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Estimation of the Tax Rates Based on Vehicle Miles Traveled Using Stochastic Models

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ESTIMATION OF THE TAX RATES BASED ON VEHICLE MILES TRAVELED USING STOCHASTIC MODELS

by

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ABSTRACT

Estimation of the Tax Rates Based on Vehicle Miles Traveled

Using Stochastic Models

by

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In this thesis, we shall study the alternative revenue collection system which is based on the vehicle miles traveled (VMT). In various studies, it has been found that the existing revenue collection system based on gas/fuel tax is not an appropriate option in the longer run. The main reasons include no effective tax process for vehicles based on alternative fuel vehicle, no effective changes in tax due to economical inflation, and more highway expenditure than generated revenue. Our main objective is to estimate the VMT tax rates that should be charged in order to generate same amount of revenue generated by gas tax.

It is apparent that the amount of gas consumed is dependent on the behavior of

gas prices which fluctuate daily. Also, VMT is dependent upon the amount of gas consumed and thus it is also dependent on the gas prices. Different mathematical models based on stochastic differential equations shall be developed for gas prices, the amount of gas consumed, and VMT. Parameters for all the proposed models shall be estimated by using maximum likelihood principle technique and the historical data. As result of our simulation, we have found that VMT tax rate should be approximately 2.5 cents per mile in order to generate same amount of revenue as generated by current system. This VMT tax rate is close enough to the estimated value of 2 cents per mile by Nevada Department of Transportation.

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CHAPTER 1

INTRODUCTION

1.1 Summary

In this chapter, advantages and limitations of the current road based on fuel taxes revenue system has been discussed. It also discusses the need for an alternative road revenue system based on road usage or Vehicle Miles Traveled (VMT). Both, the tax system based on VMT and current fuel-tax-based revenue system have been also compared in this chapter.

1.2 Introduction

The current road revenue system is based upon the fuel tax estimated based on fuel consumption. Due to various reasons, such as political influence and public acceptance, the fuel tax has not increased to meet the demand to maintain the existing highway infrastructure. As per many economic studies and projections, revenue generated by current fuel-tax-based system is neither sufficient to improve nor to maintain the existing road infrastructure. Therefore, various alternatives were evaluated to address the problems with the existing road revenue system. Vehicle Miles Traveled (VMT) is an potential alternative to overcome the existing shortcomings of the current road revenue system.

1.3 Need for Alternatives

Current road revenue is not sufficient to maintain and develop the highway infrastructure of United States. The past trend of the fuel tax growth demonstrates that tax rate has remained more or less the same from 1994 until 2004 [1]. Due to increment in the percentage of alternative-fuel vehicles, such as electric vehicles, fuel tax has been impacted. In addition, the increase in the total number of vehicles on the road causes more damage to the road. In a report by the American Association of State Highway and Transportation Officials (AASHTO, 2007d), future revenue requirements for the highway and transportation infrastructures in the U.S for the years from 2005 to 2021 [3] have been projected. This report has analyzed and forecasted the revenue requirements to maintain and to improve the infrastructures, based on future needs.

1.4 Current Road Revenue System

The current road revenue system is based upon the tax based on gas consumed. In this system, the amount of tax is directly proportional to the amount of gas consumed by the vehicle being used. This system is not sensitive towards fuel efficiency of the vehicle. Vehicles having high fuel efficiencies consume less gas and pay less tax in contrast to vehicles with lower fuel efficiencies; however, but road damage and infrastructure usage is same for both types of vehicles.

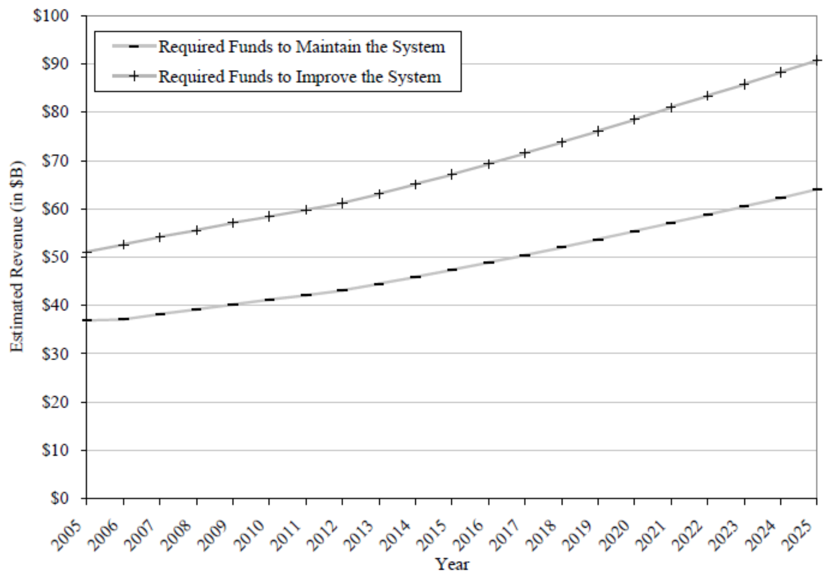

Figure 1.1: Funding required from fuel taxes for future transportation needs (Source: Policy for highway financing: Gasoline taxes and other Alternatives, Vinod Vasudevan, 2008)

1.5 Advantages to the Existing System

The current system has been proven robust enough for working in the past for decades. So far, significant development in transportation is done due to the revenue collected by this system. Some of the very useful advantages of current system are:

1. Less administrative costs due to the fuel tax is being collected directly by wholesale sellers and not by individual consumers.

2. There is no revenue loss in collection of fuel taxes due to corruption as certain type of colored dye is added to the fuel to differentiate between fuel that is taxed and not taxed.

3. The current system is working for decades.

4. The system is transparent as it is directly proportional to the individuals fuel consumption.

1.6 Problems in the Current System

There are many issues that are not taken into account in the existing system. The existing fuel tax system is more or less insensitive to impact of inflation. Figure 1.2 shows the fuel tax rate (cents/gal) from 1981 to 2004 [1], and Figure 1.3 shows the trend of fuel price from Nov-1994 to Nov-2011 [2]. Gas prices have changed over these past years while gas tax has not changed over past years. It is visible in Figure 1.3, the federal gas tax has not increased much since 1993. Figure 1.3 shows the fuel price trend from Nov-1994 to Nov 2011

Impact of Increasing Fuel Efficiency

Due to technical advances, automotive fuel efficiency has increased significantly. Increased fuel efficiency has reduced the fuel consumption related to its operating cost. Reduced operating cost is a major factor in the increment of vehicle sales. Hence, more number of vehicles on road requires more funds to maintain the same road due to more damage. Figure 1.4 describes the trend of an increase in fuel efficiency. In a report by the U.S. Energy Information Administration (EIA, 2005a), an increment in fuel efficiency is estimated from 1951 to 2021 [1]. The estimate shows increase in fuel efficiency for all types of vehicles. For most vehicles, efficiency is predicted to be

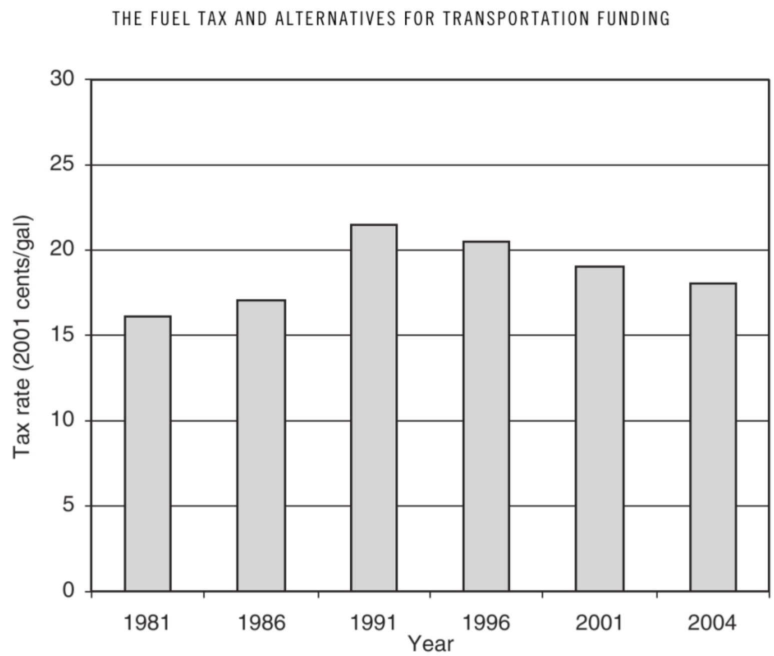

Figure 1.2: Sales-weighted constant-dollar average state gasoline tax rate (Source: FHWA 1987; FHWA 1997; FHWA 2005a) [1]

Figure 1.3: The fuel price trend from Nov-1994 to Nov 2011. (Source: NDOT 2011) [2]

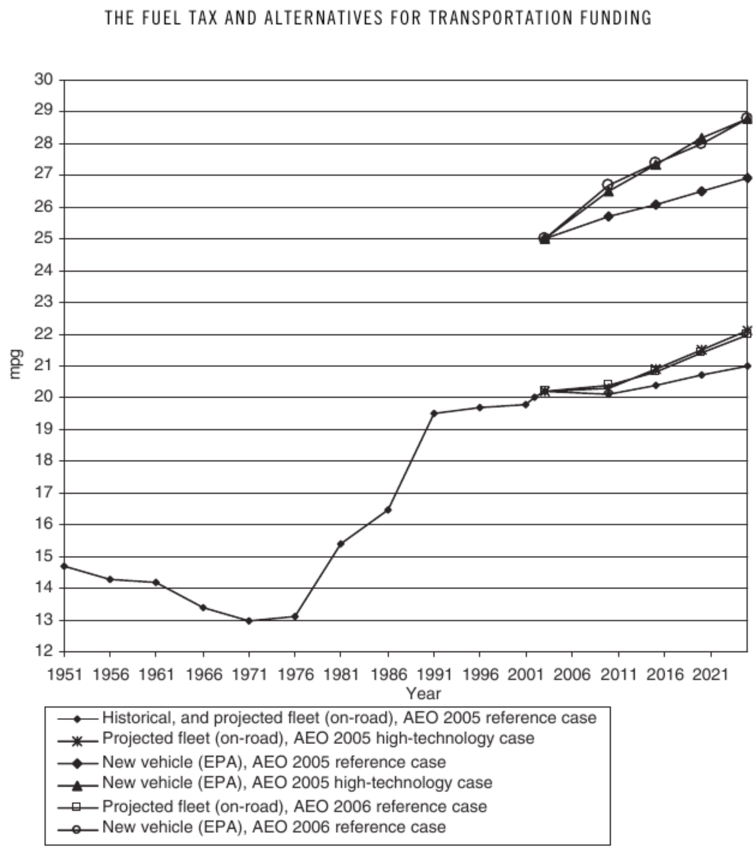

Figure 1.4: The upcoming trend of an increase in fuel efficiency (Sources: U.S. Energy Information Administration (EIA, 2005a; EIA, 2005b)[1]

approximately 29 miles per gallon in 2021 [1].

Alternative Fuel Vehicles

Nowadays, vehicles based on alternative fuel vehicles are becoming popular choices among users. For such vehicles, consumed fuel is not directly in proportional to the distance traveled on the road. However, such vehicles cause the same road damage and congestion as fuel based vehicles.

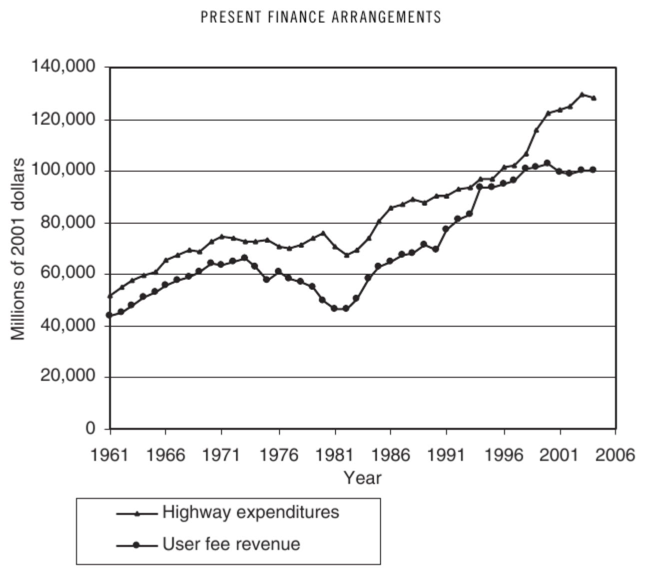

Figure 1.5: The highway user fee revenue and highway expenditures from 1961 to 2004. (Source: FHWA) [1]

Impact of Accidents, Emission, and Congestion

Figure 1.5 shows the highway user fee revenue and highway expenditures from 1961 to 2004 [1]. In the current road revenue system, revenue loss due to such factors like accidents, emissions, congestion etc. are not taken into consideration. With every accident, there is a cost of insurance, hospitalization, time of clearance etc. Similarly, increased congestion results in extended travel time and impacts productivity. This loss of productivity also has costs associated with it.

1.7 Proposed Solution: Road Revenue Based On Vehicle Miles Traveled

The Vehicle Miles Traveled (VMT) is one of the available alternatives for the road revenue collection system. As discussed in previous section, the existing revenue collection system based on a fuel tax is not an appropriate option in the long run. The main goal of the VMT study is to evaluate this alternative to collect road tax based upon road usage of a vehicle instead of fuel usage. With the VMT system, the tax will be directly proportional to the distance traveled by vehicle. In such case, vehicles with different fuel efficiencies would pay the same amount of tax based upon their road usage or miles driven on the road.

Advantage of VMT (Vehicle Mile Traveled)

VMT has potential to address the gap between required and generated revenue to maintain the current infrastructure. Some of the advantages of VMT are:

1. More transparent to users/public than any other service, for example, electric or phone utilities.

2. VMT is independent of fuel type as tax is based on travel distance.

3. State and local government will have greater ownership of the projects.

4. The expenditure for highway projects can be directly associated with revenue generated by their usage.

5. Parameters like Inflation can be taken into account in VMT fee calculations.

1.8 Conclusion

VMT is one of the most promising alternatives to the current road revenue system. It is comparatively more transparent in terms of payment as per usage than the current system. Various state and universities have identified VMT as one of the potential solutions that could replace the current fuel tax system.

CHAPTER 2

MATHEMATICAL MODELING

2.1 Summary

In this chapter, various mathematical models has been discussed to study the amount of gas consumed, gas prices and VMT. Models for each variable has been chosen intuitively based on trend of data over past years. A complete framework to estimate VMT tax rate has been proposed. The VMT tax rate has been estimated to generate same amount of revenue generated based on fuel tax.

2.2 Problem

As of now, road revenue system is based on revenue (R_g) generated using Gas Tax. If the total amount of gas consumed is G gallons and gas tax rate is (r_g) dollar/gallons, revenue generated can be given as $R_g = r_g$ x G dollars. Though gas tax rate (r_g) is more or less fixed over past 10 years but price of gas (Fixed price + Tax) has increased a lot. Price of gas also have an potential impact to the amount of gas consumption. If price of gas increases the gas consumption decreases and if price of gas decreases the consumption of gas increases. Here, a new system is being studied to collect road revenue based on number of miles driven by each vehicle. This, Vehicle

Miles Traveled (VMT) based revenue R_v can be estimated based on the number of miles driven by vehicles. The proposed new VMT tax rate is r_v dollar/mile. Hence revenue generated based on $V(VMT)$ can be estimated as $R_v = r_v$ x V. Data from the past years has information related to monthly and yearly growth of VMT. The main objectives of this work are: A) to model revenue R_g generated using gax tax based on models of gas price and amount of gas consumed, B) to model VMT based revenue R_v based on model of growth in VMT, and c) to identify VMT tax rate $r_v(t)$ such that revenue generated by VMT R_v must be greater than or equal to revenue generate by gas tax R_g.

2.3 Model Of Gas Price

In this section, we consider two popular models about the processes $P(t)$ of gas prices.

2.3.1 Model 1: Gas Model Based Mean-reverting Log-normal Process

In our first model, we assume that $P(t)$, gas prices, is given by

$$P(t) = e^{X(t)},$$

where $X(t)$ follows a mean-reverting dynamics([4]):

$$dX(t) = \kappa(X_\infty - X(t))dt + \sigma dW(t). \tag{2.1}$$

Here κ is the speed of adjustment of $X(t)$ towards to its long term level X_∞, σ is the volatility, and $W(t)$ is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$. It means that the gas price process $P(t)$ is a mean-reverting log-normal process. By Itô Lemma, we have the dynamics for the gas price process $P(t)$:

$$dP(t) = \kappa(\mu - \log(P(t)))P(t)dt + \sigma P(t)dW(t), \tag{2.2}$$

where

$$\mu = X_\infty + \frac{\sigma^2}{2\kappa}.$$

This stochastic differential equation is also known as the Schwartz model for the nature gas price [5]. Let

$$Z(t) = e^{\kappa t}X(t). \tag{2.3}$$

By Itô Lemma again, we have the following dynamics for process $Z(t)$:

$$dZ(t) = e^{\kappa t}(\kappa X_\infty dt + \sigma dW(t)). \tag{2.4}$$

Thus for $t \geq t_0 \geq 0$, we have

$$Z(t) = Z(t_0) + \int_{t_0}^{t} e^{\kappa s}(\kappa X_\infty ds + \sigma dW(s)),$$

where $Z(t_0) = X(t_0) = \log(P(t_0))$. Hence,

$$X(t) = X(t_0)e^{-\kappa(t-t_0)} + X_\infty \left(1 - e^{-\kappa(t-t_0)}\right) + \sigma \int_{t_0}^{t} e^{-\kappa(t-s)} dW(s). \tag{2.5}$$

Finally, we have

$$P(t) = e^{\log(P(t_0))e^{-\kappa(t-t_0)} + X_\infty\left(1-e^{-\kappa(t-t_0)}\right) + \sigma\int_{t_0}^{t} e^{-\kappa(t-s)}dW(s)}. \tag{2.6}$$

By simple calculation, we obtain

$$\begin{aligned}
\mathrm{E}[P(t)] &= e^{\log(P(t_0))e^{-\kappa(t-t_0)} + X_\infty\left(1-e^{-\kappa(t-t_0)}\right) + \frac{\sigma^2}{2}\int_{t_0}^{t} e^{-2\kappa(t-s)}ds} \\
&= e^{\log(P(t_0))e^{-\kappa(t-t_0)} + X_\infty\left(1-e^{-\kappa(t-t_0)}\right) + \frac{\sigma^2}{4\kappa}\left(1-e^{-2\kappa(t-t_0)}\right)}.
\end{aligned}$$

It is obvious that

$$\mathrm{E}[P(t)] \to e^{X_\infty + \frac{\sigma^2}{4\kappa}}, \quad t \to \infty,$$

which means that the expected long-term value of gas price

$$P_\infty = e^{X_\infty + \frac{\sigma^2}{4\kappa}}.$$

2.3.2 Model 2: Gas Price Model Based on GBM

For the dynamics of gas price, process $P(t)$ based on geometric Brownian motion (GBM), we have the following stochastic differential equation [6]:

$$dP(t) = P(t)(\mu dt + \sigma dW(t)), \tag{2.7}$$

where $W(t)$ is a Brownian motion under the risk-neutral measure $\mathbb{Q}$, μ is the constant percentage drift term, and σ is the percentage volatility constant. Again, by Itô Lemma, we have for $Y(t) = \log(P(t))$

$$dY(t) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW(t).$$

Thus, for $t \geq t_0 \geq 0$,

$$Y(t) = Y(t_0) + \left(\mu - \frac{1}{2}\sigma^2\right)(t - t_0) + \sigma(W(t) - W(t_0)).$$

Hence,

$$P(t) = P(t_0)e^{\left(\mu - \frac{1}{2}\sigma^2\right)(t-t_0)+\sigma(W(t)-W(t_0))}. \tag{2.8}$$

We also have by simple calculation

$$\begin{aligned} \mathrm{E}[P(t)] &= P(t_0)e^{\mu(t-t_0)}, \\ \mathrm{Var}[P(t)] &= P^2(t_0)e^{2\mu(t-t0)}\left(e^{\sigma^2(t-t_0)} - 1\right). \end{aligned}$$

2.4 Model for Gas Consumed

Since the gas prices are stochastic processes, the amount $G(t)$ of gas consumed in gallons $G(t)$ should also be stochastic processes. It is intuitive that $G(t)$ will increase or decrease as the gas prices fluctuate. Also, the gas consumption should increase due to increasing number of cars. Based on these arguments, we propose the following model for $G(t)$:

$$dG(t) = \rho G(t)dt - \nu G(t)dP(t), \tag{2.9}$$

where ρ and ν are two constants. Let

$$\zeta(t) = \log(G(t)).$$

Using Itô's formula, we have for both of the models for gas price process:

$$d\zeta(t) = \left(\rho - \frac{1}{2}\sigma^2\nu^2 P(t)\right) dt - \nu dP(t).$$

Thus,

$$\zeta(t) = \zeta(0) + \int_0^t \left(\rho - \frac{1}{2}\sigma^2\nu^2 P(s)\right) ds - \nu(P(t) - P(0)).$$

Hence,

$$G(t) = G(0)e^{\int_0^t \left(\rho - \frac{1}{2}\sigma^2\nu^2 P(s)\right) ds - \nu(P(t) - P(0))}.$$

2.5 Model fo VMT

As for the amount $G(t)$ of gas consumed, we propose the following model for $V(t)$, the total vehicle miles traveled:

$$dV(t)/V(t) = \theta dt - \eta dP(t), \tag{2.10}$$

where θ and η are two constants, which can be interpreted as ρ and ν for the model for $G(t)$. Similarly, we can obtain

$$V(t) = V(0)e^{\int_0^t \left(\theta - \frac{1}{2}\sigma^2\eta^2 P(s)\right)ds - \nu(P(t)-P(0))}. \tag{2.11}$$

2.6 Revenue (R_g) based on Gas Tax

Gas Tax rate has been assumed constant r_g dollar/gallon, as it has not changed much over past 10 years [1]. Hence total revenue generated based on above gas consumption.

$$R_g(t) = \int_0^t r_g dG(t) = r_g(G(t) - G(0)). \tag{2.12}$$

2.7 Revenue (R_v) Based on VMT

Let $r_v(t)$ be the tax rate based on VMT at time t. It is apparent that the unit for r_v is dollar per mile. Then the total revenue (R_v) generated based on VMT can be given by:

$$R_v = r_v(t)V(t), \tag{2.13}$$

where $V(t)$ is the total number of miles modeled in §2.5.

2.8 VMT Tax rate (r_v) Estimation

The VMT tax rate r_v should be estimated according to the desired revenue, for example, the revenue generated by Gas Tax. Both the revenue R_g, R_v are random variables as they are based on gas price and VMT stochastic processes respectively. Hence, the expectation of R_g must be equal to the expectation of R_v for two types of revenue to be equal.

$$E[R_v] = E[R_g].$$

Then by (2.13), we have

$$E[r_v(t)V] = E[r_g G].$$

Since r_g is a constant and $r_v(t)$ is a deterministic function, we get the estimation formula for r_v:

$$r_v(t) = r_g \frac{E[G]}{E[V]} \tag{2.14}$$

2.9 Conclusion

In this chapter, we have identified a high-level overview to estimate VMT tax rate. As of now, the goal is to generate same amount of revenue using VMT as it being generated by gas tax. Expectation of revenue based on gas tax R_g and expectation of revenue generated using VMT R_v have been considered equal in order to estimate VMT tax rate r_v. The estimated VMT tax rate r_v can be regulated in order to address the game between gas tax revenue R_g and revenue required to maintain the existing infrastructure.

