MATH 376: Mathematical Statistics

MIDTERM EXAMINATION 1

A

March 09, 2017

FILL OUT THE INFORMATION IN THE BOX AND SIGN.

Name:______________________ Campus Email:______________

I pledge that the work on this exam is entirely my own.

Student signature: ______________________________

READ THE FOLLOWING INFORMATION.

- This is a **90-minute** exam.
- Please find the tables of standard distributions and a formula sheet at the end of the exam book.
- Calculators, books, notes, and other aids are **not** allowed.
- You may use the backs of the pages or the extra pages for scratch work. **Do not unstaple or remove pages as they can be lost in the grading process.**
- Please do not put your name on any page besides the first page.
- **Read and follow the instructions for every problem**.
- Show all of your work for purposes of partial credit. **Full credit may not be given for an answer alone**.
- Justify your answers. **Full sentences are not necessary** but English words help. When in doubt, do as much you think is necessary to demonstrate that you understand the problem, keeping in mind that your grader will be necessarily skeptical.

DO NOT BEGIN THE EXAM UNTIL SIGNALED TO DO SO.

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You can use it for scratch work. Please do not remove it.)

FR1 (3+4=7 points) Suppose (X_1, X_2) has $Multinomial(n; p_1, p_2)$ distribution with joint pmf

$$f(x_1, x_2) = \frac{n!}{x_1!x_2!(n - x_1 - x_2)!} p_1^{x_1} p_2^{x_2} (1 - p_1 - p_2)^{n - x_1 - x_2}, \text{ where } x_1, x_2 \in \mathbb{Z}, x_1, x_2 \geq 0, x_1 + x_2 \leq n.$$

Derive (a) $E(X_1)$ (b) $cov(X_1, X_2)$.

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FR2 (3+3+3=9 points) Suppose $X_1, X_2, \ldots, X_9$ is a random sample from $N(\mu, \sigma^2)$. If $\bar{X}$ and S^2 denote the sample mean and sample variance respectively, then

1. find $P(|\bar{X} - \mu| > \sigma/2)$.
2. find c such that $P(S^2 > c\sigma^2) = 0.1$.
3. find h such that $P(\bar{X} > \mu + hS) = 0.01$.

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FR3 (5+4+6=15 points) A random sample $X_1, \ldots, X_n$ is drawn from the pdf $f(x;\theta) = 2x/\theta^2$, $0 \leq x \leq \theta$.

1. Find the Method of Moments Estimator of θ.
2. Find the Maximum Likelihood Estimator of θ.
3. Find the number C such that $C(\max_{1\leq i\leq n} X_i)$ is an unbiased estimator for θ.

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FR4 (4 points) Suppose $\mathbf{X}$ has $N_3(\mathbf{0}, \Sigma)$ distribution, where

$$\Sigma = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

Let $Y_1 = X_1 + 2X_2$ and $Y_2 = X_1 + CX_3$. For what values of C are Y_1 and Y_2 independent?

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FR5 (4+1=5 points) Let $X_1, X_2, \ldots, X_n$ be a random sample from $Gamma(1, \theta)$ distribution.

1. Find the Maximum Likelihood Estimator of θ and $1/\theta$.
2. (Extra Credit, 3 pts) Find the Bias of the MLE of $1/\theta$.

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