HEAT TRANSFER

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6

EMPIRICAL AND PRACTICAL RELATIONS FOR FORCED-CONVECTION HEAT TRANSFER

6-1 INTRODUCTION

The discussion and analyses of Chap. 5 have shown how forced-convection heat transfer may be calculated for several cases of practical interest; the problems considered, however, were those which could be solved in an analytical fashion. In this way, the principles of the convection process and their relation to fluid dynamics were demonstrated, primary emphasis being devoted to a clear understanding of physical mechanism. Regrettably, it is not always possible to obtain analytical solutions to convection problems, and the individual is forced to resort to experimental methods to obtain design information, as well as to secure the more elusive data which increase his physical understanding of the heat-transfer processes.

Results of experimental data are usually expressed in the form of either empirical formulas or graphical charts so that they may be utilized with a maximum of generality. It is in the process of trying to generalize the results of his experiments, in the form of some empirical correlation, that one encounters difficulty. If an analytical solution is available for a similar problem, the correlation of data is much easier, since one may guess at the functional form of the results, and hence use the experimental data to obtain values of constants or exponents on certain significant parameters such as the Reynolds or Prandtl numbers. If an analytical solution for a similar problem is not available, the individual must resort to intuition based on his physical understanding of the problem, or shrewd inferences which he may be able to draw from the differential equations of the flow processes based upon dimensional or order-of-magnitude estimates. In any event, there is no substitute for physical insight and understanding.

To show how one might proceed to analyze a new problem to obtain an important functional relationship from the differential equations, consider the problem of determining the hydrodynamic-boundary-layer thickness for flow over a flat plate. This problem was solved in the preceding chapter, but we now wish to make an order-of-magnitude analysis of the differential equations to obtain the functional form of the solution. The momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

must be solved in conjunction with the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Within the boundary layer we may say that the velocity u is of the order of the free-stream velocity u_∞. Similarly, the y dimension is of the order of the boundary-layer thickness δ. Thus

$$u \sim u_\infty$$

$$y \sim \delta$$

and we might write the continuity equation in an approximate form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u_\infty}{x} + \frac{v}{\delta} \approx 0$$

or

$$v \sim \frac{u_\infty \delta}{x}$$

Then, using this order of magnitude for v, the analysis of the momentum equation would yield

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$u_\infty\frac{u_\infty}{x} + \frac{u_\infty \delta}{x}\frac{u_\infty}{\delta} \approx \nu\frac{u_\infty}{\delta^2}$$

or

$$\delta^2 \sim \frac{\nu x}{u_\infty}$$

$$\delta \sim \sqrt{\frac{\nu x}{u_\infty}}$$

Dividing by x to express the result in dimensionless form gives

$$\frac{\delta}{x} \sim \sqrt{\frac{\nu}{u_\infty x}} = \frac{1}{\sqrt{\mathrm{Re}_x}}$$

This functional variation of the boundary-layer thickness with the Reynolds number and x position is precisely that which was obtained in Sec. 5-4. Although this analysis is rather straightforward and does indeed yield correct results, the order-of-magnitude analysis may not always be so fortunate when applied to more complex problems, particularly those involving turbulent- or separated-flow regions. Nevertheless, one may often obtain valuable information and physical insight by examining the order of magnitude of various terms in a governing differential equation for the particular problem at hand.

A conventional technique used in correlation of experimental data is that of dimensional analysis, in which appropriate dimensionless groups such as the Reynolds and Prandtl numbers are derived from purely dimensional and functional considerations. There is, of course, the assumption of flow-field and temperature-profile similarity for geometrically similar heating surfaces. Generally speaking, the application of dimensional analysis to any new problem is extremely difficult when a previous analytical solution of some sort is not available. It is usually best to attempt an order-of-magnitude analysis such as the one above if the governing differential equations are known. In this way it may be possible to determine the significant dimensionless variables for correlating experimental data. In some complex flow and heat-transfer problems a clear physical model of the processes may not be available, and the engineer must first try to establish this model before he can correlate his experimental data.

Schlichting [6], Giedt [7], and Kline [28] discuss similarity considerations and their use in boundary-layer and heat-transfer problems.

The purpose of the foregoing discussion has not been to emphasize or even to imply any new method for solving problems, but rather to indicate the necessity of applying intuitive physical reasoning to a difficult problem and to point out the obvious advantage of using any and all information which may be available. When the problem of correlation of experimental data for a previously unsolved situation is encountered, one must frequently adopt devious methods to accomplish the task.

6-2 EMPIRICAL RELATIONS FOR PIPE AND TUBE FLOW

The analysis of Sec. 5-10 has shown how one might analytically attack the problem of heat transfer in fully developed laminar tube flow. The cases of undeveloped laminar flow, flow systems where the fluid properties vary widely with temperature, and turbulent-flow systems are considerably more complicated but are of very important practical interest in the design of heat exchangers and associated heat-transfer equipment. These more complicated problems may sometimes be solved analytically, but the solutions, when possible, are very tedious. For design and

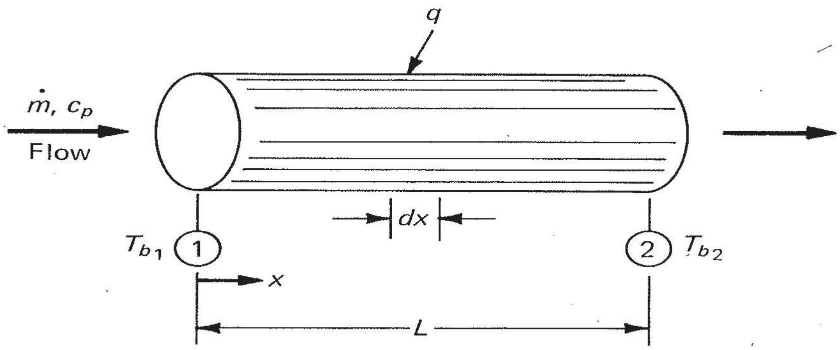


Fig. 6-1 Total heat transfer in terms of bulk-temperature difference.

engineering purposes empirical correlations are usually of greatest practical utility. In this section we present some of the more important and useful empirical relations and point out their limitations.

First let us give some further consideration to the bulk-temperature concept which is important in all heat-transfer problems involving flow inside closed channels. In Chap. 5 we noted that the bulk temperature represents energy average or "mixing-cup" conditions. Thus, for the tube flow depicted in Fig. 6-1 the total energy added can be expressed in terms of a bulk-temperature difference by

$$q = \dot{m}c_p(T_{b_2} - T_{b_1}) \tag{6-1}$$

provided c_p is reasonably constant over the length. In some differential length dx the heat added dq can be expressed either in terms of a bulk-temperature difference or in terms of the heat-transfer coefficient.

$$dq = \dot{m}c_p\, dT_b = h(2\pi r)\, dx\, (T_w - T_b) \tag{6-2}$$

where T_w and T_b are the wall and bulk temperatures at the particular x location. The total heat transfer can also be expressed

$$q = hA(T_w - T_b)_{\text{av}} \tag{6-3}$$

where A is the total surface area for heat transfer. Because both T_w and T_b can vary along the length of the tube, a suitable averaging process must be adopted for use with Eq. (6-3). In this chapter most of our attention will be focused on methods for determining h, the convection heat-transfer coefficient. Chapter 10 will discuss different methods for taking proper account of temperature variations in heat exchangers.

For fully developed turbulent flow in smooth tubes the following relation is recommended by Dittus and Boelter [1]:

$$\text{Nu}_d = 0.023\, \text{Re}_d^{\,0.8}\, \text{Pr}^n \tag{6-4}$$

The properties in this equation are evaluated at the fluid bulk temperature, and the exponent n has the following values:

$$n = \begin{cases} 0.4 & \text{for heating} \\ 0.3 & \text{for cooling} \end{cases}$$

One may ask the reason for the functional form of Eq. (6-4). Physical reasoning, based on the experience gained with the analyses of Chap. 5, would certainly indicate a dependence of the heat-transfer process on the flow field, and hence on the Reynolds number. The relative rates of diffusion of heat and momentum are related by the Prandtl number, so that the Prandtl number is expected to be a significant parameter in the final solution. We can be rather confident of dependence of the heat transfer on the Reynolds and Prandtl numbers. But the question arises as to the correct functional form of the relation; i.e., would one necessarily expect a product of two exponential functions of the Reynolds and Prandtl numbers? The answer is that one might expect this functional form since it appears in the flat-plate analytical solutions of Chap. 5, as well as the Reynolds analogy for turbulent flow. In addition, this type of functional relation is convenient to use when correlating experimental data, as described below.

Suppose a number of experiments are conducted taking measurements of heat-transfer rates of various fluids in turbulent flow inside smooth tubes under different temperature conditions. Different-diameter tubes may be used to vary the range of the Reynolds number in addition to variations in the mass-flow rate. We wish to generalize the results of these experiments by arriving at one empirical equation which represents all the data. As described above, we may anticipate that the heat-transfer data will be dependent on the Reynolds and Prandtl numbers. An exponential function for each of these parameters is perhaps the simplest type of relation to use, so we assume

$$\mathrm{Nu}_d = C\,\mathrm{Re}_d{}^m\,\mathrm{Pr}^n$$

where C, m, and n are constants to be determined from the experimental data.

A log-log plot of Nu_d versus Re_d is first made for one fluid to estimate the dependence of the heat transfer on the Reynolds number, i.e., to find an approximate value of the exponent m. This plot is made for one fluid at a constant temperature, so that the influence of the Prandtl number will be small, since the Prandtl number will be approximately constant for the one fluid. Using this first estimate for the exponent m, the data for all fluids are plotted as log $(\mathrm{Nu}_d/\mathrm{Re}_d{}^m)$ versus log Pr, and a value for the exponent n is determined. Then, using this value of n, all the data are plotted once again as log $(\mathrm{Nu}_d/\mathrm{Pr}^n)$ versus log Re_d, and a final value of the exponent m is determined as well as a value for the constant C. An example of this final type of data plot is shown in Fig. 6-2. The final correlation equation usually represents the data within ± 25 percent.

Equation (6-4) is valid for fully developed turbulent flow in smooth tubes for fluids with Prandtl numbers ranging from about 0.6 to 100 and with moderate temperature differences between wall and fluid conditions.