CHAPTER 3

ESTIMATION OF MODEL PARAMETERS

3.1 Summary

In this chapter, the maximum lilkehood estimation (MLE) technique will be employed to estimate the parameters of the various models proposed in previous chapter.

3.2 Estimation of the Parameters for Gas-Price model

Let p_j be the observation of $P(t)$ at time $t = t_j$ where $t_j = j\Delta t$ for $j = 0, 1, \ldots, M$ and $\Delta t = T/M$. In order to apply the maximum likehood estimation for parameters κ, μ, and σ in (2.2), we need to know the conditional pdf $f(p_j|p_{j-1})$ of $P(t_j)$ for given $P(t_{j-1}) = p_{j-1}$. It is not difficult to find $f(p_j|p_{j-1})$ analytically by (2.6). However, for simplicity, we shall use an approximation of $f(p_j|p_{j-1})$ obtained by discretizing the SDEs (2.2) and (2.7) for the gas models.

3.2.1 Model 1

. Discretaizing SDE (2.2) by Euler-Maruyama Scheme, we have

$$P(t_j) \approx P(t_{j-1}) + \kappa\Delta t(\mu - \log(P(t_{j-1})))P(t_{j-1}) + \sigma P(t_{j-1})Z_j, \tag{3.1}$$

where $Z_j = (W(t_j) - W(t_{j-1}) \sim N(0, \Delta t)$. Hence,

$$f(p_j|p_{j-1}) \approx \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{\left(p_j-\mu_j\right)^2}{2\sigma_j^2}},$$

where

$$\mu_j = p_{j-1} + \kappa\Delta t(\mu - \log(p_{j-1}))p_{j-1}, \quad \sigma_j = \sigma\sqrt{\Delta t}p_{j-1}.$$

Then the approximate log-likelihood function is

$$\begin{aligned} L(\sigma^2, \mu, \kappa) &= \sum_{i=1}^{M} \log(f(p_j|p_{j-1})) \\ &= -\sum_{i=1}^{M} \log(\sqrt{2\pi\Delta t}p_{j-1}) - \frac{M}{2}\log(\sigma^2) - \sum_{j=1}^{M} \frac{\left(p_j - \mu_j\right)^2}{2\sigma_j^2}. \end{aligned}$$

For

$$\frac{\partial}{\partial\sigma^2} L(\sigma^2, \mu, \kappa) = 0, \quad \frac{\partial}{\partial\mu} L(\sigma^2, \mu, \kappa) = 0, \quad \frac{\partial}{\partial\kappa} L(\sigma^2, \mu, \kappa) = 0,$$

we get the following linear system for κ, μ, and σ:

$$\begin{aligned} &\sigma^2 = \frac{1}{T} \sum_{i=1}^{M} \left(u_j - \kappa\Delta t(\mu - v_{j-1})\right)^2, \\ &\kappa\mu\Delta t \sum_{j=1}^{M} v_j - \kappa\Delta t \sum_{j=1}^{M} v_j^2 = \sum_{j=1}^{M} u_j v_j, \\ &T\kappa\mu - \kappa\Delta t \sum_{j=1}^{M} v_j = \sum_{j=1}^{M} u_j. \end{aligned}$$

where

$$u_j = \frac{p_j - p_{j-1}}{p_{j-1}}, \quad v_j = \log(p_{j-1}).$$

Solving the above system, we get the following estimators for κ, μ, and σ:

$$\hat{\kappa} = -\frac{\text{cov}(u, v)}{\Delta t \text{var}(v)},$$
$$\hat{\mu} = \frac{\bar{u} + \hat{\kappa}\Delta t \bar{v}}{\hat{\kappa}\Delta t},$$
$$\hat{\sigma} = \frac{\text{std}(u + \hat{\kappa}\Delta t v)}{\sqrt{\Delta t}}.$$

3.2.2 Model 2

Discretaizing SDE (2.7) by Euler-Maruyama Scheme, we have

$$P(t_j) \approx P(t_{j-1}) + P(t_{j-1})(\mu \Delta t + \sigma Z_j), \quad j = 1, 2, \ldots, M. \tag{3.2}$$

Hence,

$$f(p_j|p_{j-1}) \approx \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{\left(p_j - \mu_j\right)^2}{2\sigma_j^2}},$$

where

$$\mu_j = p_{j-1}(1 + \mu \Delta t), \quad \sigma_j = \sigma\sqrt{\Delta t} p_{j-1}.$$

By MLE, we get the following estimators for σ and μ:

$$\hat{\mu} = \frac{1}{T}\sum_{j=1}^{M} u_j,$$
$$\hat{\sigma}^2 = \frac{1}{T}\sum_{j=1}^{M} (u_j - \hat{\mu})^2,$$

where

$$u_j = \frac{p_j - p_{j-1}}{p_{j-1}}.$$

3.3 Estimation of the Parameters for Fuel Consumption

In this section, technique to estimate ρ and ν will be discussed for the amount of gas consumed $G(t)$. At this point of time, μ, σ and κ are known variables from the previous section for gas price models.

Discretaizing (2.9) by Euler-Maruyama Scheme, we have

$$G(t_j) \approx G(t_{j-1})(1 + \rho\Delta t - \nu(P(t_j) - P(t_{j-1}))).$$

Then by (3.1) or (3.2), we

$$G(t_j) \approx G(t_{j-1})\left(1 + \rho\Delta t - \nu\Delta t\phi_j P(t_{j-1})\right) - \sigma\nu G(t_{j-1})P(t_{j-1})Z_j.$$

where

$$\phi_j = \begin{cases} \kappa(\mu - \log(P(t_{j-1}))), & \text{for (3.1)} \\ \mu, & \text{for (3.2).} \end{cases}$$

Hence, the conditional pdf for $G(t_j)$ given $G(t_{j-1}) = g_{j-1}$ and $P(t_{j-1}) = p_{j-1}$ is

$$f(g_j|g_{j-1}, p_{j-1}) \approx \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{\left(g_j-\mu_j\right)^2}{2\sigma_j^2}},$$

where

$$\mu_j = g_{j-1}\left(1 + \rho\Delta t - \nu\Delta t\phi_j p_{j-1}\right), \quad \sigma_j = \sigma\nu\sqrt{\Delta t} g_{j-1} p_{j-1}.$$

Hence the approximate log-likelihood function is

$$L(\nu, \rho) = \sum_{i=1}^{M} \log\left(f(g_j|g_{j-1}, p_{j-1}\right) = -\sum_{i=1}^{M} \log\left(\sqrt{2\pi}\sigma_j\right) - \sum_{j=1}^{M} \frac{\left(g_j - \mu_j\right)^2}{2\sigma_j^2}.$$

For

$$\frac{\partial}{\partial\nu} L(\nu, \rho) = 0, \quad \frac{\partial}{\partial\rho} L(\nu, \rho) = 0,$$

we get the following linear system for ν and ρ:

$$-\frac{M}{\nu} + \frac{1}{\Delta t\sigma^2\nu^3} \sum_{j=1}^{M} \frac{\left(u_j - \rho\Delta t + \nu\Delta t\phi_j p_{j-1}\right)^2}{p_{j-1}^2} - \frac{1}{\sigma^2\nu^2} \sum_{j=1}^{M} \frac{\phi_j\left(u_j - \rho\Delta t + \nu\Delta t\phi_j p_{j-1}\right)}{p_{j-1}} = 0,$$

$$\sum_{j=1}^{M} \frac{\left(u_j - \rho\Delta t + \nu\Delta t\phi_j p_{j-1}\right)}{p_{j-1}^2} = 0,$$

where

$$u_j = \frac{g_j - g_{j-1}}{g_{j-1}}.$$

After eliminating ρ, we obtain

$$\left(\sigma^2 T x x'\right) \nu^2 + (MT\bar{v}\bar{w})\,\nu + (xy')^2 - (xx')\,(yy') = 0,$$

where

$$x = \left(\frac{1}{p_0}, \ldots, \frac{1}{p_{M-1}}\right), \quad y = \left(\frac{u_1}{p_0}, \ldots, \frac{u_m}{p_{M-1}}\right),$$
$$v = \left(\frac{\phi_1}{p_0}, \ldots, \frac{\phi_m}{p_{M-1}}\right), \quad w = \left(\frac{u_1}{p_0^2}, \ldots, \frac{u_m}{p_{M-1}^2}\right).$$

Solving the above quadratic equation for its positive root, we get the estimator for ν

$$\hat{\nu} = \frac{1}{2T\sigma^2 x x'}\left(-MT\bar{v}\bar{w} + \sqrt{(MT\bar{v}\bar{w})^2 + 4T\sigma^2 x x'\left((xx')\,(yy') - (xy')^2\right)}\right).$$

Then by the last equation of the system, we obtain the estimator for ρ

$$\hat{\rho} = \frac{\bar{w} + \hat{\nu}\Delta t\bar{v}}{\Delta t\bar{x}^2}.$$

3.4 Estimation of the Parameters for VMT

As discussed in previous chapter, the model for VMT is similar to the model of gas consumed but with different parameters $\theta,\ \eta$. Hence, similarly, we have the following estimators for θ and η:

$$\hat{\eta} = \frac{1}{2T\sigma^2 x x'}\left(-MT\bar{v}\bar{w} + \sqrt{(MT\bar{v}\bar{w})^2 + 4T\sigma^2 x x'\left((xx')\,(yy') - (xy')^2\right)}\right).$$

$$\hat{\theta} = \frac{\bar{w} + \hat{\nu}\Delta t \bar{v}}{\Delta t \bar{x^2}}.$$

3.5 Conclusion

In this chapter, we have developed the estimators for the all unknown parameter for the models presented in Chapter 2. Using these estimators and real time historical data, we can numerically compute the values of these parameters. Then these estimated values can be used to forecast future values of gas prices, gas consumed, and VMT.

CHAPTER 4

NUMERICAL RESULTS FOR PARATMETER ESTIMATION

4.1 Summary

In this chapter, we shall present numerical results for the estimated parameters of the models in Chapter 2 using their estimators developed in the previous Chapter.

4.2 Parameters Estimation for P

4.2.1 Data for P

The monthly gas price data for Nevada since Jan 1983 till Feb 2011 and the Annual gas price data for Nevada since 1984 till 2011 are displayed in Figs. 4.1–4.2, respectively.

4.2.2 Parameters for Model 1

Parameters for Model 1 from Monthly Data

Monthly data has been used since starting year 1983 till 2010. Starting from 1983 till 2010 whole data has been been divided into multiple sets shifting starting point by 5 years. Using these sets of data, parameters have been estimated in order to have better understanding. As per our SDE model 1, parameters $(\rho,\ \kappa)$ and variance (σ^2)

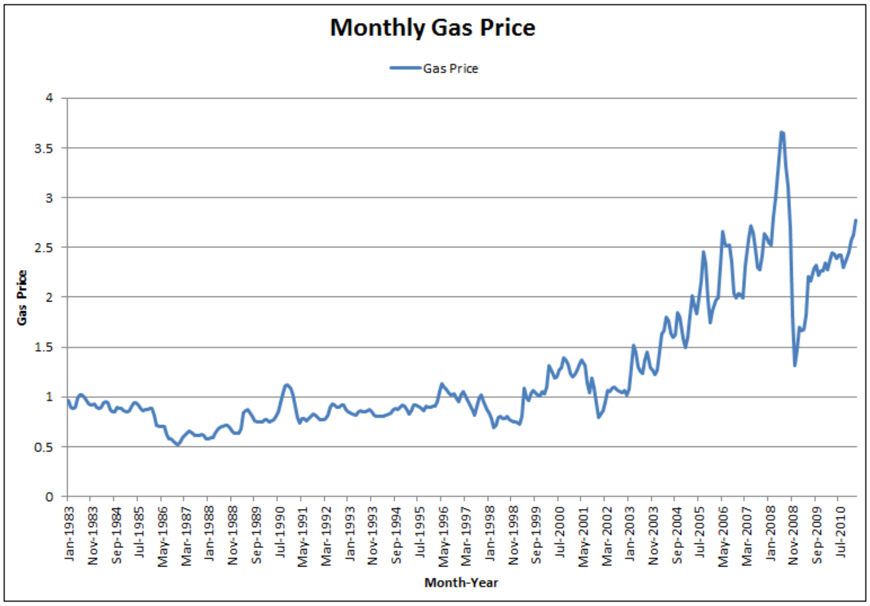

Figure 4.1: Monthly Gas Price Data For Nevada

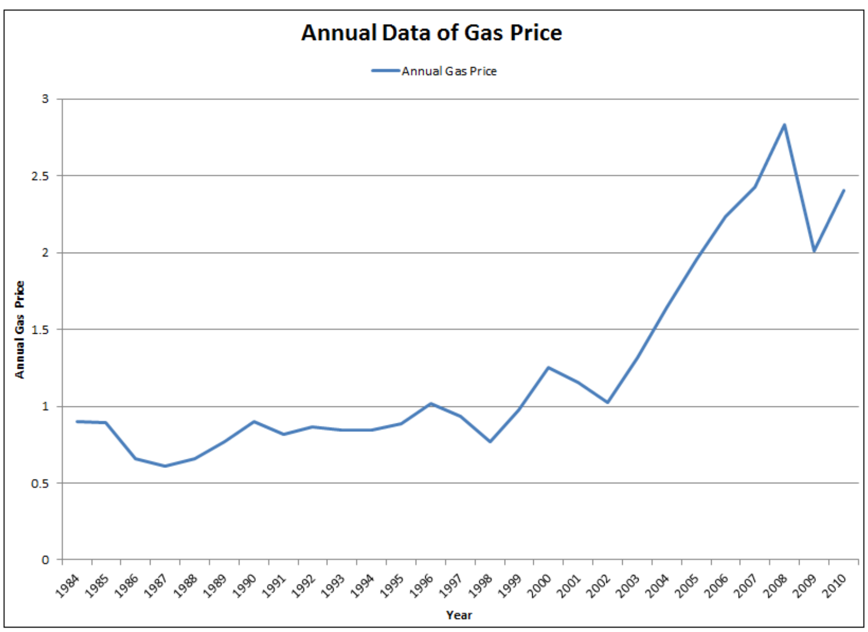

Figure 4.2: Anual Gas Price Data for Nevada

needs to be estimated from price data. Parameters have estimated using data starting from various years upto end of year 2010. Below table 4.1 is the list of parameters estimated based on equation identified in Chapter 3 for parameters μ, σ and κ.

Table 4.1: Estimated Parameters for P for Model 1 using monthly data upto 2010

Data Since Year	$\hat{\sigma}$	$\hat{\kappa}$	P_∞	$\hat{\mu}$
1983	0.23875607	0.05392849	2.75367451	1.27719572
1988	0.25361420	0.13012703	2.21924428	0.92073860
1993	0.26994668	0.13907087	2.36142417	0.99026144
1998	0.30328081	0.29674424	2.24494083	0.88616950
2003	0.30823934	1.09490686	2.29382705	0.85191559
2008	0.34232382	1.40015682	2.40908354	0.90017006

Parameters For Model 1 from Annual Data

Anual data has been used since starting year 1984 till 2010. Starting from 1984 till 2010 whole data has been been divided into multiple sets shifting starting point by 5 years. Using these sets of data, parameters have been estimated in order to have better understanding. As per our SDE model, The required ($\hat{\rho}$, κ) and variance (σ^2) needs to be estimated from price data. Below table 4.2 is the list of parameters estimated based on Chapter 3

Table 4.2: Estimated Parameters for P for Model 1 using anual data upto 2010

Data Since Year	$\hat{\sigma}$	$\hat{\kappa}$	P_∞	$\hat{\mu}$
1984	0.16348406	0.00261193	31721739.29181880	19.83068691
1989	0.16270301	0.03191323	8.33026114	2.32727173
1994	0.17592812	0.07356135	3.70753388	1.41555364
2004	0.12070089	0.75713220	2.38378082	0.87349829

Table 4.3: Estimated Parameters for P for Model 2 using monthly data upto 2010

Data Since Year	$\hat{\mu}$	$\hat{\sigma}$
1983	0.23885361	0.06355887
1988	0.25415615	0.09737292
1993	0.27049619	0.09796706
1998	0.30536041	0.13376941
2003	0.31953304	0.16320500
2008	0.35516297	0.07076299

4.2.3 Parameters for Model 2

Parameters for Model 2 from Monthly Price Data

Monthly data has been used since starting year 1983 till 2010. Starting from 1983 till 2010 whole data has been been divided into multiple sets shifting starting point by 5 years. Using these sets of data, parameters have been estimated in order to have better understanding. As per our SDE model, The required ($\hat{\mu}$) and variance (σ^2) needs to be estimated from price data. Table 4.3 is the list of parameters estimated:

Parameters For Model 2 from Annual Data

Anual data has been used since starting year 1984 till 2010. Starting from 1984 till 2010 whole data has been been divided into multiple sets shifting starting point by

Table 4.4: Estimated Parameters for P for model 2 using anual data upto 2010

Data since year	$\hat{\mu}$	$\hat{\sigma}$
1984	0.16348785	0.05157431
1989	0.16324186	0.06856462
1994	0.17858070	0.08281398
1999	0.18550085	0.10166766
2004	0.18683924	0.08205130

5 years. Using these sets of data, parameters have been estimated in order to have better understanding. As per our SDE model, The required ($\hat{\mu}$) and variance (σ^2) needs to be estimated from price data. Table 4.4 is the list of parameters estimated:

4.3 Parameter Estimation Gas Model

As per Chapter 2, parameters ν, ρ will be estimated for model G. Estimation of parameters ν, ρ is based MLE principle discussed in Chapter 3

4.3.1 Actual Gas Data

Monthly data of gas consumed for Nevada since 1983 till 2010, published on US Energy Information Administration website ($http://www.eia.gov/$) has been used for parameter estimation of Gas model. Annual data published on EIA website from 1984 to 2010 has been used to estimate parameters ν, ρ of G. Hence ν, ρ can be estimated by equations identified in Chapter 3 using μ, σ and κ estimated in previous section for both model 1 and model 2 respectively.

Table 4.5: Estimated Parameters for G for model 1 using monthly Federal data upto 2010

Data since year	$\hat{\nu}$	$\hat{\rho}$
1983	1.058799069880243	0.131117716309321
1988	0.819647933708540	0.153817914796462
1993	0.705092847186378	0.142099109104935
1998	0.554607754530127	0.174515867778997
2003	0.256377638735234	0.161880939662438
2008	0.214149809970730	0.112375070459068

4.3.2 Parameter estimation based μ, σ for Model 1

As μ, σ and κ are already known for model 1 from previous section. Hence, ν, ρ for model G have been estimated based on known estimated parameters for P.

Estimation of ν, ρ based on monthly data of Gas

Monthly data has been used since starting year 1983 till 2010. Starting from 1983 till 2010 whole data has been been divided into multiple sets shifting starting point by 5 years. Using these sets of data, parameters have been estimated in order to have better understanding. For G, Parameters have been estimated using data sets for G and estimated parameters μ, σ for P during same set of data for p. For example, ν, ρ are estimated for G using data between 1998 till 2010 and μ, σ has been estimated for year data 1998 to 2010.

Estimation of ν, ρ based on anual data of Gas

Anual data has been used since starting year 1984 till 2010. Starting from 1984 till 2010 whole data has been been divided into multiple sets shifting starting point by

Table 4.6: Estimated Parameters for G for model 1 using Nevada anual data upto 2010

Data since year	$\hat{\nu}$	$\hat{\rho}$
1984	0.135936319924288	0.048384053625804
1989	0.116521377719240	0.044195061392643
1994	0.115119573179191	0.047817757043188
1999	0.061926663678223	0.038169748232970
2004	0.108362752300796	0.030005614653113

Table 4.7: Estimated Parameters for G for model 2 using monthly Federal data upto 2010

Data since year	$\hat{\nu}$	$\hat{\rho}$
1983	1.058750334652111	0.123118896559033
1988	0.818667038830149	0.137251518761778
1993	0.704195338861509	0.125307797771148
1998	0.551645129163001	0.140809523184518
2003	0.248201321264334	0.124126193056815
2008	0.206925499506852	0.077984561001597

5 years. Using these sets of data, parameters have been estimated in order to have better understanding.

4.3.3 Parameter estimation based μ, σ for Model 2

As μ, σ are already known for model 1 from previous section. Hence, ν, ρ for model G have been estimated based on known estimated parameters for P.

Estimation of ν, ρ based on monthly data of Gas

Similar y, as in above Section parameters for Gas has been estimated for model 2 using monthly data divided into various datasets.

Table 4.8: Estimated Parameters for G for model 2 using Nevada anual data upto 2010

Data since year	$\hat{\nu}$	$\hat{\rho}$
1984	0.136132500468255	0.048351792053172
1989	0.117879505420606	0.043844880760014
1994	0.116837727054027	0.046826555280256
1999	0.061981973273421	0.036709212516266
2004	0.073324780980565	0.022061868131690

Estimation of ν, ρ based on anual data of Gas

Similar y, as in above Section, parameters for Gas has been estimated for model 2 using anual data divided into various datasets.

4.4 Parameter Estimation for VMT model

4.4.1 Actual VMT Data

Monthly Federal VMT Data since 1971 till 2011, published on USA federal Highway Administration website. has been used for parameter estimation of VMT model. Anaul VMT Data for Nevada since 2004 till 2010, published on Nevada Department of Transportation (NDOT) website. Figure 4.3 shows the monthly VMT data for Nevada since 1971 till 2011. Hence θ, η can be estimated by equations identified in Chapter 3 using μ, σ and κ estimated in previous section for both model 1 and model 2 respectively. Hence θ, η can be estimated as following.

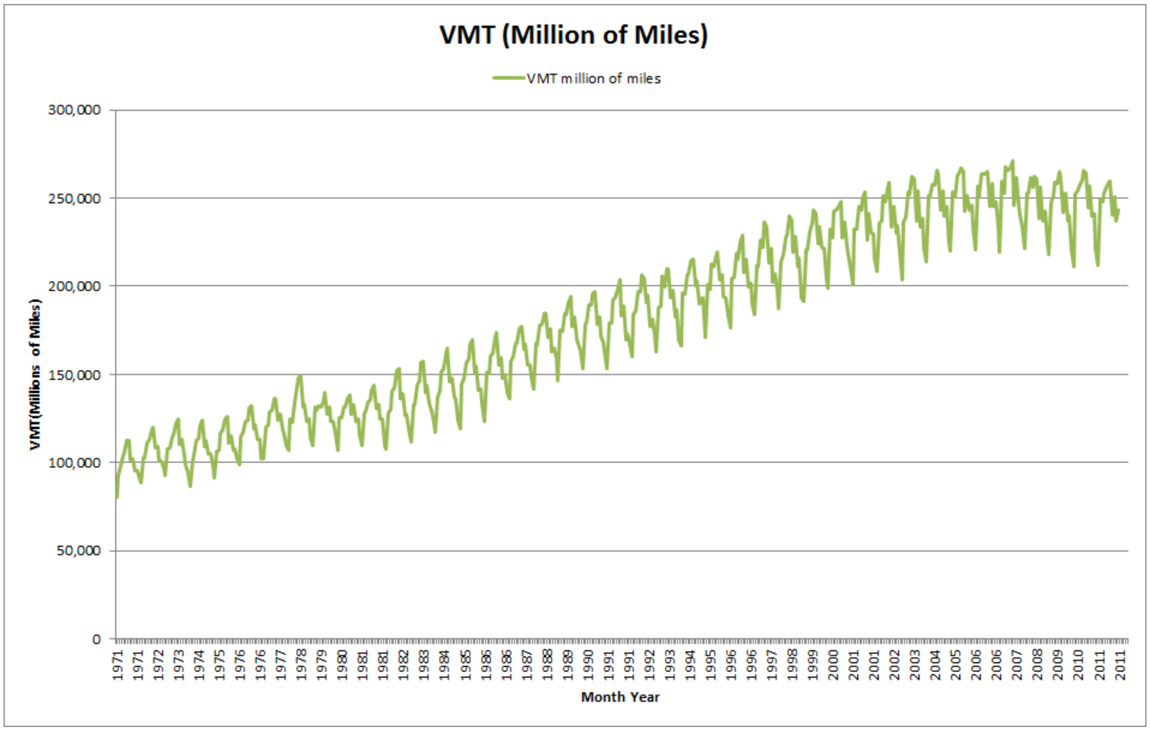

Figure 4.3: Federal monthly VMT data

4.4.2 Parameter estimation based μ, σ for Model 1

As μ, σ and κ are already known for model 1 from previous section. Hence, ν, ρ for model V have been estimated based on known estimated parameters for P.

