{ m}\approx\boxed{3.7\times10^{-34}\text{ m}}$$

(*b*) Use Eq. 28–1 to find the uncertainty in momentum.

$$\Delta p_x\ge\frac{\hbar}{\Delta x}=\frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(0.0060\text{ m})}=1.758\times10^{-32}\text{ kg}\cdot\text{m/s}\approx\boxed{1.8\times10^{-32}\text{ kg}\cdot\text{m/s}}$$

49. Use Eq. 28–1 (the uncertainty principle) and Eq. 27–8 (the de Broglie wavelength).

$$\Delta p\ge\frac{\hbar}{\Delta x}=\frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{(2.0\times10^{-8}\text{ m})}=5.275\times10^{-27}\text{ kg}\cdot\text{m/s}\approx\boxed{5.3\times10^{-27}\text{ kg}\cdot\text{m/s}}$$

$$\lambda=\frac{h}{p}=\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{5.275\times10^{-27}\text{ kg}\cdot\text{m/s}}=1.257\times10^{-7}\text{ m}\approx\boxed{1.3\times10^{-7}\text{ m}}$$

50. (*a*) Boron has $Z=4$, with an electron configuration of $1s^2 2s^2$, so the outermost electron has $n=2$. We use the Bohr result with an effective *Z*. We might naively expect to get $Z_{\text{eff}}=1$, indicating that the other three electrons shield the outer electron from the nucleus, or $Z_{\text{eff}}=2$, indicating that only the inner two electrons accomplish the shielding.

$$E_2=-\frac{(13.6\text{ eV})(Z_{\text{eff}})^2}{n^2}\rightarrow-8.26\text{ eV}=-\frac{(13.6\text{ eV})(Z_{\text{eff}})^2}{2^2}\rightarrow Z_{\text{eff}}=\boxed{1.56}$$

This indicates that the other electron in the $n=2$ shell does partially shield the electron that is to be removed.

(*b*) We find the average radius from Eq. 27–14.

$$r=\frac{n^2r_1}{Z_{\text{eff}}}=\frac{2^2(0.529\times10^{-10}\text{ m})}{(1.56)}=\boxed{1.36\times10^{-10}\text{ m}}$$

51. Use Eq. 27–14, which says that the radius of a Bohr orbit is inversely proportional to the atomic number. Also use Eq. 27–15b, which says that the energy of a Bohr orbit is proportional to the square of the atomic number. The energy to remove the electron is the opposite of the total energy. A simple approximation is used in which the electron "feels" the full effect of all 92 protons in the nucleus.

$$r_n = \frac{n^2}{Z}(0.529\times10^{-10}\text{ m}) = \frac{1}{92}(0.529\times10^{-10}\text{ m}) = \boxed{5.75\times10^{-13}\text{ m}}$$
$$|E_n| = (13.6\text{ eV})\frac{Z^2}{n^2} = (13.6\text{ eV})\frac{92^2}{1^2} = \boxed{1.15\times10^5\text{ eV}} = 115\text{ keV}$$

52. We find the wavelength of the protons from their kinetic energy, using nonrelativistic mechanics and Eq. 27–8, and then use Eq. 24–21 for two-slit interference, with a small-angle approximation. If the protons were accelerated by a 480-V potential difference, then they will have 480 eV of kinetic energy.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_\text{p}\text{KE}}};\quad d\sin\theta = m\lambda,\quad m = 1,\ 2, \ldots;\quad x = \ell\tan\theta$$
$$\sin\theta = \tan\theta \;\rightarrow\; \frac{m\lambda}{d} = \frac{x}{\ell} \;\rightarrow\; x = \frac{m\lambda\ell}{d},\quad m = 1,\ 2, \ldots \;\rightarrow$$
$$\Delta x = \frac{\lambda\ell}{d} = \frac{h\ell}{d\sqrt{2m_\text{p}\text{KE}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(28\text{ m})}{(7.0\times10^{-4}\text{ m})\sqrt{2(1.67\times10^{-27}\text{ kg})(480\text{ eV})(1.60\times10^{-19}\text{ J/ev})}}$$
$$= \boxed{5.2\times10^{-8}\text{ m}}$$

53. An *h* subshell has $\ell = 5$. For a given ℓ value, m_ℓ ranges from $-\ell$ to $+\ell$, taking on $2\ell+1$ different values. For each m_ℓ, there are two values of m_s. Thus the number of states for a given ℓ value is $2(2\ell+1)$. Thus there are $2(2\ell+1) = 2(11) = \boxed{22}$ possible electron states.

54. (*a*) For each ℓ, the value of m_ℓ can range from $-\ell$ to $+\ell$, or $2\ell+1$ values. For each of these, there are two values of m_s. Thus the total number of states in a subshell is $N = \boxed{2(2\ell+1)}$.

(*b*) For $\ell = 0, 1, 2, 3, 4, 5$, and 6, $N = \boxed{2, 6, 10, 14, 18, 22, \text{ and } 26}$, respectively.

55. For a given n, $0 \le \ell \le n-1$. Since for each ℓ, the number of possible states is $2(2\ell+1)$, as shown in Problem 54, the number of possible states for a given *n* is as follows:

$$\sum_{\ell=0}^{n-1} 2(2\ell+1) = 4\sum_{\ell=0}^{n-1}\ell + \sum_{\ell=0}^{n-1} 2 = 4\left(\frac{n(n-1)}{2}\right) + 2n = \boxed{2n^2}$$

The summation formulas can be found (and proven) in a pre-calculus mathematics textbook.

56. We find the wavelength of the electrons from their kinetic energy, using nonrelativistic mechanics. Classical formulas are justified because the kinetic energy of $45\text{ keV} = 0.045\text{ MeV}$ is much less than the rest energy of 0.511 MeV. Eq. 27–8 is used to find the wavelength, and Eq. 24–21 is used for the two-slit interference, with a small-angle approximation.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e\text{KE}}};\quad d\sin\theta = m\lambda,\quad m = 1,\ 2,\ldots;\quad x = \ell\tan\theta$$

$$\sin\theta = \tan\theta \quad\rightarrow\quad \frac{m\lambda}{d} = \frac{x}{\ell} \quad\rightarrow\quad x = \frac{m\lambda\ell}{d},\quad m = 1,\ 2,\ \ldots \quad\rightarrow$$

$$\Delta x = \frac{\lambda\ell}{d} = \frac{h\ell}{d\sqrt{2m_e\text{KE}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(0.450\ \text{m})}{(2.0\times10^{-6}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(45\times10^{3}\ \text{eV})(1.60\times10^{-19}\ \text{J/ev})}}$$

$$= 1.30\times10^{-6}\ \text{m} \approx \boxed{1.3\ \mu\text{m}}$$

57. In the Bohr model, $L_{\text{Bohr}} = n\dfrac{h}{2\pi} = \boxed{2\hbar}$. In quantum mechanics, $L_{\text{qm}} = \sqrt{\ell(\ell+1)}\,\hbar$. For $n = 2$, $\ell = 0$ or $\ell = 1$, so that $L_{\text{qm}} = \sqrt{0(0+1)}\,\hbar = \boxed{0}$ or $L_{\text{qm}} = \sqrt{1(1+1)}\,\hbar = \boxed{\sqrt{2}\,\hbar}$.

58. The fractional uncertainty in the energy is found from the lifetime and the uncertainty principle, Eq. 28–2. Also use Eq. 27–4.

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{h}{2\pi\Delta t};\quad E = hf = \frac{hc}{\lambda}$$

$$\frac{\Delta E}{E} = \frac{\dfrac{h}{2\pi\Delta t}}{\dfrac{hc}{\lambda}} = \frac{\lambda}{2\pi c\Delta t} = \frac{500\times10^{-9}\ \text{m}}{2\pi(3.00\times10^{8}\ \text{m/s})(10\times10^{-9}\ \text{s})} = 2.65\times10^{-8} \approx \boxed{3\times10^{-8}}$$

Use the binomial expansion to find the wavelength's fractional uncertainty.

$$\lambda + \Delta\lambda = \frac{hc}{(E+\Delta E)} = \frac{hc}{E\left(1+\dfrac{\Delta E}{E}\right)} = \frac{hc}{E}\left(1+\frac{\Delta E}{E}\right)^{-1} \approx \frac{hc}{E}\left(1-\frac{\Delta E}{E}\right) = \lambda\left(1-\frac{\Delta E}{E}\right) \quad\rightarrow$$

$$|\Delta\lambda| = \lambda\frac{\Delta E}{E} \quad\rightarrow\quad \left|\frac{\Delta\lambda}{\lambda}\right| = \frac{\Delta E}{E} = \boxed{3\times10^{-8}}$$

59. Find the uncertainty in the position from Eq. 28–1.

$$\Delta x = \frac{\hbar}{m\Delta \upsilon} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{(1300\ \text{kg})(0.22\ \text{m/s})} = \boxed{3.7\times10^{-37}\ \text{m}}$$

60. The difference in measured energies is found from Eq. 27–4.

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$= (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})\left(\frac{1}{487\times10^{-9}\ \text{m}} - \frac{1}{489\times10^{-9}\ \text{m}}\right) = 1.670\times10^{-21}\ \text{J}$$

The lifetime of the excited state is determined from Eq. 28–2.

$$\Delta E\Delta t \ge \hbar \quad\rightarrow\quad \Delta t \ge \frac{\hbar}{\Delta E} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{1.670\times10^{-21}\ \text{J}} = 6.317\times10^{-14}\ \text{s} \approx \boxed{6.32\times10^{-14}\ \text{s}}$$

This solution assumes that the entire spread in energy is the uncertainty.

61. We assume that the particles are not relativistic. Conservation of energy is used to find the speed of each particle. That speed then can be used to find the momentum and finally the de Broglie wavelength. We let the magnitude of the accelerating potential difference be V.

$$\mathrm{PE_{initial}} = \mathrm{KE_{final}} \quad \rightarrow \quad eV = \tfrac{1}{2}m\upsilon^2 \quad \rightarrow \quad \upsilon = \sqrt{\frac{2eV}{m}}; \quad \lambda = \frac{h}{p} = \frac{h}{m\upsilon} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} = \Delta x$$

$$\Delta x \Delta p = \frac{h}{2\pi} \quad \rightarrow \quad \Delta p = \frac{2\pi h}{\Delta x}$$

$$\frac{\Delta p_{\text{proton}}}{\Delta p_{\text{electron}}} = \frac{\dfrac{2\pi h}{\Delta x_{\text{proton}}}}{\dfrac{2\pi h}{\Delta x_{\text{electron}}}} = \frac{\Delta x_{\text{electron}}}{\Delta x_{\text{proton}}} = \frac{\dfrac{h}{\sqrt{2m_{\text{electron}}eV}}}{\dfrac{h}{\sqrt{2m_{\text{proton}}eV}}} = \sqrt{\frac{m_{\text{proton}}}{m_{\text{electron}}}} = \sqrt{\frac{1.67\times10^{-27}\text{ kg}}{9.11\times10^{-31}\text{ kg}}} = \boxed{42.8}$$

62. Limiting the number of electron shells to six would mean that the Periodic Table stops with radon (Rn), since the next element, francium (Fr), begins filling the seventh shell. Including all elements up through radon, there would be $\boxed{86}$ elements.

63. The de Broglie wavelength is determined by Eq. 27–8.

$$\lambda = \frac{h}{p} = \frac{h}{m\upsilon} \quad \rightarrow \quad \upsilon = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(68.0\text{ kg})(0.50\text{ m})} = 1.95\times10^{-35}\text{ m/s} \approx \boxed{2.0\times10^{-35}\text{ m/s}}$$

Since λ is comparable to the width of a typical doorway, $\boxed{\text{yes,}}$ you would notice diffraction effects.

However, assuming that walking through the doorway requires travel through a distance of 0.2 m, the time Δt required is huge.

$$\upsilon\Delta t = d \quad \rightarrow \quad \Delta t = \frac{d}{\upsilon} = \frac{0.20\text{ m}}{1.95\times10^{-35}\text{ m/s}} = \boxed{1.0\times10^{34}\text{ s}} \approx 3\times10^{26}\text{ yr}$$

64. Since from Eq. 27–16, $\frac{1}{\lambda} \propto Z^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$, we seek Z and n'_{X} so that for every fourth value of n_{X} there is an n_{H} such that the Lyman formula equals the formula for the unknown element.

$$\frac{1}{1} - \frac{1}{n_{\text{H}}^2} = Z^2\left(\frac{1}{n'^2_{\text{X}}} - \frac{1}{n_{\text{X}}^2}\right) = \frac{Z^2}{n'^2_{\text{X}}} - \frac{Z^2}{n_{\text{X}}^2}$$

Consider atoms (with $Z > 1$) for which $Z^2/n'^2_{\text{X}} = 1 \quad \rightarrow \quad n'_{\text{X}} = Z$, and for which $n_{\text{X}}^2 = Z^2 n_{\text{H}}^2 \quad \rightarrow \quad n_{\text{X}} = Zn_{\text{H}}$ for every fourth line. This last condition is obviously met when $Z = 4$ and $n'_{\text{X}} = 4$. The resulting spectral lines for $n_{\text{X}} = 4, 8, 12, \ldots$ match the hydrogen Lyman series lines for $n = 1, 2, 3, \ldots$. The element is $\boxed{\text{beryllium.}}$

65. The K_α line is from the $n=2$ to $n=1$ transition. We use the energies of the hydrogen atom with Z replaced by $Z-1$, as in Example 28–7, using Eq. 27–16.

$$\frac{1}{\lambda}=\left(\frac{2\pi^2e^4mk^2}{h^3c}\right)(Z-1)^2\left(\frac{1}{n'^2}-\frac{1}{n^2}\right) \rightarrow$$

$$Z=1+\sqrt{\frac{1}{\lambda}\left(\frac{2\pi^2e^4mk^2}{h^3c}\right)^{-1}\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)^{-1}}=1+\sqrt{\frac{1}{(0.154\times10^{-9}\text{ m})}(1.097\times10^7\text{ m}^{-1})^{-1}\left(\frac{1}{1^2}-\frac{1}{2^2}\right)^{-1}}$$

$$=1+28.09\approx29$$

The element is $\boxed{\text{copper.}}$

Solutions to Search and Learn Problems

1. We use Eq. 27–11, Bohr's quantum condition, with n = 1 for the ground state.

$$m\upsilon r_n=n\frac{h}{2\pi} \rightarrow m\upsilon r_1=\hbar \rightarrow m\upsilon=p=\frac{\hbar}{r_1}=\Delta p$$

$$\Delta x\Delta p\approx\hbar \rightarrow \Delta x\approx\frac{\hbar}{\Delta p}=\frac{\hbar}{\hbar/r_1}=r_1$$

The uncertainty in position is comparable to the Bohr radius.

2. The periodicity of the Periodic Table depends on the number and arrangement of the electrons in the atom. The Pauli exclusion principle requires that each electron have distinct values for the principal quantum number, angular orbital quantum number, magnetic quantum number, and spin quantum number. As electrons are added to the atom, the electrons fill the lowest available energy states. Atoms with the same number of electrons in the outer shells have similar properties, which accounts for the periodicity of the Periodic Table. The quantization of the angular momentum determines the number of angular momentum states allowed in each shell. These are represented by the letters *s, p, d, f,* etc. The magnitude of the angular momentum also limits the number of magnetic quantum states available for each orbital angular state. Finally, the electron spin allows for two electrons (one spin up and one spin down) to exist in each magnetic quantum state. So the Pauli exclusion principle, quantization of angular momentum, direction of angular momentum, and spin all play a role in determining the periodicity of the Periodic Table.

3. (*a*) The quantum-mechanical theory retained the aspect that electrons only exist in discrete states of definite energy levels. It also retained the feature that an electron absorbs or emits a photon when it changes from one energy state to another.

   (*b*) In the Bohr model the electrons moved in circular orbits around the nucleus. In the quantum-mechanical model the electrons occupy orbitals about the nucleus and do not have exact, well-defined orbits.

4. (*a*) We treat the Earth as a particle in rotation about the Sun. The angular momentum of a particle is given in Eq. 27–11 as $L=m\upsilon r_n$, where *r* is the orbit radius. We equate this to the quantum-mechanical expression in Eq. 28–3. We anticipate that the quantum number will be very large, so we approximate $\sqrt{\ell(\ell+1)}$ as ℓ.

$$L = M_{\text{Earth}} v r_{\text{Sun-Earth}} = M\frac{2\pi r}{T} r = \hbar[\ell(\ell+1)]^{\frac{1}{2}} = \hbar\ell \quad \rightarrow$$

$$\ell = \frac{M_{\text{Earth}} 2\pi r^2}{\hbar T} = \frac{(5.98\times10^{24}\ \text{kg})2\pi(1.496\times10^{11}\ \text{m})^2}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})(3.156\times10^{7}\ \text{s})} = 2.5255\times10^{74} \approx \boxed{2.53\times10^{74}}$$

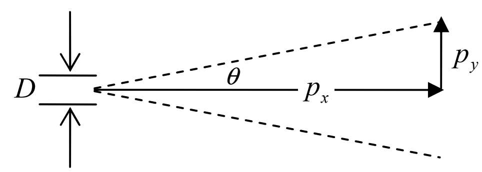
(*b*) There are $2\ell+1$ values of m_ℓ for a value of ℓ, so the number of orientations is as follows:

$$N = 2\ell+1 = 2(2.5255\times10^{74})+1 = 5.051\times10^{74} \approx \boxed{5.05\times10^{74}}$$

5. The "location" of the beam is uncertain in the transverse (y) direction by an amount equal to the aperture opening, D. This gives a value for the uncertainty in the transverse momentum. The momentum must be at least as big as its uncertainty, so we obtain a value for the transverse momentum.

$$\Delta p_y \Delta y \ge \hbar \quad \rightarrow \quad \Delta p_y \ge \frac{\hbar}{\Delta y} = \frac{\hbar}{D} \quad \rightarrow \quad p_y \approx \frac{\hbar}{D}$$

The momentum in the forward direction is related to the wavelength of the light by $p_x = \frac{h}{\lambda}$. See the diagram to relate the momentum to the angle.



$$\theta \approx \frac{p_y}{p_x} \approx \frac{\hbar/D}{h/\lambda} = \frac{\lambda}{2\pi D}; \text{ "spread"} = 2\theta = \frac{\lambda}{\pi D} \approx \boxed{\frac{\lambda}{D}}$$

6. For noble gases, the electrons fill all available states in the outermost shell. Helium has two electrons which fill the 1*s* shell. For the other noble gases, there are six electrons in the *p* subshells, filling each of those shells. For each of these gases, the next electron would have to be added to an *s* subshell at much higher energy. Since the noble gases have a filled shell, they do not react chemically with other elements.

Lithium, which has three electrons and is the first alkali metal, has its third electron in the 2*s* shell. This shell has significantly higher energy than the 1*s* shell. All of the alkali metals have one electron in their outer *s* subshell and therefore react well with halogens.

Halogens all have five electrons in their outer *p* subshell. They react well with atoms that have a donor electron in their outer shell, like the alkali metals.

7. The uncertainty in the electron position is $\Delta x = r_1$. The minimum uncertainty in the velocity, Δv, can be found by solving Eq. 28–1, where the uncertainty in the momentum is the product of the electron mass and the uncertainty in the velocity.

$$\Delta x \Delta p_{\min} = r_1(m\Delta v) \ge \hbar$$

$$\Delta v \ge \frac{\hbar}{m r_1} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.529\times10^{-10}\ \text{m})} = \boxed{2.19\times10^{6}\ \text{m/s}}$$

8. According to the uncertainty principle, it is impossible to measure a system without disturbing, or affecting, the system. The more precisely you measure the position, the greater the disturbance in the momentum, and hence the greater the uncertainty in the momentum. The more precisely you measure the momentum, the greater the disturbance in the position, and hence the greater the uncertainty in the position. When measuring position and momentum, there is a minimum value for the product of the uncertainties in momentum and position.

29

MOLECULES AND SOLIDS

Responses to Questions

$\boxed{1.}$ (*a*) The bond in an N_2 molecule is expected to be covalent.

(*b*) The bond in the HCl molecule is expected to be ionic.

(*c*) The bond between Fe atoms in a solid is expected to be metallic.

2. Ca has two *s* subshell electrons in the outer shell, and each Cl is only missing one electron from its outer shell. These three atoms share their electrons in such a way as to have filled outer shells. Look at Fig. 29–4 and Fig. 29–5. Each of the two outer electrons of Ca will fit into the "extra electron" position of the two Cl atoms, forming strong ionic bonds.

3. Neither the H_2 nor the O_2 molecule has a permanent dipole moment. The outer electrons are shared equally between the two atoms in each molecule, so there are no polar ends that are more positively or negatively charged. The H_2O molecule does have a permanent dipole moment. The electrons associated with the hydrogen atoms are pulled toward the oxygen atom, leaving each hydrogen with a small net positive charge and the oxygen with a small net negative charge. Because of the shape of the H_2O molecule (see Fig. 29–6), one end of the molecule will be positive and the other end will be negative, resulting in a permanent dipole moment.

4. The H_3 molecule has three electrons, and only two of them can be in the 1*s* state (and then only if they have opposite spins, according to the Pauli exclusion principle). Accordingly, the third electron cannot be in the 1*s* state, so it is farther from the nucleus and not held as tightly as the other two electrons. This contributes to the instability of H_3. On the other hand, the H_3^+ ion only has two electrons, and, if they have opposite spins, the Pauli exclusion principle will allow them to both be in the lower energy 1*s* state, resulting in a $1s^2$ closed shell and a spherically symmetric distribution. This makes H_3^+ relatively more stable than H_3.

5. Yes, H_2^+ should be stable. The two positive nuclei share the one negative electron. The electron spends most of its time between the two positive nuclei (basically holding them together).

6. The electron configuration of carbon is $1s^2 2s^2 2p^2$. The inner two electrons are held tightly and closely bound to the nucleus. The four remaining electrons are basically spread around the outside of the atom in four different directions (they repel each other). These four electrons can each form a simple hydrogen-like bond with four atoms that each have only one electron in an *s* orbital. Another way to consider this is that the C atom has two electrons in the outer 2*p* subshell. That subshell would be "full," so would be stable, if it had six electrons in it. Therefore, the C atom has the capability to share electrons in its outer shell with four hydrogen atoms.

7. The four categories of molecular energy are translational kinetic energy, electrostatic potential energy, rotational kinetic energy, and vibrational kinetic energy.

8. The conduction electrons are not strongly bound to particular nuclei, so a metal can be viewed as a collection of positive ions and a negative electron "gas." (The positive ions are just the metal atoms without their outermost electrons, since these "free" electrons make up the gas.) The electrostatic attraction between the freely roaming electrons and the positive ions keeps the electrons from leaving the metal.

9. As temperature increases, the thermal motion of ions in a metal lattice will increase. More electrons collide with the ions, increasing the resistivity of the metal. When the temperature of a semiconductor increases, more electrons are able to move into the conduction band, making more charge carriers available and thus decreasing the resistivity. The thermal motion also increases the resistance in semiconductors, but the increase in the number of charge carriers is a larger effect.

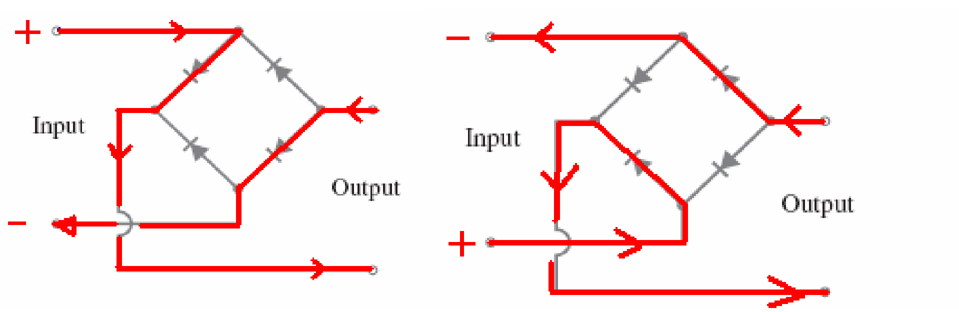
10. From Fig. 29–30, it takes about 0.6 V to get a significant current to flow through the diode in the forward bias direction and about 12 V to get current to flow through the diode in the reverse bias direction. Thus, to get the same current to flow in either direction:

$$V_{\text{forward}} = IR_{\text{forward}} \qquad V_{\text{reverse}} = IR_{\text{reverse}}$$
$$R_{\text{reverse}}/R_{\text{forward}} = 12\text{ V}/0.6\text{ V} = 20$$

Thus, reverse bias resistance is approximately $20\times\text{larger}$ than the forward bias resistance. This is very approximate based on estimates from reading the graph.

11. The base current (between the base and the emitter) controls the collector current (between the collector and the emitter). If there is no base current, then no collector current flows. Thus, controlling the relatively small base current allows the transistor to act as a switch, turning the larger collector current on and off.

12. When the top branch of the input circuit is at the high voltage (current is flowing in this direction for half the cycle), the bottom branch of the output is at the high voltage. The current follows the path through the bridge in the diagram on the left. When the bottom branch of the input circuit is at the high voltage (current is flowing in this direction during the other half of the cycle), the bottom branch of the output is still at high voltage. The current follows the path through the bridge in the diagram on the right.



13. The main difference between *n*-type and *p*-type semiconductors is the type of atom used for the doping impurity. When a semiconductor such as Si or Ge, each atom of which has four electrons to share, is doped with an element that has five electrons to share (such as As or P), then it is an *n*-type semiconductor since an extra electron has been inserted into the lattice. When a semiconductor is doped with an element that has three electrons to share (such as Ga or In), then it is a *p*-type semiconductor since an extra hole (the lack of an electron) has been inserted into the lattice.

14. The partially filled shell in Na is the 3*s* shell, which has one electron in it. The partially filled shell in Cl is the 2*p* shell, which has five electrons in it. In NaCl the electron from the 3*s* shell in Na is transferred to the 2*p* shell in Cl, which results in filled shells for both ions. Thus when many ions are considered, the resulting bands are either completely filled (the valence band) or completely empty (the conduction band). Thus a large energy is required to create a conduction electron by raising an electron from the valence band to the conduction band.

15. In the circuit shown in Fig. 29–41, the base–emitter junction is forward biased (the current will easily flow from the base to the emitter) and the base–collector junction is reverse biased (the current will not easily flow from the base to the collector).

16. The energy comes from the power supplied by the collector/emitter voltage source, $\mathscr{E}_C$. The input signal to the base just regulates how much current, and therefore how much power, can be drawn from the collector's voltage source.

17. The phosphorus atoms will be donor atoms. Phosphorus has five valence electrons. It will form four covalent bonds with the silicon atoms around it and will have one "extra" electron that is weakly bound to the atom and can be easily excited up to the conduction band. This process results in extra electrons in the conduction band, so silicon doped with phosphorus is an *n*-type semiconductor.

18. They do not obey Ohm's law. Ohmic devices (those that obey Ohm's law) have a constant resistance and therefore a linear relationship between voltage and current. The voltage–current relationship for diodes is not linear (Fig. 29–30). The resistance of a diode operated in reverse bias is very large. The same diode operated in a forward-bias mode has a much smaller resistance. Since a transistor can be thought of as made up of diodes, it is also non-ohmic.

19. A diode cannot be used to amplify a signal. A diode does let current flow through it in one direction easily (forward biased) and it does not let current flow through it in the other direction (reverse bias), but there is no way to connect a source of power to use it to amplify a signal (which is how a transistor amplifies a signal).

Responses to MisConceptual Questions

1. (*c*) Due to the small masses of the atoms, the gravitational force is much too weak to hold them together. Most atoms have very small magnetic fields and therefore could not be held together by magnetic forces. Nuclear forces hold the nuclei of the atoms together but do not act on a molecular scale. The exchange and sharing of electrons by the electric force hold molecules together.

2. (*c*) In the H_2 molecule the two electrons orbit both atoms. In order for those two electrons to not violate the Pauli exclusion principle, they must have different quantum numbers. Each atom initially had one electron and the molecule has two, so no electrons are lost. When the atoms are separated by one bond length, the energy is a minimum (not a maximum) and the molecule has less total energy than the two atoms separately. This decrease in energy is the binding energy.

3. (*c*) The shared electrons cannot have the same spin state. One electron must be spin up and the other spin down.

4. (*b*) Covalent bonding is the sharing of atoms between molecules. When one atom has excess electrons in its outer shell and another atom lacks electrons in the outer shell, the atom with excess transfers the electrons to the other atom, making a positive ion and a negative ion, and thus creating an ionic "bond"—an attraction between the two ions.

5. (*a*, *d*) Because the ADP molecule has a positive activation energy, as the phosphate group approaches the ADP molecule it is first repelled and then attracted. The phosphate group must initially have kinetic energy to overcome the repulsion. Some of this kinetic energy is stored as potential energy in the ATP molecule. The binding energy is negative, as some of the initial kinetic energy is stored as positive potential energy. When the ATP molecule is broken apart, this energy is released and is available to instigate other reactions.

6. (*d*) For the DNA to replicate properly, the bond holding the two strands together must be a very weak bond. Ionic and covalent bonds are strong bonds. The van der Waals bond is a weak bond that hold the DNA together.

7. (*d*) A hole is a positive region in the semiconductor that is formed when an electron is missing from the periodic molecular structure.

8. (*c*) A common misconception is that the resistance of materials decreases because fewer electrons collide with the crystal lattice. As a conductor is heated, the electrons actually collide more frequently with the lattice, which results in a greater resistance. In a semiconductor, the resistance also increases with heating because the collisions are more frequent. However, as a semiconductor is heated, additional electrons can jump the band gap and increase the number of conduction electrons available. This has the net effect of decreasing the resistance.

9. (*a*) To be used for doping silicon, the element should have one more or one less electron in its outer shell than silicon. Silicon has two electrons in its outer *p* shell. Boron and gallium each have one electron in their outer *p* shell, one fewer than silicon. Phosphorus and arsenic each have three electrons in their outer *p* shells, one more than silicon. Germanium has two electrons in its outer *p* shell (the same as silicon), so it would not be a good choice as a doping impurity.

10. (*a*) A common misconception is that metals have free electrons because they have more electrons than protons. Actually, metals are electrically neutral in that they have the same number of electrons as protons. In metals, the outer electrons are not tightly bound to a single atom but can move between atoms in the metal lattice. Since the electrons can move easily between atoms, metals make good conductors of electricity.

Solutions to Problems

Note: The following factor appears in the analysis of electron energies:

$$\frac{e^2}{4\pi\varepsilon_0} = ke^2 = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2 = 2.30\times10^{-28}\ \text{J}\cdot\text{m}$$

1. We calculate the binding energy as the opposite of the electrostatic potential energy. We use a relationship from Section 29–2 for the potential energy.

$$\text{Binding energy} = -\text{PE} = -k\frac{q_1\, q_2}{r} = -k\frac{(+1.0e)(-1.0e)}{0.28\times 10^{-9}\ \text{m}} = \frac{2.30\times 10^{-28}\ \text{J}\cdot\text{m}}{0.28\times 10^{-9}\ \text{m}}$$
$$= 8.214\times 10^{-19}\ \text{J} \approx \boxed{8.2\times 10^{-19}\ \text{J}}$$
$$= 8.214\times 10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right) = 5.134\ \text{eV} \approx \boxed{5.1\ \text{eV}}$$

2. From Problem 1, the "point electron" binding energy is 5.1 eV (only 2 significant figures). With the repulsion of the electron clouds included, the actual binding energy is 4.43 eV. Use these values to calculate the contribution of the electron clouds.

$$5.134\ \text{eV} - 4.43\ \text{eV} = 0.704\ \text{eV} \approx \boxed{0.7\ \text{eV}}$$

3. We follow the procedure outlined in the statement of the Problem.

$$\text{HN:}\quad \tfrac{1}{2}(d_{H_2} + d_{N_2}) = \tfrac{1}{2}(74\ \text{pm} + 145\ \text{pm}) = \boxed{110\ \text{pm}}$$
$$\text{CN:}\quad \tfrac{1}{2}(d_{C_2} + d_{N_2}) = \tfrac{1}{2}(154\ \text{pm} + 145\ \text{pm}) = \boxed{150\ \text{pm}}$$
$$\text{NO:}\quad \tfrac{1}{2}(d_{N_2} + d_{O_2}) = \tfrac{1}{2}(145\ \text{pm} + 121\ \text{pm}) = \boxed{133\ \text{pm}}$$

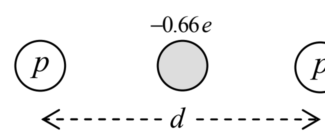
4. We convert the units from kcal/mole to eV/molecule.

$$1\frac{\text{kcal}}{\text{mole}}\times\frac{4186\ \text{J}}{1\ \text{kcal}}\times\frac{1\ \text{eV}}{1.602\times 10^{-19}\ \text{J}}\times\frac{1\ \text{mole}}{6.022\times 10^{23}\ \text{molecules}} = \boxed{4.339\times 10^{-2}\ \frac{\text{eV}}{\text{molecule}}}$$

Now convert 4.43 eV per molecule of KCl into kcal per mole.

$$4.43\frac{\text{eV}}{\text{molecule}}\times\frac{1\ \text{kcal/mol}}{4.339\times 10^{-2}\ \text{eV/molecule}} = \boxed{102\ \text{kcal/mol}}$$

5. We calculate the binding energy as the difference between the energy of two isolated hydrogen atoms and the energy of the bonded combination of particles. We estimate the energy of the bonded combination as the negative potential energy of the two electron–proton combinations plus the positive potential energy of the proton–proton combination. We approximate the electrons as a single object with a charge of 0.33 of the normal charge of two electrons, since the electrons only spend that fraction of time between the nuclei. A simple picture illustrating our bonded model is shown. When the electrons are midway between the protons, each electron will have a potential energy PE_{ep} due to the two protons.



$$\text{PE}_{\text{ep}} = \frac{-(2)(0.33)ke^2}{\left(\frac{1}{2}d\right)} = -\frac{(4)(0.33)(2.30\times 10^{-28}\ \text{J}\cdot\text{m})}{(0.074\times 10^{-9}\ \text{m})(1.60\times 10^{-19}\ \text{J/eV})} = -25.6\ \text{eV}$$

The protons themselves have this potential energy:

$$\text{PE}_{\text{pp}} = +\frac{ke^2}{r} = +\frac{(2.30\times 10^{-28}\ \text{J}\cdot\text{m})}{(0.074\times 10^{-9}\ \text{m})(1.60\times 10^{-19}\ \text{J/eV})} = +19.4\ \text{eV}$$

When the bond breaks, each hydrogen atom will be in the ground state with an energy $E_1 = -13.6$ eV. Thus, the binding energy is as follows:

$$\text{Binding energy} = 2E_1 - (2\,\text{PE}_{\text{ep}} + \text{PE}_{\text{pp}}) = 2(-13.6\text{ eV}) - [2(-25.6\text{ eV}) + 19.4\text{ eV}] = \boxed{4.6\text{ eV}}$$

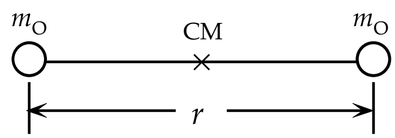
This is close to the actual value of 4.5 eV quoted in the text.

6. (*a*) The neutral He atom has two electrons in the ground state, $n = 1,\ \ell = 0,\ m_\ell = 0$. Thus the two electrons have opposite spins, $m_s = \pm\frac{1}{2}$. If we try to form a covalent bond, then we see that an electron from one of the atoms will have the same quantum numbers as one of the electrons on the other atom. From the exclusion principle, this is not allowed, so the electrons cannot be shared.

   (*b*) We consider the He_2^+ molecular ion to be formed from a neutral He atom and an He^+ ion. It will have three electrons. If the electron on the ion has a certain spin value, then it can have the opposite spin of one of the electrons on the neutral atom. Thus those two electrons can be in the same spatial region (because their quantum numbers are not identical), so a bond can be formed.

7. The MKS units of $\dfrac{\hbar^2}{I}$ are $\dfrac{(\text{J}\cdot\text{s})^2}{(\text{kg}\cdot\text{m}^2)} = \dfrac{\text{J}^2}{(\text{kg}\cdot\text{m/s}^2)\text{m}} = \dfrac{\text{J}^2}{(\text{N}\cdot\text{m})} = \dfrac{\text{J}^2}{\text{J}} = \text{J}$. The final unit is joules, which is an energy unit.

8. (*a*) We first write an expression for the moment of inertia of O_2 about its CM and then calculate the characteristic rotational energy.



$$I = 2m_O\left(\tfrac{1}{2}r\right)^2 = \tfrac{1}{2}m_O r^2$$

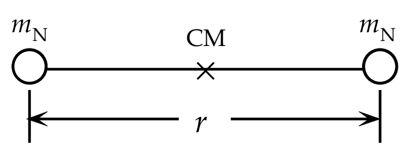
$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_O r^2} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{(16.0\text{ u})(1.66\times10^{-27}\text{ kg/u})(0.121\times10^{-9}\text{ m})^2(1.60\times10^{-19}\text{ J/eV})}$$
$$= 1.789\times10^{-4}\text{ eV} \approx \boxed{1.79\times10^{-4}\text{ eV}}$$

   (*b*) From Fig. 29–16, the energy involved in the $\ell = 3$ to $\ell = 2$ transition is $3\hbar^2/I$.

$$\Delta E = \frac{3\hbar^2}{I} = 6\frac{\hbar^2}{2I} = 6(1.789\times10^{-4}\text{ eV}) = 1.0734\times10^{-3}\text{ eV} \approx \boxed{1.07\times10^{-3}\text{ eV}}$$

$$\Delta E = h\frac{c}{\lambda} \rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.0734\times10^{-3}\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{1.16\times10^{-3}\text{ m}}$$

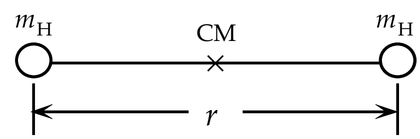
$\boxed{9.}$ Use the rotational energy and the moment of inertia of N_2 about its CM to find the bond length. See the adjacent diagram.



$$I = 2m_N\left(\tfrac{1}{2}r\right)^2 = \tfrac{1}{2}m_N r^2;\quad E_{\text{rot}} = \frac{\hbar^2}{2I} = \frac{\hbar^2}{m_N r^2} \rightarrow$$

$$r = \frac{\hbar}{\sqrt{E_{\text{rot}} m_N}} = \frac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{(2.48\times10^{-4}\text{ eV})(1.60\times10^{-19}\text{ J/eV})(14.01\text{ u})(1.66\times10^{-27}\text{ kg/u})}} = \boxed{1.10\times10^{-10}\text{ m}}$$

10. The moment of inertia of H_2 about its CM is $I = 2m_H\left(\tfrac{1}{2}r\right)^2 = \tfrac{1}{2}m_H r^2$. See the adjacent diagram. The characteristic rotational energy, as mentioned in Problem 9, is $\dfrac{\hbar^2}{2I}$.



$$\frac{\hbar^2}{2I} = \frac{\hbar^2}{m_H r^2} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{(1.67\times10^{-27}\ \text{kg})(0.074\times10^{-9}\ \text{m})^2} = 1.217\times10^{-23}\ \text{J} = 7.607\times10^{-3}\,\text{eV}$$

The rotational energy change (equal to the photon energy) is given in Eq. 29–2, $\Delta E_{\text{rot}} = \ell\dfrac{\hbar^2}{I} = 2\ell\left(\dfrac{\hbar^2}{2I}\right)$, where ℓ is the value for the upper state. Note that $\lambda = \dfrac{c}{f} = \dfrac{hc}{hf} = \dfrac{hc}{\Delta E}$.

(*a*) $\ell = 1 \rightarrow \ell = 0$:

$$\Delta E = 2(1)\left(\frac{\hbar^2}{2I}\right) = 2(7.607\times10^{-3}\ \text{eV}) = \boxed{1.5\times10^{-2}\ \text{eV}}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(1.52\times10^{-2}\ \text{eV})} = 8.17\times10^{-5}\ \text{m} = \boxed{0.082\ \text{mm}}$$

(*b*) $\ell = 2 \rightarrow \ell = 1$:

$$\Delta E = 2(2)\left(\frac{\hbar^2}{2I}\right) = 4(7.607\times10^{-3}\ \text{eV}) = \boxed{3.0\times10^{-2}\ \text{eV}}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(3.04\times10^{-2}\ \text{eV})} = 4.08\times10^{-5}\ \text{m} = \boxed{0.041\ \text{mm}}$$

(*c*) $\ell = 3 \rightarrow \ell = 2$:

$$\Delta E = 2(3)\left(\frac{\hbar^2}{2I}\right) = 6(7.607\times10^{-3}\ \text{eV}) = \boxed{4.6\times10^{-2}\ \text{eV}}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(4.56\times10^{-2}\ \text{eV})} = 2.73\times10^{-5}\ \text{m} = \boxed{0.027\ \text{mm}}$$

11. Use the value of the rotational inertia as calculated in Example 29–2. We also use Eq. 29–2.

$$\Delta E = \frac{\hbar^2}{I}\ell = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{4\pi^2(1.46\times10^{-46}\ \text{kg}\cdot\text{m}^2)}(5) = 3.813\times10^{-22}\ \text{J}$$

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(3.813\times10^{-22}\ \text{J})} = \boxed{5.22\times10^{-4}\ \text{m}}$$

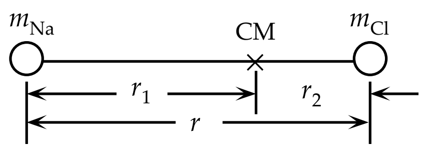
12. First find the energies of the transitions represented by the wavelengths.

$$\Delta E_1 = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(23.1\times10^{-3}\ \text{m})} = 5.381\times10^{-5}\ \text{eV}$$

$$\Delta E_2 = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(11.6\times10^{-3}\ \text{m})} = 10.717\times10^{-5}\ \text{eV}$$

$$\Delta E_3 = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(7.71\times10^{-3}\ \text{m})} = 16.124\times10^{-5}\ \text{eV}$$

Since $\frac{\Delta E_2}{\Delta E_1} = \frac{10.717}{5.381} = 1.992 \approx 2$ and $\frac{\Delta E_3}{\Delta E_1} = \frac{16.124}{5.381} = 2.996 \approx 3$, from the energy levels indicated in Fig. 29–16, and from the selection rule that $\Delta\ell = \pm\ell$, we see that these three transitions must represent the $\ell = 1$ to $\ell = 0$ transition, the $\ell = 2$ to $\ell = 1$ transition, and the $\ell = 3$ to $\ell = 2$ transition. Thus $\Delta E_1 = \hbar^2/I$. We use that relationship along with the rotational inertia about the center of mass to calculate the bond length. Use the adjacent diagram to help in the calculation of the rotational inertia.



$$r_1 = \frac{[m_{Na}(0) + m_{Cl}r]}{(m_{Na} + m_{Cl})} = \frac{(35.5\ \text{u})r}{(23.0\ \text{u} + 35.5\ \text{u})} = 0.607r$$

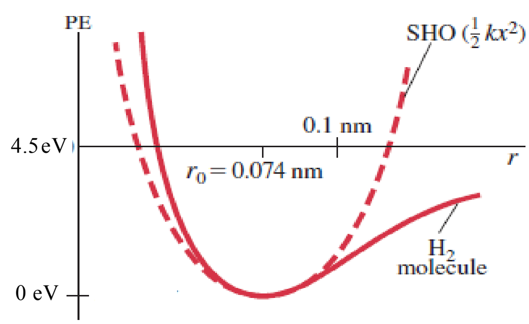
$$r_2 = r - r_1 = 0.393r$$

$$I = m_{Na}r_1^2 + m_{Cl}r_2^2 = \left[(23.0\ \text{u})(0.607r)^2 + (35.5\ \text{u})(0.393r)^2\right](1.66\times10^{-27}\ \text{kg/u})$$
$$= (2.317\times10^{-26}r^2)\ \text{kg}\cdot\text{m}^2$$

$$\Delta E_1 = \hbar^2/I \quad\rightarrow\quad (5.381\times10^{-5}\text{eV})(1.60\times10^{-19}\ \text{J/eV}) = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{(2.317\times10^{-26}r^2)\text{kg}\cdot\text{m}^2} \quad\rightarrow$$

$$r = \sqrt{\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{(2.317\times10^{-26}\ \text{kg})(5.381\times10^{-5}\text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = \boxed{2.36\times10^{-10}\text{m}}$$

13. (*a*) The curve for $\text{PE} = \frac{1}{2}kx^2$ is shown in Fig. 29–17 as a dashed line. This line crosses the $\text{PE} = 0$ axis at about 0.120 nm. To fit the expression $\text{PE} = \frac{1}{2}kx^2$, which is always positive, we shift the graph upward by 4.5 eV so that the potential energy is 0 at the lowest point and 4.5 eV for the radial positions where it currently crosses the axis. We also need to make the vertex point be at $r = r_0$.



Thus the equation should be parameterized as $\text{PE} = \frac{1}{2}k(r - r_0)^2$. A data point that fits this graph is $r = 0.120$ nm and $\text{PE} = 4.5$ eV. See the diagram, which is a modified version of Fig. 29–17. There may be some variance due to differences in reading the graph.

$$\text{PE} = \frac{1}{2}k(r - r_0)^2 \quad\rightarrow$$

$$k = \frac{2\ \text{PE}}{(r - r_0)^2} = \frac{2(4.5\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{\left[(0.120\ \text{nm} - 0.074\ \text{nm})(10^{-9}\ \text{m/nm})\right]^2} = \boxed{680\ \text{N/m}}$$

(*b*) The classical formula for the frequency of the oscillator is $\omega = 2\pi f = \sqrt{k/m}$. The statement of the problem says to only use half the mass of a hydrogen atom. One H atom has a mass of one atomic mass unit.

$$\lambda = \frac{c}{f} = \frac{c}{\frac{1}{2\pi}\sqrt{\frac{k}{m}}} = 2\pi c\sqrt{\frac{m}{k}} = 2\pi(3.00\times10^{8}\ \text{m/s})\sqrt{\frac{\frac{1}{2}(1.66\times10^{-27}\ \text{kg})}{680\ \text{N/m}}} = \boxed{2.1\times10^{-6}\ \text{m}}$$

14. Each ion is at the corner of a cube of side length *s*, the distance between ions. From Fig. 29–20, a cube of side length *s* would have four NaCl pairs. But each ion is part of eight cubes that share a common corner. So any one "cube" has only the equivalent of one-half of an NaCl molecule. Use this to find the density, which is mass per unit volume.

$$\rho = \frac{\frac{1}{2}(m_{\text{NaCl}})}{s^3} \rightarrow$$

$$s = \left(\frac{\frac{1}{2}m_{\text{NaCl}}}{\rho}\right)^{1/3} = \left[\frac{\left(\frac{1}{2}\ \text{molecule/cube}\right)(58.44\ \text{g/mole})}{(2.165\ \text{g/cm}^3)}\left(\frac{1\ \text{mole}}{6.02\times10^{23}\ \text{molecules}}\right)\right]^{1/3} = \boxed{2.82\times10^{-8}\ \text{m}}$$

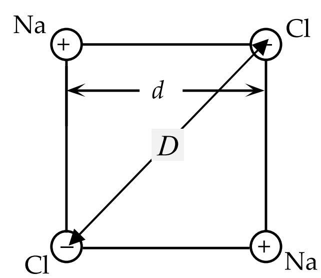
Note that Problem 16 quotes this value as 2.4×10^{-8} m.

15. Each ion is at the corner of a cube of side length *s*, the distance between ions. From Fig. 29–20, a cube of side length *s* would have four KCl pairs. But each ion is part of eight cubes that share a common corner. So any one "cube" has only the equivalent of one-half of a KCl molecule. Use this to find the density, which is mass per unit volume. From the Periodic Table, the molecular weight of KCl is 74.55.

$$\rho = \frac{\frac{1}{2}(m_{\text{KCl}})}{s^3} \rightarrow$$

$$s = \left(\frac{\frac{1}{2}m_{\text{KCl}}}{\rho}\right)^{1/3} = \left[\frac{\left(\frac{1}{2}\ \text{molecule/cube}\right)(74.55\ \text{g/mole})}{(1.99\ \text{g/cm}^3)}\left(\frac{1\ \text{mole}}{6.02\times10^{23}\ \text{molecules}}\right)\right]^{1/3} = \boxed{3.15\times10^{-8}\ \text{m}}$$

16. The NaCl crystal is illustrated in Fig. 29–20. Consider four of the labeled ions from Fig. 29–20. See the adjacent diagram. The distance from an Na ion to a Cl ion is labeled as *d*, and the distance from an Na ion to the nearest neighbor Na ion is the diagonal distance *D*.



$$D = d\sqrt{2} = (0.24\ \text{nm})\sqrt{2} = \boxed{0.34\ \text{nm}}$$

17. The photon with the minimum frequency for conduction must have an energy equal to the energy gap.

$$E_g = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(620\times10^{-9}\ \text{m})} = \boxed{2.0\ \text{eV}}$$

18. The photon with the longest wavelength or minimum frequency for conduction must have an energy equal to the energy gap.

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(1.12\ \text{eV})} = 1.11\times10^{-6}\ \text{m} = \boxed{1.11\ \mu\text{m}}$$

$\boxed{19.}$ The energy of the photon must be greater than or equal to the energy gap. The lowest energy corresponds to the longest wavelength that will excite an electron.

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(0.72\ \text{eV})} = 1.7\times10^{-6}\ \text{m} = 1.7\ \mu\text{m}$$

Thus, the wavelength range is $\boxed{\lambda \le 1.7\ \mu\text{m}}$.

20. (*a*) For the glass to be transparent to the photon, the photon's energy must be < 1.12 eV, so the wavelength of the photon must be longer than the wavelength corresponding to 1.12 eV.

$$E_{\text{band gap}} = \frac{hc}{\lambda_{\min}} \quad \rightarrow$$

$$\lambda_{\min} = \frac{hc}{E_{\text{band gap}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.12\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 1.11\times10^{-6}\ \text{m} \quad \rightarrow \quad \boxed{\lambda > 1.11\times10^{-6}\ \text{m}}$$

The minimum wavelength for transparency is in the $\boxed{\text{infrared}}$ region of the spectrum. Since IR has longer wavelengths than visible light, the silicon would not be transparent for visible light. The silicon would be opaque, as in Example 29–5.

