6-1

1. Plan

Objectives
1 To define and classify special types of quadrilaterals

Examples
1 Classifying a Quadrilateral
2 Classifying Coordinate Methods
3 Using the Properties of Special Quadrilaterals

Professional Development

Math Background

The classification in this lesson categorizes quadrilaterals first by the number of pairs of parallel sides, and then shows their subsets. Other hierarchies are possible.

More Math Background: p. 304C

Lesson Planning and Resources

See p. 304E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Finding Distance on the Coordinate Plane
Lesson 1-8: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 1

Slope
Algebra Review, p. 165: Example 1

Classifying Quadrilaterals

What You'll Learn
- To define and classify special types of quadrilaterals

. . . And Why
To use the properties of special quadrilaterals with a kite, as in Example 3

Check Skills You'll Need GO for Help Lesson 1-8 and page 165

Find the distance between the points to the nearest tenth.

1. $M(2, -5), N(-7, 1)$ 10.8
2. $P(-1, -3), Q(-6, -9)$ 7.8
3. $C(-4, 6), D(5, -3)$ 12.7

Find the slope of the line through each pair of points.

4. $X(0, 6), Y(4, 9)$ $\frac{3}{4}$
5. $R(3, 8), S(6, 0)$ $-\frac{8}{3}$
6. $A(4, 3), B(2, 1)$ 1

New Vocabulary • parallelogram • rhombus • rectangle • square • kite • trapezoid • isosceles trapezoid

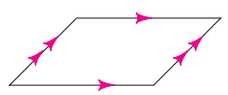
1 Classifying Special Quadrilaterals

Seven important types of quadrilaterals are defined below.

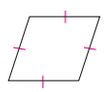
Key Concepts

Definitions **Special Quadrilaterals**

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

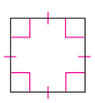


A **rhombus** is a parallelogram with four congruent sides.

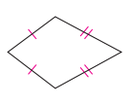


A **rectangle** is a parallelogram with four right angles.

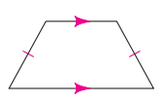
A **square** is a parallelogram with four congruent sides and four right angles.



A **kite** is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The **isosceles trapezoid** at the right is a trapezoid whose nonparallel opposite sides are congruent.



Real-World Connection

A "kite" is not the only special quadrilateral used to make a kite!

Differentiated Instruction Solutions for All Learners

Special Needs L1
Students may assume that all quadrilaterals can be classified by one of the special names. Draw examples of quadrilaterals that cannot be given any more specific name than quadrilateral.

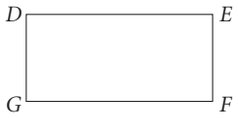
learning style: visual

Below Level L2
Students can use geoboards to model the quadrilaterals in this lesson.

learning style: tactile

1 EXAMPLE Classifying a Quadrilateral

Judging by appearance, classify $DEFG$ in as many ways as possible.



$DEFG$ is a quadrilateral because it has four sides.

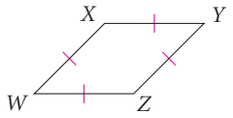
It is a parallelogram because both pairs of opposite sides are parallel.

It is a rectangle because it has four right angles.

Quick Check

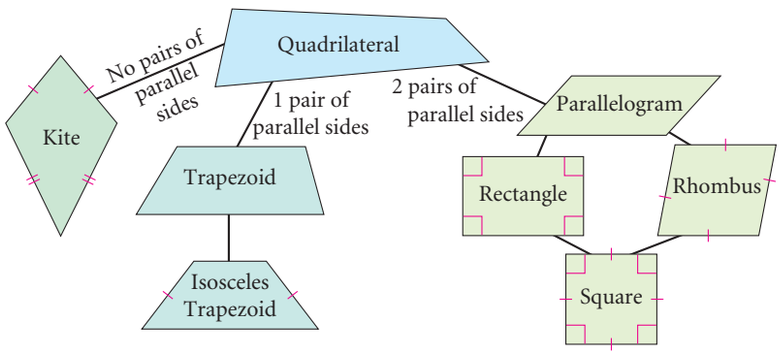
1 **a.** Judging by appearance, classify $WXYZ$ at the right in as many ways as possible. **a–b. See below left.**

b. Critical Thinking Which name gives the most information about $WXYZ$? Explain.



For: Quadrilateral Activity
Use: Interactive Textbook, 6-1

The diagram below shows the relationships among special quadrilaterals.



You can use what you know about slope and distance to classify a quadrilateral.

