Foundations of International Macroeconomics[1]

Workbook[2]

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Chapter 4 Solutions

1. We follow closely the steps outlined in section 4.3.2. Equations (12) and (17) in the book remain unaltered. So does steady-state budget constraint (11), so that $\hat{\overline{Z}} = \psi_{\rm L}\hat{w}$, as before. In the traded goods sector the zero profit condition can be written as

$$f\left(\frac{k_{\rm T}}{E_{\rm T}}\right) \equiv F\left(\frac{k_{\rm T}}{E_{\rm T}}, 1\right) = r\frac{k_{\rm T}}{E_{\rm T}} + \frac{w}{E_{\rm T}} \tag{1}$$

where $k \equiv K/L$. Log-differentiation of eq. (1) yields,

$$f'\left(\frac{k_{\rm T}}{E_{\rm T}}\right)\frac{k_{\rm T}}{E_{\rm T}}\left(\hat{k}_{\rm T} - \hat{E}_{\rm T}\right) = r\frac{k_{\rm T}}{E_{\rm T}}\left(\hat{k}_{\rm T} - \hat{E}_{\rm T}\right) + \frac{w}{E_{\rm T}}\left(\hat{w} - \hat{E}_{\rm T}\right). \tag{2}$$

The first order condition for capital in the traded goods is $f'(k_{\rm T}/E_{\rm T}) = r$. Substitution of this equality into eq. (2) above reduces it to

$$\hat{w} = \hat{E}_{\rm T}. \tag{3}$$

Since $\hat{\overline{Z}} = \psi_{\rm L}\hat{w}$, the last equality implies

$$\hat{\overline{Z}} = \psi_{\rm L}\hat{E}_{\rm T}. \tag{4}$$

Log differentiating the zero-profit condition for nontradables, $pA_{\rm N}g(k_{\rm N}) = rk_{\rm N} + w$, with $A_{\rm N}$ and r constant, we obtain $\hat{p} = \mu_{\rm LN}\hat{w}$ [recall the displayed

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equation following eq. (8) in Chapter 4]. Since $\mu_{\text{LN}} = 1 - \alpha$ in section 4.3.2, eq. (3) above implies

$$\hat{p} = (1 - \alpha)\hat{E}_{\text{T}}$$

Substituting eqs. (4) and (5) above into equation (17) of Chapter 4, we express the change in steady state nontradables consumption as

$$\hat{\bar{C}}_{\text{N}} = \{\psi_{\text{L}} - (1 - \alpha)\,[\gamma\theta + (1 - \gamma)]\}\,\hat{E}_{\text{T}}.$$

Similarly we can solve for $\hat{\bar{L}}_{\text{N}}$ as,

$$\hat{\bar{L}}_{\text{N}} = \{\psi_{\text{L}} - (1 - \alpha)\,[\gamma\theta + (1 - \gamma)] - \alpha\}\,\hat{E}_{\text{T}}. \qquad (6)$$

The sign of $\hat{\bar{L}}_{\text{N}}$ is still ambiguous, as discussed on p. 224 in the book. Indeed, eq. (6) here looks precisely like eq. (18) of Chapter 4, with the only difference that $\hat{E}_{\text{T}}$ appears in place of $\hat{A}_{\text{T}}/\mu_{\text{LT}}$. Thus, the assumption of "Harrod-neutral" instead of Hicks-neutral technological change makes no difference for the likelihood that faster productivity growth causes a labor exodus from the tradables sector.