If wide temperature differences are present in the flow, there may be an appreciable change in the fluid properties between the wall of the tube and the central flow. These property variations may be evidenced by a change in the velocity profile as indicated in Fig. 6-3. The deviations from the velocity profile for isothermal flow as shown in this figure are a result of the fact that the viscosity of gases

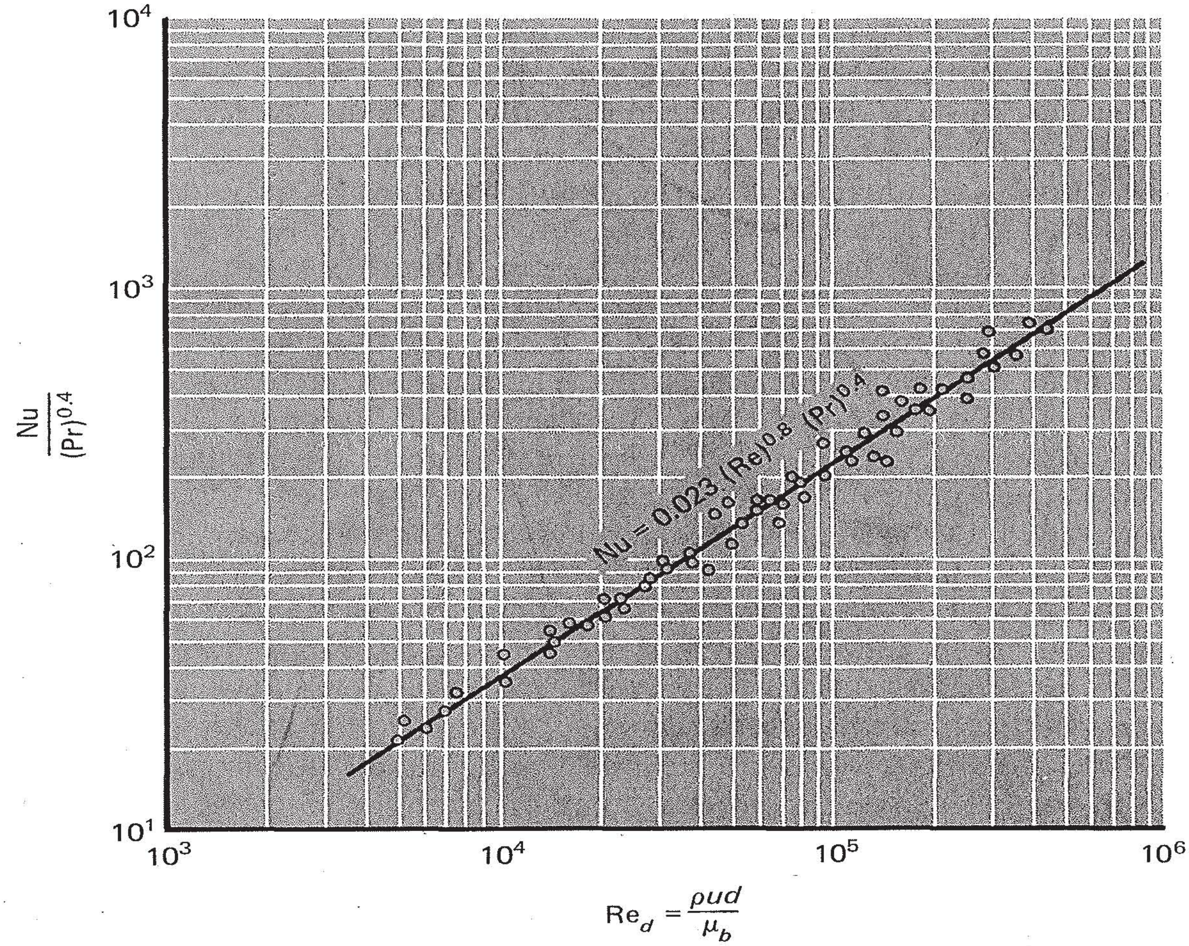


Fig. 6-2 Typical data correlation for forced convection in smooth tubes, turbulent flow.

increases with an increase in temperature, while the viscosities of liquids decrease with an increase in temperature.

To take into account the property variations, Sieder and Tate [2] recommend the following relation:

$$Nu_d = 0.027\ Re_d^{0.8}\ Pr^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{6-5}$$

All properties are evaluated at bulk-temperature conditions, except μ_w, which is evaluated at the wall temperature.

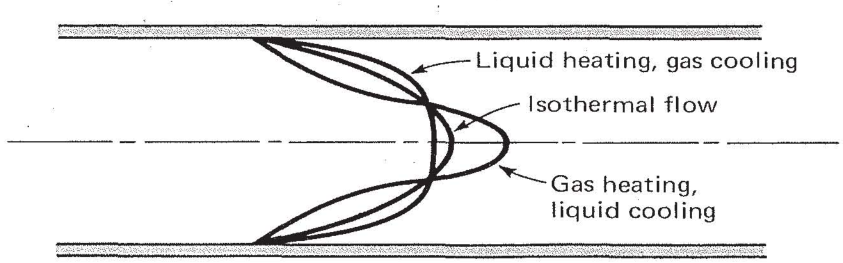


Fig. 6-3 Influence of heating on velocity profile in laminar tube flow.

Equations (6-4) and (6-5) apply to fully developed turbulent flow in tubes. In the entrance region the flow is not developed, and Nusselt [3] recommended the following equation:

$$\mathrm{Nu}_d = 0.036\,\mathrm{Re}_d^{\,0.8}\,\mathrm{Pr}^{1/3}\left(\frac{d}{L}\right)^{0.055} \qquad \text{for } 10 < \frac{L}{d} < 400 \tag{6-6}$$

where L is the length of the tube and d is the tube diameter. The properties in Eq. (6-6) are evaluated at the mean bulk temperature. Hartnett [24] has given experimental data on the thermal entrance region for water and oils. Definitive studies of turbulent heat transfer with water in smooth tubes and at uniform heat flux have been presented by Allen and Eckert [25].

Hausen [4] presents the following empirical relation for fully developed laminar flow in tubes at constant wall temperature:

$$\mathrm{Nu}_d = 3.66 + \frac{0.0668(d/L)\,\mathrm{Re}_d\,\mathrm{Pr}}{1 + 0.04[(d/L)\,\mathrm{Re}_d\,\mathrm{Pr}]^{2/3}} \tag{6-7}$$

The heat-transfer coefficient calculated from this relation is the average value over the entire length of tube. Note that the Nusselt number approaches a constant value of 3.66 when the tube is sufficiently long. This situation is similar to that encountered in the constant-heat-flux problem analyzed in Chap. 5 [Eq. (5-101)], except that in this case we have a constant wall temperature instead of a linear variation with length. The temperature profile is fully developed when the Nusselt number approaches a constant value.

A somewhat simpler empirical relation was proposed by Sieder and Tate [2] for laminar heat transfer in tubes.

$$\mathrm{Nu}_d = 1.86\,(\mathrm{Re}_d\,\mathrm{Pr})^{1/3}\left(\frac{d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{6-8}$$

In this formula the average heat-transfer coefficient is based on the arithmetic average of the inlet and outlet temperature differences, and all fluid properties are evaluated at the mean bulk temperature of the fluid, except μ_w, which is evaluated at the wall temperature. Equation (6-8) obviously cannot be used for extremely long tubes since it would yield a zero heat-transfer coefficient. A comparison by Knudsen and Katz [Ref. 9, p. 377] of Eq. (6-8) with other relationships indicates that it is valid for

$$\mathrm{Re}_d\,\mathrm{Pr}\,\frac{d}{L} > 10$$

The product of the Reynolds and Prandtl numbers which occurs in the laminar-flow correlations is called the Peclet number.

$$\mathrm{Pe} = \frac{du\,\rho c_p}{k} = \mathrm{Re}_d\,\mathrm{Pr} \tag{6-9}$$

The calculation of laminar heat-transfer coefficients is frequently complicated by the presence of natural-convection effects which are superimposed on the forced-convection effects. The treatment of combined forced- and free-convection problems is discussed in Chap. 7.

The empirical correlations presented above apply to smooth tubes. Correlations are, in general, rather sparse where rough tubes are concerned, and it is recommended that the Reynolds analogy between fluid friction and heat transfer be used to effect a solution under these circumstances. Expressed in terms of the Stanton number,

$$\mathrm{St}_b\, \mathrm{Pr}_f^{\,2/3} = \frac{f}{8} \tag{6-10}$$

The friction coefficient f is defined by

$$\Delta p = f \frac{L}{d} \rho \frac{u_m^{\,2}}{2g_c} \tag{6-11}$$

where u_m is the mean flow velocity. Values of the friction coefficient for different roughness conditions are shown in Fig. 6-4.

Note that the relation in Eq. (6-10) is the same as Eq. (5-108), except that the Stanton number has been multiplied by $\mathrm{Pr}^{2/3}$ to take into account the variation of the thermal properties of different fluids. This correction follows the recommendation of Colburn [15], and is based on the reasoning that fluid friction and heat transfer in tube flow are related to the Prandtl number in the same way as they are related in flat-plate flow [Eq. (5-52)]. In Eq. (6-10) the Stanton number is based on bulk temperature, while the Prandtl number and friction factor are based on properties evaluated at the film temperature. Further information on the effects of tube roughness on heat transfer is given in Refs. 27, 29, 30, and 31.

If the channel through which the fluid flows is not of circular cross section, it is recommended that the heat-transfer correlations be based on the hydraulic diameter D_H, defined by

$$D_H = \frac{4A}{P} \tag{6-12}$$

where A is the cross-sectional area of the flow and P is the wetted perimeter. This particular grouping of terms is used because it yields the value of the physical diameter when applied to a circular cross section. The hydraulic diameter should be used in calculating the Nusselt and Reynolds numbers, and also in establishing the friction coefficient for use with the Reynolds analogy.

Although the hydraulic-diameter concept frequently yields satisfactory relations for fluid friction and heat transfer in many practical problems, there are some notable exceptions where the method does not work. Some of the problems involved in heat transfer in noncircular channels have been summarized by Irvine [20] and Knudsen and Katz [9]. The interested reader should consult these discussions for additional information.

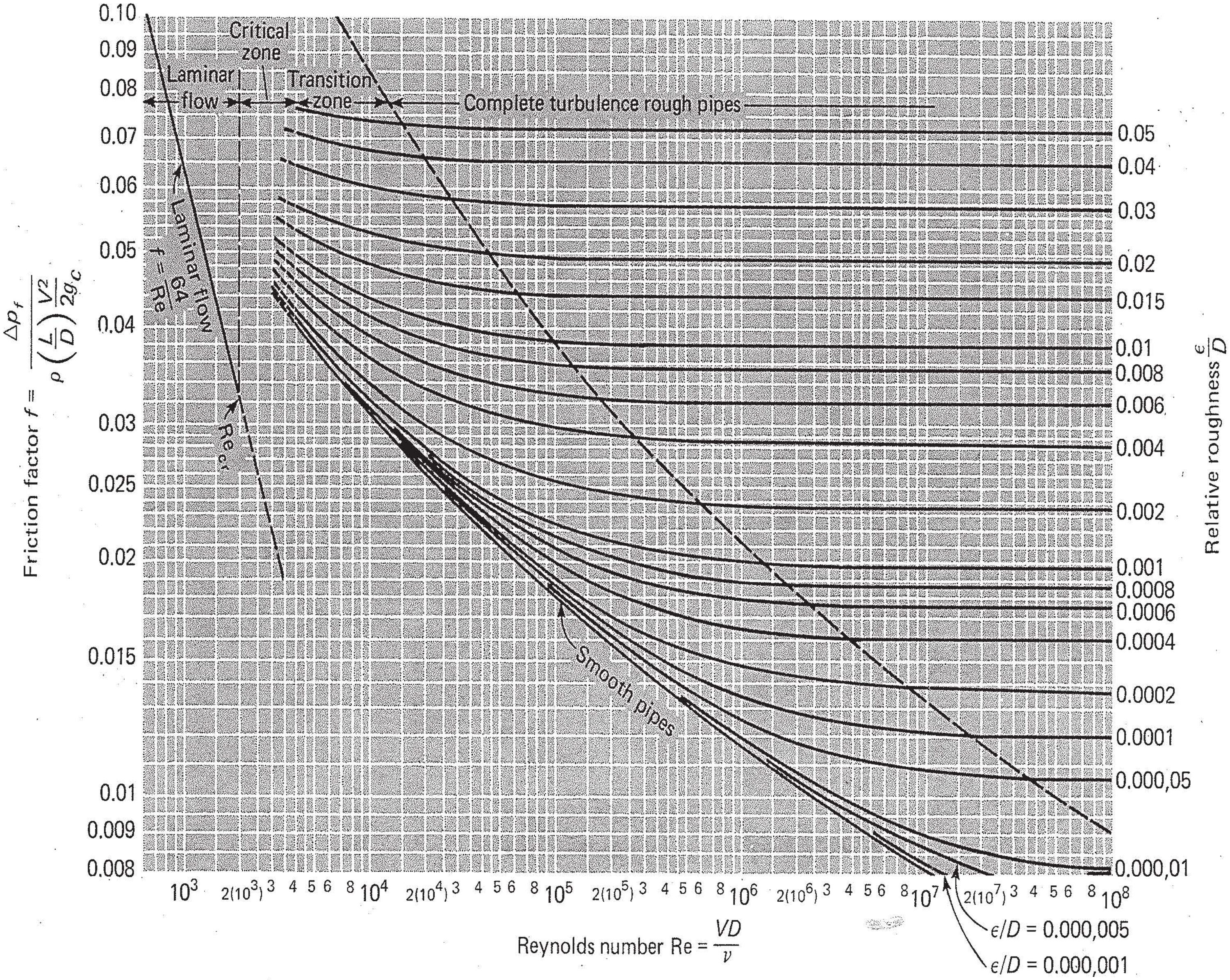


Fig. 6-4 Friction factors for pipes, from Ref. 5.

