PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN CHAPTER 14

EXERCISE 14.1

(a) The conditional mean $E(e_t \mid I_{t-1}) = 0$ because:

$$E_{t-1}(e_t) = E_{t-1}\left(z_t\sqrt{h_t}\right) \qquad \text{where } E_{t-1}(\bullet) \text{ is an alternative way of writing } E(\bullet \mid I_{t-1})$$

$$= E_{t-1}(z_t)E_{t-1}\left(\sqrt{h_t}\right) \qquad \text{since } z_t \text{ is independent of } h_t$$

$$= 0 \qquad \text{since } E_{t-1}[z_t] = 0$$

(b) The conditional variance $E(e_t^2 \mid I_{t-1}) = h_t$ because:

$$E_{t-1}\left(e_t^2\right) = E_{t-1}\left(\left(z_t\sqrt{h_t}\right)^2\right)$$

$$= E_{t-1}\left(z_t^2\right)E_{t-1}\left(h_t\right)$$

$$= h_t \qquad \text{since } E_{t-1}\left(z_t^2\right) = 1 \text{ and } E_{t-1}\left(h_t\right) = \alpha_0 + \alpha_1 e_{t-1}^2 = h_t$$

(c) $e_t \mid I_{t-1} \sim N(0, h_t)$ because $z_t \sim N(0,1)$ and hence $z_t\sqrt{h_t} \sim N(0, h_t)$ since $\sqrt{h_t}$ is known at time $t-1$.

EXERCISE 14.3

(a) If $\gamma = 0$, and $h_t = \delta + \alpha_1 e_{t-1}^2$, then,

when $e_{t-1} = -1$, $\quad h_t = \delta + \alpha_1(-1)^2 = \delta + \alpha_1$

when $e_{t-1} = 0$, $\quad h_t = \delta + \alpha_1(0)^2 = \delta$

when $e_{t-1} = 1$, $\quad h_t = \delta + \alpha_1(1)^2 = \delta + \alpha_1$

(b) If $\gamma \neq 0$, and $h_t = \delta + \alpha_1 e_{t-1}^2 + \gamma d_{t-1} e_{t-1}^2$, then

when $e_{t-1} = -1$, $d_{t-1} = 1 \quad \Rightarrow \quad h_t = \delta + \alpha_1(-1)^2 + \gamma(-1)^2 = \delta + \alpha_1 + \gamma$

when $e_{t-1} = 0$, $d_{t-1} = 0 \quad \Rightarrow \quad h_t = \delta + \alpha_1(0)^2 = \delta$

when $e_{t-1} = 1$, $d_{t-1} = 0 \quad \Rightarrow \quad h_t = \delta + \alpha_1(1)^2 = \delta + \alpha_1$

The key difference between the $\gamma = 0$ and $\gamma \neq 0$ cases lies with the contribution of the asymmetric factor.

EXERCISE 14.5

(a) Because $E\left(e_t \mid I_{t-1}\right) = E\left(e_t \mid e_{t-1,} e_{t-2}, \ldots\right)$, we have

$$E\left(e_t\right) = E_{e_{t-1}}E_{e_{t-2}}\cdots\left[E\left(e_t \mid e_{t-1,} e_{t-2}, \ldots\right)\right] = E_{e_{t-1}}E_{e_{t-2}}\cdots[0] = 0$$

