A Heat Transfer Textbook

Fifth Edition

Solutions Manual for Chapters 4–11

by

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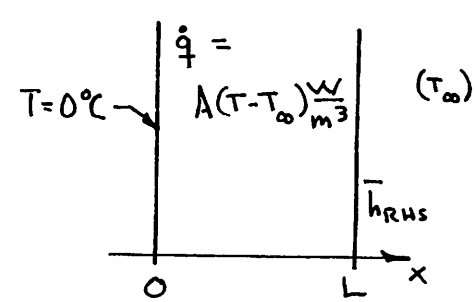
4.1) Make a table listing the general solutions of all steady, uni-dimensional, constant properties, heat conduction problems in Cartesian, cylindrical, and spherical coordinates, with and without uniform heat generation. (This table should prove a very useful tool in future problem work. It should include 18 solutions, all told.)

Geometry		Solution w/o heat generation	Solution with heat generation
cartesian	x-dir.	$T = C_1 x + C_2$	$T = C_1 x + C_2 - \frac{\dot q}{2k} x^2$
	y-dir.	$T = C_1 y + C_2$	$T = C_1 y + C_2 - \frac{\dot q}{2k} y^2$
	z-dir.	$T = C_1 z + C_2$	$T = C_1 z + C_2 - \frac{\dot q}{2k} z^2$
cylindrical	r-dir.	$T = C_1 \ln r + C_2$	$T = C_1 \ln r + C_2 - \frac{\dot q}{4k} r^2$
	θ-dir.	$T = C_1 \theta + C_2$	$T = C_1 \theta + C_2 - \frac{\dot q r^2}{2k} \theta^2$ (where r is some constant value)
	z-dir.	$T = C_1 z + C_2$	$T = C_1 z + C_2 - \frac{\dot q}{2k} z^2$
spherical	r-dir.	$T = C_1 + \frac{C_2}{r}$	$T = C_1 + \frac{C_2}{r} - \frac{\dot q}{6k} r^2$
	θ-dir.	$T = C_1 \ln \tan \frac{\theta}{2} + C_2$	$T = C_1 \ln \tan \frac{\theta}{2} + C_2 + \frac{\dot q r^2}{k} \ln \sin \theta$
	ϕ-dir.	$T = C_1 \phi + C_2$	$T = C_1 \phi + C_1 - \left(\frac{\dot q}{2k} r^2 \sin^2\theta\right) \phi^2$
			where in the last two equations r, and r&θ, respectively, must be constants

Some of these solutions will have limited practical value. For example, the θ-dir. solutions will be applicable only to thin cylindrical and spherical shells whose radius is virtually constant. This must also be the case for the ϕ-dir. solution in spherical configurations, but it is also restricted to a narrow longitudinal swath.

4.2 Develop a dimensionless equation for the temperature in the wall shown:



General solution:

$$\frac{d^2T}{dx^2} = -\frac{A}{k}(T - T_\infty)$$

so

$$T - T_\infty = C_1 \cos\sqrt{A/k}\, x + C_2 \sin\sqrt{A/k}\, x$$

b.c.'s: LHS: $-T_\infty = C_1$

RHS: $-k\frac{\partial T}{\partial x}\Big|_L = \bar h_{RHS}(T - T_\infty)$ or $T_\infty \sin\sqrt{\frac{A}{k}}L - C_2\cos\sqrt{\frac{A}{k}}L = \underbrace{\frac{\bar h_{RHS}\sqrt{\frac{k}{A}}}{k}}_{Bi_{eff.}}\left(T_\infty \cos\sqrt{\frac{A}{k}}L + C_2 \sin\sqrt{\frac{A}{k}}L\right)$

4.2 (Continued)

$$C_2\left(Bi_e \sin\sqrt{\tfrac{A}{k}}L + \cos\sqrt{\tfrac{A}{k}}L\right) = Bi_e T_\infty \cos\sqrt{\tfrac{A}{k}}L - T_\infty \sin\sqrt{\tfrac{A}{k}}L$$

$$C_2 = T_\infty \frac{Bi_e \cos\sqrt{\frac{A}{k}}L - \sin\sqrt{\frac{A}{k}}L}{Bi_e \sin\sqrt{\frac{A}{k}}L + \cos\sqrt{\frac{A}{k}}L}$$

Thus:

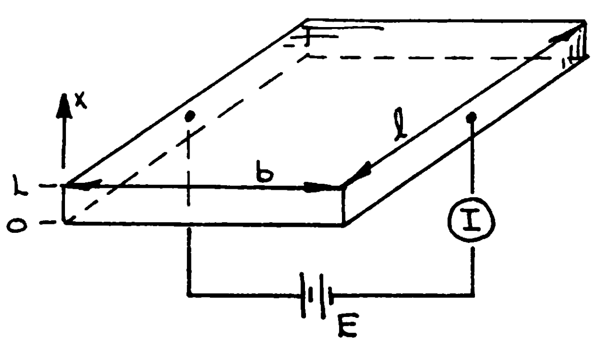
$$\frac{T_\infty - T}{T_\infty} = \cos\sqrt{\frac{A}{k}}x - \frac{Bi_e \cos\sqrt{\frac{A}{k}}L - \sin\sqrt{\frac{A}{k}}L}{Bi_e \sin\sqrt{\frac{A}{k}}L + \cos\sqrt{\frac{A}{k}}L} \sin\sqrt{\frac{A}{k}}x$$

Check the limit as $A \Rightarrow 0$:

$$\frac{T_\infty - T}{T_\infty} = 1 - \frac{Bi_e}{Bi_e\sqrt{\frac{A}{k}}L + 1}\sqrt{\frac{A}{k}}x = 1 - \frac{\frac{\bar{h}_{RHS}L}{k\ L}}{\frac{\bar{h}_{RHS}L}{k} + 1}x = 1 - \frac{1}{1 + \frac{1}{Bi}}\xi \quad \text{(ok)}$$

4.3) A long wide plate of known size, material, and thickness, L, is connected across the terminals of a power supply and serves as a resistance heater. The voltage, current, and T_∞ are known. The plate is insulated on the bottom and transfers heat out the top by convection. The temperature, T_{tc}, of the bottom is measured with a thermocouple. Obtain expressions for a) temperature distribution in the plate, b) h, at the top, c) temperature at the top. (Note that your answers must depend on known information, only.)

$$\dot{q} = \frac{EI}{Lb\ell} \equiv Bk$$



So: $\dfrac{d^2T}{dx^2} + B = 0$

general solution:

$$T = C_1\left(\frac{x}{L}\right) + C_2 - \frac{BL^2}{2}\left(\frac{x}{L}\right)^2$$

b.c.'s: $T(x=0) = T_{tc}$, the thermocouple reading

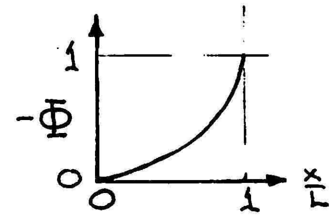
$$\left.\frac{dT}{dx}\right|_{x=0} = 0 \quad \text{, since insulated.}$$

(The b.c.'s are interesting in that both are at x=o. We might have replaced the second one with: $-k\left.\dfrac{dT}{dx}\right|_{x=\delta} = \dfrac{EI}{b\ell}$

apply the b.c.'s:
$$\begin{cases} T_{tc} = 0 + C_2 - 0; & \underline{C_2 = T_{tc}} \\ 0 = C_1 - 0 & ; \underline{C_1 = 0} \end{cases}$$

4.3 (continued)

Therefore: a.) $$\frac{T - T_{tc}}{BL^2/2} = -\left(\frac{x}{L}\right)^2$$



b.) $$h = \frac{-k(dT/dx)x = L}{T_{x=L} - T_\infty} = \underline{\underline{\frac{EI/bl}{T_{top} - T_w}}},$$

c.) $$T_{top} = T(x=L) = \underline{\underline{T_{tc} - \frac{BL^2}{2}}}$$

4.4, 4.5, 4.6 Write the dimensionless functional equation for each of the following situations.

4.4 Heat transfer to a fluid flowing over a plate of length, L.

$$\bar{h} = \bar{h}(u_\infty, \quad \mu, \quad \rho, \quad c_p, \quad k, \quad L)$$

$$\frac{W}{m^2-°C} \quad \frac{m}{s} \quad \frac{kg}{m-s} \quad \frac{kg}{m^3} \quad \frac{J}{kg-°C} \quad \frac{W}{m-°C} \quad m$$

7 var. in 4 dimensions → 7-4 or 3 pi-groups. We choose:

$$\underline{\underline{\frac{\bar{h}L}{k} = fn\left(\frac{\rho u_\infty L}{\mu}, \frac{\mu c_p}{k}\right)}}$$ see eqn. (6.58) and others that follow it.

4.5 Vapor condensing from a pipe. (Call the wavelength, λ.)

$$\lambda = \lambda([\rho_f - \rho_g], \quad \sigma, \quad g)$$

$$m \quad kg/m^3 \quad N/m \quad m/s^2;$$ 4 var in 3 dim → 1 pi-group.

$$\underline{\underline{\lambda\sqrt{\frac{g(\rho_f-\rho_g)}{\sigma}} = fn\ (\text{nothing else}) = \text{constant}}}$$

see equation (9.6b)

4.6 Velocity in a condensate film

$$u\ m/s = u(y\ m,\ g\ m/s^2,\ \nu\ m^2/s,\ \delta\ m)$$

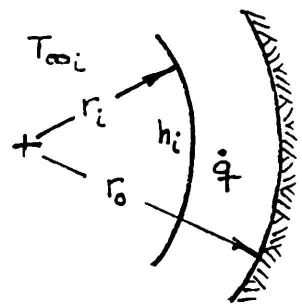
5 variables in 2 dimensions → 3 pi-groups

so

$$\frac{u}{\sqrt{g\delta}} = fn\left(\frac{y}{\delta}, \frac{\nu}{\sqrt{g\delta^3}}\right)$$

We find this situation described by eqn. (8.51) which takes this form when the vapor density is negligible.

4.7 Find the dimensionless temperature distribution in the cylindrical shell shown and plot it for $r_i/r_o = 2/3$. Establish criteria for neglecting convection and internal resistance.



General solution: $T = C_1 \ln r + C_2 - \frac{\dot{q}}{4k} r^2$

if $\Theta \equiv \dfrac{T - T_{\infty_i}}{\dot{q} r_o^2/4k}$ & $\rho \equiv \dfrac{r}{r_o}$ this becomes $\underline{\Theta = C_3 \ln \rho + C_4 - \rho^2}$

with b.c.'s: $\left.\dfrac{\partial T}{\partial r}\right|_{r=r_o} = 0$ or $\underline{\left.\dfrac{d\Theta}{d\rho}\right|_{\rho=1} = 0}$

and: $\left.\dfrac{\partial (T - T_{\infty_i})}{\partial r}\right|_{r=r_i} = \dfrac{h_i}{k}(T - T_{\infty_i})_{r=r_i}$ or $\underline{\left.\dfrac{d\Theta}{d\rho}\right|_{\rho=\rho_i} = Bi\,\Theta_{\rho=\rho_i}}$

impose the first b.c. on the gen'l. sol'n.: $0 = C_3 - 2$, $\underline{C_3 = 2}$

impose 2nd b.c.: $\dfrac{C_3}{\rho_i} - 2\rho_i = Bi\left(C_3 \ln \rho_i + C_4 - \rho_i^2\right)$, but $C_3 = 2$

so $\underline{C_4 = \dfrac{2}{Bi}\left(\dfrac{1}{\rho_i} - \rho_i\right) - 2\ln\rho_i + \rho_i^2}$

Return to the gen'l solution with these constants:

$$\Theta = 2\ln\rho + \frac{2}{Bi}\left(\frac{1-\rho_i^2}{\rho_i}\right) - 2\ln\rho_i + \rho_i^2 - \rho^2$$

or

$$\underline{\underline{\Theta = -(\rho^2 - \rho_i^2) + 2\ln\frac{\rho}{\rho_i} + \frac{2}{Bi}\left(\frac{1-\rho_i^2}{\rho_i}\right)}}$$

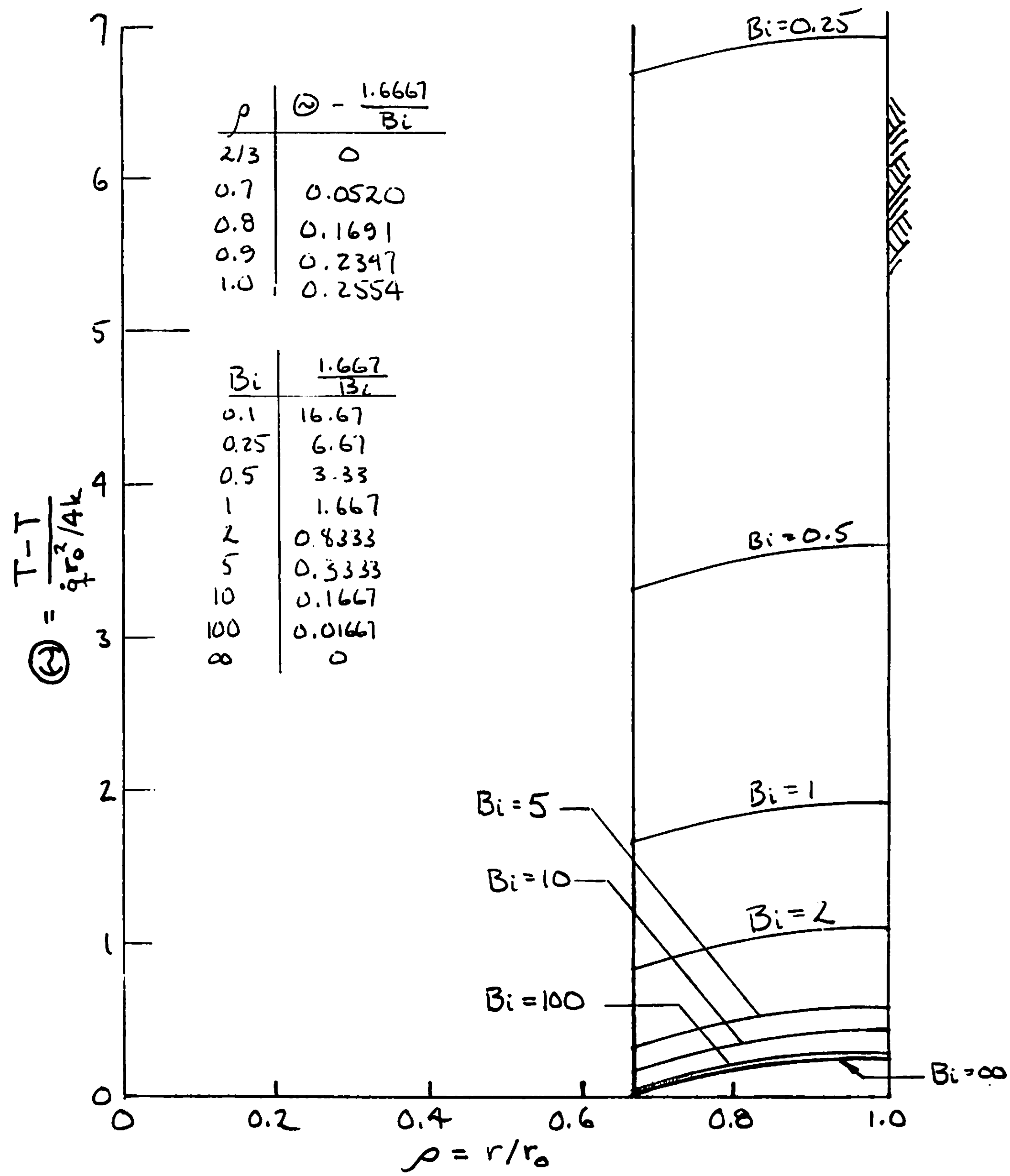
Notice that for Bi large this approaches $\Theta = -(\rho^2 - \rho_i^2) + 2\ln\dfrac{\rho}{\rho_i}$

and " " small " " $\Theta = \dfrac{2}{Bi}\left(\dfrac{1-\rho_i^2}{\rho_i}\right)$

4.7 (continued)

for $\rho_i = 2/3$ the equation reduces to:

$$\Theta = -(\rho^2 - 0.4444) + 2\ln 1.5\rho + \frac{1.6667}{Bi}$$



ρ	$\Theta - \frac{1.6667}{Bi}$
2/3	0
0.7	0.0520
0.8	0.1691
0.9	0.2347
1.0	0.2554

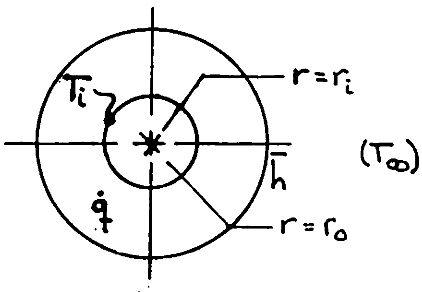
Bi	$\frac{1.667}{Bi}$
0.1	16.67
0.25	6.67
0.5	3.33
1	1.667
2	0.8333
5	0.3333
10	0.1667
100	0.01667
∞	0

When Bi = 0.25, the temperature distribution within the tube wall is within 4 percent of uniform (0.8 percent at Bi = 0.1, etc.)

When Bi = 100, the temperature drop through h_i is 6 percent of that inside the tube wall (3 percent at Bi = 200, etc.)

4.8 Steam condenses in a small pipe keeping the inside at a temperature, T_i. The pipe releases $\dot{q}$ W/m^3 within its walls as a result of electric current flowing through it. The outside temperature is T_∞ and there is a heat transfer coefficient $\bar{h}$ on the outside. a) Evaluate the dimensionless temperature distribution in the pipe. b) Plot the result for an inside radius that is 2/3 of the outside radius. c) Discuss interesting aspects of the result.

a)



1) $T = T(r)$

2) $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) = -\dfrac{\dot{q}}{k}$

3) $T = -\dfrac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$

4) $T(r = r_i) = T_i$

$$\bar{h}\,(T - T_\infty)_{r=r_o} = -k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}$$

5) $\underline{C_2 = T_i + \dfrac{\dot{q}}{4k}r_i^2 - C_1 \ln r_i}$

$$-\frac{\dot{q}}{4k}r_o^2 + C_1 \ln r_o + C_2 - T_\infty = +\frac{k}{\bar{h}}\left(\frac{\dot{q}\,r_o}{2k} - \frac{C_1}{r_o}\right)$$

Combine: $\dfrac{\dot{q}}{4k}(r_i^2 - r_o^2) + C_1 \ln r_o/r_i + \underbrace{(T_i - T_\infty)}_{\equiv \Delta T} = \dfrac{\dot{q}\,r_o}{2\bar{h}} - \dfrac{C_1}{Bi}$

$$\therefore\ C_1 = \underline{\frac{\dfrac{\dot{q}\,r_o^2}{4k\Delta T}\left[\dfrac{2}{Bi} - \left(\dfrac{r_i}{r_o}\right)^2 + 1\right] - 1}{\ln\dfrac{r_o}{r_i} + \dfrac{1}{Bi}}\,\Delta T}$$

6) $$\frac{T - T_i}{T_i - T_\infty} = -\frac{\dot{q}\,r_o^2}{4k\Delta T}\left[\left(\frac{r}{r_o}\right)^2 - \frac{\frac{2}{Bi} - \left(\frac{r_i}{r_o}\right)^2 + 1}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}\ln r - \left(\frac{r_i}{r_o}\right)^2 + \frac{\frac{2}{Bi} - \left(\frac{r_i}{r_o}\right)^2 + 1}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}\ln r_i\right] - \frac{\ln r/r_i}{\ln\frac{r_o}{r_i} + \frac{1}{Bi}}$$

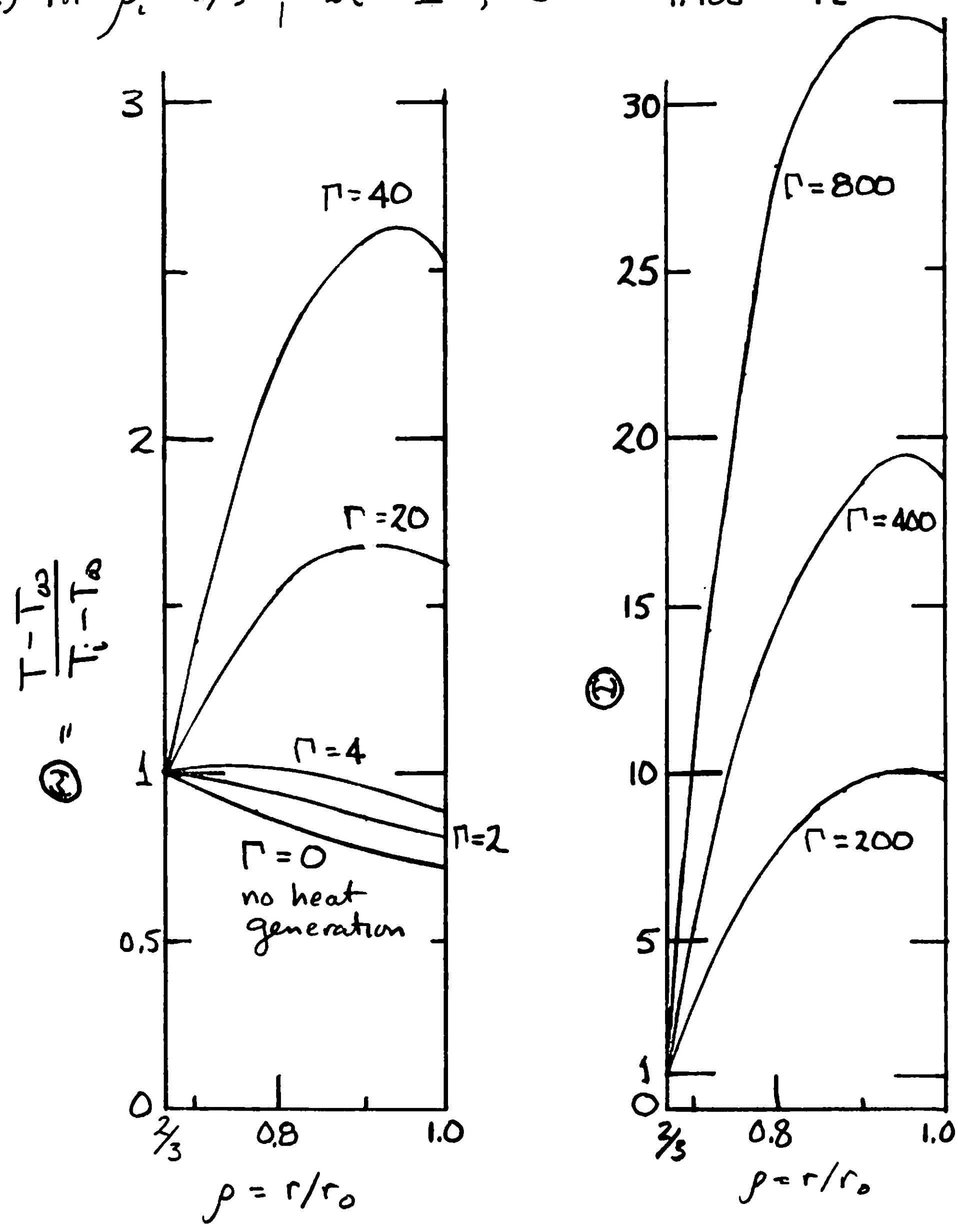
$$\frac{T - T_\infty + T_\infty - T_i}{T_i - T_\infty} = \frac{\Gamma}{4}\left(\rho^2 - \rho_i^2 - \frac{\frac{2}{Bi} - \rho_i^2 + 1}{-\ln\rho_i + \frac{1}{Bi}}\ln(\rho/\rho_i)\right) - \frac{\ln\rho/\rho_i}{\frac{1}{Bi} - \ln\rho_i}$$

4.8 (continued)

This can be rewritten as

$$\Theta = 1 - \frac{\Gamma}{4}\left[\rho^2 - \rho_i^2 + \frac{\frac{2}{Bi} + 1 - \rho_i^2}{\ln \rho_i - \frac{1}{Bi}} \ln \frac{\rho}{\rho_i}\right] - \frac{\ln \rho/\rho_i}{\frac{1}{Bi} - \ln \rho_i}$$

b.) for $\rho_i = 2/3$ & $Bi = 1$, $\Theta = 1 - \frac{\ln 3\rho/2}{1.405} - \frac{\Gamma}{4}\left[\rho^2 - \frac{4}{9} - 1.818 \ln \frac{3\rho_i}{2}\right]$



We see that internal heat generation becomes important for $\Gamma > O(1)$. For $\Gamma \gtrsim 2$, that the temperature maximizes within the shell.

4.9 Solve Problem 2.5, putting it in dimensionless form first. With reference to the Problem 2.5 solution, we repeat the steps as follows.

Step 3) $T = C_1 \ln r + C_2$ becomes $\underline{\dfrac{T - T_{\infty_i}}{T_{\infty_o} - T_{\infty_i}} \equiv \Theta = C_3 \ln \rho + C_4}$, $\rho \equiv \dfrac{r}{r_i}$

step 4) $\left.\dfrac{\partial \Theta}{\partial \rho}\right|_{\rho=1} = Bi_i \, \Theta_{\rho=1}$

$\left.\dfrac{\partial \Theta}{\partial \rho}\right|_{\rho_o} = Bi_o (1 - \Theta_{\rho_o}) / \rho_o$

step 5) $C_3 = Bi_i C_4$ & $\dfrac{C_3}{\rho_o} = Bi_o(1 - C_3 \ln \rho_o - \underbrace{C_4}_{= C_3/Bi_i})/\rho_o$

or

$C_4 = \dfrac{C_3}{Bi_i}$ & $C_3 = 1 \Big/ \left(\dfrac{1}{Bi_o} + \dfrac{1}{Bi_i} + \ln \rho_o \right)$

step 6) $$\Theta = \frac{\ln \rho + 1/Bi_i}{\dfrac{1}{Bi_o} + \dfrac{1}{Bi_i} + \ln \rho_o}$$ Same result as in Prob. 2.5 with a lot less algebra

when we allow Bi_i and $Bi_o \Rightarrow \infty$, $\Theta \Rightarrow \dfrac{\ln r/r_i}{\ln r/r_o}$ cf. Ex. 2.4

4.10 Complete the algebra leading to equation (4.41).

we have: $\dfrac{d^2(T - T_\infty)}{dx^2} = \dfrac{\bar{h}P}{kA}(T - T_\infty)$ or $\dfrac{d^2\Theta}{d\xi^2} = (mL)^2 \Theta$, so $\underline{\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}}$

subject to: $(T - T_\infty)_{x=0} = (T_o - T_\infty)$ or $\Theta(\xi = 0) = 1$, so $\underline{1 = C_1 + C_2}$

and to: $\left.\dfrac{d(T - T_\infty)}{dx}\right|_{x=L} = 0$ or $\left.\dfrac{d\Theta}{d\xi}\right|_{\xi=1} = 0$, so $\underline{0 = C_1 e^{mL} - C_2 e^{-mL}}$

Now put $C_1 = 1 - C_2$ from 1st b.c., in 2nd b.c. & get: $\underline{C_2 = \dfrac{e^{mL}}{e^{mL} + e^{-mL}} = \dfrac{1}{2} \dfrac{e^{mL}}{\cosh mL}}$

Then:
$$\Theta = \frac{2 e^{mL\xi} \cosh mL - e^{mL(1+\xi)} + e^{mL(1-\xi)}}{2 \cosh mL} = \frac{\cancel{e^{mL(1+\xi)}} + e^{-mL(1-\xi)} - \cancel{e^{mL(1+\xi)}} + e^{mL(1-\xi)}}{2 \cosh mL}$$

so:
$$\Theta = \frac{\cosh mL(1-\xi)}{\cosh mL}$$

Problem 4.11 Derive eqn. (4.48)

Solution

We already have the dimensionless form of the general solution of eqn. (4.30) in eqn. (4.35)

$$\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$$

and the dimensionless form of the b.c.s. (eqn. 4.31a) given in eqn. (4.46).