2. This is an intricate problem, so it is useful to start with an exceedingly simple case and gradually generalize it. First, suppose that consumption preferences are of the Leontief type (with fixed consumption proportions of the two goods) and that technologies also are Leontief. Since nontradables are relatively labor intensive, a rise in r lowers their relative price in terms of tradables, p, and lowers the tradables wage, w. In the special case we're now considering, the sole effect of that change is to flatten the GNP line (by lowering p) and to shift its vertical intercept: upward if $w'(r)L + \bar{Q} > 0$, downward in the opposite case (the first case occurring, despite the inequality $w'(r) < 0$, if financial wealth is sufficiently high). The income expansion path stays put due to the assumed Leontief preferences. How can we be certain the GDP line doesn't shift? Equations (7) and (10) of Chapter 4 allow us to write the GDP line as:

$$\bar{Y}_{\text{T}} = \left\{\frac{A_{\text{T}} f[k_{\text{T}}(r)]}{A_{\text{N}} g[k_{\text{N}}(r)]}\right\} \bar{Y}_{\text{N}} + A_{\text{T}} f[k_{\text{T}}(r)]L. \qquad (7)$$

Since $k'_{\text{T}}(r) = k'_{\text{N}}(r) = 0$ with a Leontief technology, the interest-rate change doesn't shift the GDP line. The diagram therefore predicts that in the case of an upward shift in the GNP line (a positive national income effect), consumption of tradables and nontradables expands as the economy reduces its capital stock and acquires the corresponding amount of foreign assets. In the case of a downward shift in the GNP line's vertical intercept, the effects are opposite. (Henceforth assume an upward shift to avoid a proliferation of cases.) Now, as a second, slightly more general case, allow substitution in consumption between tradables and nontradables. In that case a rise in r, by lowering p, makes the income expansion path rotate clockwise from the origin. This movement, other things the same, induces more consumption of nontradables, less of tradables, and a bigger shift out of domestic capital into foreign assets. Finally, what if we now relax the assumption of Leontief technologies? It is easy to see from eq. (7) above that while the change in the GDP line's slope is ambiguous, its vertical and horizontal intercepts both decrease (because $k'_{\text{T}}(r) < 0$ and $k'_{\text{N}}(r) < 0$ now). Thus, the GDP line shifts inward. This last shift has no effect on the economy's consumption point, which remains at the intersection of the GNP line and income expansion path. However, the shift implies a reduction in tradables output, and hence a further fall in domestic capital and rise in net foreign assets $\bar{B}$. Because production is now becoming less capital-intensive in both sectors but output (equals consumption) of nontradables is the same, labor must flow out of tradables, reinforcing the economy's disinvestment of capital. Accordingly, more of the economy's tradables consumption is financed out of foreign investment income rather than domestic production. The three-step solution offered here corresponds to three distinct effects of the rise in r: a budget-constraint effect, a consumption switching effect, and a capital-intensity effect.

3. (There is a typo in the statement of this question. The second to last sentence should ask for the change in B_{t+1}/P_t, not the change in B_t/P_t.) Divide both sides of eq. (30) in the book by P_t. Next, manipulating the resulting equality as is done on p. 230, section 4.4.1.4, we obtain an alternative version of eq. (27):

$$\sum_{s=t}^{\infty} R^{\mathrm{C}}_{t,s} C_s = \frac{(1+r^{\mathrm{C}}_t)B_t}{P_{t-1}} + \sum_{s=t}^{\infty} R^{\mathrm{C}}_{t,s} \frac{Y_{\mathrm{T},s} + p_s Y_{\mathrm{N},s} - I_s - G_s}{P_s}. \tag{8}$$

The Euler equation for C, when period utility is isoelastic with intertemporal substitution elasticity σ, is eq. (33) in the chapter. Through recursive substitution, optimal C_s follows:

$$C_s = (R^{\mathrm{C}}_{t,s})^{-\sigma} \beta^{\sigma(s-t)} C_t.$$

Now substitute the preceding equation into (8) to obtain the equation for optimal real consumption:

$$C_t = \frac{\frac{(1+r^{\mathrm{C}}_t)B_t}{P_{t-1}} + \sum_{s=t}^{\infty} R^{\mathrm{C}}_{t,s} \frac{Y_{\mathrm{T},s}+p_s Y_{\mathrm{N},s} - I_s - G_s}{P_s}}{\sum_{s=t}^{\infty} \beta^{\sigma(s-t)} \left(R^{\mathrm{C}}_{t,s}\right)^{1-\sigma}}.$$