(*b*) The minimum possible band gap energy for light to be transparent would mean that the band gap energy would have to be larger than the most energetic visible photon. The most energetic photon corresponds to the shortest wavelength, which is 400 nm in this Problem. We treat the wavelength as being accurate to 2 significant figures.

$$E_{\text{band gap}} > E_{\lambda_{\min}} = \frac{hc}{\lambda_{\min}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(400\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J/eV})} = 3.1078\ \text{eV} \approx \boxed{3.1\ \text{eV}}$$

$\boxed{21.}$ The minimum energy provided to an electron must be equal to the energy gap. Divide the total available energy by the energy gap to estimate the maximum number of electrons that can be made to jump.

$$N = \frac{hf}{E_g} = \frac{(830\times10^{3}\ \text{eV})}{(0.72\ \text{eV})} = \boxed{1.2\times10^{6}\ \text{electrons}}$$

22. (*a*) In the 2*s* shell of an atom, $\ell = 0$, so there are two states: $m_s = \pm\frac{1}{2}$. When *N* atoms form bands, each atom provides two states, so the total number of states in the band is $\boxed{2N}$.

(*b*) In the 2*p* shell of an atom, $\ell = 1$, so there are three states from the m_ℓ values: $m_\ell = 0, \pm1$; each of which has two states from the m_s values: $m_s = \pm\frac{1}{2}$, for a total of six states. When *N* atoms form bands, each atom provides six states, so the total number of states in the band is $\boxed{6N}$.

(*c*) In the 3*p* shell of an atom, $\ell = 1$, so there are three states from the m_ℓ values: $m_\ell = 0, \pm 1$; each of which has two states from the m_s values: $m_s = \pm\frac{1}{2}$, for a total of six states. When *N* atoms form bands, each atom provides six states, so the total number of states in the band is $\boxed{6N}$.

(*d*) In general, for a value of ℓ, there are $2\ell + 1$ states from the m_ℓ values: $m_\ell = 0, \pm 1, \ldots, \pm\ell$. For each of these, there are two states from the m_s values: $m_s = \pm\frac{1}{2}$, for a total of $2(2\ell + 1)$ states. When *N* atoms form bands, each atom provides $2(2\ell + 1)$ states, so the total number of states in the band is $\boxed{2N(2\ell + 1).}$

23. Calculate the number of conduction electrons in a mole of pure silicon. Also calculate the additional conduction electrons provided by the doping, and then take the ratio of those two numbers of conduction electrons.

$$N_{\text{Si}} = \left[\frac{(28.09\times10^{-3}\text{ kg/mol})}{(2330\text{ kg/m}^3)}\right](10^{16}\text{ electrons/m}^3) = 1.206\times10^{11}\text{ electrons/mol}$$

$$N_{\text{doping}} = \frac{(6.02\times10^{23}\text{ atoms})}{1.5\times10^6} = 4.013\times10^{17}\text{ added conduction electrons/mol}$$

$$\frac{N_{\text{doping}}}{N_{\text{Si}}} = \frac{(4.013\times10^{17})}{(1.206\times10^{11})} = 3.33\times10^6 \approx \boxed{3\times10^6}$$

$\boxed{24.}$ The wavelength is found from the energy gap.

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{E_{\text{g}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(1.3\text{ eV})} = 9.56\times10^{-7}\text{ m} = \boxed{0.96\ \mu\text{m}}$$

25. The photon will have an energy equal to the energy gap.

$$E_{\text{g}} = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(730\times10^{-9}\text{ m})} = \boxed{1.7\text{ eV}}$$

26. The band gap is the energy corresponding to the emitted wavelength.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.3\times10^{-6}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = \boxed{0.96\text{ eV}}$$

$\boxed{27.}$ From the current–voltage characteristic graph in Fig. 29–30, we see that a current of 14 mA means a voltage of about 0.68 V across the diode. The battery voltage is the sum of the voltages across the diode and the resistor.

$$V_{\text{battery}} = V_{\text{diode}} + V_{\text{R}} = 0.68\text{ V} + (0.014\text{ A})(960\ \Omega) = 14.12\text{ V} \approx \boxed{14\text{ V}}$$

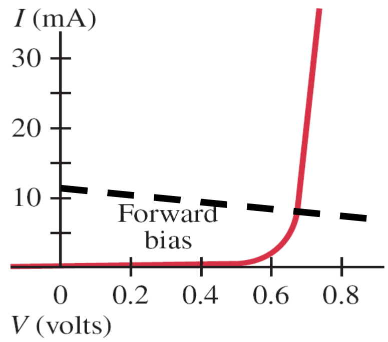
$\boxed{28.}$ (*a*) For a half-wave rectifier without a capacitor, the current is zero for half the time. We approximate the average current as half of the full rms current.

$$I_{\text{avg}} = \frac{1}{2}\frac{V_{\text{rms}}}{R} = \frac{1}{2}\frac{(120\text{ V})}{(31\text{ k}\Omega)} = \boxed{1.9\text{ mA}}$$

(*b*) For a full-wave rectifier without a capacitor, the current is positive all the time. We approximate the average current as equal to the full rms current.

$$I_{\text{avg}} = \frac{V_{\text{rms}}}{R} = \frac{(120\text{ V})}{(31\text{ k}\Omega)} = \boxed{3.9\text{ mA}}$$

29. The battery voltage is the sum of the voltages across the diode and the resistor.

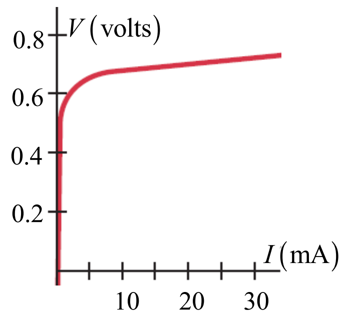


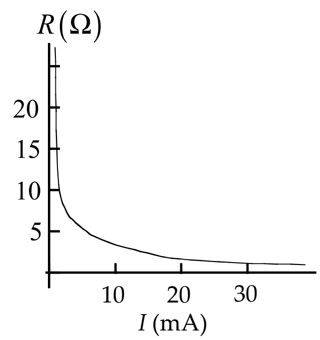
$$V_{\text{battery}} = V_{\text{diode}} + V_{\text{R}};$$

$$2.0\text{ V} = V_{\text{diode}} + I(180\ \Omega) \quad \rightarrow \quad I = -\frac{V_{\text{diode}}}{180\ \Omega} + \frac{2.0\text{ V}}{180\ \Omega}$$

This is the equation for a straight line passing through the points (0 V, 11.1 mA) and (0.8 V, 6.67 mA). The line has a *y* intercept of 11.1 mA and a slope of –5.6 mA/V. If the operating voltage of the diode is about 0.68 V, then the current is about $\boxed{7.3\text{ mA}}$. There is some approximation involved in reading the graph.

30. In the first diagram we have copied the graph for $V > 0$ and rotated it so that it shows V as a function of I. The resistance is the slope of that first graph. The slope, and thus the resistance, is very high for low currents and decreases for larger currents, approaching 0. As an approximate value, we see that the voltage changes from about 0.55 V to 0.65 V as the current goes from 0 to 10 mA. That makes the resistance about 10 ohms when the current is about 5 mA. The second diagram is a sketch of the resistance.





31. (*a*) The time constant for the circuit is $\tau_1 = RC_1 = (33\times10^3\ \Omega)(28\times10^{-6}\text{ F}) = 0.924\text{ s}$. As seen in Fig. 29–32c, there are two peaks per cycle. The period of the rectified voltage is $T = \frac{1}{120}\text{ s} = 0.0083\text{ s}$. Because $\tau_1 \gg T$, the voltage across the capacitor will be essentially constant during a cycle, so the average voltage is the same as the peak voltage. The average current is basically constant.

$$I_{\text{avg}} = \frac{V_{\text{avg}}}{R} = \frac{V_{\text{peak}}}{R} = \frac{\sqrt{2}V_{\text{rms}}}{R} = \frac{\sqrt{2}(120\text{ V})}{(33\times10^3\ \Omega)} = \boxed{5.1\text{ mA}}$$

(*b*) With a different capacitor, the time constant for the circuit changes.

$$\tau_2 = RC_2 = (33\times10^3\ \Omega)(0.10\times10^{-6}\text{ F}) = 0.0033\text{ s}$$

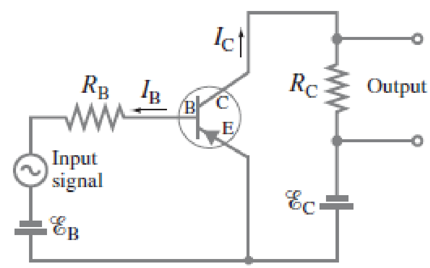
Now the period of the rectified voltage is about 2.5 time constants, so the voltage across the capacitor will be "rippled," decreasing to almost 0 during each half cycle, so that the average voltage will be close to the rms voltage.

$$I_{\text{avg}} = \frac{V_{\text{rms}}}{R} = \frac{(120\text{ V})}{(33\times10^3\ \Omega)} = \boxed{3.6\text{ mA}}$$

32. The arrow at the emitter terminal, E, indicates the direction of current I_E. The current into the transistor must equal the current out of the transistor.

$$\boxed{I_B + I_C = I_E}$$

33. For a *pnp* transistor, both the collector and the base voltages are negative, and holes move from the emitter to the collector. The diagram for a *pnp* amplifier looks just like Fig. 29–41, with the polarity of $\mathscr{E}_B$ and $\mathscr{E}_C$ reversed, I_B and I_C flowing in opposite directions, and the emitter arrow pointing toward the base.



34. By Ohm's law, the ac output (collector) current times the output resistor will be the ac output voltage.

$$V_{out} = i_C R_C \quad \rightarrow \quad R_C = \frac{V_{out}}{i_C} = \frac{V_{out}}{\beta_I i_B} = \frac{0.42\ \text{V}}{95(1.0\times10^{-6}\ \text{A})} = 4421\ \Omega \approx \boxed{4400\ \Omega}$$

35. By Ohm's law, the ac output (collector) current times the output resistor will be the ac output voltage.

$$V_{out} = i_C R_C = \beta_I i_B R_C = (85)(2.0\times10^{-6}\ \text{A})(3800\ \Omega) = 0.646\ \text{V} \approx \boxed{0.65\ \text{V}}$$

36. By Ohm's law, the ac output (collector) current times the output resistor will be the ac output voltage.

$$V_{out} = i_C R \quad \rightarrow \quad i_C = \frac{V_{out}}{R} = \frac{\beta_V V_{input}}{R} = \frac{75(0.080\ \text{V})}{25\times10^3\ \Omega} = 2.4\times10^{-4}\ \text{A} = \boxed{0.24\ \text{mA}}$$

37. (*a*) The voltage gain is the collector ac voltage divided by the base ac voltage.

$$\beta_V = \frac{V_C}{V_B} = \frac{i_C R_C}{i_B R_B} = \beta_I \frac{R_C}{R_B} = 65\left(\frac{7.8\ \text{k}\Omega}{3.8\ \text{k}\Omega}\right) = 133.4 \approx \boxed{130}$$

(*b*) The power amplification is the output power divided by the input power.

$$\beta_P = \frac{i_C V_C}{i_B V_B} = \beta_I \beta_V = (65)(133.4) = 8672 \approx \boxed{8700}$$

38. For an electron confined in one dimension, we find the uncertainty in the momentum from the uncertainty principle, $\Delta p \approx \frac{\hbar}{\Delta x}$. The momentum of the electron must be at least as big as the uncertainty in the momentum, so we approximate $p \approx \frac{\hbar}{\Delta x}$. Finally, we calculate the kinetic energy by $\text{KE} = \frac{p^2}{2m}$. Find the difference in the two kinetic energies based on the two position uncertainties.

$$\text{KE} = \frac{p^2}{2m} = \frac{\hbar^2}{2m(\Delta x)^2}$$

$$\Delta\text{KE} = \text{KE}_{\text{in atoms}} - \text{KE}_{\text{molecule}} = \frac{\hbar^2}{2m}\left[\frac{1}{(\Delta x)^2_{\text{in atoms}}} - \frac{1}{(\Delta x)^2_{\text{molecule}}}\right]$$

$$= \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})}\left[\frac{1}{(0.053\times10^{-9}\ \text{m})^2_{\text{in atoms}}} - \frac{1}{(0.074\times10^{-9}\ \text{m})^2_{\text{molecule}}}\right]\left(\frac{1}{1.60\times10^{-19}\ \text{J/eV}}\right)$$

$$= 6.62\ \text{eV}$$

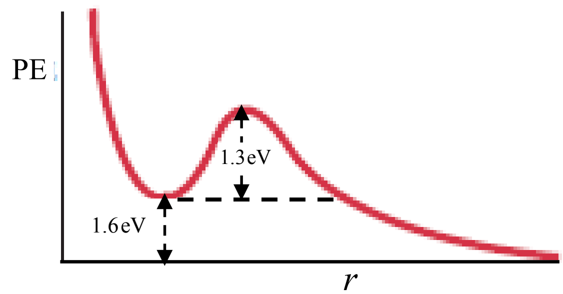
There are two electrons, and each one has this kinetic energy difference, so the total kinetic energy difference is $2(6.62\text{ eV}) = 13.2\text{ eV} \approx \boxed{13\text{ eV}}$.

39. We find the temperature from the given relationship.

(*a*) $$\text{KE} = \tfrac{3}{2}kT \quad \rightarrow \quad T = \frac{2\text{ KE}}{3k} = \frac{2(4.0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{3.1\times10^{4}\text{ K}}$$

(*b*) $$\text{KE} = \tfrac{3}{2}kT \quad \rightarrow \quad T = \frac{2\text{ KE}}{3k} = \frac{2(0.12\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{930\text{ K}}$$

40. The diagram here is similar to Fig. 29–10. The activation energy is the energy needed to get the (initially) stable system over the barrier in the potential energy. The activation energy is 1.4 eV for this molecule. The dissociation energy is the energy that is released when the bond is broken. The dissociation energy is 1.6 eV for this molecule.



41. (*a*) The potential energy for the point charges is found in Section 29–2.

$$\text{PE} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{(2.30\times10^{-28}\text{ J}\cdot\text{m})}{(0.27\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = -5.32\text{ eV} \approx \boxed{-5.3\text{ eV}}$$

(*b*) Because the potential energy of the ions is negative, 5.32 eV is released when the ions are brought together. The other energies quoted involve the transfer of the electron from the K atom to the F atom. A total of 3.41 eV is released, and 4.34 eV is absorbed in the individual electron transfer processes. Thus, the total binding energy is as follows:

$$\text{Binding energy} = 5.32\text{ eV} + 3.41\text{ eV} - 4.34\text{ eV} = 4.39\text{ eV} \approx \boxed{4.4\text{ eV}}$$

42. From Fig. 29–16, a rotational absorption spectrum would show peaks at energies of $\hbar^2/I$, $2\hbar^2/I$, $3\hbar^2/I$, etc. Adjacent peaks are separated by an energy of $\hbar^2/I$. We use the photon frequency at that energy to determine the rotational inertia.

$$\Delta E = \frac{\hbar^2}{I} \quad \rightarrow \quad I = \frac{\hbar^2}{\Delta E} = \frac{\hbar^2}{hf} = \frac{h}{4\pi^2 f} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{4\pi^2(8.9\times10^{11}\text{ Hz})} = \boxed{1.9\times10^{-47}\text{ kg}\cdot\text{m}^2}$$

43. An O_2 molecule can be treated as two point masses, 16 u each, and each having a distance of $6.05\times10^{-11}\text{ m}$ from the molecule's center of mass.

$$I = \sum mr^2 = 2[(16\text{ u})(1.66\times10^{-27}\text{ kg/u})](6.05\times10^{-11}\text{ m})^2 = \boxed{1.94\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

44. The kinetic energy of the baton is $\frac{1}{2}I\omega^2$, and the quantum number can be found from Eq. 29–1. Let the length of the baton be d. We assume that the quantum number will be very large. The rotational

inertia about the center of mass is the sum of the inertias for a uniform rod $\left(\frac{1}{12}m_{\text{bar}}d^2\right)$ and two point masses $2m_{\text{end}}\left(\frac{1}{2}d\right)^2$.

$$\tfrac{1}{2}I\omega^2 = \frac{\ell(\ell+1)\hbar^2}{2I} \approx \frac{\ell^2\hbar^2}{2I} \quad \rightarrow$$

$$\ell = \frac{I\omega}{\hbar} = \left[2m_{\text{end}}\left(\tfrac{1}{2}d\right)^2 + \tfrac{1}{12}m_{\text{bar}}d^2\right]\frac{2\pi f}{\hbar} =$$

$$= \left[2(0.38\text{ kg})(0.16\text{ m})^2 + \tfrac{1}{12}(0.23\text{ kg})(0.32\text{ m})^2\right]\frac{2\pi(1.8\text{ s}^{-1})}{(1.055\times10^{-34}\text{ J}\cdot\text{s})} = 2.30\times10^{33}$$

The spacing between rotational energy levels is given by Eq. 29–2. We compare that value to the rotational kinetic energy.

$$\frac{\Delta E}{E} = \frac{\dfrac{\ell\hbar^2}{I}}{\dfrac{\ell^2\hbar^2}{2I}} = \frac{2}{\ell} = \frac{2}{(2.30\times10^{33})} = 8.7\times10^{-34}$$

This is such a small difference that it would not be detectable, so $\boxed{\text{no}}$, we do not need to consider quantum effects.

45. The photon with the maximum wavelength for absorption has an energy equal to the energy gap.

$$E_g = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(2.06\times10^{-3}\text{ m})} = \boxed{6.03\times10^{-4}\text{ eV}}$$

$\boxed{46.}$ The photon with the maximum wavelength for conduction has an energy equal to the energy gap.

$$E_g = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(226\times10^{-9}\text{ m})} = \boxed{5.50\text{ eV}}$$

47. The longest wavelength will be for the photon with the minimum energy, which corresponds to the gap energy.

$$E_g = \frac{hc}{\lambda_{\max}} \quad \rightarrow \quad \lambda_{\max} = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(3.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 3.5\times10^{-7}\text{ m}$$

So the photon must have $\boxed{\lambda \le 3.5\times10^{-7}\text{ m}}$.

48. The photon with the longest wavelength has the minimum energy, which should be equal to the gap energy.

$$E_g = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(1100\times10^{-9}\text{ m})} = 1.130\text{ eV} \approx \boxed{1.1\text{ eV}}$$

If the energy gap is any larger than this, then some solar photons will not have enough energy to cause an electron to jump levels. Those photons will not be absorbed, making the solar cell less efficient.

49. To use silicon to filter the wavelengths, wavelengths below the IR should cause the electron to be raised to the conduction band, so the photon is absorbed in the silicon. We find the shortest wavelength that will cause the electron to jump.

$$\lambda = \frac{c}{f} = \frac{hc}{hf} > \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(1.12\ \text{eV})} = 1.11\times10^{-6}\ \text{m} = \boxed{1.11\ \mu\text{m}}$$

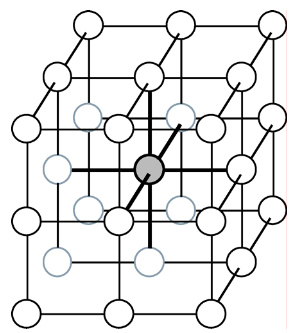
Because this is in the IR region of the spectrum, the shorter wavelengths of visible light will excite the electron and the photon would be absorbed. So silicon $\boxed{\text{can be used}}$ as a window.

50. The energy gap is related to photon wavelength by $E_g = hf = hc/\lambda.$ Use this for both colors of LED.

Green: $$E_g = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(525\times10^{-9}\ \text{m})} = \boxed{2.37\ \text{eV}}$$

Blue: $$E_g = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{J/eV})(465\times10^{-9}\ \text{m})} = \boxed{2.67\ \text{eV}}$$

51. From the diagram of the cubic lattice, we see that an atom inside the cube is bonded to the six nearest neighbors. Because each bond is shared by two atoms, the number of bonds per atom is three (as long as the sample is large enough that most atoms are in the interior and not on the boundary surface). We find the heat of fusion from the energy required to break the bonds:



$$\begin{aligned} L_F &= \left(\frac{\text{number of bonds}}{\text{atom}}\right)\left(\frac{\text{number of atoms}}{\text{mol}}\right)E_{\text{bond}} \\ &= (3)(6.02\times10^{23}\ \text{atoms/mol})(3.4\times10^{-3}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) \\ &= 982\ \text{J/mol} \approx \boxed{980\ \text{J/mol}} \end{aligned}$$

Solutions to Search and Learn Problems

1. Metallic bond theory says that the free electrons in metallic elements can vibrate at any frequency, so when light of a range of frequencies falls on a metal, the electrons can vibrate in response and re-emit light of those same frequencies. Hence, the reflected light will consist largely of the same frequencies as the incident light, so it will not have a distinct color.

2. The H_2 molecule does not have an activation energy. The two hydrogen atoms do not need an initial kinetic energy to get close enough to bond. The potential energy of the H_2 molecule is less than the potential energy of the atoms when they are infinitely apart. For the formation of ATP from ADP and P, an activation energy is necessary. The two initial molecules must have an initial kinetic energy to overcome the repulsion between them. It is part of the initial kinetic energy that is released causing the ATP molecule to bond. Part of the initial kinetic energy of the atoms remains as potential energy of the ATP molecule. That is, the potential energy of the ATP is greater than the potential energy when the ADP and P are far apart.

3. (*a*) Weak bonds enable molecular structure to be modified easily, even by simple molecular collisions. These can be accomplished at relatively low temperatures.

   (*b*) Heating the protein excessively will disassociate many of the weak bonds and may even disassociate some of the stronger bonds, thereby changing the shape of the protein.

(*c*) The hydrogen bond is the strongest because the hydrogen atom is the smallest atom and can therefore be approached the closest.

(*d*) If these bonds were stronger, then the bonds could not be broken by simple molecular collisions. That is, it would be difficult to break apart the DNA chain, which would prevent protein synthesis from occurring at low temperatures.

4. (*a*) The electrons will not be moving fast enough at this low temperature to use relativistic expressions, so the momentum is just the mass times the speed. The kinetic energy of the electrons can be found from the temperature, by Eq. 13–8. The kinetic energy is used to calculate the momentum, and the momentum is used to calculate the wavelength.

$$\text{KE} = \tfrac{3}{2}kT = \frac{p^2}{2m} \quad \rightarrow \quad p = \sqrt{3mkT}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{3(9\times10^{-31}\ \text{kg})(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}} = 6.27\times10^{-9}\ \text{m} \approx \boxed{6\ \text{nm}}$$

(*b*) The wavelength is much longer than the opening, so electrons at this temperature would experience $\boxed{\text{diffraction}}$ when passing through the lattice.

5. From Eq. 18–10, the number of charge carriers per unit volume in a current is given by $n = \dfrac{I}{ev_\text{d}A}$, where v_d is the drift speed of the charge carriers and A is the cross-sectional area through which the carriers move. From Section 20–4, the drift speed is given by $v_\text{d} = \dfrac{V_\text{Hall}}{Bd}$, where V_Hall is the Hall effect voltage and d is the width of the strip carrying the current (see Fig. 20–21). The distance d is the shorter dimension on the "top" of Fig. 29–45. We combine these equations to find the density of charge carriers. We define the thickness of the current-carrying strip as $t = A/d$.

$$n = \frac{I}{ev_\text{d}A} = \frac{IBd}{eV_\text{Hall}A} = \frac{IB}{eV_\text{Hall}t} = \frac{(0.28\times10^{-3}\ \text{A})(1.5\ \text{T})}{(1.60\times10^{-19}\ \text{C})(0.018\ \text{V})(1.0\times10^{-3}\ \text{m})}$$
$$= 1.458\times10^{20}\ \text{electrons/m}^3$$

The actual density of atoms per unit volume in the silicon is found from the density and the atomic weight. We let that density be represented by *N*. That density is used to find the number of charge carriers per atom.

$$N = (2330\ \text{kg/m}^3)\left(\frac{1\ \text{mole}}{28.0855\times10^{-3}\ \text{kg}}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{1\ \text{mole}}\right) = 4.994\times10^{28}\ \text{atoms/m}^3$$

$$\frac{n}{N} = \frac{1.458\times10^{20}\ \text{electrons/m}^3}{4.994\times10^{28}\ \text{atoms/m}^3} = \boxed{2.9\times10^{-9}\ \text{electrons/atom}}$$

6. In a dielectric, Coulomb's law becomes $F = \dfrac{ke^2}{r^2} = \dfrac{e^2}{4\pi K\varepsilon_0 r^2}$. Thus wherever e^2 appears in an equation, we divide by *K*. The arsenic ion has a charge of +1, since we consider the ion as having been formed by removing one electron from the arsenic atom. Thus the effective *Z* will be 1, and we can use the Bohr theory results for hydrogen.

(*a*) The energy of the electron is calculated by taking Eq. 27–15a and Eq. 27–15b and dividing the expression by K^2 since it has a factor of e^4 in the numerator.

$$E = -\frac{2\pi^2 Z^2 e^4 mk^2}{K^2 h^2 n^2} = -\frac{(13.6\text{ eV})Z^2}{K^2 n^2} = -\frac{(13.6\text{ eV})(1)^2}{(12)^2(1)^2} = -0.094\text{ eV}$$

Thus the binding energy is $\boxed{0.094\text{ eV}}$.

(*b*) The radius of the electron orbit is found from Eqs. 27–12 and 27–13, with the expression multiplied by *K*.

$$r = \frac{Kh^2 n^2}{4\pi^2 Ze^2 mk} = \frac{Kn^2(0.0529\text{ nm})}{Z} = \frac{(12)(1)^2(0.0529\text{ nm})}{(1)^2} = \boxed{0.63\text{ nm}}$$

30

NUCLEAR PHYSICS AND RADIOACTIVITY

Responses to Questions

$\boxed{1.}$ Different isotopes of a given element have the same number of protons and electrons. Because they have the same number of electrons, they have almost identical chemical properties. Each isotope has a different number of neutrons from other isotopes of the same element. Accordingly, they have different atomic masses and mass numbers. Since the number of neutrons is different, they may have different nuclear properties, such as whether they are radioactive or not.

$\boxed{2.}$ Identify the element based on the atomic number.

(*a*) Uranium ($Z = 92$)
(*b*) Nitrogen ($Z = 7$)
(*c*) Hydrogen ($Z = 1$)
(*d*) Strontium ($Z = 38$)
(*e*) Fermium ($Z = 100$)

3. The number of protons is the same as the atomic number, and the number of neutrons is the mass number minus the number of protons.

(*a*) Uranium: 92 protons, 140 neutrons
(*b*) Nitrogen: 7 protons, 11 neutrons
(*c*) Hydrogen: 1 proton, 0 neutrons
(*d*) Strontium: 38 protons, 48 neutrons
(*e*) Fermium: 100 protons, 152 neutrons

4. With 87 nucleons and 50 neutrons, there must be 37 protons. This is the atomic number, so the element is rubidium. The nuclear symbol is ${}^{87}_{37}\mathrm{Rb}$.

$\boxed{5.}$ The atomic mass of an element as shown in the Periodic Table is the average atomic mass of all naturally occurring isotopes. For example, chlorine occurs as roughly 75% ${}^{35}_{17}\mathrm{Cl}$ and 25% ${}^{37}_{17}\mathrm{Cl}$, so its atomic mass is about $35.5\ (= 0.75 \times 35 + 0.25 \times 37)$. Other smaller effects would include the fact that the masses of the nucleons are not exactly 1 atomic mass unit and that some small fraction of the mass energy of the total set of nucleons is in the form of binding energy.

6. The alpha particle is a very stable nucleus. It has less energy when bound together than when split apart into separate nucleons.

7. The strong force and the electromagnetic (EM) force are two of the four fundamental forces in nature. They are both involved in holding atoms together: the strong force binds quarks into nucleons and binds nucleons together in the nucleus; the EM force is responsible for binding negatively charged electrons to positively charged nuclei and for binding atoms into molecules. The strong force is the strongest fundamental force; the EM force is about 100 times weaker at distances on the order of 10^{-17} m. The strong force operates at short range and is negligible for distances greater than about the size of the nucleus. The EM force is a long-range force that decreases as the inverse square of the distance between the two interacting charged particles. The EM force operates only between charged particles. The strong force is always attractive; the EM force can be attractive or repulsive. Both of these forces have mediating field particles associated with them—the gluon for the strong force and the photon for the EM force.

8. Quoting from Section 30–3, "… radioactivity was found in every case to be unaffected by the strongest physical and chemical treatments, including strong heating or cooling and the action of strong chemical reagents." Chemical reactions are a result of electron interactions, not nuclear processes. The absence of effects caused by chemical reactions is evidence that the radioactivity is not due to electron interactions. Another piece of evidence is the fact that the α particle emitted in many radioactive decays is much heavier than an electron and has a different charge than the electron, so it can't be an electron. Therefore, it must be from the nucleus. Finally, the energies of the electrons or photons emitted from radioactivity are much higher than those corresponding to electron orbital transitions. All of these observations support radioactivity being a nuclear process.

9. The resulting nuclide for gamma decay is the same isotope in a lower energy state.

$$^{64}_{29}\mathrm{Cu}^* \;\rightarrow\; ^{64}_{29}\mathrm{Cu}+\gamma$$

The resulting nuclide for β^- decay is an isotope of zinc, $^{64}_{30}\mathrm{Zn}$.

$$^{64}_{29}\mathrm{Cu} \;\rightarrow\; ^{64}_{30}\mathrm{Zn}+\mathrm{e}^-+\overline{\nu}$$

The resulting nuclide for β^+ decay is an isotope of nickel, $^{64}_{28}\mathrm{Ni}$.

$$^{64}_{29}\mathrm{Cu} \;\rightarrow\; ^{64}_{28}\mathrm{Ni}+\mathrm{e}^++\nu$$

10. $^{238}_{92}\mathrm{U}$ decays by alpha emission into $^{234}_{90}\mathrm{Th}$, which has 144 neutrons.

11. Alpha (α) particles are helium nuclei. Each α particle consists of 2 protons and 2 neutrons and therefore has a charge of +2*e* and an atomic mass value of 4 u. They are the most massive of the three. Beta (β) particles are electrons (β^-) or positrons (β^+). Electrons have a charge of –*e* and positrons have a charge of +*e*. In terms of mass, beta particles are much lighter than protons or neutrons, by a factor of about 2000, so are lighter than alpha particles by a factor of about 8000. Their emission is always accompanied by either an antineutrino (β^-) or a neutrino (β^+). Gamma (γ) particle are photons. They have no mass and no charge.

12. (*a*) $^{45}_{20}\text{Ca} \rightarrow {}^{45}_{21}\text{Sc} + \text{e}^- + \overline{\nu}$ Scandium-45 is the missing nucleus.

(*b*) $^{58}_{29}\text{Cu} \rightarrow {}^{58}_{29}\text{Cu} + \gamma$ Copper-58 is the missing nucleus.

(*c*) $^{46}_{24}\text{Cr} \rightarrow {}^{46}_{23}\text{V} + \text{e}^+ + \nu$ The positron and the neutrino are the missing particles.

(*d*) $^{234}_{94}\text{Pu} \rightarrow {}^{230}_{92}\text{U} + \alpha$ Uranium-230 is the missing nucleus.

(*e*) $^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + \text{e}^- + \overline{\nu}$ The electron and the antineutrino are the missing particles.

13. The two extra electrons held by the newly formed thorium will be very loosely held, as the number of protons in the nucleus will have been reduced from 92 to 90, reducing the nuclear charge. It will be easy for these extra two electrons to escape from the thorium atom through a variety of mechanisms. They are in essence "free" electrons. They do not gain kinetic energy from the decay. They might get captured by the alpha nucleus, for example.

14. When a nucleus undergoes either β^- or β^+ decay, it becomes a different element, since it has converted either a neutron to a proton or a proton to a neutron. Thus its atomic number (*Z*) has changed. The energy levels of the electrons are dependent on *Z*, so all of those energy levels change to become the energy levels of the new element. Photons (with energies on the order of tens of eV) are likely to be emitted from the atom as electrons change energies to occupy the new levels.

15. In alpha decay, assuming the energy of the parent nucleus is known, the unknowns after the decay are the energies of the daughter nucleus and the alpha. These two values can be determined by energy and momentum conservation. Since there are two unknowns and two conditions, the values are uniquely determined. In beta decay, there are three unknown postdecay energies since there are three particles present after the decay. The conditions of energy and momentum conservation are not sufficient to exactly determine the energy of each particle, so a range of values is possible.

16. In electron capture, the nucleus will effectively have a proton change to a neutron. This isotope will then lie to the left and above the original isotope. Since the process would only occur if it made the nucleus more stable, it must lie BELOW the line of stability in Fig. 30–2.

17. Neither hydrogen nor deuterium can emit an α particle. Hydrogen has only one nucleon (a proton) in its nucleus, and deuterium has only two nucleons (one proton and one neutron) in its nucleus. Neither one has the necessary four nucleons (two protons and two neutrons) to emit an α particle.

18. Many artificially produced radioactive isotopes are rare in nature because they have decayed away over time. If the half-lives of these isotopes are relatively short in comparison with the age of Earth (which is typical for these isotopes), then there won't be any significant amount of these isotopes left to be found in nature. Also, many of these isotopes have a very high energy of formation, which is generally not available under natural circumstances.

19. After two months the sample will not have completely decayed. After one month, half of the sample will remain, and after two months, one-fourth of the sample will remain. Each month, half of the remaining atoms decay.

20. For $Z > 92$, the short range of the attractive strong nuclear force means that no number of neutrons is able to overcome the long-range electrostatic repulsion of the large concentration of protons.

$\boxed{21.}$ There are a total of 4 protons and 3 neutrons in the reactant. The α particle has 2 protons and 2 neutrons, so 2 protons and 1 neutron are in the other product particle. It must be ${}^{3}_{2}\mathrm{He}$.

$${}^{6}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{p} \quad \rightarrow \quad {}^{4}_{2}\alpha + {}^{3}_{2}\mathrm{He}$$

$\boxed{22.}$ The technique of ${}^{14}_{6}\mathrm{C}$ would not be used to measure the age of stone walls and tablets. Carbon-14 dating is only useful for measuring the age of objects that were living at some earlier time.

23. The decay series of Fig. 30–11 begins with a nucleus that has many more neutrons than protons and lies far above the line of stability in Fig. 30–2. The alpha decays remove both 2 protons and 2 neutrons, but smaller stable nuclei have a smaller percentage of neutrons. In β^+ decay, a proton is converted to a neutron, which would take the nuclei in this decay series even farther from the line of stability. Thus, β^+ decay is not energetically preferred.

24. There are four alpha particles and four β^- particles (electrons) emitted, no matter which decay path is chosen. The nucleon number drops by 16 as ${}^{222}_{86}\mathrm{Rn}$ decays into ${}^{206}_{82}\mathrm{Pb},$ indicating that four alpha decays occurred. The proton number only drops by four, from $Z = 86$ to $Z = 82$, but four alpha decays would result in a decrease of eight protons. Four β^- decays will convert four neutrons into protons, making the decrease in the number of protons only four, as required. (See Fig. 30–11.)

25. (i) Since the momentum before the decay was 0, the total momentum after the decay will also be 0. Since there are only two decay products, they must move in opposite directions with equal magnitude of momentum. Thus, (*c*) is the correct choice: both the same.

(ii) Since the products have the same momentum, the one with the smallest mass will have the greater velocity. Thus (*b*) is the correct choice: the alpha particle.

(iii) We assume that the products are moving slowly enough that classical mechanics can be used. In that case, $\mathrm{KE} = p^2/2m.$ Since the particles have the same momentum, the one with the smallest mass will have the greater kinetic energy. Thus, (*b*) is the correct choice: the alpha particle.

Responses to MisConceptual Questions

1. (*a*) A common misconception is that the elements of the Periodic Table are distinguished by the number of electrons in the atom. This misconception arises because the number of electrons in a neutral atom is the same as the number of protons in its nucleus. Elements can be ionized by adding or removing electrons, but this does not change what type of element it is. When an element undergoes a nuclear reaction that changes the number of protons in the nucleus, the element does transform into a different element.

2. (*b*) The role of energy in binding nuclei together is often misunderstood. As protons and neutrons are added to the nucleus, they release some mass energy, which usually appears as radiation or kinetic energy. This lack of energy is what binds the nucleus together. To break the nucleus apart, the energy must be added back in. As a result, a nucleus will have less energy than the protons and neutrons that made up the nucleus.

3. (*c*) Large nuclei are typically unstable, so increasing the number of nuclei does not necessarily make the nucleus more stable. Nuclei such as ${}^{14}_{8}\mathrm{O}$ have more protons than neutrons, yet ${}^{14}_{8}\mathrm{O}$ is not as stable as ${}^{16}_{8}\mathrm{O}$. Therefore, having more protons than neutrons does not necessarily make a nucleus more stable. Large unstable nuclei, such as ${}^{238}_{94}\mathrm{Pu}$, have a much larger total binding energy than small stable nuclei such as ${}^{4}_{2}\mathrm{He}$, so the total binding energy is not a measure of stability. However, stable nuclei typically have large binding energies per nucleon, which means that each nucleon is more tightly bound to the others.

4. (*e*) The Coulomb repulsive force does act inside the nucleus, pushing the protons apart. Another larger attractive force is thus necessary to keep the nucleus together. The force of gravity is far too small to hold the nucleus together. Neutrons are not negatively charged. It is the attractive strong nuclear force that overcomes the Coulomb force to hold the nucleus together.

5. (*b*) The exponential nature of radioactive decay is a concept that can be misunderstood. It is sometimes thought that the decay is linear, such that the time for a substance to decay completely is twice the time for half of the substance to decay. However, radioactive decay is not linear but exponential. That is, during each half-life, half of the remaining substance decays. If half the original decays in the first half-life, then half remains. During the second half-life, half of what is left decays, which would be one-quarter of the initial substance. In each subsequent half-life, half of the remaining substance decays, so it takes many half-lives for a substance to effectively decay away. The decay constant is inversely related to the half-life.

6. (*e*) The half-life is the time it takes for half of the substance to decay away. The half-life is a constant determined by the composition of the substance and not by the quantity of the initial substance. As the substance decays away, the number of nuclei is decreasing and the activity (number of decays per second) is decreasing, but the half-life remains constant.

7. (*d*) A common misconception is that it would take twice the half-life, or 20 years, for the substance to completely decay. This is incorrect because radioactive decay is an exponential process. That is, during each half-life, 1/2 of the remaining substance decays. After the first 10 years, 1/2 remains. After the second 10-year period, 1/4 remains. After each succeeding half-life another half of the remaining substance decays, leaving 1/8, then 1/16, then 1/32, and so forth. The decays stop when none of the substance remains, and this time cannot be exactly determined.

8. (*e*) During each half-life, 1/2 of the substance decays, leaving 1/2 remaining. After the first day, 1/2 remains. After the second day, 1/2 of 1/2, or 1/4, remains. After the third day, 1/2 of 1/4, or 1/8, remains. Since 1/8 remains, 7/8 of the substance has decayed.

9. (*c*) A common misconception is that after the second half-life none of the substance remains. However, during each half-life, 1/2 of the remaining substance decays. After one half-life, 1/2 remains. After two half-lives, 1/4 remains. After three half-lives, 1/8 remains.

10. (*a*) The decay constant is proportional to the probability of a particle decaying and is inversely proportional to the half-life. Therefore, the substance with the larger half-life (Tc) will have the smaller decay constant and smaller probability of decaying. The activity is proportional to the amount of the substance (number of atoms) and the decay constant, so the activity of Tc will be smaller than the activity of Sr.

11. (*d*) The element with the largest decay constant will have the shortest half-life. Converting each of the choices to decays per second yields the following: (*a*) 100/s, (*b*) 1.6×10^{8}/s, (*c*) 2.5×10^{10}/s, (*d*) 8.6×10^{13}/s. Answer (*d*) has the largest decay rate, so it will have the smallest half-life.

12. (*d*) If U-238 only decayed by beta decay, the number of nucleons would not change. Pb-206 has 32 nucleons less than U-238, so all decays cannot be beta decays. In each alpha decay, the number of nucleons decreases by four and the number of protons decreases by two. U-238 has 92 protons and Pb-206 has 82 protons, so if only alpha decays occurred, then the number of nucleons would need to decrease by 20. But Pb-206 has 32 fewer nucleons than U-238. Therefore, the sequence is a combination of alpha and beta decays. Gamma decays are common, since some of the alpha and beta decays leave the nuclei in excited states where energy needs to be released.

13. (*d*) A common misconception is that carbon dating is useful for very long time periods. C-14 has a half-life about 6000 years. After about 10 half-lives only about 1/1000 of the substance remains. It is difficult to obtain accurate measurements for longer time periods. For example, after 600,000 years, or about 100 half-lives, only one part in 10^{30} remains.

14. (*b*) The half-life of radon remains constant and is not affected by temperature. Radon that existed several billion years ago will have completely decayed away. Small isotopes, such as carbon-14, can be created from cosmic rays, but radon is a heavy element. Lightning is not energetic enough to affect the nucleus of the atoms. Heavy elements such as plutonium and uranium can decay into radon and thus are the source of present-day radon gas.

15. (*d*) The gravitational force between nucleons is much weaker (about 50 orders of magnitude weaker) than the repulsive Coulomb force; therefore, gravity cannot hold the nucleus together. Neutrons are electrically neutral and therefore cannot overcome the Coulomb force. Covalent bonds exist between the electrons in molecules, not among the nucleons. The actual force that holds the nucleus together is the strong nuclear force, but this was not one of the options.

16. (*a*) The nature of mass and energy in nuclear physics is often misunderstood. When the neutron and proton are close together, they bind together by releasing mass energy that is equivalent to the binding energy. This energy comes from a reduction in their mass. Therefore, when the neutron and proton are far from each other, their net mass is greater than their net mass when they are bound together.

Solutions to Problems

1. Convert the units from MeV/c^2 to atomic mass units.

$$m = (139\ \text{MeV}/c^2)\left(\frac{1\ \text{u}}{931.5\ \text{Me V}/c^2}\right) = \boxed{0.149\ \text{u}}$$

$\boxed{2.}$ The α particle is a helium nucleus and has $A = 4$. Use Eq. 30–1.

$$r = (1.2\times10^{-15}\ \text{m})A^{\frac{1}{3}} = (1.2\times10^{-15}\ \text{m})(4)^{\frac{1}{3}} = \boxed{1.9\times10^{-15}\ \text{m}} = 1.9\ \text{fm}$$

3. The radii of the two nuclei can be calculated with Eq. 30–1. Take the ratio of the two radii.

$$\frac{r_{238}}{r_{232}} = \frac{(1.2\times10^{-15}\text{ m})(238)^{1/3}}{(1.2\times10^{-15}\text{ m})(232)^{1/3}} = \left(\frac{238}{232}\right)^{1/3} = 1.00855$$

So the radius of ${}^{238}_{92}\text{U}$ is $\boxed{0.855\%}$ larger than the radius of ${}^{232}_{92}\text{U}$.

4. Use Eq. 30–1 for both parts of this problem.

(*a*) $r = (1.2\times10^{-15}\text{ m})A^{1/3} = (1.2\times10^{-15}\text{ m})(112)^{1/3} = \boxed{5.8\times10^{-15}\text{ m}} = 5.8\text{ fm}$

(*b*) $r = (1.2\times10^{-15}\text{ m})A^{1/3} \rightarrow A = \left(\frac{r}{1.2\times10^{-15}\text{ m}}\right)^3 = \left(\frac{3.7\times10^{-15}\text{ m}}{1.2\times10^{-15}\text{ m}}\right)^3 = 29.3 \approx \boxed{29}$

5. To find the mass of an α particle, subtract the mass of the two electrons from the mass of a helium atom:

$$m_\alpha = m_{\text{He}} - 2m_{\text{e}}$$
$$= (4.002603\text{ u})\left(\frac{931.5\text{ MeV}/c^2}{1\text{ u}}\right) - 2(0.511\text{ MeV}/c^2) = \boxed{3727\text{ MeV}/c^2}$$

6. Each particle would exert a force on the other through the Coulomb electrostatic force (given by Eq. 16–1). The distance between the particles is twice the radius of one of the particles.

$$F = k\frac{Q_\alpha Q_\alpha}{(2r_\alpha)^2} = \frac{(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left[(2)(1.60\times10^{-19}\text{ C})\right]^2}{\left[(2)\left(4^{1/3}\right)(1.2\times10^{-15}\text{ m})\right]^2} = 63.41\text{ N} \approx \boxed{63\text{ N}}$$

The acceleration is found from Newton's second law. We use the mass of a "bare" alpha particle as calculated in Problem 5.

$$F = ma \rightarrow a = \frac{F}{m} = \frac{63.41\text{ N}}{3727\text{ MeV}/c^2\left(\frac{1.6605\times10^{-27}\text{ kg}}{931.5\text{ MeV}/c^2}\right)} = \boxed{9.5\times10^{27}\text{ m/s}^2}$$

7. First, we calculate the density of nuclear matter. The mass of a nucleus with mass number A is approximately (A u) and its radius is $r = (1.2\times10^{-15}\text{ m})\,A^{1/3}$. Calculate the density.

$$\rho = \frac{m}{V} = \frac{A(1.6605\times10^{-27}\text{ kg/u})}{\frac{4}{3}\pi r^3} = \frac{A(1.6605\times10^{-27}\text{ kg/u})}{\frac{4}{3}\pi(1.2\times10^{-15}\text{ m})^3 A} = 2.294\times10^{17}\text{ kg/m}^3$$

We see that this is independent of A. The value has 2 significant figures.

(*a*) We set the density of the Earth equal to the density of nuclear matter.

$$\rho_{\text{Earth}} = \rho_{\substack{\text{nuclear}\\\text{matter}}} = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}^3} \rightarrow$$

$$R_{\text{Earth}} = \left(\frac{M_{\text{Earth}}}{\frac{4}{3}\pi\rho_{\substack{\text{nuclear}\\\text{matter}}}}\right)^{1/3} = \left(\frac{5.98\times10^{24}\text{ kg}}{\frac{4}{3}\pi(2.294\times10^{17}\text{ kg/m}^3)}\right)^{1/3} = 183.9\text{ m} \approx \boxed{180\text{ m}}$$

(*b*) Set the density of Earth equal to the density of uranium, and solve for the radius of the uranium. Then compare that with the actual radius of uranium, using Eq. 30–1, with $A = 238$.

$$\rho = \frac{M_{\text{Earth}}}{\frac{4}{3}\pi R_{\text{Earth}}^3} = \frac{m_{\text{U}}}{\frac{4}{3}\pi r_{\text{U}}^3} \rightarrow \frac{M_{\text{Earth}}}{R_{\text{Earth}}^3} = \frac{m_{\text{U}}}{r_{\text{U}}^3} \rightarrow$$

$$r_{\text{U}} = R_{\text{Earth}}\left(\frac{m_{\text{U}}}{M_{\text{Earth}}}\right)^{1/3} = (6.38\times10^6 \text{ m})\left[\frac{(238 \text{ u})(1.6605\times10^{-27} \text{ kg/u})}{(5.98\times10^{24} \text{ kg})}\right]^{1/3} = 2.58\times10^{-10} \text{ m}$$

$$\frac{2.58\times10^{-10} \text{ m}}{(1.2\times10^{-15})(238)^{1/3}} = \boxed{3.5\times10^4}$$

8. Use Eq. 30–1 to find the value for A. We use uranium-238 since it is the most common isotope.

$$\frac{r_{\text{unknown}}}{r_{\text{U}}} = \frac{(1.2\times10^{-15} \text{ m})A^{1/3}}{(1.2\times10^{-15} \text{ m})(238)^{1/3}} = 0.5 \rightarrow A = 238(0.5)^3 = 29.75 \approx 30$$

From Appendix B, a stable nucleus with $A \approx 30$ is $\boxed{{}^{31}_{15}\text{P}}$.

9. Use conservation of energy. Assume that the centers of the two particles are located a distance from each other equal to the sum of their radii, and use that distance to calculate the initial electrical PE. Then we also assume, since the nucleus is much heavier than the alpha, that the alpha has all of the final KE when the particles are far apart from each other (so they have no PE).

$$\text{KE}_{\text{i}} + \text{PE}_{\text{i}} = \text{KE}_{\text{f}} + \text{PE}_{\text{f}} \rightarrow 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Fm}}}{(r_\alpha + r_{\text{Fm}})} = \text{KE}_\alpha + 0 \rightarrow$$

$$\text{KE}_\alpha = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2)(100)(1.60\times10^{-19} \text{ C})^2}{(4^{1/3}+257^{1/3})(1.2\times10^{-15} \text{ m})(1.60\times10^{-19} \text{ J/eV})} = 3.017\times10^7 \text{ eV}$$

$$\approx \boxed{3.0\times10^7 \text{ eV}} = 30 \text{ MeV}$$

10. (*a*) The hydrogen atom is made of a proton and an electron. Use values from Appendix B.

$$\frac{m_{\text{p}}}{m_{\text{H}}} = \frac{1.007276 \text{ u}}{1.007825 \text{ u}} = \boxed{0.9994553} \approx 99.95\%$$

(*b*) Compare the volume of the nucleus to the volume of the atom. The nuclear radius is given by Eq. 30–1. For the atomic radius we use the Bohr radius, given in Eq. 27–13.

$$\frac{V_{\text{nucleus}}}{V_{\text{atom}}} = \frac{\frac{4}{3}\pi r_{\text{nucleus}}^3}{\frac{4}{3}\pi r_{\text{atom}}^3} = \left(\frac{r_{\text{nucleus}}}{r_{\text{atom}}}\right)^3 = \left[\frac{(1.2\times10^{-15} \text{ m})}{(0.53\times10^{-10} \text{ m})}\right]^3 = \boxed{1.2\times10^{-14}}$$

11. Electron mass is negligible compared to nucleon mass, and one nucleon weighs about 1.0 atomic mass unit. Therefore, in a 1.0-kg object, we find the following:

$$N = \frac{(1.0 \text{ kg})(6.02\times10^{26} \text{ u/kg})}{1.0 \text{ u/nucleon}} \approx \boxed{6.0\times10^{26} \text{ nucleons}}$$

No, it does not matter what the element is, because the mass of one nucleon is essentially the same for all elements.

12. The initial kinetic energy of the alpha must be equal to the electrical potential energy when the alpha just touches the uranium. The distance between the two particles is the sum of their radii.