Estimation of θ, η based on monthly data of VMT

Similarly, as gas model, parameters for VMT has been estimated for model 1 using monthly data devised into various datasets.

Table 4.9: Estimated Parameters for V for model 1 using monthly Federal data upto 2010

Data since year	$\hat{\theta}$	$\hat{\eta}$
1983	1.058799069880243	0.131117716309321
1988	0.910787441608703	0.175952205245861
1993	0.749306305179710	0.152417328210087
1998	0.575332012342734	0.178049417614565
2003	0.405070336161383	0.211008626182020
2008	0.289002848053881	0.114202089034893

Data since year	$\hat{\theta}$	$\hat{\eta}$
2004	0.110722623913606	0.039104565518444

Table 4.10: Estimated Parameters for V for model 1 using Nevada anual data upto 2010

Estimation of θ, η based on anual data of VMT

Anual data has been used since starting year 2004 till 2010. Using these sets of data, parameters have been estimated in order to have better understanding.

4.4.3 Parameter estimation based μ, σ for Model 2

As μ, σ are already known for model 1 from previous section. Hence, ν, ρ for model V have been estimated based on known estimated parameters for P.

Estimation of θ, η based on monthly data of VMT

Similarly, as gas model, parameters for VMT has been estimated for model 2 using monthly data divided into various datasets.

Table 4.11: Estimated Parameters for V for model 2 using monthly Federal data upto 2010

Data since year	$\hat{\theta}$	$\hat{\eta}$
1983	1.058750334652111	0.123118896559033
1988	0.909760220536034	0.157549564445772
1993	0.748367059385680	0.134574625546193
1998	0.572211548749443	0.143075914932765
2003	0.391370811861368	0.151125426545720
2008	0.278836288115707	0.067727149626408

Table 4.12: Estimated Parameters for V for model 2 using Nevada anual data upto 2010

Data since year	$\hat{\theta}$	$\hat{\eta}$
2004	0.077447333939959	0.031422298400719

Estimation of θ, η based on annual data of VMT

Anual data has been used since starting year 2004 till 2010. Using these sets fo data, parameters have been estimated in order to have better understanding.

4.5 Conclusion

In this chapter, parameters for the three variables $P(t)$, $V(t)$ and $G(t)$ have been identified using MLE technique as discussed in Chapter 3. Various numerical experiments and simulations were conducted in order to verify the estimate of parameters. All $P(t)$, $V(t)$ and $G(t)$ can be forcasted based on simulated values using numerical methods using these estimated parameters. Expected values of $V(t)$ and $G(t)$ can be easily calculated once variables $P(t)$, $V(t)$ and $G(t)$ are simulated.

CHAPTER 5

ESTIMATION OF VMT TAX RATE $r(t)$

5.1 Summary

In this chapter, all variables $P(t), V(t)$ and $G(t)$ are being simulated using the estimated parameters in Chapter 4. Values for $P(t), V(t)$ and $G(t)$ and projected for future years based on numerical simulations of our models. Expectation of variable $V(t)$ and $G(t)$ are calculated based of numerical simulation. Once we have the expectation of $V(t)$ and $G(t)$, VMT tax rate $r(t)$ can be calculated to generate same amount of revenue as generated by current gas tax using equation (2.14). Current gas tax rate r_g has been assumed constant approximately 0.52 *cents/mile* in order to estimate gas tax based road revenue.

5.2 Simulation for Model 1

In Chapter 4 parameters have been estimated considering various set of data starting from various year upto 2010. Simulation for $P(t), V(t)$ and $G(t)$ are done for all the set of parameters. Initial values P_0, V_0 and G_0 of $P(t), V(t)$ and $G(t)$ are considered based on past data. To project the future data using monthly data, P_0, V_0 and G_0 have been considered values of $P(t), V(t)$ and $G(t)$ in Dec 2010. While in

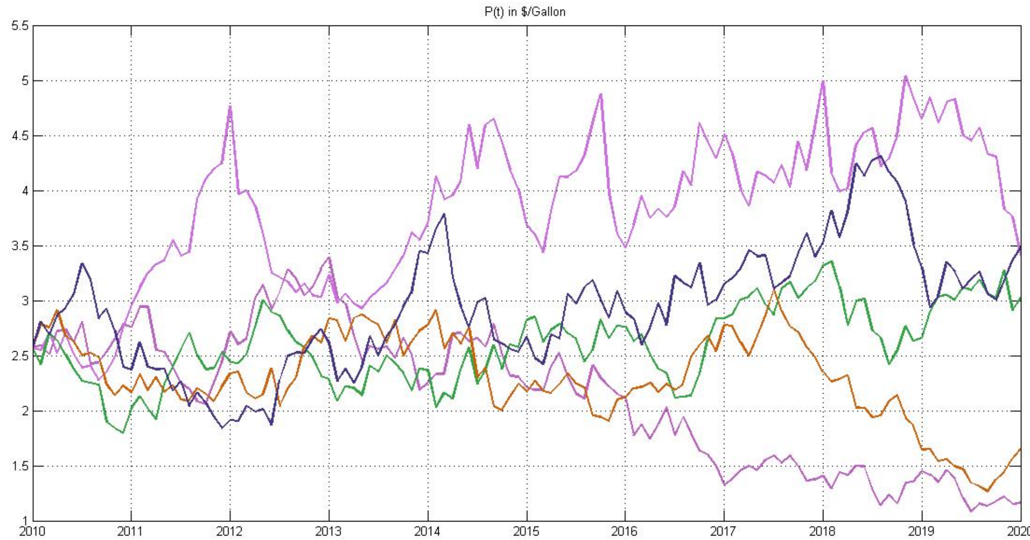

Figure 5.1: Trajectories of P(t) based on model 1 using μ, σ since 1983

case of anual projection, values of P_0, V_0 and G_0 have been considered for year 2004.

5.2.1 Simulation for Model 1 Using monthly Data

Simulation using estimated parameters for data since 1983

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.1–5.4 display five different trajectories of the forecoasted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

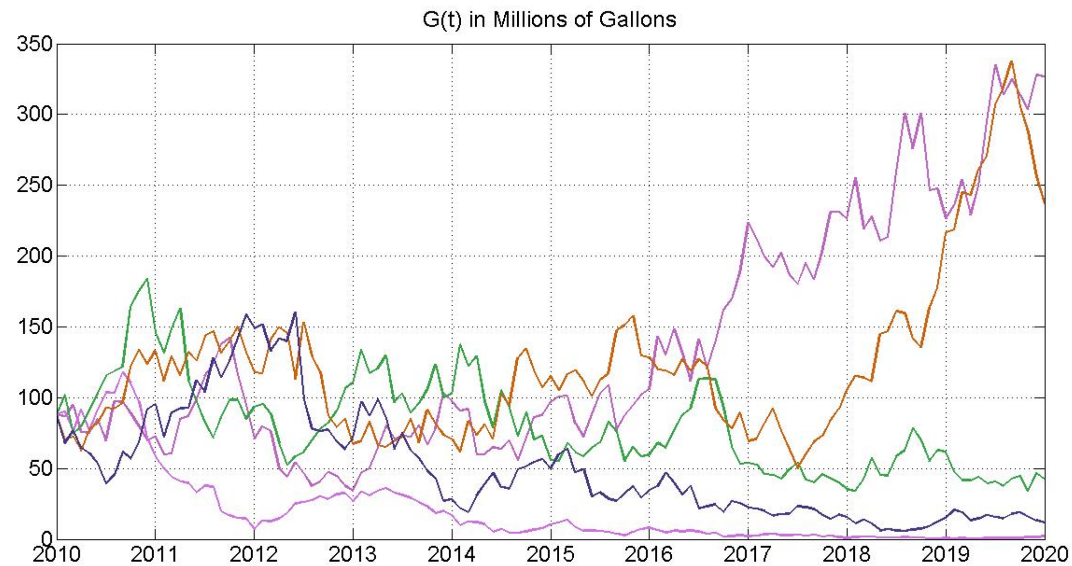

Figure 5.2: Trajectories of G(t) based on model 1 using ν, ρ since 1983

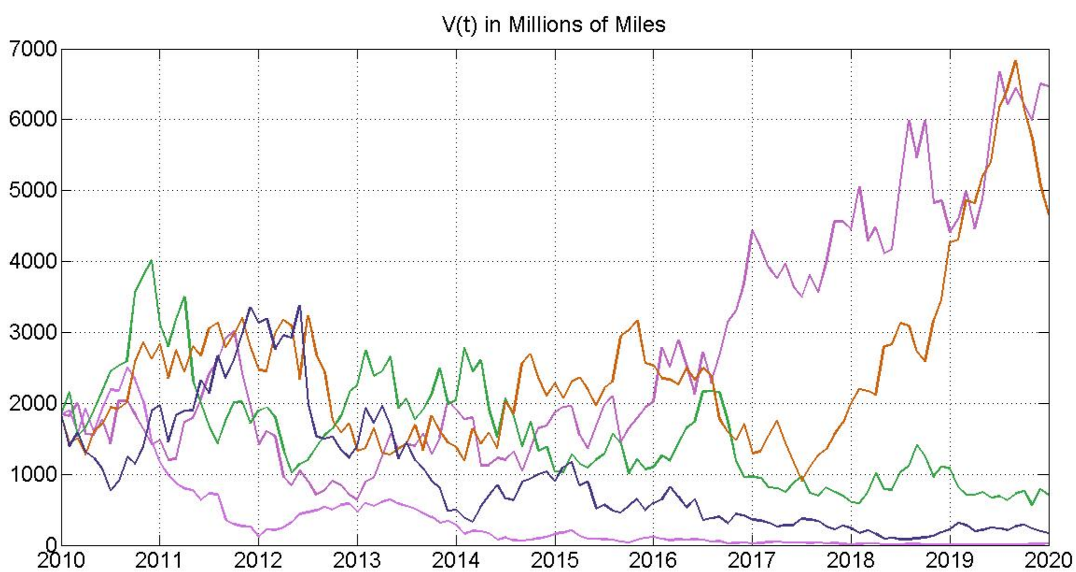

Figure 5.3: Trajectories of V(t) based on model 1 using $\theta,\ \eta$ since 1983

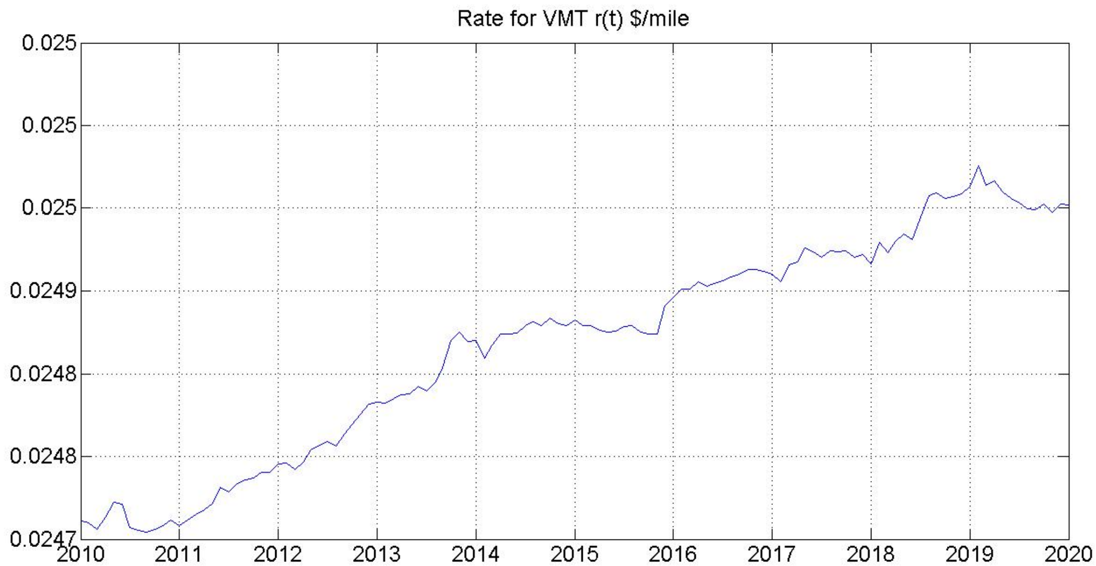

Figure 5.4: r(t) based on model 1 using parameters for data since 1983

Simulation using estimated parameters for data since 1988

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 808.873$ millions of Gallon. Figures 5.5–5.8 displays five different trajectories of the fore casted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 10 years

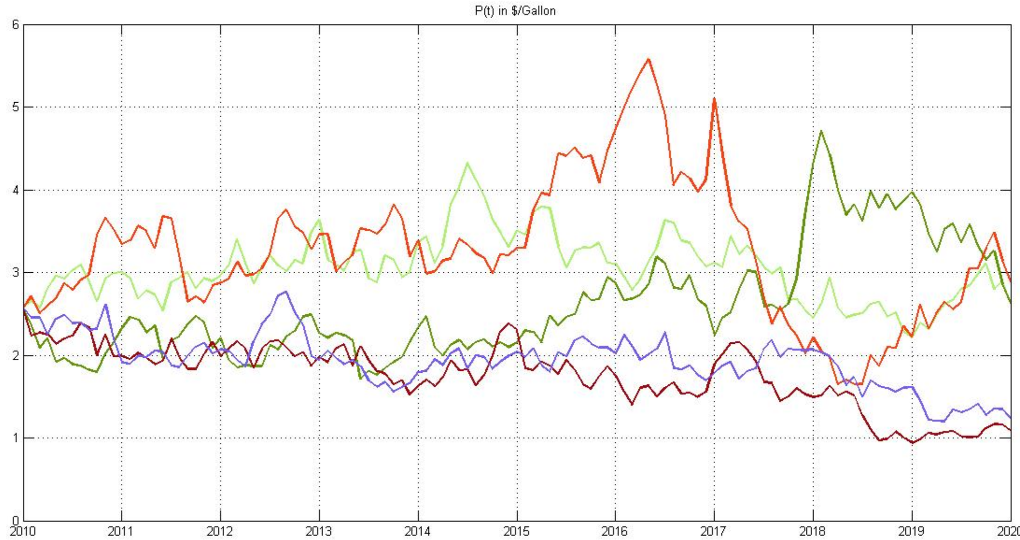

Figure 5.5: Trajectories of P(t) based on model 1 using μ, σ since 1988

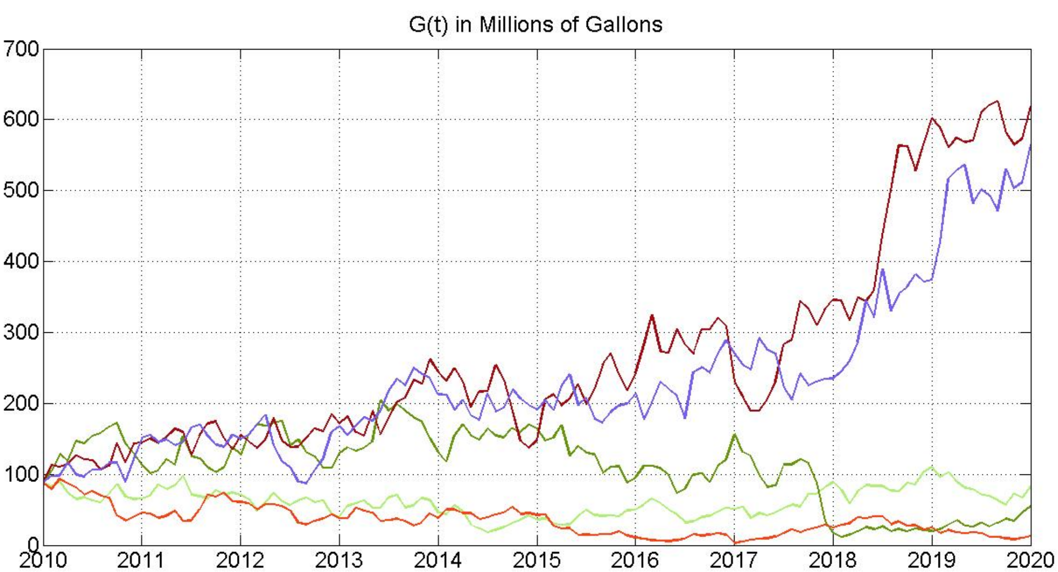

Figure 5.6: Trajectories of G(t) based on model 1 using ν, ρ since 1988

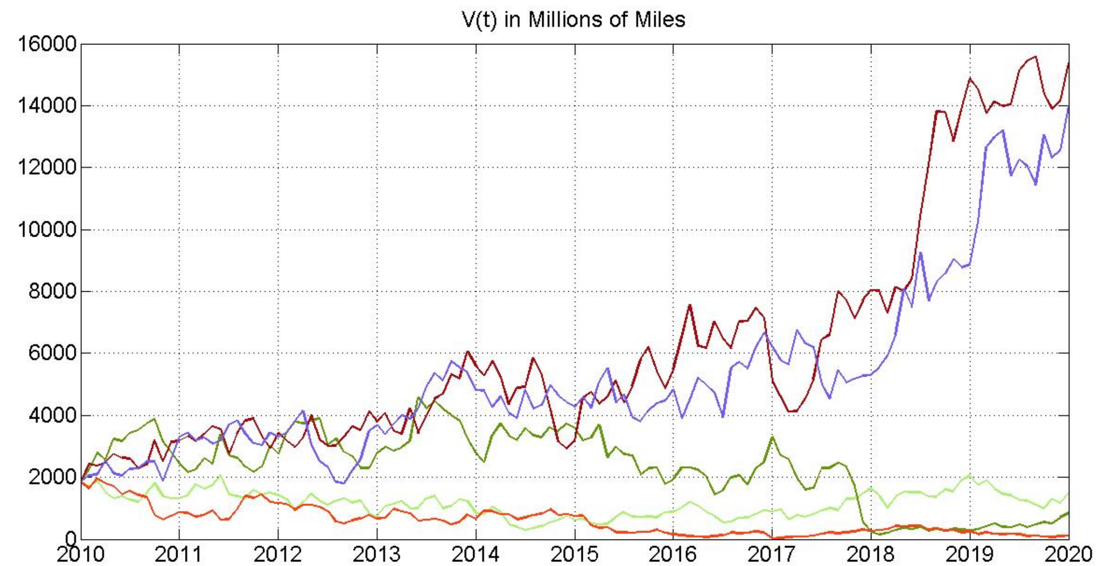

Figure 5.7: Trajectories of V(t) based on model 1 using θ, η since 1988

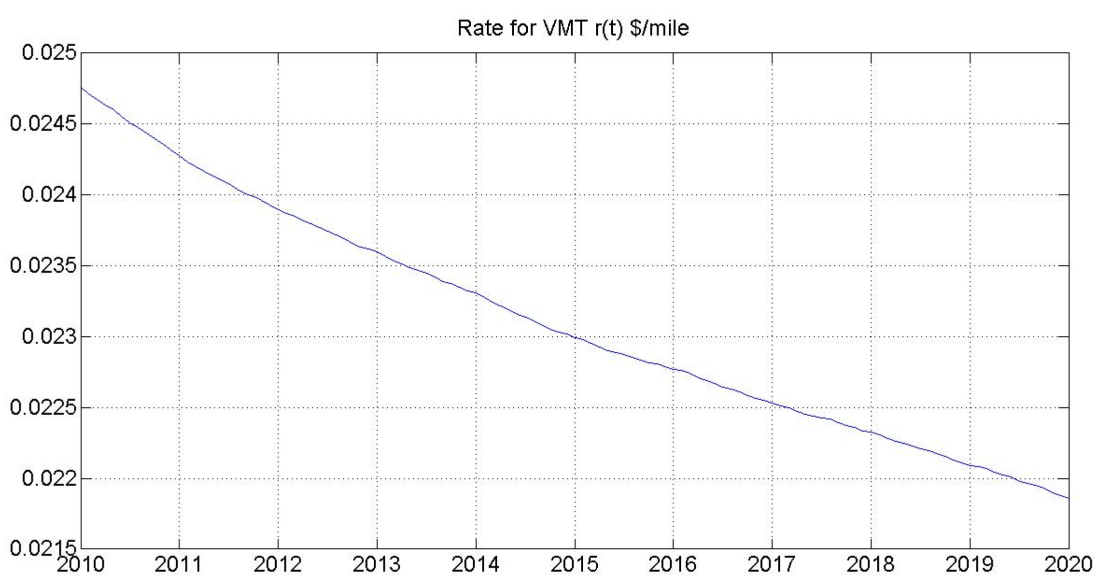

Figure 5.8: r(t) based on model 1 using parameters for data since 1988

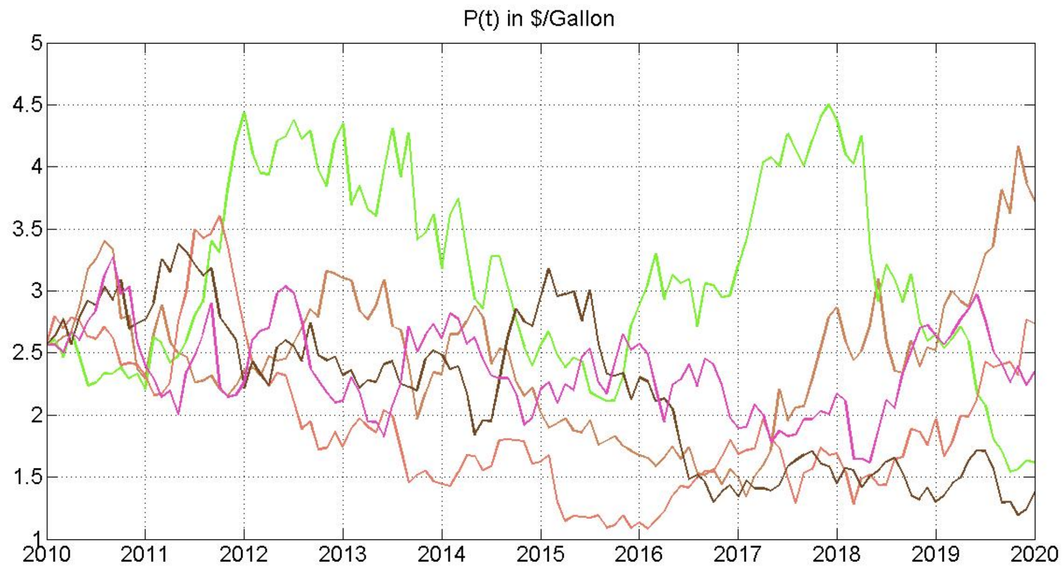

Figure 5.9: Trajectories of P(t) based on model 1 using μ, σ since 1993

Simulation using estimated parameters for data since 1993

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.9–5.12 displays five different trajectories of the fore casted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 10 years

