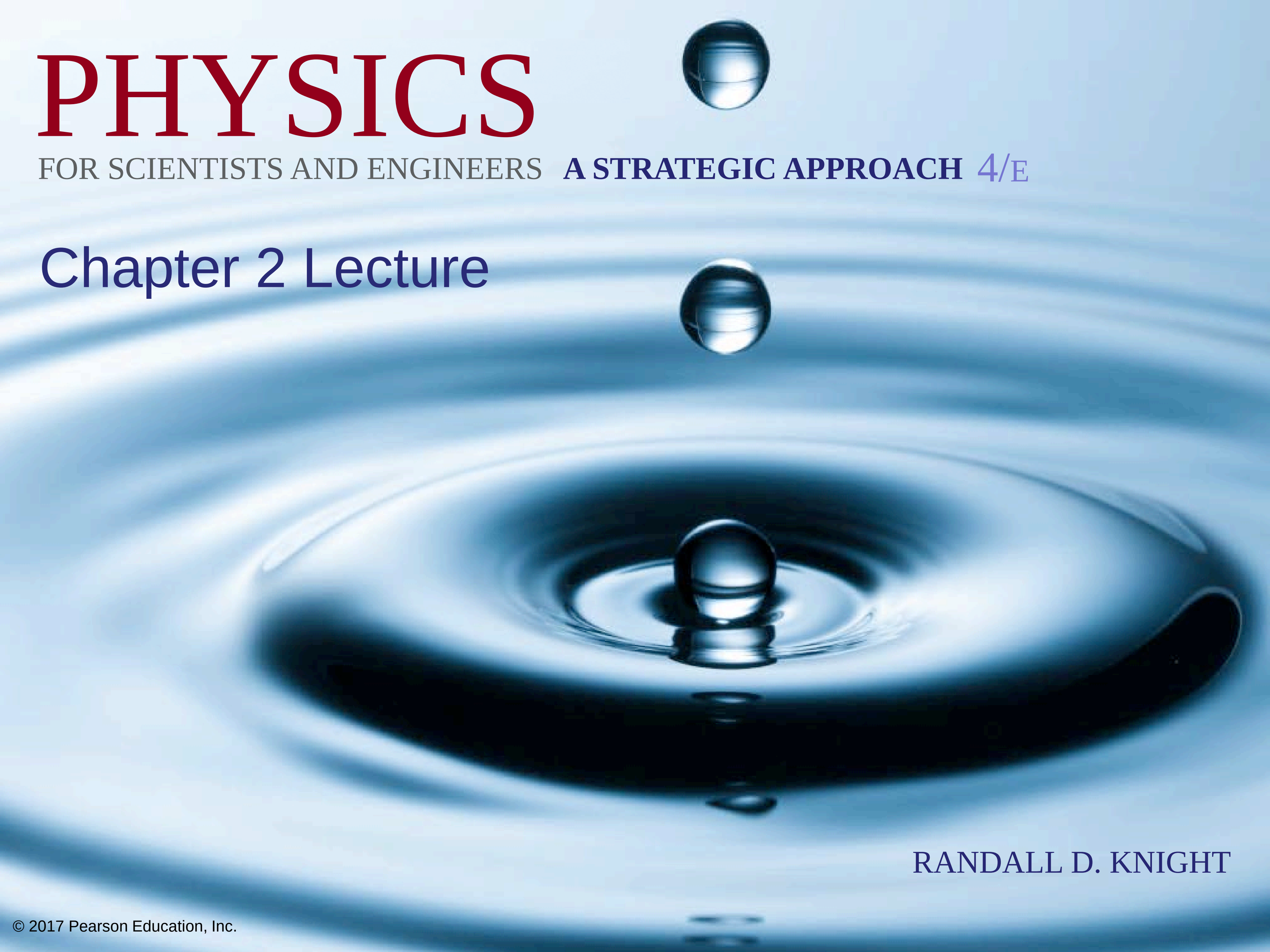


PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

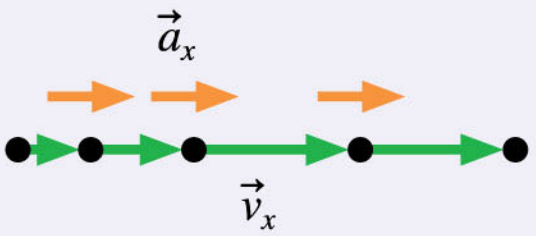
Chapter 2 Lecture

RANDALL D. KNIGHT

IN THIS CHAPTER, you will learn to solve problems about motion along a straight line.

What is kinematics?

Kinematics is the mathematical description of motion. We begin with motion along a straight line. Our primary tools will be an object's position, velocity, and acceleration.

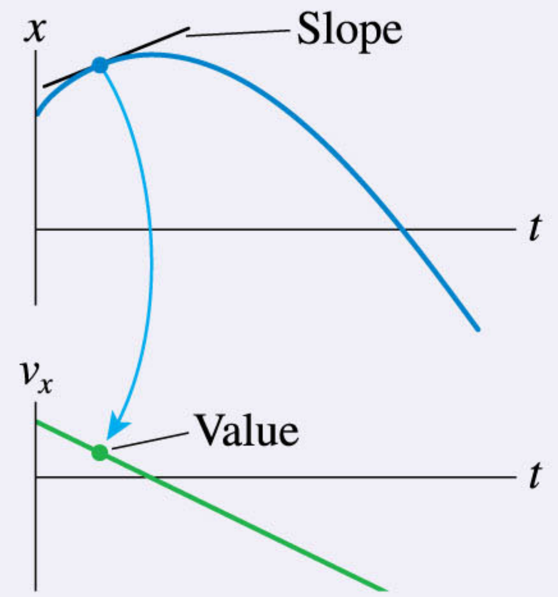


« LOOKING BACK Sections 1.4–1.6 Velocity, acceleration, and Tactics Box 1.4 about signs

How are graphs used in kinematics?

Graphs are a very important visual representation of motion, and learning to "think graphically" is one of our goals. We'll work with graphs showing how position, velocity, and acceleration change with time. These graphs are related to each other:

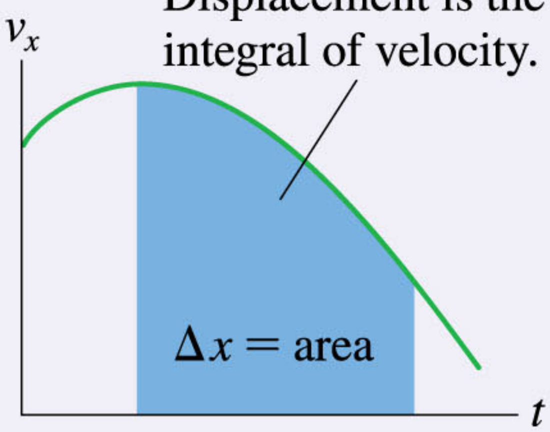
- Velocity is the slope of the position graph.
- Acceleration is the slope of the velocity graph.



How is calculus used in kinematics?

Motion is change, and calculus is the mathematical tool for describing a quantity's rate of change. We'll find that

- Velocity is the time derivative of position.
- Acceleration is the time derivative of velocity.



What are models?

A model is a simplified description of a situation that focuses on essential features while ignoring many details. Models allow us to make sense of complex situations by seeing them as variations on a common theme, all with the same underlying physics.

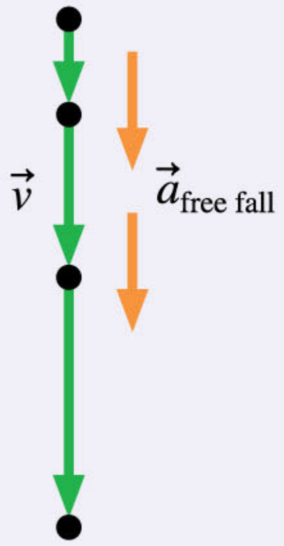
MODEL 2.1

Look for model boxes like this throughout the book.

- Key figures
- Key equations
- Model limitations

What is free fall?

Free fall is motion under the influence of gravity only. Free fall is not literally "falling" because it also applies to objects thrown straight up and to projectiles. Surprisingly, all objects in free fall, *regardless of their mass,* have the same acceleration. Motion on a frictionless inclined plane is closely related to free-fall motion.



How will I use kinematics?

The equations of motion that you learn in this chapter will be used throughout the entire book. In Part I, we'll see how an object's motion is related to forces acting on the object. We'll later apply these kinematic equations to the motion of waves and to the motion of charged particles in electric and magnetic fields.

Chapter 2 Reading Questions

Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
B. The object's average velocity at that point.
C. The object's instantaneous velocity at that point.
D. The object's acceleration at that point.
E. The distance traveled by the object to that point.

Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
B. The object's average velocity at that point.
✔ **C. The object's instantaneous velocity at that point.**
D. The object's acceleration at that point.
E. The distance traveled by the object to that point.

Reading Question 2.2

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
D. The displacement of the object.
E. This topic was not covered in this chapter.

Reading Question 2.2

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
✔ **D. The displacement of the object.**
E. This topic was not covered in this chapter.

Reading Question 2.3

The slope at a point on a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's instantaneous acceleration at that point.
C. The distance traveled by the object.
D. The displacement of the object.
E. The object's instantaneous velocity at that point.

Reading Question 2.3

The slope at a point on a velocity-versus-time graph of an object is

A. The object's speed at that point.
✔ **B. The object's instantaneous acceleration at that point.**
C. The distance traveled by the object.
D. The displacement of the object.
E. The object's instantaneous velocity at that point.

Reading Question 2.4

Suppose we define the *y*-axis to point vertically upward. When an object is in **free fall**, it has acceleration in the *y*-direction

A. $a_y = -g$, where $g = +9.80 \text{ m/s}^2$
B. $a_y = g$, where $g = -9.80 \text{ m/s}^2$
C. Which is negative and increases in magnitude as it falls.
D. Which is negative and decreases in magnitude as it falls.
E. Which depends on the mass of the object.

Reading Question 2.4

Suppose we define the y-axis to point vertically upward. When an object is in **free fall**, it has acceleration in the y-direction

✔ **A.** $\boldsymbol{a_y = -g}$**, where** $\boldsymbol{g = +9.80\ \mathrm{m/s^2}}$

B. $a_y = g$, where $g = -9.80\ \mathrm{m/s^2}$

C. Which is negative and increases in magnitude as it falls.

D. Which is negative and decreases in magnitude as it falls.

E. Which depends on the mass of the object.

Reading Question 2.5

At the turning point of an object,

A. The instantaneous velocity is zero.
B. The acceleration is zero.
C. Both A and B are true.
D. Neither A nor B is true.
E. This topic was not covered in this chapter.

Reading Question 2.5

At the turning point of an object,

A. **The instantaneous velocity is zero.**
B. The acceleration is zero.
C. Both A and B are true.
D. Neither A nor B is true.
E. This topic was not covered in this chapter.

Reading Question 2.6

A 1-pound block and a 100-pound block are placed side by side at the top of a frictionless hill. Each is given a very light tap to begin their race to the bottom of the hill. In the absence of air resistance

A. The 1-pound block wins the race.
B. The 100-pound block wins the race.
C. The two blocks end in a tie.
D. There's not enough information to determine which block wins the race.

Reading Question 2.6

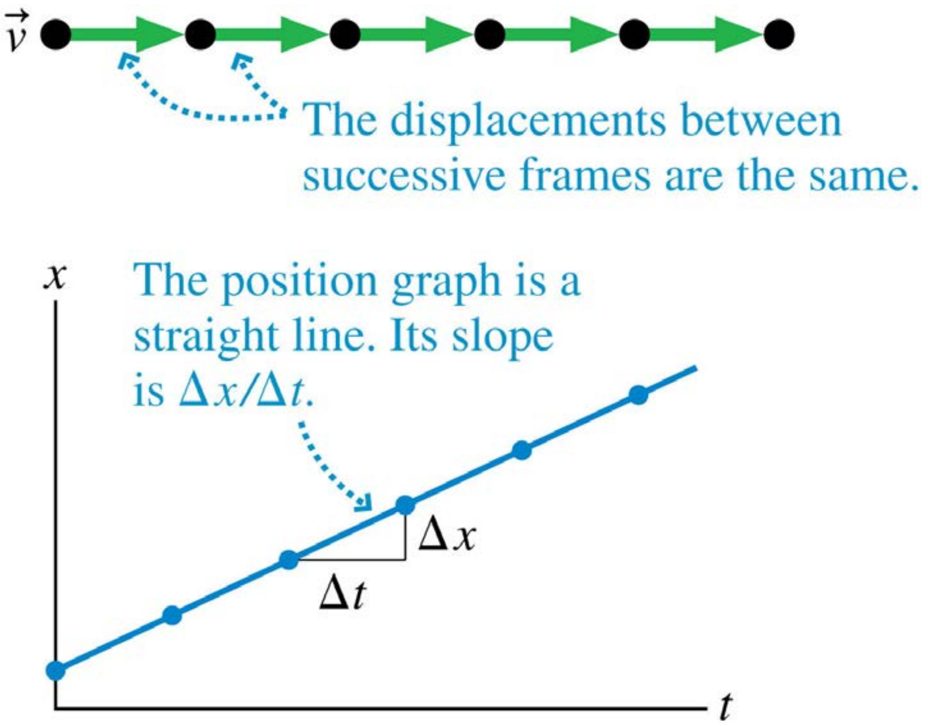
A 1-pound block and a 100-pound block are placed side by side at the top of a frictionless hill. Each is given a very light tap to begin their race to the bottom of the hill. In the absence of air resistance

A. The 1-pound block wins the race.
B. The 100-pound block wins the race.
✔ **C. The two blocks end in a tie.**
D. There's not enough information to determine which block wins the race.

Chapter 2 Content, Examples, and QuickCheck Questions

Uniform Motion

- The simplest possible motion is motion along a straight line at a constant, unvarying speed.
- We call this **uniform motion.**
- An object's motion is uniform if and only if its position-versus-time graph is a straight line.



Uniform Motion

- For one-dimensional motion, the average velocity is simply $\Delta x / \Delta t$ (for horizontal motion) or $\Delta y / \Delta t$ (for vertical motion).
- On a horizontal position-versus-time graph, Δx and Δt are, respectively, the "rise" and "run".
- Because rise over run is the slope of a line, **the average velocity is the slope of the position-versus-time graph**.
- The SI units of velocity are meters per second, abbreviated m/s.

$$v_{\text{avg}} \equiv \frac{\Delta x}{\Delta t} \text{ or } \frac{\Delta y}{\Delta t} = \text{slope of the position-versus-time graph}$$

Example 2.1 Relating a velocity graph to a position graph

EXAMPLE 2.1 | Relating a velocity graph to a position graph

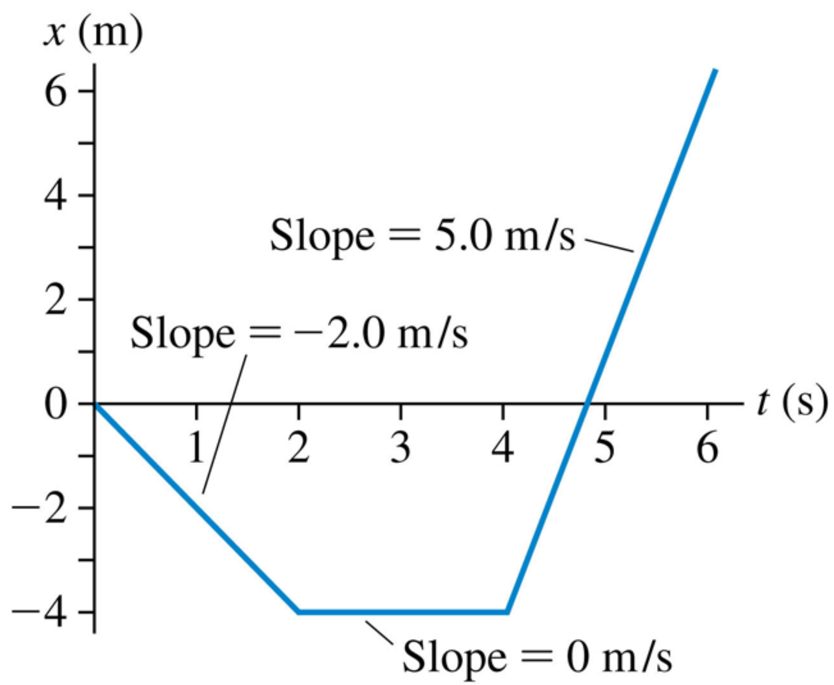
FIGURE 2.2 is the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.

b. Describe the car's motion.

MODEL Model the car as a particle, with a well-defined position at each instant of time.

VISUALIZE Figure 2.2 is the graphical representation.



Example 2.1 Relating a velocity graph to a position graph

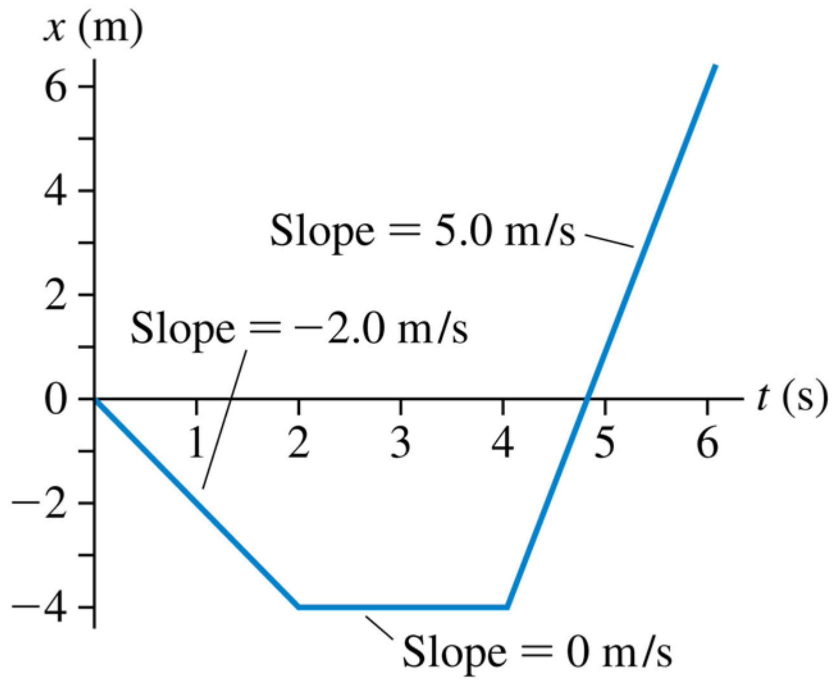
EXAMPLE 2.1 | **Relating a velocity graph to a position graph**

SOLVE a. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

The position graph starts out sloping downward—a negative slope. Although the car moves a distance of 4.0 m during the first 2.0 s, its *displacement* is

$$\Delta x = x_{\text{at } 2.0\text{ s}} - x_{\text{at } 0.0\text{ s}} = -4.0\text{ m} - 0.0\text{ m} = -4.0\text{ m}$$

The time interval for this displacement is $\Delta t = 2.0$ s, so the velocity during this interval is



Example 2.1 Relating a velocity graph to a position graph

EXAMPLE 2.1 | **Relating a velocity graph to a position graph**

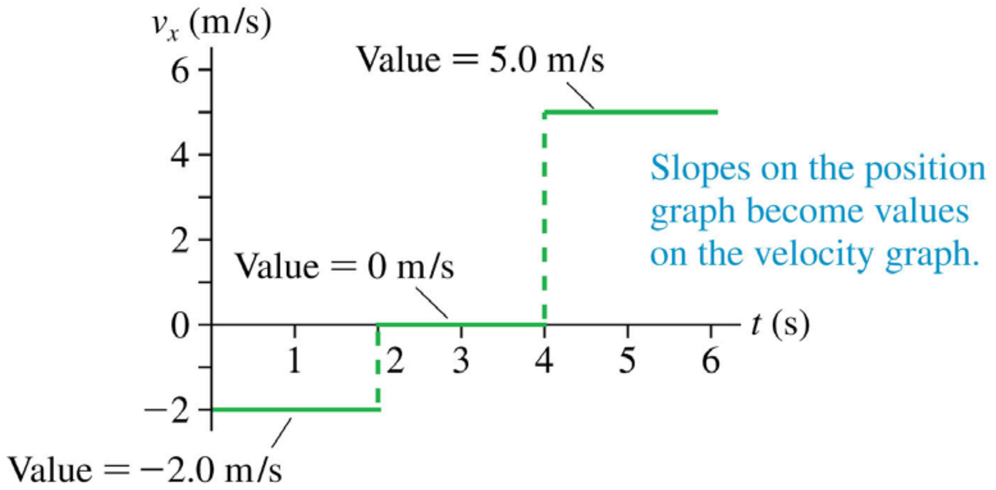
SOLVE

$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4.0\text{ m}}{2.0\text{ s}} = -2.0\text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$), so $v_x = 0$. Finally, the displacement between $t = 4$ s and $t = 6$ s is $\Delta x = 10.0$ m. Thus the velocity during this interval is

$$v_x = \frac{10.0\text{ m}}{2.0\text{ s}} = 5.0\text{ m/s}$$

These velocities are shown on the velocity-versus-time graph of **FIGURE 2.3**.

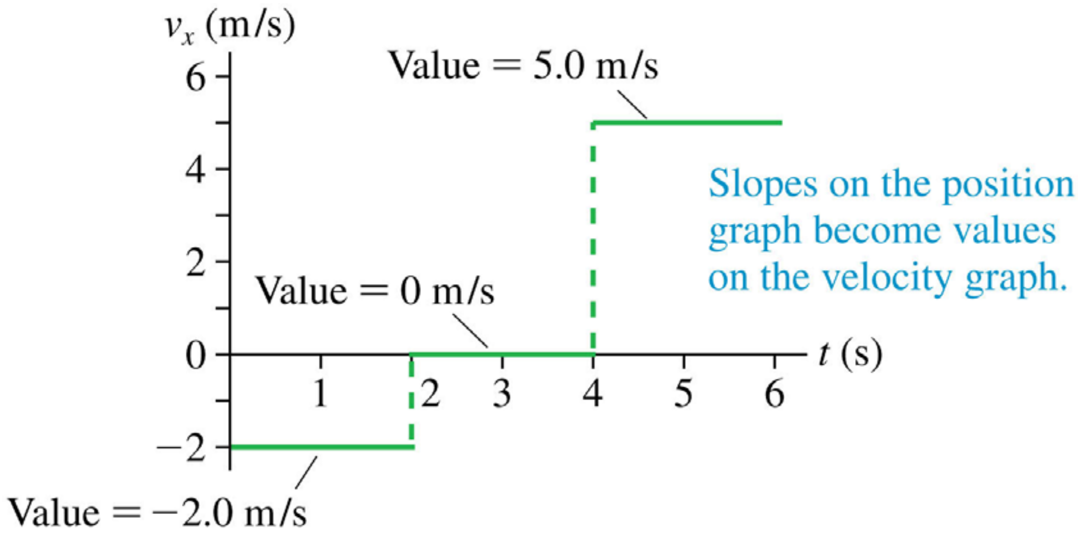


Example 2.1 Relating a velocity graph to a position graph

EXAMPLE 2.1 **Relating a velocity graph to a position graph**

SOLVE b. The car backs up for 2 s at 2.0 m/s, sits at rest for 2 s, then drives forward at 5.0 m/s for at least 2 s. We can't tell from the graph what happens for $t > 6$ s.

ASSESS The velocity graph and the position graph look completely different. The *value* of the velocity graph at any instant of time equals the *slope* of the position graph.



Tactics: Interpreting Position-versus-Time Graphs

TACTICS BOX 2.1

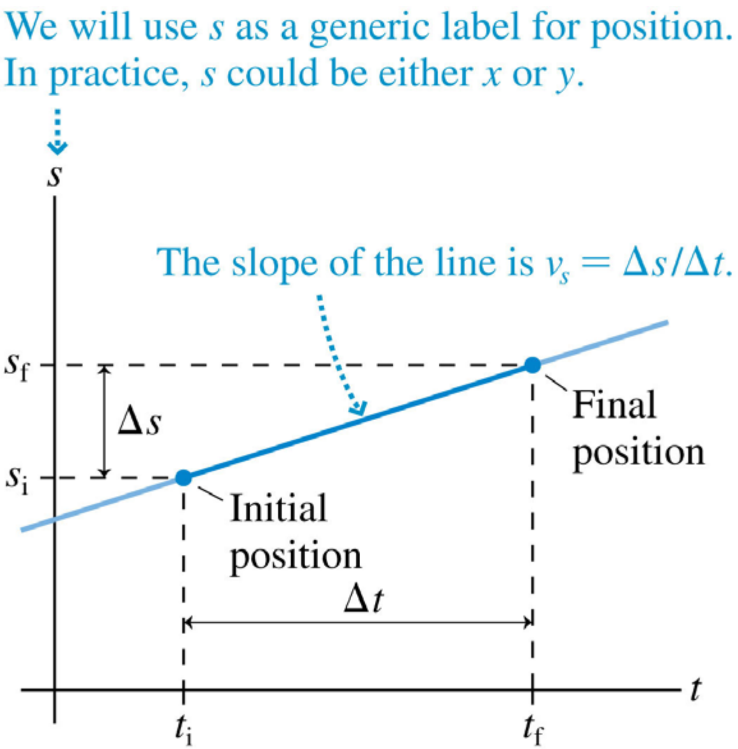
Interpreting position-versus-time graphs

1. Steeper slopes correspond to faster speeds.
2. Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).
3. The slope is a ratio of intervals, $\Delta x / \Delta t$, not a ratio of coordinates. That is, the slope is *not* simply x / t.

Exercises 1–3

The Mathematics of Uniform Motion

- Consider an object in uniform motion along the s-axis, as shown in the graph.
- The object's **initial position** is s_i at time t_i.
- At a later time t_f the object's **final position** is s_f.
- The change in time is $\Delta t = t_f - t_i$.
- The final position can be found as



$$s_f = s_i + v_s \, \Delta t \qquad \text{(uniform motion)}$$

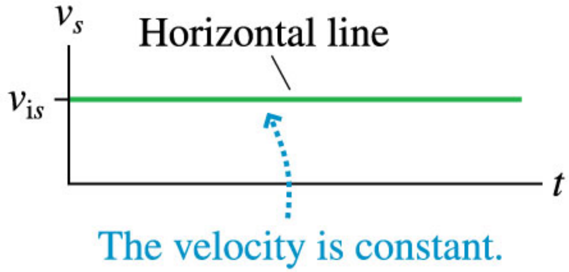
MODEL 2.1

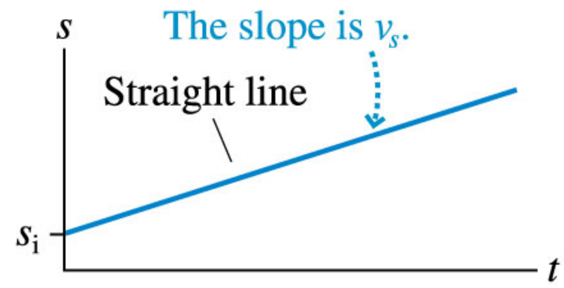
Uniform motion

For motion with constant velocity.