We put the solution in the two b.c.'s and get:

$$\Theta(\xi=0) = 1 \Rightarrow 1 = C_1 + C_2 \quad \text{or} \quad \underline{C_1 = 1 - C_2}$$

$$\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = -Bi_{ax}\,\Theta(\xi=1) \Rightarrow mLe^{mL}C_1 - mLe^{-mL}C_2 = -Bi_{ax}(C_1 e^{mL} + C_2 e^{-mL})$$

We put $C_1 = 1 - C_2$ in this, rearrange it, and get:
$$C_2 = \frac{e^{mL} + \dfrac{Bi_{ax}}{mL} e^{mL}}{2\left(\cosh mL + \dfrac{Bi_{ax}}{mL} \sinh mL\right)}$$

Put this C_2 in $\Theta = (1-C_2)^{mL\xi} + C_2 e^{-mL\xi}$ and get:

$$\Theta = \frac{2e^{mL\xi}\left(\cosh mL + \dfrac{Bi_{ax}}{mL}\sinh mL\right) - \left(e^{mL} + \dfrac{Bi_{ax}}{mL} e^{mL}\right)e^{mL\xi} + \left(e^{mL} + \dfrac{Bi_{ax}}{mL}\right)e^{-mL\xi}}{2\left(\cosh mL \quad \dfrac{Bi_{ax}}{mL}\sinh mL\right)}$$

$$= \frac{\cancel{e^{mL(1+\xi)}} + e^{-mL(1-\xi)} + \cancel{\dfrac{Bi_{ax}}{mL}}\left(\cancel{e^{mL(1+\xi)}}\right) - \cancel{e^{mL(1+\xi)}} - \cancel{\dfrac{Bi_{ax}}{mL}}\cancel{e^{mL(1+\xi)}}}{}$$

(continued)

$$\frac{e^{mL(1-\xi)} + \dfrac{Bi_{ax}}{mL} e^{mL(1-\xi)}}{2\left(\cosh mL + \dfrac{Bi_{ax}}{mL}\sinh mL\right)}$$

$$\Theta = \frac{\frac{1}{2}\left[e^{mL(1-\xi)} + e^{mL(1-\xi)}\right] + \frac{1}{2}\dfrac{Bi_{ax}}{mL}\left[e^{mL(1-\xi)} - e^{-mL(1-\xi)}\right]}{\cosh mL + \dfrac{Bi_{ax}}{mL}\sinh mL}$$

or, finally:

$$\underline{\underline{\Theta = \frac{\cosh mL(1-\xi) + \dfrac{Bi_{ax}}{mL}\sinh mL(1-\xi)}{\cosh mL + \dfrac{Bi_{ax}}{mL}\sinh mL(1-\xi)}}}$$

4.12 Obtain the infinite fin result: $\Theta = e^{-mL\xi}$ by starting with the general fin solution: $\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$ and using the b.c.'s $T(x=0) = T_o$ or $\Theta(\xi=0) = 1$ and $T(x=L) = T_\infty$ or $\Theta(\xi=L) = 0$. Discuss fully.

From the first b.c. we get $C_1 = 1-C_2$ as before. From the second b.c. we obtain: $0 = C_1 e^{mL} + C_2 e^{-mL}$

$$\text{or} \quad 0 = e^{mL} - C_2 \underbrace{(e^{mL} - e^{-mL})}_{\equiv\, 2 \sinh mL}$$

$$\text{or} \quad C_2 = \frac{e^{mL}}{2 \sinh mL}$$

Then the general solution becomes:

$$\Theta = e^{-mL\xi} - C_2 (e^{mL\xi} - e^{-mL\xi}) = \frac{e^{mL\xi}(e^{mL} - e^{-mL}) - e^{mL(1+\xi)} + e^{mL(1-\xi)}}{2 \sinh mL}$$

$$\Theta = \frac{\cancel{e^{mL(1+\xi)}} - e^{-mL(1-\xi)} - \cancel{e^{mL(1+\xi)}} + e^{mL(1-\xi)}}{2 \sinh mL}$$

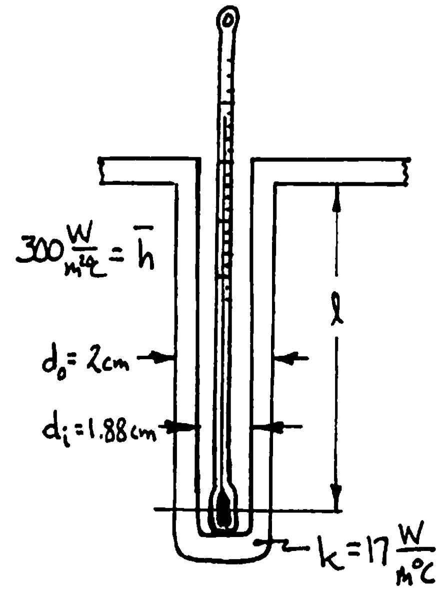
$$\Theta = \frac{\sinh mL(1-\xi)}{\sinh mL}$$

When x is large, $\sinh x = \dfrac{e^x - e^{-x}}{2} \to \dfrac{e^x}{2}$. Therefore, when mL is large

$$\Theta \to \frac{e^{mL(1-\xi)}}{e^{mL}} = e^{-mL\xi}$$ which is eqn. (4.50)

It follows that these b.c.s are not enough, by themselves, to get the infinite fin result. We must also require that the fin be very long.

4.13 How long must l be to guarantee an error less than than 0.5 percent in the thermometer well shown.



Find ml

$$ml=\sqrt{\frac{\bar{h}Pl^2}{kA}}=\sqrt{\frac{\bar{h}\,\pi d_o l^2 4}{k\,\pi(d_o^2-d_i^2)}}$$

$$=\sqrt{\frac{300(0.02)4}{17(0.0004-0.00035\,34)}}\;l=174\,l$$

As long as $l>0.0172$ m, ml will be greater than 3 and we can use the "finite fin" approximation.

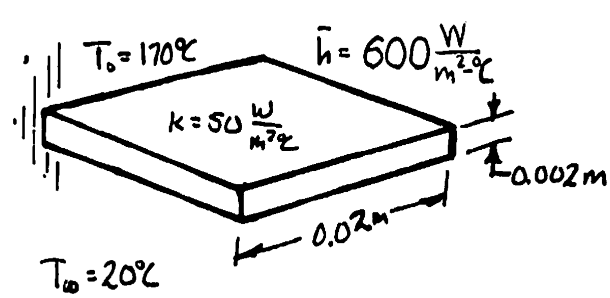
$$\Theta=\frac{1}{\cosh ml}\leqslant 0.005\quad,\quad\therefore\; e^{ml}+e^{-ml}=400$$

This is true for $ml=5.992$.

Therefore $\quad 5.992=174\,l \quad$ so $\quad l=0.03444$ m

This means that the well must only be <u>3.44 cm</u> in length to guarantee the required accuracy.

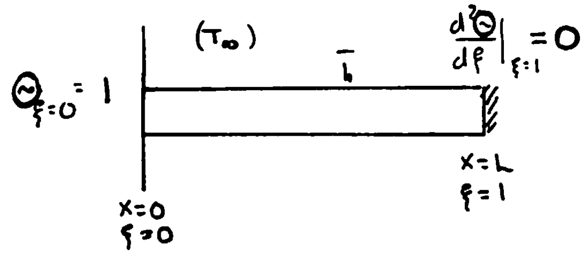
4.14 What is the maximum possible heat flux from the fin shown.



$$Q=\sqrt{kA\bar{h}P}\,(T_o-T_\infty)\underbrace{\tanh mL}_{\text{max value for long L is 1}}=\sqrt{50(.00004)(600)2(.002+.02)}\,(170-20)$$

$$=\underline{\underline{34.5\text{ W}}}$$

4.15) A thin rod is anchored at a wall at $T=T_0$ on one end, and is insulated at the other end. Plot the dimensionless temperature distribution in the rod as a function of dimensionless length: a) if the rod is exposed to an environment at T_∞ through a heat transfer coefficient, and b) if the rod is insulated but heat is consumed in it at the uniform rate $-\dot{q}=\bar{h}P(T_0-T_\infty)/A$. Comment on the implications of the comparison.



$$\frac{d^2\Theta}{d\xi^2}=(mL)^2\Theta \quad \text{in case a)}$$
$$=(mL)^2 \quad \text{in case b)}$$

case a) We already know the solution. It is $\underline{\Theta_a=\dfrac{\cosh mL(1-\xi)}{\cosh mL}}$

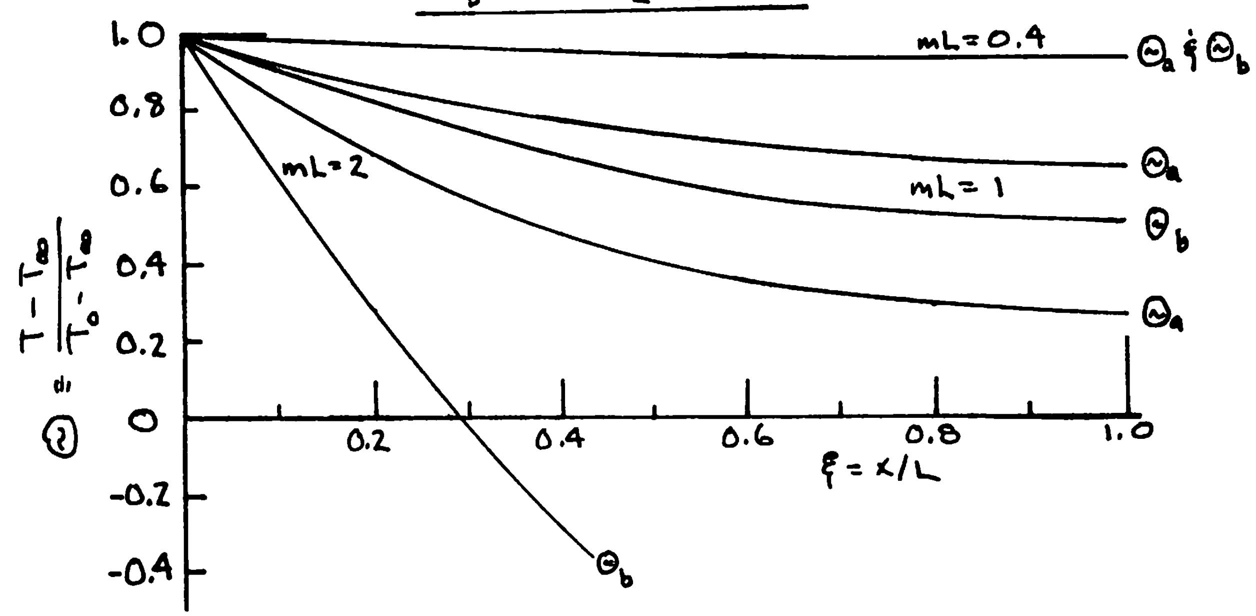
case b) $\dfrac{d^2\Theta}{d\xi^2}=(mL)^2 = \underline{\text{constant}}$ so we integrate twice and get:

the general solution: $\Theta=\dfrac{(mL)^2}{2}\xi^2+C_1\xi+C_2$

Apply 1st b.c.: $\Theta(\xi=0)=\underline{C_2=1}$

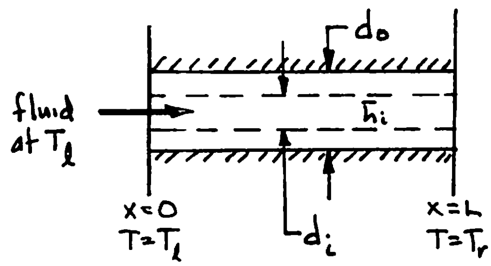
Apply 2nd b.c.: $\left.\dfrac{d^2\Theta}{d\xi^2}\right|_{\xi=1}=(mL)^2+C_1=0$, $\underline{C_1=-(mL)^2}$

Therefore $\underline{\Theta_b=(mL)^2\left(\frac{\xi^2}{2}-\xi\right)+1}$



When Θ is close to unity, $\bar{h}P(T_0-T_\infty)/A \cong \bar{h}P(T-T_\infty)/A$, or $(mL)^2\Theta \cong (mL)^2$, and the problems (and their solutions) become identical. As Θ becomes < 1 the solutions diverge. the energy consumption in case b is unabated and Θ_b is generally $< \Theta_a$.

4.16 Consider the tube shown below. Fluid enters the tube on the left at $T = T_\ell$. Assume its temperature to remain constant. Evaluate and plot the temperature distribution in the tube.



$$\frac{d^2(T-T_\ell)}{dx^2} = -\frac{\dot{q}}{k} = \frac{\pi d_i \bar{h}_i (T_\ell - T)}{\frac{\pi}{4}(d_o^2 - d_i^2)\,k}$$

or $$\frac{d^2\Theta}{d\xi^2} = (mL)^2\Theta$$

where $\Theta \equiv \dfrac{T-T_\ell}{T_r-T_\ell}$, $m \equiv \sqrt{\dfrac{4\bar{h}_i}{k d_i\left(\frac{d_o^2}{d_i^2}-1\right)}}$

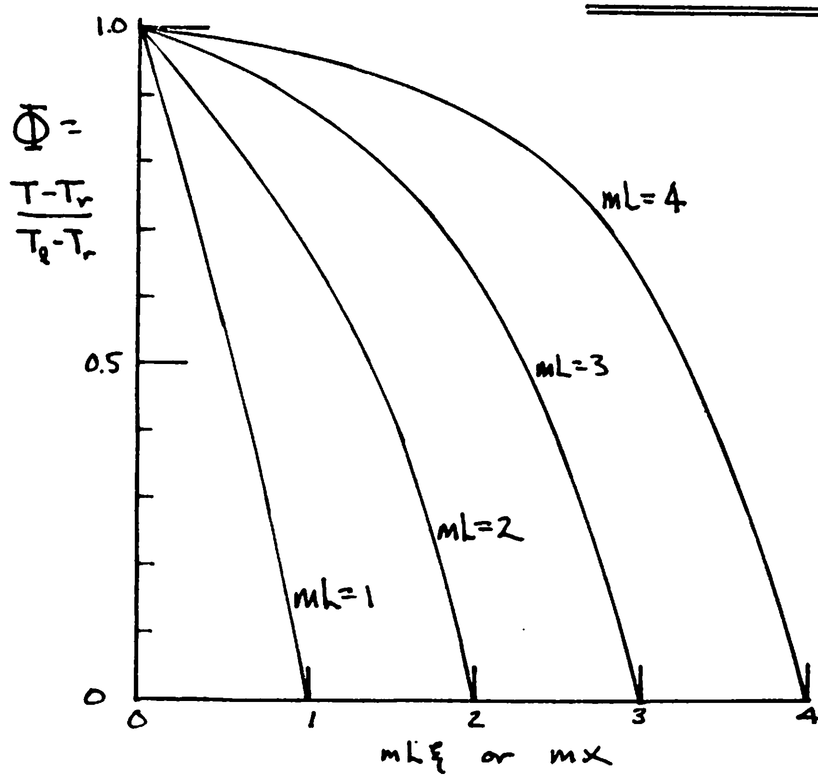
Gen'l. soln.: $\Theta = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$

left b.c.: $\Theta(\xi=0) = 0 = C_1 + C_2$ so $C_1 = -C_2$

right b.c.: $\Theta(\xi=1) = 1 = C_2(e^{-mL} - e^{mL})$ so $C_2 = -1/2\sinh mL$

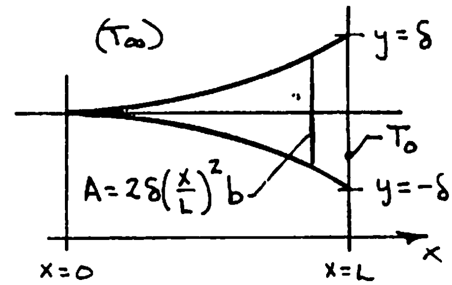
Thus: $$\underline{\underline{\Theta = \frac{\sinh mL\xi}{\sinh mL}}}$$ or we might better define $\Phi \equiv \dfrac{T-T_r}{T_\ell - T_r}$

so: $$\underline{\underline{\Phi = 1 - \Theta = 1 - \frac{\sinh mL\xi}{\sinh mL}}}$$



Notice that when m is small, the influence of convection is also small and the temperature distribution is almost linear as it would be in pure conduction. In the other extreme -- the convection dominated or large m case -- the temperature distribution remains near T_ℓ, except as it approaches the right-hand wall.

4.17 Plot the temperature distribution in the fin shown and evaluate η_t.



Eqn. (4.57) becomes:

$$\frac{d}{d\xi}\left[2\delta(\xi^2)b\,\frac{d\frac{T-T_\infty}{T_0-T_\infty}}{d\xi}\right] = \frac{\bar{h}P}{k}L^2 \underbrace{\frac{T-T_\infty}{T_0-T_\infty}}_{\equiv \Theta}$$

or: $$\xi^2\frac{d^2\Theta}{d\xi^2} + 2\xi\frac{d\Theta}{d\xi} - \underbrace{\frac{\bar{h}PL^2}{k(2\delta b)}}_{\equiv (mL)^2}\Theta = 0$$

To solve this (Euler's d.e.) we look for a solution of the form: $\Theta = c\xi^P$

so: $$P(P-1)\xi^P + 2P\xi^P - (mL)^2\xi^P = 0 \quad \text{or} \quad \underline{P^2 + P - (mL)^2 = 0}$$

This has two solutions: $$P_1 \text{ and } P_2 = \pm\sqrt{\tfrac{1}{4}+(mL)^2} - \tfrac{1}{2}$$

so the general solution is: $$\Theta = C_1\xi^{P_1} + C_2\xi^{P_2}$$

and the usual b.c.'s give: $$\Theta(\xi=1) = 1 = C_1 + C_2$$
$$\Theta(\xi=0) = 0 = \left[C_1\xi^{P_1} + C_2\xi^{P_2}\right]_{\xi=0}$$

Notice that P_2 must be negative so C_2 must be zero to satisfy this. Therefore $C_2 = 0$ and $C_1 = 1$, and:

$$\Theta = \xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}$$

The efficiency is:

$$\eta_f = \frac{\int_0^L \bar{h}(T-T_\infty)\,b\,dx}{\bar{h}(T_r-T_\infty)\,bL} = \int_0^1 \Theta(\xi)\,d\xi$$

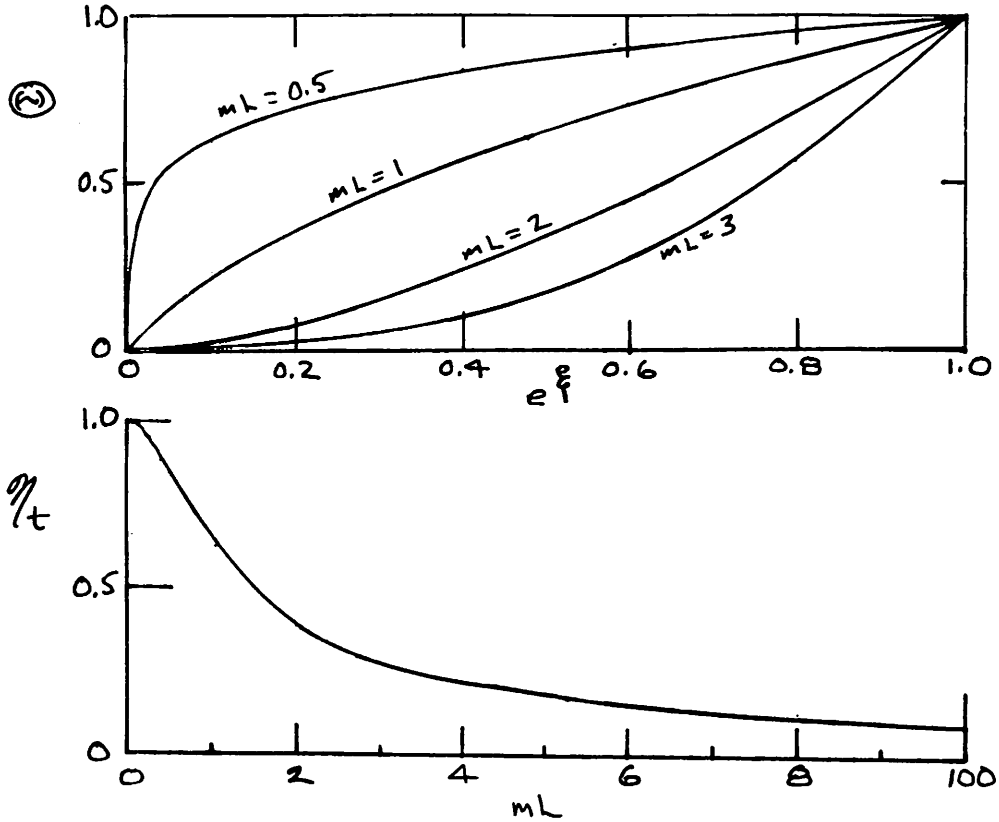
$$= \int_0^1 \xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}\,d\xi = \frac{1}{\sqrt{\frac{1}{4}+(mL)^2}+\frac{1}{2}}\,\xi^{\sqrt{\frac{1}{4}+(mL)^2}-\frac{1}{2}}\Bigg]_0^1$$

so:

$$\eta_f = \frac{2}{\sqrt{1+4(mL)^2}+1}$$

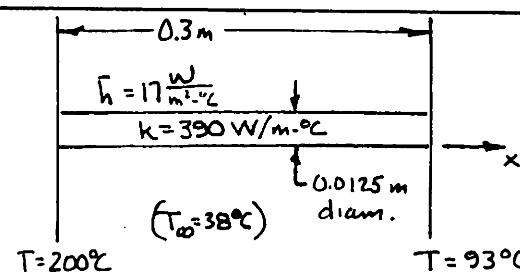
(cont'd.)

Before we plot these results, we note that mL in this case is the same as $mL(L/P)^{1/2}$ in Fig. 4.13.



4.18 Problem 4.18 was solved under a full nondimensionalization in the solution given for 2.21. We do not repeat it here.

4.19 A fin connects two walls as shown. How much heat is removed from its surface?



$$\Theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\left[\Theta \equiv \frac{T - T_\infty}{T_{left} - T_\infty}\right] \qquad \text{b.c.'s:} \quad \underline{\Theta_l = 1 = C_1 + C_2} \quad ; \quad C_1 = 1 - C_2$$

$$\underline{\Theta_r = C_1 e^{ml} + C_2 e^{-ml}}$$

Then: $\Theta_r = (1 - C_2)e^{ml} + C_2 e^{-ml}$; $\underline{C_2 = \dfrac{e^{ml} - \Theta_r}{2\sinh ml}}$ $\quad \underline{C_1 = 1 - \dfrac{e^{ml} - \Theta_r}{2\sinh ml}}$

so:

$$\underline{\underline{\Theta = \left(1 - \frac{e^{ml} - \Theta_r}{2\sinh ml}\right)e^{mx} + \frac{e^{ml} - \Theta_r}{2\sinh ml}\, e^{-mx}}}$$

4.19 (continued)

$$Q = -kA\Delta T\left[\frac{\partial \Theta}{\partial x}\bigg|_{x=0} - \frac{\partial \Theta}{\partial x}\bigg|_{x=L}\right] = -kmA\Delta T\left[\left(1-\frac{e^{-ml}-\Theta_r}{2\sinh ml}\right)(1-e^{mL}) + \frac{e^{ml}-\Theta_r}{2\sinh mL}(1-e^{-mL})\right]$$

$$= 390\,\pi(.00625)^2(200-38)\underbrace{\sqrt{\frac{17(4)}{390(.0125)}}}_{m=3.735}\left[\left(1-\underbrace{\frac{3.065-\frac{53-38}{200-38}}{e^{1.12}-e^{-1.12}}}_{0.9952}\right)\underbrace{(1-e^{1.12})}_{-2.065}+0.9952\underbrace{(1-e^{-1.12})}_{0.6705}\right]$$

$$= \underline{\underline{19.13\ W}}$$

4.20 How much error does the insulated tip assumption giverise to in example 4.8?

Calculate $\%\,error = \dfrac{Q_{ins.} - Q_{unins.}}{Q_{unins.}}$ using eqns. (4.44) and (4.48):

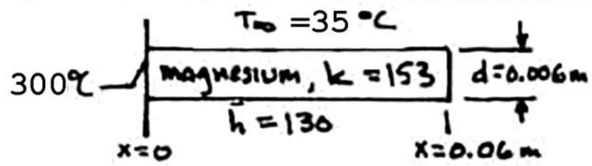
$$\%\,error = \frac{\tanh mL\left(1+\frac{Bi_{ax}}{mL}\tanh mL\right) - \frac{Bi_{ax}}{mL} - \tanh mL}{\frac{Bi_{ax}}{mL} + \tanh mL}\,100$$

$$= \frac{\frac{Bi_{ax}}{mL}\left(\tanh^2 mL - 1\right)}{\frac{Bi_{ax}}{mL} + \tanh mL}\,100$$

From Example 4.8, $mL = 0.8656$, $\dfrac{Bi_{ax}}{mL} = \dfrac{0.0468}{0.8656} = 0.0541$, so

the error $= \underline{\underline{3.67\ \%}}$

4.21 Compute the heat removed from the fin shown, con- root depression. Assume the tip to be insulated.



$$mL = \sqrt{\frac{130(\pi)(0.006)}{153(\pi)(.003)^2}}\,(0.06) = 1.428$$

$$Q = \sqrt{k\bar{A}\bar{h}P}\,(\Delta T)\tanh mL = \sqrt{153\,\pi^2(.003)^2(.006)130}\,(265)(0.8913) = \underline{\underline{24.38\ W}}$$

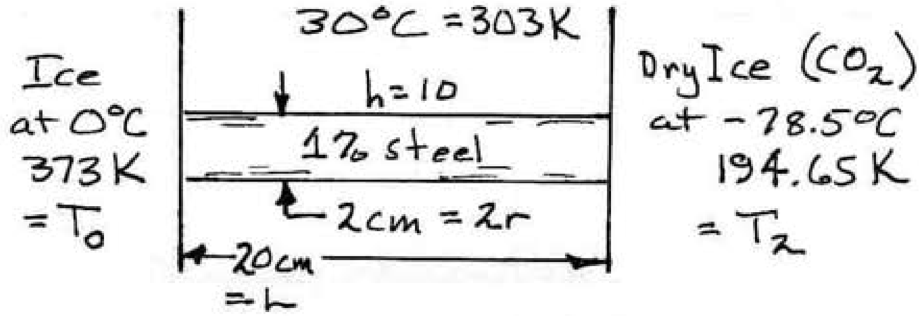
The fin efficiency, $\eta_f = \tanh(mL)/mL = 0.8913/1.428 = 0.624 = \underline{62.4\%}$

The fin effectiveness, $\epsilon = \eta_f$ (fin surface area)/fin cross-sectional area

$\epsilon = 0.624(2\pi rL/\pi r^2) = 1.248L/r = \underline{\underline{25}}$

4.22 A 2 cm dia. horizontal 1.0% steel rod connects a block of ice with a block of dry ice (CO_2) in a 30°C room. The frozen blocks are insulated from the room. The rod is embedded in each block with a 20 cm span between the blocks. The heat transfer coefficient between the rod and the room is 10 W/m²K. Will the ice begin to melt when the rod is at steady state?

Solution We need to determine whether the temperature gradient in rod is positive or negative where it enters the ice on the left. If it is positive, heat will flow into the ice and it will begin to melt. Fortunately, we have already solved for the temperature distribution in a "fin" with specified temperatures at two ends, in Problem 4.19. We need only differentiate that expression for temperature, and determine whether the slope is positive or negative.



$$mL = \sqrt{\frac{hPL^2}{kA}} \qquad \text{but } P/A = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

$$= \sqrt{\frac{2hL^2}{kr}} = \sqrt{\frac{2(10)(0.2)^2}{43(0.01)}} = \underline{1.364}$$

We obtain from the solution of Problem 4.19:

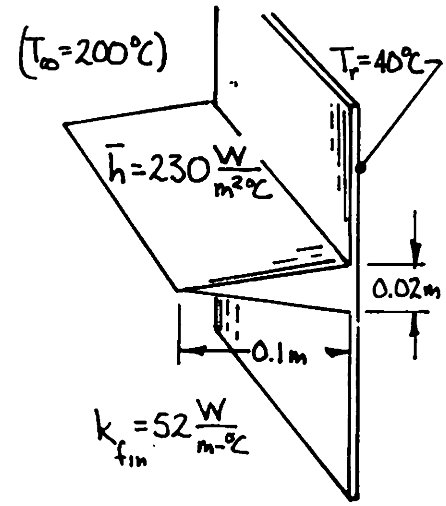
$$\Theta = \left(1 - \frac{e^{mL} - \Theta_2}{2\sinh mL}\right)e^{mx} + \left(\frac{e^{mL} - \Theta_2}{2\sinh mL}\right)e^{-mx}$$

Where $\Theta = \dfrac{T - T_\infty}{T_0 - T_\infty} = \dfrac{T - 303}{273 - 303}$ so $\Theta_0 = 1$, $\Theta_2 = 3.612$

Then $\left.\dfrac{d\Theta}{dx}\right|_{x=0} = m\left[1 - \dfrac{e^{mL} - 3.612}{\sinh(1.364)}\right]$ = 0.836m

The slope is thus positive & the ice will melt.