EXAMPLE 6-1

Air at 2 atm and 200°C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20°C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-in length of the tube?

Solution

We first calculate the Reynolds number to determine if the flow is laminar or turbulent, and then select the appropriate empirical correlation to calculate the heat transfer. The properties of air at a bulk temperature of 200°C are

$$\rho = \frac{p}{RT} = \frac{(2)(1.0132 \times 10^5)}{(287)(473)} = 1.493 \text{ kg/m}^3 \ (0.0932 \text{ lb}_m/\text{ft}^3)$$

$$\text{Pr} = 0.681$$

$$\mu = 2.57 \times 10^{-5} \text{ kg/m·s} \ (0.0622 \text{ lb}_m/\text{h·ft})$$

$$k = 0.0386 \text{ W/m·°C} \ (0.0223 \text{ Btu/h·ft·°F})$$

$$c_p = 1.025 \text{ kJ/kg·°C}$$

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57 \times 10^{-5}} = 14{,}756$$

so that the flow is turbulent. We therefore use Eq. (6-4) to calculate the heat-transfer coefficient.

$$\text{Nu}_d = \frac{hd}{k} = 0.023 \text{ Re}_d^{0.8} \text{ Pr}^{0.4} = (0.023)(14{,}756)^{0.8}(0.681)^{0.4} = 42.67$$

$$h = \frac{k}{d}\text{Nu}_d = \frac{(0.0386)(42.67)}{0.0254} = 64.85 \text{ W/m}^2\text{·°C} \ (11.42 \text{ Btu/h·ft}^2\text{·°F})$$

The heat flow per unit length is then

$$\frac{q}{L} = h\pi d\,(T_w - T_b) = (64.85)\pi(0.0254)(20) = 103.5 \text{ W/m} \ (107.7 \text{ Btu/ft})$$

We can now make an energy balance to calculate the increase in bulk temperature in a 3.0-m length of tube

$$q = \dot{m} c_p \, \Delta T_b = L\left(\frac{q}{L}\right)$$

We also have

$$\dot{m} = \rho u_m \frac{\pi d^2}{4} = (1.493)(10)\pi \frac{(0.0254)^2}{4}$$

$$= 7.565 \times 10^{-3} \text{ kg/s} \ (0.0167 \text{ lb}_m/\text{s})$$

so that we insert the numerical values in the energy balance to obtain

$$(7.565 \times 10^{-3})(1025)\,\Delta T_b = (3.0)(103.5)$$

and

$$\Delta T_b = 40.04\text{°C} \ (72.07\text{°F})$$

EXAMPLE 6-2

Water at 60°C enters a tube of 1-in (2.54 cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80°C.

Solution

We first evaluate the Reynolds number at the inlet bulk temperature to determine the flow regime. The properties of water at 60°C are

$$\rho = 985 \text{ kg/m}^3 \qquad c_p = 4.18 \text{ kJ/kg}\cdot{}^\circ\text{C}$$
$$\mu = 4.71 \times 10^{-4} \text{ kg/m}\cdot\text{s } (1.139 \text{ lb}_m/\text{h}\cdot\text{ft})$$
$$k = 0.651 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 3.02$$

Then

$$\text{Re}_d = \frac{\rho u_m d}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71 \times 10^{-4}} = 1062$$

so the flow is laminar. Calculating the additional parameter, we have

$$\text{Re}_d \text{ Pr} \frac{d}{L} = \frac{(1062)(3.02)(0.0254)}{3} = 27.15 > 10$$

so Eq. (6-8) is applicable. We do not yet know the mean bulk temperature to evaluate properties so we first make the calculation on the basis of 60°C, determine an exit bulk temperature, and then make a second iteration to obtain a more precise value. When inlet and outlet conditions are designated with the subscripts 1 and 2, respectively, the energy balance becomes

$$q = h\pi\, dL\left(T_w - \frac{T_{b1} + T_{b2}}{2}\right) = \dot{m}c_p(T_{b2} - T_{b1}) \tag{a}$$

At the wall temperature of 80°C we have

$$\mu_w = 3.55 \times 10^{-4} \text{ kg/m}\cdot\text{s}$$

From Eq. (6-8)

$$\text{Nu}_d = (1.86)\left[\frac{(1062)(3.02)(0.0254)}{3}\right]^{1/3}\left(\frac{4.71}{3.55}\right)^{0.14} = 5.816$$

$$h = \frac{k\,\text{Nu}_d}{d} = \frac{(0.651)(5.816)}{0.0254} = 149.1 \text{ W/m}^2\cdot{}^\circ\text{C } (26.26 \text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})$$

The mass flow rate is

$$\dot{m} = \rho\frac{\pi d^2}{4}u_m = \frac{(985)\pi(0.0254)^2(0.02)}{4} = 9.982 \times 10^{-3} \text{ kg/s}$$

Inserting the value for h into Eq. (a) along with $\dot{m}$ and $T_{b1} = 60$°C and $T_w = 80$°C gives

$$(149.1)\pi(0.0254)(3.0)\left(80 - \frac{T_{b2} + 60}{2}\right) = (9.982 \times 10^{-3})(4180)(T_{b2} - 60) \tag{b}$$

This equation can be solved to give

$$T_{b2} = 71.98^\circ\text{C}$$

Thus, we should go back and evaluate properties at

$$T_{b,\text{ mean}} = \frac{71.98 + 60}{2} = 66^\circ\text{C}$$

We obtain

$$\rho = 982 \text{ kg/m}^3 \qquad c_p = 4185 \text{ J/kg}\cdot{}^\circ\text{C} \qquad \mu = 4.36 \times 10^{-4} \text{ kg/m}\cdot\text{s}$$
$$k = 0.656 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 2.78$$

and
$$\mathrm{Re}_d = \frac{(982)(0.02)(0.0254)}{4.36 \times 10^{-4}} = 1144$$

$$\mathrm{Re}\,\mathrm{Pr}\,\frac{d}{L} = \frac{(1144)(2.78)(0.0254)}{3} = 26.93$$

$$\mathrm{Nu}_d = (1.83)(26.93)^{1/3}\left(\frac{4.36}{3.55}\right)^{0.14} = 5.645$$

$$h = \frac{(0.656)(5.645)}{0.0254} = 145.8 \ \mathrm{W/m^2 \cdot ^\circ C}$$

We insert this value of h back into Eq. (a) to obtain

$$T_{b2} = 71.78^\circ\mathrm{C}\ (161.2^\circ\mathrm{F})$$

The iteration makes very little difference in this problem. If a large bulk-temperature difference had been encountered, the change in properties could have had a larger effect.

6-3 FLOW ACROSS CYLINDERS AND SPHERES

While the engineer may frequently be interested in the heat-transfer characteristics of flow systems inside tubes or over flat plates, equal importance must be placed on the heat transfer which may be achieved by a cylinder in cross flow, as shown in Fig. 6-5. As would be expected, the boundary-layer development on the cylinder determines the heat-transfer characteristics. As long as the boundary layer remains laminar and well behaved, it is possible to compute the heat transfer by a method similar to the boundary-layer analysis of Chap. 5. It is necessary, however, to include the pressure gradient in the analysis since this influences the boundary-layer velocity profile to an appreciable extent. In fact, it is this pressure gradient which causes a separated-flow region to develop on the back side of the cylinder when the free-stream velocity is sufficiently large.

The phenomenon of boundary-layer separation is indicated in Fig. 6-6. The physical reasoning which explains the phenomenon in a qualitative way is as follows. Consistent with boundary-layer theory, the pressure through the boundary layer is essentially constant at any x position on the body. In the case of the cylinder, one might measure x distance from the front stagnation point of the cylinder. Thus the pressure in the boundary layer should follow that of the free stream for potential flow around a cylinder, provided this behavior would not contradict some basic principle which must apply in the boundary layer. As the flow pro-

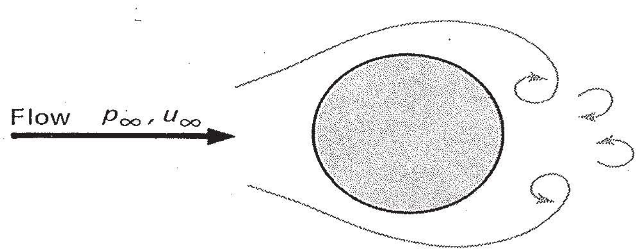


Fig. 6-5 Cylinder in cross flow.

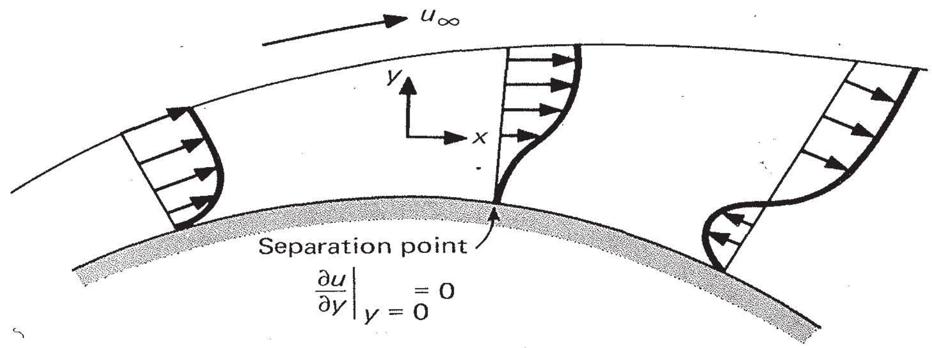


Fig. 6-6 Velocity distributions indicating flow separation on a cylinder in cross flow.

gresses along the front side of the cylinder, the pressure would decrease and then increase along the back side of the cylinder, resulting in an increase in free-stream velocity on the front side of the cylinder and a decrease on the back side. The transverse velocity (that velocity parallel to the surface) would decrease from a value of u_∞ at the outer edge of the boundary layer to zero at the surface. As the flow proceeds to the back side of the cylinder, the pressure increase causes a reduction in velocity in the free stream and throughout the boundary layer. The pressure increase and reduction in velocity are related through the Bernoulli equation written along a streamline.

$$\frac{dp}{\rho} = -d\left(\frac{u^2}{2g_c}\right)$$

Since the pressure is assumed constant throughout the boundary layer, we note that reverse flow may begin in the boundary layer near the surface; i.e., the momentum of the fluid layers near the surface is not sufficiently high to overcome the increase in pressure. When the velocity gradient at the surface becomes zero, the flow is said to have reached a separation point.

$$\text{Separation point at } \left.\frac{\partial u}{\partial y}\right)_{y=0} = 0$$

This separation point is indicated in Fig. 6-6. As the flow proceeds past the separation point, reverse-flow phenomena may occur, as also shown in Fig. 6-6. Eventually, the separated-flow region on the back side of the cylinder becomes turbulent and random in motion.

The drag coefficient for bluff bodies is defined by

$$\text{Drag force} = F_D = C_D A \frac{\rho u_\infty^2}{2g_c} \tag{6-13}$$

where C_D is the drag coefficient and A is the *frontal area* of the body exposed to the flow, which, for a cylinder, is the product of diameter and length. The values of the drag coefficient for cylinders and spheres are given as a function of the Reynolds number in Figs. 6-7 and 6-8.