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \quad \to \quad \mathrm{KE}_\alpha + 0 = 0 + k\frac{Q_\alpha Q_\mathrm{U}}{(r_\alpha + r_\mathrm{U})} \quad \to$$

$$\mathrm{KE}_\alpha = (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(2)(92)(1.60\times10^{-19}\ \mathrm{C})^2}{(4^{1/3}+232^{1/3})(1.2\times10^{-15}\ \mathrm{m})(1.60\times10^{-19}\ \mathrm{J/eV})} = 2.852\times10^7\ \mathrm{eV}$$

$$\approx \boxed{29\ \mathrm{MeV}}$$

13. The text states that the average binding energy per nucleon between $A \approx 40$ and $A \approx 80$ is about 8.7 MeV. Multiply this by the number of nucleons in the nucleus.

$$(63)(8.7\ \mathrm{MeV}) = 548.1\ \mathrm{MeV} \approx \boxed{550\ \mathrm{MeV}}$$

14. (*a*) From Fig. 30–1, we see that the average binding energy per nucleon at $A = 238$ is 7.5 MeV. Multiply this by the 238 nucleons.

$$(238)(7.5\ \mathrm{MeV}) = \boxed{1.8\times10^3\ \mathrm{MeV}}$$

(*b*) From Fig. 30–1, we see that the average binding energy per nucleon at $A = 84$ is 8.7 MeV. Multiply this by the 84 nucleons.

$$(84)(8.7\ \mathrm{MeV}) = \boxed{730\ \mathrm{MeV}}$$

15. ${}^{15}_{7}\mathrm{N}$ consists of seven protons and eight neutrons. We find the binding energy from the masses of the components and the mass of the nucleus, from Appendix B.

$$\text{Binding energy} = \left[7m\left({}^{1}_{1}\mathrm{H}\right) + 8m\left({}^{1}_{0}\mathrm{n}\right) - m\left({}^{15}_{7}\mathrm{N}\right)\right]c^2$$

$$= [7(1.007825\ \mathrm{u}) + 8(1.008665\ \mathrm{u}) - (15.000109\ \mathrm{u})]c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right)$$

$$= 115.49\ \mathrm{MeV}$$

$$\text{Binding energy per nucleon} = (115.49\ \mathrm{MeV})/15 = \boxed{7.699\ \mathrm{MeV}}$$

16. Deuterium consists of one proton, one neutron, and one electron. Ordinary hydrogen consists of one proton and one electron. We use the atomic masses from Appendix B, and the electron masses cancel.

$$\text{Binding energy} = \left[m\left({}^{1}_{1}\mathrm{H}\right) + m\left({}^{1}_{0}\mathrm{n}\right) - m\left({}^{2}_{1}\mathrm{H}\right)\right]c^2$$

$$= [(1.007825\ \mathrm{u}) + (1.008665\ \mathrm{u}) - (2.014102\ \mathrm{u})]c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right)$$

$$= \boxed{2.224\ \mathrm{MeV}}$$

17. We find the binding energy of the last neutron from the masses of the isotopes.

$$\text{Binding energy} = \left[m\left({}^{22}_{11}\text{Na}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2$$
$$= [(21.994437\text{ u}) + (1.008665\text{ u}) - (22.989769\text{ u})]c^2(931.5\text{ MeV}/c^2)$$
$$= \boxed{12.42\text{ MeV}}$$

$\boxed{18.}$ (*a*) ${}^{7}_{3}\text{Li}$ consists of three protons and three neutrons. We find the binding energy from the masses, using hydrogen atoms in place of protons so that we account for the mass of the electrons.

$$\text{Binding energy} = \left[3m\left({}^{1}_{1}\text{H}\right) + 4m\left({}^{1}_{0}\text{n}\right) - m\left({}^{7}_{3}\text{Li}\right)\right]c^2$$
$$= [3(1.007825\text{ u}) + 4(1.008665\text{ u}) - (7.016003\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$
$$= \boxed{39.25\text{ MeV}}$$
$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{39.25\text{ MeV}}{7\text{ nucleons}} = \boxed{5.607\text{ MeV/nucleon}}$$

(*b*) ${}^{195}_{78}\text{Pt}$ consists of 78 protons and 117 neutrons. We find the binding energy as in part (*a*).

$$\text{Binding energy} = \left[78m\left({}^{1}_{1}\text{H}\right) + 117m\left({}^{1}_{0}\text{n}\right) - m\left({}^{195}_{78}\text{Pt}\right)\right]c^2$$
$$= [78(1.007825\text{ u}) + 117(1.008665\text{ u}) - (194.964792\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$
$$= 1545.7\text{ MeV} \approx \boxed{1546\text{ MeV}}$$
$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{1545.7\text{ MeV}}{195\text{ nucleons}} = \boxed{7.927\text{ MeV/nucleon}}$$

19. ${}^{23}_{11}\text{Na}$ consists of 11 protons and 12 neutrons. We find the binding energy from the masses.

$$\text{Binding energy} = \left[11m\left({}^{1}_{1}\text{H}\right) + 12m\left({}^{1}_{0}\text{n}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2$$
$$= [11(1.007825\text{ u}) + 12(1.008665\text{ u}) - (22.989769\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$
$$= 186.6\text{ MeV}$$
$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{186.6\text{ MeV}}{23} = \boxed{8.113\text{ MeV/nucleon}}$$

We do a similar calculation for ${}^{24}_{11}\text{Na}$, consisting of 11 protons and 13 neutrons.

$$\text{Binding energy} = \left[11m\left({}^{1}_{1}\text{H}\right) + 13m\left({}^{1}_{0}\text{n}\right) - m\left({}^{24}_{11}\text{Na}\right)\right]c^2$$
$$= [11(1.007825\text{ u}) + 13(1.008665\text{ u}) - (23.990963\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$
$$= 193.5\text{ MeV}$$
$$\frac{\text{Binding energy}}{\text{nucleon}} = \frac{193.5\text{ MeV}}{24} = \boxed{8.063\text{ MeV/nucleon}}$$

By this measure, the nucleons in ${}^{23}_{11}\text{Na}$ are more tightly bound than those in ${}^{24}_{11}\text{Na}$. Thus we expect ${}^{23}_{11}\text{Na}$ to be more stable than ${}^{24}_{11}\text{Na}$.

20. We find the required energy by calculating the difference in the masses.

(*a*) Removal of a proton creates an isotope of carbon. To balance electrons, the proton is included as a hydrogen atom: ${}^{15}_{7}\text{N} \rightarrow {}^{1}_{1}\text{H} + {}^{14}_{6}\text{C}$.

$$\begin{aligned}\text{Energy needed} &= \left[m\left({}^{14}_{6}\text{C}\right) + m\left({}^{1}_{1}\text{H}\right) - m\left({}^{15}_{7}\text{N}\right)\right]c^2 \\ &= [(14.003242\ \text{u}) + (1.007825\ \text{u}) - (15.000109\ \text{u})]\left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) \\ &= \boxed{10.21\ \text{MeV}}\end{aligned}$$

(*b*) Removal of a neutron creates another isotope of nitrogen: ${}^{15}_{7}\text{N} \rightarrow {}^{1}_{0}\text{n} + {}^{14}_{7}\text{N}$.

$$\begin{aligned}\text{Energy needed} &= \left[m\left({}^{14}_{7}\text{N}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{15}_{7}\text{N}\right)\right]c^2 \\ &= [(14.003074\ \text{u}) + (1.008665\ \text{u}) - (15.000109\ \text{u})]\left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) \\ &= \boxed{10.83\ \text{MeV}}\end{aligned}$$

The nucleons are held by the attractive strong nuclear force. It takes less energy to remove the proton because there is also the $\boxed{\text{repulsive electric force}}$ from the other protons.

21. (*a*) We find the binding energy from the masses.

$$\begin{aligned}\text{Binding energy} &= \left[2m\left({}^{4}_{2}\text{He}\right) - m\left({}^{8}_{4}\text{Be}\right)\right]c^2 \\ &= [2(4.002603\ \text{u}) - (8.005305\ \text{u})]c^2\left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) \\ &= -0.092\ \text{MeV}\end{aligned}$$

Because the binding energy is negative, the nucleus is unstable. It will be in a lower energy state as two alphas instead of a beryllium.

(*b*) We find the binding energy from the masses.

$$\begin{aligned}\text{Binding energy} &= \left[3m\left({}^{4}_{2}\text{He}\right) - m\left({}^{12}_{6}\text{C}\right)\right]c^2 \\ &= [3(4.002603\ \text{u}) - (12.000000\ \text{u})]c^2\left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) = +7.3\ \text{MeV}\end{aligned}$$

Because the binding energy is positive, the nucleus is $\boxed{\text{stable}}$.

22. The wavelength is determined from the energy change between the states.

$$\Delta E = hf = h\frac{c}{\lambda} \quad \rightarrow \quad \lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.48\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})} = \boxed{2.6\times10^{-12}\ \text{m}}$$

23. For the decay ${}^{11}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + {}^{1}_{1}\text{p},$ we find the difference of the initial and the final masses. We use hydrogen so that the electrons are balanced.

$$\begin{aligned}\Delta m &= m\left({}^{11}_{6}\text{C}\right) - m\left({}^{10}_{5}\text{B}\right) - m\left({}^{1}_{1}\text{H}\right) \\ &= (11.011434\text{ u}) - (10.012937\text{ u}) - (1.007825\text{ u}) = -0.009328\text{ u}\end{aligned}$$

Since $\boxed{\text{the final masses are more than the original mass}}$, energy would not be conserved.

24. The decay is ${}^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{v}.$ When we add one electron to both sides to use atomic masses, we see that the mass of the emitted β particle is included in the atomic mass of ${}^{3}_{2}\text{He}.$ The energy released is the difference in the masses.

$$\text{Energy released} = \left[m\left({}^{3}_{1}\text{H}\right) - m\left({}^{3}_{2}\text{He}\right)\right]c^2$$

$$= [(3.016049\text{ u}) - (3.016029\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right) = \boxed{0.019\text{ MeV}}$$

25. The decay is ${}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{0}_{-1}\text{e} + \bar{v}.$ The electron mass is accounted for if we use the atomic mass of ${}^{1}_{1}\text{H}$ as a product. If we ignore the recoil of the proton and the neutrino, and any possible mass of the neutrino, then we get the maximum kinetic energy.

$$\begin{aligned}\text{KE}_{\text{max}} &= \left[m\left({}^{1}_{0}\text{n}\right) - m\left({}^{1}_{1}\text{H}\right)\right]c^2 = [(1.008665\text{ u}) - (1.007825\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right) \\ &= \boxed{0.782\text{ MeV}}\end{aligned}$$

$\boxed{26.}$ For each decay, we find the difference of the final masses and the initial mass. If the final mass is more than the initial mass, then the decay is not possible.

(*a*) $\Delta m = m\left({}^{232}_{92}\text{U}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{232}_{92}\text{U}\right) = 232.037156\text{ u} + 1.008665\text{ u} - 233.039636\text{ u} = 0.006185\text{ u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible}}$.

(*b*) $\Delta m = m\left({}^{13}_{7}\text{N}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{13}_{7}\text{N}\right) = 13.005739\text{ u} + 1.008665\text{ u} - 14.003074\text{ u} = 0.011330\text{ u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible}}$.

(*c*) $\Delta m = m\left({}^{39}_{19}\text{K}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{40}_{19}\text{K}\right) = 38.963706\text{ u} + 1.008665\text{ u} - 39.963998\text{ u} = 0.008373\text{ u}$

Because an increase in mass is required, the decay is $\boxed{\text{not possible}}$.

27. (*a*) From Appendix B, ${}^{24}_{11}\text{Na}$ is a $\boxed{\beta^- \text{ emitter}}$.

(*b*) The decay reaction is $\boxed{{}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + \beta^- + \bar{v}}$. We add 11 electrons to both sides in order to use atomic masses. Then the mass of the beta is accounted for in the mass of the magnesium. The maximum kinetic energy of the β^- corresponds to the neutrino having no kinetic energy (a limiting case). We also ignore the recoil of the magnesium.

$$\mathrm{KE}_{\beta^-} = \left[m\left({}^{24}_{11}\mathrm{Na}\right) - m\left({}^{24}_{12}\mathrm{Mg}\right)\right]c^2$$
$$= [(23.990963\ \mathrm{u}) - (23.985042\ \mathrm{u})]c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right) = \boxed{5.515\ \mathrm{MeV}}$$

$\boxed{28.}$ (*a*) We find the final nucleus by balancing the mass and charge numbers.

$$Z(X) = Z(\mathrm{U}) - Z(\mathrm{He}) = 92 - 2 = 90$$
$$A(X) = A(\mathrm{U}) - A(\mathrm{He}) = 238 - 4 = 234$$

Thus the final nucleus is $\boxed{{}^{234}_{90}\mathrm{Th}}$.

(*b*) If we ignore the recoil of the thorium, then the kinetic energy of the α particle is equal to the difference in the mass energies of the components of the reaction. The electrons are balanced.

$$\mathrm{KE} = \left[m\left({}^{238}_{92}\mathrm{U}\right) - m\left({}^{234}_{90}\mathrm{Th}\right) - m\left({}^{4}_{2}\mathrm{He}\right)\right]c^2 \quad \rightarrow$$
$$m\left({}^{234}_{90}\mathrm{Th}\right) = m\left({}^{238}_{92}\mathrm{U}\right) - m\left({}^{4}_{2}\mathrm{He}\right) - \frac{\mathrm{KE}}{c^2}$$
$$= \left[238.050788\ \mathrm{u} - 4.002603\ \mathrm{u} - \frac{4.20\ \mathrm{MeV}}{c^2}\left(\frac{1\ \mathrm{u}}{931.5\ \mathrm{MeV}/c^2}\right)\right]$$
$$= \boxed{234.043676\ \mathrm{u}}$$

This answer assumes that the 4.20-MeV value does not limit the significant figures of the answer.

29. The reaction is ${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + \beta^- + \bar{v}$. The kinetic energy of the β^- will be maximum if the (essentially) massless neutrino has no kinetic energy. We also ignore the recoil of the nickel.

$$\mathrm{KE}_{\beta} = \left[m\left({}^{60}_{27}\mathrm{Co}\right) - m\left({}^{60}_{28}\mathrm{Ni}\right)\right]c^2$$
$$= [(59.933816\ \mathrm{u}) - (59.930786\ \mathrm{u})]c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right) = \boxed{2.822\ \mathrm{MeV}}$$

30. We add three electron masses to each side of the reaction ${}^{7}_{4}\mathrm{Be} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{7}_{3}\mathrm{Li} + v$. Then for the mass of the product side, we may use the atomic mass of ${}^{7}_{3}\mathrm{Li}$. For the reactant side, including the three electron masses and the mass of the emitted electron, we may use the atomic mass of ${}^{7}_{4}\mathrm{Be}$. The energy released is the *Q*-value.

$$Q = \left[m\left({}^{7}_{4}\mathrm{Be}\right) - m\left({}^{7}_{3}\mathrm{Li}\right)\right]c^2$$
$$= [(7.016929\ \mathrm{u}) - (7.016003\ \mathrm{u})]c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{\mathrm{u}}\right) = \boxed{0.863\ \mathrm{MeV}}$$

31. For alpha decay we have ${}^{218}_{84}\text{Po} \to {}^{214}_{82}\text{Pb} + {}^{4}_{2}\text{He}$. We find the Q-value, which is the energy released.

$$Q = \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{214}_{82}\text{Pb}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2$$
$$= [218.008973\text{ u} - 213.999806\text{ u} - 4.002603\text{ u}]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$
$$= \boxed{6.114\text{ MeV}}$$

For beta decay we have ${}^{218}_{84}\text{Po} \to {}^{218}_{82}\text{At} + {}^{0}_{-1}\text{e} + \bar{\nu}$. We assume that the neutrino is massless, and find the Q-value. The original 84 electrons plus the extra electron created in the beta decay means that there are 85 total electrons on the right side of the reaction, so we can use the mass of the astatine atom and "automatically" include the mass of the beta decay electron.

$$Q = \left[m\left({}^{218}_{84}\text{Po}\right) - m\left({}^{218}_{85}\text{At}\right)\right]c^2$$
$$= [218.008973\text{ u} - 218.008695\text{ u}]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right) = \boxed{0.259\text{ MeV}}$$

32. (*a*) We find the final nucleus by balancing the mass and charge numbers.

$$Z(X) = Z(\text{P}) - Z(\text{e}) = 15 - (-1) = 16$$
$$A(X) = A(\text{P}) - A(\text{e}) = 32 - 0 = 32$$

Thus the final nucleus is $\boxed{{}^{32}_{16}\text{S}}$.

(*b*) If we ignore the recoil of the sulfur and the energy of the neutrino, then the maximum kinetic energy of the electron is the Q-value of the reaction. The reaction is ${}^{32}_{15}\text{P} \to {}^{32}_{16}\text{S} + \beta^- + \bar{\nu}$. We add 15 electrons to each side of the reaction, and then we may use atomic masses. The mass of the emitted beta is accounted for in the mass of the sulfur.

$$\text{KE} = \left[m\left({}^{32}_{15}\text{P}\right) - m\left({}^{32}_{16}\text{S}\right)\right]c^2 \quad \to$$
$$m\left({}^{32}_{16}\text{S}\right) = m\left({}^{32}_{15}\text{P}\right) - \frac{\text{KE}}{c^2} = \left[(31.973908\text{ u}) - \frac{1.71\text{ MeV}}{c^2}\frac{1\text{ u}}{931.5\text{ MeV}/c^2}\right] = \boxed{31.97207\text{ u}}$$

33. We find the energy from the wavelength.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.15\times10^{-13}\text{ m})(1.602\times10^{-13}\text{ J/MeV})} = \boxed{10.8\text{ MeV}}$$

This has to be a $\boxed{\gamma\text{ ray from the nucleus}}$ rather than a photon from the atom. Electron transitions do not involve this much energy. Electron transitions involve energies on the order of a few eV.

34. The emitted photon and the recoiling nucleus have the same magnitude of momentum. We find the recoil energy from the momentum. We assume that the energy is small enough that we can use classical relationships.

$$p_\gamma = \frac{E_\gamma}{c} = p_{\text{K}} = \sqrt{2m_{\text{K}}\text{KE}_{\text{K}}} \quad \to$$

$$\text{KE}_{\text{K}} = \frac{E_\gamma^2}{2m_{\text{K}}c^2} = \frac{(1.46\text{ MeV})^2}{2(39.963998\text{ u})\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2} = 2.86\times10^{-5}\text{ MeV} = \boxed{28.6\text{ eV}}$$

35. The kinetic energy of the β^+ particle will be its maximum if the (almost massless) neutrino has no kinetic energy. We ignore the recoil of the boron. Note that if the mass of one electron is added to the mass of the boron, then we may use atomic masses. We also must include the mass of the β^+. (See Problem 36 for details.)

$$^{11}_{6}\text{C} \to \left(^{11}_{5}\text{B} + {}^{0}_{-1}\text{e}\right) + {}^{0}_{1}\beta^+ + v$$

$$\text{KE} = \left[m\left(^{11}_{6}\text{C}\right) - m\left(^{11}_{5}\text{B}\right) - m\left(^{0}_{-1}\text{e}\right) - m\left(^{0}_{1}\beta^+\right)\right]c^2 = \left[m\left(^{11}_{6}\text{C}\right) - m\left(^{11}_{5}\text{B}\right) - 2m\left(^{0}_{-1}\text{e}\right)\right]c^2$$

$$= [(11.011434\text{ u}) - (11.009305\text{ u}) - 2(0.00054858\text{ u})]c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right) = \boxed{0.9612\text{ MeV}}$$

If the β^+ has no kinetic energy, then the maximum kinetic energy of the neutrino is also $\boxed{0.9612\text{ MeV}}$. The minimum energy of each is $\boxed{0}$, when the other has the maximum.

36. For the positron emission process, $^{A}_{Z}\text{N} \to {}^{A}_{Z-1}\text{N}' + \text{e}^+ + v$. We must add Z electrons to the nuclear mass of N to be able to use the atomic mass, so we must also add Z electrons to the reactant side. On the reactant side, we use $Z-1$ electrons to be able to use the atomic mass of N′. Thus we have 1 "extra" electron mass and the β particle mass, which means that we must include 2 electron masses on the right-hand side. We find the Q-value given this constraint.

$$Q = [M_{\text{P}} - (M_{\text{D}} + 2m_{\text{e}})]c^2 = (M_{\text{P}} - M_{\text{D}} - 2m_{\text{e}})c^2$$

37. We assume that the energies are low enough that we may use classical kinematics. In particular, we will use $p = \sqrt{2m\,\text{KE}}$. The decay is $^{238}_{92}\text{U} \to {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$. If the uranium nucleus is at rest when it decays, then the magnitude of the momentum of the two daughter particles must be the same.

$$p_\alpha = p_{\text{Th}};\ \text{KE}_{\text{Th}} = \frac{p_{\text{Th}}{}^2}{2m_{\text{Th}}} = \frac{p_\alpha{}^2}{2m_{\text{Th}}} = \frac{2m_\alpha K_\alpha}{2m_{\text{Th}}} = \frac{m_\alpha}{m_{\text{Th}}}\text{KE}_\alpha = \left(\frac{4\text{ u}}{234\text{ u}}\right)(4.20\text{ MeV}) = \boxed{0.0718\text{ MeV}}$$

The Q-value is the total kinetic energy produced.

$$Q = \text{KE}_\alpha + \text{KE}_{\text{Th}} = 4.20\text{ MeV} + 0.0718\text{ MeV} = \boxed{4.27\text{ MeV}}$$

38. (*a*) The decay constant can be found from the half-life, using Eq. 30–6.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.5\times10^{9}\ \text{yr}} = \boxed{1.5\times10^{-10}\ \text{yr}^{-1}} = 4.9\times10^{-18}\ \text{s}^{-1}$$

(*b*) The half-life can be found from the decay constant, using Eq. 30–6.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3.2\times10^{-5}\ \text{s}^{-1}} = 21{,}660\ \text{s} = \boxed{6.0\ \text{h}}$$

39. We find the half-life from Eq. 30–5 and Eq. 30–6.

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{\ln 2}{T_{1/2}}t} \quad\rightarrow\quad T_{1/2} = -\frac{\ln 2}{\ln\frac{R}{R_0}}t = -\frac{\ln 2}{\ln\frac{140}{1120}}(3.6\ \text{h}) = \boxed{1.2\ \text{h}}$$

40. We use Eq. 30–4 and Eq. 30–6 to find the fraction remaining.

$$N = N_0 e^{-\lambda t} \quad\rightarrow\quad \frac{N}{N_0} = e^{-\lambda t} = e^{-\frac{\ln 2}{T_{1/2}}t} = e^{-\left[\frac{(\ln 2)(2.5\ \text{yr})(12\ \text{mo/yr})}{9\ \text{mo}}\right]} = 0.0992 \approx \boxed{0.1} \approx 10\%$$

Only 1 significant figure was kept since the Problem said “about” 9 months.

41. The activity at a given time is given by Eq. 30–3b. We also use Eq. 30–6. The half-life is found in Appendix B.

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{(5730\ \text{yr})(3.16\times10^{7}\ \text{s/yr})}(6.5\times10^{20}\ \text{nuclei}) = \boxed{2.5\times10^{9}\ \text{decays/s}}$$

42. For every half-life, the sample is multiplied by one-half.

$$\frac{N}{N_0} = \left(\tfrac{1}{2}\right)^{n} = \left(\tfrac{1}{2}\right)^{5} = \boxed{0.03125 = 1/32 = 3.125\%}$$

43. We need the decay constant and the initial number of nuclei. The half-life is found in Appendix B. Use Eq. 30–6 to find the decay constant.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(8.0252\ \text{days})(24\ \text{h/day})(3600\ \text{s/h})} = 9.99668\times10^{-7}\ \text{s}^{-1}$$

$$N_0 = \left[\frac{(782\times10^{-6}\ \text{g})}{(130.906\ \text{g/mol})}\right](6.02\times10^{23}\ \text{atoms/mol}) = 3.5962\times10^{18}\ \text{nuclei}$$

(*a*) We use Eq. 30–3b and Eq. 30–4 to evaluate the initial activity.

$$\text{Activity} = \lambda N = \lambda N_0 e^{-\lambda t} = (9.99668\times10^{-7}\ \text{s}^{-1})(3.5962\times10^{18})e^{-0} = 3.5950\times10^{12}\ \text{decays/s}$$
$$\approx \boxed{3.60\times10^{12}\ \text{decays/s}}$$

(*b*) We evaluate Eq. 30–5 at $t = 1.5\text{ h}$.

$$R = R_0 e^{-\lambda t} = (3.5950\times10^{12}\text{ decays/s})e^{-(9.99668\times10^{-7}\text{ s}^{-1})(1.5\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)}$$
$$\approx \boxed{3.58\times10^{12}\text{ decays/s}}$$

(*c*) We evaluate Eq. 30–5 at t = 3.0 months. We use a time of 1/4 year for the 3.0 months.

$$R = R_0 e^{-\lambda t} = (3.5950\times10^{12}\text{ decays/s})e^{-(9.99668\times10^{-7}\text{ s}^{-1})(0.25\text{yr})(3.156\times10^{7}\text{ s/yr})}$$
$$= 1.3497\text{ decays/s} \approx \boxed{1.34\times10^{9}\text{ decays/s}}$$

44. We find the number of nuclei from the activity of the sample, using Eq. 30–3b and Eq. 30–6. The half-life is found in Appendix B.

$$\left|\frac{\Delta N}{\Delta t}\right| = \lambda N = \frac{\ln 2}{T_{1/2}}N \quad \rightarrow$$
$$N = \frac{T_{1/2}}{\ln 2}\left|\frac{\Delta N}{\Delta t}\right| = \frac{(4.468\times10^{9}\text{ yr})(3.156\times10^{7}\text{ s/yr})}{\ln 2}(420\text{ decays/s}) = \boxed{8.5\times10^{19}\text{ nuclei}}$$

45. Each α emission decreases the mass number by 4 and the atomic number by 2. The mass number changes from 235 to 207, a change of 28. Thus there must be $\boxed{7\ \alpha\text{ particles}}$ emitted. With the 7 α emissions, the atomic number would have changed from 92 to 78. Each β^- emission increases the atomic number by 1, so to have a final atomic number of 82, there must be $\boxed{4\ \beta^-\text{ particles}}$ emitted.

46. We use the decay constant often, so we calculate it here: $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{30.8\text{ s}} = 0.022505\text{ s}^{-1}$.

(*a*) We find the initial number of nuclei from an estimate of the atomic mass.

$$N_0 = \frac{(8.7\times10^{-6}\text{ g})}{(124\text{ g/mol})}(6.02\times10^{23}\text{ atoms/mol}) = 4.223\times10^{16} \approx \boxed{4.2\times10^{16}\text{ nuclei}}$$

(*b*) Evaluate Eq. 30–4 at $t = 2.6\text{ min}$.

$$N = N_0 e^{-\lambda t} = (4.223\times10^{16})e^{-(0.022505\text{ s}^{-1})(2.6\text{ min})(60\text{ s/min})} = 1.262\times10^{15} \approx \boxed{1.3\times10^{15}\text{ nuclei}}$$

(*c*) The activity is found from using the absolute value of Eq. 30–3b.

$$\lambda N = (0.022505\text{ s}^{-1})(1.262\times10^{15}) = 2.840\times10^{13}\text{ decays/s} \approx \boxed{2.8\times10^{13}\text{ decays/s}}$$

(*d*) We find the time from Eq. 30–4.

$$\lambda N = \lambda N_0 e^{-\lambda t} \quad \rightarrow$$
$$t = -\frac{\ln\left(\dfrac{\lambda N}{\lambda N_0}\right)}{\lambda} = -\frac{\ln\left[\dfrac{1\text{ decay/s}}{(0.022505\text{ s}^{-1})(4.223\times10^{16})\text{ decays/s}}\right]}{0.022505\text{ s}^{-1}} = 1532\text{ s} \approx \boxed{26\text{ min}}$$

47. Find the initial number of nuclei from the initial decay rate (activity) and then the mass from the number of nuclei.

$$\text{initial decay rate} = 2.0\times10^5\,\text{decays/s} = \lambda N_0 = \frac{\ln 2}{T_{1/2}}N_0 \quad\rightarrow$$

$$N_0 = \frac{T_{1/2}}{\ln 2}(2.4\times10^5\ \text{s}^{-1}) = \frac{(1.248\times10^9\ \text{yr})(3.156\times10^7\ \text{s/yr})}{\ln 2}(2.4\times10^5\ \text{s}^{-1}) = 1.364\times10^{22}\ \text{nuclei}$$

$$m = N_0\frac{(\text{atomic weight})\ \text{g/mol}}{6.02\times10^{23}\ \text{nuclei/mol}} = (1.364\times10^{22}\ \text{nuclei})\frac{(39.963998\ \text{g})}{(6.02\times10^{23})} = \boxed{0.91\ \text{g}}$$

48. The number of nuclei is found from the mass and the atomic weight. The activity is then found from the number of nuclei and the half-life.

$$N = \left[\frac{(6.7\times10^{-6}\ \text{g})}{(31.9739\ \text{g/mol})}\right](6.02\times10^{23}\ \text{atoms/mol}) = 1.261\times10^{17}\ \text{nuclei}$$

$$\lambda N = \frac{\ln 2}{T_{1/2}}N = \left[\frac{\ln 2}{(1.23\times10^6\ \text{s})}\right](1.261\times10^{17}) = \boxed{7.1\times10^{10}\ \text{decays/s}}$$

49. (*a*) The decay constant is found from Eq. 30–6.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(1.59\times10^5\ \text{yr})(3.156\times10^7\ \text{s/yr})} = 1.381\times10^{-13}\ \text{s}^{-1} \approx \boxed{1.38\times10^{-13}\ \text{s}^{-1}}$$

(*b*) The activity is the decay constant times the number of nuclei.

$$\lambda N = (1.381\times10^{-13}\ \text{s}^{-1})(4.50\times10^{18}) = 6.215\times10^5\ \text{decays/s}\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{3.73\times10^7\ \text{decays/min}}$$

50. We use Eq. 30–5.

$$R = \tfrac{1}{6}R_0 = R_0e^{-\lambda t} \quad\rightarrow\quad \tfrac{1}{6} = e^{-\lambda t} \quad\rightarrow\quad \ln\left(\tfrac{1}{6}\right) = -\lambda t = -\frac{\ln 2}{T_{1/2}}t \quad\rightarrow$$

$$T_{1/2} = -\frac{\ln 2}{\ln\left(\frac{1}{6}\right)}t = -\frac{\ln 2}{\ln\left(\frac{1}{6}\right)}(9.4\ \text{min}) = \boxed{3.6\ \text{min}}$$

51. Because the fraction of atoms that are ${}^{14}_{6}\text{C}$ is so small, we use the atomic weight of ${}^{12}_{6}\text{C}$ to find the number of carbon atoms in the sample. The activity is found from Eq. 30–3b.

$$N_{{}^{14}_{6}\text{C}} = \left(\frac{1.3}{10^{12}}\right)N_{{}^{12}_{6}\text{C}} = \left(\frac{1.3}{10^{12}}\right)\left[\frac{(345\ \text{g})}{(12\ \text{g/mol})}\right](6.02\times10^{23}\ \text{nuclei/mol}) = 2.250\times10^{13}\ \text{nuclei}$$

$$\lambda N = \frac{\ln 2}{T_{1/2}}N = \left[\frac{\ln 2}{(5730\ \text{yr})(3.156\times10^7\ \text{s/yr})}\right](2.250\times10^{13}) = \boxed{86\ \text{decays/s}}$$

52. We first find the number of nuclei from the activity and then find the mass from the number of nuclei and the atomic weight. The half-life is found in Appendix B.

$$\text{Activity} = \lambda N = \frac{\ln 2}{T_{1/2}} N = 4.20\times10^2 \text{ decays/s} \rightarrow$$

$$N = \frac{(4.468\times10^9 \text{ yr})(3.156\times10^7 \text{ s/yr})(4.20\times10^2 \text{ decays/s})}{\ln 2} = 8.544\times10^{19} \text{ nuclei}$$

$$m = \left[\frac{(8.544\times10^{19} \text{nuclei})}{(6.02\times10^{23} \text{ atoms/mol})}\right](238.05 \text{ g/mol}) = 3.38\times10^{-2} \text{ g} = \boxed{33.8 \text{ mg}}$$

53. We assume that the elapsed time is much smaller than the half-life, so we can approximate the decay rate as being constant. We also assume that the ${}^{87}_{38}\text{Sr}$ is stable, and there was none present when the rocks were formed. Thus every atom of ${}^{87}_{37}\text{Rb}$ that decayed is now an atom of ${}^{87}_{38}\text{Sr}$.

$$N_{\text{Sr}} = -\Delta N_{\text{Rb}} = \lambda N_{\text{Rb}} \, \Delta t \quad \rightarrow \quad \Delta t = \frac{N_{\text{Sr}}}{N_{\text{Rb}}} \frac{T_{1/2}}{\ln 2} = (0.0260)\frac{4.75\times10^{10} \text{ yr}}{\ln 2} = \boxed{1.78\times10^9 \text{ yr}}$$

This is $\approx 4\%$ of the half-life, so our original assumption is valid.

54. We are not including the neutrinos that will be emitted during beta decay.

First sequence: ${}^{232}_{90}\text{Th} \rightarrow \boxed{{}^{228}_{88}\text{Ra}} + {}^{4}_{2}\alpha$; ${}^{228}_{88}\text{Ra} \rightarrow \boxed{{}^{228}_{89}\text{Ac}} + \beta^-$; ${}^{228}_{89}\text{Ac} \rightarrow \boxed{{}^{228}_{90}\text{Th}} + \beta^-$;

${}^{228}_{90}\text{Th} \rightarrow \boxed{{}^{224}_{88}\text{Ra}} + {}^{4}_{2}\alpha$; ${}^{224}_{88}\text{Ra} \rightarrow \boxed{{}^{220}_{86}\text{Rn}} + {}^{4}_{2}\alpha$

Second sequence: ${}^{235}_{92}\text{U} \rightarrow \boxed{{}^{231}_{90}\text{Th}} + {}^{4}_{2}\alpha$; ${}^{231}_{90}\text{Th} \rightarrow \boxed{{}^{231}_{91}\text{Pa}} + \beta^-$; ${}^{231}_{91}\text{Pa} \rightarrow \boxed{{}^{227}_{89}\text{Ac}} + {}^{4}_{2}\alpha$;

${}^{227}_{89}\text{Ac} \rightarrow \boxed{{}^{227}_{90}\text{Th}} + \beta^-$; ${}^{227}_{90}\text{Th} \rightarrow \boxed{{}^{223}_{88}\text{Ra}} + {}^{4}_{2}\alpha$

$\boxed{55.}$ Because the fraction of atoms that are ${}^{14}_{6}\text{C}$ is so small, we use the atomic weight of ${}^{12}_{6}\text{C}$ to find the number of carbon atoms in 73 g. We then use the given ratio to find the number of ${}^{14}_{6}\text{C}$ atoms present when the club was made. Finally, we use the activity as given in Eq. 30–5 to find the age of the club.

$$N_{{}^{12}_{6}\text{C}} = \left[\frac{(73 \text{ g})}{(12 \text{ g/mol})}\right](6.02\times10^{23} \text{ atoms/mol}) = 3.662\times10^{24} \text{ atoms}$$

$$N_{{}^{14}_{6}\text{C}} = (1.3\times10^{-12})(3.662\times10^{24}) = 4.761\times10^{12} \text{ nuclei}$$

$$\left(\lambda N_{{}^{14}_{6}\text{C}}\right)_{\text{today}} = \left(\lambda N_{{}^{14}_{6}\text{C}}\right)_0 e^{-\lambda t} \quad \rightarrow$$

$$t = -\frac{1}{\lambda} \ln \frac{\left(\lambda N_{{}^{14}_{6}\text{C}}\right)_{\text{today}}}{\left(\lambda N_{{}^{14}_{6}\text{C}}\right)_0} = -\frac{T_{1/2}}{\ln 2} \ln \frac{\left(\lambda N_{{}^{14}_{6}\text{C}}\right)_{\text{today}}}{\left(\frac{\ln 2}{T_{1/2}} N_{{}^{14}_{6}\text{C}}\right)_0}$$

$$= -\frac{5730 \text{ yr}}{\ln 2} \ln \frac{(7.0 \text{ decays/s})}{\left[\frac{\ln 2}{(5730 \text{ yr})(3.156\times10^7 \text{ s/yr})}(4.761\times10^{12} \text{ nuclei})\right]_0} = 7921 \text{ yr} \approx \boxed{7900 \text{ yr}}$$

56. The decay rate is given by Eq. 30–3b, $\frac{\Delta N}{\Delta t}=-\lambda N.$ We assume equal numbers of nuclei decaying by α emission.

$$\frac{\left(\frac{\Delta N}{\Delta t}\right)_{218}}{\left(\frac{\Delta N}{\Delta t}\right)_{214}}=\frac{-\lambda_{218}N_{218}}{-\lambda_{214}N_{214}}=\frac{\lambda_{218}}{\lambda_{214}}=\frac{T_{1/2\,214}}{T_{1/2\,218}}=\frac{(1.6\times10^{-4}\text{ s})}{(3.1\text{ min})(60\text{ s/min})}=\boxed{8.6\times10^{-7}}$$

57. The activity is given by Eq. 30–5. The original activity is λN_0, so the activity 31.0 hours later is $0.945\lambda N_0$.

$$0.945\lambda N_0=\lambda N_0 e^{-\lambda t}\quad\rightarrow\quad \ln 0.945=-\lambda t=-\frac{\ln 2}{T_{1/2}}t\quad\rightarrow$$

$$T_{1/2}=-\frac{\ln 2}{\ln 0.945}(31.0\text{ h})=379.84\text{ h}\left(\frac{1\text{ d}}{24\text{ h}}\right)=\boxed{15.8\text{ d}}$$

58. The activity is given by Eq. 30–5.

(*a*) $R=R_0e^{-\lambda t}\quad\rightarrow\quad t=-\frac{1}{\lambda}\ln\frac{R}{R_0}=-\frac{T_{1/2}}{\ln 2}\ln\frac{R}{R_0}=-\frac{53\text{ d}}{\ln 2}\left(\ln\frac{25\text{ decays/s}}{350\text{ decays/s}}\right)=201.79\text{ d}\approx\boxed{2.0\times10^{2}\text{ d}}$

(*b*) We find the mass from the activity. Note that N_{A} is used to represent Avogadro's number, and A is the atomic weight.

$$R_0=\lambda N_0=\frac{\ln 2}{T_{1/2}}\frac{m_0N_{\text{A}}}{A}\quad\rightarrow$$

$$m=\frac{R_0T_{1/2}A}{N_{\text{A}}\ln 2}=\frac{(350\text{ decays/s})(53\text{ d})(86{,}400\text{ s/d})(7.017\text{ g/mole})}{(6.02\times10^{23}\text{ nuclei/mole})\ln 2}=\boxed{2.7\times10^{-14}\text{ g}}$$

59. The number of radioactive nuclei decreases exponentially, and every radioactive nucleus that decays becomes a daughter nucleus.

$$N=N_0e^{-\lambda t};\quad N_{\text{D}}=N_0-N=\boxed{N_0(1-e^{-\lambda t})}$$

60. The activity is given by Eq. 30–5, with $R=0.01050R_0$.

$$R=R_0e^{-\lambda t}\quad\rightarrow\quad\lambda=-\frac{\ln R/R_0}{t}=\frac{\ln 2}{T_{1/2}}\quad\rightarrow\quad T_{1/2}=-\frac{t\ln 2}{\ln R/R_0}=-\frac{(4.00\text{ h})\ln 2}{\ln 0.01050}=0.6085\text{ h}=36.5\text{ min}$$

From Appendix B we see that the isotope is $\boxed{{}^{211}_{82}\text{Pb}}$.

61. Because the carbon is being replenished in living trees, we assume that the amount of ${}^{14}_{6}\text{C}$ is constant until the wood is cut, and then it decays. Use Eq. 30–4 and Eq. 30–7 to find the age of the tool.

$$N=N_0e^{-\lambda t}=N_0e^{-\frac{0.693t}{T_{1/2}}}=0.045N_0\quad\rightarrow\quad t=-\frac{T_{1/2}\ln(0.045)}{0.693}=-\frac{(5730\text{ yr})\ln(0.045)}{0.693}=\boxed{2.6\times10^{4}\text{ yr}}$$

62. (*a*) The mass number is found from the radius, using Eq. 30–1.

$$r=(1.2\times10^{-15}\text{ m})A^{1/3} \quad\rightarrow\quad A=\left(\frac{r}{1.2\times10^{-15}\text{ m}}\right)^3=\left(\frac{5000\text{ m}}{1.2\times10^{-15}\text{ m}}\right)^3=7.23\times10^{55}\approx\boxed{7\times10^{55}}$$

(*b*) The mass of the neutron star is the mass number times the atomic mass unit conversion in kg.

$$m=A(1.66\times10^{-27}\text{ kg/u})=(7.23\times10^{55}\text{ u})(1.66\times10^{-27}\text{ kg/u})=1.20\times10^{29}\text{ kg}\approx\boxed{1\times10^{29}\text{ kg}}$$

Note that this is about 6% of the mass of the Sun.

(*c*) The acceleration of gravity on the surface of the neutron star is found from Eq. 5–5 applied to the neutron star.

$$g=\frac{Gm}{r^2}=\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.20\times10^{29}\text{ kg})}{(5000\text{ m})^2}=3.20\times10^{11}\text{ m/s}^2\approx\boxed{3\times10^{11}\text{ m/s}^2}$$

63. Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We use Eq. 30–4.

$$N=N_0e^{-\lambda t} \quad\rightarrow\quad \lambda=-\frac{\ln\frac{N}{N_0}}{t}=\frac{\ln 2}{T_{1/2}} \quad\rightarrow\quad t=-\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0}=-\frac{(12.3\text{ yr})\ln 0.10}{\ln 2}=\boxed{41\text{ yr}}$$

64. (*a*) We assume a mass of 70 kg of water and find the number of protons, given that there are 10 protons in a water molecule.

$$N_{\text{protons}}=\left[\frac{(70\times10^3\text{ g water})}{(18\text{ g water/mol water})}\right]\left(6.02\times10^{23}\,\frac{\text{molecules water}}{\text{mol water}}\right)\left(\frac{10\text{ protons}}{\text{water molecule}}\right)$$
$$=2.34\times10^{28}\text{ protons}$$

We assume that the time is much less than the half-life so that the rate of decay is constant.

$$\frac{\Delta N}{\Delta t}=\lambda N=\left(\frac{\ln 2}{T_{1/2}}\right)N \quad\rightarrow$$
$$\Delta t=\frac{\Delta N}{N}\left(\frac{T_{1/2}}{\ln 2}\right)=\frac{1\text{ proton}}{2.34\times10^{28}\text{ protons}}\left(\frac{10^{33}\text{ yr}}{\ln 2}\right)=6.165\times10^4\text{ yr}\approx\boxed{60,000\text{ yr}}$$

This is about 880 times a normal life expectancy.

(*b*) Instead of waiting 61,650 consecutive years for one person to experience a proton decay, we could interpret that number as there being 880 people, each living 70 years, to make that 61,650 years (since $880\times70\approx61,650$). We would then expect one person out of every 880 to experience a proton decay during their lifetime. Divide 7 billion by 880 to find out how many people on Earth would experience proton decay during their lifetime.

$$\frac{7\times10^9}{880}=7.95\times10^6\approx\boxed{8\text{ million people}}$$

65. We assume that all of the kinetic energy of the alpha particle becomes electrostatic potential energy at the distance of closest approach. Note that the distance found is the distance from the center of the alpha to the center of the gold nucleus.

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \quad \to \quad \mathrm{KE}_\alpha + 0 = 0 + k\frac{Q_\alpha Q_{\mathrm{Au}}}{r} \quad \to$$

$$r = k\frac{Q_\alpha Q_{\mathrm{Au}}}{\mathrm{KE}_\alpha} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(2)(79)(1.60\times10^{-19}\ \mathrm{C})^2}{(7.7\ \mathrm{MeV})(1.60\times10^{-13}\ \mathrm{J/MeV})} = 2.951\times10^{-14}\ \mathrm{m}$$

$$\approx \boxed{3.0\times10^{-14}\ \mathrm{m}}$$

We use Eq. 30–1 to compare to the size of the gold nucleus.

$$\frac{r_{\mathrm{approach}}}{r_{\mathrm{Au}}} = \frac{2.951\times10^{-14}\ \mathrm{m}}{197^{1/3}(1.2\times10^{-15}\ \mathrm{m})} = 4.2$$

So the distance of approach is about $\boxed{4.2\times}$ the radius of the gold nucleus.

66. Use Eq. 30–5 and Eq. 30–6.

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{t \ln 2}{T_{1/2}}} = 0.0200 R_0 \quad \to \quad \frac{t}{T_{1/2}} = -\frac{\ln(0.0200)}{\ln 2} = 5.64$$

It takes 5.64 half-lives for a sample to drop to 2.00% of its original activity.

67. The number of ${}^{40}_{19}\mathrm{K}$ nuclei can be calculated from the activity, using Eq. 30–3b and Eq. 30–5. The half-life is found in Appendix B. A subscript is used on the variable N to indicate the isotope.

$$R = \lambda N_{40} \quad \to \quad N_{40} = \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} = \frac{\left(42\frac{\mathrm{decays}}{\mathrm{s}}\right)(1.248\times10^9\ \mathrm{yr})\left(\frac{3.156\times10^7\ \mathrm{s}}{1\ \mathrm{yr}}\right)}{\ln 2} = 2.387\times10^{18}\ \mathrm{nuclei}$$

The mass of ${}^{40}_{19}\mathrm{K}$ in the milk is found from the atomic mass and the number of nuclei.

$$m_{40} = (2.387\times10^{18})(39.964\ \mathrm{u})(1.66\times10^{-27}\ \mathrm{kg/u}) = 1.583\times10^{-7}\ \mathrm{kg} \approx \boxed{0.16\ \mathrm{mg}}$$

From Appendix B, in a natural sample of potassium, 0.0117% is ${}^{40}_{19}\mathrm{K}$ and 93.2581% is ${}^{39}_{19}\mathrm{K}$. Find the number of ${}^{39}_{19}\mathrm{K}$ nuclei and then use the atomic mass to find the mass of ${}^{39}_{19}\mathrm{K}$ in the milk.

$$N_{40} = (0.0117\%)N_{\mathrm{total}}; \quad N_{39} = (93.2581\%)N_{\mathrm{total}} \quad \to$$

$$N_{39} = \left[\frac{(93.2581\%)}{(0.0117\%)}\right]N_{40} = \left[\frac{(93.2581\%)}{(0.0117\%)}\right](2.387\times10^{18}\ \mathrm{nuclei}) = 1.903\times10^{22}\ \mathrm{nuclei}$$

The mass of ${}^{39}_{19}\mathrm{K}$ is the number of nuclei times the atomic mass.

$$m_{39} = (1.903\times10^{22})(38.964\ \mathrm{u})(1.66\times10^{-27}\ \mathrm{kg/u}) = 1.231\times10^{-3}\ \mathrm{kg} \approx \boxed{1.2\ \mathrm{g}}$$

68. In the Periodic Table, Sr is in the same column as $\boxed{\text{Ca}}$. If Sr is ingested, then the body may treat it chemically as if it were Ca, which means it might be $\boxed{\text{stored by the body in bones}}$. Use Eq. 30–4 to find the time to reach a 1% level.

$$N = N_0 e^{-\lambda t} \quad \rightarrow \quad \lambda = -\frac{\ln \dfrac{N}{N_0}}{t} = \frac{\ln 2}{T_{1/2}} \quad \rightarrow \quad t = -\frac{T_{1/2}}{\ln 2} \ln \frac{N}{N_0} = -\frac{(29\text{ yr}) \ln 0.01}{\ln 2} = 192.67\text{ yr} \approx \boxed{200\text{ yr}}$$

Assume that both the Sr and its daughter undergo beta decay, since the Sr has too many neutrons.

$$\boxed{{}^{90}_{38}\text{Sr} \;\rightarrow\; {}^{90}_{39}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}; \qquad {}^{90}_{39}\text{Y} \;\rightarrow\; {}^{90}_{40}\text{Zr} + {}^{0}_{-1}\text{e} + \bar{\nu}}$$

69. The number of nuclei is found from Eq. 30–5 and Eq. 30–3b. The mass is then found from the number of nuclei and the atomic weight. The half-life is given.

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} N \quad \rightarrow \quad N = \frac{T_{1/2}}{\ln 2} R; \quad m = N(\text{atomic weight}) = \frac{T_{1/2}}{\ln 2} R\,(\text{atomic weight})$$

$$m = \frac{(87.37\text{ d})\left(\dfrac{24\text{ h}}{1\text{ d}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)}{\ln 2}(4.28\times10^{4}\text{ decays/s})(34.969\text{ u})(1.66\times10^{-27}\text{ kg/u})$$

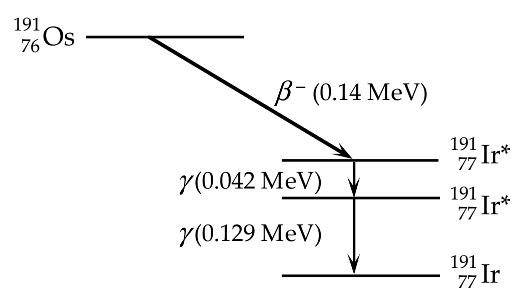
$$= 2.71\times10^{-14}\text{ kg} = \boxed{2.71\times10^{-11}\text{ g}}$$

70. (*a*) We find the daughter nucleus by balancing the mass and charge numbers.

$$Z(X) = Z(\text{Os}) - Z(\text{e}^-) = 76 - (-1) = 77$$

$$A(X) = A(\text{Os}) - A(\text{e}^-) = 191 - 0 = 191$$

The daughter nucleus is $\boxed{{}^{191}_{77}\text{Ir}}$.



(*b*) See the diagram.

(*c*) Because there is only one β energy, the β decay must be to the higher excited state.

