Direct displacement-based seismic design

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ABSTRACT: There has been considerable discussion recently in New Zealand about the relative merits of displacement-based, and displacement-focused force-based seismic design. This paper puts the case for direct displacement-based seismic design. It is shown that the emphasis on secant stiffness to maximum displacement, rather than initial stiffness (as in force-based seismic design) is important for rational force-distribution to different seismic-resisting structural elements, and in most cases obviates the need for iteration in the design process, which is inherent in displacement-focused force-based seismic design. It is shown that the influence of hysteretic characteristics has been underestimated in recent force-based studies. These assertions are supported by results of recent analytical studies, which have included refinement of ductility/equivalent-viscous damping relationships, and an examination of the important (and largely ignored) role of "elastic" damping in inelastic time-history analyses.

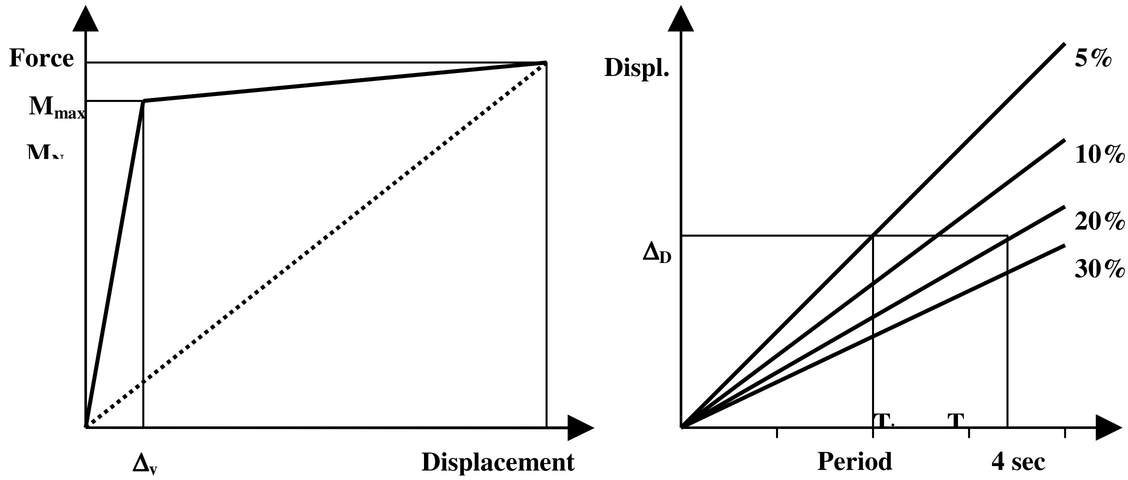
1 INTRODUCTION

1.1 A brief comparison of displacement-based, and displacement-focused force-based design

The concept of seismic design based on limit displacements has been gaining credence over the past 15 years, as it has become appreciated that structural damage can be directly related to strain (and hence by integration to displacement), and non-structural damage, in buildings at least, can be related to drift. The inverse relationship between damage potential and strength, long held to be self evident, has proven to be illusory.

Different approaches have been proposed to increase the emphasis on displacement. Current seismic design normally requires a rather approximate check that peak displacements or drifts do not exceed specified code limits, and no attempt is made to obtain a uniformity of risk of structural or non-structural damage. Direct displacement-based seismic design (DDBD) (Priestley,1993,2000) has been developed as a simple method for designing to achieve, rather than be bounded by, displacement limits that could be strain-based or code drift-limit based. The essence of the approach is to characterise the structure by the effective stiffness (k_e) to the design displacement, rather than the initial stiffness (k_i) , and by a level of equivalent elastic damping (ξ_e) that represents the combined effects of elastic and hysteretic damping, rather than the 5% elastic damping normally assumed to be appropriate in force-based design. Since the design approach has been fully described elsewhere, it will not be repeated here.

Initial estimates of the relationship between expected ductility demand and equivalent elastic damping were based on the area of the steady-state hysteretic response, which worked well for reinforced concrete structures, but overestimated the equivalent elastic damping for "fat" hysteresis loops such as bilinear or elasto-plastic. This fact was originally noted by Gulkan and Sozen(1974), and further elaborated by Judi et al (2002), who developed relationships based on energy considerations first suggested by Gulkan and Sozen. Subsequent work, reported briefly in this paper and elsewhere (Grant et al,2004), has calibrated the ductility/displacement relationships for a number of different hysteretic loop shapes.



(a) Design Force-Displacement Response (b) Design Displacement Response Spectra

Figure 1. Essential differences between force-based and direct displacement-based design

The essential differences between force-based and direct displacement-based design are summarized in Figure 1. Force-based design uses an initial stiffness k_i, a nominal strength F_N, and an acceleration response spectrum, (not shown in Figure 1) based on 5% elastic damping. Direct displacement-based design uses an effective secant stiffness k_e to the design displacement Δ_D, the strength F_{max} corresponding to the design displacement, and displacement spectra for different levels of equivalent viscous damping.