The drag force on the cylinder is a result of a combination of frictional resistance and so-called form, or pressure drag, resulting from a low-pressure region on the rear of the cylinder created by the flow-separation process. At low Reynolds

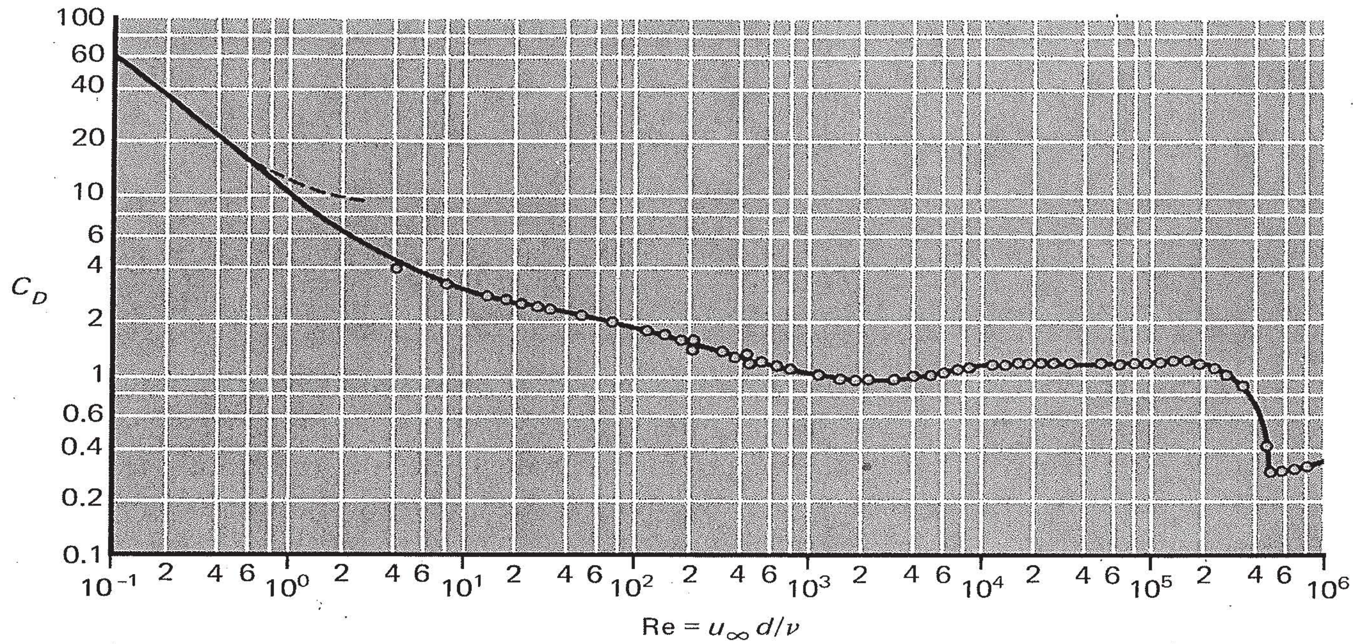


Fig. 6-7 Drag coefficient for circular cylinders as a function of the Reynolds number, from Ref. 6.

numbers of the order of unity, there is no flow separation, and all the drag results from viscous friction. At Reynolds numbers of the order of 10, the friction and form drag are of the same order, while the form drag resulting from the turbulent separated-flow region predominates at Reynolds numbers greater than 1000. At Reynolds numbers of approximately 10^5, based on diameter, the boundary-layer flow may become turbulent, resulting in a steeper velocity profile and extremely

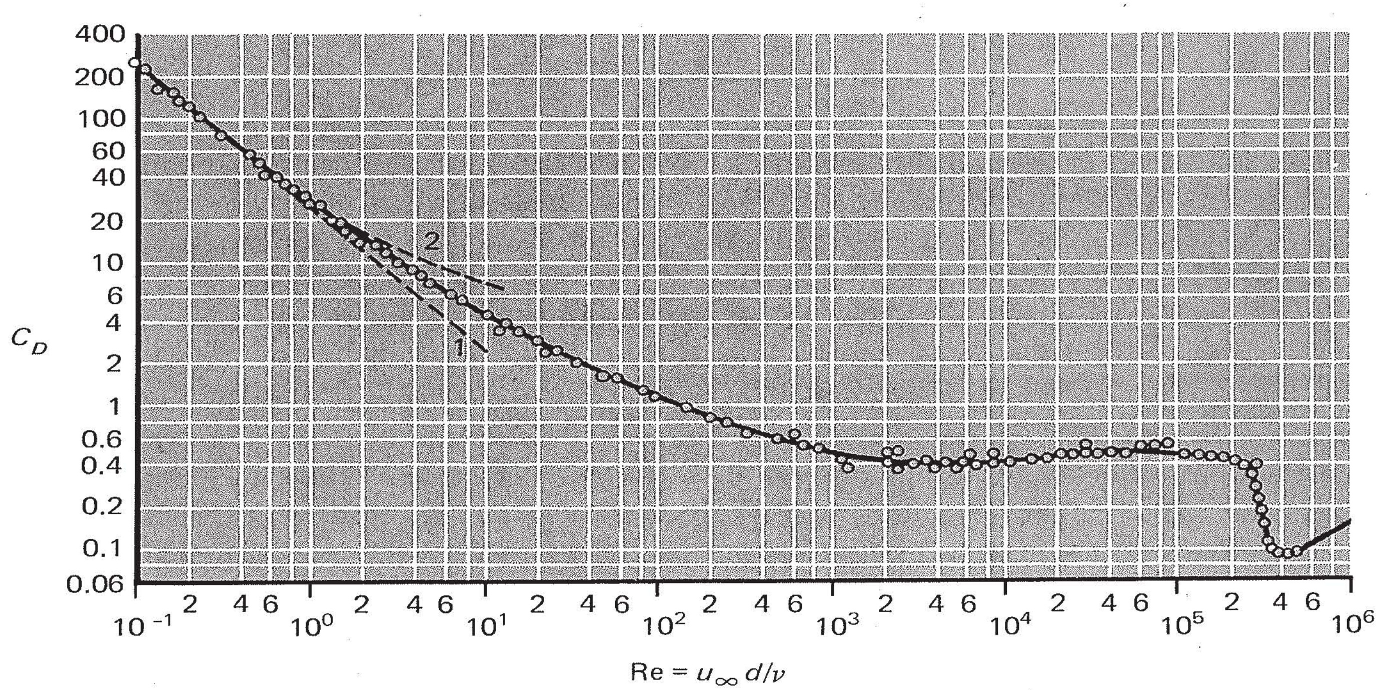


Fig. 6-8 Drag coefficient for spheres as a function of the Reynolds number, from Ref. 6.

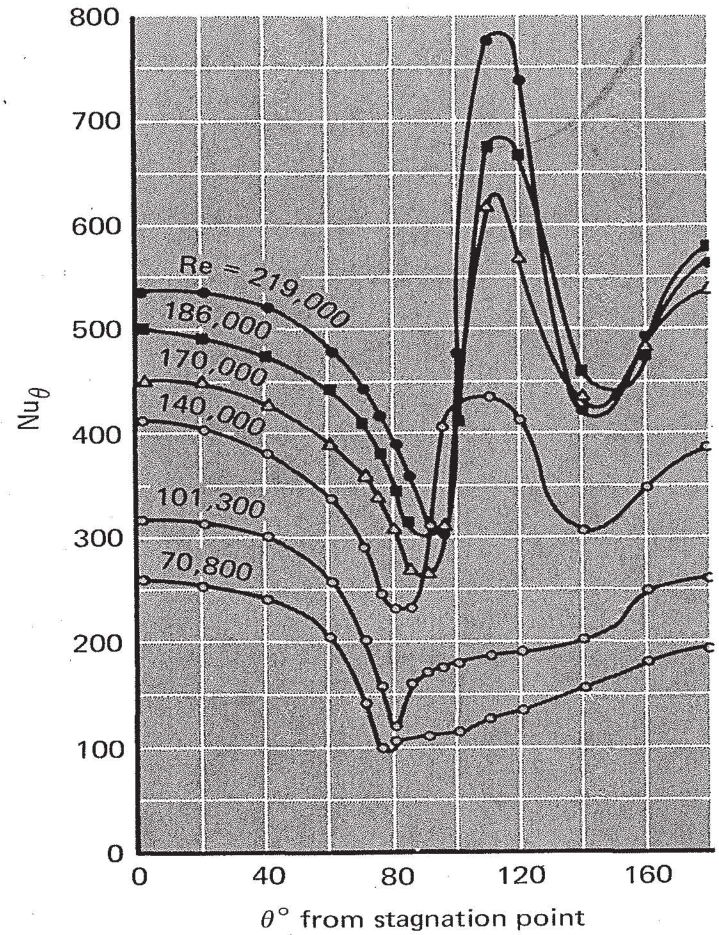


Fig .6-9 Local Nusselt number for heat transfer from a cylinder in cross flow, from Ref.7.

late flow separation. Consequently, the form drag is reduced, and this is represented by the break in the drag-coefficient curve at about Re $= 3 \times 10^5$. The same reasoning applies to the sphere as to the circular cylinder. Similar behavior is observed with other bluff bodies, such as elliptic cylinders and airfoils.

The flow processes discussed above obviously influence the heat transfer from a heated cylinder to a fluid stream. The detailed behavior of the heat transfer from a heated cylinder to air has been investigated by Giedt [7], and the results are summarized in Fig. 6-9. At the lower Reynolds numbers (70,800 and 101,300) a minimum point in the heat-transfer coefficient occurs at approximately the point of separation. There is a subsequent increase in the heat-transfer coefficient on the rear side of the cylinder, resulting from the turbulent eddy motion in the separated flow. At the higher Reynolds numbers two minimum points are observed. The first occurs at the point of transition from laminar to turbulent boundary layer, and the second minimum occurs when the turbulent boundary layer separates. There is a rapid increase in heat transfer when the boundary layer becomes turbulent, and another when the increased eddy motion at separation is encountered.

Because of the complicated nature of the flow-separation processes, it is not possible to calculate analytically the average heat-transfer coefficients in cross flow; however, correlations of the experimental data of Hilpert [8] for gases and Knudsen and Katz [9] for liquids indicate that the average heat-transfer coefficients may be calculated with

$$\frac{hd}{k_f} = C\left(\frac{u_\infty d}{\nu_f}\right)^n \mathrm{Pr}^{1/3} \tag{6-14}$$

where the constants C and n are tabulated in Table 6-1. The heat-transfer data for

TABLE 6-1 Constants for Use with Eq. (6-14), Based on Refs. 8 and 9

Re_{df}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

air are plotted in Fig. 6-10. Properties for use with Eq. (6-14) are evaluated at the film temperature as indicated by the subscript f.

Figure 6-11 shows the temperature field around heated cylinders placed in a transverse airstream. The dark lines are lines of constant temperature, made visible through the use of an interferometer. Note the separated-flow region which develops on the back side of the cylinder at the higher Reynolds numbers and the turbulent field which is present in that region.

We may note that the original correlation for gases omitted the Prandtl number term in Eq. (6-14) with little error because most diatomic gases have $\mathrm{Pr} \sim 0.7$. The introduction of the $\mathrm{Pr}^{1/3}$ factor follows from the previous reasoning in Chap. 5.

Fand [21] has shown that the heat-transfer coefficients from liquids to cylinders in cross flow may be better represented by the relation

$$\mathrm{Nu}_f = (0.35 + 0.56\,\mathrm{Re}_f^{0.52})\,\mathrm{Pr}_f^{0.3} \tag{6-15}$$

This relation is valid for $10^{-1} < \mathrm{Re}_f < 10^5$ provided excessive free-stream turbulence is not encountered.

