



Non-linear stability of a rolling stock moving on a flexible rail-track with random imperfections

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Abstract

The non-linear interaction of a moving rolling stock with a deformable railway track with random imperfections is analysed. The stochastic stability and the asymptotic stability of the perturbed motion, using Itô's differential system and solving the corresponding Fokker - Planck - Kolmogorov equation, is estimated.

1.Introduction

The phenomenon of the loss of stability of vehicle motion has been the object of a number of periodical articles and monographs, beginning e.g. with [1],[2]. All older papers are restricted practically to the assessment of the state in the first approach. That means that the behaviour of the perturbed state of the system is described by a linear differential system. The linear part certainly is most important, as it answers the qualitative question as to when it is at all necessary to deal with the problem.

The influence of at least the most important non-linearities has been introduced into mathematical models by some authors only in recent years, which is probably due to computer development. Next to many others mention should be made of [3],[4],[5]. With reference to stresses and possible danger to the track structure the state after the loss of linear stability is the most important. The origin of instability and its further manifestations, however, are substantially influenced by the compliance of the track, its foundation, as well as by the character and extent of track unevenness.

The influence of the compliance of the track and its foundation have not been introduced adequately into the analysis of the whole problem. Also

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the influence of irregular imperfections affecting the horizontal transverse movements of the vehicle cannot be considered as properly treated in the above mentioned sources. A certain attempt was made in [5], but still with the assumption of a perfectly rigid track. As the problem of contact and movement of a rolling stock along a compliant track is very extensive in volume, it has been possible to deal with its individual sections only in the form of samples without a claim to the completeness of the individual differential systems and their derivation. All mathematical details, figures and tables can be found in the series of research reports [6],[7],[8].

2. Track Compliance

The first group of the mathematical models consider the subsoil as a two- or three-dimensional continuum with regularly distributed mass filling the half-plane or half-space respectively or as a strip or layer on a perfectly rigid foundation. This model results in an extensive spectrum of critical velocities of various types, see e.g. [6],[9],[10]. In this phase, however, they make the mechanism of rolling stock - rail - subsoil interaction incomprehensible.

The second group of models uses the same track description by means of infinite beams of different types. The foundation, however, is modelled by various types of "simple elastic foundations" of Winkler, Pasternak and other types, see [9]. While the vertical movement has been dealt in a relatively great number of books and papers, the horizontal movement perpendicular to the track axis has been dealt with only very rarely. If we neglect the effects introducing disturbances to the longitudinal velocity of the rolling stock (lead of wheels due to the rotation of the rolling stock about its vertical axis, disturbances of operating velocity c , etc. (and some further less important phenomena) we can write for flexure:

vertical direction:

$$EJ_y w_p^{IV} + 2\mu b \ddot{w}_p + 4\mu \omega_{bs} b \dot{w}_p + 2K_s b w_p = p_p(t) \delta(x_s - ct) \quad (1)$$

horizontal direction (beam pliable in shear):

$$-GF^*(v''_p - \psi'_p) + 2\mu b \ddot{v}_p + 4\mu \omega_{bv} b \dot{v}_p + 2K_v b v_p = h_p(t) \delta(x_s - ct) \\ EJ_z \psi''_p + GF^*(v'_p - \psi_p) - J_z \rho \ddot{\psi} = 0 \quad (2)$$

torsion:

$$-GJ_k \varphi''_p + J_p \rho \ddot{\varphi}_p + \frac{4}{3} \mu \omega_{bs} b^3 \dot{\varphi}_p + \frac{3}{2} K_s b^3 \varphi_p = m_p(t) \delta(x_s - ct) \quad (3)$$

where:

ρ - specific mass of the "track strip"

μ - mass of "track strip" per unit length

$w_p = w_p(x_s, t)$, $v_p = v_p(x_s, t)$ - displacement of track axis in vertical or horizontal direction respectively

$\psi_p = \psi_p(x_s, t)$ - shear skew

GF^* - cross section rigidity in shear (incl. the correction coefficient in F^* , e.g. 2/3 for a rectangular cross section)