The current account identity is

$$\begin{aligned} CA_t &= B_{t+1} - B_t = rB_t + Y_{\mathrm{T},t} + p_t Y_{\mathrm{N},t} - P_t C_t - I_t - G_t \\ &= rB_t + Z_t - P_t C_t, \end{aligned}$$

where $Z_t \equiv Y_{\mathrm{T},t} + p_t Y_{\mathrm{N},t} - I_t - G_t$. Divide the current account identity by P_t (to put it in terms of real consumption). The result is

$$\frac{B_{t+1}}{P_t} = \frac{(1+r)B_t}{P_t} + \frac{Z_t}{P_t} - C_t = \frac{(1+r)P_{t-1}B_t}{P_t P_{t-1}} + \frac{Z_t}{P_t} - C_t,$$

which can be rewritten as

$$\frac{B_{t+1}}{P_t} - \frac{B_t}{P_{t-1}} = \frac{r^{\mathrm{C}}_t B_t}{P_{t-1}} + \frac{Z_t}{P_t} - C_t.$$

Substitute into this the equation for consumption derived above. The result is

$$\frac{B_{t+1}}{P_t} - \frac{B_t}{P_{t-1}} = \frac{r^{\mathrm{C}}_t B_t}{P_{t-1}} + \frac{Z_t}{P_t} - \frac{\frac{(1+r^{\mathrm{C}}_t)B_t}{P_{t-1}} + \sum_{s=t}^{\infty} R^{\mathrm{C}}_{t,s} \frac{Z_s}{P_s}}{\sum_{s=t}^{\infty} \beta^{\sigma(s-t)} \left(R^{\mathrm{C}}_{t,s}\right)^{1-\sigma}}.$$

As in section 2.2.2 of the book, let us define

$$\tilde{\Gamma}_t \equiv \frac{\sum_{s=t}^{\infty}\left(R_{t,s}^{\mathrm{C}}\right)^{1-\sigma}\beta^{\sigma(s-t)}}{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}}.$$

Then we can write the last equation as

$$\begin{aligned}\frac{B_{t+1}}{P_t}-\frac{B_t}{P_{t-1}} &= \frac{r_t^{\mathrm{C}}B_t}{P_{t-1}}+\frac{Z_t}{P_t}-\frac{\frac{(1+r_t^{\mathrm{C}})B_t}{P_{t-1}}+\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}\frac{Z_s}{P_s}}{\tilde{\Gamma}_t\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}} \\ &= \frac{r_t^{\mathrm{C}}B_t}{P_{t-1}}+\frac{Z_t}{P_t}-\frac{\frac{(1+r_t^{\mathrm{C}})B_t}{P_{t-1}}+\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}\frac{Z_s}{P_s}}{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}} \\ &\quad +\left(\frac{\tilde{\Gamma}_t-1}{\tilde{\Gamma}_t}\right)\frac{\frac{(1+r_t^{\mathrm{C}})B_t}{P_{t-1}}+\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}\frac{Z_s}{P_s}}{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}}.\end{aligned}$$

Define the *permanent* level of a variable as

$$\tilde{X}_t = \frac{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}X_s}{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}}.$$

Then the last equation can be expressed as the analog of eq. (26) in section 2.2.2,

$$\frac{B_{t+1}}{P_t}-\frac{B_t}{P_{t-1}} = \left(r_t^{\mathrm{C}}-\tilde{r}_t^{\mathrm{C}}\right)\frac{B_t}{P_{t-1}}+\frac{Z_t}{P_t}-\widetilde{\left(\frac{Z}{P}\right)}_t+\left(\frac{\tilde{\Gamma}_t-1}{\tilde{\Gamma}_t}\right)\left[\frac{\tilde{r}_t^{\mathrm{C}}B_t}{P_{t-1}}+\widetilde{\left(\frac{Z}{P}\right)}_t\right],$$

where we have used the following relation (from footnote 13 on p. 78 of the book):

$$\frac{1+r_t^{\mathrm{C}}}{\sum_{s=t}^{\infty}R_{t,s}^{\mathrm{C}}} = \tilde{r}_t^{\mathrm{C}}.$$