71. (*a*) The number of nuclei is found from the mass of the sample and the atomic mass. The activity is found from the half-life and the number of nuclei, using Eq. 30–3b and Eq. 30–5.

$$N = \left(\frac{1.0\text{ g}}{130.91\text{ g/mol}}\right)(6.02\times10^{23}\text{ nuclei/mol}) = 4.599\times10^{21}\text{ nuclei}$$

$$R = \lambda N = \left[\frac{0.693}{(8.02\text{ d})(8.64\times10^{4}\text{ s/d})}\right](4.599\times10^{21}) = \boxed{4.6\times10^{15}\text{ decays/s}}$$

(*b*) Follow the same procedure as in part (*a*).

$$N = \left(\frac{1.0\text{ g}}{238.05\text{ g/mol}}\right)(6.02\times10^{23}\text{ nuclei/mol}) = 2.529\times10^{21}\text{ nuclei}$$

$$R = \lambda N = \left[\frac{0.693}{(4.47\times10^{9}\text{ yr})(3.16\times10^{7}\text{ s/yr})}\right](2.529\times10^{21}) = \boxed{1.2\times10^{4}\text{ decays/s}}$$

72. From Fig. 30–1, the average binding energy per nucleon at $A = 63$ is ~8.6 MeV. Use the average atomic weight as the average number of nucleons for the two stable isotopes of copper to find the total binding energy for one copper atom.

$$(63.5 \text{ nucleons})(8.6 \text{ MeV/nucleon}) = 546.1 \text{ MeV} \approx \boxed{550 \text{ MeV}}$$

Convert the mass of the penny to a number of atoms and then use the above value to calculate the energy needed.

$$\begin{aligned}\text{Energy} &= (\#\text{ atoms})(\text{energy/atom})\\ &= \left[\frac{(3.0\text{ g})}{(63.5\text{ g/mol})}\right](6.02\times10^{23}\text{ atoms/mol})(546.1\times10^{6}\text{ eV/atom})(1.60\times10^{-19}\text{ J/eV})\\ &= \boxed{2.5\times10^{12}\text{ J}}\end{aligned}$$

73. (*a*) $\Delta\left({}^{4}_{2}\text{He}\right) = m\left({}^{4}_{2}\text{He}\right) - A\left({}^{4}_{2}\text{He}\right) = 4.002603\text{ u} - 4 = \boxed{0.002603\text{ u}}$

$$= (0.002603\text{ u})(931.5\text{ MeV/u}c^2) = \boxed{2.425\text{ MeV}/c^2}$$

(*b*) $\Delta\left({}^{12}_{6}\text{C}\right) = m\left({}^{12}_{6}\text{C}\right) - A\left({}^{12}_{6}\text{C}\right) = 12.000000\text{ u} - 12 = \boxed{0}$

(*c*) $\Delta\left({}^{86}_{38}\text{Sr}\right) = m\left({}^{86}_{38}\text{Sr}\right) - A\left({}^{86}_{38}\text{Sr}\right) = 85.909261\text{ u} - 86 = \boxed{-0.090739\text{ u}}$

$$= (-0.090739\text{ u})(931.5\text{ MeV/u}c^2) = \boxed{-84.52\text{ MeV}/c^2}$$

(*d*) $\Delta\left({}^{235}_{92}\text{U}\right) = m\left({}^{235}_{92}\text{U}\right) - A\left({}^{235}_{92}\text{U}\right) = 235.043930\text{ u} - 235 = \boxed{0.043930\text{ u}}$

$$= (0.043930\text{ u})(931.5\text{ MeV/u}c^2) = \boxed{40.92\text{ MeV}/c^2}$$

(*e*) From Appendix B, we see the following:

$$\boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le Z \le 8 \text{ and } Z \ge 85;\\ \Delta < 0 \text{ for } 9 \le Z \le 84\end{array}} \qquad \boxed{\begin{array}{l}\Delta \ge 0 \text{ for } 0 \le A \le 15 \text{ and } A \ge 218;\\ \Delta < 0 \text{ for } 16 \le A < 218\end{array}}$$

74. The reaction is ${}^{1}_{1}\text{H} + {}^{1}_{0}\text{n} \rightarrow {}^{2}_{1}\text{H}$. If we assume that the initial kinetic energies are small, then the energy of the gamma is the Q-value of the reaction.

$$\begin{aligned}Q &= \left[m\left({}^{1}_{1}\text{H}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{2}_{1}\text{H}\right)\right]c^2\\ &= [(1.007825\text{ u}) + (1.008665\text{ u}) - (2.014102\text{ u})]c^2(931.5\text{ MeV/u}c^2) = \boxed{2.224\text{ MeV}}\end{aligned}$$

75. The mass of carbon 60,000 years ago was 1.0 kg. Find the number of carbon atoms at that time and then find the number of ${}^{14}_{6}\text{C}$ atoms at that time. Use that with the half-life to find the present activity, using Eq. 30–5 and Eq. 30–6.

$$N=\frac{(1.0\times10^3\text{ g})(6.02\times10^{23}\text{ atoms/mol})}{12\text{ g/mol}}=5.017\times10^{25}\text{ C atoms}$$

$$N_{14}=(5.017\times10^{25})(1.3\times10^{-12})=6.522\times10^{13}\text{ nuclei of }{}^{14}_{6}\text{C}=N_0$$

$$R=\lambda N=\frac{\ln 2}{T_{1/2}}N=\frac{\ln 2}{T_{1/2}}N_0e^{-\frac{\ln 2}{T_{1/2}}t}=\frac{\ln 2}{(5730\text{ yr})(3.16\times10^7\text{ s/yr})}(6.522\times10^{13})e^{-\frac{\ln 2(60{,}000\text{ yr})}{(5730\text{ yr})}}$$

$$=0.1759\text{ decays/s}\approx\boxed{0.2\text{ decays/s}}$$

76. The energy to remove the neutron would be the difference in the masses of the ${}^{4}_{2}\text{He}$ and the combination of ${}^{3}_{2}\text{He}+\text{n}$. It is also the opposite of the Q-value for the reaction. The number of electrons doesn't change, so atomic masses can be used for the helium isotopes.

$$-Q_{\text{He}}=(m_{\text{He-3}}+m_{\text{n}}-m_{\text{He-4}})c^2=(3.016029\text{ u}+1.008665\text{ u}-4.002603\text{ u})c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$

$$=\boxed{20.58\text{ MeV}}$$

Repeat the calculation for the carbon isotopes.

$$-Q_{\text{C}}=(m_{\text{C-12}}+m_{\text{n}}-m_{\text{C-13}})c^2=(12.000000\text{ u}+1.008665\text{ u}-13.003355\text{ u})c^2\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)$$

$$=4.946\text{ MeV}$$

The helium value is $\boxed{4.16\times\text{ greater}}$ than the carbon value.

77. (*a*) Take the mass of the Earth and divide it by the mass of a nucleon to find the number of nucleons. Then use Eq. 30–1 to find the radius.

$$r=(1.2\times10^{-15}\text{ m})A^{1/3}=(1.2\times10^{-15}\text{ m})\left[\frac{5.98\times10^{24}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right]^{1/3}=183.6\text{ m}\approx\boxed{180\text{ m}}$$

(*b*) Follow the same process as above, but this time use the Sun's mass.

$$r=(1.2\times10^{-15}\text{ m})A^{1/3}=(1.2\times10^{-15}\text{ m})\left[\frac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right]^{1/3}=12{,}700\text{ m}\approx\boxed{1.3\times10^4\text{ m}}$$

78. (*a*) From Section 30–11, the usual fraction of ${}^{14}_{6}\mathrm{C}$ is 1.3×10^{-12}. Because the fraction of atoms that are ${}^{14}_{6}\mathrm{C}$ is so small, use the atomic weight of ${}^{12}_{6}\mathrm{C}$ to find the number of carbon atoms in 72 g. Use Eq. 30–4 to find the time.

$$N_{12} = \left(\frac{72\text{ g}}{12\text{ g/mol}}\right)(6.02\times10^{23}\text{ atoms/mol}) = 3.612\times10^{24}\text{ atoms}$$

$$N_{14} = (3.612\times10^{24}\text{ atoms})(1.3\times10^{-12}) = 4.6956\times10^{12}\text{ atoms}$$

$$N = N_0 e^{-\lambda t} \to t = -\frac{1}{\lambda}\ln\frac{N}{N_0} = -\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0} = -\frac{(5730\text{ yr})}{\ln 2}\ln\frac{1}{4.6956\times10^{12}} = \boxed{2.4\times10^5\text{ yr}}$$

(*b*) Do a similar calculation for an initial mass of 340 g.

$$N_{14} = \left(\frac{340\text{ g}}{12\text{ g/mol}}\right)(6.02\times10^{23}\text{ atoms/mol})(1.3\times10^{-12}) = 2.217\times10^{13}\text{ atoms}$$

$$N = N_0 e^{-\lambda t} \to t = -\frac{1}{\lambda}\ln\frac{N}{N_0} = -\frac{T_{1/2}}{\ln 2}\ln\frac{N}{N_0} = -\frac{(5730\text{ yr})}{\ln 2}\ln\frac{1}{2.217\times10^{13}} = \boxed{2.5\times10^5\text{ yr}}$$

For times on the order of 10^5 yr, the sample amount has fairly little effect on the age determined. Thus, times of this magnitude are not accurately measured by carbon dating.

79. (*a*) This reaction would turn the protons and electrons in atoms into neutrons. This would eliminate chemical reactions and thus eliminate life as we know it.

(*b*) We assume that there is no kinetic energy brought into the reaction and solve for the increase of mass necessary to make the reaction energetically possible. For calculating energies, we write the reaction as ${}^{1}_{1}\mathrm{H} \to {}^{1}_{0}\mathrm{n} + \nu$, and we assume that the neutrino has no mass or kinetic energy.

$$Q = \left[m\left({}^{1}_{1}\mathrm{H}\right) - m\left({}^{1}_{0}\mathrm{n}\right)\right]c^2 = [(1.007825\text{ u}) - (1.008665\text{ u})]c^2(931.5\text{ MeV/u}c^2)$$
$$= -0.782\text{ MeV}$$

This is the amount that the proton would have to increase in order to make this energetically possible. We find the percentage change.

$$\left(\frac{\Delta m}{m}\right)(100) = \left[\frac{(0.782\text{ MeV}/c^2)}{(938.27\text{ MeV}/c^2)}\right](100) = \boxed{0.083\%}$$

80. We assume that the particles are not relativistic, so that $p = \sqrt{2m\text{KE}}$. The radius is given in Example 20–6 as $r = \dfrac{mv}{qB}$. Set the radii of the two particles equal. Note that the charge of the alpha particle is twice that of the electron (in absolute value). We also use the “bare” alpha particle mass, subtracting the two electrons from the helium atomic mass.

$$\frac{m_\alpha v_\alpha}{2eB} = \frac{m_\beta v_\beta}{eB} \quad\to\quad m_\alpha v_\alpha = 2m_\beta v_\beta \quad\to\quad p_\alpha = 2p_\beta$$

$$\frac{\mathrm{KE}_\alpha}{\mathrm{KE}_\beta}=\frac{\dfrac{p_\alpha^2}{2m_\alpha}}{\dfrac{p_\beta^2}{2m_\beta}}=\frac{\dfrac{4p_\beta^2}{2m_\alpha}}{\dfrac{p_\beta^2}{2m_\beta}}=\frac{4m_\beta}{m_\alpha}=\frac{4(0.000549\text{ u})}{4.002603\text{ u}-2(0.000549\text{ u})}=\boxed{5.48\times10^{-4}}$$

81. Natural samarium has an atomic mass of 150.36 grams per mole. We find the number of nuclei in the natural sample and then take 15% of that to find the number of ${}^{147}_{62}\mathrm{Sm}$ nuclei. We first find the number of ${}^{147}\mathrm{Sm}$ nuclei from the mass and proportion information.

$$N_{{}^{147}_{62}\mathrm{Sm}}=(0.15)N_{\text{natural}}=\frac{(0.15)(1.00\text{ g})(6.02\times10^{23}\text{ nuclei/mol})}{150.36\text{ g/mol}}=6.006\times10^{20}\text{ nuclei of }{}^{147}_{62}\mathrm{Sm}$$

The activity level is used to calculate the half-life.

$$\text{Activity}=R=\lambda N=\frac{\ln 2}{T_{1/2}}N\quad\rightarrow$$

$$T_{1/2}=\frac{\ln 2}{R}N=\frac{\ln 2}{120\text{ decays/s}}(6.006\times10^{20})=3.469\times10^{18}\text{ s}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right)=\boxed{1.1\times10^{11}\text{ yr}}$$

82. Since amounts are not specified, assume that "today" there is 0.720 g of ${}^{235}_{92}\mathrm{U}$ in our sample, and $100.000-0.720=99.280\text{ g}$ of ${}^{238}_{92}\mathrm{U}$. Use Eq. 30–4.

(*a*) Relate the amounts today to the amounts 1.0×10^{9} years ago.

$$N=N_0e^{-\lambda t}\rightarrow N_0=Ne^{\lambda t}=Ne^{\frac{t}{T_{1/2}}\ln 2}$$

$$(N_0)_{235}=(N_{235})e^{\frac{t}{T_{1/2}}\ln 2}=(0720\text{ g})e^{\frac{(1.0\times10^{9}\text{ yr})}{(7.04\times10^{8}\text{ yr})}\ln 2}=1.927\text{ g}$$

$$(N_0)_{238}=(N_{238})e^{\frac{t}{T_{1/2}}\ln 2}=(99.280\text{ g})e^{\frac{(1.0\times10^{9})}{(4.468\times10^{9})}\ln 2}=115.94\text{ g}$$

The percentage of ${}^{235}_{92}\mathrm{U}$ was $\dfrac{1.927}{1.927+115.94}\times100\%=\boxed{1.63\%}$.

(*b*) Relate the amounts today to the amounts 100×10^{6} years from now.

$$N=N_0e^{-\lambda t}\rightarrow(N_{235})=(N_0)_{235}e^{-\frac{t}{T_{1/2}}\ln 2}=(0.720\text{ g})e^{-\frac{(100\times10^{6}\text{ yr})}{(7.04\times10^{8}\text{ yr})}\ln 2}=0.6525\text{ g}$$

$$(N_{238})=(N_0)_{238}e^{-\frac{t}{T_{1/2}}\ln 2}=(99.280\text{ g})e^{-\frac{(100\times10^{6}\text{ yr})}{(4.468\times10^{9}\text{ yr})}\ln 2}=97.752\text{ g}$$

The percentage of ${}^{235}_{92}\mathrm{U}$ will be $\dfrac{0.6525}{0.6525+97.752}\times100\%=\boxed{0.663\%}$.

83. We determine the number of ${}^{40}_{19}\text{K}$ nuclei in the sample and then use the half-life to determine the activity.

$$N_{{}^{40}_{19}\text{K}} = (0.000117)N_{\substack{\text{naturally}\\ \text{occurring K}}} = (0.000117)\frac{(420\times10^{-3}\ \text{g})(6.02\times10^{23}\ \text{atoms/mol})}{39.0983\ \text{g/mol}} = 7.566\times10^{17}$$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}}N = \frac{\ln 2}{1.265\times10^{9}\ \text{yr}}(7.566\times10^{17})\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right) = 13.14\ \text{decays/s} \approx \boxed{13\ \text{decays/s}}$$

84. The kinetic energy of the electron will be at its maximum if the (essentially) massless neutrino has no kinetic energy. We also ignore the recoil energy of the sodium. The maximum kinetic energy of the reaction is then the Q-value of the reaction. Note that the emitted electron mass is accounted for by using atomic masses.

$$\text{KE} = Q = \left[m\left({}^{23}_{10}\text{Ne}\right) - m\left({}^{23}_{11}\text{Na}\right)\right]c^2 = [(22.9947\ \text{u}) - (22.9898\ \text{u})]c^2(931.5\ \text{MeV/u}c^2)$$
$$= \boxed{4.6\ \text{MeV}}$$

If the neutrino were to have all of the kinetic energy, then the minimum kinetic energy of the electron is $\boxed{0}$. The sum of the kinetic energy of the electron and the energy of the neutrino must be the Q-value, so the neutrino energies are $\boxed{0}$ and $\boxed{4.6\ \text{MeV}}$, respectively.

85. (*a*) If the initial nucleus is at rest when it decays, then momentum conservation says that the magnitude of the momentum of the alpha particle will be equal to the magnitude of the momentum of the daughter particle. We use that to calculate the (nonrelativistic) kinetic energy of the daughter particle. The mass of each particle is essentially equal to its atomic mass number, in atomic mass units. Note that classically, $p = \sqrt{2m\text{KE}}$.

$$p_\alpha = p_\text{D};\quad \text{KE}_\text{D} = \frac{p_\text{D}^2}{2m_\text{D}} = \frac{p_\alpha^2}{2m_\text{D}} = \frac{2m_\alpha \text{KE}_\alpha}{2m_\text{D}} = \frac{m_\alpha}{m_\text{D}}\text{KE}_\alpha = \frac{A_\alpha}{A_\text{D}}\text{KE}_\alpha = \frac{4}{A_\text{D}}\text{KE}_\alpha$$

$$\frac{\text{KE}_\text{D}}{\text{KE}_\alpha + \text{KE}_\text{D}} = \frac{\frac{4}{A_\text{D}}\text{KE}_\alpha}{\left[\text{KE}_\alpha + \frac{4}{A_\text{D}}\text{KE}_\alpha\right]} = \frac{\text{KE}_\alpha}{\left[\frac{A_\text{D}}{4}\text{KE}_\alpha + \text{KE}_\alpha\right]} = \boxed{\frac{1}{1+\frac{1}{4}A_\text{D}}}$$

(*b*) We specifically consider the decay of ${}^{226}_{88}\text{Ra}$. The daughter has $A_\text{D} = 222$.

$$\frac{\text{KE}_\text{D}}{\text{KE}_\alpha + \text{KE}_\text{D}} = \frac{1}{1+\frac{1}{4}A_\text{D}} = \frac{1}{1+\frac{1}{4}(222)} = 0.017699 \approx 1.8\%$$

Thus, the alpha particle carries away $1 - 0.0177 = 0.9823 = \boxed{98.2\%}$.

86. The mass number changes only with α decay, and it changes by -4. If the mass number is $4n$, then the new number is $4n - 4 = 4(n-1) = 4n'$. There is a similar result for each family, as shown here.

$$4n \quad \rightarrow \quad 4n - 4 = 4(n-1) = 4n'$$

$$4n+1 \quad \rightarrow \quad 4n - 4 + 1 = 4(n-1) + 1 = 4n' + 1$$

$$4n+2 \quad \rightarrow \quad 4n-4+2 = 4(n-1)+2 = 4n'+2$$

$$4n+3 \quad \rightarrow \quad 4n-4+3 = 4(n-1)+3 = 4n'+3$$

Thus, the daughter nuclides are always in the same family.

Solutions to Search and Learn Problems

1. The nucleus of an atom consists of protons (which carry a positive electric charge) and neutrons (which are electrically neutral). The electric force between protons is repulsive and much larger than the force of gravity. If the electric and gravitational forces are the only two forces present in the nucleus, then the nucleus would be unstable, as the electric force would push the protons away from each other. Nuclei are stable; therefore, another force must be present in the nucleus to overcome the electric force. This force is the strong nuclear force.

2. (*a*) An einsteinium nucleus has 99 protons and a fermium nucleus has 100 protons. If the fermium undergoes either electron capture or β^+ decay, then a proton would in effect be converted into a neutron. The nucleus would now have 99 protons and be an einsteinium nucleus.

   (*b*) If the einsteinium undergoes β^- decay, then a neutron would be converted into a proton. The nucleus would now have 100 protons and be a fermium nucleus.

3. Take the momentum of the nucleon to be equal to the uncertainty in the momentum of the nucleon, as given by the uncertainty principle. The uncertainty in position is estimated as the radius of the nucleus. With that momentum, calculate the kinetic energy, using the classical formula. Iron has about 56 nucleons (depending on the isotope).

$$\Delta p \Delta x \approx \hbar \quad \rightarrow \quad p \approx \Delta p \approx \frac{\hbar}{\Delta x} = \frac{\hbar}{r}$$

$$\text{KE} = \frac{p^2}{2m} = \frac{\hbar^2}{2mr^2} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.67\times10^{-27}\ \text{kg})\left[(56^{1/3})(1.2\times10^{-15}\ \text{m})\right]^2(1.60\times10^{-13}\ \text{J/MeV})}$$

$$= 0.988\ \text{MeV} \approx \boxed{1\ \text{MeV}}$$

4. (*a*) From Fig. 30–17, the initial force on the detected particle is down. Using the right-hand rule, the force on a positive particle would be upward. Thus the particle must be negative, and the decay is $\boxed{\beta^-\ \text{decay}}$.

   (*b*) The magnetic force is producing circular motion. Set the expression for the magnetic force equal to the expression for centripetal force and solve for the velocity.

$$qvB = \frac{mv^2}{r} \quad \rightarrow \quad v = \frac{qBr}{m} = \frac{(1.60\times10^{-19}\ \text{C})(0.012\ \text{T})(4.7\times10^{-3}\ \text{m})}{9.11\times10^{-31}\ \text{kg}} = \boxed{9.9\times10^6\ \text{m/s}}$$

5. In β decay, an electron is ejected from the nucleus of the atom, and a neutron is converted into a proton. The atomic number of the nucleus increases by one, and the element now has different chemical properties. In internal conversion, an orbital electron is ejected from the atom. This does not change either the atomic number of the nucleus or its chemical properties.

Also, in β decay, both a neutrino and an electron will be emitted from the nucleus. Because there are three decay products (the neutrino, the β particle, and the nucleus), the momentum of the β particle can have a range of values. In internal conversion, since there are only two decay products (the electron and the nucleus), the electron will have a unique momentum and, therefore, a unique energy.

6. If the Earth had been bombarded with additional radiation several thousand years ago, then there would have been a larger abundance of carbon-14 in the atmosphere at that time. Organisms that died in that time period would have had a greater percentage of carbon-14 in them than organisms that die today. Since we assumed the amount of starting carbon-14 was the same, we would have previously underestimated the age of the organisms. With the new discovery, we would re-evaluate the organisms as older than previously calculated.

7. (*a*) The decay constant is calculated from the half-life using Eq. 30–6.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(5730\text{ yr})(3.156\times10^{7}\text{ s/yr})} = \boxed{3.83\times10^{-12}\text{ s}^{-1}}$$

(*b*) In a living organism, the abundance of ${}^{14}_{6}\text{C}$ atoms is 1.3×10^{-12} per carbon atom. Multiply this abundance by Avogadro's number and divide by the molar mass of carbon to find the number of carbon-14 atoms per gram of carbon.

$$N = \left(1.3\times10^{-12}\,\frac{{}^{14}_{6}\text{C atoms}}{{}^{12}_{6}\text{C atoms}}\right)\frac{N_{\text{A}}}{M} = \left(1.3\times10^{-12}\,\frac{{}^{14}_{6}\text{C atoms}}{{}^{12}_{6}\text{C atoms}}\right)\frac{(6.02\times10^{23}\text{ atoms }{}^{12}_{6}\text{C/mol})}{12.0107\text{ g }{}^{12}_{6}\text{C/mol}}$$

$$= 6.516\text{ atoms }{}^{14}_{6}\text{C/g }{}^{12}_{6}\text{C} \approx \boxed{6.5\times10^{10}\text{ atoms }{}^{14}_{6}\text{C/ g }{}^{12}_{6}\text{C}}$$

(*c*) The activity in natural carbon for a living organism is the product of the decay constant and the number of ${}^{14}_{6}\text{C}$ atoms per gram of ${}^{12}_{6}\text{C}$. Use Eq. 30–5 and Eq. 30–3b.

$$R = \lambda N = (3.83\times10^{-12}\text{ s}^{-1})(6.5\times10^{10}\text{ atoms }{}^{14}_{6}\text{C/g }{}^{12}_{6}\text{C}) = 0.249\text{ decays/s/g }{}^{12}_{6}\text{C}$$

$$\approx \boxed{0.25\text{ decays/s/g }{}^{12}_{6}\text{C}}$$

(*d*) Take the above result as the initial decay rate (while Otzi was alive) and use Eq. 30–5 to find the time elapsed since he died.

$$R = R_0 e^{-\lambda t} \quad \rightarrow$$

$$t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -\frac{1}{3.83\times10^{-12}\text{ s}^{-1}}\ln\left(\frac{0.121}{0.249}\right) = 1.884\times10^{11}\text{ s}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right)$$

$$= 5970\text{ yr} \approx \boxed{6000\text{ yr}}$$

Otzi lived approximately 6000 years ago.

8. Use Eq. 30–5 and 30–3b to relate the activity to the half-life, along with the atomic weight.

$$\left|\frac{\Delta N}{\Delta t}\right| = R = \lambda N = \frac{\ln 2}{T_{1/2}}N \quad \rightarrow$$

$$T_{1/2} = \frac{\ln 2}{R}N = \frac{\ln 2}{1\text{ decay/s}}(1.5\times10^{7}\text{ g})\left(\frac{6.02\times10^{23}\text{ nuclei}}{152\text{ g}}\right)\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = \boxed{1.3\times10^{21}\text{ yr}}$$

31

NUCLEAR ENERGY; EFFECTS AND USES OF RADIATION

Responses to Questions

1. (*a*) $n + {}^{137}_{56}\text{Ba} \rightarrow ? + \gamma$

   Conserve nucleon number: $1+137 = A+0$. Thus, $A = 138$. Conserve charge: $0+56 = Z+0$. Thus, $Z = 56$. This is ${}^{138}_{56}\text{Ba}$, or barium-138.

   (b) $n + {}^{137}_{56}\text{Ba} \rightarrow {}^{137}_{55}\text{Cs} + ?$

   Conserve nucleon number: $1+137 = 137+A$. Thus, $A = 1$. Conserve charge: $0+56 = 55+Z$. Thus, $Z = 1$. This is ${}^{1}_{1}\text{H}$, or p (a proton).

   (*c*) $d + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + ?$

   Conserve nucleon number: $2+2 = 4+A$. Thus, $A = 0$. Conserve charge: $1+1 = 2+Z$. Thus, $Z = 0$. This is γ, or a gamma ray (a photon).

   (d) $\alpha + {}^{197}_{79}\text{Au} \rightarrow ? + d$

   Conserve nucleon number: $4+197 = A+2$. Thus, $A = 199$. Conserve charge: $2+79 = Z+1$. Thus, $Z = 80$. This is ${}^{199}_{80}\text{Hg}$, or mercury-199.

2. The reaction is ${}^{22}_{11}\text{Na} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + ?$. The reactants have 12 protons and 24 nucleons, so they must have 12 neutrons. Thus the products must have 12 protons and 12 neutrons as well. Since the alpha has 2 protons and 2 neutrons, the other product must have 10 protons and 10 neutrons. That nucleus is ${}^{20}_{10}\text{Ne}$. Thus, the full equation is $\boxed{{}^{22}_{11}\text{Na} + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{20}_{10}\text{Ne}}$.

3. Neutrons are good projectiles for producing nuclear reactions because they are neutral and they are massive. If you want a particle to hit the nucleus with a lot of energy, then a more massive particle is the better choice. A light electron would not be as effective. Using a positively charged projectile like an alpha or a proton means that the projectile will have to overcome the large electrical repulsion from the positively charged nucleus. Neutrons can penetrate directly to the nucleus and cause nuclear reactions.

4. For spontaneous radioactive decay reactions, $Q > 0$, answer (*b*). Otherwise, more energy must be added to make the reaction proceed.

5. The thermal energy from nuclear fission appears in the kinetic energy of the fission products (daughter nuclei and neutrons). In other words, the fission products are moving very fast (especially the neutrons, due to the conservation of momentum).

6. (*a*) Yes, since the multiplication factor is greater than 1 ($f = 1.5$), a chain reaction can be sustained. (*b*) The difference would be that the chain reaction would proceed more slowly, and to make sure the chain reaction continued, you would need to be very careful about leakage of neutrons to the surroundings.

7. Uranium can't be enriched by chemical means because chemical reactions occur similarly with all of the isotopes of a given element. The number of neutrons does not influence the chemistry, which is primarily due to the valence electrons. Thus, trying to enrich uranium by chemical means, which means trying to increase the percentage of ${}^{235}_{92}\mathrm{U}$ in the sample compared with ${}^{238}_{92}\mathrm{U}$, is impossible.

8. First of all, the neutron can get close to the nucleus at such slow speeds due to the fact that it is neutral and will not be electrically repelled by either the electron cloud or the protons in the nucleus. Then, once it hits a nucleus, it is held due to the strong nuclear force. It adds more energy than just kinetic energy to the nucleus due to $E = mc^2$. This extra amount of energy leaves the nucleus in an excited state. To decay back to a lower energy state, the nucleus may undergo fission.

9. For a nuclear chain reaction to occur in a block of porous uranium, the neutrons being emitted by the decays must be slowed down. If the neutrons are too fast, then they will pass through the block of uranium without interacting, effectively prohibiting a chain reaction. Water contains a much higher density of protons and neutrons than does air, and those protons and neutrons will slow down (moderate) the neutrons, enabling them to take part in nuclear reactions. Thus, if water is filling all of the porous cavities, then the water will slow the neutrons, allowing them to be captured by other uranium nuclei, and allow the chain reaction to continue, which might lead to an explosion.

10. Ordinary water does not moderate, or slow down, neutrons as well as heavy water; more neutrons will also be lost to absorption in ordinary water. However, if the uranium in a reactor is highly enriched, then there will be many fissionable nuclei available in the fuel rods. It will be likely that the few moderated neutrons will be absorbed by a fissionable nucleus, and it will be possible for a chain reaction to occur.

11. A useful fission reaction is one that is self-sustaining. The neutrons released from an initial fission process can go on to initiate further fission reactions, creating a self-sustaining reaction. If no neutrons were released, then the process would end after a single reaction and not be very useful.

12. Heavy nuclei decay because they are neutron-rich, especially after neutron capture. After fission, the smaller daughter nuclei will still be neutron-rich and relatively unstable and will emit neutrons in order to move to a more stable configuration. Lighter nuclei are generally more stable with approximately equal numbers of protons and neutrons; heavier nuclei need additional neutrons in order to decrease Coulomb repulsion between the protons.

13. The water in the primary system flows through the core of the reactor and therefore could contain radioactive materials, including deuterium, tritium, and radioactive oxygen isotopes. The use of a secondary system provides for isolation of these potentially hazardous materials from the external environment. Also, it is desirable to keep the radioactive materials as confined as possible to avoid

accidental leakage and contamination. In the electric generating portion of the system, waste heat must be given off to the surroundings, and keeping all of the radioactive portions of the energy generation process as far away from this "release point" as possible is a major safety concern.

14. Fission is the process in which a larger nucleus splits into two or more fragments, roughly equal in size, releasing energy. Fusion is the process in which smaller nuclei combine to form larger nuclei, also releasing energy.

15. Fossil fuel power plants are less expensive to construct, and the technology is well known. However, the mining of coal is dangerous and can be environmentally destructive, the transportation of oil can be damaging to the environment through spills, the production of power from both coal and oil contributes to air pollution and the release of greenhouse gases into the environment, and there is a limited supply of both coal and oil. Fission power plants produce no greenhouse gases and virtually no air pollution, and the technology is well known. However, they are expensive to build, they produce thermal pollution and radioactive waste, and when accidents occur they tend to be very destructive. Uranium is also dangerous to mine. Fusion power plants produce very little radioactive waste and virtually no air pollution or greenhouse gases. Unfortunately, the technology for large-scale sustainable power production is not yet known, and the pilot plants are very expensive to build.

$\boxed{16.}$ Gamma particles penetrate better than beta particles because they are neutral and have no mass. Thus, gamma particles do not interact with matter as easily or as often as beta particles, allowing them to better penetrate matter.

$\boxed{17.}$ The large amount of mass of a star creates an enormous gravitational attraction, which causes the gas to be compressed to a very high density. This high density creates very high pressure and high temperature. The high temperatures give the gas particles a large amount of kinetic energy, which allows them to overcome the Coulomb repulsion and then fuse when they collide. These conditions at the center of the Sun and other stars make the fusion process possible.

18. Stars, which include our Sun, maintain confinement of the fusion plasma with gravity. The huge amount of mass in a star creates an enormous gravitational attraction on the gas molecules, and this attractive force overcomes the outward repulsive forces from electrostatics and radiation pressure.

19. Alpha particles are relatively large and are generally emitted with relatively low kinetic energies. They are not able to penetrate the skin, so they are not very destructive or dangerous as long as they stay outside the body. If alpha emitters are ingested or inhaled, however, then the protective layer of skin is bypassed, and the alpha particles, which are charged, can do tremendous amounts of damage to the lungs and other delicate internal tissues due to ionizing effects. Thus, there are strong rules against eating and drinking around alpha emitters, and the machining of such materials, which produces fine dust particles that could be inhaled, can be done only in sealed conditions.

20. The absorbed dose measures the amount of energy deposited per unit mass of absorbing material and is measured in grays (Gy) or rads. The gray is the SI unit and is 1 J/kg. A rad is 0.01 Gy. The effective dose takes into account the type of radiation depositing the energy and is used to determine the biological damage done by the radiation. The effective dose is the absorbed dose multiplied by a relative biological effectiveness (RBE) factor. The effective dose is measured in rem, which are rad × RBE, or sieverts (Sv), which are gray × RBE. Sieverts is the SI unit, and 1 Sv = 100 rem. 1 Sv of any type of radiation does approximately the same amount of biological damage.

$\boxed{21.}$ Radiation can kill or deactivate bacteria and viruses on medical supplies and even in food. Thus, the radiation will sterilize these items, making them safer for humans to use.

22. Allow a radioactive tracer to be introduced into the liquid that flows through the pipe. Choose a tracer that emits particles that cannot penetrate the walls of the pipe. Then check the pipe with a Geiger counter. When tracer radiation is found on the outside of the pipe (radiation levels will be higher at that point), the leak will have been located.

Responses to MisConceptual Questions

1. (*e*) In nuclear reactions, energy is converted between mass, kinetic energy, and radiant energy, but the total energy is conserved. The momentum is conserved as particles are absorbed and released in the reactions. The net electric charge is conserved. That is, the total net charge before the reaction will always equal the net charge after the reaction. The new conserved quantity in this chapter is the conservation of nucleon number. In any reaction, the number of nucleons will remain unchanged.

2. (*b*) Large nuclei have more neutrons per proton than smaller nuclei. In a fission reaction, the daughter nuclei are neutron-rich and emit β^- particles to convert excess neutrons into protons.

3. (*c*) The melting point, boiling point, and valence shells do not affect the critical mass for a chain reaction. The nuclear density is relatively constant for all substances. The critical mass is determined by the mass necessary for each reaction to produce, on average, one neutron that creates an additional reaction. If each fission can produce additional neutrons, then it is more likely that one of those neutrons will create an additional reaction and the critical mass can be smaller.

4. (*b*) Fusion occurs in small nuclei because the binding energy per nucleon of larger nuclei is greater than the binding energies of the small nuclei. Fission occurs in very large nuclei because the binding energy per nucleon of the smaller nuclei is greater than the binding energy per nucleon of the very large nuclei. If the binding energy per nucleon always increased, then fusion could occur through all elements, but fission could not occur.

5. (*a*) Nuclear reactions occur when uranium-235 absorbs a neutron. Thermal neutrons are more likely to be absorbed by uranium-235, so the moderator is used to slow down the neutrons.

6. (*e*) Fission and fusion are sometimes thought of as different names for the same physical process, but this is incorrect. Fission occurs when energy is released as large nuclei are broken apart. Fusion occurs when energy is released as small nuclei are put together.

7. (*c*) The primary fuel source for fusion is hydrogen, which is very plentiful. The by-product of nuclear fusion is helium, which is not radioactive. The problem with fusion is that the hydrogen nuclei repel each other; therefore, very high temperatures are necessary for them to get close enough to fuse.

8. (*a*) When the two hydrogen nuclei bind together to form helium, energy is released. This energy comes from the mass of the hydrogen nuclei, so the resulting helium nucleus has less mass than the mass of the two hydrogen nuclei.

9. (*a*) Gamma and beta radiation induce about the same amount of biological damage. Alpha radiation induces much more damage in a concentrated area than either gamma or beta radiation due to its relatively large mass and the slow speed of alpha radiation.

10. (*a*) A single fission reaction releases about 200 MeV of energy. A single fusion reaction produces less than 25 MeV. Therefore, the fission reaction releases more energy.

11. (*a*) The fuel for fusion is hydrogen, which is a component of water.

12. (*a*, *b*) Radiation is composed of high-energy particles that can disrupt cells. If a small amount of radiation hits the correct molecules of a cell (such as DNA), then it can alter the cell, causing cancer. Therefore, any amount of radiation can be harmful. Radiation is produced in natural nuclear decays and is therefore part of the normal environment. Gamma radiation can be very penetrating, but alpha and beta radiation do not penetrate very far into living tissue.

13. (*e*) Cell damage due to radiation occurs from extended exposure to high levels of radiation. The damage can be minimized by having the technician work farther away from the radiation, work for a shorter time, and be shielded from the radiation. A radiation badge can warn the worker when the allowed radiation exposure has been reached. Therefore, all of the answers can help reduce radiation damage.

14. (*d*) X-rays, gamma rays, and beta rays all produce about the same amount of radiation damage. Neutrons and alpha particles produce more cell damage per unit energy than X-rays, gamma rays, or beta rays due to their larger masses and slower speeds.

15. (*b*) The critical mass is the mass that must be present so that on average, a neutron from each fission is absorbed by another nucleus to instigate another fission. When more neutrons are released during each fission, another reaction is more likely with less mass present. Therefore, the plutonium has a smaller critical mass.

Solutions to Problems

1. When aluminum absorbs a neutron, the mass number increases by one and the atomic number is unchanged. The product nucleus is $\boxed{{}^{28}_{13}\text{Al}}$. Since the nucleus now has an "extra" neutron, it will decay by $\boxed{\beta^-}$, according to this reaction: ${}^{28}_{13}\text{Al} \rightarrow {}^{28}_{14}\text{Si} + \beta^- + \bar{\nu}_\text{e}$. Thus the product is $\boxed{{}^{28}_{14}\text{Si}}$.

2. If the Q-value is positive, then no threshold energy is needed.

$$Q = 2m_{{}^2_1\text{H}}c^2 - m_{{}^3_2\text{He}}c^2 - m_\text{n}c^2 = [2(2.014102\text{ u}) - 3.016029\text{ u} - 1.008665\text{ u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= 3.270\text{ MeV}$$

Thus, $\boxed{\text{no threshold energy is required}}$, since the Q-value is positive. The final mass is less than the initial mass, so energy is released in this process.

$\boxed{3.}$ A "slow" neutron has negligible kinetic energy. If the Q-value is positive, then the reaction is possible.

$$Q = m_{{}^{238}_{92}\text{U}}c^2 + m_\text{n}c^2 - m_{{}^{239}_{92}\text{U}}c^2 = [238.050788\text{ u} + 1.008665\text{ u} - 239.054294\text{ u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$
$$= 4.806\text{ MeV}$$

Thus, $\boxed{\text{the reaction is possible}}$.

4. (*a*) The product has 16 protons and 16 neutrons. Thus the reactants must have 16 protons and 16 neutrons. Thus, the missing nucleus has 15 protons and 16 neutrons, so it is $\boxed{{}^{31}_{15}\mathrm{P}}$.

   (*b*) The *Q*-value tells us whether the reaction requires or releases energy. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton.

$$Q = m_{\mathrm{p}}c^2 + m_{{}^{31}_{15}\mathrm{S}}c^2 - m_{{}^{32}_{16}\mathrm{S}}c^2$$
$$= [1.007825\ \mathrm{u} + 30.973762\ \mathrm{u} - 31.972071\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{8.864\ \mathrm{MeV}}$$

5. We assume that all of the particles are essentially at rest, so we ignore conservation of momentum.

   To make just the fluorine nucleus, the *Q*-value plus the incoming kinetic energy should add to 0. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton.

$$\mathrm{KE} + Q = \mathrm{KE} + m_{\mathrm{p}}c^2 + m_{{}^{18}_{8}\mathrm{O}}c^2 - m_{{}^{18}_{9}\mathrm{F}}c^2 - m_{\mathrm{n}}c^2 = 0 \quad \rightarrow$$

$$m_{{}^{18}_{9}\mathrm{F}}c^2 = \mathrm{KE} + m_{\mathrm{p}}c^2 + m_{{}^{18}_{8}\mathrm{O}}c^2 - m_{\mathrm{n}}c^2 =$$
$$= 2.438\ \mathrm{MeV} + [1.007825\ \mathrm{u} + 17.999160\ \mathrm{u} - 1.008665\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$
$$= 1.6767873 \times 10^4\ \mathrm{MeV}$$
$$m_{{}^{18}_{9}\mathrm{F}} = (1.6767873 \times 10^4\ \mathrm{MeV}/c^2)\left(\frac{1\ \mathrm{u}}{931.5\ \mathrm{MeV}/c^2}\right) = \boxed{18.000937\ \mathrm{u}}$$

6. (*a*) If the *Q*-value is positive, then no threshold energy is needed.

$$Q = m_{\mathrm{n}}c^2 + m_{{}^{24}_{12}\mathrm{Mg}}c^2 - m_{{}^{23}_{11}\mathrm{Na}}c^2 - m_{\mathrm{d}}c^2$$
$$= [1.008665\ \mathrm{u} + 23.985042\ \mathrm{u} - 22.989769\ \mathrm{u} - 2.014102\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = -9.468\ \mathrm{MeV}$$

   Thus more energy is required if this reaction is to occur. The 18.00 MeV of kinetic energy is more than sufficient, so $\boxed{\text{the reaction can occur}}$.

   (*b*) $18.00\ \mathrm{MeV} - 9.468\ \mathrm{MeV} = \boxed{8.53\ \text{MeV of energy is released}}$

7. (*a*) If the *Q*-value is positive, then no threshold energy is needed. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton and the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_{\mathrm{p}}c^2 + m_{{}^{7}_{3}\mathrm{Li}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - m_{\alpha}c^2$$
$$= [1.007825\ \mathrm{u} + 7.016003\ \mathrm{u} - 2(4.002603\ \mathrm{u})]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 17.346\ \mathrm{MeV}$$

   Since the *Q*-value is positive, $\boxed{\text{the reaction can occur}}$.

(*b*) The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$\mathrm{KE_{total}} = \mathrm{KE_{reactants}} + Q = 3.1\ \mathrm{MeV} + 17.346\ \mathrm{MeV} = \boxed{20.4\ \mathrm{MeV}}$$

8. (*a*) If the Q-value is positive, then no threshold energy is needed. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton and the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_\alpha c^2 + m_{{}^{14}_{7}\mathrm{N}}c^2 - m_{{}^{17}_{8}\mathrm{O}}c^2 - m_\mathrm{p}c^2$$
$$= [4.002603\ \mathrm{u} + 14.003074\ \mathrm{u} - 16.999132\ \mathrm{u} - 1.007825\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = -1.192\ \mathrm{MeV}$$

Thus, more energy is required if this reaction is to occur. The 9.85 MeV of kinetic energy is more than sufficient, so $\boxed{\text{the reaction can occur}}$.

(*b*) The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$\mathrm{KE_{total}} = \mathrm{KE_{reactants}} + Q = 9.85\ \mathrm{MeV} - 1.192\ \mathrm{MeV} = \boxed{8.66\ \mathrm{MeV}}$$

9. In order to balance the electrons, we use the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_\alpha c^2 + m_{{}^{16}_{8}\mathrm{O}}c^2 - m_{{}^{20}_{10}\mathrm{Ne}}$$
$$= [4.002603\ \mathrm{u} + 15.994915\ \mathrm{u} - 19.992440\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.730\ \mathrm{MeV}}$$

10. The Q-value tells us whether the reaction requires or releases energy.

$$Q = m_\mathrm{d}c^2 + m_{{}^{13}_{6}\mathrm{C}}c^2 - m_{{}^{14}_{7}\mathrm{N}}c^2 - m_\mathrm{n}c^2$$
$$= [2.014102\ \mathrm{u} + 13.003355\ \mathrm{u} - 14.003074\ \mathrm{u} - 1.008665\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 5.326\ \mathrm{MeV}$$

The total kinetic energy of the products will be the Q-value plus the incoming kinetic energy.

$$\mathrm{KE_{total}} = \mathrm{KE_{reactants}} + Q = 41.4\ \mathrm{MeV} + 5.326\ \mathrm{MeV} = \boxed{46.7\ \mathrm{MeV}}$$

11. The ${}^{14}_{7}\mathrm{N}$ absorbs a neutron, and ${}^{14}_{6}\mathrm{C}$ is a product. Thus the reaction is $\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + ?$. The reactants have 7 protons and 15 nucleons, which means 8 neutrons. Thus the products also have 7 protons and 8 neutrons, so the unknown product must be a proton. The reaction is $\boxed{\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}}$. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton.

$$Q = m_\mathrm{n}c^2 + m_{{}^{14}_{7}\mathrm{N}}c^2 - m_{{}^{14}_{6}\mathrm{C}}c^2 - m_\mathrm{p}c^2$$
$$= [1.008665\ \mathrm{u} + 14.003074\ \mathrm{u} - 14.003242\ \mathrm{u} - 1.007825\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{0.626\ \mathrm{MeV}}$$

12. (*a*) The deuteron is ${}^{2}_{1}\mathrm{H}$, so the reactants have 4 protons and 8 nucleons. Therefore, the reactants have 4 neutrons. Thus the products must have 4 protons and 4 neutrons. That means that X must have 3 protons and 4 neutrons, so X is $\boxed{{}^{7}_{3}\mathrm{Li}}$.

(*b*) This is called a "stripping" reaction because the lithium nucleus has "stripped" a neutron from the deuteron.

(*c*) The *Q*-value tells us whether the reaction requires or releases energy. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton

$$Q = m_{\mathrm{d}}c^2 + m_{{}^{6}_{3}\mathrm{Li}}c^2 - m_{{}^{7}_{3}\mathrm{Li}}c^2 - m_{\mathrm{p}}c^2$$
$$= [2.014102\ \mathrm{u} + 6.015123\ \mathrm{u} - 7.016003\ \mathrm{u} - 1.007825\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{5.027\ \mathrm{MeV}}$$

Since the *Q*-value is positive, the reaction is $\boxed{\text{exothermic}}$.

13. (*a*) This is called a "pickup" reaction because the helium has "picked up" a neutron from the carbon nucleus.

(*b*) The alpha is ${}^{4}_{2}\mathrm{He}$. The reactants have 8 protons and 15 nucleons, so they have 7 neutrons. Thus, the products must also have 8 protons and 7 neutrons. The alpha has 2 protons and 2 neutrons, so X must have 6 protons and 5 neutrons. Thus, X is $\boxed{{}^{11}_{6}\mathrm{C}}$.

(*c*) The *Q*-value tells us whether the reaction requires or releases energy. In order to balance the electrons, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton and the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_{{}^{3}_{2}\mathrm{He}}c^2 + m_{{}^{12}_{6}\mathrm{C}}c^2 - m_{{}^{11}_{6}\mathrm{C}}c^2 - m_{\alpha}c^2$$
$$= [3.016029\ \mathrm{u} + 12.000000\ \mathrm{u} - 11.011434\ \mathrm{u} - 4.002603\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.856\ \mathrm{MeV}}$$

Since the *Q*-value is positive, the reaction is $\boxed{\text{exothermic}}$.

14. The *Q*-value tells us whether the reaction requires or releases energy. In order to balance the electron count, we use the mass of ${}^{1}_{1}\mathrm{H}$ for the proton and the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_{\mathrm{p}}c^2 + m_{{}^{7}_{3}\mathrm{Li}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - m_{\alpha}c^2 = [1.007825\ \mathrm{u} + 7.016003\ \mathrm{u} - 2(4.002603)\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$
$$= 17.35\ \mathrm{MeV}$$

The reaction $\boxed{\text{releases 17.35 MeV}}$.

15. The *Q*-value tells us whether the reaction requires or releases energy. In order to balance the electron count, we use the mass of ${}^{4}_{2}\mathrm{He}$ for the alpha particle.

$$Q = m_{\alpha}c^2 + m_{{}^{9}_{4}\mathrm{Be}}c^2 - m_{{}^{12}_{6}\mathrm{C}}c^2 - m_{\mathrm{n}}c^2$$
$$= [4.002603\ \mathrm{u} + 9.012183\ \mathrm{u} - 12.000000\ \mathrm{u} - 1.008665\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 5.702\ \mathrm{MeV}$$

The reaction $\boxed{\text{releases 5.702 MeV}}$.

16. The Q-value gives the energy released in the reaction, assuming that the initial kinetic energy of the neutron is very small.

$$Q = m_{\text{n}}c^2 + m_{^{235}_{92}\text{U}}c^2 - m_{^{141}_{56}\text{Ba}}c^2 - m_{^{92}_{36}\text{Kr}}c^2 - 3m_{\text{n}}c^2$$

$$= [1.008665\text{ u} + 235.043930\text{ u} - 140.914411\text{ u} - 91.926156\text{ u} - 3(1.008665\text{ u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= \boxed{173.3\text{ MeV}}$$

17. The Q-value gives the energy released in the reaction, assuming the initial kinetic energy of the neutron is very small.

$$Q = m_{\text{n}}c^2 + m_{^{235}_{92}\text{U}}c^2 - m_{^{88}_{38}\text{Sr}}c^2 - m_{^{136}_{54}\text{Xe}}c^2 - 12m_{\text{n}}c^2$$

$$= [1.008665\text{ u} + 235.043930\text{ u} - 87.905612\text{ u} - 135.907214\text{ u} - 12(1.008665\text{ u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= \boxed{126.5\text{ MeV}}$$

18. The power released is the energy released per reaction times the number of reactions per second.

$$P = \frac{\text{energy}}{\text{reaction}} \times \frac{\#\text{ reactions}}{\text{s}} \quad \rightarrow$$

$$\frac{\#\text{ reactions}}{\text{s}} = \frac{P}{\dfrac{\text{energy}}{\text{reaction}}} = \frac{240\times10^6\text{ W}}{(200\times10^6\text{ eV/reaction})(1.60\times10^{-19}\text{ J/eV})} = 7.5\times10^{18}\text{ reactions/s}$$

$$\approx \boxed{8\times10^{18}\text{ reactions/s}}$$

19. Compare the energy per fission with the mass energy.

$$\frac{\text{energy per fission}}{\text{mass energy } mc^2} = \frac{200\text{ MeV}}{(235\text{ u})(931.5\text{ MeV}/c^2)c^2} = 9.1\times10^{-4} \approx \boxed{\frac{1}{1100}} \approx (9\times10^{-2})\%$$

20. Convert the 960 watts over a year's time to a mass of uranium.

$$\left(\frac{960\text{ J}}{1\text{ s}}\right)\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)\left(\frac{1\text{ fission}}{200\text{ MeV}}\right)\left(\frac{0.235\text{ kg }^{235}_{92}\text{U}}{6.02\times10^{23}\text{ atoms}}\right) = 3.696\times10^{-4}\text{ kg }^{235}_{92}\text{U}$$

$$\approx \boxed{0.4\text{ g }^{235}_{92}\text{U}}$$

21. (*a*) The total number of nucleons for the reactants is 236, so the total number of nucleons for the products must also be 236. The two daughter nuclei have a total of 231 nucleons, so $\boxed{\text{5 neutrons}}$ must be produced in the reaction: $^{235}_{92}\text{U} + \text{n} \rightarrow {}^{133}_{51}\text{Sb} + {}^{98}_{41}\text{Nb} + 5\text{n}$.

