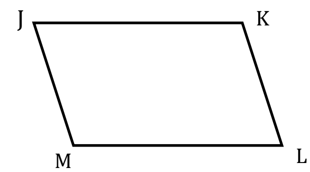
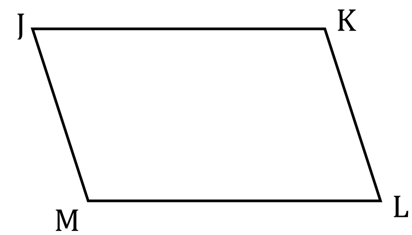
# Unit 6 – Quadrilaterals

| Day | Classwork | Day | Homework |
|---|---|---|---|
| Friday 11/9 | **Unit 5 Test** | | |
| Monday 11/12 | ***No School*** | | |
| Tuesday 11/13 | Properties of a Parallelogram | 1 | HW 6.1 |
| Wednesday 11/14 | Proving a Parallelogram | 2 | HW 6.2 |
| Thursday 11/15 | Rectangle | 3 | HW 6.3 |
| Friday 11/16 | Rhombus & Square<br>*Unit 6 Quiz 1* | 4 | HW 6.4 |
| Monday 11/19 | Trapezoid & Isosceles Trapezoid | 5 | HW 6.5 |
| Tuesday 11/20 | Kites<br>*Unit 6 Quiz 2* | 6 | HW 6.6 |
| 11/21 - 11/23 | **Thanksgiving Break** | | |
| Monday 11/26 | Coordinate Proof Formulas | 7 | HW 6.7 |
| Tuesday 11/27 | Symmetry in Quadrilaterals<br>*Unit 6 Quiz 3* | 9 | HW 6.9 |
| Wednesday 11/28 | Coordinate Proofs | 8 | HW 6.8 |
| Thursday 11/29 | Review | 10 | Review Sheet |
| Friday 11/30 | Review | 11 | Review Sheet |
| Monday 12/3 | Review | 12 | Review Sheet |
| Tuesday 12/4 | **Unit 6 Test** | 13 | |

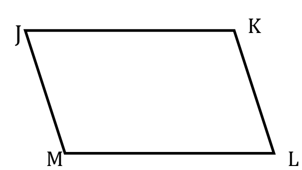
# PARALLELOGRAMS

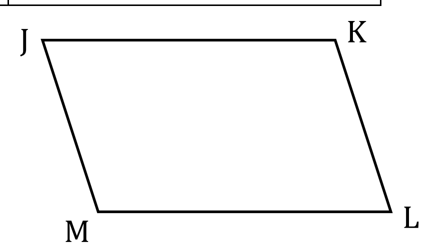
*A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.*
*In parallelogram ABCD, $\overline{BC} \parallel \overline{AD}$ and $\overline{AB} \parallel \overline{DC}$ by definition.*

| Properties of Parallelograms | | |
|---|---|---|
| *Theorems* | *Example* | *Figure* |
| Opposite sides of a parallelogram are congruent. | |   |

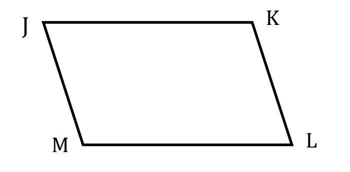



**Given**: *JKLM* is a parallelogram
**Prove:** $\overline{JK} \cong \overline{LM}, \overline{JM} \cong \overline{LK}$

| | | |
|---|---|---|
| Opposite angles of a parallelogram are congruent. | |   |




**Given**: *JKLM* is a parallelogram
**Prove:** $\angle J \cong \angle L, \angle K \cong \angle M$

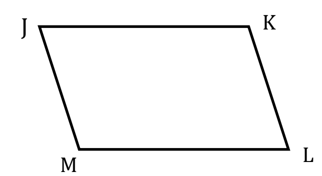
| | | |
|---|---|---|
| Consecutive Angles in a parallelogram are supplementary. | |  |

**Examples**

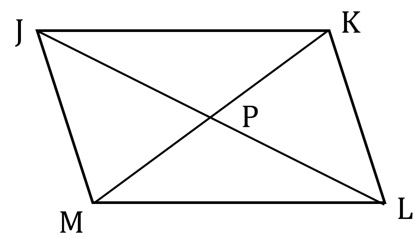
1. In parallelogram *ABCD*, $m\angle A = (2x - 60)^{\circ}$ and $m\angle B = (x + 30)^{\circ}$. Find $m\angle A$.

2. In parallelogram *ABCD*, $m\angle A = (x + 20)^{\circ}$ and $m\angle C = (6x - 50)^{\circ}$. Find *x*.

3. In parallelogram *ABCD*, *AB* = 7*x* – 3 and *CD* = 2*x* + 22. Find the value of *x*.

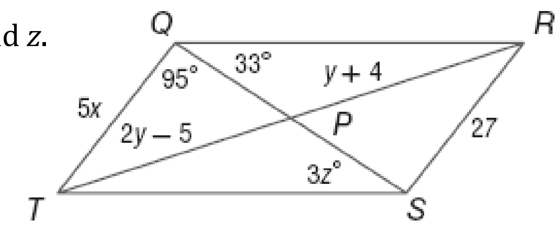
| | | |
|---|---|---|
| Diagonals of a parallelogram bisect each other | |  |

**Given**: *JKLM* is a parallelogram
**Prove**: $\overline{JL}$ and $\overline{KM}$ bisect each other

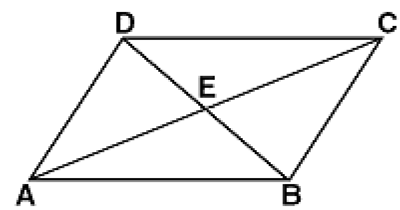


**Examples**

1. If $QRST$ is a parallelogram, find the value of $x$, $y$, and $z$.

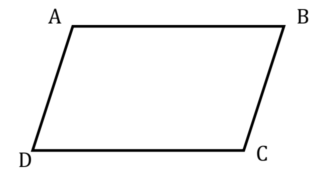



2. In parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at $E$.
   If $AE = x + 4$ and $AC = 5x - 10$, find the value of $x$.

