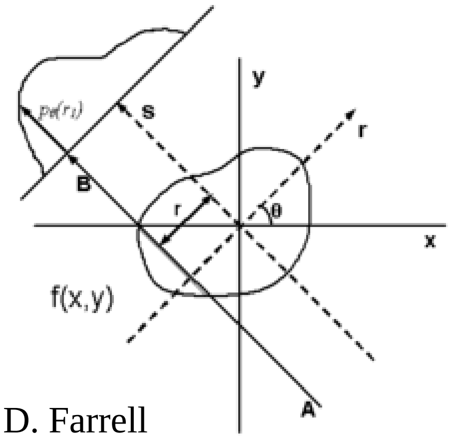
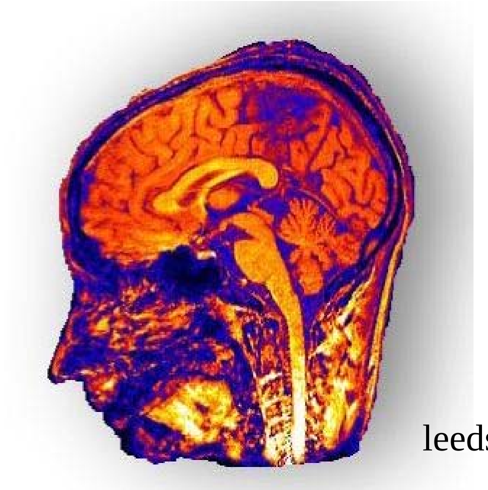
Strategies for Learning to Solve Physics Problems



D. Farrell



leeds, medical physics

"I understand the concepts, I just can't solve the problems."

Ken Heller

School of Physics and Astronomy
University of Minnesota

20 year continuing project to improve undergraduate education with contributions by:
Many faculty and graduate students of U of M Physics Department
In collaboration with U of M Physics Education Group

Details at http://groups.physics.umn.edu/physed/

Supported in part by Department of Education (FIPSE), NSF,
and the University of Minnesota

TASK

Discuss why you assign problems in your courses.

List the common goals of the problems.

Report the single most important goal

TIME ALLOTTED

5 minutes

PROCEDURES

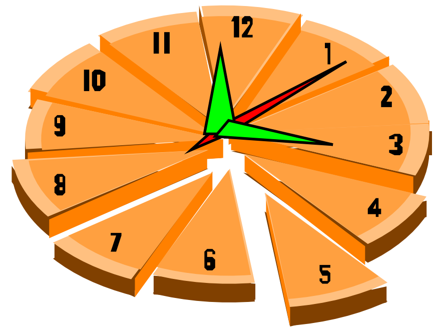
Form a group of 3 people
Choose one person as a recorder

Formulate a response individually.
Discuss your response with your partners.
Listen to your partners' responses.
Create a new group response through discussion.

Learning Problem Solving

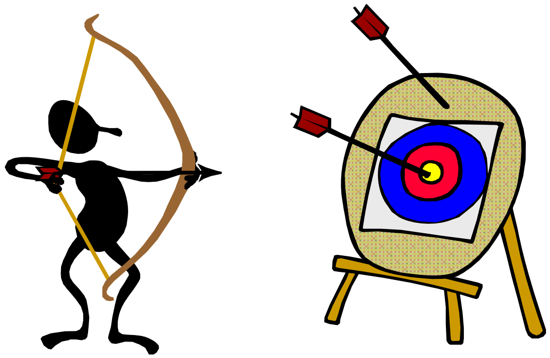
Using Cooperative Group Problem Solving

A Guide for Discussion



1. What is Problem Solving?
2. What is Learning?
3. Why Learn Problem Solving?
4. Strategies for Teaching Problem Solving
    1. Goals
    2. Guidance from learning theory
    3. Logical framework
    4. Useful problems
    5. Cooperative groups
    6. Grading and assessment

Some Goals of Problem Solving



- Students can make both qualitative and quantitative predictions about the real world from basic, well-understood principles.

- Students will know the difference between fundamental principles, special cases, and specific applications.

- Students can make decisions, know the assumptions that underlie them, and be able to evaluate them.

- Students can construct and communicate a long chain of logic (including mathematics) to themselves and others.

What Is Problem Solving?

"Process of Moving Toward a Goal When Path is Uncertain"

- **If you know how to do it, its not a problem.**

Problems are solved using general purpose tools

Heuristics

Not specific algorithms

"Problem Solving Involves Error and Uncertainty"

A problem for your student is not a problem for you

Exercise vs Problem

M. Martinez, Phi Delta Kappan, April, 1998

Metacognition – Reflecting on Your Own Thought Process

- Managing time and direction
- Determining next step
- Monitoring understanding
- Asking skeptical questions

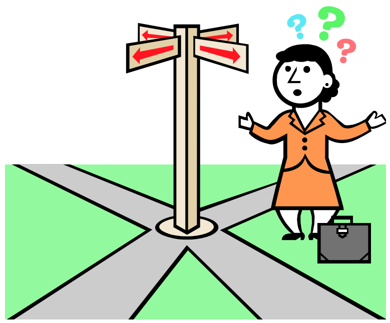
Some General Tools (Heuristics)

- Means - Ends Analysis (identifying goals and subgoals)
- Working Backwards (step by step planning from desired result)
- Successive Approximations (idealization, approximation, evaluation)
- External Representations (pictures, diagrams, mathematics)
- General Principles of Physics

M. Martinez, Phi Delta Kappan, April, 1998

Problem Solving Is an Organized Framework for Making Decisions

- Visualize situation
- Determine goal
- Choose applicable principles
- Choose relevant information
- Make necessary simplifications
- Construct a plan
- Arrive at an answer
- Evaluate the solution



This is a process not a linear sequence. It requires students to reflect on their work

Not natural for most students – must be explicitly taught in every new academic environment

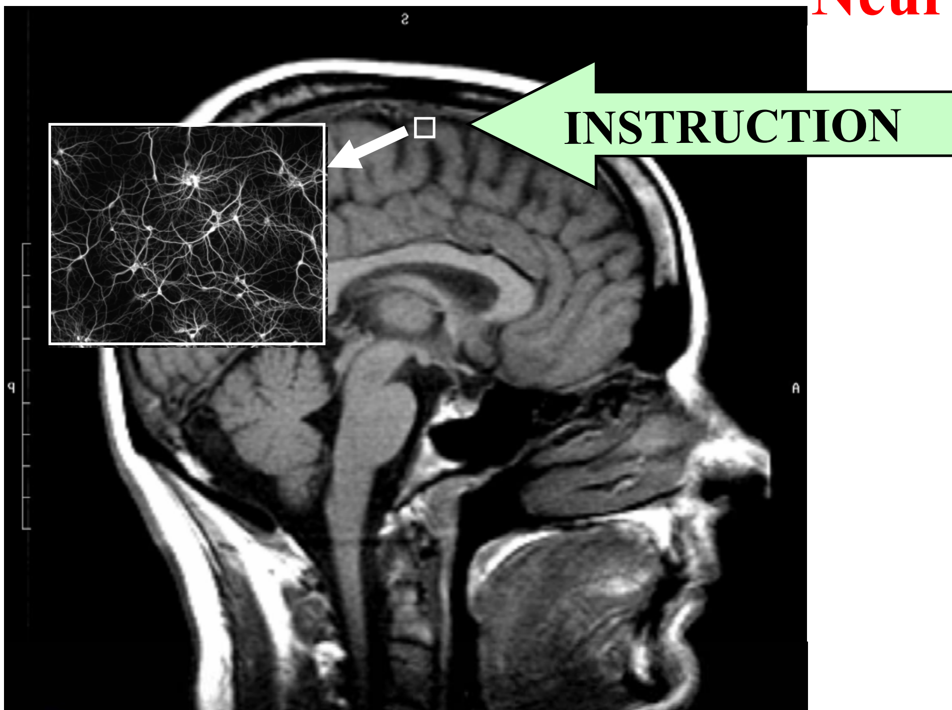
Problems that facilitate learning should

Explicitly connect important concepts

Explicitly connect to reality

Learning is a Biological Process

Neural Science Gives the Constraints



Neurons that fire together, wire together

Simplification of Hebbian theory:
Hebb, D (1949). *The organization of behavior*. New York: Wiley.

Brain MRI from Yale Medical School
Neuron image from Ecole Polytechnique Lausanne

Knowing is an individual's neural interconnections

> **Knowing something means a student can use it in novel (for them) situations and communicate that usage.**

Learning is expanding the network of neural connections by linking and changing existing ones and establishing new ones

Teaching is putting the student in a situation that stimulates neural activity that renovates the relevant network of neural connections.

Teaching requires Interactive Engagement (~~Active Learning~~, ~~Activities~~)
Cognitive Apprenticeship

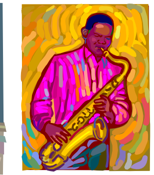
Strategies for Learning Problem Solving (or anything else)

- **Watch experts solve problems** – Ask yourself (and them) what are they doing and why. Don' t be surprised if they don't know.
- **Develop expert problem solving skills by repeated practice.** – Always use an organized framework for your problem solving. "Practice does not make perfect, only perfect practice makes perfect" – Vince Lombardy.
- **Practice problem solving in different contexts that are meaningful to you** – Solving the same problem over and over until it becomes automatic will not help in learning how to solve problems.
- **Practice isolated skills by doing exercises** – However you can't learn to solve problems by doing exercises.
- **Get a coach that will make sure you engage in perfect practice.** - A good practitioner is not necessarily a good coach.
- **Work with others solving problems.** - Learn from their successes and struggles as you solve problems together.
- **Work on your own to solve problems.** - Get coaching only after you have tried your best and failed. The help should never be directed at how to solve that specific problem.
- **Don't get discouraged.** - Applying newly learned skills will lead to slower and more error prone practice. Get through that in your practice.

Problem-solving Framework

Used by experts in all fields

G. Polya, 1945



STEP 1

Recognize the Problem

What's going on and what do I want?

STEP 2

Describe the problem in terms of the field

What does this have to do with ...... ?

STEP 3

Plan a solution

How do I get what I want?

STEP 4

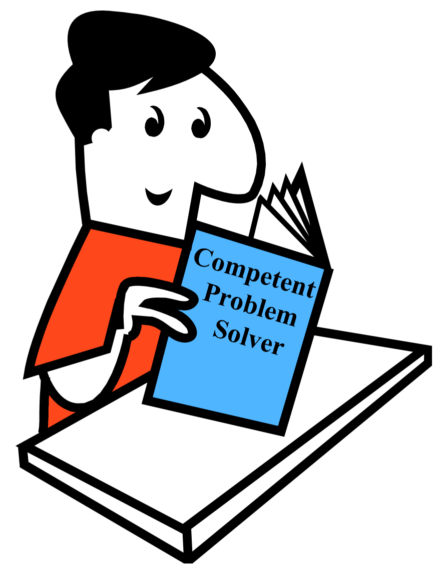
Execute the plan

Let's get it.

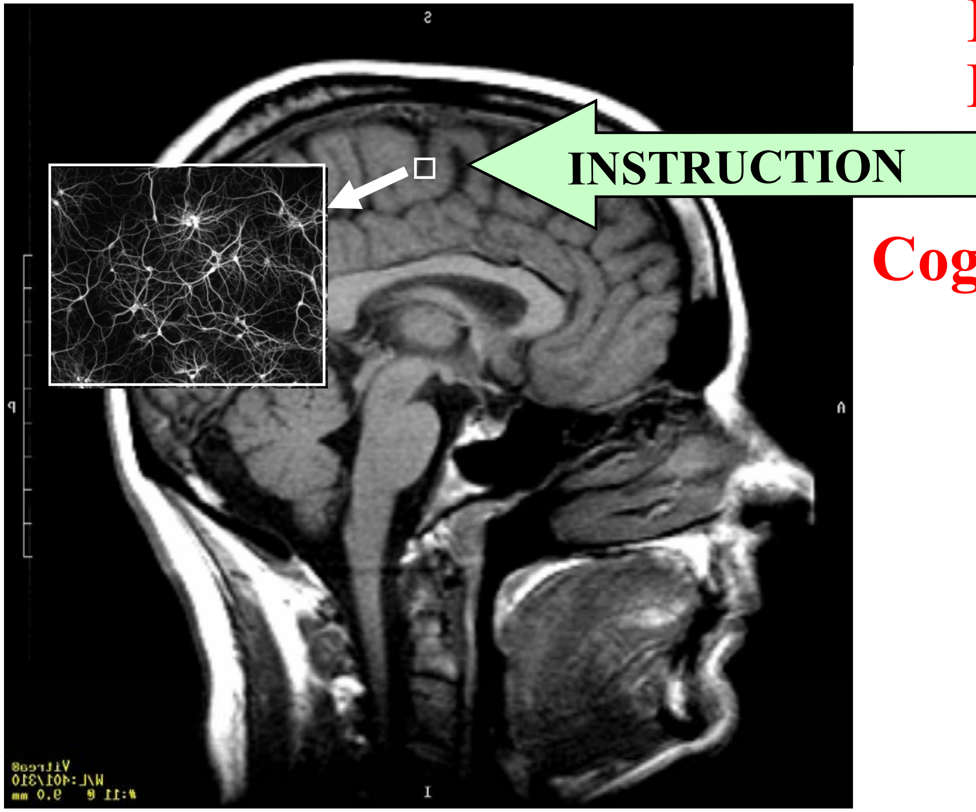
STEP 5

Evaluate the solution

Can this be true?



Learning is Too Complex to Predetermine



Brain MRI from Yale Medical School
Neuron image from Ecole Polytechnique Lausanne

Phenomenological Learning Theory

Apprenticeship Works

Cognitive Apprenticeship

Learning in the environment of expert practice

- Why it is important
- How it is used
- How is it related to a student's existing knowledge

model

coach

Collins, Brown, & Newman (1990)

fade

Learning Requires Scaffolding

Additional structure used to support the construction of a complex structure.

Removed as the structure is built

Examples of Scaffolding in teaching Introductory Physics using problem solving

- **Problems that discourage novice problem solving**
- **An explicit problem solving framework**
- **Cooperative group structure that facilitates peer coaching by encouraging productive group interactions**
  - **Grouping rules**
  - **Group roles**
- **A worksheet that structures the framework**
- **Limit use of formulas by giving an equation sheet (only allowed equations)**
- **Explicit grading rubric to encourage expert-like behavior**

Cooperative Group Problem Solving is an Implementation of Cognitive Apprenticeship

Essential Elements

1. Organized Framework for Problem Solving
2. Problems that Require Using an Organized Framework
3. Cooperative Groups to provide coaching to students while solving problems

Peer coaching

Instructor coaching

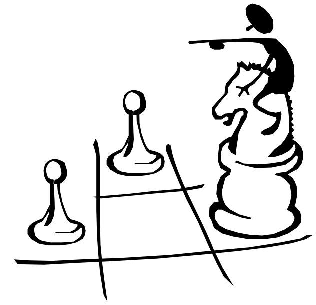
Appropriate Problems for Practicing Problem Solving

The problems must be challenging enough so there is a *real* advantage to using a problem solving framework.

1. **The problem must be complex enough so the best student in the class is not certain how to solve it.**

   The problem must be simple enough so that the solution, once arrived at, can be understood and appreciated by everyone.





2. The problems must be designed so that

- **the major problem solving heuristics are required (e.g. physics understood, a situation requiring an external representation);**
- **there are decisions to make in order to do the problem (e.g. several different quantities that could be calculated to answer the question; several ways to approach the problem);**
- **the problem cannot be resolved in one or two steps by copying a pattern.**



3. The task problem must connect to each student's mental processes

- the situation is real to the student so other information is connected;
- there is a reasonable goal on which to base decision making.

This is not what is called Problem Based Learning (PBL). These are closed ended problems with a definite answer (or a few possible answers) appropriate for novice problem solvers and directed toward a specific learning goal.

Context Rich Problem

You are investigating using MRI to identify cancer cells. To do this, you have constructed a 3.0 cm diameter solenoid into which you can place a tissue sample. You will then change the magnetic field at your sample by changing the current through the solenoid. You need to monitor the magnetic field inside the solenoid but its size makes inserting your Hall probe impractical. Instead you put the solenoid through the center of a 5 cm diameter, 10 turn coil of wire and measure the voltage across that coil. To decide if this gives enough precision, you calculate the change in the coil voltage as a function of time as you change the solenoid current. The solenoid is 20 cm long and consists of 2000 turns of wire. Your signal generator varies the current through the solenoid as a sine function at a frequency of 500 Hz with a maximum of 15 A.

Gives a motivation – allows some students to access their mental connections.
Gives a realistic situation – allows some students to visualize the situation.
Does not give a picture – students must practice visualization.
Uses the character "you" –more easily connects to student's mental framework.

Decisions must be made

Context Rich Problem

You are investigating using MRI to identify cancer cells. To do this, you have constructed a 3.0 cm diameter solenoid into which you can place a tissue sample. You will then change the magnetic field at your sample by changing the current through the solenoid. You need to monitor the magnetic field inside the solenoid but its size makes inserting your Hall probe impractical. Instead you put the solenoid through the center of a 5 cm diameter, 10 turn coil of wire and measure the voltage across that coil. To decide if this gives enough precision, you calculate the change in the coil voltage as a function of time as you change the solenoid current. The solenoid is 20 cm long and consists of 2000 turns of wire. Your signal generator varies the current through the solenoid as a sine function at a frequency of 500 Hz with a maximum of 15 A.

What is happening? – you need a picture.

What is the question? – it is not in the last line.

What quantities are important and what should I name them? – choose symbols.

What physics is important and what is not? – Faraday's Law, definition of flux

What assumptions are necessary? – Can you ignore the field outside the solenoid?

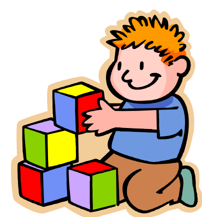
Is all the information necessary? – There is a lot of information.

The Dilemma

Start with complex problems
so novice framework fails

Difficulty using strange new framework
with challenging problems.

Why change?



Start with simple problems to
learn expert-like framework.

Success using novice framework.

Why change?

Coaching is the necessary ingredient that allows students to work complex problems that require an expert-like framework.

Cooperative Groups

Provide peer coaching and facilitates expert coaching.

Allow success solving complex problems with an organized framework from the beginning.

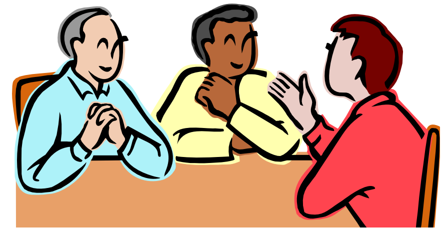
- **Positive Interdependence**
- **Face-to-Face Interaction**
- **Individual Accountability**
- **Explicit Collaborative Skills**
- **Group Functioning Assessment**

Johnson & Johnson, 1978

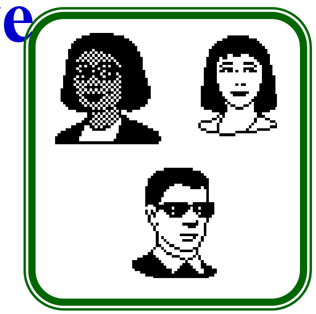
Scaffolding

Structure and Management of Groups

1. What is the "optimal" group size?
    - three (or occasionally four) for novices



2. What should be the gender and performance composition of cooperative groups?



    - heterogeneous groups:
        - one from top third
        - one from middle third
        - one from bottom third

      based on past test performance.

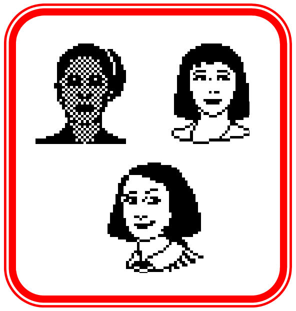
    - two women with one man, or same-gender groups

Structure and Management of Groups

3. **How often should the groups be changed?**

   For most groups:

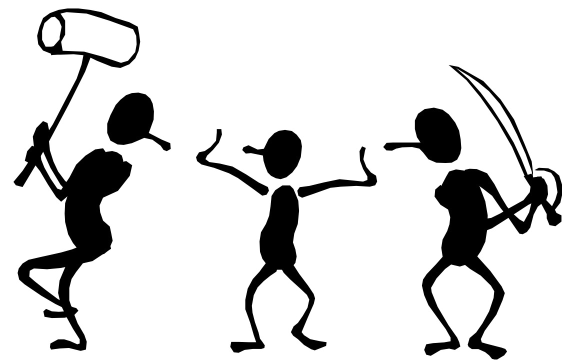
   - **stay together long enough to be successful**
   - **enough change so students know that success is due to them, not to a "magic" group.**
   - **about four times per semester**



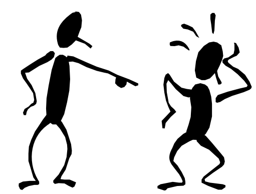
Structure and Management of Groups

4. How can problems of dominance by one student and conflict avoidance within a group be addressed?

- Group problems are part of each test. One common solution for all members.
- Assign and rotate roles:
  - Manager
  - Skeptic
  - Checker/Recorder
  - Summarizer

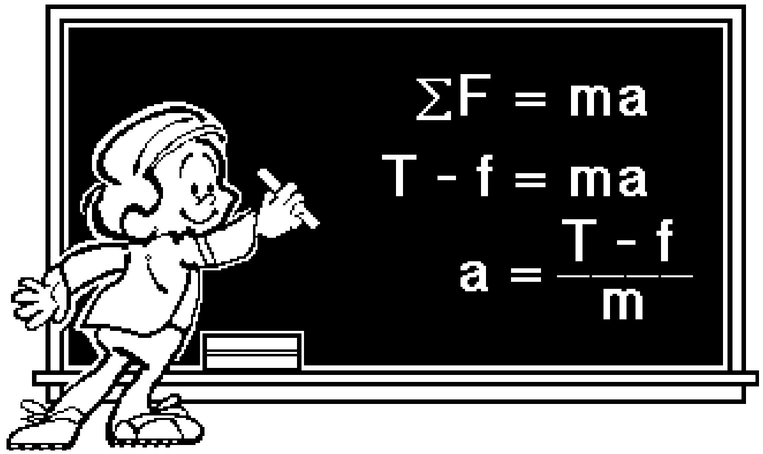


- Most of grade is based on individual problem solving.



- Students discuss how they worked together and how they could be more effective.

Structure and Management of Groups



5. How can individual accountability be addressed?

- assign and rotate roles, group functioning;
- seat arrangement -- eye-to-eye, knee-to-knee;
- individual students randomly called on to present group results;
- a group problem counts as a test question --if group member was absent the week before, he or she cannot take group test;
- most of the test is taken individually. The final exam is all individual. All lab reports are individual

Identify Critical Failure Points

Fail Gracefully

Non-optimal implementation gives some success

1. Inappropriate Tasks

Engage all group members (not just one who knows how to do it)

2. Inappropriate Grading

Don't penalize those who help others (no grading on the curve)

Reward for individual learning

3. Poor structure and management of Groups

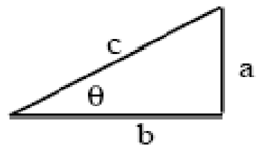
Scaffolding

Control of Equations that are Allowed

Equation sheet on the final exam for 1st semester: Calculus Based Physics for Biology Majors

40 equations

Useful Mathematical Relationships:



For a right triangle: $\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$, $\tan\theta = \frac{a}{b}$,

$a^2 + b^2 = c^2$, $\sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R$, $A = \pi R^2$

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\frac{d(z^n)}{dz} = nz^{n-1}$, $\frac{d(\cos z)}{dz} = -\sin z$, $\frac{d(\sin z)}{dz} = \cos z$, $\frac{d(e^{az})}{dz} = ae^{az}$, $\frac{d(\ln z)}{dz} = \frac{1}{z}$,

$\frac{df(z)}{dt} = \frac{df(z)}{dz}\frac{dz}{dt}$,

Fundamental Concepts, Principles, and Definitions:

$\sum \vec{F} = m\vec{a}$	$\rho = \frac{m}{V}$	$E_f - E_i = E_{in} - E_{out}$	$KE = \frac{1}{2}mv^2$	$P = \frac{F}{A}$	$e = \frac{E_{desired}}{E_{input}}$
$\tau = rF_\perp$	$\frac{dW}{d\ell} = F_\ell$	$f = \frac{1}{T}$	$\frac{dU}{dx} = -F_{internal}$	$S = k\ln\Omega$	$\theta = \frac{\delta C}{r}$
$F = U - TS$	$\frac{dx}{dt} = v_x$	$\frac{dv_x}{dt} = a_x$	$s_{av} = \frac{distance}{\Delta t}$	$v_{x\,av} = \frac{\Delta x}{\Delta t}$	$a_{x\,av} = \frac{\Delta v_x}{\Delta t}$

Under Certain Conditions:

$F = mg$	$F = kx$	$F = \mu_k n$	$F \le \mu_s n$	$F = G\frac{m_1 m_2}{r^2}$	$F = k_e\frac{q_1 q_2}{r^2}$
$\sum \tau = 0$	$\Delta E_{internal} = mL$	$\Delta E_{internal} = Cm\Delta T$	$PV = NkT$	$PV = nRT$	$\frac{dW}{dV} = P$
$U = mgh$	$U = \frac{1}{2}kx^2$	$W = -T\Delta S$	$\Delta S = \frac{Q}{T}$	$F = bv$	$a = \frac{v^2}{r}$
$x = A\cos(2\pi ft + \phi)$	$x = \frac{1}{2}at^2 + v_o t + x_o$	$\frac{1}{2}\rho v^2 + \rho gy + P = constant$			

Useful constants: 1 mile = 5280 ft, $g = 9.8\ m/s^2 = 32\ ft/s^2$, $k_B = 1.4 \times 10^{-23}$ J/K, $N_{av} = 6 \times 10^{23}$, $R = 8.3$ J/(mol K), $\rho_{water} = 1\ g/cm^3$

Scaffolding

Grading Guidance

This is a closed book, closed notes quiz. Calculators are permitted. The ONLY formulas that may be used are those given below. Define all symbols and justify all mathematical expressions used. Make sure to state all of the assumptions used to solve a problem. Credit will be given only for a logical and complete solution that is clearly communicated with correct units. Partial credit will be given for a well communicated problem solving strategy based on correct physics. MAKE SURE YOUR NAME, ID #, SECTION #, and TAs NAME ARE ON EACH PAGE!! START EACH PROBLEM ON A NEW PAGE. Each problem is worth 25 points:

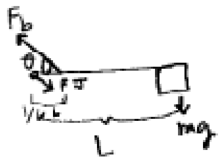
In the context of a unified solution, partial credit will be awarded as follows:

- **a useful picture, defining the question, and giving your approach is worth 6 points;**
- **a complete physics diagram defining the relevant quantities, identifying the target quantity, and specifying the relevant equations with reasons is worth 6 points;**
- **planning the solution by constructing the mathematics leading to an algebraic answer and checking the units of that answer is worth 7 points;**
- **calculating a numerical value with correct units is worth 3 points; and**
- **evaluating the validity of the answer is worth 3 points.**

The multiple choice questions are each worth 1.5 points.

