Sample Problems and Solutions

The content of the questions on the Kennesaw State Mathematics Competition include all topics in the high school mathematics curriculum up through, but not including, Calculus.

1. In a magic square, each row, column, and diagonal has the same sum. All the odd numbered years from 1987 to 2003 are used to form a magic square. Given the locations of 1999 and 2003, what is the year that belongs in the box marked *X*?

	1999	
2003		
		X

(A) 1987 (B) 1991 (C) 1995 (D) 1997 (E) 2001

2. Three distinct digits are selected from the set {1,2,3,4,5,6,7,8,9} and then the three digits are multiplied together to form a number. What is the probability that this number is a multiple of 10?

(A) $\frac{13}{36}$ (B) $\frac{1}{7}$ (C) $\frac{11}{42}$ (D) $\frac{2}{7}$ (E) $\frac{13}{42}$

3. One four-digit number is subtracted from another four-digit number. The digits of one number are 1, 3, 5, and 7 in some order. The digits of the other number are 2, 4, 6, and 8 in some order. Compute the smallest positive difference that can be obtained.

(A) 267 (B) 275 (C) 279 (D) 283 (E) None of these

4. In triangle ABC, AB = 12″, AC = 16″, and BC = 20″. Point P is chosen in the interior of triangle ABC so that its distance from both sides AB and BC is 1”. Compute the distance from point P to side AC.

(A) 6″ (B) 7″ (C) 8″ (D) 9″ (E) 10″

5. If $\log(\sin x)+\log(\cos x)+\log(\tan x)=-1$, where $0° < x < 90°$, compute the value of $\tan x$. (Logarithms are to base 10.)

(A) 10 (B) $\frac{1}{10}$ (C) 3 (D) $\frac{1}{3}$ (E) 1

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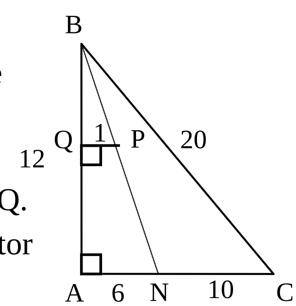
1. (E) The average of the nine years is 1995. Thus the sum of the years in any row, column, or diagonal of the magic square is 3x1995 = 5985. Let N(r,c) represent the year in row r and column c. Then N(1,2) = 1999 and N(2,1) = 2003. Let N(1,1) = x. Then N(1,3) = 3986-x and N(3,1) = 3982-x. Now N(2,2) = 2x-1983. Finally, N(2,3) = 5965-2x. Since N(1,1) + N(2,2) = N(1,3) + N(2,3), we have x + (2x-1983) = (3986-x) + (5965 - 2x) or 6x = 11934 or x = 1989. Now the remaining years can be filled in, as shown, making **2001** the answer.

1989	1999	1997
2003	1995	1987
1993	1991	2001

2. (C) One of the three digits must be a 5. At least one of the remaining two digits must be even. If both are even, there are ${}_4C_2 = 6$ choices. If one is even and the other odd, there are ${}_4C_1 \cdot {}_4C_1 = 16$ choices. Therefore, there are 22 numbers divisible by 10. Since there are ${}_9C_3 = 84$ ways of choosing three digits, the desired probability is $\frac{22}{84} = \frac{11}{42}$.

3. (B) (i) The thousands digits must be consecutive. (ii) In the hundreds place, the largest digit of one set (7 or 8) must be subtracted from the smallest digit (2 or 1) of the other set. Thus the subtraction is ?1??-?8?? and the difference is less than 300. (iii) In the tens place, the second largest digit of the first set is subtracted from the second smallest digit of the second set. Thus the subtraction is ?13?-?86?. (iv) The remaining digits are 2, 4, 5, and 7. Since 4 and 5 are the only consecutive pair, the subtraction is 5137-4862 = **275**.

4. (E) Clearly, ΔABC is a right triangle with right angle at A . Since point P is equidistant from $\overline{AB}$ and $\overline{BC}$, it lies on the bisector of $\angle B$. Let the angle bisector intersect $\overline{AC}$ at N. Then, $\frac{AN}{CN} = \frac{12}{20}$, and since AC = 16, this makes AN = 6 and CN = 10. Let the perpendicular from P to $\overline{AB}$ intersect $\overline{AB}$ at Q. Because P is 1” from $\overline{AB}$, PQ = 1. Since $\Delta PQB \sim \Delta NAB$, with a scale factor of $\frac{1}{6}$, QB = 2 and AQ = 10. Therefore, the distance from P to $\overline{AC}$ is **10″**.



5. (D) Using the properties of logs and well known trigonometric identities,

$$\log(\sin x) + \log(\cos x) + \log(\tan x) = \log(\sin x \cdot \cos x \cdot \tan x) = \log(\sin x \cdot \cos x \cdot \frac{\sin x}{\cos x}) = \log(\sin^2 x)$$

Therefore, $\log(\sin^2 x) = -1 \Rightarrow \sin^2 x = \frac{1}{10}$ and $\sin x = \frac{1}{\sqrt{10}}$. Thus

$$\sin^2 x = 1 - \cos^2 x = \frac{1}{10} \text{ and } \cos x = \frac{3}{\sqrt{10}} \text{ and finally } \tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{\sqrt{10}}}{\frac{3}{\sqrt{10}}} = \mathbf{\frac{1}{3}}.$$