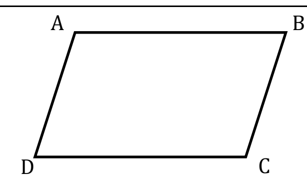


**PROVING PARALLELOGRAMS**

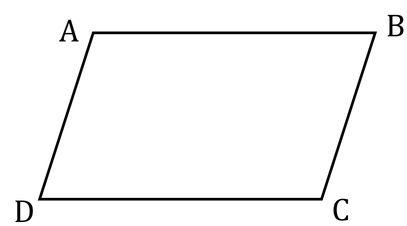
*If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram by definition. This is not the only test, however, that can be used to determine if a quadrilateral is a parallelogram.*

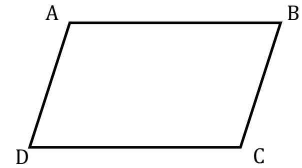
| **Conditions for Parallelograms** | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram. | |  |

**Given**: $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$
**Prove**: $ABCD$ is a parallelogram

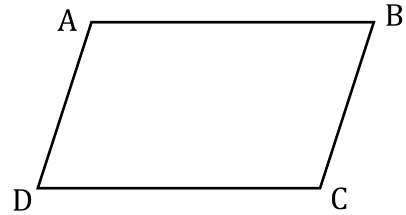

| | | |
|---|---|---|
| If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram. | |  |

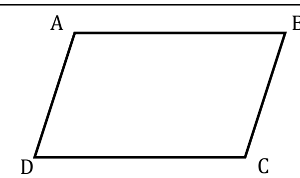
**Given**: $\angle A \cong \angle C, \angle B \cong \angle D$
**Prove**: $ABCD$ is a parallelogram



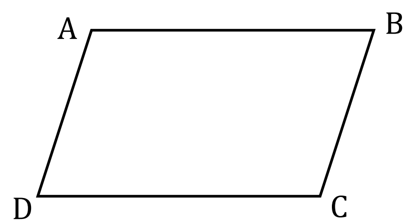
| | | |
|---|---|---|
| If diagonals bisect each other, then the quadrilateral is a parallelogram. | |  |

**Given**: $\overline{AB}$ and $\overline{CD}$ bisect each other
**Prove**: *ABCD* is a parallelogram



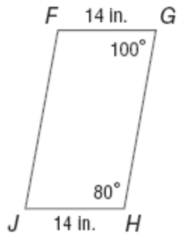

| | | |
|---|---|---|
| If one pair of opposite sides is congruent and parallel, then the quadrilateral is a parallelogram. | |  |

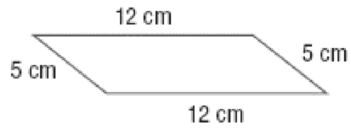
**Given**: $\overline{AB} \cong \overline{CD}$ , $\overline{AB} \parallel \overline{CD}$
**Prove**: *ABCD* is a parallelogram

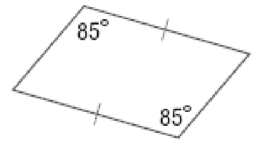


**Examples**

*Determine whether each quadrilateral is a parallelogram. Justify your answer.*

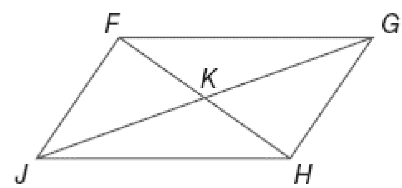
1. 

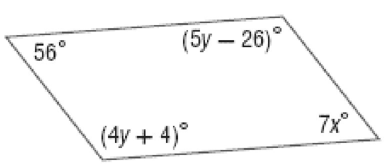

2. 

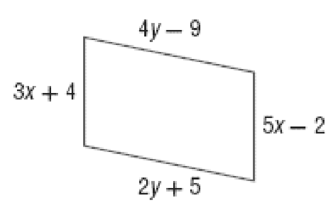

3. 


*Find x and y so that each of the following quadrilaterals are parallelograms.*

4. $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$



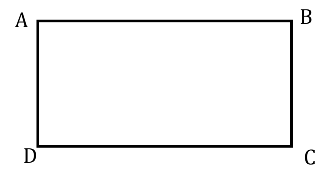

5. 

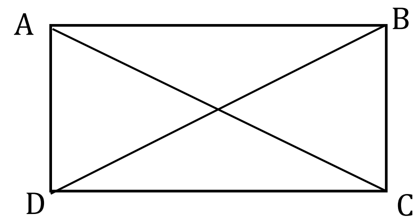

6. 