J_y, J_z - moment of inertia of the cross section with regard to axes y, z

J_k - modulus of resistance to torsion

J_p - polar moment of inertia of the cross section

$p_p(t), h_p(t), m_p(t)$ - vertical load, horizontal load, torque respectively, forces applied to the ideal cross section centroid of the track at $x_s = 0$

K_s, K_v - subsoil rigidity in vertical and horizontal direction respectively per unit area

$(\omega_{bs}, \omega_{bv})$ - subsoil viscosity in vertical and horizontal direction respectively per unit area

b - width of ideal cross section

$\delta(\cdot)$ - Dirac function

Boundary conditions for $w_p, w'_p, v_p, \psi_p, \varphi_p$ are homogeneous for $x_s \rightarrow \pm\infty$. The mobile longitudinal coordinate $x = x_s - ct$ is introduced into Eqs.(1),(2),(3) to which the Fourier transformation $x \rightarrow s$ is applied subsequently. This gives rise to a system of ordinary equations

$$\Theta_1 \ddot{W}_p + 2\Theta_1(\omega_{bs} - i c s) \dot{W}_p + [s^2(s^2 - \Theta_1 c^2) + \frac{\Theta_1 K_s}{\mu} - i 2\Theta_1 \omega_{bs} c s] W_p = \frac{p_p(t)}{E J_y} \quad (4)$$

$$\begin{aligned} \Theta_2 \ddot{V}_p + 2\Theta_2(\omega_{bs} - i c s) \dot{V}_p + [(1 - \Theta_2 c^2) s^2 + \frac{\Theta_2 K_v}{\mu} - i 2\Theta_2 \omega_{bs} c s] V_p + i s \Psi_p = \frac{h_p(t)}{G F^*} \\ - \Theta_4 \ddot{\Psi}_p + i 2\Theta_4 c s \dot{\Psi}_p + [-(1 - \Theta_4 c^2) s^2 - \Theta_3] \Psi_p + i \Theta_3 s V_p = 0 \end{aligned} \quad (5)$$

$$\Theta_5 \ddot{\Phi}_p + 2(\omega_{bc} \Theta_6 - i \Theta_5 c s) \dot{\Phi}_p + [(1 - \Theta_5 c^2) s^2 + \frac{\Theta_5 K_s}{\mu} - i 2\Theta_6 \omega_{bc} c s] \Phi_p = \frac{m_p(t)}{G F^*} \quad (6)$$

where:

W_p, V_p, Ψ_p, Φ_p - Fourier transforms of functions $w_p, v_p, \psi_p, \varphi_p$

Θ_1 to Θ_6 - functions of the above described track and subsoil parameters

For the problems which are linear with reference to the rolling stock movement its cooperation with the track it is advantageous to convert the set of Eqs.(4)-(6) into the Fourier picture according to time. The problem is thus transformed into an analysis of a linear algebraic system. Eqs.(4),(5),(6) become part of a whole system of non-linear differential equations. The relations between the displacements w_p, v_p, φ_p in the centroid of the "track cross section" and displacements in the axis halving the distance between railhead tops $w_r, v_r, \varphi_r, \psi_r$ or at railhead tops $w_{1r}, w_{2r}, v_{1r}, v_{2r}$ are determined by the expressions (7). Every line presents always the first approach (linear case) and, next to it, the second approach:

$$w_{1r} = w_r - a \varphi_r ; \quad w_{1r} = w_r - a \varphi_r$$

$$w_{2r} = w_r + a \varphi_r ; \quad w_{2r} = w_r + a \varphi_r$$

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$$\begin{aligned}
 v_{1r} &= v_r ; & v_{1r} &= v_r + a\varphi_r^2/2 \\
 v_{2r} &= v_r ; & v_{2r} &= v_r - a\varphi_r^2/2 \\
 w_p &= w_r ; & w_p &= w_r + e\varphi_r^2/2 \\
 v_p &= v_r + e\varphi_r ; & v_p &= v_r + e\varphi_r \\
 \varphi_p &= \varphi_r ; & \varphi_p &= \varphi_r \\
 \psi_p &= \psi_r ; & \psi_p &= \psi_r
 \end{aligned} \tag{7}$$

where:

$2a$ - distance between the basic points of railhead tops

e - vertical distance between the centroid of the track body O_p and body O_r of the line connecting the basic points of the railhead tops.