4.23 Compute the heat removed by the fin shown.



$m = \sqrt{\frac{\bar{h}P}{kA}}$ where $A = \frac{1}{2}(0.1)(0.02) = 0.001\,m^2$

$mL\sqrt{\frac{L}{P}} = \sqrt{\frac{\bar{h}L}{kA}}\,L = \sqrt{\frac{230(0.1)}{52(0.001)}}\,0.1 = 2.1$

This gives an efficiency, from Fig. 4.13b, of: $\underline{\eta_t = 0.415}$

$$Q = \eta_t\left(A\,\bar{h}\,[200-40]\right) = 0.415\left(2\sqrt{0.1^2+0.01^2}\right)(230)(160) = 3070\ \frac{W}{m}$$

4.24 The initial temperature distribution in a slab of width, L, is:
$\frac{T-T_w}{\dot{q}L^2/k} = \frac{1}{2}\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right)$ where $\dot{q}L^2/k$ can be viewed as a constant, A, with the units of temperature. The sides are kept at T_w and the slab is permitted to cool. Predict $(T-T_w)/A$ as a function of x & t.

The maximum amplitude of the parabolic distribution is $\frac{T-T_w}{A} = \frac{1}{8}$ so we approximate the initial distribution with $\frac{T-T_w}{A} = \frac{\sin\pi(x/L)}{8}$.
The heat diffusion equation can be written as: $\frac{d^2\left(\frac{T-T_w}{A}\right)}{dx^2} = \frac{1}{\alpha}\frac{\partial\left(\frac{T-T_w}{A}\right)}{\partial t}$
so the general solution (eqn. (4.11)) becomes:

$\frac{T-T_w}{A} = (D\sin\lambda x + E\cos\lambda x)\,e^{-\alpha\lambda^2 t}$. Then:

b.c. at $x=0$: $\frac{T-T_w}{A} = 0 = (0+E)e^{-\alpha\lambda^2 t}$ so $\underline{E=0}$

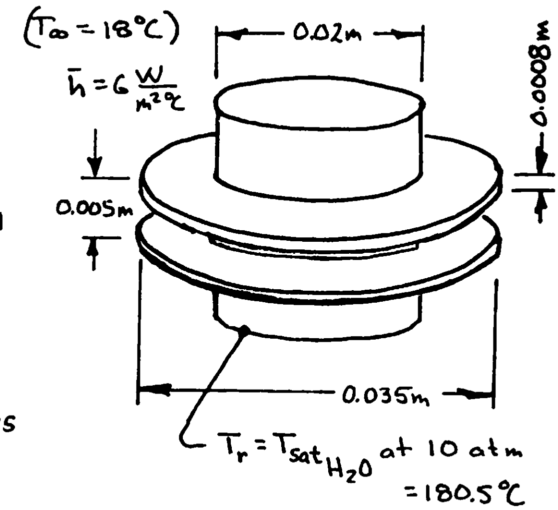
b.c. at $x=L$: $\frac{T-T_w}{A} = 0 = (D\sin\lambda L)e^{-\alpha\lambda^2 t}$ so $\underline{\lambda = \pi/L}$

i.c.: $\frac{T-T_w}{A} \simeq \frac{\sin\pi(x/L)}{8} = D\sin\pi(x/L)$ so $D \simeq \frac{1}{8}$

Thus:

$$\frac{T-T_w}{A} \simeq \frac{\sin\pi(x/L)}{8}\,e^{-\pi^2\frac{\alpha t}{L^2}}$$

4.25 A 1.5 m length of pipe is finned as shown. Find the rate at which steam at 10 atm, within the tube, will be condensed.



$k_{fin} \simeq 190 \frac{W}{m\text{-}°C}$

First evaluate the heat removal

$r_2/r_1 = 3.5/2 = 1.75$

$$mL\sqrt{\frac{L}{P}} = \sqrt{\frac{\bar{h}L}{kA}}\,L$$

$$= \sqrt{\frac{6(0.0075)}{190(0.0008 \times 0.0075}}\;0.0075$$

$$= 0.04712$$

So from Fig. 4.13a we read, $\eta_t = 0.98$

Then: $Q_{fin} = 0.98\left[(6)\,2\frac{\pi}{4}(0.035^2 - 0.02^2)(180.5-18)\right] = 1.24\ W$

and: $$Q_{pipe} = Q_{fin}\left(\frac{1.5}{0.005}\right) + \bar{h}A_{pipe}\left(\frac{0.50-0.08}{0.50}\right)\Delta T$$

$$= 1.24(300) + 6(\pi[0.02]1.5)(0.84)(180.5-18)$$

$$\underline{= 448\ W}$$

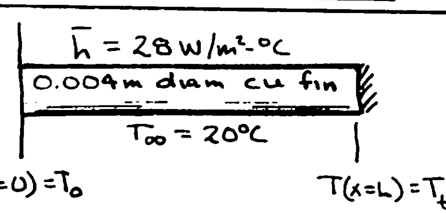
The mass rate of condensate is $\dot{m}_{cond} = \dfrac{Q_{pipe}}{h_{fg}}$

At 10 atm, $h_{fg} = 2.013 \times 10^6\ J/kg$ so

$$\dot{m}_{cond} = \frac{448\ J/s}{2.013 \times 10^6\ J/kg} = \underline{\underline{0.000223\ \frac{kg}{s} = 0.802\ \frac{kg}{hr}}}$$

4.26 How long must the fin shown be, if:

$$\left.\frac{T - T_\infty}{T_0 - T_\infty}\right|_{x=L} = 0.2\ ?$$



Using eqn. (4.45): $$0.2 = 1/\cosh\sqrt{\frac{\bar{h}P}{kA}}\,L = 1/\cosh\sqrt{\frac{2\bar{h}}{kR}}\,L$$

or: $$5 = \cosh\sqrt{\frac{2(28)}{398(0.002)}}\,L$$

solving by trial and error we obtain $\underline{\underline{L = 0.2734\ m}}$

4.27 A 2 cm ice cube sits on a shelf of aluminum rods, 3 mm in diam., in a refrigerator at 10°C. How rapidly, in mm/min, does the ice cube melt through the wires if $\bar{h}$ between the wires and the air is 10 W/m^2-°C. (Be careful that you understand the physical mechanism before you make the calculation.) Check your result experimentally. (h_{fs} = 333,300 J/kg.)

Solution. The rods act as infinite fins. Each carries heat off in both directions at a rate given by eqn. (4.42). This is balanced by the rate of melt:

$$2\sqrt{(kA)(\bar{h}P)}\ (T_r - T_{sat}) = (2R)(2\text{cm})\dot{\ell}\ \rho_{ice}\ h_{fs}$$

where $\dot{\ell}$ is the rate the rod advances. Thus

$$\dot{\ell} \simeq \frac{2\sqrt{209\,(0.0015)^3\,10\,(2)\pi^2(10-0)}}{2(0.0015)(0.02)(917)333{,}300} = 12.9\times10^{-6}\ \frac{m}{s}$$

$$= \underline{\underline{0.772\ \text{mm/min}}}$$

I did this in my refrigerator and found about 1 cm advance after 15 min. This gave <u>0.667 mm/min</u> which is a reasonable comparison.

4.28 The highest heat flux that can be achieved in nucleate boiling (called q_{max}--see the qualitative discussion in Section 9.1) depends upon: ρ_g, the saturated vapor density; h_{fg}, the latent heat of vaporization; σ, the surface tension; a characteristic length, L; and the gravity force per unit volume, $g(\rho_f - \rho_g)$, where ρ_f is the saturated liquid density. Develop the dimensionless functional equation for q_{max} in terms of a dimensionless length.

$$\underset{\frac{J}{m^2\text{-}s}}{q_{max}} = fn\Big(\underset{\frac{kg}{m^2}}{\rho_g},\ \underset{\frac{J}{kg}}{h_{fg}},\ \underset{\frac{kg}{s^2}}{\sigma},\ \underset{\frac{kg}{m^2\text{-}s^2}}{g(\rho_f-\rho_g)},\ \underset{m}{L}\Big)$$

There are 6 variables in 4 dimensions (J, m, kg, s). This gives 2 Π groups. To find them we first eliminate J from the dimensional functional equation:

only term with J, must go out

$$\underset{kg/m^2\text{-}s}{\frac{q_{max}}{h_{fg}}} = fn\Big(\underset{kg/m^3}{\rho_g},\ \underset{J/kg}{\cancel{h_{fg}}},\ \underset{kg/s^2}{\sigma},\ \underset{kg/m^2\text{-}s^2}{g(\rho_f-\rho_g)},\ \underset{m}{L}\Big)$$

4.28 (continued)

Next, get rid of m:

$$\underset{s}{\frac{q_{max}}{h_{fg}\, g(\rho_f-\rho_g)}} = fn\left(\underset{s^3/kg^{1/2}}{\frac{\rho_g}{[g(\rho_f-\rho_g)]^{3/2}}},\ \underset{kg/s^2}{\sigma},\ \underset{kg/m^2\text{-}s}{\cancel{g(\rho_f/\rho_g)}},\ \underset{kg^{1/2}/s}{L\sqrt{g(\rho_f-\rho_g)}}\right)$$

→ delete

Get rid of kg:

$$\underset{s}{\frac{q_{max}}{h_{fg}\, g(\rho_f-\rho_g)}} = fn\left(\underset{s^2}{\frac{\rho_g\sqrt{\sigma}}{[g(\rho_f-\rho_g)]^{3/2}}},\ \underset{kg/s^2}{\cancel{\sigma}},\ \frac{L}{\sqrt{\sigma/g(\rho_f-\rho_g)}}\right)$$

→ delete

Finally we get rid of s and the term ↑, & obtain the result:

$$\underbrace{\frac{q_{max}}{\rho_g^{1/2}\, h_{fg}\sqrt[4]{\sigma g(\rho_f-\rho_g)}}}_{\text{This is called the Kutateladze No. after S. S. Kutateladze, head of the heat transfer lab. in Novosibirsk, Siberia}} = fn\left(\underbrace{\frac{L}{\sqrt{\sigma/g(\rho_f-\rho_g)}}}_{\text{This is the square root of what is called a Bond No.}}\right) \longleftarrow$$

4.29 You want to rig a handle for a door in the wall of a furnace. The door is at 160°C. You consider bending a 16 in. length of 1/4 in. mild steel rod into a U-shape, and welding the ends to the door. Surrounding air at 24°C will cool the handle ($\bar{h}$ = 12 W/m^2-°C). What is the coolest temperature of the handle? How close to the door can you grasp it without being burned? How might you improve the handle?

This handle is like 2, 8 in. (0.2032 m) long fins with insulated tips.

$$(mL)^2 = \frac{\bar{h}PL^2}{kA} = \frac{\bar{h}\pi D L^2}{k\pi D^2/4} = \frac{4(12)(0.2032)^2}{52(0.00635)} = 6.00$$

$$\Theta_{tip} = \frac{1}{\cosh mL} = \frac{1}{\cosh 2.45} = 0.1713\ ,\quad T_{tip} = 24 + 0.1713(160-24)$$

$$= T_{coolest} = \underline{\underline{47.3\,°C}} \longleftarrow$$

$$\Theta_{burn} = \frac{65-24}{160-24} = 0.3015 = \frac{\cosh 2.45(1-\xi)}{\cosh 2.45}$$

4.29 (continued)

or $\cosh 2.45(1-\xi) = 1.76$, $2.45(1-\xi) = 1.165$, $\xi \doteq 0.5245$

Therefore if you touch this handle within 0.5245(8) or 4.2 inches of the door, you'll be burned.

To improve the design you need a far smaller rod diameter -- maybe a mere wire loop. Better still, weld on two short steel studs and connect them with a (low conductivity) piece of wood. The proposed design is not a good one.

4.30 A 14 cm long, 1 cm by 1 cm square brass rod is supplied with 25 W at its base. The other end is insulated. It is cooled by air at 20°C with h = 68 W/m²-°C. Develop a dimensionless expression for Θ as a function of ξ and other known information. Calculate the base temperature.

We know that: $T-T_\infty = C_3 e^{mL\xi} + C_4 e^{-mL\xi}$ (cf. eqn. (4.35)) with b.c. s:

$$-\left.\frac{d(T-T_\infty)}{d\xi}\right|_{\xi=0} = \frac{Q_{base}L}{Ak} \quad ; \quad \left.\frac{d(T-T_\infty)}{d\xi}\right|_{\xi=1} = 0$$

so:

$$C_3 - C_4 = -\frac{Q_b L}{AkmL} \quad ; \quad C_3 e^{mL} = C_4 e^{-mL}$$

so $$C_3 = C_4 e^{-2mL} = C_3 e^{-2mL} + \frac{Q_b L}{AkmL} e^{-2mL}$$

or $$C_3 = \frac{Q_b L}{AkmL} \underbrace{\frac{e^{-2mL}}{1-e^{-2mL}}}_{} = C_4 - \frac{Q_b L}{kAmL}$$

$$= \frac{e^{-mL}}{2\sinh mL}$$

$$\frac{T-T_\infty}{Q_b L/kAmL} = e^{-mL\xi} + \frac{e^{-mL}\left(e^{mL\xi} + e^{-mL\xi}\right)}{2\sinh mL}$$

$$= \frac{e^{mL(1-\xi)} - \cancel{e^{-mL(1+\xi)}} + e^{-mL(1-\xi)} + \cancel{e^{-mL(1+\xi)}}}{2\sinh mL}$$

$$\Theta \equiv \frac{T-T_\infty}{Q_b L/kA} = \frac{\cosh mL(1-\xi)}{mL \sinh mL}$$

then:

$$\frac{T_{base}-T_\infty}{Q_b L/kA} = \frac{1}{mL\tanh mL} \quad ; \quad T_{base} = T_\infty + \frac{Q_b L}{kAmL\tanh mL}$$

so

$$T_{base} = 20 + \frac{25(0.14)}{109(0.01)^2 \underbrace{\sqrt{\frac{68\times4}{109(0.01)}}\,0.14}_{2.2116}\ \underbrace{\tanh 2.2116}_{0.9763}}$$

$$= 20 + 148.7 = 168.7\,^\circ\mathrm{C}$$

4.31 A cylindrical fin has a constant imposed heat flux of q_1 at one end and q_2 at the other end, and it is cooled convectively along its length. Develop the dimensionless temperature distribution in the fin. Specialize this result for $q_2 = 0$ and $L\to\infty$, and compare it with equation (4.50).

The general solution is $T - T_\infty = C_1 e^{mL\xi} + C_2 e^{-mL\xi}$, with b.c.'s:

$$q_1 = -\frac{k}{L}\frac{d(T-T_\infty)}{d\xi}\bigg|_{\xi=0} \quad \& \quad q_2 = +\frac{k}{L}\frac{d(T-T_\infty)}{d\xi}\bigg|_{\xi=1}$$

or:

$$\frac{q_1}{km} = -C_1 + C_2 \quad \& \quad \frac{q_2}{km} = C_1 e^{mL} - C_2 e^{-mL}$$

so

$$C_2 = \frac{q_1}{km} + C_1 \quad \& \quad \frac{q_2}{km} = -\frac{q_1}{km}e^{-mL} + C_1\, 2\sinh mL$$

thus: $\underline{C_1 = \frac{q_1}{km}\left(e^{-mL} + \frac{q_2}{q_1}\right)\Big/ 2\sinh mL}$

And we have:

$$\frac{T-T_\infty}{q_1/km} = \frac{\left(e^{-mL}+\frac{q_2}{q_1}\right)e^{mL\xi} + 2\sinh mL\, e^{-mL\xi} + \left(e^{-mL}+\frac{q_2}{q_1}\right)e^{-mL\xi}}{2\sinh mL}$$

$$= \frac{(q_2/q_1)\,2\cosh mL\xi + e^{-mL(1-\xi)} + e^{mL(1-\xi)} \cancel{- e^{-mL(1+\xi)}} \cancel{+ e^{-mL(1+\xi)}}}{2\sinh mL}$$

so

$$\boxed{\frac{T-T_\infty}{q_1/km} = \frac{q_2}{q_1}\frac{\cosh mL\xi}{\sinh mL} + \frac{\cosh mL(1-\xi)}{\sinh mL}}$$

for $q_2 = 0$, the insulated tip, $\frac{T-T_\infty}{q_1/km} = \frac{\cosh mL(1-\xi)}{\sinh mL}$ (which is the solution of Problem 4.30.). As $mL\to\infty$ this becomes:

$$\underline{\underline{\frac{T-T_\infty}{q_1/km} = e^{-mL\xi}}}$$

which is equation (4.50) with q_1/km serving in lieu of the characteristic temperature: $T_0 - T_\infty$.

4.32 A thin metal cylinder of radius, r_0, serves as an electrical resistance heater. One axial line in one side is kept at T_1. Another line, θ_2 radians away, is kept at T_2. Develop a dimensionless expressions for the temperature distributions in the two sections.

$$\frac{1}{r_0^2}\frac{d^2(T-T_1)}{d\theta^2} = -\frac{\dot q}{k} \quad \text{so} \quad T-T_1 = -\frac{\dot q r_0^2}{2k}\theta^2 + C_1\theta + C_2$$

b.c.'s: $T-T_1 = 0$ at $\theta = 0$ so $\underline{C_2 = 0}$

$T-T_1 = T_2-T_1$ at $\theta=\theta_2$ so $C_1 = \dfrac{T_2-T_1}{\theta_2} + \dfrac{\dot q r_0^2}{2k}\theta_2$

Then:
$$\frac{T-T_1}{T_2-T_1} = -\frac{\dot q r_0^2}{2k\Delta T}\theta^2 + \frac{\theta}{\theta_2} + \frac{\dot q r_0^2}{2k\Delta T}\theta_2\theta$$

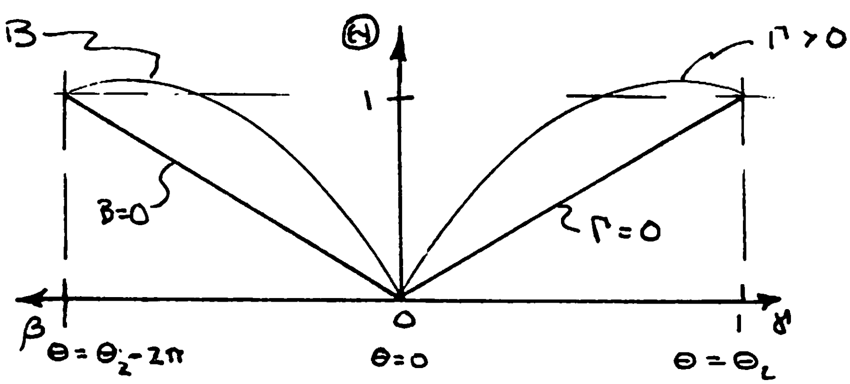
Call: $\Theta \equiv \dfrac{T-T_1}{T_2-T_1}$; $\dfrac{\dot q r_0^2 \theta_2^2}{2k\Delta T} \equiv \Gamma$; $\dfrac{\theta}{\theta_2} \equiv \gamma$. Then
$$\boxed{\Theta = \Gamma(\gamma-\gamma^2) + \gamma \quad (0 \le \theta \le \theta_2)}$$

For the other segment $-\theta = 2\pi - \theta_2$ at $T-T_1 = T_2-T_1$ so the solution becomes:

$$\Theta = -\frac{\dot q r_0^2(\theta_2-2\pi)^2}{2k\Delta T}\beta^2 - \beta - \frac{\dot q r_0^2(\theta_2-2\pi)^2}{2k\Delta T}\beta$$

where $\beta \equiv \theta/[\theta_2-2\pi]$. so if we define $B \equiv \dot q r_0^2(\theta_2-2\pi)^2/2k\Delta T$,

$$\boxed{\Theta = B(-\beta-\beta^2) - \beta \quad (\theta_2 \le \theta \le 0)}$$

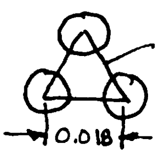


4.33 Heat transfer is augmented, in a particular heat exchanger, with a field of 0.007 m diameter fins protruding 0.02 m into a flow. The fins are arranged in a hexagonal array with a minimum spacing of 1.8 cm. The fins are bronze and $\bar{h}_f$ around the fins is 168 W/m^2-°C. On the wall itself, $\bar{h}_w$ is only 54 W/m^2-°C. Calculate $\bar{h}_{eff}$ for the wall with its fins ($\bar{h}_{eff}$ = Q_{wall} divided by A_{wall} and $[T_{wall} - T_\infty]$.)

In this case: $mL = \sqrt{\dfrac{\bar{h}\pi D}{k \frac{\pi}{4} D^2}}\, L = \sqrt{\dfrac{168 \times 4}{(0.007)}}\, 0.02 = \underline{1.215}$

Next define $\bar{h}_A \equiv \dfrac{Q_{fin}}{A} \dfrac{1}{T_w - T_\infty} = k(mL)\tanh(mL)/L = \dfrac{26}{0.02}\, 1.215 \underbrace{\tanh 1.215}_{0.8382}$

$\underline{\bar{h}_A = 1324\ W/m^2\text{-}°C}$ This characterizes heat removal where the fin replaces the wall.



$A_{triangle} = \frac{1}{2}(0.018)^2 \cos 60° = 0.0001403\ m^2$

$A_{fin\ within\ \Delta} = 3\left[\frac{1}{6}\frac{\pi}{4}(0.007)^2\right] = 1.924 \times 10^{-5}\ m^2$

$$h_{eff} = \frac{1}{A_\Delta}\left[\bar{h}_A \times A_{fin} + \bar{h}_w \times (A_\Delta - A_{fin})\right]$$

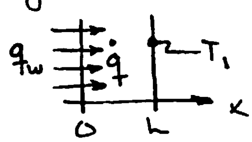
$$= \left[1324 \times 1.924 (10)^{-5} + 54(0.0001403 - 1.924\,(10)^{-5}\right]/0.0001403$$

$$\underline{\bar{h}_{eff} = 228\ W/m^2\text{-}°C}$$

The fins therefore yield a considerably improved heat removal.

4.34 An engineer seeks to study the effect of temperature on the curing of concrete by controlling the temperature of curing in the following way. A sample slab of thickness, L, is subjected to a heat flux, q_w, on one side, and it is cooled to temperature, T_1, on the other. Derive a dimensionless expression for the steady temperature in the slab. Plot the expression and offer a criterion for neglecting the internal heat generation in the slab.

general solution: $T - T_1 = -\frac{\dot q L^2}{2k}\xi^2 + C_1\xi + C_2$ where $\xi \equiv \frac{x}{L}$



b.c.'s: $-\frac{k}{L}\frac{d(T-T_1)}{d\xi}\Big|_{\xi=0} = q_w = \left[\frac{\dot q L}{k}\xi - k\frac{C_1}{L}\right]_{\xi=0}$

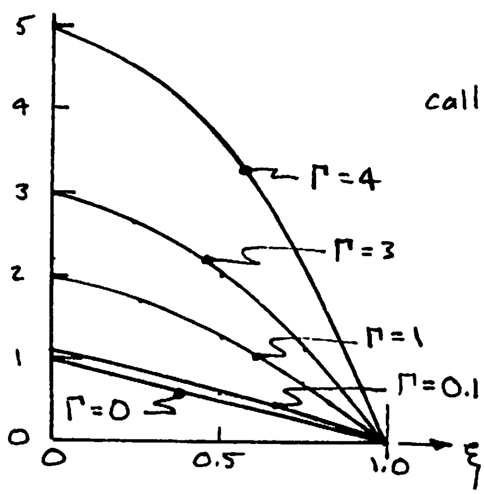
so $C_1 = -q_w L/k$

$\left[T-T_1\right]_{\xi=1} = 0 = -\frac{\dot q L^2}{2k} - \frac{q_w L}{k} + C_2$

so $\underline{C_2 = \frac{\dot q L^2}{2k} + \frac{q_w L}{k}}$

So: $$T - T_1 = -\frac{\dot q L^2}{2k}\xi^2 + \frac{\dot q L^2}{2k} - \frac{q_w L}{k}\xi + \frac{q_w L}{k}$$

or: $$\frac{T-T_1}{q_w L/k} \equiv \underline{\underline{\Phi = \frac{\dot q L}{2q_w}(1-\xi^2) + (1-\xi)}}$$



call $\underline{\frac{\dot q L}{2q_w} \equiv \Gamma}$

When $\Gamma < 0.1$ we can neglect internal heat generation with only 10 percent error.

4.35 Develop the dimensionless temperature distribution in a spherical shell with the inside wall kept at one temperature, and the outside wall at a second temperature. Reduce your solution to the limiting cases in which $r_{outside} >> r_{inside}$ and in which $r_{outside}$ is very close to r_{inside}. Discuss these limits.

The general solution is: $T = \dfrac{C_1}{r} + C_2$ with b.c.'s $T(r_i) = T_i$, $T(r_o) = T_o$

Then:
$$\left.\begin{aligned} T_i &= \frac{C_1}{r_i} + C_2 \\ T_o &= \frac{C_1}{r_o} + C_2 \end{aligned}\right\} \quad T_i - T_o = C_1\left[\frac{r_o - r_i}{r_o r_i}\right]; \quad C_1 = \Delta T \frac{r_o r_i}{\Delta r}$$
$$T_i = \frac{C_1}{r_i} + C_2; \quad C_2 = T_i - \Delta T \frac{r_o}{\Delta r}$$

so:
$$T - T_i = \Delta T \frac{r_o}{\Delta r}\left[\frac{r_i}{r} - 1\right]; \qquad \underline{\underline{\Theta = \frac{r_o}{r_o - r_i}\left[\frac{r - r_i}{r}\right]}}$$

where $\Theta \equiv (T_i - T)/(T_i - T_o)$ and we have switched the signs to make everything positive. Then:

$$\lim_{r_o \to \infty} \Theta = \frac{r - r_i}{r}$$
which is the result for a a semi-infinite region. (See eg., the solution to 2.15)

[Note: This could also be written as:
$$-\frac{T - T_o}{T_i - T_o} - \frac{T_i - T_o}{T_i - T_o} = \frac{r - r_i}{r} \quad \text{or as} \quad \underline{\frac{T - T_o}{T_i - T_o} = \frac{r_i}{r}}\ ]$$

And:
$$\lim_{r_o \to r_i} \Theta = \frac{r_o}{r_o - r_i}\left[\frac{r - r_i}{r_o}\right] = \underline{\frac{r - r_i}{r_o - r_i}}$$
which is the result for a plane slab of thickness $r_o - r_i$

4.36 Does the temperature distribution during steady heat transfer in an object, with b.c.'s of only the first kind, depend on k? Explain.

For such a problem we have: $\nabla^2 T = 0$ or $-\dot{q}/k$ and $T(x = L_1) = T_1$, $T(x = L_2) = T_2$, etc. Thus $T - T_1 = fn([T_2 - T_1], x, L_1, L_2, \dot{q}/k)$. This gives 6 var. in °C, m, only so with $\dot{q}$ there are 6 − 2 or 4 π groups:
$$\frac{T - T_1}{T_2 - T_1} = fn\left(\frac{x}{L_1}, \frac{L_2}{L_1}, \frac{\dot{q}L}{k(T_2 - T_1)}\right)$$
k can only enter if $\dot{q}$ is in the problem also.

