Answer Key

Chapter 1 Limits and Continuity

1.1 Rate of Change

1. C 2. B 3. (a) -1.3 ft/sec (b) 0.9 ft/sec

1.2 The Limit of a Function and One Sided Limits

1. C 2. D 3. D 4. B 5. 1 6. 2 7. 4 8. DNE 9. 3 10. $\frac{\pi}{4}$

11. 10

1.3 Calculating Limits Using the Limit Laws

1. D 2. C 3. B 4. A 5. D 6. D 7. A 8. 4 9. $\frac{1}{\sqrt{2x+1}}$

10. $5/2$ 11. 2

1.4 Properties of Continuity and Intermediate Value Theorem

1. D 2. C 3. B 4. D 5. $a=\pi$, $b=\frac{3\pi}{4}$ 6. $\frac{1}{2}$ 7. 6

1.5 Limits and Asymptotes

1. B 2. C 3. C 4. A 5. D 6. C 7. (a) $x=2$ (b) $y=0$

8. (a) $x=-2$ (b) $y=0$

2.4 Derivatives of Trigonometric Functions

1. B 2. C 3. D 4. A 5. C 6. B 7. B 8. $-4\sqrt{2}+3$ 9. $a=-1$, $b=\pi$

2.5 Derivatives of Exponential and Logarithmic Functions

1. C 2. C 3. A 4. B 5. D 6. D 7. C 8. B 9. 10 10. x

2.6 Tangent Lines and Normal Lines

1. C 2. C 3. B 4. C 5. D 6. A 7. (a) 30 (b) $y=\frac{1}{2}x+\frac{15}{2}$ (c) 7.5

2.7 Implicit Differentiation

1. A 2. C 3. B 4. D 5. D 6. B 7. C 8. (a) $\frac{dy}{dx}=\frac{3x^2-y}{x-2y}$

(b) At $(1,-1)$ $y+1=\frac{4}{3}(x-1)$, at $(1,2)$ $y-2=-\frac{1}{3}(x-1)$ (c) $x=0.822$ and $x=-0.709$

9. (a) $\frac{dy}{dx}=\frac{y-2x}{2y-x}$ (b) At $(2,-1)$ $y+1=\frac{5}{4}(x-2)$, at $(2,3)$ $y-3=-\frac{1}{4}(x-2)$ (c) $x=\pm\frac{2\sqrt{21}}{3}$

2.8 Derivatives of an Inverse Function

1. C 2. B 3. A 4. D 5. D 6. (a) $y-2=\frac{1}{4}(x+1)$ (b) $h(1)=3$, $h'(1)=-4$ (c) $-\frac{1}{4}$

2.9 Derivatives of Inverse Trigonometric Functions

1. D 2. C 3. A 4. B 5. (a) $x^{\tan^{-1}x}(\frac{\tan^{-1}x}{x}+\frac{\ln x}{1+x^2})$ (b) $y-1=\frac{\pi}{4}(x-1)$

2.10 Approximating a Derivative

1. C 2. (a) $19.2^{\circ}F/\text{mon}$ (b) $23.5^{\circ}F/\text{mon}$ (c) $26.472^{\circ}F/\text{mon}$

Chapter 2 Differentiation

2.1 Definition of Derivatives and the Power Rule

1. A 2. B 3. D 4. D 5. C 6. C 7. B 8. $m=-2/3$, $k=-8/3$

9. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left[f(x)+x^3h-xh^3-f(h)\right]-f(x)}{h}=\lim_{h\to 0}\frac{x^3h-xh^3-f(h)}{h}$

$=\lim_{h\to 0}\frac{h(x^3-xh^2)}{h}-\lim_{h\to 0}\frac{f(h)}{h}=\lim_{h\to 0}\frac{\not{h}(x^3-xh^2)}{\not{h}}-1=x^3-1$

10. (a) 1 (b) 2 (c) No (d) $a=1/8$, $b=4$

2.2 The Product and Quotient Rules and Higher Derivatives

1. A 2. C 3. B 4. B 5. C 6. D 7. D 8. -4 9. $1/8$

2.3 The Chain Rule and the Composite Functions

1. D 2. D 3. C 4. A 5. 1 6. 13 7. $\frac{13}{16}$ 8. 20 9. $-\frac{5}{16}$

10. (a) $f(g(x))=2x$, $\frac{d}{dx}\left[f(g(x))\right]=\frac{d}{dx}\left[2x\right]\Rightarrow f'(g(x))\cdot g'(x)=2\Rightarrow g'(x)=\frac{2}{f'(g(x))}$

(b) $f'(x)=1+\left[f(x)\right]^2\Rightarrow f'(g(x))=1+\left[f(g(x))\right]^2=1+\left[2x\right]^2=1+4x^2$

Therefore, $g'(x)=\frac{2}{f'(g(x))}=\frac{2}{1+4x^2}$.