3. Forces and Geometric Conditions at the Rolling Stock-Rail Contact

At the point of contact of the rolling stock and the rail highly complex processes take place in the course of real movement, arising from a number of causes: firm connection of both wheels, conical contact surface between the wheel and the rail which is mildly inclined about its axis, lateral movements in the rail channel, etc. While in the case of road vehicles it is sufficient to describe the origin of instability on the basis of classical non-holonomous rolling relations, in case of the railway rolling stock such simplification is impossible for principal reasons, see e.g. [11], [12].

The most important factor at the contact is, on the one hand, the shear force, the vector of which has a general direction in the contact plane (S_x, S_y), on the other hand the moment in the direction of the normal to this plane M_z . Many authors have dealt with this problem. However, most frequently they adopt linearized constitutive relations according to Kalker [13], see also [3].

$$\begin{aligned}
 S_x &= -c_{xx}\epsilon_x \\
 S_y &= -c_{yy}\epsilon_y - c_{yz}\gamma_z \\
 M_z &= c_{yz}\epsilon_y - c_{zz}\gamma_z
 \end{aligned} \tag{8}$$

The quantities $\epsilon_x, \epsilon_y, \gamma_z$ are certain analogies of relative deformations and are derived from the geometric and kinematic analyses of the processes taking place at the wheel/rail contact. For instance, it holds for the right-hand wheel

$$\gamma_{z1} \approx ((\dot{\varphi}_z - \dot{\varphi}'_r) + \Omega \alpha_1)/c \tag{9}$$

The coefficients c_{ij} are considered constants in most works, incl. [3]. In others, such as [5], the authors consider them per partes broken linear functions.

The coefficients c_{ij} , consequently, must be considered as the functions of the ratio of the normal force to the resultant of shear forces with the inclusion of the influence of the moment M_z . Like in the theory of plasticity, it is necessary to select a certain criterion of the termination of the "plastic"

state [13]. Beyond that the wheel moves "irreversibly by skid" and if the force which has produced this motion drops, the motion proceeds around a different mean value. This can be coped with by the solution of the problem in time coordinate by numerical direct integration when the coefficients can be defined in a certain sequential manner.

If we aim at qualitative estimates by an analytical route, we can base our considerations on the assumption that the process is approximately centered and that, consequently, the most probable position of the neutral point is in the very places of basic points. In that case we can consider the coefficients c_{ij} as approximately non-linear functions with fixed position of origin from which it is possible to separate the linear part which results in the expressions (8) according to Kalker. However, what ever the definition of the coefficients c_{ij} , we can formulate S_x, S_y, M_z on the contact of both wheels with the rails by using the expressions of the Eq.(9) type as the functions of the components of motion characterizing the motion of the rolling stock, track and the geometry of the wheels/rail contact. For instance

$$M_{z1} = \frac{c_{yz}}{c} (\dot{v}_{1g} + \dot{w}_{1g} \alpha_2 + c(1 - \frac{\Delta r_1}{r})(\varphi_z - v'_r)) - \frac{c_{2z}}{c} ((\dot{\varphi}_z - \dot{v}'_r) + \Omega \alpha_1) \quad (10)$$

where:

α_1, α_2 -inclination of the R.H. and L.H. rail respectively

$\Delta r_1 = \Delta r_1(v_g, \varphi_g)$ -profile of the contact surface of the R.H. wheel

r -radius of the contact surface in the basic rolling stock position

Ω -circular frequency of rolling stock revolutions

φ_z -rolling stock rotation about vertical axis

w_{1g}, v_{1g} -displacement of contact point on the R.H. rail with regard to the track bed (motion in the rail channel)

The displacements $v_{1g}, w_{1g}, v_{2g}, w_{2g}$ are directly linked with movements of the track and the subsoil, which are described by Eqs. (1),(2),(3) or (4),(5),(6) together with the links with the movements $w_{1r}, w_{2r}, v_{1r}, v_{2r}$, describing the movements of the basic points of railhead tops according to (7) in the approximation for the 1st and 2nd approaches.