**RECTANGLES**

*By definition, a **rectangle** is a parallelogram with four right angles.*

1. All four angles are right angles
2. Opposite sides are $\parallel$ and $\cong$
3. Opposite angles are $\cong$
4. Consecutive angles are supplementary
5. Diagonals bisect each other
6. Diagonals of a rectangle are $\cong$

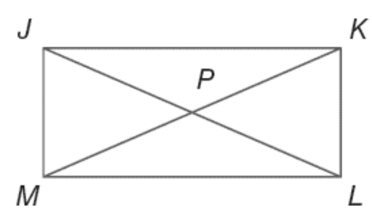
| Diagonals of a Rectangle | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| A rectangle is a parallelogram with congruent diagonals | |  |



**Given**: *ABCD* is a rectangle
**Prove**: $\overline{AC} \cong \overline{BD}$

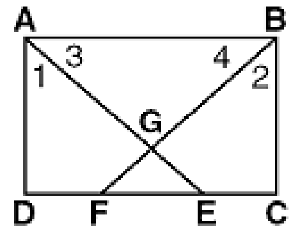
**Examples**

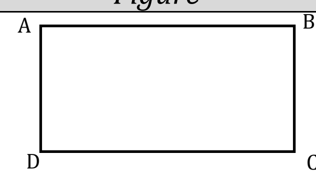
1. In rectangle *JKLM*, *JL* = 2*x* + 15 and *KM* = 4*x* – 5. Find MP.



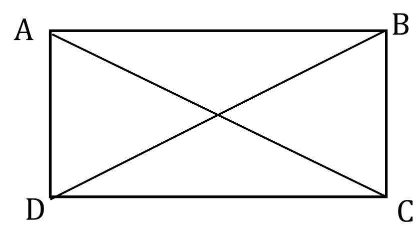
2. *Quadrilateral JKLM is a rectangle. If* $m\angle KJL$ *= 2x + 4 and* $m\angle JLK$ *= 7x + 5, find x.*

3. **Given**: *ABCD* is a rectangle.
$\overline{DF} \cong \overline{EC}$
**Prove:** $\angle 1 \cong \angle 2$
$\overline{AG} \cong \overline{GB}$



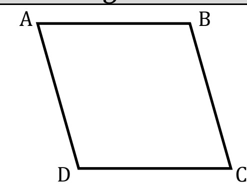

| Proving Parallelograms are Rectangles | | |
|---|---|---|
| *Abbreviation* | *Example* | *Figure* |
| If a parallelogram has one right angle, then it has four right angles. | |  |
| If diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. | |  |

**Given**: $\overline{AC} \cong \overline{BD}$, *ABCD* is a parallelogram
**Prove**: *ABCD* is a rectangle

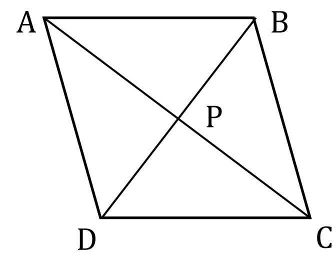


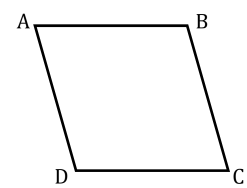
## RHOMBI AND SQUARES

*A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram.*

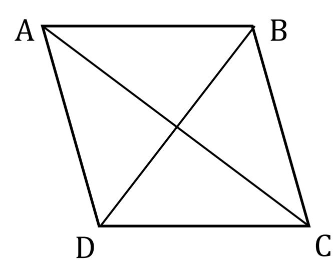
| Diagonals of a Rhombus | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| If a parallelogram is a rhombus, then its diagonals are perpendicular | |   |

**Given**: *ABCD* is a rhombus
**Prove**: $\overline{AC} \perp \overline{BD}$



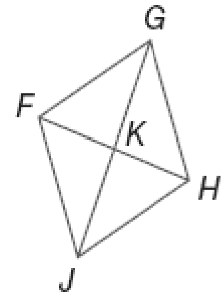

| | | |
|---|---|---|
| If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. | |   |

**Given**: *ABCD* is a rhombus
**Prove**: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$
$\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$



**Examples**

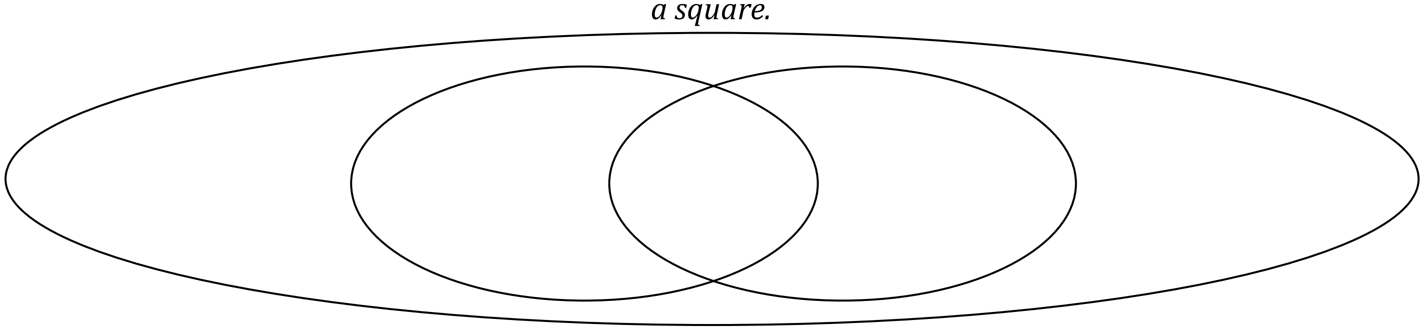
*The diagonals of rhombus FGHI intersect at K. Use the given information to find each measure or value.*

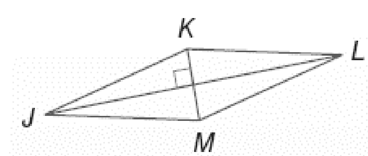
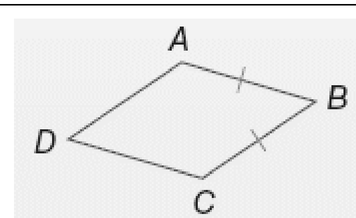



a. If $m\angle FJH$ = 82, find $m\angle KHJ$.

b. If $KH$ = $x$ + 5, $KG$ =x – 2, and $FG$ = 17. Find $KH$.