(b) Also, for the variance,

$$\begin{aligned}
E\left(e_t^2\right) &= E_{e_{t-1}} E_{e_{t-2}} \cdots \left[E\left(e_t^2 \mid e_{t-1}, e_{t-2}, \ldots\right)\right] \\
&= E_{e_{t-1}} E_{e_{t-2}} \cdots \left[\alpha_0 + \alpha_1 e_{t-1}^2\right] \\
&= E_{e_{t-2}} E_{e_{t-3}} \cdots \left[\alpha_0 + \alpha_1 E\left(e_{t-1}^2 \mid e_{t-2}, e_{t-3}, \ldots\right)\right] \\
&= E_{e_{t-2}} E_{e_{t-3}} \cdots \left[\alpha_0 + \alpha_1\left(\alpha_0 + \alpha_1 e_{t-2}^2\right)\right] \\
&= E_{e_{t-3}} \cdots \left[\alpha_0 + \alpha_1\alpha_0 + \alpha_1^2 E\left(e_{t-2}^2 \mid e_{t-3}, \ldots\right)\right] \\
&= E_{e_{t-3}} \cdots \left[\alpha_0 + \alpha_1\alpha_0 + \alpha_1^2\left(\alpha_0 + \alpha_1 e_{t-3}^2\right)\right] \\
&= E_{I_{t-3}} \left[\alpha_0 + \alpha_1\alpha_0 + \alpha_1^2\alpha_0 + \alpha_1^3 e_{t-3}^2\right] \\
&= E_{I_{t-s}} \left[\alpha_0 + \alpha_1\alpha_0 + \alpha_1^2\alpha_0 + \cdots + \alpha_1^{s-1}\alpha_0 + \alpha_1^s e_{t-s}^2\right]
\end{aligned}$$

As $s \to \infty$, with $0 < \alpha_1 < 1$,

$$E\left(e_t^2\right) == \alpha_0 + \alpha_1\alpha_0 + \alpha_1^2\alpha_0 + \cdots = \frac{\alpha_0}{1-\alpha_1}$$

(c) For this model,

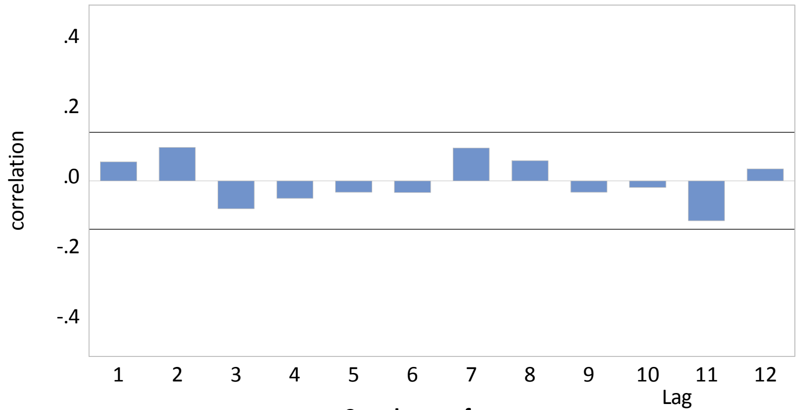
$$\begin{aligned}
E\left(e_t^2\right) &= E_{I_{t-1}}\left[E\left(e_t^2 \mid I_{t-1}\right)\right] \\
&= E_{I_{t-1}}\left[\delta + \alpha_1 e_{t-1}^2 + \beta_1 h_{t-1}\right] \\
&= E_{I_{t-2}}\left[\delta + \alpha_1 E\left(e_{t-1}^2 \mid I_{t-2}\right) + \beta_1 h_{t-1}\right] \\
&= E_{I_{t-2}}\left[\delta + \alpha_1\left(\delta + \alpha_1 e_{t-2}^2 + \beta_1 h_{t-2}\right) + \beta_1\left(\delta + \alpha_1 e_{t-2}^2 + \beta_1 h_{t-2}\right)\right] \\
&= E_{I_{t-2}}\left[\delta + \left(\alpha_1 + \beta_1\right)\left(\delta + \alpha_1 e_{t-2}^2 + \beta_1 h_{t-2}\right)\right] \\
&= E_{I_{t-3}}\left[\delta + \delta\left(\alpha_1 + \beta_1\right) + \alpha_1\left(\alpha_1 + \beta_1\right) E\left(e_{t-2}^2 \mid I_{t-3}\right) + \beta_1\left(\alpha_1 + \beta_1\right) h_{t-2}\right] \\
&= E_{I_{t-3}}\left[\delta + \delta\left(\alpha_1 + \beta_1\right) + \alpha_1\left(\alpha_1 + \beta_1\right)\left(\delta + \alpha_1 e_{t-3}^2 + \beta_1 h_{t-3}\right) + \beta_1\left(\alpha_1 + \beta_1\right)\left(\delta + \alpha_1 e_{t-3}^2 + \beta_1 h_{t-3}\right)\right] \\
&= E_{I_{t-3}}\left[\delta + \delta\left(\alpha_1 + \beta_1\right) + \left(\alpha_1 + \beta_1\right)^2\left(\delta + \alpha_1 e_{t-3}^2 + \beta_1 h_{t-3}\right)\right] \\
&= E_{I_{t-s}}\left[\delta + \delta\left(\alpha_1 + \beta_1\right) + \delta\left(\alpha_1 + \beta_1\right)^2 + \cdots + \delta\left(\alpha_1 + \beta_1\right)^{s-1} + \left(\alpha_1 + \beta_1\right)^{s-1}\left(\alpha_1 e_{t-s}^2 + \beta_1 h_{t-s}\right)\right]
\end{aligned}$$