Student Solution for this Question

Your task is to design an artificial joint to replace arthritic elbow joints. After healing, the patient should be able to hold at least a gallon of milk while the lower arm is horizontal. The biceps muscle is attached to the bone at the distance 1/6 of the bone length from the elbow joint, and makes an angle of 80^o with the horizontal bone. How strong should you design the artificial joint if you can assume the weight of the bone is negligible.



Knowns

$\theta = 80^\circ$

$V_m = 3.76\ L = 3760\ mL$

Target: F_j = Force of joint

Approach: Use Forces

$\Sigma F_x = 0$

$\Sigma F_y = 0$

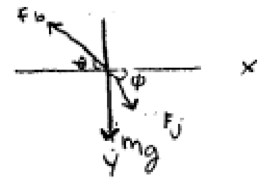
Use Torque

$\Sigma \tau = 0$

$\rho = \frac{m}{V}$

$\rho V_m = m$

Force Diagram



Student 1

assume density of milk is similar to water. $= 1\,g/cm^3 = 1\,g/mL = .001\ kg/mL$

$\Sigma F_x = 0$

$F_{jx} - F_{bx} = 0$

$\Sigma F_y = 0$

$F_{by} - mg - F_{jy} = 0$

$\Sigma \tau = 0$ (joint is pivot point)

$F_{by}(1/6 L) - mgL = 0$

$F_b \cos\theta = F_{bx}$

$F_b \sin\theta = F_{by}$

$F_{jy} = F_j \sin\phi$

$F_{jx} = F_j \cos\phi$

$F_j^2 = F_{jx}^2 + F_{jy}^2$

Plug in ϕ, ρV for m

$F_j = \sqrt{\left(\frac{6\rho V_g}{\sin\theta}\cos\theta\right)^2 + \left(5\rho V_g\right)^2}$

$= \sqrt{\left(\frac{6(.001)3760 g}{\sin 80}\cos 80\right)^2 + \left(5(.001)(3760 g)\right)^2} =$

43.3 N

units $\left(\frac{kg}{mL}\cdot mL\frac{m}{s^2}\right)^2 + \left(\frac{kg}{mL}\cdot m\cdot\frac{m}{s^2}\right)^2 = \sqrt{\frac{kg^2 m^2}{s^4}} = kg\,m/s^2 = N$ ✓

43.3N is the amount needed to lift an ≈ 4.3 kg object straight off the ground this is reasonable

---

equation		unknown
$F_j^2 = F_{jx}^2 + F_{jy}^2$	1	F_{jx}[1], F_{jy}[2]
$F_{by} - mg - F_{jy} = 0$	2	F_{by}[3]
$F_{jx} - F_{bx} = 0$	3	F_{bx}[4]
$F_{by}(1/6 L) - mgL = 0$	4	
$F_{bx} = F_b\cos\theta$	5	F_b[5]
$F_{by} = F_b\sin\theta$	6	F_j[6]

$F_b = \frac{F_{by}}{\sin\theta}$

$\frac{F_{by}}{\sin\theta}\cos\theta = F_{bx}$

$6mg = F_{by}$

$F_{jx} = \frac{6mg}{\sin\theta}\cos\theta$

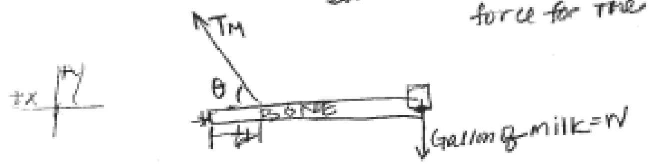
$F_{jy} = F_{by} - mg = 6mg - mg$

$F_j = \sqrt{\left(\frac{6mg}{\sin\theta}\cos\theta\right)^2 + \left(6mg - mg\right)^2}$

PROBLEM #1 – Page 1 of 1

Student 2

Question: What is the minimum force for the artificial joint?



L = length of bone

$\theta = 80°$

Let * be the pivot point where elbow bends

use: ΣF_x, ΣF_y, $\Sigma \tau$

$$\Rightarrow \Sigma \tau = T_M \sin\theta \left(\tfrac{1}{6}L\right) - WL = 0$$

$$\Rightarrow T_M (\sin\theta)\left(\tfrac{L}{6}\right) = WL$$

$$\Leftrightarrow W = \tfrac{1}{6} T_M \sin\theta$$

$$F_y = T_M \sin\theta - W$$

$$\Rightarrow F_y = T_M \sin\theta - \tfrac{1}{6} T_M \sin\theta$$

$$= \tfrac{5}{6} T_M \sin\theta$$

$$F_x = + T_M \cos\theta$$

$$F_{net} = \frac{F_y}{F_x} = \frac{\frac{5}{6} T_M \sin\theta}{T_M \cos\theta}$$

$$= \tfrac{5}{6} \tan\theta$$

∴ Force of the bone should be $\frac{5}{6}\tan\theta$ (in this case, $\frac{5}{6}\tan 80° = 4.7$ N)

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Problem Solving Assessment – Not Grading

Almost Independent Dimensions

- **Useful Description**
  - organize information from the problem statement symbolically, visually, and/or in writing.
- **Physics Approach**
  - select appropriate physics concepts and principles
- **Specific Application of Physics**
  - apply physics approach to the specific conditions in problem
- **Mathematical Procedures**
  - follow appropriate & correct math rules/procedures
- **Logical Progression**
  - overall the solution progresses logically; it is coherent, focused toward a goal, and consistent (not necessarily linear)

Based on previous work at Minnesota by:

J. Blue (1997); T. Foster (2000); T. Thaden-Koch (2005);

P. Heller, R. Keith, S. Anderson (1992)

Problem solving rubric at a glance

CATEGORY:
(based on literature)

SCORE (arrow pointing left)

CATEGORY	5	4	3	2	1	0	NA (P)	NA (S)
Useful Description								
Physics Approach								
Specific Application								
Math Procedures								
Logical Progression								

Want

- **Minimum** number of categories that include relevant aspects of problem solving
- **Minimum** number of scores that give enough information to improve instruction

Rubric Scores (in general)

5	4	3	2	1	0
Complete & appro-priate	Minor omission or errors	Parts missing and/or contain errors	Most missing and/or contain errors	All inappro-priate	No evidence of category

NOT APPLICABLE (NA):

NA - Problem	NA - Solver
Not necessary for this problem *(i.e. visualization or physics principles given)*	Not necessary for this solver *(i.e. able to solve without explicit statement)*

Useful Description **assesses a solver's skill at organizing information from the problem statement into an appropriate and useful representation that summarizes essential information symbolically and visually. The description is considered "useful" if it guides further steps in the solution process. A** ***problem description*** **could include restating known and unknown information, assigning appropriate symbols for quantities, stating a goal or target quantity, a visualization (sketch or picture), stating qualitative expectations, an abstracted physics diagram (force, energy, motion, momentum, ray, etc.), drawing a graph, stating a coordinate system, and choosing a system.**

5 The description is useful, appropriate, and complete

4 The description is useful but contains minor omissions or errors.

3 Parts of the description are not useful, missing, and/or contain errors.

2 Most of the description is not useful, missing, and/or contains errors.

1 The entire description is not useful and/or contains errors.

0 The solution does not include a description and it is necessary for this problem /solver.

NA (P) A description is not necessary for this problem. (i.e., it is given in the problem statement)

NA (S) A description is not necessary for this solver.

Physics Approach **assesses a solver's skill at selecting appropriate physics concepts and principle(s) to use in solving the problem. Here the term *concept* is defined to be a general physics idea, such as the basic concept of "vector" or specific concepts of "momentum" and "average velocity". The term *principle* is defined to be a fundamental physics rule or law used to describe objects and their interactions, such as the law of conservation of energy, Newton's second law, or Ohm's law.**

5 The physics approach is appropriate, and complete

4 Some concepts and principles of the physics approach are missing and/or inappropriate.

3 Most of the physics approach is missing and/or inappropriate.

2 All of the chosen concepts and principles are inappropriate.

1 The entire description is not useful and/or contains errors.

0 The solution does not indicate an approach, and it is necessary for this problem/ solver.

NA (P) A physics approach is not necessary for this problem. (i.e., it is given in the problem statement)

NA (S) An explicit physics approach is not necessary for this solver.

Specific Application of Physics **assesses a solver's skill at applying the physics concepts and principles from their selected approach to the specific conditions in the problem. If necessary, the solver has set up specific equations for the problem that are consistent with the chosen approach. A *specific application of physics* could include a statement of definitions, relationships between the defined quantities, initial conditions, and assumptions or constraints in the problem (i.e., friction negligible, massless spring, massless pulley, inextensible string, etc.)**

5 The specific application of physics is appropriate and complete.

4 The specific application of physics contains minor omissions or errors.

3 Parts of the specific application of physics are missing and/or contain errors.

2 Most of the specific application of physics is missing and/or contains errors.

1 All of the application of physics is inappropriate and/or contains errors.

0 The solution does not indicate an application of physics and it is necessary.

NA (P) A specific application of physics is not necessary for this <u>problem</u>.

NA (S) A specific application of physics is not necessary for this <u>solver</u>.

Mathematical Procedures **assesses a solver's skill at following appropriate and correct mathematical rules and procedures during the solution execution. The term *mathematical procedures* refers to techniques that are employed to solve for target quantities from specific equations of physics, such as isolate and reduce strategies from algebra, substitution, use of the quadratic formula, or matrix operations. The term *mathematical rules* refers to conventions from mathematics, such as appropriate use of parentheses, square roots, and trigonometric identities. If the course instructor or researcher using the rubric expects a symbolic answer prior to numerical calculations, this could be considered an appropriate mathematical procedure.**

- **5** **The mathematical procedures are appropriate and complete.**
- **4** **Appropriate mathematical procedures are used with minor omissions or errors.**
- **3** **Parts of the mathematical procedures are missing and/or contain errors.**
- **2** **Most of the mathematical procedures are missing and/or contain errors.**
- **1** **All mathematical procedures are inappropriate and/or contain errors.**
- **0** **There is no evidence of mathematical procedures, and they are necessary.**
- **NA (P) Mathematical procedures are not necessary for this <u>problem</u> or are very simple.**
- **NA (S) Mathematical procedures are not necessary for this <u>solver</u>.**

Logical Progression **assesses the solver's skills at communicating reasoning, staying focused toward a goal, and evaluating the solution for consistency (implicitly or explicitly). It checks whether the entire problem solution is clear, focused, and organized logically. The term *logical* means that the solution is coherent (the solution order and solver's reasoning can be understood from what is written), internally consistent (parts do not contradict), and externally consistent (agrees with physics expectations).**

5 The entire problem solution is clear, focused, and logically connected.

4 The solution is clear and focused with minor inconsistencies.

3 Parts of the solution are unclear, unfocused, and/or inconsistent.

2 Most of the solution parts are unclear, unfocused, and/or inconsistent.

1 The entire solution unclear, unfocused, and/or inconsistent.

0 There is no evidence of logical progression, and it is necessary.

NA (P) Logical progression is not necessary for this <u>problem</u>. (i.e., one-step)

NA (S) Logical progression is not necessary for this <u>solver</u>.

Some References

M.E. Martinez, "What is problem solving?" *Phi Delta Kappan*, *79*, 605-609 (1998).

J.R. Hayes, *The complete problem solver* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates (1989).

A.H. Schoenfeld, *Mathematical problem solving*. Orlando, FL: Academic Press, Inc. (1985).

G. Pólya, *How to solve it*. Princeton, NJ: Princeton University Press (1945).

F. Reif & J.I. Heller, "Knowledge structure and problem solving in physics," *Educational Psychologist*, *17*(2), 102-127 (1982).

P. Heller, R. Keith, and S. Anderson, "Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving," Am. J. Phys., 60(7), 627-636 (1992).

J.M. Blue, *Sex differences in physics learning and evaluations in an introductory course.* Unpublished doctoral dissertation, University of Minnesota, Twin Cities (1997).

T. Foster, *The development of students' problem-solving skills from instruction emphasizing qualitative problem-solving.* Unpublished doctoral dissertation, University of Minnesota, Twin Cities (2000).

J.H. Larkin, J. McDermott, D.P. Simon, and H.A. Simon, "Expert and novice performance in solving physics problems," Science 208 (4450), 1335-1342.

C. Henderson, E. Yerushalmi, V. Kuo, P. Heller, K. Heller, "Grading student problem solutions: The challenge of sending a consistent message" *Am. J. Phys*. 72(2), 164-169 (2004).

P.A. Moss, "Shifting conceptions of validity in educational measurement: Implications for performance assessment," Review of Educational Research 62(3), 229-258 (1992).

The End

Please visit our website for more information:

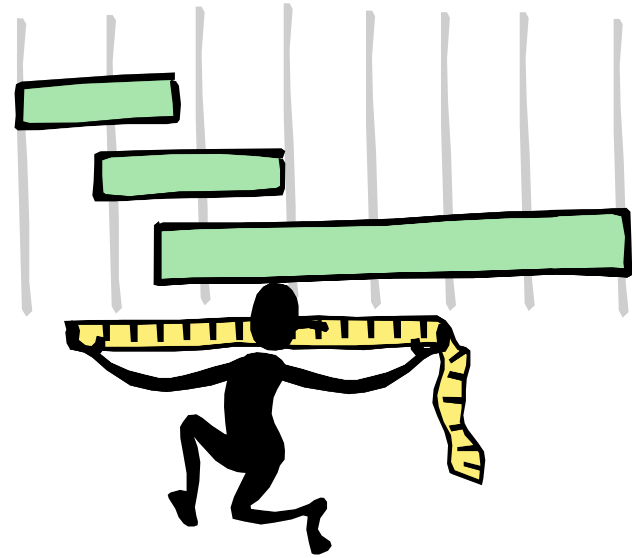
http://groups.physics.umn.edu/physed/

The best is the enemy of the good.

"le mieux est l'ennemi du bien"

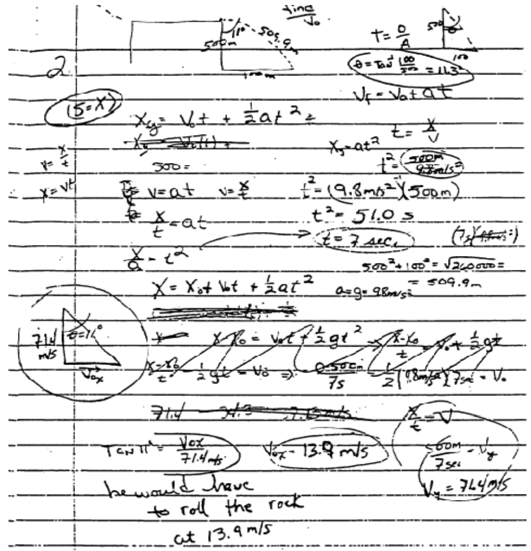
Voltaire

Assessment

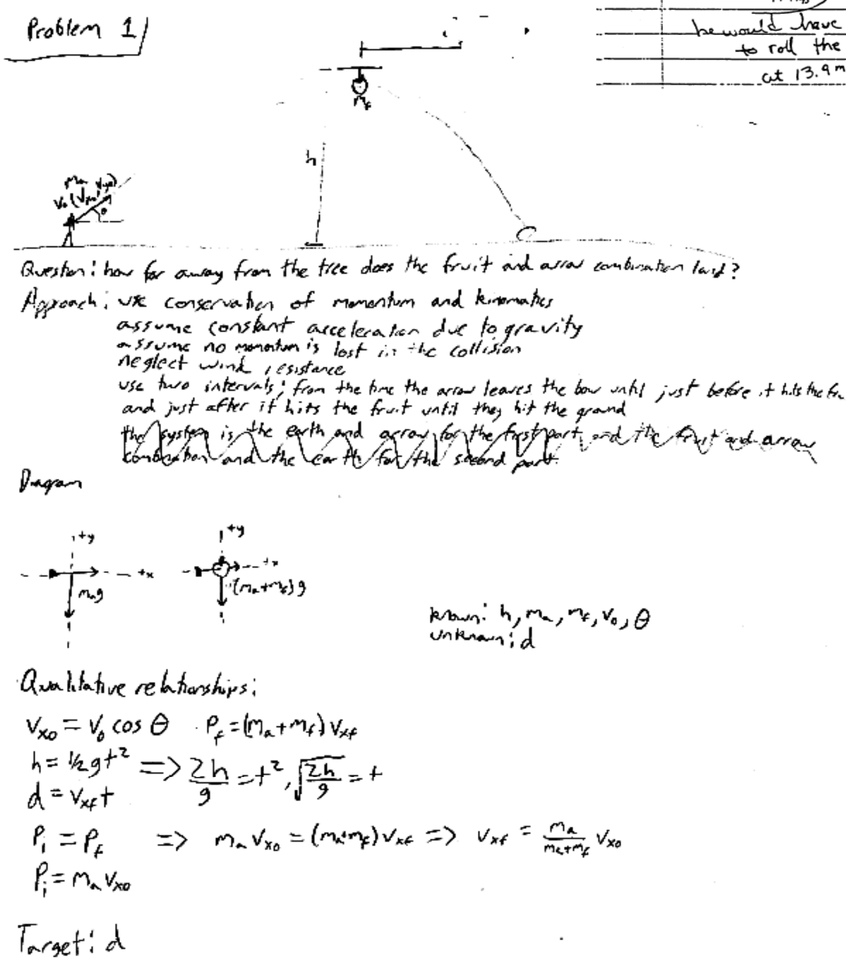


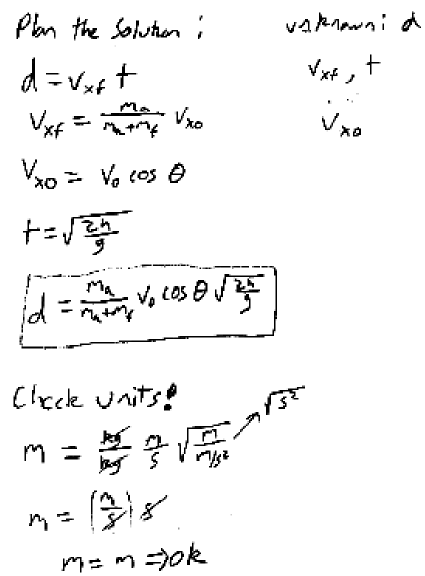
- Problem Solving Skill
- Drop out rate
- Failure rate
- FCI – some mechanics concepts
- BEMA – some E&M concepts
- CLASS – attitudes toward learning physics
- Math Skills
- What students value in the course
- Engineering student longitudinal study
- Faculty use
- Adoption by other institutions and other disciplines

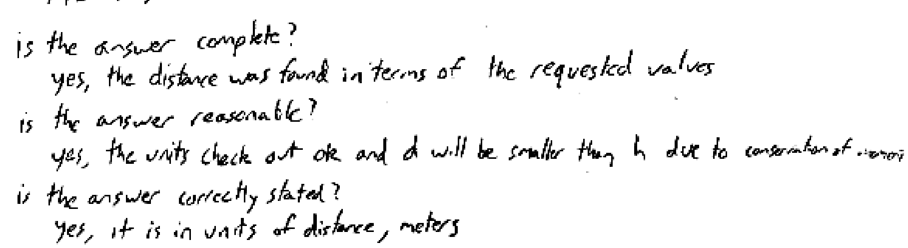
Student Problem Solutions



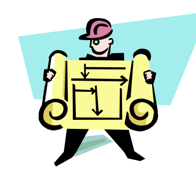
Initial State



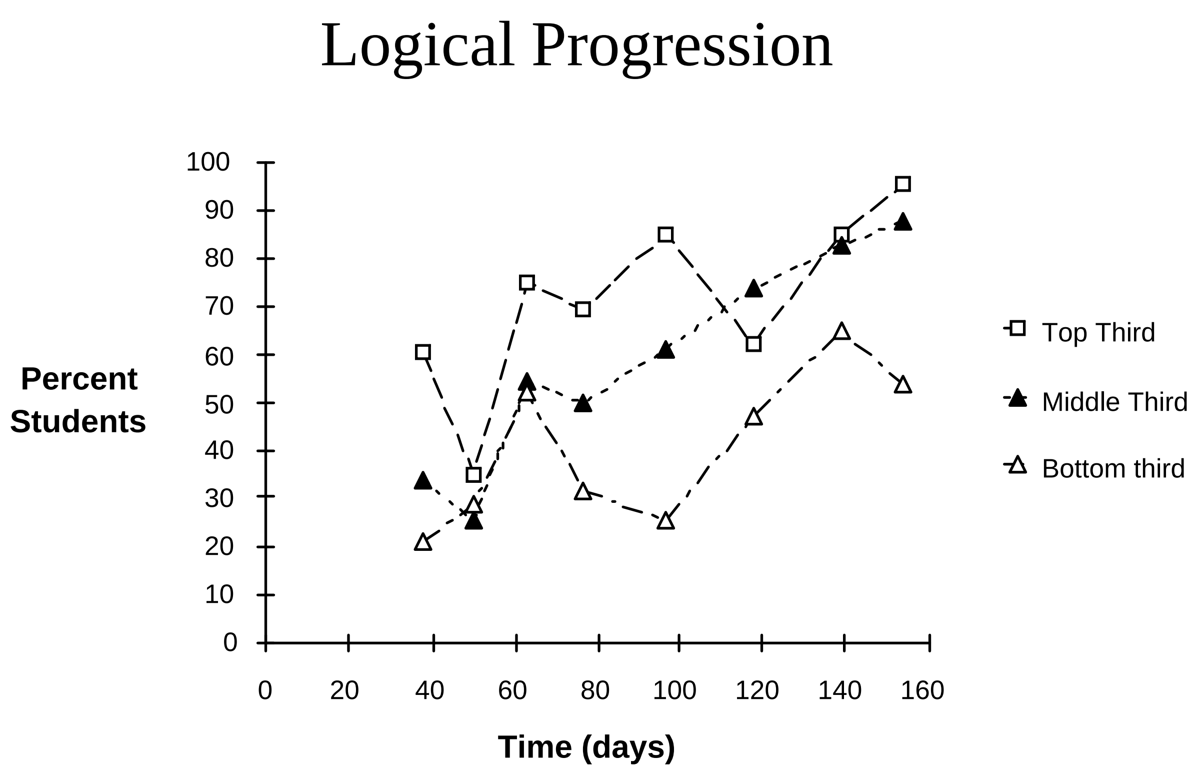




Final State



Improvement in Problem Solving



Algebra based physics 1991

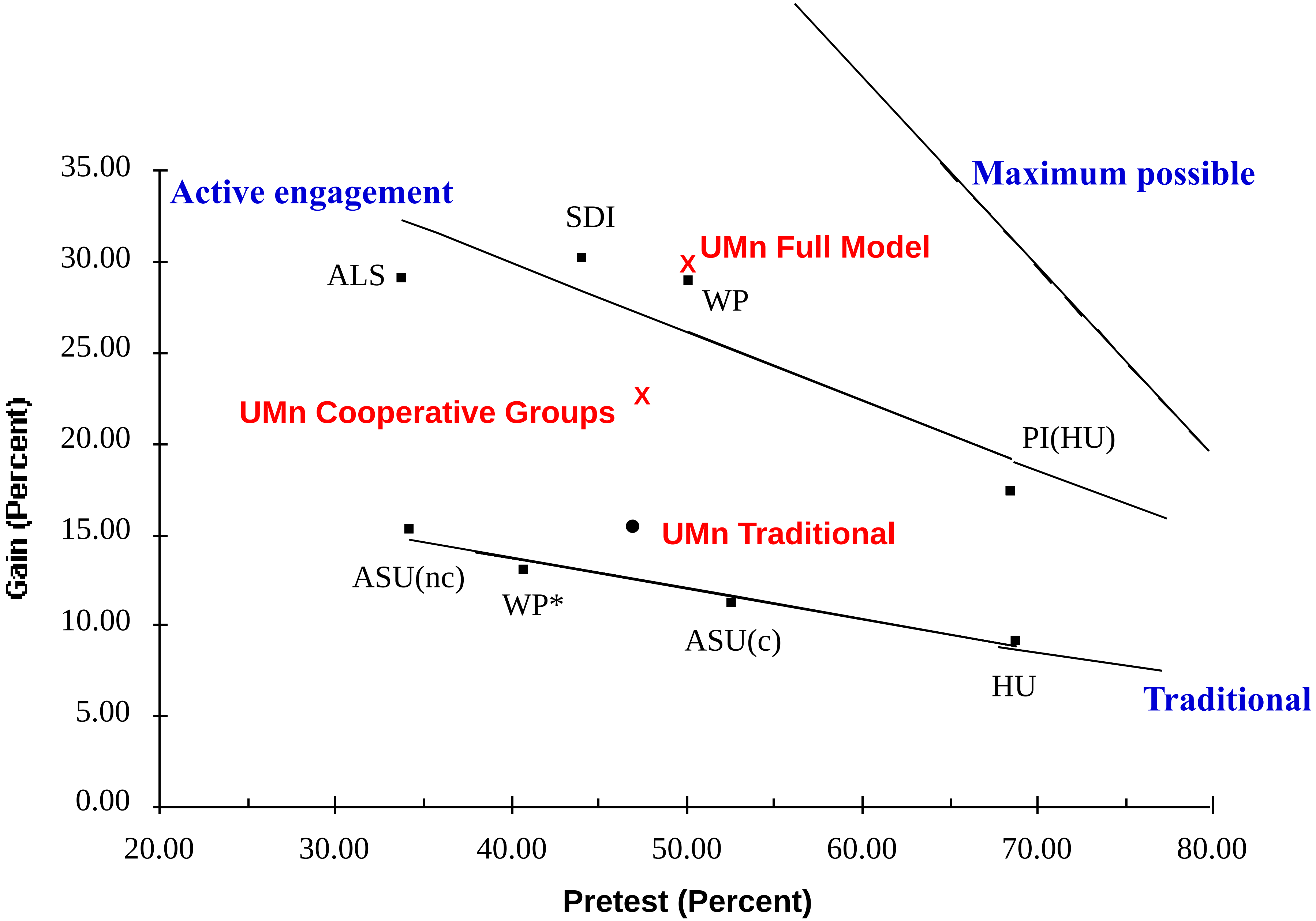
General Approach - does the student understand the physics