Recently, emphasis has been given in New Zealand to what has been called "displacement-focused force-based design" (DFFBD) (Davidson et al, 2002), which attempts to account for some of the problems that have been identified in force-based design philosophy, while still maintaining its familiar framework. A problem with conventional force-based design has been that for reinforced concrete and masonry structures, the stiffness of sections depends on the strength, as influenced by axial force level, and, particularly, reinforcement content. Consequently, the stiffness cannot be determined correctly until the structure is designed. In order to correctly account for this, and also to provide the ability to design to achieve, rather than be bounded by design displacement limits, Davidson et al describe an iterative force-based design approach, compared in Table 1 with DDBD.

Table 1. Comparison of DDBD and DFFBD Design Steps

Direct Displacement-Based Design	Displacement-Focused Force-Based Design
1. Assume Structure Geometry (spans, heights, sections)	1. Assume Structure Geometry (spans, heights, sections)
2. Determine design displacement (normally drift based)	2. Estimate member stiffnesses (assumed rebar)
3. Calculate yield displacement, hence ductility	3. Analyse structure for dynamic characteristics (periods)
4. Determine equivalent viscous damping	3. Select design ductility (normally code-specified)
5. Determine effective period from displacement spectra	4. Determine design base shear strength
6. Determine effective stiffness from SDOF Eqn	5. Analyse structure for required member strengths
7. Determine design base shear strength	6. Determine reinforcement contents; revise stiffness
8. Distribute base shear and analyse structure	7. Cycle 2 to 6
	8. Determine design displacement limit (DDBD, step 2)
	9. Calculate limit elastic period T_e (5% damping)
	10. Check structure period $T<T_e$
	11. Revise stiffness if necessary or desired so that $T=T_e$

Direct displacement-based design requires no iteration, since the yield displacement can be directly related to section dimensions, independent of strength, (Priestley,2003) and hence is known as soon as the structural geometry is selected. As described by (Davidson et al, 2002), two levels of iteration are required in DFFBD to determine the final design. The only difference from conventional design is improved estimation of elastic stiffness, through the iterative approach, and the final (optional, in Davidson et al, 2002) iterative optimization to achieve a design where the design limit is achieved.

The approach is very similar to that proposed by Chopra (Chopra, 2001), which directly uses the inelastic acceleration spectra, and one level of iteration. In fact, both levels of iteration in (Davidson et al, 2002), and the single level in (Chopra 2001) are unnecessary if advantage is taken of the known value for yield displacement, as in DDBD. This enables the design ductility corresponding to the displacement limit to be defined in the force-based design approach at the start of the design, as in DDBD. It would thus appear, with this modification, that the two approaches would have identical design effort, and presumably identical results.

There are, however, a number of assumptions made in (Davidson, et al, 2002) and (Chopra, 2001) that need examination. These are:

- Relative elastic stiffnesses of members are appropriate for seismic force distribution.
- Inelastic displacements can be predicted from elastic displacements by simple rules.
- Influence of hysteretic rule (shape of force-displacement response envelope) is insignificant.

These are discussed in the remainder of this paper.

2 FORCE DISTRIBUTION BASED ON ELASTIC STIFFNESS

Two examples are given where design to the DFFBD will result in poor designs. In the first, the building shown in Figure 2, for which seismic resistance is provided by cantilever walls of different length, is considered. Force-based design (and DFFBD) will allocate strength between the walls in

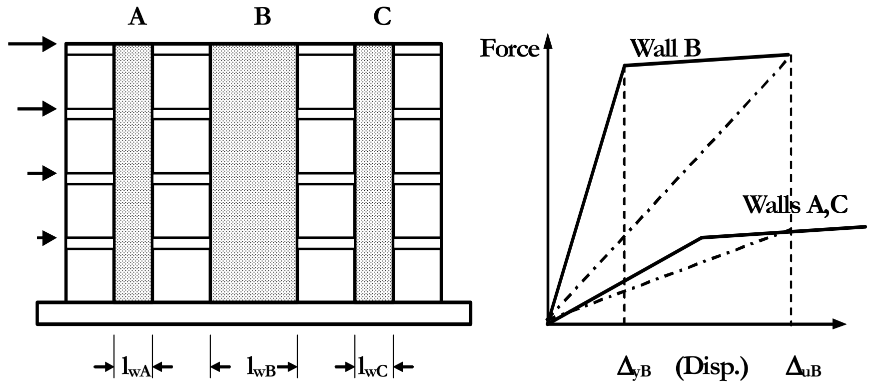


Figure 2. Building with unequal length cantilever walls

proportion to elastic stiffness (that is, in proportion to l_w^3). This results in higher reinforcement ratios for the longer walls. Refinement of the elastic stiffness to reflect the different reinforcement ratios of the longer and shorter walls will increase the stiffness discrepancy between the walls, further concentrating reinforcement, and strength, in the longer walls, underutilising the shorter walls and possibly making the longer walls shear-critical. Note further that since the yield displacements of the walls can be shown to be inversely proportional to wall length as shown in Figure 2 (Priestley,2003) the ductility demand on the walls is different. In DFFBD this makes selection of the design ductility factor complex.