4. Rolling Stock Dynamics and Cooperation with the Track

We shall consider the rolling stock as a rigid body of 6 degrees of freedom. The deviations from its basic straight line movement in the direction x_s are perturbations the increase of which beyond all limits or beyond the limits determined with regard to damping and the non-linear character of the problems in the 2nd approach we shall call unstable or asymptotically unstable state. We shall denominate these deviations from the basic motion u, v, w and the rotations $\varphi_\xi, \varphi_\eta, \varphi_z$ (ξ, η - axes x, y , rotated by x against φ_z and by y against φ_ξ). The reference point is the centre of the line connecting the contact points of the rolling stock in basic position (point O_r), i.e. between the basic points of the railhead tops. The distance of basic points is $2a$; the distance between the reference point O_r and the

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centroid of the wheel set is r (the contact surface radius in the basic wheel set position).

For the construction of the equations of motion we shall use Lagrange equations of 2nd order. Since the perturbations $u, v, w, \varphi_\xi, \varphi_\eta, \varphi_z$ are small in comparison with the basic motion, these equations can be modified for the 1st approach - linear effect of perturbations, and for the 2nd approach - neglect of terms of the 3rd and higher orders. At the same time it is possible to eliminate from the system the terms of "zero" approach for which equations describing the basic motion can be obtained. For instance, the dynamic equilibrium of moments about axis ξ can be expressed, on the basis of these considerations and with regard to the results from last chapter, as follows:

1st approach:

$$\begin{aligned} mr\ddot{v} + (mr^2 + J_r)\ddot{\varphi}_\xi - J_h\Omega\dot{\varphi}_z + mgr\varphi_\xi = \\ = N_{z1}^o a - N_{z2}^o a + (N_{z1}^o \delta_1 + N_{z1} a) - (N_{z2}^o \delta_2 + N_{z2} a) - C_\xi^M \varphi_\xi - D_\xi^M \dot{\varphi}_\xi \end{aligned} \quad (11)$$

2nd approach:

$$\begin{aligned} mr\ddot{v} + (mr^2 + J_v)\ddot{\varphi}_\xi - J_h\Omega\dot{\varphi}_z + mgr\varphi_\xi + J_h\dot{\varphi}_\eta\dot{\varphi}_z + J_h(\varphi_\xi\dot{\varphi}_z) = \\ = N_{z1}^o a - N_{z2}^o a + (N_{z1}^o \delta_1 + N_{z1} a) - (N_{z2}^o \delta_2 + N_{z2} a) - C_\xi^M \varphi_\xi - D_\xi^M \dot{\varphi}_\xi \\ + \left(\frac{C_{yy}}{c} (v_{1n} + c(\varphi_z - v_r'))\right) + \frac{C_{yz}}{c} ((\dot{\varphi}_z - \dot{v}_r') + \Omega\alpha_1))a\alpha_1 - \frac{1}{2} N_{z1}^o a\alpha_1^2 \\ + \left(\frac{C_{yy}}{c} (v_{2n} + c(\varphi_z - v_r'))\right) + \frac{C_{yz}}{c} ((\dot{\varphi}_z - \dot{v}_r') - \Omega\alpha_1))a\alpha_2 + \frac{1}{2} N_{z2}^o a\alpha_2^2 \end{aligned} \quad (12)$$

where:

m - rolling stock mass

J_h, J_v - moment of inertia of the rolling stock about axis of rotation and perpendicularly to the axis of rotation respectively

δ_1, δ_2 - description of geometry of the contact surface profile of the wheel (right, left), together with $\Delta r_1, \Delta r_2$

N_{z1}^o, N_{z2}^o - nominal normal force at the wheel/rail contact

N_{z1}, N_{z2} - perturbation of these normal forces

C_ξ^M, D_ξ^M - spring constant, viscous resistance of the rolling stock connection with the carriage body

Under certain circumstances it is possible to eliminate certain terms from the system described by Eq.(11) or Eq.(12) and thus define simplified systems specified e.g. in [3]. The simplification, however, must be always made on a certain level with reference to the small parameter and avoid the confusion of the linear and non-linear theories.