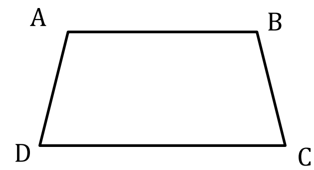
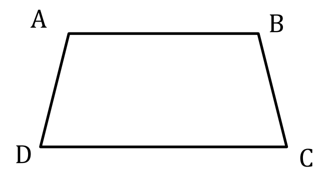
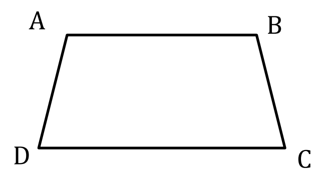
*A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.*



| **Conditions for Rhombi and Squares** | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. | |  |
| If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. | |  |
| If a quadrilateral is both a rectangle and a rhombus, then it is a square. | | |

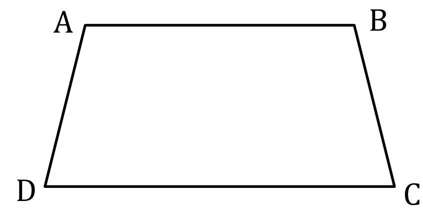
## TRAPEZOIDS

*A **trapezoid** is a quadrilateral with **at least** one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**. The **base angles** are formed by the base and one of the legs. By a definition, an isosceles trapezoid is a trapezoid with at least one pair of opposite sides congruent.*

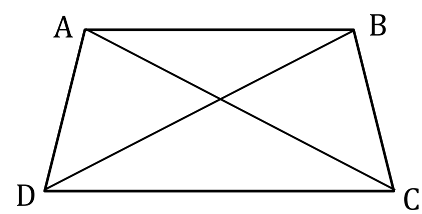
| **Isosceles Trapezoids** | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| If a trapezoid is isosceles, then each pair of base angles are congruent | |  |
| If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid. | |  |
| A trapezoid is isosceles if and only if its diagonals are congruent. | |  |

**Given**: *ABCD* is an isosceles trapezoid with $\overline{AD} \cong \overline{BC}$
**Prove**: $\angle A \cong \angle B, \angle C \cong \angle D$

**Given**: *ABCD* is a trapezoid and $\angle C \cong \angle D$
**Prove**: *ABCD* is an isosceles trapezoid

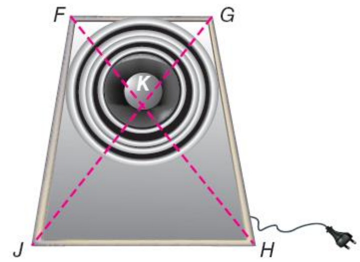



**Given**: *ABCD* is a trapezoid and $\overline{AC} \cong \overline{BD}$
**Prove**: *ABCD* is an isosceles trapezoid



**Examples**

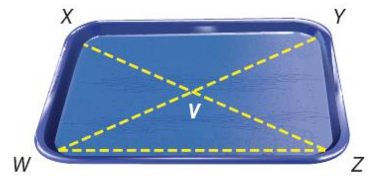
1. The speaker shown is an isosceles trapezoid. If $m\angle FJH$ = 85, *FK* = 8 inches, and *JG* = 19 inches, find each measure.




   c. $m\angle FGH$

   d. *KH*

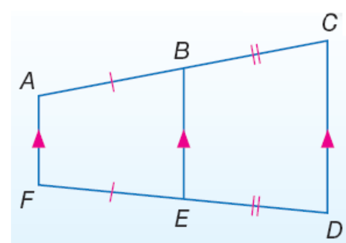
2. To save space at a square table, cafeteria trays often incorcporate trapezoids into their design. If *WXYZ* is an isosceles trapezoid and $m\angle YZW$ = 45, *WV* = 15 cm, and *VY* = 10 cm, find each measure below.




   a. $m\angle XWZ$ b. $m\angle WXY$

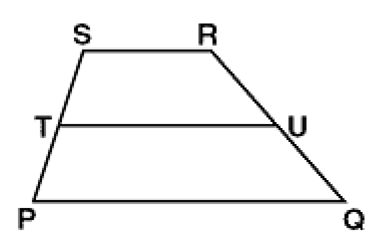
   c. *XZ* d. *XV*

*The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid. The theorem below relates the midsegment and the bases of a trapezoid.*

| **Trapezoid Midsegment Theorem** | | |
|---|---|---|
| *Theorem* | *Example* | *Figure* |
| The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. | |   |

**Examples**

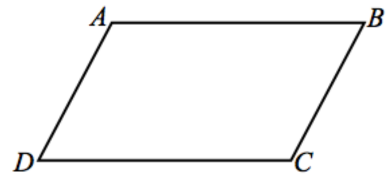
1. In the figure, *PQRS* is a trapezoid. $\overline{TU}$ is the median. If *SR* = 2*x* – 3, *PQ* = 2*x* + 11, and *TU* = 14, what is the length of $\overline{SR}$?



**ADDITIONAL PRACTICE PROOFS**

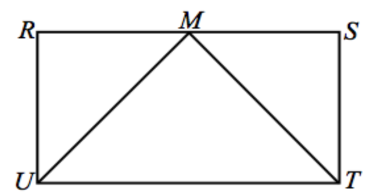
1. **Given:** $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \cong \overline{AB}$, $\overline{CD} \cong \overline{CB}$.

   **Prove:** $ABCD$ is a rhombus




2. **Given:** Rectangle $RSTU$, $M$ is the midpoint of $\overline{RS}$.