4. (a) With a constant world interest rate r equal to $(1-\beta)/\beta$ and a constant net supply of nontradables, consumption of tradables is constant too, and equal to the level specified in eq. (10) of Chapter 2, provided we reinterpret Y, G, and I in that equation as output, government consumption, and investment of tradable goods. (The book already assumes that only tradable goods can be transformed into capital.) To see this, consider eq. (35) in Chapter 4, and note that the result claimed follows [with $r = (1-\beta)/\beta$] provided the price index P is constant over time. By eq. (20) in Chapter 4, however, P depends only on p (the price of nontradables in terms of tradables); furthermore, the individual's static first-order condition for optimal consumption is

$$\frac{\Omega_{C_{\mathrm{N}}}(C_{\mathrm{T}}, C_{\mathrm{N}})}{\Omega_{C_{\mathrm{T}}}(C_{\mathrm{T}}, C_{\mathrm{N}})} = p.$$

Equation (34) (in the present case) can be written as $P_{s+1}^{\sigma-\theta} C_{\mathrm{T},s+1} = P_s^{\sigma-\theta} C_{\mathrm{T},s}$. Since C_{N} is constant over time (at $Y_{\mathrm{N}} - G_{\mathrm{N}}$), eq. (34) therefore can be written as

$$\phi(C_{\mathrm{T},s+1}, Y_{\mathrm{N}} - G_{\mathrm{N}}) = \phi(C_{\mathrm{T},s}, Y_{\mathrm{N}} - G_{\mathrm{N}})$$

for the appropriately defined function $\phi(C_{\mathrm{T}}, C_{\mathrm{N}})$. But this last equation implies that C_{T} is constant over time, as must also be p and P. In this case, therefore, eq. (35) implies that optimal consumption of tradables is the annuity value of the net endowment of tradables after investment.

(b) The previous result no longer holds when $Y_{\mathrm{N}} - G_{\mathrm{N}}$ is subject to anticipated changes, except in the special case $\sigma = \theta$. Let's call that case the "benchmark" case; all changes in consumption and the current account in the rest of this answer are relative to the economy's path in the benchmark case. The argument in the answer to part a shows that consumption of tradables is constant until date $t+T$, when $Y_{\mathrm{N}} - G_{\mathrm{N}}$ rises permanently to $Y'_{\mathrm{N}} - G'_{\mathrm{N}}$. From date $t+T$ on consumption of tradables is constant again, after possibly jumping between dates $t+T-1$ and $t+T$. The current account is zero from date $t+T$ on. There are two cases to consider in solving for the pre-$t+T$ path (see p. 234 of the text for a discussion of the tension between intertemporal and intratemporal substitution effects):

$\sigma > \theta$. The Euler equation for C_{T} between dates $t+T-1$ and $t+T$ is

$$C_{\mathrm{T},t+T} = \left(\frac{P_{t+T-1}}{P_{t+T}}\right)^{\sigma-\theta} C_{\mathrm{T},t+T-1}; \tag{9}$$

see eq. (34) in Chapter 4. Since $Y_{\mathrm{N}} - G_{\mathrm{N}}$ rises on date $t+T$, it is reasonable to make the tentative hypothesis that P_{t+T} is below P_{t+T-1}, which implies via (9) above that $C_{\mathrm{T},t+T} > C_{\mathrm{T},t+T-1}$. In that case C_{T} is constant and below its benchmark level prior to $t+T$, as the economy runs a current account surplus. The foreign assets accumulated in this way permit a higher level of tradables consumption from $t+T$ on. Before concluding that we have solved this case we must ask whether P can ever *rise* on date $t+T$. This is impossible: a rise in P would require a (very large) upward jump in C_{T}, which would imply a violation of Euler eq. (33) in the chapter. (That equation, the Euler equation for total real consumption, implies that both C_{T} and C_{N} can rise between dates $t+T-1$ and $t+T$ only if P *falls*.)