As $s \to \infty$, with $0 < \alpha_1 + \beta_1 < 1$,

$$E\left(e_t^2\right) = \delta + \delta\left(\alpha_1 + \beta_1\right) + \delta\left(\alpha_1 + \beta_1\right)^2 + \cdots = \frac{\delta}{1 - \alpha_1 - \beta_1}$$

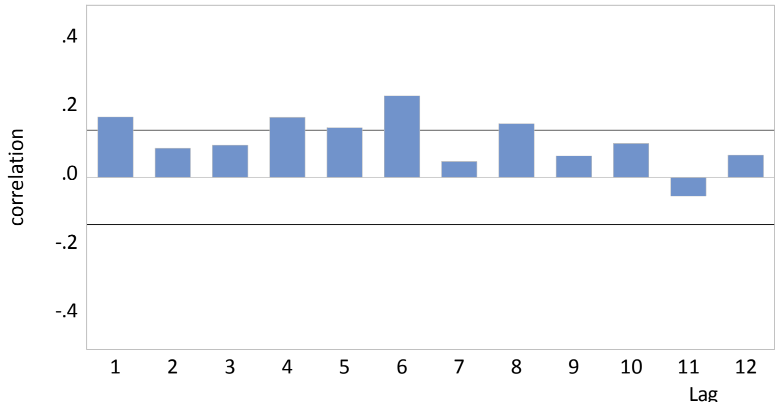
EXERCISE 14.7

(a) The correlogram of returns (up to order 12) is presented below, with 5% significance bounds drawn at $\pm 1.96/\sqrt{203} = \pm 0.138$. There is no evidence of autocorrelation since none of the autocorrelations exceed their significance bounds. In other words, there is no indication of significant lagged mean effects.



Correlogram for r

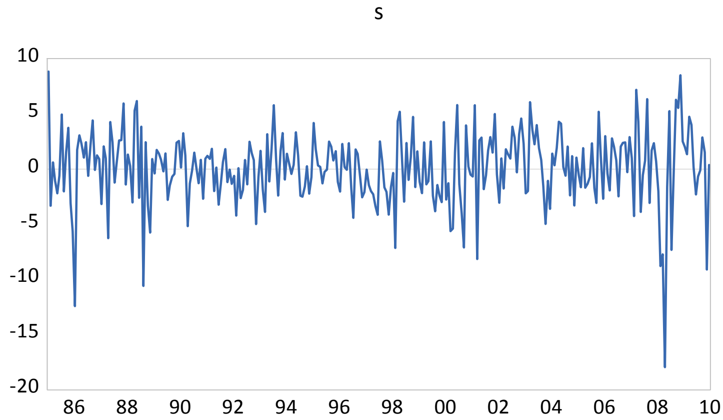
(b) The correlogram of squared returns (up to order 12) is given below, with 5% significance bounds drawn at $\pm 1.96/\sqrt{203} = \pm 0.138$. Although they are not large in magnitude, there is evidence of significant autocorrelation at lags 1, 4, 5, 6 and 8. In other words, there is indication of significant lagged variance effects.