In some instances, particularly those involving calculations on a computer, it may be more convenient to utilize a more complicated expression than Eq. (6-14) if it can be applied over a wider range of Reynolds numbers. Eckert and Drake [34] recommend the following relations for heat transfer from tubes in cross flow, based on the extensive study of Ref. 33:

$$\mathrm{Nu} = (0.43 + 0.50\,\mathrm{Re}^{0.5})\,\mathrm{Pr}^{0.38}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25} \quad \text{for } 1 < \mathrm{Re} < 10^3 \tag{6-16}$$

$$\mathrm{Nu} = 0.25\,\mathrm{Re}^{0.6}\,\mathrm{Pr}^{0.38}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{0.25} \quad \text{for } 10^3 < \mathrm{Re}_f < 2\times10^5 \tag{6-17}$$

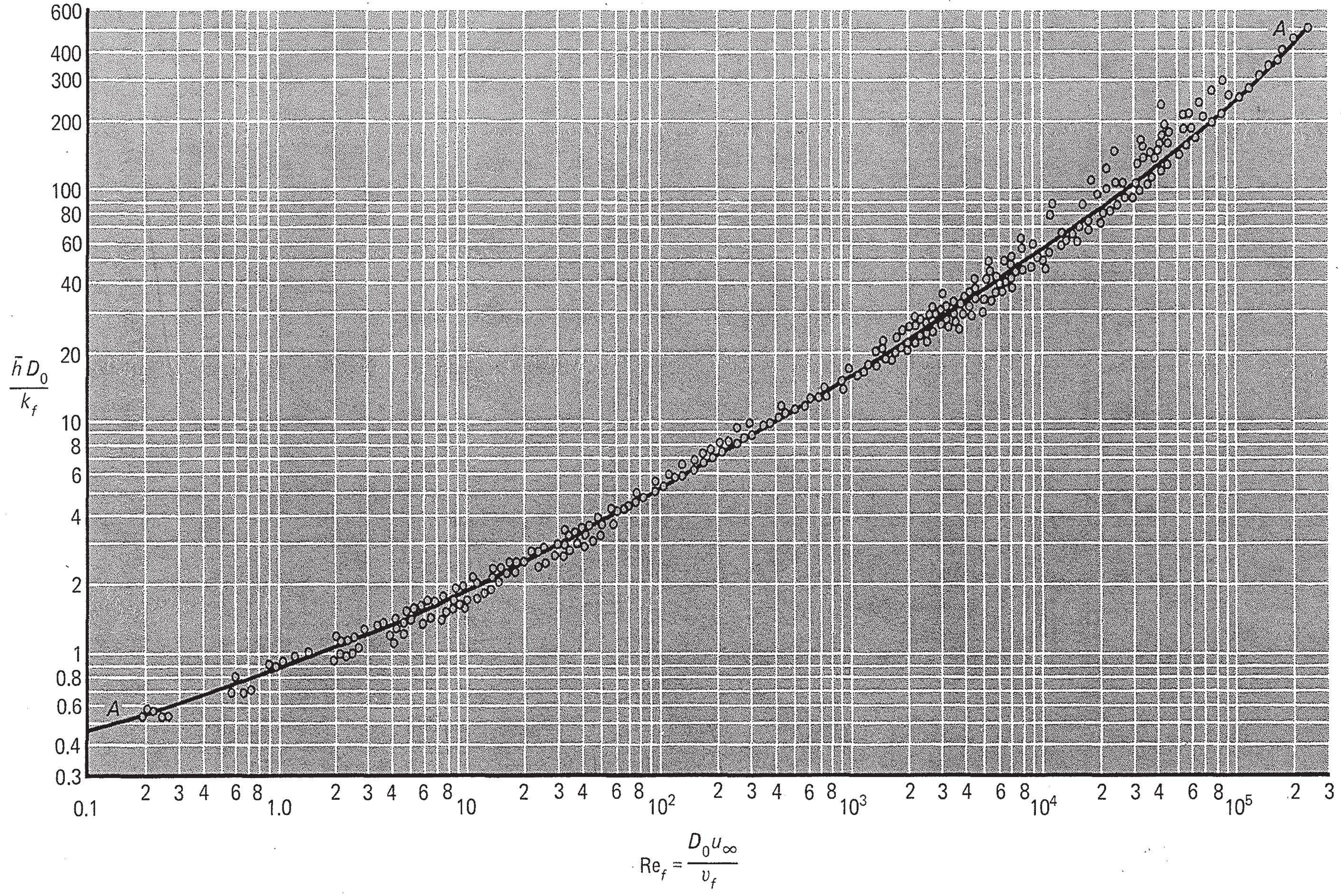


Fig. 6-10 Dataf or heating and cooling of air flowing normal to single cylinders, from Ref. 10.

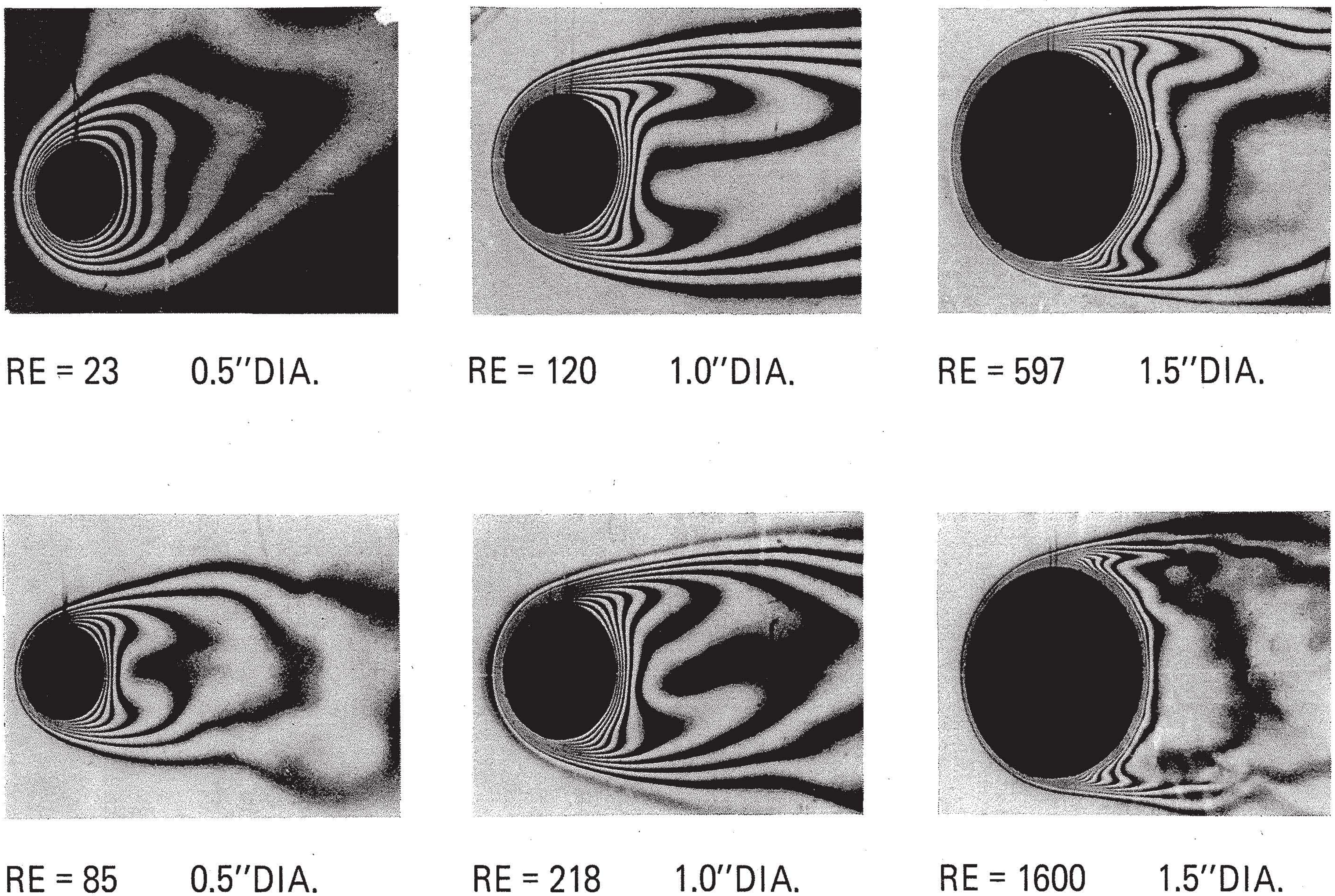
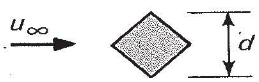
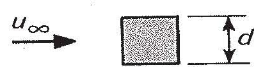
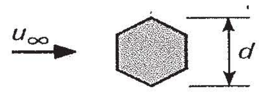
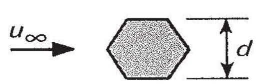


Fig. 6-11 Interferometer photograph showing isotherms around heated horizontal cylinders placed in a transverse airstream. Photograph courtesy E. Soehngen.

For gases the Prandtl number ratio may be dropped, and fluid properties are evaluated at the film temperature. For liquids the ratio is retained, and fluid properties are evaluated at the free-stream temperature. Equations (6-16) and (6-17) are in agreement with results obtained using Eq. (6-14) within 5 to 10 percent.

Jakob [22] has summarized the results of experiments with heat transfer from noncircular cylinders. Equation (6-14) is employed in order to obtain an empirical correlation for gases, and the constants for use with this equation are summarized in Table 6-2.

TABLE 6-2 Constants for Heat Transfer from Noncircular Cylinders According to Ref. 22

Geometry	Re_{df}	C	n
	$5 \times 10^3 - 10^5$	0.246	0.588
	$5 \times 10^3 - 10^5$	0.102	0.675
	$5 \times 10^3 - 1.95 \times 10^4$ $1.95 \times 10^4 - 10^5$	0.160 0.0385	0.638 0.782
	$5 \times 10^3 - 10^5$	0.153	0.638
	$4 \times 10^3 - 1.5 \times 10^4$	0.228	0.731

McAdams [10] recommends the following relation for heat transfer from spheres to a flowing gas:

$$\frac{hd}{k_f} = 0.37 \left(\frac{u_\infty d}{\nu_f}\right)^{0.6} \quad \text{for } 17 < Re_d < 70{,}000 \tag{6-18}$$

For flow of liquids past spheres, the data of Kramers [11] may be used to obtain the correlation

$$\frac{hd}{k_f} Pr_f^{-0.3} = 0.97 + 0.68 \left(\frac{u_\infty d}{\nu_f}\right)^{0.5} \quad \text{for } 1 < Re_d < 2000 \tag{6-19}$$

Vliet and Leppert [19] recommend the following expression for heat transfer from spheres to oil and water over a more extended range of Reynolds numbers from 1 to 200,000:

$$\text{Nu}\ \text{Pr}^{-0.3}\left(\frac{\mu_w}{\mu}\right)^{0.25} = 1.2 + 0.53\ \text{Re}_d^{0.54} \tag{6-20}$$

where all properties are evaluated at free-stream conditions, except μ_w, which is evaluated at the surface temperature of the sphere. Equation (6-20) represents the data of Ref. 11, as well as the more recent data of Ref. 19.

All of the above data have been brought together by Whitaker [35] to develop a single equation for gases and liquids flowing past spheres:

$$\text{Nu} = 2 + (0.4\ \text{Re}_d^{1/2} + 0.06\ \text{Re}_d^{2/3})\ \text{Pr}^{0.4}\ (\mu_\infty/\mu_w)^{1/4} \tag{6-20a}$$

which is valid for the range $3.5 < \text{Re}_d < 8 \times 10^4$ and $0.7 < \text{Pr} < 380$. Properties in Eq. (6-20a) are evaluated at the free-stream temperature.