It is obvious from Eq.(12) that the non-linearity has three reasons. The first arises from the six-dimensional motion of the rolling stock as a rigid body - see the left-hand side of Eq.(12). In case of major displacements these

interactions influence significantly the gyroscopic forces and may reverse entirely the picture of the problem which is the state after the loss of linear stability.

The second source of non-linearity are kinematic links of the rolling stock with the rails. This arises from the non-linear shape of rail head tops and the contact surface of the wheels (parameters $\delta_1, \delta_2, \Delta r_1, \Delta r_2, \alpha_1, \alpha_2$ etc.). If the loss of linear stability takes place, these very terms play a highly significant part.

The third source of non-linearity are contacts of the rolling stock with the rails - see last chapter. These non-linearities are concealed in the parameters c_{ij} . The moment the motion loses asymptotic stability skids take place and the parameters c_{ij} must be considered as strongly non-linear, as suggested in last chapter.

The equations of Eq.(11) or Eq.(12) type, further the constitutive equations of Eq.(10) type, the equations of motion of the track and the foundation (4),(5),(6) the geometric links (7) and others give rise to a system of linear or non-linear equations of 42 unknown quantities describing the rolling stock motions (or actually the perturbations of its useful motion) and the motion of the foundation (in the meaning of Fourier transforms according to x) and the force conditions at their contact. Particularly in the linear state some groups of these equations are independent. In non-linear state it is possible to use either a purely numerical solution in the time coordinate or follow the route which could be called "semi-analytical". When selecting concrete methods, however, it is always necessary to take account of the investigated operator due to the influence of gyroscopic terms and terms originating because of the moving load is asymmetric and non-selfadjoint.

5. Motion Stability of the Rolling Stock

The effective procedure for an estimation of the motion stability of the described system could be based on an analysis of stochastic moments describing the generalised solution of the Fokker - Planck - Kolmogorov (FPK) equation, see e.g.[14]. In such a case, however, the excitation cannot be fully arbitrary. It should have the form of a result of a white noise filtered through a fictitious system whose transmission function is of a rational fraction type. Taking into account the usually adopted form of a spectral density of a track geometrical imperfections a diffusion filtration (differential equation of the 1st order could be acceptable. It means that the system of 42 equations of the (4) to (12) type should be accomplished by three equations:

$$\left. \begin{aligned} \dot{p}_p(t) &= -a_p p_p(t) + \sqrt{s_p} \cdot w_p(t) \\ \dot{h}_p(t) &= -a_h h_p(t) + \sqrt{s_h} \cdot w_h(t) \\ \dot{m}_p(t) &= -a_m m_p(t) + \sqrt{s_m} \cdot w_m(t) \end{aligned} \right\} \quad (13)$$

The system (4) to (12) must be rewritten into a normal form so that the original system should be expanded by unknown functions describing velocities $\dot{W}_p, \dot{V}_p, \dot{\Phi}_p, \dot{u}, \dot{v}, \dot{w}, \dot{\varphi}_\xi, \dot{\varphi}_\eta, \dot{\varphi}_z$. Altogether with equations (13) an Ito's system of 54 state variables is assembled and it could be written in the form:

$$\dot{\mathbf{Y}}(z) = \mathbf{A}(\mathbf{Y}, t) + \mathbf{B}(\mathbf{Y}, t) \cdot \mathbf{w}(t) \quad (14)$$

$\mathbf{Y}(t)$ - vector of state variables



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$\mathbf{A}(t)$ - vector of functions depending on state variables and time

$\mathbf{B}(\mathbf{Y}, t)$ - rectangular matrix (54×3); functions putting together random excitation by white noise vector $\mathbf{w}(t) = [w_p(t), w_h(t), w_m(t)]$ and the system itself

Using eq.(14) we can set up FPK equation for an unknown mutual density of probability $p(\mathbf{Y}, t)$ of all $\mathbf{Y}(t)$ vector components:

$$\frac{\partial p(\mathbf{Y}, t)}{\partial t} = - \sum_{i=1}^n \frac{\partial}{\partial y_i} (\kappa_i(\mathbf{Y}, t) p(\mathbf{Y}, t)) + \frac{1}{2} \sum_{ij=1}^n \frac{\partial^2}{\partial y_i \partial y_j} (\kappa_{ij}(\mathbf{Y}, t) p(\mathbf{Y}, t)) \quad (15)$$