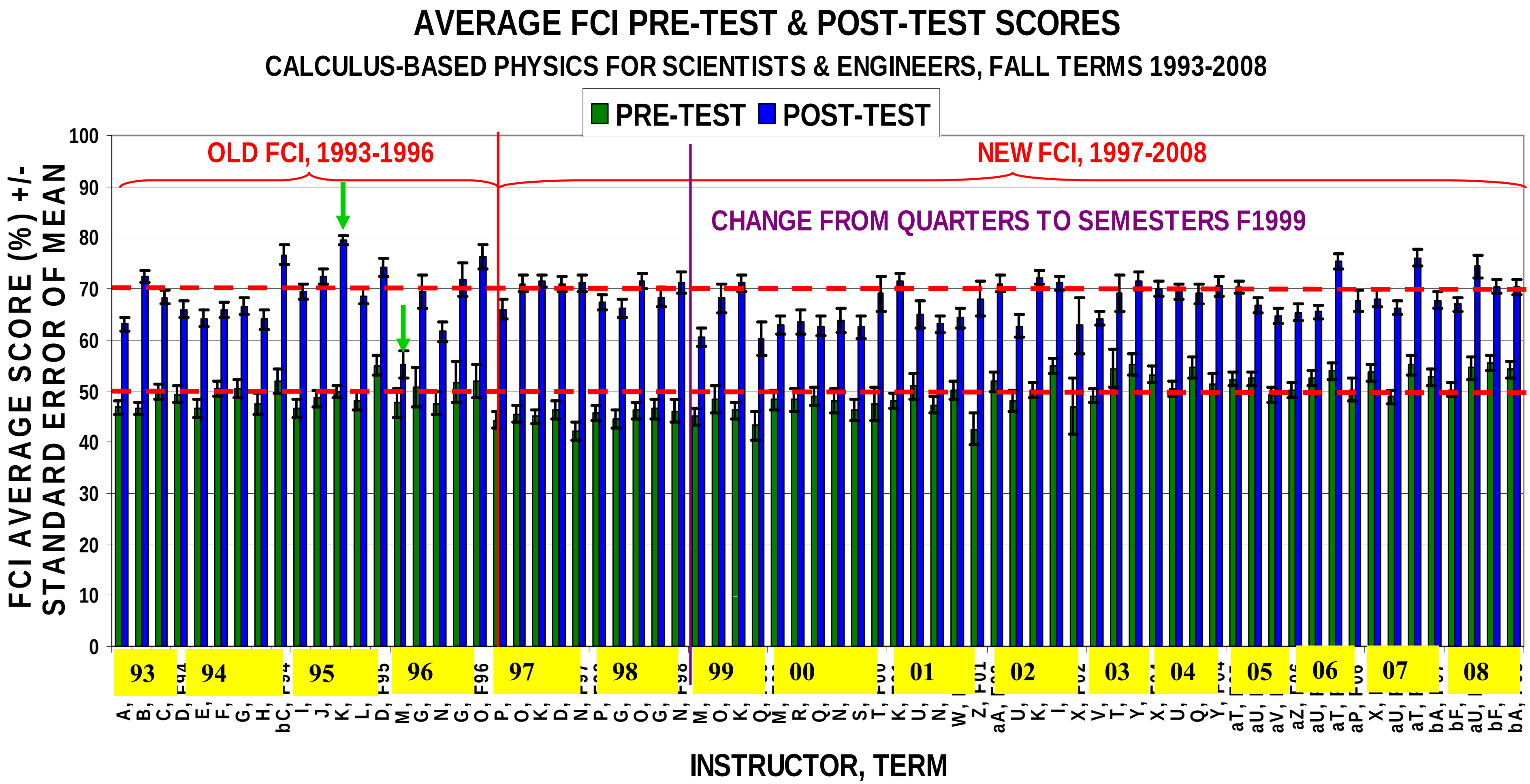
Specific Application of the Physics - starting from the physics they used, how did the student apply this knowledge?

Logical Progression - is the solution logically presented?

Appropriate Mathematics - is the math correct and useful?

Gain on FCI (Hake plot)

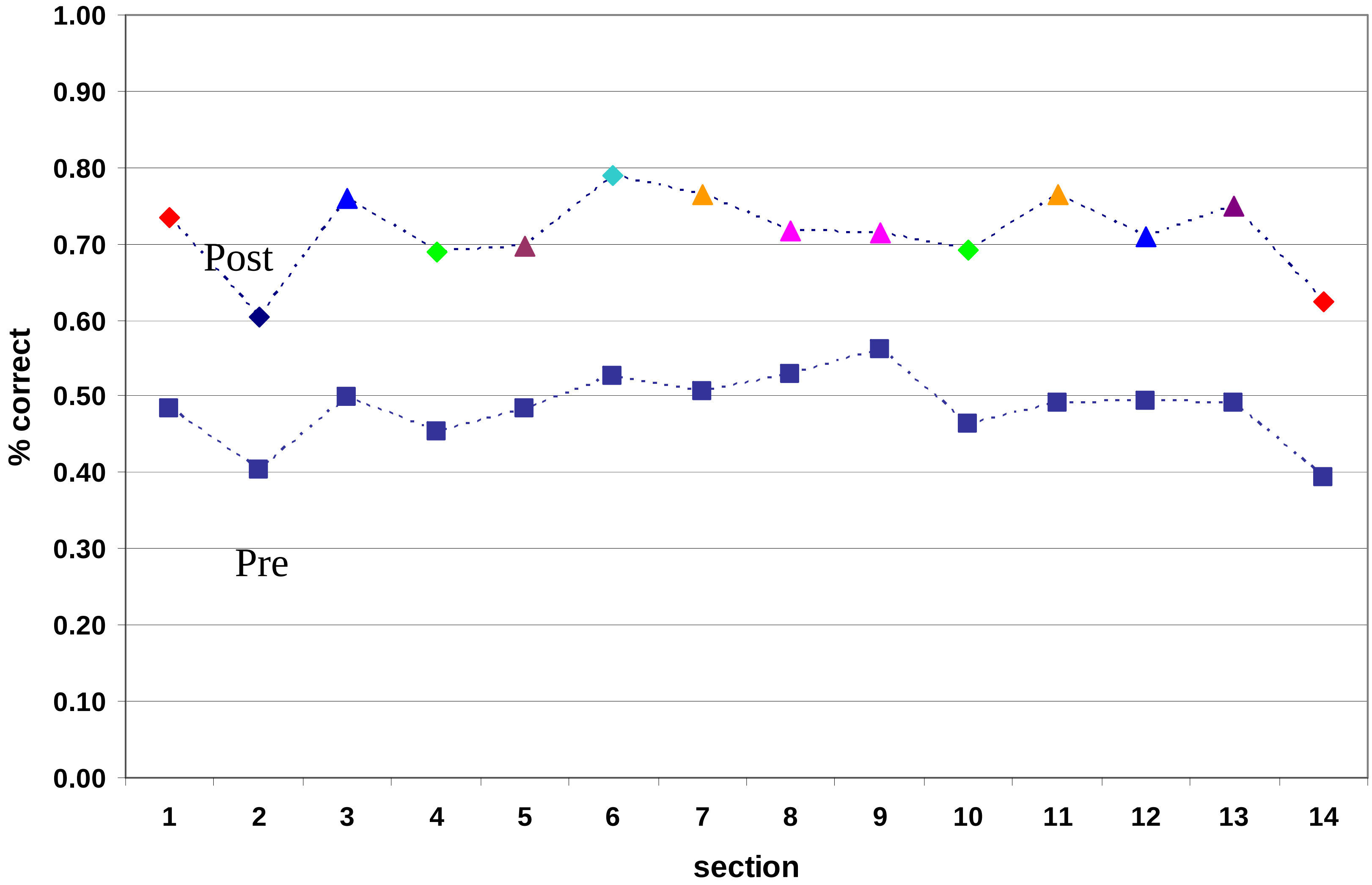




Each letter represents a different professor (39 different ones)

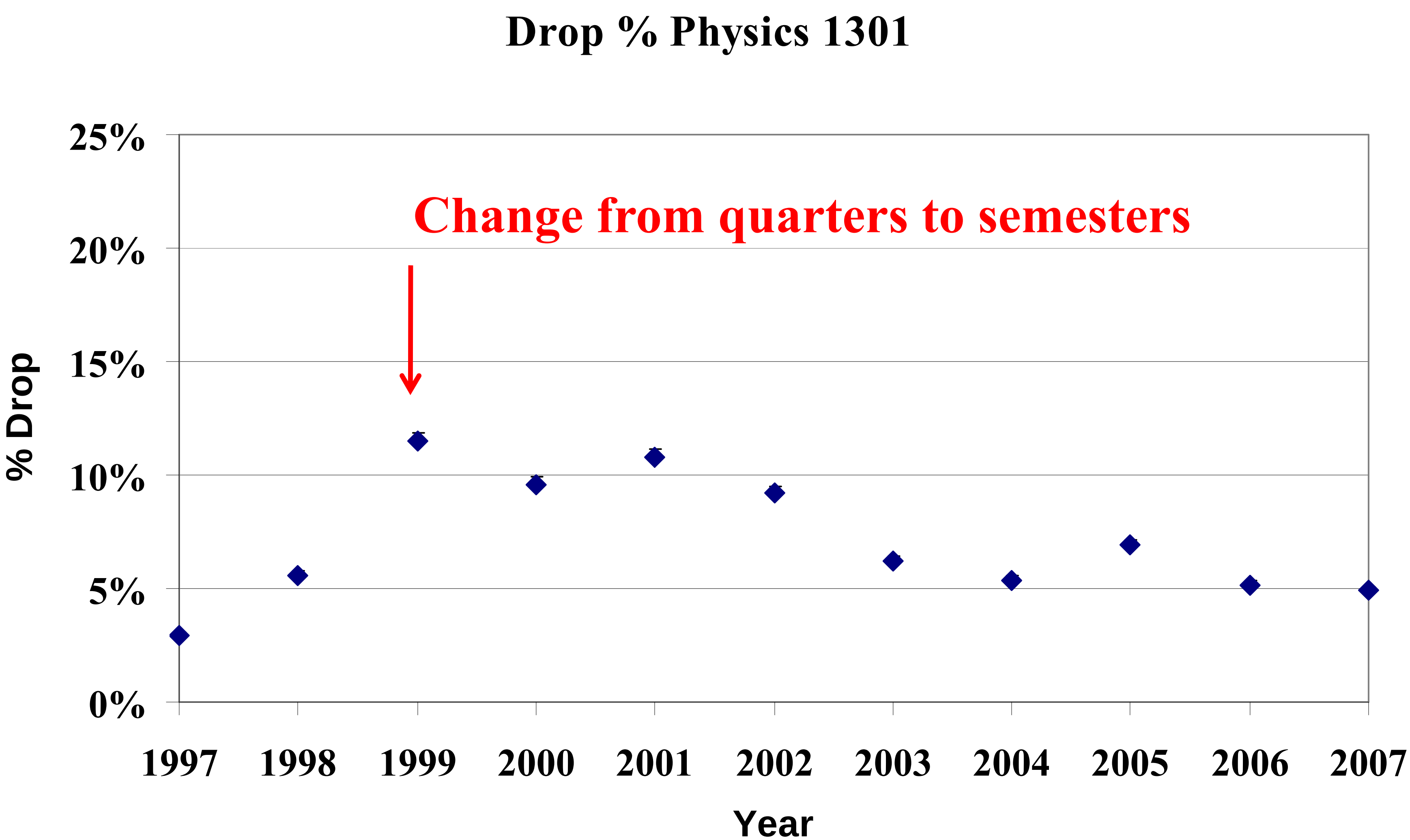
- **Incoming student scores are slowly rising (better high school preparation)**
- **Our standard course (CGPS) achieves average FCI ~70%**
- **Our “best practices” course achieves average FCI ~80%**
- **Not executing any cooperative group procedures achieves average FCI ~50%**

FCI by discussion/lab section

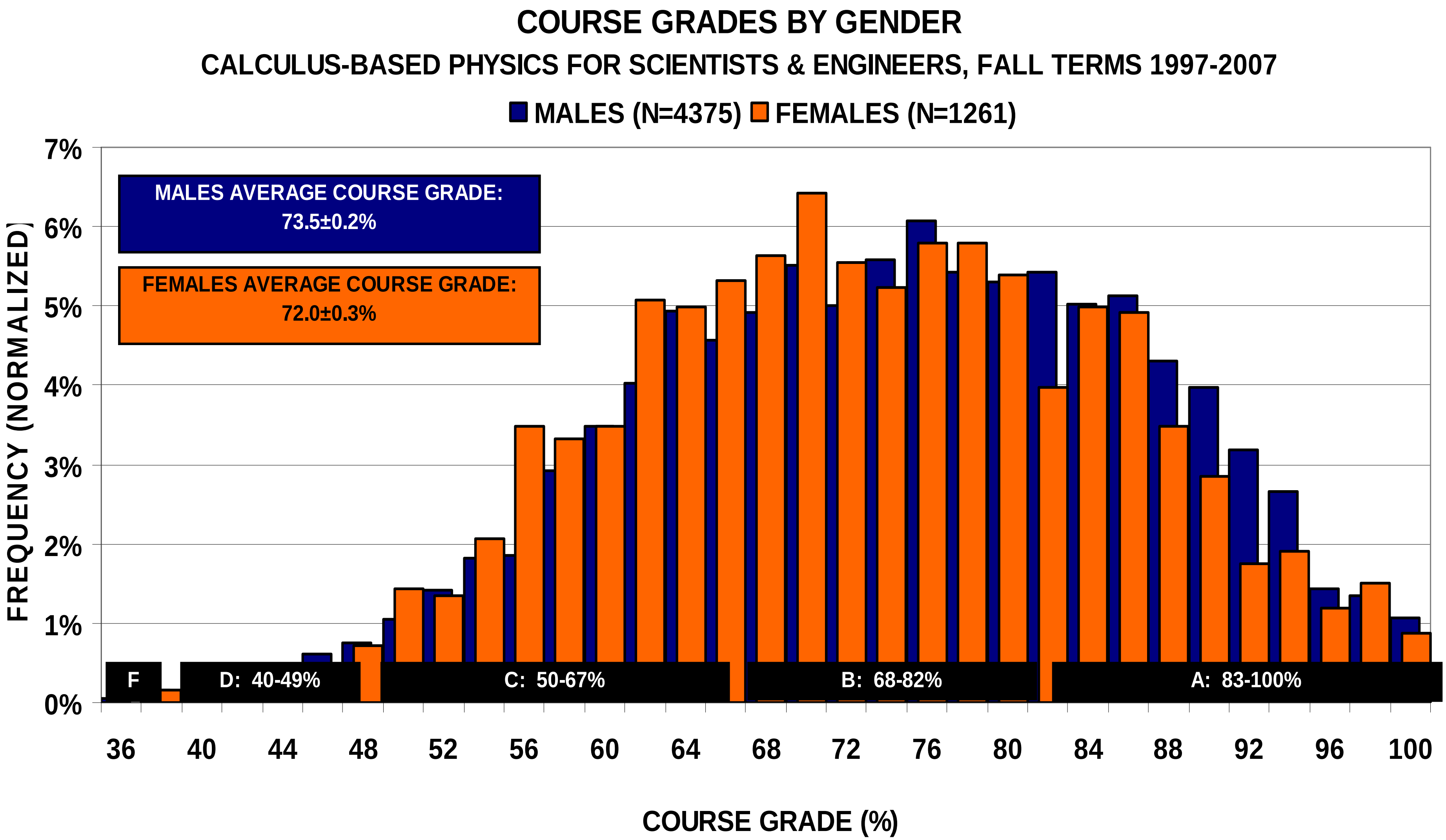


Same symbol (color and shape) is the same TA

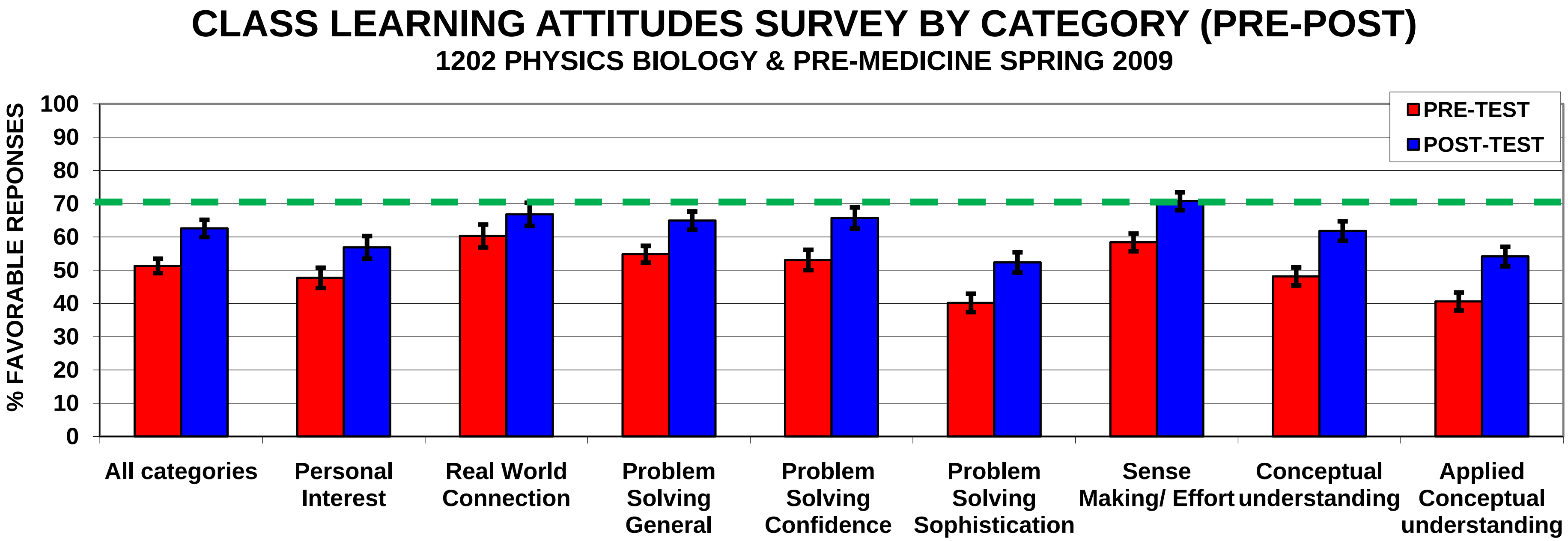
Retention

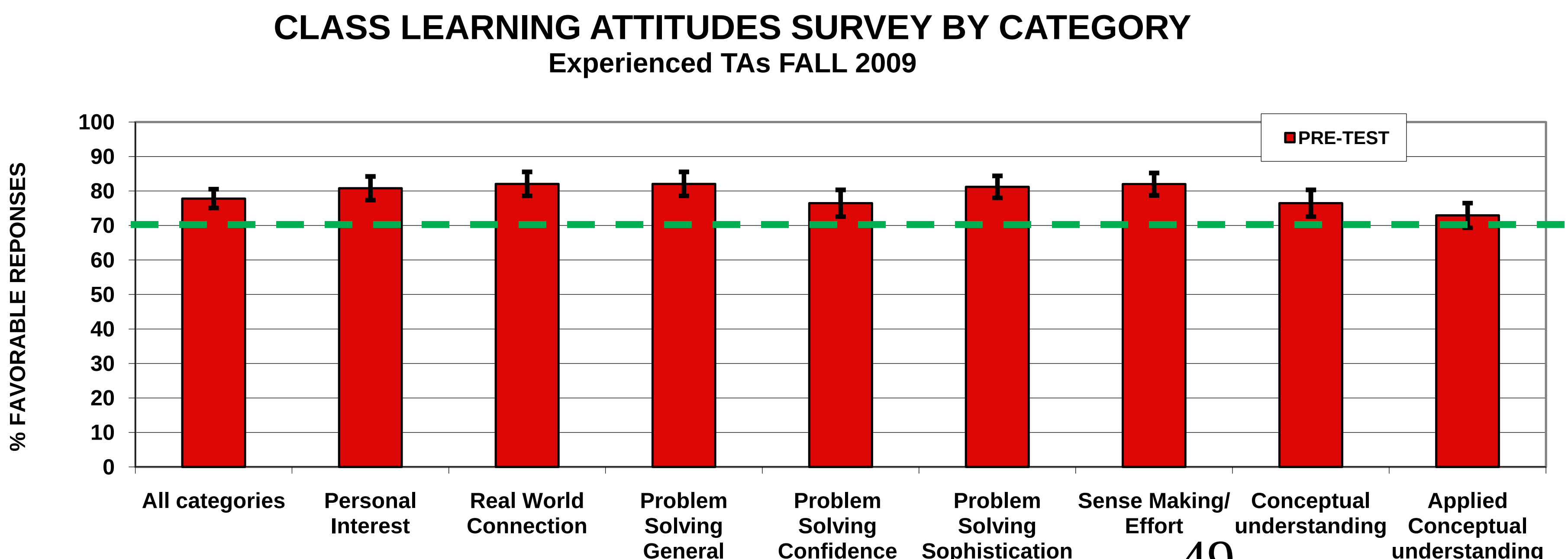


Dropout rate ~ 6%, F/D rate ~ 3% in all classes



Males and females do about as well in the course.


CLASS LEARNING ATTITUDES SURVEY BY CATEGORY (PRE-POST)
1202 PHYSICS BIOLOGY & PRE-MEDICINE SPRING 2009
% FAVORABLE REPONSES
100
90
80
70
60
50
40
30
20
10
0
PRE-TEST
POST-TEST
All categories
Personal Interest
Real World Connection
Problem Solving General
Problem Solving Confidence
Problem Solving Sophistication
Sense Making/ Effort
Conceptual understanding
Applied Conceptual understanding


CLASS LEARNING ATTITUDES SURVEY BY CATEGORY
Experienced TAs FALL 2009
% FAVORABLE REPONSES
100
90
80
70
60
50
40
30
20
10
0
PRE-TEST
All categories
Personal Interest
Real World Connection
Problem Solving General
Problem Solving Confidence
Problem Solving Sophistication
Sense Making/ Effort
Conceptual understanding
Applied Conceptual understanding

Student Opinion Data: Algebra-based Physics 1998

Rate the usefulness of the following components of the course.
Use a scale from 1 to 10
with 10 being extremely useful and 1 being completely useless
in helping you learn physics in this course.

	Ave. All Sections (N = 393)	Rank
108. Textbook	6.6 ± 0.13	1
106. Discussion Sessions (CGPS)	6.5 ± 0.13	2
101. Homework (not graded)	6.4 ± 0.14	3
105. Quizzes and Exams	6.1 ± 0.12	4
103. Lectures	6.1 ± 0.13	5
102. Laboratory	5.5 ± 0.12	6
109. Material on Class Web Pages	5.3 ± 0.14	7
107. TA's in tutoring room	4.6 ± 0.14	8
110. University tutors in Lind Hall	4.2 ± 0.14	9
104. Lecturer Office Hours	3.9 ± 0.12	10

CGPS Propagates Through the Department

Algebra-based Course (24 different majors) 1987

Goals: Calculus-based Course (88% engineering majors) 1993

- **4.5 Basic principles behind all physics**
- **4.5 General qualitative problem solving skills**
- **4.4 General quantitative problem solving skills**
- **4.2 Apply physics topics covered to new situations**
- **4.2 *Use with confidence***

Goals: Biology Majors Course 2003

- **4.9 Basic principles behind all physics**
- **4.4 General qualitative problem solving skills**
- **4.3 *Use biological examples of physical principles***
- **4.2 *Overcome misconceptions about physical world***
- **4.1 General quantitative problem solving skills**
- **4.0 *Real world application of mathematical concepts and techniques***

Upper Division Physics Major Courses 2002

- **Analytic Mechanics**
- **Electricity & Magnetism**
- **Quantum Mechanics**

Graduate Courses 2007

- **Quantum Mechanics**

RECORDER/CHECKER _______ SKEPTIC _________________ TA _____________

MANAGER _______________ SUMMARIZER _____________ Sec # __________

FOCUS the PROBLEM
Picture and Given Information

Question(s)

Approach

DESCRIBE the PHYSICS
Diagram and Define Quantities

Target Quantity(s)

Quantitative Relationships

PLAN the SOLUTION
Construct specific equations

EXECUTE the PLAN
Follow the Plan

Scaffolding

EVALUATE the ANSWER
Is Answer Properly Stated?

Is Answer Uneasonable?

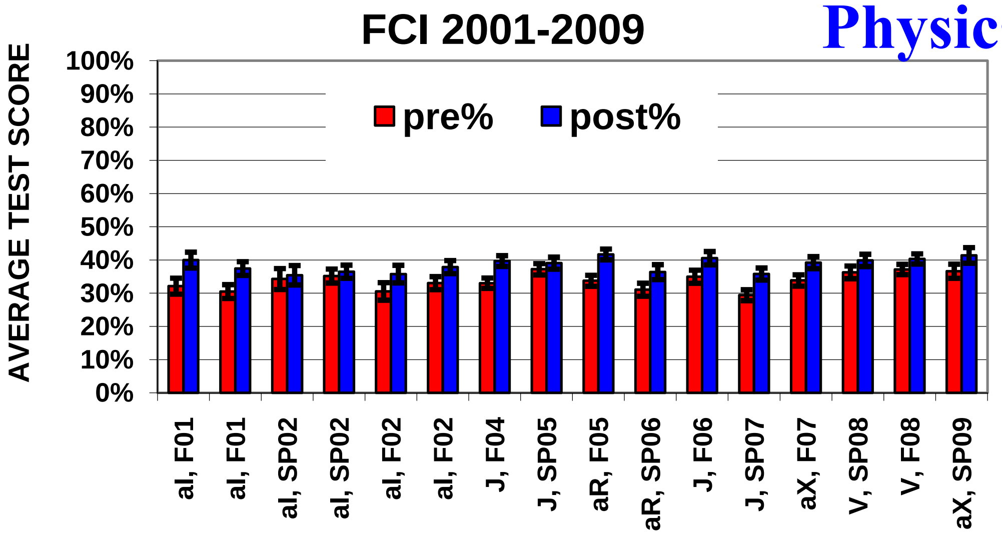
Is Answer Complete?