With DDBD, the effective stiffness ratios of the walls at the design displacement are used (see Figure 2). Since the elastic stiffness is of only minor significance, the rational decision to reinforce both walls with the same reinforcement ratio (or even to allocate a higher reinforcement ratio to the shorter walls) can be made, improving the structural efficiency.

The second example, shown in Figure 3, represents a bridge crossing a valley, and hence having piers of different height. Under longitudinal seismic response the deflections at the top of the piers will be equal. Assuming an effectively rigid superstructure, force-based design will allocate the seismic force between piers in proportion to H^3, again resulting in higher flexural reinforcement ratios for the shorter piers, which, when stiffness is adjusted in DFFBD will result in further separation of the pier stiffnesses and reinforcement ratios, with similar consequences to the wall building discussed above.

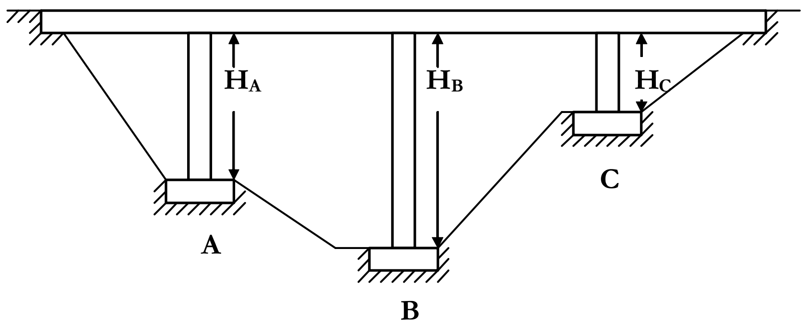


Figure 3. Bridge with unequal column heights

Again DDBD frees the designer from elastic stiffness considerations, and allows the different ductility levels for the piers to be directly considered in the design process. Note that under transverse response, a portion of the seismic loads will be transmitted back to the abutments by superstructure elastic bending. The rational distribution of force between the elastic (superstructure flexure) and inelastic (column flexure) load paths is straightforward in DDBD, but difficult in elastic-stiffness based design.

Further examples of problems with in initial-stiffness based design are considered elsewhere (Priestley et al, 2005).

3 RELATIONSHIP BETWEEN ELASTIC AND INELASTIC DISPLACEMENTS

The second and third assumptions of DFFBD noted in section 1.1 are related and will be considered together. In both (Davidson et al, 2002) and (Chopra,2001), relationships between the elastic displacement corresponding to the initial displacement and a 5% elastic damping, and the inelastic displacement, are based loosely on the Newmark and Hall (Newmark and Hall, 1982) relationships. Key amongst these is the adoption of the "equal-displacement" approximation for structures with initial periods greater than about 0.7 seconds. At lower periods, the inelastic displacement is amplified when compared with the corresponding elastic displacement, though the dependency on hysteretic shape is held to be insignificant. Based on the large number of analytical studies (e.g. Miranda and Bertero, 1994, Judi et al, 2002) involving thousands of analyses using real earthquake records, these relationships would seem to be reasonable.

There are, however, two grounds for concern relating to these analyses. The first relates to the relationship between the earthquake records chosen for analysis, and the code-specified design spectra, and the second relates to the way in which initial elastic damping is considered in the time-history analyses validating the elastic/inelastic relationships.

3.1 Choice of earthquake records

When using real, unmodified earthquake records for inelastic time-history analysis, it is common to scale the records in amplitude so that it matches the code spectrum over the elastic period range of interest, as illustrated in the displacement spectrum of Figure 4. With a single degree of freedom structure, the matching is made at a single period. It is important to consider, however, the spectrum for a period range that includes the period shift expected as the structure responds inelastically. This is also shown in Figure 4 for a displacement ductility of $\mu = 3$. In this example, the scaled displacement spectrum becomes increasingly conservative as the period increases above the elastic period, resulting in an unconservative estimate of the inelastic displacement from the time-history analysis. Of course, this argument applies to a single record. If a large number of records are used such that the average of the displacement spectra over the full range from elastic to inelastic period matches the design spectrum, valid average results can be expected. Unfortunately this is unlikely to be the case for longer period structures, since the large majority of records used in the analyses tend to have peak spectral