(*b*)
$$Q = m_{^{235}_{92}\text{U}}c^2 + m_{\text{n}}c^2 - m_{^{133}_{51}\text{Sb}}c^2 - m_{^{98}_{41}\text{Nb}}c^2 - 5m_{\text{n}}c^2$$

$$= [235.043930\text{ u} + 1.008665\text{ u} - 132.915250\text{ u} - 97.910328\text{ u} - 5(1.008665\text{ u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2$$

$$= \boxed{171.1\text{ MeV}}$$

22. We assume as stated in Problems 18, 19, and 20 that an average of 200 MeV is released per fission of a uranium nucleus.

$$(3\times10^{7}\ \text{J})\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{MeV}}{1\times10^{6}\ \text{eV}}\right)\left(\frac{1\ \text{nucleus}}{200\ \text{MeV}}\right)\left(\frac{0.235\ \text{kg}}{6.02\times10^{23}\ \text{nuclei}}\right)=\boxed{3.7\times10^{-7}\ \text{kg}\ {}^{235}_{92}\text{U}}$$

23. Since the reaction is 34% efficient, the fission needs to generate $(950/0.34)\ \text{MW}$ of power. Convert the power rating to a mass of uranium using the factor-label method. We assume that 200 MeV is released per fission, as in other Problems.

$$\frac{950\times10^{6}}{0.34}\frac{\text{J}}{\text{s}}\times\frac{1\ \text{atom}}{200\times10^{6}\ \text{eV}}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\times\frac{0.235\ \text{kg U}}{6.02\times10^{23}\ \text{atoms}}\times\frac{3.156\times10^{7}\ \text{s}}{1\ \text{yr}}=1075\ \text{kg}\ {}^{235}_{92}\text{U}$$

$$\approx\boxed{1100\ \text{kg}\ {}^{235}_{92}\text{U}}$$

24. We find the number of collisions from the relationship $E_n=E_0\left(\frac{1}{2}\right)^n$, where n is the number of collisions.

$$E_n=E_0\left(\tfrac{1}{2}\right)^n\quad\rightarrow\quad n=\frac{\ln\frac{E_n}{E_0}}{\ln\frac{1}{2}}=\frac{\ln\frac{0.040\ \text{eV}}{1.0\times10^{6}\ \text{eV}}}{\ln\frac{1}{2}}=24.58\approx\boxed{25\ \text{collisions}}$$

25. If the uranium splits into two roughly equal fragments, then each will have an atomic mass number of half of 236, or 118. Each will have a nuclear charge of half of 92, or 46. Calculate the electrical potential energy using a relationship derived in Example 17–7. The distance between the nuclei will be twice the radius of a nucleus, and the radius is given in Eq. 30–1.

$$\text{PE}=\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}=(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(46)^2(1.60\times10^{-19}\ \text{C})^2}{2(1.2\times10^{-15}\ \text{m})(118)^{1/3}}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)=\boxed{260\ \text{MeV}}$$

This is about $\boxed{\text{30\% larger}}$ than the nuclear fission energy released.

26. The reaction rate is proportional to the number of neutrons causing the reactions. For each fission, the number of neutrons will increase by a factor of 1.0004, so in 1000 milliseconds, the number of neutrons will increase by a factor of $(1.0004)^{1000}=\boxed{1.5}$.

$\boxed{27.}$ $$\text{KE}=\tfrac{3}{2}kT=\tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})(2\times10^{7}\ \text{K})=\boxed{4\times10^{-16}\ \text{J}}=4\times10^{-16}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$

$$\approx\boxed{3000\ \text{eV}}$$

28. The Q-value gives the energy released in the reaction.

$$Q=m_{{}^2_1\text{H}}c^2+m_{{}^3_1\text{H}}c^2-m_{{}^4_2\text{He}}c^2-m_{\text{n}}c^2$$

$$=[2.014102\ \text{u}+3.016049\ \text{u}-4.002603\ \text{u}-1.008665\ \text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{17.59\ \text{MeV}}$$

29. Calculate the Q-value for the reaction ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \text{n}$.

$$Q = 2m_{{}^2_1\text{H}}c^2 - m_{{}^3_2\text{He}}c^2 - m_{\text{n}}c^2$$
$$= [2(2.014102\text{ u}) - 3.016029\text{ u} - 1.008665\text{ u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{3.27\text{ MeV}}$$

30. For the reaction in Eq. 31–6a, if atomic masses are to be used, then one more electron needs to be added to the products side of the equation. Notice that charge is not balanced in the equation as written. The balanced reaction is ${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + \text{e}^+ + \nu + \text{e}^-$.

$$Q = 2m_{{}^1_1\text{H}}c^2 - m_{{}^2_1\text{H}}c^2 - m_{\text{e}^+}c^2 - m_{\text{e}^-}c^2$$
$$= [2(1.007825\text{ u}) - 2.014102\text{ u} - 2(0.000549\text{ u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 0.4192\text{ MeV} \approx \boxed{0.42\text{ MeV}}$$

For the reaction in Eq. 31–6b, use atomic masses, since there would be two electrons on each side.

$$Q = m_{{}^1_1\text{H}}c^2 + m_{{}^2_1\text{H}}c^2 - m_{{}^3_2\text{He}}c^2$$
$$= [1.007825\text{ u} + 2.014102\text{ u} - 3.016029\text{ u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 5.4940\text{ MeV} \approx \boxed{5.49\text{ MeV}}$$

For the reaction in Eq. 31–6c, use atomic masses, since there would be two electrons on each side.

$$Q = 2m_{{}^3_2\text{He}}c^2 - m_{{}^4_2\text{He}}c^2 - 2m_{{}^1_1\text{H}}c^2$$
$$= [2(3.016029\text{ u}) - 4.002603 - 2(1.007825\text{ u})]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = 12.8594\text{ MeV} \approx \boxed{12.86\text{ MeV}}$$

31. (*a*) Eq. 31–8a: $\dfrac{4.03\text{ MeV}}{2(2.014102\text{ u})} \times \dfrac{1\text{ u}}{1.66\times10^{-27}\text{ kg}} \times \dfrac{1\text{ kg}}{1000\text{ g}} = \boxed{6.03\times10^{23}\text{ MeV/g}}$

Eq. 31–8b: $\dfrac{3.27\text{ MeV}}{2(2.014102\text{ u})} \times \dfrac{1\text{ u}}{1.66\times10^{-27}\text{ kg}} \times \dfrac{1\text{ kg}}{1000\text{ g}} = \boxed{4.89\times10^{23}\text{ MeV/g}}$

Eq. 31–8c: $\dfrac{17.59\text{ MeV}}{(2.014102\text{ u} + 3.016049\text{ u})} \times \dfrac{1\text{ u}}{1.66\times10^{-27}\text{ kg}} \times \dfrac{1\text{ kg}}{1000\text{ g}} = \boxed{2.11\times10^{24}\text{ MeV/g}}$

(*b*) Uranium fission (200 MeV per nucleus):

$$\frac{200\text{ MeV}}{(235\text{ u})} \times \frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}} \times \frac{1\text{ kg}}{1000\text{ g}} = \boxed{5.13\times10^{23}\text{ MeV/g}}$$

Eq. 31–8a: $\dfrac{5.13\times10^{23}}{6.03\times10^{23}} = \boxed{0.851}$

Eq. 31–8b: $\dfrac{5.13\times10^{23}}{4.89\times10^{23}} = \boxed{1.05}$

Eq. 31–8c: $\dfrac{5.13\times10^{23}}{2.11\times10^{24}} = \boxed{0.243}$

32. Calculate the Q-value for the reaction ${}^{238}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U}$.

$$Q = m_{{}^{238}_{92}\mathrm{U}}c^2 + m_{\mathrm{n}}c^2 - m_{{}^{239}_{92}\mathrm{U}}c^2$$
$$= [238.050788\ \mathrm{u} + 1.008665\ \mathrm{u} - 239.054294\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.806\ \mathrm{MeV}}$$

33. The reaction of Eq. 31–8b consumes 2 deuterons and releases 3.23 MeV of energy. The amount of energy needed is the power times the elapsed time, and the energy can be related to the mass of deuterium by the reaction.

$$\left(960\frac{\mathrm{J}}{\mathrm{s}}\right)(1\ \mathrm{yr})\left(3.156\times10^{7}\ \frac{\mathrm{s}}{\mathrm{yr}}\right)\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right)\left(\frac{2\ \mathrm{d}}{3.23\ \mathrm{MeV}}\right)\left(\frac{2.014\times10^{-3}\ \mathrm{kg}}{6.02\times10^{23}\ \mathrm{d}}\right)$$
$$= 3.923\times10^{-4}\ \mathrm{kg} = \boxed{0.39\ \mathrm{g}}$$

34. (*a*) The reactants have a total of 3 protons and 7 neutrons, so the products should have the same. After accounting for the helium, there are 3 neutrons and 1 proton in the other product, so it must be tritium, $\boxed{{}^{3}_{1}\mathrm{H}}$. The reaction is ${}^{6}_{3}\mathrm{Li} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H}$.

(*b*) The Q-value gives the energy released.

$$Q = m_{{}^{6}_{3}\mathrm{Li}}c^2 + m_{{}^{1}_{0}\mathrm{n}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - m_{{}^{3}_{1}\mathrm{H}}c^2$$
$$= [6.015123\ \mathrm{u} + 1.008665\ \mathrm{u} - 4.002603\ \mathrm{u} - 3.016049\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{4.784\ \mathrm{MeV}}$$

35. Assume that the two reactions take place at equal rates, so they are both equally likely. Then from the reaction of 4 deuterons, there would be a total of 7.30 MeV of energy released, or an average of 1.825 MeV per deuteron. A total power of $\dfrac{1150\ \mathrm{MW}}{0.33} = 3485\ \mathrm{MW}$ must be obtained from the fusion reactions to provide the required 1150-MW output, because of the 33% efficiency. We convert the power to a number of deuterons based on the energy released per reacting deuteron and then convert that to an amount of water using the natural abundance of deuterium.

$$3485\ \mathrm{MW} \rightarrow \left[\begin{array}{l}\left(3485\times10^{6}\ \dfrac{\mathrm{J}}{\mathrm{s}}\right)\left(\dfrac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right)\left(\dfrac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right)\left(\dfrac{1\ \mathrm{d}}{1.825\ \mathrm{MeV}}\right)\left(\dfrac{1\ \mathrm{H\ atom}}{0.000115\ \mathrm{d's}}\right)\times \\ \left(\dfrac{1\ \mathrm{H_2O\ molecule}}{2\ \mathrm{H\ atoms}}\right)\left(\dfrac{0.018\ \mathrm{kg\ H_2O}}{6.02\times10^{23}\ \mathrm{molecules}}\right)\end{array}\right]$$
$$= 5586\ \mathrm{kg/h} \approx \boxed{5600\ \mathrm{kg/h}}$$

36. We assume that the reactants are at rest when they react, so the total momentum of the system is 0. As a result, the momenta of the two products are equal in magnitude. The available energy of 17.57 MeV is much smaller than the masses involved, so we use the nonrelativistic relationship between momentum and kinetic energy, $\mathrm{KE} = \dfrac{p^2}{2m} \rightarrow p = \sqrt{2m\mathrm{KE}}$.

$$\mathrm{KE}_{^4_2\mathrm{He}} + \mathrm{KE}_\mathrm{n} = \mathrm{KE}_\mathrm{total} = 17.57\ \mathrm{MeV} \qquad p_{^4_2\mathrm{He}} = p_\mathrm{n} \ \rightarrow\ \sqrt{2m_{^4_2\mathrm{He}}\mathrm{KE}_{^4_2\mathrm{He}}} = \sqrt{2m_\mathrm{n}\mathrm{KE}_\mathrm{n}} \ \rightarrow$$

$$m_{^4_2\mathrm{He}}\mathrm{KE}_{^4_2\mathrm{He}} = m_\mathrm{n}\mathrm{KE}_\mathrm{n} \ \rightarrow\ m_{^4_2\mathrm{He}}\mathrm{KE}_{^4_2\mathrm{He}} = m_\mathrm{n}\left(\mathrm{KE}_\mathrm{total} - \mathrm{KE}_{^4_2\mathrm{He}}\right) \ \rightarrow$$

$$\mathrm{KE}_{^4_2\mathrm{He}} = \frac{m_\mathrm{n}}{m_{^4_2\mathrm{He}} + m_\mathrm{n}}\mathrm{KE}_\mathrm{total} = \left(\frac{1.008665}{4.002603 + 1.008665}\right)17.57\ \mathrm{MeV} = 3.536\ \mathrm{MeV} \approx \boxed{3.5\ \mathrm{MeV}}$$

$$\mathrm{KE}_\mathrm{n} = \mathrm{KE}_\mathrm{total} - \mathrm{KE}_{^4_2\mathrm{He}} = 17.57\ \mathrm{MeV} - 3.54\ \mathrm{MeV} = 14.03\ \mathrm{MeV} \approx \boxed{14\ \mathrm{MeV}}$$

If the plasma temperature were significantly higher, then the approximation of 0 kinetic energy being brought into the reaction would not be reasonable. Thus the results would depend on plasma temperature. A higher plasma temperature would result in higher values for the energies.

$\boxed{37.}$ In Eq. 31–8a, 4.00 MeV of energy is released for every 2 deuterium atoms. The mass of water can be converted to a number of deuterium atoms.

$$(1.00\ \mathrm{kg\ H_2O})\left(\frac{6.02\times10^{23}\ \mathrm{H_2O}}{0.018\ \mathrm{kg\ H_2O}}\right)\left(\frac{2\ \mathrm{H}}{1\ \mathrm{H_2O}}\right)\left(\frac{1.15\times10^{-4}\ \mathrm{d}}{1\ \mathrm{H}}\right) = 7.692\times10^{21}\ \mathrm{d\ nuclei} \ \rightarrow$$

$$(7.692\times10^{21}\ \mathrm{d\ nuclei})\left(\frac{4.00\times10^{6}\ \mathrm{eV}}{2\ \mathrm{d\ atoms}}\right)\left(\frac{1.60\times10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}\right) = \boxed{2.46\times10^{9}\ \mathrm{J}}$$

As compared to gasoline: $\dfrac{2.46\times10^{9}\ \mathrm{J}}{5\times10^{7}\ \mathrm{J}} \approx \boxed{50\times \text{more than gasoline}}$

38. (*a*) We follow the method of Example 31–9. The reaction is $^{12}_{6}\mathrm{C} + ^{2}_{1}\mathrm{H} \rightarrow ^{13}_{7}\mathrm{N} + \gamma.$ We calculate the potential energy of the particles when they are separated by the sum of their radii. The radii are calculated from Eq. 30–1.

$$\mathrm{KE}_\mathrm{total} = \frac{1}{4\pi\varepsilon_0}\frac{Q_\mathrm{C}Q_\mathrm{H}}{r_\mathrm{C} + r_\mathrm{H}} = (8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})\frac{(6)(1)(1.60\times10^{-19}\ \mathrm{C})^2}{(1.2\times10^{-15}\ \mathrm{m})(1^{1/3} + 12^{1/3})}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right)$$
$$= 2.19\ \mathrm{MeV}$$

For the d–t reaction, Example 31–9 shows that $\mathrm{KE}_\mathrm{total} = 0.45\ \mathrm{MeV}.$ Find the ratio of the two energies.

$$\frac{\mathrm{KE}_\mathrm{C}}{\mathrm{KE}_\mathrm{d-t}} = \frac{2.19\ \mathrm{MeV}}{0.45\ \mathrm{MeV}} = 4.9$$

$\boxed{\text{The carbon reaction requires about 5 times more energy than the d–t reaction.}}$

(*b*) The kinetic energy is proportional to the temperature by $\overline{\mathrm{KE}} = \frac{3}{2}kT.$ Since the kinetic energy has to increase by a factor of 5, so does the temperature. Thus we estimate $\boxed{T \approx 1.5\times10^{9}\ \mathrm{K}}$.

39. Because the RBE of alpha particles is up to 20 and the RBE of X-rays is 1, it takes 20 times as many rads of X-rays to cause the same biological damage as alpha particles. Thus the 350 rads of alpha particles is equivalent to $350\ \mathrm{rads}\times20 = \boxed{7000\ \mathrm{rads}}$ of X-rays.

40. Use Eq. 31–10b to relate Sv to Gy. From Table 31–1, the RBE of gamma rays is 1, so the number of Sv is equal to the number of Gy. Thus $4.0\text{ Sv} = \boxed{4.0\text{ Gy}}$.

41. The biological damage is measured by the effective dose, Eq. 31–10a, using Table 31–1.

$$(72\text{ rads fast neutrons})\times 10 = (x\text{ rads slow neutrons})\times 5 \quad \to$$
$$x = \frac{72\text{ rads}\times 10}{5} = \boxed{144\text{ rads slow neutrons}}$$

$\boxed{42.}$ A gray is 1 joule per kilogram, according to Eq. 31–9.

$$(2.5\text{ J/kg})\times 65\text{ kg} = 162.5\text{ J} \approx \boxed{160\text{ J}}$$

43. (*a*) Since the RBE is 1, the effective dose (in rem) is the same as the absorbed dose (in rad). Thus the absorbed dose is $\boxed{1.0\text{ rad or }0.010\text{ Gy}}$.

(*b*) A Gy is 1 J/ kg.

$$(0.01\text{ Gy})\left(\frac{1\text{ J/kg}}{1\text{ Gy}}\right)(0.20\text{ kg})\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right)\left(\frac{1\text{ p}}{1.2\times 10^{6}\text{ eV}}\right) = \boxed{1.0\times 10^{10}\text{ p}}$$

44. The counting rate will be 85% of 35% of the activity.

$$(0.035\times 10^{-6}\text{ Ci})\left(\frac{3.7\times 10^{10}\text{ decays/s}}{1\text{ Ci}}\right)\left(\frac{1\,\beta}{1\text{ decay}}\right)(0.35)(0.85) = 385.3\text{ counts/s} \approx \boxed{390\text{ counts/s}}$$

45. The two definitions of roentgen are 1.6×10^{12} ion pairs/g produced by the radiation and the newer definition of 0.878×10^{-2} J/kg deposited by the radiation. Start with the current definition and relate them by the value of 35 eV per ion pair.

$$1\text{ R} = (0.878\times 10^{-2}\text{ J/kg})(1\text{ kg/1000 g})(1\text{ eV}/1.60\times 10^{-19}\text{ J})(1\text{ ion pair/35 eV})$$
$$= 1.567\times 10^{12}\text{ ion pairs/g}$$

The two values of ion pairs per gram are within about 2% of each other.

46. We approximate the decay rate as constant and find the time to administer 32 Gy. If that calculated time is significantly shorter than the half-life of the isotope, then the approximation is reasonable. If 1.0 mCi delivers about 10 mGy/min, then 1.6 mCi would deliver 16 mGy/min.

$$\text{dose} = \text{rate}\times\text{time} \quad \to \quad \text{time} = \frac{\text{dose}}{\text{rate}} = \frac{32\text{ Gy}}{16\times 10^{-3}\text{ Gy/min}}\left(\frac{1\text{ day}}{1440\text{ min}}\right) = 1.39\text{ days} \approx \boxed{1.4\text{ days}}$$

This is only about 10% of a half-life, so our approximation is reasonable.

47. Since the half-life is long (5730 yr), we can consider the activity as constant over a short period of time. Use the definition of the curie from Section 31–5.

$$(2.50\times10^{-6}\text{ Ci})\left(\frac{3.70\times10^{10}\text{ decays/s}}{1\text{ Ci}}\right)=9.25\times10^{4}\text{ decays/s}=\frac{\Delta N}{\Delta t}=\frac{0.693}{T_{\frac{1}{2}}}N\quad\rightarrow$$

$$N=\frac{\Delta N}{\Delta t}\frac{T_{\frac{1}{2}}}{0.693}=(9.25\times10^{4}\text{ decays/s})\frac{5730\text{ yr}}{0.693}(3.156\times10^{7}\text{ s/yr})=2.414\times10^{16}\text{ nuclei}$$

$$2.414\times10^{16}\text{ nuclei}\left(\frac{0.0140\text{ kg}}{6.02\times10^{23}\text{ nuclei}}\right)=\boxed{5.61\times10^{-10}\text{ kg}}=0.561\ \mu\text{g}$$

48. Each decay releases one gamma ray of energy 122 keV. Half of that energy is deposited in the body. The activity tells at what rate the gamma rays are released into the body. We assume that the activity is constant.

$$(1.55\times10^{-6}\text{ Ci})\left(3.70\times10^{10}\frac{\gamma/\text{s}}{\text{Ci}}\right)(86{,}400\text{ s/day})(0.50)(122\text{ keV}/\gamma)(1.60\times10^{-16}\text{ J/keV})\left(\frac{1}{65\text{ kg}}\right)$$

$$=7.44\times10^{-7}\frac{\text{J/kg}}{\text{day}}\approx\boxed{7.4\times10^{-7}\frac{\text{Gy}}{\text{day}}}$$

49. Use the dose, the mass of the beef, and the energy per electron to find the number of electrons.

$$(4.5\times10^{3}\text{ Gy})\left(\frac{1\text{ J/kg}}{1\text{ Gy}}\right)(5\text{ kg})\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)\left(\frac{1\text{ e}^{-}}{1.6\text{ MeV}}\right)=8.78\times10^{16}\text{ e}^{-}\approx\boxed{9\times10^{16}\text{ e}^{-}}$$

50. (*a*) According to Appendix B, ${}^{131}_{53}\text{I}$ decays by beta decay.

$$\boxed{{}^{131}_{53}\text{I}\ \rightarrow\ {}^{131}_{54}\text{Xe}+\beta^{-}+\bar{\nu}}$$

(*b*) The number of nuclei present is given by Eq. 30–4, with $N=0.05N_0$.

$$N=N_0e^{-\lambda t}\quad\rightarrow\quad t=-\frac{\ln N/N_0}{\lambda}=-\frac{T_{1/2}\ln N/N_0}{0.693}=-\frac{(8.0\text{ d})\ln 0.05}{0.693}=34.58\text{ d}\approx\boxed{35\text{ d}}$$

(*c*) The activity is given by $\Delta N/\Delta t=\lambda N$. This can be used to find the number of nuclei, and then the mass can be found.

$$\Delta N/\Delta t=\lambda N\quad\rightarrow$$

$$N=\frac{\Delta N/\Delta t}{\lambda}=\frac{(T_{1/2})(\Delta N/\Delta t)}{0.693}=\frac{(8.0\text{ d})(86{,}400\text{ s/d})(1\times10^{-3}\text{ Ci})(3.70\times10^{10}\text{ decays/s})}{0.693}$$

$$=3.69\times10^{13}\text{ nuclei};\quad m=3.69\times10^{13}\text{ nuclei}\left(\frac{0.131\text{ kg}}{6.02\times10^{23}\text{ nuclei}}\right)=\boxed{8\times10^{-12}\text{ kg}}$$

51. The activity is converted to decays per day, then to energy per year, and finally to a dose per year. The potassium decays by gammas and betas, according to Appendix F. Gammas and betas have an RBE of 1, so the number of Sv is the same as the number of Gy, and the number of rem is the same as the number of rad.

$$\left(2000\times10^{-12}\,\frac{\text{Ci}}{\text{L}}\right)\left(3.70\times10^{10}\,\frac{\text{decays/s}}{1\,\text{Ci}}\right)(12\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)\left(0.5\frac{\text{L}}{\text{day}}\right)=1.598\times10^{6}\,\frac{\text{decays}}{\text{day}}$$

$$\left(1.598\times10^{6}\,\frac{\text{decays}}{\text{day}}\right)\left(365\frac{\text{days}}{\text{yr}}\right)(0.10)\left(1.5\frac{\text{MeV}}{\text{decay}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{\text{MeV}}\right)=1.40\times10^{-5}\,\frac{\text{J}}{\text{yr}}$$

(*a*) For the adult, use a mass of 60 kg.

$$\text{Effective dose}=\left(1.40\times10^{-5}\,\frac{\text{J}}{\text{yr}}\right)\left(\frac{1}{60\text{ kg}}\right)\left(\frac{1\text{ Gy}}{1\text{ J/kg}}\right)\left(\frac{1\text{ Sv}}{1\text{ Gy}}\right)$$
$$=2.33\times10^{-7}\text{ Sv/yr}\left(\frac{10^{5}\text{ mrem}}{\text{Sv}}\right)=2.33\times10^{-2}\text{ mrem/yr}$$
$$\approx\boxed{2\times10^{-7}\text{ Sv/yr}}\text{ or }\boxed{2\times10^{-2}\text{ mrem/yr}}$$

$$\text{Fraction of allowed dose}=\frac{2.33\times10^{-2}\,\frac{\text{mrem}}{\text{year}}}{100\,\frac{\text{mrem}}{\text{year}}}\approx\boxed{2\times10^{-4}\text{ times the allowed dose}}$$

(*b*) For the baby, the only difference is that the mass is 10 times smaller, so the effective dose is 10 times bigger. The results are as follows:

$$\approx\boxed{2\times10^{-6}\text{ Sv/yr}},\ \boxed{0.2\text{ mrem/yr}},\ \text{and }\boxed{2\times10^{-3}\text{ times the allowed dose}}$$

52. (*a*) The reaction has $Z = 86$ and $A = 222$ for the parent nucleus. The alpha has $Z = 2$ and $A = 4$, so the daughter nucleus must have $Z = 84$ and $A = 218$. That makes the daughter nucleus $\boxed{{}^{218}_{84}\text{Po}}$.

(*b*) From Fig. 30–11, polonium-218 is $\boxed{\text{radioactive}}$. It decays via both $\boxed{\text{alpha and beta decay}}$. The half-life for the alpha decay is $\boxed{\text{3.1 minutes}}$.

(*c*) The daughter nucleus is not a noble gas, so it is $\boxed{\text{chemically reacting}}$. It is in the same group as oxygen, so it might react with many other elements chemically.

(*d*) The activity is given by Eq. 30–3b, $R=\lambda N=\dfrac{\ln 2}{T_{1/2}}N$.

$$\frac{\Delta N}{\Delta t}=\frac{0.693}{T_{\frac{1}{2}}}N=\frac{0.693}{(3.8235\text{ d})(86{,}400\text{ s/d})}(1.4\times10^{-9}\text{ g})\frac{6.02\times10^{23}\text{ nuclei}}{222\text{ g}}$$
$$=7.964\times10^{6}\text{ decays/s}\approx\boxed{8.0\times10^{6}\text{ Bq}}\times\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}=\boxed{2.2\times10^{-4}\text{ Ci}}$$

To find the activity after 1 month, use Eq. 30–5.

$$\frac{\Delta N}{\Delta t}=\left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t}=(7.964\times10^{6}\text{ decays/s})e^{-\frac{0.693}{(3.8235\text{ d})}(30\text{ d})}=3.465\times10^{4}\text{ decays/s}$$
$$\approx\boxed{3.5\times10^{4}\text{ Bq}}\times\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}=\boxed{9.4\times10^{-7}\text{ Ci}}$$

53. The frequency is given in Section 31–9 as 42.58 MHz. Use that to find the wavelength.

$$c = f\lambda \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{42.58\times10^{6}\ \text{Hz}} = \boxed{7.041\ \text{m}}$$

This lies in the $\boxed{\text{radio wave}}$ portion of the spectrum.

54. (*a*) The mass of fuel can be found by converting the power to energy to number of nuclei to mass.

$$(2100\times10^{6}\ \text{J/s})(1\ \text{yr})\left(\frac{3.156\times10^{7}\ \text{s}}{1\ \text{yr}}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)\left(\frac{1\ \text{fission atom}}{200\ \text{MeV}}\right)\left(\frac{0.235\ \text{kg}}{6.02\times10^{23}\ \text{atom}}\right)$$
$$= 808.5\ \text{kg} \approx \boxed{810\ \text{kg}}$$

(*b*) The product of the first five factors above gives the number of U atoms that fission.

$$\#\ {}^{90}_{38}\text{Sr nuclei} = 0.06(2100\times10^{6}\ \text{J/s})(1\ \text{yr})\left(\frac{3.156\times10^{7}\ \text{s}}{1\ \text{yr}}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)\left(\frac{1\ \text{fission atom}}{200\ \text{MeV}}\right)$$
$$= (1.24\times10^{26})\ {}^{90}_{38}\text{Sr nuclei}$$

The activity is given by the absolute value of Eq. 30–3b.

$$\frac{\Delta N}{\Delta t} = \lambda N = \frac{0.693}{T_{1/2}}N = \frac{0.693}{(29\ \text{yr})(3.156\times10^{7}\ \text{s/yr})}(1.24\times10^{26}) = 9.389\times10^{16}\ \frac{\text{decays}}{\text{s}}$$
$$= (9.389\times10^{16}\ \text{decays/s})\left(\frac{1\ \text{Ci}}{3.70\times10^{10}\ \text{decays/s}}\right) = \boxed{2.5\times10^{6}\ \text{Ci}}$$

55. (*a*) The reaction is ${}^{9}_{4}\text{Be} + {}^{4}_{2}\text{He} \rightarrow \text{n} + ?$. There are 6 protons and 13 nucleons in the reactants, so there must be 6 protons and 13 nucleons in the products. The neutron is 1 nucleon, so the other product must have 6 protons and 12 nucleons. Thus, it is $\boxed{{}^{12}_{6}\text{C}}$.

(*b*)
$$Q = m_{{}^{9}_{4}\text{Be}}c^2 + m_{{}^{4}_{2}\text{He}}c^2 - m_{\text{n}}c^2 - m_{{}^{12}_{6}\text{C}}c^2$$
$$= [9.012183\ \text{u} + 4.002603\ \text{u} - 1.008665\ \text{u} - 12.000000\ \text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{5.702\ \text{MeV}}$$

56. If $\overline{\text{KE}} = kT$, then to get from kelvins to keV, use the Boltzmann constant. It must be put in the proper units.

$$k = 1.381\times10^{-23}\ \frac{\text{J}}{\text{K}}\times\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\times\frac{1\ \text{keV}}{1000\ \text{eV}} = \boxed{8.620\times10^{-8}\ \text{keV/K}}$$

$\boxed{57.}$ From Eq. 13–9, the average speed of a gas molecule (root-mean-square speed) is inversely proportional to the square root of the mass of the molecule, if the temperature is constant. We assume that the two gases are in the same environment and therefore at the same temperature. We use UF_6 molecules for the calculations.

$$\frac{v_{{}^{235}_{92}\text{UF}_6}}{v_{{}^{238}_{92}\text{UF}_6}} = \sqrt{\frac{m_{{}^{238}_{92}\text{UF}_6}}{m_{{}^{235}_{92}\text{UF}_6}}} = \sqrt{\frac{238.05 + 6(19.00)}{235.04 + 6(19.00)}} = \boxed{1.0043:1}$$

58. (*a*) We assume that the energy produced by the fission was 200 MeV per fission, as in Problems 18, 19, and 20.

$$(20\text{ kilotons TNT})\left(\frac{5\times10^{12}\text{ J}}{1\text{ kiloton}}\right)\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)\left(\frac{1\text{ fission atom}}{200\text{ MeV}}\right)\left(\frac{0.235\text{ kg}}{6.02\times10^{23}\text{ atoms}}\right)$$
$$=1.220\text{ kg}\approx\boxed{1\text{ kg}}$$

(*b*) Use $E=mc^2$.

$$E=mc^2 \quad\rightarrow\quad m=\frac{E}{c^2}=\frac{(20\text{ kilotons TNT})\left(\dfrac{5\times10^{12}\text{ J}}{1\text{ kiloton}}\right)}{(3.0\times10^{8}\text{ m/s})^2}=1.11\times10^{-3}\text{ kg}\approx\boxed{1\text{ g}}$$

59. The effective dose (in rem) is equal to the actual dose (in rad) times the RBE factor.

$$\text{dose(rem)}=(32\text{ mrad/yr X-ray, }\gamma\text{-ray})(1)+(3.4\text{ mrad/yr})(10)=\boxed{66\text{ mrem/yr}}$$

60. Because the RBE factor for gamma rays is 1, the dose in rem is equal in number to the dose in rad. Since the intensity falls off as r^2 (the square of the distance), the exposure rate times r^2 will be constant.

$$\text{Allowed dose}=\frac{5.0\text{ rem}}{\text{yr}}\left(\frac{1\text{ rad}}{1\text{ rem}}\right)\left(\frac{1\text{ yr}}{52\text{ weeks}}\right)\left(\frac{1\text{ week}}{35\text{ h}}\right)=2.747\times10^{-3}\,\frac{\text{rad}}{\text{h}}$$

$$\left(2.747\times10^{-3}\,\frac{\text{rad}}{\text{h}}\right)r^2=\left(4.8\times10^{-2}\,\frac{\text{rad}}{\text{h}}\right)(1\text{ m})^2\quad\rightarrow$$

$$r=\sqrt{\frac{\left(4.8\times10^{-2}\,\dfrac{\text{rad}}{\text{h}}\right)(1\text{ m})^2}{\left(2.747\times10^{-3}\,\dfrac{\text{rad}}{\text{h}}\right)}}=4.180\text{ m}\approx\boxed{4.2\text{ m}}$$

61. (*a*) The reaction is of the form $?\rightarrow {}^{4}_{2}\text{He}+{}^{222}_{86}\text{Rn}$. There are 88 protons and 226 nucleons as products, so there must be 88 protons and 226 nucleons as reactants. The parent nucleus is ${}^{226}_{88}\text{Ra}$.

$$\boxed{{}^{226}_{88}\text{Ra}\ \rightarrow\ {}^{4}_{2}\text{He}+{}^{222}_{86}\text{Rn}}$$

(*b*) If we ignore the KE of the daughter nucleus, then the KE of the alpha particle is the *Q*-value of the reaction.

$$\text{KE}_\alpha=m_{{}^{226}_{88}\text{Ra}}c^2-m_{{}^{4}_{2}\text{He}}c^2-m_{{}^{222}_{86}\text{Rn}}c^2$$
$$=\left[226.025410\text{ u}-4.002603\text{ u}-222.017578\text{ u}\right]\left(931.5\,\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{4.871\text{ MeV}}$$

(*c*) From momentum conservation, the momentum of the alpha particle will be equal in magnitude to the momentum of the daughter particle. At the energy above, the alpha particle is not relativistic, so $\text{KE}_\alpha = \frac{p_\alpha^2}{2m_\alpha} \rightarrow p_a = \sqrt{2m_a\text{KE}_a}$.

$$p_\alpha = \sqrt{2m_a\text{KE}_a} = \sqrt{2(4.002603\text{ u})\left(\frac{931.5\text{ MeV}/c^2}{1\text{ u}}\right)(4.871\text{ MeV})} = \boxed{190.6\text{ MeV}/c}$$

The momentum of the daughter nucleus is the same as that of the alpha, $\boxed{190.6\text{ MeV}/c}$.

(*d*) Since $p_\alpha = p_{\text{daughter}}$, $\text{KE}_{\text{daughter}} = \frac{p_{\text{daughter}}^2}{2m_{\text{daughter}}} = \frac{p_\alpha^2}{2m_{\text{daughter}}}$.

$$\text{KE}_{\text{daughter}} = \frac{p_\alpha^2}{2m_{\text{daughter}}} = \frac{(190.6\text{ MeV}/c)^2}{2(222.0\text{ u})\left(\frac{931.5\text{ MeV}/c^2}{1\text{ u}}\right)} = \boxed{8.78\times10^{-2}\text{ MeV}}$$

Thus we see that our original assumption of ignoring the kinetic energy of the daughter nucleus is valid. The kinetic energy of the daughter is less than 2% of the *Q*-value.

62. This “heat of combustion” is 26.2 MeV/4 hydrogen atoms.

$$\frac{26.2\text{ MeV}}{4\text{ H atoms}}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)\left(\frac{1\text{ H atom}}{1.007825\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{6.26\times10^{14}\text{ J/kg}}$$

This is about $\boxed{2\times10^7}$ times the heat of combustion of coal.

63. (*a*) The energy is radiated uniformly over a sphere with a radius equal to the orbit radius of the Earth.

$$(1300\text{ W/m}^2)4\pi(1.496\times10^{11}\text{ m})^2 = 3.656\times10^{26}\text{ W} \approx \boxed{3.7\times10^{26}\text{ W}}$$

(*b*) After subtracting the energy of the neutrinos, the reaction of Eq. 31–7 releases 26.2 MeV for every 4 protons consumed.

$$\left(3.656\times10^{26}\frac{\text{J}}{\text{s}}\right)\left(\frac{4\text{ protons}}{26.2\text{ MeV}}\right)\left(\frac{1\text{ MeV}}{1.6\times10^{-13}\text{ J}}\right) = 3.489\times10^{38}\text{ protons/s}$$

$$\approx \boxed{3.5\times10^{38}\text{ protons/s}}$$

(*c*) Convert the Sun’s mass to a number of protons and then use the above result to estimate the Sun’s lifetime.

$$2.0\times10^{30}\text{ kg}\left(\frac{1\text{ proton}}{1.673\times10^{-27}\text{ kg}}\right)\left(\frac{1\text{ s}}{3.489\times10^{38}\text{ protons}}\right)\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right) \approx \boxed{1.1\times10^{11}\text{ yr}}$$

64. For the net proton cycle, Eq. 31–7, there are two neutrinos produced for every four protons consumed. Thus the net number of neutrinos generated per second from the Sun is just half the value of protons consumed per second. That proton consumption rate is calculated in Problem 63b.

$$(3.423\times10^{38}\text{ protons/s})(2\nu/4\text{p}) = 1.712\times10^{38}\ \nu/\text{s}$$

Assume that the neutrino distribution is spherically symmetric, centered at the Sun. The fraction that would pass through the area of the ceiling can be found by a ratio of areas, assuming that the ceiling is perpendicular to the neutrino flux. But since the ceiling is not perpendicular, a cosine factor is included to account for the angle difference, as discussed in Eq. 21–1. Finally, we adjust for the one-hour duration, assuming that the relative angle is constant over that hour.

$$(1.712\times10^{38}\ \nu/\text{s})\frac{180\ \text{m}^2}{4\pi(1.496\times10^{11}\ \text{m})^2}(\cos 44°)(3600\ \text{s}) = \boxed{2.8\times10^{20}\ \nu}$$

65. Use the common value of 200 MeV of energy released per fission, as used in Problems 18, 19, and 20. Multiply that by the number of fissions, which is 5.0% of the number of U-238 atoms.

$$\text{Total energy} = \left[\begin{array}{l}\dfrac{200\ \text{MeV}}{1\ \text{nucleus of }{}^{235}_{92}\text{U}}\left(\dfrac{1.602\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)\left(0.05\dfrac{{}^{235}_{92}\text{U nuclei}}{{}^{238}_{92}\text{U nuclei}}\right)(2.0\ \text{kg}\ {}^{238}_{92}\text{U})\\ \times\left(\dfrac{6.022\times10^{23}\ \text{nuclei of }{}^{238}_{92}\text{U nuclei}}{0.238\ \text{kg}\ {}^{238}_{92}\text{U}}\right)\end{array}\right]$$

$$= 8.107\times10^{12}\ \text{J} \approx \boxed{8\times10^{12}\ \text{J}}$$

66. (*a*) The energy released is given by the *Q*-value.

$$Q = 2m_{{}^{12}_{6}\text{C}}c^2 - m_{{}^{24}_{12}\text{Mg}}c^2 = [2(12.000000\ \text{u}) - 23.985042\ \text{u}]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2 = \boxed{13.93\ \text{MeV}}$$

(*b*) The total KE of the two nuclei must equal their PE when separated by 6.0 fm.

$$2\text{KE} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} \rightarrow$$

$$\text{KE} = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} = \tfrac{1}{2}(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{\left[(6)(1.60\times10^{-19}\ \text{C})\right]^2}{6.0\times10^{-15}\ \text{m}} = 6.912\times10^{-13}\ \text{J}$$

$$= 6.912\times10^{-13}\ \text{J}(1\ \text{eV}/1.60\times10^{-19}\ \text{J})(1\ \text{MeV}/10^6\ \text{eV}) = 4.23\ \text{MeV} \approx \boxed{4.3\ \text{MeV}}$$

(*c*) The kinetic energy and temperature are related by Eq.13–8.

$$\text{KE} = \tfrac{3}{2}kT \quad\rightarrow\quad T = \tfrac{2}{3}\frac{\text{KE}}{k} = \tfrac{2}{3}\frac{6.912\times10^{-13}\ \text{J}}{1.38\times10^{-23}\ \text{J/K}} = \boxed{3.3\times10^{10}\ \text{K}}$$

67. (*a*) A Curie is 3.7×10^{10} decays/s.

$$(0.10\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{decays/s}) = \boxed{3700\ \text{decays/s}}$$

(*b*) The beta particles have an RBE factor of 1. We calculate the dose in Gy and then convert to Sv. The half-life is over a billion years, so we assume that the activity is constant.

$$(3700\text{ decays/s})(1.4\text{ MeV/decay})(1.60\times10^{-13}\text{ J/MeV})(3.156\times10^{7}\text{ s/yr})\left(\frac{1}{65\text{ kg}}\right)$$

$$=4.024\times10^{-4}\text{ J/kg/yr}=4.024\times10^{-4}\text{ Gy/yr}=\boxed{4.0\times10^{-4}\text{ Sv/yr}}$$

This is about $\dfrac{4.024\times10^{-4}\text{ Sv/yr}}{3.6\times10^{-3}\text{ Sv/yr}}=0.11$ or $\boxed{\text{11\% of the background rate}}$.

68. The surface area of a sphere is $4\pi r^2$.

$$\frac{\text{Activity}}{\text{m}^2}=\frac{2.0\times10^{7}\text{ Ci}}{4\pi r_{\text{Earth}}^2}=\frac{(2.0\times10^{7}\text{ Ci})(3.7\times10^{10}\text{ decays/s})}{4\pi(6.38\times10^{6}\text{ m})^2}=\boxed{1.4\times10^{3}\ \frac{\text{decays/s}}{\text{m}^2}}$$

69. $$Q=3m_{{}^4_2\text{He}}c^2-m_{{}^{12}_{6}\text{C}}c^2=\left[3(4.002603\text{ u})-12.000000\text{ u}\right]\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right)c^2=\boxed{7.274\text{ MeV}}$$

70. Since the half-life is 30 years, we assume that the activity does not change during the 1.4 hours of exposure. We calculate the total energy absorbed and then calculate the effective dose. The two energies can be added directly since the RBE factor for both gammas and betas is about 1.

$$\text{Energy}=\left[\begin{array}{c}(1.2\times10^{-6}\text{ Ci})\left(3.7\times10^{10}\,\dfrac{\text{decays}}{\text{s}}\right)(1.4\text{ h})\left(3600\,\dfrac{\text{s}}{1\text{ h}}\right)\\ \times\left(850\times10^{3}\,\dfrac{\text{eV}}{\text{decay}}\right)\left(1.60\times10^{-19}\,\dfrac{\text{J}}{\text{eV}}\right)\end{array}\right]=3.043\times10^{-5}\text{ J}$$

$$\text{dose}=\frac{3.043\times10^{-5}\text{ J}}{62\text{ kg}}\times\frac{100\text{ rad}}{1\text{ J/kg}}=4.909\times10^{-5}\text{ rad}\approx\boxed{4.9\times10^{-5}\text{ rem}}$$

71. The actual power created by the reaction of Eq. 31–8a must be 1000 MW/0.33 = 3030 MW. We convert that to grams of deuterium using Eq. 31–8a.

$$\text{Mass d}=\left(3.03\times10^{9}\,\frac{\text{J}}{\text{s}}\right)\left(\frac{2\text{ d nuclei}}{4.03\text{ MeV}}\right)\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)\left(\frac{2\times10^{-3}\text{ kg d}}{6.02\times10^{23}\text{ nuclei}}\right)\left(\frac{3.156\times10^{7}\text{ s}}{1\text{ yr}}\right)$$

$$=985.4\text{ kg d}\approx\boxed{990\text{ kg d}}$$

72. In the first scenario, slow neutrons have an RBE of 5, from Table 31–1. The maximum dose for a radiation worker is given in the text as 50 mSv/yr, so this worker has been exposed to 50 mSv. Use Eq. 31–10b to convert the Sv to Gy and then convert that to J by the definition of Gy.

$$\text{Energy}=\text{Gy}\times\text{mass}=(50\times10^{-3}\text{ Sv})\left(\frac{1\text{ Gy}}{5\text{ Sv}}\right)\left(\frac{1\text{ J/kg}}{1\text{ Gy}}\right)(65\text{ kg})=\boxed{0.65\text{ J}}$$

In the second scenario, the RBE changes to a value of 2 for the protons.

$$\text{Energy}=\text{Gy}\times\text{mass}=(50\times10^{-3}\text{ Sv})\left(\frac{1\text{ Gy}}{2\text{ Sv}}\right)\left(\frac{1\text{ J/kg}}{1\text{ Gy}}\right)(65\text{ kg})=\boxed{1.6\text{ J}}$$

73. (*a*) There are 92 protons in the reactants, so X must have 92 – 38 = 54 protons. There are 236 total nucleons in the reactants, so X must have 236 – 92 – 3 = 141 total nucleons. Thus, X is $\boxed{{}^{141}_{54}\text{Xe}}$.

(*b*) If the reaction is "barely critical," then every nucleus that fissions leads to another nucleus that fissions. Thus one of the produced neutrons causes another fission. The other two either escape the reactor vessel or are absorbed by some nucleus without causing a fission.

(*c*) The masses needed are ${}^{141}_{54}\text{Xe}$: 140.92665 u and ${}^{92}_{38}\text{Sr}$: 91.911038 u. Use those to calculate the *Q*-value.

$$\begin{aligned} Q &= (m_{{}^{235}_{92}\text{U}} + m_{{}^{1}_{0}\text{n}} - m_{{}^{92}_{38}\text{Sr}} - m_{{}^{141}_{54}\text{Xe}} - 3m_{{}^{1}_{0}\text{n}})c^2 \\ &= [235.043930\text{ u} + 1.008665\text{ u} - 91.911038\text{ u} - 140.92665\text{ u} - 3(1.008665\text{ u})]c^2 \\ &= (0.188912\text{ u})\left(931.5\frac{\text{MeV}/c^2}{\text{u}}\right) = \boxed{176.0\text{ MeV}} \end{aligned}$$

74. The half-life of the strontium isotope is 28.90 years. Use that with Eq. 30–5 to find the time for the activity to be reduced to 15% of its initial value.

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t} \rightarrow 0.15\left(\frac{\Delta N}{\Delta t}\right)_0 = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\frac{0.693}{T_{1/2}}t} \rightarrow 0.15 = e^{-\frac{0.693}{T_{1/2}}t} \rightarrow$$

$$\ln(0.15) = -\frac{0.693}{T_{1/2}}t \rightarrow t = -\frac{T_{1/2}\ln(0.15)}{0.693} = -\frac{(28.90\text{ yr})\ln(0.15)}{0.693} \approx \boxed{79\text{ yr}}$$

75. Source B is more dangerous than source A because of its higher energy. Since the sources have the same activity, they both emit the same number of gammas. Source B can deposit twice as much energy per gamma and therefore causes more biological damage.

Source C is more dangerous than source B because the alphas have an RBE factor up to 20 times larger than the gammas. Thus a number of alphas may have an effective dose up to 20 times higher than the effective dose of the same number of like-energy gammas.

So from most dangerous to least dangerous, the ranking of the sources is $\boxed{\text{C} > \text{B} > \text{A.}}$

We might say that source B is twice as dangerous as source A, and source C is 20 times more dangerous than source B.

76. The whole-body dose can be converted into a number of decays, which would be the maximum number of nuclei that could be in the Tc sample. The RBE factor of gammas is 1.

$$50\text{ mrem} = 50\text{ mrad} \rightarrow (50\times10^{-3}\text{ rad})\left(\frac{1\text{ kg}}{100\text{ rad}}\right)(55\text{ kg})\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.719\times10^{17}\text{ eV}$$

$$(1.719\times10^{17}\text{ eV})\left(\frac{1\text{ effective }\gamma}{140\times10^3\text{ eV}}\right)\left(\frac{2\ \gamma\text{ decays}}{1\text{ effective }\gamma}\right)\left(\frac{1\text{ nucleus}}{1\ \gamma\text{ decay}}\right) = 2.455\times10^{12}\text{ nuclei}$$

This then is the total number of decays that will occur. The activity for this number of nuclei can be calculated from Eq. 30–3b.

$$\begin{aligned} \text{Activity} &= \frac{\Delta N}{\Delta t} = \lambda N = \frac{0.693N}{T_{1/2}} = \frac{0.693(2.455\times10^{12}\text{ decays})}{(6\text{ h})(3600\text{ s/h})}\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ decays/s}}\right) \\ &= 2.129\times10^{-3}\text{ Ci} \simeq \boxed{2\text{ mCi}} \end{aligned}$$

Solutions to Search and Learn Problems

1. (*a*) Three problems that must be overcome to make a functioning fission nuclear reactor:
   (i) the neutrons must be moderated (slowed) so they will react with the fissionable nuclei;
   (ii) the neutrons must be prevented from being absorbed by the wrong nuclei;
   (iii) the neutrons must be prevented from escaping the reaction vessel.

   (*b*) Five environmental problems or dangers resulting from a nuclear fission reactor:
   (i) thermal pollution—the warming of the environment around the nuclear reactor;
   (ii) the need for safe storage of fuel before using it in the reactor;
   (iii) the safe disposal of radioactive waste;
   (iv) radiation damage to the power plant itself;
   (v) accidental release of radiation into the environment.

   (*c*) Breeder reactors make fissionable material that can be used in fission bombs.

2. (*a*) For small nuclei, the binding energy per nucleon increases as the number of nucleons increases. When two small nuclei are fused together, the total energy of the resulting nucleus is smaller than the energy of the two initial nuclei, with the excess energy released in the fusion process.