Page 1

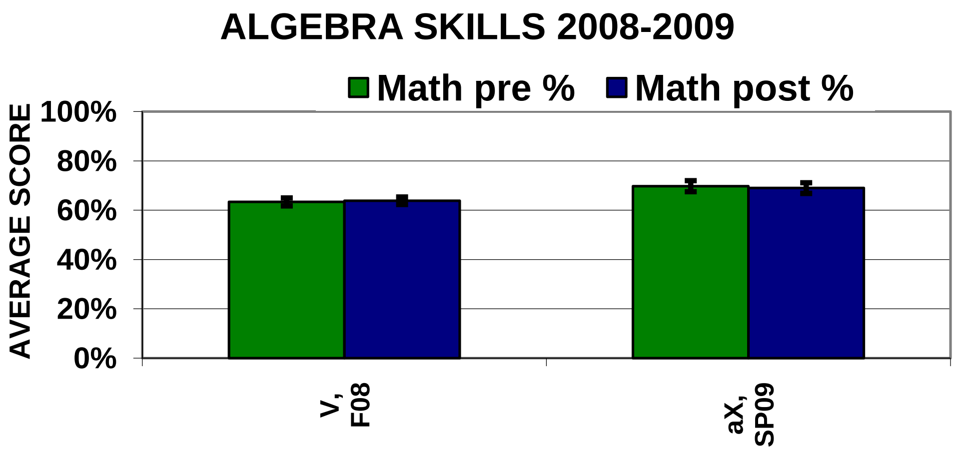
Problem Solving Worksheet used at the beginning of the course

Page 2

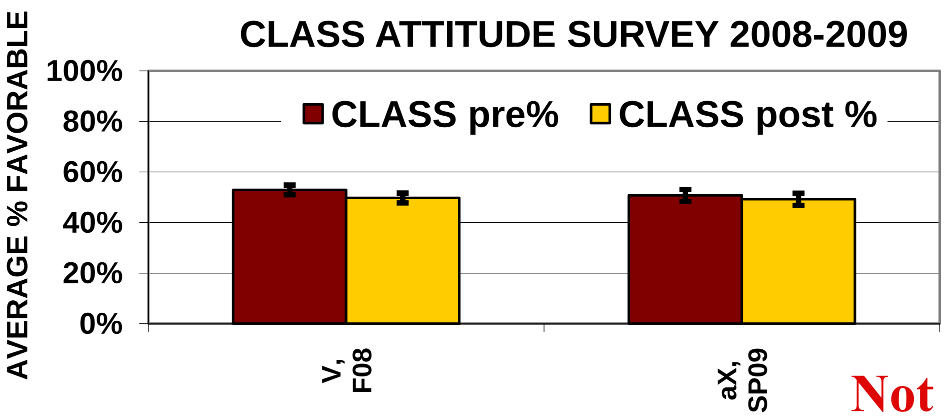
Physics 1001 (Energy & Environment)
Conceptual course



Small increase in force and motion concepts



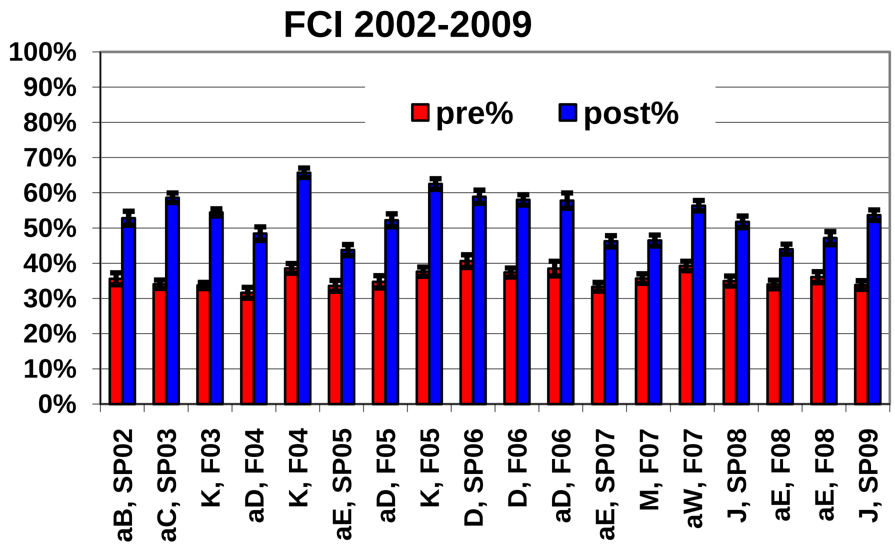
Students have reasonably high math skills



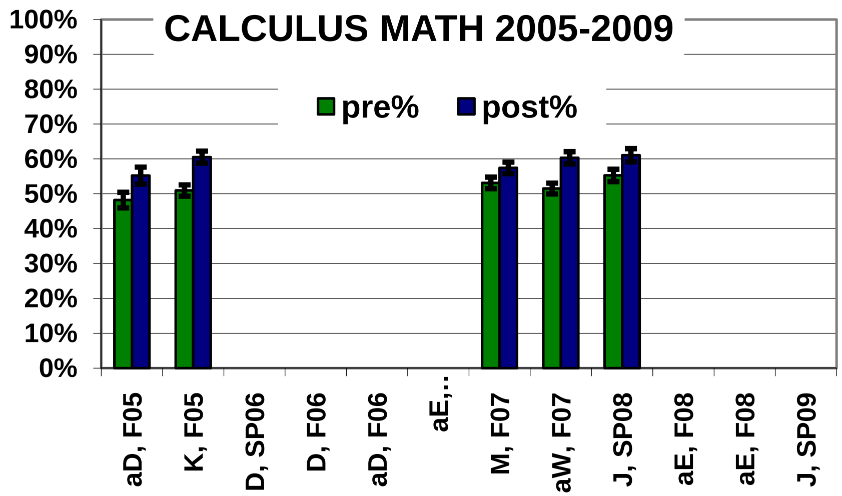
Students do not significantly change attitude toward science

Not using cooperative group problem solving

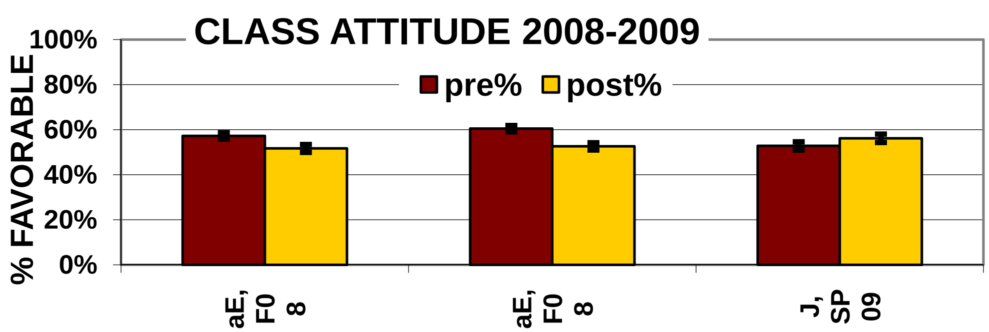
Physics 1201 (Biology & Pre-Meds) Calculus Based



Significant gain in force and motion concepts

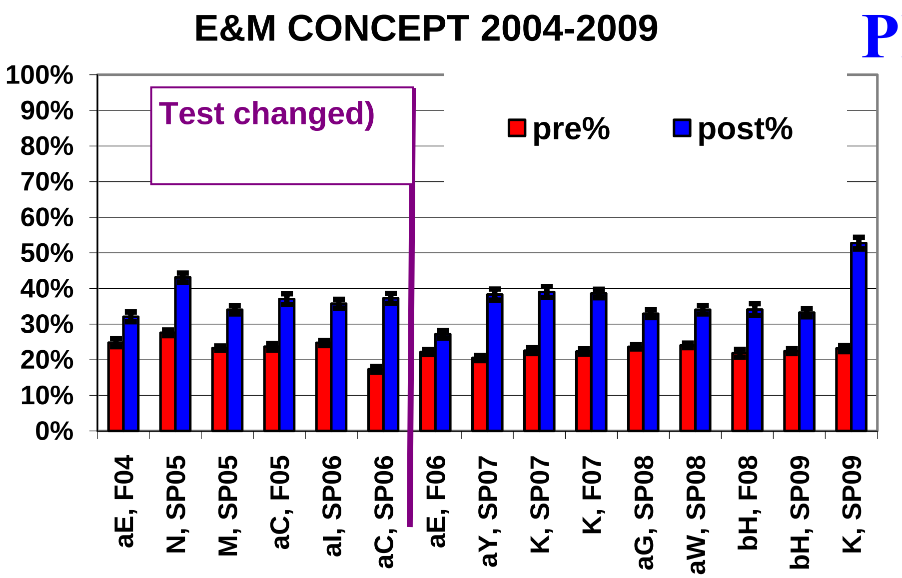


Students have reasonably high math skills

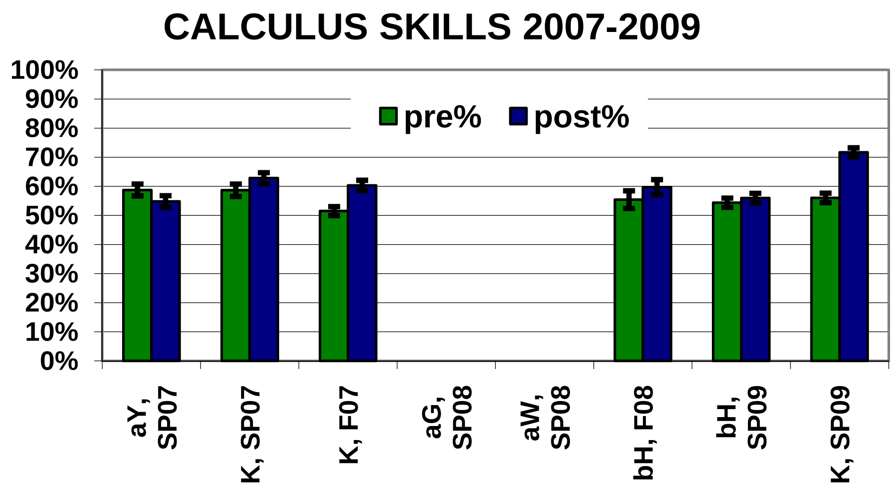


Students can decrease or increase their attitude toward science

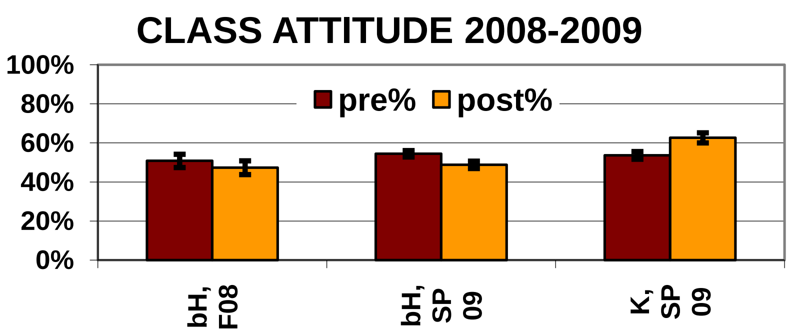
Physics 1202 (Biology & Pre-Meds) Calculus Based



Significant gain in E&M concepts

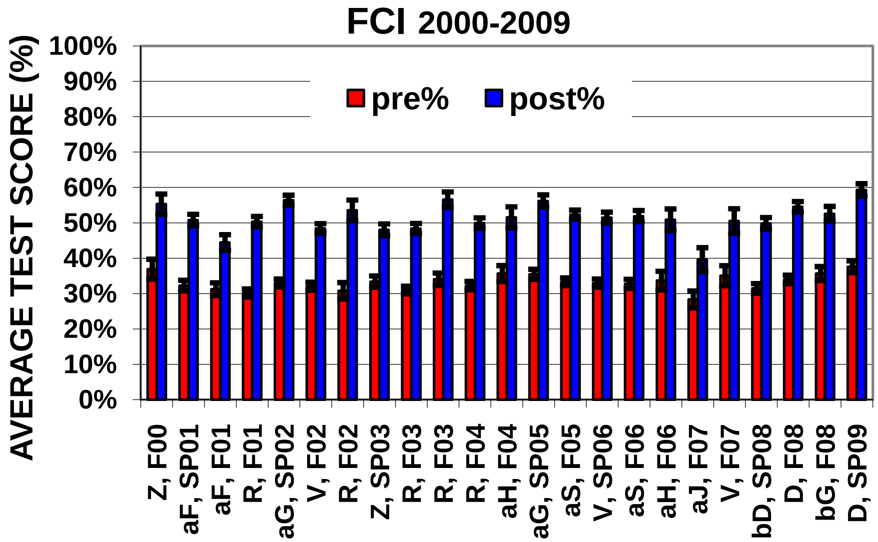


Students have reasonably high math skills

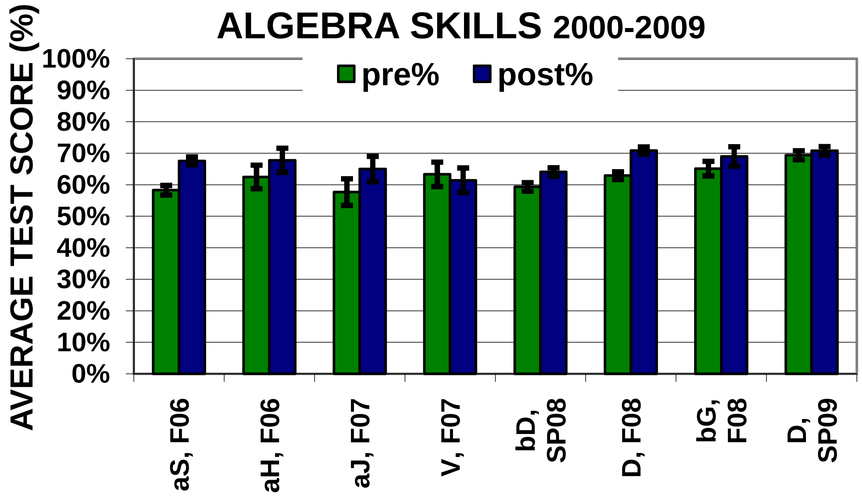


Students can decrease or increase their attitude toward science

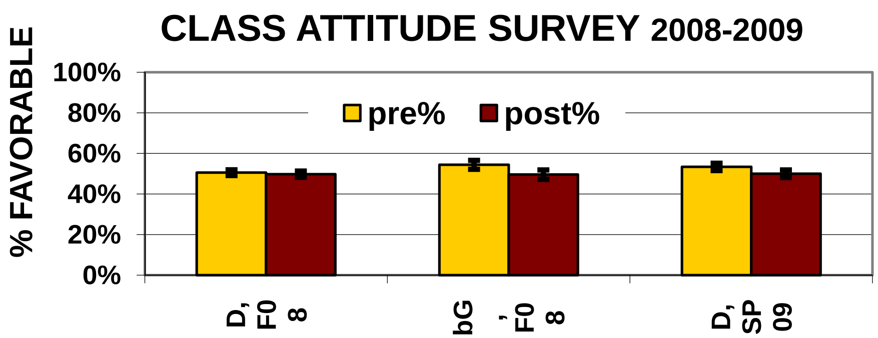
Physics 1101
Algebra Based



Significant gain in force and motion concepts

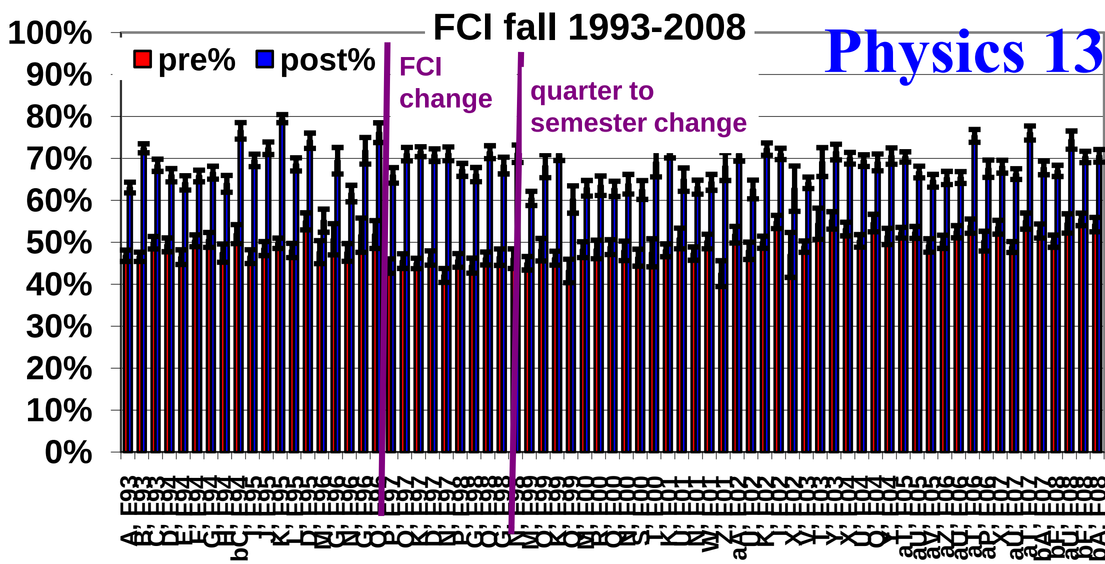


Students have reasonably high math skills

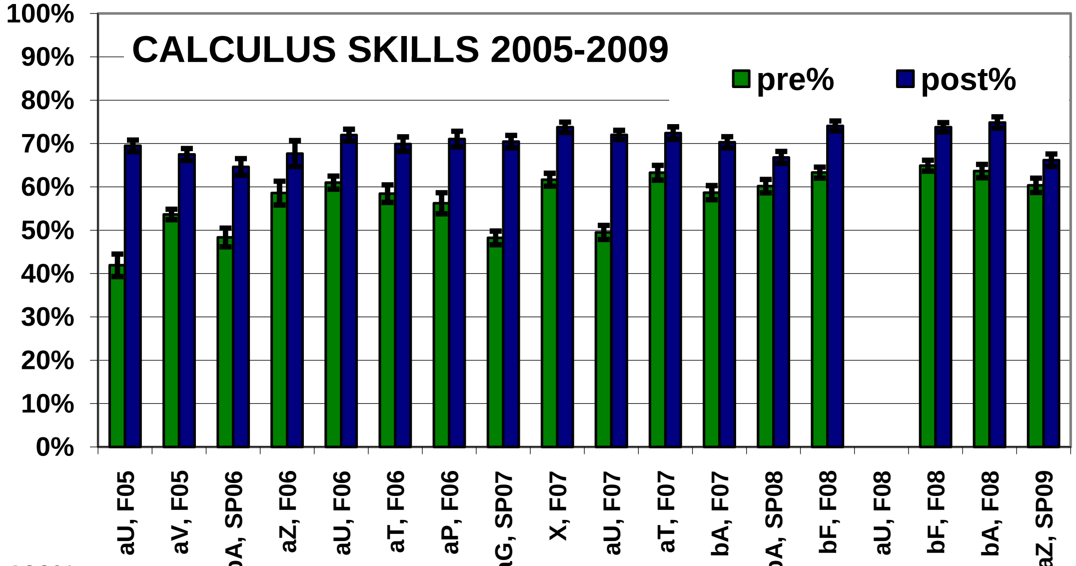


Students perhaps decrease their attitude toward science

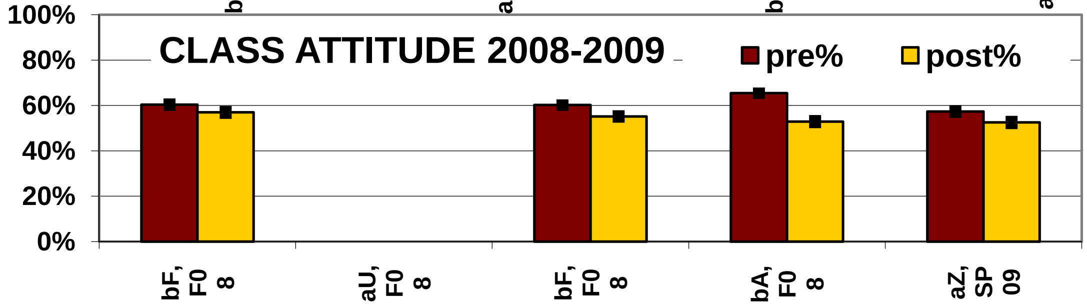
Physics 1301 (Engineer & Physical Sci) Calculus Based



Significant gain in force & motion concepts

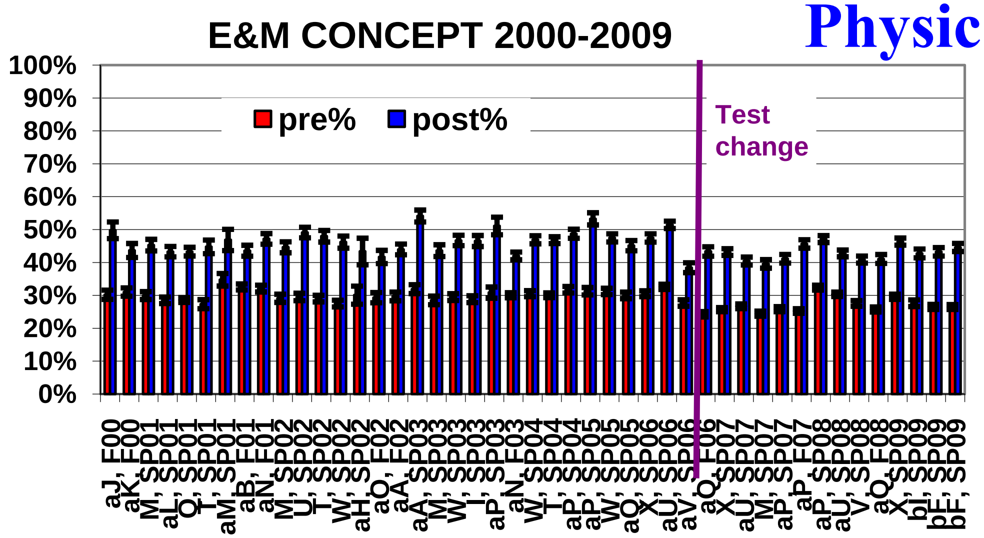


Students have reasonably high math skills

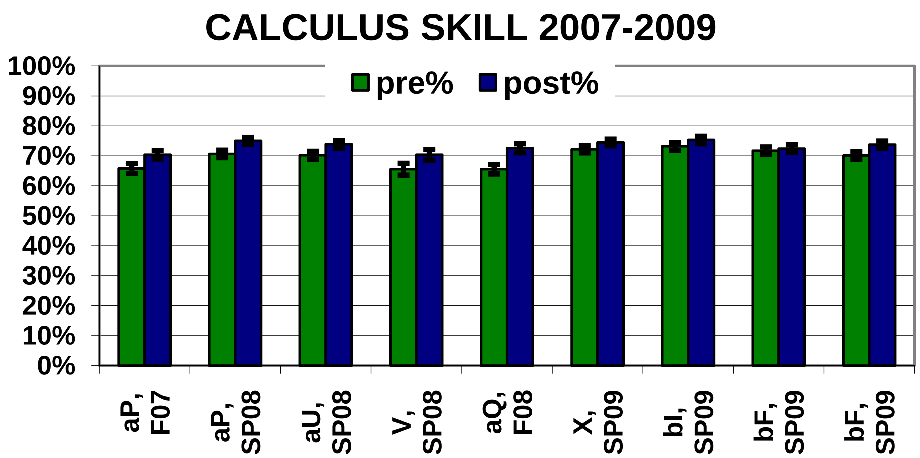


Students decrease their attitude toward science

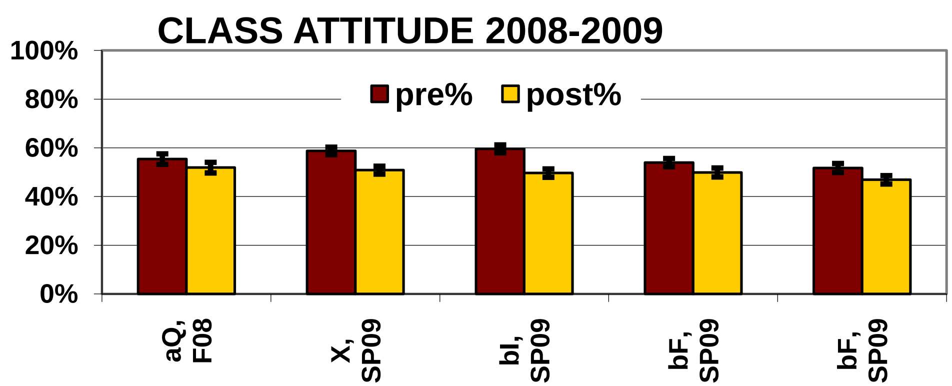
Physics 1302 (Engineer & Physical Sci)
Calculus Based



Significant gain in E&M concepts

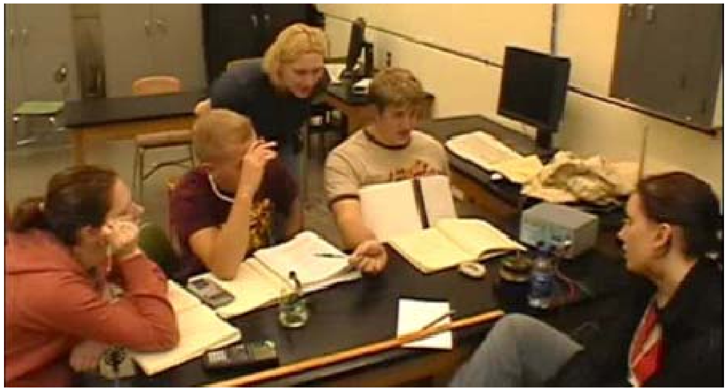


Students have reasonably high math skills



Students decrease their attitude toward science

The Advantages of Using Cooperative Group Problem Solving



1. Using a problem solving framework seems too long and complex for most students.

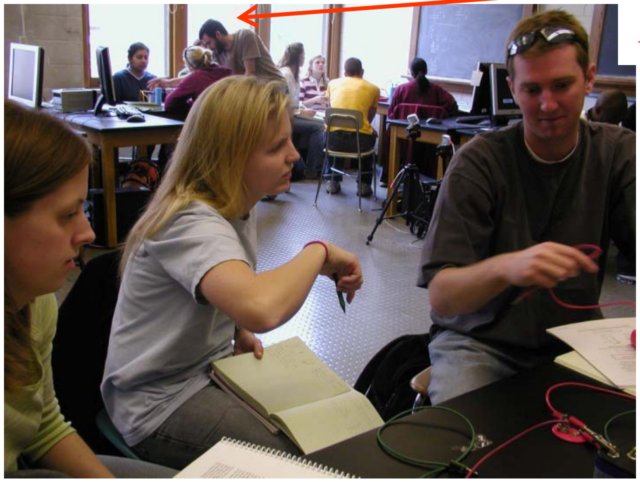
   The cooperative-group provides the motivation and knowledge to practice the parts until the framework becomes more natural.