4.37 A long, 0.005 m diameter, duralumin rod is wrapped with an electrical resistor over 3 cm of its length. The resistor imparts a surface flux of 40 kW/m^2. Evaluate the temperature of the rod on either side of the heated section, if $\bar{h}$ = 150 W/m^2-^{o}C, and $T_{ambient}$ = 27^{o}C.

Q to either side $= \frac{1}{2}(0.03[\pi(0.005)])40{,}000 = 9.42$ W

q_o at the base of the rod is $\frac{Q}{A} = \frac{9.42}{(\pi/4)0.005^2} = \underline{480 \text{ kW/m}^2}$

From equation (4.51) we have: $q_o = k\, m\, \Delta T$, but

$$m = \sqrt{\frac{4\bar{h}}{kD}} = \sqrt{\frac{4(150)}{164(0.005)}} = 27.05 \text{ m}^{-1} \quad \text{so} \quad \Delta T = \frac{4.8(10)^5}{164(27.05)} = 108.2^\circ\text{C}$$

Therefore the base temperature is $T_o = 108.2 + 27 = \underline{\underline{135.2^\circ\text{C}}}$

4.38 **The heat transfer coefficient between a cool surface and a saturated vapor, when the vapor condenses in a film on the surface, depends on: the liquid density and specific heat, the temperature difference, the buoyant force per unit volume ($g[\rho_f - \rho_g]$), the latent heat, the liquid conductivity and kinematic viscosity, and the position (x) on the cooler. Develop the dimensionless functional equation for h.**

$$h = h(\rho_f, c_{p_f}, \underbrace{(T_{sat.} - T_w)}_{\Delta T}, h_{fg}, g(\rho_f - \rho_g), k, \nu, x)$$

$$\frac{J}{m^2\text{-}s\text{-}^\circ C} \quad \frac{kg}{m^3} \quad \frac{J}{kg\text{-}^\circ C} \quad {}^\circ C \quad \frac{J}{kg} \quad \frac{kg}{s^2\text{-}m^2} \quad \frac{J}{m\text{-}^\circ C\text{-}s} \quad \frac{m^2}{s} \quad m$$

We have 9 variables in 5 dimensions (J, m, s, °C, kg)
This gives 4 Π-groups. The method in the text will -- if used in the correct sequence -- give:

$Nu_x = fn(\Pi, Pr, Ja)$ where: $Pr \equiv \mu c_p/k$, $Ja \equiv \frac{c_p \Delta T}{h_{fg}}$

$Nu_x = \frac{hx}{k}$, $\Pi = \frac{\rho_f(\rho_f - \rho_g) g h_{fg} x^3}{\mu k \Delta T}$

(Of course other combinations are also acceptable. See details in Section 8.5.)

4.39 A duralumin pipe through a cold room has a 4 cm ID and a 5 cm OD. It carries water which sometimes sits stationary. It is proposed to put electric heating rings around the pipe to protect against freezing during cold periods of -7°C. The heat transfer coefficient outside the pipe is 9W/m^2-°C. Neglect the presence of the water in the conduction calculation, and determine how far apart the heaters would have to be if they brought the pipe temperature to 40°C, locally. How much heat do they require?

Find m: $m = \sqrt{\dfrac{\bar{h}P}{kA}} = \left(\dfrac{9\,\pi(0.05)}{164\frac{\pi}{4}(0.05^2-0.04^2)}\right)^{1/2} = 3.49$

$\Theta = \dfrac{0-(-7)}{40-(-7)} = 0.149 = \dfrac{1}{\cosh mL}$ at the midpoint

so, by trial and error, $\underline{mL = 2.592}$ or $\underline{L = 0.743\text{ m}}$

Thus the heaters must be spaced every $\underline{\underline{1.486\text{ m}}}$

and: $\dfrac{Q}{2} = \sqrt{164\frac{\pi}{4}(0.05^2-0.04^2)\,9\,\pi(0.05)}\,[40-(-7)]\underbrace{\tanh 2.592}_{0.989} = 18.82\text{W}$

For heat flow both left & right, $Q = 2(18.82) = \underline{\underline{37.64}}$

4.40 Evaluate d(tanhx)/dx.

$$\frac{d\tanh x}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = \frac{e^x}{e^x+e^{-x}} + \frac{e^{-x}}{e^x+e^{-x}} - \frac{e^x - e^{-x}}{(e^x+e^{-x})^2}\left[e^x - e^{-x}\right]$$

$$= \frac{(e^x+e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}$$

$$= \underline{\underline{\frac{1}{(\cosh x)^2}}}$$

4.41 The specific entropy of an ideal gas depends on its specific heat at constant pressure, its temperature and pressure, the ideal gas constant and reference values of the temperature and pressure. Obtain the dimensionless functional equation for the specific entropy and compare it with the known equation.

$$\underset{\frac{J}{kg\,^\circ K}}{s} = s(\underset{\frac{J}{kg\,^\circ K}}{c_p}, \underset{^\circ K}{T}, \underset{^\circ K}{T_{ref}}, \underset{\frac{N}{m^2}}{p}, \underset{\frac{N}{m^2}}{p_{ref}}, \underset{\frac{J}{kg^\circ}}{R})$$

7 var in $\frac{J}{kg}$, $^\circ C$, $\frac{N}{m^2}$ $\Rightarrow$ 7-3 or 4 Π-groups

Thus: $\underline{\underline{\frac{s}{R} = fn\left(\frac{c_p}{R}, \frac{T}{T_{ref}}, \frac{p}{p_{ref}}\right)}}$

This is in the form of the known result: $\underline{\frac{s}{R} = \frac{c_p}{R}\ln\frac{T}{T_{ref}} - \ln\frac{p}{p_{ref}}}$

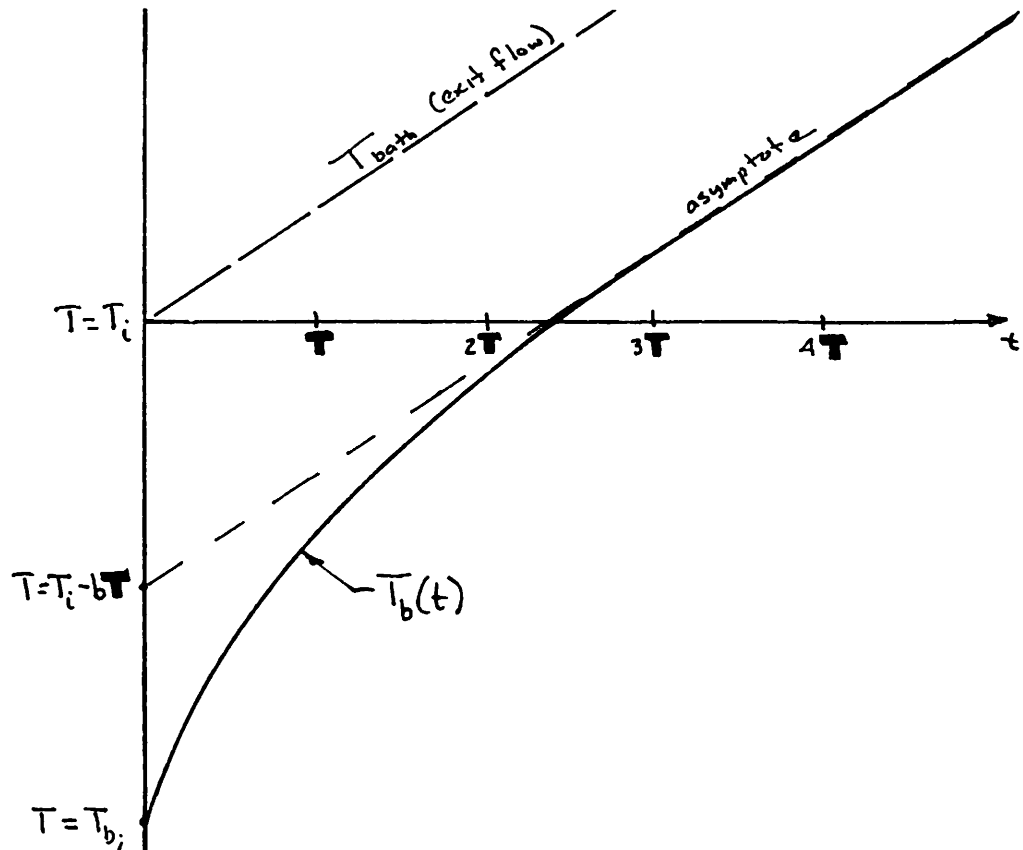
5.1 A body at $T_b = T_i$ with $Bi \ll 1$ is immersed in a bath at $t = 0$. If $T_{bath} = T_i + bt$, plot $T_{body} = f(t)$ for the case in which $T_{b_i} < T_i - b\mathbf{T}$

As in Example 5.1, the general solution is given by eqn. (5.13)

$$T_b - T_i = C_1 e^{-t/\mathbf{T}} + b(t - \mathbf{T})$$

and $C_1 = T_{b_i} - T_i + b\mathbf{T}$ so the particular solution is

$$T_b = T_i + (t - \mathbf{T}) + (T_{b_i} - T_i + b\mathbf{T})e^{-t/\mathbf{T}}$$



5.2 A body of known volume and temperature, initially at T_i, is suddenly immersed in a bath for which

$$T_{bath} = T_i + (T_o - T_i)e^{-t/\tau}$$

where $\tau = 10\mathbf{T}$. Plot T_{body} from $t = 0$ to $t = 2\tau = 20\mathbf{T}$.

$$\frac{dT}{dt} = \frac{T - T_b}{\mathbf{T}} \quad \text{or} \quad \frac{d(T-T_i)}{dt} = -\frac{T-T_i}{\mathbf{T}} + (T_o - T_i)\,e^{t/\tau}$$

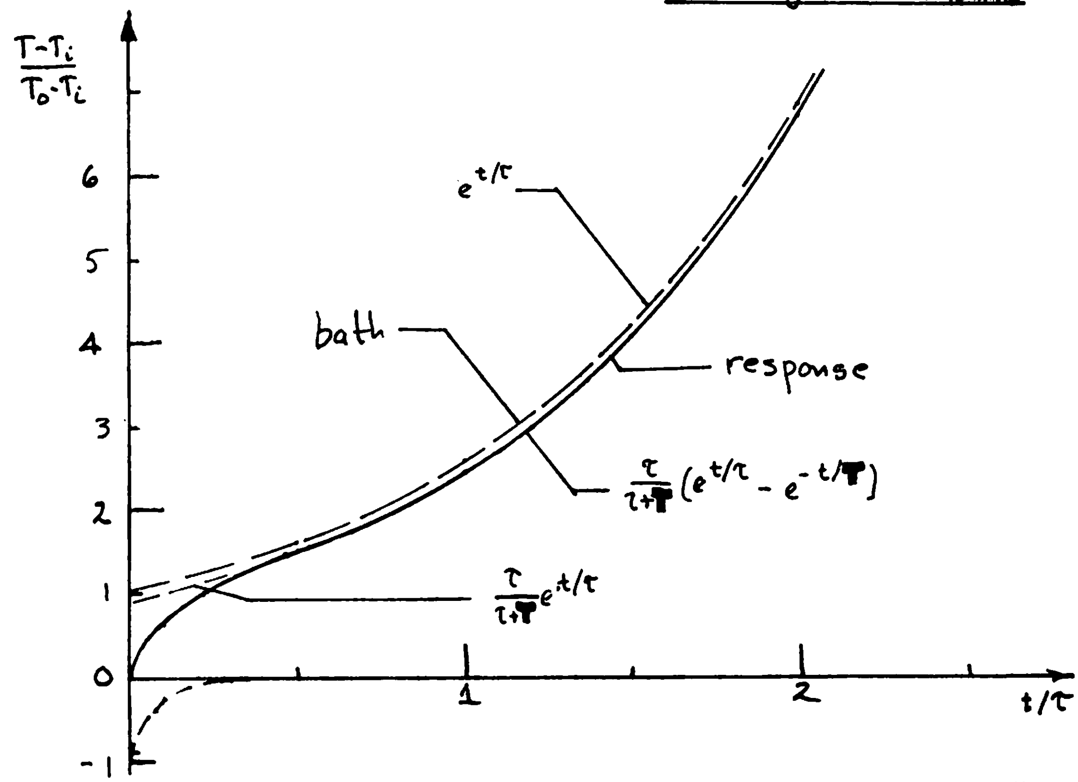
The gen'l. soln. of the homo. eqn. is $T - T_i = C_1 e^{-t/\mathbf{T}}$ and the particular soln. of the complete eqn. might be found by substituting $T - T_i = Ae^{t/\tau} + Be^{-t/\tau}$ in the d.e. and adjusting A & B to satisfy it. We get $B = 0$ and $A = (T_o - T_i)/(\tau^{-1} + \mathbf{T}^{-1})\mathbf{T}$ so

$$\frac{T - T_i}{T_o - T_i} = \frac{T_o - T_i}{\left(\frac{1}{\tau} + \frac{1}{\mathbf{T}}\right)\mathbf{T}}\, e^{t/\tau} + C_1 e^{-t/\mathbf{T}}$$

5.2 (continued)

The i.c., $(T-T_i)_{t=0} = 0$, gives $C_1 = -\dfrac{T_0 - T_i}{\frac{\mathbf{T}}{\tau} + 1}$

$$\underline{\underline{\frac{T-T_i}{T_0-T_i} = \frac{\tau}{\mathbf{T}+\tau}\left(e^{t/\tau} - e^{-t/\mathbf{T}}\right)}} \quad \& \quad \underline{\underline{\left.\frac{T-T_i}{T_0-T_i}\right|_{\text{long time}} = \frac{\tau}{\mathbf{T}+\tau} e^{t/\tau}}}$$



5.3 A body of known volume and area is immersed in a bath whose temperature varies $T_\infty = T_{mean} + A \sin\omega t$. Find the steady periodic response of the body if its Biot number is small.

Define: $\Theta \equiv \frac{T-T_m}{A}$, $\mathbf{T} \equiv \rho c V/\bar{h}A$, $\tau = t/\mathbf{T}$, $\Omega \equiv \omega \mathbf{T}$. Then the d.e.

$$\frac{dT}{dt} = -\frac{T-T_m}{\mathbf{T}} + A\sin\omega t \quad \text{becomes} \quad \underline{\frac{d\Theta}{d\tau} + \Theta = \sin\Omega\tau}$$

The general sol'n of the homogeneous eqn. is: $\underline{\Theta = C_1 e^{-\tau}}$. The particular sol'n. of the complete eqn. can be found by trying $\underline{\Theta = C_2\cos\Omega\tau + C_3\sin\Omega\tau}$ in the complete eqn. This gives

$$-\Omega C_2 \sin\Omega\tau + \Omega C_3 \cos\Omega\tau + C_2\cos\Omega\tau + C_3\sin\Omega\tau = \sin\Omega\tau$$

or

$$(-\Omega C_2 + C_3 - 1)\sin\Omega\tau + (\Omega C_3 + C_2)\cos\Omega\tau = 0$$

This will be true if $\underline{C_2 = -\Omega C_3}$ and $\underline{C_3 = \frac{1}{\Omega^2+1}}$. Then the

5.3 (continued)

particular solution of the complete equation is

$$\Theta = C_1 e^{-\tau} - \frac{\Omega}{\Omega^2+1}\cos\Omega\tau + \frac{1}{\Omega^2+1}\sin\Omega\tau$$

or

$$\Theta = C_1 e^{-\tau} - \frac{1}{\Omega^2+1}\left[\Omega\cos\Omega\tau - \sin\Omega\tau\right]$$

At time $\tau = 0$, $\Theta = \Theta_0 = C_1 - \frac{\Omega}{\Omega^2+1}$ so $C_1 = \Theta_0 + \frac{\Omega}{\Omega^2+1}$

where Θ_0 might be anything, between 0 and 1, we might wish to specify. Thus

$$\Theta = \Theta_0 e^{-\tau} - \frac{1}{\Omega^2+1}\left(\Omega\cos\Omega\tau - \sin\Omega\tau - \Omega e^{-\tau}\right)$$

After a long time ($t > 3T_w$ or $\tau > 3$) this reduces to the steady periodic solution:

$$\Theta \Rightarrow \frac{-1}{\Omega^2+1}\left(\Omega\cos\Omega\tau - \sin\Omega\tau\right)$$

Now use the trigonometric identity

$$A\cos x + B\sin x = \sqrt{A^2+B^2}\,\sin\left(x + \cos^{-1}\frac{1}{\sqrt{A^2+B^2}}\right)$$

In this case:

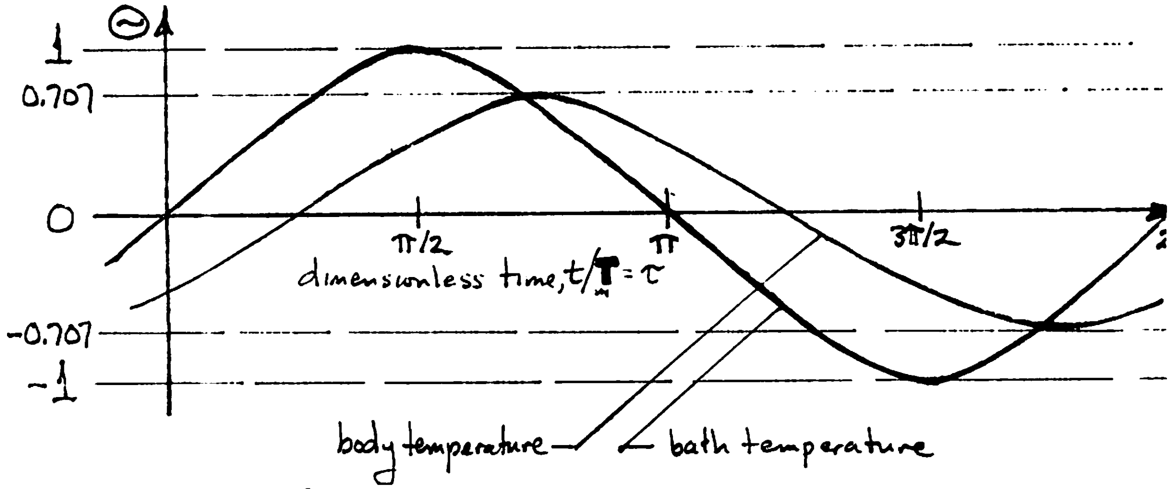
$$\Omega\cos\Omega\tau - \sin\Omega\tau = \sqrt{\Omega^2+1}\,\sin\Big(\Omega\tau - \underbrace{\cos^{-1}\frac{1}{\sqrt{\Omega^2+1}}}_{\beta}\Big)$$

we identify $\beta \equiv$ the phase lag angle. Then

$$\Theta_{periodic} = -\underbrace{\frac{1}{\sqrt{\Omega^2+1}}}_{0<\text{amplitude}<1}\sin(\Omega\tau - \underbrace{\beta}_{0<\beta<90^\circ})$$

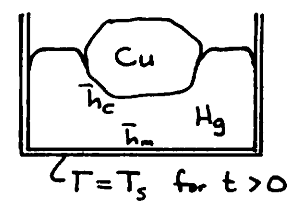
Suppose, for example, that $\omega T_w = \Omega = 1$. Then $\beta = \cos^{-1} 0.707 = 45^\circ$ or $\pi/4$ radians and the amplitude is 0.707.

5.3 (continued)



Notice that $\Theta_{periodic} = f(\tau \text{ and } \Omega)$. When Ω (or ωT) is large, the process can be regarded as slow and $\Theta \Rightarrow \Theta_{bath} = \sin\Omega\tau$. When Ω is small the process is rapid, $\beta \Rightarrow 90^\circ$ and the amplitude of the response goes to zero. In a rapid oscillation $\Theta \Rightarrow 0$.

5.4 A copper block of volume, V, floats in mercury contacting it over an area, A_c, and exchanging heat with it by convection. The mercury container also exchanges heat with the mercury itself by convection. The entire system is initially in equilibrium at temperature, T_i.



Predict the response of the copper if the container temp. is suddenly raised to T_s and if Bi_{cu} and $Bi_{hg} \ll 1$.

For the copper: $(\rho c V)_c \dfrac{dT_c}{dt} = (\overline{h}A)_c (T_m - T_c)$ or $\dfrac{dT_c}{dt} = \dfrac{T_m - T_c}{T_c}$ ①

for the mercury: $\dfrac{dT_m}{dt} = \dfrac{T_s - T_m}{T_m}$ ②

This is exactly the second-order lumped capacitance problem solved in the text. The solution is eqn. (5.23) which we paraphrase as follows:

$$\frac{T_c - T_s}{T_i - T_s} = \frac{\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}}{2\sqrt{\left(\frac{b}{2}\right)^2 - c}}\, e^{\left(-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t} + \frac{-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 - c}}{2\sqrt{\left(\frac{b}{2}\right)^2 - c}}\, e^{\left(-\frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - c}\right)t}$$

where $b = \dfrac{1}{T_c} + \dfrac{1}{T_m} + \dfrac{\overline{h}_c}{\overline{h}_m T_m}$ and $c = 1/T_c T_m$

This can be rewritten as $\dfrac{T_c - T_s}{T_i - T_s} = A_1 e^{a_1 t} + A_2 e^{a_2 t}$

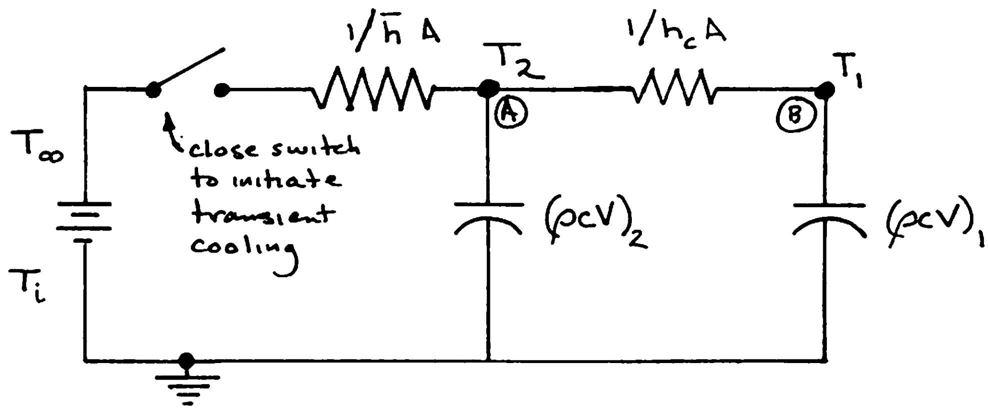
5.4 (continued)

at $t=0$, $\dfrac{T_c-T_s}{T_i-T_s} = A_1+A_2 = \dfrac{2\sqrt{(\frac{b}{2})^2-c}}{2\sqrt{(\frac{b}{2})^2-c}} = 1 \quad \therefore \quad T_c = T_i$ ✔

at $t=0$, $T_m = T_c$ so from eqn. ① $\dfrac{dT_c}{dt} = 0 = (T_i - T_\infty)(A_1 a_1 + A_2 a_2)$

Thus: $\dfrac{A_1}{A_2} = -\dfrac{a_2}{a_1}$. We see that both sides equal $\dfrac{\frac{b}{2}+\sqrt{\;}}{-\frac{b}{2}+\sqrt{\;}}$ so the second b.e. is also satisfied. Finally, we expect $T_c \Rightarrow T_s$ or $\dfrac{T_c-T_s}{T_i-T_s} \Rightarrow 0$ as $t \Rightarrow \infty$. This means that a_1 & a_2 must both be negative. a_2 obviously is. a_1 is also negative because b is a positive number greater than c.

5.5 Sketch the electrical circuit that is analogous to the second order lumped capacity system shown in Fig. 5.5.



To see that this is valid we write the nodal equations for nodes Ⓐ and Ⓑ as an E.E. might.

node Ⓐ

$$(\rho c V)_2 \frac{dT_2}{dt} + \frac{T_2 - T_1}{1/h_c A} + \frac{T_2 - T_\infty}{1/\bar{h} A} = 0$$

node Ⓑ

$$(\rho c V)_1 \frac{dT_1}{dt} + \frac{T_1 - T_2}{1/h_c A} = 0$$

These equations are identical to equations (5.16) and (5.15), respectively, so the circuit is correct.

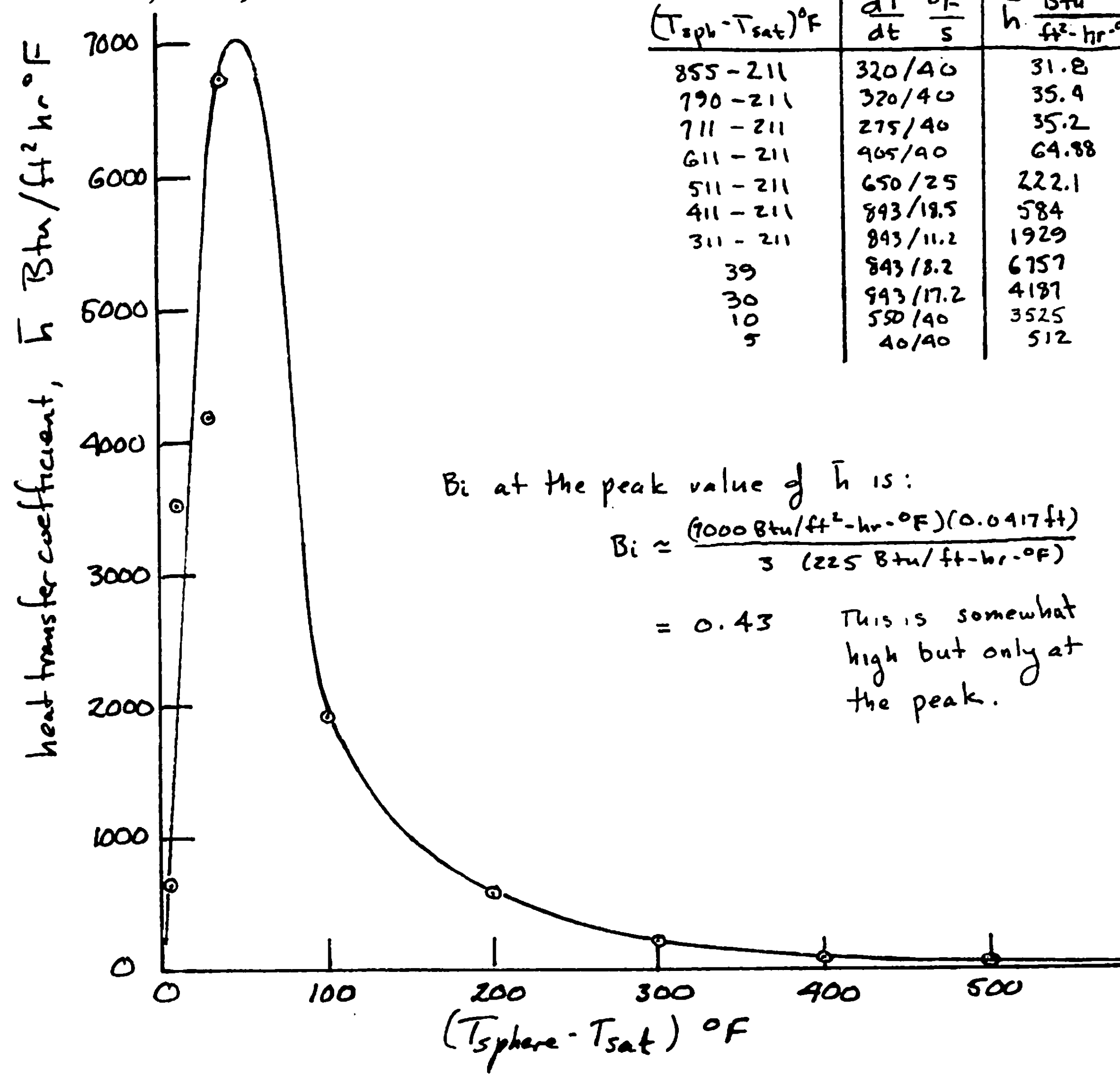
5.6 Plot $\bar{h}$ vs. $(T_{sph} - T_{sat})$ for the sphere quench in the figure with the problem in the text.

$$\frac{dU}{dt} = \rho c V \frac{dT_{sph}}{dt} = \bar{h} A (T_{sph} - T_{sat}) \; ; \quad \bar{h} = \rho c \frac{R}{3} \frac{dT_{sph}/dt}{T_{sph} - T_{sat}}$$

$$\rho c \frac{R}{3} = 8954(384) \frac{0.0254}{2(3)} \frac{\text{J}}{\text{m}^2\text{-}^\circ\text{K}} \, 0.0009478 \frac{\text{Btu}}{\text{J}} \frac{0.3048^2 \text{m}^2}{\text{ft}^2} \frac{^\circ\text{K}}{1.8^\circ\text{F}} = 0.712 \frac{\text{Btu}}{\text{ft}^2{}^\circ\text{F}}$$

Then, scaling points from the Figure:

$(T_{sph} - T_{sat})^\circ$F	$\frac{dT}{dt}$ $\frac{^\circ\text{F}}{\text{s}}$	$\bar{h}$ $\frac{\text{Btu}}{\text{ft}^2\text{-hr-}^\circ\text{F}}$
855 - 211	320/40	31.8
790 - 211	320/40	35.4
711 - 211	275/40	35.2
611 - 211	405/40	64.88
511 - 211	650/25	222.1
411 - 211	843/18.5	584
311 - 211	843/11.2	1929
39	843/8.2	6757
30	843/17.2	4187
10	550/40	3525
5	40/40	512



Bi at the peak value of $\bar{h}$ is:

$$Bi \simeq \frac{(7000 \text{ Btu/ft}^2\text{-hr-}^\circ\text{F})(0.0417 \text{ ft})}{3 \;(225 \text{ Btu/ft-hr-}^\circ\text{F})}$$

$$= 0.43$$

This is somewhat high but only at the peak.