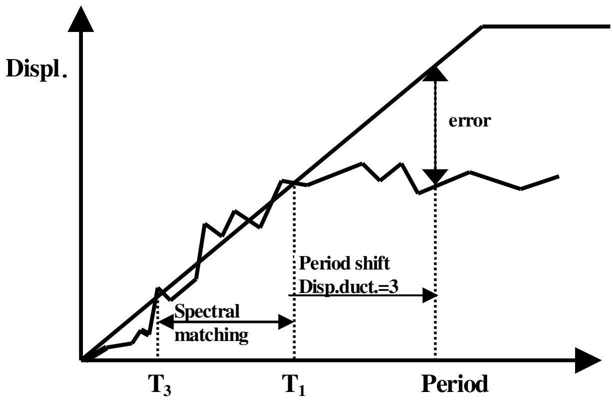


Figure 4. Displacement spectrum matching for inelastic time-history analysis

displacements at periods from 1.0-2.0 seconds. Thus conclusions about structural response for structures with expected displacement ductilities of (say) $\mu = 4$ and elastic periods of T >0.75 seconds can be expected to be suspect, unless the earthquake records are very carefully chosen. Note that though the argument above has related to code spectrum matching, it also applies to more general studies related to investigation of the relationship between elastic and inelastic displacement.

3.2 Modelling elastic damping in time-history analyses

The second issue relates to how elastic damping is modelled in time-history analyses. Typically research papers reporting results on single-degree-of-freedom (SDOF) inelastic time-history analysis (ITHA) state that 5% initial elastic damping was used, without clarifying whether initial-stiffness or tangent-stiffness proportional damping was assumed. It appears that many analysts consider the choice of initial elastic damping model to be rather insignificant, as the effects are expected to be masked by the much greater energy dissipation associated with hysteretic response. This is despite evidence by others [e.g. Otani, 1981] that the choice of elastic damping model between mass-proportional (essentially identical to initial-stiffness proportional) and tangent-stiffness proportional damping could be significant, particularly for short period structures.

There appear to be three main reasons for incorporating initial elastic damping in ITHA:

- The assumption of linear elastic response at force-levels less than yield: Many hysteretic rules make this assumption, and hence do not represent the nonlinearity, and hence hysteretic damping, within the elastic range for concrete and masonry structures, unless additional damping is provided.
- Foundation damping: Soil flexibility, nonlinearity and radiation damping are not normally incorporated in structural time-history analyses, and may provide additional damping to the structural response.
- Nonstructural damping: Hysteretic response of nonstructural elements, and relative movement between structural and nonstructural elements in a building may result in an effective additional damping force.

Discussing these reasons in turn, it should be recognised that hysteretic rules are generally calibrated to structural response in the inelastic phase of response. Therefore additional elastic damping should not be used in the post-yield state to represent structural response except when the structure is unloading and reloading elastically. If the hysteretic rule models the elastic range nonlinearly then no additional damping should be used in ITHA for structural representation. It is thus clear that the elastic damping of hysteretic rules with linear elastic representation would be best modelled with tangent-stiffness damping, since the elastic damping force will greatly reduce when the stiffness drops to the post-yield level.

If the structure deforms with perfect plasticity, while the foundation remains elastic, then foundation forces will remain constant, and foundation damping will cease. It is thus clear that the effects of foundation damping are best represented by tangent stiffness related to the structural response, unless the foundation response is separately modelled by springs and dashpots.

It is conceivable that the non-structural damping force is displacement-dependent rather than force-dependent, and hence initial-stiffness damping might be appropriate for the portion of "elastic" damping that is attributable to non-structural elements. There are two possible contributions to non-structural damping that should be considered separately:

- Energy dissipation due to hysteretic response of the nonstructural elements
- Energy dissipation due to sliding between nonstructural and structural elements

For a modern frame building, separation between structural and non-structural elements is required, and hence they should not contribute significantly to damping. Further, even if not separated, the lateral strength of all non-structural elements is likely to be less than 5% of the structural lateral strength (unless the non-structural elements are masonry infill). If we assume 10% viscous damping in these elements, an upper bound of about 0.5% equivalent viscous damping related to the structural response seems reasonable. Sliding will normally relate to a frictional coefficient, and the weight of the non-structural element. Unless the non-structural elements are masonry, the frictional force is likely to be negligible. Nonstructural elements are unlikely to play a significant role in the response of bridges.