2. Complex problems that need organization are initially difficult.

   Groups can successfully solve them so students see the advantage of a logical problem-solving framework early in the course.

3. The group interaction allows individuals to observe the planning and monitoring skills needed to solve problems. (Metacognition)

TA Coaching Another Group



4. Students practice the language of physics -- "talking physics."

5. Students must deal with and resolve their misconceptions.

6. Coaching by instructors is more effective – student groups are not sufficient, a more knowledgeable coach for the groups is required.

External clues of group difficulties
Group processing of instructor input

Competent Problem Solving

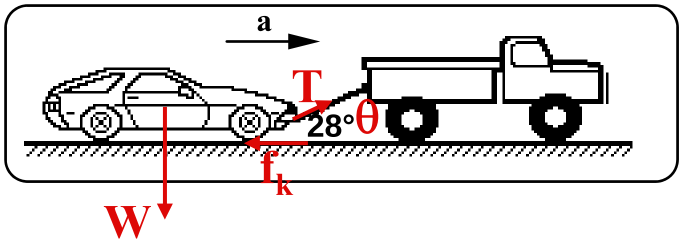
Step | **Bridge**

1. Focus on the Problem

Translate the words into an image of the situation.

Know T_{max}, θ, μ, W

What is a_{max}

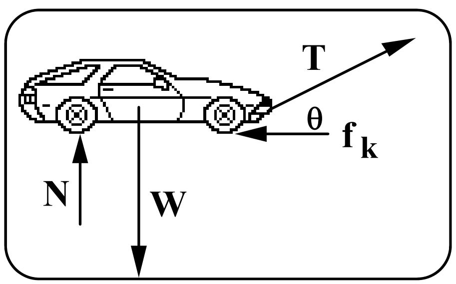


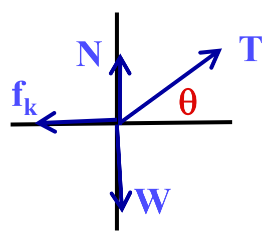
Identify an approach to the problem.

Relate forces on car to acceleration using Newton's Second Law

2. Describe the Physics

Translate the mental image into a physics representation of the problem (e.g., idealized diagram, symbols for important quantities).





Assemble mathematical tools (equations).

$$\sum F = ma$$

$$f_k = \mu N$$

$$W = mg$$

3. Plan a Solution

Step **Bridge**

3. Plan a Solution

Translate the physics description into a mathematical representation of the problem.

Find a:

[1] $\sum F_x = ma_x$

Find $\sum F_x$:

[2] $\sum F_x = T_x - f_k$

Find T_x

[3] $T_x = T\cos\theta$

⋮

Outline the mathematical solution steps.

Solve[3] for T_x and put into [2].

Solve[2] for $\sum F_x$ and put into [1].

Solve[1] for a_x.

4. Execute the Plan

Translate the plan into a series of appropriate mathematical actions.

$\Sigma F_x = T\cos\theta - \mu(W - T\sin\theta)$

$(W/g)a_x = T\cos\theta - \mu(W - T\sin\theta)$

$a_x = (T\cos\theta - \mu(W - T\sin\theta))\, g/W$

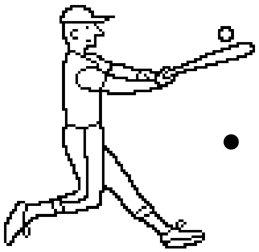
Check units of algebraic solution.

$$\frac{\left[\frac{m}{s^2}\right][N]}{[N]} - \left[\frac{m}{s^2}\right] = \left[\frac{m}{s^2}\right] \quad \text{OK}$$

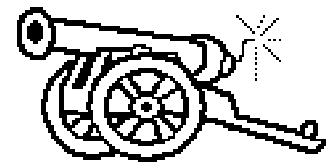
5. Evaluate the Solution

Scaffolding

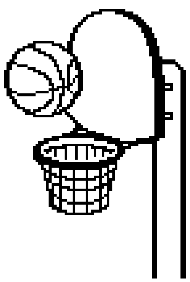
Context-rich Problems



- **Each problem is a short story in which the major character is the student. The problem statement uses the personal pronoun "you."**

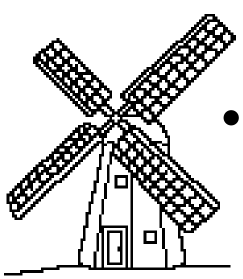


- **Some decisions are necessary to proceed.**

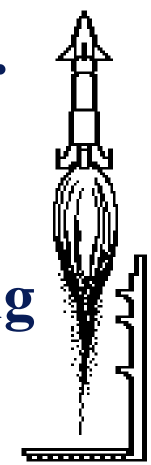


- **The problem statement includes a plausible motivation or reason for "you" to calculate something.**

- **The objects in the problems are real (or can be imagined) – students must practice idealization.**



- **No pictures or diagrams are given with the problems. Students must visualize the situation by using their own experiences.**



- **The problem can not be solved in one step by plugging numbers into a formula.**

Context-rich Problems

In addition, more difficult context-rich problems can have one or more of the following characteristics:

- The **unknown quantity is not explicitly specified** in the problem statement (e.g., Will this design work?).
- **More information** may be given in the problem statement than is required to solve the problems, or relevant **information may be missing**.
- **Assumptions** may need to be made to solve the problem.
- The problem may **require more than one fundamental principle** for a solution (e.g., Newton's 2nd Law and the Conservation of Energy).
- The **context can be very unfamiliar** (i.e., involve the interactions in the nucleus of atoms, quarks, quasars, etc.)

Solving This

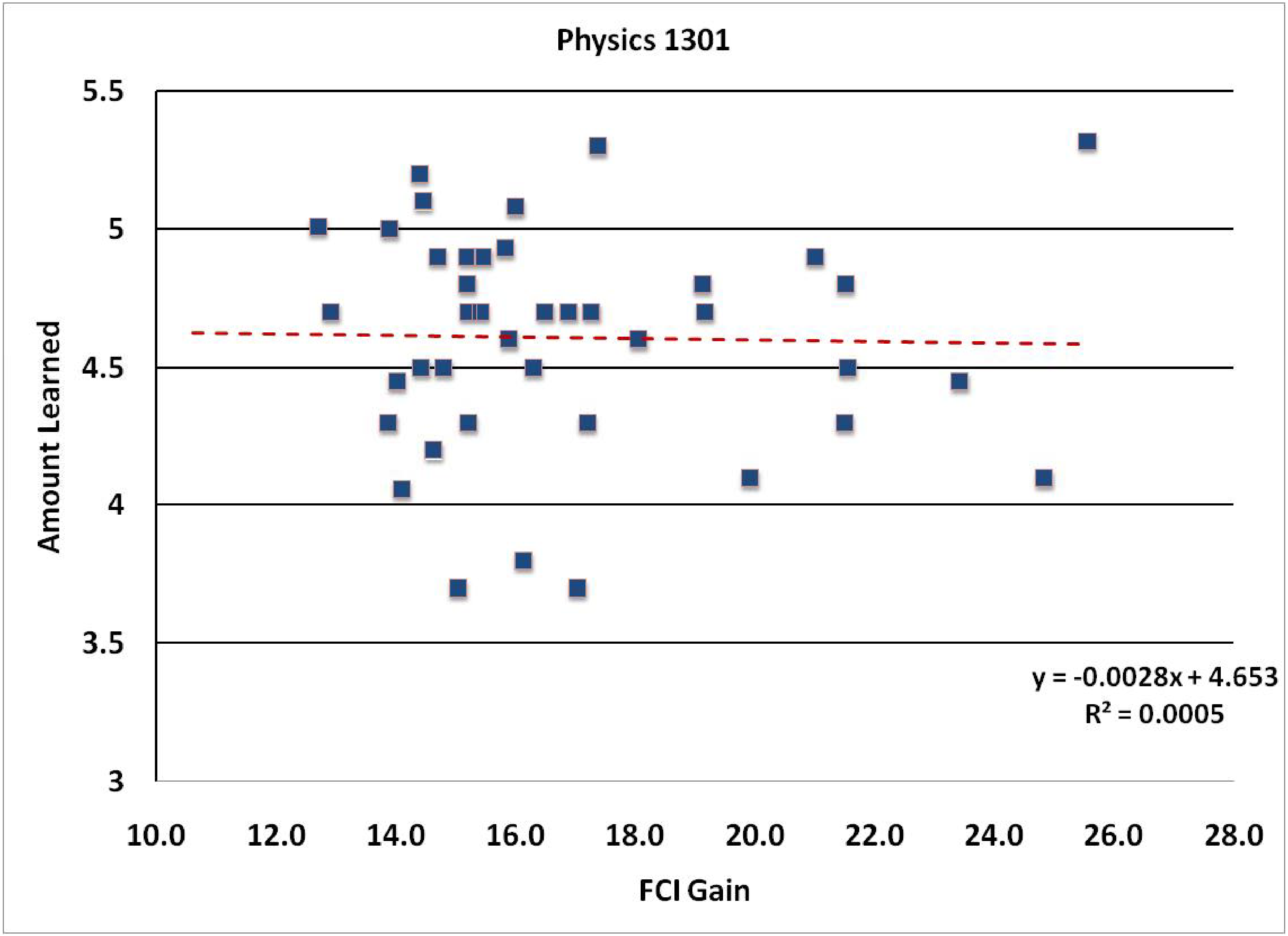
An infinitely long cylinder of radius R carries a uniform (volume) charge density r. Use Gauss' Law to calculate the field everywhere inside the cylinder.

is NOT Problem Solving?

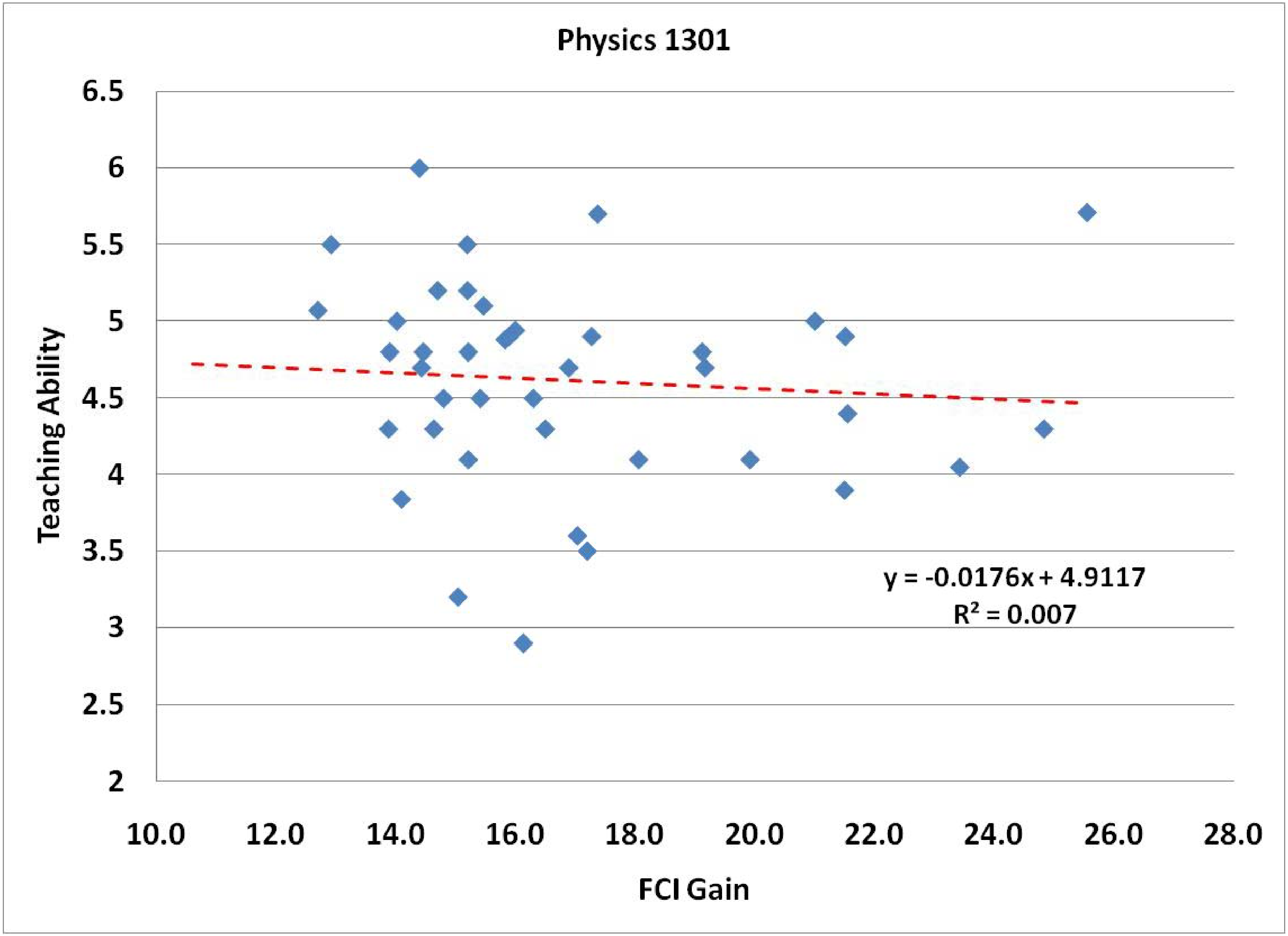
This is Problem Solving

You are investigating the possibility of producing power from fusion. The device being designed confines a hot gas of positively charged ions in a very long cylinder with a radius of 2.0 cm. The charge density of the ions in the cylinder is 6.0 x 10^{-5} C/m^3. Positively charged Tritium ions are to be injected perpendicular to the axis of the cylinder in a direction toward the center of the cylinder. Your job is to determine the speed that a Tritium ion should have when it enters the cylinder so that its velocity is zero when it reaches the axis of the cylinder. Tritium is an isotope of Hydrogen with one proton and two neutrons. You look up the charge of a proton and mass of the tritium in your Physics text and find them to be 1.6 x 10^{-19} C and 5.0 x 10^{-27} Kg.

Student Evaluations



Student Evaluations

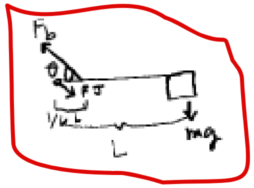


Student C

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

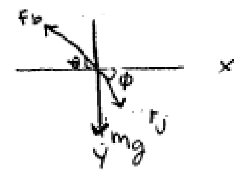
Physics
- Forces as interaction
- Newton's 2nd Law

Math
- Trig
- Algebra
- Calculus



Knowns
$\theta = 80°$
$V_m = 3.76\ L = 3760\ mL$

force Diagram



Target: F_j = Force of joint

Approach: Use Forces
$\Sigma F_x = 0$
$\Sigma F_y = 0$
Use Torque
$\Sigma \tau = 0$

$\rho = \frac{m}{V}$

$\rho V_m = m$

assume density of milk is similar to water. $= 1\ g/cm^3 = 1\ g/mL = .001\ kg/mL$

$\Sigma F_x = 0$

$F_{jx} - F_{bx} = 0$

$\Sigma F_y = 0$

$F_{by} - mg - F_{jy} = 0$

$\Sigma \tau = 0$ (joint is pivot point)

$F_{by}(1/6 L) - mgL = 0$

$F_b \cos\theta = F_{bx}$
$F_b \sin\theta = F_{by}$
$F_{jy} = F_j \sin\phi$
$F_{jx} = F_j \cos\phi$
$F_j^2 = F_{jx}^2 + F_{jy}^2$

equation		unknown
$F_j^2 = F_{jx}^2 + F_{jy}^2$	1	F_{jx} 1, F_{jy} 2
$F_{by} - mg - F_{jy} = 0$	2	F_{by} 3
$F_{jx} - F_{bx} = 0$	3	F_{bx} 4
$F_{by}(1/6 L) - mgL = 0$	4	
$F_{bx} = F_b \cos\theta$	5	F_b 5
$F_{by} = F_b \sin\theta$	6	

$F_b = \frac{F_{by}}{\sin\theta}$

$\frac{F_{by}}{\sin\theta} \cos\theta = F_{bx}$

$6mg = F_{by}$

$F_{jx} = \frac{6mg}{\sin\theta} \cos\theta$

$F_{jy} = F_{by} - mg = 6mg - mg$

$F_j = \sqrt{\left(\frac{6mg}{\sin\theta}\cos\theta\right)^2 + (6mg - mg)^2}$

Plug in ρV for m

$F_j = \sqrt{\left(\frac{6\rho V g}{\sin\theta}\cos\theta\right)^2 + (5\rho V g)^2}$

$= \sqrt{\left(\frac{6(.001)3760 g}{\sin 80}\cos 80\right)^2 + (5(.001)(3760 g))^2} =$

43.3 N

units $\left(\frac{kg}{mL} \cdot mL \cdot \frac{m}{s^2}\right)^2 + \left(\frac{kg}{mL} \cdot m \cdot \frac{m}{s^2}\right)^2 = \sqrt{\frac{kg^2 m^2}{s^4}} = kg\,m/s^2 = N$ ✓

43.3 N is the amount needed to lift and ≈ 4.3 kg object straight off the ground this is reasonable

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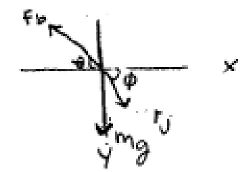

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Plug in ρV for m

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units $\left(\frac{kg}{mL} \cdot mL \frac{m}{s^2}\right)^2 + \left(\frac{kg}{mL} \cdot m \cdot \frac{m}{s^2}\right)^2 = \sqrt{\frac{kg^2 m^2}{s^4}} = kg\,m/s^2 = N$ ✓

43.3N is the amount needed to lift and ≈ 4.3 kg object straight up the ground. this is reasonable

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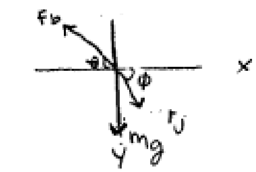

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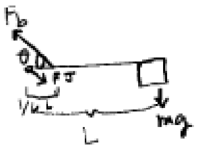

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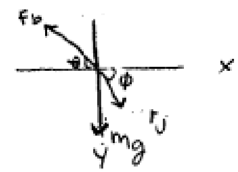

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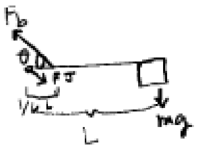

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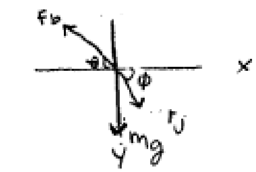

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$F_{jy} = F_{by} - mg = 6mg - mg$

$F_j = \sqrt{\left(\frac{6mg}{\sin\theta}\cos\theta\right)^2 + (6mg - mg)^2}$

correct

Plug in ρV for m

$F_j = \sqrt{\left(\frac{6\rho V g}{\sin\theta}\cos\theta\right)^2 + (5\rho V g)^2}$

$= \sqrt{\left(\frac{6(.001)3760 g}{\sin 80}\cos 80\right)^2 + (5(.001)(3760)g)^2} =$ 43.3 N

wrong

units $\left(\frac{kg}{mL} \cdot mL \frac{m}{s^2}\right)^2 + \left(\frac{kg}{mL} \cdot m \cdot \frac{m}{s^2}\right)^2 = \sqrt{\frac{kg^2 m^2}{s^4}} = kg\,m/s^2 = N$ ✓

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Coaching this student

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Physics
- Forces as interactions
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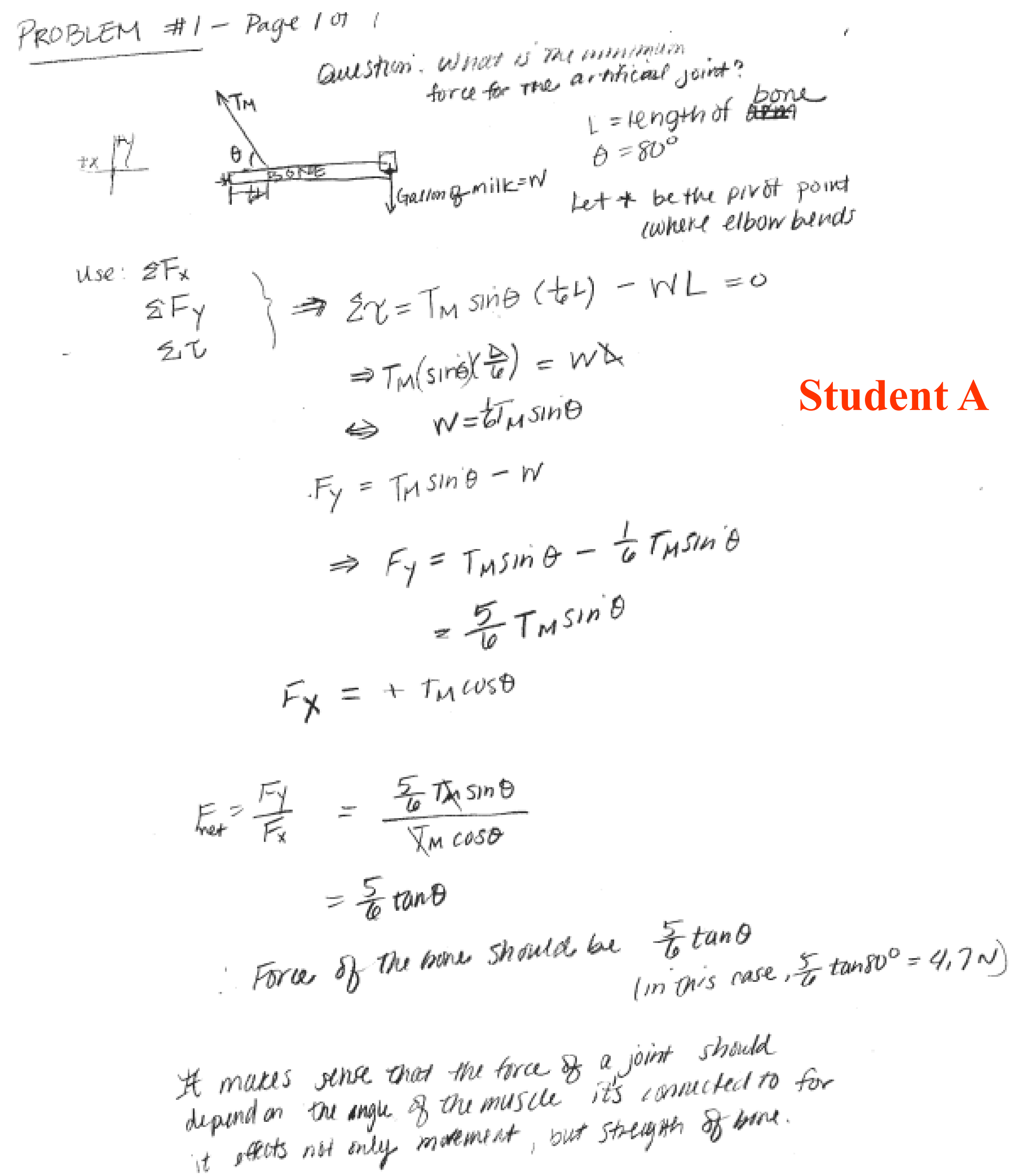

PROBLEM #1 – Page 1 of 1

Question: What is the minimum force for the artificial joint?

L = length of bone

θ = 80°

Let * be the pivot point where elbow bends

use: ΣF_x, ΣF_y, $\Sigma \tau$ $\Rightarrow \Sigma\tau = T_M \sin\theta\,(\tfrac{1}{6}L) - WL = 0$

$\Rightarrow T_M(\sin\theta)(\tfrac{\Delta}{6}) = W\Delta$

$\Leftrightarrow W = \tfrac{1}{6}T_M \sin\theta$

$F_y = T_M \sin\theta - W$

$\Rightarrow F_y = T_M \sin\theta - \tfrac{1}{6} T_M \sin\theta$

$= \tfrac{5}{6} T_M \sin\theta$

$F_x = + T_M \cos\theta$

$F_{net} = \frac{F_y}{F_x} = \frac{\frac{5}{6} T_M \sin\theta}{T_M \cos\theta}$

$= \tfrac{5}{6}\tan\theta$

∴ Force of the bone should be $\tfrac{5}{6}\tan\theta$ (in this case, $\tfrac{5}{6}\tan 80° = 4.7\,N$)

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

Good visualization except incomplete interaction of elbow with the upper arm.

Organizational Framework

- **Visualization**
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Physics

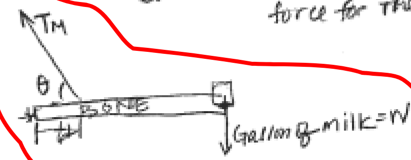
- **Forces as interactions**
- **Newton's 2nd Law**

Math

- **Trig**
- **Algebra**
- **Calculus**

PROBLEM #1 – Page 1 of 1

Question: What is the minimum force for the artificial joint?