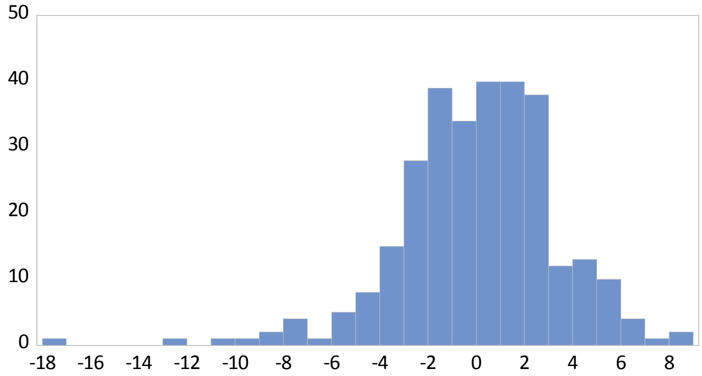


Correlogram for r-squared

EXERCISE 14.9

(a) A plot of the series and a histogram of all the observations are presented below. Volatility is relatively high at the beginning of the sample and towards the end of the sample, and relatively low in the middle years. The unconditional distribution of the series is not normal. It is skewed to the left and has a kurtosis of 6.484 which is very different from the kurtosis of 3 for normality. Furthermore, the Jarque-Bera statistic value of 192 is significantly greater than the 5% critical value of $\chi_{(0.95,2)} = 5.99$.





Series: S	
Sample 1985M07 2010M06	
Observations 300	
Mean	0.082467
Median	0.254433
Maximum	8.852538
Minimum	-17.98234
Std. Dev.	3.280140
Skewness	-0.900014
Kurtosis	6.486620
Jarque-Bera	192.4578
Probability	0.000000

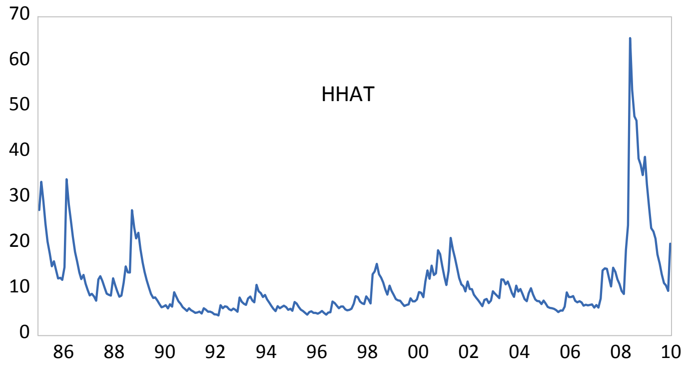
(b) GARCH(1,1) estimates for $S_t = \beta_0 + e_t$, $(e_t \mid I_{t-1}) \sim N(0, h_t)$, $h_t = \delta + \alpha_1 e_{t-1}^2 + \beta_1 h_{t-1}$ follow.