2 EXAMPLE Classifying by Coordinate Methods

Coordinate Geometry Determine the most precise name for quadrilateral $LMNP$.

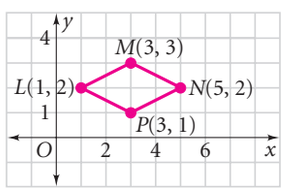
Step 1 Find the slope of each side.

$$\text{slope of } \overline{LM} = \frac{3-2}{3-1} = \frac{1}{2}$$

$$\text{slope of } \overline{NP} = \frac{2-1}{5-3} = \frac{1}{2}$$

$$\text{slope of } \overline{MN} = \frac{3-2}{3-5} = -\frac{1}{2}$$

$$\text{slope of } \overline{LP} = \frac{2-1}{1-3} = -\frac{1}{2}$$



Both pairs of opposite sides are parallel, so $LMNP$ is a parallelogram. No sides are perpendicular, so $LMNP$ is not a rectangle.

Step 2 Use the Distance Formula to see if any pairs of sides are congruent.

$$LM = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{5} \qquad MN = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{5}$$

$$NP = \sqrt{(5-3)^2 + (2-1)^2} = \sqrt{5} \qquad LP = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{5}$$

All sides are congruent, so $LMNP$ is a rhombus.

Vocabulary Tip

Although *LMNP* is a parallelogram, rhombus is the more *precise* name because it gives more information about the quadrilateral.

Quick Check

2 Determine the most precise name for quadrilateral $ABCD$ with vertices $A(-3, 3)$, $B(2, 4)$, $C(3, -1)$, and $D(-2, -2)$. **square**

1a. ***WXYZ*** **is a quad. because it has 4 sides; it is a ▱ because both pairs of opp. sides are ∥; it is a rhombus because all 4 sides are ≅.**

b. rhombus, because that means it is a ▱ and quad. with 4 sides that are ≅

2. Teach

Guided Instruction

1 EXAMPLE Tactile Learners

Use geoboards to model quadrilaterals.

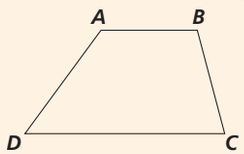
2 EXAMPLE Connection to Coordinate Geometry

If necessary, display the formulas for slope and distance.

PowerPoint

Additional Examples

1 Judging by appearance, classify $ABCD$ in as many ways as possible.



quadrilateral, trapezoid

2 Determine the most precise name for the quadrilateral with vertices $Q(-4, 4)$, $B(-2, 9)$, $H(8, 9)$, and $A(10, 4)$. **isosceles trapezoid**

3 In parallelogram $RSTU$, $m\angle R = 2x - 10$ and $m\angle S = 3x + 50$. Find x. **28**

Resources

- Daily Notetaking Guide 6-1 L3
- Daily Notetaking Guide 6-1—Adapted Instruction L1

Closure

$ABCD$ is a square. Which classifications from this lesson also apply? Which do *not* apply? **parallelogram, rectangle, rhombus; trapezoid, isosceles trapezoid, kite**

Advanced Learners L4
Have students explain why the word *exactly* is necessary in the definition of a trapezoid.

learning style: verbal

English Language Learners ELL
Have students use magazines and newspapers to find real-world examples for each special quadrilateral. Have students present their examples speaking each name with correct pronunciation.

learning style: tactile

3. Practice

Assignment Guide

1 A B	1-55
C Challenge	56-59
Test Prep	60-64
Mixed Review	65-74

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 13, 21, 25, 35, 44.

Auditory Learners

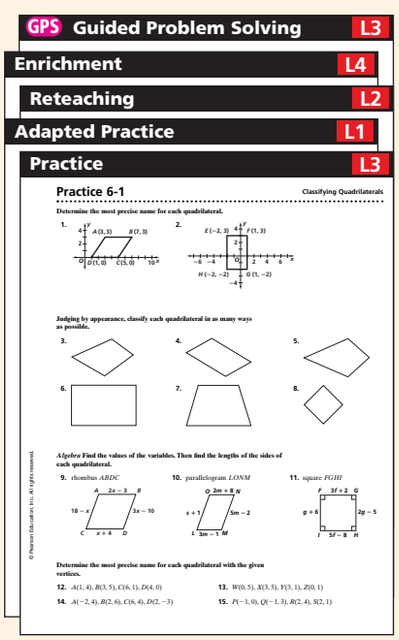
Exercises 1–6 Have students work with partners to discuss the appearance of each quadrilateral to help reinforce the classifications and establish visual-verbal cues.