Analyses of the steady-state response of SDOF oscillators under sinusoidal excitation (Priestley and Grant, 2005) have shown that at displacement ductility levels of about 4 to 6, the energy dissipated by 5% initial-stiffness damping is approximately the same as the hysteretic energy absorption of concrete structures represented by the modified Takeda hysteretic characteristic. This intuitively seems unrealistic. The energy absorbed by tangent-stiffness damping for the same conditions is only about 15% of the hysteretic energy. Analyses of SDOF systems subjected to real earthquake records show the significance of the elastic damping model is not just limited to steady-state response. Figure 5 shows a typical comparison of the displacement response for a SDOF oscillator with initial stiffness and tangent stiffness elastic damping. In this example the El Centro 1940 NS record has been used, the initial period was 0.5 seconds, a Takeda hysteretic rule with second slope stiffness of 5% was adopted, and the force-reduction factor was approximately 4. The peak displacement for the tangent-

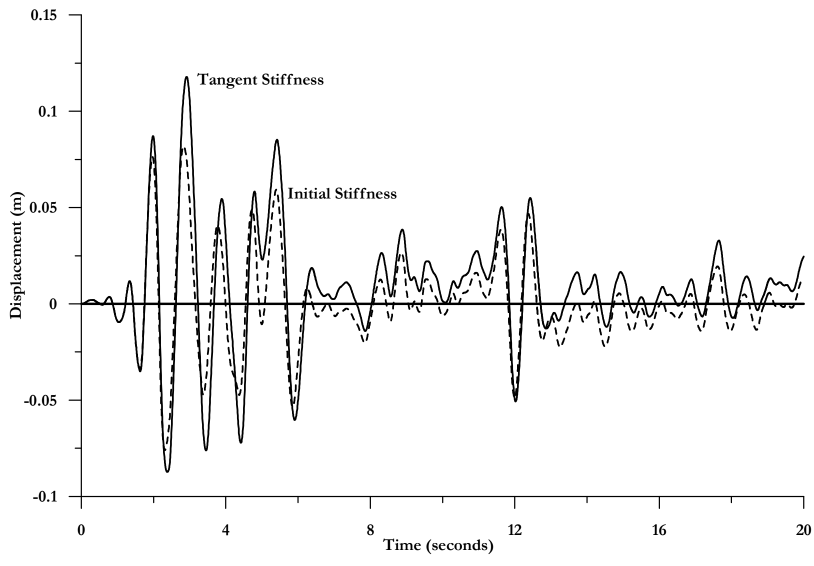


Figure 5. Displacement response of initial-stiffness and tangent-stiffness SDOF to El Centro 1940NS

stiffness elastic damping case is 44% larger than for the initial-stiffness damping case, indicating a very significant influence.

In order to investigate the effect of elastic damping assumption on displacement response further, a series of analyses were carried out on SDOF oscillators with initial periods between 0.25 sec. and 2.0 sec. when subjected to five artificial accelerograms spectrum-compatible with the ATC32 spectrum for Class C (moderate stiffness) soils. Elastic analyses, using 5% damping were first carried out, then inelastic analyses where either initial-stiffness or tangent-stiffness 5% damping were specified, with yield strengths based on force-reduction factors of R=2,4 and 6 based on the average elastic response peak force. Modified Takeda (representing concrete response), Bilinear (approximating steel) and Flag (representing hybrid prestressed precast concrete) hysteresis rules were considered. Full details are available in (Priestley and Grant, 2005, and Grant et al, 2004).

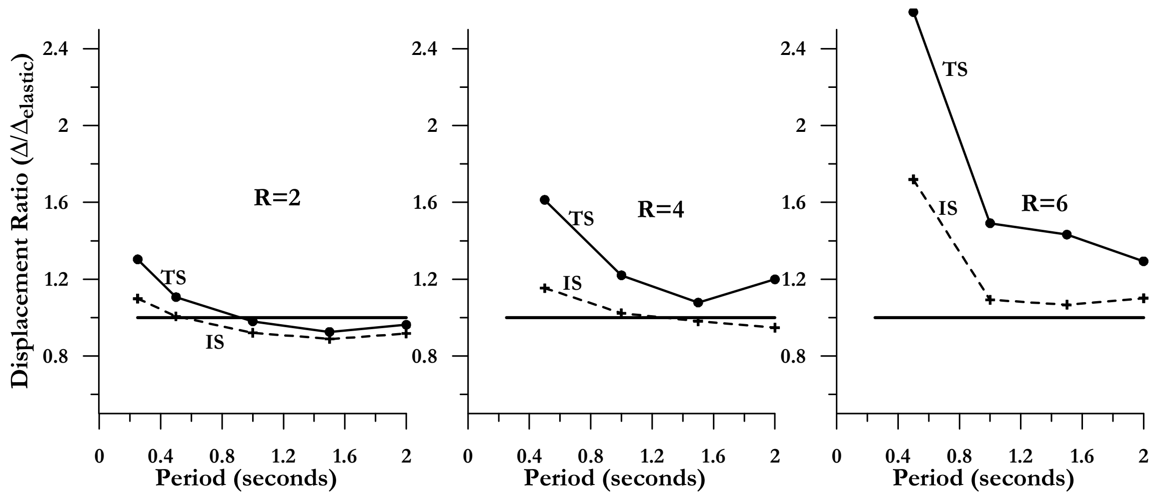
A selection of results is shown in Figure 6, which plots the ratio of peak inelastic response to elastic response. The second-slope stiffness ratio for the modified-Takeda and bilinear rules for these analyses was r=0.002 (the minimum value considered), but similar results were obtained for r=0.05 (the maximum considered). The flag hysteresis represented in Figure 6 represents the minimum additional damping considered in the hybrid precast modelling. Note that in some cases the displacement ratios at T = 0.25 sec. have not been plotted as they exceed the range included by the graph axes.