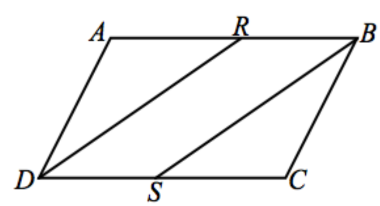
   **Prove:** $\triangle UMT$ is isosceles.



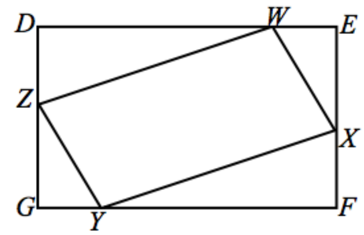

3. **Given:** $ABCD$ is a parallelogram, $\overline{RD}$ bisects $\angle ADC$,

   $\overline{SB}$ bisects $\angle CBA$.

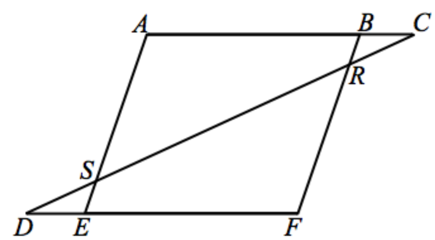
   **Prove:** $DRBS$ is a parallelogram.



4. **Given:** $DEFG$ is a rectangle, $\overline{WE} \cong \overline{YG}$, $\overline{WX} \cong \overline{YZ}$.
   **Prove:** $WXYZ$ is a parallelogram.

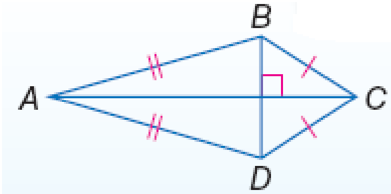
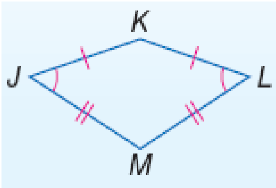



5. **Given:** Parallelogram $ABFE$, $\overline{CR} \cong \overline{DS}$.
   **Prove:** $\overline{BR} \cong \overline{SE}$.



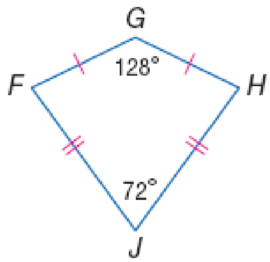
**KITES**

*A **kite** is a quadrilateral with at least two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.*

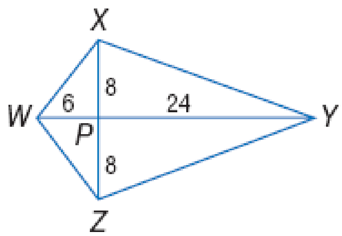
| *Theorem* | *Example* | *Figure* |
| --- | --- | --- |
| If a quadrilateral is a kite, then its diagonals are perpendicular. | |   |
| If a quadrilateral is a kite, then at least one pair of opposite angles are congruent. | |   |

**Examples**

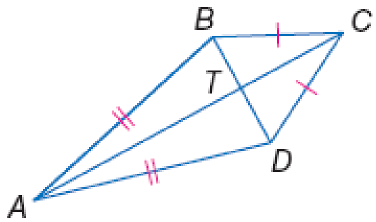
a. If $FGHJ$ is a kite, find $m\angle GFJ$.




b. If $WXYZ$ is a kite, find $ZY$.




c. If $m\angle BAD = 38$ and $m\angle BCD = 50$, find $m\angle ADC$.

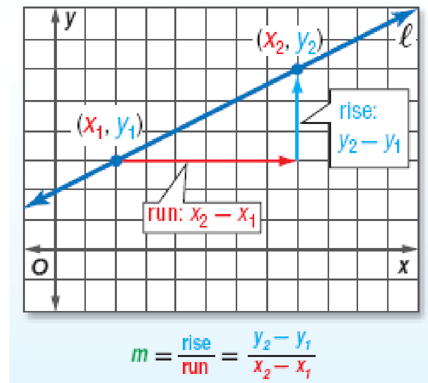



d. If $BT = 5$ and $TC = 8$, find $CD$.

## SLOPES OF LINES

*Slope can be interpreted as **rate of change**, describing how a quantity y changes in relationship to quantity x. The slope of a line can also be used to identify the coordinates of any point on the line.*

| Parallel and Perpendicular Lines | | |
|---|---|---|
| | Description | Example |
| Slopes of Parallel Lines | | |
| Slopes of Perpendicular Lines | | |




**Examples**

1. Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither* for $A(1, 1)$, $B(-1, -5)$, $C(3, 2)$, and $D(6, 1)$.

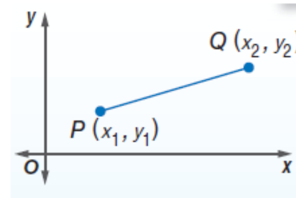
2. Given $A(1,1)$, $B(2,4)$, $C(4,1)$, and $D(3, k)$
   a) Find the slope of $\overleftrightarrow{AB}$.

   b) Express the slope of $\overleftrightarrow{CD}$ in terms of $k$.

   c) If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, find the value of $k$.