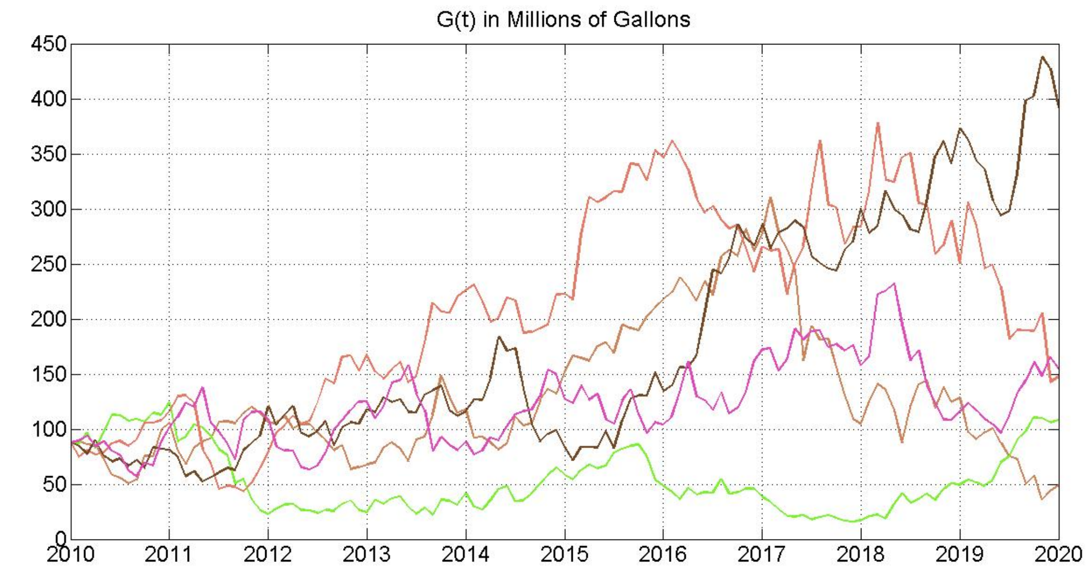

Figure 5.10: Trajectories of G(t) based on model 1 using ν, ρ since 1993

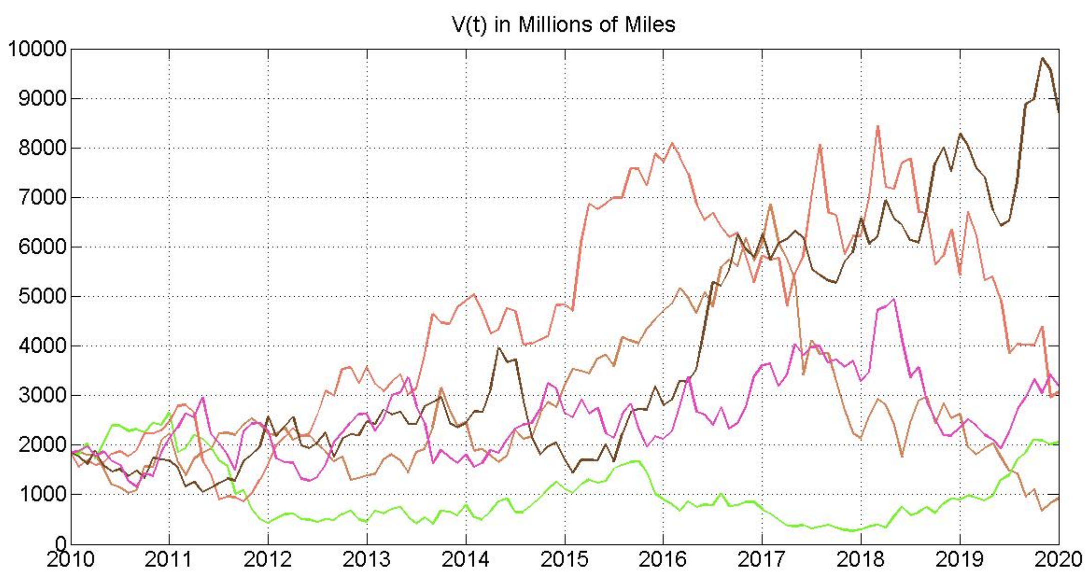

Figure 5.11: Trajectories of V(t) based on model 1 using θ, η since 1993

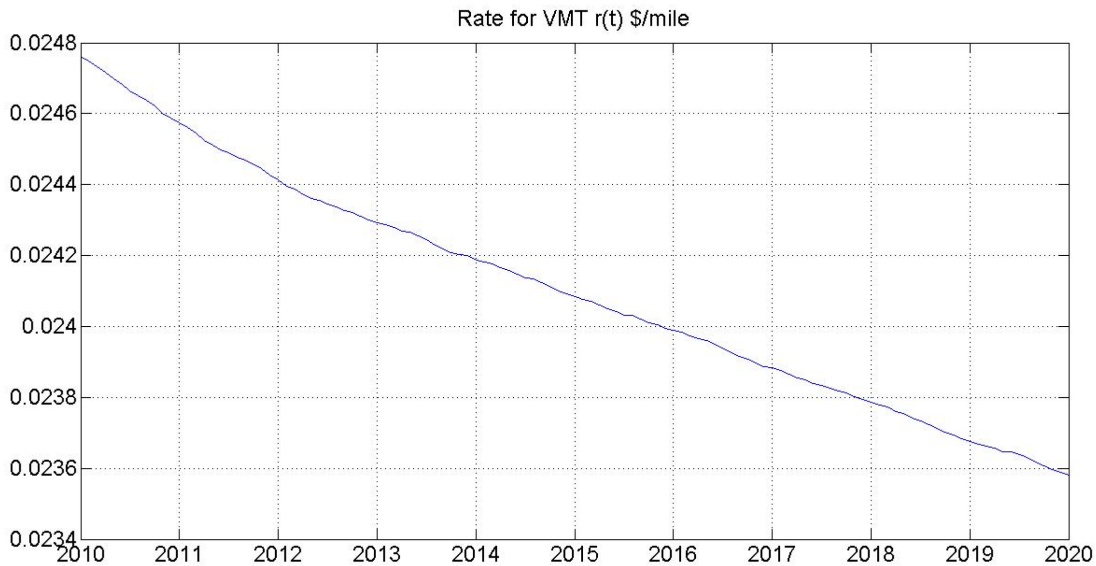

Figure 5.12: r(t) based on model 1 using parameters for data since 1993

Simulation using estimated parameters for data since 1998

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.13–5.16 displays five different trajectories of the fore casted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 10 years

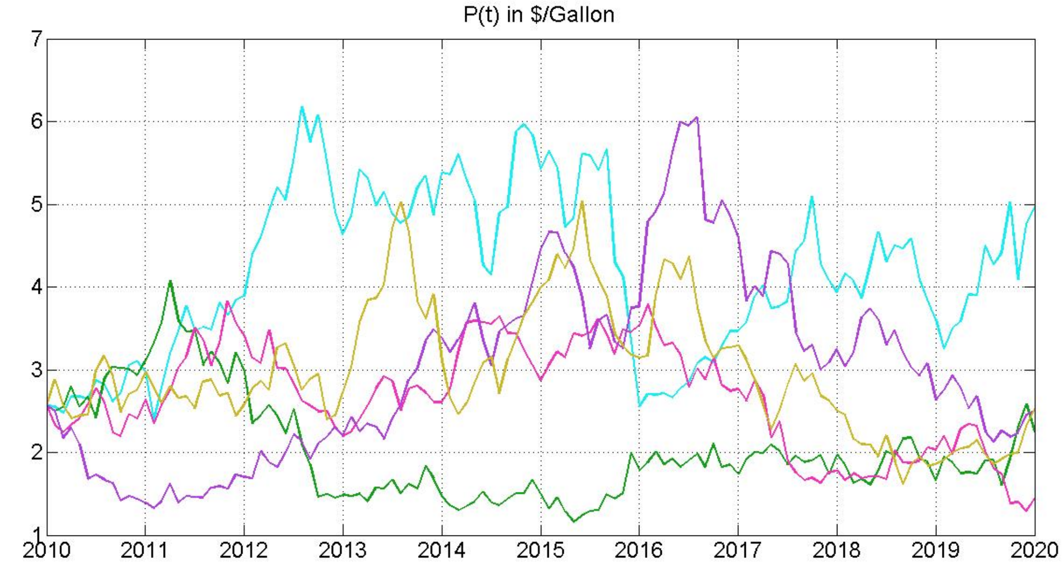

Figure 5.13: Trajectories of P(t) based on model 1 using μ, σ since 1998

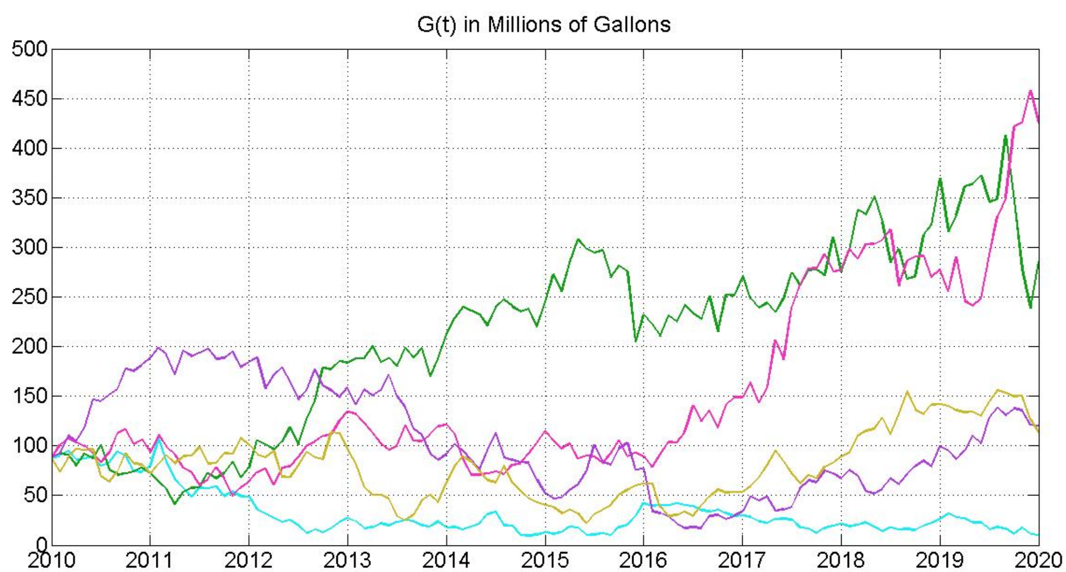

Figure 5.14: Trajectories of G(t) based on model 1 using ν, ρ since 1998

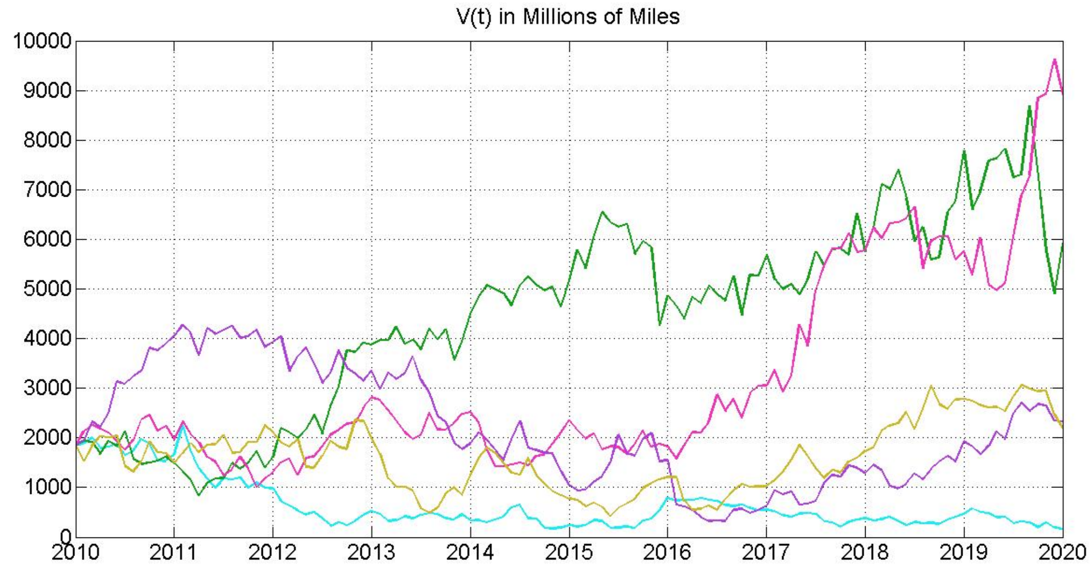

Figure 5.15: Trajectories of V(t) based on model 1 using θ, η since 1998

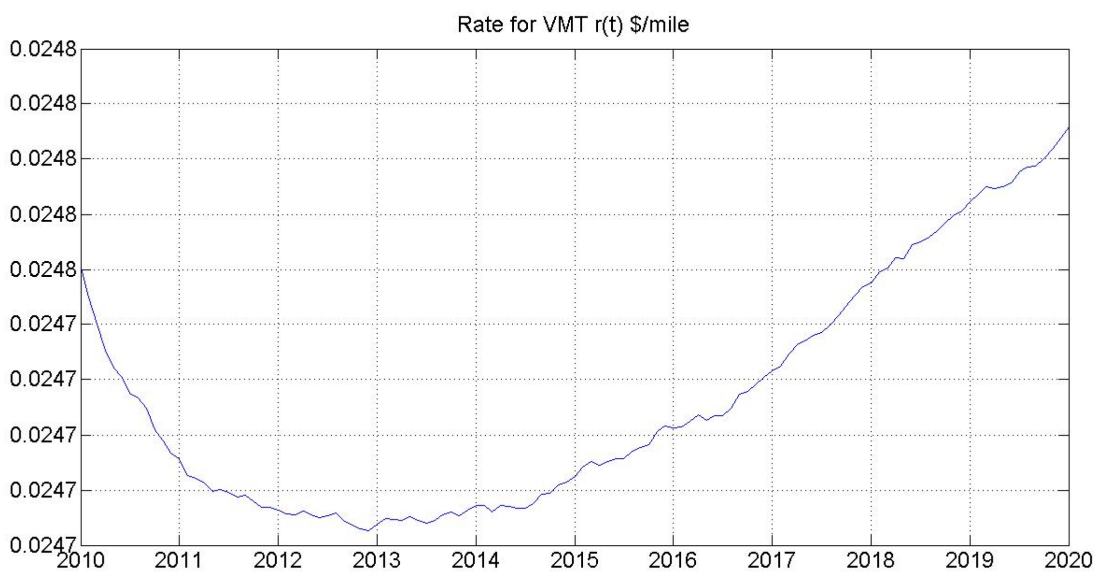

Figure 5.16: r(t) based on model 1 using parameters for data since 1998

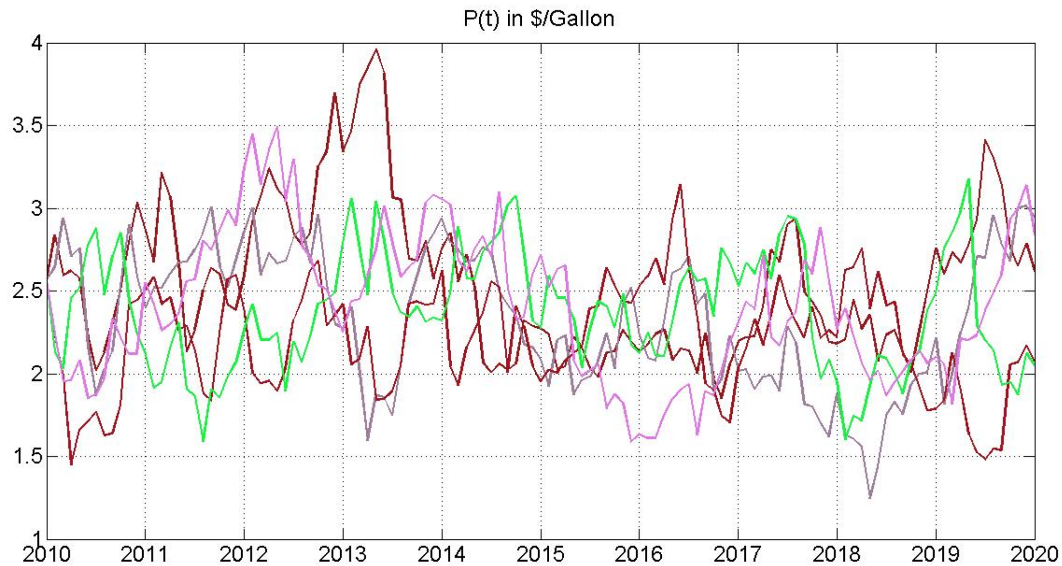

Figure 5.17: Trajectories of P(t) based on model 1 using μ, σ since 2003

Simulation using estimated parameters for data since 2003

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.17–5.20 displays five different trajectories of the fore casted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 10 years

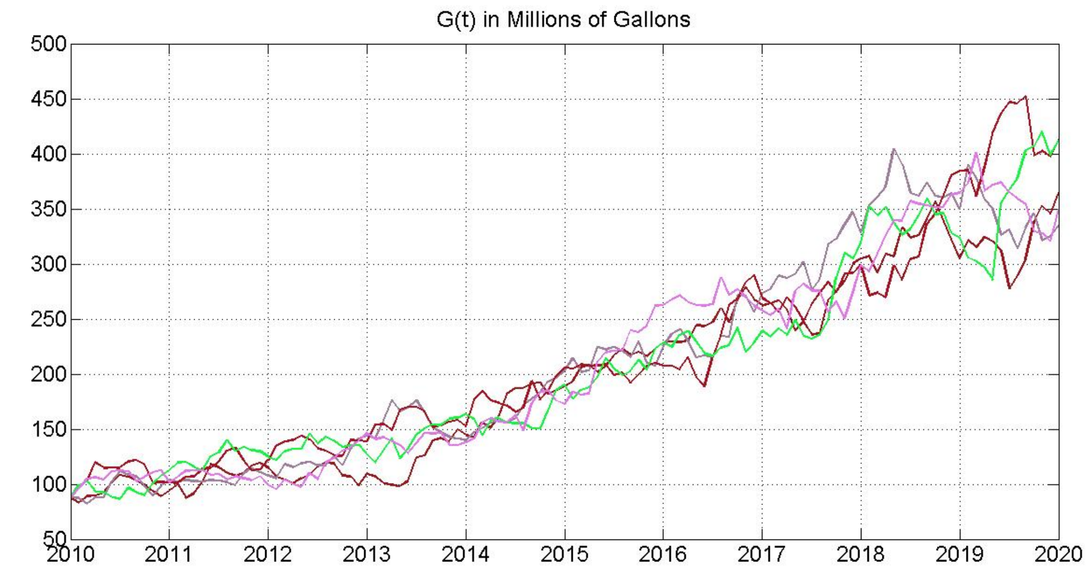

Figure 5.18: Trajectories of G(t) based on model 1 using ν, ρ since 2003

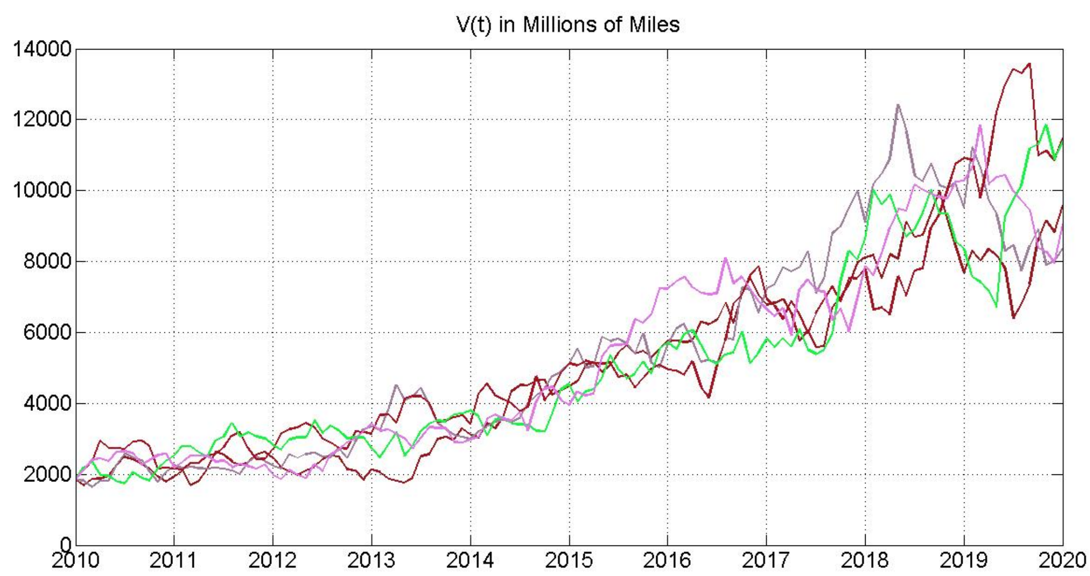

Figure 5.19: Trajectories of V(t) based on model 1 using $\theta,\ \eta$ since 2003

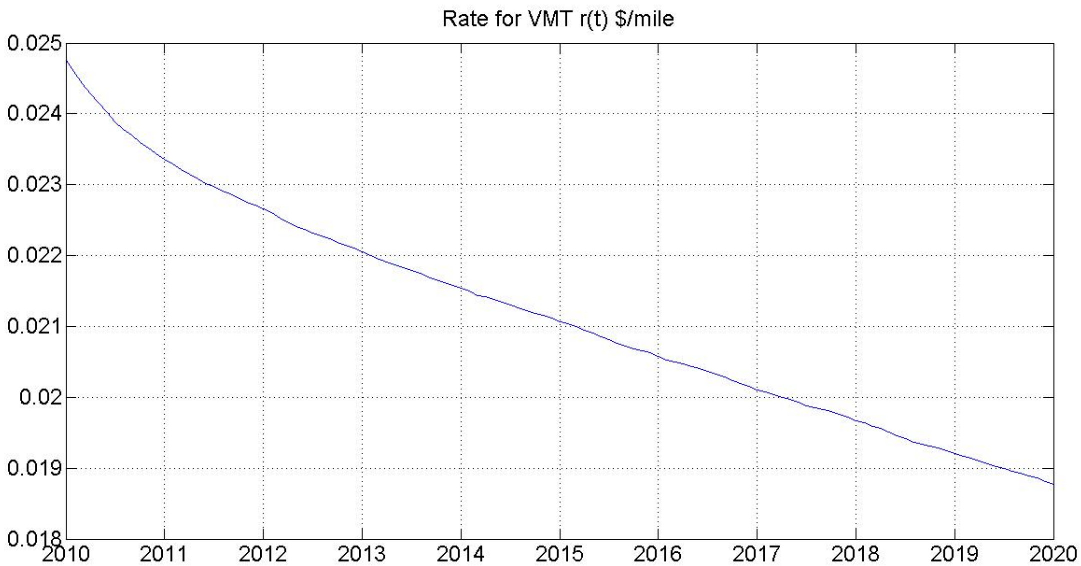

Figure 5.20: r(t) based on model 1 using parameters for data since 2003

Simulation using estimated parameters for data since 2008

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.21–5.24 displays five different trajectories of the fore casted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