From examination of Figure 6 it will be noted that there is significant difference between the response of the initial-stiffness and tangent-stiffness (represented by IS and TS respectively) models, that this difference is rather independent of initial period for T > 0.5 seconds, that the difference increases with force-reduction factor, and is dependent on the hysteretic rule assumed. It will also be noted that though the "equal displacement" approximation (represented by a displacement ratio of 1.0 in Figure 6) is reasonable for initial stiffness damping and initial periods greater than T = 1.0 seconds, it is significantly non-conservative for tangent-stiffness elastic damping. This calls into question the assumptions for DFFBD, discussed earlier.

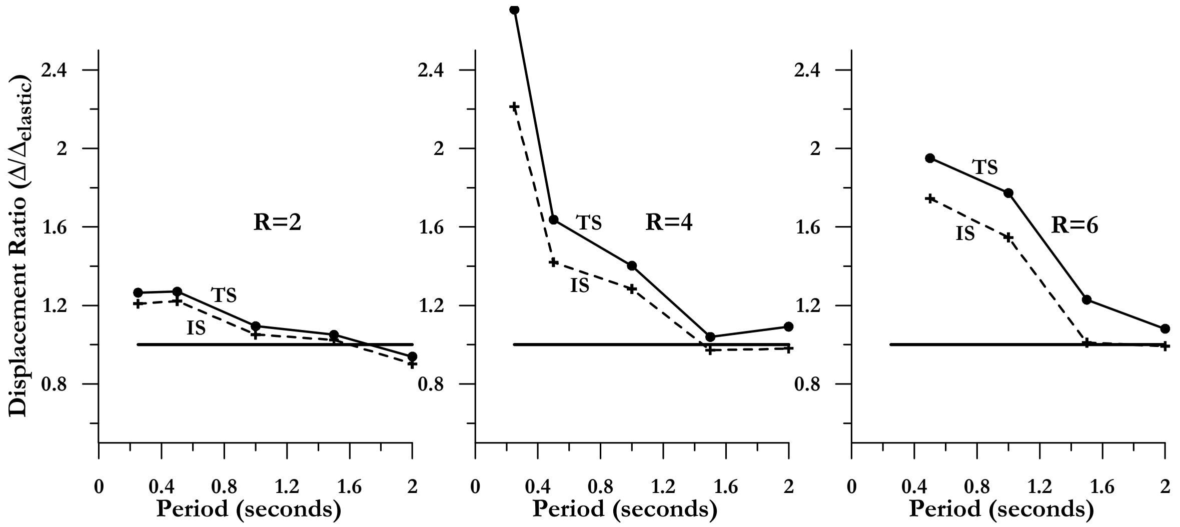
There are additional questions relating to the treatment of elastic damping that have particular significance for DDBD that have not, perhaps, been recognized by its critics. In DDBD some elastic damping is normally added to the hysteretic component, as in time-history analysis. However, in DDBD, the initial elastic damping is related to the secant stiffness to maximum displacement, whereas it is normal in seismic analysis to relate the elastic damping to the initial (elastic) stiffness, or more correctly, as noted above, to a stiffness that varies as the structural stiffness degrades with inelastic action (tangent stiffness). Since the response velocities of the "real" and "substitute" structures are expected to be similar under seismic response, the damping force, which is proportional to the product of the stiffness and the velocity, will differ significantly, since the effective stiffness k_e of the substitute structure is approximately equal to $k_e = k_i/\mu$ (for low post-yield stiffness). Grant (Grant et al,2004) has determined the adjustment that would be needed to the value of the elastic damping assumed in DDBD (based on either initial-stiffness or tangent-stiffness proportional damping) to ensure compatibility between the "real" and "substitute" structures. Without such an adjustment, the verification of DDBD by inelastic time-history analysis would be based on incompatible assumptions.

