Index

associated Laguerre functions, 29, 171
associated Legendre functions, 29, 170

Banach space, **105**
Bloch decomposition, 36
Brownian scaling, 43
Burger's equation solution map, 198

Cauchy–Riemann equations, 3
Cesaro integrability, 116
complete orthonormal basis, 26
conformal mapping, 2, 8, 99
conservation law, 15, 34
constituitive law, 31
convex dual, 187
convex duality, 187
convolution, 46
convolution algebra, 114

d'Alembert's formula, 17
Darboux equation, 127
diffusion equation, 56
Dirac δ-function, 49, 109, 110
Dirichlet–Neumann operator, **86**, 88, 94
dispersion, 151
dispersion relation, 160
dispersive equation, 34
distributions, 110
double layer potential, 85, 87
Duhamel's principle, 19
  heat equation, 53
  wave equation, 19

Einstein's equations, 6
electrical impedence tomography, 91
entire functions, 134
entire functions of exponential type, 135
entropy, 62
entropy, relative, 64
Euler equations, 196
Euler–Poisson–Darboux equation, 129

Féjer kernel, 117
first-order quasilinear equation, *178*
  arrow of time, 185
  Burger's equation, 178
  characteristic curves, 179
  characteristic equations, 179
  conservation laws, 178
  entropy condition, 185
  Hamilton–Jacobi equation, 188
  jump condition, 182
  Lax–Olenik solutions, 186
  Lax–Olenik variational principle, 189
  Lax–Olenik weak solutions, 190
  noncharacteristic Cauchy surface, 180
  Riemann problem, 183
  shock curves, 192
  shocks, 181
  uniqueness, 194
  weak solutions, 181
flow of a dynamical system, 200
fluid dynamics, 196
Fourier multiplier, 154
Fourier transform, 10, **105**
  Fourier inversion formula, 10
  isometry, 105

Schwartz class, 110
Fréchet derivative, 93
Fréchet spaces, 108

Gâteau derivative, 66
group velocity, 161

Hadamard variational formula, **93**, 99
Hadamard well posed criterion, 198
Hamiltonian PDE, **139**, 140, 141
action integral, 139
Euler–Lagrange equations, 139
Hamilton's canonical equations, 142
Kortweg–de Vries equation, 144
Lagrangian, 139
nonlinear Schrödinger equation, 144
nonlinear wave equation, 143
principle of least action, 139
harmonic function, 3, 72
heat equation, 4, **43**
conservation laws and moments, 56
contraction on L^1, 60
contraction on L^2, 60
Dirichlet boundary conditions, 52
energy functional, 65
entropy, 68
Fourier's law, 57
Green's identities, 55
heat kernel, 45
initial value problem, 49
initial-boundary value problem, 55
maximum principle, 48
method of images, 51, 67
Neumann boundary conditions, 52
nonuniqueness of solutions, 69
semigroup property, 46, 66
separation of variables, 68
weak maximum principle, 48
Heisenberg uncertainty principle, **158**, 167
Hermite functions, 28
Hilbert space, 104, **105**
Hölder continuity, 106
Hölder continuity, lack of for solution map of PDE, 200
Hölder inequality, 119
Hölder spaces, 106
Huygens' principle, 18, 132, 138
hydrogen atom, 28, 40, 156
orbitals, 169
hyperbolic rotation, 123

integrable functions, 113

John, F., 145
joke, 2

Klein–Gordon equation, 145

L^p spaces, 105
Laplace's equation, 3, **71**
attenuation of oscillatory data, 91
conformal transformations, 99
Dirichlet problem, 72
eigenfunctions, 97
fundamental solution, 74, 95
Gauss' law of arithmetic mean, 77, 98
Green's function, 81, 98
Green's function for $\mathbb{R}^n_+$, 81
Green's function for $B_R(0)$, 82
Green's identities, 72, 73
interior regularity, 76
Laplacian operator, 3
maximum principle for subharmonic functions, 79
maximum principle, strong, 79, 90
maximum principle, weak, 77
minimum principle for superharmonic functions, 79
mixed boundary conditions, 98
Neumann problem, 72, 84
Poisson kernel, **83**, 90
Poisson kernel for $\mathbb{R}^n_+$, 84, 87
Poisson kernel for $B_R(0)$, 84
Poisson problem, 72
Robin problem, 72
separation of variables, 95, 96
subharmonic function, 79
superharmonic function, 79
Lax–Olenik solutions, 186
Legendre transform, 187
Lipschitz continuity, 106

Maxwell's equations, 4
method of characteristics, 12
method of stationary phase, **162**, 163
metric space, 108
minimal surfaces, 7
Monge–Ampère equations, 5

Navier–Stokes equations, 8

Paley–Wiener theory, **134**, 136
phase velocity, 161
physics of music, **29**

Beals, R., 32
clarinet, 31
DeTurck, D., 33
drum, 33
oboe, 32, 37
saxophone, 32
strings, 29
synthesizers, 33
woodwinds, 30
Plancherel identity, 105

quantum harmonic oscillator, 27, 39, 156
quantum mechanics, 26

real analytic functions, 108
Riemann–Lebesgue Lemma, 114

Schauder spaces, 106
Schrödinger's equation, 6, **152**
bounds on moments, 168
complex probability amplitude, 152
conservation laws, 154, 166
Dirichlet boundary conditions, 167
Gaussian wave packets, 168
kinetic energy, 154
momentum distribution, 154
periodic boundary conditions, 167
position distribution, 154
potential, 166
quantum mechanical flux, 156
Schrödinger kernel, 152
separation of variables, 167, 169
Sobolev estimates, 155
wave function, 152
Schwartz class, **107**
self-adjoint operator, 27
separation of variables, **24**
heat equation, 68
Laplace's equation, 95
Schrödinger's equation, 167
wave equation, 24
shear flow, 196
single layer potential, 84
Sobolev embedding theorem, 106
Sobolev space, 105
special theory of relativity, 126
strong Huygens' principle, 18
symbol, 160

tempered distributions, 110
transport equations, 10

variation, 93
variation of constants, 21

water waves, **162**
dispersion relation, 173
Kelvin ship wake, 172
wave equation, 4, **16**, 121
Cauchy problem, 16, 121
Cauchy problem in Friedmann–Robertson–Walker space-time, 148
conservation of energy, 35, 122
Dirichlet boundary conditions, 21
domain of dependence, 134
domain of influence, 134
equipartition of energy, 35, 147
finite propagation speed, 18, 133
focusing singularity, 145
Fourier synthesis, 122
global existence, 145
Huygens' principle, **132**, 138
Huygens' principle, strong, 132
inhomogeneous problem, 19
initial value problem, 121
John, F., 145
Kirchhoff's formula, 130
light cone, 17, 125
Lorentz transformations, 123
Minkowski space, 126, 148
Neumann boundary conditions, 21
propagation of singularities, 36
spherical means, 127
spherical means in $\mathbb{R}^2$, 146
spherical means in $\mathbb{R}^3$, 129
spherical means in $\mathbb{R}^n$ for n odd, 130
spherical means in $\mathbb{R}^{2m}$, 147
traveling waves, 35
weak solutions, 17