5.7 The temperature of a butt-welded 36 gage (0.127 mm diam.) thermocouple in a gas flow rises at 20°C/s, and stays 2.4°C below the gas flow temperature. Find $\bar{h}$ between the wire and the gas if ρc = 3800 kJ/m^3-°C.

$$T = \frac{\rho c V}{\bar{h} A} = \frac{\rho c R}{2\bar{h}} = \frac{3.8(10)^6(0.000127)}{4\bar{h}} = \frac{120.7}{\bar{h}}$$

but

$$\frac{dT_w}{dt} = 20 = \frac{T_g - T_w}{T} = \frac{2.4}{120.7}\bar{h} \; ; \quad \bar{h} = 1006 \frac{\text{W}}{\text{m}^2\text{-}^\circ\text{C}}$$

5.8 Predict the temperature at the point Fo = 0.2, Bi = 10 or Bi^{-1} = 0.1, and x/L = 0, and compare it with the graphical value in Fig. 5.7.

To do this we use eqn. (5.34) with (λL) values generated by eqn. (5.35): ctn(λL) = λL/Bi = 0.1(λL). By trail and error we get: $(\lambda L)_1$ = 1.42887, $(\lambda L)_2$= 4.30580, $(\lambda L)_3$ = 7.22811, etc. Using these numbers in eqn. (5.34) we get:

$$\Theta = e^{-1.42887^2(0.2)} \frac{2\sin(1.42887)\cos(1.42887[0])}{1.42887+\sin(1.42887)\cos(1.42887)}$$

$$+ e^{-4.3058^2(0.2)} \frac{2\sin(4.3058)\cos 0}{4.3058+\sin(4.3058)\cos(4.3058)} + \cdots$$

$$\Theta = 0.8389 - 0.00965 + O\left(e^{-7.23^2(0.2)}\right) \simeq \underline{\underline{0.8293}}$$

From Fig. 7 we read $\Theta \cong$ 0.82 or 0.83 so the results agree within the accuracy with which we can read the graphs.

5.9 Prove that when B_i is large, and the b.c. of the 3rd kind therefore reduces to a b.c. of the 1st kind, eqn. (5.34) reduces to (5.33)

The eigen value eqn. (5.35) becomes ctnλL = 0 or tanλL = ∞, so $\lambda L = \frac{\pi}{2}, 3\frac{\pi}{2}, \ldots, n\frac{\pi}{2}$ where n is odd. Therefore eqn. (5.34) becomes:

$$\Theta = \sum_{n=odd}^{\infty} e^{-\left(\frac{n\pi}{2}\right)^2 F_o} \frac{2\overbrace{\sin\frac{n\pi}{2}}^{=1}\overbrace{\cos\frac{n\pi}{2}(\xi-1)}^{=\sin\frac{n\pi}{2}\xi}}{\frac{n\pi}{2}+\sin\frac{n\pi}{2}\underbrace{\cos\frac{n\pi}{2}}_{=0}}$$

$$\underline{\underline{\Theta = \frac{4}{\pi}\sum_{n=odd}^{\infty} e^{-\left(\frac{n\pi}{2}\right)^2 F_o} \frac{\sin\frac{n\pi}{2}\xi}{n}}} \qquad (5.33)$$

5.10 Check the point $B_i = 0.1$, $F_o = 2.5$ on the graph for slabs in Fig. 5.10.

First we go to eqn. (5.35) $\text{ctn}\,\lambda L = \frac{\lambda L}{0.1}$ and get $\lambda L = 0.31105$, 3.1731, etc., by trial and error. Then we put eqn. (5.34) in eqn. (5.36) and get:

$$\Phi = + \int_0^{F_o} \lambda L e^{-(\lambda L)^2 F_o} \frac{2 \sin\lambda L \sin\lambda L(\xi-1)}{\lambda L+\sin\lambda L \cos\lambda L}\Bigg|_{\xi=2} dF_o = - \frac{e^{-(\lambda L)^2 F_o}-1}{\lambda L} \times \frac{2(\sin^2\lambda L)}{\lambda L+\sin\lambda L \cos\lambda L}$$

so

$$\Phi = \frac{1-e^{-.31105^2(2.5)}}{0.31105} \frac{2\sin^2(.31105)}{.31105+\sin(.31105)\cos(.31105)} + \frac{1-e^{-3.173^3(2.5)}}{3.173} \times \frac{2\sin^2(3.173)}{3.173+\sin(.31105)\cos(.31105)} + \cdots$$

or

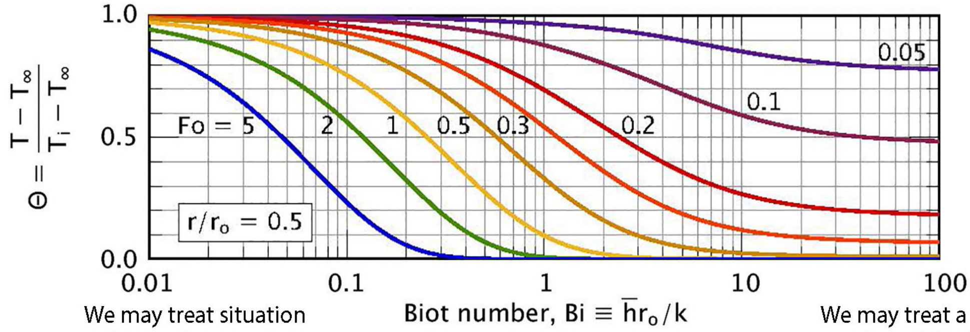
$$\Phi = 0.2148 - 0.0002 + \ldots = \underline{\underline{0.2146}}$$

From Fig. 5.10 we read $\Phi = 0.22$, which agrees within graphical accuracy.

5.11 Show, in Fig. 5.7, 5.8, or 5.9, where b.c.s of the third kind may be replaced with b.c.s of the first kind, where we can assume lumped capacity, and where the solid may be seen as semi-infinite.

Solution We choose the chart for a point midway between the center and surface of a sphere.

This region will effectively be semi-infinite as long as the change of surface temperature does not penetrate all the way to the center. That will be true for very low values of Fo.

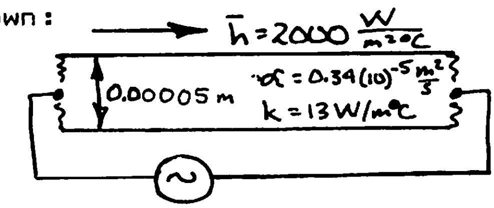


We may treat situation as lumped capacity in this region of low Biot numbers (less than 0.1)

We may treat a situation as a b.c. of the first kind in this region of high Biot numbers (greater than 50 or so.)

5.12 A ribbon is heated by a.c. as shown:

How much does its temperature fluctuate?



$$Bi = \frac{\bar{h}\delta}{k} = \frac{2000(0.00005)}{13} = 0.00769$$

$$\phi = \frac{\omega\delta^2}{\alpha} = \frac{(2\pi 60)(0.00005)^2}{0.34(10)^{-5}} = 0.2772$$

From Fig. 5.11 we then read $\dfrac{T_{max}-T_{avg}}{T_{avg}-T_\infty} \simeq \underline{\underline{0.014}}$

so the temperature fluctuation is just a little over one percent of the average temperature difference between the wall and the stream.

5.13 Resolve eqn. (5.58) into appropriate dimensionless groups.

In this case: $R = R(k, \Delta T, \rho_g h_{fg}, \rho_f c_{p_f}, t)$. Thus there are 6 basic variables in J, m, kg, °C so we look for two Π-groups:

$$\frac{R}{\sqrt{\alpha t}} = fn\left(\frac{\rho_f c_{p_f} \Delta T}{\rho_g h_{fg}}, \text{ a modified } Ja\right)$$

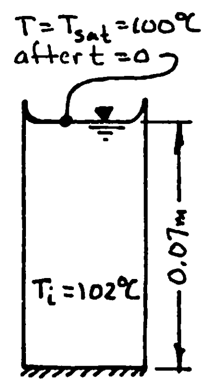
The eqn. (5.52) can be rearranged as: $\underline{\underline{\dfrac{R}{\sqrt{\alpha t}} = \dfrac{2}{\sqrt{\pi}} Ja_{mod}}}$

which conforms to the dim. analysis.

5.14 The water column shown is initially at 102°C. Then it is suddenly depressurized to 1 atm.

a) When will the temperature reach 101.95 at the bottom?

b) Plot the height of the column vs. time, up to this time.



a) $\Theta = \dfrac{101.95-100}{102-100} = 0.975$ and $\dfrac{k}{\bar{h}L} = \dfrac{k/L}{\infty} = 0$

from Fig 5.7, $Fo = 0.06 = \dfrac{\alpha t}{L^2}$; $t = \dfrac{0.06(0.07)^2}{1.69 \times 10^{-7}}$

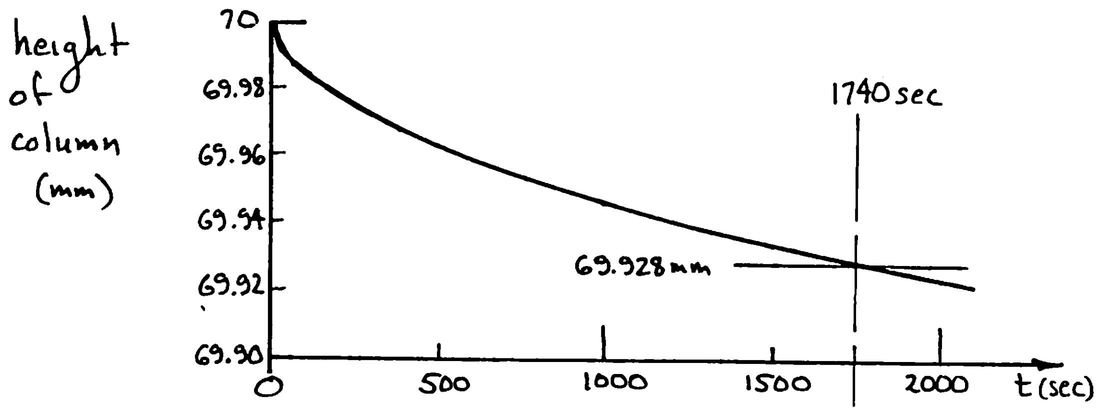
$\underline{\underline{t = 1740 \text{ sec or } 29 \text{ minutes}}}$

b) $\displaystyle\int_0^t q\,dt = \frac{k\Delta T}{\sqrt{\pi\alpha}}\int_0^t \frac{dt}{\sqrt{t}} = \frac{2k\Delta T}{\sqrt{\pi\alpha}}\sqrt{t}$ where we have used eqn. (5.48) for q

$$\text{reduction in height} = \frac{\left(\int_0^t q\,dt\right)\frac{J}{m^2}}{\rho_g h_{fg}\ \frac{J}{m^3}} = \frac{2k\Delta T}{\rho_g h_{fg}\sqrt{\pi\alpha}}\sqrt{t} = \frac{2(0.68)2}{961(2.26)10^6}\frac{\sqrt{t}}{\sqrt{\pi(1.68)10^{-7}}}$$

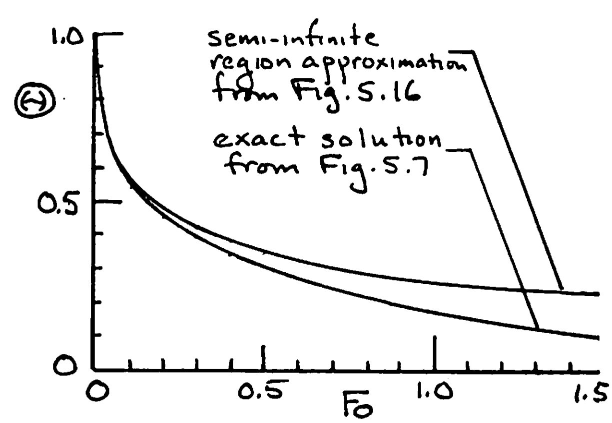
5.14 (continued)

We therefore obtain: $\underline{\underline{\text{height} = \left(0.07 - 1.72(10)^{-6}\sqrt{t}\right)\,\text{m}}}$



5.15 A slab with Bi = 2 is cooled. Compare the exact and semi-infinite region solutions for Θ, on the surface.

Fo	$\beta^2 = Bi^2\,Fo$	$\Theta_{Fig. 5.7}$	$\Theta_{Fig. 5.16}$
0	0	1	1
0.1	0.4	0.54	0.56
0.2	0.8	0.46	0.465
0.3	1.2	0.395	0.41
0.4	1.6	0.345	0.38
0.5	2.0	0.305	0.34
0.75	3.0	0.23	0.30
1.0	4.0	0.175	0.26
1.5	6.0	0.098	0.218



Since the semi-infinite approximation does not reflect the influence of the insulated wall at x/L = 0, it eventually shows a slower cooling than the correct solution.

5.16 Derive eqn. (5.62) from: $\dfrac{1}{2}\dfrac{d^2\Theta}{d\xi^2} = \dfrac{d\Theta}{d\Omega}$, $\Theta(\xi=0) = \cos\Omega$; $\Theta(\xi>0) = \text{finite}$

Assume: $\Theta = f(\xi)\,e^{i\Omega}$ so: $\dfrac{d^2 f}{d\xi^2} = 2i(f)$, hence $f = C_1 e^{\sqrt{2i}\,\xi} + C_2 e^{-\sqrt{2i}\,\xi}$

but $\sqrt{2i} = 1 + i$ & $e^{ix} = \cos x + i\sin x$ so: $\Theta = \underbrace{(\cos\xi\cos\Omega + \sin\xi\sin\Omega)}_{=\cos(\xi-\Omega)}\underbrace{(C_3 e^{\xi} + C_4 e^{-\xi})}_{\text{new consts.}}$

To accomodate the second b.c. we must get $C_3 = 0$.

To accomodate the first b.c.: $\Theta(\xi=0) = \cos(-\Omega)C_4 e^{-0} = \cos\Omega$

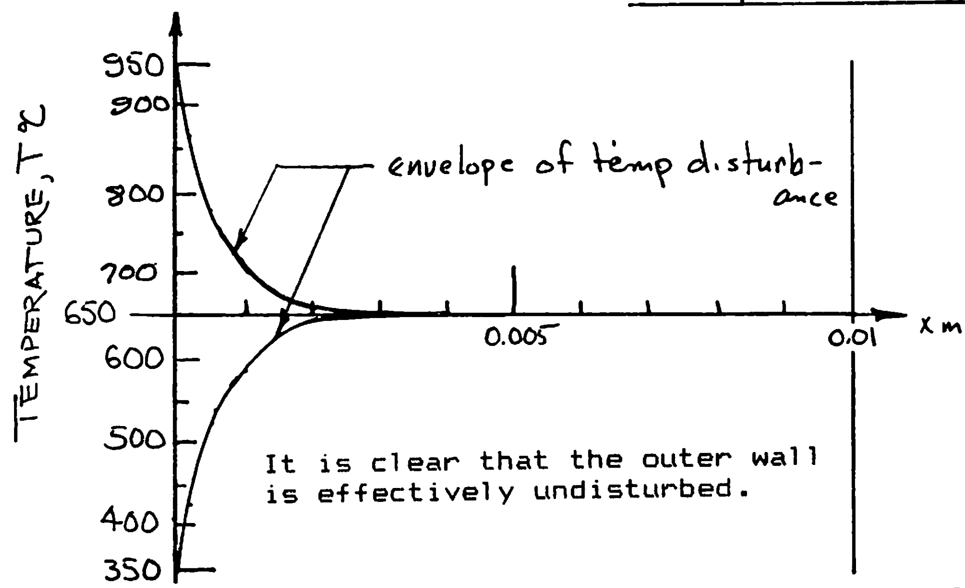
It follows that $C_4 = 1$ so we get: $\underline{\underline{\Theta = e^{-\xi}\cos(\xi-\Omega)}}$

5.17 A "steel" cylinder wall is 1 cm thick. (Take $\alpha = k/\rho c$ = 32/(7800)(473) = 0.000008.67m²/s.) The inside wall temp. is (650 + 100cosωt)°C and ω = 2π8 = 50.26 rad/sec. Plot the envelope of the temperature disturbance in the wall.

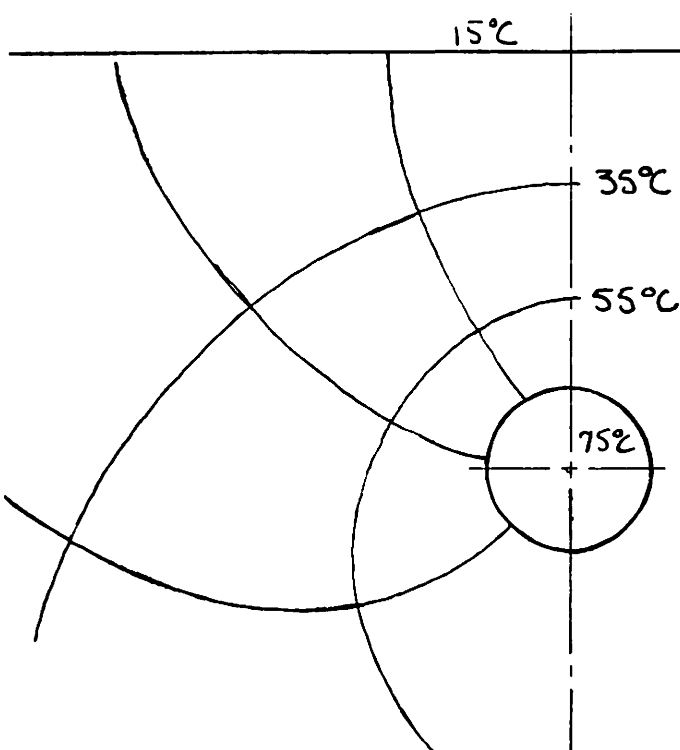
$$\Theta = e^{-\xi}\cos(\Omega - \xi)$$

so the envelope is given by $\Theta = \pm e^{-\xi}$ where $\xi = x\sqrt{\frac{\omega}{2\alpha}} = 170.25x$

and where: $\Theta = \frac{T - \overline{T}}{\Delta T} = \frac{T-650}{300}$, so $\underline{T_{envelope} = 650 \pm 300 e^{-170.25(x\,m)}}$



5.18 A 75°C, 0.4 m dia. pipe is buried in Portland cement (k = 17.) It is parallel to a 15°C surface and 1 m away from it. Plot T along a vertical line through the center of the pipe and compute the heat loss per meter of pipe.



$$Q = k\Delta T\,S = 1.7(75-15)S$$
$$= 102\,S$$

where: $S_{graphical} = \frac{N}{I} = \frac{8}{3}$
$$= \underline{\underline{2.67}}$$

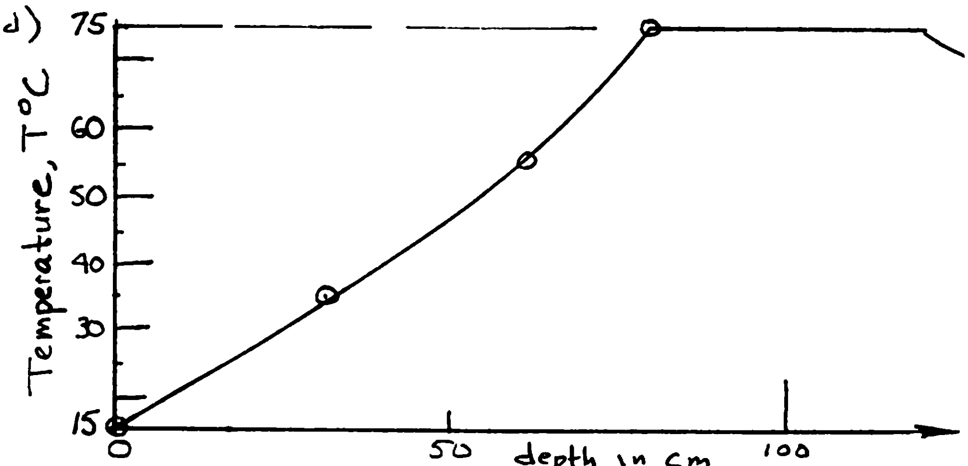
$$S_{analyt.} = \frac{2\pi}{\cosh^{-1}(5)} = \frac{2\pi}{2.292}$$
$$= \underline{2.74}$$

$$\left[\text{error} = \frac{2.74-2.67}{2.74} = 2.6\%\right]$$

$Q_{graphical}$ = 272 W/m

$Q_{analytical}$ = 279.5 W/m

5.18 (continued)



The variation of temp. with depth obtained from the flux plot.

5.19 Obtain S for a sphere buried in an infinite medium.

general solution for this case (cf. Example 5.10) is $T = C_1/r + C_2$

w/ b.c.'s: $T(r=\infty) = T_\infty$, so $C_2 = T_\infty$; and $T(r=R) = T_w$ so $C_1 = (T_w - T_\infty)R$

Thus: $T - T_\infty = (T_w - T_\infty)\frac{R}{r}$ and $Q = 4\pi R^2\left(-k\left.\frac{\partial T}{\partial r}\right|_{r=R}\right) = +4\pi R^2 k(T_w - T_\infty)\frac{1}{R}$

It follows that $\underline{\underline{S = \frac{Q}{k\Delta T} = 4\pi R}}$

5.20 Find S for parallel cylinders in an infinite medium. One has twice the diameter of the other. The centers are one diameter of the larger cylinder, apart.

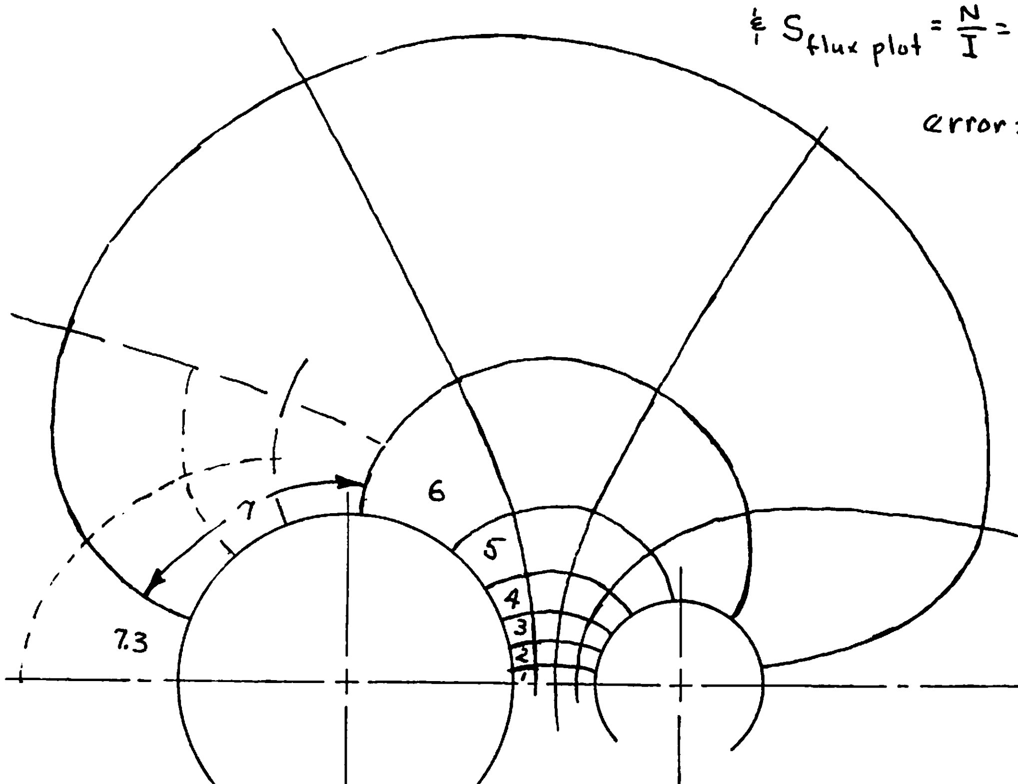
According to #9 in Table 5.2, $m_1/R_1 = 1 - \frac{1}{4} + \frac{1}{16} = \frac{19}{16}$ & $m_2/R_2 = 2\left[1 - \frac{1}{4} + \frac{1}{16}\right] = \frac{13}{8}$

Then $S_{analytical} = \dfrac{2\pi}{\cosh^{-1}\frac{19}{16} + \cosh^{-1}\frac{13}{8}} = \dfrac{2\pi}{0.60 + 1.07} = \underline{\underline{3.76}}$

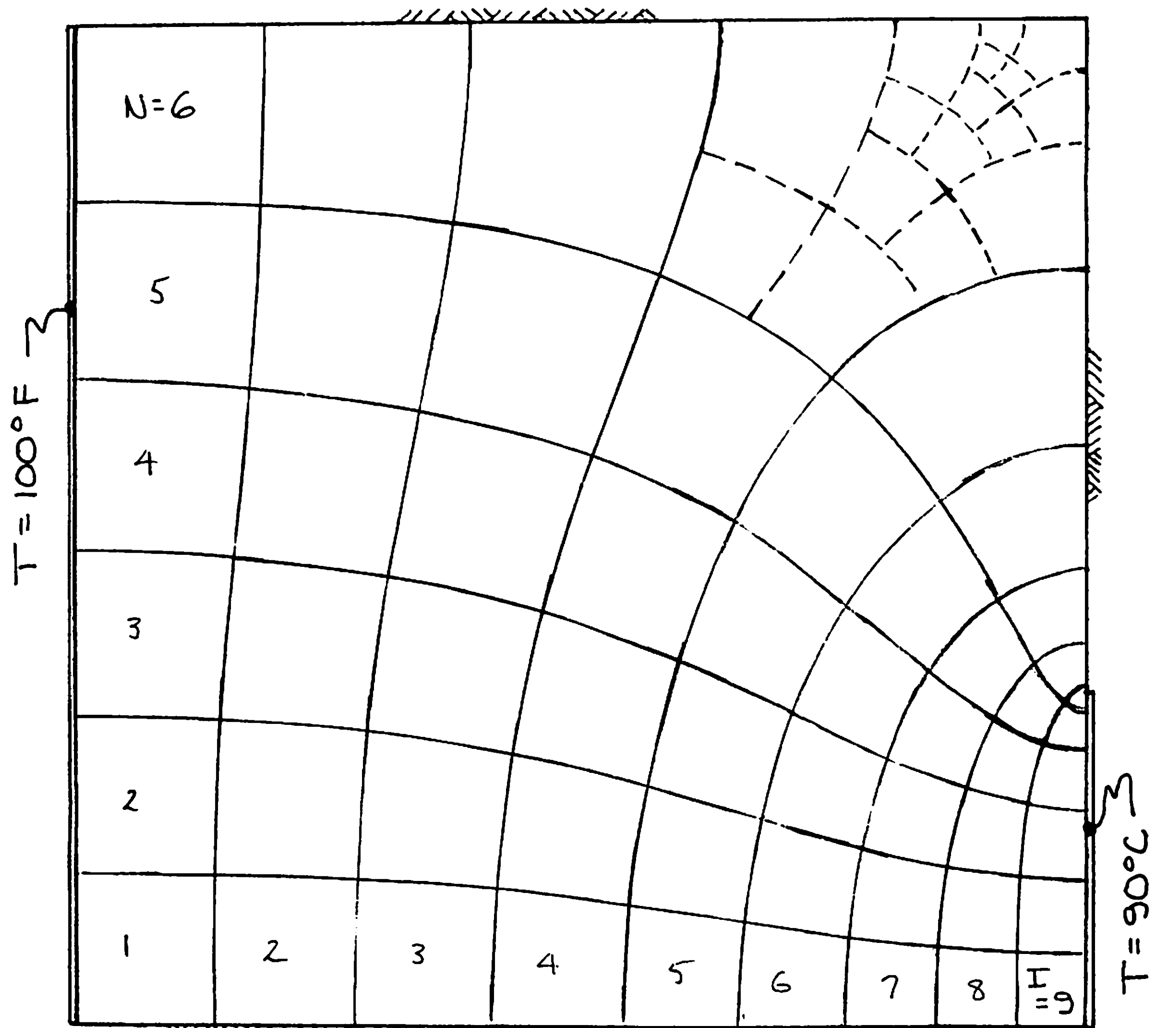
& $S_{flux\ plot} = \frac{N}{I} = \frac{2(7.3)}{4} = \underline{\underline{3.65}}$

$error = \dfrac{3.76 - 3.65}{3.76}$

$= \underline{\underline{2.9\%}}$



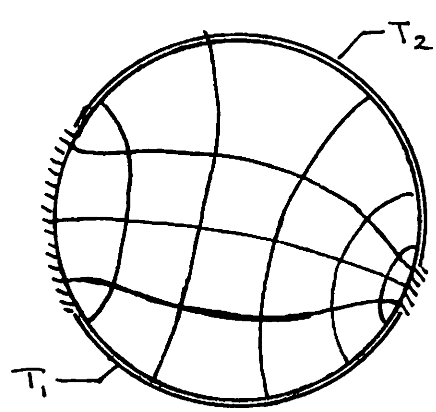
5.21 The 3 in. by 3 in. copper slab (1 in. thick) shown below conducts heat from the 100°F surface on the left to the 1 in. portion of the right side which is kept at 90°F.



$$Q = S k \Delta T = \frac{6}{9}\, 226 \frac{\text{Btu}}{\text{ft}^2\text{-hr-}^\circ\text{F}} \left((100-90)^\circ\text{F}\right) = 1507 \frac{\text{Btu}}{\text{ft-hr}}$$

$$1507 \frac{\text{Btu}}{\text{ft-hr}} \cdot \frac{\text{ft}}{12} = \underline{\underline{125.6 \frac{\text{Btu}}{\text{hr}}}} = \underline{\underline{38.8\ \text{W}}}$$

5.22a Obtain the shape factor for the following shape.