$\kappa_i(\mathbf{Y}, t), \kappa_{ij}(\mathbf{Y}, t)$ - intensities of a Markov multidimensional process

As the processes considered could be regarded, with sufficient accuracy, as continuous, Ito's definition of the stochastic differential should be used. In this case:

$$\kappa_i(\mathbf{Y}, t) = A_i(\mathbf{Y}, t); \quad \kappa_{ij}(\mathbf{Y}, t) = \sum_{k=1}^m B_{ik}(\mathbf{Y}, t) B_{jk}(\mathbf{Y}, t) \quad (16)$$

If it is desired to comprehend more carefully the influence of real skidding on the contact wheel - rail, the expressions $\kappa_i(\mathbf{Y}, t)$ in (16) should be accomplished by terms resulting from the Stratonovitch stochastic differential definition. It is evident that equations and forms (14), (15), (16) comprise all linear as well as non-linear approximations of a real system.

No stationary solution of eq.(15) exist, unless the system is of a potential type with a corresponding distribution of excitation intensity, so that $\partial p(\mathbf{Y}, t)/\partial t \neq 0$. For example, in a numerical solution to eq.(17), the random response due to non-linearities will be strongly non-stationary, and density of probability $p(\mathbf{Y}, t)$ will be dependent on time, although an excitation remains stationary and transition effects have already disappeared.

A classical solution of eq.(15) does not exist either. It is possible, however, to find a generalised solution, and in a stochastic meaning of the word, which is characterised by a set of stochastic moments. A corresponding system of ordinary equations, which is also of a normal type, could be derived from eq.(15), by successive multiplication of (15) by phase variables in appropriate degrees and by integration around the whole phase space using integration by parts. Finally, we obtain:

$$\frac{d\mathbf{U}_\kappa}{dt} = \Phi_\kappa(\mathbf{U}_1, \dots, \mathbf{U}_\kappa, \dots, t); \quad \kappa = 1, \dots, \infty \quad (17)$$

$$\mathbf{U}_\kappa(t) = |U_\kappa^1, \dots, U_\kappa^{j_{max}}|^t$$

$$U_\kappa^j(t) = \int_{-\infty}^{\infty} Y_1^{k_1}(t) \cdots Y_n^{k_n} \cdot p(\mathbf{Y}, t) dY_1 \cdots dY_n$$

$$k_1 + \cdots + k_n = \kappa; \quad j - \text{combination of } k_1, \dots, k_n$$

Due to eq.(15), the system (17) is linear. All the non-linearities presented in the original system (14) are described in (17), inside of the coefficients involved. The system (17) is, however, of an infinite size. It cannot be rounded off on the $\kappa = 2$ level adopting a supposition of a pseudo - Gaussian

character of the response, if the problem on the level of the second approach (non linear) is analysed. In addition to it, system (17) would lose its linear character. A procedure neglecting higher order moments, e.g. for $\kappa > 4$, could be used.

On that assumption the stability of a trivial solution $\mathbf{Y}(t) \equiv 0$ of the non-linear system could be estimated using the Rous - Hurwitz determinant test for all the levels of κ which has been conserved in (17). This procedure means that the probability of

$$\sigma_{\kappa} = \sup_{0 < t < \infty} \mathbf{E} \left\{ \left(\sum_{j=1}^n |Y_j|^{\kappa} \right)^{1/\kappa} \right\} < \varepsilon \quad (18)$$

is determined. If $\kappa = 2$, the stability on the random mean square level has been estimated. If $P(\sigma_{\kappa} < \varepsilon) = 1$, a motion perturbed by imperfections of the track conserves an asymptotic stability, whereas for $P(\sigma_{\kappa} < \varepsilon) > 1 - \phi$ (ϕ is a small real number) only its stability (not its asymptotic stability) has been detected. In this case a reaction of the system could be considerable, but always remaining within certain limits. Using only the first approach (linear model), the loss of asymptotic stability brings about the collapse the whole system. On the other hand the loss of the asymptotic stability of a linear system cannot be compensated by any non-linearities.