Dependent Variable: S
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample: 1985M07 2010M06
Included observations: 300
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

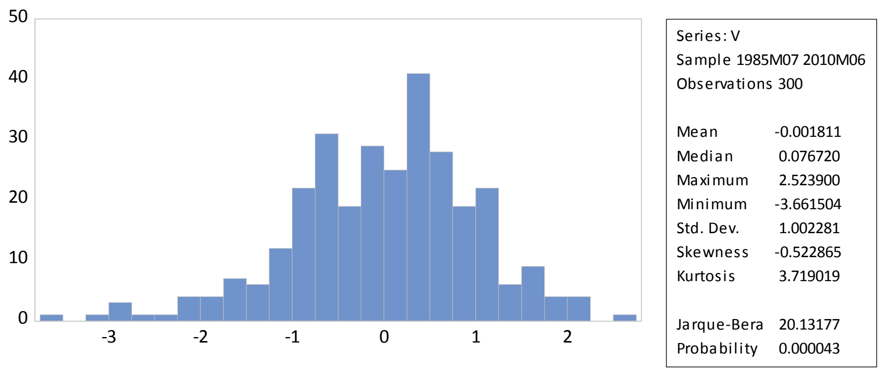
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.028717	0.171610	0.167338	0.8671
Variance Equation				
C	0.564709	0.310367	1.819486	0.0688
RESID(-1)^2	0.139344	0.030771	4.528368	0.0000
GARCH(-1)	0.811055	0.041467	19.55911	0.0000

The average value of the change in the exchange rate *s* is estimated as 0.0287, but it is not significantly different from zero. From the variance equation, the significance of the coefficient of the lagged squared residual term (0.139) indicates that lagged news/shocks affect volatility. The significance of the coefficient of h_{t-1} (0.811) indicates persistence of the lagged volatility effects. The GARCH estimation procedure uses a backcasting procedure which requires the setting of a backcast parameter. As indicated in the output, the following results were obtained with a backcast parameter of 0.7. Using a different parameter will change the results.

(c) A graph of the one-step ahead within-sample estimates $\hat{h}_t$ follows.



A histogram for the observations on $v_t = \hat{e}_t \big/ \sqrt{\hat{h}_t}$ where $\hat{e}_t$ are the residuals $\hat{e}_t = S_t - \hat{\beta}_0$ is given below. The distribution is skewed to the left with a Jarque-Bera *p*-value of 0.0000, casting doubt on one of the assumptions of the GARCH model.



(d) The forecast for the conditional mean in both 2010M7 and 2010M8 is 0.028717. The forecasts for the conditional variances are

$$\hat{h}_{2010\text{M7}} = 0.564709 + 0.139344 \times (0.359222)^2 + 0.811055 \times 20.14901$$
$$= 16.9246$$

$$\hat{h}_{2010\text{M8}} = 0.564709 + 0.139344 \times (0.0)^2 + 0.811055 \times 16.9246$$
$$= 14.2915$$

EXERCISE 14.11

(a) The estimated GARCH model, obtained using estimation with an unconditional pre-sample variance, is as follows

Dependent Variable: R
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample: 1 651
Included observations: 651
Convergence achieved after 23 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: unconditional
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-2.272414	0.121685	-18.67459	0.0000
Variance Equation				
C	1.728698	0.275643	6.271518	0.0000
RESID(-1)^2	0.719279	0.114485	6.282737	0.0000
GARCH(-1)	0.224034	0.056107	3.992951	0.0001

(b) The estimated GARCH-in-mean model, obtained using estimation with an unconditional pre-sample variance, is given below. The estimates differ from those reported in early printings of *POE5*.

Dependent Variable: R
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample: 1 651
Included observations: 651
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: unconditional
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.219279	0.074070	2.960438	0.0031
C	-3.440489	0.200192	-17.18597	0.0000
Variance Equation				
C	1.481698	0.283175	5.232453	0.0000
RESID(-1)^2	0.722388	0.114072	6.332718	0.0000
GARCH(-1)	0.249303	0.054052	4.612286	0.0000

The contribution of volatility to the term premium is captured in the term $0.219\sqrt{h_t}$.

(c) The significance of the GARCH-in-mean term $\left(0.211\sqrt{h_t}\right)$ suggests that the GARCH-in-mean model is better than the GARCH model in a financial econometric sense.

The positive sign suggests that returns increase when volatility rises which is consistent with financial economic theory.