- Model the object as a particle moving in a straight line at constant speed:

- Mathematically:
  - $v_s = \Delta s/\Delta t$
  - $s_f = s_i + v_s \Delta t$
- Limitations: Model fails if the particle has a significant change of speed or direction.





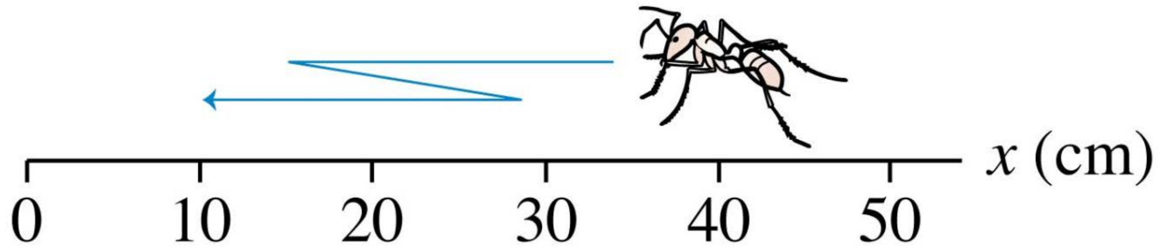
Exercise 4

Scalars and Vectors

- The **distance** an object travels is a scalar quantity, independent of direction.
- The **displacement** of an object is a vector quantity, equal to the final position minus the initial position.
- An object's **speed** v is scalar quantity, independent of direction.
- Speed is how fast an object is going; it is always positive.
- **Velocity** is a vector quantity that includes direction.
- In one dimension the direction of velocity is specified by the + or – sign.

QuickCheck 2.1

An ant zig-zags back and forth on a picnic table as shown.

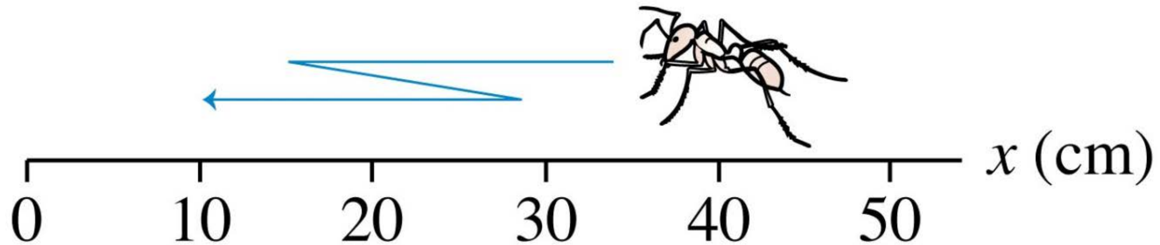


The ant's **distance traveled** and **displacement** are

A. 50 cm and 50 cm.
B. 30 cm and 50 cm.
C. 50 cm and 30 cm.
D. 50 cm and –50 cm.
E. 50 cm and –30 cm.

QuickCheck 2.1

An ant zig-zags back and forth on a picnic table as shown.



The ant's **distance traveled** and **displacement** are

A. 50 cm and 50 cm.
B. 30 cm and 50 cm.
C. 50 cm and 30 cm.
D. 50 cm and –50 cm.
✔ **E. 50 cm and –30 cm.**

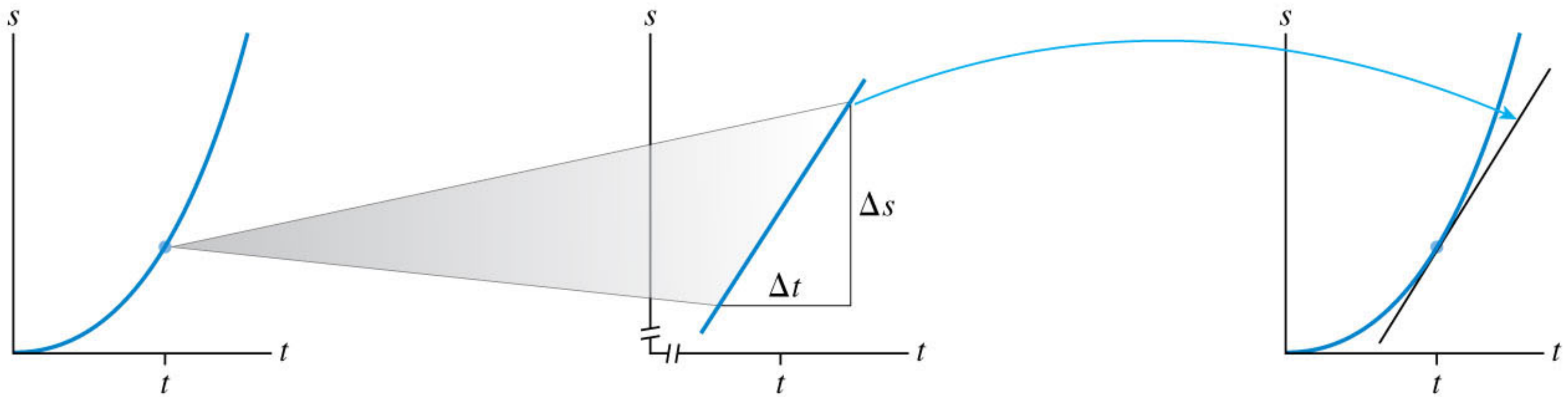
Instantaneous Velocity

- Objects rarely travel for long with a constant velocity.
- Far more common is a velocity that changes with time.
- If you watch a car's speedometer, at any instant of time, the speedometer tells you how fast the car is going *at that instant*.
- If we include directional information, we can define an object's **instantaneous velocity**—speed and direction—as its velocity at a single instant of time.
- The average velocity $v_{avg} = \Delta s/\Delta t$ becomes a better and better approximation to the instantaneous velocity as Δt gets smaller and smaller.

$$v_s \equiv \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(instantaneous velocity)}$$

Instantaneous Velocity

Motion diagrams and position graphs of an accelerating rocket.



What is the velocity at time t?

Zoom in on a *very* small segment of the curve centered on the point of interest. This little piece of the curve is essentially a straight line. Its slope $\Delta s/\Delta t$ is the average velocity during the interval Δt.

The little segment of straight line, when extended, is the tangent to the curve at time t. Its slope is the instantaneous velocity at time t.

Instantaneous Velocity

- As Δt continues to get smaller, the average velocity $v_{avg} = \Delta s/\Delta t$ reaches a constant or *limiting* value.
- The instantaneous velocity at time t is the average velocity during a time interval Δt centered on t, as Δt approaches zero.
- In calculus, this is called *the derivative of* s *with respect to* t.
- Graphically, $\Delta s/\Delta t$ is the slope of a straight line.
- In the limit $\Delta t \to 0$, the straight line is tangent to the curve.
- The instantaneous velocity at time t is the slope of the line that is tangent to the position-versus-time graph at time t.

$$v_s = \text{slope of the position-versus-time graph at time } t$$

QuickCheck 2.2

The slope at a point on a position-versus-time graph of an object is

A. The object’s speed at that point.
B. The object’s velocity at that point.
C. The object’s acceleration at that point.
D. The distance traveled by the object to that point.
E. I really have no idea.

QuickCheck 2.2

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
✔ **B. The object's velocity at that point.**
C. The object's acceleration at that point.
D. The distance traveled by the object to that point.
E. I really have no idea.

Example 2.3 Finding Velocity from Position Graphically

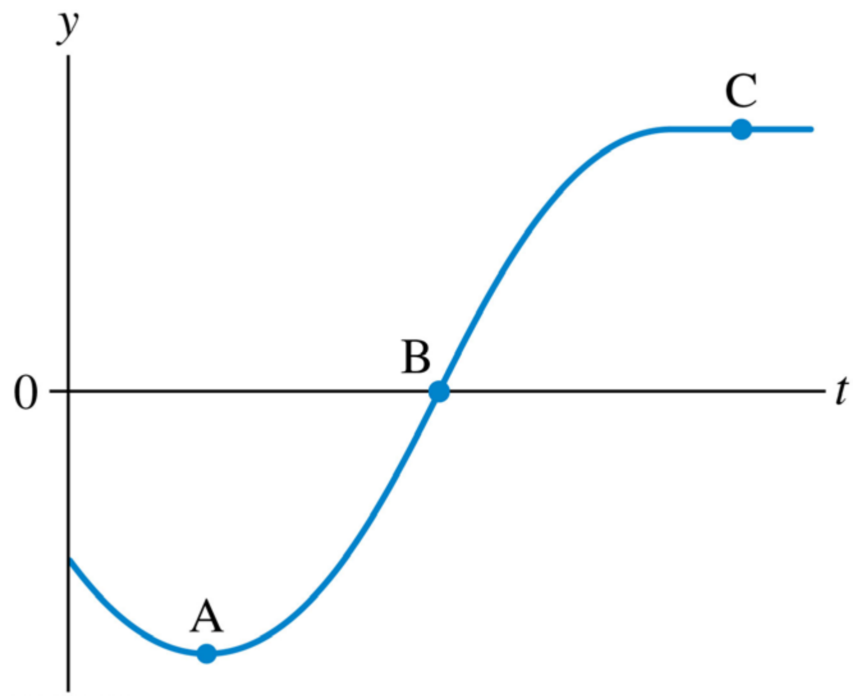
EXAMPLE 2.3 | **Finding velocity from position graphically**

FIGURE 2.9 shows the position-versus-time graph of an elevator.

a. At which labeled point or points does the elevator have the least velocity?

b. At which point or points does the elevator have maximum velocity?

c. Sketch an approximate velocity-versus-time graph for the elevator.



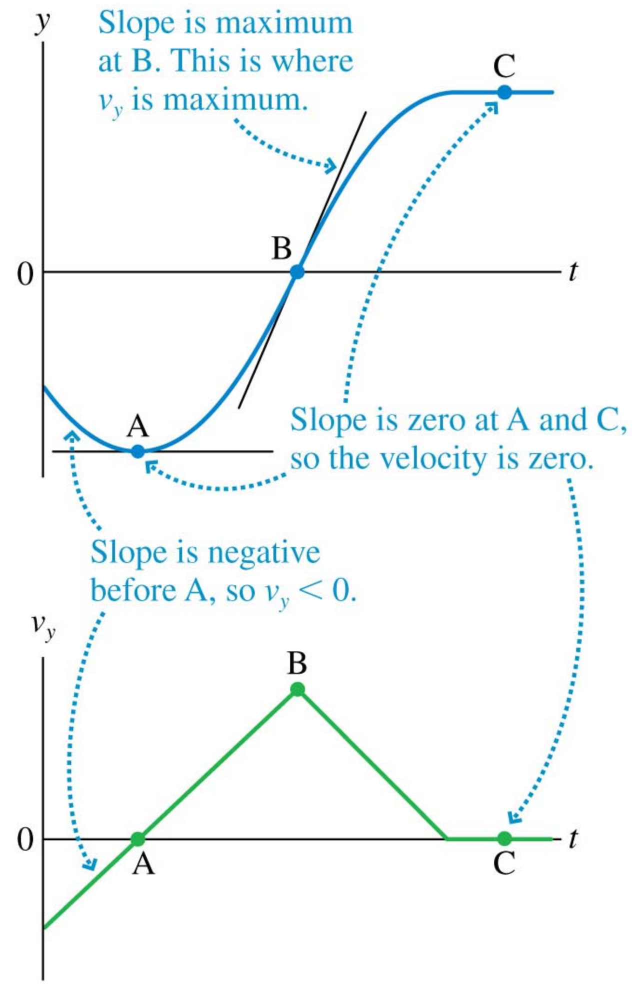
Example 2.3 Finding Velocity from Position Graphically

EXAMPLE 2.3 **Finding velocity from position graphically**

MODEL Model the elevator as a particle.

VISUALIZE Figure 2.9 is the graphical representation.

SOLVE a. At any instant, an object's velocity is the slope of its position graph. **FIGURE 2.10a** shows that the elevator has the least velocity—no velocity at all!—at points A and C where the slope is zero. At point A, the velocity is only instantaneously zero. At point C, the elevator has actually stopped and remains at rest.



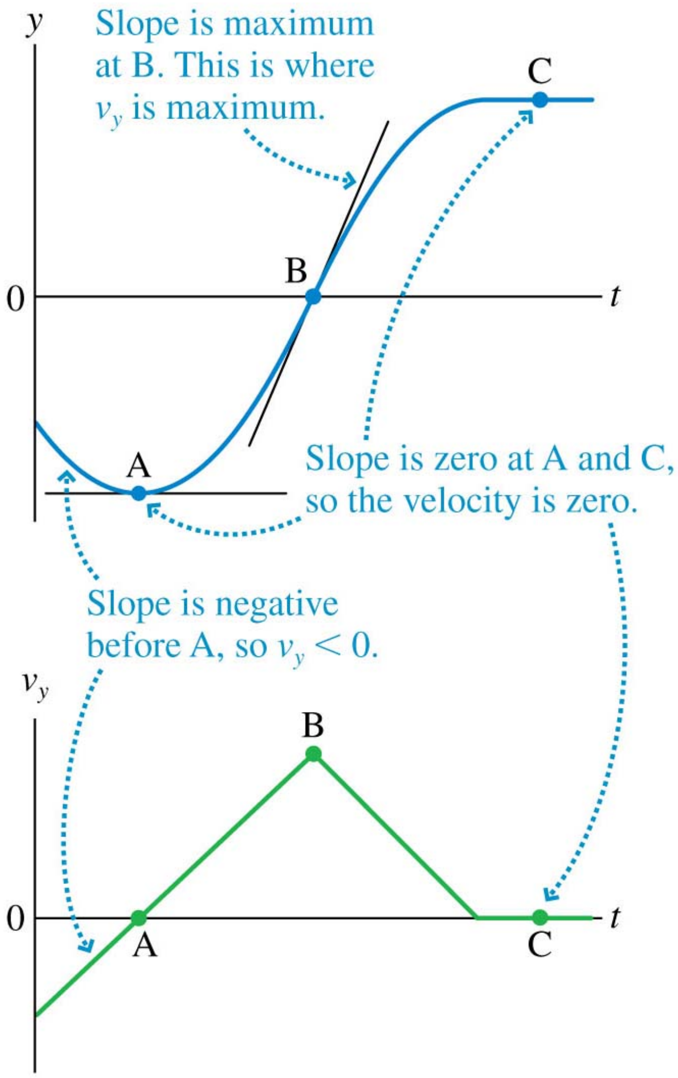
Example 2.3 Finding Velocity from Position Graphically

EXAMPLE 2.3 | **Finding velocity from position graphically**

b. The elevator has maximum velocity at B, the point of steepest slope.

c. Although we cannot find an exact velocity-versus-time graph, we can see that the slope, and hence v_y, is initially negative, becomes zero at point A, rises to a maximum value at point B, decreases back to zero a little before point C, then remains at zero thereafter.

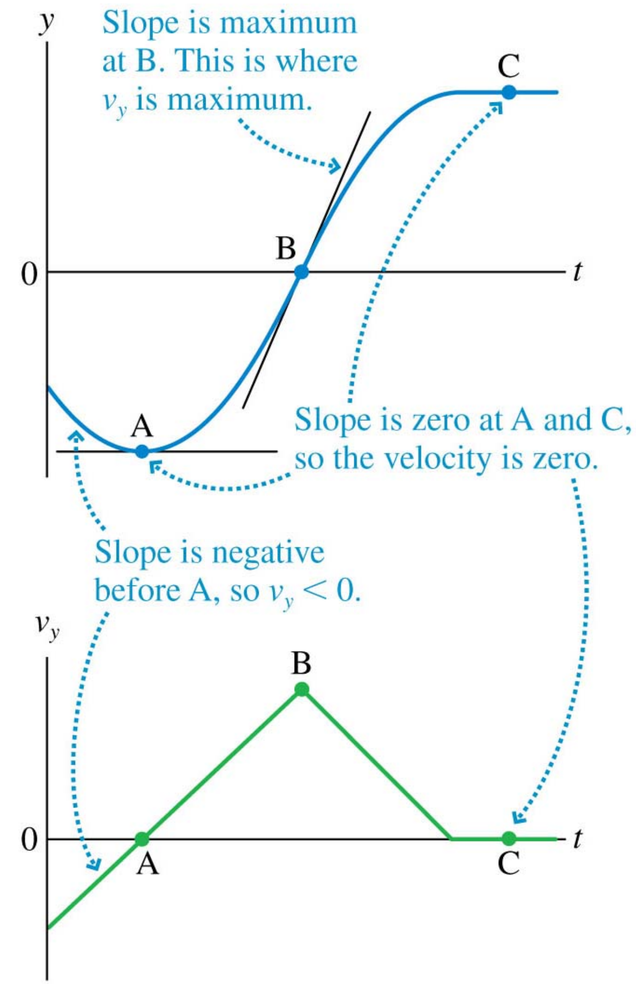
Thus **FIGURE 2.10b** shows, at least approximately, the elevator's velocity-versus-time graph.



Example 2.3 Finding Velocity from Position Graphically

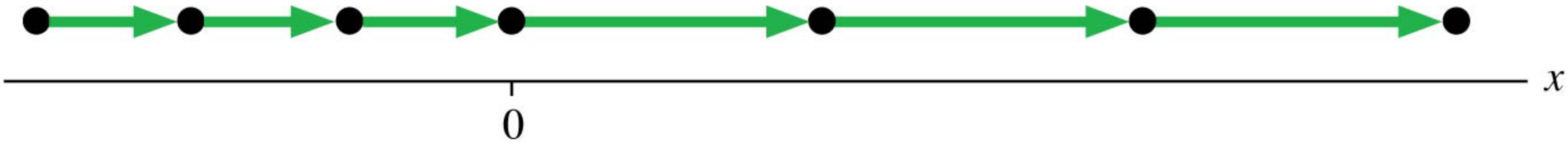
EXAMPLE 2.3 **Finding velocity from position graphically**

ASSESS Once again, the shape of the velocity graph bears no resemblance to the shape of the position graph. You must transfer *slope* information from the position graph to *value* information on the velocity graph.

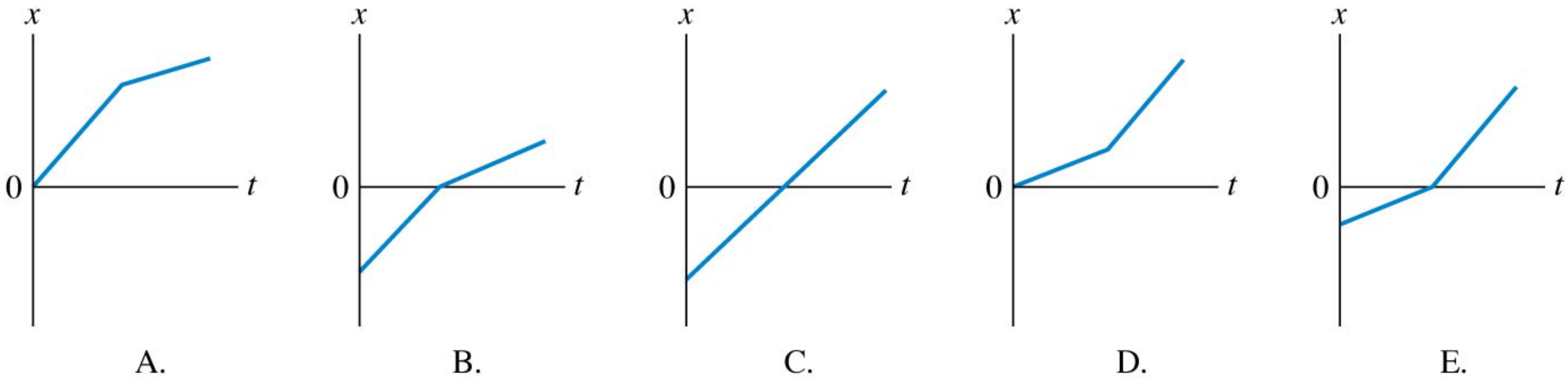


QuickCheck 2.3

Here is a motion diagram of a car moving along a straight road:



Which position-versus-time graph matches this motion diagram?



QuickCheck 2.3

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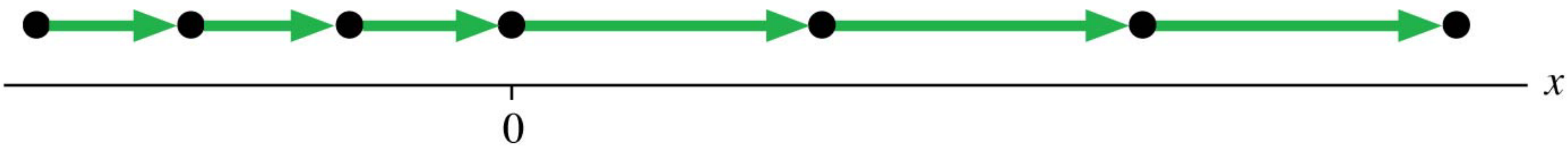
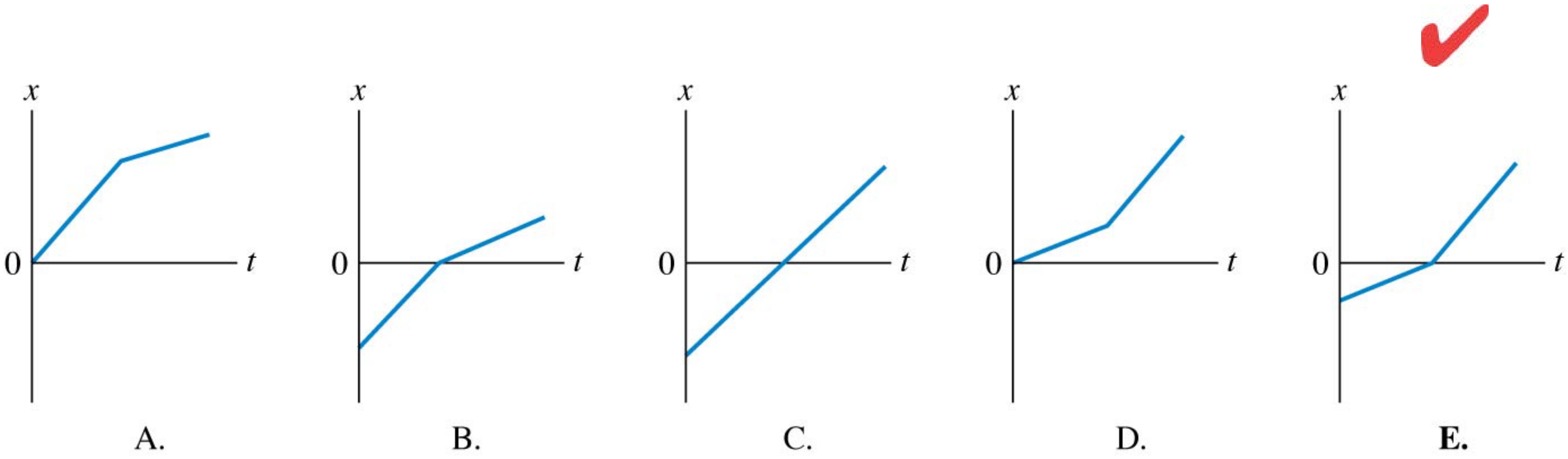
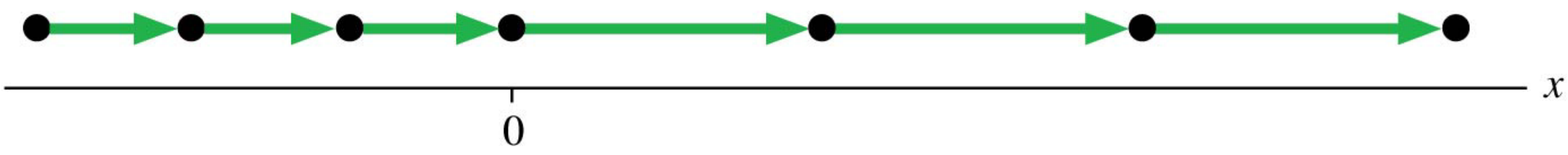


Which position-versus-time graph matches this motion diagram?

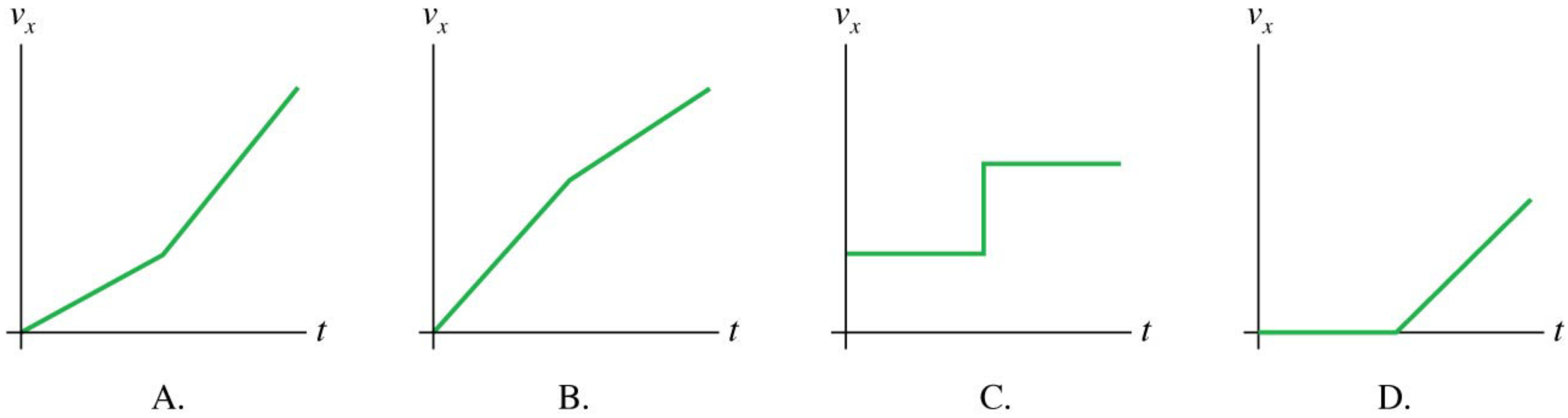


QuickCheck 2.4

Here is a motion diagram of a car moving along a straight road:



Which velocity-versus-time graph matches this motion diagram?