Chapter 3 Applications of Differentiation

3.1 Related Rates

1. B 2. D 3. C 4. C 5. A 6. D 7. (a) 1 (b) $\frac{9\sqrt{5}}{5}$ units/sec (c) $-\frac{1}{5}$

8. (a) $-\frac{1}{48\pi}$ ft /min (b) $r=\sqrt{50y-y^2}$ (c) $-\frac{7}{1152\pi}$ ft /min

9. (a) $\frac{3}{2}$ ft /sec (b) -7 ft^2 /sec (c) $-\frac{1}{8}$ rad /sec 10. (a) $\frac{dy}{dx}=-\frac{3y}{4y+3x}$ (b) $(0,-\frac{\sqrt{2}}{2})$, $(0,\frac{\sqrt{2}}{2})$ (c) $\frac{3}{2}$

3.2 Position, Velocity, and Acceleration

1. C 2. B 3. D 4. C 5. A 6. B 7. D 8. (a) $v(t)=4(t-2)^2(t-5)$
$a(t)=12(t-2)(t-4)$ (b) $t=4$ (c) $t>5$ (d) $2<t<4$ (e) $2<t<4$ or $t>5$

3.3 Roll's Theorem and the Mean Value Theorem

1. D 2. C 3. C 4. B 5. B 6. (a) Yes. Since $v(50) = -1.2 < -1 < -0.4 = v(45)$, the Intermediate Value Theorem guarantees a t in $(45, 50)$ so that $v(t) = -1$
(b) Since $v(5) = v(20)$, the MVT guarantees a t in $(5, 20)$ so that $a(t) = v'(t) = 0$. The smallest instances that the acceleration of the car could equal zero is 1.

3.4 The First Derivative Test and the Extreme Values of Functions

1. D 2. A 3. B 4. D 5. B 6. A 7. B 8. C 9. C 10.D
11. A 12. D 13. (a) f attains a relative minimum at $x = 1$, because f' changes from negative to positive at $x = 1$. (b) f attains a relative maximum at $x = -2$, because f' changes from positive to negative $x = -2$. (c) The absolute maximum occurs at $x = 7$.

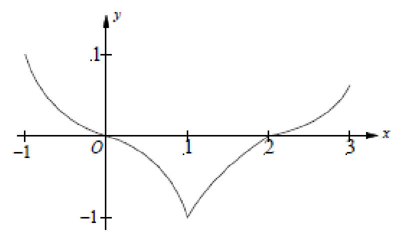
3.5 The Second Derivative Test

1. B 2. D 3. C 4. D 5. A 6. D 7. A 8. C 9. B 10. C
11. B 12. (a) $x = -1,\ 1,$ and 4 (b) $x = 6$ (c) $y - 2 = -2(x + 2)$
13. (a) $y + 1 = 2(x - 1)$ (b) We don't have sufficient information as to whether f'' changes sign at $x = 1$.
(c) $y = 3$ (d) g has a local minimum at $x = 1$.

3.6 Curves of f, f', f'' and Curve Sketching

1. C 2. C 3. B 4. A 5. D 6. C 7. C 8. (a) $x = 0,\ 2,$ and 5
(b) $0 < x < 2$ and $5 < x < 8$ (c) $0 < x < 1$ and $3 < x < 7$ (d) $x = 2$ (e) $x = 1,\ 3,$ and 7

9. (a) f has a relative minimum at $x = 1$. (b)



(c) h has a relative maximum at $x = 0$.
h has a relative minimum at $x = 2$.
(d) The graph of h has a POI at $x = 1$.