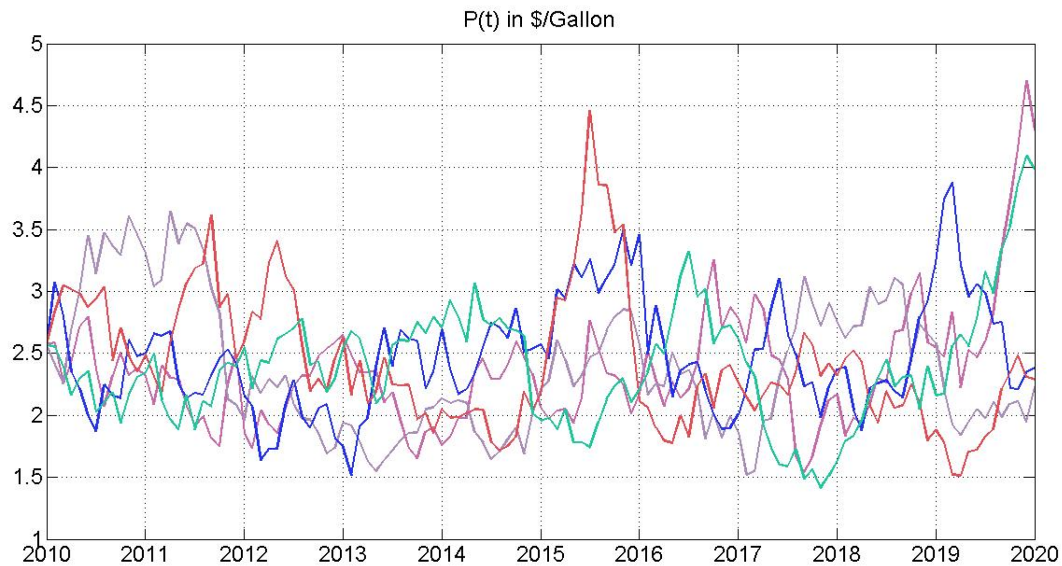

Figure 5.21: Trajectories of P(t) based on model 1 using μ, σ since 2008

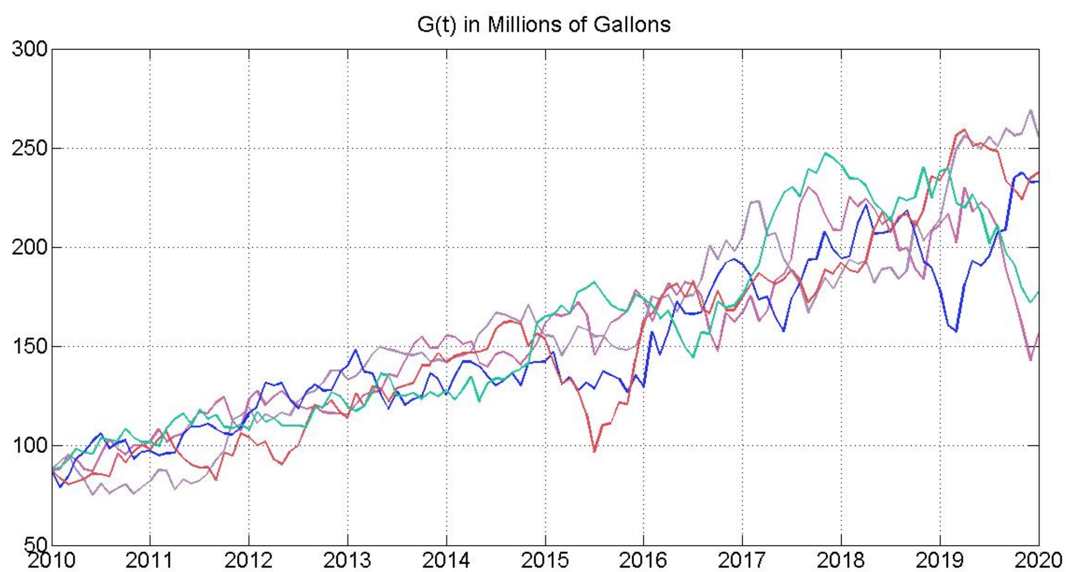

Figure 5.22: Trajectories of G(t) based on model 1 using ν, ρ since 2008

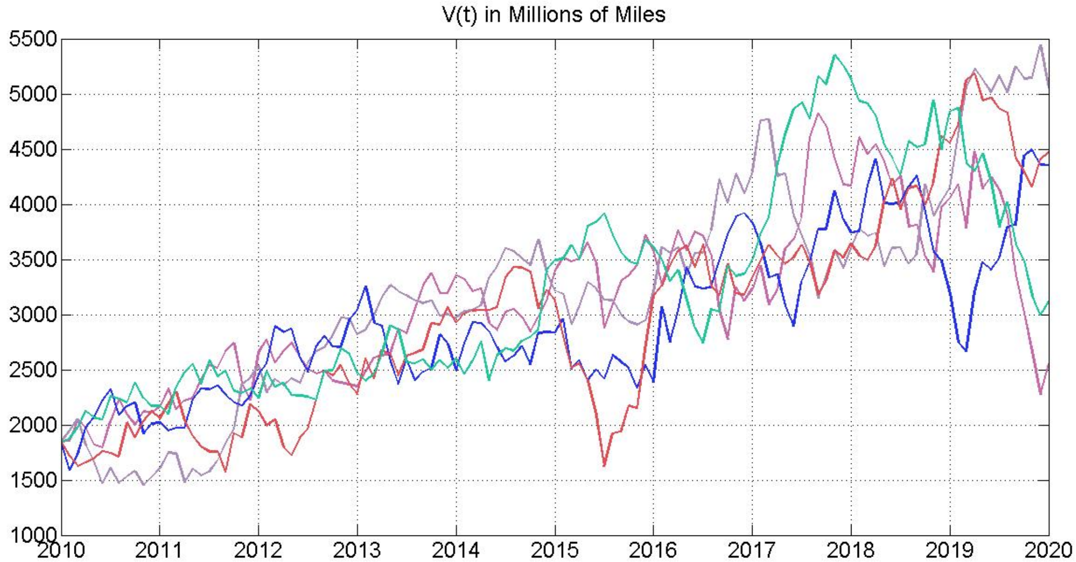

Figure 5.23: Trajectories of V(t) based on model 1 using θ, η since 2008

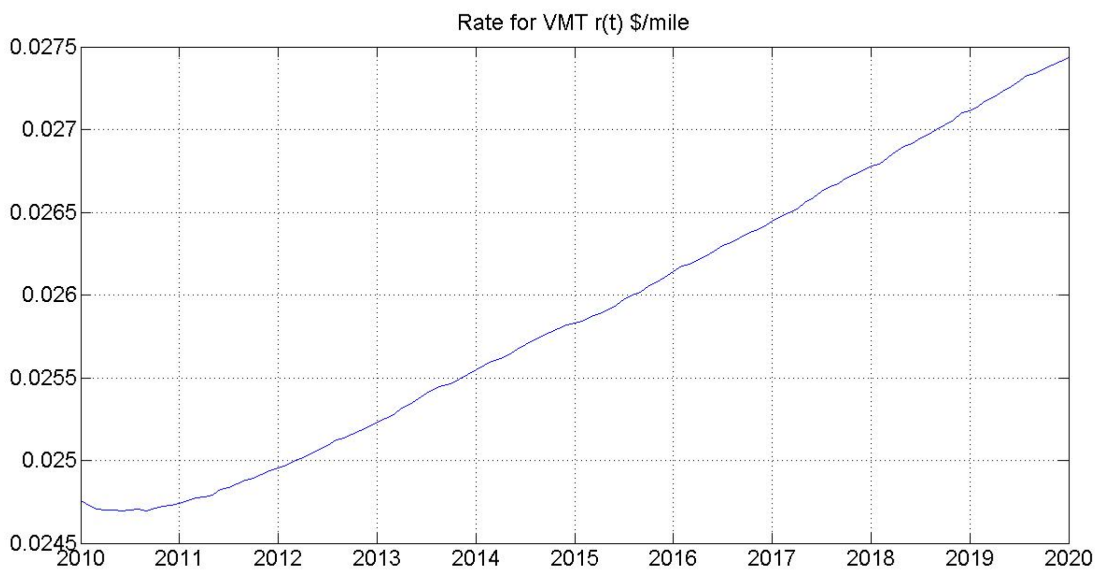

Figure 5.24: r(t) based on model 1 using parameters for data since 2008

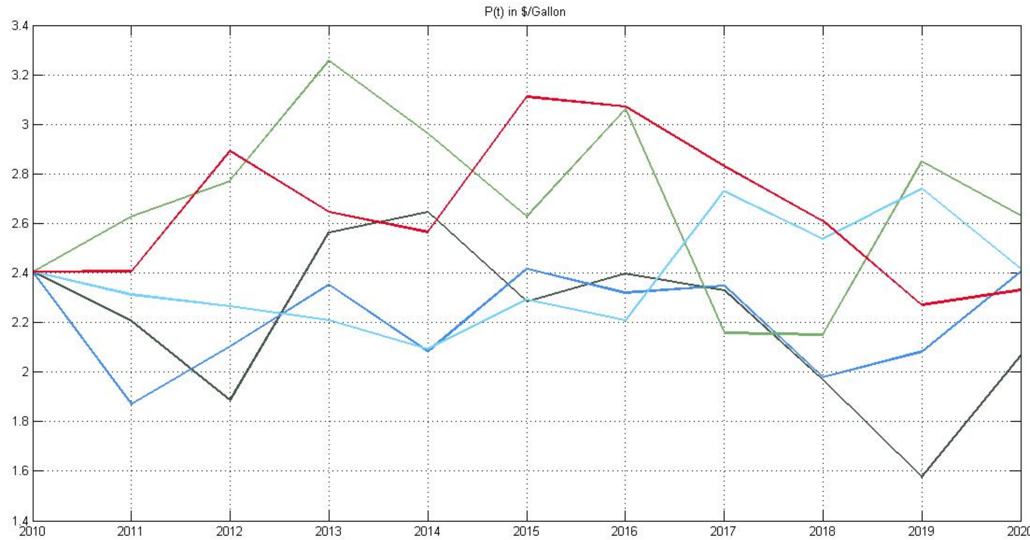

Figure 5.25: Trajectories of P(t) based on model 1 using μ, σ since 2004

5.2.2 Simulation for model 1 Using Anual Data

Simulation using estimated parameters for data since 2004

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.405$ \$ per gallon as gas price in 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for year 2010 has been considered initial values of $V_0 = 22144.76$ millions of miles and $G_0 = 809.046$ millions of Gallon. Figures 5.25–5.28 displays five different trajectories of the forecoasted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 20 years

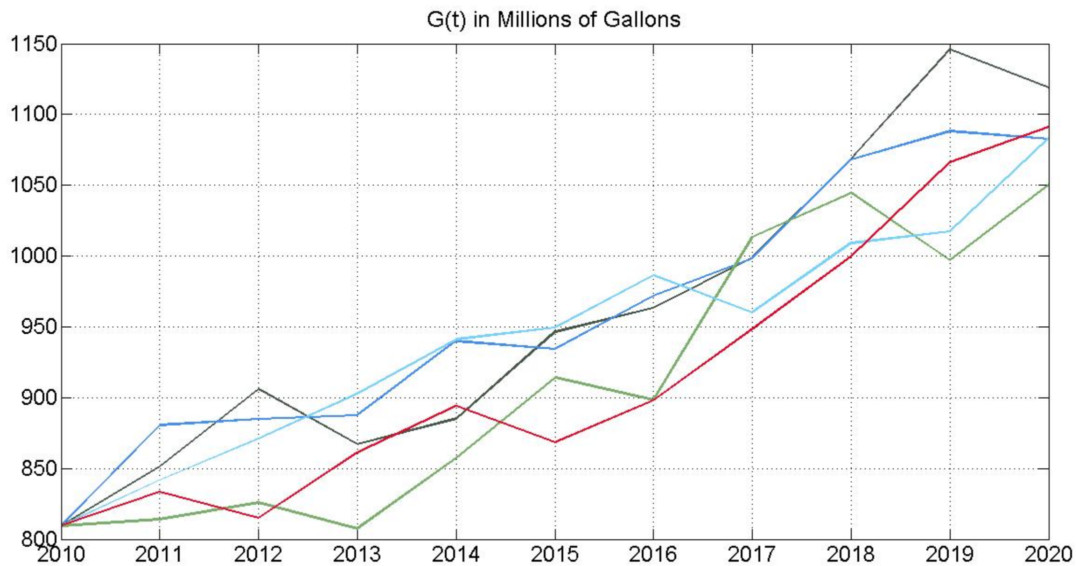

Figure 5.26: Trajectories of G(t) based on model 1 using ν, ρ since 2004

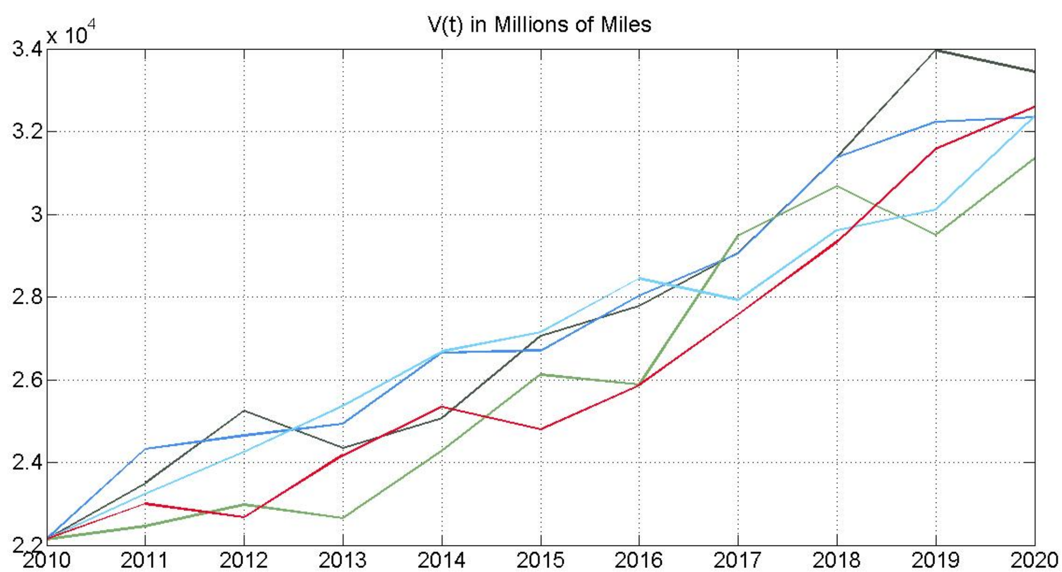

Figure 5.27: Trajectories of V(t) based on model 1 using θ, η since 2004

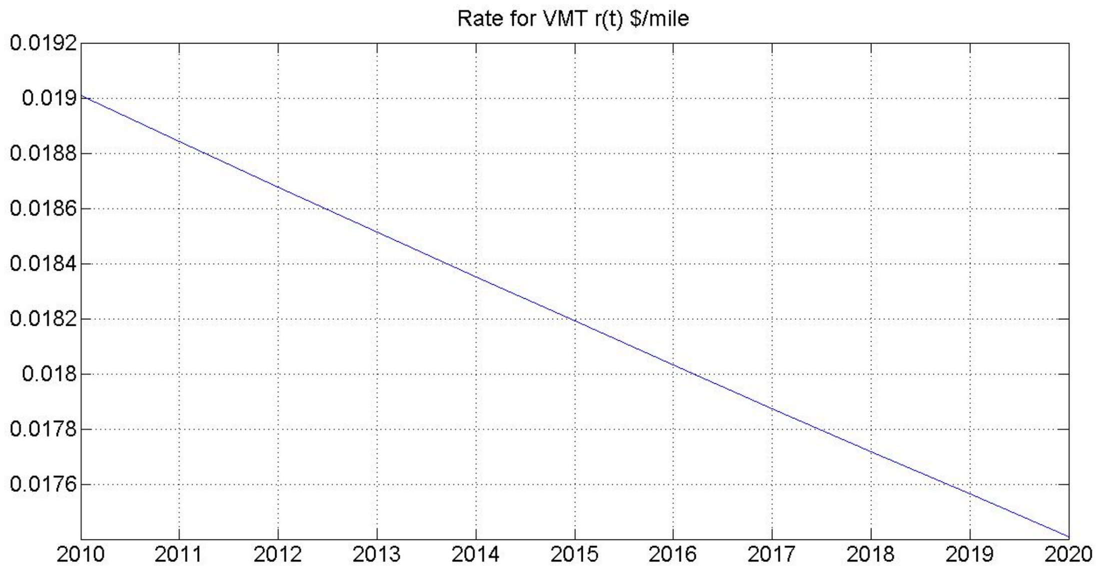

Figure 5.28: r(t) based on model 1 using parameters for data since 2004

Simulation using estimated parameters for data since 2004

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.405$ $ per gallon as gas price in 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for year 2010 has been considered initial values of $V_0 = 22144.76$ millions of miles and $G_0 = 809.046$ millions of Gallon. Figures 5.29–5.32 displays five different trajectories of the forecoasted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 20 years

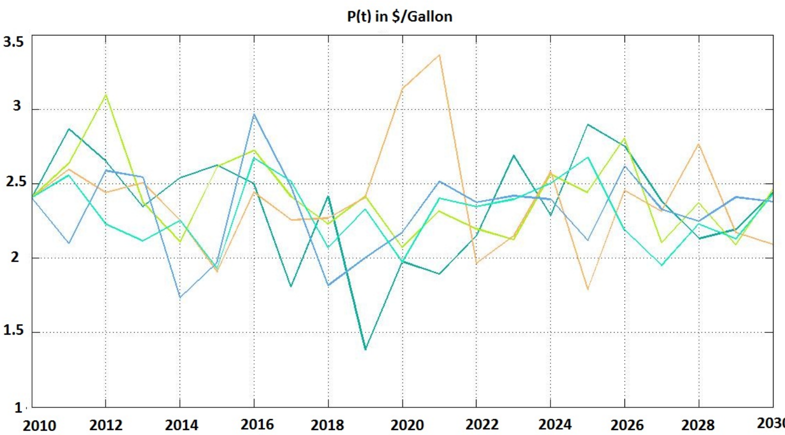

Figure 5.29: Trajectories of P(t) based on model 1 using $\mu,\ \sigma$ since 2004

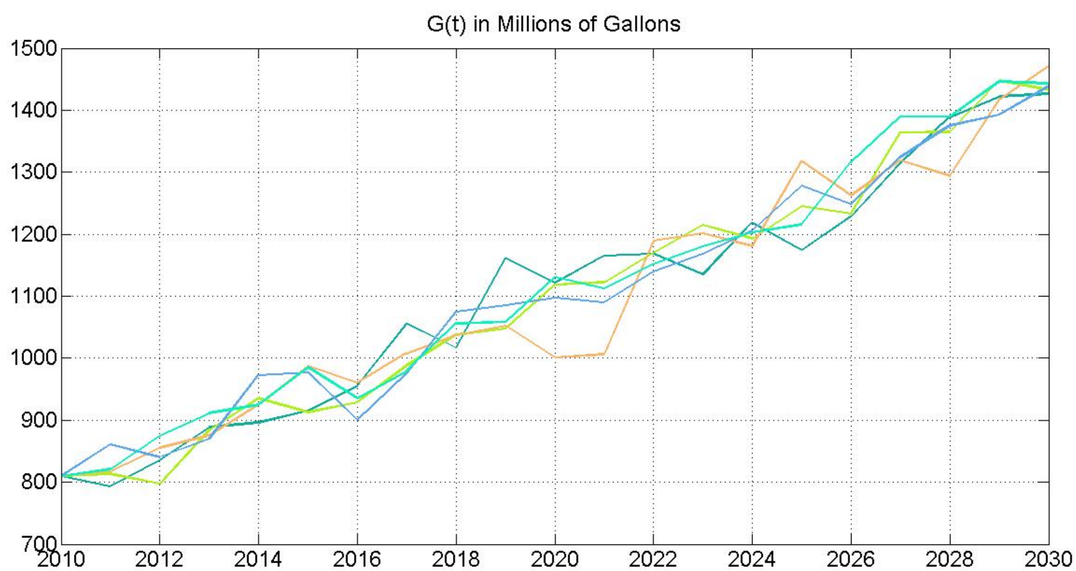

Figure 5.30: Trajectories of G(t) based on model 1 using $\nu,\ \rho$ since 2004

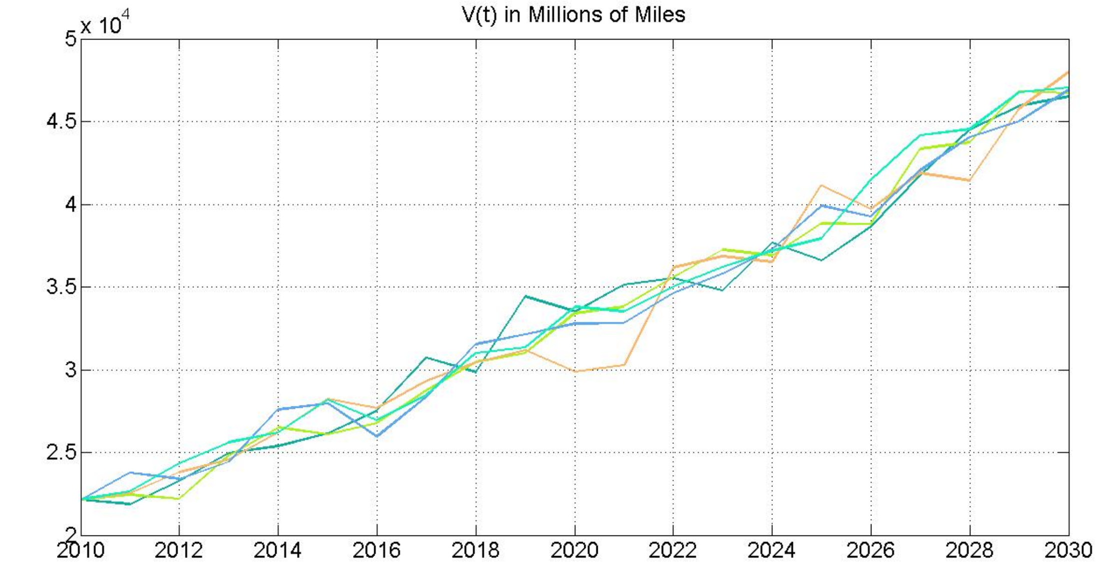

Figure 5.31: Trajectories of V(t) based on model 1 using θ, η since 2004

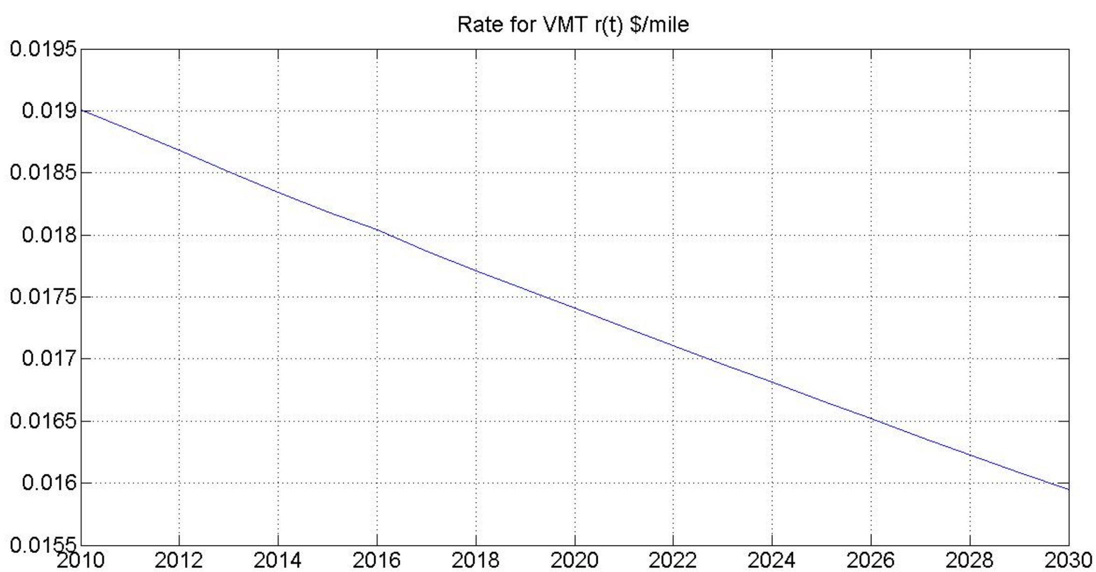

Figure 5.32: r(t) based on model 1 using parameters for data since 2004

5.3 Simulation for model 2

In Chapter 4 parameters have been estimated considering various set of data starting from various year upto 2010. Simulation for $P(t), V(t)$ and $G(t)$ are done for all thse set of parameters. Initial values P_0, V_0 and G_0 of $P(t), V(t)$ and $G(t)$ are considered based on past data. To project the future data using monthly data, P_0, V_0 and G_0 have been considered values of $P(t), V(t)$ and $G(t)$ in Dec 2010. While in case of anual projection, values of P_0, V_0 and G_0 have been considered for year 2004.