E. None of the above.

QuickCheck 2.4

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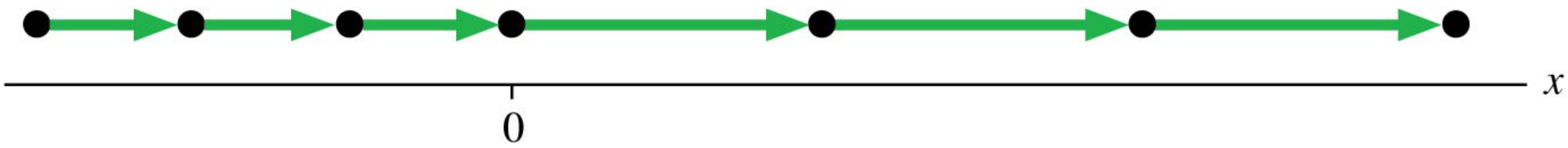
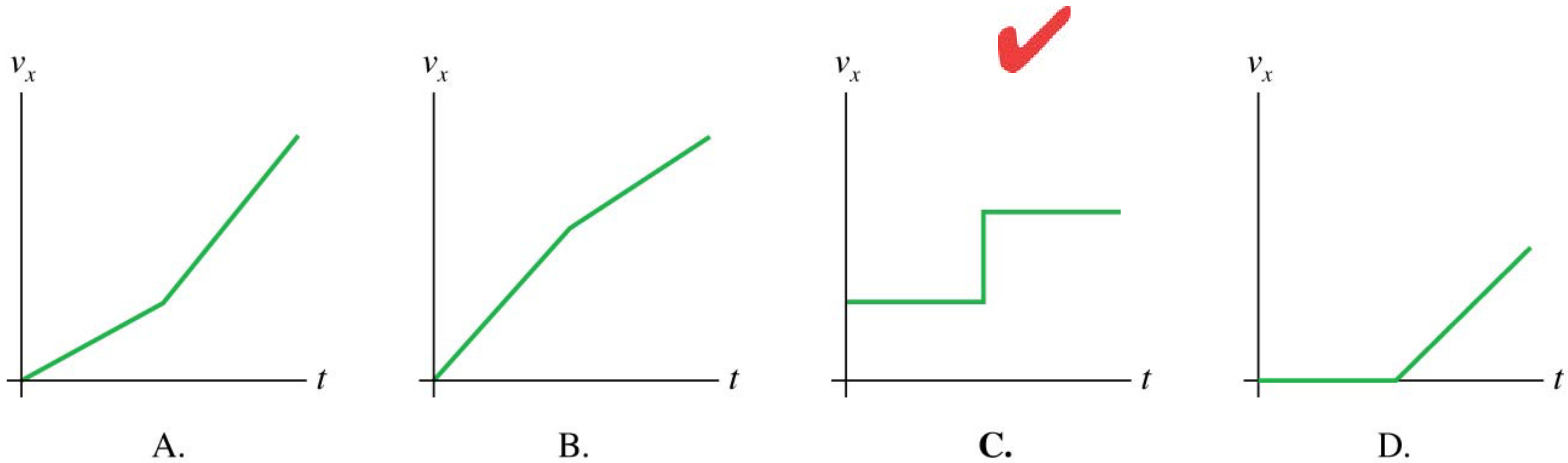


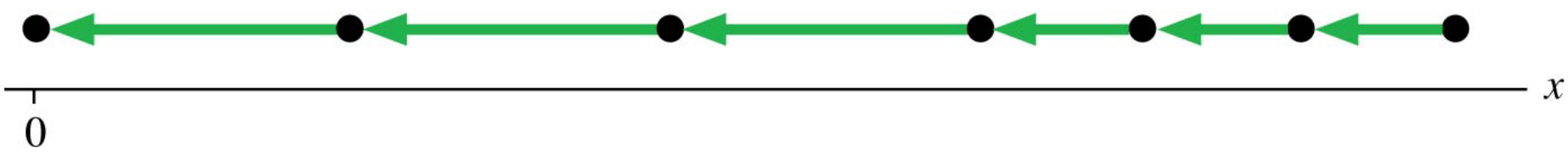
Which velocity-versus-time graph matches this motion diagram?



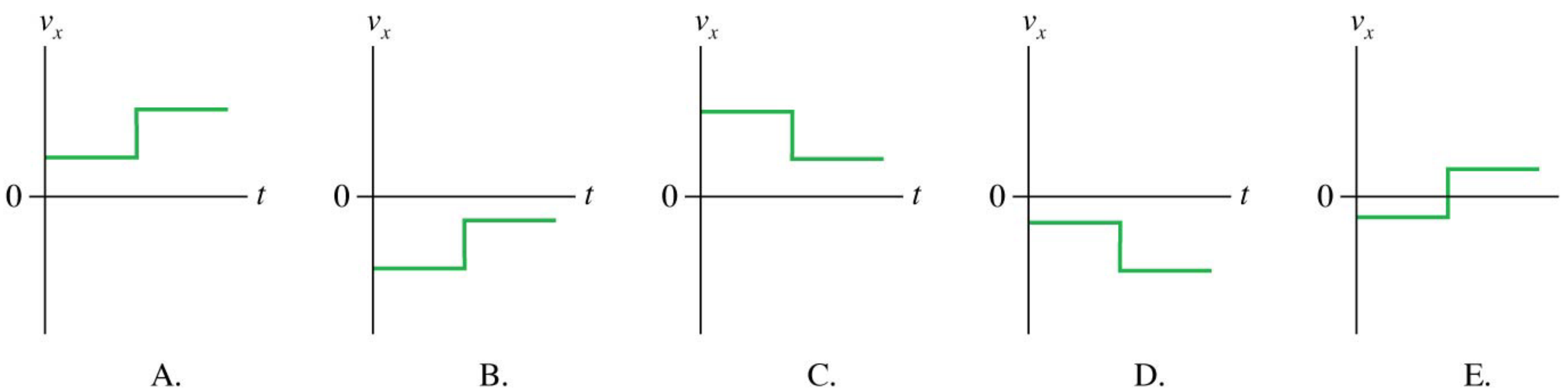
E. None of the above.

QuickCheck 2.5

Here is a motion diagram of a car moving along a straight road:



Which velocity-versus-time graph matches this motion diagram?



QuickCheck 2.5

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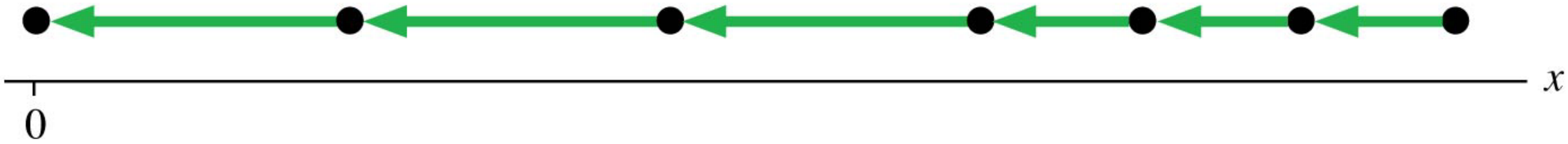
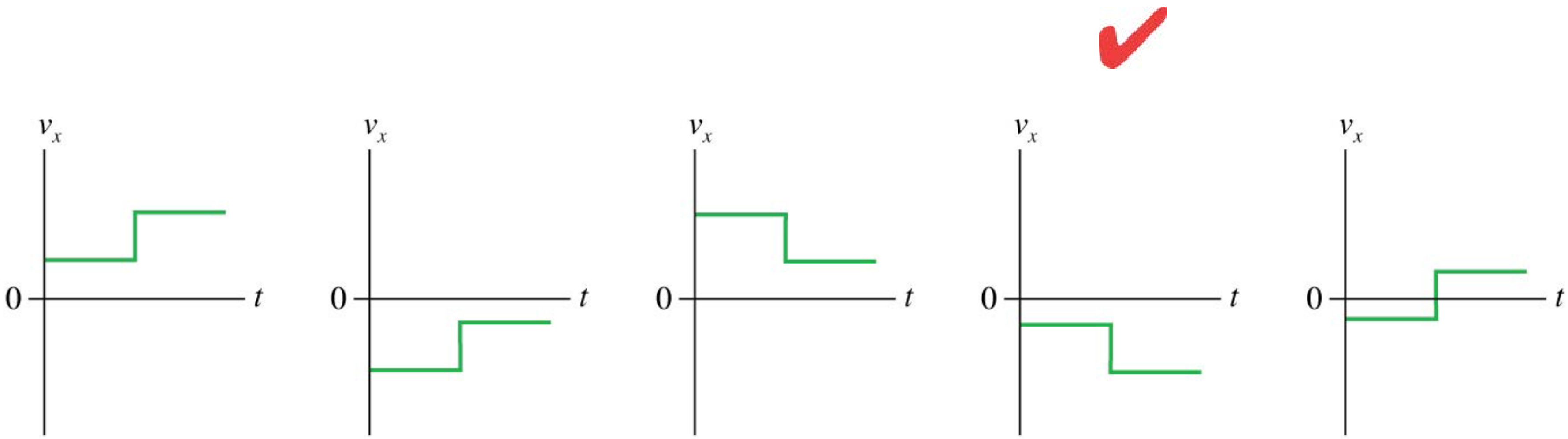


Which velocity-versus-time graph matches this motion diagram?



A. B. C. **D.** E.

A Little Calculus: Derivatives

- ds/dt is called *the derivative of* s *with respect to* t.
- ds/dt is the slope of the line that is tangent to the position-versus-time graph.
- Consider the function $u(t) = ct^n$, where c and n are constants:

$$\text{The derivative of } u = ct^n \text{ is } \frac{du}{dt} = nct^{n-1}$$

- The derivative of a constant is zero:

$$\frac{du}{dt} = 0 \text{ if } u = c = \text{constant}$$

- The derivative of a sum is the sum of the derivatives. If u and w are two separate functions of time, then

$$\frac{d}{dt}(u + w) = \frac{du}{dt} + \frac{dw}{dt}$$

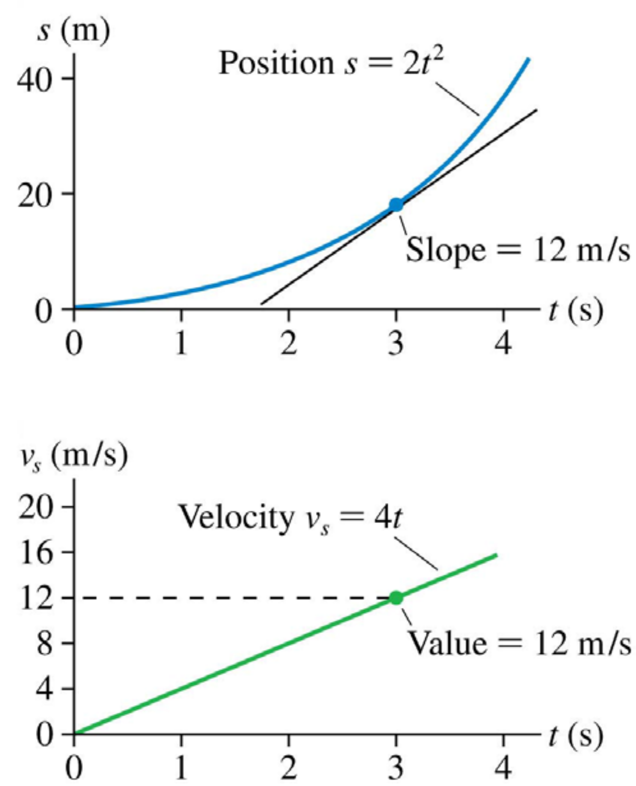
Derivative Example

Suppose the position of a particle as a function of time is $s(t) = 2t^2$ m where t is in s. What is the particle's velocity?

- Velocity is the derivative of s with respect to t:

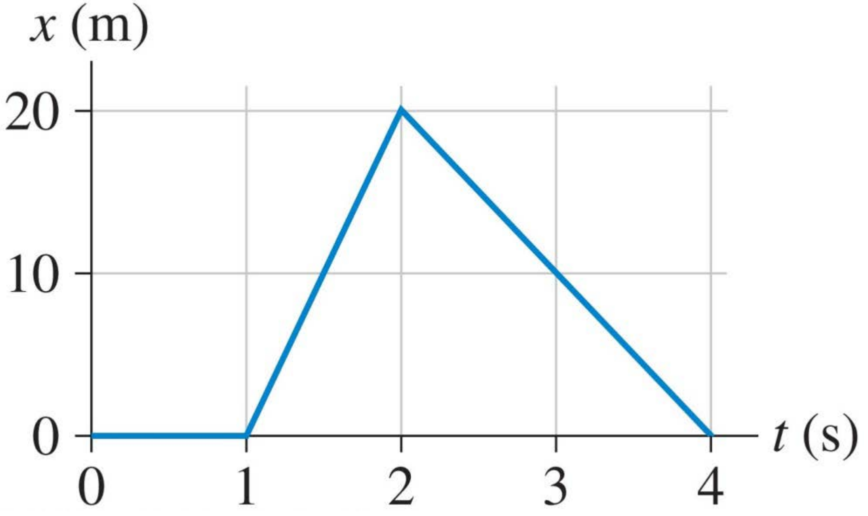
$$v_s = \frac{ds}{dt} = 2 \cdot 2t^{2-1} = 4t$$

- The figure shows the particle's position and velocity graphs.
- The *value* of the velocity graph at any instant of time is the *slope* of the position graph at that same time.



QuickCheck 2.6

Here is a position graph of an object:



At $t = 1.5$ s, the object's velocity is

A. 40 m/s

B. 20 m/s

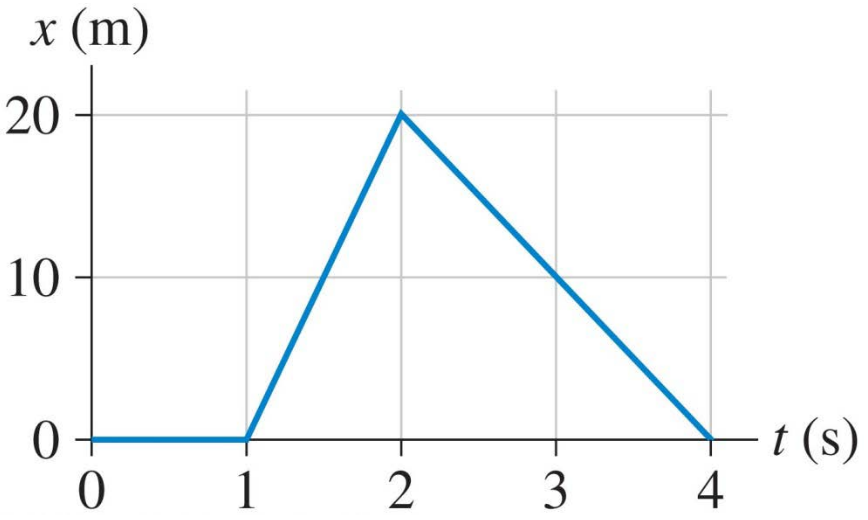
C. 10 m/s

D. –10 m/s

E. None of the above.

QuickCheck 2.6

Here is a position graph of an object:



At $t = 1.5$ s, the object's velocity is

A. 40 m/s

✔ **B. 20 m/s**

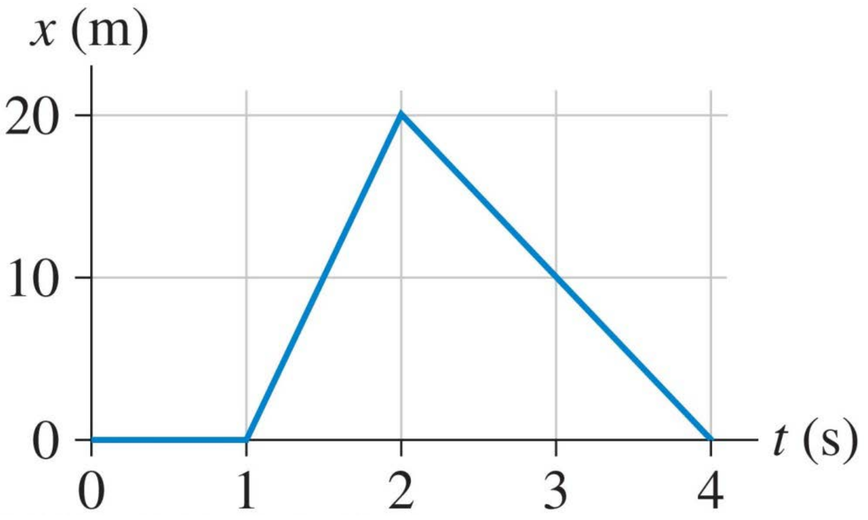
C. 10 m/s

D. –10 m/s

E. None of the above.

QuickCheck 2.7

Here is a position graph of an object:



At $t = 3.0$ s, the object's velocity is

A. 40 m/s

B. 20 m/s

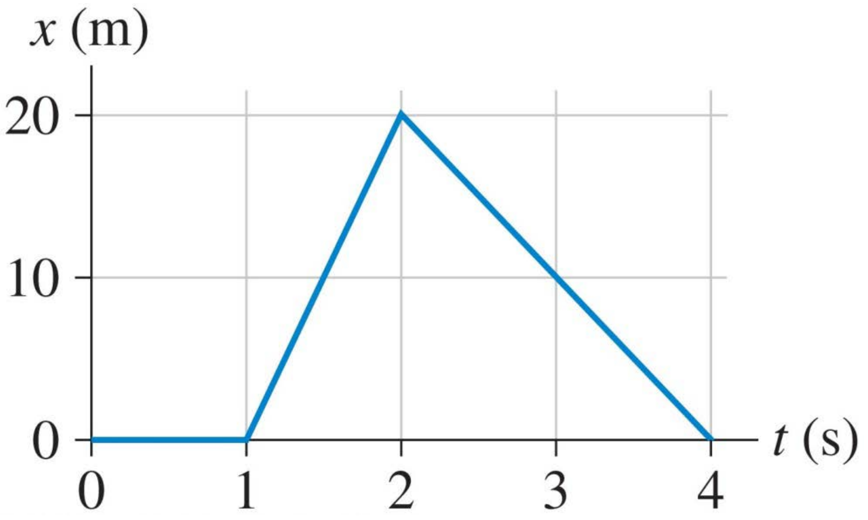
C. 10 m/s

D. –10 m/s

E. None of the above.

QuickCheck 2.7

Here is a position graph of an object:



At $t = 3.0$ s, the object's velocity is

A. 40 m/s

B. 20 m/s

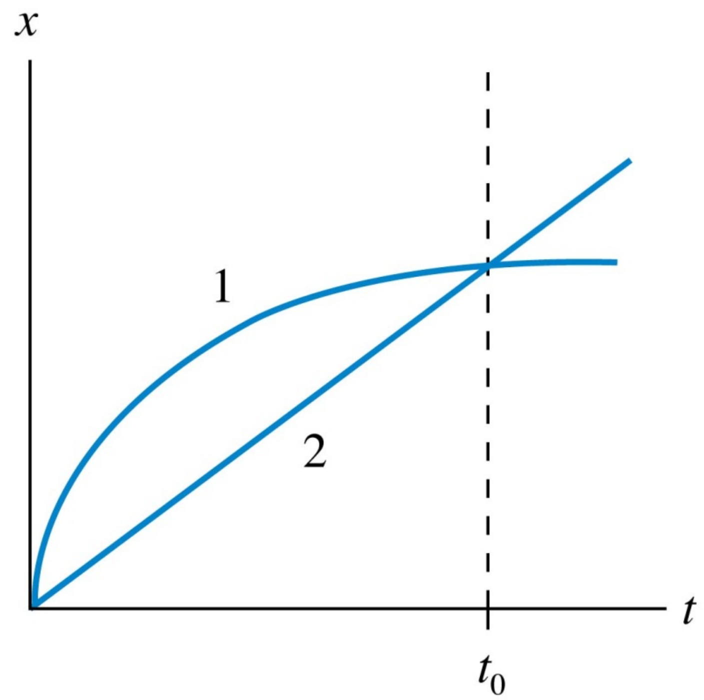
C. 10 m/s

✔ **D. –10 m/s**

E. None of the above.

QuickCheck 2.8

When do objects 1 and 2 have the same velocity?

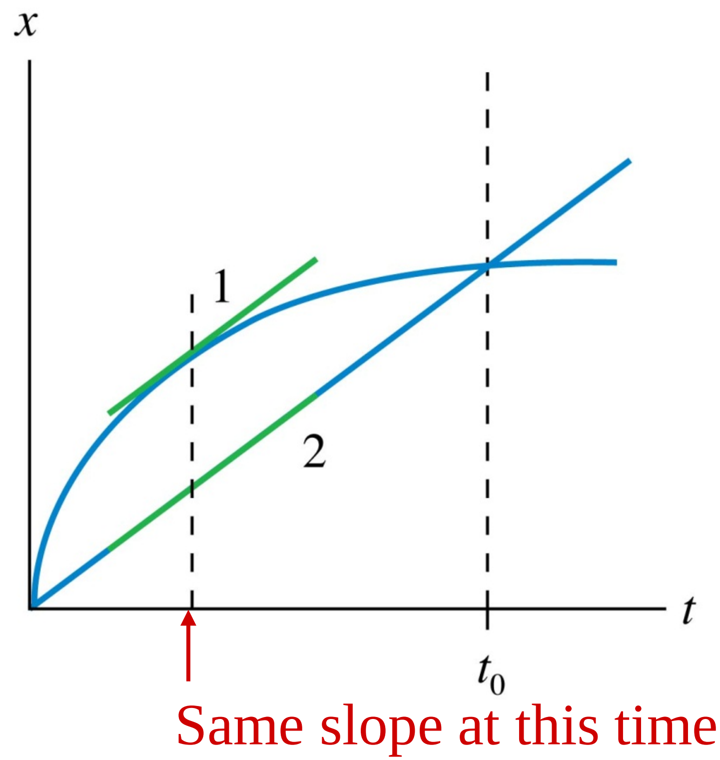


A. At some instant before time t_0.
B. At time t_0.
C. At some instant after time t_0.
D. Both A and B.
E. Never.

QuickCheck 2.8

When do objects 1 and 2 have the same velocity?

- ✔ **A. At some instant before time t_0.**
- B. At time t_0.
- C. At some instant after time t_0.
- D. Both A and B.
- E. Never.



Finding Position from Velocity

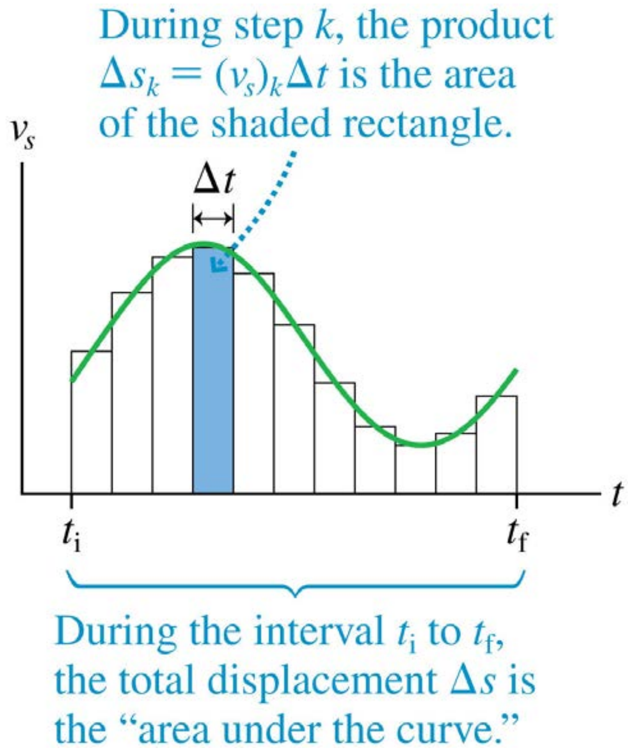
- Suppose we know an object's position to be s_i at an initial time t_i.
- We also know the velocity as a function of time between t_i and some later time t_f.
- Even if the velocity is not constant, we can divide the motion into N steps in which it is approximately constant, and compute the final position as

$$s_f = s_i + \lim_{\Delta t \to 0} \sum_{k=1}^{N} (v_s)_k \Delta t = s_i + \int_{t_i}^{t_f} v_s \, dt$$

- The curlicue symbol is called an *integral*.
- The expression on the right is read "the integral of $v_s \, dt$ from t_i to t_f."

Finding Position from Velocity

- The integral may be interpreted graphically as the total area enclosed between the t-axis and the velocity curve.
- The total displacement Δs is called the “area under the curve.”

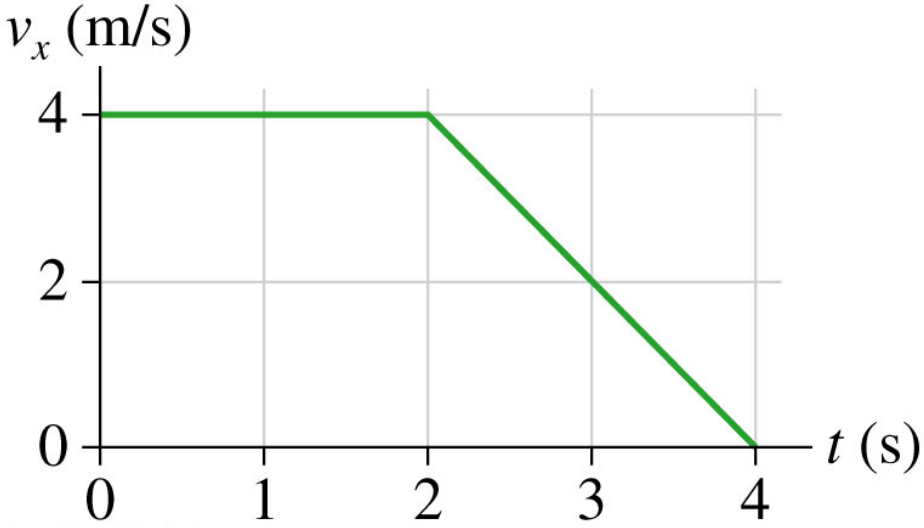


$$s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f$$

QuickCheck 2.9

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

At $t = 4.0$ s, the object's position is



A. 20 m.

B. 16 m.

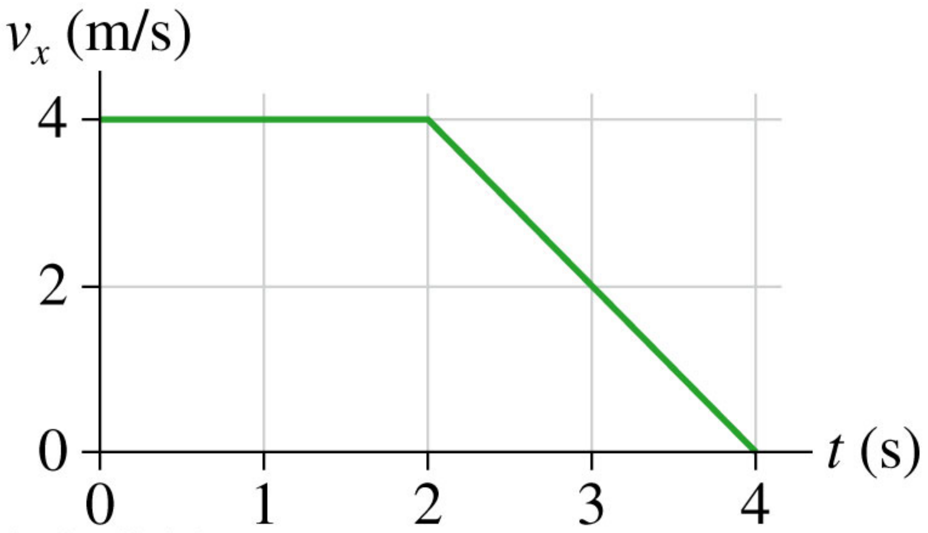
C. 12 m.

D. 8 m.

E. 4 m.

QuickCheck 2.9

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.