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$\Rightarrow T_M(\sin\theta)(\tfrac{L}{6}) = WL$

$\Leftrightarrow W = \tfrac{1}{6}T_M \sin\theta$

$F_y = T_M \sin\theta - W$

$\Rightarrow F_y = T_M \sin\theta - \tfrac{1}{6} T_M \sin\theta$

$= \tfrac{5}{6} T_M \sin\theta$

$F_x = + T_M \cos\theta$

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

Good question statement.
Should have alerted the student
to the need for forces at the joint

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PROBLEM #1 – Page 1 of 1

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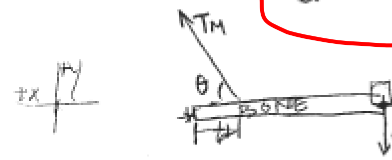

L = length of bone
$\theta = 80°$

Let * be the pivot point where elbow bends

Student A

Use: ΣF_x, ΣF_y, $\Sigma \tau$ $\Rightarrow$ $\Sigma\tau = T_M \sin\theta\,(\tfrac{1}{6}L) - WL = 0$

$\Rightarrow T_M(\sin\theta)(\tfrac{L}{6}) = WL$

$\Leftrightarrow W = \tfrac{1}{6}T_M \sin\theta$

$F_y = T_M \sin\theta - W$

$\Rightarrow F_y = T_M \sin\theta - \tfrac{1}{6}T_M \sin\theta$

$= \tfrac{5}{6}T_M \sin\theta$

$F_x = +T_M \cos\theta$

$F_{net} = \frac{F_y}{F_x} = \frac{\frac{5}{6}T_M \sin\theta}{T_M \cos\theta}$

$= \tfrac{5}{6}\tan\theta$

∴ Force of the bone should be $\tfrac{5}{6}\tan\theta$ (in this case, $\tfrac{5}{6}\tan 80° = 4.7\,N$)

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Recognized the need to sum forces and torque. No statement about relation to acceleration (Newton's 2nd Law) or about equilibrium.

Organizational Framework

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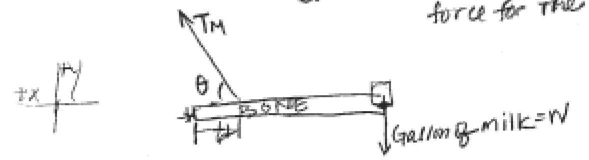

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Let ✱ be the pivot point where elbow bends

Use: ΣF_x, ΣF_y, $\Sigma \tau$

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Student A

Not clearly stated. From the drawing, the joint might be part of the system. Need to ask the student. Could account for the missing force.

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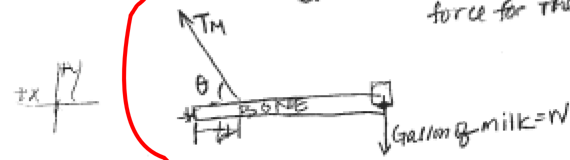

L = length of bone
θ = 80°

Let * be the pivot point (where elbow bends)

Use: ΣF_x, ΣF_y, $\Sigma \tau$

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

Could be a free-body diagram but not used to see missing elbow interaction.

Coordinate system defined but force not put on it to make missing force observable.

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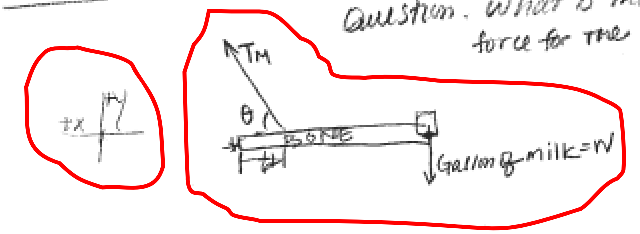

L = length of bone
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Let * be the pivot point where elbow bends

use: ΣF_x, ΣF_y, $\Sigma \tau$

$\Rightarrow \Sigma \tau = T_M \sin\theta \left(\frac{1}{6}L\right) - WL = 0$

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

Good use of symbols. Defining L even though it is not given in the problem so it can cancel out.

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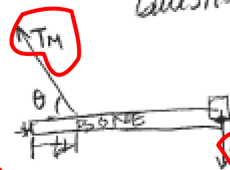

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Let * be the pivot point where elbow bends

+x, +y

Use: ΣF_x, ΣF_y, $\Sigma \tau$

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Student A

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Does not write equations for the sum of the forces or torques. This might have helped.

Recovers for torques but not forces.

Strange idea of sum = ratio.

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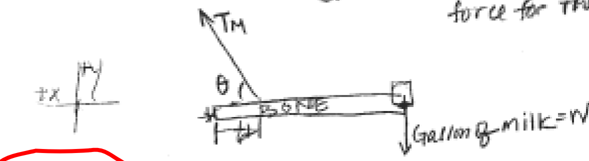

L = length of bone

$\theta = 80°$

Let * be the pivot point where elbow bends

Use: ΣF_x, ΣF_y, $\Sigma \tau$

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Student A

No goal oriented plan. Calculates F_{net}, described as force of the bone (on what?) Seems to have lost track of the question. Needs an early definition of the target.

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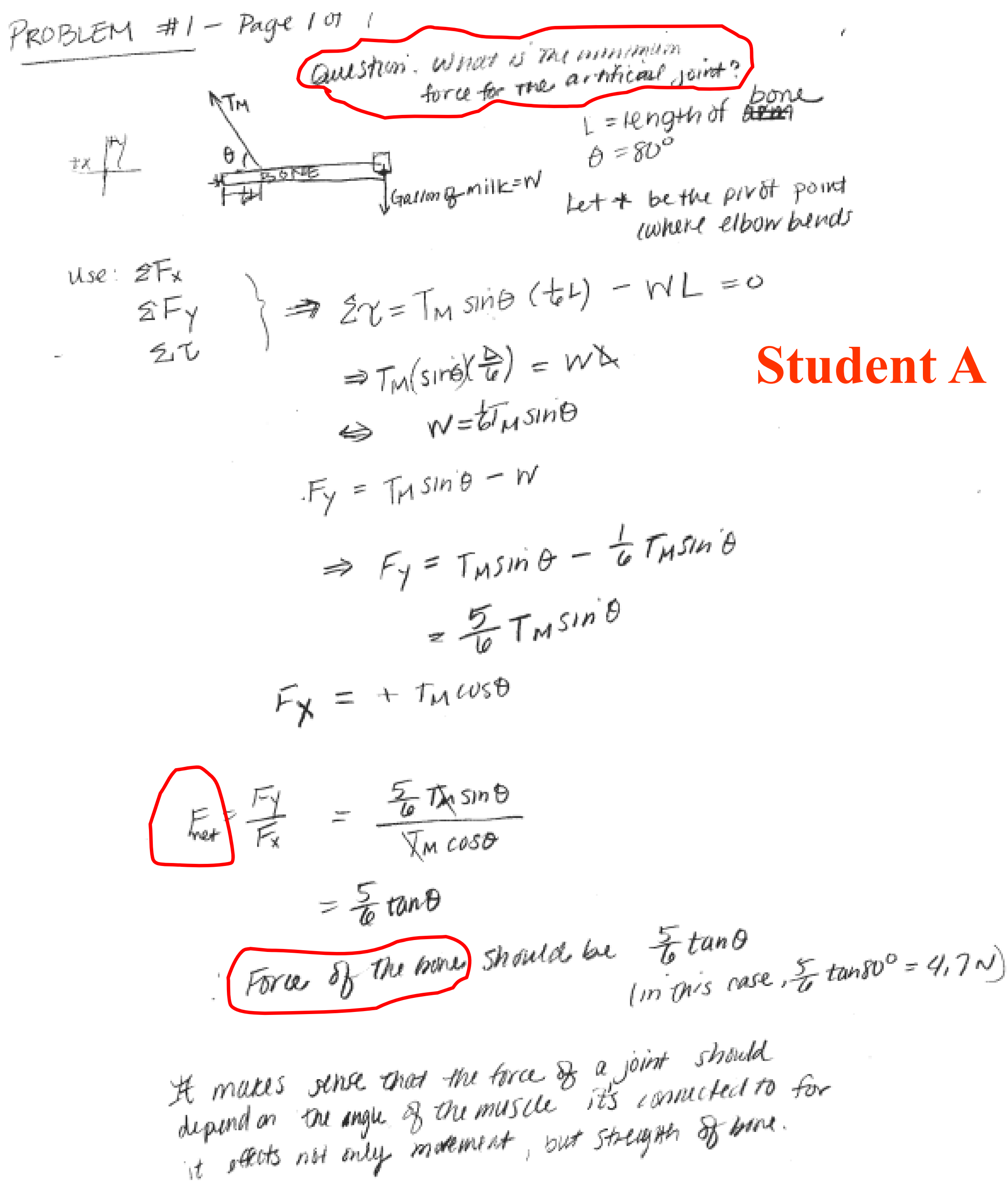

PROBLEM #1 – Page 1 of 1

Question: What is the minimum force for the artificial joint?

T_M, θ, BONE, $\frac{1}{6}L$, Gallon of milk = W, +x, +y

L = length of bone

$\theta = 80°$

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Use: ΣF_x, ΣF_y, $\Sigma \tau$

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

The math is a logical progression but without a goal.

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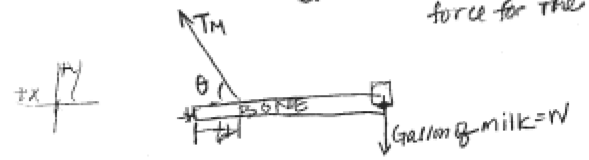

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Student A

There is a conclusion but not an answer to the question.

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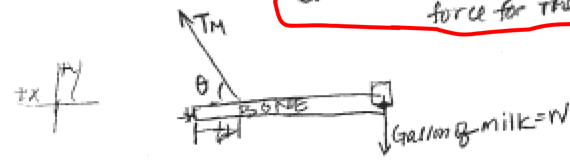

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Use: ΣF_x, ΣF_y, $\Sigma \tau$

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It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

The evaluation is confused representing the lack of connection of the calculation to a goal.

There is no checking of the units which would have revealed a difficulty

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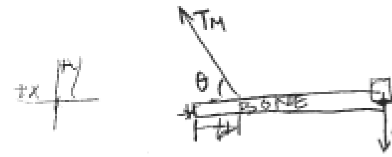

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Student A

Joint does not interact with upper arm if it is part of the system. Bone does not interact with joint if it is not.

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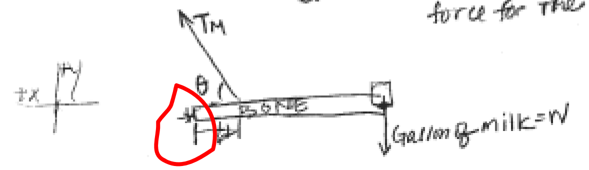

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Student A

No use of 2^{nd} Law for forces although it is used for torques.

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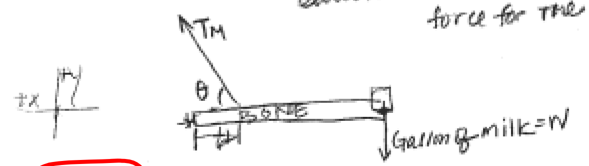

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Student A

Trigonometry is fine.

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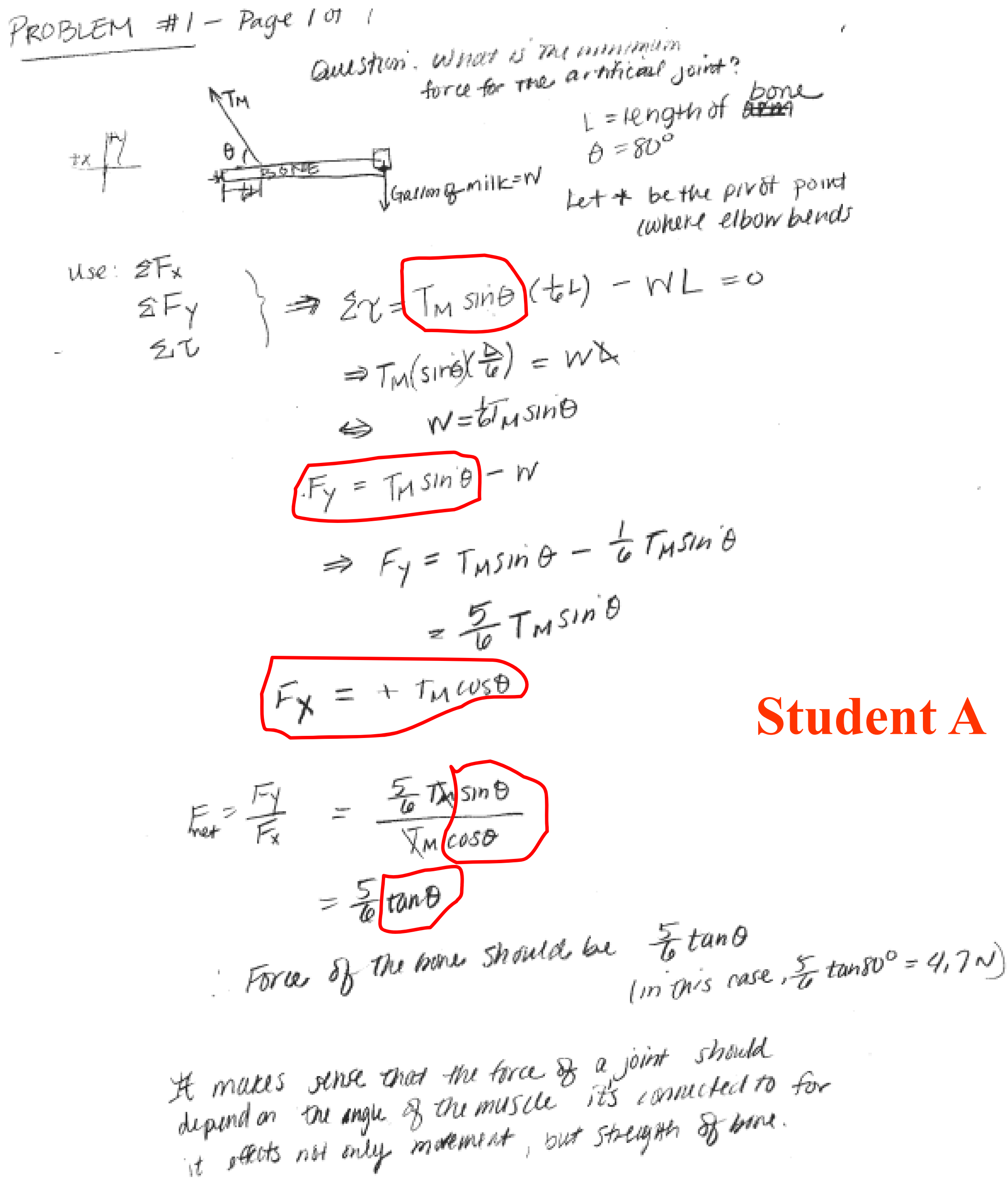
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$\theta = 80^\circ$

Let * be the pivot point where elbow bends

Gallon of milk = W

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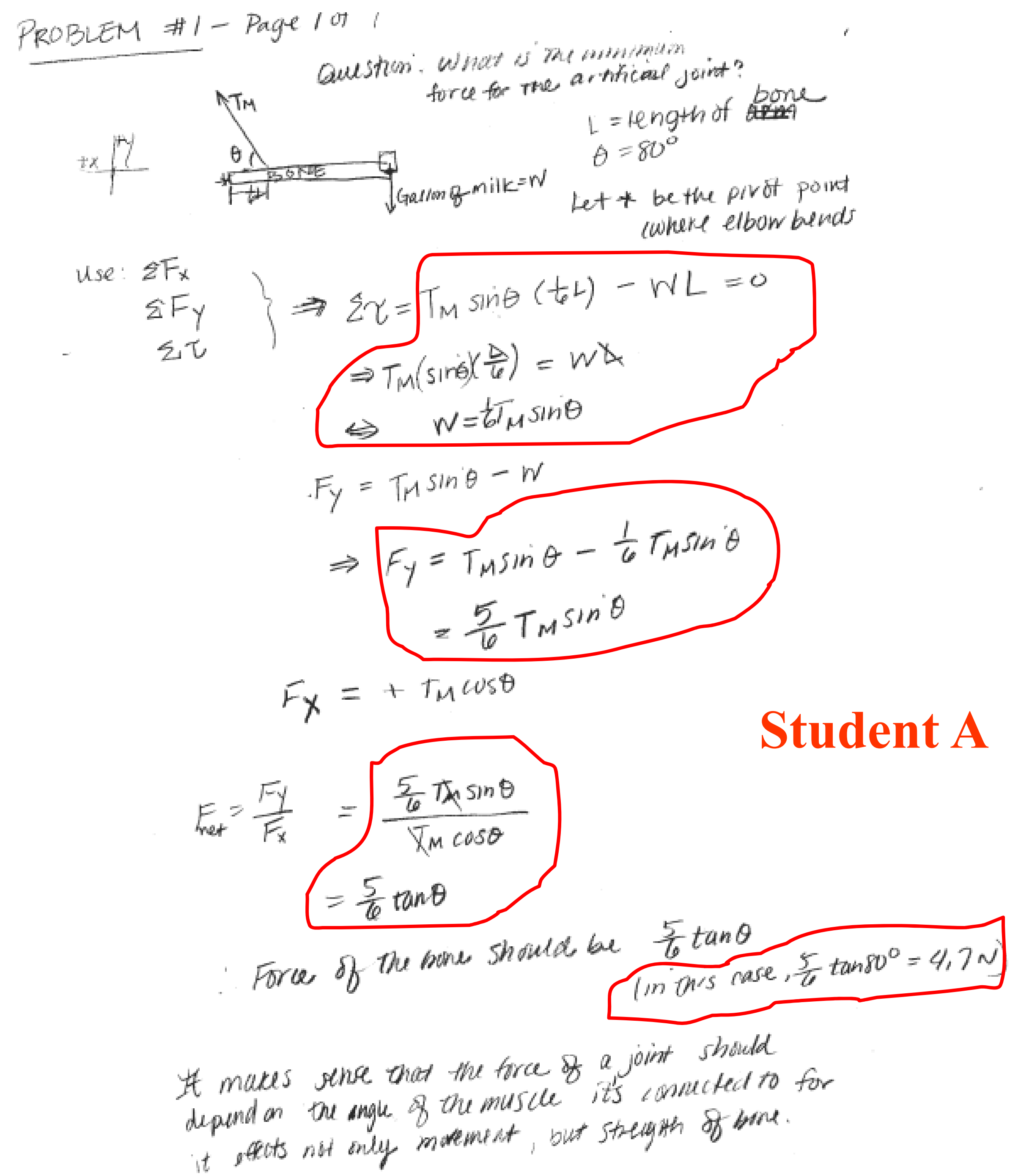
Student A

Algebra is fine except when dealing with units.



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Use: ΣF_x, ΣF_y, $\Sigma \tau$

$\Rightarrow \Sigma\tau = T_M \sin\theta\,(\frac{1}{6}L) - WL = 0$

$\Rightarrow T_M(\sin\theta)(\frac{L}{6}) = WL$

$\Leftrightarrow W = \frac{1}{6}T_M \sin\theta$

$F_y = T_M \sin\theta - W$

$\Rightarrow F_y = T_M \sin\theta - \frac{1}{6}T_M \sin\theta$

$= \frac{5}{6}T_M \sin\theta$

$F_x = +T_M\cos\theta$

$F_{net} = \frac{F_y}{F_x} = \frac{\frac{5}{6}T_M\sin\theta}{T_M\cos\theta}$

$= \frac{5}{6}\tan\theta$

∴ Force of the bone should be $\frac{5}{6}\tan\theta$ (in this case, $\frac{5}{6}\tan 80° = 4.7$ N)

It makes sense that the force of a joint should depend on the angle of the muscle it's connected to for it affects not only movement, but strength of bone.

Student A

Organizational Framework
- Visualization
- Question statement
- Appropriate physics approach
- System selected
- Useful diagram
- Useful symbols
- Useful equations
- Goal oriented plan
- Logical math
- A conclusion
- Evaluation

Physics
- Forces as interactions
- Newton's 2nd Law

Math
- Trig
- Algebra
- Calculus

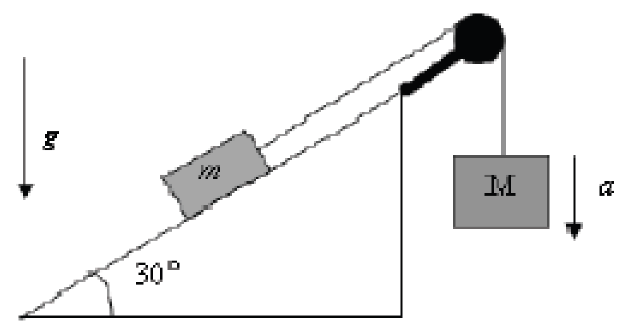
Some problems given on tests that do not help most students from learn either problem solving or physics concepts.

A block of mass m = 3 kg and a block of unknown mass M are connected by a massless rope over a frictionless pulley, as shown below. The kinetic frictional coefficient between the block m and the inclined plane is μ_k = 0.17. The plane makes an angle 30° with horizontal. The acceleration, a, of the block M is 1 m/s^2 downward.

(a) Draw free-body diagrams for both masses. [5 points]

(b) Find the tension in the rope. [5 points]

(c) If the block M drops by 0.5 m, how much work, W, is done on the block m by the tension in the rope? [15 points]



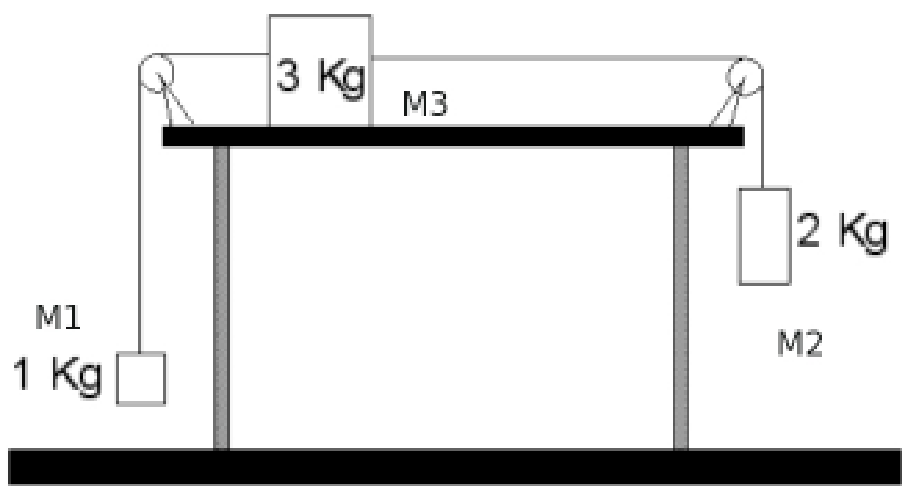
The system of three blocks shown is released from rest. The connecting strings are massless, the pulleys ideal and massless, and there is no friction between the 3kg block and the table.

a) **At the instant M_3 is moving at speed v, how far *d* has it moved from the point where it was released from rest? (answer in terms of M_1, M_2, M_3, *g* and *v*.) [10 pts]**

b) **At the instant the 3 kg block is moving with a speed of 0.8 m/s, how far, d, has it moved from the point where it was released from rest? [5 pts]**

c) **From the instant when the system was released from rest, to the instant when the 1 kg block has risen a height h, which statement (1, 2 or 3) is true for the three-block system? (1) The total mechanical energy of the system increases. (2) The total potential energy of the system increases. (3) The net work done on the system by the tension forces is 0. [5pts]**

d) **Now suppose the table is rough and has a coefficient of kinetic friction $\mu k = 0.1$. What is the speed, v, of the 3 kg block after the 2 kg block drops by 0.5 m? (Assume again that the system is released from rest.) [5pts]**



Highest Rated Goals

Scale of 1 to 5

Goals: Biology Majors Course 2003

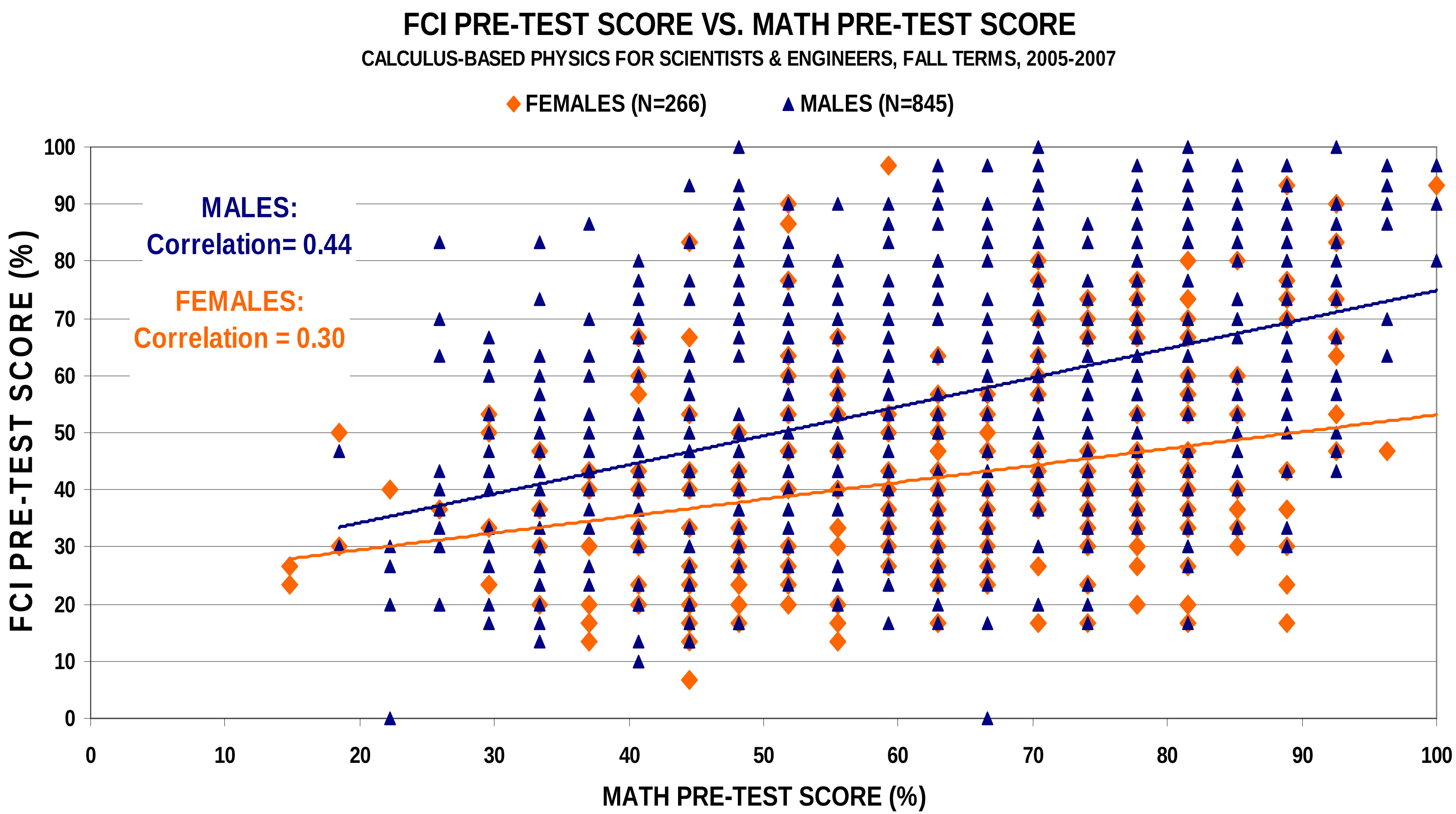
- **4.9** Basic principles behind all physics
- **4.4** General qualitative problem solving skills ←
- **4.3** *Use biological examples of physical principles*
- **4.2** *Overcome misconceptions about physical world*
- **4.1** General quantitative problem solving skills ←
- **4.0** *Real world application of mathematical concepts and techniques* ←