5.3.1 Simulation for model 2 Using monthly Data

Simulation using estimated parameters for data since 1983

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.33–5.36 displays five different trajectories of the forecoasted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

Simulation using estimated parameters for data since 1988

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value

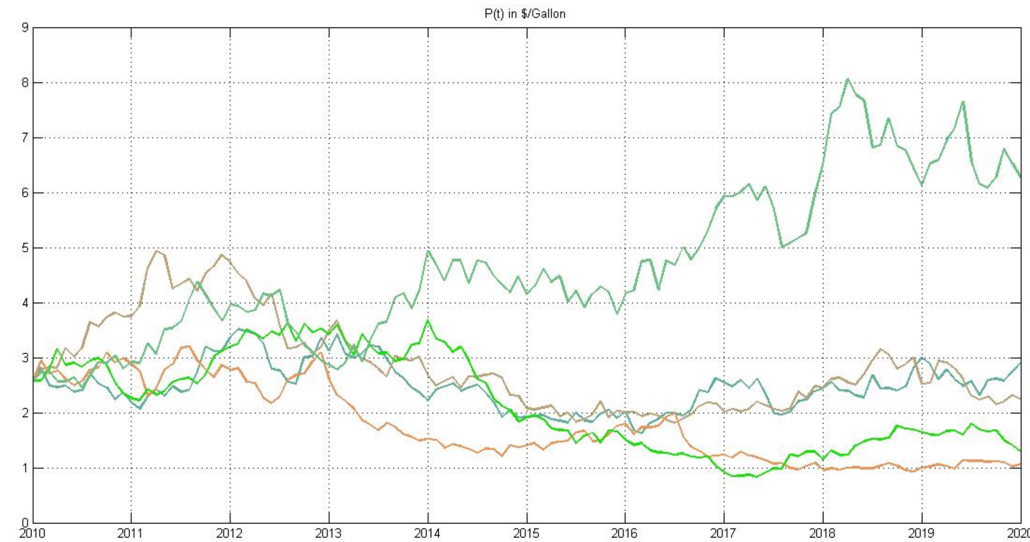

Figure 5.33: Trajectories of P(t) based on model 2 using μ, σ since 1983

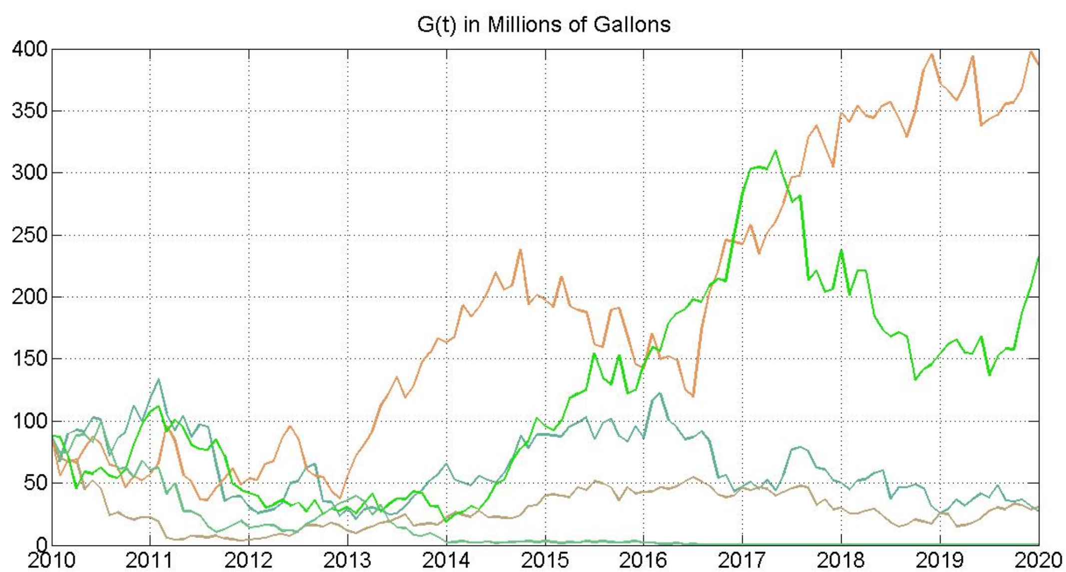

Figure 5.34: Trajectories of G(t) based on model 2 using ν, ρ since 1983

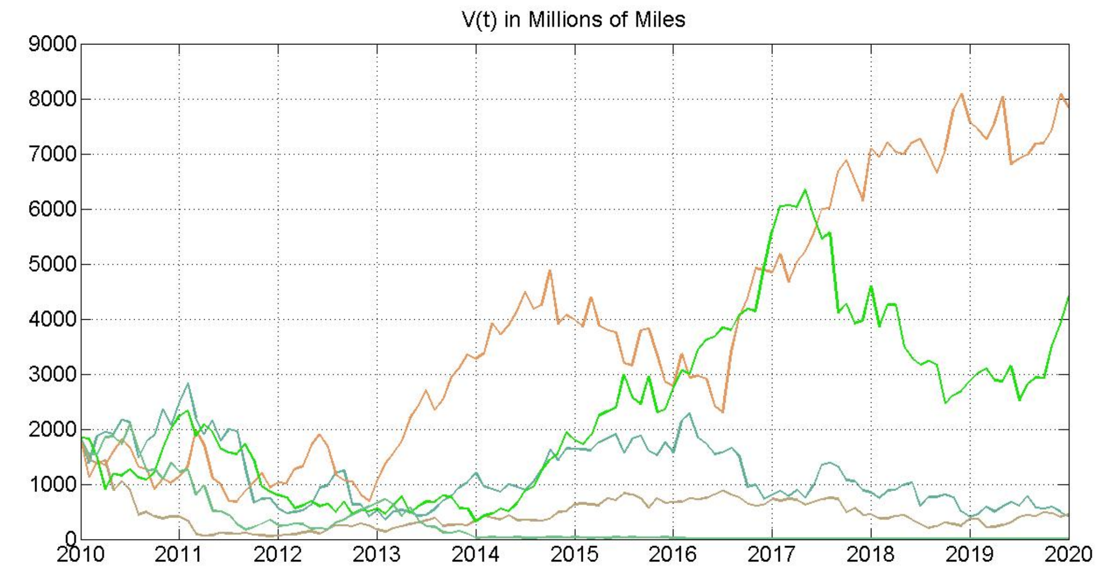

Figure 5.35: Trajectories of V(t) based on model 2 using $\theta,\ \eta$ since 1983

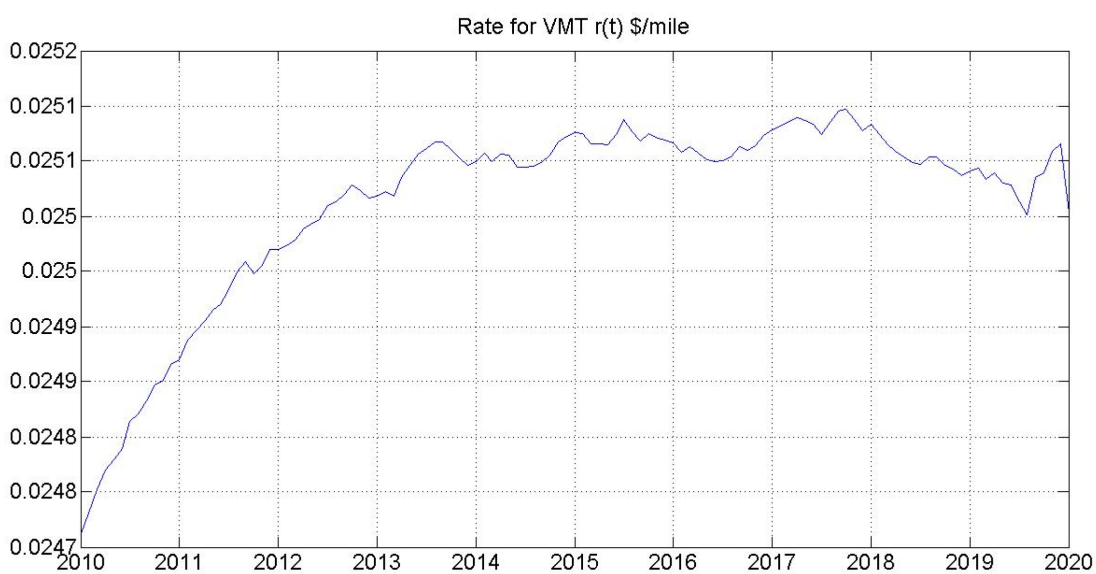

Figure 5.36: r(t) based on model 2 using parameters for data since 1983

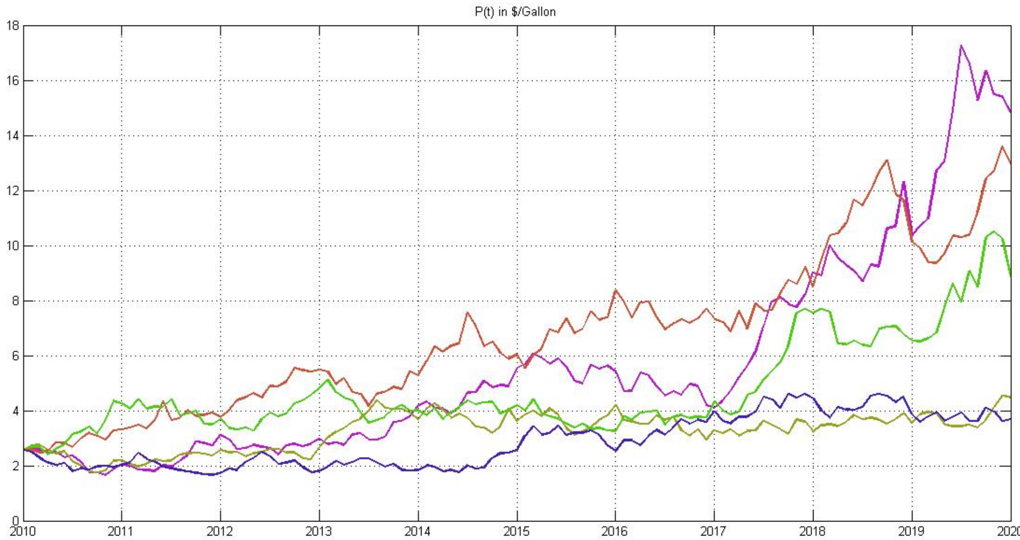

Figure 5.37: Trajectories of P(t) based on model 2 using $\mu,\ \sigma$ since 1988

of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 808.873$ millions of Gallon. Figures 5.37–5.40 displays five different trajectories of the fore casted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

Simulation using estimated parameters for data since 1993

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.41–5.44 displays five different trajectories

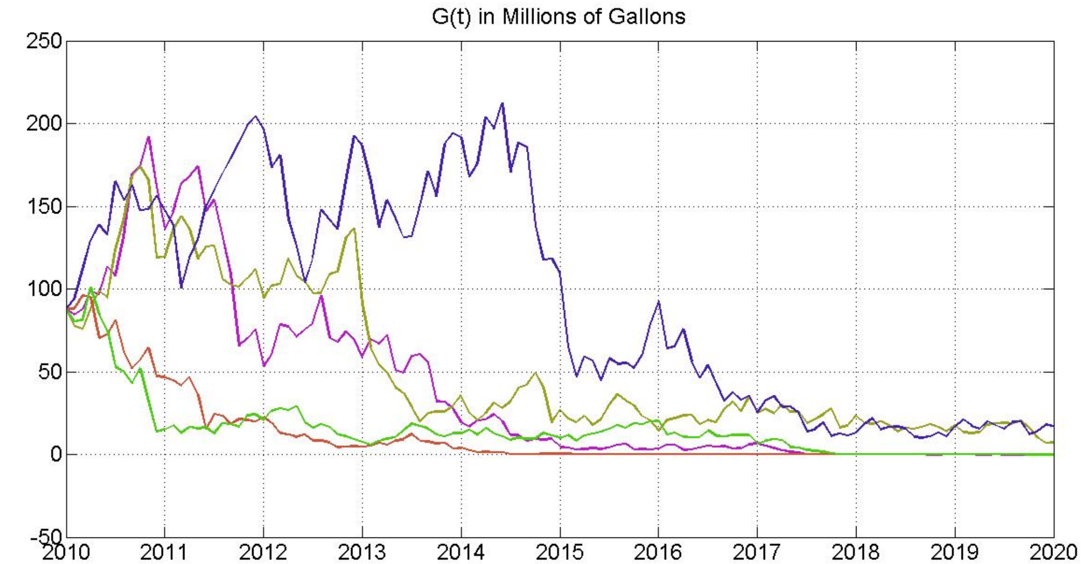

Figure 5.38: Trajectories of G(t) based on model 2 using ν, ρ since 1988

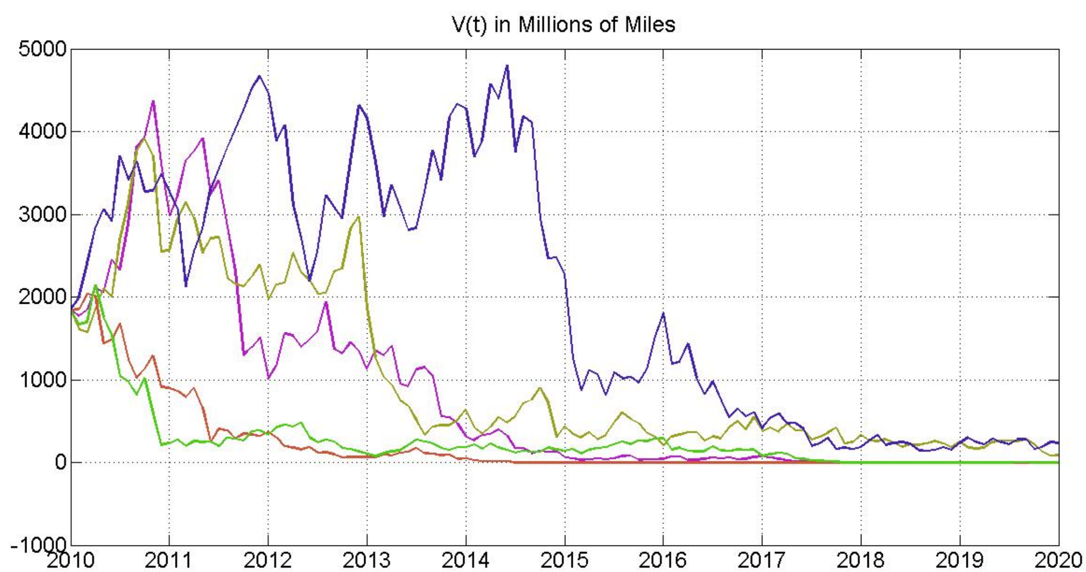

Figure 5.39: Trajectories of V(t) based on model 2 using θ, η since 1988

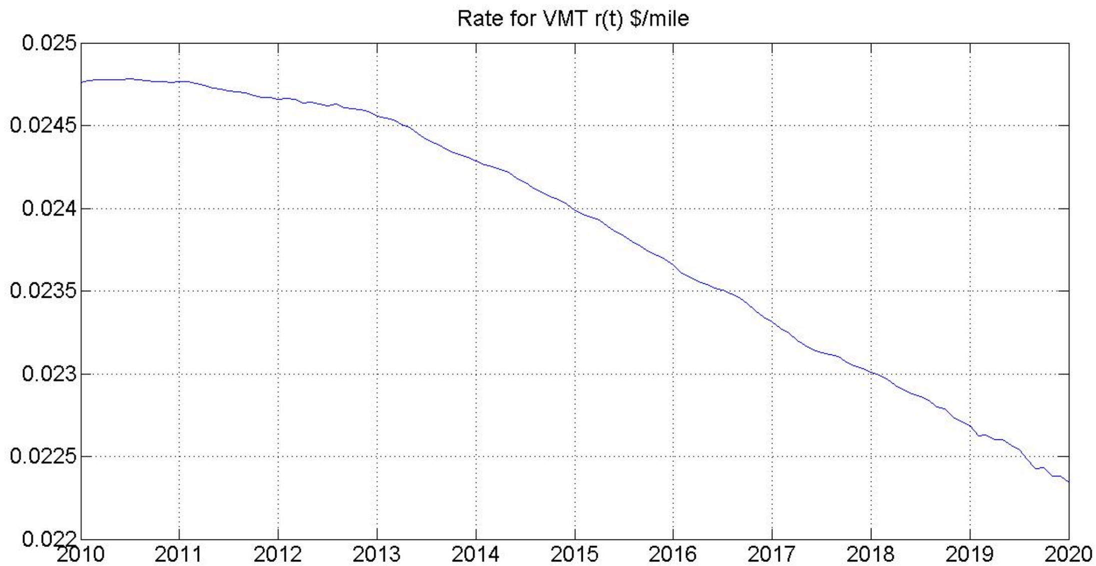

Figure 5.40: r(t) based on model 2 using parameters for data since 1988

of the fore casted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

Simulation using estimated parameters for data since 1998

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.45–5.48 displays five different trajectories of the fore casted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