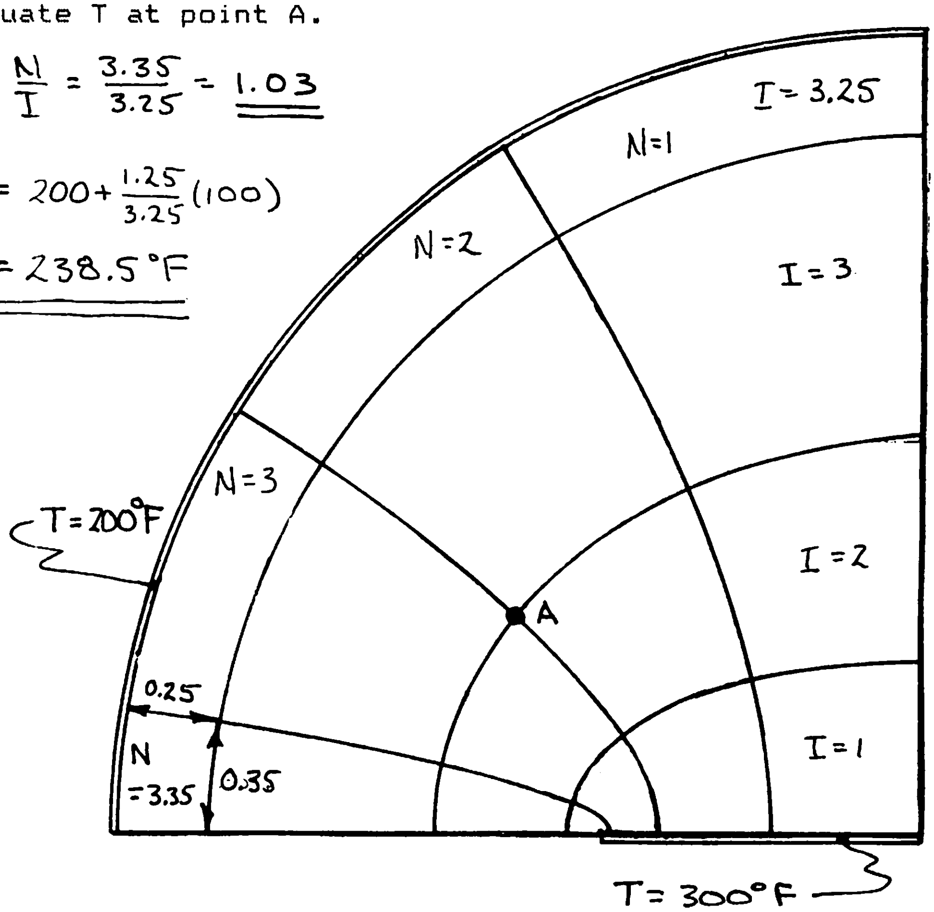
$$S = \frac{N}{I} \approx \frac{6}{4} = 1.67$$

5.22b Obtain the shape factor for the configuration shown. Evaluate T at point A.

$$S = \frac{N}{I} = \frac{3.35}{3.25} = \underline{\underline{1.03}}$$

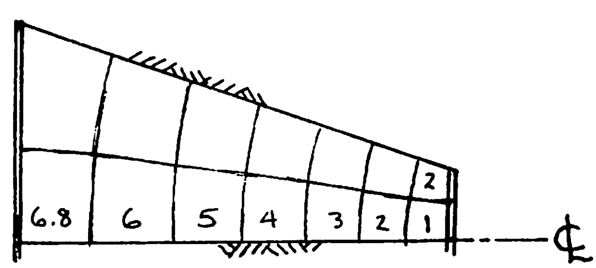
$$T_A = 200 + \frac{1.25}{3.25}(100)$$

$$\underline{\underline{T_A = 238.5\,°F}}$$



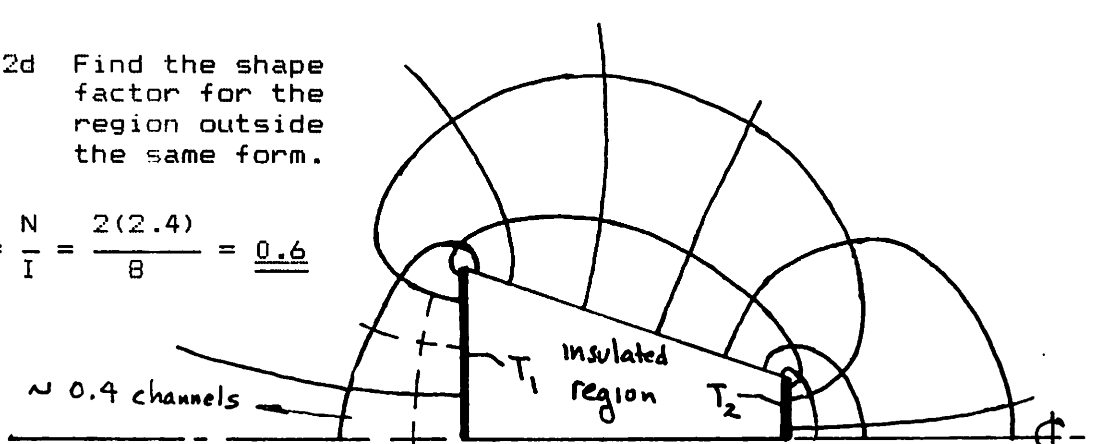
5.22c Find S for the inside of the form shown.

$$S = \frac{2(2)}{6.8} = \underline{\underline{0.59}}$$



5.22d Find the shape factor for the region outside the same form.

$$S = \frac{N}{I} = \frac{2(2.4)}{8} = \underline{\underline{0.6}}$$

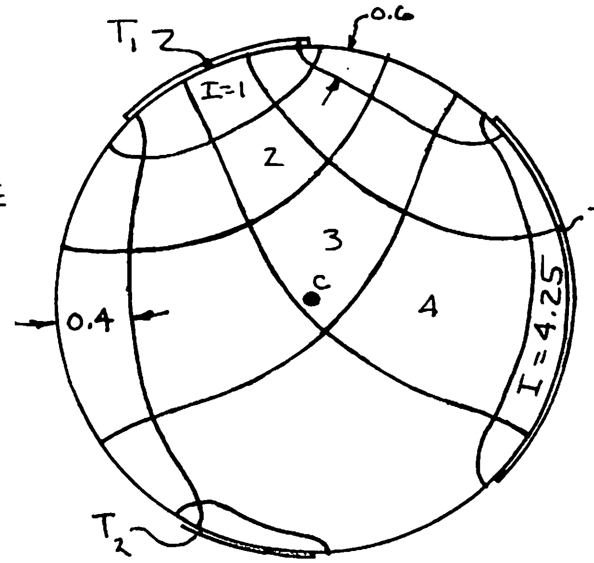


5.22e Find S for the shape shown and the center temperature.

$$S = \frac{N}{I} = \frac{0.4 + 3 + 0.6}{4.25} = \underline{\underline{0.94}}$$

$$T_c = T_1 + \frac{2.75}{4.25}(T_2 - T_1)$$

$$= \underline{\underline{T_1 + 0.65(T_2 - T_1)}}$$

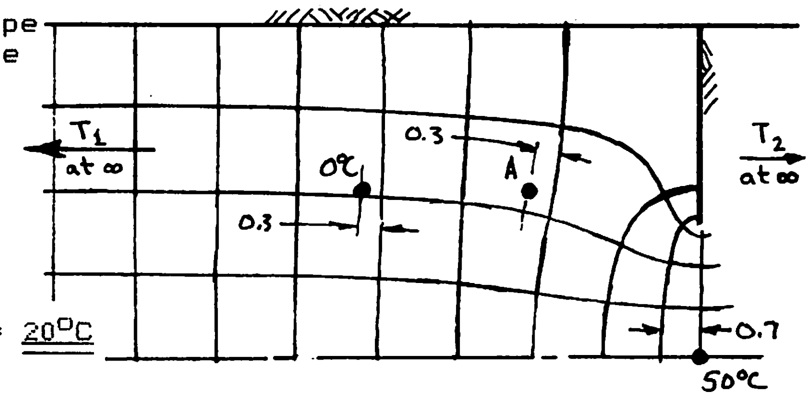


5.22f Find S for the shape below, and evaluate T at point A.

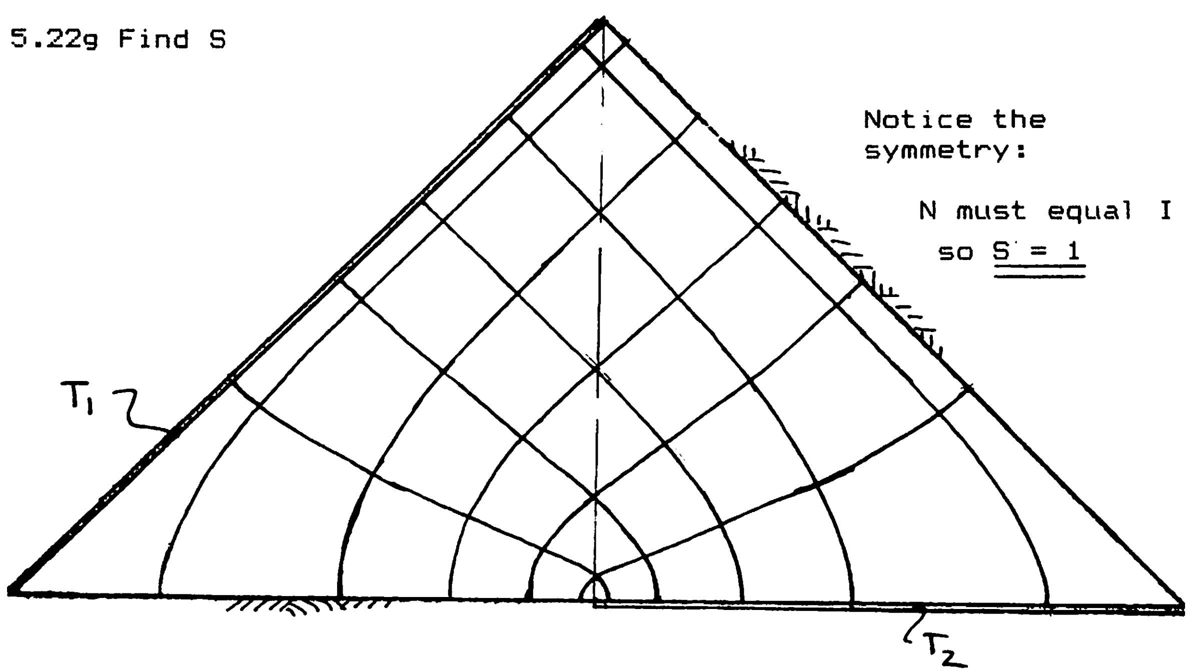
$$S = \frac{N}{I} = \frac{2(4)}{\infty} = \underline{\underline{0}}$$

because the thermal resistance is infinite in length.

$$T_A = 0 + \frac{2}{5}(50 - 0) = \underline{\underline{20^{\circ}C}}$$



5.22g Find S



Notice the symmetry:

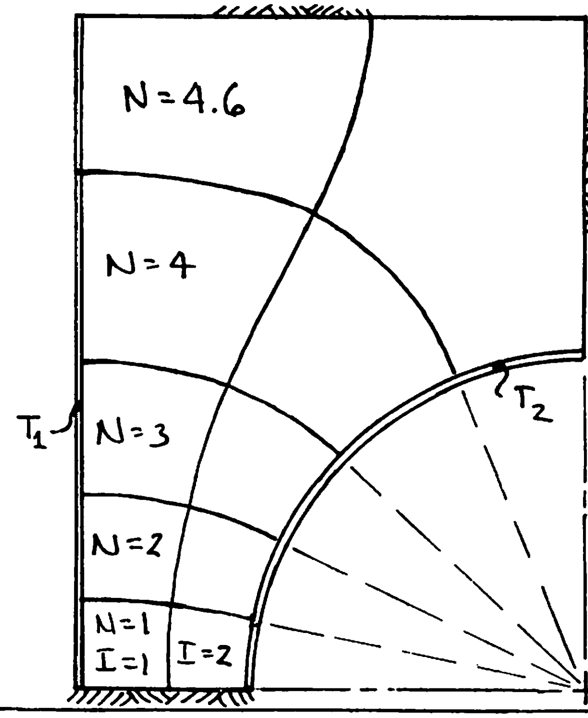
N must equal I so $\underline{\underline{S = 1}}$

(There are some interesting ramifications to this problem. See J. Heat Transfer, Vol. 103, No. 3, 1981, pp. 600-1.)

5.22h Find S

$$S = \frac{N}{I} = \frac{4.6}{2}$$

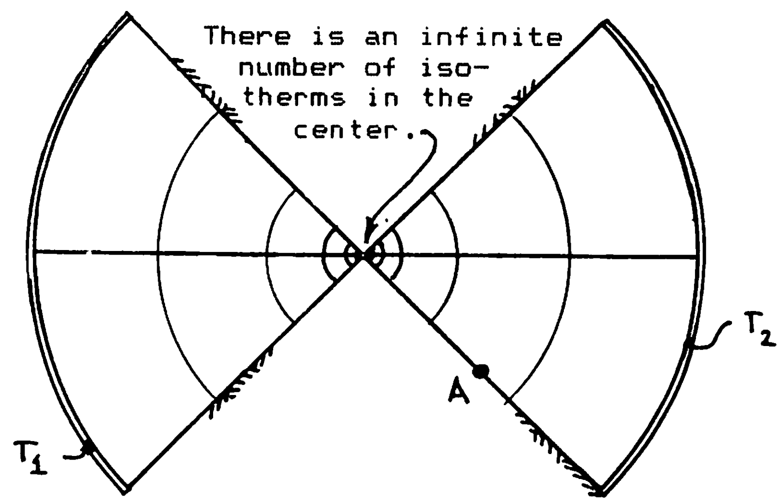
$$\underline{\underline{S = 2.3}}$$



5.22i Find S and T_A

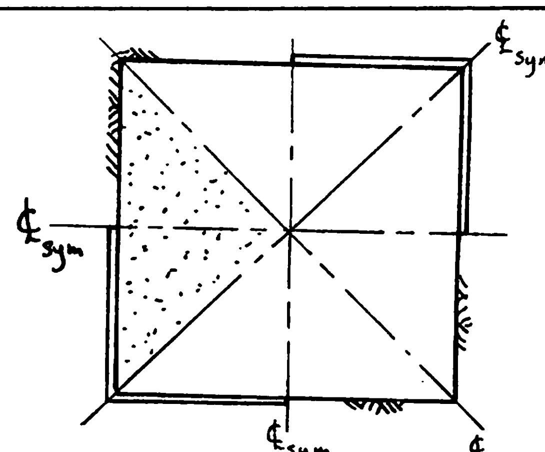
$$S = \frac{N}{I} = \frac{2}{\infty} = \underline{\underline{0}}$$

Thus the singular point of infinite resistance in the center blocks all heat flow from T_1 to T_2.



$\underline{\underline{T_A = T_2}}$ since all temperature drop occurs across the point of infinite resistance in the center.

5.22j Find S for the form shown



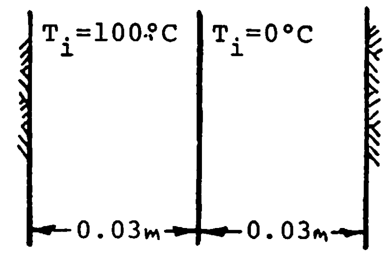
This form has 4 axes of symmetry. We therefore isolate the stippled area and do a flux plot for it. We get (see Prob. 5.22g)

$$S_{\triangleright} = 1.00$$

Then for the total shape

$$S_{\boxtimes} = \frac{\text{twice as many channels}}{\text{twice as many isotherms}} = \frac{2}{2} S_{\triangleright} = \underline{\underline{1.00}}$$

5.23 The two copper slabs shown are suddenly laid together as shown. Find the temperature of the left-hand adiabatic side after 2.3 sec elapses.



By symmetry, we see that the interface must immediately assume--and retain--the average temperature of 50°C.

$$F_o = \frac{\alpha t}{L^2} = \frac{11.57(10)^{-5}(2.3)}{0.03^2} = \underline{0.296} \quad \text{and } B_i^{-1} = \frac{k}{\bar{h}L} = \frac{k/L}{\infty} = \underline{0}$$

Then from 5.7 we read $\simeq \dfrac{T_{ad.wall} - T_{int.}}{T_{i_{left}} - T_{int.}} = 0.61$

so $T_{ad.wall} = 0.61(100-50)+50 = \underline{\underline{80.5°C}}$

5.25 Prove that temperature cannot oscillate in a 2nd order lumped capacity system.

If the system is to oscillate, $\sqrt{\left(\frac{b}{2}\right)^2 - c}$ in eqn. (5.23) must be imaginary or

$$b^2 = \left(\frac{1}{T_1} + \frac{1}{T_2} + \frac{h_c}{\bar{h}T_2}\right)^2 < \frac{4}{T_1 T_2}$$

or:

$$\sqrt{\frac{T_2}{T_1}} + \sqrt{\frac{T_1}{T_2}}\left(1 + \frac{h_c}{\bar{h}}\right) - 2 < 0$$

If we call $\sqrt{T_2/T_1} \equiv x$, then this says: $x^2 - 2x + \left(1 + \frac{h_c}{\bar{h}}\right) < 0$

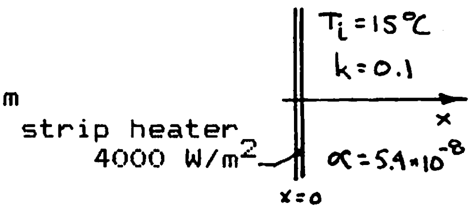
or: $(x-1)^2 + \frac{h_c}{\bar{h}} < 0$

But everything on the left must be positive so this cannot be! Therefore the system cannot oscillate.

5.26 When the isothermal and adiabatic lines in a flux plot are interchanged, N turns into I and I into N. It follows that:

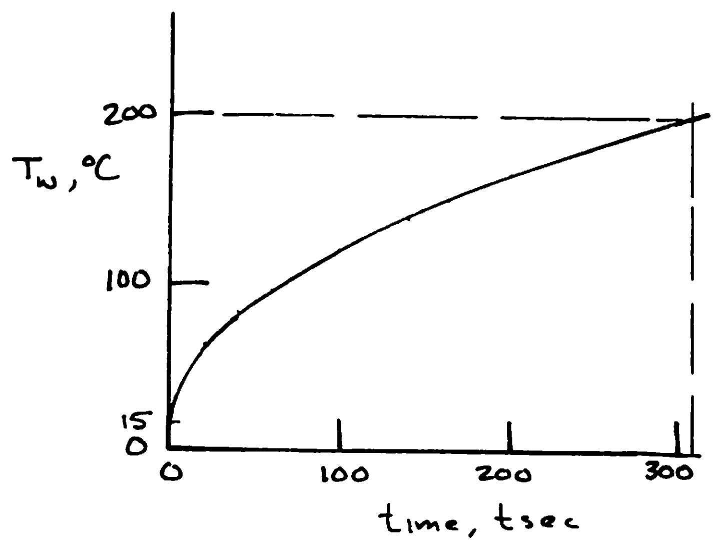
$$\underline{S_{\text{interchanged}} = I/N = 1/S_{\text{original}}}$$

5.28 Plot T(x=0) as a function of time when the strip heater shown is turned on. What is q at x = 0.01 m when T(x=0) = 200^{o}C?



from eqn. (5.50) $T_w(t) = T_\infty + 2\frac{q_w}{k}\sqrt{\frac{\alpha t}{\pi}} = 15 + \frac{8000}{0.1}\sqrt{\frac{5.4(10)^{-8}}{\pi}}\sqrt{t}$

$$= 15 + 10.49\sqrt{t}$$



$T_w = 200°C$ when
$\underline{t = 310.4 \text{ sec.}}$

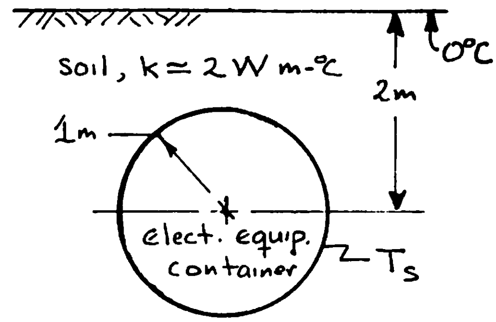
Then from eqn. (5.49):

$$\frac{q_w - q(x,t)}{q_w} = \frac{4000 - q}{4000} = \text{erf}\frac{x}{2\sqrt{\alpha t}} = \text{erf}\,1.22 = 0.9155$$

so $q(x=0.01\text{m}, t=310.4\text{s}) = \underline{\underline{338 \frac{W}{m^2}}}$

5.29 There as many answers to this problem as there are students. (The most common error students will make is that of touching items in a room that are not at room temperature -- rings on fingers, window panes, ice cubes, etc.)

5.30 What is the maximum Q from the container shown if T_s cannot exceed 30°C



$$Q = k\Delta T S = 2(30-0)S$$

And from Table 5.2, No. 7,

$$S = \frac{4\pi R}{1-\frac{R}{2h}} = \frac{4\pi}{1-\frac{1}{2(2)}} = \underline{16.76}$$

Therefore the maximum Q is 2(30)16.76 = $\underline{\underline{1005\ \text{W}}}$.

5.31 A semi-infinite slab of ice at -10°C is exposed to air at 15°C through a heat transfer coefficient of 10 W/m²-°C. What is the initial rate of melting in kg of water/m²-s? What is the asymptotic rate of melting? Describe the melting process in physical terms. (The latent heat of fusion of ice h_{fs} = 333,300 J/kg.)

Solution. The surface must first be brought up to the melting temperature. During this period $\underline{\underline{\dot{m}_{melt} = 0\ \text{kg/m}^2\text{-s}}}$.

Once the saturation temperature, 0°C, has been reached at the surface, heat will flow into the interior of the slab in accordance with equation (5.48) which shows that $q \sim 1/\sqrt{t}$. Thus, after a long time, that portion of the heat reaching the interface, which flows to the interior, becomes negligible. Then a simple energy belance yields:

$$\bar{h}\ (T_{air} - 0°\text{C}) = h_{fs}\ \dot{m}_{melt}$$

or:

$$\dot{m}_{melt} = 10(15-0)/333{,}300 \doteq \underline{\underline{0.00045\ \text{kg/m}^2\text{-s}}}$$

5.32 One side of a firebrick wall, 10 cm thick, initially at 20°C is exposed to a 1000°C flame through a heat transfer coefficient of 230 W/m²-°C. How long will it be before the other side is too hot to touch? (Estimate properties at 500°C).

Solution. $k/\bar{h}L = 0.15/230(0.1) = 0.00652$

$$\frac{T_{burn} - T_\infty}{T_i - T_\infty} = \frac{65-1000}{20-1000} = 0.954$$

Then from Fig. 5.7, upper left, we get

$$\frac{\alpha t}{L^2} = 0.075,\ t \simeq \frac{0.075(0.1)^2}{5.4\times 10^{-8}} = 13{,}890\ \text{sec}$$

$$= \underline{\underline{3\ \text{hr}\ 51\ \text{min}}}.$$

Problem 5.33 A lead bullet travels for 0.5 seconds within a shock wave that heats the air near the bullet to 300°C. Approximate the bullet as a cylinder 0.8 mm in diameter. What is its surface temperature at impact if h = 600 W/m²K and if the bullet was initially at 20°C? What is its center temperature?

Solution The Biot number 600(0.004)/35 = 0.0685, so we can first try the lumped capacity approximation. See eqn. (1.22):

$$(T_{sfc} - 300)/(20 - 300) = \exp(-t/\mathbf{T}), \text{ where } \mathbf{T} = mc/hA$$

So $\mathbf{T} = \rho c(\text{area})/h(\text{circumf.}) = 11{,}373(130)\pi(0.004)^2/h\pi(0.008) = 4.928$ seconds

And $(T_{sfc} - 300)/(20 - 300) = \exp(-0.5/4.928)$.

So $\underline{\underline{T_{sfc} = 300 - 0.903(280) = 47.0^{o}C}}$

In accordance with the lumped capacity assumption,

$\underline{\underline{47.0^{o}\text{C is also the center temperature.}}}$

Now let us see what happens when we use the exact graphical solution, Fig. 5.8:

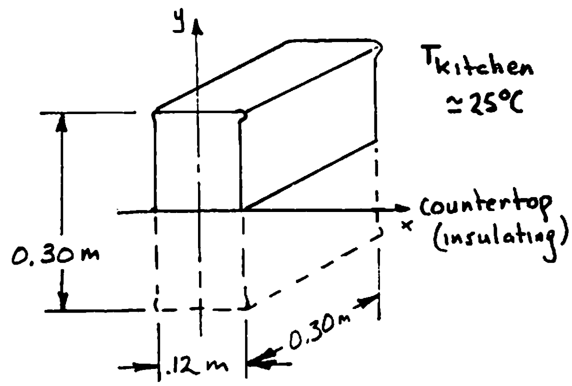
for $Fo = \alpha t/r_o^2 = 2.34(10^{-5})(0.5)/0.004^2 = 0.731$ and $r/r_o = 1$, we get:

$(T_{sfc} - 300)/(20 - 300) = 0.90,$ So $\underline{\underline{T_{sfc} = 48.0^{o}C}}$

And at $r/r_o = 0$, $(T_{ctr} - 300)/(20 - 300) = 0.92,$ & $\underline{\underline{T_{ctr} = 42.4^{o}C}}$

We thus have good agreement within the limitations of graph-reading accuracy. It also appears that the lumped capacity assumption is accurate within around 6 degrees in this situation.