At $t = 4.0$ s, the object's position is

A. 20 m.

B. 16 m.

✔ **C. 12 m.** Displacement = area under the curve

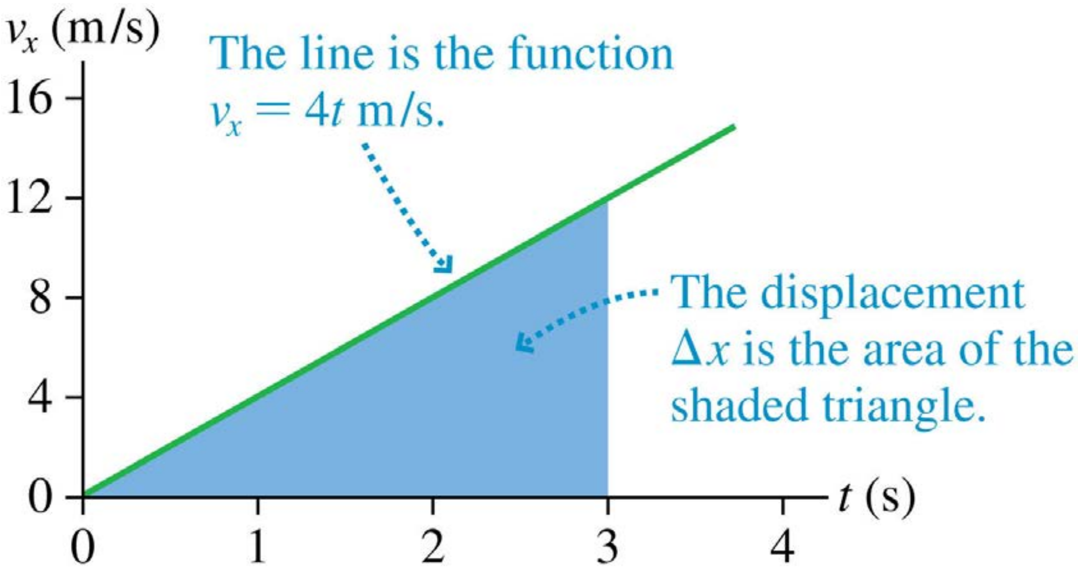
D. 8 m.

E. 4 m.

Example 2.5 The Displacement During a Drag Race

EXAMPLE 2.5 | **The displacement during a drag race**

FIGURE 2.16 shows the velocity-versus-time graph of a drag racer. How far does the racer move during the first 3.0 s?



Example 2.5 The Displacement During a Drag Race

EXAMPLE 2.5 The displacement during a drag race

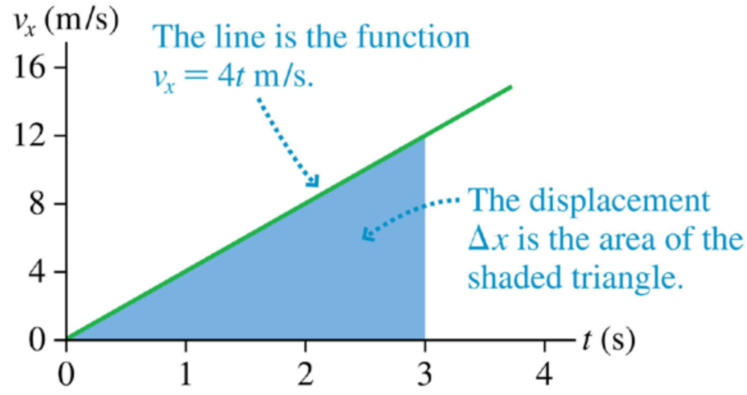
MODEL Model the drag racer as a particle with a well-defined position at all times.

VISUALIZE Figure 2.16 is the graphical representation.

SOLVE The question "How far?" indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.12, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s. The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height}$$
$$= \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The drag racer moves 18 m during the first 3 seconds.



A Little More Calculus: Integrals

- Taking the derivative of a function is equivalent to finding the slope of a graph of the function.
- Similarly, evaluating an integral is equivalent to finding the area under a graph of the function.
- For the important function $u(t) = ct^n$, the essential result from calculus is that

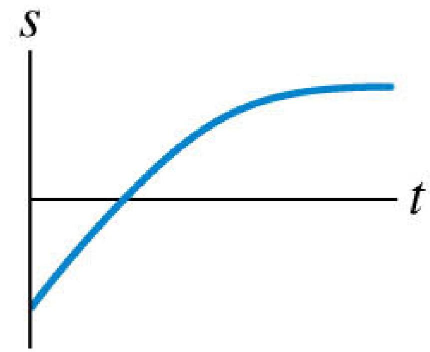
$$\int_{t_i}^{t_f} u\,dt = \int_{t_i}^{t_f} ct^n\,dt = \frac{ct^{n+1}}{n+1}\Bigg|_{t_i}^{t_f} = \frac{ct_f^{\,n+1}}{n+1} - \frac{ct_i^{\,n+1}}{n+1} \qquad (n \neq -1)$$

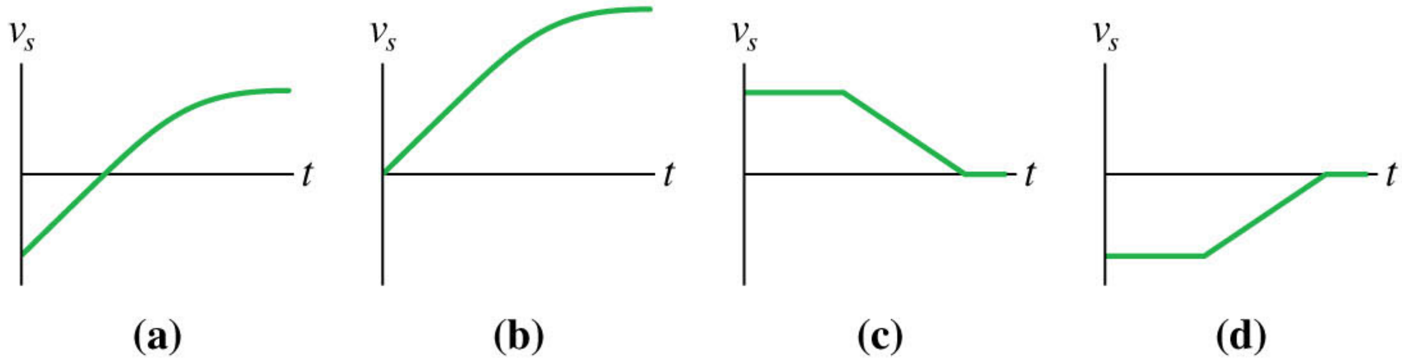
- The vertical bar in the third step means the integral evaluated at t_f minus the integral evaluated at t_i.
- The integral of a sum is the sum of the integrals. If u and w are two separate functions of time, then:

$$\int_{t_i}^{t_f} (u + w)\,dt = \int_{t_i}^{t_f} u\,dt + \int_{t_i}^{t_f} w\,dt$$

QuickCheck 2.10

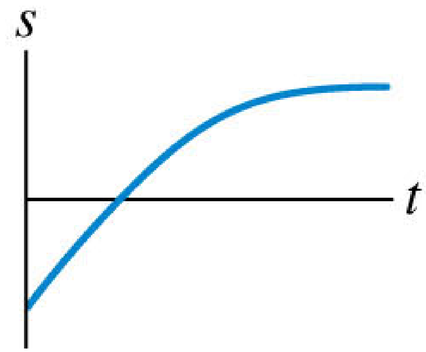
Which velocity-versus-time graph goes with this position graph?

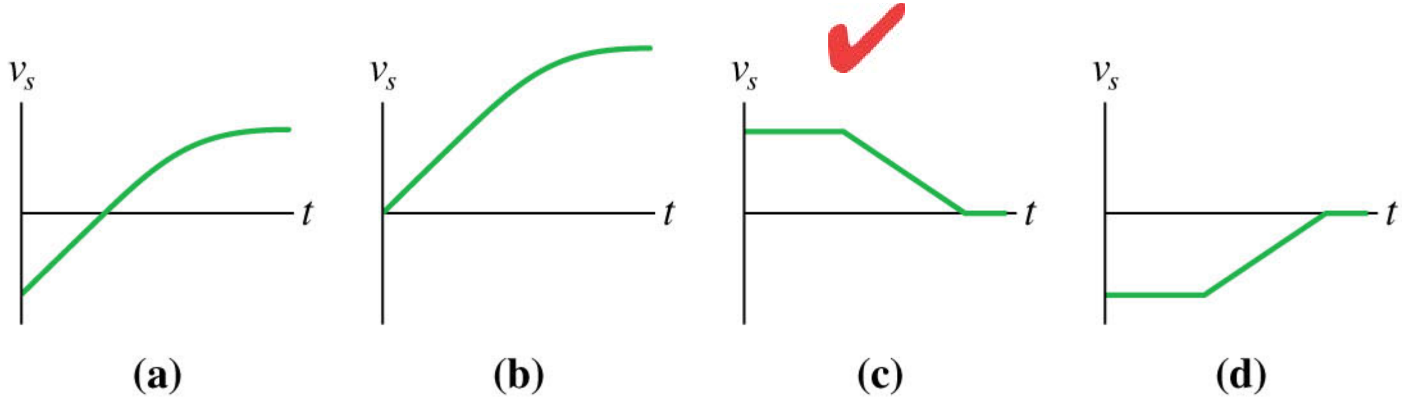




QuickCheck 2.10

Which velocity-versus-time graph goes with this position graph?





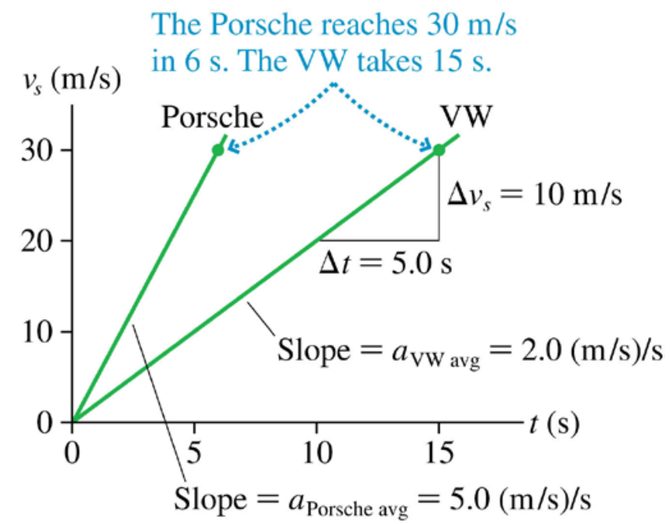
Motion with Constant Acceleration

- Imagine a competition between a Volkswagen Beetle and a Porsche to see which can achieve a velocity of 30 m/s in the shortest time.
- The table shows the velocity of each car, and the figure shows the velocity-versus-time graphs.
- Both cars achieved every velocity between 0 and 30 m/s, so neither is faster.
- But for the Porsche, the rate at which the velocity changed was

$$\text{rate of velocity change} = \frac{\Delta v_s}{\Delta t} = \frac{30\text{ m/s}}{6.0\text{ s}} = 5.0\text{ (m/s)/s}$$

TABLE 2.1 Velocities of a Porsche and a Volkswagen Beetle

t (s)	v_{Porsche} (m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
⋮	⋮	⋮



Motion with Constant Acceleration

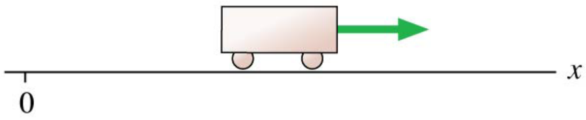
- The SI units of acceleration are $(\text{m/s})/\text{s}$, or m/s^2.
- It is the rate of change of velocity and measures how quickly or slowly an object's velocity changes.
- The **average acceleration** during a time interval Δt is

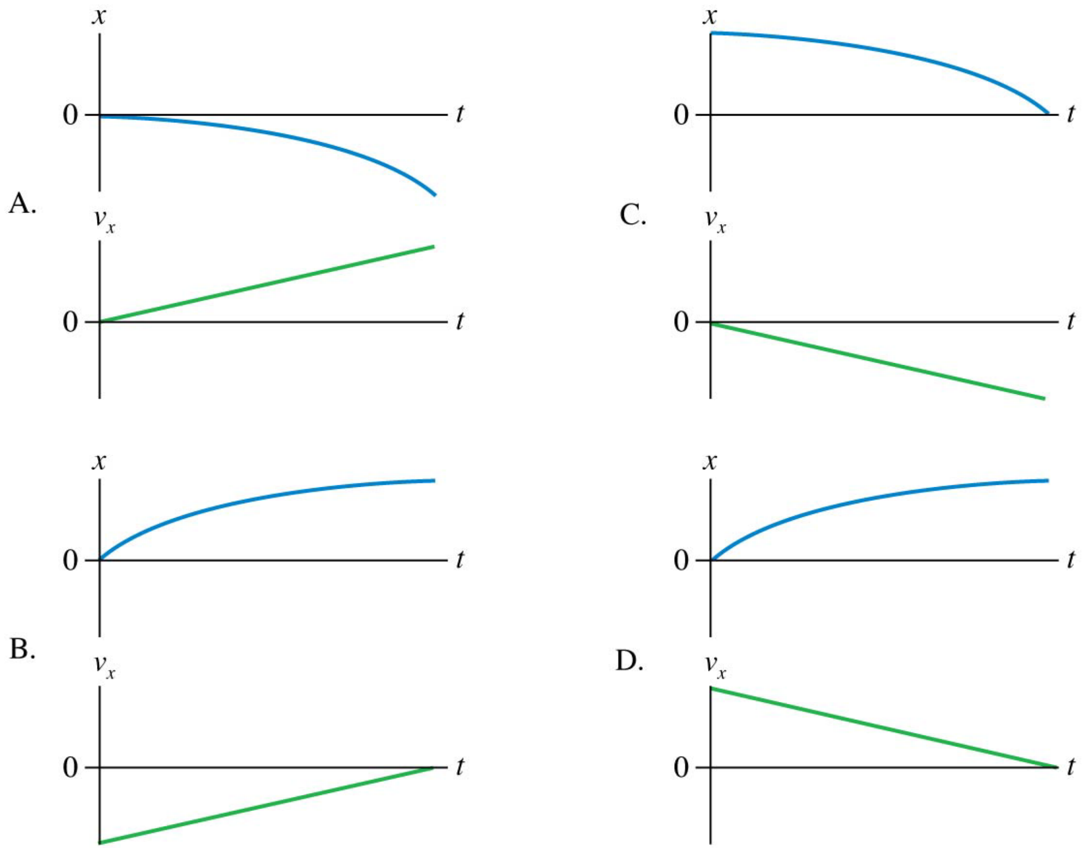
$$a_{\text{avg}} \equiv \frac{\Delta v_s}{\Delta t} \qquad \text{(average acceleration)}$$

- Graphically, a_{avg} is the *slope* of a straight-line velocity-versus-time graph.
- If acceleration is constant, the acceleration a_s is the same as a_{avg}.
- Acceleration, like velocity, is a vector quantity and has both magnitude and direction.

QuickCheck 2.11

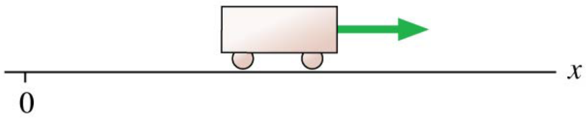
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

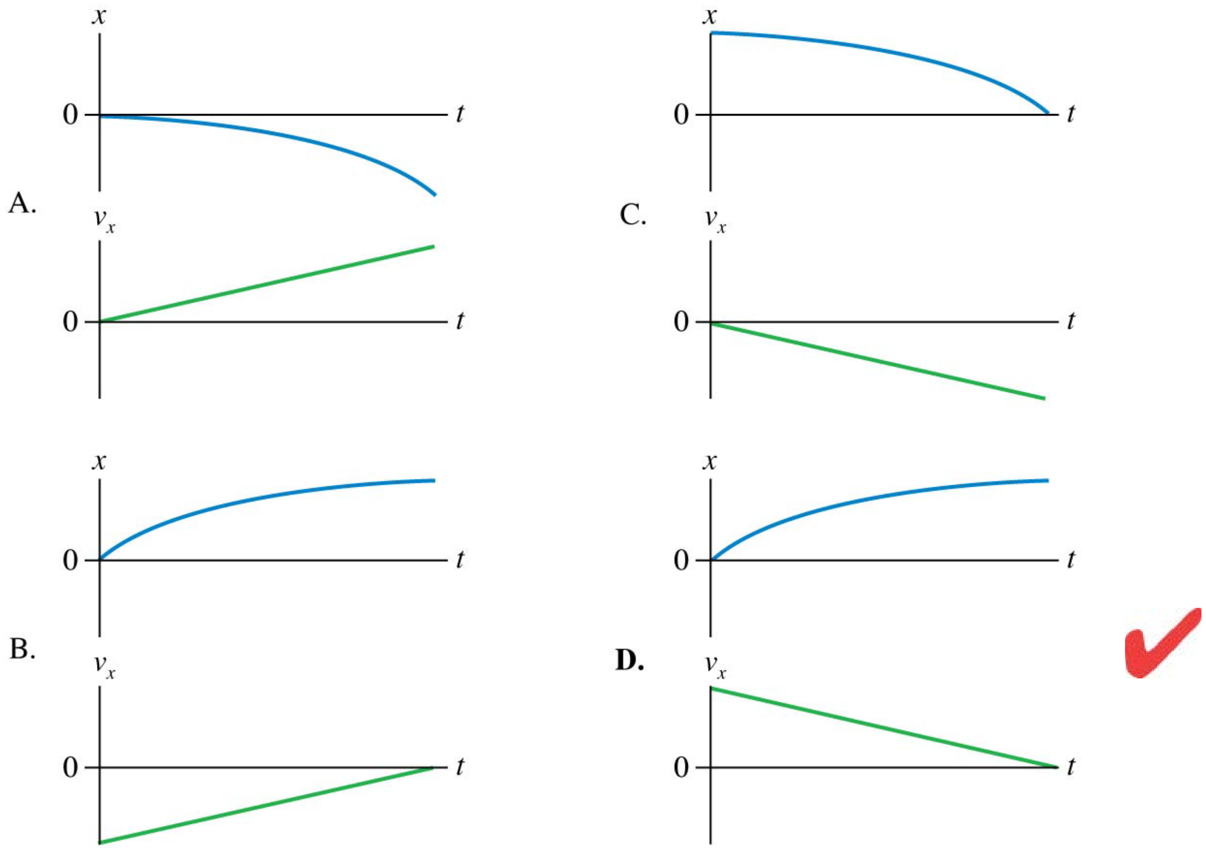




QuickCheck 2.11

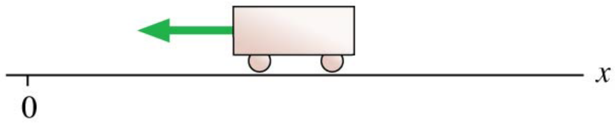
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

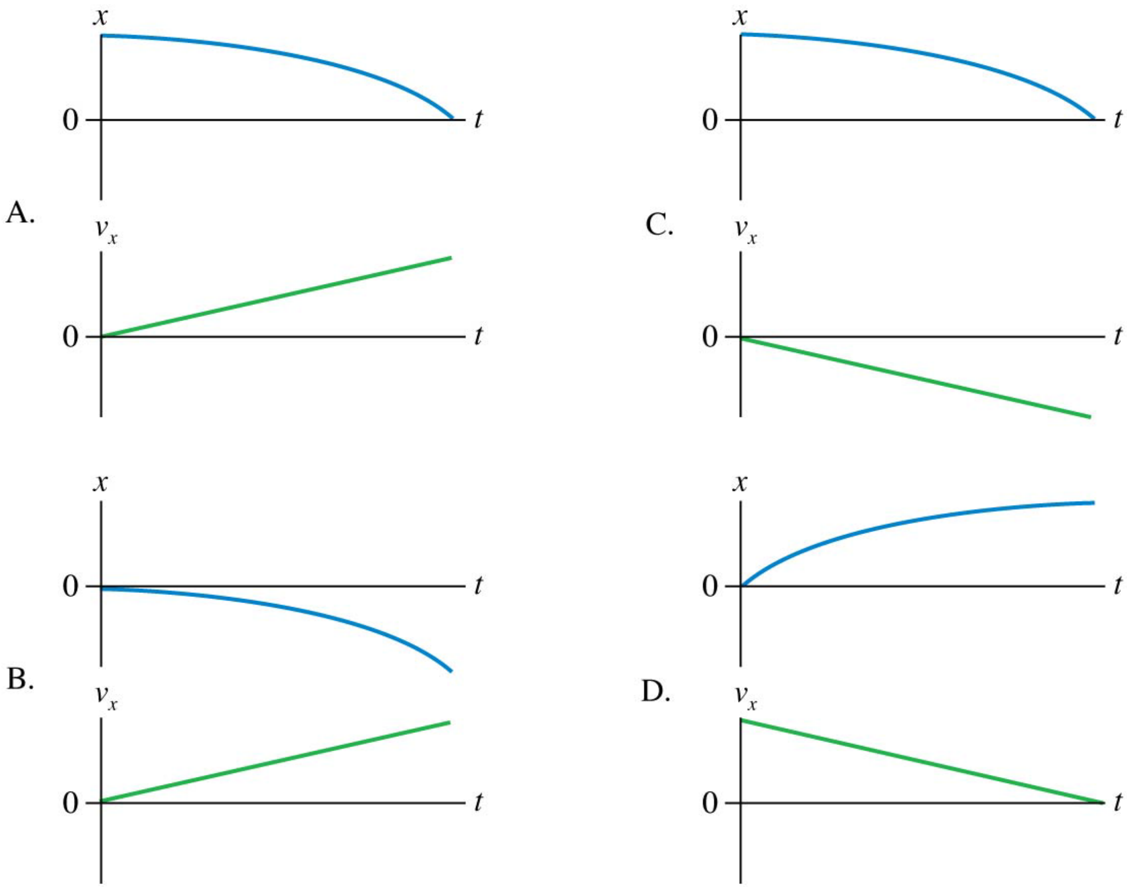




QuickCheck 2.12

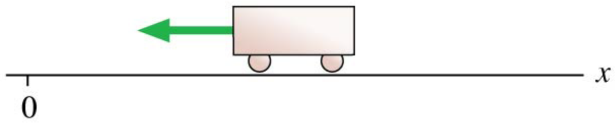
A cart speeds up toward the origin. What do the position and velocity graphs look like?

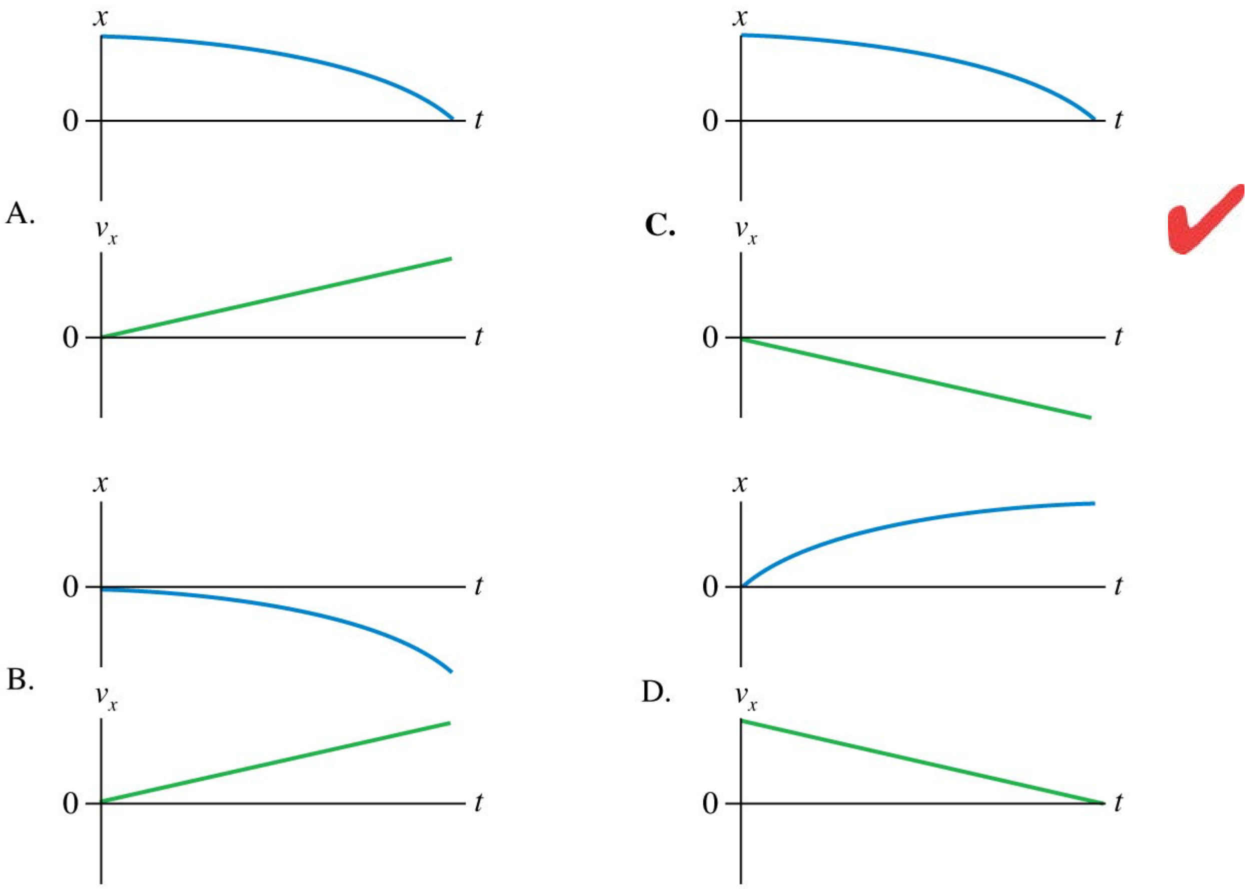




QuickCheck 2.12

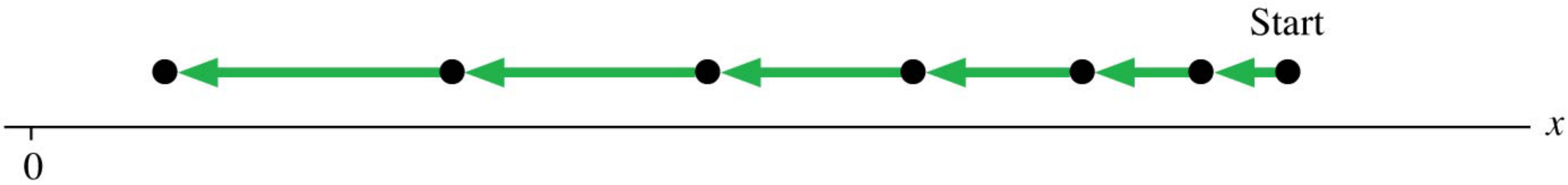
A cart speeds up toward the origin. What do the position and velocity graphs look like?