EXAMPLE 6-3

Air at 1 atm and 35°C flows across a 5.0-cm-diameter cylinder at a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150°C. Calculate the heat loss per unit length of the cylinder.

Solution

We first determine the Reynolds number and then find the applicable constants from Table 6-1 for use with Eq. (6-14). The properties of air are evaluated at the film temperature.

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°\text{C} = 365.5°\text{K}$$

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(365.5)} = 0.966\ \text{kg/m}^3\ (0.0603\ \text{lb}_m/\text{ft}^3)$$

$$\mu_f = 2.01 \times 10^{-5}\ \text{kg/m·s}\ (0.0486\ \text{lb}_m/\text{h·ft})$$

$$k_f = 0.0312\ \text{W/m·°C}\ (0.018\ \text{Btu/h·ft·°F})$$

$$\text{Pr}_f = 0.695$$

$$\text{Re}_f = \frac{\rho u_\infty d}{\mu} = \frac{(0.966)(50)(0.05)}{2.01 \times 10^{-5}} = 1.201 \times 10^5$$

From Table 6-1

$$C = 0.0266 \qquad n = 0.805$$

so that, from Eq. (6-14)

$$\frac{hd}{k_f} = (0.0266)(1.201 \times 10^5)^{0.805}\ (0.695)^{1/3} = 289.2$$

$$h = \frac{(289.2)(0.0312)}{0.05} = 180.5\ \text{W/m}^2\text{·°C}\ (31.8\ \text{Btu/h·ft}^2\text{·°F})$$

The heat transfer per unit length is therefore

$$\frac{q}{L} = h\pi d\,(T_w - T_\infty)$$

$$= (180.5)\pi(0.05)(150 - 35)$$

$$= 3260 \text{ W/m } (3391 \text{ Btu/ft})$$

6-4 FLOW ACROSS TUBE BANKS

Since many heat-exchanger arrangements involve multiple rows of tubes, the heat-transfer characteristics for tube banks are of important practical interest. The heat-transfer characteristics of staggered and in-line tube banks were studied by Grimson [12], and on the basis of a correlation of the results of various investigators, he was able to represent the data in the form of Eq. (6-14). The values of the constant C and the exponent n are given in Table 6-3 in terms of the geometric parameters used to describe the tube-bundle arrangement. The Reynolds number is based on the maximum velocity occurring in the tube bank, i.e., the velocity through the minimum-flow area. This area will depend on the geometric tube arrangement. The nomenclature for use with Table 6-3 is shown in Fig. 6-12. The data of Table 6-3 pertain to tube banks having 10 or more rows of tubes in the

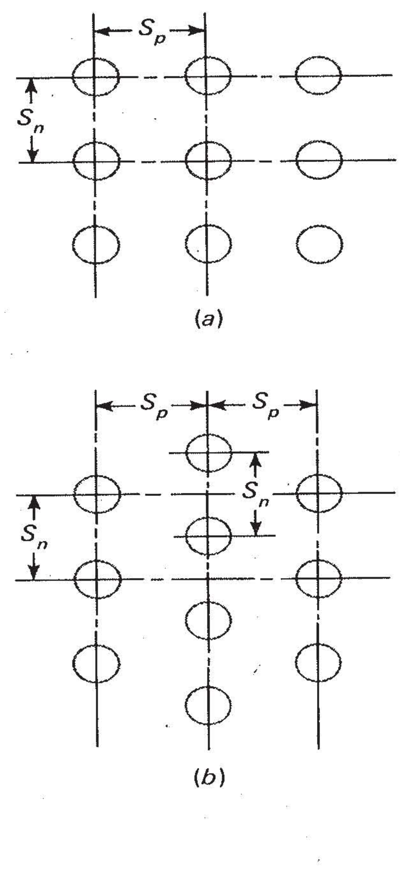


Fig. 6-12 Nomenclature for use with Table 6-3: (*a*) in-line tube rows; (*b*) staggered tube rows.

TABLE 6-3 Correlation of Grimson for Heat Transfer for Tube Banks of 10 Rows or More, from Ref. 12

	$\frac{S_n}{D}$							
	1.25		1.5		2.0		3.0	
$\frac{S_p}{D}$	C	n	C	n	C	n	C	n
				In line				
1.25	0.386	0.592	0.305	0.608	0.111	0.704	0.0703	0.752
1.5	0.407	0.586	0.278	0.620	0.112	0.702	0.0753	0.744
2.0	0.464	0.570	0.332	0.602	0.254	0.632	0.220	0.648
3.0	0.322	0.601	0.396	0.584	0.415	0.581	0.317	0.608
				Staggered				
0.6	. .	. .	. .	. .	. .	. .	0.236	0.636
0.9	. .	. .	. .	. .	0.495	0.571	0.445	0.581
1.0	. .	. .	0.552	0.558				
1.125	. .	. .	. .	. .	0.531	0.565	0.575	0.560
1.25	0.575	0.556	0.561	0.554	0.576	0.556	0.579	0.562
1.5	0.501	0.568	0.511	0.562	0.502	0.568	0.542	0.568
2.0	0.448	0.572	0.462	0.568	0.535	0.556	0.498	0.570
3.0	0.344	0.592	0.395	0.580	0.488	0.562	0.467	0.574

TABLE 6-4 Ratio of h for N Rows Deep to That for 10 Rows Deep, from Ref. 17

N	1	2	3	4	5	6	7	8	9	10
Ratio for staggered tubes	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.0
Ratio for in-line tubes	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.0

direction of flow. For fewer rows the ratio of h for N rows deep to that for 10 rows is given in Table 6-4.

Pressure drop for flow of gases over a bank of tubes may be calculated with the relation

$$\Delta p = \frac{f' G_{max}^2 N}{\rho(2.09 \times 10^8)} \left(\frac{\mu_w}{\mu_b}\right)^{0.14} \tag{6-21}$$

where G_{max} = mass velocity at minimum flow area, $lb_m/h \cdot ft^2$
ρ = density evaluated at free-stream conditions, lb_m/ft^3
N = number of transverse rows

The empirical friction factor f' is given by Jakob [18] as

$$f' = \left\{0.25 + \frac{0.118}{[(S_n - d)/d]^{1.08}}\right\} \mathrm{Re}_{max}{}^{-0.16} \tag{6-22}$$

for staggered tube arrangements, and

$$f' = \left\{0.044 + \frac{0.08 S_p/d}{[(S_n - d)/d]^{0.43 + 1.13d/S_p}}\right\} \mathrm{Re}_{max}{}^{-0.15} \tag{6-23}$$

for in-line tube arrangements.

EXAMPLE 6-4

Air at 1 atm and 10°C flows across a bank of tubes 15 rows high and 5 rows deep at a velocity of 7 m/s measured at a point in the flow before the air enters the tube bank. The surfaces of the tubes are maintained at 65°C. The diameter of the tubes is 1 in (2.54 cm); they are arranged in an in-line manner so that the spacing in both the normal and parallel directions to the flow is 1.5 in (3.81 cm). Calculate the total heat transfer per unit length for the tube bank and the exit air temperature.

Solution

The constants for use with Eq. (6-14) may be obtained from Table 6-3, using

$$\frac{S_p}{D} = \frac{3.81}{254} = 1.5 \qquad \frac{S_n}{D} = \frac{3.81}{2.54} = 1.5$$

so that

$$C = 0.278 \qquad n = 0.620$$

The properties of air are evaluated at the film temperature, which at entrance to the tube bank is

$$T_{f1} = \frac{T_w + T_\infty}{2} = \frac{65 + 10}{2} = 37.5°\mathrm{C} = 310.5°\mathrm{K}\ (558.9°\mathrm{R})$$

Then

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(310.5)} = 1.137\ \mathrm{kg/m^3}$$

$$\mu_f = 2.002 \times 10^{-5}\ \mathrm{kg/m{\cdot}s}$$

$$k_f = 0.027\ \mathrm{W/m{\cdot}°C}\ (0.0156\ \mathrm{Btu/h{\cdot}ft{\cdot}°F})$$

$$c_p = 1007\ \mathrm{J/kg{\cdot}°C}\ (0.24\ \mathrm{Btu/lb_m{\cdot}°F})$$

$$\mathrm{Pr} = 0.706$$

To calculate the maximum velocity we must determine the minimum flow area. From Fig. 6-12 we find that the ratio of the minimum flow area to the total frontal area is $(S_n - D)/S_n$. The maximum velocity is thus

$$u_{max} = u_\infty \frac{S_n}{S_n - D} = \frac{(7)(3.81)}{3.81 - 2.54} = 21\ \mathrm{m/s}\ (68.9\ \mathrm{ft/s}) \tag{a}$$

where u_∞ is the incoming velocity before entrance to the tube bank. The Reynolds number is computed using the maximum velocity:

$$\mathrm{Re} = \frac{\rho u_{max} d}{\mu} = \frac{(1.136)(21)(0.0254)}{2.002 \times 10^{-5}} = 30{,}293 \tag{b}$$

The heat-transfer coefficient is then calculated with Eq. (6-14)

$$\frac{hd}{k_f} = (0.278)(30{,}293)^{0.62}(0.706)^{1/3} = 148.6 \tag{c}$$

$$h = \frac{(148.6)(0.027)}{0.0254} = 158\ \text{W/m}^2\cdot{}^\circ\text{C}\ (27.8\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}) \tag{d}$$

This is the heat-transfer coefficient which would be obtained if there were 10 rows of tubes in the direction of the flow. Because there are only 5 rows, this value must be multiplied by the factor 0.92, as determined from Table 6-4.

The total surface area for heat transfer, considering unit length of tubes, is

$$A = N\pi d(1) = (15)(5)\pi(0.0254) = 5.985\ \text{m}^2/\text{m}$$

where N is the total number of tubes.

Before calculating the heat transfer we must recognize that the air temperature increases as the air flows through the tube bank. Therefore, this must be taken into account when using

$$q = hA(T_w - T_\infty) \tag{e}$$

As a good approximation we can use an arithmetic average value of T_∞ and write for the energy balance

$$q = hA\left(T_w - \frac{T_{\infty,1} + T_{\infty,2}}{2}\right) = \dot{m}c_p(T_{\infty,2} - T_{\infty,1}) \tag{f}$$

where now the subscripts 1 and 2 designate entrance and exit to the tube bank. The mass flow at entrance to the 15 tubes is

$$\dot{m} = \rho_\infty u_\infty (15) S_n$$

$$\rho_\infty = \frac{p}{RT_\infty} = \frac{1.0132 \times 10^5}{(287)(283)} = 1.246\ \text{kg/m}^3 \tag{g}$$

$$\dot{m} = (1.247)(7)(15)(0.0381) = 4.99\ \text{kg/s}\ (11.0\ \text{lb}_m/\text{s})$$

so that Eq. (f) becomes

$$(0.92)(158)(5.985)\left(65 - \frac{10 + T_{\infty,2}}{2}\right) = (4.99)(1006)(T_{\infty,2} - 10)$$

which may be solved to give

$$T_{\infty,2} = 18.77^\circ\text{C}$$

The heat transfer is then obtained from the right side of Eq. (f)

$$q = (4.99)(1005)(18.77 - 10) = 44.03\ \text{kW/m}$$

This answer could be improved somewhat by recalculating the air properties based on a mean value of T_∞ but the improvement would be small and well within the accuracy of the empirical heat-transfer correlation of Eq. (6-14).