5.34 A loaf of bread as shown, is at 125°C when it is removed from and oven and put to cool on an insulating counter. k = 0.05 W/m-°C and 5×10^{-7} m²/s. $\bar{h}$ = 10 W/m-°C. When will the bottom center reach 60°C.



$$\Theta = \frac{60-25}{125-25} = 0.35$$
$$= \Theta_x \Theta_y \Theta_z$$

Let us guess times, calculate Θ_x, Θ_y, Θ_z, and see how close to 0.35 we come.

$$F_o = \frac{\alpha t}{L^2} \quad ; \quad k/\bar{h}L = 0.005/L$$

	for x, L = 0.06m			for y, L = 0.15m			$\Theta_z = \Theta_y$	
Guess t	F_o	$k/\bar{h}L$	Θ_x	F_o	$k/\bar{h}L$	Θ_y		$\Theta_x\Theta_y\Theta_z$
3600 sec	0.5	0.0833	0.92	0.08	0.0333	0.95	0.95	0.38
4000	0.556	"	0.38	0.089	"	0.95	0.95	0.34

It looks like about 3900 seconds or <u>1hr and 5 min.</u>

5.35 A lead cube, 50 cm on each side, is initially at 20°C. The surroundings are suddenly raised to 200°C and h around the cube is 272 W/m²–°C. Plot the cube temperature along a line from the center to the middle of one face, after 20 minutes have elapsed.

$$Bi^{-1} = (\bar{h}L/k)^{-1} = \frac{34}{272(0.25)} = \underline{0.5} \quad ; \quad F_o = \frac{\alpha t}{L^2} = \frac{2.35\times10^{-5}(20\times60)}{0.25^2} = \underline{0.451}$$

Then:

$$\Theta = \underbrace{\frac{T-T_\infty}{T_i-T_\infty}}_{(T-200)/-180} = \underbrace{\Theta^2\left(Bi=0.5,\ F_o=0.451,\ \frac{x}{L}=0\right)}_{0.7^2} \times \Theta\left(Bi=0.5,\ F_o=0.451,\ \frac{x}{L}\right)$$

so:

$T = 200 - 88.2\times0.70 = 138.3°C$ at $x/L = 0$

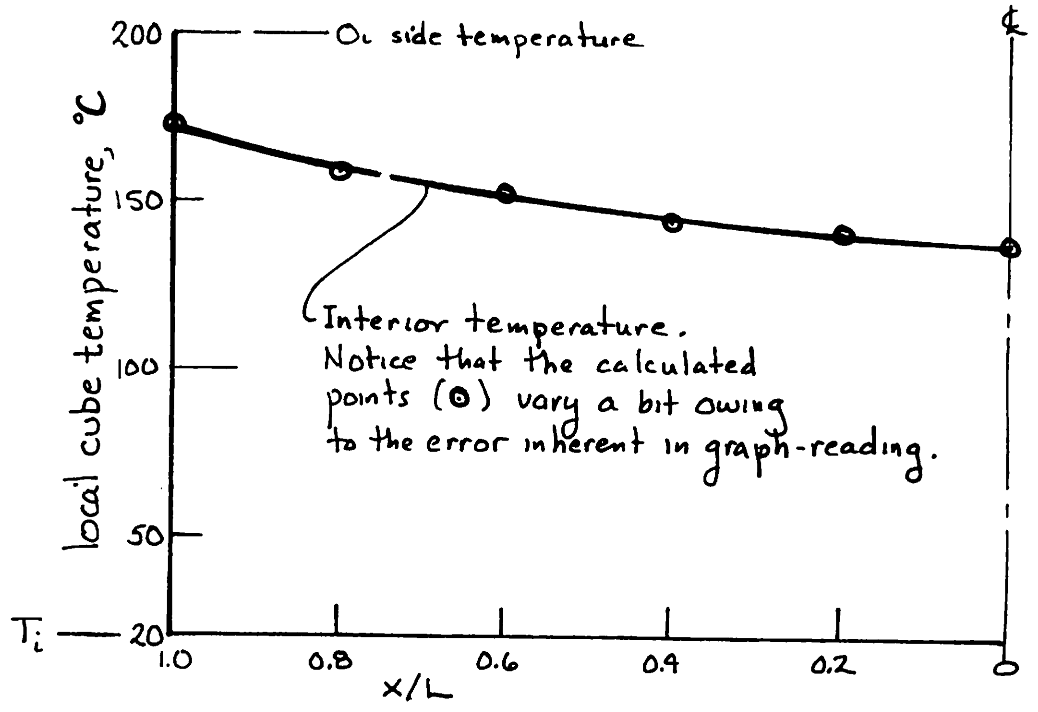
$T = 200 - 88.2\times0.67 = 140.9°C$ at $x/L = 0.2$

$T = 200 - 88.2\times0.63 = 144.4°C$ at $x/L = 0.4$

$T = 200 - 88.2\times0.55 = 151.5°C$ at $x/L = 0.6$

$T = 200 - 88.2\times0.46 = 159.4°C$ at $x/L = 0.8$

$T = 200 - 88.2\times0.32 = 171.8°C$ at $x/L = 1.0$

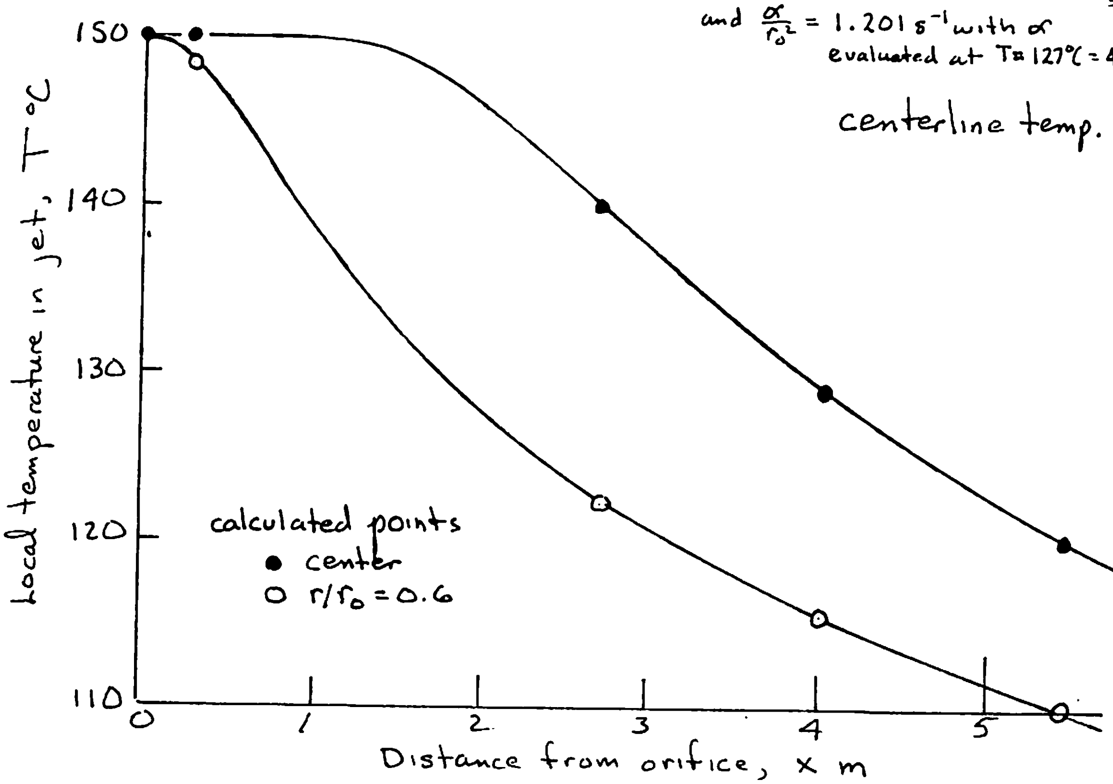


5.36 A jet of clean water superheated to 150°C issues from a (1/16)in. diameter sharp-edged orifice into air at 1 atm., moving at 27 m/s. The coefficient of contraction of the jet is 0.611. Evaporation at $T=T_{sat}$ begins immediately the outside of the jet. Plot the centerline temperature of the jet, and $T(r/r_o=0.6)$, as functions of distance from the orifice, up to about 5 m. Neglect any axial conduction and any dynamic interactions between the jet and the air.

Any element of the jet cools approximately as an infinite cylinder would, while it moves. Therefore we can use Fig. 5.8 with $r/r_o=0$ and $r/r_o = 0.6$, and (since we have a b.c. of the first kind, $T(r=r_o)=T_{sat}$) Bi^{-1} is 0. Then:

t sec	$x\,m = u_{jet}\,t$	$Fo = \frac{\alpha t}{r_o^2}$	Θ_{c_L}	$\Theta(\frac{r}{r_o}=0.6)$	$T=[(T_i-T_\infty)\Theta+T_\infty]$°C Θ_{c_L}	$\Theta(\frac{r}{r_o}=0.6)$
0.01	0.27	0.0120	1.00	0.99	150°C	148 °C
0.1	2.7	0.1201	0.80	0.44	140	122
0.15	4.05	0.1801	0.158	0.29	129	114.5
0.2	5.4	0.2402	0.42	0.20	121	110

where $r_o = \frac{1}{2}(0.0254/16)\sqrt{0.611} = 0.000$; $u_{jet} \doteq 27\,m/s$; $T_\infty = T_{sat} = 100°C$; $T_i - T_\infty = 150-100 = 50°C$;
and $\frac{\alpha}{r_o^2} = 1.201\,s^{-1}$ with α evaluated at $T \simeq 127°C = 400°K$



5.37 A 3 cm thick slab of aluminum (initially at 50°C) is slapped tightly against a 5 cm slab of copper (initially at 20°C). The outsides are both insulated and the contact resistance is negligible. What is the initial interfacial temperature? Estimate how long the interface will keep its initial temperature.

In accordance with equation (5.60), we get:

$$\frac{T_i - T_c}{T_a - T_c} = \frac{k_a/\sqrt{\alpha_a}}{k_c/\sqrt{\alpha_c} + k_a/\sqrt{\alpha_a}}$$

or

$$\frac{T_i - 20}{50-20} = \frac{237/\sqrt{9.61(10)^{-5}}}{398/\sqrt{11.57\times10^{-5}} + 237/\sqrt{9.61(10)^{-5}}} = 0.39$$

The solution is $\underline{\underline{T_i = 31.86°C}}$

and this will be valid as long as the slabs behave as though they were semi-infinite regions. This will be as long as $\xi > 3.65$ (See eqn. (5.45))

$$\xi = \frac{x}{\sqrt{\alpha t}} \quad \text{so} \quad 3.65 = \xi_a = \frac{0.02}{\sqrt{9.61(10)^{-5} t}} \quad ; \quad t < 0.703 \text{ sec.}$$

$$3.65 = \xi_{Cu} = \frac{0.03}{\sqrt{11.57(10)^{-5} t}} \quad ; \quad t < 1.622 \text{ sec.}$$

The shorter of the two times dictates how long it will take the first insulated wall to be felt. Consequently the interface temperature of $T_i = 31.86°C$ will remain constant for:

$\underline{\underline{0.703 \text{ sec}}}$

(at least)

5.38 A cylindrical underground gasoline tank, 2m in diameter and 4m long, is embedded in 10°C soil with k=0.8 W/m-°C and $\alpha = 1.3(10)^{-6}$ m²/s. Water at 27°C is injected into the tank to test it for leaks. It is well-stirred with a submerged, (1/2)kW pump. We observe the water level in a 10cm ID transparent standpipe and measure its rate of rise or fall. What rate of change of height will occur after one hour if there is no leakage? Will the level rise or fall? Neglect thermal expansion and deformation of the tank which should be complete by the time the tank is filled. (Hint: see eqn. (8.7)

$$\text{Area} = \frac{4}{3}\pi(2)^2 + 2\pi(4) = \underline{58.64\,m^2}, \quad V = \frac{4}{3}\pi(2)^2 4 = \underline{67.02\,m^3}$$

There are two energy transfers: $\underline{500\,J/s}$ of work $\underline{\text{to}}$ the water.

and: $$Q = \frac{kA\Delta T}{\sqrt{\pi\alpha t}} = \frac{0.8(58.64)(27-10)}{\sqrt{\pi(1.3)(10)^{-6}(3600)}} = 6577\,\frac{J}{s}$$ of heat $\underline{\text{from}}$ the water.

Now, using eqn. (8.7),

$$\beta = \frac{1}{v}\frac{\partial v}{\partial T}\bigg|_p = \frac{1}{V}\frac{\partial V}{\partial T}\bigg|_p \;;\quad \frac{dV}{dT} = 0.000275(67.02) = \underline{0.01843\,\frac{m^3}{°C}}$$

And: $$\frac{dt}{dT} = \frac{\rho c V}{Q_{net}} = \frac{996.6(4177)(67.02)}{(500-6577)} = \underline{-45{,}909\,\frac{sec}{°C}}$$

so $$\frac{dV}{dt} = \frac{dV}{dT}\frac{dT}{dt} = -\frac{0.01843}{45{,}909} = -0.401\times10^{-6}\,\frac{m^3}{s} = \underline{-0.001445\,\frac{m^3}{hr}}$$

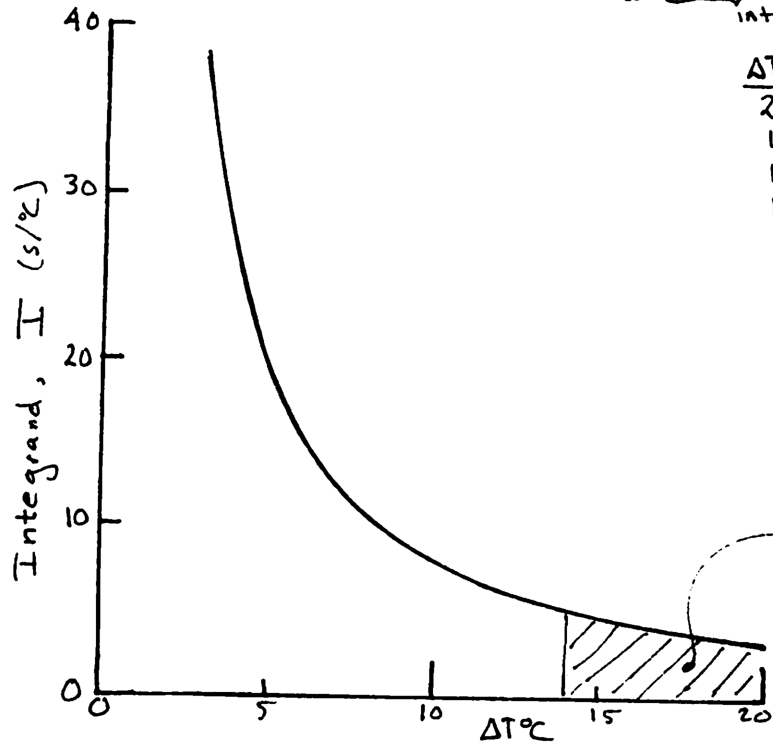
Then: $$\frac{dh}{dt} = \frac{d(V/A_{pipe})}{dt} = \frac{-0.001445}{\frac{\pi}{4}(0.1)^2} = -0.184\,\frac{m}{hr} = \underline{\underline{-18.4\,\frac{cm}{hr}}}$$

Thus the tank is leak-free when the standpipe drops at the rather substantial rate of 18.4 cm/hr. The drop is the result of thermal contraction.

5.39 A 47°C copper cylinder, 3 cm in diameter, is suddenly immersed horizontally in water at 27°C. Plot $T_{cyl.}$ as a function of time if $g = 0.76\ m/s^2$, $\bar{h} = [2.733 + 10.448(\Delta T°C)^{1/6}]^2\ W/m^2\text{-}°C$. (Do it numerically if you cannot integrate the resulting equation analytically.)

IN THIS CASE: $\dfrac{d(T_{cyl}-T_\infty)}{dt} = -\dfrac{\bar{h}A}{\rho c V}(T_{cyl}-T_\infty)$ or $\dfrac{\rho c D}{4}\dfrac{1}{\bar{h}\Delta T}\dfrac{d\Delta T}{dt} = -1$

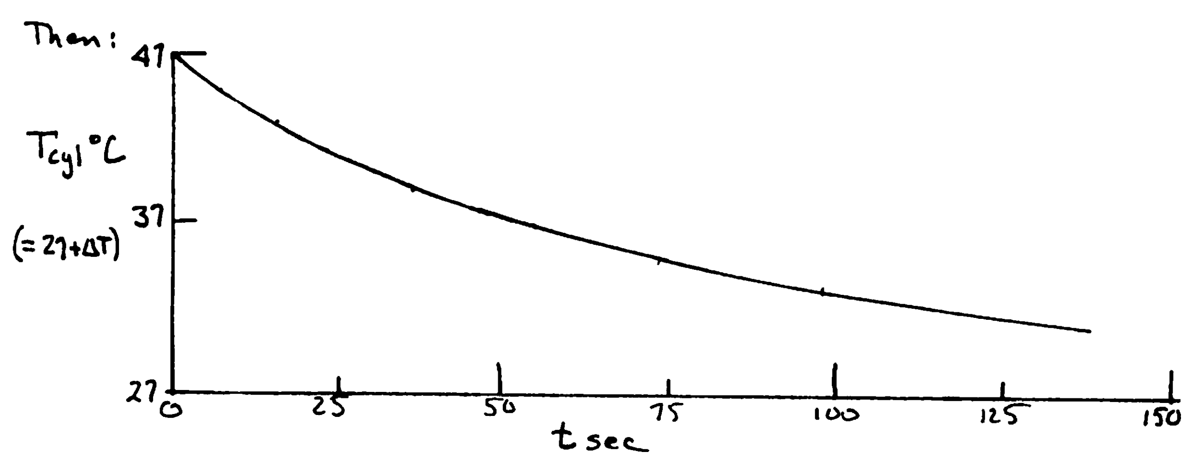
SO: $$\int_{\Delta T_i}^{\Delta T} \frac{8954(384)(0.03)}{4\Delta T(2.733+10.448\Delta T^{1/6})^2}\, d\Delta T = \int_{20}^{\Delta T} \underbrace{\frac{236.2\, d\Delta T}{\Delta T(0.2616+\Delta T^{1/6})^2}}_{\text{integrand} \equiv I} = -t$$



ΔT°C	I	$-\int_{20}^{\Delta T} I\, d\Delta T = t$ sec
20	3.24	0
18	3.71	7.32
16	4.32	15.44
14	5.13	24.84
12	6.25	36.44
10	7.90	54.44
8	10.51	72.64
6	15.20	97.36
4	25.50	136.4
3	61.66	167.4

Area between 20 & 14°C (at 2 sec/$\frac{1}{5}$ in. square) is 24.84 sec

Then:



5.40 The mechanical engineers at the University of Utah end Spring semester by roasting a pig and having a picnic. The pig is roughly cylindrical and about 26 cm in diameter. It is roasted over a propane flame, whose products have properties similar to those of air, at 280°C. The hot gas flows across the pig at about 2 m/s. If the meat is cooked when it reaches 95°C, and if it is to be served at 2:00 P.M., what time should cooking commence? Assume Bi to be large, but note Problem 7.40. The pig is initially at 25°C.

In this case, $\Theta = \dfrac{95-280}{25-280} = 0.725$, $Bi^{-1} \simeq 0$, so from Fig 5.8 we read: $F_o = \underline{0.13}$

using α for beef we get: $t = F_o \dfrac{r_o^2}{\alpha} = 0.13 \dfrac{0.13^2}{1.35(10)^{-7}} = \underline{16274 \text{ sec}}$

So the pig must be cooked for 4½ hrs. Cooking should begin at about 9:30 AM in the morning. ← If Bi is not large, this will be low.

5.41 People from cold Northern climates know not to grasp metal with their bare hands in subzero weather. A very slightly frosted piece of, say, cast iron will stick to your hand like glue in, say, -20°C weather, and you can tear off patches of skin. Explain this quantitatively.

Equation (5.60) tells us what to expect for the interfacial temperature when we touch ice. (Take $k_{body} \simeq k_{H_2O}$, $\alpha_{body} \simeq \alpha_{beef}$)

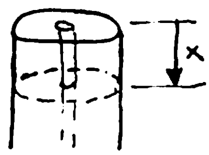
$$\frac{T_{sfc} - (-20)}{37 - (-20)} = \frac{0.6/\sqrt{1.35(10)^{-7}}}{2.215/\sqrt{1.15(10)^{-6}} + 1850.7} = 0.473 \qquad \text{so } \underline{T_{sfc} \simeq 7\,°C}$$

Thus, on immediate contact the ice (or frost) will melt to water (very quickly because it is thin.) Then:

$$\frac{T_{sfc} - (-20)}{0°C - (-20)} = \frac{2.215/\sqrt{1.15(10)^{-6}}}{\underbrace{52/\sqrt{1.7(10)^{-5}}}_{\text{iron}} + 2065} = 0.128 \qquad \text{so } \underline{T_{sfc} = -17.4\,°C}$$

Thus the iron will immediately refreeze the water, causing it to "glue" your hand to the iron.

5.42 A 4 cm dia. No. 304 stainless steel rod has a very small hole down its center. The hole is clogged with wax that has a melting point of 60°C. The rod is at 20°C. In an attempt to free the hole, a workman swirls the end of the rod -- and about a meter of its length -- in a tank of water at 80°C. If $\bar{h}$ is 688 W/m^2-C° on both the end and the sides of the rod, plot the depth of the melt front as a function of time, up to, say, 4 cm.



$$\Theta(r=0, x) = \underset{\text{semi-infinite}}{\Theta(x)} \cdot \underset{\text{cyl.}}{\Theta(r=0)}$$

At melt front $\Theta = \dfrac{40-80}{20-80} = 0.667$

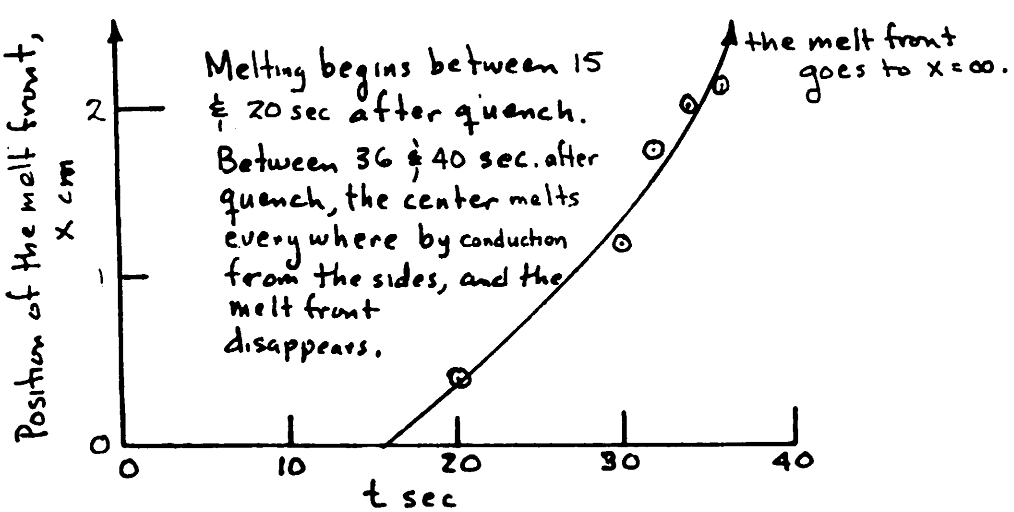
The rod: $Bi^{-1} = \dfrac{k}{\bar{h} r_o} = \dfrac{13.8}{688(0.02)} = 1.00$, $F_o = \dfrac{\alpha t}{r_o^2} = \dfrac{0.000004}{0.02^2} t = 0.01t$

The semi-inf. region: $\beta^2 = \dfrac{\bar{h}^2 \alpha t}{k^2} = \dfrac{688^2(0.000004)}{13.8^2} t = 0.00994t$

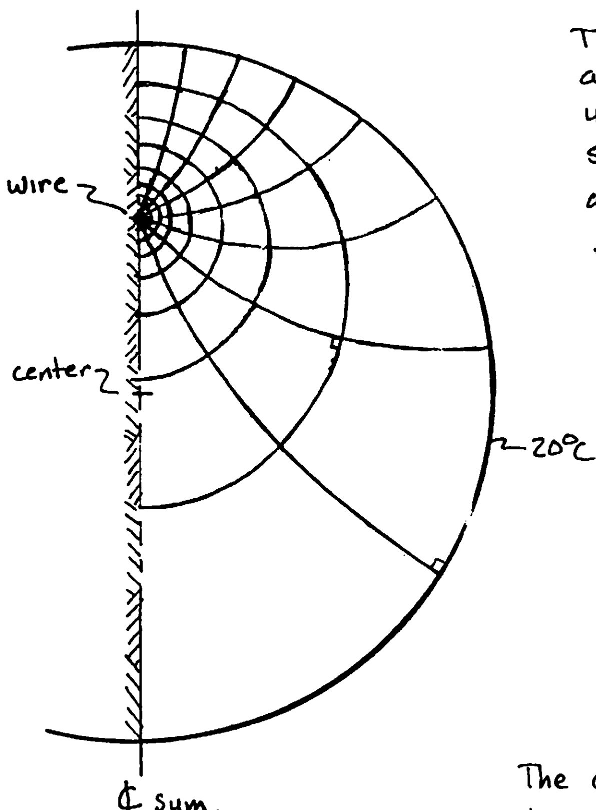
$$\beta\xi = \bar{h}x/k = \frac{688}{13.8}x = 49.86x$$

Then

t (s)	Fo_{cyl}	from Fig. 5.8 $\Theta_{cyl}(\frac{r}{r_o}=0)$	$\Theta_{semi\text{-}inf} = \dfrac{0.667}{\Theta_{cyl}}$	$\beta^2 = 0.00994t$	from Fig. 5.16 $\beta\xi$	x (m) $= \dfrac{\beta\xi}{49.86}$
15	0.15	0.94	0.71	off-scale, Melt has not begun		
20	0.20	0.88	0.76	0.20	0.20	0.004
30	0.30	0.79	0.894	0.30	0.56	0.0112
32	0.32	0.73	0.913	0.32	0.87	0.0174
34	0.34	0.715	0.932	0.338	~1.00	0.020
36	0.36	0.690	0.966	0.358	1.04	0.021
40	0.40	0.640	1.04	off-scale. Melt is complete for all x's.		



5.43 A cylindrical insulator contains a single, very thin, electrical resistor wire that runs along a line halfway between the center and the outside. The wire liberates 480 W/m. The thermal conductivity of the insulation is 3 W/m-°C, and the outside perimeter is held at 20°C. Develop a flux plot for the cross section, considering carefully how the field should look in the neighborhood of the point through which the wire passes. Evaluate the temperature at the center of the insulation.



The wire emits heat equally in all directions. Therefore we set up 16 adiabatic lines -- 8 on one symmetrical side -- all converging at equal angles (22.5°) on the wire.

There are an infinite number of isotherms -- T goes to infinity at the wire, (A real wire with finite diameter would alleviate this problem.)

Now if the wire liberates Q W Then through each square:

$$\frac{Q}{16} = k\Delta T \quad , \quad \Delta T = \frac{480}{16k} = \underline{10°C}$$

The center is 1.85 squares in from the perimeter. Therefore it is at

$$T_{center} = 20 + 1.85(10) = \underline{\underline{38.5°C}}$$

5.45 Lord Kelvin made an interesting estimate of the age of the earth in 1864. He assumed the earth originated as a mass of molten rock at 4144°K (7000°F) and that it has been cooled by outer space at 0°K, ever since. To do this, he assumed that Bi for the earth is very large, and that cooling has thus far penetrated only through a relatively thin (one-dimensional) layer. Using $\alpha_{rock} = 1.18 \times 10^{-6}$ m/s^2 and the measured surface temperature gradient of the earth, (1/27)°C/m, find Kelvin's value of Earth's age.