QuickCheck 2.13

Here is a motion diagram of a car speeding up on a straight road:

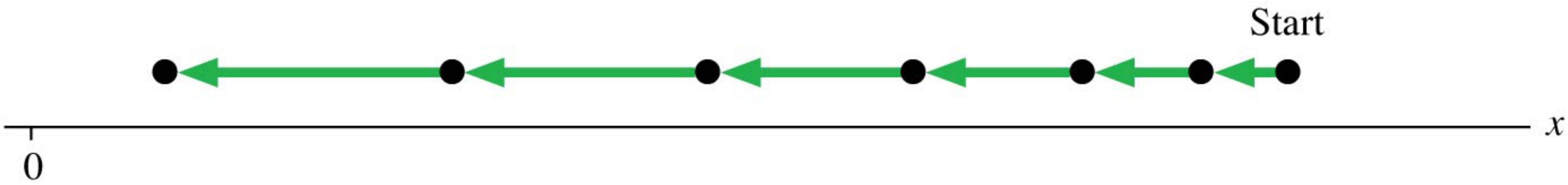


The sign of the acceleration a_x is

A. Positive.
B. Negative.
C. Zero.

QuickCheck 2.13

Here is a motion diagram of a car speeding up on a straight road:



The sign of the acceleration a_x is

A. Positive.

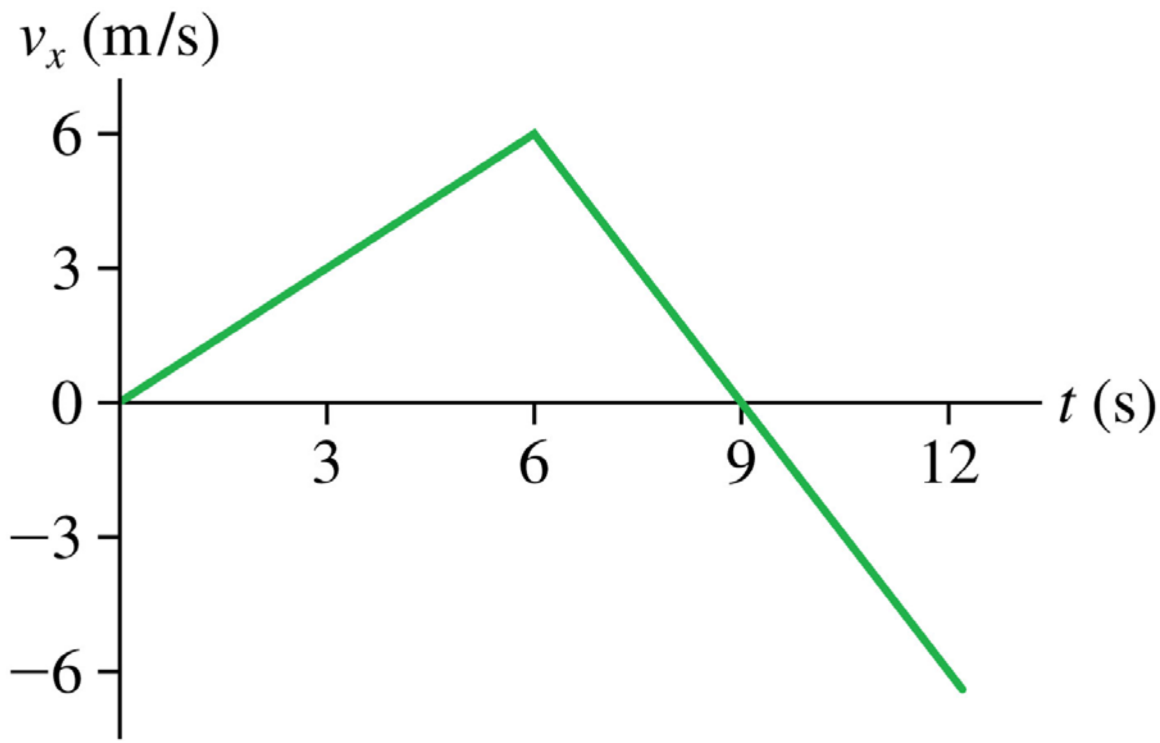
✔ **B. Negative.** Speeding up means v_x and a_x have the same sign.

C. Zero.

Example 2.9 Running the Court

EXAMPLE 2.9 | **Running the court**

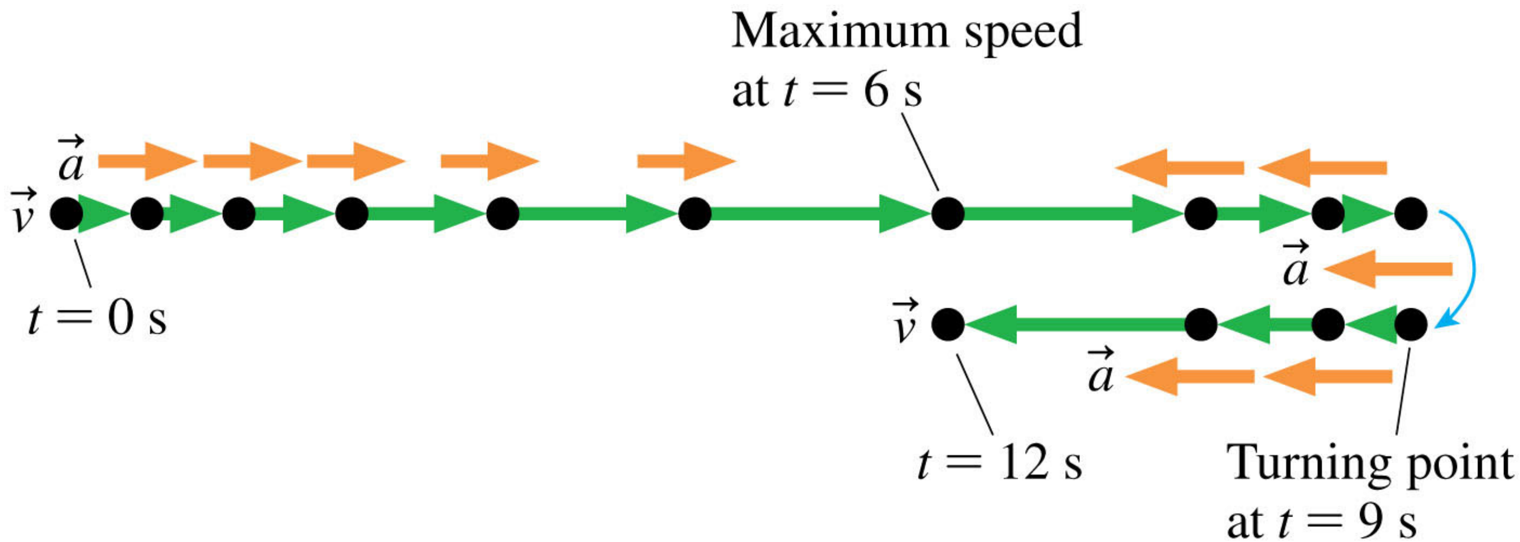
A basketball player starts at the left end of the court and moves with the velocity shown in FIGURE 2.20. Draw a motion diagram and an acceleration-versus-time graph for the basketball player.



Example 2.9 Running the Court

EXAMPLE 2.9 Running the court

VISUALIZE The velocity is positive (motion to the right) and increasing for the first 6 s, so the velocity arrows in the motion diagram are to the right and getting longer. From $t = 6$ s to 9 s the motion is still to the right (v_x is still positive), but the arrows are getting shorter because v_x is decreasing. There's a turning point at $t = 9$ s, when $v_x = 0$, and after that the motion is to the left (v_x is negative) and getting faster. The motion diagram of FIGURE 2.21a shows the velocity and the acceleration vectors.



Example 2.9 Running the Court

EXAMPLE 2.9 Running the court

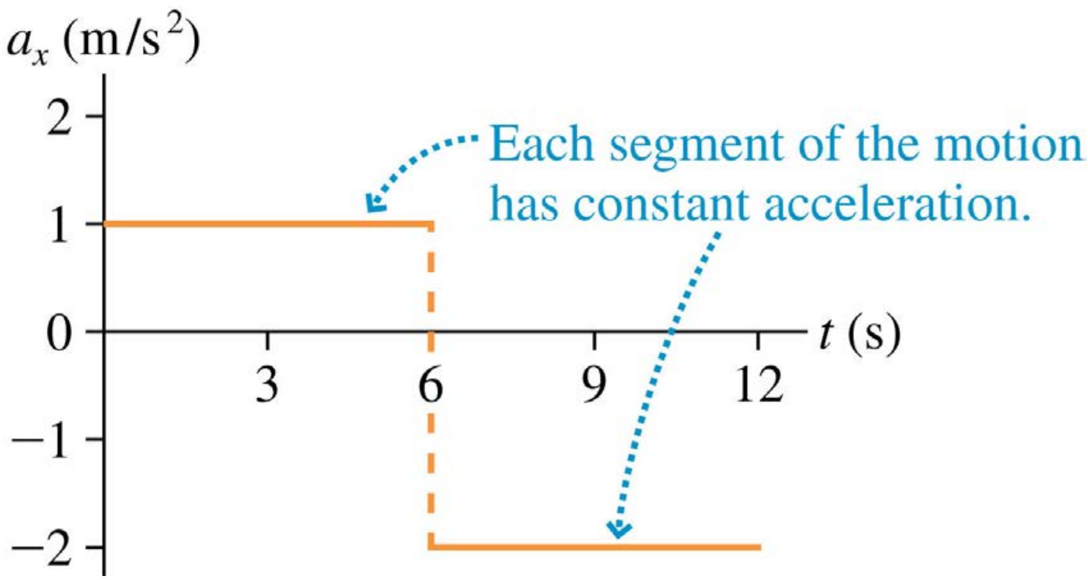
SOLVE Acceleration is the slope of the velocity graph. For the first 6 s, the slope has the constant value

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{6.0 \text{ m/s}}{6.0 \text{ s}} = 1.0 \text{ m/s}^2$$

The velocity then decreases by 12 m/s during the 6 s interval from $t = 6$ s to $t = 12$ s, so

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{-12 \text{ m/s}}{6.0 \text{ s}} = -2.0 \text{ m/s}^2$$

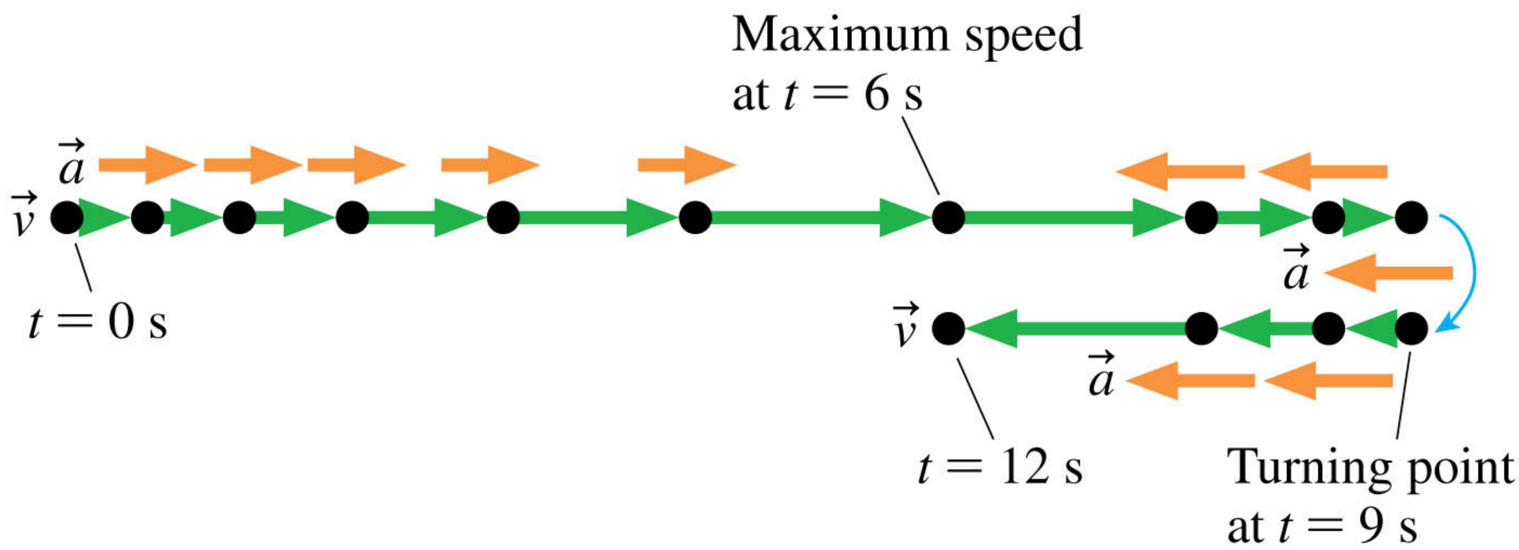
The acceleration graph for these 12 s is shown in **FIGURE 2.21b**. Notice that there is no change in the acceleration at $t = 9$ s, the turning point.



Example 2.9 Running the Court

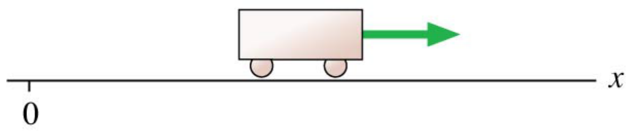
EXAMPLE 2.9 **Running the court**

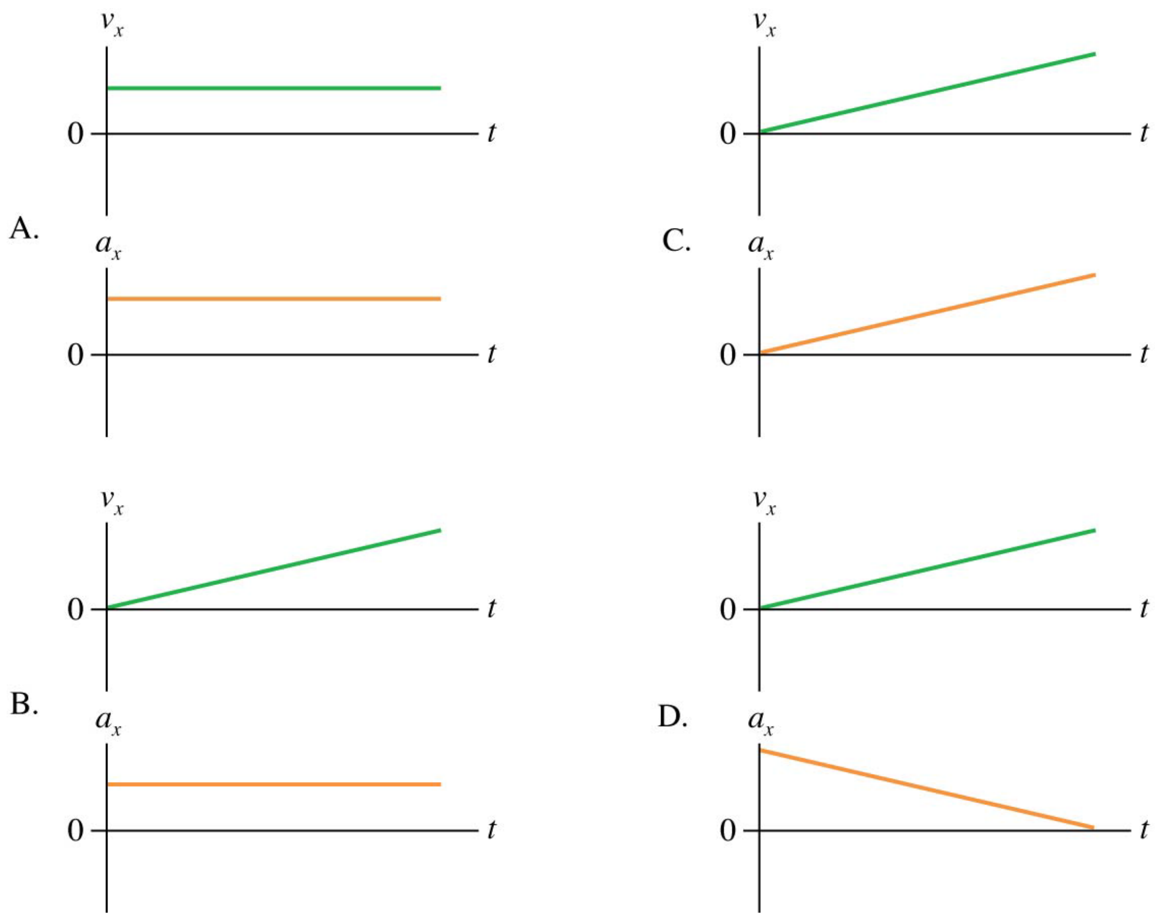
ASSESS The *sign* of a_x does *not* tell us whether the object is speeding up or slowing down. The basketball player is slowing down from $t = 6$ s to $t = 9$ s, then speeding up from $t = 9$ s to $t = 12$ s. Nonetheless, his acceleration is negative during this entire interval because his acceleration vector, as seen in the motion diagram, always points to the left.



QuickCheck 2.14

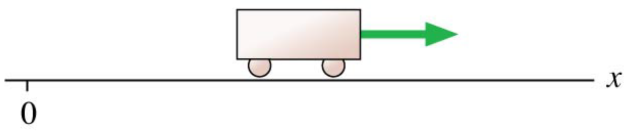
A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?

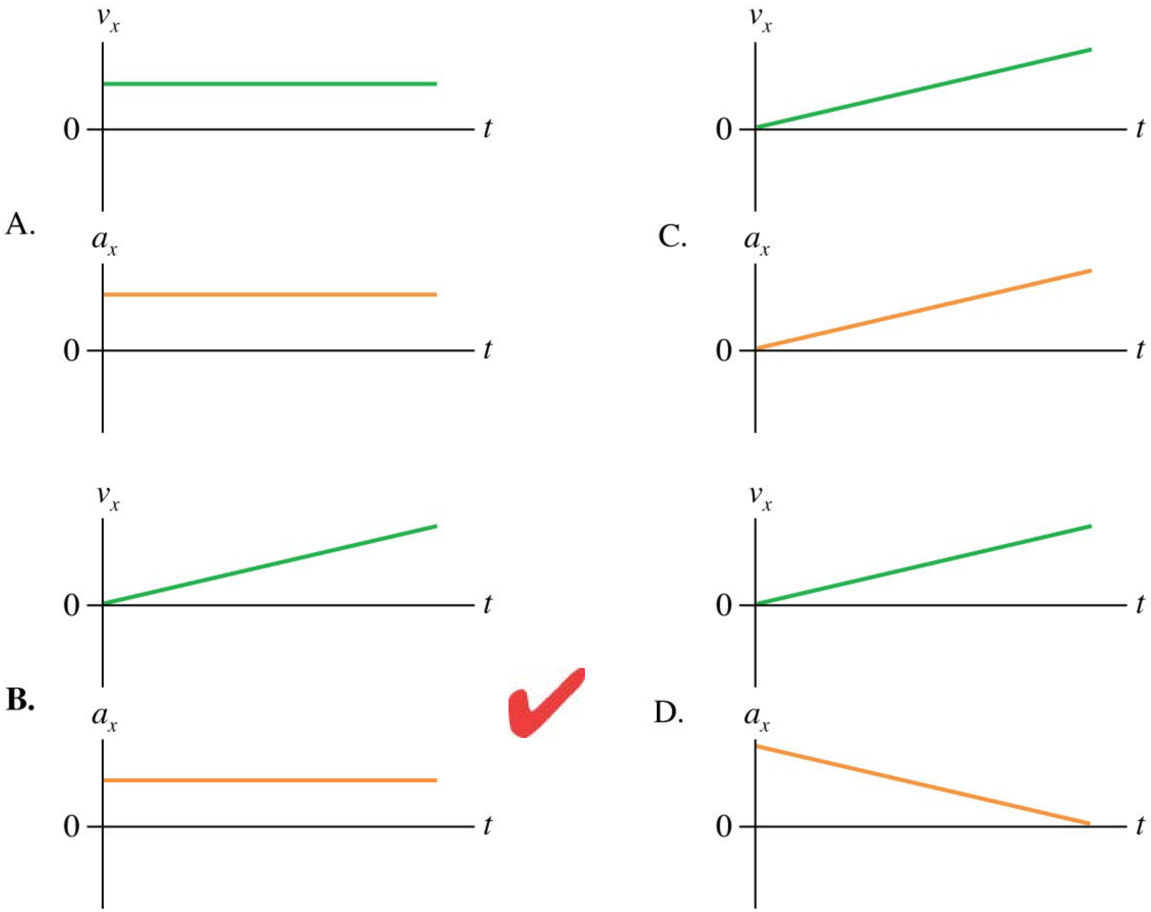




QuickCheck 2.14

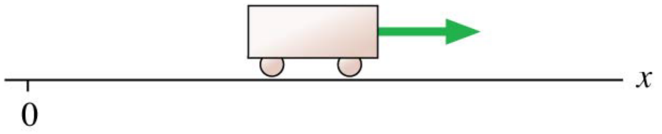
A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?

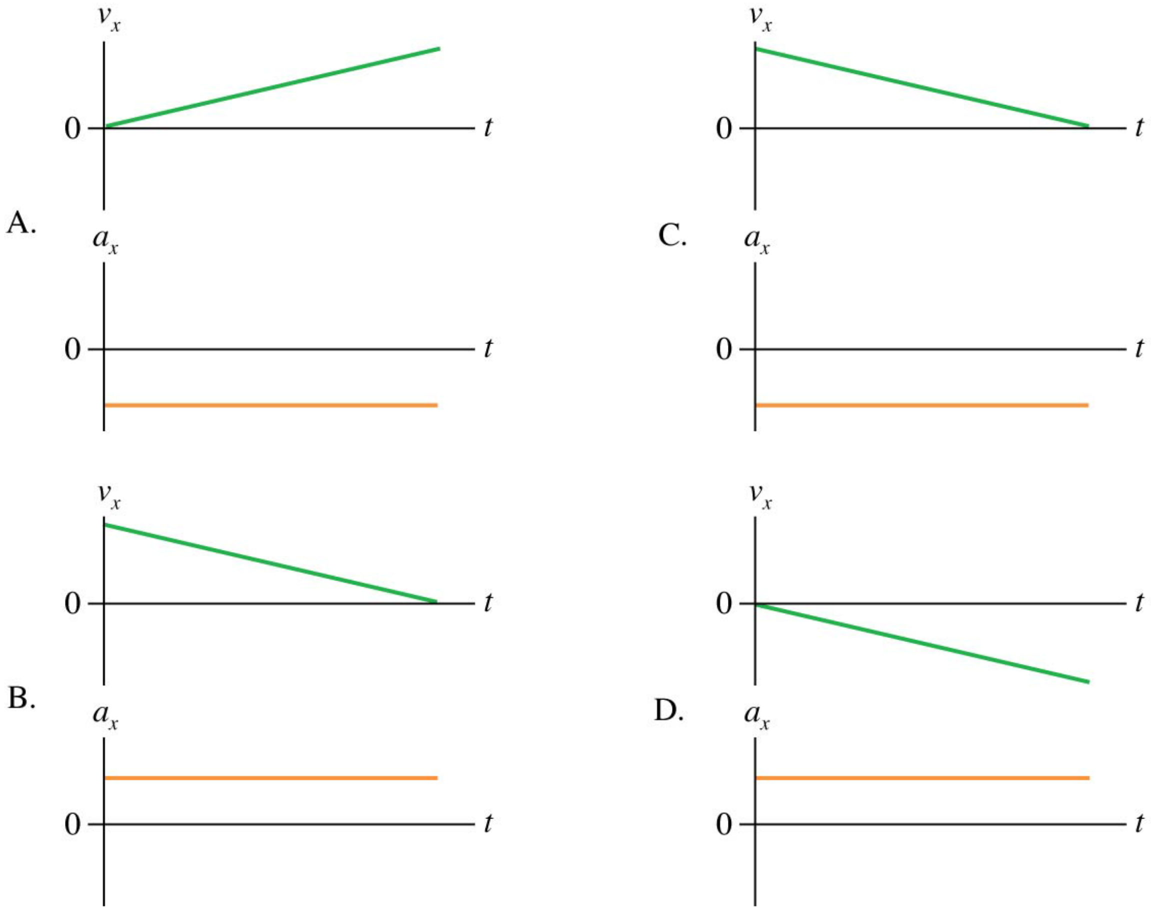




QuickCheck 2.15

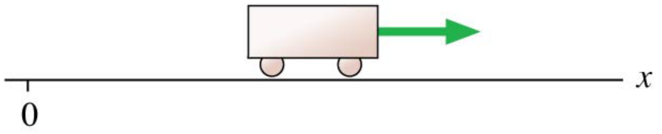
A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

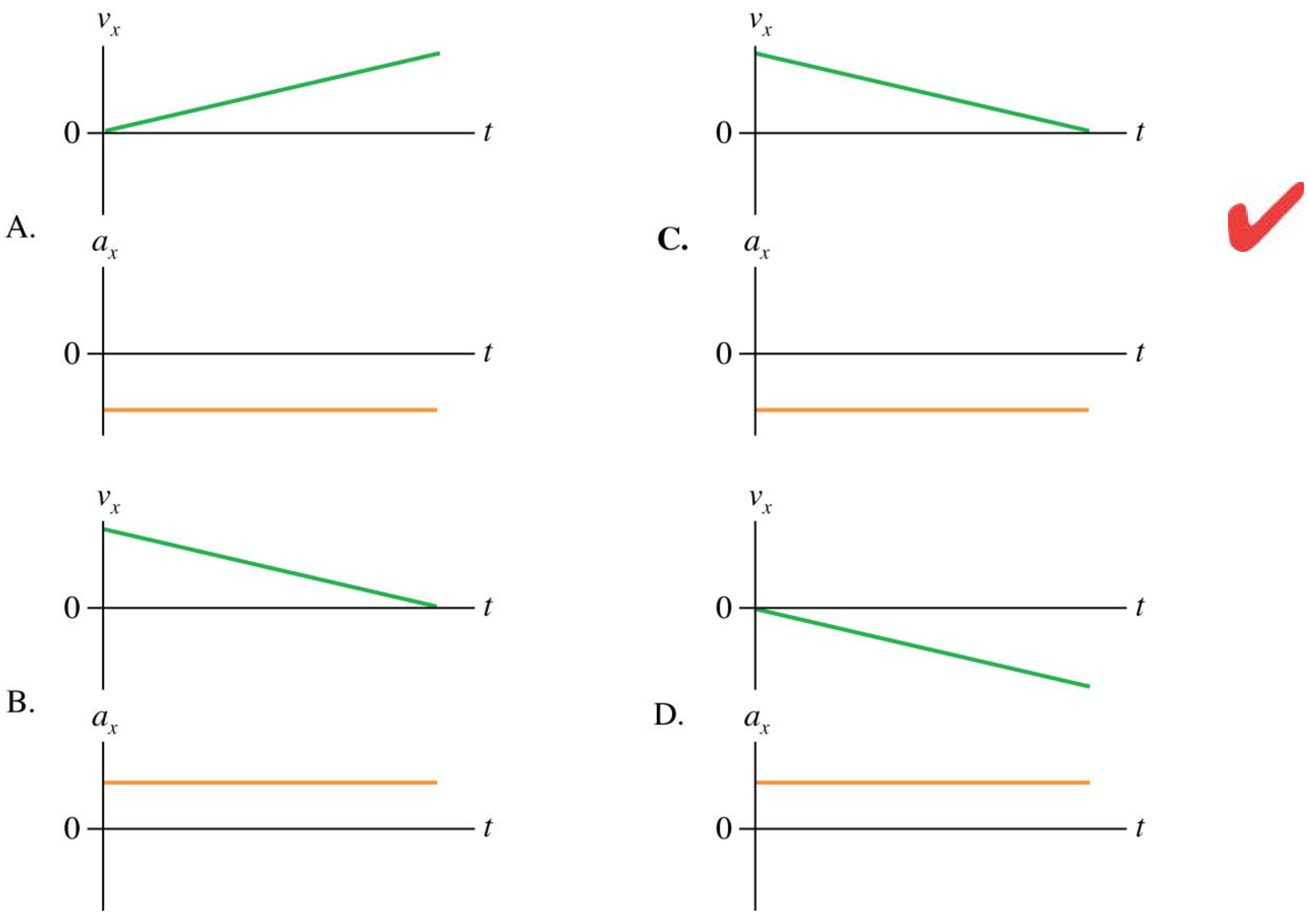




QuickCheck 2.15

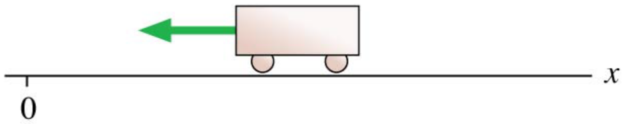
A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