6. Conclusion

The carried out studies show the field of application of the linear and non-linear formulations of the rolling stock motion and the cooperation of the rolling stock with the deformable track and its subsoil. The principal significance of the interaction of the rolling stock with the deformable track has been revealed. For instance, the influence of gyroscopic forces cannot much develop in the case of a rigid track, as the gravity "pseudo-compliance" is much lower than the compliance of the track itself. The number of critical velocities endangering the very existence of the track is much higher than is revealed by track analyses not including the dynamic and kinematic characteristics of the wheel set and the whole carriage. Entirely different is also the character and influence of contact forces wheel - rail.

A numerical evaluation of the derived formulae and equations shows that the areas of instability of the analysed system and those of the parametrically excited systems are similar. The critical stages are not isolated points of operating velocity, they constitute some critical zones of instability. The width of these zones is very variable depending on parameters and on immediate state of the system. Non-linear terms originate various internal effects of local instabilities only in some components of the motion. These results could be used for the technical rules governing the design and operation of the railway cars and track.

It has been shown that more critical velocities or zones exist in comparison with non-deformable track model and, moreover, these velocities can become near to those used in the normal operation of railways.

Three described types of non-linearities are decisive for possibly conserving the motion stability after the asymptotic stability having been lost. Linear models and models with non-deformable track lead to useless conservative results at points of basic critical speeds. They also underestimate the width of critical zones and overlook local effects.



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As the principal solution method, a stochastic stability analysis of the non-linear system has been used. For this purpose, a generalised solution of the corresponding Fokker - Planck - Kolmogorov equation has been derived. Via the Rous - Hurwitz determinant its stability according to the probability density of the response in a non-linear state has been checked.

References

1. Carter F.W., On the Stability of Running of Locomotives, *Proc. of the Royal Soc. of London. ser.A*, v. 121, 1928.
2. Rocard Y., Les méfaits du coulement autooscillations et instabilités de route, *La revue scientifique*, t.84, No.15, 1946.
3. Freibauer L., Rus L., Zahradka J., *Dynamics of Railway Cars* (in Czech), NADAS, Praha 1988.
4. Gark V.K., Dukkipati R.V., *Dynamics of Railway Vehicle Systems*, Academic Press, New York, 1984.
5. Verigo M.F., Kogan A.Ja., *Cooperation of Way and Moving Vehicle* (in Russian), Transport, Moskva 1986.
6. Náprstek J., Turek J., *Modeling of Dynamic Effects of a Railway Car and Track Cooperation onto the Track and its Subsoil* (in Czech), vol.1.2. Res.Rep. of Inst.of Th.and Appl.Mech., Praha 1992.
7. Náprstek J., Turek J., *Motion Stability of a Rolling Stock moving on a randomly perturbed Track* (in Czech), Res.Rep. Inst.of Th.and Appl.Mech., Praha 1992.
8. Izer I., *Research of Leading Safety of a Car in Railway Channel* (in Czech), Res.Rep. VSDS-KKVZM, Zilina 1987.
9. Frýba L., *Vibrations of Solids and Structures under Moving Loads*, Academia, Praha 1972.
10. Náprstek J., Čenský J., Dynamic Response of a Beam on Twodimensional Continuum to Moving Periodic Loading, *Proc. "4th Int.Conf. on Recent Adv. in Structural Dynamics"*, Inst. of Sound and Vibration, Southampton 1991.
11. Nejmark Ju.I., Fufaev N.A., *Dynamics of Non-holonomous Systems* (in Russian), Nauka, Moskva 1967.
12. Malkin I.G., *Theory of Motion Stability* (in Russian), Gostekhnizdat, Moskva 1952.
13. Kalker J.J., Über die Mechanik der Kontaktes zwischen Rad und Schiene, *Glaser's Annalen* 102, 1978.
14. Pugachev V.S., Sinitsyn I.N., *Stochastic Differential Systems* (in Russian), Nauka, Moskva, 1990.