6-5 LIQUID-METAL HEAT TRANSFER

In recent years concentrated interest has been placed on liquid-metal heat transfer because of the high heat-transfer rates which may be achieved with these media. These high heat-transfer rates result from the high thermal conductivities of

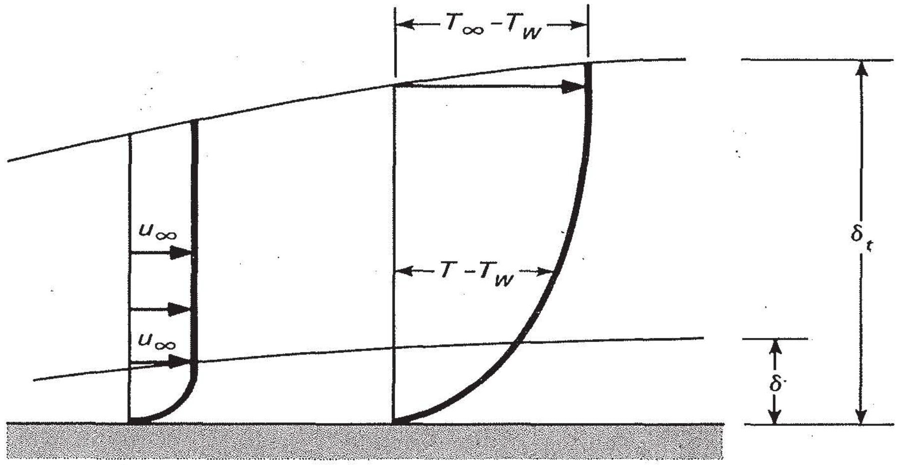


Fig. 6-13 Boundary-layer regimes for analysis of liquid-metal heat transfer.

liquid metals as compared with other fluids; as a consequence, they are particularly applicable to situations where large energy quantities must be removed from a relatively small space, as in a nuclear reactor. In addition, the liquid metals remain in the liquid state at higher temperatures than conventional fluids like water and various organic coolants. This also makes more compact heat-exchanger design possible. Liquid metals are difficult to handle because of their corrosive nature and the violent action which may result when they come into contact with water or air; even so, their advantages in certain heat-transfer applications have overshadowed their shortcomings, and suitable techniques for handling them have been developed.

Let us first consider the simple flat plate with a liquid metal flowing across it. The Prandtl number for liquid metals is very low, of the order of 0.01, so that the thermal-boundary-layer thickness should be substantially larger than the hydrodynamic-boundary-layer thickness. The situation results from the high values of thermal conductivity for liquid metals, and is depicted in Fig. 6-13. Since the ratio of δ/δ_t is small, the velocity profile has a very blunt shape over most of the thermal boundary layer. As a first approximation, then, we might assume a slug flow model for calculation of the heat transfer; i.e., we take

$$u = u_\infty \tag{6-23}$$

throughout the thermal boundary layer for purposes of computing the energy-transport term in the integral energy equation (Sec. 5-6)

$$\frac{d}{dx}\left[\int_0^{\delta_t} (T_\infty - T)u\, dy\right] = \alpha \left.\frac{dT}{dy}\right)_w \tag{6-24}$$

The conditions on the temperature profile are the same as those in Sec. 5-6, so that we use the cubic parabola as before.

$$\frac{\theta}{\theta_\infty} = \frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \tag{6-25}$$

Inserting Eqs. (6-23) and (6-25) in (6-24) gives

$$\theta_\infty u_\infty \frac{d}{dx}\left\{\int_0^{\delta_t}\left[1 - \frac{3}{2}\frac{y}{\delta_t} + \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3\right]dy\right\} = \frac{3\alpha\,\theta_\infty}{2\delta_t} \tag{6-26}$$

which may be integrated to give

$$2\delta_t\, d\delta_t = \frac{8\alpha}{u_\infty}\, dx \tag{6-27}$$

The solution to this differential equation is

$$\delta_t = \sqrt{\frac{8\alpha x}{u_\infty}} \tag{6-28}$$

for a plate heated over its entire length.

The heat-transfer coefficient may be expressed by

$$h_x = \frac{-k(\partial T/\partial y)_w}{T_w - T_\infty} = \frac{3k}{2\delta_t} = \frac{3\sqrt{2}}{8}\,k\sqrt{\frac{u_\infty}{\alpha x}} \tag{6-29}$$

This relationship may be put in dimensionless form as

$$\text{Nu}_x = \frac{h_x x}{k} = 0.530(\text{Re}_x\,\text{Pr})^{1/2} = 0.530\,\text{Pe}^{1/2} \tag{6-30}$$

where we have now introduced the new dimensionless grouping called the Peclet number,

$$\text{Pe} = \text{Re}\,\text{Pr} = \frac{u_\infty x}{\alpha} \tag{6-30a}$$

Using Eq. (5-21) for the hydrodynamic-boundary-layer thickness,

$$\frac{\delta}{x} = \frac{4.64}{\text{Re}_x^{\,1/2}} \tag{6-31}$$

we may compute the ratio δ/δ_t.

$$\frac{\delta}{\delta_t} = \frac{4.64}{\sqrt{8}}\sqrt{\text{Pr}} = 1.64\sqrt{\text{Pr}} \tag{6-32}$$

Using Pr $\sim$ 0.01, we obtain

$$\frac{\delta}{\delta_t} \sim 0.16$$

which is in reasonable agreement with our slug-flow model.

The flow model discussed above serves to illustrate the general nature of liquid-metal heat transfer and it is important to note that the heat transfer is dependent on the Peclet number. Empirical correlations are usually expressed in terms of this parameter, three of which we present below.

Extensive data on liquid metals are given in Ref. 13, and the heat-transfer characteristics are summarized in Ref. 23. Lubarsky and Kaufman [14] recommended the following relation for calculation of heat-transfer coefficients in fully developed turbulent flow of liquid metals in smooth tubes with uniform heat flux at the wall:

$$\mathrm{Nu}_d = \frac{hd}{k} = 0.625\,(\mathrm{Re}_d\,\mathrm{Pr})^{0.4} \tag{6-33}$$

All properties for use in Eq. (6-33) are evaluated at the bulk temperature. Equation (6-33) is valid for $10^2 < \mathrm{Pe} < 10^4$ and for $L/D > 60$. Seban and Shimazaki [16] propose the following relation for calculation of heat transfer to liquid metals in tubes with constant wall temperature:

$$\mathrm{Nu}_d = 5.0 + 0.025\,(\mathrm{Re}_d\,\mathrm{Pr})^{0.8} \tag{6-34}$$

where all properties are evaluated at the bulk temperature. Equation (6-34) is valid for $\mathrm{Pe} > 10^2$ and $L/D > 60$.

More recent data by Skupinshi et al. [26] with sodium-potassium mixtures indicates that the following relation may be preferable to that of Eq. (6-33) for constant-heat-flux conditions:

$$\mathrm{Nu} = 4.82 + 0.0185\,\mathrm{Pe}^{0.827} \tag{6-35}$$

This relation is valid for $3.6 \times 10^3 < \mathrm{Re} < 9.05 \times 10^5$ and $10^2 < \mathrm{Pe} < 10^4$.

Witte [32] has measured the heat transfer from a sphere to liquid sodium during forced convection, with the data being correlated by

$$\mathrm{Nu} = 2 + 0.386\,(\mathrm{Re}\,\mathrm{Pr})^{0.5} \tag{6-36}$$

for the Reynolds number range $3.56 \times 10^4 < \mathrm{Re} < 1.525 \times 10^5$.

In general, there are many open questions concerning liquid-metal heat transfer, and the reader is referred to Refs. 13 and 23 for more information.

EXAMPLE 6-5

Liquid bismuth flows at a rate of 4.5 kg/s through a 5.0-cm-diameter stainless-steel tube. The bismuth enters at 415°C and is heated to 440°C as it passes through the tube. If a constant heat flux is maintained along the tube and the tube wall is at a temperature 20°C higher than the bismuth bulk temperature, calculate the length of tube required to effect the heat transfer.

Solution

Because a constant heat flux is maintained, we may use Eq. (6-35) to calculate the heat-transfer coefficient. The properties of bismuth are evaluated at the average bulk temperature of (415 + 440)/2 = 427.5°C.

$$\begin{aligned}
\mu &= 1.34 \times 10^{-3}\ \mathrm{kg/m\cdot s}\ (3.242\ \mathrm{lb}_m/\mathrm{h\cdot ft}) \\
c_p &= 0.149\ \mathrm{kJ/kg\cdot{}^\circ C}\ (0.0356\ \mathrm{Btu/lb}_m\cdot{}^\circ\mathrm{F}) \\
k &= 15.6\ \mathrm{W/m\cdot{}^\circ C}\ (9.014\ \mathrm{Btu/h\cdot ft\cdot{}^\circ F}) \\
\mathrm{Pr} &= 0.013
\end{aligned}$$

The total heat transfer is calculated from

$$q = \dot{m} c_p\,\Delta T_b = (4.5)(149)(440 - 415) = 16.76\ \mathrm{kW}\ (57{,}186\ \mathrm{Btu/h}) \tag{a}$$

We calculate the Reynolds and Peclet numbers as

$$\mathrm{Re}_d = \frac{dG}{\mu} = \frac{(0.05)(4.5)}{[\pi(0.05)^2/4](1.34 \times 10^{-3})} = 85{,}520 \qquad (b)$$

$$\mathrm{Pe} = \mathrm{Re\,Pr} = (85{,}520)(0.013) = 1111$$

The heat-transfer coefficient is then calculated from Eq. (6-35)

$$\mathrm{Nu}_d = 4.82 + (0.0185)(1111)^{0.827} = 10.93 \qquad (c)$$

$$h = \frac{(10.93)(15.6)}{0.05} = 3410\ \mathrm{W/m^2 \cdot {}^\circ C}\ (600\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F})$$

The total required surface area of the tube may now be computed from

$$q = hA(T_w - T_b) \qquad (d)$$

where we use the temperature difference of 20°C.

$$A = \frac{16{,}760}{(3410)(20)} = 0.246\ \mathrm{m^2}\ (2.65\ \mathrm{ft^2})$$

This area in turn can be expressed in terms of the tube length

$$A = \pi d L \qquad \text{and} \qquad L = \frac{0.246}{\pi(0.05)} = 1.57\ \mathrm{m}\ (5.15\ \mathrm{ft})$$

REVIEW QUESTIONS

1. What is the Dittus-Boelter equation? When does it apply?
2. How may heat-transfer coefficients be calculated for flow in rough pipes?
3. What is the hydraulic diameter? When is it used?
4. What is the form of equation used to calculate heat transfer for flow over cylinders and bluff bodies?
5. Why does a slug-flow model yield reasonable results when applied to liquid-metal heat transfer?
6. What is the Peclet number?

PROBLEMS

6-1. Water at the rate of 0.8 kg/s is heated from 35 to 40°C in a 2.5-cm-diameter tube whose surface is at 90°C. How long must the tube be to accomplish this heating?

6-2. Water at the rate of 0.5 kg/s is forced through a smooth 2.5-cm-ID tube 15 m long. The inlet water temperature is 10°C, and the tube wall temperature is 15°C higher than the water temperature all along the length of the tube. What is the exit water temperature?

6-3. Water at an average temperature of 20°C flows at 0.7 kg/s in a 2.5-cm-diameter tube 6 m long. The pressure drop is measured as 2 kN/m^2. A constant heat flux is imposed, and the average wall temperature is 55°C. Estimate the exit temperature of the water.

6-4. Water at the rate of 3 kg/s is heated from 5 to 15°C by passing it through a 5-cm-ID copper tube. The tube wall temperature is maintained at 90°C. What is the length of the tube?

6-5. Air at 1400 kN/m^2 enters a duct 7.5 cm in diameter and 6 m long at a rate of 0.5 kg/s. The duct wall is maintained at an average temperature of 200°C. The average air temperature in the duct is 250°C. Estimate the decrease in temperature of the air as it passes through the duct.

6-6. Water at an average temperature of 10°C flows in a 2.5-cm-diameter tube 6 m long at a rate of 0.4 kg/s. The pressure drop is measured as 3 kN/m^2. A constant heat flux is imposed, and the average wall temperature is 50°C. Estimate the exit temperature of the water.