$\sigma < \theta$. This case is a mirror image of the last one, with a current account deficit prior to date $t+T$ and C_{T} falling between dates $t+T-1$ and $t+T$ as a result of the fall in P. [Once again, eq. (33) in the chapter precludes a rise in P between dates $t+T-1$ and $t+T$.]

5. Departing from the notation in appendix 4A, let C denote the real consumption index depending on consumption of tradables and leisure, $C = \Omega(C_{\mathrm{T}}, \bar{L} - L)$. With leisure interpreted as consumption of nontradables, budget constraint (23) in the chapter can then be written as

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left[C_{\mathrm{T},s} + w_s(\bar{L} - L_s)\right] = (1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s\bar{L} - G_{\mathrm{T},s})$$

(recall that all government consumption is of tradables now). Constraint (24) therefore takes the form

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} P_s C_s = (1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s\bar{L} - G_{\mathrm{T},s}).$$

Combining this equation with Euler eq. (33) from the chapter yields a real consumption function along the lines of (29):

$$C_t = \frac{(1+r)Q_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(w_s\bar{L} - G_{\mathrm{T},s})}{P_t\sum_{s=t}^{\infty}\left[(1+r)^{s-t}\left(\frac{P_t}{P_s}\right)\right]^{\sigma-1}\beta^{\sigma(s-t)}}.$$

Applying eq. (22) in the chapter, we arrive at eq. (67).

6. (a) Think of the individual as maximizing the "profit" from undertaking education, which equals earnings less the unskilled labor income foregone. The latter is $w/(r+\pi)$, the former, $\int_T^\infty e^{-(r+\pi)t} A T^\alpha h \mathrm{d}t$, equal to the discounted value of earnings from human capital starting on the first date after graduation, T. The first-order condition is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}T}\int_T^\infty e^{-(r+\pi)t} A T^\alpha h \mathrm{d}t &= -e^{-(r+\pi)T} A T^\alpha h + \alpha \int_T^\infty e^{-(r+\pi)t} A T^{\alpha-1} h \mathrm{d}t \\ &= -e^{-(r+\pi)T} A T^\alpha h + \frac{\alpha A T^{\alpha-1} h}{r+\pi} e^{-(r+\pi)T} = 0,\end{aligned}$$

from which $T^* = \alpha/(r+\pi)$ follows. Less sharply decreasing returns to education (α closer to 1) lengthens the optimal time in school, but a higher discount rate shortens optimal schooling. The reward h doesn't affect T here because it multiplies the term in the profit function that depends on T.

(b) The first-order condition is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}T}\int_T^\infty e^{-(r+\pi)t} A T^\alpha h \mathrm{d}t &= -e^{-(r+\pi)T} A T^\alpha h + \alpha \int_T^\infty e^{-(r+\pi)t} A T^{\alpha-1} h \mathrm{d}t \\ &= -e^{-(r+\pi)T} A T^\alpha h + \frac{\alpha A T^{\alpha-1} h}{r+\pi} e^{-(r+\pi)T} = 0,\end{aligned}$$

from which $T^* = \alpha/(r+\pi)$ follows. Less sharply decreasing returns to education (α closer to 1) lengthens the optimal time in school, but a higher discount rate shortens optimal schooling. The reward h doesn't affect T here because it multiplies the term in the profit function that depends on T.