Connection to Coordinate Geometry

Exercise 16 Remind students that lines are perpendicular if the product of their slopes is -1.

Exercise 25 Before students begin, ask: *What is the relationship between* $\angle F$ *and* $\angle G$? **They are supplementary.**

Differentiated Instruction Resources


GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3

Practice 6-1 Classifying Quadrilaterals

Determine the most precise name for each quadrilateral.

Judging by appearance, classify each quadrilateral in as many ways as possible.

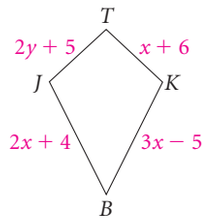
Algebra Find the values of the variables. Then find the lengths of the sides of each quadrilateral.

Determine the most precise name for each quadrilateral with the given vertices.

You can use the definitions of special quadrilaterals and algebra to find lengths of sides.

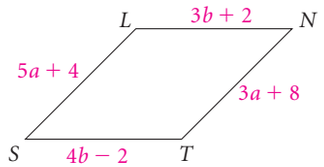
3 EXAMPLE Using the Properties of Special Quadrilaterals

Algebra Find the values of the variables for the kite.



$KB = JB$	Definition of kite
$3x - 5 = 2x + 4$	Substitute.
$x - 5 = 4$	Subtract $2x$ from each side.
$x = 9$	Add 5 to each side.
$KT = x + 6 = 15$	Substitute 9 for x.
$KT = JT$	Definition of kite
$15 = 2y + 5$	Substitute.
$10 = 2y$	Subtract 5 from each side.
$5 = y$	Divide each side by 2.

Quick Check 3 Find the values of the variables for the rhombus. Then find the lengths of the sides.
***a* = 2, *b* = 4; *LN* = *ST* = *NT* = *SL* = 14**



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

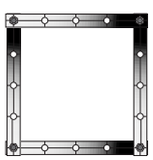
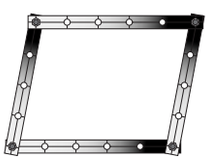
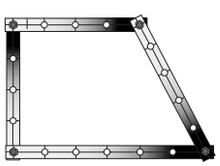
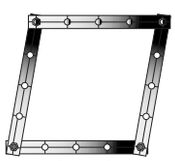
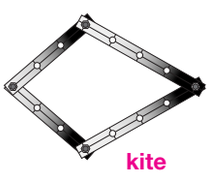
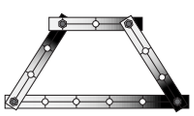
Practice and Problem Solving

A Practice by Example

Example 1 (page 307)

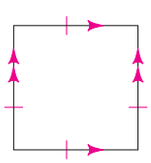
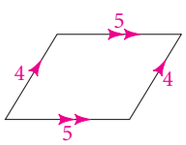
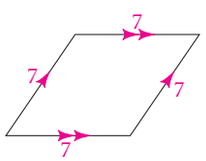
GO for Help

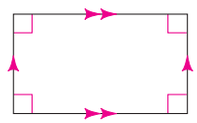
These quadrilaterals are made from a toy building set. Judging by appearance, classify each quadrilateral in as many ways as possible.

1.  ▱, rectangle, rhombus, square
2.  parallelogram
3.  trapezoid
4.  ▱, rhombus
5.  kite
6.  trapezoid, isosc. trapezoid

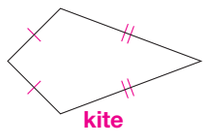
Example 2 (page 307)

Determine the most precise name for each quadrilateral.

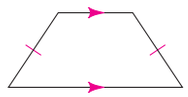
7.  rhombus
8.  parallelogram
9.  rhombus

10. 

rectangle

11. 

kite

12. 

isosc. trapezoid

Coordinate Geometry **Graph and label each quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral.** 13–18. See back of book.