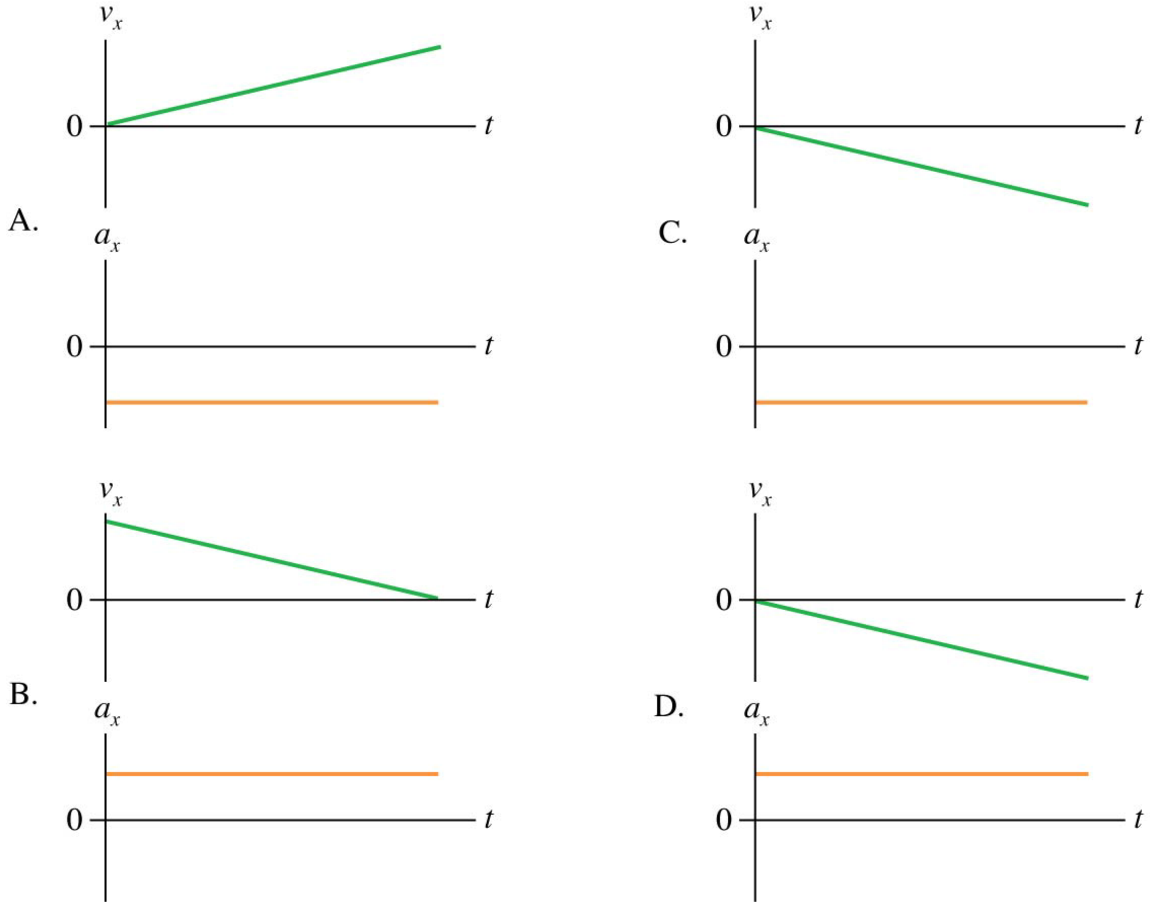




QuickCheck 2.16

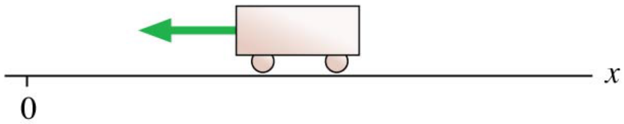
A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?

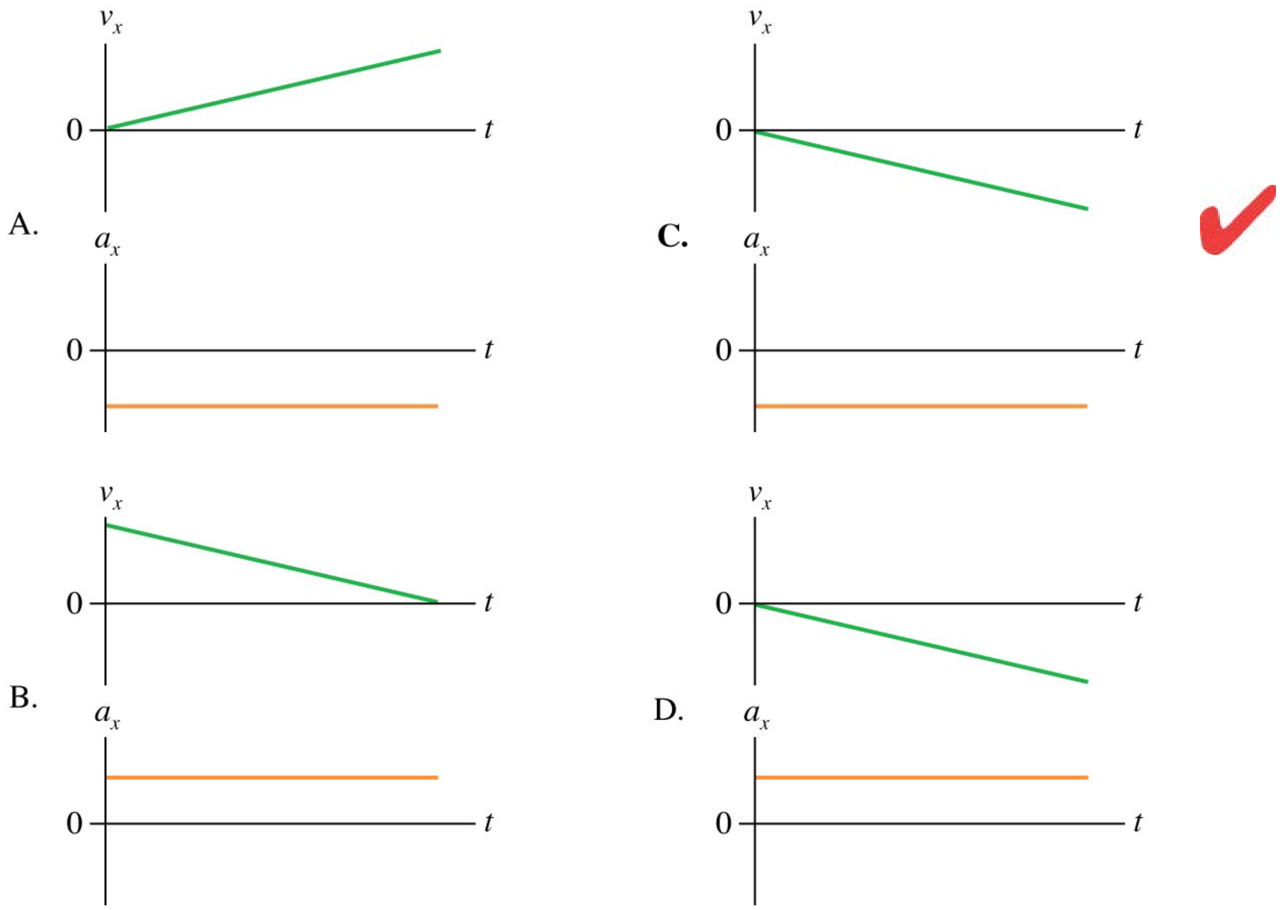




QuickCheck 2.16

A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?





The Kinematic Equations of Constant Acceleration

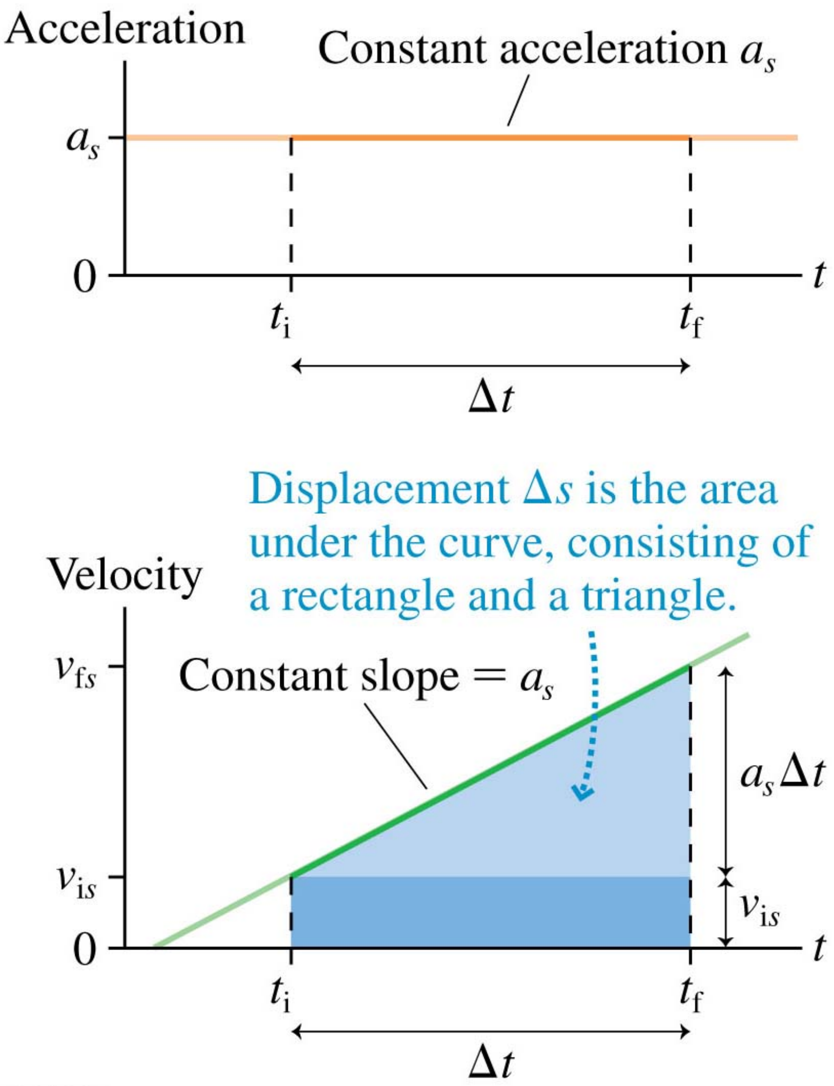
- Suppose we know an object's velocity to be v_{is} at an initial time t_i.
- We also know the object has a constant acceleration of a_s over the time interval $\Delta t = t_f - t_i$.
- We can then find the object's velocity at the later time t_f as

$$v_{fs} = v_{is} + a_s \Delta t$$

The Kinematic Equations of Constant Acceleration

- Suppose we know an object's position to be s_i at an initial time t_i.
- It's constant acceleration a_s is shown in graph (a).
- The velocity-versus-time graph is shown in graph (b).
- The final position s_f is s_i plus the area under the curve of v_s between t_i and t_f:

$$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$$



The Kinematic Equations of Constant Acceleration

- Suppose we know an object's velocity to be v_{is} at an initial position s_i.
- We also know the object has a constant acceleration of a_s while it travels a total displacement of $\Delta s = s_f - s_i$.
- We can then find the object's velocity at the final position s_f:

$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

The Constant-Acceleration Model

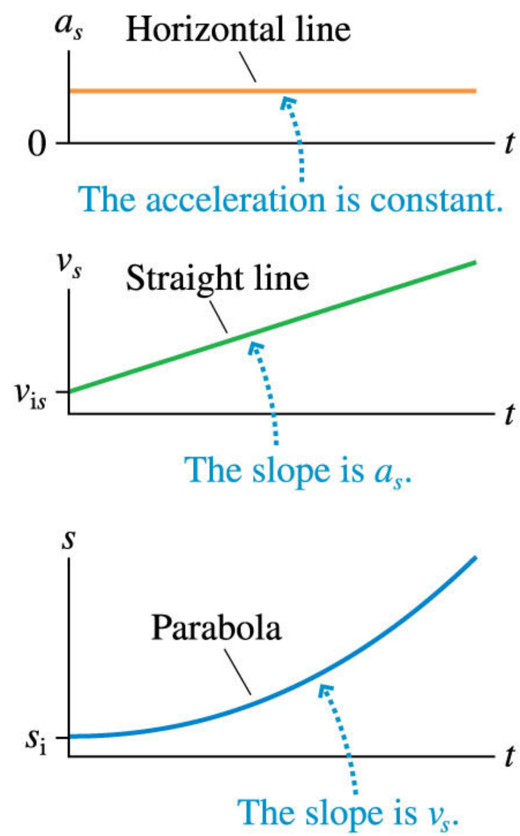
MODEL 2.2

Constant acceleration

For motion with constant acceleration.

- Model the object as a particle moving in a straight line with constant acceleration.

- Mathematically:
  - $v_{fs} = v_{is} + a_s \Delta t$
  - $s_f = s_i + v_{is} \Delta t + \frac{1}{2} a_s (\Delta t)^2$
  - $v_{fs}^2 = v_{is}^2 + 2 a_s \Delta s$
- Limitations: Model fails if the particle's acceleration changes.



Exercise 16

The Kinematic Equations of Constant Acceleration

PROBLEM-SOLVING STRATEGY 2.1

Kinematics with constant acceleration

MODEL Model the object as having constant acceleration.

VISUALIZE Use different representations of the information in the problem.

- Draw a *pictorial representation.* This helps you assess the information you are given and starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these two representations as needed.

SOLVE The mathematical representation is based on the three kinematic equations:

$$v_{fs} = v_{is} + a_s \Delta t$$
$$s_f = s_i + v_{is} \Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$
$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$

- Use x or y, as appropriate to the problem, rather than the generic s.
- Replace i and f with numerical subscripts defined in the pictorial representation.

ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

Example 2.11 A Two-Car Race

EXAMPLE 2.11 | **A two-car race**

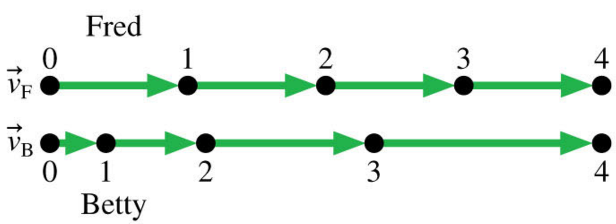
Fred is driving his Volkswagen Beetle at a steady 20 m/s when he passes Betty sitting at rest in her Porsche. Betty instantly begins accelerating at 5.0 m/s^2. How far does Betty have to drive to overtake Fred?

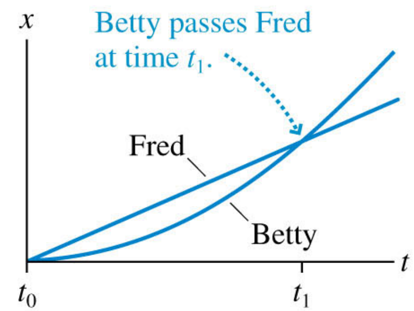
MODEL Model the VW as a particle in uniform motion and the Porsche as a particle with constant acceleration.

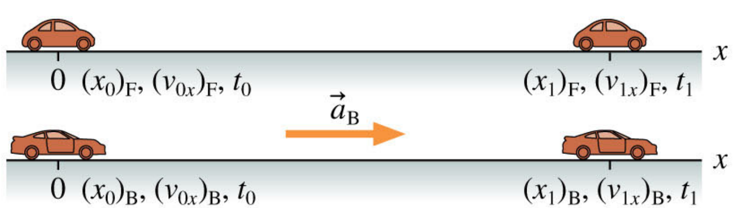
Example 2.11 A Two-Car Race

EXAMPLE 2.11 | **A two-car race**

VISUALIZE **FIGURE 2.24** is the pictorial representation. Fred's motion diagram is one of uniform motion, while Betty's shows uniform acceleration. Fred is ahead in frames 1, 2, and 3, but Betty catches up with him in frame 4. The coordinate system shows the cars with the same position at the start and at the end—but with the important difference that Betty's Porsche has an acceleration while Fred's VW does not.







Known

$(x_0)_F = 0$ m $(x_0)_B = 0$ m $t_0 = 0$ s
$(v_{0x})_F = 20$ m/s $(v_{0x})_B = 0$ m/s
$(a_{0x})_B = 5.0$ m/s^2 $(v_{1x})_F = 20$ m/s

Find

$(x_1)_B$ at t_1 when $(x_1)_B = (x_1)_F$

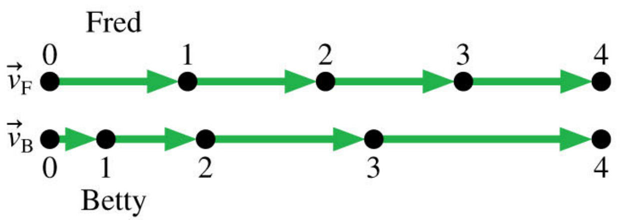
Example 2.11 A Two-Car Race

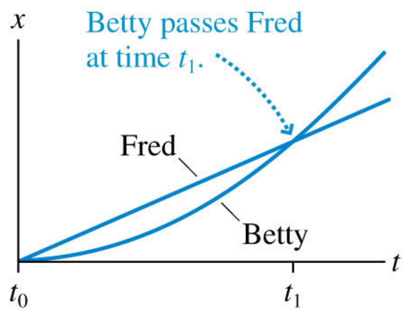
EXAMPLE 2.11 | **A two-car race**

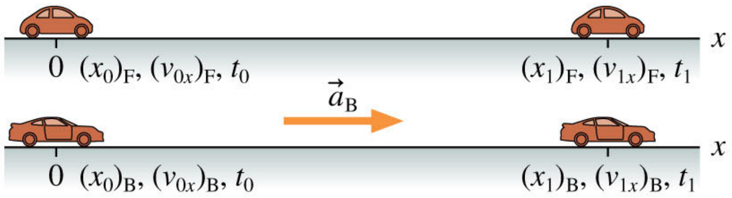
SOLVE This problem is similar to Example 2.2, in which Bob and Susan met for lunch. As we did there, we want to find Betty's position $(x_1)_B$ at the instant t_1 when $(x_1)_B = (x_1)_F$. We know, from the models of uniform motion and uniform acceleration, that Fred's position graph is a straight line but Betty's is a parabola. The position graphs in Figure 2.24 show that we're solving for the intersection point of the line and the parabola.

Fred's and Betty's positions at t_1 are

$$(x_1)_F = (x_0)_F + (v_{0x})_F(t_1 - t_0) = (v_{0x})_F t_1$$
$$(x_1)_B = (x_0)_B + (v_{0x})_B(t_1 - t_0) + \tfrac{1}{2}(a_{0x})_B(t_1 - t_0)^2 = \tfrac{1}{2}(a_{0x})_B t_1^2$$







Known

$(x_0)_F = 0$ m $(x_0)_B = 0$ m $t_0 = 0$ s
$(v_{0x})_F = 20$ m/s $(v_{0x})_B = 0$ m/s
$(a_{0x})_B = 5.0$ m/s^2 $(v_{1x})_F = 20$ m/s

Find

$(x_1)_B$ at t_1 when $(x_1)_B = (x_1)_F$

Example 2.11 A Two-Car Race

EXAMPLE 2.11 | **A two-car race**

By equating these,

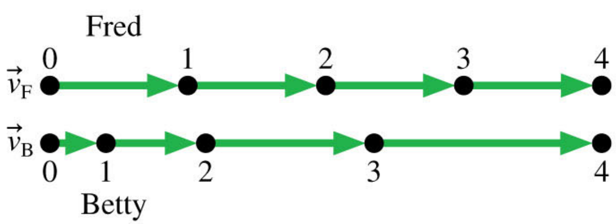
$$(v_{0x})_F t_1 = \tfrac{1}{2}(a_{0x})_B t_1^2$$

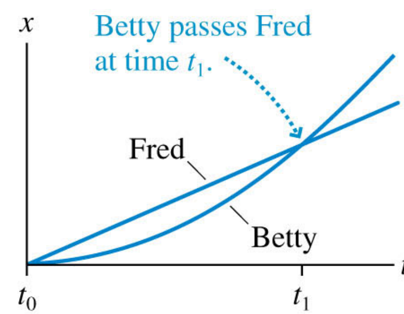
we can solve for the time when Betty passes Fred:

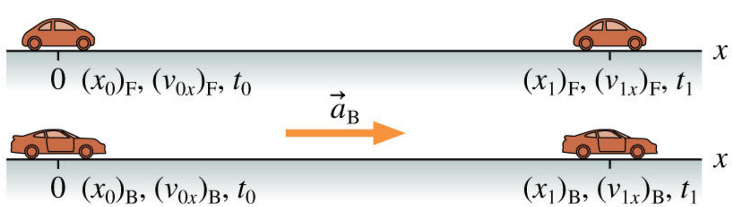
$$t_1\left[\tfrac{1}{2}(a_{0x})_B t_1 - (v_{0x})_F\right] = 0$$

$$t_1 = \begin{cases} 0 \text{ s} \\ 2(v_{0x})_F/(a_{0x})_B = 8.0 \text{ s} \end{cases}$$

Interestingly, there are two solutions. That's not surprising, when you think about it, because the line and the parabola of the position graphs have *two* intersection points: when Fred first passes Betty, and 8.0 s later when Betty passes Fred. We're interested in only the second of these points. We can now use either of the distance equations to find $(x_1)_B = (x_1)_F = 160$ m. Betty has to drive 160 m to overtake Fred.







Known

$(x_0)_F = 0$ m $(x_0)_B = 0$ m $t_0 = 0$ s
$(v_{0x})_F = 20$ m/s $(v_{0x})_B = 0$ m/s
$(a_{0x})_B = 5.0$ m/s^2 $(v_{1x})_F = 20$ m/s

Find

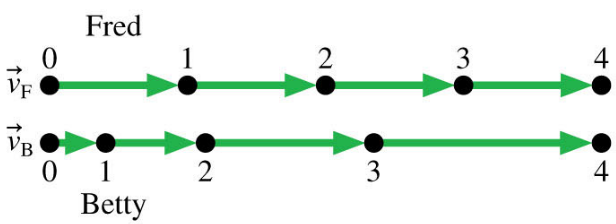
$(x_1)_B$ at t_1 when $(x_1)_B = (x_1)_F$

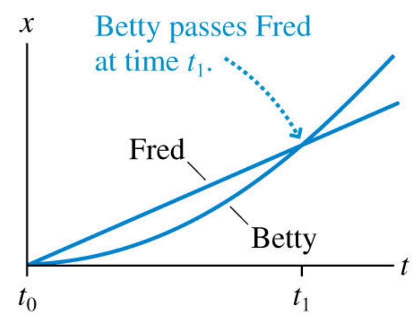
Example 2.11 A Two-Car Race

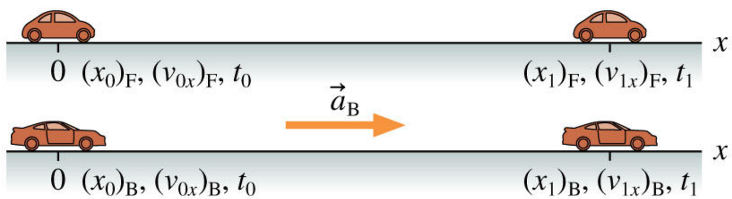
EXAMPLE 2.11 **A two-car race**

ASSESS 160 m ≈ 160 yards. Because Betty starts from rest while Fred is moving at 20 m/s ≈ 40 mph, needing 160 yards to catch him seems reasonable.

NOTE The purpose of the Assess step is not to prove that an answer must be right but to rule out answers that, with a little thought, are clearly wrong.







Known

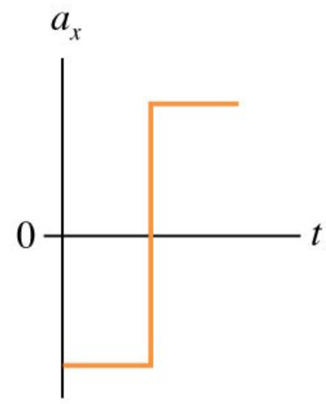
$(x_0)_F = 0$ m $(x_0)_B = 0$ m $t_0 = 0$ s
$(v_{0x})_F = 20$ m/s $(v_{0x})_B = 0$ m/s
$(a_{0x})_B = 5.0$ m/s^2 $(v_{1x})_F = 20$ m/s

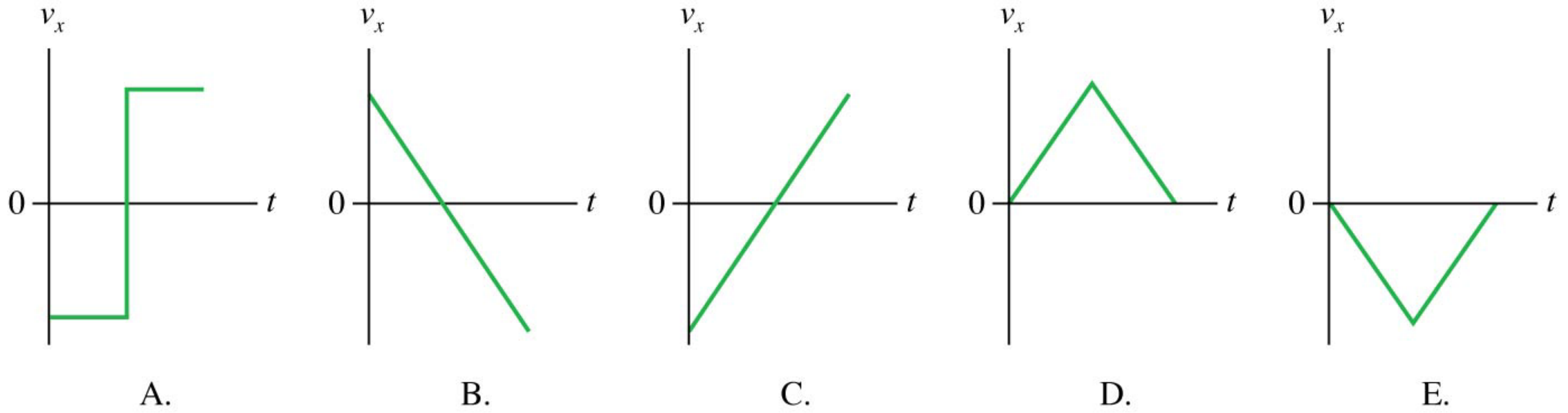
Find

$(x_1)_B$ at t_1 when $(x_1)_B = (x_1)_F$

QuickCheck 2.17

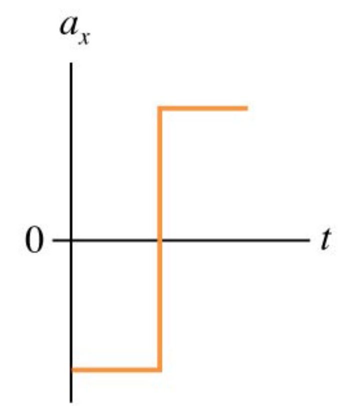
Which velocity-versus-time graph goes with this acceleration graph?