(c) Using the solution for T^*, lifetime earnings are

$$\begin{aligned}\int_{T^*}^\infty e^{-(r+\pi)t} A (T^*)^\alpha h \mathrm{d}t &= \int_{\alpha/(r+\pi)}^\infty e^{-(r+\pi)t} A \left(\frac{\alpha}{r+\pi}\right)^\alpha h \mathrm{d}t \\ &= \frac{1}{r+\pi} A \left(\frac{\alpha}{r+\pi}\right)^\alpha h e^{-\alpha}.\end{aligned}$$

Equating this to $w/(r+\pi)$ yields the answer that was claimed.

(d) Taking derivatives,

$$\frac{\mathrm{d}w}{\mathrm{d}\alpha} = -w + w + w \log\left(\frac{\alpha}{r+\pi}\right) > 0$$

provided, as assumed, $\alpha > r+\pi$. (Hint: $[\alpha/(r+\pi)]^{\alpha} = \exp\{\alpha \log[\alpha/(r+\pi)]\}$.) The other derivatives are obvious, and so is the intuition.

(e) If wages w are higher for the reasons in d, then since r is given, the relative price of nontradables p is higher. Since tradables sell at the same price worldwide, the real exchange rate is higher. This corresponds to the Harrod-Balassa-Samuelson result, as measured labor productivity in tradables is higher.

7. The representative individual in each country maximizes

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s,$$

so

$$\mathrm{d}U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\mathrm{d}C_s}{C_s} = \sum_{s=t}^{\infty} \beta^{s-t} \hat{C}_s.$$

As discussed in the chapter, the economy reaches its new steady state after one period. The change in lifetime utility relative to the baseline steady state path can therefore be written as

$$\mathrm{d}U_t = \hat{C}_t + \frac{\beta}{1-\beta} \widehat{\overline{C}}.$$

Using eq. (51) in Chapter 4, we can write this sum as

$$\mathrm{d}U_t = -(1-\beta)\hat{r} + \frac{1}{1-\beta} \widehat{\overline{C}}.$$

Equations (55) and (59) in the chapter now allow us to substitute for $\hat{r}$ and $\widehat{\overline{C}}$:

$$\begin{aligned} \mathrm{d}U_t &= \frac{\hat{v}}{2} - \left\{ \left(\frac{1}{1-\beta}\right) \left(\frac{\bar{r}}{1+\bar{r}}\right) \left[\frac{\hat{v}}{2 - \frac{1}{2} A'\left(\frac{1}{2}\right)} \right] \right\} \\ &= \left\{ 1 - \left[\frac{1}{1 - \frac{1}{4} A'\left(\frac{1}{2}\right)} \right] \right\} \frac{\hat{v}}{2}. \end{aligned}$$

[The last equality follows from $\bar{r} = (1-\beta)/\beta$.] Because, $A'(\frac{1}{2}) < 0$, however, the preceding equation shows that $\mathrm{d}U_t > 0$. The temporary increase in Foreign productivity raises Home's lifetime utility.

8. (a) If Z is expenditure in imports, then demands are $X = \gamma Z/p$, $M = (1-\gamma)Z$. The price index P therefore satisfies

$$(\gamma P/p)^{\gamma} \left[(1-\gamma)P\right]^{1-\gamma} = 1,$$

which implies the answer.

(b) The proposed identity states that the current account (in real consumption units) equals output less consumption, both measured in like units. The intertemporal constraint is

$$(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{p_s Y}{P_s} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{p_s X_s + M_s}{P_s}.$$

(c) In terms of C the problem is to maximize

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

subject to

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{p_s Y}{P_s}.$$

The first-order Euler condition therefore is the usual $C_{s+1} = (1+r)^{\sigma}\beta^{\sigma}C_s$. In the notation of Chapter 2, we find that

$$C_t = \frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \frac{p_s Y}{P_s}\right].$$

Of course, optimal $X = \gamma Z/p = \gamma PC/p$, optimal $M = (1-\gamma)PC$, and, under the assumption made in the exercise, $\vartheta = 0$.