3.7 Optimization Problems

1. D 2. A 3. C 4. B
5. (a) $A = \frac{1}{4}k\sqrt{64 - k^2}$ (b) $k = 4\sqrt{2}$ 6. (a) $y = -2zx + z^2 + 3$ (b) $z = 1$

3.8 Tangent Line Approximation and Differentials

1. C 2. D 3. C 4. A 5.B 6. C 7. D 8. (a) $y = -\frac{1}{2}x + 1$ (b) 0.95
(c) $f^{-1} = \sin^{-1}\left[\ln\left|\frac{2}{x} - 1\right|\right]$
9. (a) $f'(0) \approx \frac{f(1) - f(-2)}{1 - (-2)} = -\frac{2}{3}$ (b) $y = \frac{9}{5}x - \frac{27}{5}$ (c) $y = 2x - 5$ (d) If the graph is CU, the tangent line approximation is smaller than the real value. Therefore, $f(5) \ge \frac{9}{5}(5) - \frac{27}{5} = \frac{18}{5}$.
(e) The secant line connecting $(1, -3)$ and $(6, 7)$ lies above the graph for $1 \le x \le 6$.
Therefore, $f(5) \le 2(5) - 5 = 5$.
10. (a) $y = \frac{5}{12}x + \frac{7}{4}$ (b) $h''(3) = \frac{1}{4}\left[\frac{2f''(2) - f'(2)}{8}\right]$. $h''(3)$ is negative, since $f''(2) < 0$ and $f'(2) > 0$.
(c) If the curve is CD, tangent line lies above the curve and the secant line lies below the curve.
Therefore, $h(2) \le \frac{5}{12}(2) + \frac{7}{4} = \frac{31}{12}$. Equation of secant line is $y = x$. Thus, $h(2) \ge 2$.

4.1 Antiderivatives and Indefinite Integrals

1. B 2. D 3. C 4. C 5. -7

4.2 Riemann Sums

1. C 2. D 3. C 4. A 5. A 6. B

4.3 Definite Integral and Area Under a Curve

1. A 2. C 3. B 4. D 5. D 6. A 7. C 8. C 9. B 10. D

11. (a) $A = \frac{1}{\sqrt{k}} \int_1^2 \sqrt{x-1}\, dx$ (b) $k = \frac{1}{9}$

(c) The tangent line is $y = \frac{1}{2\sqrt{k}} x$. Thus the tangent line passes through $(0,0)$.

12. (a) 1 (b) 2π (c) $-\frac{5}{2}$ (d) $2\pi + \frac{11}{2}$

4.4 Properties of Definite Integral

1. C 2. D 3. C 4. B 5. (a) 10 (b) 3 (c) -18 (d) -10 (e) 7

6. (a) $3n$ (b) $2n-1$ (c) $k = \frac{3}{2}$

4.5 Trapezoidal Rule

1. A 2. C 3. D 4. C 5. B 6. 0.527 7. 21

4.6 Fundamental Theorem of Calculus Part 1

1. C 2. C 3. B 4. C 5. B 6. A 7. (a) $-1 \le x \le 3$ (b) 2 (c) $x = 3$

8. (a) 0 (b) $x = 2$ and 6 (c) $\frac{1}{2}$ (d) There are two values of c.

4.7 Fundamental Theorem of Calculus Part 2

1. B 2. C 3. A 4. C 5. (a) 15 (b) 12 (c) 19 (d) 17

6. (a) $f(-3) = \frac{13}{2}$, $f(7) = \frac{15}{2} + 2\pi$ (b) $y = x + 1$ (c) $1 < x < 7$ (d) $-1 < x < 4$

7. (a) $g(0) = 2$, $g'(0) = 2$, $g''(0) = 1$ (b) $-2 < x < 1$ and $7 < x < 9$ (c) $-2 < x < 4$ and $8 < x < 9$

8. (a) $h(8) = 2$, $h'(6) = -2$, $h''(4) = -2$ (b) h has a relative minimum at $x = 1$.
h has a relative maximum at $x = 5$. (c) $y - 11 = 4(x - 35)$

4.8 Integration by Substitution

1. D 2. D 3. C 4. A 5. B 6. B 7. C 8. 5

4.9 Integration of Exponential and Logarithmic Functions

1. B 2. C 3. D 4. B 5. A 6. A 7. D

8. (a) $C = 45{,}000$, $k = -1.861$ (b) 174,069

Chapter 5 Applications of Integration

5.1 Area of a Region Between Two Curves

1. C 2. B 3. C 4. D Q 5. (a) $\frac{e}{2}+\frac{1}{2e}-1$ (b) $\frac{e}{2}-\frac{1}{2e}-1$

5.2 Volumes by Disk and Washers

1. C 2. D 3. B 4. C 5. B 6. A 7. D 8. A

9. (a) $y=-x+1$ (b) 2.670 (c) $V=\pi\int_0^{2.313}\left(\left[(2-(x^3-2x^2-x+\cos x)\right]^2-\left[2-(-x+1)\right]^2\right)dx$