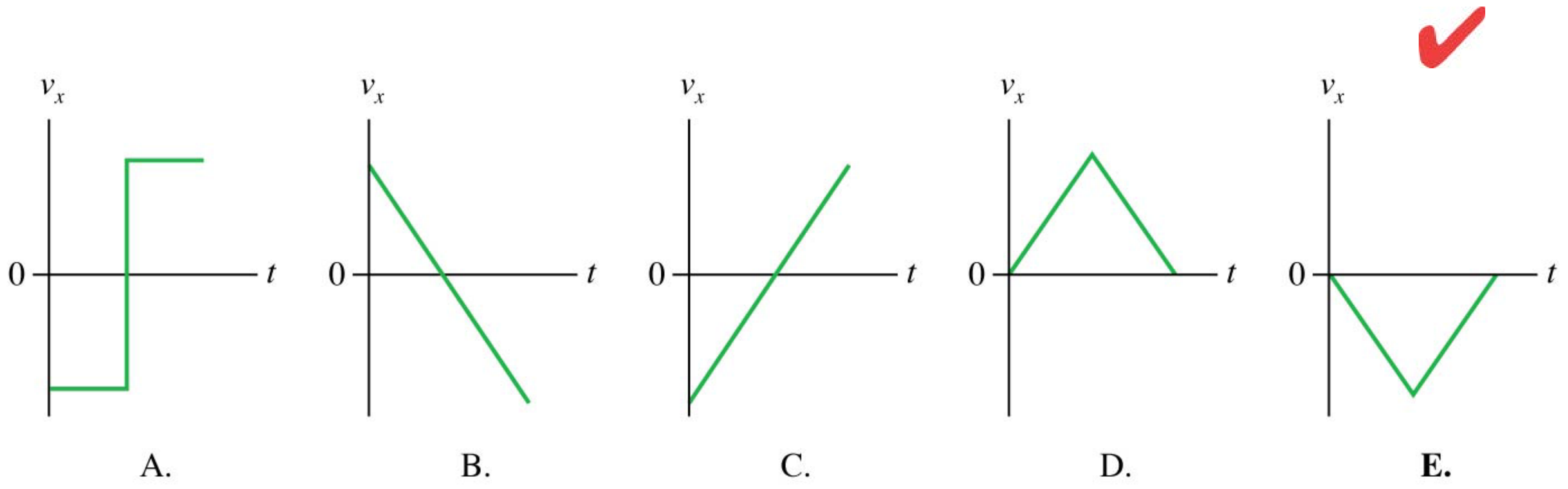




QuickCheck 2.17

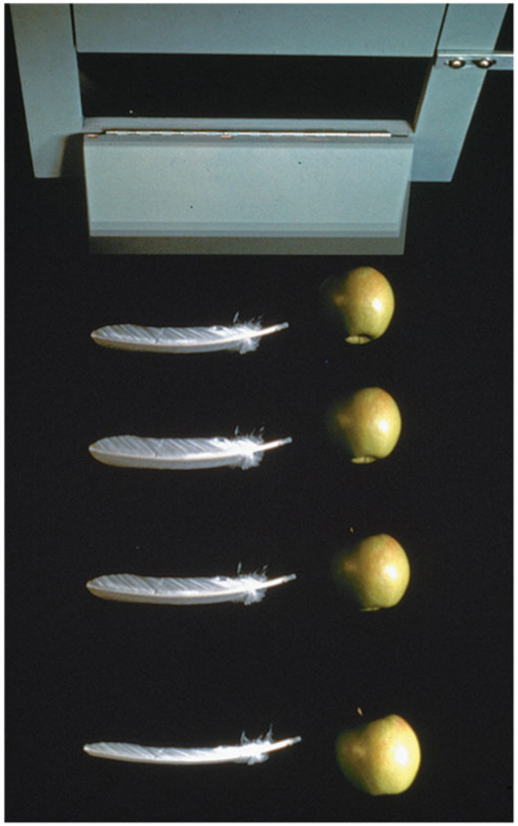
Which velocity-versus-time graph goes with this acceleration graph?





Free Fall

- The motion of an object moving under the influence of gravity only, and no other forces, is called **free fall**.
- Two objects dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed.
- Consequently, any two objects in free fall, regardless of their mass, have the same acceleration:



In a vacuum, the apple and feather fall at the same rate and hit the ground at the same time.

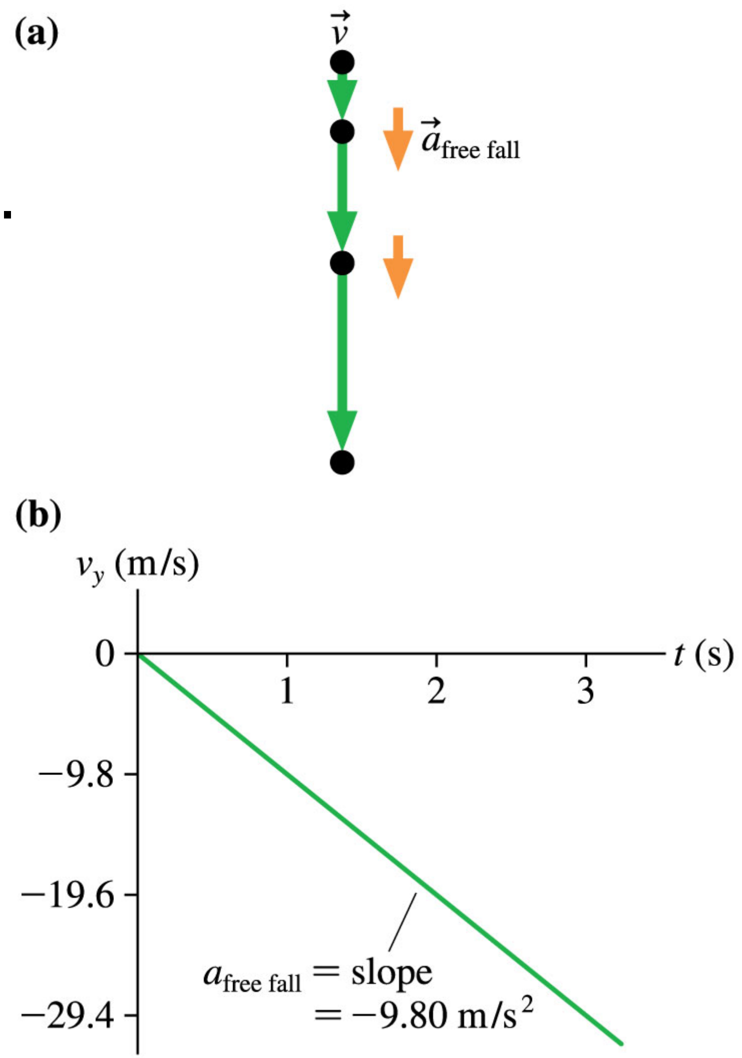
$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{vertically downward})$$

Free Fall

- Figure (a) shows the motion diagram of an object that was released from rest and falls freely.
- Figure (b) shows the object's velocity graph.
- The velocity graph is a straight line with a slope:

$$a_y = a_{\text{free fall}} = -g$$

- Where g is a positive number which is equal to $9.80\ \text{m/s}^2$ on the surface of the earth.
- Other planets have different values of g.



QuickCheck 2.18

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration a_y is

A. Positive.
B. Negative.
C. Zero.

QuickCheck 2.18

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration a_y is

A. Positive.

✔ **B. Negative.**

C. Zero.

Example 2.13 Finding the Height of a Leap

EXAMPLE 2.13 | **Finding the height of a leap**

The springbok, an antelope found in Africa, gets its name from its remarkable jumping ability. When startled, a springbok will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 m/s^2 for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air. How high does it go?

Example 2.13 Finding the Height of a Leap

EXAMPLE 2.13 | **Finding the height of a leap**

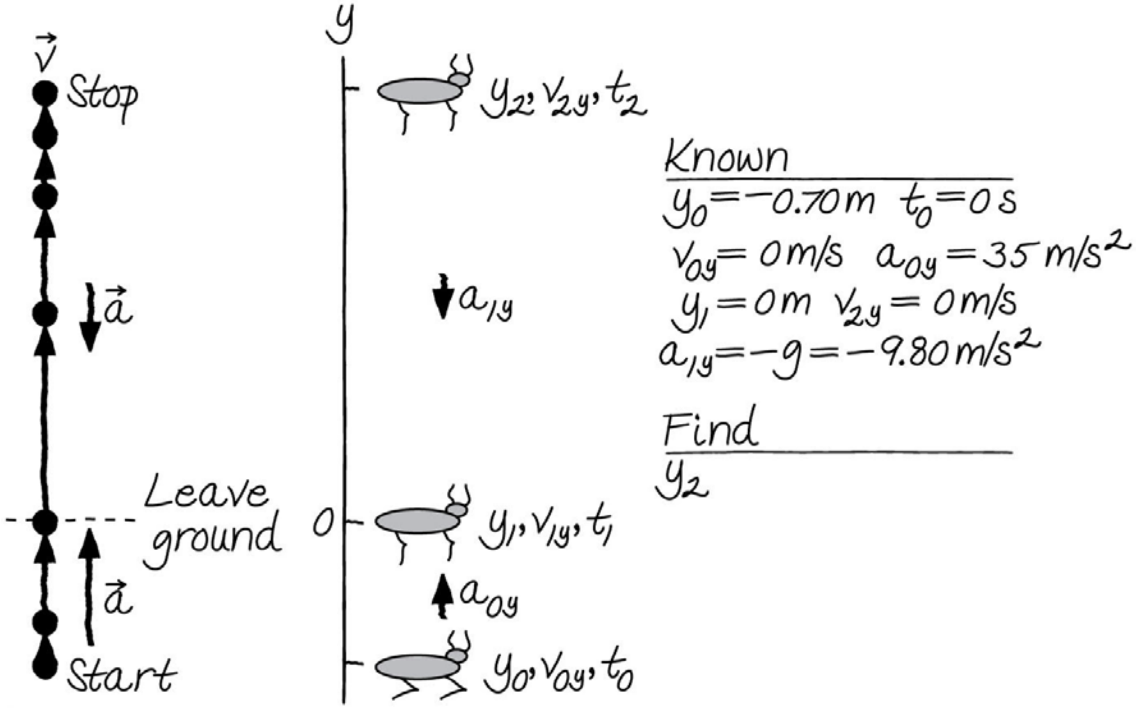
MODEL The springbok is changing shape as it leaps, so can we reasonably model it as a particle? We can if we focus on the *body* of the springbok, treating the expanding legs like external springs. Initially, the body of the springbok is driven upward by its legs. We'll model this as a particle—the body—undergoing constant acceleration. Once the springbok's feet leave the ground, we'll model the motion of the springbok's body as a particle in free fall.

Example 2.13 Finding the Height of a Leap

EXAMPLE 2.13 **Finding the height of a leap**

VISUALIZE **FIGURE 2.27** shows the pictorial representation. This is a problem with a beginning point, an end point, and a point in between where the nature of the motion changes. We've identified these points with subscripts 0, 1, and 2. The motion from 0 to 1 is a rapid upward acceleration until the springbok's feet leave the ground at 1. Even though the springbok is moving upward from 1 to 2, this is free-fall motion because the springbok is now moving under the influence of gravity only.

How do we put "How high?" into symbols? The clue is that the very top point of the trajectory is a *turning point,* and we've seen that the instantaneous velocity at a turning point is $v_{2y} = 0$. This was not explicitly stated but is part of our interpretation of the problem.



Example 2.13 Finding the Height of a Leap

EXAMPLE 2.13 | **Finding the height of a leap**

SOLVE For the first part of the motion, pushing off, we know a displacement but not a time interval. We can use

$$v_{1y}^2 = v_{0y}^2 + 2a_{0y}\,\Delta y = 2(35\text{ m/s}^2)(0.70\text{ m}) = 49\text{ m}^2/\text{s}^2$$

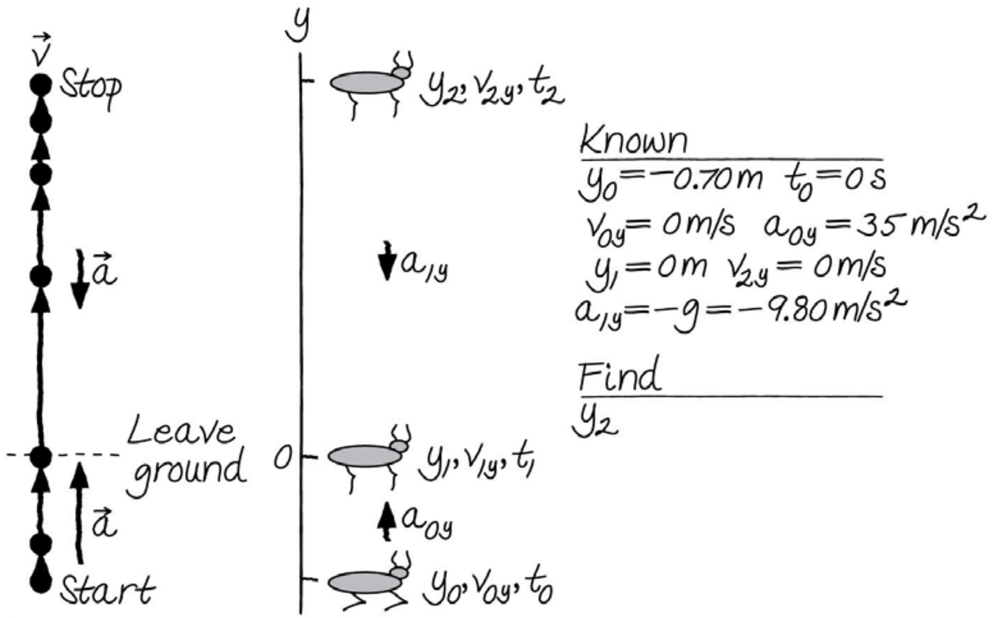
$$v_{1y} = \sqrt{49\text{ m}^2/\text{s}^2} = 7.0\text{ m/s}$$

The springbok leaves the ground with a velocity of 7.0 m/s. This is the starting point for the problem of a projectile launched straight up from the ground. One possible solution is to use the velocity equation to find how long it takes to reach maximum height, then the position equation to calculate the maximum height. But that takes two separate calculations. It is easier to make another use of the velocity-displacement equation:

$$v_{2y}^2 = 0 = v_{1y}^2 + 2a_{1y}\,\Delta y = v_{1y}^2 - 2g(y_2 - y_1)$$

where now the acceleration is $a_{1y} = -g$. Using $y_1 = 0$, we can solve for y_2, the height of the leap:

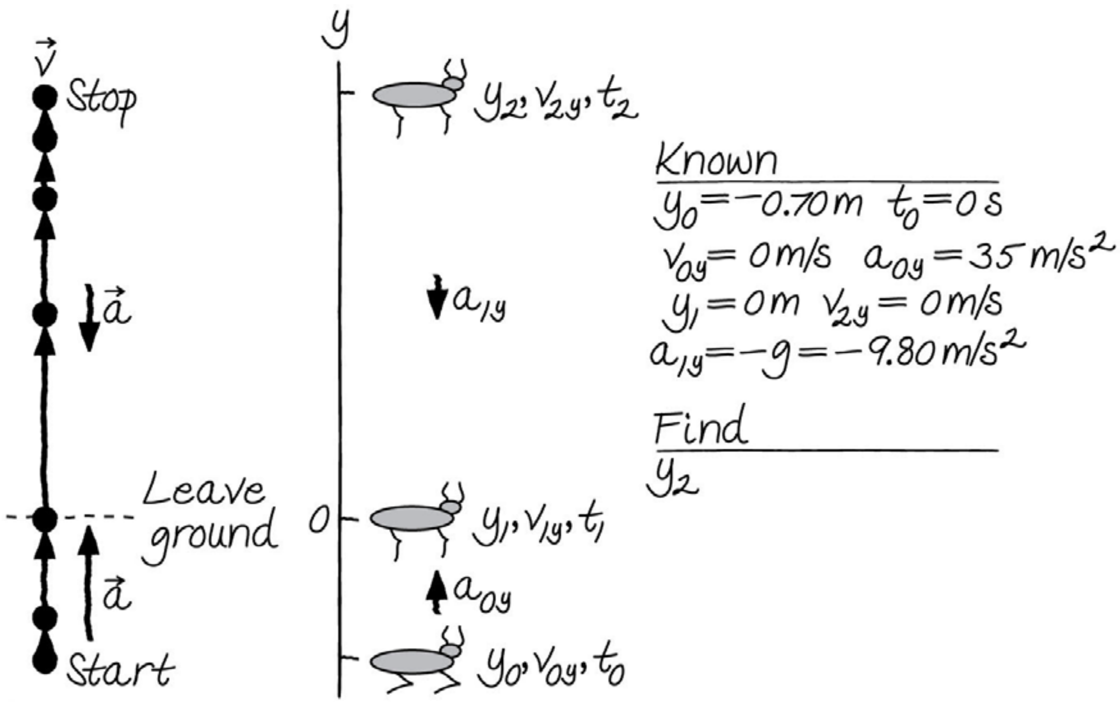
$$y_2 = \frac{v_{1y}^2}{2g} = \frac{(7.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 2.5\text{ m}$$



Example 2.13 Finding the Height of a Leap

EXAMPLE 2.13 | **Finding the height of a leap**

ASSESS 2.5 m is a bit over 8 feet, a remarkable vertical jump. But these animals are known for their jumping ability, so the answer seems reasonable. Note that it is especially important in a multipart problem like this to use numerical subscripts to distinguish different points in the motion.

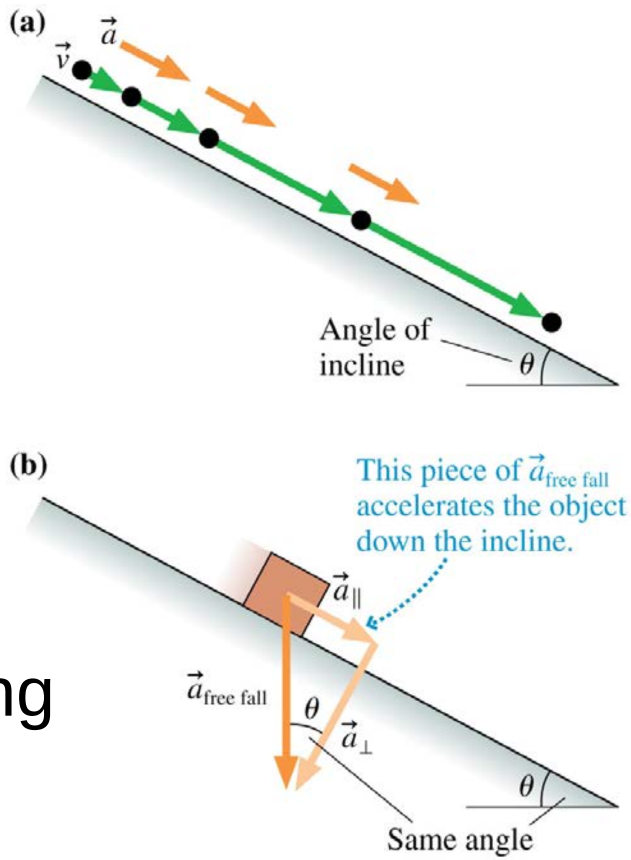


Motion on an Inclined Plane

- Figure (a) shows the motion diagram of an object sliding down a straight, frictionless inclined plane.
- Figure (b) shows the free-fall acceleration $\vec{a}_{\text{free fall}}$ the object would have if the incline suddenly vanished.
- This vector can be broken into two pieces: $\vec{a}_{||}$ and $\vec{a}_{\perp}$.
- The surface somehow "blocks" $\vec{a}_{\perp}$, so the one-dimensional acceleration along the incline is

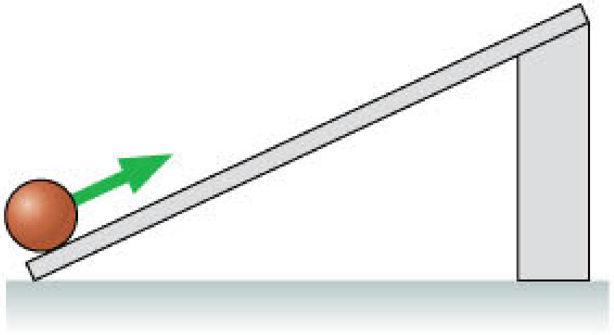
$$a_s = \pm g \sin\theta$$

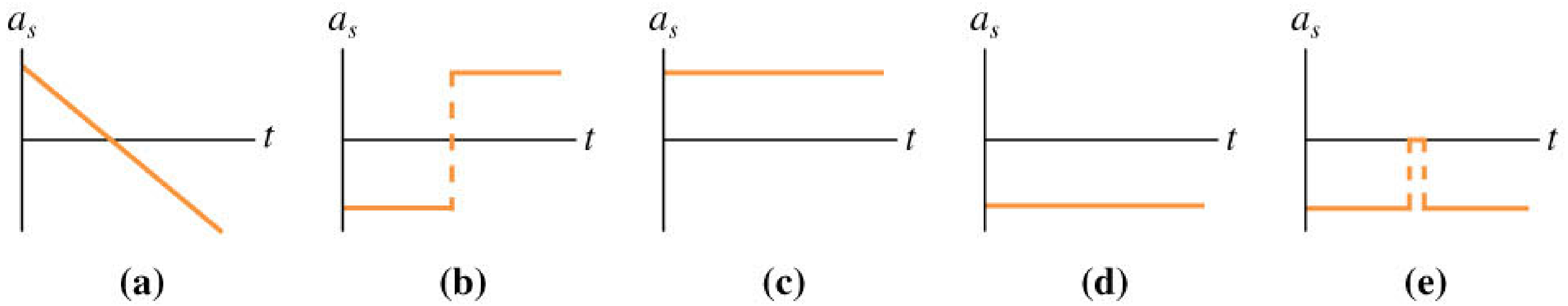
- The correct sign depends on the direction the ramp is tilted.



QuickCheck 2.19

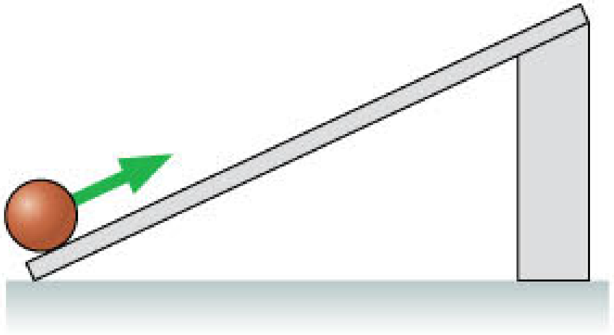
The ball rolls up the ramp, then back down. Which is the correct acceleration graph?

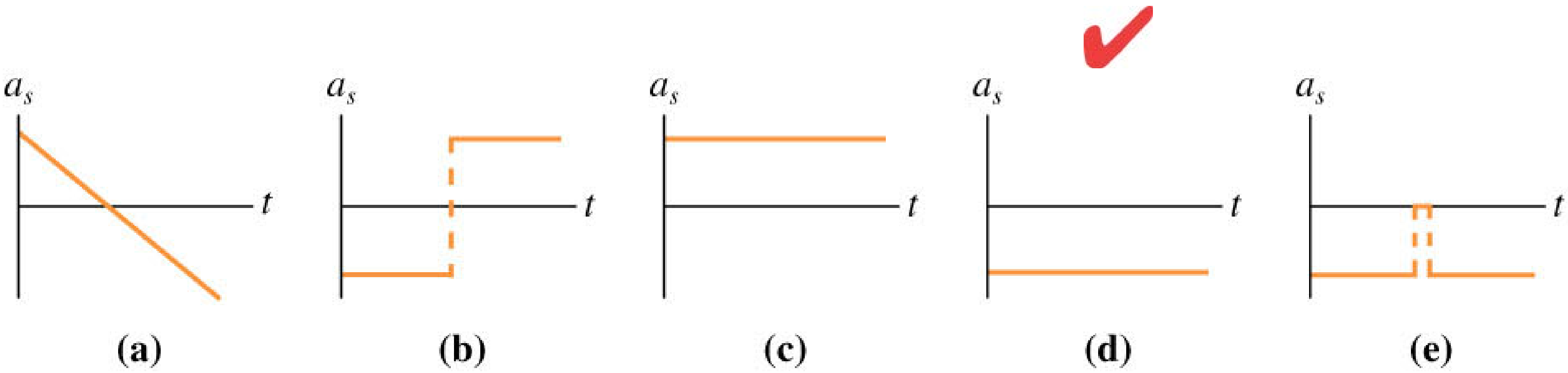




QuickCheck 2.19

The ball rolls up the ramp, then back down. Which is the correct acceleration graph?





Thinking Graphically

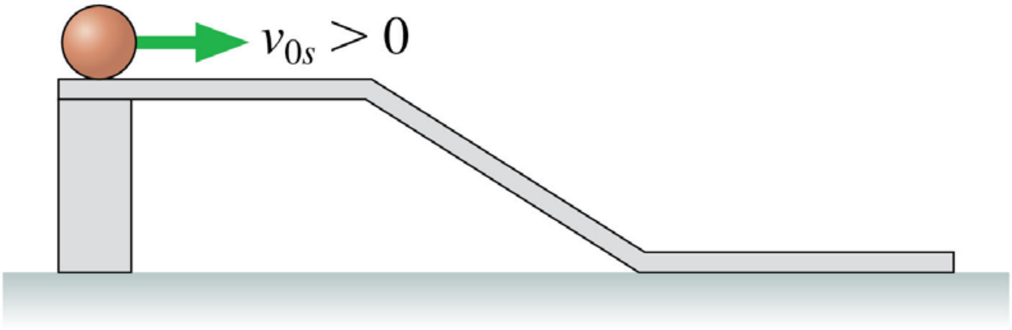
- Consider the problem of a hard, smooth ball rolling on a smooth track made up of several straight segments connected together.
- Your task is to analyze the ball's motion graphically.
- There are a small number of rules to follow:

1. Assume that the ball passes smoothly from one segment of the track to the next, with no abrupt change of speed and without ever leaving the track.
2. The graphs have no numbers, but they should show the correct relationships. For example, the position graph should be steeper in regions of higher speed.
3. The position s is the position measured along the track. Similarly, v_s and a_s are the velocity and acceleration parallel to the track.