As part of an on-going study into direct displacement-based design at the European School for Advanced Studies in Reduction of Seismic Risk, the relationship between displacement ductility and equivalent damping has been investigated for a range of common hysteretic models, using spectrum-compatible non-linear time-history analyses. The work has consisted of three stages:

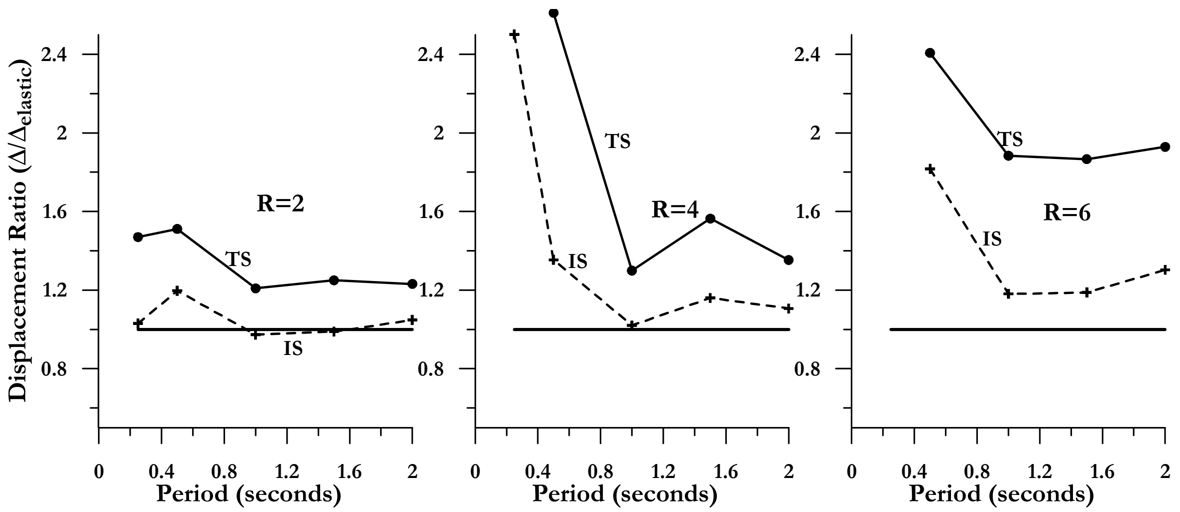
1. Initial determination of the relationship between ductility and equivalent viscous damping, where the effects of initial elastic damping were ignored.
2. Investigation of the relationship between elastic damping in hysteretic and substitute structure models, as discussed above.
3. Combination of the first two phases and recalibration to provide final ductility/damping relationships for direct displacement-based design, combining the effects of tangent stiffness and hysteretic damping.



(a) Takeda (concrete) hysteresis



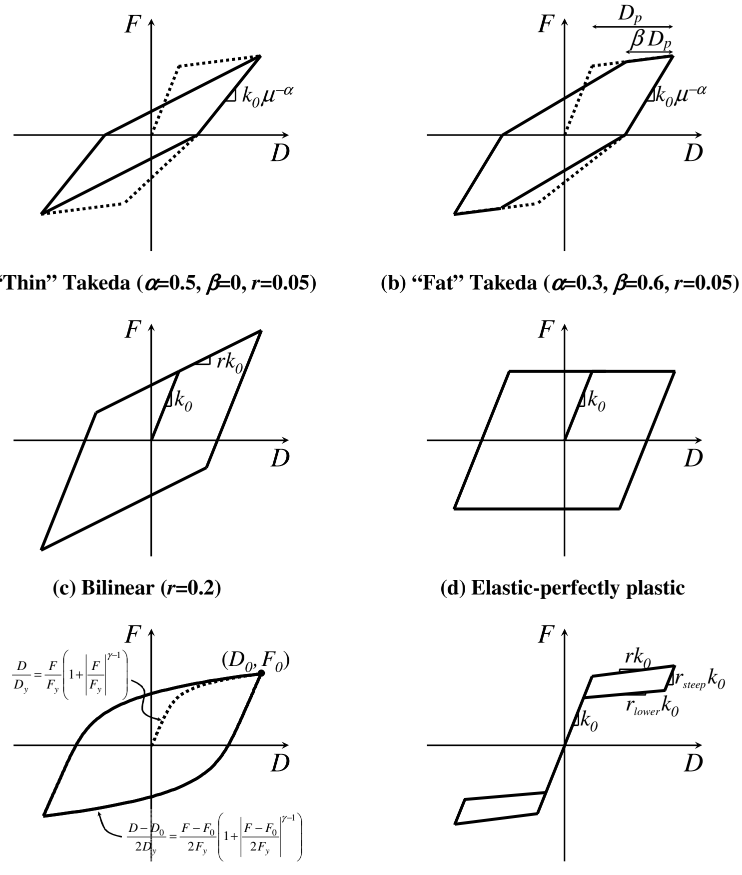
(b) Bilinear hysteresis



(c) Flag Hysteresis

Figure 6 Response of SDOF oscillators to ATC32 spectrum-compatible accelerograms

Because of space limitations, only the final stage will be briefly presented in this paper. Full details are available in (Grant et al,2004). Hysteresis rules considered in the analyses are shown in Fig.7.