(d) A temporary fall in p has effects on C qualitatively identical here to those of a temporary fall in Y, since $p/P = \gamma^{\gamma}(1-\gamma)^{1-\gamma}p^{1-\gamma}$ falls when p falls. The current account effect can be seen from

$$B_{t+1} - B_t = rB_t + p_t Y/P_t - C_t.$$

Consumption C is constant after the initial fall in the terms of trade, and the country runs a current account deficit while its terms of trade are temporarily depressed.

(e) In this case the consumer maximizes

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

subject to

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} P_s C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} p_s Y.$$

The first-order Euler condition for C is now

$$C_{s+1} = \left[\frac{(1+r)P_s}{P_{s+1}}\right]^{\sigma} \beta^{\sigma} C_s,$$

which is formally identical to eq. (33) in Chapter 4. The difference here, in comparison to the case analyzed in parts c and d, is that the expected future terms of trade change (occurring on date T, say) affects the path of the consumption-based real interest rate. We thus have a situation reminiscent of that in exercise 4 above. Since $r = (1-\beta)/\beta$, the Euler equation is $C_{s+1} = (P_s/P_{s+1})^{\sigma} C_s$, so that P rises and C falls discretely the day the terms of trade switch back to their initial level. The current account (measured in imports) is given by

$$B_{t+1} - B_t = rB_t + p_t Y - P_t C_t. \tag{10}$$

To understand current-account behavior, notice that the Euler equation can be written as in terms of expenditure as:

$$P_{s+1}C_{s+1} = \left(\frac{P_s}{P_{s+1}}\right)^{\sigma-1} P_sC_s.$$

When $\sigma \geq 1$, PC falls or remains unchanged (if $\sigma = 0$) when the terms of trade make their expected return up to their initial level on date T (that is, $P_TC_T \leq P_{T-1}C_{T-1}$). Since pY rises at the same time and the current account balance is zero starting on date T (because the expected future terms-of-trade path is again constant), we can infer from eq. (10) that the current account must have been in deficit during the interval before T when the terms of trade were expected to improve. When $\sigma < 1$, however, we cannot make this argument, because PC rises on date T, and if PC rises then by more than pY, we would have to conclude that the current account was moving from a *surplus* to zero. Can this ever occur? It cannot, as we now show, provided it is assumed that the temporary fall in p *harms* the home country. (This need not be so, as we will discuss in closing.) Suppose that the economy runs a surplus between date 0, the date the terms of trade fall to p', and date T, when they return to p. Since foreign assets have risen between 0 and T, the economy's residents are better off from T on than they were before date 0. The Euler equation tells, us, however, that $C_T = (P_{T-1}/P_T)^{\sigma} C_{T-1}$; and, since $P_T > P_{T-1}$, we see that $C_{T-1} > C_T$, where C_T is in turn higher than its level prior to date 0 as a result of the external surplus between dates 0 and T. So we get the following picture of what must happen if the economy's response is a current account surplus: C rises when p first falls and remains constant until date T, when C falls, but to a level permanently higher than before the initial fall in p. The implication is that the economy is better off as a result of the fall in p! If it is worse off, it must run a current account *deficit* between dates 0 and T, as in the $\sigma > 1$ case. But how can a fall in the relative price of an economy's export good ever make it better off? This is a logical possibility, it turns out, albeit a far-fetched one. If the economy initially has a positive stock of net foreign assets B, then it effectively has a positive endowment rB of the import good. In that circumstance, a very large fall in p (the relative price of exports in terms of imports) can reduce the economy's consumption of the initial import good and raise that of the initial export good enough that the economy becomes a net exporter of its previously imported good. In this implausible case of a trade pattern reversal, the fall in p can make the economy better off and result in a temporary surplus in the current account. Otherwise the more intuitive prediction of a deficit is correct.