EXERCISE 14.13

(a) The monthly rate of inflation is shown below.

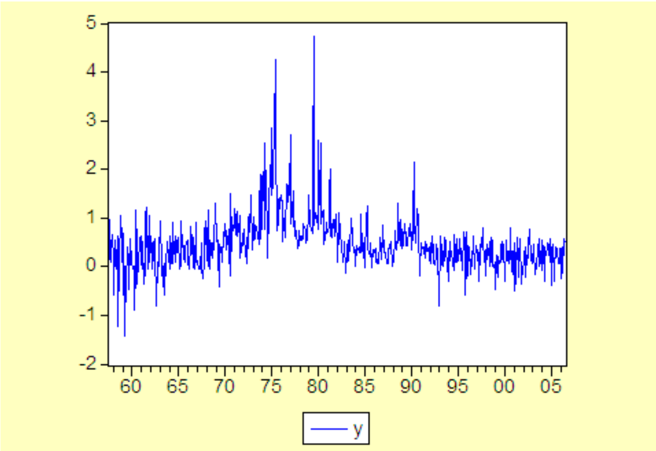


Figure xr14.13(a) Plot of monthly rate of inflation.

(b) The estimated T-GARCH-in-mean model, obtained using estimation with an unconditional pre-sample variance, is given below. The estimates differ from those reported in early printings of *POE5*.

Dependent Variable: Y
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 08/24/18 Time: 11:31
Sample (adjusted): 2 589
Included observations: 588 after adjustments
Convergence achieved after 58 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: unconditional
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	1.950757	0.373534	5.222429	0.0000
C	-0.395760	0.140487	-2.817066	0.0048
Variance Equation				
C	0.022384	0.004795	4.668679	0.0000
RESID(-1)^2	0.214566	0.024089	8.907176	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.224670	0.025881	-8.680902	0.0000
GARCH(-1)	0.779964	0.028662	27.21294	0.0000

(c) The negative asymmetric effect (–0.225) suggests that negative shocks (such as falls in prices) reduce volatility in inflation. This result is consistent with an economic hypothesis that volatility tends to be low when inflation rates are low.

(d) The positive in-mean effect (1.951) means that inflation in the UK increases when volatility in prices increases.

EXERCISE 14.15

The variables stored in the file *gfc* are *LUSA* and *LEURO*, the logs of GDP for the USA and the Euro area, respectively. We define the rates of growth for each case as

$$DUSA_t = LUSA_t - LUSA_{t-1} \qquad DEURO_t = LEURO_t - LEURO_{t-1}$$

(a) A model for the Euro area when only its own lagged effects matter and no special variance term for the expected effect of shocks is an autoregressive model

$$GEURO_t = \theta_0 + \theta_1 GEURO_{t-1} + \theta_2 GEURO_{t-2} + \cdots + \theta_p GEURO_{t-p} + e_t; \qquad e_t \sim N\left(0, \sigma^2\right)$$

Estimating this model, we find an AR(1) model is adequate. The results follow.

Dependent Variable: GEURO
Method: Least Squares
Sample (adjusted): 3 60
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001430	0.000771	1.854761	0.0689
GEURO(-1)	0.631619	0.103491	6.103117	0.0000

(b) Adding a GARCH model to the model in part (a) to accommodate a time-varying variance,

$$GEURO_t = \theta_0 + \theta_1 GEURO_{t-1} + e_t; \qquad \left(e_t \mid I_{t-1}\right) \sim N\left(0, h_t\right)$$

$$h_t = \delta + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}$$

Beginning with a GARCH(1,1) model, the estimate for β_1 was not significantly different from zero, and so pure ARCH models were considered, and the following ARCH(2) model was estimated. Including lags beyond 2 led to convergence problems. A backcasting parameter of 0.7 was used.