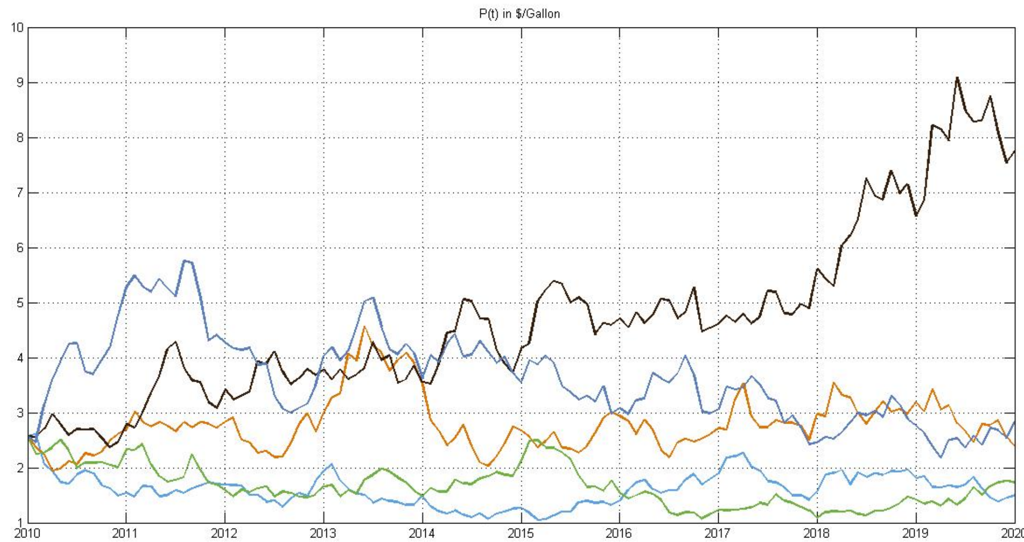

Figure 5.41: Trajectories of P(t) based on model 2 using μ, σ since 1993

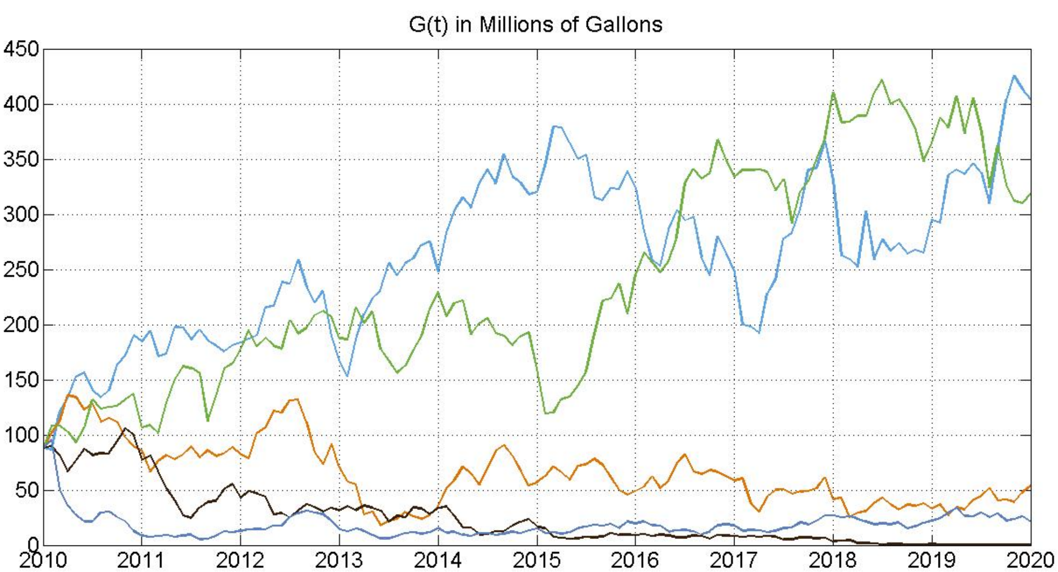

Figure 5.42: Trajectories of G(t) based on model 2 using ν, ρ since 1993

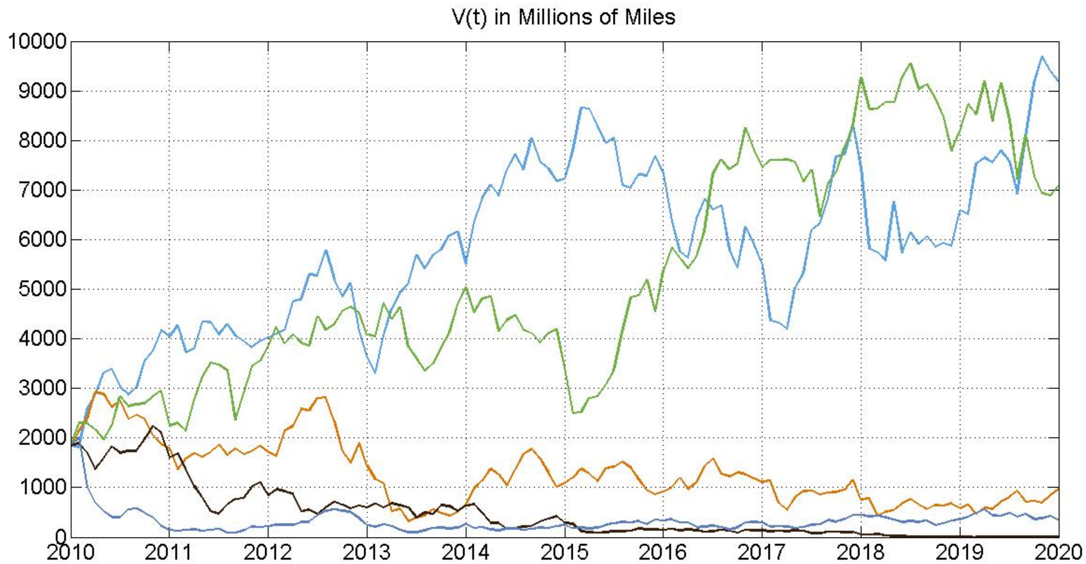

Figure 5.43: Trajectories of V(t) based on model 2 using θ, η since 1993

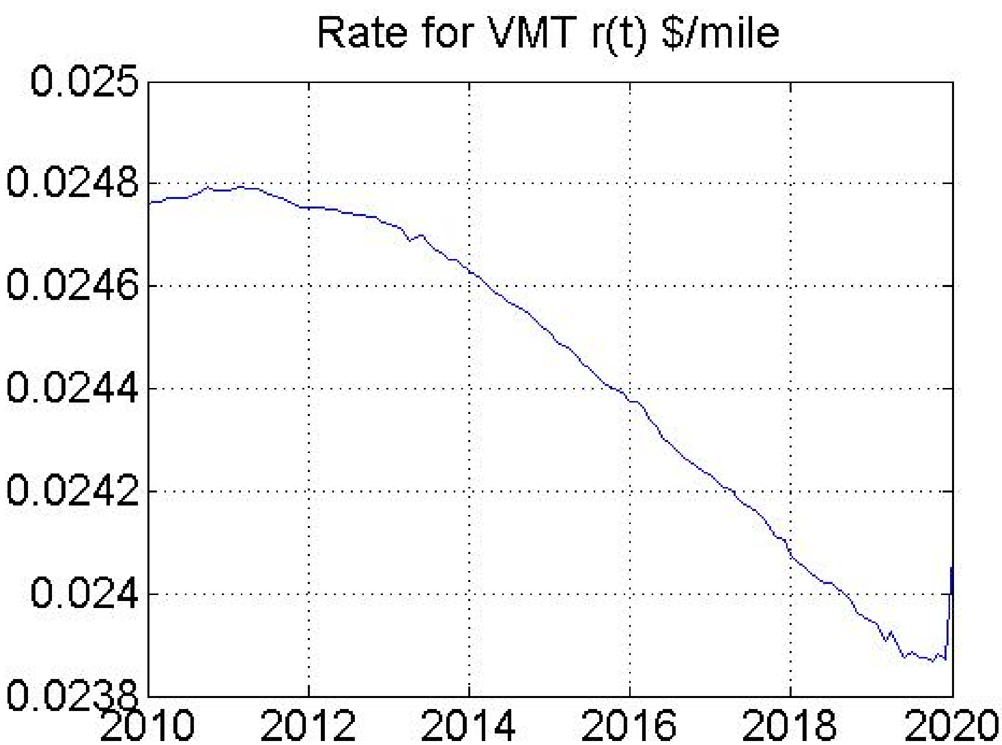

Figure 5.44: r(t) based on model 2 using parameters for data since 1993

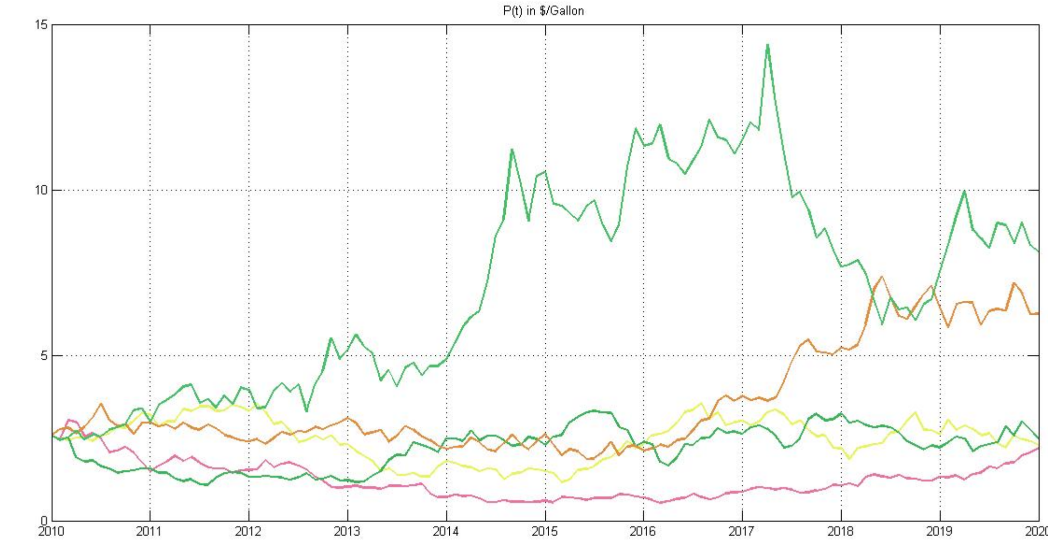

Figure 5.45: Trajectories of P(t) based on model 2 using μ, σ since 1998

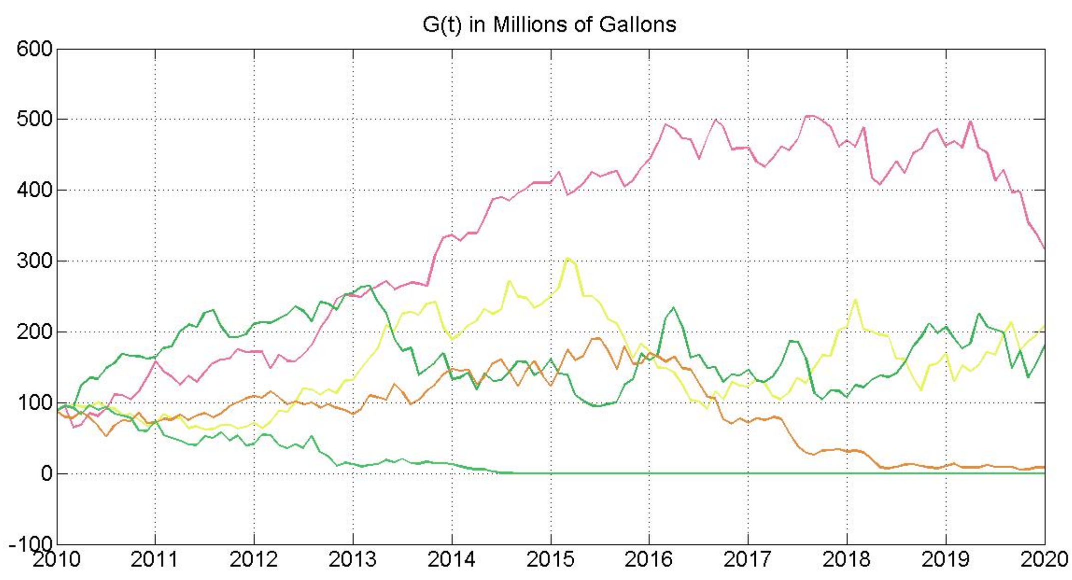

Figure 5.46: Trajectories of G(t) based on model 2 using ν, ρ since 1998

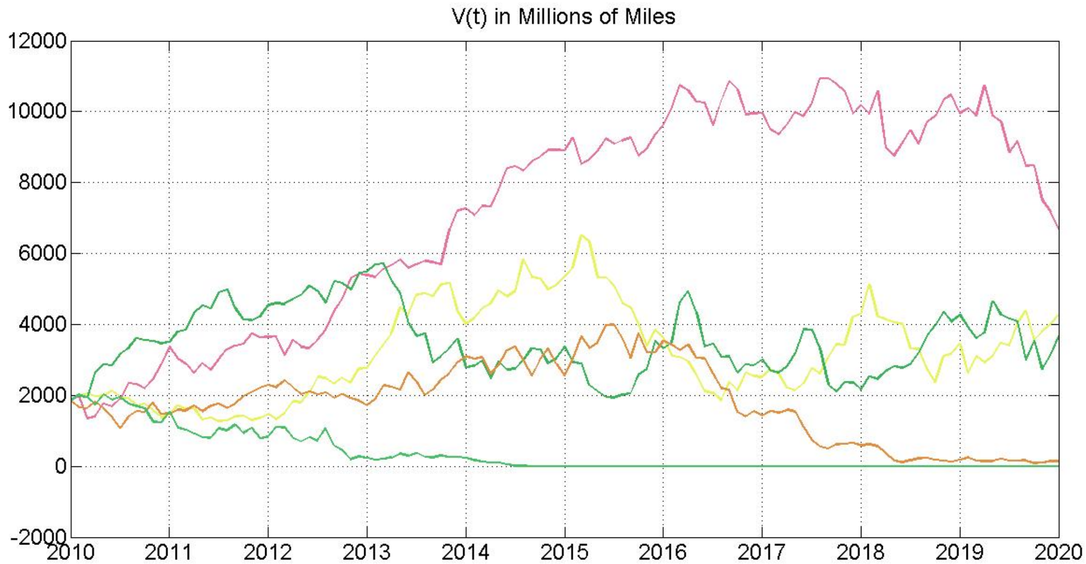

Figure 5.47: Trajectories of V(t) based on model 2 using $\theta,\ \eta$ since 1998

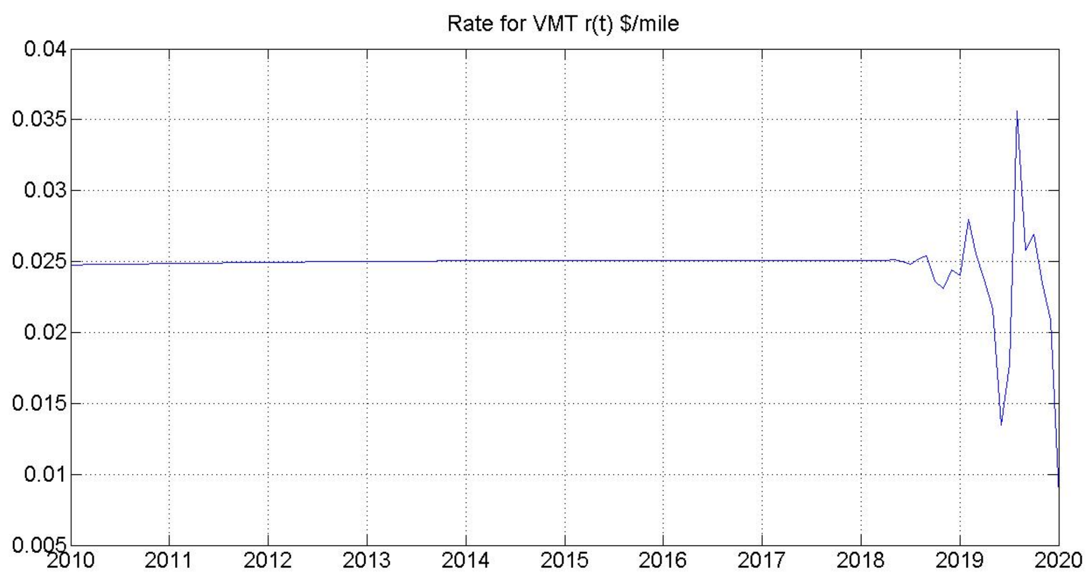

Figure 5.48: r(t) based on model 2 using parameters for data since 1998

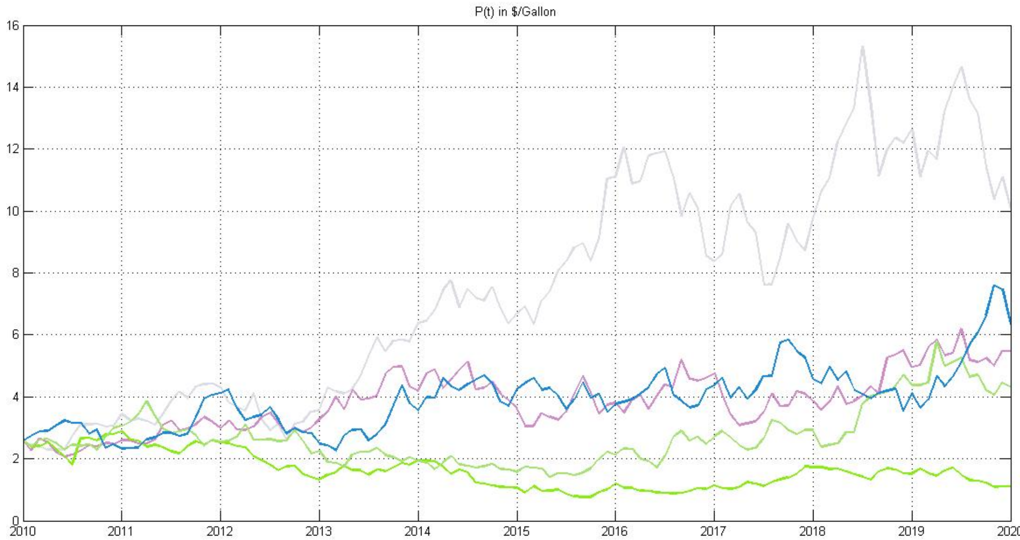

Figure 5.49: Trajectories of P(t) based on model 2 using $\mu,\ \sigma$ since 2003

Simulation using estimated parameters for data since 2003

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ \$ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ s\eta$ and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.49– 5.52 displays five different trajectories of the fore casted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

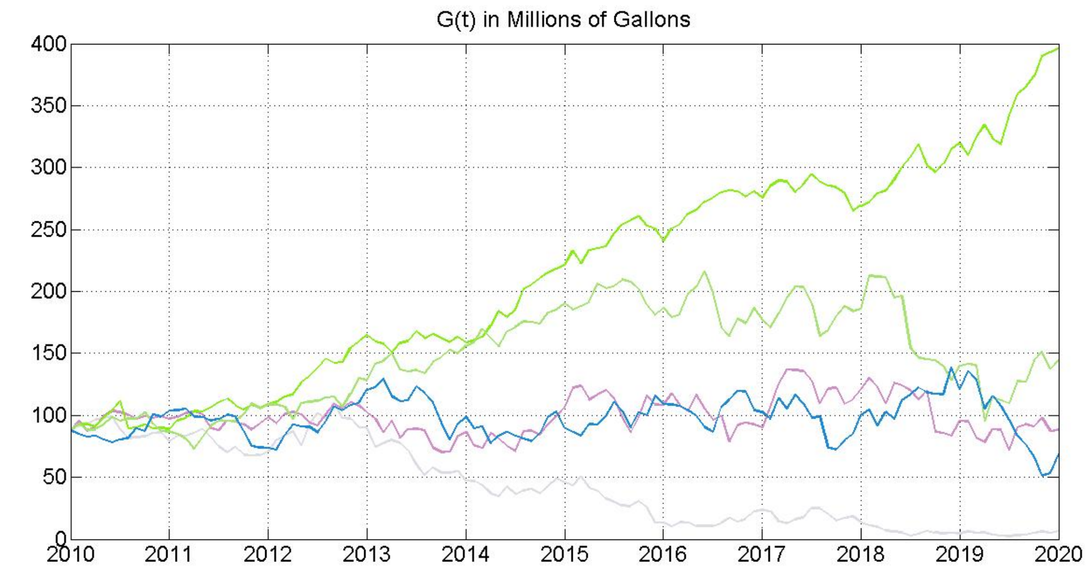

Figure 5.50: Trajectories of G(t) based on model 2 using ν, ρ since 2003

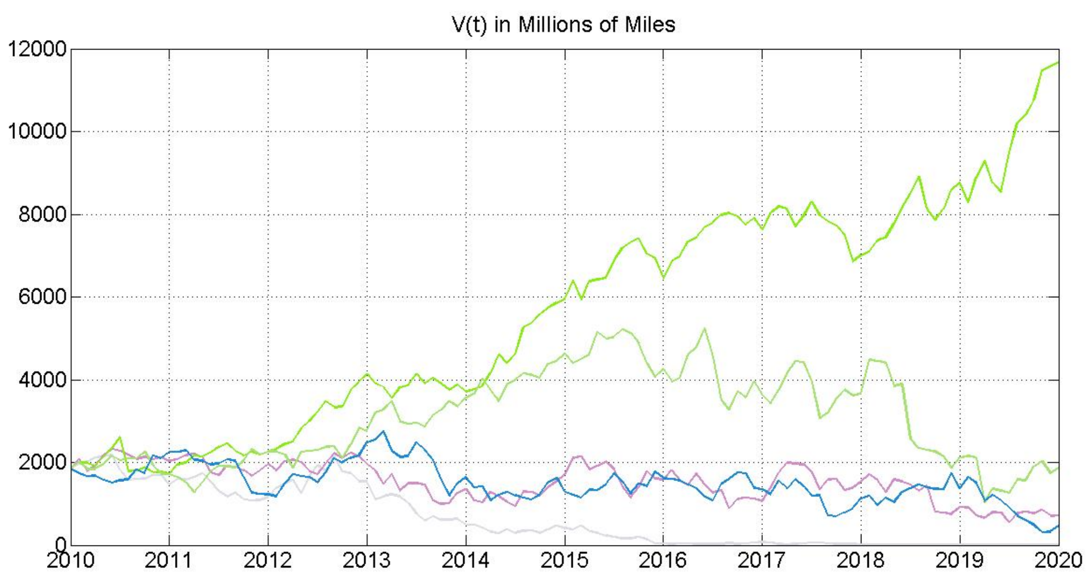

Figure 5.51: Trajectories of V(t) based on model 2 using θ, η since 2003

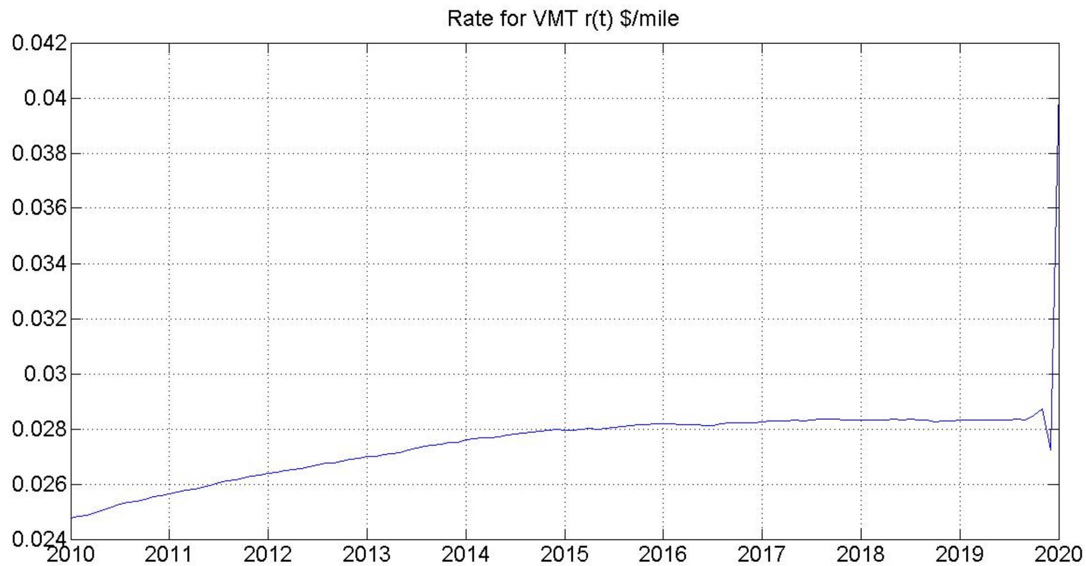

Figure 5.52: r(t) based on model 2 using parameters for data since 2003

Simulation using estimated parameters for data since 2008

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.567$ $ per gallon as gas price in Dec 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for Dec 2010 has been considered initial values of $V_0 = 1845.397$ millions of miles and $G_0 = 87.873$ millions of Gallon. Figures 5.53–5.56 displays five different trajectories of the fore casted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 10 years

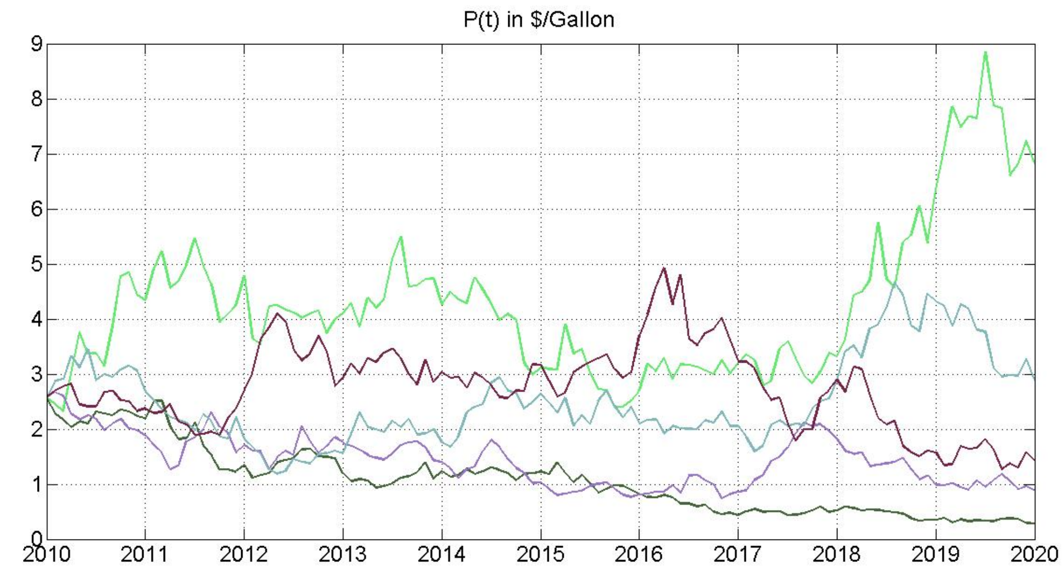

Figure 5.53: Trajectories of P(t) based on model 2 using μ, σ since 2008

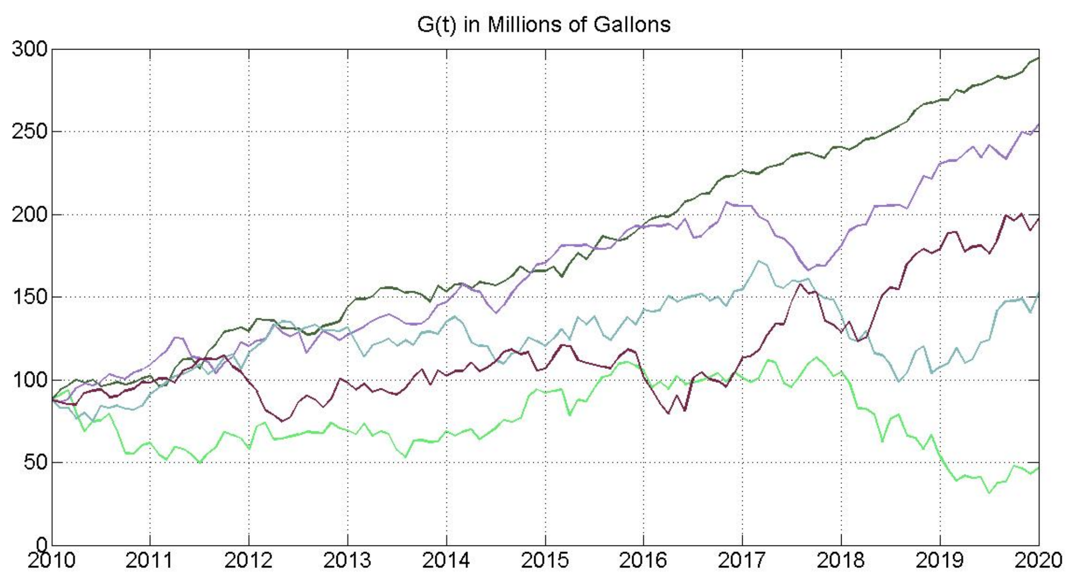

Figure 5.54: Trajectories of G(t) based on model 2 using ν, ρ since 2008

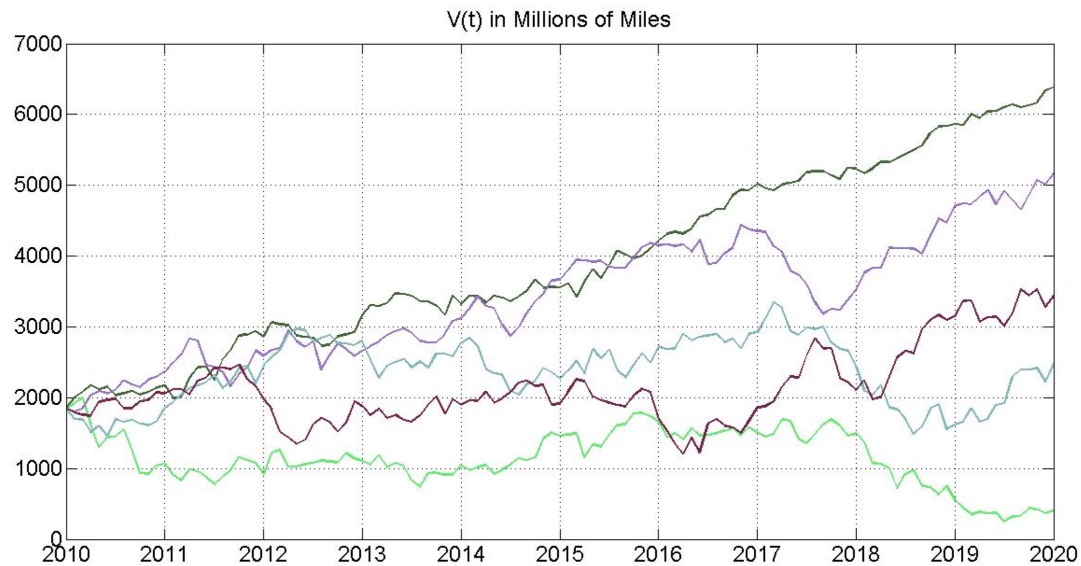

Figure 5.55: Trajectories of V(t) based on model 2 using $\theta,\ \eta$ since 2008