10. (a) $\ln 4+\frac{1}{e}$ (b) 2.225π (c) 8.348π 11. (a) $V=\pi\left[3-\frac{3}{k^3+1}\right]$ (b) 1

5.3 Volumes of Solids with Known Cross Sections

1. B 2. D 3. A 4. C 5. A 6. C 7. D 8. (a) 4 (b) 24π (c) 3

9. (a) Volume $=\int_0^{12}\pi\left[\frac{D(x)}{2}\right]^2 dx$ (b) Volume $\approx 19.386\text{ m}^3$

(c) Yes. Since $D(2)=D(8)=1.5$, MVT guarantees that there is at least one x in $(2,8)$ such that $D'(x)=0$.

10. (a) $\int_0^{1.5}[f(x)-g(x)]\,dx$ (b) $\int_{1.5}^{4}[g(x)-f(x)]\,dx$ (c) 3.776 (d) 16.584

5.4 The Total Change Theorem (Application of FTC)

1. B 2. C 3. C 4. D 5. A 6. (a) 13.925 (b) 116.075

(c) $f(t)=50+8t-\int_0^t \frac{1}{2}t^{2/3}\,dt$ (d) $t=64$ 7. (a) $P'(3)=-2.041$. The rate at which granules of plastic is changing is decreasing by 2.041 tons per hour per hour at time $t=3$ hours.

(b) $t=4.550$ (c) 11.532 tons

5.5 Distance Traveled by a Particle Along a Line

1. B 2. D 3. D 4. C 5. A 6. B 7. B 8. A 9. C

10. (a) The particle is moving to the left at time $t=2$. (b) $a(2)=-8\sin(3)+\cos(3)$. No. The velocity of the particle is decreasing at time $t=2$, since $a(2)<0$.

(c) Yes (d) 3.991 (e) 4.921 (f) 1.992

11. (a) -14 (b) $2/3<t<4$ (c) -4 (d) -5 (e) -10 (f) 17.037

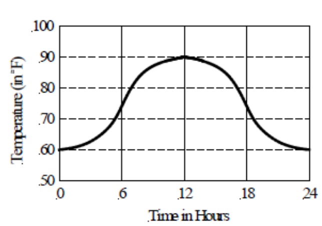
12. (a) 0 (b) 1 (c) $2<t<6$ (d) $0<t<3$ (e) On the interval $2<t<3$, $v\ (=s')<0$ and $a\ (=s'')<0$. Since v and a have the same sign on $2<t<3$, the speed of particle is increasing.

(f) $a\ (=s'')>0$, on the interval $3<t<8$ since on this interval the curve of s is concave upward.

5.6 Average Value of a Function

1. C 2. D 3. C 4. B 5. B 6. A 7. (a) -1 (b) 0 (c) -1

8. (a)



(b) 78° F
(c) $7.298\le t\le 16.702$
(d) \$7.32

5.7 Length of a Curve (Distance Traveled Along a Curve) BC

1. B 2. C 3. A 4. D 5. C 6. D

7. (a) $P(x,y)=(\cot^2\theta, \cot\theta)$ (b) 3.168 (c) 1.442 units /min

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Chapter 6 Techniques of Integration