Goals: Calculus-based Course (88% engineering majors) 1993

- **4.5** Basic principles behind all physics
- **4.5** General qualitative problem solving skills ←
- **4.4** General quantitative problem solving skills ←
- **4.2** Apply physics topics covered to new situations ←
- **4.2** *Use with confidence*

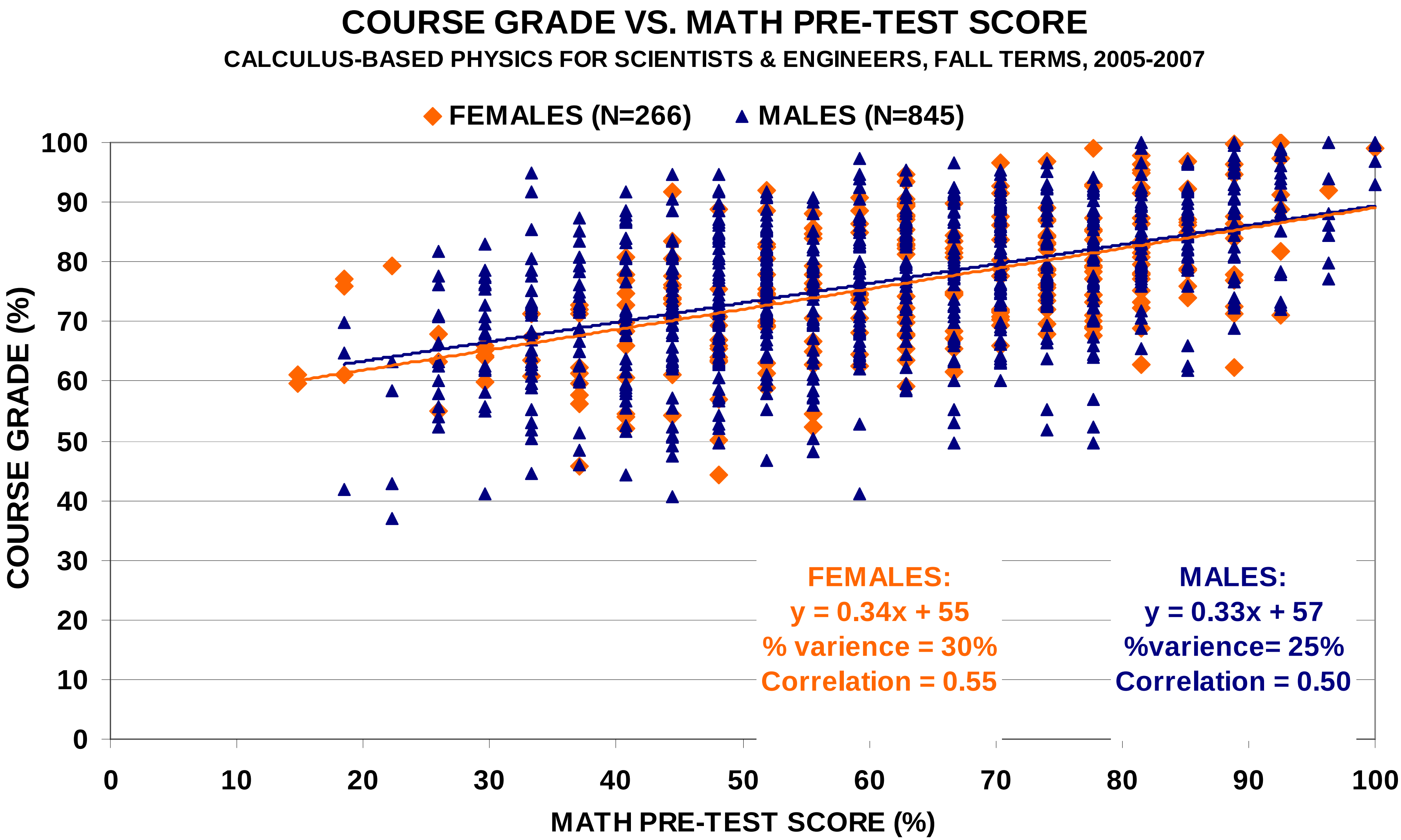
Goals: Algebra-based Course (24 different majors) 1987

- **4.7** Basic principles behind all physics
- **4.2** General qualitative problem solving skills ←
- **4.2** *Overcome misconceptions about physical world*
- **4.0** General quantitative problem solving skills ←
- **4.0** Apply physics topics covered to new situations ←



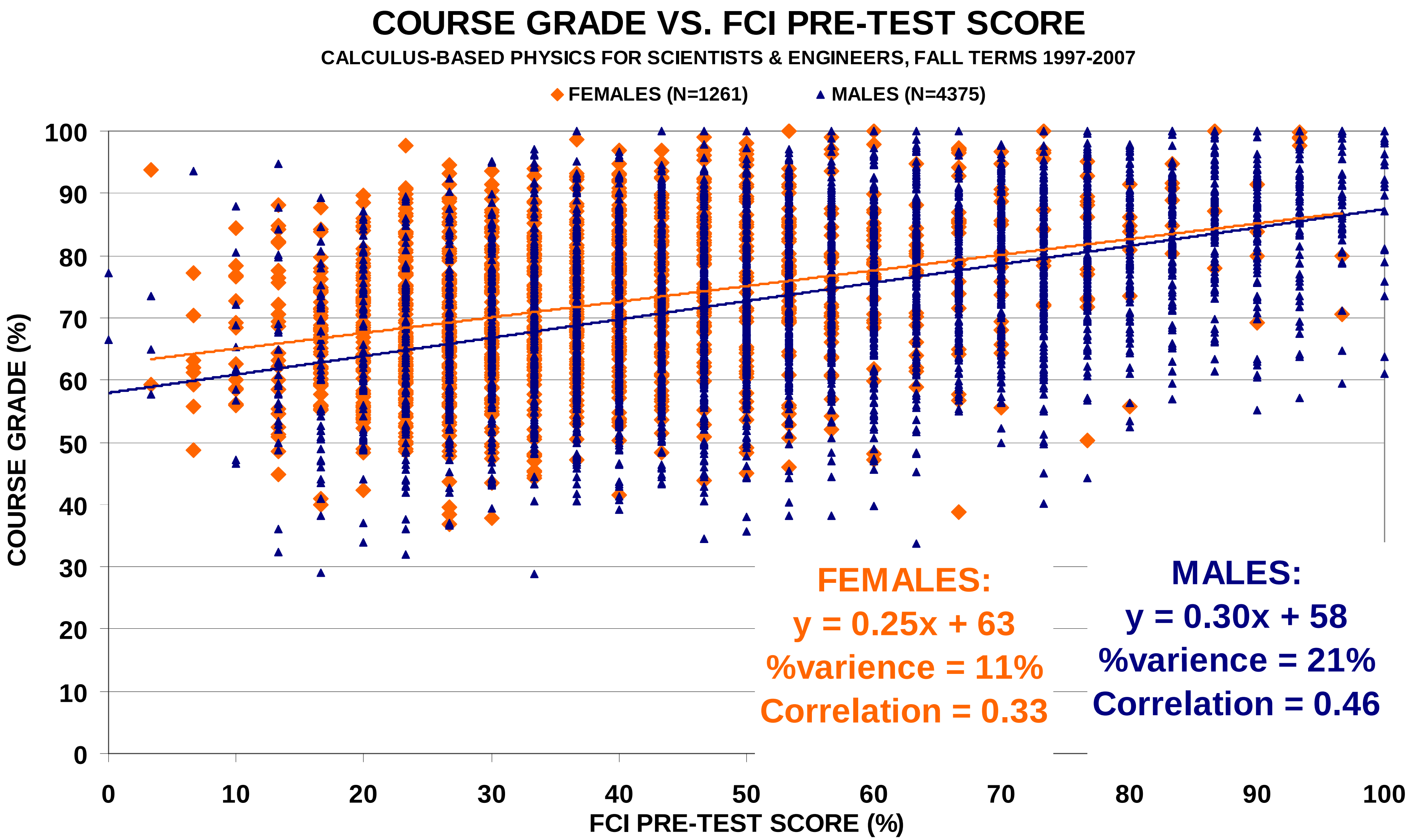
The concept test is correlated with the math skills test.

Can a Math Skills Test be used as a placement test?

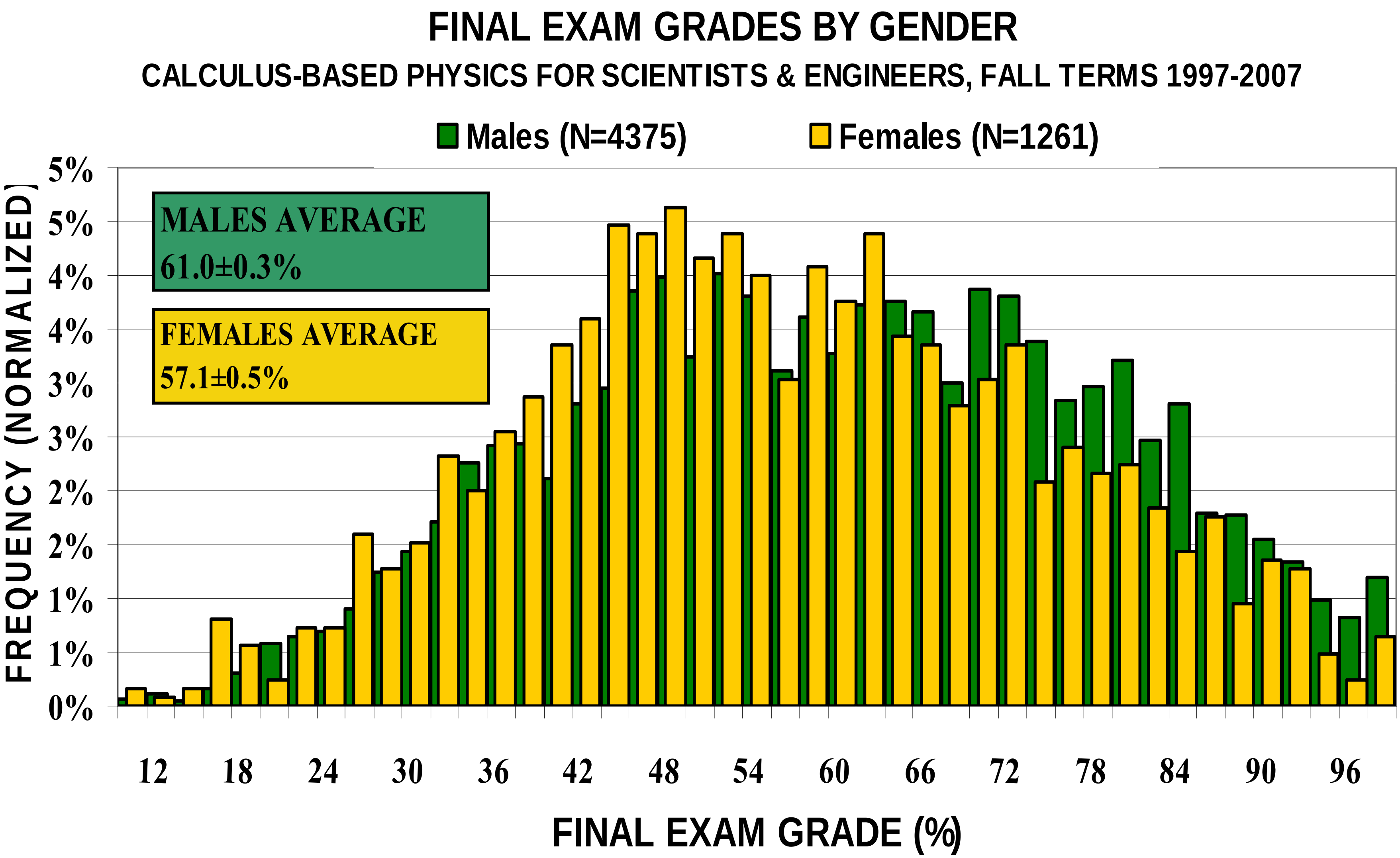


The Math Skills Test is not a good predictor of performance.

Can the FCI be used as a placement test?



The FCI is not a good predictor of performance.


FINAL EXAM GRADES BY GENDER
CALCULUS-BASED PHYSICS FOR SCIENTISTS & ENGINEERS, FALL TERMS 1997-2007
Males (N=4375)
Females (N=1261)
MALES AVERAGE
61.0±0.3%
FEMALES AVERAGE
57.1±0.5%
FREQUENCY (NORMALIZED)
5%
5%
4%
4%
3%
3%
2%
2%
1%
1%
0%
12
18
24
30
36
42
48
54
60
66
72
78
84
90
96
FINAL EXAM GRADE (%)

Math Diagnostic Test

Powers of ten

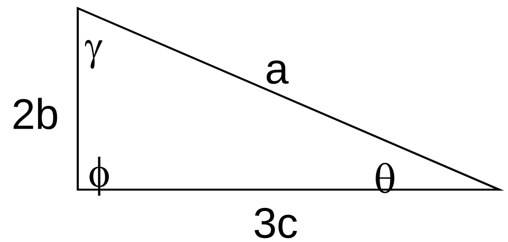
$$\frac{4\times10^{-3}}{10^{-4}} = ?$$

(a) 4×10^{-7} [10-20%] (b) $4\times10^{-3/4}$ (c) 4 [20-28%]

(d) 40 [51-63%] (e) 4×10^{7}

Triangles

For this right triangle, cos θ = ?



(a) 2b/3c (b) a/3c (c) 2b/a [7-16%]

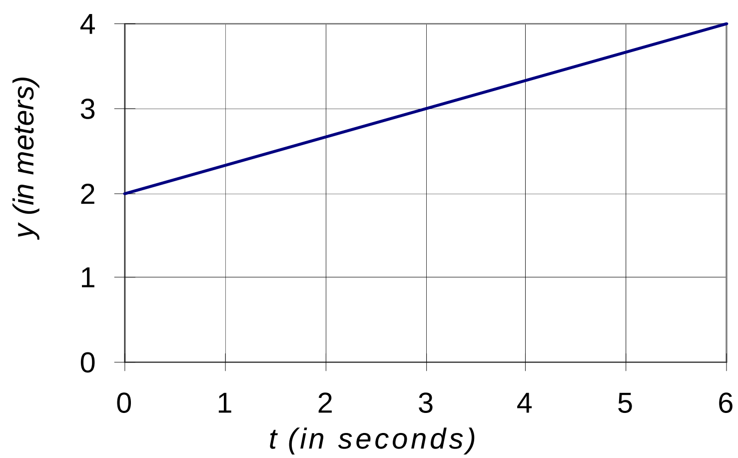
(d) 3c/a [69-89%] (e) a/2b

Graphs

The slope of the curve pictured is equal to:

(a) 0 m/s ***(b) 1/3 m/s [85-96%]*** (c) 2 m/s(d)

(d) 3 m/s [4-12%] (e) 6 m/s



Algebra

Solve for **a in the equation $a^2x + cy = t$**

(a) $\pm\sqrt{t - cy - x}$ ***(b)*** $\pm\sqrt{\dfrac{t - cy}{x}}$ ***[95-99%]*** (c) $\pm\dfrac{1}{a}\sqrt{t - cy}$

(d) $\dfrac{t - cy}{2x}$ (e) $(cy - t)(cy + t)$

Solve for **y in the equation** $\mathbf{\dfrac{ax + b}{cy + d} = f}$

(a) $\dfrac{ax + b - df}{cf} = y$ ***[49-72%]*** (b) $\dfrac{ax + b}{f + d}$ (c) $\dfrac{ax + b}{d}\left(\dfrac{1}{cf}\right)$

(d) $\dfrac{ax + b}{cf + d}$ (e) $\dfrac{1}{c}\left(\dfrac{f}{ax + b} - d\right)$ [15-34%]

Simultaneous Equations If you know **at = b** and **cx + dt = f** and the values of **a, b, c, d** and **f**, but you don't know the value of **t**, solve for the value of **x**.

(a) $\frac{f + dt}{c}$ (b) $\frac{b + f}{c(a + d)}$ ***(c)*** $\frac{f}{c} - \frac{db}{ac}$ ***[65-88%]***

(d) $\frac{f}{c} - \frac{db}{a}$ (e) $\frac{b}{a}$

If you know $\frac{b}{2}y^2 - cd^2 = 0$, **ax+y=d** and the values of **a**, **b**, **c** and **d** but you don't know the value of **y**, solve for the value of **x**.

(a) $\frac{y - d}{a}$ ***(b)*** $\frac{d}{a}\left(1 \pm \sqrt{\frac{2c}{b}}\right)$ ***[22-40%]*** (c) $\frac{d}{a} \pm \frac{1}{a}\sqrt{\frac{2cd}{b}}$ [31-45%]

(d) $\frac{b}{2}(d - ax)^2 - cd^2$ [9-28%] (e) $\frac{d}{a} - \frac{2cd^2}{ab}$

Derivatives

If $\mathbf{z = ax^3 + bx + c}$, then $\dfrac{\mathbf{dz}}{\mathbf{dx}} = \;\;?$

(a) $ax^2 + b$ (b) $a + b + c$ (c) $3ax^2 + 2b$

(d) $3ax^2 + b + c$ ***(e) $3ax^2 + b$ [73-93%]***

If $\mathbf{z = ae^{bt}}$, where **a** and **b** are not functions of **t**, then $\dfrac{\mathbf{dz}}{\mathbf{dt}} = ?$

(a) bz [4-15%] (b) ae^b [7-27%] (c) az

(d) abe^t [39-58%] (e) abe^b [6-21%]

Anti-Derivatives

If $\frac{\mathbf{dx}}{\mathbf{dt}}$ = **$5at^3 + b$**, where **a** and **b** are constants, then **x** =?

(a) $15at^2$ [7-19%] ***(b)*** $\frac{5}{4}at^4 + bt + c$ ***[60-88%]*** (c) $\frac{5}{4}at^4 + b$

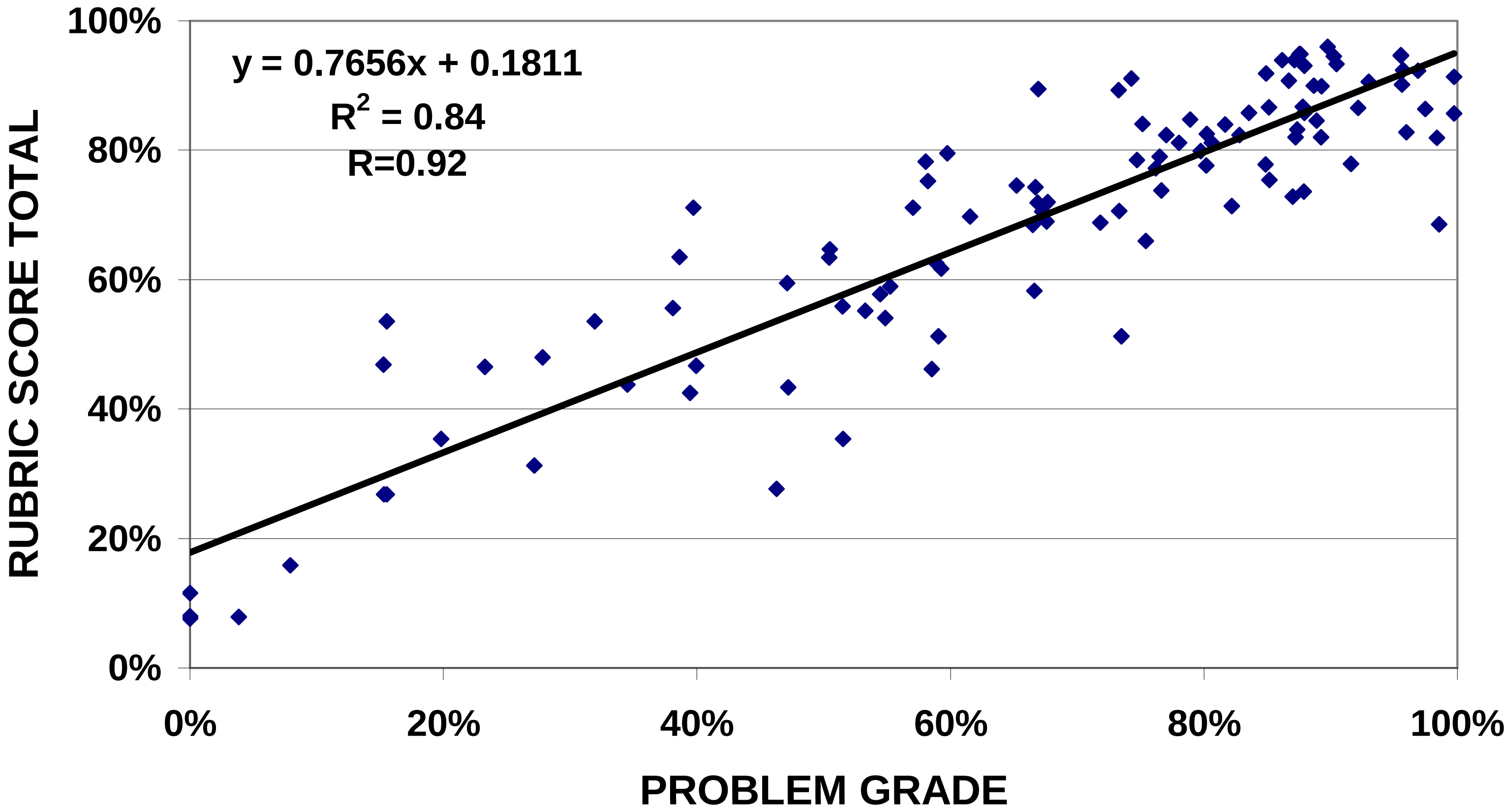
(d) $5at^2$ (e) $\frac{5}{4}at^4$

If $\frac{\mathbf{dz}}{\mathbf{dt}}$ = **$-ab^2 \sin(b^2 t)$**, where **a** and **b** are constants, then **z** = ?