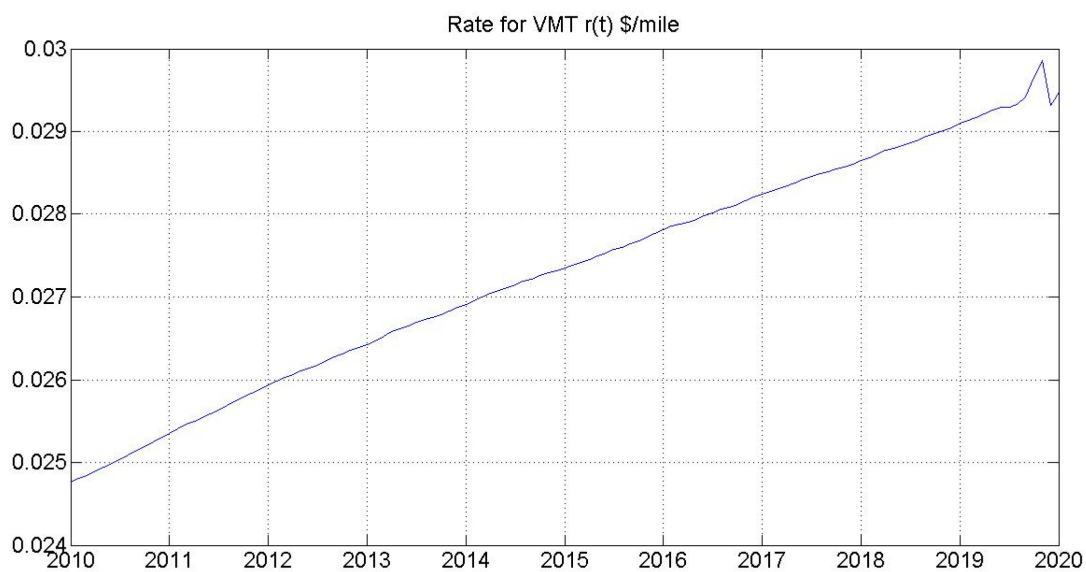

Figure 5.56: r(t) based on model 2 using parameters for data since 2008

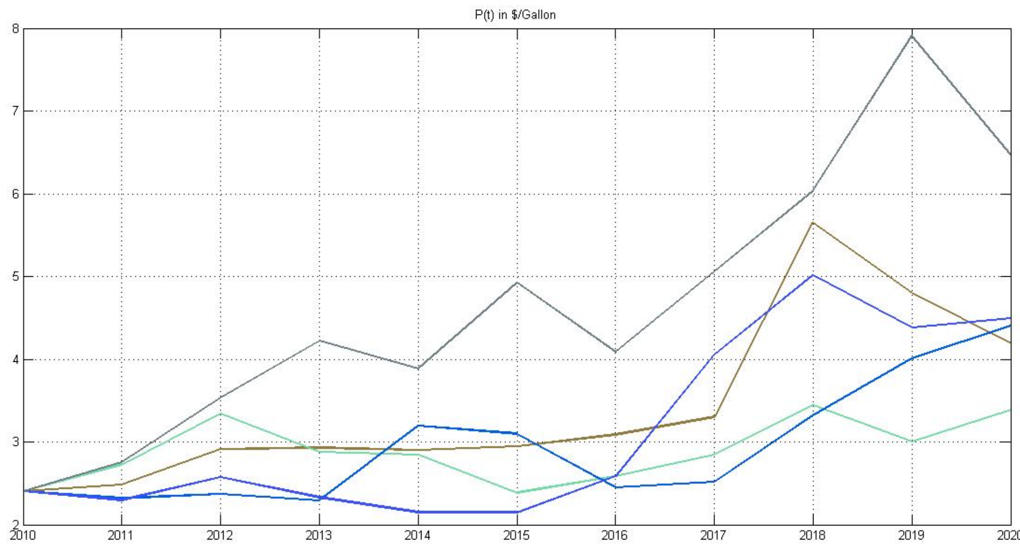

Figure 5.57: Trajectories of P(t) based on model 2 using $\mu,\ \sigma$ since 2004

5.3.2 Simulation for model 2 Using Anual Data

Simulation using estimated parameters for data since 2004

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.405$ \$ per gallon as gas price in 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on $\nu,\ \rho,\ \theta,\ \eta$ and simulated $P(t)$. Values for year 2010 has been considered initial values of $V_0 = 22144.76$ millions of miles and $G_0 = 809.046$ millions of Gallon. Figures 5.57–5.60 displays five different trajectories of the forecoasted values of $P(t),\ G(t),\ V(t)$ and $r(t)$ for next 10 years

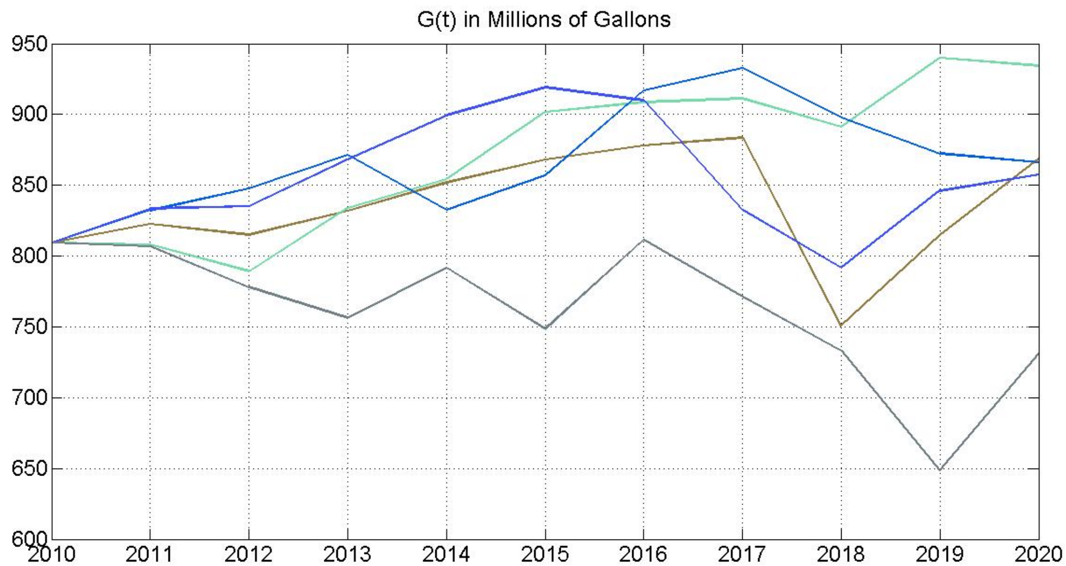

Figure 5.58: Trajectories of G(t) based on model 2 using ν, ρ since 2004

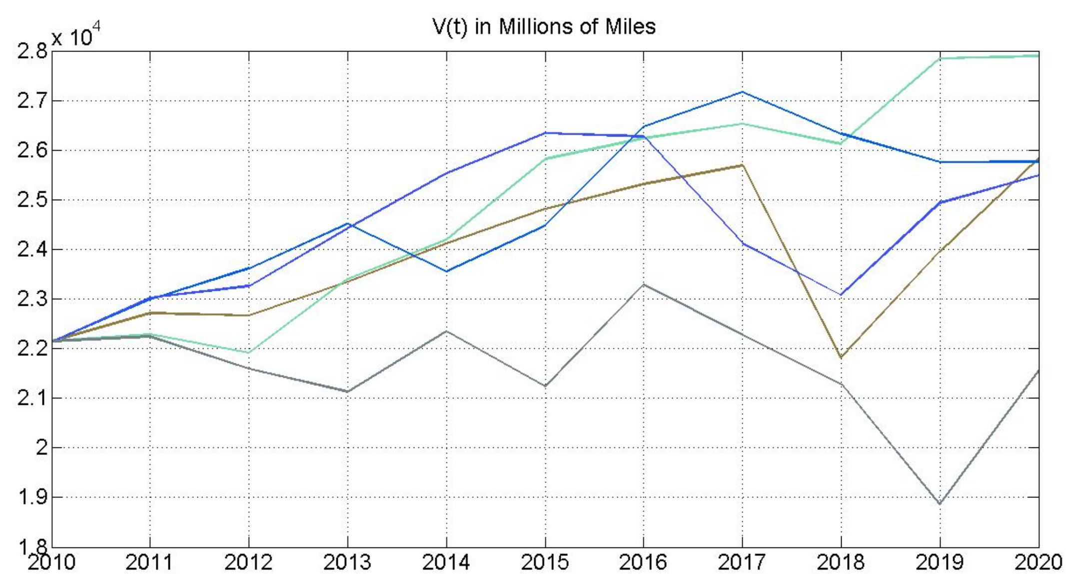

Figure 5.59: Trajectories of V(t) based on model 2 using θ, η since 2004

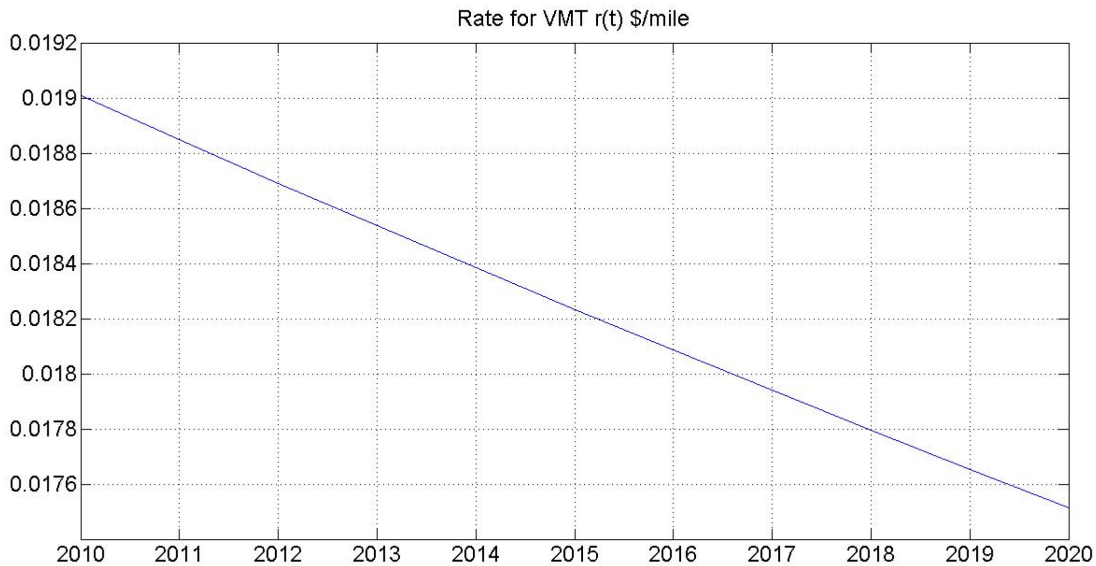

Figure 5.60: r(t) based on model 2 using parameters for data since 2004

Simulation using estimated parameters for data since 2004

Below plots represent the simulated value of $P(t)$, $G(t)$ and $V(t)$. $P(t)$ has been simulated using estimated parameters for the corresponding year and initial value of $P_0 = 2.405$ \$ per gallon as gas price in 2010. Once we have the simulated value of $P(t)$, $G(t)$ and $V(t)$ are simulated based on ν, ρ, θ, η and simulated $P(t)$. Values for year 2010 has been considered initial values of $V_0 = 22144.76$ millions of miles and $G_0 = 809.046$ millions of Gallon. Figures 5.61–5.64 displays five different trajectories of the forecoasted values of $P(t)$, $G(t)$, $V(t)$ and $r(t)$ for next 20 years

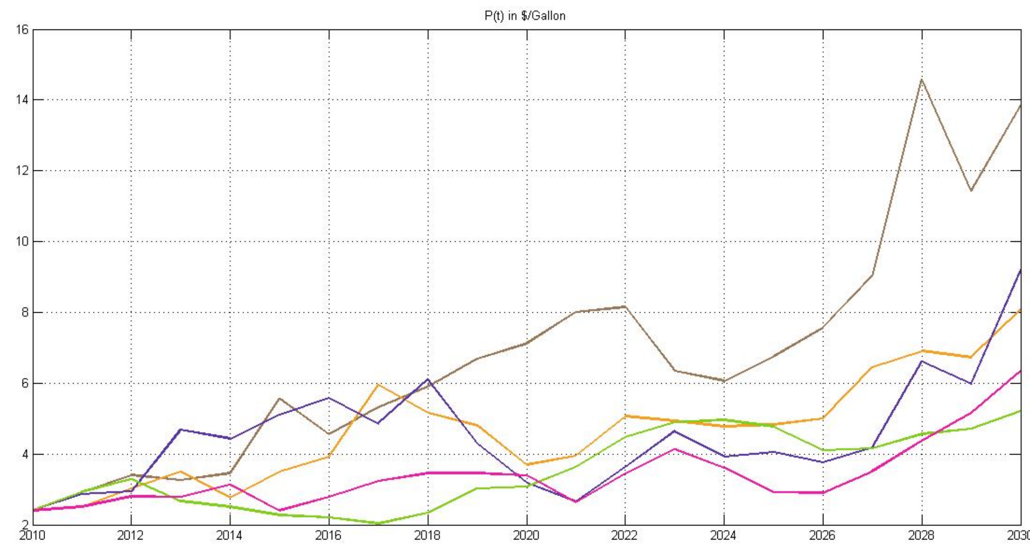

Figure 5.61: Trajectories of P(t) based on model 2 using $\mu,\ \sigma$ since 2004

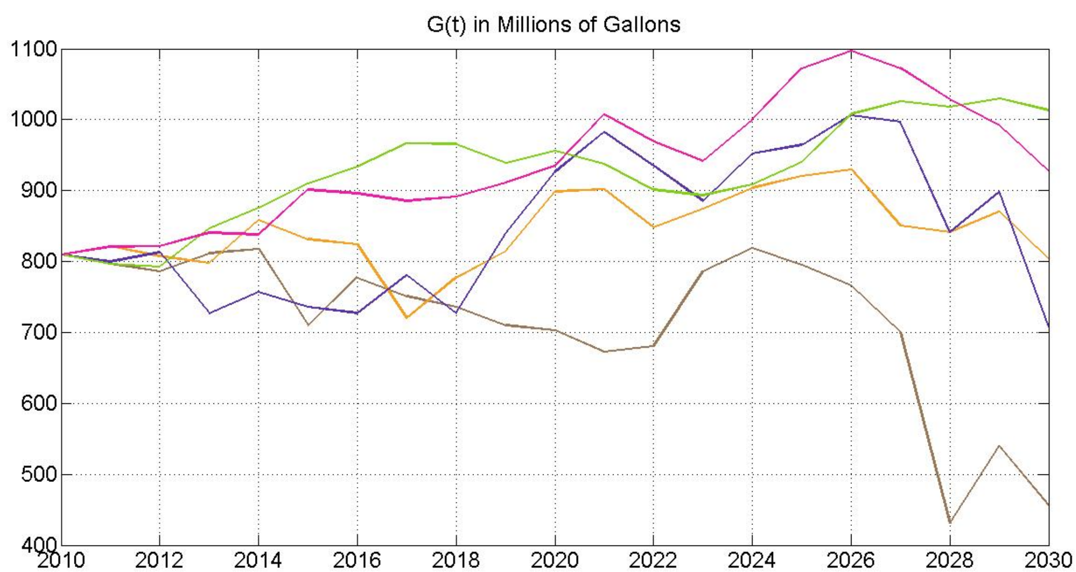

Figure 5.62: Trajectories of G(t) based on model 2 using $\nu,\ \rho$ since 2004

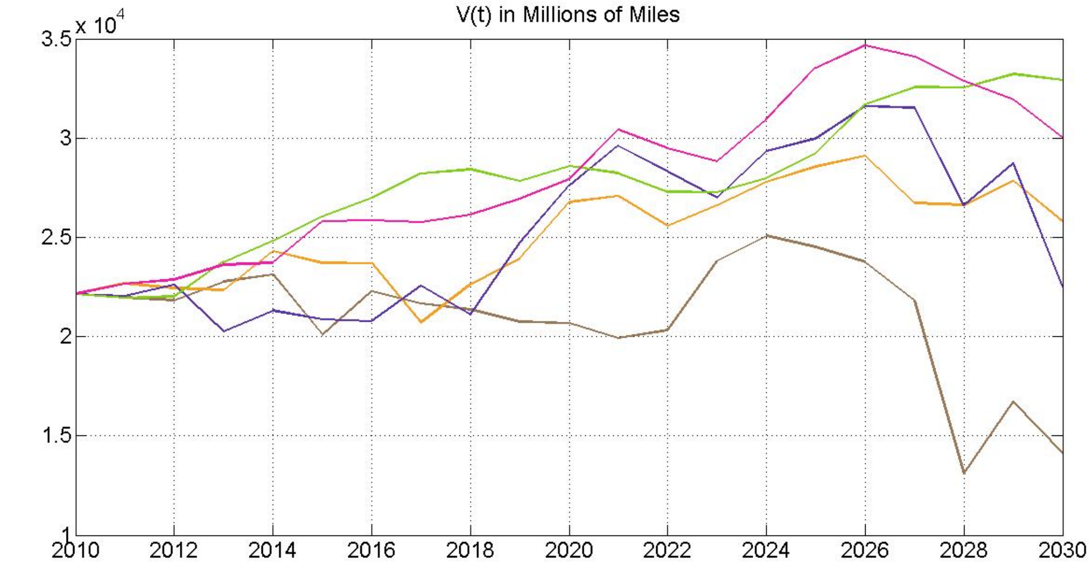

Figure 5.63: Trajectories of V(t) based on model 2 using $\theta,\ \eta$ since 2004

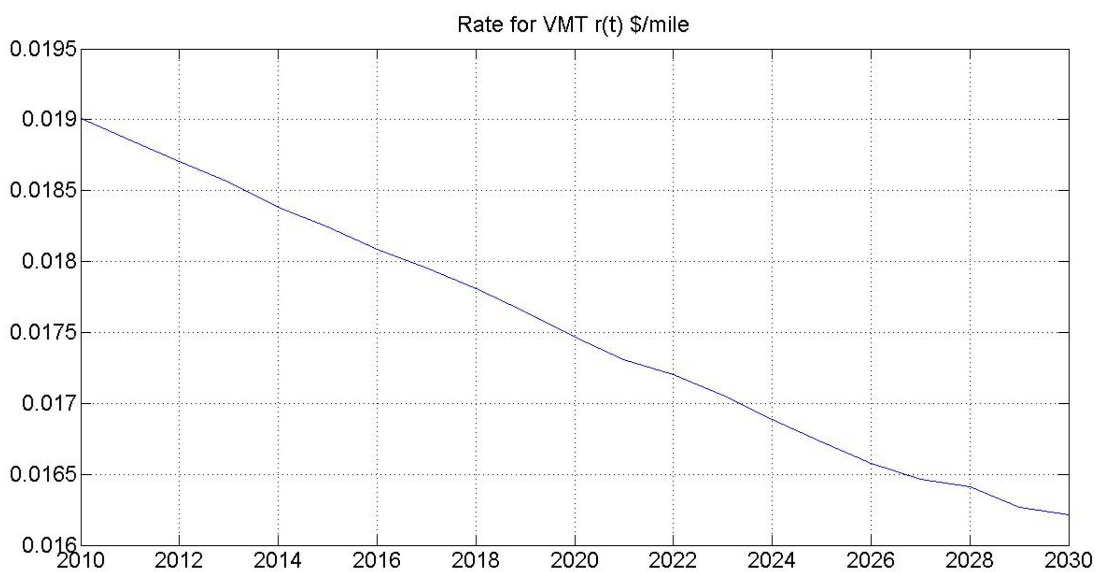

Figure 5.64: r(t) based on model 2 using parameters for data since 2004

5.4 Conclusion

VMT tax rate $r(t)$ is estimated approximately 2.5 $cents/mile$ from the results of simulation using various parameters calculated from data of various years. Current estimate of 2.5 $cents/mile$ for VMT tax rate $r(t)$ is to generate equal amount of revenue generated by fuel-tax. VMT tax rate $r(t)$ can be controlled in order to generate higher revenue to address the gap between required revenue and generated revenue by current system. Running various simulations for all the combinations, it is observed that model 1 of gas price with parameters estimated from datasets 2003-2010 and 2008-2010 have better converging results for $P(t)$, $G(t)$ and $V(t)$. It is also observed the identified value 2.5 $cents/mile$ of $r(t)$ is close to the estimated value of 2 $cents/mile$ of VMT tax rate by Nevada Department of Transportation (NDOT). Using stochastic models for variable $P(t)$, $G(t)$ and $V(t)$ over all system incorporates the affect due to inflation and $r(t)$ can be controlled in order to minimize its impact on generated revenue.

CHAPTER 6

CONCLUSION AND DISCUSSION

In various studies, VMT has been proven a potential alternative to the current gas tax based road revenue system. Apart from various technical challenges, it is extremely important to understand the mathematical aspect of VMT. In order to be sure that VMT will address the gap between required and generated revenue, it is necessary to develop and study mathematical models considering all possible factors such as inflation, variation of gas price etc. As the system has impact of lots of unpredictable factors like inflation, gas price, economy etc, a model based on stochastic differential equation (SDE) has been chosen to have better understanding of its behaviour. While modeling amount of gas consumed and vehicle miles traveled impact of change in gas price has been considered in both the models. Intuitively, increment in gas price will cause negative impact on increment of gas consumed and corresponding VMT while decrement in gas price might be an encouraging factor in increment of gas consumed and in corresponding VMT. Parameters of models have been estimated using real time past data of various years. Once parameters were estimated, future predictions were made based on current values. As result of simulation, VMT tax rate is estimated approximately 2.5 cents per mile in order to generate same amount of revenue as it is being generated by current system. Value of VMT tax rate is close enough to the estimated value of 2 cents per mile by Nevada Department of Transportation. In order to address the gap between required revenue and generated revenue, VMT tax rate can be adjusted accordingly based on estimated value.

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