   (*b*) The initial proton–proton reaction has a much smaller probability of occurring than the other reactions. Since the other reactions can take place relatively quickly compared with the first reaction, it determines the time scale for the full fusion process.

   (*c*) Iron (Fe).

   (*d*) The force of gravity between all of the atoms in the Sun (or star) holds it together.

   (*e*) (i) A tokamak uses magnetic fields to confine the fusion material.
   (ii) Lasers are used to heat the fusion material so quickly that the nuclei cannot move away before they fuse (inertial confinement).

3. The oceans cover about 70% of the Earth, to an average depth of approximately 4 km. The density of the water is approximately $1000\ \text{kg/m}^3$. The mass of water is found by multiplying the volume of water by its density.

$$\begin{aligned}\text{Mass of water} &= 0.7(\text{surface area})(\text{depth})(\text{density})\\ &= (0.7)4\pi(6.38\times10^6\ \text{m})^2(4000\ \text{m})(1000\ \text{kg/m}^3) = 1.43\times10^{21}\ \text{kg water}\end{aligned}$$

Dividing the mass by the molar mass of water and multiplying by Avogadro's number gives the number of water molecules. There are two hydrogen atoms per water molecule. Multiplying the number of hydrogen atoms by the percentage of deuterium gives the number of deuterium atoms.

If the reactions of Eqs. 31–8a and 31–8b are carried out at the same rate, then 4 deuterons would produce 7.30 MeV of energy. Use that relationship to convert the number of deuterons in the oceans to energy.

$$(1.10\times10^{43}\ \text{d})\times\frac{7.30\ \text{MeV}}{4\ \text{d}}\times\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}} = 3.21\times10^{30}\ \text{J} \approx \boxed{3\times10^{30}\ \text{J}}$$

Based on Chapter 26, Problem 23, this is about 30 billion times the annual energy consumption of the United States.

4. (*a*) No carbon is consumed in this cycle because one ${}^{12}_{6}\mathrm{C}$ nucleus is required in the first step of the cycle, and one ${}^{12}_{6}\mathrm{C}$ nucleus is produced in the last step of the cycle. The net effect of the cycle can be found by adding all the reactants and all the products together and canceling those that appear on both sides of the reaction.

$$\begin{aligned}
{}^{12}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H} &\rightarrow {}^{13}_{7}\mathrm{N}+\gamma \\
{}^{13}_{7}\mathrm{N} &\rightarrow {}^{13}_{6}\mathrm{C}+\mathrm{e}^{+}+\nu \\
{}^{13}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H} &\rightarrow {}^{14}_{7}\mathrm{N}+\gamma \\
{}^{14}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H} &\rightarrow {}^{15}_{8}\mathrm{O}+\gamma \\
{}^{15}_{8}\mathrm{O} &\rightarrow {}^{15}_{7}\mathrm{N}+\mathrm{e}^{+}+\nu \\
{}^{15}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H} &\rightarrow {}^{12}_{6}\mathrm{C}+{}^{4}_{2}\mathrm{He}
\end{aligned}$$

$$\begin{aligned}
&{}^{12}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}+{}^{13}_{7}\mathrm{N}+{}^{13}_{6}\mathrm{C}+{}^{1}_{1}\mathrm{H}+{}^{14}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H}+{}^{15}_{8}\mathrm{O}+{}^{15}_{7}\mathrm{N}+{}^{1}_{1}\mathrm{H} \\
&\rightarrow {}^{13}_{7}\mathrm{N}+\gamma+{}^{13}_{6}\mathrm{C}+\mathrm{e}^{+}+\nu+{}^{14}_{7}\mathrm{N}+\gamma+{}^{15}_{8}\mathrm{O}+\gamma+{}^{15}_{7}\mathrm{N}+\mathrm{e}^{+}+\nu+{}^{12}_{6}\mathrm{C}+{}^{4}_{2}\mathrm{He}
\end{aligned}$$

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He}+2\,\mathrm{e}^{+}+2\nu+3\gamma$$

Eq. 31–7 shows the proton–proton chain, which is only different from the carbon cycle by the emission of one additional gamma ray in the carbon cycle.

(*b*) To use the values from Appendix B, we must be sure that the number of electrons is balanced as well as the number of protons and neutrons. The above "net" equation does not consider the electrons that neutral nuclei would have, because it does not conserve charge. What the above reaction really represents (ignoring the gammas and neutrinos) is the following:

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He}+2\,\mathrm{e}^{+} \Rightarrow 4\,{}^{1}_{1}\mathrm{p} \rightarrow 2\,{}^{1}_{1}\mathrm{p}+2\,{}^{1}_{0}\mathrm{n}+2\mathrm{e}^{+}$$

To use the values from Appendix B, we must add 4 electrons to each side of the reaction.

$$(4{}^{1}_{1}\mathrm{p}+4\mathrm{e}^{-}) \rightarrow (2{}^{1}_{1}\mathrm{p}+2\mathrm{e}^{-}+2{}^{1}_{0}\mathrm{n})+2\mathrm{e}^{+}+2\mathrm{e}^{-} \Rightarrow 4{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He}+2\mathrm{e}^{+}+2\mathrm{e}^{-}$$

The energy produced in the reaction is the Q-value.

$$\begin{aligned}
Q &= 4m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2 - 4m_{\mathrm{e}} \\
&= [4(1.007825\ \mathrm{u}) - 4.002603\ \mathrm{u} - 4(0.000549\ \mathrm{u})]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 24.69\ \mathrm{MeV}
\end{aligned}$$

As mentioned in the text, each positron–electron annihilation produces another 1.02 MeV, so the total energy released is $24.69\ \mathrm{MeV}+2(1.02\ \mathrm{MeV})=\boxed{26.73\ \mathrm{MeV}}$.

(*c*) In some reactions extra electrons must be added in order to use the values from Appendix B.

The <u>first equation</u> in the carbon cycle is electron-balanced, so Appendix B can be used directly.

$$\begin{aligned}
Q &= m_{{}^{12}_{6}\mathrm{C}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{13}_{7}\mathrm{N}}c^2 \\
&= [12.000000\ \mathrm{u}+1.007825\ \mathrm{u}-13.005739\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{1.943\ \mathrm{MeV}}
\end{aligned}$$

The <u>second equation</u> needs to have another electron, so that ${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + \mathrm{e}^- + \mathrm{e}^+ + v.$

$$Q = m_{{}^{13}_{7}\mathrm{N}}c^2 - m_{{}^{13}_{6}\mathrm{C}}c^2 - 2m_\mathrm{e}c^2$$
$$= [13.005739\ \mathrm{u} - 13.003355\ \mathrm{u} - 2(0.000549\ \mathrm{u})]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 1.198\ \mathrm{MeV}$$

We must include an electron–positron annihilation in this reaction.

$$1.198\ \mathrm{MeV} + 1.02\ \mathrm{MeV} = \boxed{2.218\ \mathrm{MeV}}$$

The <u>third equation</u> of the carbon cycle is electron-balanced.

$$Q = m_{{}^{13}_{6}\mathrm{C}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{14}_{7}\mathrm{N}}c^2$$
$$= [13.003355\ \mathrm{u} + 1.007825\ \mathrm{u} - 14.003074\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.551\ \mathrm{MeV}}$$

The <u>fourth equation</u> of the carbon cycle is also electron-balanced.

$$Q = m_{{}^{14}_{7}\mathrm{N}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{15}_{8}\mathrm{O}}c^2$$
$$= [14.003074\ \mathrm{u} + 1.007825\ \mathrm{u} - 15.003066\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = \boxed{7.296\ \mathrm{MeV}}$$

The <u>fifth equation</u> needs to have another electron, so ${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \mathrm{e}^- + \mathrm{e}^+ + v.$

$$Q = m_{{}^{15}_{8}\mathrm{O}}c^2 - m_{{}^{15}_{7}\mathrm{N}}c^2 - 2m_\mathrm{e}c^2$$
$$= [15.003066\ \mathrm{u} - 15.000109\ \mathrm{u} - 2(0.000549\ \mathrm{u})]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2 = 1.732\ \mathrm{MeV}$$

We must include an electron–positron annihilation in this reaction.

$$1.732\ \mathrm{MeV} + 1.02\ \mathrm{MeV} = \boxed{2.752\ \mathrm{MeV}}$$

The <u>sixth equation</u> is electron-balanced.

$$Q = m_{{}^{15}_{7}\mathrm{N}}c^2 + m_{{}^{1}_{1}\mathrm{H}}c^2 - m_{{}^{12}_{6}\mathrm{C}}c^2 - m_{{}^{4}_{2}\mathrm{He}}c^2$$
$$= [15.000109\ \mathrm{u} + 1.007825\ \mathrm{u} - 12.000000\ \mathrm{u} - 4.002603\ \mathrm{u}]\left(931.5\frac{\mathrm{MeV}/c^2}{\mathrm{u}}\right)c^2$$
$$= \boxed{4.966\ \mathrm{MeV}}$$

The total energy released is found by summing the energy released in each process.

$$1.943\ \mathrm{MeV} + 2.218\ \mathrm{MeV} + 7.551\ \mathrm{MeV} + 7.296\ \mathrm{MeV} + 2.752\ \mathrm{MeV} + 4.966\ \mathrm{MeV}$$
$$= \boxed{26.73\ \mathrm{MeV}}$$

(*d*) This reaction requires a higher temperature than the proton–proton reaction because the reactants need to have more initial kinetic energy to overcome the Coulomb repulsion between the nuclei. In particular, the carbon and nitrogen nuclei have higher *Z* values, leading to a greater Coulomb repulsion that must be overcome by a higher temperature (so that the nuclei are moving faster).

5. (*a*) The advantages of nuclear fusion include that its by-products are not radioactive, but chemically inert helium. Also, the source for fusion is hydrogen, which is a plentiful resource.
   (*b*) The major technological problem with fusion is developing a way to confine the nuclei at a sufficiently high temperature and density for fusion to occur.
   (*c*) (i) Magnetic confinement (such as in a tokamak): A region with circular magnetic field lines is created such that the charged particles are confined to orbit the field lines as they are heated to very high temperatures.
   (ii) Inertial confinement: A pellet containing a hydrogen-rich compound is bombarded with lasers that heat the pellet to very high temperatures. The heating occurs so rapidly that the inertia of the hydrogen holds it in place long enough for fusion to occur.
   (*d*) The preferred fuel for a fusion reactor is deuterium, a hydrogen isotope with one proton and one neutron.
   (*e*) ${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \quad \rightarrow \quad {}^{4}_{2}\mathrm{He}$
   (*f*) When using the data in Appendix B, it is important that the electrons of the atoms be correctly accounted for. The two hydrogen atoms each have one electron. The resulting helium atom has two electrons. Since there are two electrons before the reaction and two after, the equation is balanced, and the data from Appendix B can be used.

$$Q = \left[2m\left({}^{2}_{1}\mathrm{H}\right) - m\left({}^{4}_{2}\mathrm{He}\right)\right]c^2$$

$$= \left[2(2.014102\text{ u}) - 4.002603\text{ u}\right]\left(\frac{931.5\text{ MeV}/c^2}{1\text{ u}}\right) = \boxed{23.85\text{ MeV}}$$

This agrees with the estimate made in Example 31–7.

6. (*a*) Alpha and beta particles are charged particles and can ionize materials through electrical forces. Gamma rays and energetic neutrons are not charged, but they can ionize materials by Compton scattering off of the bound electrons in the materials.
   (*b*) Metals become brittle, and their strength is weakened by intense radiation, mostly due to atom dislocations in the metal's crystal structure.
   (*c*) Living cells can be damaged in two ways. First, the radiation can produce ions that interfere with the normal function of the cell. When a few cells are damaged, they die and the body replaces them. When many cells are damaged simultaneously, the body cannot replace them fast enough and the person experiences radiation poisoning. In the second way, the radiation can ionize, and therefore alter, the DNA within the cell. This can mutate genes within the cell and affect how the cell functions. The result can be tumorous growths and/or cancer.

32

ELEMENTARY PARTICLES

Responses to Questions

1. A reaction between two nucleons that would produce a π^- is $p + n \rightarrow p + p + \pi^-$.

2. No, the decay is still impossible. In the rest frame of the proton, this decay is energetically impossible, because the proton's mass is less than the mass of the products. Since it is impossible in the rest frame, it is impossible in every other frame as well. In a frame in which the proton is moving very fast, the decay products must be moving very fast as well to conserve momentum. With this constraint, there will still not be enough energy to make the decay energetically possible.

3. Antiatoms would be made up of $(-)$ charged antiprotons and neutral antineutrons in the nucleus with $(+)$ charged positrons surrounding the nucleus. If antimatter and matter came into contact, then the particle–antiparticle pairs would annihilate, converting their mass into energetic photons.

4. The photon signals the electromagnetic interaction.

5. (*a*) Yes. If a neutrino is produced during a decay, then the weak interaction is responsible.
   (*b*) No. For example, a weak interaction decay could produce a Z^0 instead of a neutrino.

6. The neutron decay process also produces an electron and an antineutrino; the antineutrino will only be present in a weak interaction.

7. An electron takes part in the electromagnetic, weak, and gravitational interactions. A neutrino takes part in the weak and gravitational interactions. A proton takes part in the strong, electromagnetic, weak, and gravitational interactions.

8. The following chart shows charge and baryon conservation checks for many of the decays in Table 32–2 of the textbook.

Particle	Decay	Charge conservation	Baryon conservation
W	$W^+ \rightarrow e^+ + \nu_e$ (others are similar)	$+1 = +1 + 0$	$0 = 0 + 0$
Z^0	$Z^0 \rightarrow e^+ + e^-$ (others are similar)	$0 = +1 + (-1)$	$0 = 0 + 0$

Particle	Decay	Charge conservation	Baryon conservation
Higgs	$H^0 \rightarrow b+\overline{b}$ $H^0 \rightarrow W^+ + W^-$ $H^0 \rightarrow Z^0 + Z^0$ (others are similar)	$0=\left(-\frac{1}{3}\right)+\frac{1}{3}$ $0=+1+(-1)$ $0=0+0$	$0=\frac{1}{3}+\left(-\frac{1}{3}\right)$ $0=0+0$ $0=0+0$
muon	$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$	$-1=(-1)+0+0$	$0=0+0+0$
tau	$\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$ (other is similar)	$-1=(-1)+0+0$	$0=0+0+0$
pion	$\pi^+ \rightarrow \mu^+ + \nu_\mu$ $\pi^0 \rightarrow \gamma+\gamma$	$+1=+1+0$ $0=0+0$	$0=0+0$ $0=0+0$
kaon	$K^+ \rightarrow \mu^+ + \nu_\mu$ $K^+ \rightarrow \pi^+ + \pi^0$ $K_S^0 \rightarrow \pi^+ + \pi^-$ $K_S^0 \rightarrow \pi^0 + \pi^0$ $K_L^0 \rightarrow \pi^+ + e^- + \overline{\nu}_e$ $K_L^0 \rightarrow \pi^+ + \mu^- + \overline{\nu}_\mu$ $K_L^0 \rightarrow \pi^+ + \pi^- + \pi^0$ $K_L^0 \rightarrow \pi^0 + \pi^0 + \pi^0$	$+1=+1+0$ $+1=+1+0$ $0=+1+(-1)$ $0=0+0$ $0=+1+(-1)+0$ $0=+1+(-1)+0$ $0=+1+(-1)+0$ $0=0+0+0$	$0=0+0$ $0=0+0$ $0=0+0$ $0=0+0$ $0=0+0+0$ $0=0+0+0$ $0=0+0+0$ $0=0+0+0$
eta	$\eta^0 \rightarrow \gamma+\gamma$ $\eta^0 \rightarrow \pi^0+\pi^0+\pi^0$ $\eta^0 \rightarrow \pi^+ + \pi^- + \pi^0$	$0=0+0$ $0=0+0+0$ $0=+1+(-1)+0$	$0=0+0$ $0=0+0+0$ $0=0+0+0$
rho	$\rho^0 \rightarrow \pi^+ + \pi^-$ $\rho^0 \rightarrow \pi^0 + \pi^0$ $\rho^+ \rightarrow \pi^+ + \pi^0$	$0=+1+(-1)$ $0=0+0$ $+1=+1+0$	$0=0+0$ $0=0+0$ $0=0+0$
neutron	$n \rightarrow p + e^- + \overline{\nu}_e$	$0=+1+(-1)+0$	$+1=+1+0+0$
lambda	$\Lambda^0 \rightarrow p+\pi^-$ $\Lambda^0 \rightarrow n+\pi^0$	$0=+1+(-1)$ $0=0+0$	$+1=+1+0$ $+1=+1+0$
sigma	$\Sigma^+ \rightarrow p+\pi^0$ $\Sigma^+ \rightarrow n+\pi^+$ $\Sigma^0 \rightarrow \Lambda^0+\gamma$ $\Sigma^- \rightarrow n+\pi^-$	$+1=+1+0$ $+1=0+1$ $0=0+0$ $-1=0+(-1)$	$+1=+1+0$ $+1=+1+0$ $+1=+1+0$ $+1=+1+0$

Particle	Decay	Charge conservation	Baryon conservation
xi	$\Xi^0 \rightarrow \Lambda^0 + \pi^0$ $\Xi^- \rightarrow \Lambda^0 + \pi^-$	$0 = 0 + 0$ $-1 = 0 + (-1)$	$+1 = +1 + 0$ $+1 = +1 + 0$
omega	$\Omega^- \rightarrow \Xi^0 + \pi^-$ $\Omega^- \rightarrow \Lambda^0 + K^-$ $\Omega^- \rightarrow \Xi^- + \pi^0$	$-1 = 0 + (-1)$ $-1 = 0 + (-1)$ $-1 = (-1) + 0$	$+1 = +1 + 0$ $+1 = +1 + 0$ $+1 = +1 + 0$

9. Since decays via the electromagnetic interaction are indicated by the production of photons, the decays in Table 32–2 that occur via the electromagnetic interaction are those of the Higgs boson, the π^0, the Σ^0, and the η^0.

10. All of the decays listed in Table 32–2 with a neutrino or antineutrino as a decay product occur via the weak interaction. These include the W, muon, tau, pion, kaon, K_L^0, and neutron. In addition, the Z particle and the Higgs boson both decay via the weak interaction. In each case, include both the particle and the corresponding antiparticle.

11. Since the Δ baryon has $B = 1$, it is made of three quarks. Since the spin of the Δ baryon is 3/2, none of these quarks can be antiquarks. Thus, since the charges of quarks are either $+2/3$ or $-1/3$, the only charges that can be created with this combination are $q = -1(= -1/3 - 1/3 - 1/3)$, $0\ (= +2/3 - 1/3 - 1/3)$, $+1\ (= +2/3 + 2/3 - 1/3)$, and $+2\ (= +2/3 + 2/3 + 2/3)$.

12. Based on the lifetimes shown in Table 32–4 of the textbook, the particle decays that occur via the electromagnetic interaction are $J/\psi(3097)$ and $\Upsilon(9460)$.

13. All of the particles in Table 32–4 except for $J/\psi(3097)$, $\Upsilon(9460)$, and the sigma particles decay via the weak interaction, based on their lifetimes.

14. Baryons are formed from three quarks or antiquarks, each of spin $\frac{1}{2}$ or $-\frac{1}{2}$, respectively. Any combination of quarks and antiquarks will yield a spin magnitude of either $\frac{1}{2}$ or $\frac{3}{2}$. Mesons are formed from two quarks or antiquarks. Any combination of two quarks or antiquarks will yield a spin magnitude of either 0 or 1.

$\boxed{15.}$ If a neutrinolet was massless, then it would not interact via the gravitation force; if it had no electrical charge, then it would not interact via the electromagnetic force; if it had no color charge, then it would not interact via the strong force; and if it does not interact via the weak force, then it would not interact with matter at all. It would be very difficult to say that it even exists at all. However, a similar argument could be made for photons. Photons have no color, no mass, and no charge, but they do exist.

16. (*a*) No. Leptons are fundamental particles with no known internal structure. Baryons are made up of three quarks.
(*b*) Yes. All baryons are hadrons.
(*c*) No. A meson is a quark–antiquark pair.
(*d*) No. Hadrons are made up of quarks, and leptons are fundamental particles.

17. No. A particle made up of two quarks would have a particular color. Three quarks or a quark–antiquark pair is necessary for the particle to be white or colorless. A combination of two quarks and two antiquarks is possible, as the resulting particle could be white or colorless. The neutral pion can in some ways be considered a four-quark combination.

18. In the nucleus, the strong interaction with the other nucleons does not allow the neutron to decay. When a neutron is free, the weak interaction is the dominant force and can cause the neutron to decay.

19. No, the reaction $e^- + p \rightarrow n + \overline{\nu}_e$ is not possible. The electron lepton number is not conserved: The reactants have $L_e = 1 + 0 = 1$, but the products have $L_e = 0 - 1 = -1$. Thus, this reaction is not possible. If the product were an electron neutrino (instead of an antineutrino), then the reaction would be possible.

20. The reaction $\Lambda^0 \rightarrow p^+ + e^- + \overline{\nu}_e$ proceeds via the weak force. We know that this is the case since an antineutrino is emitted, which only happens in reactions governed by the weak interaction.

Responses to MisConceptual Questions

1. (*a*) A common misconception is that all six quarks make up most of the known matter. However, charmed, strange, top, and bottom quarks are unstable. Protons and neutrons are made of up and down quarks, so they make up most of the known matter.

2. (*b*, *e*) Quarks combine together, forming mesons (such as the π meson) and baryons (such as the proton and the neutron). The electron and Higgs boson are fundamental particles and therefore cannot be made of quarks.

3. (*c*) All of the fundamental forces act on a variety of objects, including our bodies. Although the strong and weak forces are very short range, the electromagnetic force is a long-range force, just like gravity. One reason we notice the "weak" gravity force more than the electromagnetic force is that most objects are electrically neutral, so they do not have significant net electromagnetic forces on them. It is true that the gravitational force between people and other objects of similar size is too small for us to notice, but due to the huge mass of the Earth, we are always aware of the influence of the Earth's gravitational force on us.

4. (*d*) A tau lepton has a tau lepton number of one. When it decays, the lepton number must be conserved, so it cannot decay into only hadrons; at least a tau neutrino would have to be one of the by-products.

5. (*d*) In a circular accelerator, the beam can repeatedly pass through the same physical location each time around. A single accelerating potential difference at one location can then be used to increase the beam energy multiple times. In a linear accelerator, the beam passes each point only once, so the beam path needs to be very long with many accelerating potential differences along its path.

6. (*b*, *e*, *f*, *g*) Atoms are not fundamental, because they are made up of protons, neutrons, and electrons. Protons and neutrons are not fundamental, as they are made up of quarks. The fundamental particles are leptons (including electrons), quarks, and gauge bosons (including the photon and Higgs boson).

7. (*a*) Unlike the electron, the positron has positive charge and a negative lepton number. Unlike the charge and lepton number, the mass of the positron has the same sign (and magnitude) as the mass of the electron.

8. (*e*) A common misconception is that the strong force is a result of just the exchange of π mesons between the protons and neutrons. This is correct on the scale of the nucleons. However, when

the quark composition of the protons, neutrons, and π mesons is considered at the elementary particle scale, it is seen that the transfer is due to the exchange of gluons. Therefore, both answers can be considered correct at different scales. Students who answer (*d*) should be given credit for their answer as well.

9. (*c*) Pions are not fundamental particles and are made up of quark and antiquark pairs. Leptons and bosons (including photons) are fundamental particles, but are not a constituent of protons and neutrons. Protons and neutrons are composed of up and down quarks.

10. (*d*) Quarks, gluons, neutrons, and the Higgs boson interact through the strong force. Electrons and muons are charged particles and interact through the electromagnetic force. Neutrinos only interact through the weak force.

Solutions to Problems

1. The total energy is given by Eq. 26–6a.

$$E = mc^2 + \text{KE} = 0.938 \text{ GeV} + 4.65 \text{ GeV} = \boxed{5.59 \text{ GeV}}$$

$\boxed{2.}$ Because the energy of the electrons is much greater than their mass, we have $\text{KE} = E = pc.$ Combine that with Eq. 32–1 for the de Broglie wavelength.

$$E = pc;\ p = \frac{h}{\lambda} \quad\to\quad E = \frac{hc}{\lambda} \quad\to\quad \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(28\times10^{9}\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{4.4\times10^{-17}\text{ m}}$$

3. Use Eq. 32–2 to calculate the frequency.

$$f = \frac{qB}{2\pi m} = \frac{2(1.60\times10^{-19}\text{ C})(1.7\text{ T})}{2\pi[4(1.67\times10^{-27}\text{ kg})]} = \boxed{1.3\times10^{7}\text{ Hz}} = 13\text{ MHz}$$

4. The time for one revolution is the period of revolution, which is the circumference of the orbit divided by the speed of the protons. Since the protons have very high energy, their speed is essentially the speed of light.

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.0\times10^{3}\text{ m})}{3.0\times10^{8}\text{ m/s}} = \boxed{2.1\times10^{-5}\text{ s}}$$

$\boxed{5.}$ The frequency is related to the magnetic field by Eq. 32–2.

$$f = \frac{qB}{2\pi m} \quad\to\quad B = \frac{2\pi mf}{q} = \frac{2\pi(1.67\times10^{-27}\text{ kg})(3.1\times10^{7}\text{ Hz})}{1.60\times10^{-19}\text{ C}} = \boxed{2.0\text{ T}}$$

6. (*a*) The maximum kinetic energy is $\text{KE} = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2.$ Compared to Example 32–2, the charge has been doubled and the mass has been multiplied by 4. These two effects cancel each other in the equation, so the maximum kinetic energy is unchanged (8.653 MeV).

$$\text{KE} = \boxed{8.7\text{ MeV}} \qquad v = \sqrt{\frac{2\,\text{KE}}{m}} = \sqrt{\frac{2(8.653\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{4(1.66\times10^{-27}\text{ kg})}} = \boxed{2.0\times10^{7}\text{ m/s}}$$

(*b*) The maximum kinetic energy is $\text{KE} = \frac{q^2B^2R^2}{2m} = \frac{1}{2}mv^2$. Compared to Example 32–2, the charge is unchanged and the mass has been multiplied by 2. Thus the kinetic energy will be half of what it was in Example 32–2 (8.654 MeV).

$$\text{KE} = \boxed{4.3\text{ MeV}} \quad v = \sqrt{\frac{2\text{ KE}}{m}} = \sqrt{\frac{2(4.327\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{2(1.66\times10^{-27}\text{ kg})}} = \boxed{2.0\times10^{7}\text{ m/s}}$$

The alpha and the deuteron have the same charge to mass ratio, so they move at the same speed.

(*c*) The frequency is given by $f = \frac{qB}{2\pi m}$. Since the charge to mass ratio of both the alpha and the deuteron is half that of the proton, the frequency for both the alpha and the deuteron will be half the frequency found in Example 32–2 for the proton (25.94 MHz).

$$\boxed{f = 13\text{ MHz}}$$

7. From Eq. 30–1, the diameter of a nucleon is about $d_{\text{nucleon}} = 2.4\times10^{-15}$ m. The 25-MeV alpha particles and protons are not relativistic, so their momentum is given by $p = mv = \sqrt{2m\text{KE}}$. The wavelength is given by Eq. 32–1, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m\text{KE}}}$.

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha\text{KE}}} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(4)(1.66\times10^{-27}\text{ kg})(25\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV})}} = 2.88\times10^{-15}\text{ m}$$

$$\lambda_\text{p} = \frac{h}{\sqrt{2m_\text{p}\text{KE}}} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(1.67\times10^{-27}\text{ kg})(25\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV})}} = 5.75\times10^{-15}\text{ m}$$

We see that $\boxed{\lambda_\alpha \approx d_{\text{nucleon}}}$ and $\boxed{\lambda_\text{p} \approx 2d_{\text{nucleon}}}$. Thus, the $\boxed{\text{alpha particle will be better}}$ for picking out details in the nucleus.

$\boxed{8.}$ Because the energy of the protons is much greater than their mass, we have $\text{KE} = E = pc$. Combine that with Eq. 32–1 for the de Broglie wavelength. That is the minimum size that protons of that energy could resolve.

$$E = pc; \quad p = \frac{h}{\lambda} \;\rightarrow\; E = \frac{hc}{\lambda} \;\rightarrow\; \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.0\times10^{8}\text{ m/s})}{(7.0\times10^{12}\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{1.8\times10^{-19}\text{ m}}$$

9. If the speed of the protons is *c*, then the time for one revolution is found from uniform circular motion. The number of revolutions is the total time divided by the time for one revolution. The energy per revolution is the total energy gained divided by the number of revolutions.

$$v = \frac{2\pi r}{T} \;\rightarrow\; T = \frac{2\pi r}{v} = \frac{2\pi r}{c} \qquad n = \frac{t}{T} = \frac{ct}{2\pi r}$$

$$\text{Engergy/revolution} = \frac{\Delta E}{n} = \frac{(\Delta E)2\pi r}{ct} = \frac{(1.0\times10^{12}\text{ eV} - 150\times10^{9}\text{ eV})2\pi(1.0\times10^{3}\text{ m})}{(3.00\times10^{8}\text{ m/s})(20\text{ s})}$$

$$= 8.9\times10^{5}\text{ eV/rev} \approx \boxed{0.9\text{ MeV/rev}}$$

10. (*a*) The magnetic field is found from the maximum kinetic energy as derived in Example 32–2.

$$\text{KE} = \frac{q^2B^2R^2}{2m} \rightarrow B = \frac{\sqrt{2m\text{KE}}}{qR} \rightarrow$$

$$B = \frac{\sqrt{2(2.014)(1.66\times10^{-27}\ \text{kg})(12\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}}{(1.60\times10^{-19}\ \text{C})(1.0\ \text{m})} = 0.7082\ \text{T} \approx \boxed{0.71\ \text{T}}$$

(*b*) The cyclotron frequency is given by Eq. 32–2.

$$f = \frac{qB}{2\pi m} = \frac{(1.60\times10^{-19}\ \text{C})(0.7082\ \text{T})}{2\pi(2.014)(1.66\times10^{-27}\ \text{kg})} = 5.394\times10^{6}\ \text{Hz} \approx \boxed{5.4\ \text{MHz}}$$

(*c*) The deuteron will be accelerated twice per revolution, so it will gain energy equal to twice its charge times the voltage on each revolution.

$$\text{number of revolutions} = n = \frac{12\times10^{6}\ \text{eV}}{2(1.60\times10^{-19}\ \text{C})(22\times10^{3}\ \text{V})}(1.60\times10^{-19}\ \text{J/eV})$$

$$= 273\ \text{revolutions} \approx \boxed{270\ \text{revolutions}}$$

(*d*) The time is the number of revolutions divided by the frequency (which is revolutions per second).

$$\Delta t = \frac{n}{f} = \frac{273\ \text{revolutions}}{5.394\times10^{6}\ \text{rev/s}} = \boxed{5.1\times10^{-5}\ \text{s}}$$

(*e*) If we use an average radius of half the radius of the cyclotron, then the distance traveled is the average circumference times the number of revolutions.

$$\text{distance} = \tfrac{1}{2}2\pi rn = \pi(1.0\ \text{m})(273) = \boxed{860\ \text{m}}$$

11. Start with an expression from Section 32–1, relating the momentum and radius of curvature for a particle in a magnetic field, with q replaced by e.

$$v = \frac{eBr}{m} \rightarrow mv = eBr \rightarrow p = eBr$$

In the relativistic limit, $p = E/c$, so $\frac{E}{c} = eBr.$ To get the energy in electron volts, divide the energy by the charge of the object.

$$\frac{E}{c} = eBr \rightarrow \boxed{\frac{E}{e} = Brc}$$

12. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\Lambda^0}c^2 - m_nc^2 - m_{\pi^0}c^2 = 1115.7\ \text{MeV} - 939.6\ \text{MeV} - 135.0\ \text{MeV} = \boxed{41.1\ \text{MeV}}$$

13. The energy released is the difference in the mass energy between the products and the reactant.

$$\Delta E = m_{\pi^+}c^2 - m_{\mu^+}c^2 - m_{\nu_\mu}c^2 = 139.6\ \text{MeV} - 105.7\ \text{MeV} - 0 = \boxed{33.9\ \text{MeV}}$$

14. Use Eq. 32–3 to estimate the range of the force based on the mass of the mediating particle.

$$mc^2 \approx \frac{hc}{2\pi d} \rightarrow d \approx \frac{hc}{2\pi mc^2} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{2\pi(497.7\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} \approx \boxed{3.98\times10^{-16}\ \text{m}}$$

15. The energy required is the mass energy of the two particles.

$$E = 2m_n c^2 = 2(939.6\text{ MeV}) = \boxed{1879.2\text{ MeV}}$$

16. The reaction is multistep and can be written as shown here. The energy released is the initial rest energy minus the final rest energy of the proton and pion, using Table 32–2.

$$\begin{array}{lll} \Sigma^0 & \rightarrow & \Lambda^0 + \gamma \\ & \downarrow & \rightarrow \quad p + \pi^- \end{array}$$

$$\Delta E = (m_{\Sigma^0} - m_{\text{p}} - m_{\pi^-})c^2 = 1192.6\text{ MeV} - 938.3\text{ MeV} - 139.6\text{ MeV} = \boxed{114.7\text{ MeV}}$$

17. Because the two protons are heading toward each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all three particles at rest, so the minimum kinetic energy of the collision must be equal to the mass energy of the π^0. Each proton will have half of that kinetic energy. From Table 32–2, the mass of the π^0 is $135.0\text{ MeV}/c^2$.

$$2(\text{KE}_{\text{proton}}) = m_{\pi^0}c^2 = 135.0\text{ MeV} \quad \rightarrow \quad \text{KE}_{\text{proton}} = \boxed{67.5\text{ MeV}}$$

18. Because the two particles have the same mass and they are traveling toward each other with the same speed, the total momentum of the system is 0. The minimum kinetic energy for the collision would result in all four particles at rest, so the minimum kinetic energy of the collision must be equal to the mass energy of the K^+K^- pair. Each initial particle will have half of that kinetic energy. From Table 32–2, the mass of each K^+ and K^- is $493.7\text{ MeV}/c^2$.

$$2(\text{KE}_{\text{p or }\overline{\text{p}}}) = 2m_{\text{K}}c^2 \quad \rightarrow \quad \text{KE}_{\text{p or }\overline{\text{p}}} = m_{\text{K}}c^2 = \boxed{493.7\text{ MeV}}$$

19. The energy of the two photons (assumed to be equal so that momentum is conserved) must be the combined mass energy of the proton and antiproton.

$$2mc^2 = 2hf = 2h\frac{c}{\lambda} \quad \rightarrow \quad \lambda = \frac{hc}{mc^2} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(938.3\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{1.32\times10^{-15}\text{ m}}$$

20. (*a*) $\Lambda^0 \rightarrow \text{n} + \pi^-$ Charge conservation is violated, since $0 \neq 0 - 1$.
Strangeness is violated, since $-1 \neq 0 + 0$.

(*b*) $\Lambda^0 \rightarrow \text{p} + \text{K}^-$ Energy conservation is violated, since
$1115.7\text{ MeV}/c^2 < 938.3\text{ MeV}/c^2 + 493.7\text{ MeV}/c^2 = 1432.0\text{ MeV}/c^2$.

(*c*) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number conservation is violated, since $1 \neq 0 + 0$.
Strangeness is violated, since $-1 \neq 0 + 0$.
Spin is violated, since $\frac{1}{2} \neq 0 + 0$.

$\boxed{21.}$ The total momentum of the electron and positron is 0, so the total momentum of the two photons must be 0. Thus each photon has the same momentum, so each photon also has the same energy. The total energy of the photons must be the total energy of the electron–positron pair.

$$E_{\text{e}^+/\text{e}^-\text{ pair}} = E_{\text{photons}} \quad \rightarrow \quad 2(mc^2 + \text{KE}) = 2hf = 2h\frac{c}{\lambda} \quad \rightarrow$$

$$\lambda = \frac{hc}{mc^2 + \text{KE}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(0.511\times10^6\text{ eV} + 420\times10^3\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 1.335\times10^{-12}\text{ m} \approx \boxed{1.3\times10^{-12}\text{ m}}$$

22. (*a*) For the reaction $\pi^- + \mathrm{p} \rightarrow \mathrm{n} + \eta^0$, the conservation laws are as follows:

Charge: $-1+1=0+0$ Charge is conserved.
Baryon number: $0+1=1+0$ Baryon number is conserved.
Lepton number: $0+0=0+0$ Lepton number is conserved.
Strangeness: $0+0=0+0$ Strangeness is conserved.

$\boxed{\text{The reaction is possible.}}$

(*b*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \mathrm{n} + \pi^0$, the conservation laws are as follows:

Charge: $1+1 \neq 0+0$ Charge is NOT conserved.

$\boxed{\text{The reaction is forbidden, because charge is not conserved.}}$

(*c*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{e}^+$, the conservation laws are as follows:

Charge: $1+1=1+1$ Charge is conserved.
Baryon number: $0+1=1+0$ Baryon number is conserved.
Lepton number: $0+0 \neq 0+1$ Lepton number is NOT conserved.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*d*) For the reaction $\mathrm{p} \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows:

Charge: $1=1+0$ Charge is conserved.
Baryon number: $1 \neq 0+0$ Baryon number NOT conserved.
Mass energy is fine, because $m_\mathrm{p} > m_\mathrm{e} + m_\nu$.

$\boxed{\text{The reaction is forbidden, because baryon number is not conserved.}}$

(*e*) For the reaction $\mu^+ \rightarrow \mathrm{e}^+ + \overline{\nu}_\mu$, the conservation laws are as follows:

Charge: $1=1+0$ Charge is conserved.
Baryon number: $0=0+0$ Baryon number is conserved.
Electron lepton number: $0 \neq -1+0$ Lepton number is NOT conserved.
Mass energy is fine, because $m_\mu > m_\mathrm{e} + m_\nu$.

$\boxed{\text{The reaction is forbidden, because lepton number is not conserved.}}$

(*f*) For the reaction $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$, the conservation laws are as follows:

Mass energy: $938.3\ \mathrm{MeV}/c^2 < 939.6\ \mathrm{MeV}/c^2 + 0.511\ \mathrm{MeV}/c^2$

Mass energy is NOT conserved.

$\boxed{\text{The reaction is forbidden, because energy is not conserved.}}$

23. $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\overline{\mathrm{p}}$: This reaction will not happen because charge is not conserved $(2 \neq 0)$, and baryon number is not conserved $(2 \neq 0)$.

$\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+\overline{\mathrm{p}}$: This reaction will not happen because charge is not conserved $(2 \neq 1)$, and baryon number is not conserved $(2 \neq 1)$.

$p + p \rightarrow p + p + p + \bar{p}$: This reaction is possible. All conservation laws are satisfied.

$p + p \rightarrow p + e^+ + e^+ + \bar{p}$: This reaction will not happen. Baryon number is not conserved $(2 \neq 0)$, and lepton number is not conserved $(0 \neq -2)$.

24. Since the pion decays from rest, the momentum before the decay is zero. Thus the momentum after the decay is also zero, so the magnitudes of the momenta of the positron and the neutrino are equal. We also treat the neutrino as massless. Use energy and momentum conservation along with the relativistic relationship between energy and momentum.

$$m_{\pi^+}c^2 = E_{e^+} + E_\nu;\quad p_{e^+} = p_\nu \quad\rightarrow\quad (p_{e^+}^2c^2) = (p_\nu^2c^2) \quad\rightarrow\quad E_{e^+}^2 - m_{e^+}^2c^4 = E_\nu^2$$

$$E_{e^+}^2 - m_{e^+}^2c^4 = (m_{\pi^+}c^2 - E_{e^+})^2 = m_{\pi^+}^2c^4 - 2E_{e^+}m_{\pi^+}c^2 + E_{e^+}^2 \quad\rightarrow\quad 2E_{e^+}m_{\pi^+}c^2 = m_{\pi^+}^2c^4 + m_{e^+}^2c^4$$

$$E_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \quad\rightarrow\quad \mathrm{KE}_{e^+} + m_{e^+}c^2 = \tfrac{1}{2}m_{\pi^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} \quad\rightarrow$$

$$\mathrm{KE}_{e^+} = \tfrac{1}{2}m_{\pi^+}c^2 - m_{e^+}c^2 + \frac{m_{e^+}^2c^2}{2m_{\pi^+}} = \tfrac{1}{2}(139.6\ \mathrm{MeV}) - 0.511\ \mathrm{MeV} + \frac{(0.511\ \mathrm{MeV}/c^2)(0.511\ \mathrm{MeV})}{2(139.6\ \mathrm{MeV}/c^2)}$$

$$= \boxed{69.3\ \mathrm{MeV}}$$

Here is an alternate solution, using the momentum.

$$Q = [m_{\pi^+} - (m_{e^+} + m_\nu)]c^2 = 139.6\ \mathrm{MeV} - 0.511\ \mathrm{MeV} = 139.1\ \mathrm{MeV}$$

$$\mathrm{KE}_{e^+} = E_{e^+} - m_{e^+}c^2 = \sqrt{(p_{e^+}c)^2 + (m_{e^+}c^2)^2} - m_{e^+}c^2;\quad \mathrm{KE}_\nu = E_\nu = p_\nu c;\quad p_{e^+} = p_\nu = p$$

$$Q = \mathrm{KE}_{e^+} + \mathrm{KE}_\nu \quad\rightarrow\quad 139.1\ \mathrm{MeV} = \sqrt{(pc)^2 + (0.511\ \mathrm{MeV})^2} - (0.511\ \mathrm{MeV}) + pc$$

$$139.6 - pc = \sqrt{(pc)^2 + (0.511\ \mathrm{MeV})^2} \quad\rightarrow\quad (139.6)^2 - 2(139.6)pc + p^2c^2 = (pc)^2 + (0.511\ \mathrm{MeV})^2$$

$$\frac{(139.6\ \mathrm{MeV})^2 - (0.511\ \mathrm{MeV})^2}{2(139.6\ \mathrm{MeV})} = pc = 69.8\ \mathrm{MeV}$$

$$E_\nu = \mathrm{KE}_\nu = 69.8\ \mathrm{MeV};\quad Q - \mathrm{KE}_\nu = \mathrm{KE}_{e^+} = 139.1\ \mathrm{MeV} - 69.8\ \mathrm{MeV} = \boxed{69.3\ \mathrm{MeV}}$$

25. (*a*) The Q-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\Lambda^0}c^2 - (m_p c^2 + m_{\pi^-}c^2) = 1115.7\ \mathrm{MeV} - (938.3\ \mathrm{MeV} + 139.6\ \mathrm{MeV}) = \boxed{37.8\ \mathrm{MeV}}$$

(*b*) Energy conservation for the decay gives the following.

$$m_{\Lambda^0}c^2 = E_p + E_{\pi^-} \quad\rightarrow\quad E_{\pi^-} = m_{\Lambda^0}c^2 - E_p$$

Momentum conservation says that the magnitudes of the momenta of the two products are equal. Then convert that relationship to energy using $E^2 = p^2c^2 + m^2c^4$, with energy conservation.

$$p_p = p_{\pi^-} \quad\rightarrow\quad (p_p c)^2 = (p_{\pi^-}c)^2 \quad\rightarrow$$

$$E_p^2 - m_p^2c^4 = E_{\pi^-}^2 - m_{\pi^-}^2c^4 = (m_{\Lambda^0}c^2 - E_p)^2 - m_{\pi^-}^2c^4$$

$$E_p^2 - m_p^2c^4 = (m_{\Lambda^0}^2c^4 - 2E_p m_{\Lambda^0}c^2 + E_p^2) - m_{\pi^-}^2c^4 \quad\rightarrow$$

$$E_p = \frac{m_{\Lambda^0}^2c^4 + m_p^2c^4 - m_{\pi^-}^2c^4}{2m_{\Lambda^0}c^2} = \frac{(1115.7\ \mathrm{MeV})^2 + (938.3\ \mathrm{MeV})^2 - (139.6\ \mathrm{MeV})^2}{2(1115.7\ \mathrm{MeV})} = 943.7\ \mathrm{MeV}$$

$$E_{\pi^-} = m_{\Lambda^0}c^2 - E_p = 1115.7\text{ MeV} - 943.7\text{ MeV} = 172.0\text{ MeV}$$
$$\text{KE}_p = E_p - m_p c^2 = 943.7\text{ MeV} - 938.3\text{ MeV} = \boxed{5.4\text{ MeV}}$$
$$\text{KE}_{\pi^-} = E_{\pi^-} - m_{\pi^-}c^2 = 172.0\text{ MeV} - 139.6\text{ MeV} = \boxed{32.4\text{ MeV}}$$

26. The two neutrinos must move together, in the opposite direction of the electron, in order for the electron to have the maximum kinetic energy, and thus the total momentum of the neutrinos will be equal in magnitude to the momentum of the electron. Since a neutrino is (essentially) massless, we have $E_\nu = p_\nu c.$ The muon is at rest when it decays. Use conservation of energy and momentum, along with their relativistic relationship.

$$p_{e^-} = p_{\bar{\nu}_e} + p_{\nu_\mu}$$
$$m_{\mu^-}c^2 = E_{e^-} + E_{\bar{\nu}_e} + E_{\nu_\mu} = E_{e^-} + p_{\bar{\nu}_e}c + p_{\nu_\mu}c = E_{e^-} + (p_{\bar{\nu}_e} + p_{\nu_\mu})c = E_{e^-} + p_{e^-}c \quad \rightarrow$$
$$m_{\mu^-}c^2 - E_{e^-} = p_{e^-}c \quad \rightarrow \quad (m_{\mu^-}c^2 - E_{e^-})^2 = (p_{e^-}c)^2 = E_{e^-}^2 - m_{e^-}^2c^4 \quad \rightarrow$$
$$m_{\mu^-}c^4 - 2m_{\mu^-}c^2E_{e^-} + E_{e^-}^2 = E_{e^-}^2 - m_{e^-}^2c^4 \quad \rightarrow \quad E_{e^-} = \frac{m_{\mu^-}^2c^4 + m_{e^-}^2c^4}{2m_{\mu^-}c^2} = \text{KE}_{e^-} + m_{e^-}c^2 \quad \rightarrow$$
$$\text{KE}_{e^-} = \frac{m_{\mu^-}^2c^4 + m_{e^-}^2c^4}{2m_{\mu^-}c^2} - m_{e^-}c^2 = \frac{(105.7\text{ MeV})^2 + (0.511\text{ MeV})^2}{2(105.7\text{ MeV})} - (0.511\text{ MeV}) = \boxed{52.3\text{ MeV}}$$

27. We use the uncertainty principle to estimate the uncertainty in rest energy.

$$\Delta E \approx \frac{h}{2\pi\,\Delta t} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{2\pi(7\times10^{-20}\text{ s})(1.60\times10^{-16}\text{ J/eV})} = 9420\text{ eV} \approx \boxed{0.009\text{ MeV}}$$

28. We estimate the lifetime from the energy width and the uncertainty principle.

$$\Delta E \approx \frac{h}{2\pi\,\Delta t} \quad \rightarrow \quad \Delta t \approx \frac{h}{2\pi\,\Delta E} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(300\times10^3\text{ eV})\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{2\times10^{-21}\text{ s}}$$

29. Apply the uncertainty principle, which says that $\Delta E \approx \dfrac{h}{2\pi\Delta t}.$

$$\Delta E \approx \frac{h}{2\pi\,\Delta t} \quad \rightarrow \quad \Delta t \approx \frac{h}{2\pi\,\Delta E} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(88\times10^3\text{ eV})\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{7.5\times10^{-21}\text{ s}}$$

30. (*a*) For $\text{B}^- = \text{b}\,\bar{\text{u}},$ we have

Charge:	$-1 = -\frac{1}{3} - \frac{2}{3}$	Spin:	$0 = \frac{1}{2} - \frac{1}{2}$
Baryon number:	$0 = \frac{1}{3} - \frac{1}{3}$	Strangeness:	$0 = 0 + 0$
Charm:	$0 = 0 + 0$	Bottomness:	$-1 = -1 + 0$
Topness:	$0 = 0 + 0$		

(*b*) Because B^+ is the antiparticle of B^-, $\boxed{\mathrm{B}^+ = \bar{\mathrm{b}}\,\mathrm{u}}$. The B^0 still must have a bottom quark but must be neutral. Therefore, $\boxed{\mathrm{B}^0 = \mathrm{b}\,\bar{\mathrm{d}}}$. Because $\overline{\mathrm{B}}^0$ is the antiparticle to B^0, we must have $\boxed{\overline{\mathrm{B}}^0 = \bar{\mathrm{b}}\,\mathrm{d}}$.

31. We find the energy width from the lifetime in Table 32–2 and the uncertainty principle.

(*a*) $\Delta t = 5.1\times10^{-19}\ \text{s} \quad \Delta E \approx \dfrac{h}{2\pi\,\Delta t} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(5.1\times10^{-19}\ \text{s})(1.60\times10^{-19}\ \text{J/eV})} = 1293\ \text{eV} \approx \boxed{1300\ \text{eV}}$

(*b*) $\Delta t = 4.4\times10^{-24}\ \text{s} \quad \Delta E \approx \dfrac{h}{2\pi\,\Delta t} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(4.4\times10^{-24}\ \text{s})(1.60\times10^{-19}\ \text{J/eV})} = 1.499\times10^{8}\ \text{eV} \approx \boxed{150\ \text{MeV}}$

32. (*a*)

Charge:	$(0) = (+1) + (-1)$	Charge is conserved.
Baryon number:	$(+1) = (+1) + (0)$	Baryon number is conserved.
Lepton number:	$(0) = (0) + (0)$	Lepton number is conserved.
Strangeness:	$(-2) \neq (-1) + (0)$	Strangeness is NOT conserved.
Spin:	$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) + (0)$	Spin is conserved.

Energy: $1314.9\ \text{Mev}/c^2 > 1189.4\ \text{Mev}/c^2 + 139.6\text{Mev}/c^2 = 1329\text{Mev}/c^2$

Energy is NOT conserved.

$\boxed{\text{The decay is not possible. Neither strangeness nor energy is conserved}}$.

(*b*)

Charge:	$(-1) = (0) + (-1) + (0)$	Charge is conserved.
Baryon number:	$(+1) = (+1) + (0) + (0)$	Baryon number is conserved.
Lepton number:	$(0) = (0) + (0) + (1)$	Lepton number is NOT conserved.
Strangeness:	$(-3) \neq (-1) + (0) + (0)$	Strangeness is NOT conserved.
Spin:	$\left(\frac{3}{2}\right) \neq \left(\frac{1}{2}\right) + (0) + \left(\frac{1}{2}\right)$	Spin is NOT conserved.