(a) $2ab\cos(t) + k$ (b) $-2ab\sin(b^2 t) + k$ (c) $-2ab\sin(bt) + k$

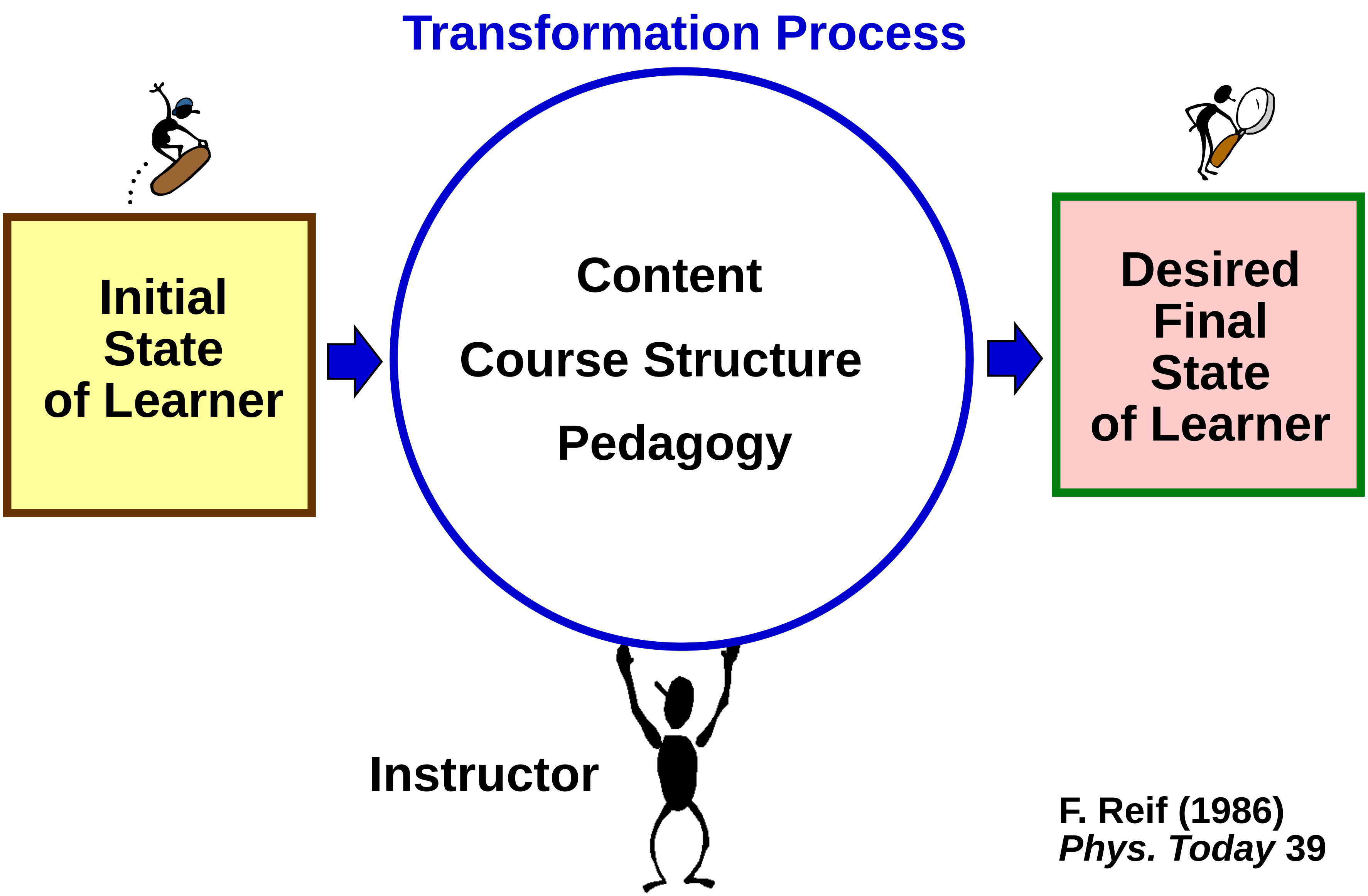
(d) $a\cos(b^2 t) + k$ [33-63%] (e) $-2ab\cos(bt) + k$ [17-30%]

RUBRIC SCORE VS. PROBLEM GRADE
TEST 1 PROBLEM 2 (SECTION 2, N=110)



The Teaching Process – A Physicist View

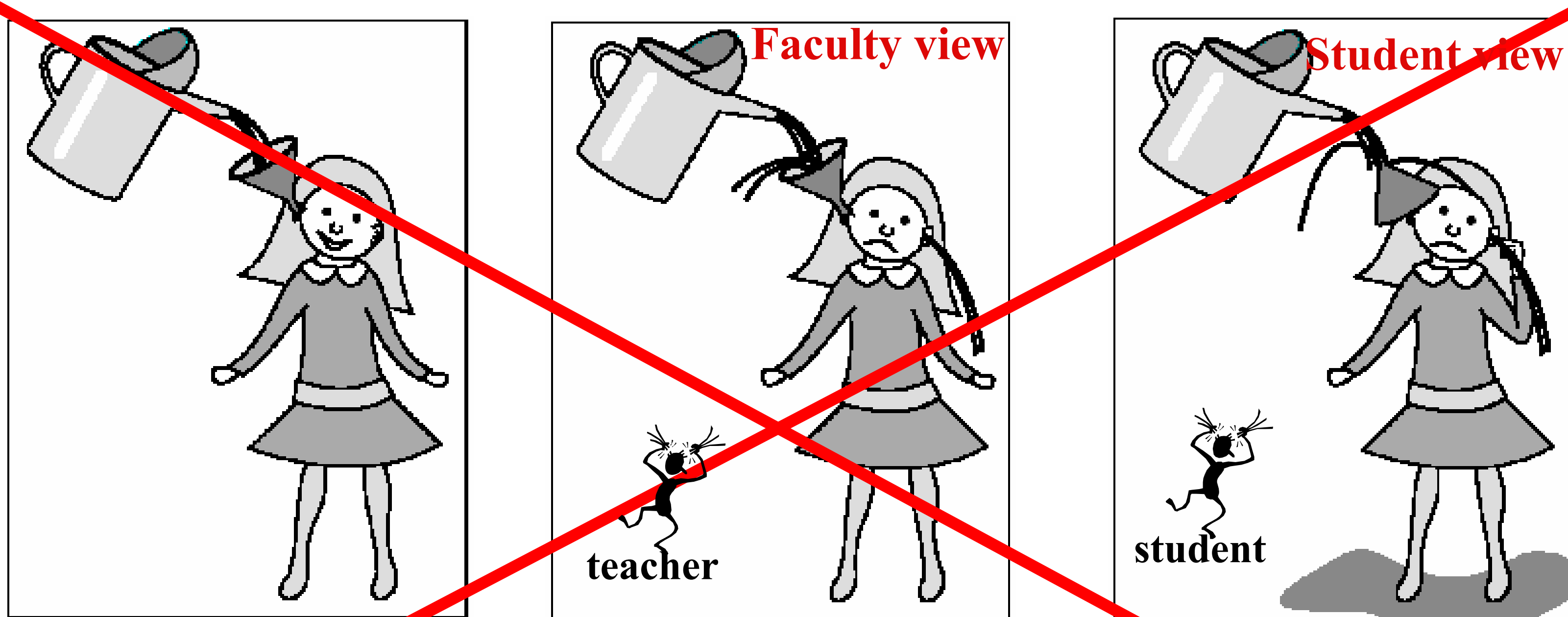
<final | T | initial>



F. Reif (1986)
Phys. Today 39

The Teaching Process
The Clear Explanation Misconception

Common Source of Frustration of Faculty, TAs, Students, & Administrators



Instructor pours knowledge into students by explaining things clearly.

Little knowledge is retained.

Student's Fault

Impedance mismatch between student and instructor.

Instructor's Fault

Learning is much more complicated

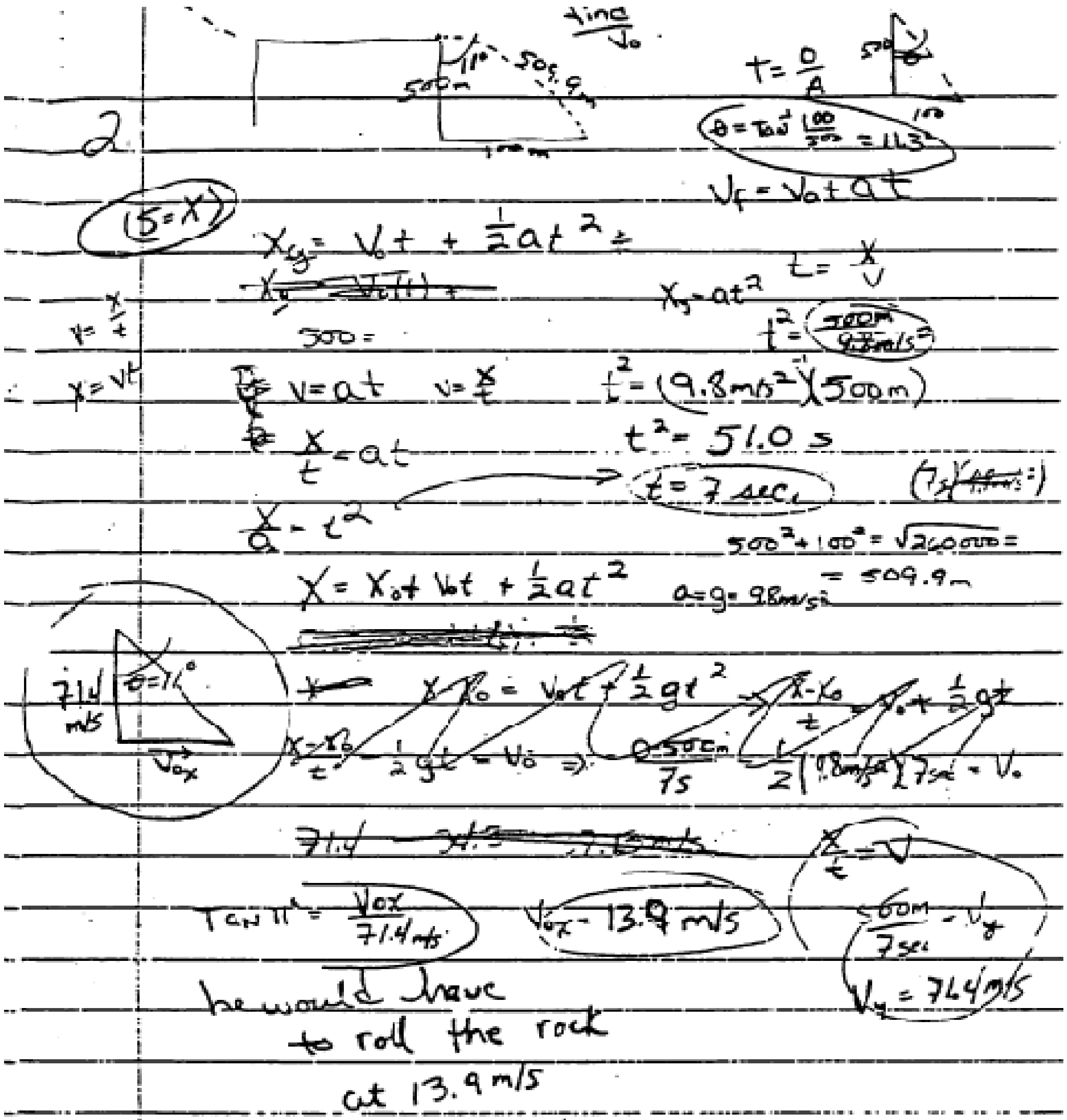
Leonard et. al. (1999). Concept-Based Problem Solving.

An Appropriate Problem

Your task is to design an artificial joint to replace arthritic elbow joints. After healing, the patient should be able to hold at least a gallon of milk while the lower arm is horizontal. The biceps muscle is attached to the bone at the distance 1/6 of the bone length from the elbow joint, and makes an angle of 80^o with the horizontal bone. How strong should you design the artificial joint if you can assume the weight of the bone is negligible.

Gives a motivation – allows some students to access their mental connections.
Gives a realistic situation – allows some students to visualize the situation.
Does not give a picture – students must practice visualization.
Uses the character "you" – allows some students to visualize the situation.
Requires decisions – students practice decision making.

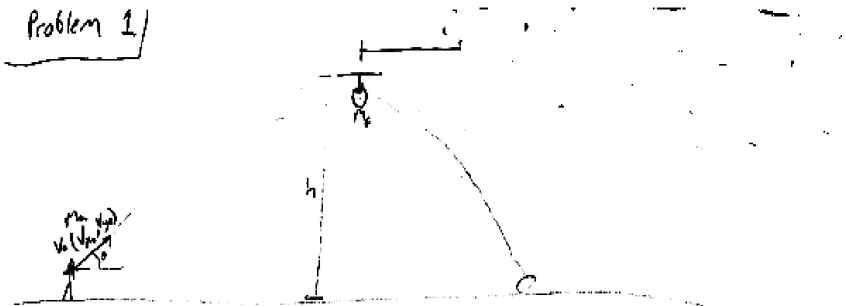
The result of students "natural" problem solving inclinations



Circled work by evaluators

Desired Student Solution

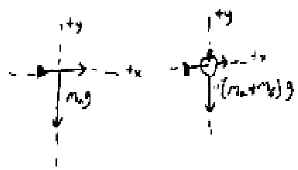
Problem 1



Question: how far away from the tree does the fruit and arrow combination land?

Approach: use conservation of momentum and kinematics
assume constant acceleration due to gravity
assume no momentum is lost in the collision
neglect wind resistance
use two intervals; from the time the arrow leaves the bow until just before it hits the fruit and just after it hits the fruit until they hit the ground
~~the system is the earth and arrow for the first part, and the fruit and arrow combination and the earth for the second part.~~

Diagram



known: h, m_a, m_f, v_0, θ
unknown: d

Qualitative relationships:

$V_{x0} = V_0 \cos\theta \quad P_f = (m_a + m_f) V_{xf}$

$h = \frac{1}{2} g t^2 \Rightarrow \frac{2h}{g} = t^2, \sqrt{\frac{2h}{g}} = t$

$d = V_{xf} t$

$P_i = P_f \quad \Rightarrow \quad m_a V_{x0} = (m_a + m_f) V_{xf} \Rightarrow V_{xf} = \frac{m_a}{m_a + m_f} V_{x0}$

$P_i = m_a V_{x0}$

Target: d

Plan the Solution; unknown: d

$d = V_{xf} t$ — V_{xf}, t

$V_{xf} = \frac{m_a}{m_a + m_f} V_{x0}$ — V_{x0}

$V_{x0} = V_0 \cos\theta$

$t = \sqrt{\frac{2h}{g}}$

$$\boxed{d = \frac{m_a}{m_a + m_f} V_0 \cos\theta \sqrt{\frac{2h}{g}}}$$

Check units!

$m = \frac{kg}{kg} \frac{m}{s} \sqrt{\frac{m}{m/s^2}} \quad \nearrow \sqrt{s^2}$

$m = \left(\frac{m}{s}\right) s$

$m = m \Rightarrow ok$

is the answer complete?
yes, the distance was found in terms of the requested values

is the answer reasonable?
yes, the units check out ok and d will be smaller than h due to conservation of momentum

is the answer correctly stated?
yes, it is in units of distance, meters

Quiz 1 – kinematics and forces (calc based for engineers & physical science

A block of mass m = 2.5 kg starts from rest and slides down a frictionless ramp that makes an angle of θ= 25º with respect to the horizontal floor. The block slides a distance *d* down the ramp to reach the bottom. At the bottom of the ramp, the speed of the block is measured to be *v* = 12 m/s.

(a) Draw a diagram, labeling θ and d. [5 points]

(b) What is the acceleration of the block, in terms of g? [5 points]

(c) What is the distance *d*, in meters? [15 points]

From a test

Better

A 2.5 kg block starts from rest and slides down a frictionless ramp at 25º to the horizontal floor. At the bottom of the ramp, the speed of the block is measured to be 12 m/s.

(a) Draw a diagram, with appropriate labeling. [5 points]

(b) What is the acceleration of the block, in terms of g? [5 points]

(c) What is the distance the block slides, in meters? [15 points]

Allow students to practice making simple decisions.

Better

A 2.5 kg block starts from rest and slides down a frictionless ramp at 25º to the horizontal floor. At the bottom of the ramp, the speed of the block is measured to be 12 m/s. How far did the block slide?

Allow students to practice making decisions about structuring their solution and connecting physics concepts.

Better

A 2.5 kg block starts from the top and slides down a slippery ramp reaching 12 m/s at the bottom. How long is the ramp? The ramp is at 25º to the horizontal floor .

Allow students to practice making assumptions.

Original

A block of mass m = 2.5 kg starts from rest and slides down a frictionless ramp that makes an angle of θ= 25º with respect to the horizontal floor. The block slides a distance *d* down the ramp to reach the bottom. At the bottom of the ramp, the speed of the block is measured to be *v* = 12 m/s.

(a) Draw a diagram, labeling θ and d. [5 points]

(b) What is the acceleration of the block, in terms of g? [5 points]

(c) What is the distance *d*, in meters? [15 points]

Better

A 2.5 kg block starts from the top and slides down a slippery ramp reaching 12 m/s at the bottom. How long is the ramp? The ramp is at 25º to the horizontal floor .

Better

You have been asked to design a simple system to transport boxes from one part of a warehouse to another. The design has boxes placed on the top of the ramp so that they slide to their destination. A box slides easily because the ramp is covered with rollers. Your job is to calculate the maximum length of the ramp if the heaviest box is 25 kg and the ramp is at 5.0º to the horizontal. To be safe, no box should go faster than 3.0 m/s when it reaches the end of the ramp.

Context Rich Problem

Allows student decisions.
Practice making assumptions.
Connects to student reality.
Has a motivation (why should I care?).

Beginning Context-Rich Problem

You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Gives a motivation – allows some students to access their mental connections.
Gives a realistic situation – allows some students to visualize the situation.
Does not give a picture – students must practice visualization.
Uses the character "you" –more easily connects to student's mental framework.

Decisions must be made

Decisions

You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

What is happening? – you need a picture.

What is the question? – it is not in the last line.

What quantities are important and what should I name them? – choose symbols.

What physics is important? – difference between average and instantaneous.

What assumptions are necessary? – should friction be ignored?

Is all the information necessary? – the angle? The vertical drop? The time?

Stop surface feature pattern matching

You are working with an insurance company to help investigate a tragic accident. At the scene, you see a road running straight down a hill at 10° to the horizontal. At the bottom of the hill, the road widens into a small, level parking lot overlooking a cliff. The cliff has a vertical drop of 400 feet to the horizontal ground below where a car is wrecked 30 feet from the base of the cliff. A witness claims that the car was parked on the hill and began coasting down the road, taking about 3 seconds to get down the hill. Your boss drops a stone from the edge of the cliff and, from the sound of it hitting the ground below, determines that it takes 5.0 seconds to fall to the bottom. You are told to calculate the car's average acceleration coming down the hill based on the statement of the witness and the other facts in the case. Obviously, your boss suspects foul play.

Not an inclined plane problem

Not a projectile motion problem

Same as this textbook question except students must engage with the content

A block starts from rest and accelerates for 3.0 seconds. It then goes 30 ft. in 5.0 seconds at a constant velocity.

a. What was the final velocity of the block?
b. What was the acceleration of the block?

Competent Problem Solving

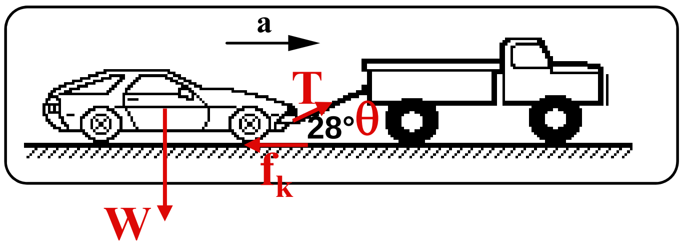
Step | **Bridge**

1. Focus on the Problem

Translate the words into a useful image of the situation. Decide on what you know and what you don't. Decide on the question.

Know T_{max}, θ, μ, W

What is a_{max}

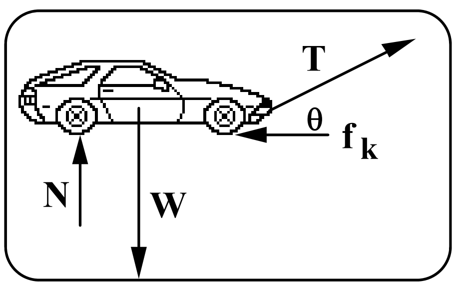


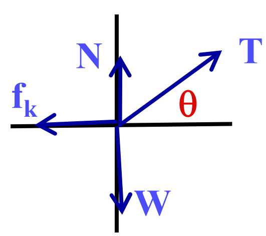
Identify an approach to the problem.

Relate forces on car to acceleration using Newton's Second Law

2. Describe the Physics

Translate the image into a physics representation of the problem (e.g., idealized diagram, symbols for important quantities).





Assemble mathematical tools (equations).

$$\sum F = ma$$

$$f_k = \mu N$$

$$W = mg$$

3. Plan a Solution

Step **Bridge**

3. Plan a Solution

Decide on the order of using a mathematical representation of the problem.

Find a:

[1] $\sum F_x = ma_x$

Find $\sum F_x$:

[2] $\sum F_x = T_x - f_k$

Find T_x

[3] $T_x = T\cos\theta$

⋮

Outline the mathematical solution steps.

Solve[3] for T_x and put into [2].

Solve[2] for $\sum F_x$ and put into [1].

Solve[1] for a_x.

4. Execute the Plan

Translate the plan into a series of appropriate mathematical actions.

$$\Sigma F_x = T\cos\theta - \mu(W - T\sin\theta)$$

$$(W/g)a_x = T\cos\theta - \mu(W - T\sin\theta)$$

$$a_x = (T\cos\theta - \mu(W - T\sin\theta))\, g/W$$

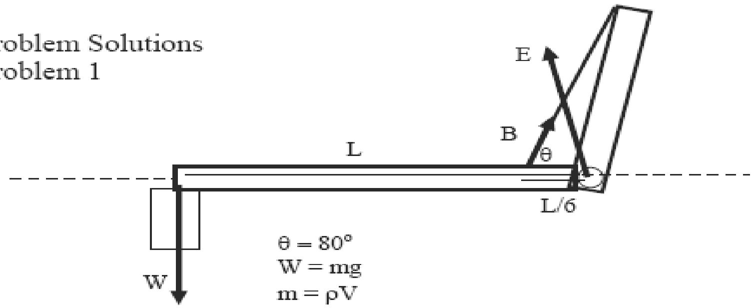
Check units of algebraic solution.

$$\frac{\left[\frac{m}{s^2}\right][N]}{[N]} - \left[\frac{m}{s^2}\right] = \left[\frac{m}{s^2}\right] \quad \text{OK}$$

5. Evaluate the Solution

Model of a Solution

Problem Solutions
Problem 1



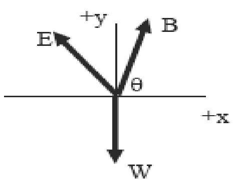
$\theta = 80°$
$W = mg$
$m = \rho V$
$V = 3.76$ liters

What is the distance magnitude of the force on the elbow joint?

Approach:

The arm is in equilibrium
Sum of the horizontal forces = 0.
Sum of the vertical forces = 0.
Sum of the torques = 0.
Choose the pivot point at the elbow joint.
Assume density of milk = density of water = $1 g/cm^3$

Force Diagram of the arm bone



$$\sum F_x = B\cos\theta - E_x = 0$$
$$\sum F_y = E_y + B\sin\theta - W = 0$$

$$E_x^2 + E_x^2 = E^2$$

Torques: $\sum \tau = WL - B_{\perp}\frac{L}{6} = 0$

$$B_{\perp} = B\sin\theta$$

Target: E

		Unknowns
Find E		E
$E_x^2 + E_y^2 = E^2$	[1]	E_x , E_y
Find E_x		
$B\cos\theta - E_x = 0$	[2]	B
Find B		
$WL - B\sin\theta\frac{L}{6} = 0$		

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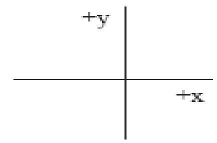


$W - B\sin\theta\frac{1}{6} = 0$ [3] W

Find W
$W = \rho V g$ [4]

Find E_y
$E_y + B\sin\theta - W = 0$ [5]

5 unknown, 5 equations OK

Solve [5] for E_y
$$E_y + B\sin\theta - W = 0$$
$$E_y = W - B\sin\theta \quad \text{put into [1]}$$
$$E_x^2 + E_y^2 = E^2$$
$$E_x^2 + (W - B\sin\theta)^2 = E^2 \quad [1]$$

Put [4] into [1] and [3]
$$E_x^2 + (\rho V g - B\sin\theta)^2 = E^2 \quad [1]$$

Solve [3] for B
$$\rho V g - B\sin\theta\frac{1}{6} = 0$$
$$\rho V g = B\sin\theta\frac{1}{6}$$
$$\frac{6\rho V g}{\sin\theta} = B \quad \text{put into [1] and [2]}$$

$$E_x^2 + \left(\rho V g - \frac{6\rho V g}{\sin\theta}\sin\theta\right)^2 = E^2 \quad [1]$$

Solve [2] for E_x
$$\frac{6\rho V g}{\sin\theta}\cos\theta - E_x = 0$$
$$\frac{6\rho V g}{\sin\theta}\cos\theta = E_x \quad \text{put into [1]}$$

Solve [1] for E
$$\left(\frac{6\rho V g}{\sin\theta}\cos\theta\right)^2 + \left(\rho V g - \frac{6\rho V g}{\sin\theta}\sin\theta\right)^2 = E^2$$
$$\left(\frac{6\rho V g}{\sin\theta}\cos\theta\right)^2 + (\rho V g - 6\rho V g)^2 = E^2$$
$$\rho V g\sqrt{\left(\frac{6}{\tan\theta}\right)^2 + (-5)^2} = E$$

Page 2

$$\rho V g\sqrt{\frac{36}{\tan^2\theta}+25}=E$$

Check units

$$\left[\frac{kg}{m^3}\right]\left[m^3\right]\left[\frac{m}{s^2}\right]=[kg]\left[\frac{m}{s^2}\right]=[N]=[E]$$

Units are correct since newtons is a unit of force and E is the force of the elbow joint on the arm.

Put in numbers

$$\left(1\frac{g}{cm^3}\right)(3.67L)\left(9.8\frac{m}{s^2}\right)\sqrt{\frac{36}{\tan^2 80^o}+25}=E$$

$$\left(1\frac{g}{cm^3}\right)(3.67L)\left(9.8\frac{m}{s^2}\right)\sqrt{\frac{36}{\tan^2 80^o}+25}=E$$

$$184\left(\frac{g}{cm^3}\right)(L)\left(\frac{m}{s^2}\right)=E$$

$$184\left(\frac{1kg}{1000g}\right)\left(\frac{g}{cm^3}\right)\left(\frac{1000cm^3}{L}\right)(L)\left(\frac{m}{s^2}\right)=E$$

$$184(kg)\left(\frac{m}{s^2}\right)=E$$

$$\boxed{184N=E}$$

By the 3rd law this is equal to the force of the arm on the elbow joint.

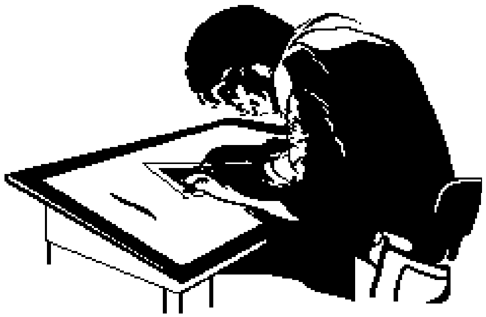
The milk has a weight of 38 N. This means the force on the elbow joint is 5 times greater than the weight of the milk. This seems large but not impossible.

Evaluating the answer equation:

The force on the elbow is greater if the volume of the milk is greater. This is reasonable since then the milk would exert a larger force on the arm.
The force on the elbow is greater if the density of the milk is greater. This is reasonable since then the milk would exert a larger force on the arm.

Student Difficulties Solving Problems

- Lack of an Organizational Framework
  - Random walk (knowledge fragments + math)
  - Situation specific (memorized pattern)
- Physics Misknowledge
  - Incomplete (lack of a concept)
  - Misunderstanding (weak misknowledge)
  - Misconceptions (strong misknowledge)



- No Understanding of Range of Applicability – Mathematics & Physics
  - Always true
  - True under a broad range of well-defined circumstances
  - True in very special cases
  - Never true
- Lack of internal monitoring skills (reflection on what they did and why, asking skeptical questions about their actions)

Students must be taught a problem solving framework that addresses these *explicitly*