(a) "Thin" Takeda (α=0.5, β=0, r=0.05) (b) "Fat" Takeda (α=0.3, β=0.6, r=0.05)

(c) Bilinear (r=0.2) (d) Elastic-perfectly plastic

(e) Ramberg-Osgood (γ=7) (f) Ring-spring (r_{lower}=0.035, r_{steep}=1, r=0.04)

Figure 7. Hysteresis Rules considered in Direct displacement-based design calibration

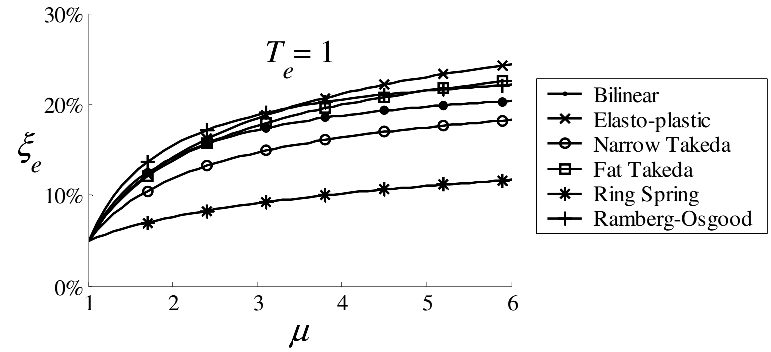
The "thin" and "fat" Takeda models represent the reasonable range for reinforced concrete and masonry structures. The bilinear oscillator (Figure 7c) is typical for a bridge structure with FPS or lead-rubber bearing isolators; the EPP system is an unrealistic idealized response (except perhaps for structures isolated for coulomb friction sliders; the Ramberg Osgood is reasonably appropriate for steel structures, while the ring-spring (similar to the flag-shaped hysteresis discussed earlier) is appropriate for a precast concrete structure, with unbonded post-tensioning, and additional energy absorption.

Relationships between damping and ductility were developed to optimize the fit from the analysis results, in the form:

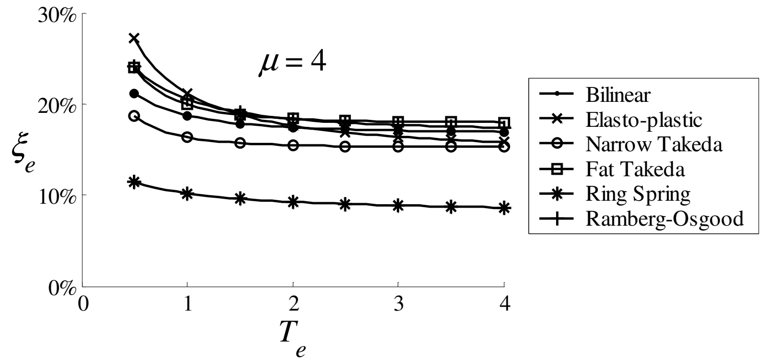
$$\xi_e = 0.05 + a\left(1 - \frac{1}{\mu^b}\right)\left(1 + \frac{1}{(T_e + c)^d}\right) \tag{1}$$

where a,b,c,d are constants, T_e is the effective period at the design displacement, and ξ_e is the equivalent viscous damping corresponding to the design ductility μ. Equation (1) was calibrated with inelastic time-history analysis assuming 5% tangent stiffness proportional damping. The 0.05 represents the initial level of elastic damping for μ = 1; for higher ductility levels, the secant-stiffness modification (from step 2 of the project described above), is incorporated directly into the calibration of the second half of the equation. In design applications for which 5% tangent-stiffness proportional damping is not considered to be appropriate, the parameters in equation (1) would require re-calibration.

Figure 8 shows average results from the calibration in two forms: variation with displacement ductility



(a) Equivalent damping vs ductility for effective period T_e = 1.0 seconds



(b) Equivalent damping vs period for displacement ductility μ = 4

Figure 8 Equivalent viscous damping for different hysteresis rules and 5% tangent-stiffness damping

and variation with effective period. It will be seen that there are significant differences in effective damping for the different hysteresis rules – particularly between the three rules representing reinforced and prestressed concrete. Designing on the assumption that one relationship would be appropriate for all hysteresis rules would clearly be very crude.

4 CONCLUSIONS

There are significant differences between DFFBD and DDBD. It was shown that, as currently formulated, DFFBD requires significantly more design effort than DDBD, though DFFBD can be reformulated so that the design effort is similar. Rational reasons were advanced for distributing seismic forces between structural elements based on secant stiffness to the design displacement, (as in DDBD) rather than on initial stiffness (as in DFFBD). It was shown that conclusions from earlier time-history analyses may be suspect because of the use of initial-stiffness proportional elastic damping, rather than tangent-stiffness proportional damping. Analyses using tangent-stiffness damping indicate that commonly accepted relationships between elastic and inelastic displacements are inappropriate. Finally, a brief summary of results calibrating damping/ductility relationships for DDBD show that, contrary to the results of earlier studies, the results are significantly affected by the hysteresis rule, and hence by structural material type.

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