Example 2.15 From Track to Graphs

EXAMPLE 2.15 | **From track to graphs**

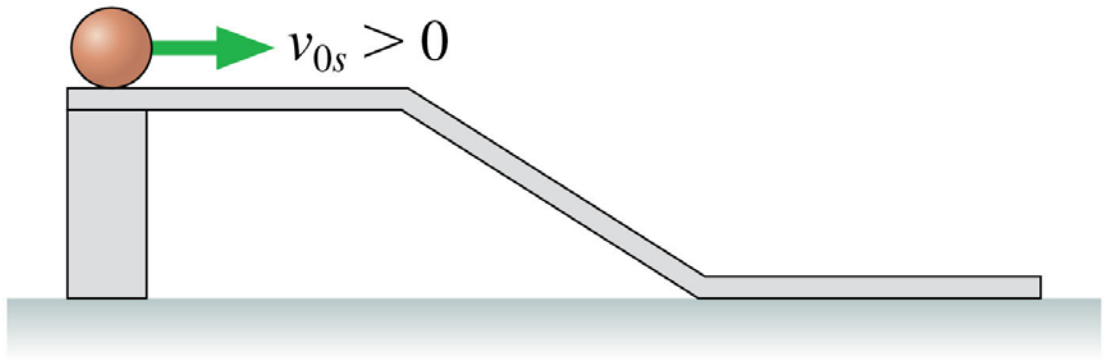
Draw position, velocity, and acceleration graphs for the ball on the smooth track of FIGURE 2.32.

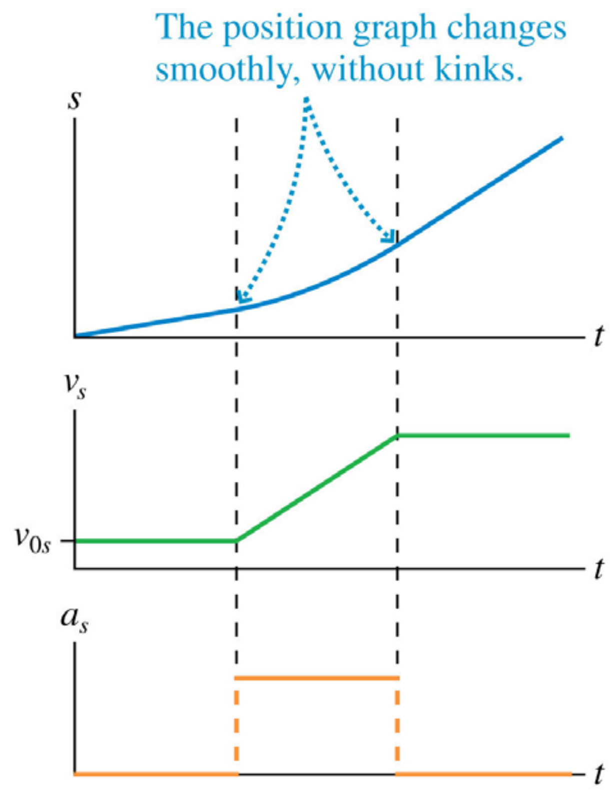


Example 2.15 From Track to Graphs

EXAMPLE 2.15 | **From track to graphs**

VISUALIZE It is often easiest to begin with the velocity. There is no acceleration on the horizontal surface ($a_s = 0$ if $\theta = 0°$), so the velocity remains constant at v_{0s} until the ball reaches the slope. The slope is an inclined plane where the ball has constant acceleration. The velocity increases linearly with time during constant-acceleration motion. The ball returns to constant-velocity motion after reaching the bottom horizontal segment. The middle graph of **FIGURE 2.33** shows the velocity.

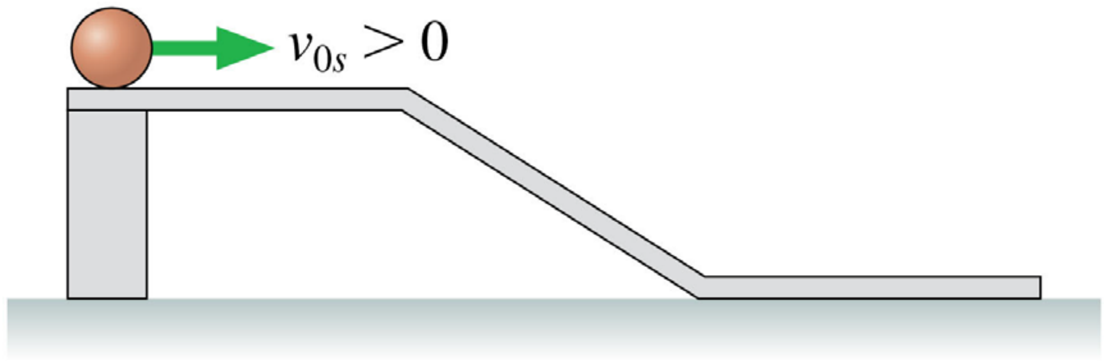


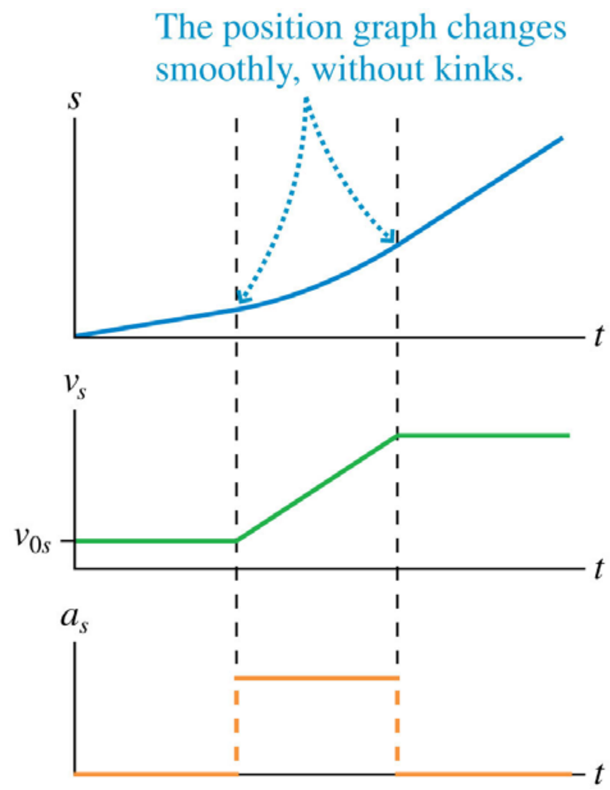


Example 2.15 From Track to Graphs

EXAMPLE 2.15 | **From track to graphs**

VISUALIZE We can easily draw the acceleration graph. The acceleration is zero while the ball is on the horizontal segments and has a constant positive value on the slope. These accelerations are consistent with the slope of the velocity graph: zero slope, then positive slope, then a return to zero. The acceleration cannot *really* change instantly from zero to a nonzero value, but the change can be so quick that we do not see it on the time scale of the graph. That is what the vertical dotted lines imply.

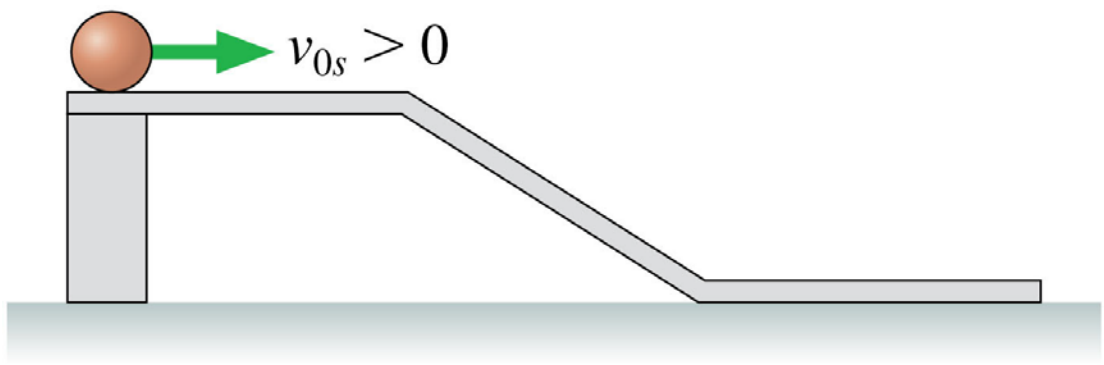


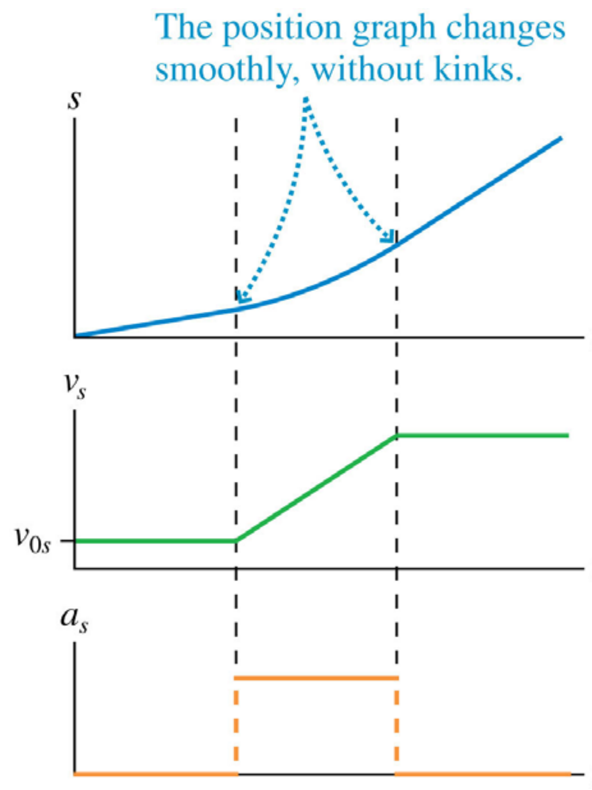


Example 2.15 From Track to Graphs

EXAMPLE 2.15 | **From track to graphs**

VISUALIZE Finally, we need to find the position-versus-time graph. The position increases linearly with time during the first segment at constant velocity. It also does so during the third segment of motion, but with a steeper slope to indicate a faster velocity. In between, while the acceleration is nonzero but constant, the position graph has a *parabolic* shape. Notice that the parabolic section blends *smoothly* into the straight lines on either side. An abrupt change of slope (a "kink") would indicate an abrupt change in velocity and would violate rule 1.

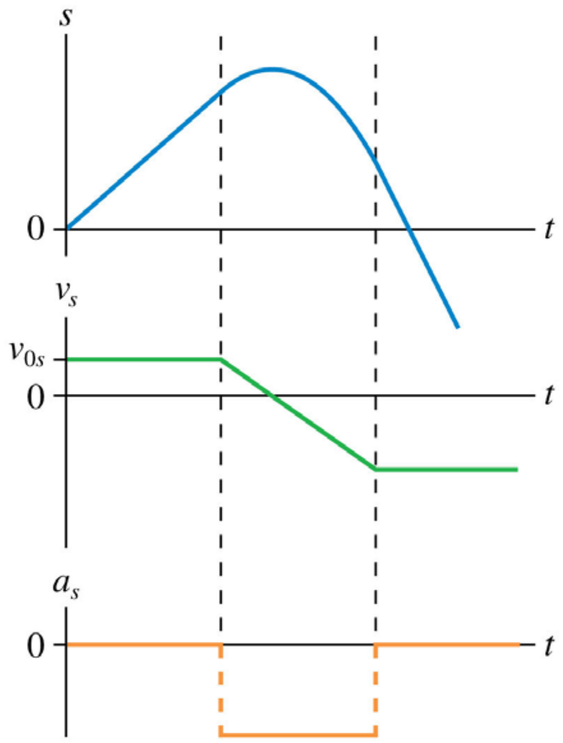




Example 2.16 From Graphs to Track

EXAMPLE 2.16 | **From graphs to track**

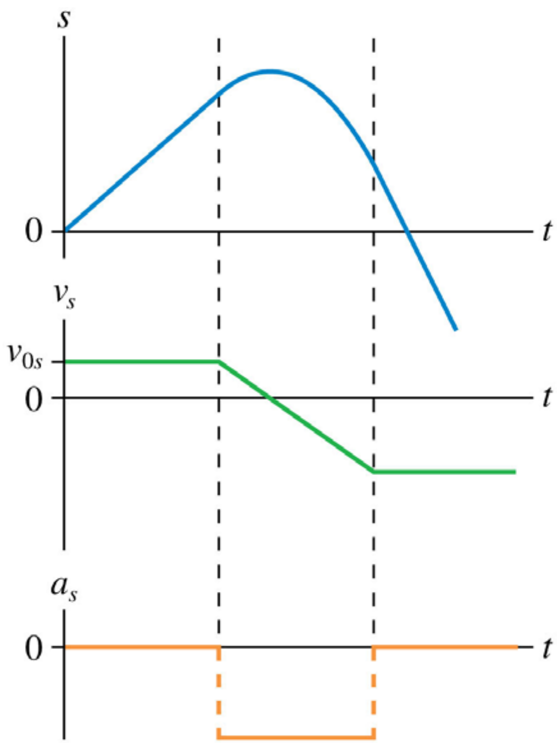
FIGURE 2.34 shows a set of motion graphs for a ball moving on a track. Draw a picture of the track and describe the ball's initial condition. Each segment of the track is *straight*, but the segments may be tilted.

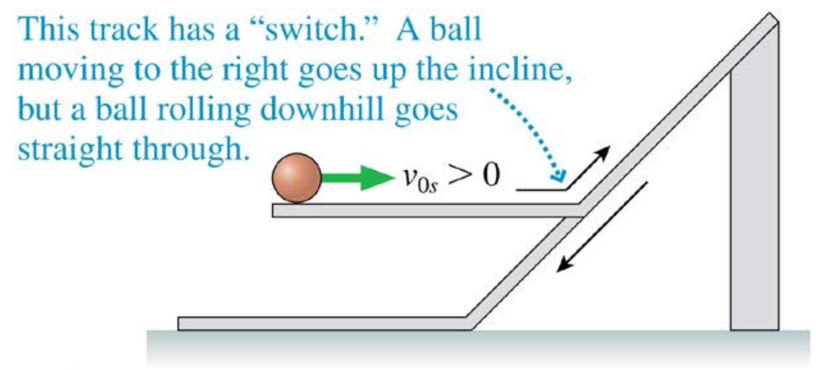


Example 2.16 From Graphs to Track

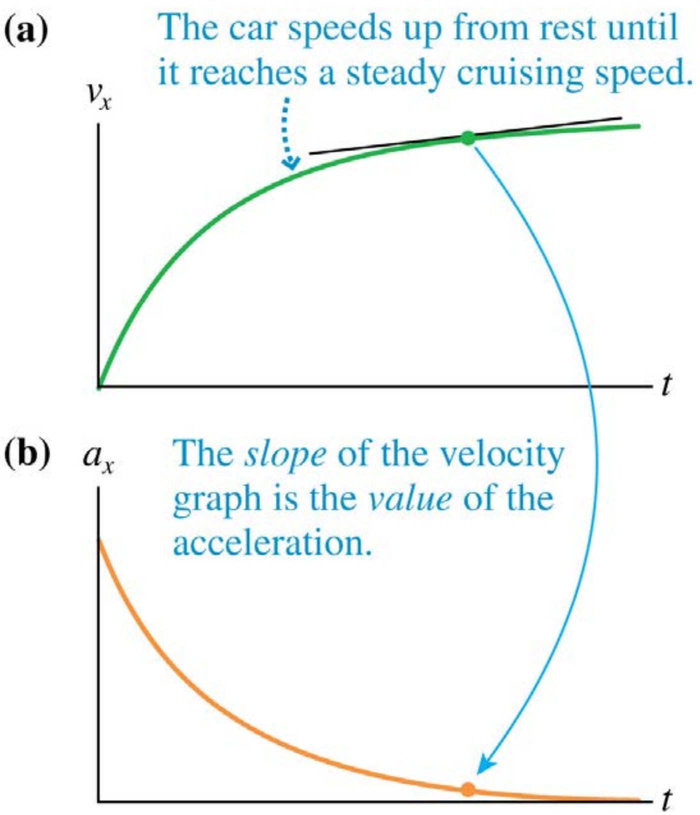
EXAMPLE 2.16 | **From graphs to track**

VISUALIZE The ball starts with initial velocity $v_{0s} > 0$ and maintains this velocity for awhile; there's no acceleration. Thus the ball must start out rolling to the right on a horizontal track. At the end of the motion, the ball is again rolling on a horizontal track (no acceleration, constant velocity), but it's rolling to the *left* because v_s is negative. Further, the final speed ($|v_s|$) is greater than the initial speed. The middle section of the graph shows us what happens. The ball starts slowing with constant acceleration (rolling uphill), reaches a turning point (s is maximum, $v_s = 0$), then speeds up in the opposite direction (rolling downhill). This is still a negative acceleration because the ball is speeding up in the negative s-direction. It must roll farther downhill than it had rolled uphill before reaching a horizontal section of track. **FIGURE 2.35** shows the track and the initial conditions that are responsible for the graphs of Figure 2.34.





Advanced Topic: Instantaneous Acceleration



- Figure (a) shows a realistic velocity-versus-time graph for a car leaving a stop sign.
- The graph is not a straight line, so this is *not* motion with a constant acceleration.
- Figure (b) shows the car's acceleration graph.
- The **instantaneous acceleration** a_s is the slope of the line that is tangent to the velocity-versus-time curve at time t:

$$a_s = \frac{dv_s}{dt} = \text{slope of the velocity-versus-time graph at time } t$$

Advanced Topic: Instantaneous Acceleration

- Suppose we know an object's velocity to be v_{is} at an initial time t_i.
- We also know the acceleration as a function of time between t_i and some later time t_f.
- Even if the acceleration is not constant, we can divide the motion into N steps of length Δt in which it is approximately constant.
- In the limit $\Delta t \rightarrow 0$ we can compute the final velocity as

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s \, dt$$

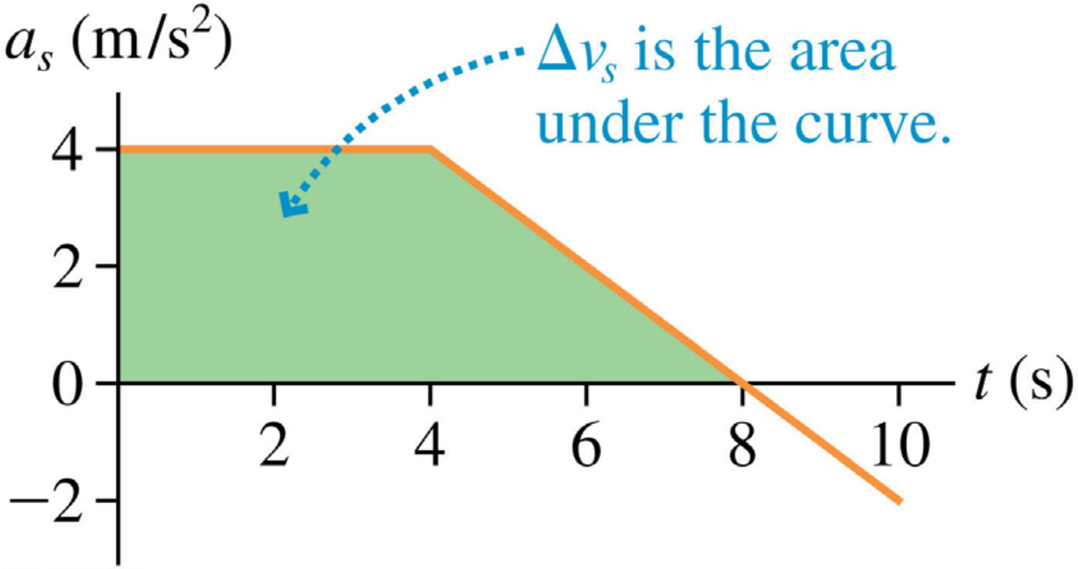
- The graphical interpretation of this equation is

$v_{fs} = v_{is}$ + area under the acceleration curve a_s between t_i and t_f

Example 2.17 Finding Velocity from Acceleration

EXAMPLE 2.17 | **Finding velocity from acceleration**

FIGURE 2.37 shows the acceleration graph for a particle with an initial velocity of 10 m/s. What is the particle's velocity at $t = 8$ s?



Example 2.17 Finding Velocity from Acceleration

EXAMPLE 2.17 | **Finding velocity from acceleration**

MODEL We're told this is the motion of a particle.

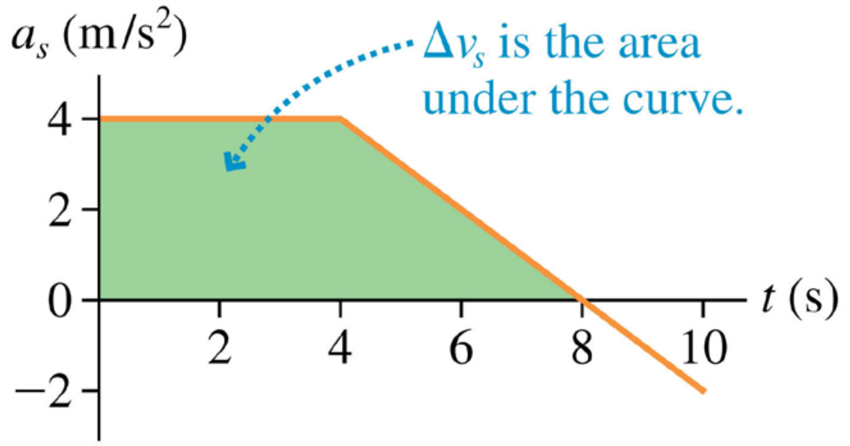
VISUALIZE Figure 2.37 is a graphical representation of the motion.

SOLVE The change in velocity is found as the area under the acceleration curve:

$$v_{fs} = v_{is} + \text{area under the acceleration curve } a_s \text{ between } t_i \text{ and } t_f$$

The area under the curve between $t_i = 0$ s and $t_f = 8$ s can be subdivided into a rectangle ($0 \text{ s} \leq t \leq 4 \text{ s}$) and a triangle ($4 \text{ s} \leq t \leq 8 \text{ s}$). These areas are easily computed. Thus

$$\begin{aligned} v_s(\text{at } t = 8 \text{ s}) &= 10 \text{ m/s} + (4 \text{ (m/s)/s})(4 \text{ s}) \\ &\quad + \tfrac{1}{2}(4 \text{ (m/s)/s})(4 \text{ s}) \\ &= 34 \text{ m/s} \end{aligned}$$



Chapter 2 Summary Slides

General Principles

Kinematics describes motion in terms of position, velocity, and acceleration.

General kinematic relationships are given **mathematically** by:

Instantaneous velocity $v_s = ds/dt =$ slope of position graph

Instantaneous acceleration $a_s = dv_s/dt =$ slope of velocity graph

Final position $s_f = s_i + \int_{t_i}^{t_f} v_s\,dt = s_i + \begin{cases}\text{area under the velocity}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Final velocity $v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\,dt = v_{is} + \begin{cases}\text{area under the acceleration}\\ \text{curve from } t_i \text{ to } t_f\end{cases}$

Solving Kinematics Problems

MODEL Uniform motion or constant acceleration.

VISUALIZE Draw a pictorial representation.

SOLVE

- Uniform motion $s_f = s_i + v_s \Delta t$
- Constant acceleration $v_{fs} = v_{is} + a_s \Delta t$

$$s_f = s_i + v_s \Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$

$$v_{fs}^{\,2} = v_{is}^{\,2} + 2 a_s \Delta s$$

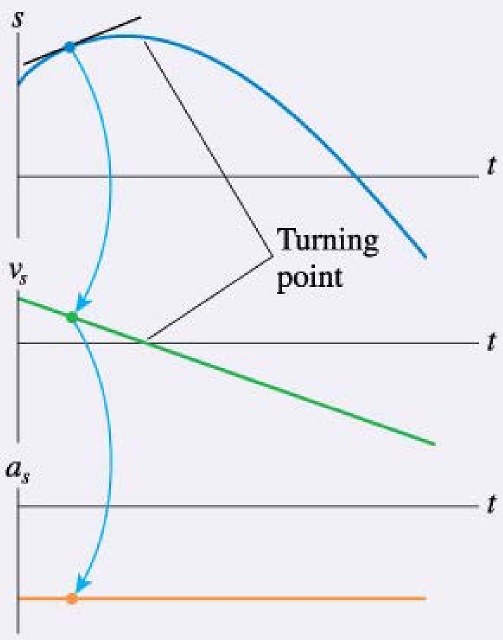
ASSESS Is the result reasonable?

Important Concepts

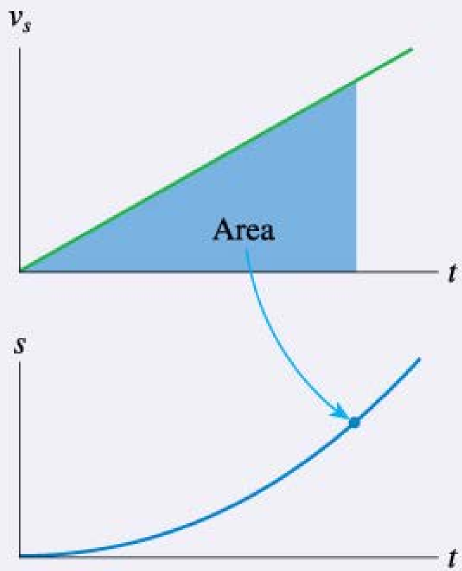
Position, velocity, and acceleration are related graphically.

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- s is a maximum or minimum at a turning point, and $v_s = 0$.

Important Concepts



- Displacement is the area under the velocity curve.



Applications

The **sign of v_s** indicates the direction of motion.

- $v_s > 0$ is motion to the right or up.
- $v_s < 0$ is motion to the left or down.

The **sign of a_s** indicates which way $\vec{a}$ points, *not* whether the object is speeding up or slowing down.

- $a_s > 0$ if $\vec{a}$ points to the right or up.
- $a_s < 0$ if $\vec{a}$ points to the left or down.
- The direction of $\vec{a}$ is found with a motion diagram.

Applications

An object is **speeding up** if and only if v_s and a_s have the same sign.

An object is **slowing down** if and only if v_s and a_s have opposite signs.

Free fall is constant-acceleration motion with

$$a_y = -g = -9.80 \text{ m/s}^2$$

Motion on an inclined plane has $a_s = \pm g \sin\theta$. The sign depends on the direction of the tilt.