

Main Notation Used in This Book

z	Direction normal to the surface
x, y	Directions in the plane of the surface
$\parallel$	Used to describe a component parallel to the interface plane
xOz	Plane of incidence
j	Label of layer. Numbering of layers goes from 0 (upper medium) to N the last layer. s is the substrate
Z_j	Average location of the $j - 1, j$ interface
$z_j(x, y)$	Fluctuations of the interface location around Z_j
$\mathbf{k}$	Wave-vector
$\mathbf{k}_{\text{in}}, \mathbf{k}_{\text{r}}, \mathbf{k}_{\text{tr}}, \mathbf{k}_{\text{sc}}$	Incident, reflected, transmitted and scattered wave vectors
$k_{\text{in } z, j}$	z component of the incident wavevector in the j th layer
$k_{z, j}$	when unambiguous
$\mathbf{q}$	Wave vector transfer
q	Modulus of the wave vector transfer
$q_x, q_{\parallel}, q_z$	Components of the wave vector
$\mathbf{u}$	Scattering direction
r, t	Reflection and transmission coefficients in amplitude
R, T	Intensity reflection and transmission coefficients
$r_{j-1, j}$	Reflection coefficient in amplitude when passing from medium $j - 1$ to medium j
$t_{j-1, j}$	Transmission coefficient in amplitude when passing from medium $j - 1$ to medium j
$\mathbf{E}$	Electric field
$\hat{\mathbf{e}}_{\text{in}}, \hat{\mathbf{e}}_{\text{sc}}$	Polarisation vectors of the incident and scattered fields
$\mathbf{B}$	Magnetic field
$\mathbf{j}$	Current density
$\mathbf{P}$	Electric polarisation
$\mathbf{A}$	Vector potential
$\mathbf{S}$	Poynting's vector

$A_j^\pm$	Amplitude of the upwards and downwards propagating electric fields in layer j
$U(\pm k_{\text{in } z, j}, z)$	$A_j^\pm e^{\pm k_{\text{in } z, j} z}$
$\mathcal{M}$	Transfer matrix
p_n	n -point probability distribution
σ	rms roughness. $\sigma^2 = \langle z^2 \rangle$
$C_{zz}(x_1, x_2, y_1, y_2)$	Height–height correlation function Also denoted $\langle z(x_1, y_1) z(x_2, y_2) \rangle$
$g(r)$	$2\sigma^2 - 2C_{zz}(x_1, x_2, y_1, y_2)$
$\overline{G}$	Green function
$\overline{\mathcal{G}}$	Green tensor (electromagnetic case)

$e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}$ waves are used except in Chap. 5 devoted to neutron reflectivity (see Sect. 1.2.1 for details related to the conventions used in this book, and Sect. 5.1 for the notation used in Chap. 5).

Table 7.1 Typical length scales for x-ray reflectivity experiments

	Definition	Value
Wavelength λ		1 Å
Scattering length	b	$r_e = 2.818 \times 10^{-15}$ m for 1 electron
Extinction length	$L_e = \frac{\lambda}{2\pi n-1 }$	1 μm
Longitudinal coherence length	$\lambda^2 / \delta\lambda$	1 μm
Incidence slit opening		0.1 mm
Detector slit opening normal to the plane of incidence (y)	h_y	10 mm
Detector slit opening in the plane of incidence (x)	h_x	0.1–1 mm
Sample-to-detector distance	L	1 m
Transverse coherence length normal to the plane of incidence (y) (when fixed by the detector)	$\lambda / \Delta\theta_y$ with $\Delta\theta_y = h_y / L$	10 nm
Transverse coherence length in the plane of incidence projected on the surface (x) (when fixed by the detector)	$\lambda / (\theta \Delta\theta)$ with $\Delta\theta_x = h_x / L$	100 μm for $\theta = 10$ mrad
Illuminated area (length $\times$ width)		$(0.1 \text{ mm} / \theta) \times$ (1–10 mm)
Absorption length	$\mu = \lambda / 4\pi\beta$	0.1–1 mm for $\beta = 10^{-7}$ – 10^{-8}

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