Energy: $1672.5\ \text{Mev}/c^2 > 1192.6\text{Mev}/c^2 + 139.6\ \text{Mev}/c^2 + 0 = 1332.2\ \text{Mev}/c^2$

Energy is conserved.

$\boxed{\text{The decay is not possible. Lepton number, strangeness, and spin are not conserved}}$.

(*c*)

Charge:	$(0) = (0) + (0) + (0)$	Charge is conserved.
Baryon number:	$(0) = (0) + (0) + (0)$	Baryon number is conserved.
Lepton number:	$(0) = (0) + (0) + (0)$	Lepton number is conserved.
Strangeness:	$(-1) = (-1) + (0) + (0)$	Strangeness is conserved.
Spin:	$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) + (1) + (-1)$	Spin is conserved, if the gammas have opposite spins.

Energy: $1192.6\ \text{Mev}/c^2 > 1115.7\ \text{Mev}/c^2 + 0 + 0 = 1115.7\ \text{Mev}/c^2$

Energy is conserved.

$\boxed{\text{The decay is possible.}}$

33. The expected lifetime of the virtual W particle is found from the Heisenberg uncertainty principle.

$$\Delta E\,\Delta t \approx \hbar \quad \rightarrow \quad \Delta t \approx \frac{\hbar}{\Delta E} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(80.385\times10^{9}\ \text{eV})} \times \frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}} = 8.20\times10^{-27}\ \text{s} \approx \boxed{8\times10^{-27}\ \text{s}}$$

34. (*a*) The Ξ^0 has a strangeness of –2, so it must contain two strange quarks. In order to make a neutral particle, the third quark must be an up quark. So $\boxed{\Xi^0 = \text{u s s}}$.

(*b*) The Ξ^- has a strangeness of –2, so it must contain two strange quarks. In order to make a particle with a total charge of –1, the third quark must be a down quark. So, $\boxed{\Xi^- = \text{d s s}}$.

35. (*a*) The neutron has a baryon number of 1, so there must be three quarks. The charge must be 0, as must be the strangeness, the charm, the bottomness, and the topness. Thus, $\boxed{n = \text{u d d}}$.

(*b*) The antineutron is the antiparticle of the neutron, so $\boxed{\overline{n} = \overline{\text{u}}\,\overline{\text{d}}\,\overline{\text{d}}}$.

(*c*) The Λ^0 has a strangeness of –1, so it must contain an s quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\Lambda^0 = \text{u d s}}$.

(*d*) The $\overline{\Sigma}^0$ has a strangeness of +1, so it must contain an $\overline{\text{s}}$ quark. It is a baryon, so it must contain three quarks. And it must have charge, charm, bottomness, and topness equal to 0. Thus $\boxed{\overline{\Sigma}^0 = \overline{\text{u}}\,\overline{\text{d}}\,\overline{\text{s}}}$.

36. (*a*) The combination u u d has charge = +1, baryon number = +1, and strangeness, charm, bottomness, and topness all equal to 0. Thus, $\boxed{\text{u u d} = \text{p}}$.

(b) The combination $\overline{\text{u}}\,\overline{\text{u}}\,\overline{\text{s}}$ has charge = –1, baryon number = –1, strangeness = +1, and charm, bottomness, and topness all equal to 0. Thus, $\boxed{\overline{\text{u}}\,\overline{\text{u}}\,\overline{\text{s}} = \overline{\Sigma}^-}$.

(c) The combination $\overline{\text{u}}\,\text{s}$ has charge = –1, baryon number = 0, strangeness = –1, and charm, bottomness, and topness all equal to 0. Thus, $\boxed{\overline{\text{u}}\,\text{s} = \text{K}^-}$.

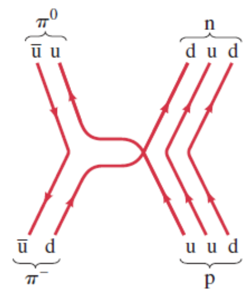
(d) The combination $\text{d}\,\overline{\text{u}}$ has charge = –1, baryon number = 0, and strangeness, charm, bottomness, and topness all equal to 0. Thus, $\boxed{\text{d}\,\overline{\text{u}} = \pi^-}$.

(e) The combination $\overline{\text{c}}\,\text{s}$ has charge = –1, baryon number = 0, strangeness = –1, charm = –1, and bottomness and topness of 0. Thus, $\boxed{\overline{\text{c}}\,\text{s} = \text{D}_S^-}$.

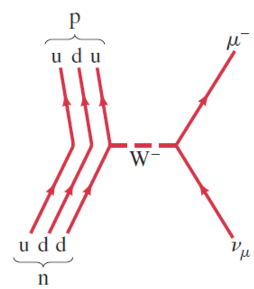
37. To form the D^0 meson, we must have a total charge of 0, a baryon number of 0, a strangeness of 0, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a c quark with a charge of $+\frac{2}{3}e$. To have a neutral meson, there must be another quark with a charge of $-\frac{2}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. The only candidate with those properties is an anti-up quark. Thus, $\boxed{D^0 = \text{c}\,\overline{\text{u}}}$.

38. To form the D_S^+ meson, we must have a total charge of +1, a baryon number of 0, a strangeness of +1, and a charm of +1. We assume that there is no topness or bottomness. To get the charm, we must have a c quark with a charge of $+\frac{2}{3}e$. To have a total charge of +1, there must be another quark with a charge of $+\frac{1}{3}e$. To have a baryon number of 0, that second quark must be an antiquark. To have a strangeness of +1, the other quark must be an antistrange. Thus, $\boxed{D_S^+ = \mathrm{c\,\overline{s}}}$.

39. Here is a Feynman diagram for the reaction $\pi^- + \mathrm{p} \rightarrow \pi^0 + \mathrm{n}$. There are other possibilities, since the π^0 also can be represented as a $\mathrm{d\,\overline{d}}$ combination or as a mixture of $\mathrm{d\,\overline{d}}$ and $\mathrm{u\,\overline{u}}$ combinations.



40. Since leptons are involved, the reaction $\mathrm{n} + \nu_\mu \rightarrow \mathrm{p} + \mu^-$ is a weak interaction. Since there is a charge change in the lepton, a W boson must be involved in the interaction. If we consider the neutron as having emitted the boson, then it is a W^-, which interacts with the neutrino. If we consider the neutrino as having emitted the boson, then it is a W^+, which interacts with the neutron.



41. The total energy is the sum of the kinetic energy and the mass energy. The wavelength is found from the relativistic momentum.

$$E = \mathrm{KE} + mc^2 = 15\times10^9\ \mathrm{eV} + 938.3\times10^6\ \mathrm{eV} = 1.5938\times10^{10}\ \mathrm{eV} \approx \boxed{16\,\mathrm{GeV}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\dfrac{\sqrt{E^2-(mc^2)^2}}{c}} = \frac{hc}{\sqrt{E^2-(mc^2)^2}}$$

$$= \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^8\ \mathrm{m/s})}{\sqrt{(1.5938\times10^{10}\ \mathrm{eV})^2-(938.3\times10^6\ \mathrm{eV})^2}}\,\frac{1}{(1.60\times10^{-19}\ \mathrm{J/eV})} = \boxed{7.8\times10^{-17}\ \mathrm{m}}$$

42. To find the length in the lab, we need to know the speed of the particle that is moving relativistically. Start with Eq. 26–5a.

$$\mathrm{KE} = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) \rightarrow v = c\sqrt{1-\frac{1}{\left(\dfrac{\mathrm{KE}}{mc^2}+1\right)^2}} = c\sqrt{1-\frac{1}{\left(\dfrac{950\ \mathrm{MeV}}{1777\ \mathrm{MeV}}+1\right)^2}} = 0.7585c$$

$$\Delta t_{\mathrm{lab}} = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{2.91\times10^{-13}\ \mathrm{s}}{\sqrt{1-(0.7585)^2}} = 4.465\times10^{-13}\ \mathrm{s}$$

$$\Delta x_{\mathrm{lab}} = v\Delta t_{\mathrm{lab}} = (0.7585)(3.00\times10^8\ \mathrm{m/s})(4.465\times10^{-13}\ \mathrm{s}) = \boxed{1.02\times10^{-4}\ \mathrm{m}}$$

$\boxed{43.}$ By assuming that the initial kinetic energy is approximately 0, the total energy released is the mass energy of the annihilating pair of particles.

(*a*) $E_{\text{total}} = 2mc^2 = 2(0.511\text{ MeV}) = \boxed{1.022\text{ MeV}}$

(*b*) $E_{\text{total}} = 2mc^2 = 2(938.3\text{ MeV}) = \boxed{1876.6\text{ MeV}}$

$\boxed{44.}$ (*a*) At an energy of 4.0 TeV, the protons are moving at practically the speed of light. From uniform circular motion, we find the time for the protons to complete one revolution around the ring. Then the total charge that passes any point in the ring during that time is the charge of the entire group of stored protons. The current is then the total charge divided by the period.

$$v = \frac{2\pi R}{T} \quad \rightarrow \quad T = \frac{2\pi R}{v} = \frac{2\pi R}{c}$$

$$I = \frac{Ne}{T} = \frac{Nec}{2\pi R} = \frac{(2\times10^{14}\text{ protons})(1.60\times10^{-19}\text{ C/proton})(3.0\times10^{8}\text{ m/s})}{2\pi(4.3\times10^{3}\text{ m})} = 0.355\text{A} \approx \boxed{0.4\text{ A}}$$

(*b*) The 4.0 TeV is equal to the KE of the proton beam. We assume that the car would not be moving relativistically.

$$\text{KE}_{\text{beam}} = \text{KE}_{\text{car}} \quad \rightarrow \quad \text{KE}_{\text{beam}} = \tfrac{1}{2}mv^2 \quad \rightarrow$$

$$v = \sqrt{\frac{2\text{KE}_{\text{beam}}}{m}} = \sqrt{\frac{2(4.0\times10^{12}\text{ eV/proton})(2\times10^{14}\text{ protons})(1.60\times10^{-19}\text{ J/eV})}{1500\text{ kg}}} = 413\text{ m/s}$$

$$\approx \boxed{400\text{ m/s}}$$

Our assumption that the car is not relativistic is confirmed. This is about 900 mi/h.

45. These protons will be moving at essentially the speed of light for the entire time of acceleration. The number of revolutions is the total gain in energy divided by the energy gain per revolution. Then the distance is the number of revolutions times the circumference of the ring, and the time is the distance of travel divided by the speed of the protons.

$$N = \frac{\Delta E}{\Delta E/\text{rev}} = \frac{(4.0\times10^{12}\text{ eV} - 450\times10^{9}\text{ eV})}{8.0\times10^{6}\text{ eV/rev}} = 4.44\times10^{5}\text{ rev}$$

$$d = N(2\pi R) = (4.44\times10^{5})2\pi(4.3\times10^{3}\text{ m}) = 1.199\times10^{10}\text{ m} \approx \boxed{1.2\times10^{10}\text{ m}}$$

$$t = \frac{d}{c} = \frac{1.199\times10^{10}\text{ m}}{3.00\times10^{8}\text{ m/s}} = \boxed{4.0\times10^{1}\text{ s}}$$

46. (*a*) For the reaction $\pi^- + \text{p} \rightarrow \text{K}^0 + \text{p} + \pi^0$, the conservation laws are as follows:

Charge: $-1 + 1 \neq 0 + 1 + 0$ Charge is NOT conserved.

$\boxed{\text{The reaction is not possible, because charge is not conserved.}}$

Also, we note that the reactants would have to have significant kinetic energy to be able to "create" the K^0.

(*b*) For the reaction $\text{K}^- + \text{p} \rightarrow \Lambda^0 + \pi^0$, the conservation laws are as follows:

Charge: $-1 + 1 = 0 + 0$ Charge is conserved.

Spin: $0 + \tfrac{1}{2} = \tfrac{1}{2} + 0$ Spin is conserved.

Baryon number: $0+1=1+0$ Baryon number is conserved.

Lepton number: $0+0=0+0$ Lepton number is conserved.

Strangeness: $-1+0=-1+0$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*c*) For the reaction $K^+ + n \rightarrow \Sigma^+ + \pi^0 + \gamma$, the conservation laws are as follows:

Charge: $1+0=1+0+0$ Charge is conserved.

Spin: $0+\frac{1}{2}=-\frac{1}{2}+0+1$ Spin is conserved.

Baryon number: $0+1=1+0+0$ Baryon number is conserved.

Lepton number: $0+0=0+0+0$ Lepton number is conserved.

Strangeness: $1+0 \neq -1+0+0$ Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*d*) For the reaction $K^+ \rightarrow \pi^0 + \pi^0 + \pi^+$, the conservation laws are as follows:

Charge: $1=0+0+1$ Charge is conserved.

Spin: $0=0+0+0$ Spin is conserved.

Baryon number: $0=0+0+0$ Baryon number is conserved.

Lepton number: $0=0+0+0$ Lepton number is conserved.

Strangeness: $1 \neq 0+0+0$ Strangeness is NOT conserved.

The reaction is not possible via the strong interaction because strangeness is not conserved. It is possible via the weak interaction.

(*e*) For the reaction $\pi^+ \rightarrow e^+ + \nu_e$, the conservation laws are as follows:

Charge: $1=1+0$ Charge is conserved.

Spin: $0=-\frac{1}{2}+\frac{1}{2}$ Spin is conserved.

Baryon number: $0=0+0$ Baryon number is conserved.

Lepton number: $0=-1+1$ Lepton number is conserved.

Strangeness: $0+0=0+0+0$ Strangeness is conserved.

The reaction is possible, via the weak interaction.

47. (*a*) For the reaction $\pi^- + p \rightarrow K^+ + \Sigma^-$, the conservation laws are as follows:

Charge: $-1+1=1-1$ Charge is conserved.

Spin: $0+\frac{1}{2}=0+\frac{1}{2}$ Spin is conserved.

Baryon number: $0+1=0+1$ Baryon number is conserved.

Lepton number: $0+0=0+0$ Lepton number is conserved.

Strangeness: $0+0=1-1$ Strangeness is conserved.

The reaction is possible, via the strong interaction.

(*b*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \mathrm{K}^+ + \Sigma^+$, the conservation laws are as follows:

Charge: $1+1=1+1$	Charge is conserved.
Spin: $0+\frac{1}{2}=0+\frac{1}{2}$	Spin is conserved.
Baryon number: $0+1=0+1$	Baryon number is conserved.
Lepton number: $0+0=0+0$	Lepton number is conserved.
Strangeness: $0+0=1-1$	Strangeness is conserved.

$\boxed{\text{The reaction is possible, via the strong interaction.}}$

(*c*) For the reaction $\pi^- + \mathrm{p} \rightarrow \Lambda^0 + \mathrm{K}^0 + \pi^0$, the conservation laws are as follows:

Charge: $-1+1=0+0+0$	Charge is conserved.
Spin: $0+\frac{1}{2}=\frac{1}{2}+0+0$	Spin is conserved.
Baryon number: $0+1=1+0+0$	Baryon number is conserved.
Lepton number: $0+0=0+0+0$	Lepton number is conserved.
Strangeness: $0+0=-1+1+0$	Strangeness is conserved.

$\boxed{\text{The reaction is possible, via the strong interaction.}}$

(*d*) For the reaction $\pi^+ + \mathrm{p} \rightarrow \Sigma^0 + \pi^0$, the conservation laws are as follows:

Charge: $1+1 \neq 0+0$	Charge is NOT conserved.

$\boxed{\text{The reaction is not possible, because charge is not conserved.}}$

(*e*) For the reaction $\pi^- + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e}$, the conservation laws are as follows:

Charge: $-1+1=1-1+0$	Charge is conserved.
Spin: $0+\frac{1}{2}=\frac{1}{2}+\frac{1}{2}-\frac{1}{2}$	Spin is conserved.
Baryon number: $0+1=1+0+0$	Baryon number is conserved.
Lepton number: $0+0=0+1-1$	Lepton number is conserved.
Strangeness: $0+0=0+0+0$	Strangeness is conserved.

$\boxed{\text{The reaction is possible, via the weak interaction.}}$

Note that we did not check mass conservation, because in a collision, there is always some kinetic energy brought into the reaction. Thus the products can be heavier than the reactants.

48. The π^- is the antiparticle of the π^+, so the reaction is $\boxed{\pi^- \rightarrow \mu^- + \overline{\nu}_\mu}$. The conservation rules are as follows:

Charge: $-1=-1+0$	Charge is conserved.
Baryon number: $0=0+0$	Baryon number is conserved.
Lepton number: $0=1-1$	Lepton number is conserved.
Strangeness: $0=0+0$	Strangeness is conserved.
Spin: $0=\frac{1}{2}-\frac{1}{2}$	Spin is conserved.

49. Use Eq. 32–3 to estimate the mass of the particle based on the given distance.

$$mc^2 \approx \frac{hc}{2\pi d} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^8\ \text{m/s})}{2\pi(10^{-18}\ \text{m})}\left(\frac{1}{1.60\times10^{-19}\ \text{J/eV}}\right) = 1.98\times10^{11}\ \text{eV} \approx \boxed{200\ \text{GeV}}$$

This value is of the same order of magnitude as the mass of the $\text{W}^{\pm}$.

50. The Q-value is the mass energy of the reactants minus the mass energy of the products.

For the first reaction, $\text{p}+\text{p} \rightarrow \text{p}+\text{p}+\pi^0$:

$$Q = 2m_{\text{p}}c^2 - (2m_{\text{p}}c^2 + m_{\pi^0}c^2) = -m_{\pi^0}c^2 = \boxed{-135.0\ \text{MeV}}$$

For the second reaction, $\text{p}+\text{p} \rightarrow \text{p}+\text{n}+\pi^+$:

$$Q = 2m_{\text{p}}c^2 - (m_{\text{p}}c^2 + m_{\text{n}}c^2 + m_{\pi^+}c^2) = m_{\text{p}}c^2 - m_{\text{n}}c^2 - m_{\pi^+}c^2$$
$$= 938.3\ \text{MeV} - 939.6\ \text{MeV} - 139.6\ \text{MeV} = \boxed{-140.9\ \text{MeV}}$$

51. The fundamental fermions are the quarks and electrons. In a water molecule there are 2 hydrogen atoms consisting of 1 electron and 1 proton each, and 1 oxygen atom, consisting of 8 electrons, 8 protons, and 8 neutrons. Thus there are 18 nucleons, consisting of 3 quarks each, and 10 electrons. The total number of fermions is thus $18\times3+10 = \boxed{64\ \text{fundamental fermions}}$.

52. We assume that the interaction happens essentially at rest, so that there is no initial kinetic energy or momentum. Thus the momentum of the neutron and the momentum of the π^0 will have the same magnitude. From energy conservation we find the total energy of the π^0.

$$m_{\pi^-}c^2 + m_{\text{p}}c^2 = E_{\pi^0} + m_{\text{n}}c^2 + \text{KE}_{\text{n}} \quad \rightarrow$$
$$E_{\pi^0} = m_{\pi^-}c^2 + m_{\text{p}}c^2 - (m_{\text{n}}c^2 + \text{KE}_{\text{n}}) = 139.6\ \text{MeV} + 938.3\ \text{MeV} - (939.6\ \text{MeV} + 0.60\ \text{MeV})$$
$$= 137.7\ \text{MeV}$$

From momentum conservation, we can find the mass energy of the π^0. We utilize Eq. 26–9 to relate momentum and energy.

$$p_{\text{n}} = p_{\pi^0} \quad \rightarrow \quad (p_{\text{n}}c)^2 = (p_{\pi^0}c)^2 \quad \rightarrow \quad E_{\text{n}}^2 - m_{\text{n}}^2c^4 = E_{\pi^0}^2 - m_{\pi^0}^2c^4 \quad \rightarrow \quad m_{\pi^0}^2c^4 = E_{\pi^0}^2 - E_{\text{n}}^2 + m_{\text{n}}^2c^4 \quad \rightarrow$$
$$m_{\pi^0}c^2 = \sqrt{E_{\pi^0}^2 - E_{\text{n}}^2 + m_{\pi^0}^2c^4} = [(137.7\ \text{MeV})^2 - (939.6\ \text{MeV} + 0.60\ \text{MeV})^2 + (939.6\ \text{MeV})^2]^{1/2}$$
$$= 133.5\ \text{MeV} \quad \rightarrow \quad m_{\pi^0} = \boxed{133.5\ \text{MeV}/c^2}$$

The reference value is $135.0\ \text{MeV}$.

53. (*a*) First we use the uncertainty principle, Eq. 28–1. The energy is so high that we assume $E = pc$, so $\Delta p = \dfrac{\Delta E}{c}$.

$$\Delta x\,\Delta p \approx \frac{h}{2\pi} \quad \rightarrow \quad \Delta x\frac{\Delta E}{c} \approx \frac{h}{2\pi} \quad \rightarrow$$
$$\Delta E \approx \frac{hc}{2\pi\,\Delta x} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{2\pi(10^{-31}\ \text{m})}\frac{(1\ \text{GeV}/10^9\ \text{eV})}{(1.60\times10^{-19}\ \text{J/eV})} = \boxed{2\times10^{15}\ \text{GeV}}$$

Next, we use de Broglie's wavelength formula. We take the de Broglie wavelength as the unification distance.

$$\lambda = \frac{h}{p} = \frac{h}{Ec} \quad \rightarrow$$

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(10^{-31}\ \text{m})}\frac{(1\ \text{GeV}/10^{9}\ \text{eV})}{(1.60\times10^{-19}\ \text{J/eV})} = \boxed{1\times10^{16}\ \text{GeV}}$$

Both energies are reasonably close to 10^{16} GeV. This energy is the amount that could be violated in conservation of energy if the universe were the size of the unification distance.

(*b*) From Eq. 13–8, we have $E = \frac{3}{2}kT$.

$$E = \tfrac{3}{2}kT \quad \rightarrow \quad T = \frac{2E}{3k} = \frac{2(10^{25}\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}{3(1.38\times10^{-23}\ \text{J/K})} = 7.7\times10^{28}\ \text{K} \approx \boxed{10^{29}\ \text{K}}$$

54. The Q-value is the mass energy of the reactants minus the mass energy of the products.

$$Q = m_{\pi^-}c^2 + m_{\text{p}}c^2 - (m_{\Lambda^0}c^2 + m_{\text{K}^0}c^2) = 139.6\,\text{MeV} + 938.3\,\text{MeV} - (1115.7\,\text{MeV} + 497.6\,\text{MeV})$$
$$= \boxed{-535.4\,\text{MeV}}$$

We consider the products to be one mass $M = m_{\Lambda^0} + m_{\text{K}^0} = 1613.3\ \text{MeV}/c^2$ since they have the same velocity. Energy conservation gives the following: $E_{\pi^-} + m_{\text{p}}c^2 = E_M$. Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship that $E^2 = p^2c^2 + m^2c^4$.

$$p_{\pi^-} = p_{\text{M}} \quad \rightarrow \quad (p_{\pi^-}c)^2 = (p_{\text{M}}c)^2 \quad \rightarrow \quad E_{\pi^-}^2 - m_{\pi^-}^2c^4 = E_M^2 - M^2c^4 \quad \rightarrow$$
$$E_{\pi^-}^2 - m_{\pi^-}^2c^4 = (E_{\pi^-} + m_{\text{p}}c^2)^2 - M^2c^4 = E_{\pi^-}^2 + 2E_{\pi^-}m_{\text{p}}c^2 + m_{\text{p}}^2c^4 - M^2c^4 \quad \rightarrow$$
$$E_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\text{p}}^2c^4}{2m_{\text{p}}c^2} = \text{KE}_{\pi^-} + m_{\pi^-}c^2 \quad \rightarrow$$
$$\text{KE}_{\pi^-} = \frac{M^2c^4 - m_{\pi^-}^2c^4 - m_{\text{p}}^2c^4}{2m_{\text{p}}c^2} - m_{\pi^-}c^2$$
$$= \frac{(1613.3\ \text{MeV})^2 - (139.6\ \text{MeV})^2 - (938.3\ \text{MeV})^2}{2(938.3\ \text{MeV})} - (139.6\ \text{MeV}) = \boxed{767.8\,\text{MeV}}$$

55. Since there is no initial momentum, the final momentum must add to zero. Thus each of the pions must have the same magnitude of momentum and therefore the same kinetic energy. Use energy conservation to find the kinetic energy of each pion.

$$2m_{\text{p}}c^2 = 2\text{KE}_\pi + 2m_\pi c^2 \quad \rightarrow \quad \text{KE}_\pi = m_{\text{p}}c^2 - m_\pi c^2 = 938.3\ \text{MeV} - 139.6\ \text{MeV} = \boxed{798.7\ \text{MeV}}$$

56. The Q-value is the energy of the reactants minus the energy of the products. We assume that one of the initial protons is at rest and that all four final particles have the same speed and therefore the same kinetic energy, since they all have the same mass. We consider the products to be one mass $M = 4m_{\text{p}}$ since they all have the same speed.

$$Q = 2m_{\text{p}}c^2 - 4m_{\text{p}}c^2 = 2m_{\text{p}}c^2 - Mc^2 = -2m_{\text{p}}c^2$$

Energy conservation gives the following, where $\mathrm{KE}_{\mathrm{th}}$ is the threshold energy.

$$(\mathrm{KE}_{\mathrm{th}} + m_{\mathrm{p}}c^2) + m_{\mathrm{p}}c^2 = E_M = \mathrm{KE}_M + Mc^2$$

Momentum conservation says that the incoming momentum is equal to the outgoing momentum. Then convert that relationship to energy using the relativistic relationship $E^2 = p^2c^2 + m^2c^4$.

$$p_{\mathrm{p}} = p_{\mathrm{M}} \;\rightarrow\; (p_{\mathrm{p}}c)^2 = (p_{\mathrm{M}}c)^2 \;\rightarrow\; (\mathrm{KE}_{\mathrm{th}} + m_{\mathrm{p}}c^2)^2 - m_{\mathrm{p}}^2c^4 = (\mathrm{KE}_M + Mc^2)^2 - M^2c^4 \;\rightarrow$$
$$\mathrm{KE}_{\mathrm{th}}^2 + 2\mathrm{KE}_{\mathrm{th}}m_{\mathrm{p}}c^2 + m_{\mathrm{p}}^2c^4 - m_{\mathrm{p}}^2c^4 = \mathrm{KE}_{\mathrm{th}}^2 + 4\mathrm{KE}_{\mathrm{th}}m_{\mathrm{p}}c^2 + 4m_{\mathrm{p}}^2c^4 - (4m_{\mathrm{p}})^2c^4 \;\rightarrow$$
$$2\mathrm{KE}_{\mathrm{th}}m_{\mathrm{p}}c^2 = 4\mathrm{KE}_{\mathrm{th}}m_{\mathrm{p}}c^2 + 4m_{\mathrm{p}}^2c^4 - 16m_{\mathrm{p}}^2c^4 \;\rightarrow\; 2\mathrm{KE}_{\mathrm{th}}m_{\mathrm{p}}c^2 = 12m_{\mathrm{p}}^2c^4 \;\rightarrow$$
$$\mathrm{KE}_{\mathrm{th}} = 6m_{\mathrm{p}}c^2 = 3|Q|$$

57. We use λ_0 to represent the actual wavelength and λ to define the approximate wavelength. The approximation is to ignore the mass in the expression for the total energy, $E = \mathrm{KE} + mc^2$. We also use Eqs. 26–5a, 26–6b, 26–9, and 32–1.

$$p^2c^2 = E^2 - (mc^2)^2 = (\mathrm{KE} + mc^2)^2 - (mc^2)^2 = \mathrm{KE}^2\left(1 + 2\frac{mc^2}{\mathrm{KE}}\right)$$

$$\lambda_0 = \frac{h}{p} = \frac{hc}{\mathrm{KE}\left(1 + \frac{2mc^2}{\mathrm{KE}}\right)^{1/2}};\quad \lambda = \frac{hc}{\mathrm{KE}} > \lambda_0;\quad \lambda = 1.01\lambda_0 \;\rightarrow\; \frac{hc}{\mathrm{KE}} = 1.01\frac{hc}{\mathrm{KE}\left(1 + \frac{2mc^2}{\mathrm{KE}}\right)^{1/2}} \;\rightarrow$$

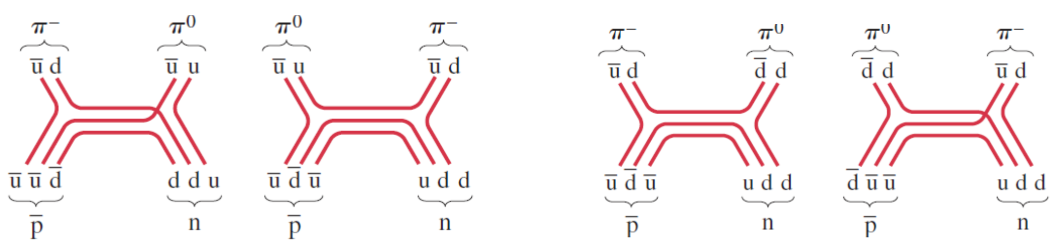
$$\mathrm{KE} = \frac{2mc^2}{(1.01)^2 - 1} = \frac{2(9.38\times10^8\ \mathrm{eV})}{0.0201} = 9.333\times10^{10}\ \mathrm{eV} \approx \boxed{9.3\times10^{10}\ \mathrm{eV}}$$

$$\lambda = \frac{hc}{\mathrm{KE}} = \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^8\ \mathrm{m/s})}{(9.33\times10^{10}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})} = 1.332\times10^{-17}\ \mathrm{m} \approx \boxed{1.3\times10^{-17}\ \mathrm{m}}$$

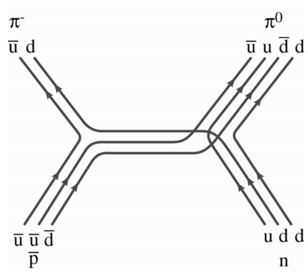
$$\mathrm{KE} = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 \;\rightarrow$$

$$v = c\sqrt{1 - \left(\frac{\mathrm{KE}}{mc^2} + 1\right)^{-2}} = c\sqrt{1 - \left(\frac{9.333\times10^{10}\ \mathrm{eV}}{9.38\times10^8\ \mathrm{eV}} + 1\right)^{-2}} = 0.99995c$$

58. As mentioned in Example 32–8, the π^0 can be considered as either $\mathrm{u\bar{u}}$ or $\mathrm{d\bar{d}}$. There are various models to describe this reaction. Four are shown here.



Another model that shows the π^0 as a combination of both $\mathrm{u\bar{u}}$ and $\mathrm{d\bar{d}}$ is also shown.



59. (*a*) To conserve charge, the missing particle must be neutral. To conserve baryon number, the missing particle must be a meson. To conserve strangeness, charm, topness, and bottomness, the missing particle must be made of up and down quarks and antiquarks only. With all this information, the missing particle is $\boxed{\pi^0}$.

(*b*) This is a weak interaction since one product is a lepton. To conserve charge, the missing particle must be neutral. To conserve the muon lepton number, the missing particle must be an antiparticle in the muon family. With this information, the missing particle is $\boxed{\bar{\nu}_\mu}$.

60. A relationship between total energy and speed is given by Eq. 26–6b.

$$E = \frac{mc^2}{\sqrt{1-v^2/c^2}} \;\rightarrow\; \sqrt{1-v^2/c^2} = \frac{mc^2}{E} \;\rightarrow\; 1-\frac{v^2}{c^2} = \left(\frac{mc^2}{E}\right)^2 \;\rightarrow$$

$$\frac{v}{c} = \sqrt{1-\left(\frac{mc^2}{E}\right)^2} = \sqrt{1-\left(\frac{9.38\times10^8 \text{ eV}}{7.0\times10^{12} \text{ eV}}\right)^2} = 1 \text{ (to 8 digits)} \;\rightarrow\; \boxed{v = c}$$

Use the binomial expansion to express the answer differently.

$$\frac{v}{c} = \sqrt{1-\left(\frac{9.38\times10^8 \text{ eV}}{7.0\times10^{12} \text{ eV}}\right)^2} \approx 1-\tfrac{1}{2}\left(\frac{9.38\times10^8 \text{ eV}}{7.0\times10^{12} \text{ eV}}\right)^2 = 1-9.0\times10^9 \;\rightarrow\; v = c(1-9.0\times10^9)$$

This is about 3 m/s slower than the speed of light.

61. According to Section 32–1, the energy involved in the LHC's collisions will reach 14 TeV. Use that with the analysis used in Example 32–1. From Eq. 30–1, nuclear sizes are on the order of 1.2×10^{-15} m.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^8 \text{ m/s})}{(14\times10^{12} \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = 8.9\times10^{-20} \text{ m}$$

$$\frac{\lambda}{r_{\text{nucleus}}} = \frac{8.9\times10^{-20} \text{ m}}{1.2\times10^{-15} \text{ m}} = 7.4\times10^{-5} \approx \frac{1}{13{,}500}$$

62. (*a*) The nucleus undergoes two beta decays, so in the nucleus 2 neutrons change into protons. The daughter nucleus must have 2 more protons than the parent but the same number of nucleons. Thus the daughter nucleus is $\boxed{^{96}_{42}\text{Mo}}$. The reaction would be $^{96}_{40}\text{Zr} \rightarrow {}^{96}_{42}\text{Mo} + 2\,^{0}_{-1}\text{e}^-$.

(*b*) Since the reaction is neutrinoless, $\boxed{\text{lepton conservation}}$ would be violated in this decay. Checking isotope mass values at www.nist.gov/pml/data/comp.cfm shows that the reaction is energetically possible.

(*c*) If the $^{96}_{40}\text{Zr}$ would undergo two beta decays simultaneously and thereby emit 2 electron antineutrinos, then it could decay to $^{96}_{42}\text{Mo}$ without violating any conservation laws.

63. The width of the "bump" in Fig. 32–19 is about $5\ \text{GeV}/c^2$. Use that value with the uncertainty principle to estimate the lifetime of the Higgs boson.

$$\Delta E\,\Delta t \approx \frac{h}{2\pi} \quad\rightarrow\quad \Delta t \approx \frac{h}{2\pi\,\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(5\times10^{9}\ \text{eV})}\frac{1}{(1.60\times10^{-19}\ \text{J/eV})} \approx \boxed{1\times10^{-25}\ \text{s}}$$

Solutions to Search and Learn Problems

1. (*a*) The two major classes of fundamental particles are quarks and leptons.
   (*b*) Quarks: up, down, strange, charm, bottom, and top.
   Leptons: electron, muon, tau, electron neutrino, muon neutrino, and tau neutrino.
   (*c*) Gravity, electromagnetic, weak nuclear, and strong nuclear.
   (*d*) Gravity is carried by the graviton; the electromagnetic force is carried by the photon; the weak nuclear force is carried by the W^+, W^-, and Z^0 bosons; the strong nuclear force is carried by the gluon. The gravitational force is much weaker than the other three forces.

2. (*a*) Hadrons interact via the strong nuclear force (as well as the other three fundamental forces) and are made up of quarks.
   (*b*) Baryons are hadrons, are made of three quarks, and have a baryon number of either +1 or −1.
   (*c*) Mesons are hadrons, are made of one quark and one antiquark, and have a baryon number of 0.

3. The conservation laws that must hold are the conservation of momentum, angular momentum (spin), mass–energy, charge, baryon number, and lepton number. Momentum can be conserved by the two decay products having equal but opposite momentum in the rest frame of the parent particle. There are no measurements given to enable us to check conservation of momentum, so we assume that it holds. The other conservation laws are evaluated for each decay. Spin can of course be positive or negative, so our "check" means seeing if there is a way for spins to add up, using either positive or negative values for the spin. For the mass–energy to be conserved, the mass of the initial particle must be greater than the mass of the resulting particles, with the remaining mass becoming kinetic energy of the particles. Those values are in GeV/c^2, but units are omitted here.

   For the reaction $\text{t} \rightarrow \text{W}^+ + \text{b}$:

Spin:	$\frac{1}{2} = 1 + \left(-\frac{1}{2}\right)$	Spin is conserved.
Mass–energy:	$173 > 80.4 + 4.18$	Mass–energy is conserved.
Charge:	$\frac{2}{3} = 1 + \left(-\frac{1}{3}\right)$	Charge is conserved.
Baryon number:	$\frac{1}{3} = 0 + \frac{1}{3}$	Baryon number is conserved.
Lepton number:	$0 = 0 + 0$	Lepton number is conserved.

For the reaction $\overline{t} \rightarrow W^{-} + \overline{b}$:

Spin:	$\frac{1}{2} = 1 + \left(-\frac{1}{2}\right)$	Spin is conserved.
Mass–energy:	$173 > 80.4 + 4.18$	Mass–energy is conserved.
Charge:	$\left(-\frac{2}{3}\right) = (-1) + \frac{1}{3}$	Charge is conserved.
Baryon number:	$-\frac{1}{3} = 0 + \left(-\frac{1}{3}\right)$	Baryon number is conserved.
Lepton number:	$0 = 0 + 0$	Lepton number is conserved.

For the reaction $W^{+} \rightarrow u + \overline{d}$:

Spin:	$1 = \frac{1}{2} + \frac{1}{2}$	Spin is conserved.
Mass–energy:	$80.4 > 0.0023 + 0.0048$	Mass–energy is conserved.
Charge:	$1 = \frac{2}{3} + \frac{1}{3}$	Charge is conserved.
Baryon number:	$0 = \frac{1}{3} + \left(-\frac{1}{3}\right)$	Baryon number is conserved.
Lepton number:	$0 = 0 + 0$	Lepton number is conserved.

For the reaction $W^{-} \rightarrow \mu^{-} + \overline{\nu}_{\mu}$:

Spin:	$1 = \frac{1}{2} + \frac{1}{2}$	Spin is conserved.
Mass–energy:	$80.4 > 0.1057 + 0$	Mass–energy is conserved.
Charge:	$-1 = (-1) + 0$	Charge is conserved.
Baryon number:	$0 = 0 + 0$	Baryon number is conserved.
Lepton number:	$0 = 1 + (-1)$	Lepton number is conserved.

4. (*a*) Each tau will carry off half of the released kinetic energy. The kinetic energy is the difference in mass–energy of the two tau particles and the original Higgs boson.

$$Q = (m_{H^0} - 2m_{\tau})c^2 = 125\times10^3 \text{ MeV} - 2(1777 \text{ MeV}) = 121.4\times10^3 \text{ MeV} = 121.4 \text{ GeV}$$

$$\text{KE}_{\tau} = \tfrac{1}{2}Q = \boxed{60.7 \text{ GeV}}$$

(*b*) The total charge must be zero, so one will have a positive charge and the other will have a negative charge.

(*c*) No. The mass of the two Z bosons is greater than the mass of the initial Higgs boson, so the decay would violate the conservation of energy.

5. (*a*) We work in the rest frame of the isolated electron, so that it is initially at rest. Energy conservation gives the following.

$$m_e c^2 = \text{KE}_e + m_e c^2 + E_{\gamma} \quad\rightarrow\quad \text{KE}_e = -E_{\gamma} \quad\rightarrow\quad \text{KE}_e = E_{\gamma} = 0$$

Since the photon has no energy, it does not exist, so it has not been emitted.

Here is an alternate explanation: Consider the electron in its rest frame. If it were to emit a single photon of energy E_{γ}, then the photon would have a momentum $p_{\gamma} = E_{\gamma}/c$. For momentum to be conserved, the electron would move away in the opposite direction as the photon with the same magnitude of momentum. Since the electron is moving (relative to the initial frame of reference), the electron will now have kinetic energy in that frame of reference, with a total energy given by Eq. 26–9, $E=\sqrt{(mc^2)^2+(p_\gamma c)^2}=\sqrt{(mc^2)^2+E_\gamma^2}$. With the added kinetic energy, the electron now has more energy than it had at rest, and the total energy of the photon and electron is greater than the initial rest energy of the electron. This is a violation of the conservation of energy, so an electron cannot emit a single photon.

(*b*) For the photon exchange in Fig. 32–8, the photon exists for such a short time that the Heisenberg uncertainty principle allows energy conservation to be violated during the exchange. So if Δt is the duration of the interaction and ΔE is the amount by which energy is not conserved during the interaction, then as long as $\Delta E \Delta t > \hbar$, the process can happen.

6. Because the energy of the protons is much greater than their mass, we have $\text{KE}=E=pc$. Combine this with the expression from Problem 11 that relates the momentum and radius of curvature for a charged particle in a synchrotron.

$$B\ (\text{in T})=\frac{E\ (\text{in eV})}{rc}=\frac{(7.0\times10^{12}\ \text{eV})}{(4.25\times10^{3}\ \text{m})(3.00\times10^{8}\ \text{m/s})}=\boxed{5.5\ \text{T}}$$

Alternate derivation: In Chapter 26, Problem 46, it is stated that $\gamma m\upsilon^2/r=q\upsilon B$ for a relativistic charge executing circular motion in a magnetic field. It is also stated in Eq. 26–4 that $p=\gamma m\upsilon$. Combine those relationships with the fact that $E=pc$ for highly relativistic particles.

$$\frac{\gamma m\upsilon^2}{r}=q\upsilon B \quad\rightarrow\quad \frac{p}{r}=qB \quad\rightarrow\quad \frac{E}{cr}=qB \quad\rightarrow$$

$$B=\frac{E}{qrc}=\frac{(7.0\times10^{12}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(1.60\times10^{-19}\ \text{C})(4.25\times10^{3}\ \text{m})(3.00\times10^{8}\ \text{m/s})}=\boxed{5.5\ \text{T}}$$

33

ASTROPHYSICS AND COSMOLOGY

Responses to Questions

$\boxed{1.}$ Long ago, without telescopes, it was difficult to see the individual stars in the Milky Way. The stars in this region of the sky were so numerous and so close together and so tiny that they all blended together to form a cloudy or milky stripe across the night sky. Now, using more powerful telescopes, we can see the individual stars that make up the Milky Way Galaxy.

2. If a star generates more energy in its interior than it radiates away, then its temperature will increase. Consequently, there will be greater outward pressure opposing the gravitational force directed inward. To regain equilibrium, the star will expand. If a star generates less energy than it radiates away, then its temperature will decrease. There will be a smaller outward pressure opposing the gravitational force directed inward, and, in order to regain equilibrium, the star will contract.

3. Red giants are extremely large stars with relatively cool surface temperatures, resulting in their reddish colors. These stars are very luminous because they are so large. When the Sun becomes a red giant, for instance, its radius will be on the order of the distance from the Earth to the Sun. A red giant has run out of hydrogen in its inner core and is fusing hydrogen to helium in a shell surrounding the core. Red giants have left their main-sequence positions on the H–R diagram and have moved up (more luminous) and to the right (cooler).

$\boxed{4.}$ Although the H–R diagram only directly relates the surface temperature of a star to its absolute luminosity (and thus doesn't directly reveal anything about the core), the H–R diagram does provide clues regarding what is happening at the core of a star. Using the current model of stellar evolution and the H–R diagram, we can infer that the stars on the main sequence are fusing hydrogen nuclei to helium nuclei at the core and that stars in the red giant region are fusing helium and beryllium to make heavier nuclei such as carbon and that this red giant process will continue until fusion can no longer occur and the star will collapse.

$\boxed{5.}$ The initial mass of a star determines its final destiny. If, after the red giant stage of a star's life, its mass is less than 1.4 solar masses, then the star cools as it shrinks and it becomes a white dwarf. If its mass is between 1.4 and 2–3 solar masses, then the star will condense down to a neutron star, which will eventually explode as a supernova and become a white dwarf. If its mass is greater than 2–3 solar masses, then the star will collapse even more than the neutron star and form a black hole.

6. When measuring parallaxes from the Moon, there are two cases: (1) If you did the measurements two weeks apart (one at full moon and one at new moon), you would need to assume that the Earth did not move around the Sun very far, and then the *d* shown in Fig. 33–11 would be the Earth–Moon distance instead of the Sun–Earth distance. (2) If you did the measurements six months apart and at full moon, then the *d* shown in Fig. 33–11 would be the Sun–Earth distance plus the Earth–Moon distance instead of just the Sun–Earth distance. From Mars, then the *d* shown in Fig. 33–11 would be the Sun–Mars distance instead of the Sun–Earth distance. You would also need to know the length of a Mars "year" so that you could take your two measurements at the correct times.

7. Measure the period of the changing luminosity of a Cepheid variable star. Use the known relationship between period and luminosity to find its absolute luminosity. Compare its absolute luminosity to its apparent brightness (the observed brightness) to determine the distance to the galaxy in which it is located.

8. A geodesic is the shortest distance between two points. For instance, on a flat plane the shortest distance between two points is a straight line, and on the surface of a sphere the shortest distance is an arc of a great circle. According to general relativity, space–time is curved. Determining the nature of a geodesic, for instance, by observing the motion of a body or light near a large mass, will help determine the nature of the curvature of space–time near that large mass.

9. If the redshift of spectral lines of galaxies was due to something other than expansion of the universe, then the Big Bang theory and the idea that the universe is expanding would be called into question. However, the evidence of the cosmic background microwave radiation would conflict with this view, unless it too was determined to result from some cause other than expansion.

10. No, just because everything appears to be moving away from us does not mean we are at the center of the universe. Here is an analogy. If you were sitting on the surface of a balloon and more air was put into the balloon, causing it to expand, then every other point on the balloon would move away from you. The points close to you would be farther away because of the expansion of the rubber, and the points on the other side of the balloon would be farther away from you because the radius of the balloon would be larger.

11. If you were located in a galaxy near the boundary of our observable universe, galaxies in the direction of the Milky Way would be receding from you. The outer "edges" of the observable universe are expanding at a faster rate than the points more "interior." Accordingly, due to the relative velocity argument, the slower galaxies in the direction of the Milky Way would look like they are receding from your faster galaxy near the outer boundary. Also see Fig. 33–22.

12. An explosion on Earth blows pieces out into the space around it, but the Big Bang was the start of the expansion of space itself. In an explosion on Earth, the pieces that are blown outward will slow down due to air resistance, and the farther away they are, the slower they will be moving. They will eventually come to rest. But with the Big Bang, the farther away galaxies are from each other, the faster they are moving away from each other. In an explosion on Earth, the pieces with the higher initial speeds end up farther away from the explosion before coming to rest, but the Big Bang appears to be relatively uniform: the farthest galaxies are moving the fastest, and the nearest galaxies are moving the slowest. An explosion on Earth would correspond to a closed universe, since the pieces would eventually stop, but we would not see a "big crunch" due to gravity as we would with an actual closed universe.

13. To "see" a black hole in space, we need indirect evidence. If a large visible star or galaxy was rotating quickly around a nonvisible gravitational companion, then the nonvisible companion could be a massive black hole. Also, as matter begins to accelerate toward a black hole, it will emit characteristic

X-rays, which we could detect on Earth. Another way we could "see" a black hole is if it caused gravitational lensing of objects behind it. Then we would see stars and galaxies in the "wrong" place as their light is bent as it passes past the black hole on its way to Earth.

14. Both the formation of the Earth and the time during which people have lived on Earth are on the far right edge of Fig. 33–29, in the era of dark energy.

15. Atoms were unable to exist until hundreds of thousands of years after the Big Bang because the temperature of the universe was still too high. At those very high temperatures, the free electrons and nuclei were moving so fast and had so much kinetic energy, and they had so many high energy photons colliding with them, that they could never combine together to form atoms. Once the universe cooled below 3000 K, this coupling could take place, and atoms were formed.

16. (*a*) All Type Ia supernovae are expected to be of nearly the same luminosity. Thus they are a type of "standard candle" for measuring very large distances.

    (*b*) The distance to a supernova can be determined by comparing the apparent brightness with the intrinsic luminosity and then using Eq. 33–1 to find the distance.

17. If the average mass density of the universe is above the critical density, then the universe will eventually stop its expansion and contract, collapsing on itself and ending finally in a "big crunch." This scenario corresponds to a closed universe, or one with positive curvature.

18. (*a*) Gravity between galaxies should be pulling the galaxies back together, slowing the expansion of the universe.

    (*b*) Astronomers could measure the redshift of light from distant supernovae and deduce the recession velocities of the galaxies in which they lie. By obtaining data from a large number of supernovae, they could establish a history of the recessional velocity of the universe and perhaps tell whether the expansion of the universe is slowing down.

Responses to MisConceptual Questions

1. (*c*) At the end of the hydrogen-fusing part of the star's life, it will move toward the upper right of the diagram as it becomes a red giant. Low-mass stars, not large enough to end as neutron stars, then move to the lower left as they become white dwarf stars. The position of the star on the main sequence is a result of the size (mass) of the star. This does not change significantly during its main-sequence lifetime, and therefore the star does not change positions along the main sequence.

2. (*b*) Parallax requires that the star move relative to the background of distant stars. This only happens when the distance to the star is relatively small, less than about 100 light-years.

3. (*b*) If the universe were expanding in every direction, then any location in the universe would observe that all other points in the universe are moving away from it with the speed increasing with distance. This is observed when galaxy speeds are measured, implying that indeed the universe is expanding. The observation does not say anything about whether the expansion will continue forever or eventually stop.

4. (*c*) The Sun is primarily made of hydrogen, not heavy radioactive isotopes. As gravity initially compressed the Sun, its core heated sufficiently for the hydrogen atoms in the core to fuse to create helium. This is now the primary source of energy in the Sun. The Sun is too hot for molecules, such as water, to be formed by the oxidation of hydrogen. When fusion began in the

core, the energy release balanced the force of gravity to stop the gravitational collapse. Since the Sun is no longer contracting, the gravitational potential energy is no longer decreasing and is not a significant source of energy.

5. (*b*, *c, d*) Parallax is only useful for finding distances to stars that are less than 100 light-years distant, much less than intergalactic distances. The H–R diagram with luminosity and temperature of the star can tell us the absolute brightness of the distant star. The relationship between the relative brightness and absolute brightness tells us the distance to the star. Certain supernova explosions have a standard luminosity. A comparison of the apparent brightness and absolute brightness of the supernova can give the distance to the supernova. Finally, since the universe is expanding and the velocity of objects increases with distance, a measure of the redshift in light from distant stars can be used to find the speed of the stars and therefore the distance.

6. (*b*) Due to the speed of light, light takes a finite time to reach Earth from the stars, with light from more distant objects requiring more time to reach Earth. Therefore, when an object that is a million light-years away is observed, the observer sees the object as it was a million years ago. If the object is 100 million light-years away, the observer sees it as it was 100 million years ago. The farther the observed object, the earlier the time that it is being observed.