13. $A(3,5), B(7,6), C(6,2), D(2,1)$

14. $W(-1,1), X(0,2), Y(1,1), Z(0,-2)$

15. $J(2,1), K(5,4), L(7,2), M(2,-3)$

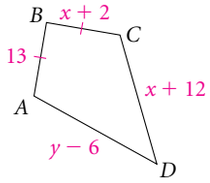
16. $R(-2,-3), S(4,0), T(3,2), V(-3,-1)$

17. $N(-6,-4), P(-3,1), Q(0,2), R(-3,5)$

18. $E(-3,1), F(-7,-3), G(6,-3), H(2,1)$

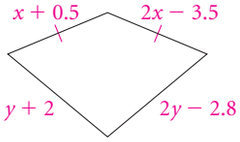
Example 3 (page 308) **Algebra** **Find the values of the variables. Then find the lengths of the sides.**

19. kite



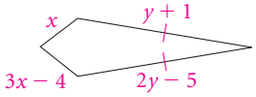
19. $x = 11, y = 29$; 13, 13, 23, 23

20. kite



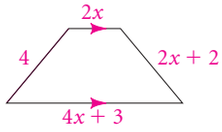
$x = 4, y = 4.8$; 4.5, 4.5, 6.8, 6.8

21. kite



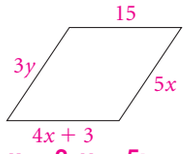
$x = 2, y = 6$; 2, 7, 7, 2

22. isosceles trapezoid



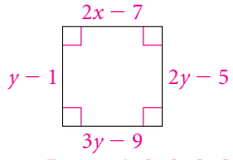
$x = 1$; 4, 2, 4, 7

23. rhombus



$x = 3, y = 5$; 15, 15, 15, 15

24. square



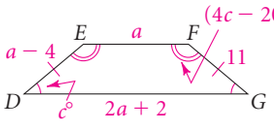
$x = 5, y = 4$; 3, 3, 3, 3

B Apply Your Skills

Algebra **In each figure, find the measures of the angles and the lengths of the sides.**

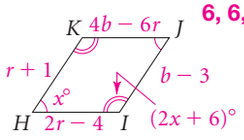
25. isosceles trapezoid $DEFG$

GPS



40, 40, 140, 140; 11, 11, 15, 32

26. rhombus $HKJI$



58, 58, 122, 122; 6, 6, 6, 6

Exercise 27

27. **Art** American artist Charles Demuth created *My Egypt*, the oil painting pictured at the left. It is in an art style called Cubism, in which subjects are made of cubes and other geometric forms. Identify the types of special quadrilaterals you see in the painting. rectangle, square, trapezoid

28. **Multiple Choice** $K(-3,0), I(0,2)$, and $T(3,0)$ are the vertices of a kite. Which point could be the fourth vertex? D

(A) $E(0,2)$ (B) $E(0,0)$ (C) $E(0,-2)$ (D) $E(0,-10)$

Draw each figure on graph paper. If not possible, explain. 29–34. See margin.

29. a parallelogram that is neither a rectangle nor a rhombus

30. an isosceles trapezoid with vertical and horizontal congruent sides

31. a trapezoid with only one right angle

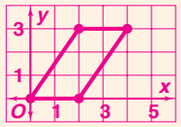
32. a trapezoid with two right angles

33. a rhombus that is not a square

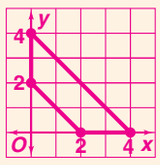
34. a kite with two right angles

29–34. Answers may vary. Samples are given.

29.

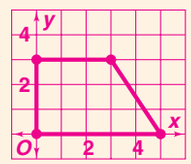


30.

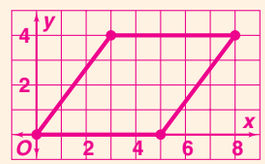


31. Impossible; a trapezoid with one rt. ∠ must have another, since two sides are ∥.

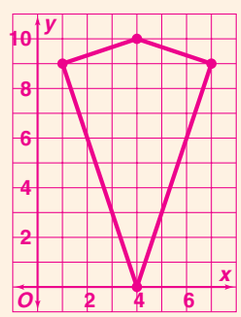
32.



33.



34.



Diversity

Exercise 27 Point out that artists have used representations of geometric figures for hundreds of years. For example, Arabian tiles show geometric shapes that tessellate. Ask students to describe other art forms that use geometric figures.

Exercise 28 Encourage students to make a sketch to aid them in finding the fourth point. Then ask: *What do you know about the missing vertex?* It is opposite *I*, but it is not (0, 2) which would form a rhombus.

Error Prevention!

Exercise 44 As students describe a kite, make sure that they include the stipulation that opposite sides are not congruent. Point out that without that condition, squares and rhombuses would be considered kites.

Tactile Learners

Exercises 50–53 For each exercise, have students cut out cardboard triangles to connect in every possible way.

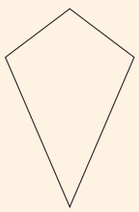
4. Assess & Reteach

PowerPoint

Lesson Quiz

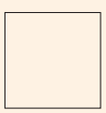
Judging by appearance, classify the quadrilaterals in Exercises 1 and 2 in as many ways as possible.