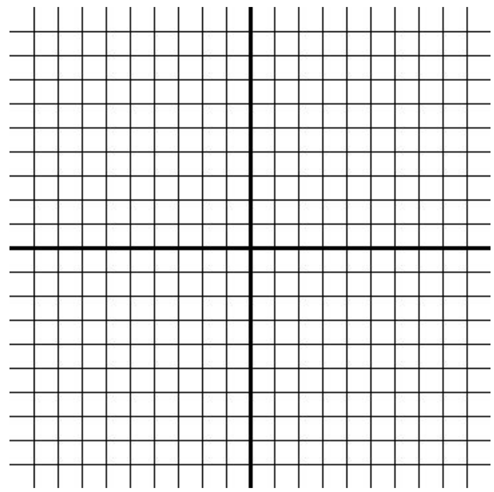
# Distance

The *distance* between two points is the length of the segment with those points as its endpoints.

| DISTANCE FORMULA (Coordinate Plane) | | |
|---|---|---|
| WORDS | SYMBOLS | PICTURE |
| If $P$ has coordinates $(x_1, y_1)$ and $Q$ has coordinates $(x_2, y_2)$, then | |   |

**Examples**

1. Find the distance between $C(1, 1)$ and $D(3, -3)$. Check using the Pythagorean Theorem.



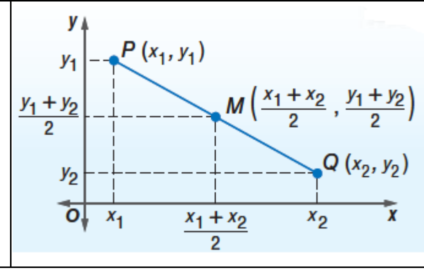
2. Given the points A(6, 7) and B(14, -1).
   Find the length of $\overline{AB}$.

3. What is the length of the diameter of a circle whose center is at (6,0) and passes through (2,-3)?

4. A triangle has vertices $D(2,3)$, $E(5,5)$, and $F(4,0)$. Determine if the triangle is scalene, isosceles, or equilateral.

## Midpoint

The *midpoint* of a segment is the point halfway between the endpoints of the segment.

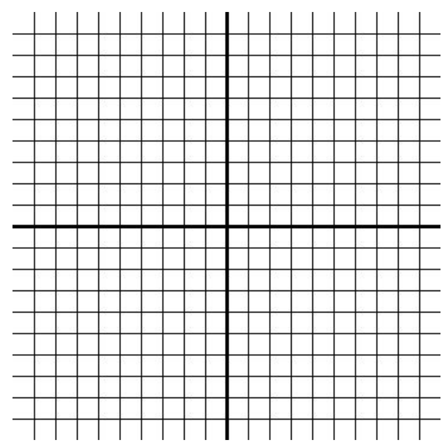
| **Midpoint Formula (Coordinate Plane)** | | |
|---|---|---|
| WORDS | SYMBOLS | PICTURE |
| If $\overline{PQ}$ has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane, then the midpoint $M$ of $\overline{PQ}$ is | |   |

**Examples**

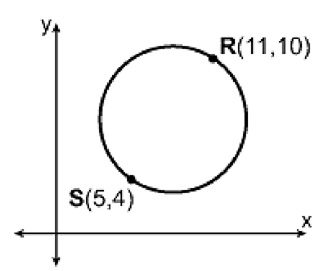
1. Find the coordinates of $M$, the midpoint of $\overline{ST}$, for $S(-6, 3)$ and $T(2, 1)$.

2. Find the midpoint when given the endpoints (-1,-4) and (3, -2).

3. Find the coordinates of $J$ if $M(-1, 2)$ is the midpoint of $\overline{JL}$ and $L$ has coordinates (3, -5).



4. $\overline{RS}$ is the diameter of the circle shown in the accompanying diagram. What are the coordinates of the center of this circle?



## COORDINATE PROOF

**Parallel Lines ( $||$ )** – If two lines have equal slopes, then the lines are parallel.

**Perpendicular Lines ( $\perp$ )** – If two non-vertical lines have slopes that are negative reciprocals of one another, then the lines are perpendicular.

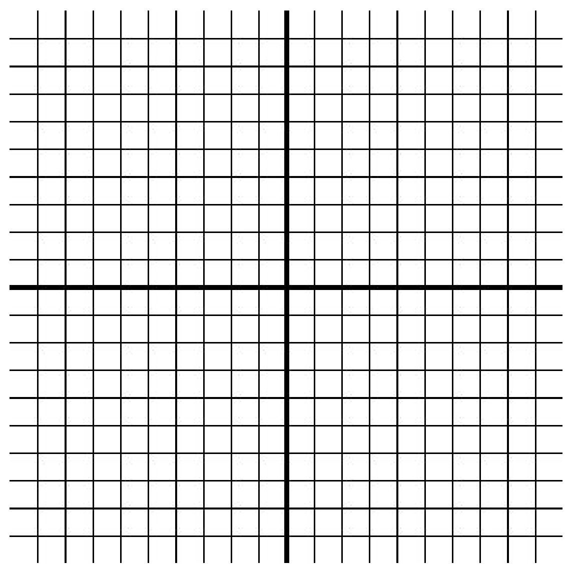
**<u>Examples:</u>**

1. Given the points A(6, 9) and B(14, -1).
   a. Find the slope of $\overline{AB}$.

   b. Find the slope of the line perpendicular to $\overline{AB}$

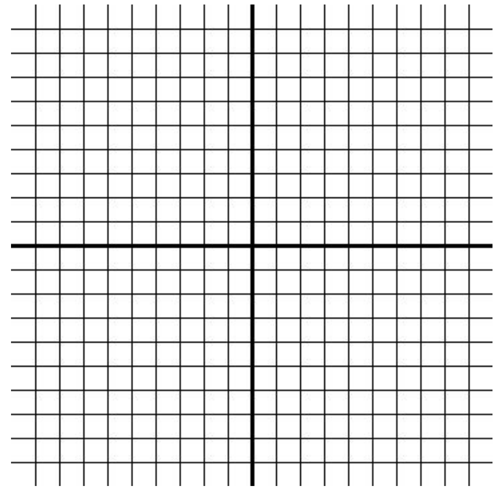
   c. Find the slope of the line parallel to $\overline{AB}$

2. Consider the line segments $\overline{AB}$ and $\overline{CD}$, with A(-3,-2), B(2,1), C(-7,-1), and D(-2,2).
   a. ***Prove*** that $\overline{AB} \parallel \overline{CD}$.