6.1 Basic Integration Rules

1. B 2. D 3. C 4. A 5. D 6. $\pi\left[\ln(\sqrt{2}+1)-\sqrt{2}/2\right]$

6.2 Trigonometric Integrals

1. B 2. C 3. B 4. D 5. A 6. $\frac{1}{3}$

6.3 Trigonometric Substitutions

1. D 2. B 3. C 4. D 5. C 6. (a) 15.904 (b) $27\int_0^{\pi/2}\cos^4\theta\, d\theta$

6.4 L'Hospital's Rule

1. B 2. D 3. A 4. D 5. C 6. C 7. B 8. 3 9. $-\frac{1}{2}$

6.5 Integration by Partial Fractions BC

1. C 2. C 3. A 4. D 5. (a) $\int \frac{-dx}{x(x-1)}$ (b) $\ln\left|\frac{\cos\theta}{\cos\theta-1}\right|+C$

6.6 Integration by Parts BC

1. C 2. B 3. B 4. C 5. C 6. D 7. A 8. D 9. A 10. $\frac{\pi}{2}-1$

6.7 Improper Integrals BC

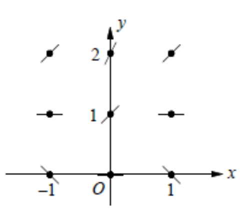
1. D 2. B 3. C 4. A 5. B 6. C

7. (a) $\int_1^\infty f(x)\,dx=\lim_{b\to\infty}\int_1^b \frac{x}{\sqrt{x^2+1}}\,dx=\lim_{b\to\infty}\left[\sqrt{x^2+1}\right]_1^b=\lim_{b\to\infty}\left[\sqrt{b^2+1}-\sqrt{1+1}\right]=\infty$ (b) 1

Chapter 7 Further Applications of Integration

7.1 Slope Field

1. B 2. C 3.



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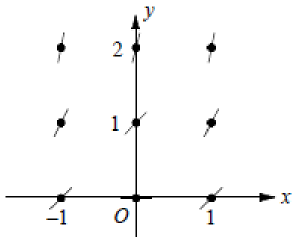

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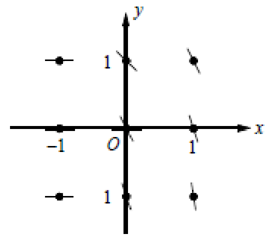
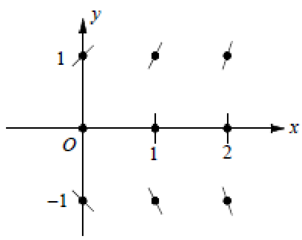


7.2 Separable Differential Equations

1\. D 2. B 3. C 4. A 5. C

6\. (a)



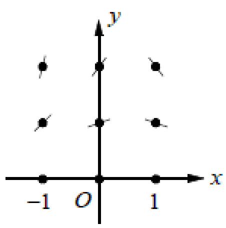
(b) $y = \frac{2\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$, $f(1.2) \approx 1.963$

(c) $y = \sqrt{x^2 + 2x}$

(d) $f(1.2) = 1.959$

7\. (a) $y = \frac{3}{e^2}x + 2$ (b) $f''(0) = \frac{2e^2 - 9}{e^4}$ (c) $y = \ln\left|x^2 + 3x + e^2\right|$.

8\. (a)



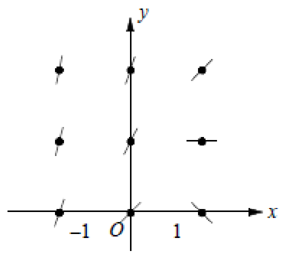
(b) $\frac{d^2y}{dx^2} = \frac{1}{3}\left[-2y^2 + (2y - 4xy)\frac{y^2(1-2x)}{3}\right]$

(c) $\left.\frac{dy}{dx}\right|_{(\frac{1}{2},4)} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(\frac{1}{2},4)} = -\frac{32}{3} < 0$

Therefore, f has a relative maximum at $x = 1/2$.

(d) $y = \frac{3}{x^2 - x + 1}$

9. (a)



(b) $\dfrac{d^2y}{dx^2} = -2x + y - 1$ If the curve is CD, $y'' < 0$.

$-2x + y - 1 < 0 \;\Rightarrow\; y < 2x + 1$

Therefore, solution curves will be concave down on the half-plane below the line $y = 2x + 1$.

(c) $\left.\dfrac{dy}{dx}\right|_{(0,-1)} = 0$ and $\left.\dfrac{d^2y}{dx^2}\right|_{(0,-1)} < 0$. Therefore, f has a relative maximum at $(0,-1)$.

(d) $m = 2$, $b = 1$

7.3 Exponential Growth and Decay

1. D 2. B 3. C 4. A 5. (a) $V = 16e^{-0.00866t}$ (b) 0.03464 oz/sec (c) $t = 240$ seconds

7.4 Logistic Equations BC

1. D 2. C 3. B 4. A 5. D

6. (a) $y = e^{2t - \frac{t^2}{4} - 3}$ (b) 0 (c) $\lim_{t\to\infty} g(t) = 5 \;\Rightarrow\; \lim_{t\to\infty} g'(t) = 0$ (d) POI at $y = 5/2$, Slope $= 5/4$