1.



quadrilateral, kite

2.

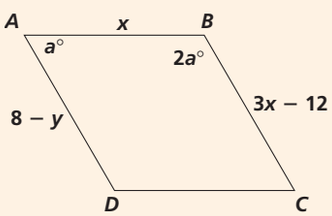


quadrilateral, parallelogram, rectangle, rhombus, square

3. What is the most precise name for the figure in Exercise 1?
kite

4. What is the most precise name for the figure in Exercise 2?
square

5. Find the values of the variables in the rhombus below.

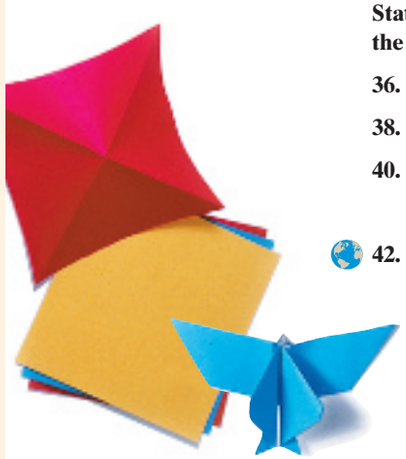


$a = 60, x = 6, y = 2$

Alternative Assessment

Have students work in pairs to identify examples in the school building of four of the special quadrilaterals in Lesson 6-1 and then justify each classification.

GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0601



Real-World Connection

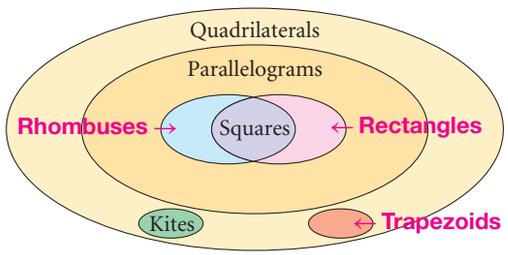
To make this butterfly, an origami square was folded first on its diagonals.

44. A rhombus has 4 ≅ sides, while a kite has 2 pairs of adj. sides ≅, but no opp. sides are ≅. Opp. sides of a rhombus are ∥, while opp. sides of a kite are not ∥.

Problem Solving Hint

In Exercises 50–53, if you flip one of the two triangles, you may find different quadrilaterals.

35. Copy the Venn diagram. Add the labels *Rectangles, Rhombuses,* and *Trapezoids* to the diagram in the appropriate places.



State whether each statement is *true* or *false*. Justify your response. You may find the diagram from Exercise 35 helpful. 36–41. See margin.

36. All squares are rectangles.

37. A trapezoid is a parallelogram.

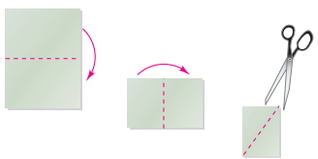
38. A rhombus can be a kite.

39. Some parallelograms are squares.

40. Every quadrilateral is a parallelogram.

41. All rhombuses are squares.

42. Paper Folding Fold a nonsquare, rectangular piece of paper in half horizontally and then vertically, as shown at the right. Draw and then cut along the line connecting the two opposite corners containing a fold. What quadrilateral do you find when you unfold the paper? Why doesn't it matter what size rectangle you start with?



Rhombus; all 4 sides are ≅ because they come from the same cut.

43. Identify a parallelogram, rhombus, rectangle, square, kite, and trapezoid in your classroom. State whether your trapezoid is isosceles. Check students' work.

44. Writing Describe the difference between a rhombus and a kite. See left.

Name each type of special quadrilateral that can meet the given condition. Make sketches to support your answers. 45–48. See margin.

45. exactly one pair of congruent sides

46. two pairs of parallel sides

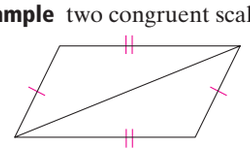
47. four right angles

48. adjacent sides that are congruent

49. Error Analysis Lauren argues, "A parallelogram has two pairs of parallel sides, so it certainly has one pair of parallel sides. Therefore a parallelogram must also be a trapezoid." What is the error in Lauren's argument?
A trapezoid has only one pair of ∥ sides.

Name the type(s) of special quadrilateral(s) it appears that you can form by joining the triangles in each pair. Make sketches to support your answers.

