APPENDIX

# Glossary of Unfamiliar Terms Used in this Work

*A posteriori stable parameters*
Parameters whose stability can only be established after the model has been fitted in terms of other parameters (Section 2.3).

*A priori stable parameters*
Parameters for which stability is expected on informal grounds before the model is fitted.

*Analogous parameters*
Parameters in different models which have the same practical interpretation, such as asymptotes of curves (Section 2.5).

*Analysis of deviance*
Generalization of analysis of variance which uses the deviances obtained by fitting hierarchically related models with different numbers of parameters (Section 1.2.1).

*Calibration model*
A type of parallel model in which the same response function is expected for each set of data, but the position in each set relative to the first is to be estimated (Section 1.1.2).

*Common estimate locus*
The locus in data space of all data sets for which a given parameter value is a maximum likelihood estimate (MLE) (Section 4.3.1).

*Computing parameters*
Parameters chosen for efficient computation of maximum likelihood estimates (MLEs) (Section 2.1).

*Convergence locus*
The locus in parameter space of all initial estimates which converge to the solution in a given number of iterations, given the estimation procedure and the convergence criterion (Section 4.4.8).

*Defining parameters*
Parameters that define a model in standard algebraic terms (Section 2.1).

*Design (nonlinear)*
Controllable choice of observations, such as timing and frequency of making a measurement (Section 3.3).

*Deviance*
Measure of discrepancy between data and model based on the likelihood ratio test statistic (Section 1.2.1).

*Deviance residual*
Residual computed from the square root of the contribution to the deviance attributed to any given observation, taking the same sign as the simple residual (Section 2.4).

*Discrepancy plot*
Plot of difference between a general function and an approximation to the function (Section 4.4.2).

*Effective design matrix*
Matrix of first derivatives of expectations with respect to parameters, being the analogue of the design matrix in a linear model (Section 3.3.1).

*Effective replication*
The number of observations (not necessarily an integer) required to estimate a particular expected value with the same precision as the fitted value from the given model and data (Section 3.3).

*Function loadings*
The relative effects of each observation in a data set in estimating a function of parameters (Section 4.2).

*Function locus*
The locus in data space of functions of parameters: a subspace of the solution locus (q.v.) (Section 4.3.1).

*Hat matrix*
In a linear model, the matrix $H$ such that $\hat{y} = Hy$, the matrix that puts on the "hat." In a nonlinear model, an approximation may be computed using the effective design matrix (q.v.) (Section 3.3).

*Interpretable parameters*
Parameters that relate directly to quantities of interest in the model and the data (Section 2.1).

*Intrinsic nonlinearity*
Nonlinearity in the model that cannot be removed by transforming parameters, explained in terms of curvature of the solution locus (Section 2.4.4).

*Likelihood-based confidence regions*
Regions of parameter space bounded by contours of equal likelihood. Given a particular contour, corresponding to a likelihood ratio test level of significance, upper and lower limits for particular parameters or functions of parameters may be found numerically (Section 1.2.2).

*Linear parameters*
Parameters $\beta_k$ in models of the form $E(y) = \sum \beta_k f_k$ where $f_k$ may be a constant, an independent variable, or a function of nonlinear parameters. When the error distribution is normal, Poisson, or gamma some linear parameters may be estimated directly in terms of functions of nonlinear parameters. Such parameters are called *separable* (Section 4.4.3).

*Parallel models*
By analogy with parallel lines, models fitted to several sets of data with some parameters common to all sets, the rest specific for each set (Section 1.1.2).

*Parameter-effects curvature*
Nonlinearity due to choice of parameter system, additional to intrinsic nonlinearity (q.v.) (Section 2.4.4.)

*Parameter loadings*
The relative effects of particular observations on the estimates of each parameter (Section 3.3.2).

*Parameter locus*
The locus in parameter space of all parameter sets predicting a given fitted value (Section 4.4.5).

*Pseudomodel*
A model introduced to extend the range of expectations so that data may be fitted and likelihood contours evaluated. Parameters in pseudomodels may not be interpretable in the terms of the original model, but in combination with estimates from other data sets may lead to acceptable inferences (Section 3.1.4).

*Self-estimating observations*
Observations with effective replication close to unity, so that their expectations derive almost no information from other observations via the model (Section 3.3.1).

*Separable linear parameters*, See *Linear parameters*

*Similar models*
Models expressible in terms of *analogous parameters* (q.v.), such as curves of the same general shape over a range of data values (Section 2.5).

*Solution locus*
The locus in data space of expectations generated by different values of the parameters: the set of data values exactly fitted by the model. Also called the *expectation surface* (Section 4.3.1).

*Stable ordinates*
In curve fitting, sets of points at which the expectations are uncorrelated, or nearly so (Section 2.3).

*Stable parameters*
Parameters not greatly affected by uncertainties in the values of other parameters (Section 2.1.3).

*Statistic locus*
In data space the locus of all data values sharing the same value of some statistic such as the mean (Section 4.3.2).

*Working constant*
A constant estimated *a priori* to relate a general class of transformations to the particular locations, scales or other properties of data sets (Section 2.2).

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