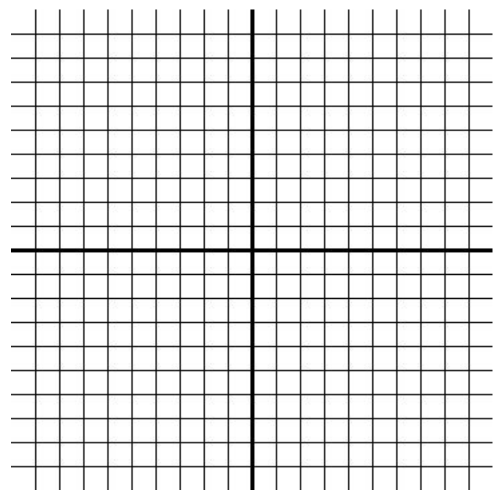


   b. Determine if $\overline{BC} \parallel \overline{AB}$.

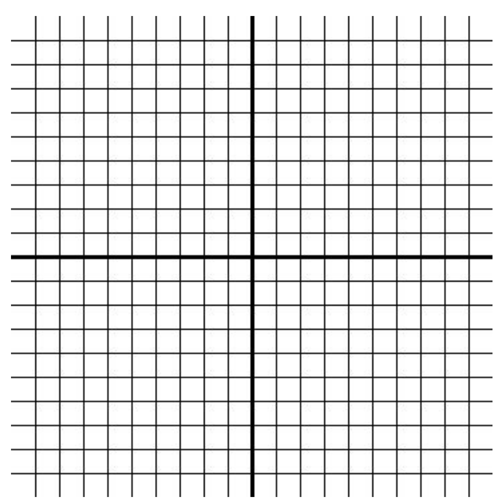
3. If X(5,0), Y(3,4), and Z(-1,2), prove $\overline{XY} \perp \overline{YZ}$ .



4. The vertices of triangle WIN are W(2, 1), I(4, 7) and N(8, 3). Using coordinate geometry, show that ΔWIN is an isosceles triangle and state the reasons for your conclusion.

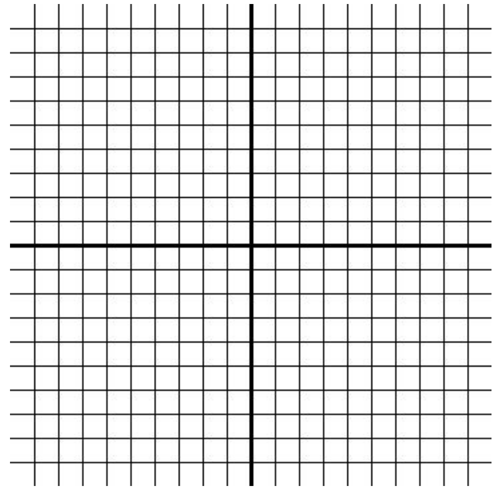


5. Triangle NAQ has coordinates N(2, 3), A(6, 0) and Q(12, 8). Using coordinate geometry, show that ΔNAQ is a right triangle and state the reasons for your conclusion

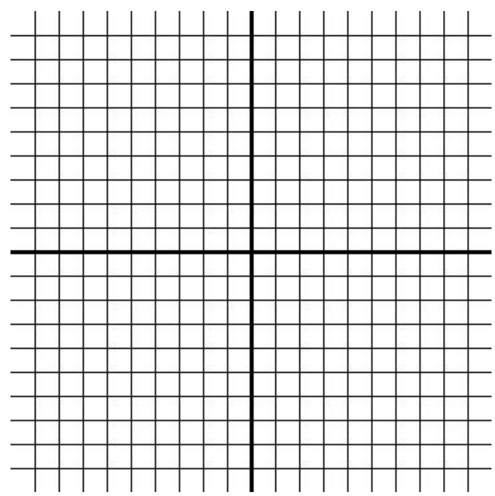


**CLASSIFYING QUADRILATERALS USING COORDINATE GEOMETRY**

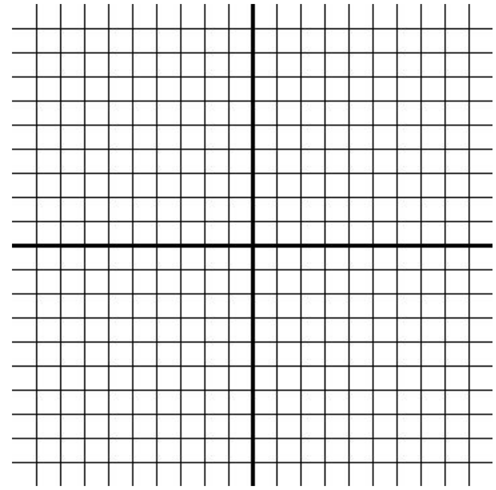
1. Determine the coordinates of the intersection of the diagonals of parallelogram *FGHI* with vertices *F*(-2, 4), *G*(3, 5), *H*(2, -3), and *J*(-3, -4). Prove *FGHI* is a parallelogram.