Dependent Variable: GEURO
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample (adjusted): 3 60
Included observations: 58 after adjustments
Convergence achieved after 21 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002715	0.000935	2.903082	0.0037
GEURO(-1)	0.488027	0.162417	3.004772	0.0027
Variance Equation				
C	6.67E-06	2.62E-06	2.541973	0.0110
RESID(-1)^2	0.245590	0.253277	0.969653	0.3322
RESID(-2)^2	0.502589	0.213527	2.353748	0.0186

(c) When lagged values for USA growth are added to the model in part (a), one lag was found to be sufficient, and so the following model was estimated.

$$GEURO_t = \theta_0 + \theta_1 GEURO_{t-1} + \delta_1 GUSA_{t-1} + e_t; \qquad e_t \sim N\left(0, \sigma^2\right)$$

Dependent Variable: GEURO
Method: Least Squares
Sample (adjusted): 3 60
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000223	0.000823	0.270489	0.7878
GEURO(-1)	0.375193	0.128371	2.922726	0.0050
GUSA(-1)	0.361593	0.119139	3.035052	0.0037

(d) To model Euro area growth dependent on its history and with an allowance for shocks to have a nonzero effect on the rate of growth, we add the term $\sqrt{h_t}$ to the model in part (b). The model and estimates (using a backcasting parameter of 0.7) follow.

$$GEURO_t = \theta_0 + \theta_1 GEURO_{t-1} + \lambda\sqrt{h_t} + e_t; \qquad \left(e_t \mid I_{t-1}\right) \sim N\left(0, h_t\right)$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2$$

Dependent Variable: GEURO
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample (adjusted): 3 60
Included observations: 58 after adjustments
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.218086	0.690215	-0.315969	0.7520
C	0.003440	0.002497	1.378043	0.1682
GEURO(-1)	0.491338	0.174662	2.813074	0.0049
Variance Equation				
C	6.84E-06	2.77E-06	2.467264	0.0136
RESID(-1)^2	0.221354	0.234581	0.943615	0.3454
RESID(-2)^2	0.488218	0.239474	2.038707	0.0415

(e) To model Euro area growth dependent on its past history, and the history of growth in the U.S., and with shocks in both countries affecting Euro area growth, we combine models (c) and (d) and include squared residuals from an equation for USA growth in the variance function. Specifically,

$$GUSA_t = \theta_0^U + \theta_1^U GUSA_{t-1} + e_{2t}; \qquad e_{2t} \sim N\left(0, \sigma_2^2\right)$$

$$GEURO_t = \theta_0 + \theta_1 GEURO_{t-1} + \delta_1 GUSA_{t-1} + \lambda\sqrt{h_t} + e_{1t}; \qquad \left(e_{1t} \mid I_{t-1}\right) \sim N\left(0, h_t\right)$$

$$h_t = \alpha_0 + \alpha_1 e_{1,t-1}^2 + \alpha_2 e_{1,t-2}^2 + \alpha_U e_{2,t-1}^2$$

Estimates for this model (using a backcasting parameter of 0.7) follow.

Dependent Variable: GEURO
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample (adjusted): 4 60
Included observations: 57 after adjustments
Convergence achieved after 30 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)
*E_USA_SQ_LAG

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.455128	0.267170	1.703510	0.0885
C	-7.24E-05	0.001120	-0.064656	0.9484
GEURO(-1)	0.411852	0.116375	3.539014	0.0004
GUSA(-1)	0.246744	0.066488	3.711091	0.0002
Variance Equation				
C	1.36E-06	2.43E-06	0.558640	0.5764
RESID(-1)^2	0.543568	0.513060	1.059462	0.2894
RESID(-2)^2	0.766892	0.444678	1.724599	0.0846
E_USA_SQ_LAG	0.045673	0.097742	0.467276	0.6403

Dependent Variable: GUSA
Method: Least Squares
Sample (adjusted): 3 60
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003457	0.001085	3.185250	0.0024
GUSA(-1)	0.478842	0.118470	4.041876	0.0002