6-7. Water flows through a 2.5-cm-ID pipe 1.5 m long at a rate of 1.0 kg/s. The pressure drop is 7 kN/m^2 through the 1.5-m length. The pipe wall temperature is maintained at a constant temperature of 50°C by a condensing vapor, and the inlet water temperature is 20°C. Estimate the exit water temperature.

6-8. Water at the rate of 0.5 kg/s is to be cooled from 65 to 26°C. Which would result in less pressure drop: to run the water through a 12.5-mm-diameter pipe at a constant temperature of 4°C or through a constant-temperature 25-mm-diameter pipe at 20°C?

6-9. Water at the rate of 1 kg/s is forced through a tube with a 2.5-cm ID. The inlet water temperature is 15°C, and the outlet water temperature is 50°C. The tube wall temperature is 14°C higher than the water temperature all along the length of the tube. What is the length of the tube?

6-10. Air at 1 atm and 15°C flows through a long rectangular duct 7.5 by 15 cm. A 1.8-m section of the duct is maintained at 120°C, and the average air temperature at exit from this section is 65°C. Calculate the airflow rate and the total heat transfer.

6-11. Water at 38°C enters a 5-cm-ID pipe having a relative roughness of 0.002 at a rate of 6 kg/s. If the pipe is 9 m long and is maintained at 65°C, calculate the exit water temperature and the total heat transfer.

6-12. A heat exchanger is constructed so that hot flue gases at 425°C flow inside a 2.5-cm-ID copper tube with 1.6-mm wall thickness. A 5.0-cm tube is placed around the 2.5-cm-diameter tube, and high-pressure water at 150°C flows in the annular space between the tubes. If the flow rate of water is 1.5 kg/s and the total heat transfer is 17.5 kW, estimate the length of the heat exchanger for a gas mass flow of 0.8 kg/s. Assume that the properties of the flue gas are the same as those of air at atmospheric pressure and 425°C.

6-13. Engine oil enters a 1.25-cm-diameter tube 3 m long at a temperature of 38°C. The tube wall temperature is maintained at 65°C, and the flow velocity is 30 cm/s. Estimate the total heat transfer to the oil and the exit temperature of the oil.

6-14. Using the values of the local Nusselt number given in Fig. 6-9, obtain values for the average Nusselt number as a function of the Reynolds number. Plot the results as log Nu versus log Re, and obtain an equation which represents all the data. Compare this correlation with that given by Eq. (6-14) and Table 6-1.

6-15. Air at 70 kN/m^2 and 20°C flows across a 5-cm-diameter cylinder at a velocity of 20 m/s. Compute the drag force exerted on the cylinder.

6-16. Water at the rate of 0.8 kg/s at 93°C is forced through a 5-cm-ID copper tube at a velocity of 60 cm/s. The wall thickness is 0.8 mm. Air at 15°C and atmospheric pressure is forced over the outside of the tube at a velocity of 15 m/s in a direction normal to the axis of the tube. What is the heat loss per meter of length of the tube?

6-17. A heated cylinder at 150°C and 2.5 cm in diameter is placed in an atmospheric airstream at 1 atm and 38°C. The air velocity is 30 m/s. Calculate the heat loss per meter of length of the cylinder.

6-18. Assuming that a man can be approximated by a cylinder 1 ft in diameter and 6 ft high with a surface temperature of 75°F, calculate the heat he would lose while standing in a 30-mi/h wind whose temperature is 30°F.

6-19. The drag coefficient for a sphere at Reynolds numbers less than 100 may be approximated by $C_D = b\,\mathrm{Re}^{-1}$, where b is a constant. Assuming that the Colburn analogy between heat transfer and fluid friction applies, derive an expression for the heat loss from a sphere of diameter d and temperature T_s, released from rest and allowed to fall in a fluid of temperature T_∞. (Obtain an expression for the heat lost between the time the sphere is released and the time it reaches some velocity v. Assume that the Reynolds number is less than 100 during this time and that the sphere remains at a constant temperature.)

6-20. Air at 3.5 MN/m^2 and 38°C flows across a tube bank consisting of 400 1.25-cm-OD tubes arranged in a staggered manner 20 rows high. $S_p = 3.75$ cm, and $S_n = 2.5$ cm. The incoming-flow velocity is 9 m/s, and the tube wall temperatures are maintained constant at 200°C by a condensing vapor on the inside of the tubes. The length of the tubes is 1.5 m. Estimate the exit air temperature as it leaves the tube bank.

6-21. Liquid bismuth enters a 2.5-cm-diameter stainless-steel pipe at 400°C and at a rate of 1 kg/s. The tube wall temperature is maintained constant at 450°C. Calculate the bismuth exit temperature if the tube is 60 cm long.

6-22. Liquid sodium is to be heated from 120 to 149°C at a rate of 2.3 kg/s. A 2.5-cm-diameter electrically heated tube is available (constant heat flux). If the tube wall temperature is not to exceed 200°C, calculate the minimum length required.

6-23. Assume that one-half the heat transfer from a cylinder in cross flow occurs on the front half of the cylinder. On this assumption, compare the heat transfer from a cylinder in cross flow with the heat transfer from a flat plate having a length equal to the distance from the stagnation point on the cylinder. Discuss this comparison.

6-24. Using the slug-flow model, show that the boundary-layer energy equation reduces to the same form as the transient-conduction equation for the semi-infinite solid of Sec. 4-3. Solve this equation and compare the solution with the integral analysis of Sec. 6-5.

6-25. An oil with $\Pr = 1{,}960$, $\rho = 860$ kg/m³, $\nu = 1.6 \times 10^{-4}$ m²/s, and $k = 0.14$ W/m·°C enters a 2.5-mm-diameter tube 60 cm long. The oil entrance temperature is 20°C, the mean flow velocity is 30 cm/s, and the tube wall temperature is 120°C. Calculate the heat-transfer rate.

6-26. Liquid ammonia flows through a 2.5-cm-diameter smooth tube 2.5 m long at a rate of 1 lb_m/s. The ammonia enters at 10°C and leaves at 38°C, and a constant heat flux is imposed on the tube wall. Calculate the average wall temperature necessary to effect the indicated heat transfer.

6-27. Water flows over a 3-mm-diameter sphere at 6 m/s. The free-stream temperature is 38°C, and the sphere is maintained at 93°C. Calculate the heat-transfer rate.

6-28. Show that the hydraulic diameter for an annulus space is given by $D_H = d_o - d_i$.

6-29. A 0.13-mm-diameter wire is exposed to an airstream at −30°C and 54 kN/m². The flow velocity is 230 m/s. The wire is electrically heated and is 12.5 mm long. Calculate the electric power necessary to maintain the wire surface temperature at 175°C.

6-30. A spherical water droplet having a diameter of 1.3 mm is allowed to fall from rest in atmospheric air at 1 atm and 20°C. Estimate the velocities the droplet will attain after a drop of 30, 60, and 300 m.

6-31. Air at 90°C and 1 atm flows past a heated $\frac{1}{16}$-in-diameter wire at a velocity of 6 m/s. The wire is heated to a temperature of 150°C. Calculate the heat transfer per unit length of wire.

6-32. A tube bank uses an in-line arrangement with $S_n = S_p = 1.9$ cm and 6.33-mm-diameter tubes. Six rows of tubes are employed with a stack 50 tubes high. The surface temperature of the tubes is constant at 90°C, and atmospheric air is forced across them at an inlet velocity of 4.5 m/s before the flow enters the tube bank. Calculate the total heat transfer per unit length for the tube bank. Estimate the pressure drop for this arrangement.

6-33. Repeat Prob. 6-32 for a staggered tube arrangement with the same values of S_p and S_n.

6-34. A more compact version of the tube bank in Prob. 6-32 can be achieved by reducing the S_p and S_n dimensions while still retaining the same number of tubes. Investigate the effect of reducing S_p and S_n in half, that is, $S_p = S_n = 0.95$ cm. Calculate the heat transfer and pressure drop for this new arrangement.

6-35. Atmospheric air at 20°C flows across a 5-cm square rod at a velocity of 15 m/s. The velocity is normal to one of the faces of the rod. Calculate the heat transfer per unit length for a surface temperature of 90°C.

6-36. A short tube is 6.4 mm in diameter and 15 cm long. Water enters the tube at 1.5 m/s and 38°C, and a constant-heat-flux condition is maintained such that the tube wall temperature remains 28°C above the water bulk temperature. Calculate the heat-transfer rate and exit water temperature.

6-37. Helium at 1 atm and 38°C flows across a 3-mm-diameter cylinder which is heated to 150°C. The flow velocity is 9 m/s. Calculate the heat transfer per unit length of wire. How does this compare with the heat transfer for air under the same conditions?

6-38. Calculate the heat-transfer rate per unit length for flow over a 0.025-mm-diameter cylinder maintained at 65°C. Perform the calculation for (*a*) air at 20°C and 1 atm and (*b*) water at 20°C. $u_\infty = 6$ m/s.

6-39. Compare the heat-transfer results of Eqs. (6-14) and (6-15) for water at Reynolds numbers of 10^3, 10^4, and 10^5 and a film temperature of 90°C.

6-40. Liquid Freon 12 (CCl_2F_2) flows inside a 1.25-cm-diameter tube at a velocity of 3 m/s. Calculate the heat-transfer coefficient for a bulk temperature of 10°C. How does this compare with water at the same conditions?

6-41. A certain home electric heater uses thin metal strips to dissipate heat. The strips are 6 mm wide and are oriented normal to the airstream, which is produced by a small fan. The air velocity is 2 m/s, and seven 35-cm strips are employed. If the strips are heated to 870°C, estimate the total convection heat transfer to the room air at 20°C. (Note, in such a heater, that much of the *total* transfer will be by thermal radiation.)

6-42. Compare Eqs. (6-16) and (6-17) with Eq. (6-14) for a gas with Pr = 0.7 at the following Reynolds numbers: (*a*) 500, (*b*) 1000, (*c*) 2000, (*d*) 10,000, (*e*) 100,000.

6-43. A pipeline in the Arctic carries hot oil at 50°C. A strong arctic wind blows across the 50-cm-diameter pipe at a velocity of 13 m/s and a temperature of −35°C. Estimate the heat loss per meter of pipe length.

6-44. Condensing steam at 150°C is used on the inside of a bank of tubes to heat a cross-flow stream of CO_2 which enters at 3 atm, 35°C, and 5 m/s. The tube bank consists of one hundred 1.25-cm-OD tubes in a square in-line array with $S_n = S_p = 1.875$ cm. The tubes are 60 cm long. Assuming the outside tube wall temperature is constant at 150°C, calculate the overall heat transfer to the CO_2 and its exit temperature.

6-45. A fine wire 0.025 mm in diameter and 15 cm long is to be used to sense flow velocity by measuring the electrical heat which can be dissipated from the wire when placed in an airflow stream. The resistivity of the wire is 70 $\mu\Omega\cdot$cm. The temperature of the wire is determined by measuring its electric resistance relative to some reference temperature T_0 so that

$$R = R_0[1 + a(T - T_0)]$$

For this particular wire the value of the temperature coefficient a is 0.006°C^{-1}. The resistance can be determined from measurements of the current and voltage impressed on the wire, and

$$R = E/I$$

Suppose a measurement is made for air at 20°C with a flow velocity of 10 m/s and the wire temperature is 40°C. What values of voltage and current would be measured for these conditions assuming R_0 is evaluated at $T_0 = 20$°C? What values of voltage and current would be measured for the same wire temperature but flow velocities of 15 and 20 m/s?

6-46. A square duct, 30 by 30 cm, is maintained at a constant temperature of 30°C and an airstream at 50°C and 1 atm is forced across it with a velocity of 6 m/s. Calculate the heat gained by the duct. How much would the heat flow be reduced if the flow velocity were reduced in half?

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