Sample two congruent scalene triangles



parallelogram

50–53. Check students' sketches.
50. rectangle, ▱, kite
51. rhombus, ▱
52. square, rhombus, ▱
53. rhombus, ▱, kite

50. two congruent scalene right triangles

51. two congruent equilateral triangles

52. two congruent isosceles right triangles

53. two congruent isosceles acute triangles

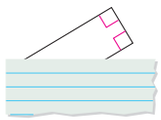
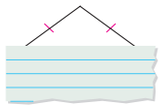
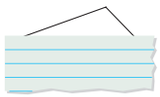
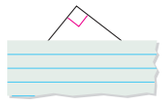
36. True; a square is both a rectangle and a rhombus.
37. False; a trapezoid only has one pair of ∥ sides.
38. False; a kite does not have ≅ opp. sides.
39. True; all squares are ▱.
40. False; kites are not ▱.
41. False; only rhombuses with rt. △ are squares.
45–48. Check students' sketches.
45. some isos. trapezoids, some trapezoids
46. ▱, rhombus, rectangle, square
47. rectangle, square
48. rhombus, square, kite, some trapezoids

Identify a parallelogram, rhombus, rectangle, square, kite and trapezoid at each site. State whether your trapezoid is isosceles. 54–55. Check students' work.

54. home **55.** somewhere other than school and home

Challenge

Reasoning **A scrap of paper covers part of each quadrilateral. Name all the special quadrilaterals that each could be. Explain each choice.** 56–59. See margin.

56.  **57.**  **58.**  **59.** 

Test Prep

Multiple Choice

60. Which statement is NEVER true? C
- **A.** Square *ABCD* is a rhombus.
- **B.** Parallelogram *PQRS* is a square.
- **C.** Trapezoid *GHJK* is a parallelogram.
- **D.** Square *WXYZ* is a parallelogram.

61. Which statement is true for some, but not all, rectangles? J
- **F.** Opposite sides are parallel.
- **G.** It is a parallelogram.
- **H.** Adjacent sides are perpendicular.
- **J.** All sides are congruent.

62. A parallelogram has four congruent sides. Which name best describes the figure? C

A. trapezoid **B.** parallelogram **C.** rhombus **D.** square

63. Which name best describes a parallelogram with four congruent angles? H

F. kite **G.** rhombus **H.** rectangle **J.** square

Short Response

64. $A(-3, 1)$, $B(-1, -2)$, and $C(2, 1)$ are three vertices of quadrilateral *ABCD*. Could *ABCD* be a rectangle? Explain. See margin.

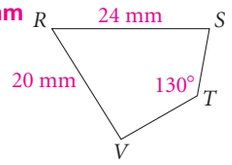
Mixed Review

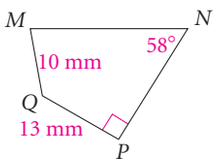
Lesson 5-5 **Can a triangle have sides with the given lengths? Explain.**

65. 8 mm, 6 mm, 3 mm See margin. **66.** 5 ft, 20 ft, 7 ft No; $5 + 7 \not> 20$. **67.** 3 m, 5 m, 8 m No; $3 + 5 \not> 8$.

Lesson 4-1 **Quadrilaterals *RSTV* and *NMQP* are congruent. Find the length of the side or the measure of the angle.**

68. $\overline{MN}$ 24 mm **69.** $\overline{VT}$ 13 mm **70.** $\overline{ST}$ 10 mm **71.** $\angle S$ 82 **72.** $\angle V$ 90 **73.** $\angle R$ 58





Lesson 3-7 **74.** Write an equation for the line parallel to $y = -3x - 5$ that contains point $(0, 4)$. $y = -3x + 4$

Online lesson quiz, PHSchool.com, Web Code: aua-0601

Test Prep

Resources

For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 361
- Test-Taking Strategies, p. 356
- Test-Taking Strategies with Transparencies

56–59. Explanations may vary.

56. ▱, rectangle, trapezoid

57. ▱, kite, rhombus, trapezoid, isos. trapezoid

58. kite, ▱, rhombus, trapezoid, isos. trapezoid

59. ▱, rectangle, square, rhombus, kite, trapezoid

64. [2] Slope of $\overline{AB}$ is $-\frac{3}{2}$. The slope of $\overline{BC}$ is 1, so $\overline{AB}$ and $\overline{BC}$ are not $\perp$. Since one $\angle$ is not a right $\angle$ and a rectangle requres all 4 $\measuredangle$ to be right $\measuredangle$, the figure could not be a rectangle.

[2] incorrect slope OR failure to recognize the information provided by the slopes