7. (*d*) At the time of the Big Bang, all of space was compacted to a small region. The expansion occurred throughout all space. So it occurred near the Earth, near the center of the Milky Way Galaxy, several billion light-years away, near the Andromeda Galaxy, and everywhere else.

8. (*d*) In the first few seconds of the universe, matter did not exist, only energy; the first atoms were not formed until many thousands of years later, and these atoms were primarily hydrogen. Stars on the main sequence and novae convert hydrogen into helium. They do not create the heavier elements. These heavy elements are typically created in the huge energy bursts of supernovae.

9. (*c*) The rotational period of a star in a galaxy is related to the mass in the galaxy. Astronomers observing the rotational periods found them to be much greater than could be accounted for by the visible matter (stars and dust clouds). Dust clouds are not dark matter, as they can reflect light and are therefore visible. The acceleration of the expansion of the universe is the foundation for the theory of dark energy, which is different from dark matter.

10. (*a*) If the Big Bang were a single large explosion that gave the universe an initial large kinetic energy, and since currently the only large-scale force in the universe is the attractive force of gravity, we would expect that the kinetic energy of the universe would be decreasing as that energy becomes gravitational potential energy. The expansion, however, is accelerating, so there must be another energy source fueling the acceleration. This source has been dubbed "dark energy."

Solutions to Problems

1. Convert the angle to seconds of arc, reciprocate to find the distance in parsecs, and then convert to light-years.

$$\phi = (2.9\times10^{-4})^\circ\left(\frac{3600''}{1^\circ}\right) = 1.044''$$

$$d\ (\text{pc}) = \frac{1}{\phi''} = \frac{1}{1.044''} = 0.958\ \text{pc}\left(\frac{3.26\ \text{ly}}{1\ \text{pc}}\right) = \boxed{3.1\ \text{ly}}$$

$\boxed{2.}$ Use the angle to calculate the distance in parsecs and then convert to light-years.

$$d\ (\text{pc}) = \frac{1}{\phi''} = \frac{1}{0.27''} = 3.704\ \text{pc} \quad \rightarrow \quad 3.704\ \text{pc}\left(\frac{3.26\ \text{ly}}{1\ \text{pc}}\right) = \boxed{12\ \text{ly}}$$

3. The parallax angle is $\boxed{\text{smaller}}$ for the farther star. Since $\tan\phi = d/D$, as the distance D to the star increases, the tangent decreases, so the angle decreases. And since for small angles, $\tan\phi \approx \phi$, we have that $\phi \approx d/D$. Thus if the distance D is doubled, then the angle ϕ will be $\boxed{\text{smaller by a factor of 2}}$.

$\boxed{4.}$ The apparent brightness of an object is inversely proportional to the observer's distance from the object, given by Eq. 33–1, $b = \dfrac{L}{4\pi d^2}$. To find the relative brightness at one location as compared with another, take a ratio of the apparent brightness at each location.

$$\frac{b_{\text{Jupiter}}}{b_{\text{Earth}}} = \frac{\dfrac{L}{4\pi d_{\text{Jupiter}}^2}}{\dfrac{L}{4\pi d_{\text{Earth}}^2}} = \frac{d_{\text{Earth}}^2}{d_{\text{Jupiter}}^2} = \left(\frac{d_{\text{Earth}}}{d_{\text{Jupiter}}}\right)^2 = \left(\frac{1}{5.2}\right)^2 = \boxed{3.7\times10^{-2}}$$

5. The density is the mass divided by the volume.

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(6\times10^{10}\ \text{m})^3} = \boxed{2\times10^{-3}\ \text{kg/m}^3}$$

6. (*a*) The apparent brightness is the solar constant, $\boxed{1.3\times10^3\ \text{W/m}^2}$.

(*b*) Use Eq. 33–1 to find the absolute luminosity.

$$b = \frac{L}{4\pi d^2} \quad \rightarrow \quad L = 4\pi d^2 b = 4\pi(1.496\times10^{11}\ \text{m}^2)^2(1.3\times10^3\ \text{W/m}^2) = \boxed{3.7\times10^{26}\ \text{W}}$$

7. The angular width is the inverse tangent of the diameter of our Galaxy divided by the distance to the nearest galaxy. According to Fig. 33–2, our Galaxy is about 100,000 ly in diameter.

$$\phi = \tan^{-1}\frac{\text{Galaxy diameter}}{\text{Distance to nearest galaxy}} = \tan^{-1}\frac{1.0\times10^5\ \text{ly}}{2.4\times10^6\ \text{ly}} = \boxed{4.2\times10^{-2}\ \text{rad}} \approx 2.4^\circ$$

$$\phi_{\text{Moon}} = \tan^{-1}\frac{\text{Moon diameter}}{\text{Distance to Moon}} = \tan^{-1}\frac{3.48\times10^6\ \text{m}}{3.84\times10^8\ \text{m}} = 9.1\times10^{-3}\ \text{rad}\ (\approx 0.52^\circ)$$

$\boxed{\text{The galaxy width is about 4.5 times the Moon's width.}}$

8. The text says that there are about 4×10^{11} stars in the galaxy and about 10^{11} galaxies, so that means there are about $\boxed{4\times10^{22}\ \text{stars}}$ in the observable universe.

$\boxed{9.}$ The density is the mass divided by the volume.

$$\rho = \frac{M}{V} = \frac{M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Earth}}^3} = \frac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(6.38\times10^6\ \text{m})^3} = \boxed{1.83\times10^9\ \text{kg/m}^3}$$

Since the volumes are the same, the ratio of the densities is the same as the ratio of the masses.

$$\frac{\rho}{\rho_{\text{Earth}}}=\frac{M}{M_{\text{Earth}}}=\frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}=\boxed{3.33\times10^{5}\text{ times larger}}$$

10. The density of the neutron star is its mass divided by its volume. Use the proton to calculate the density of nuclear matter. The radius of the proton is taken from Eq. 30–1.

$$\rho_{\substack{\text{neutron}\\\text{star}}}=\frac{M}{V}=\frac{1.5(1.99\times10^{30}\text{ kg})}{\frac{4}{3}\pi(11\times10^{3}\text{ m})^{3}}=5.354\times10^{17}\text{ kg/m}^3\approx\boxed{5.4\times10^{17}\text{ kg/m}^3}$$

$$\frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{white}\\\text{dwarf}}}}=\frac{5.354\times10^{17}\text{ kg/m}^3}{1.83\times10^{9}\text{ kg/m}^3}=\boxed{2.9\times10^{8}}\qquad\frac{\rho_{\substack{\text{neutron}\\\text{star}}}}{\rho_{\substack{\text{nuclear}\\\text{star}}}}=\frac{5.354\times10^{17}\text{ kg/m}^3}{\dfrac{1.673\times10^{-27}\text{ kg}}{\frac{4}{3}\pi(1.2\times10^{-15}\text{ m})^{3}}}=\boxed{2.3}$$

11. The reciprocal of the distance in parsecs is the angle in seconds of arc.

(*a*) $$\phi''=\frac{1}{d\text{ (pc)}}=\frac{1}{56\text{ pc}}=0.01786\approx\boxed{0.018''}$$

(*b*) $$0.01786''\left(\frac{1^\circ}{3600''}\right)=\left(4.961\times10^{-6}\right)^\circ\approx\boxed{(5.0\times10^{-6})^\circ}$$

12. Convert the light-years to parsecs and then take the reciprocal of the number of parsecs to find the parallax angle in seconds of arc.

$$65\text{ ly}\left(\frac{1\text{ pc}}{3.26\text{ ly}}\right)=19.94\text{ pc}\approx\boxed{20\text{ pc}}\text{ (2 significant figures)}\qquad\phi=\frac{1}{19.94\text{ pc}}=\boxed{0.050''}$$

$\boxed{13.}$ Find the distance in light-years. That value is also the time for light to reach us.

$$85\text{ pc}\left(\frac{3.26\text{ ly}}{1\text{ pc}}\right)=277\text{ ly}\approx280\text{ ly}\;\rightarrow\;\text{It takes light }\boxed{280\text{ years}}\text{ to reach us.}$$

14. It is given that $b_1=b_2,\ r_1=r_2,\ \lambda_{\text{P1}}=750\text{ nm},$ and $\lambda_{\text{P2}}=450\text{ nm}.$ Wien's law (Eq. 27–1) says $\lambda_{\text{P}}T=\alpha,$ where α is a constant, so $\lambda_{\text{P1}}T_1=\lambda_{\text{P2}}T_2.$ The Stefan-Boltzmann equation (Eq. 14–6) says that the power output of a star is given by $P=\beta AT^4,$ where β is a constant and *A* is the radiating area. The *P* in the Stefan-Boltzmann equation is the same as the luminosity *L* in this Chapter. The luminosity *L* is related to the apparent brightness *b* by Eq. 33–1.

$$\lambda_{\text{P1}}T_1=\lambda_{\text{P2}}T_2\;\rightarrow\;\frac{T_2}{T_1}=\frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}$$

$$b_1=b_2\;\rightarrow\;\frac{L_1}{4\pi d_1^2}=\frac{L_2}{4\pi d_2^2}\;\rightarrow\;\frac{d_2^2}{d_1^2}=\frac{L_2}{L_1}=\frac{P_2}{P_1}=\frac{\beta A_2T_2^4}{\beta A_1T_1^4}=\frac{4\pi r_2^2T_2^4}{4\pi r_1^2T_1^4}=\frac{T_2^4}{T_1^4}=\left(\frac{T_2}{T_1}\right)^4\;\rightarrow$$

$$\frac{d_2}{d_1}=\left(\frac{T_2}{T_1}\right)^2=\left(\frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}\right)^2=\left(\frac{750}{450}\right)^2=2.8$$

$\boxed{\text{The star with the peak at 450 nm is 2.8 times farther away than the star with the peak at 750 nm.}}$

15. Wien's law (Eq. 27–2) says that $\lambda_{\text{P}}T = \alpha,$ where α is a constant, so $\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2.$ The Stefan-Boltzmann equation (Eq. 14–6) says that the power output of a star is given by $P = \beta AT^4,$ where β is a constant and A is the radiating area. The P in the Stefan-Boltzmann equation is the same as the luminosity L in this Chapter. The luminosity L is related to the apparent brightness b by Eq. 33–1. It is given that $b_1/b_2 = 0.091,$ $d_1 = d_2,$ $\lambda_{\text{P1}} = 470\text{ nm},$ and $\lambda_{\text{P2}} = 720\text{ nm}.$

$$\lambda_{\text{P1}}T_1 = \lambda_{\text{P2}}T_2 \quad \rightarrow \quad \frac{T_2}{T_1} = \frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}; \quad b_1 = 0.091\,b_2 \quad \rightarrow \quad \frac{L_1}{4\pi d_1^2} = 0.091\frac{L_2}{4\pi d_2^2} \quad \rightarrow$$

$$1 = \frac{d_2^2}{d_1^2} = \frac{0.091L_2}{L_1} = \frac{0.091P_2}{P_1} = \frac{0.091A_2T_2^4}{A_1T_1^4} = \frac{(0.091)4\pi r_2^2T_2^4}{4\pi r_1^2T_1^4} = 0.091\frac{T_2^4}{T_1^4}\frac{r_2^2}{r_1^2} \quad \rightarrow$$

$$\frac{r_1}{r_2} = \sqrt{0.091}\left(\frac{T_2}{T_1}\right)^2 = \sqrt{0.091}\left(\frac{\lambda_{\text{P1}}}{\lambda_{\text{P2}}}\right)^2 = \sqrt{0.091}\left(\frac{470\text{ nm}}{720\text{ nm}}\right)^2 = 0.1285$$

The ratio of the diameters is the same as the ratio of radii, so $\frac{D_1}{D_2} = \boxed{0.13}.$

$\boxed{16.}$ The Schwarzschild radius is $2GM/c^2$.

$$R_{\text{Earth}} = \frac{2GM_{\text{Earth}}}{c^2} = \frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(3.00\times10^8\text{ m/s})^2} = 8.86\times10^{-3}\text{ m} \approx \boxed{8.9\text{ mm}}$$

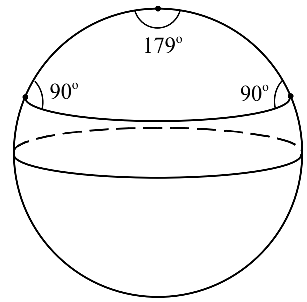
17. The Schwarzschild radius is given by $R = 2GM/c^2.$ An approximate mass of ordinary matter for our Galaxy is calculated in Example 33–1. A value of twice that mass is also quoted in the text, so the given answer may vary by a factor of 2.

$$R = \frac{2GM}{c^2} = \frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2\times10^{41}\text{ kg})}{(3.00\times10^8\text{ m/s})^2} = \boxed{3\times10^{14}\text{ m}}$$

18. The Schwarzschild radius is given by $R = 2GM/c^2,$ so $M = \frac{Rc^2}{2G}.$ The radius is given in Eq. 27–14.

$$M = \frac{Rc^2}{2G} = \frac{(5.29\times10^{-11}\text{ m})(3.00\times10^8\text{ m/s})^2}{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = \boxed{3.57\times10^{-16}\text{ kg}}$$

19. The limiting value for the angles in a triangle on a sphere is $\boxed{540°}$. Imagine drawing an equilateral triangle near the north pole, enclosing the north pole. If that triangle were small, then the surface would be approximately flat, and each angle in the triangle would be $60°.$ Then imagine "stretching" each vertex of that triangle down toward the equator, ensuring that the north pole was inside the triangle. The angle at each vertex of the triangle would expand, with a limiting value of $180°.$ The three $180°$ angles in the triangle would sum to $540°.$



20. (*a*) For the vertices of the triangle, we choose the north pole and two points on a latitude line on nearly opposite sides of the Earth, as shown on the diagram. Let the angle at the north pole be 179°.

(*b*) It is not possible to draw a triangle on a sphere with the sum of the angles less than 180°. To draw a triangle like that, a hyperbolic surface like a saddle (similar to Fig. 33–18) would be needed.

21. We find the time for the light to cross the elevator and then find how far the elevator moves during that time due to its acceleration.

$$\Delta t = \frac{\Delta x}{c}; \quad \Delta y = \tfrac{1}{2} g(\Delta t)^2 = \frac{g(\Delta x)^2}{2c^2} = \frac{(9.80\text{ m/s}^2)(2.4\text{ m})^2}{2(3.00\times10^8\text{ m/s})^2} = \boxed{3.1\times10^{-16}\text{ m}}$$

Note that this is smaller than the size of a proton.

22. Use Eq. 33–4, Hubble's law.

$$\upsilon = H_0 d \quad \rightarrow \quad d = \frac{\upsilon}{H_0} = \frac{1850\text{ km/s}}{21\text{ km/s/Mly}} = \boxed{88\text{ Mly}} = 8.8\times10^7\text{ ly}$$

23. Use Eq. 33–4, Hubble's law.

$$\upsilon = H_0 d \quad \rightarrow \quad d = \frac{\upsilon}{H_0} = \frac{(0.015)(3.00\times10^8\text{ m/s})}{2.1\times10^4\text{ m/s/Mly}} = 214.3\text{ Mly} \approx \boxed{210\text{ Mly}} = 2.1\times10^8\text{ ly}$$

$\boxed{24.}$ (*a*) Use Eq. 33–6, applicable for $\upsilon \ll c$, to solve for the speed of the galaxy.

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} \approx \frac{\upsilon}{c} \quad \rightarrow \quad \upsilon = c\left(\frac{455\text{ nm} - 434\text{ nm}}{434\text{ nm}}\right) = 0.04839c \approx \boxed{0.048c}$$

The size of the answer does meet the condition that $\upsilon \ll c$.

(*b*) Use Hubble's law, Eq. 33–4, to solve for the distance.

$$\upsilon = H_0 d \quad \rightarrow \quad d = \frac{\upsilon}{H_0} = \frac{0.04839(3.00\times10^8\text{ m/s})}{21000\text{ m/s/Mly}} = 691\text{ Mly} \approx \boxed{6.9\times10^8\text{ ly}}$$

25. We find the velocity from Hubble's law, Eq. 33–4, and the observed wavelength from the Doppler shift, Eq. 33–3.

(*a*) $$\frac{\upsilon}{c} = \frac{H_0 d}{c} = \frac{(21{,}000\text{ m/s/Mly})(7.0\text{ Mly})}{3.00\times10^8\text{ m/s}} = 4.9\times10^{-4}$$

$$\lambda = \lambda_0\sqrt{\frac{1+\upsilon/c}{1-\upsilon/c}} = (656.3\text{ nm})\sqrt{\frac{1+4.9\times10^{-4}}{1-4.9\times10^{-4}}} = 656.62\text{ nm} \quad \rightarrow \quad \text{shift} = \lambda - \lambda_0 = \boxed{0.3\text{ nm}}$$

(*b*) $$\frac{\upsilon}{c} = \frac{H_0 d}{c} = \frac{(21{,}000\text{ m/s/Mly})(70\text{ Mly})}{3.00\times10^8\text{ m/s}} = 4.9\times10^{-3}$$

$$\lambda = \lambda_0\sqrt{\frac{1+\upsilon/c}{1-\upsilon/c}} = (656.3\text{ nm})\sqrt{\frac{1+4.9\times10^{-3}}{1-4.9\times10^{-3}}} = 659.52\text{ nm} \quad \rightarrow \quad \text{shift} = \lambda - \lambda_0 = \boxed{3.2\text{ nm}}$$

26. Use Eqs. 33–3 and 33–4 to solve for the distance to the galaxy.

$$\lambda_{\text{obs}} = \lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}} \quad \rightarrow \quad v = c\frac{\left(\lambda_{\text{obs}}^2 - \lambda_{\text{rest}}^2\right)}{\left(\lambda_{\text{obs}}^2 + \lambda_{\text{rest}}^2\right)}$$

$$d = \frac{v}{H_0} = \frac{c}{H_0}\frac{\left(\lambda_{\text{obs}}^2 - \lambda_{\text{rest}}^2\right)}{\left(\lambda_{\text{obs}}^2 + \lambda_{\text{rest}}^2\right)} = \frac{(3.00\times10^8\text{ m/s})}{(2.1\times10^4\text{ m/s/Mly})}\frac{\left[(423.4\text{ nm})^2 - (393.4\text{ nm})^2\right]}{\left[(423.4\text{ nm})^2 + (393.4\text{ nm})^2\right]}$$

$$= 1.048\times10^3\text{ Mly} \approx \boxed{1.0\times10^9\text{ ly}}$$

27. Use Eqs. 33–3 and 33–5a to solve for the speed of the galaxy.

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} - 1 = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \quad \rightarrow$$

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} = \frac{1.060^2 - 1}{1.060^2 + 1} = 0.05820 \quad \rightarrow \quad v = \boxed{0.058c}$$

The approximation of Eq. 33–6 gives $v = zc = \boxed{0.060c}$.

28. Use Eqs. 33–3 and 33–5a to solve for the redshift parameter.

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} - 1 = \sqrt{\frac{1+v/c}{1-v/c}} - 1 = \sqrt{\frac{1+0.075}{1-0.075}} - 1 = \boxed{0.078}$$

Or we can use the approximation given in Eq. 33–6.

$$z \approx v/c = \boxed{0.075}$$

29. We cannot use the approximation of Eq. 33–6 for this circumstance. Instead, we use Eq. 33–5b combined with Eq. 33–3, and then use Eq. 33–4 for the distance.

$$z = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \quad \rightarrow \quad z+1 = \sqrt{\frac{1+v/c}{1-v/c}} = 2 \quad \rightarrow \quad \frac{1+v/c}{1-v/c} = 4$$

$$1 + v/c = 4(1 - v/c) \quad \rightarrow \quad 1 + v/c = 4 - 4v/c \quad \rightarrow \quad 5v/c = 3 \quad \rightarrow \quad v = 0.6c$$

$$d = \frac{v}{H_0} = \frac{0.6(3.00\times10^8\text{ m/s})}{21\times10^3\text{ m/s/Mly}} = 8571\text{ Mly} \approx \boxed{9\times10^9\text{ ly}}$$

30. Use Eqs. 33–4 and 33–5a to solve for the speed of the galaxy.

$$\Delta\lambda = 610\text{ nm} - 434\text{ nm} = 176\text{ nm}$$

$$z = \frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \quad \rightarrow \quad \frac{v}{c} = \frac{\left(\frac{\Delta\lambda}{\lambda_0}+1\right)^2 - 1}{\left(\frac{\Delta\lambda}{\lambda_0}+1\right)^2 + 1} = \frac{\left(\frac{176\text{ nm}}{434\text{ nm}}+1\right)^2 - 1}{\left(\frac{176\text{ nm}}{434\text{ nm}}+1\right)^2 + 1} = \frac{0.9755}{2.9755} = 0.3278c \quad \rightarrow \quad v \approx \boxed{0.33c}$$

Use Hubble's law, Eq. 33–4, to solve for the distance.

$$v = H_0 d \quad \rightarrow \quad d = \frac{v}{H_0} = \frac{v/c}{H_0/c} = \frac{0.3278}{\frac{(21000\text{ m/s/Mly})}{3.00\times10^8\text{ m/s}}} = 4.68\times10^3\text{ Mly} \approx \boxed{4.7\times10^9\text{ ly}}$$

31. For small relative wavelength shifts, we may use Eq. 33–6 to find the speed. We use Eq. 33–4 to find the distance.

$$\frac{v}{c}=\frac{\Delta\lambda}{\lambda_{\text{rest}}} \rightarrow v=c\frac{\Delta\lambda}{\lambda_{\text{rest}}}; \quad v=H_0 d \rightarrow$$

$$d=\frac{v}{H_0}=\frac{c}{H_0}\frac{\Delta\lambda}{\lambda_{\text{rest}}}=\left(\frac{3.00\times10^8 \text{ m/s}}{21{,}000 \text{ m/s/Mly}}\right)\left(\frac{0.10 \text{ cm}}{21 \text{ cm}}\right)=\boxed{68 \text{ Mly}}$$

We also use the more exact relationship of Eq. 33–3.

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}=\sqrt{\frac{1+v/c}{1-v/c}} \rightarrow \frac{v}{c}=\frac{\left(\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}\right)^2-1}{\left(\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}}\right)^2+1}=\frac{\left(\frac{21.1 \text{ cm}}{21 \text{cm}}\right)^2-1}{\left(\frac{21.1 \text{ cm}}{21 \text{ cm}}\right)^2+1}=4.75\times10^{-2}$$

$$v=H_0 d \rightarrow d=\frac{v}{H_0}=\frac{v/c}{H_0/c}=\frac{4.75\times10^{-2}}{\frac{(21000 \text{ m/s/Mly})}{3.00\times10^8 \text{ m/s}}}=67.86 \text{ Mly}\approx\boxed{68 \text{ Mly}}$$

32. Eq. 33–3 states $\lambda=\lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}}$.

$$\lambda=\lambda_{\text{rest}}\sqrt{\frac{1+v/c}{1-v/c}}=\lambda_{\text{rest}}\left(1+\frac{v}{c}\right)^{1/2}\left(1-\frac{v}{c}\right)^{1/2}\approx\lambda_{\text{rest}}\left(1+\tfrac{1}{2}\frac{v}{c}\right)\left(1-\left(-\tfrac{1}{2}\right)\frac{v}{c}\right)=\lambda_{\text{rest}}\left(1+\tfrac{1}{2}\frac{v}{c}\right)^2$$

$$\lambda=\lambda_{\text{rest}}\left(1+2\left(\tfrac{1}{2}\frac{v}{c}\right)\right)=\lambda_{\text{rest}}\left(1+\frac{v}{c}\right)=\lambda_{\text{rest}}+\lambda_{\text{rest}}\frac{v}{c} \rightarrow \lambda-\lambda_{\text{rest}}=\Delta\lambda=\lambda_{\text{rest}}\frac{v}{c} \rightarrow$$

$$\boxed{\frac{\Delta\lambda}{\lambda_{\text{rest}}}=\frac{v}{c}}$$

33. Wien's law is Eq. 27–2.

$$\lambda_{\text{P}}T=2.90\times10^{-3} \text{ m}\cdot\text{K} \rightarrow \lambda_{\text{P}}=\frac{2.90\times10^{-3} \text{ m}\cdot\text{K}}{T}=\frac{2.90\times10^{-3} \text{ m}\cdot\text{K}}{2.7 \text{ K}}=\boxed{1.1\times10^{-3} \text{ m}}$$

34. We use Wien's law, Eq. 27–2. From Fig. 33–29, the temperature at that time is about 10^{10} K.

$$\lambda_{\text{P}}T=2.90\times10^{-3} \text{ m}\cdot\text{K} \rightarrow \lambda_{\text{P}}=\frac{2.90\times10^{-3} \text{ m}\cdot\text{K}}{T}=\frac{2.90\times10^{-3} \text{ m}\cdot\text{K}}{10^{10} \text{ K}}=\boxed{3\times10^{-13} \text{ m}}$$

From Fig. 22–8, that wavelength is in the $\boxed{\text{gamma ray}}$ region of the EM spectrum.

35. We use the proton as typical nuclear matter.

$$\left(10^{-26}\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1 \text{ nucleon}}{1.67\times10^{-27} \text{ kg}}\right)=\boxed{6 \text{ nucleons/m}^3}$$

36. If the scale of the universe is inversely proportional to the temperature, then the scale times the temperature should be constant. If we call the current scale 1 and know the current temperature to be about 3 K, then the product of scale and temperature should be about 3. Use Fig. 33–29 to estimate the temperature at various times. For purposes of illustration, we assume that the universe has a current size of about 10^{10} ly. There will be some variation in the answer due to differences in reading the figure.

(*a*) At $t = 10^6$ yr, the temperature is about 1000 K. Thus the scale is found as follows:

$$(\text{Scale})(\text{Temperature}) = 3 \quad \to \quad \text{Scale} = \frac{3}{\text{Temperature}} = \frac{3}{1000} = \boxed{3\times10^{-3}} \quad \to$$

$$\text{Size} \approx (3\times10^{-3})(10^{10}\ \text{ly}) = \boxed{3\times10^{7}\ \text{ly}}$$

(*b*) At $t = 1$ s, the temperature is about 10^{10} K.

$$\text{Scale} = \frac{3}{\text{Temperature}} = \frac{3}{10^{10}} = \boxed{3\times10^{-10}} \quad \to \quad \text{Size} \approx (3\times10^{-10})(10^{10}\ \text{ly}) = \boxed{3\ \text{ly}}$$

(*c*) At $t = 10^{-6}$ s, the temperature is about 10^{12} K.

$$\text{Scale} = \frac{3}{\text{Temperature}} = \frac{3}{10^{12}} = \boxed{3\times10^{-12}} \quad \to$$

$$\text{Size} \approx (3\times10^{-12})(10^{10}\ \text{ly}) = \boxed{3\times10^{-2}\ \text{ly}} \approx 3\times10^{14}\ \text{m}$$

(*d*) At $t = 10^{-35}$ s, the temperature is about 10^{27} K.

$$\text{Scale} = \frac{3}{\text{Temperature}} = \frac{3}{10^{27}} = \boxed{3\times10^{-27}}$$

$$\text{Size} \approx (3\times10^{-27})(10^{10}\ \text{ly}) = 3\times10^{-17}\ \text{ly} \approx \boxed{0.3\ \text{m}}$$

37. We approximate the temperature–energy relationship by $kT = E = mc^2$ as suggested in Section 33–7.

$$kT = mc^2 \quad \to \quad T = \frac{mc^2}{k}.$$

(*a*) $$T = \frac{mc^2}{k} = \frac{(500\ \text{MeV}/c^2)c^2(1.60\times10^{-13}\ \text{J/MeV})}{1.38\times10^{-23}\ \text{J/K}} = 6\times10^{12}\ \text{K}$$

From Fig. 33–29, this corresponds to a time of $\boxed{\approx 10^{-5}\ \text{s}}$.

(*b*) $$T = \frac{mc^2}{k} = \frac{(9500\ \text{MeV}/c^2)c^2(1.60\times10^{-13}\ \text{J/MeV})}{1.38\times10^{-23}\ \text{J/K}} = 1\times10^{14}\ \text{K}$$

From Fig. 33–29, this corresponds to a time of $\boxed{\approx 10^{-7}\ \text{s}}$.

(*c*) $$T = \frac{mc^2}{k} = \frac{(100\ \text{MeV}/c^2)c^2(1.60\times10^{-13}\ \text{J/MeV})}{1.38\times10^{-23}\ \text{J/K}} = 1\times10^{12}\ \text{K}$$

From Fig. 33–29, this corresponds to a time of $\boxed{\approx 10^{-4}\ \text{s}}$.

There will be some variation in the answers due to differences in reading the figure.

38. (*a*) According to the textbook, near Fig. 33–33, the visible matter makes up about one-tenth of the total baryonic matter. The average baryonic density is therefore 10 times the density of visible matter. The data in the Problem are for visible matter only (stars and galaxies).

$$\rho_{\text{baryon}} = 10\rho_{\text{visible}} = 10\frac{M_{\text{visible}}}{\frac{4}{3}\pi R^3}$$

$$= 10\frac{(10^{11}\text{ galaxies})(10^{11}\text{ stars/galaxy})(2.0\times10^{30}\text{ kg/star})}{\frac{4}{3}\pi\left[(14\times10^9\text{ ly})(9.46\times10^{15}\text{ m/ly})\right]^3} \approx \boxed{2.1\times10^{-26}\text{ kg/m}^3}$$

(*b*) Again, according to the text, dark matter is about 5 times more plentiful than baryonic matter.

$$\rho_{\text{dark}} = 5\rho_{\text{baryon}} = 5(2.055\times10^{-26}\text{ kg/m}^3) \approx \boxed{1.0\times10^{-25}\text{ kg/m}^3}$$

39. The angular momentum is the product of the rotational inertia and the angular velocity.

$$(I\omega)_{\text{initial}} = (I\omega)_{\text{final}} \quad \rightarrow$$

$$\omega_{\text{final}} = \omega_{\text{initial}}\left(\frac{I_{\text{initial}}}{I_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{\frac{2}{5}MR^2_{\text{initial}}}{\frac{2}{5}MR^2_{\text{final}}}\right) = \omega_{\text{initial}}\left(\frac{R_{\text{initial}}}{R_{\text{final}}}\right)^2 = (1\text{ rev/month})\left(\frac{6\times10^6\text{ m}}{8\times10^3\text{ m}}\right)^2$$

$$= 5.625\times10^5\text{ rev/month} = 5.625\times10^5\frac{\text{rev}}{\text{month}}\times\frac{1\text{ month}}{30\text{ d}}\times\frac{1\text{ d}}{24\text{ h}}\times\frac{1\text{ h}}{3600\text{ s}} = 0.217\text{ rev/s}$$

$$\approx \boxed{0.2\text{ rev/s}}$$

40. The rotational kinetic energy is given by $\frac{1}{2}I\omega^2$. The final angular velocity, from Problem 39, is 5.625×10^5 rev/month.

$$\frac{\text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}} = \frac{\frac{1}{2}I_{\text{final}}\omega^2_{\text{final}}}{\frac{1}{2}I_{\text{initial}}\omega^2_{\text{initial}}} = \frac{\frac{2}{5}MR^2_{\text{final}}\omega^2_{\text{final}}}{\frac{2}{5}MR^2_{\text{initial}}\omega^2_{\text{initial}}} = \left(\frac{R_{\text{final}}\omega_{\text{final}}}{R_{\text{initial}}\omega_{\text{initial}}}\right)^2$$

$$= \left(\frac{(8\times10^3\text{ m})(5.625\times10^5\text{ rev/month})}{(6\times10^6\text{ m})(1\text{ rev/month})}\right)^2 = 5.625\times10^5 \approx \boxed{6\times10^5}$$

41. A: The temperature increases, the luminosity stays the same, and the size decreases.
B: The temperature stays the same, the luminosity decreases, and the size decreases.
C: The temperature decreases, the luminosity increases, and the size increases.

42. The apparent luminosity is given by Eq. 33–1. Use that relationship to derive an expression for the absolute luminosity, and equate the Sun's luminosity to the star's luminosity.

$$b = \frac{L}{4\pi d^2} \quad \rightarrow \quad L = 4\pi d^2 b$$

$$L_{\substack{\text{distant}\\\text{star}}} = L_{\text{Sun}} \quad \rightarrow \quad 4\pi d^2_{\substack{\text{distant}\\\text{star}}} b_{\substack{\text{distant}\\\text{star}}} = 4\pi d^2_{\text{Sun}} b_{\text{Sun}} \quad \rightarrow$$

$$d_{\substack{\text{distant}\\\text{star}}} = d_{\text{Sun}}\sqrt{\frac{l_{\text{Sun}}}{l_{\substack{\text{distant}\\\text{star}}}}} = (1.5\times10^{11}\text{ m})\sqrt{\frac{1}{10^{-11}}}\left(\frac{1\text{ ly}}{9.461\times10^{15}\text{ m}}\right) = \boxed{5\text{ ly}}$$

43. The power output is the energy loss divided by the elapsed time.

$$P = \frac{\Delta \text{KE}}{\Delta t} = \frac{\frac{1}{2} I\omega^2 (\text{fraction lost})}{\Delta t} = \frac{\frac{1}{2}\frac{2}{5} MR^2\omega^2 (\text{fraction lost})}{\Delta t}$$

$$= \frac{1}{5}\frac{(1.5)(1.99\times10^{30}\ \text{kg})(8.0\times10^3\ \text{m})^2 (2\pi \text{rad/s})^2 (1\times10^{-9})}{(1\ \text{d})(24\ \text{h/d})(3600\ \text{s/h})} = 1.74610^{25}\ \text{W} \approx \boxed{1.7\times10^{25}\ \text{W}}$$

44. Use Newton's law of universal gravitation.

$$F = G\frac{m_1 m_2}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(4\times10^{41}\ \text{kg})^2}{\left[(2\times10^6\ \text{ly})(9.46\times10^{15}\ \text{m/ly})\right]^2} = 2.98\times10^{28}\ \text{N}$$

$$\approx \boxed{3\times10^{28}\ \text{N}}$$

45. We use the Sun's mass and given density to calculate the size of the Sun.

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r_{\text{Sun}}^3} \quad \rightarrow$$

$$r_{\text{Sun}} = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left[\frac{3(1.99\times10^{30}\ \text{kg})}{4\pi(10^{-26}\ \text{kg/m}^3)}\right]^{1/3} = 3.62\times10^{18}\ \text{m}\left(\frac{1\text{ly}}{9.46\times10^{15}\ \text{m}}\right) = 382\ \text{ly} \approx \boxed{400\ \text{ly}}$$

$$\frac{r_{\text{Sun}}}{d_{\text{Earth-Sun}}} = \frac{3.62\times10^{18}\ \text{m}}{1.50\times10^{11}\ \text{m}} \approx \boxed{2\times10^7}; \quad \frac{r_{\text{Sun}}}{d_{\text{galaxy}}} = \frac{382\ \text{ly}}{100{,}000\,\text{ly}} \approx \boxed{4\times10^{-3}}$$

46. The temperature of each star can be found from Wien's law, Eq. 27–2. The peak wavelength is used as a subscript to designate each star's properties.

$$\lambda_{\text{P}} T = 2.90\times10^{-3}\ \text{m}\cdot\text{K} \quad \rightarrow$$

$$T_{660} = \frac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{660\times10^{-9}\ \text{m}} = 4394\ \text{K} \qquad T_{480} = \frac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{480\times10^{-9}\ \text{m}} = 6042\ \text{K}$$

The luminosity of each star can be found from the H–R diagram.

$$L_{660} \approx 7\times10^{25}\ \text{W} \qquad L_{480} \approx 5\times10^{26}\ \text{W}$$

The Stefan-Boltzmann equation says that the power output of a star is given by $P = \beta A T^4$, where β is a constant and A is the radiating area. The P in the Stefan-Boltzmann equation is the same as the luminosity L given in Eq. 33–1. Form the ratio of the two luminosities.

$$\frac{L_{480}}{L_{660}} = \frac{\beta A_{480} T_{480}^4}{\beta A_{660} T_{660}^4} = \frac{4\pi r_{480}^2 T_{480}^4}{4\pi r_{660}^2 T_{660}^4} \quad \rightarrow \quad \frac{r_{480}}{r_{660}} = \sqrt{\frac{L_{480}}{L_{660}}\frac{T_{660}^2}{T_{480}^2}} = \sqrt{\frac{5\times10^{26}\ \text{W}}{7\times10^{25}\ \text{W}}\frac{(4394\ \text{K})^2}{(6042\ \text{K})^2}} = 1.413$$

The diameters are in the same ratio as the radii.

$$\frac{d_{480}}{d_{660}} = 1.413 \approx \boxed{1.4}$$

The luminosities are fairly subjective, since they are read from the H–R diagram. Different answers may arise due to different readings of the H–R diagram.

47. (*a*) First, calculate the number of parsecs. Then use the fact that the number of parsecs is the reciprocal of the angular resolution in seconds of arc.

$$100\text{ ly}\left(\frac{1\text{ pc}}{3.26\text{ ly}}\right)=30.67\text{ pc}=\frac{1}{\phi''}\quad\to$$

$$\phi=\left(\frac{1}{30.67}\right)''\left(\frac{1'}{60''}\right)\left(\frac{1^\circ}{60'}\right)=(9.06\times10^{-6})^\circ\approx\boxed{(9\times10^{-6})^\circ}$$

(*b*) We use the Rayleigh criterion, Eq. 25–7, which relates the angular resolution to the diameter of the optical element. We choose a wavelength of 550 nm, in the middle of the visible range.

$$\theta=\frac{1.22\lambda}{D}\quad\to\quad D=\frac{1.22\lambda}{\theta}=\frac{1.22(550\times10^{-9}\text{ m})}{\left[(9.06\times10^{-6})^\circ\right](\pi\text{ rad}/180^\circ)}=4.24\text{ m}\approx\boxed{4\text{ m}}$$

The largest optical telescopes with single mirrors are about 8 m in diameter.

$\boxed{48.}$ (*a*) We approximate the temperature–kinetic energy relationship by $kT=\text{KE}$, as given in Section 33–7.

$$kT=\text{KE}\quad\to\quad T=\frac{\text{KE}}{k}=\frac{(14\times10^{12}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{1.38\times10^{-23}\text{ J/K}}=\boxed{1.6\times10^{17}\text{ K}}$$

(*b*) From Fig. 33–29, this is in the $\boxed{\text{hadron era}}$.

49. (*a*) Find the *Q*-value for this reaction. From Eq. 30–2, the *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$${}^{12}_{6}\text{C}+{}^{12}_{6}\text{C}\quad\to\quad{}^{24}_{12}\text{Mg}$$

$$Q=2m_\text{C}c^2-m_\text{Mg}c^2=\left[2(12.000000\text{ u})-23.985042\text{ u}\right]c^2(931.5\text{ MeV}/c^2)=\boxed{13.93\text{ MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 30–1. Each nucleus will have half the total kinetic energy.

$$r=(1.2\times10^{-15}\text{ m})(A)^{1/3}=(1.2\times10^{-15}\text{ m})(12)^{1/3};\qquad \text{PE}=\frac{1}{4\pi\varepsilon_0}\frac{q^2_\text{nucleus}}{2r};$$

$$\text{KE}_\text{nucleus}=\tfrac{1}{2}\text{PE}=\tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2_\text{nucleus}}{2r}$$

$$=\tfrac{1}{2}(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(6)^2(1.60\times10^{-19}\text{ C})^2}{2(1.2\times10^{-15}\text{ m})(12)^{1/3}}\times\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}=\boxed{4.71\text{ MeV}}$$

(*c*) Approximate the temperature–kinetic energy relationship by $kT=\text{KE}$, as given in Section 33–7.

$$kT=\text{KE}\quad\to\quad T=\frac{\text{KE}}{k}=\frac{(4.71\times10^6\text{ eV})\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)}{1.38\times10^{-23}\text{ J/K}}=\boxed{5.46\times10^{10}\text{ K}}$$

$\boxed{50.}$ (*a*) Find the *Q*-value for this reaction. From Eq. 30–2, the *Q*-value is the mass energy of the reactants minus the mass energy of the products.

$$^{16}_{8}\text{C} + {}^{16}_{8}\text{C} \quad \rightarrow \quad {}^{28}_{14}\text{Si} + {}^{4}_{2}\text{He}$$

$$Q = 2m_{\text{C}}c^2 - m_{\text{Si}}c^2 - m_{\text{He}}c^2 = [2(15.994915\text{ u}) - 27.976927\text{ u} - 4.002603]c^2(931.5\text{ Mev}/c^2)$$
$$= \boxed{9.594\text{ MeV}}$$

(*b*) The total kinetic energy should be equal to the electrical potential energy of the two nuclei when they are just touching. The distance between the two nuclei will be twice the nuclear radius, from Eq. 30–1. Each nucleus will have half the total kinetic energy.

$$r = (1.2\times10^{-15}\text{ m})(A)^{1/3} = (1.2\times10^{-15}\text{ m})(16)^{1/3}; \quad \text{PE} = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{nucleus}}^2}{2r}$$

$$\text{KE}_{\text{nucleus}} = \tfrac{1}{2}\text{PE} = \tfrac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{nucleus}}^2}{2r}$$
$$= \tfrac{1}{2}(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(8)^2(1.60\times10^{-19}\text{ C})^2}{2(1.2\times10^{-15}\text{ m})(16)^{1/3}}\times\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}} = \boxed{7.61\text{ MeV}}$$

(*c*) Approximate the temperature–kinetic energy relationship by $kT = \text{KE}$, as given in Section 33–7.

$$kT = \text{KE} \quad \rightarrow \quad T = \frac{\text{KE}}{k} = \frac{(7.61\times10^6\text{ eV})\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)}{1.38\times10^{-23}\text{ J/K}} = \boxed{8.82\times10^{10}\text{ K}}$$

51. We must find a combination of *c*, *G*, and $\hbar$ that has the dimensions of time. The dimensions of *c* are $\left[\frac{L}{T}\right]$, the dimensions of *G* are $\left[\frac{L^3}{MT^2}\right]$, and the dimensions of $\hbar$ are $\left[\frac{ML^2}{T}\right]$. Use dimensional analysis, as discussed in Chapter 1.

$$t_P = c^\alpha G^\beta \hbar^\gamma \rightarrow [T] = \left[\frac{L}{T}\right]^\alpha \left[\frac{L^3}{MT^2}\right]^\beta \left[\frac{ML^2}{T}\right]^\gamma = [L]^{\alpha+3\beta+2\gamma}[M]^{\gamma-\beta}[T]^{-\alpha-2\beta-\gamma}$$
$$\alpha + 3\beta + 2\gamma = 0; \quad \gamma - \beta = 0; \quad -\alpha - 2\beta - \gamma = 1 \quad \rightarrow \quad \alpha + 5\beta = 0; \quad \alpha = -1 - 3\beta \quad \rightarrow$$
$$-5\beta = -1 - 3\beta \quad \rightarrow \quad \beta = \tfrac{1}{2}; \quad \gamma = \tfrac{1}{2}; \quad \alpha = -\tfrac{5}{2}$$
$$t_P = c^{-5/2}G^{1/2}\hbar^{1/2} = \sqrt{\frac{G\hbar}{c^5}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{1}{2\pi}(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(3.00\times10^8\text{ m/s})^5}} = \boxed{5.38\times10^{-44}\text{ s}}$$

52. The radius of the universe is estimated as the speed of light (*c*) times the age of the universe (*T*). This radius is used to find the volume of the universe, and the critical density times the volume gives the mass.

$$r = cT; \quad V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(cT)^3;$$
$$m = V\rho = \tfrac{4}{3}\pi(cT)^3\rho = \tfrac{4}{3}\pi\left[(3.00\times10^8\text{ m/s})(13.8\times10^9\text{ yr})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\right]^3(10^{-26}\text{ kg/m}^3)$$
$$= 9.34\times10^{52}\text{ kg} \approx \boxed{10^{53}\text{ kg}}$$

Solutions to Search and Learn Problems

1. (*a*) Assume that the nucleons make up only 2% of the critical mass density, so neutrinos make up 98% of the critical mass density.

$$\text{Nucleon mass density} = 0.02(10^{-26}\ \text{kg/m}^3)$$

$$\text{Nucleon number density} = \frac{0.02(10^{-26}\ \text{kg/m}^3)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 0.12\ \text{nucleon/m}^3$$

$$\text{Neutrino number density} = 10^9\ (\text{nucleon number density}) = 1.2\times10^8\ \text{neutrino/m}^3$$

The neutrino number density times the mass per neutrino must make up 98% of the critical mass density. The mass-to- eV/c^2 conversion is in the front of the textbook.

$$(m_\nu\ \text{kg/neutrino})(1.2\times10^8\ \text{neutrino/m}^3) = 0.98(10^{-26}\ \text{kg/m}^3) \quad\to$$

$$m_\nu = \frac{0.98(10^{-26}\ \text{kg/m}^3)}{(1.2\times10^8\ \text{neutrino/m}^3)}\left(\frac{931.5\times10^6\ \text{eV}/c^2}{1.6605\times10^{-27}\ \text{kg}}\right) = 45.8\ \text{eV}/\nu \approx \boxed{50\ \text{eV}/\nu}$$

(*b*) Assume that the nucleons make up only 5% of the critical mass density.

$$\text{Nucleon mass density} = 0.05(10^{-26}\ \text{kg/m}^3)$$

$$\text{Nucleon number density} = \frac{0.05(10^{-26}\ \text{kg/m}^3)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 0.30\ \text{nucleon/m}^3$$

$$\text{Neutrino number density} = 10^9\ (\text{nucleon number density}) = 3.0\times10^8\ \text{neutrino/m}^3$$

$$(m_\nu\ \text{kg/neutrino})(3.0\times10^8\ \text{neutrino/m}^3) = 0.95(10^{-26}\ \text{kg/m}^3) \quad\to$$

$$m_\nu = \frac{0.95(10^{-26}\ \text{kg/m}^3)}{(3.0\times10^8\ \text{neutrino/m}^3)}\left(\frac{931.5\times10^6\ \text{eV}/c^2}{1.6605\times10^{-27}\ \text{kg}}\right) = 17.8\ \text{eV/v} \approx \boxed{20\ \text{eV}/\nu}$$

2. Many methods are available.

- For nearby stars (up to 500 or 100 ly away) we can use parallax. In this method we measure the angular distance that a star moves relative to the background of stars as the Earth travels around the Sun. Half of the angular displacement is then equal to the ratio of the Earth–Sun distance and the distance between the Earth and that star.
- The apparent brightness of the brightest stars in galaxies, combined with the inverse square law, can be used to estimate distances to galaxies, assuming they have the same intrinsic luminosity.
- The H–R diagram can be used for distant stars. Determine the surface temperature using its blackbody radiation spectrum and Wien's law and then estimate its luminosity from the H–R diagram. Using its apparent brightness with Eq. 33–1 will give its distance.
- Variable stars, like Cepheid variables, can be used by relating the period to its luminosity. The luminosity and apparent brightness can be used to find the distance.
- The largest distances are measured by measuring the apparent brightness of Type Ia supernovae. All supernovae are thought to have nearly the same luminosity, so the apparent brightness can be used to find the distance.
- The redshift in the spectral lines of very distant galaxies can be used to estimate distances that are farther than 10^7 to 10^8 ly.

3. (*a*) From Section 33–2, a white dwarf with a mass equal to that of the Sun has a radius about the size of the Earth's radius, $\boxed{6380\text{ km}}$, and a neutron star with a mass equal to 1.5 solar masses has a radius of about $\boxed{20\text{ km}}$. For the black hole, we use the Schwarzschild radius formula.

$$R=\frac{2GM}{c^2}=\frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left[3(1.99\times10^{30}\text{ kg})\right]}{(3.00\times10^8\text{ m/s})^2}=8849\text{ m}\approx\boxed{8.85\text{ km}}$$

(*b*) The ratio is $6380:20:8.85=721:2.26:1\approx\boxed{700:2:1}$.

4. (*a*) All distant objects in the universe are moving away from each other, as indicated by the galactic redshift, indicating that the universe is expanding. If the universe has always expanded, it must have started as a point. The 25% abundance of He supports the standard Big Bang model. The Big Bang theory predicted the presence of background radiation, which has since been observed.

(*b*) The curvature of the universe determines whether the universe will continue expanding forever (open) or eventually collapse back in on itself (closed).

(*c*) Dark energy increases the total energy of the universe, increasing the probability that it is an open universe.

5. Each helium atom requires 2 protons and 2 neutrons and has a total mass of 4u. Each hydrogen atom requires 1 proton and has a mass of 1u. If there are 7 times more protons than neutrons, then for every 2 neutrons there are 14 protons. Two protons combine with the 2 neutrons to produce a helium atom. The other 12 protons produce 12 hydrogen atoms. Therefore, there is 12 times as much hydrogen as helium, by number of atoms. Each helium atom has a mass 4 times that of the hydrogen atom, so the total mass of the hydrogen is only 3 times the total mass of the helium.

6. About 380,000 years after the Big Bang, the initially very hot temperature of the universe would have cooled down to about 3000 K. At that temperature, electrons could orbit bare nuclei and remain there, without being ejected by collisions. Thus stable atoms were able to form. This allowed photons to travel unimpeded through the universe, so the universe became transparent. The radiation at this time period would have been blackbody radiation at a temperature of about 3000 K. As the universe continued to expand and cool, the wavelengths of this radiation would have increased according to Wien's law. The peak wavelength now is much larger and corresponds to the blackbody radiation at a temperature of 2.7 K.

7. (*a*) Because the speed of the galaxy is small compared to the speed of light, we can use Eq. 33–6 instead of Eq. 33–3. The velocity of the galaxy is negative.

$$\Delta\lambda=\lambda_{\text{rest}}\left(\frac{v}{c}\right)=656\text{ nm}\left(\frac{-130\times10^3\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)=\boxed{-0.284\text{ nm}}$$

(*b*) This is a blue shift because the wavelength has decreased and because the galaxy is approaching.

(*c*) The time is equal to the distance between the galaxies divided by their relative speed. This is assuming that they continue to move at the same present speed. (They would actually accelerate due to the gravitational force between them.)

$$t=\frac{d}{v}=\frac{2.5\times10^6\text{ ly}}{130\times10^3\text{ m/s}}\left(\frac{9.46\times10^{15}\text{ m}}{\text{ly}}\right)\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right)=\boxed{5.8\times10^9\text{ yr}}$$