Book Review

A Handbook of Mathematical Discourse

Reviewed by Steven G. Krantz

A Handbook of Mathematical Discourse
Charles Wells
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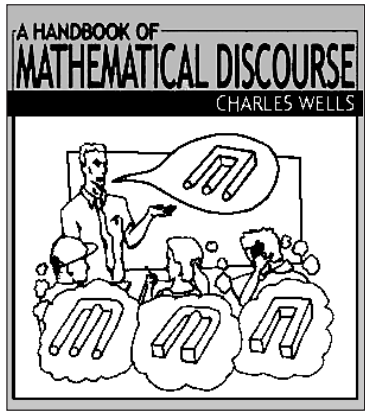
A first-class education consists of learning different modes of discourse. Certainly philosophy, history, biology, engineering, and mathematics have quite distinct and special argots. Learning these different languages causes one to turn ideas over in one's mind, and that is the purpose of education.

Thus it is a welcome breath of fresh air to see *A Handbook of Mathematical Discourse* by Charles Wells. Mathematical discourse is the heart of how we record our ideas. Our language, and mode of expression, is very special to our culture and our subject. It is well to have a book stating once and for all what we are about. This must be an ambitious project, for the language of mathematics is extensive and diverse. Even to learn all the jargon of a single area (such as algebraic topology) is a considerable challenge. The potential impact for a book such as this is considerable.

Most of us do not receive much training in mathematical writing. Perhaps, if one is lucky, one has a thesis advisor who puts one through the paces in the writing of the thesis. But after that one is on one's own. A book on mathematical discourse could be a touchstone for a young mathematician struggling to learn to articulate his thoughts clearly and effectively. It could tell the neophyte what are the issues and the challenges of mathematical writing. It could suggest finger exercises for developing writing skills. It could provide examples of good mathematical writing and bad. We note that [HIG] is a splendid example of this sort of book. [KRA] is also an effort in this direction.



Charles Wells has set for himself a weighty and potentially substantial task. His book could be the leaping-off point for the serious mathematical writer. It could be the wellspring of a future generation of fine mathematical writers. One is somewhat intimidated by the thought of even launching upon such a project. It is a ponderous responsibility and a daunting task. How does Wells succeed with his quest?

The answer is both startling and disappointing. In his introduction, Wells tells us that he will be treating the language of postcalculus mathematics and his intended audience is (i) teachers of college-level mathematics, (ii) undergraduate mathematics majors, (iii) graduate students in mathematics, and (iv) researchers in mathematics education. Thus the

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book, which is actually a lexicon of terms and concepts, contains entries such as

> **theorem proposition proof logic**
> **function superscript permutation**
> **universal quantifier if and only if**

All good and well, but most of the entries are at this level of simplicity—very naive indeed. There is no entry for **homotopy**, no entry for **spectrum**, and certainly no entry for **pseudoconvex**.

What is even more astonishing is that there are entries for

> **college synecdoche symbolitis**
> **twiddle unwind bad at math**
> **cognitive dissonance**
> **existential bigamy grasshopper**
> **enthymeme**

I have been in this business for over thirty years, and I can say with some authority that most of these words are *not* part of the standard lexicon of mathematics. I would wager that most mathematicians would have no idea what "synecdoche" or "enthymeme" means; and in fact "synecdoche" is part of the discourse of literary criticism, and "enthymeme" is part of philosophy—neither belongs to mathematics.

Wells does provide considerable and detailed discussion and examples of many of his terms, such as **function**, **Greek alphabet**, **definition**, and **convention**. He is remarkably terse in his treatment of such fundamental concepts as **theorem** and **true**. After a few hours browsing through the book, one cannot help but wonder who the audience for this book *could actually be*. Certainly a first-year graduate student or an upper-division major (Wells's professed targets) will already know the standard mathematical terminology that appears in this book. I would like to think that researchers in mathematics education would also be at least acquainted with this fundamental vocabulary. There are no deep math concepts here. Contrast Wells's book with, for example, the extremely useful and detailed *Concise Dictionary of Mathematics* [WEI] by Eric Weisstein. In Weisstein's book one can look up modern ideas such as **singular integral** or **exotic cohomology** or **spectral sequence** or **Kähler manifold** and actually find out what they are. The volume is a tremendous resource. With Wells's book one is never quite sure what one will find or not find.

I can imagine that a nonnative English speaker, newly arrived on our shores, might find something of value in the book by Wells. Reading his book might prevent such a tyro from saying something silly like "This putrescent theorem is isomorphic to a blue fish." I doubt that it would actually teach him anything of substance. And the many byways and detours that Wells takes might, in the end, prove to be confusing.

Ambrose Bierce's *Devil's Dictionary* [BIE] is a remarkable and compelling piece of writing because of its searing wit and sardonic take on life. Bierce does not define any new words. He instead gives deadly interpretations of very familiar words. Wells's book does not fit into the same category of literary effort. His book provides insipid, and often incomplete, definitions of familiar (or sometimes irrelevant) words. I would frankly be embarrassed to give this book to my students. They can learn the meaning and use of these words—at least the ones that have any bearing on the way that mathematics is practiced today—by listening to *me*; they do not need to read a book in order to internalize these ideas. And the treatment in Wells's book is so hit-and-miss that one cannot be confident that any mastery of anything worthwhile would be the result of time spent with the volume.

In sum, I find it difficult to imagine why this book was written and even more difficult to conceive of why it was published. This manuscript might be fun to circulate among friends just as a catalyst for conversation over coffee. It does not seem to have the gravitas that the title *A **Handbook** of Mathematical Discourse* might suggest. If I were to outline a book of this kind, I would suggest actual essays on why and how we formulate definitions, how we prove theorems, why we have different modes of proof, what is the difference between a lemma and a theorem, what makes for credibility in mathematical writing and what does not, what is worthwhile mathematics and what is trivia. The book *Proofs and Refutations* [LAK] by Lakatos is a step in that direction, and one well worth examining. I would give the present book a rather lower priority.

References

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