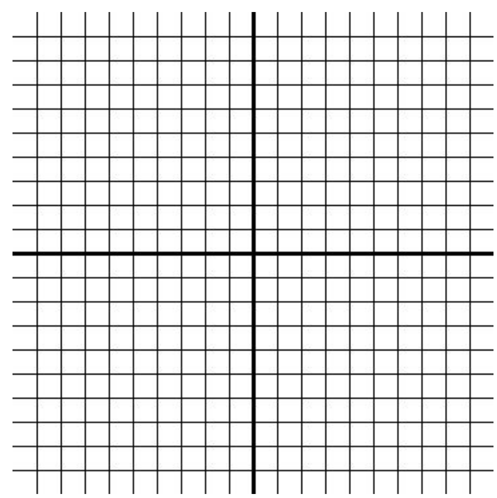
2. Graph quadrilateral *KLMN* with vertices *K*(2, 3), *L*(8, 4), *M*(7, -2), and *N*(1, -3). Prove the quadrilateral is a parallelogram. Justify your answer using the Slope Formula *and* Distance formula.



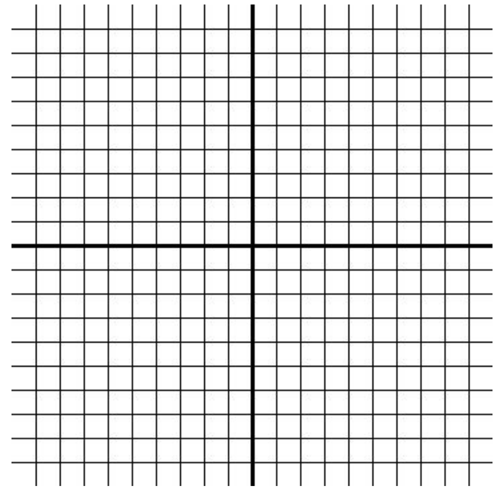
3. Quadrilateral *PQRS* has vertices *P*(-5, 3), *Q*(1, -1), *R*(-1, -4), and *S*(-7, 0). Prove *PQRS* is a rectangle.



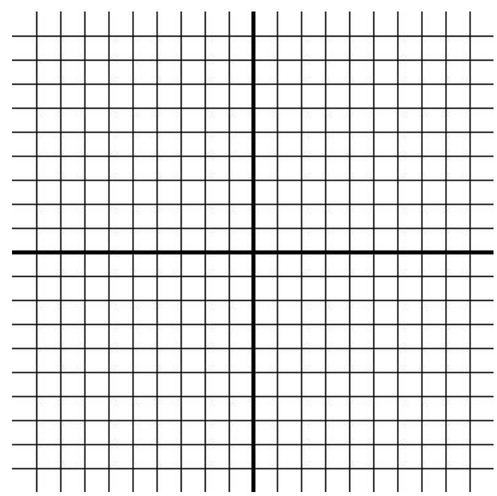
4. Prove that parallelogram *JKLM* with vertices *J* (-7, -2), *K* (0, 4), *L* (9, 2), and *M* (2, -4) is a rhombus. Is it a rectangle and/or square also?



5. $A(-3, 4)$, $B(2, 5)$, $C(3,3)$, $D(-1,0)$. Prove $ABCD$ is a trapezoid. Is the trapezoid isosceles?

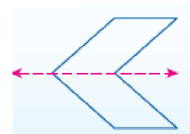
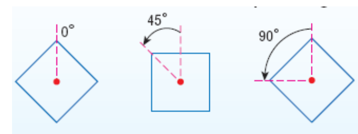


6. $F(-2, 4)$, $G(3, 5)$, $H(2, -3)$, $J(-3, -4)$. Prove that quadrilateral $FGHJ$ is a parallelogram.



## SYMMETRY IN QUADRILATERALS

***RECALL:*** *A figure has* ***symmetry*** *if there exists a rigid motion – reflection, translation, rotation, or glide-reflection – that maps the figure onto itself.*

| | |
|---|---|
| A figure in the plane has line symmetry if the figure can be mapped onto itself by a reflection in a line, called a line of symmetry. |  |
| A *nontrivial rotational symmetry* of a figure is a rotation of the plane that maps the figure back to itself such that the rotation is greater than 0˚ but less than 360˚. |  |

1. Determine whether each figure has line symmetry and/or rotational symmetry. If so, draw all lines of symmetry and/or give the angle of rotational symmetry.

   a. Rectangle

   b. Isosceles Trapezoid

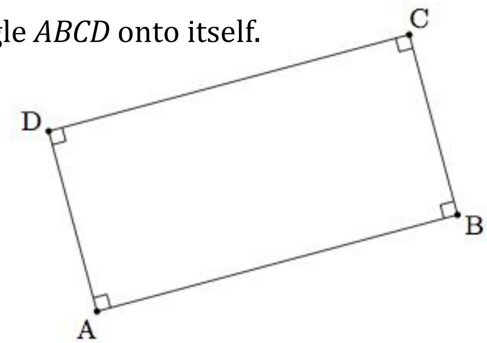
   c. Parallelogram

   d. Regular Hexagon

2. Suppose *ABCD* is a quadrilateral for which there is exactly one rotation, through an angle larger than 0 degrees and less than 360 degrees, which maps it to itself. Further, no reflections map *ABCD* to itself. What shape is *ABCD*?

3. Draw an example of a trapezoid that does *not* have line symmetry.

4. Jennifer draws the rectangle *ABCD* below.
   a. Find all rotations and reflections that carry rectangle *ABCD* onto itself.




   b. Lisa draws a different rectangle and she finds a larger number of symmetries (than Jennifer) for her rectangle. What can you conclude about Lisa's rectangle? Explain.

5. There is exactly one reflection and no rotation that sends the convex quadrilateral *ABCD* onto itself. What shape(s) could quadrilateral ABCD be? Explain.

6. Draw an example of a parallelogram that has exactly two lines of symmetry. Draw the lines of symmetry and give the most specific name for the parallelogram you drew.