(Kelvin's result turns out to be much less than the accepted value of 4 billion years. His calculation fails because internal heat generation by radioactive decay of the material in the surface layer causes the surface temperature gradient to be higher than it would otherwise be.)

Solution. Since we take the problem to be unidimensional and since, with a large Bi, we may approximate the earth's surface as 0°K (with respect to 4144°K core), we may therefore use eqn. (5.48) for the heat flux

$$q = k\frac{\partial T}{\partial x}\bigg|_{x=0} = k(T_i - T_\infty)\mathrm{erf}(\pi\alpha t)^{-1/2}$$

where the derivative is given as (1/27)°C/m. Then

$$t = \frac{(T_i - T_\infty)^2}{(1/27)^2}\frac{1}{\pi\alpha} = 27^2(4144 - 0)^2/\pi(1.18 \times 10^{-6})$$

$$= 3.38 \times 10^{15} \text{ seconds} = \underline{107 \text{ million years}}$$

It is interesting that, though Kelvin used 4144°K as the tempèrature of molten rock, he revised this estimate downward to 1473°K in the late 1890's, giving an even smaller age of the earth. Further discussion can be found in Carslaw and Jaeger [1.14] or various geophysics textbooks.

PROBLEM 5.52 Suppose that $T_\infty(t)$ is the time-dependent temperature of the environment surrounding a convectively-cooled, lumped object.

a) When T_∞ is not constant, show that eqn. (1.19) leads to

$$\frac{d}{dt}(T - T_\infty) + \frac{(T - T_\infty)}{\boldsymbol{T}} = -\frac{dT_\infty}{dt}$$

where the time constant $\boldsymbol{T}$ is defined as usual.

b) If the object's initial temperature is T_i, use either an integrating factor or Laplace transforms to show that $T(t)$ is

$$T(t) = T_\infty(t) + \big[T_i - T_\infty(0)\big]\, e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}} \frac{d}{ds} T_\infty(s)\, ds$$

SOLUTION

a) From eqn. (1.19) for constant c, with $T_\infty(t)$ not constant:

$$\begin{aligned} -\overline{h}A(T - T_\infty) &= \frac{d}{dt}\big[\rho c V(T - T_{\text{ref}})\big] = mc\frac{dT}{dt} \\ &= mc\frac{d(T - T_\infty)}{dt} + mc\frac{dT_\infty}{dt} \end{aligned}$$

Setting $\boldsymbol{T} \equiv mc \big/ \overline{h}A$ and rearranging, we obtain the desired result:

$$\boxed{\frac{d}{dt}(T - T_\infty) + \frac{(T - T_\infty)}{\boldsymbol{T}} = -\frac{dT_\infty}{dt}} \tag{1}$$

b) The integrating factor for this first-order o.d.e. is $e^{t/\boldsymbol{T}}$. Multiplying through and using the product rule, we have

$$\frac{d}{dt}\Big[e^{t/\boldsymbol{T}}(T - T_\infty)\Big] = -e^{t/\boldsymbol{T}}\, \frac{dT_\infty}{dt}$$

Next integrate from $t = 0$ to t:

$$e^{t/\boldsymbol{T}}(T - T_\infty) - \big[T_i - T_\infty(0)\big] = -\int_0^t e^{s/\boldsymbol{T}}\, \frac{dT_\infty}{ds}\, ds$$

Multiplying through by $e^{-t/\boldsymbol{T}}$ and rearranging gives the stated result:

$$\boxed{T(t) = T_\infty(t) + \big[T_i - T_\infty(0)\big]\, e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}}\, \frac{dT_\infty}{ds}\, ds}$$

ALTERNATE APPROACH: To use Laplace transforms, we first simplify eqn. (1) by defining $y(t) \equiv T - T_\infty$ and $f(t) \equiv -dT_\infty/dt$:

$$\frac{dy}{dt} + \frac{y}{\boldsymbol{T}} = f(t)$$

Next, we apply the Laplace transform $\mathscr{L}\{..\}$, with $\mathscr{L}\{y(t)\} = Y(p)$ and $\mathscr{L}\{f(t)\} = F(p)$:

$$\mathscr{L}\left\{\frac{dy}{dt}\right\} + \mathscr{L}\left\{\frac{y}{\boldsymbol{T}}\right\} = \mathscr{L}\{f(t)\}$$

$$pY(p) - y(0) + \frac{1}{\boldsymbol{T}}Y(p) = F(p)$$

Solving for $Y(p)$:

$$Y(p) = \frac{1}{p + 1/\boldsymbol{T}}\, y(0) + \frac{1}{p + 1/\boldsymbol{T}}\, F(p)$$

Now take the inverse transform, $\mathscr{L}^{-1}\{..\}$:

$$\mathscr{L}^{-1}\{Y(p)\} = \mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\right\} y(0) + \mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\, F(p)\right\} \tag{2}$$

With a table of Laplace transforms, we find

$$\mathscr{L}^{-1}\underbrace{\left\{\frac{1}{p + 1/\boldsymbol{T}}\right\}}_{\equiv G(p)} = \underbrace{e^{-t/\boldsymbol{T}}}_{\equiv g(t)}$$

and with $G(p)$ and $g(t)$ defined as shown, the last term is just a convolution integral

$$\mathscr{L}^{-1}\left\{\frac{1}{p + 1/\boldsymbol{T}}\, F(p)\right\} = \mathscr{L}^{-1}\{G(p)F(p)\} = \int_0^t g(t-s)\, f(t)\, ds$$

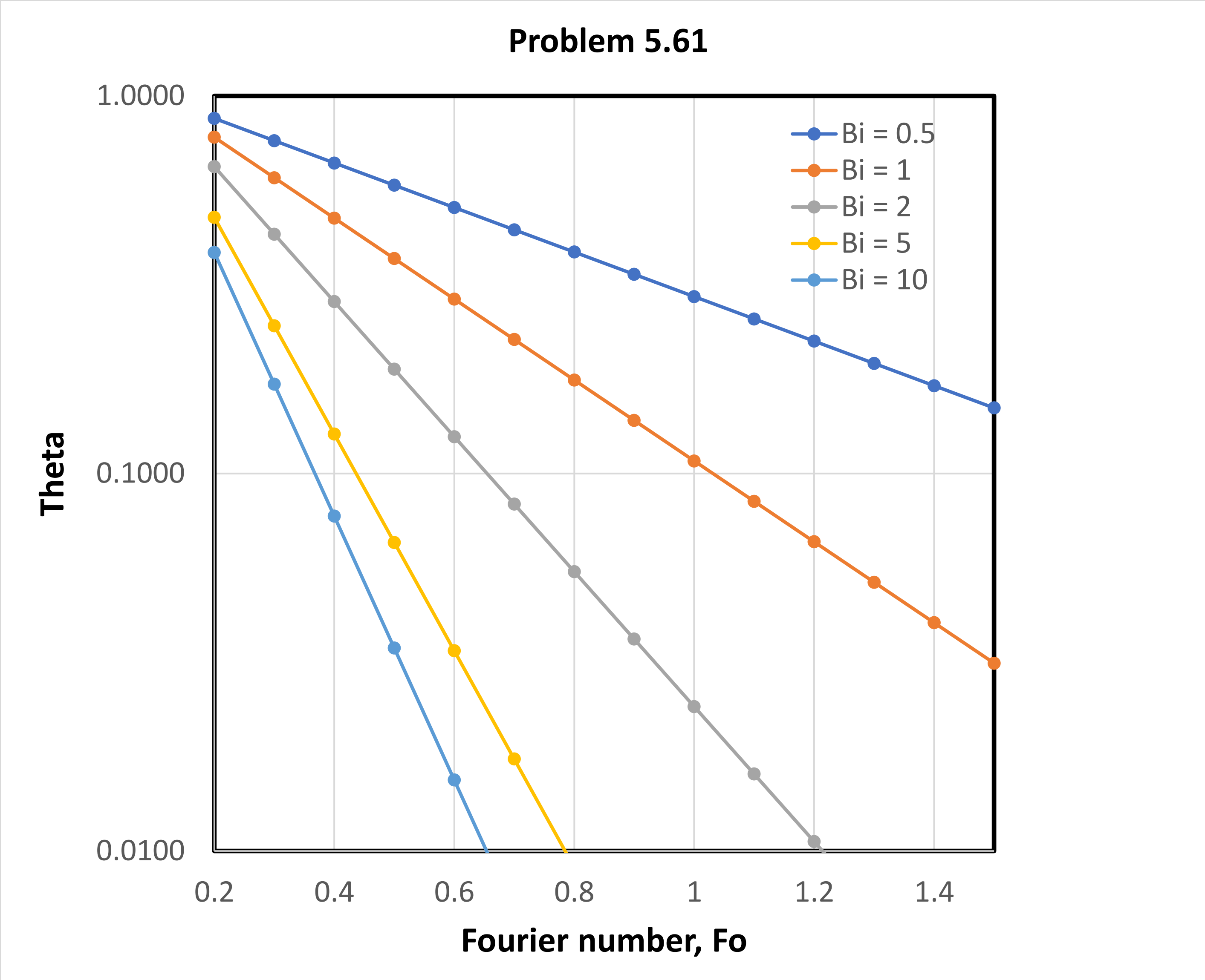
Putting all this back into eqn. (2), we find

$$y(t) = e^{-t/\boldsymbol{T}} y(0) + \int_0^t e^{-(t-s)/\boldsymbol{T}} f(t)\, ds$$

and putting back the original variables in place of y and f, we have at length obtained:

$$\boxed{T(t) = T_\infty(t) + \left[T_i - T_\infty(0)\right] e^{-t/\boldsymbol{T}} - e^{-t/\boldsymbol{T}} \int_0^t e^{s/\boldsymbol{T}}\, \frac{dT_\infty}{ds}\, ds}$$

Extra credit. State which approach is more straightforward!


Problem 5.61
1.0000
0.1000
0.0100
Theta
0.2
0.4
0.6
0.8
1
1.2
1.4
Fourier number, Fo
Bi = 0.5
Bi = 1
Bi = 2
Bi = 5
Bi = 10

Problem 6.1 Verify that eqn. (6.13) follows from eqns. (6.11a) and (6.12).

Begin with $\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$

or $u\frac{\partial u}{\partial x} + \left(u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y}\right) + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$

then multiply the continuity equation by u & get: $u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} = 0$
and subtract it to get

$$\underline{\underline{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}}}$$

Problem 6.2 Complete the algebra between eqns.(6.16) and (6.20).

Start with $\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \nu\frac{\partial^3\psi}{\partial y^3}$. Substitute $\psi(x,y) = \sqrt{u_\infty \nu x}\, f(\eta)$
and $\eta \equiv \sqrt{u_\infty/\nu x}\; y$ and get eqn. (6.18).

$$\frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y} = \left(\sqrt{u_\infty\nu x}\, f'\right)\sqrt{\frac{u_\infty}{\nu x}} = \underline{u_\infty f'} = u$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial x} = \left(\sqrt{u_\infty\nu x}\, f'\right)\sqrt{\frac{u_\infty}{\nu}}\frac{-y}{2x^{3/2}} + \frac{1}{2}\sqrt{\frac{u_\infty\nu}{x}}\, f$$
$$= \underline{-\frac{1}{2}\sqrt{\frac{u_\infty\nu}{x}}\left(f'\eta - f\right)} = v$$

$$\frac{\partial^2\psi}{\partial x\partial y} = u_\infty\frac{\partial f'}{\partial x} = u_\infty\frac{\partial f'}{\partial\eta}\frac{\partial\eta}{\partial x} = -\frac{u_\infty y}{2x\sqrt{\frac{\nu x}{u_\infty}}}f'' = -\frac{u_\infty}{2x}\eta f''$$

$$\frac{\partial^2\psi}{\partial y^2} = u_\infty\frac{\partial f'}{\partial y} = u_\infty\frac{\partial f'}{\partial\eta}\frac{\partial\eta}{\partial y} = \frac{u_\infty}{\sqrt{\frac{\nu x}{u_\infty}}}f'' \; ; \quad \frac{\partial^3\psi}{\partial y^3} = \frac{u_\infty^2}{\nu x}f'''$$

Combine these in the 3rd order mom. eqn. in ψ, and get

$$-u_\infty f'\left(\frac{u_\infty}{2x}\eta f''\right) + \frac{1}{2}\sqrt{\frac{u_\infty\nu}{x}}\left(f'\eta - f\right)\frac{u_\infty}{\sqrt{\frac{\nu x}{u_\infty}}}f'' = \nu\frac{u_\infty^2}{x}f'''$$

or

$$-\frac{u_\infty^2}{2x}\eta f'f'' + \frac{u_\infty^2}{2x}\eta f'f'' - \frac{u_\infty^2}{2x}ff'' = \frac{u_\infty^2}{x}f'''$$

so

$$\underline{\underline{ff'' + 2f''' = 0}}$$

6.3 Solve $ff'' + 2f''' = 0$ subject to the b.c.'s: $f(0) = f'(0) = 0$ and $f'(\infty) = 1$.

We begin by mapping the b.c., $f'(\infty) = 1$ into the origin, thus:

set $f = aF(\xi)$ and $\xi = a\eta$. Then
$$f' = a^2 F'$$
$$f'' = a^3 F''$$
$$f''' = a^4 F'''$$

so $ff'' + 2f''' = 0$ becomes $a^4 FF'' + 2a^4 F''' = 0$ or $\underline{FF'' + 2F''' = 0}$

with the b.c.s

$$f(0) = aF(0) = 0 \quad \text{or} \quad F(0) = 0$$
$$f'(0) = a^2 F'(0) = 0 \qquad F'(0) = 0$$
$$f'(\infty) = a^2 F'(\infty) = 1 \qquad F'(\infty) = 1/a^2$$

Now we would normally have to guess $f''(0) = a^3 F''(0)$. What we shall do is to set $F''(0) = 1$ so that $f''(0) = a^3$, and solve $FF'' + 2F''' = 0$ subject to $F(0) = F'(0) = 0$. This solution will give a certain value of $F'(\infty)$ from which we can calculate $a = [F'(\infty)]^{-1/2}$. Once we know a, we return to our calculated values of F for given values of ξ and correct these back to f and η using $f = aF$ and $\eta = \xi/a$.

There are many ways to solve the system $FF'' + 2F''' = 0$, $F(0) = F'(0) = 0$, $F''(0) = 1$. The simplest is probably to reduce it to three first order d.e.s thus: let $y_1 = F$, $y_2 = F'$, and $y_3 = F''$ so

$$\left.\begin{aligned} \frac{dy_3}{d\xi} &= -\frac{y_1 y_3}{2} \\ \frac{dy_2}{dx} &= y_3 \\ \frac{dy_1}{dx} &= y_2 \end{aligned}\right\} \qquad \begin{aligned} y_1(0) &= 0 \\ y_2(0) &= 0 \\ y_3(0) &= 1 \end{aligned}$$

Runga-Kutta integration schemes are available in computer libraries and can easily be called in to solve such systems of first-order equations.

6.4 Verify that the Blasius solution (given in Table 6.1) satisfies eqn. (7.25). Do this by showing graphically that

$$\frac{d}{dx}\left[\delta\int_0^1 \frac{u}{u_\infty}\left(\frac{u}{u_\infty}-1\right)d\left(\frac{y}{\delta}\right)\right] = -\frac{\nu}{u_\infty \delta}\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}$$

is satisfied by the numbers in Table 6.1. We begin by converting the equation with the help of $\delta = 4.92x/\sqrt{Re_x}$:

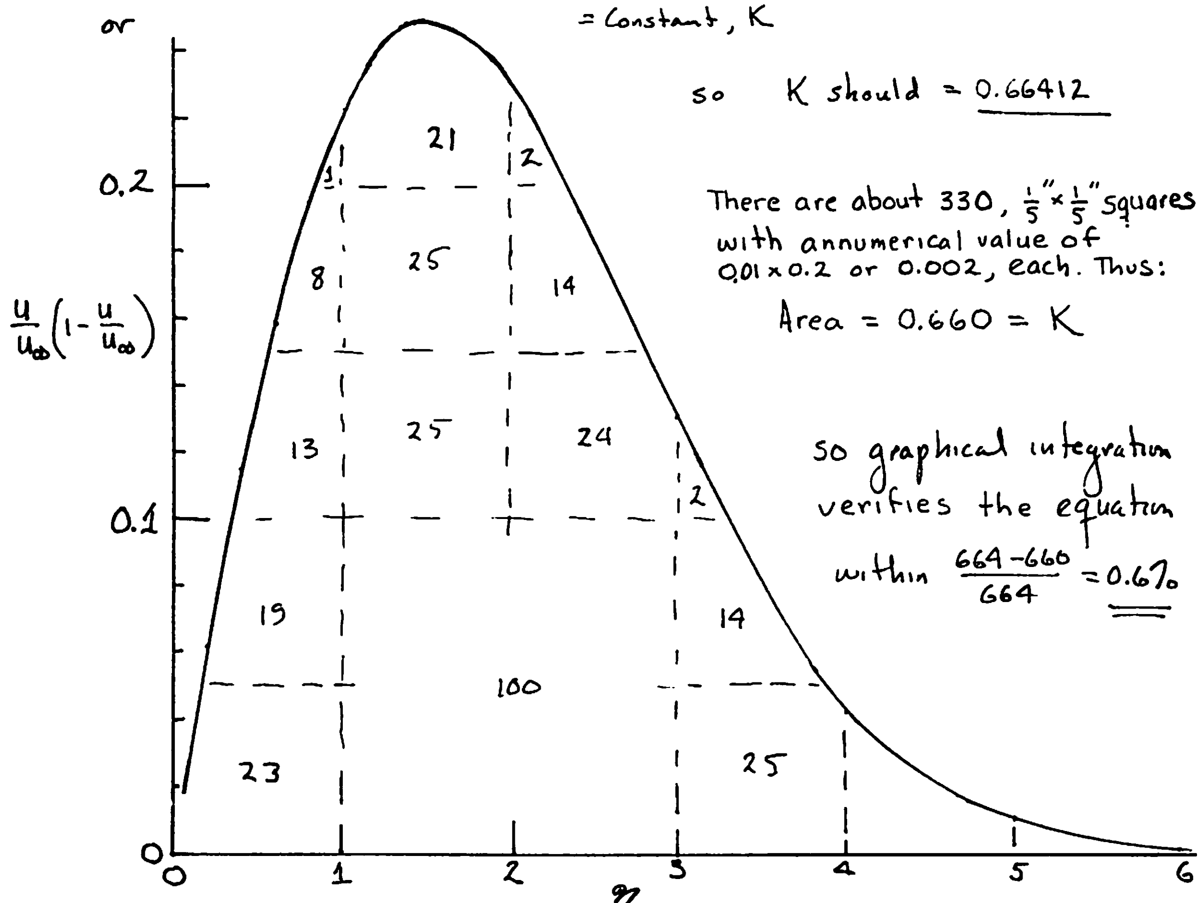
$$\frac{u_\infty}{\nu}\frac{4.92x}{\sqrt{Re_x}}\frac{d}{dx}\left[\frac{4.92x}{\sqrt{Re_x}}\int_0^1 \frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\underbrace{\left(\frac{y}{4.92\sqrt{\frac{x\nu}{u_\infty}}}\right)}_{\frac{\eta}{4.92}}\right] = +\underbrace{\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}}_{=0.33206\,\eta_\delta/\eta \text{ from Table 6.1}}$$

or

$$\frac{4.92\sqrt{u_\infty x}}{\sqrt{\nu}}\frac{d}{dx}\left[\sqrt{\frac{2\nu x}{u_\infty}}\underbrace{\int_0^{4.92}\frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)d\eta}_{=\text{Constant},\ K}\right] = 0.33206 \times 4.92$$

or

so K should $= \underline{0.66412}$



There are about 330, $\frac{1}{5}'' \times \frac{1}{5}''$ squares with annumerical value of 0.01×0.2 or 0.002, each. Thus:

Area = 0.660 = K

so graphical integration verifies the equation within $\frac{664-660}{664} = \underline{\underline{0.6\%}}$

Graphical integration based on Table 6.1

6.5 Verify eqn. (6.30)

Start with $\frac{d}{dx}\left[\delta\int_0^1 \frac{u}{u_\infty}\left(\frac{u}{u_\infty}-1\right)d\left(\frac{y}{\delta}\right)\right] = -\frac{\nu}{u_\infty\delta}\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{y=0}$

and substitute $\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$. Let us call $\xi \equiv y/\delta$. Then:

$$\frac{d}{dx}\left[\delta\underbrace{\int_0^1\left(\frac{3}{2}\xi-\frac{1}{2}\xi^3\right)\left(\frac{3}{2}\xi-\frac{1}{2}\xi^3-1\right)d\xi}_{\cancel{\frac{3}{4}}-\frac{3}{20}-\cancel{\frac{3}{4}}-\frac{3}{20}+\frac{1}{28}+\frac{1}{8}\,=\,-\frac{39}{280}}\right] = -\frac{\nu}{u_\infty\delta}\left(\frac{3}{2}\right)$$

so:

$$-\frac{39}{280}\frac{d\delta}{dx} = -\frac{3}{2}\frac{\nu}{u_\infty\delta}$$

6.6 Derive τ_w using the momentum integral method.

$$\tau_w = (\mu u_\infty/\delta)\frac{\partial(u/u_\infty)}{\partial(y/\delta)} = \frac{\mu u_\infty}{\delta}\frac{3}{2}$$

substitute eqn. (7.31), $\delta = 4.64\sqrt{\frac{\nu x}{u_\infty}}$.

$$\tau_w = \frac{3/2}{4.64}\sqrt{\frac{u_\infty}{\nu x}}\,\mu u_\infty = 0.3233\frac{\mu u_\infty}{x}Re_x^{1/2}$$

This is only 3% below the exact value, eqn. (6.32)

6.7 Find δ and τ_w using the momentum integral method and assuming $u/u_\infty = y/\delta$ for the velocity profile.

Using eqn. (7.25): $\frac{d}{dx}\left[\delta\underbrace{\int_0^1\frac{y}{\delta}\left(\frac{y}{\delta}-1\right)d\frac{y}{\delta}}_{-1/6}\right] = -\frac{\nu}{u_\infty\delta}\times 1$

so: $\int_0^\delta d\delta^2 = -\int_0^x \frac{12\nu}{u_\infty\delta}dx$ or $\frac{\delta}{x} = \sqrt{12}\frac{1}{\sqrt{Re_x}} = \frac{3.46}{\sqrt{Re_x}}$

And:

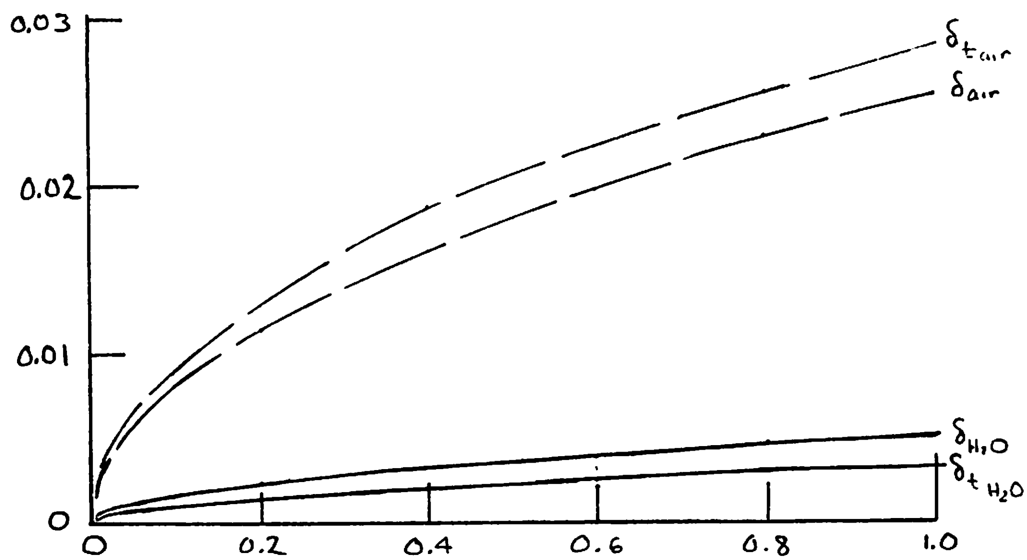
$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu u_\infty}{\delta}\underbrace{\left.\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\right|_{\frac{y}{\delta}=0}}_{=1}$$

so:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} = \frac{2\nu}{\delta u_\infty} = \frac{2}{\sqrt{12}}\frac{\nu}{u_\infty}\sqrt{\frac{u_\infty}{\nu x}} = \frac{0.577}{\sqrt{Re_x}}$$

The use of this extremely crude approximation to u/u_∞ yields a vlaue of x/δ that is low by only 30% and a C_f that is low by only 13%.

6.8 The b.l. thickness for a particular water flow (plotted below) is given by δ m = $0.005\sqrt{x\text{ m}}$. Add to the plot: δ_t for the water flow; and δ and δ_t for air at the same temperature and velocity.

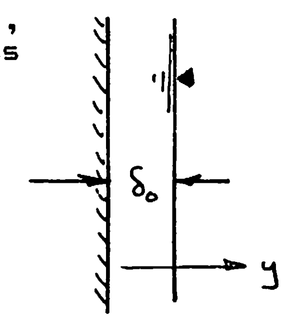


$$\delta_{t_{H_2O}} = \frac{\delta}{Pr_{H_2O}^{1/3}} = \frac{\delta}{4.34^{1/3}} = \underline{0.613\,\delta}\;; \qquad \delta_{air} = \sqrt{\frac{\nu_{air}}{\nu_{H_2O}}}\,\delta_{H_2O} = \sqrt{\frac{17\times10^{-6}}{0.658\times10^{-6}}}\,\delta_{H_2O}$$

$$= \underline{5.083\,\delta_{H_2O}}$$

$$\delta_{t_{air}} = \frac{\delta}{Pr_{air}^{1/3}} = \frac{5.083\,\delta_{H_2O}}{0.704^{1/3}} = \underline{5.714\,\delta_{H_2O}}$$

6.9 A liquid film flows down a plate as shown, at its terminal velocity, with a thickness of δ_o. Derive an expression for u(y).



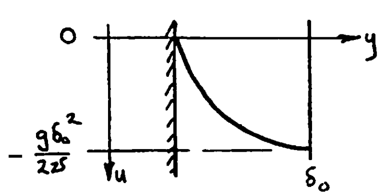
$$\underbrace{u\frac{\partial u}{\partial x}}_{=0} + \underbrace{v\frac{\partial u}{\partial y}}_{=0} = \underbrace{-\frac{1}{\rho}\frac{dp}{dx}}_{=0} - g + \nu\frac{\partial^2 u}{\partial y^2}$$

so $\dfrac{d^2u}{dy^2} = \dfrac{g}{\nu}$; $u = \dfrac{gy^2}{2\nu} + C_1 y + C_2$ w/b.c.'s $u(y=0)=0 \Rightarrow \underline{C_2 = 0}$

$$\left.\frac{\partial u}{\partial y}\right|_{y=\delta_o} = 0 \Rightarrow \underline{C_1 = -\frac{g\delta_o}{\nu}}$$

Then: $$\underline{\underline{u = \frac{gy^2}{2\nu} - \frac{g\delta_o^2}{\nu}\frac{y}{\delta_o}}}$$

or:



$$\underline{\underline{\frac{u}{g\delta_o^2/\nu} = \frac{1}{2}\left(\frac{y}{\delta_o}\right)^2 - \frac{y}{\delta_o}}} \quad \left(\begin{array}{c}\text{= negative}\\ \text{since it is}\\ \text{downward}\end{array}\right)$$

6.10 Evaluate $\overline{Nu_L}$ for laminar flow over an isothermal flat plate.

We know that $Nu_x = A Re_x^{1/2}$, where $A = \dfrac{0.3387\,Pr^{1/3}}{[1+0.0468/Pr^{2/3}]^{1/4}}$, from eqn. (6.63) so:

$$q(x) = \frac{k\Delta T}{x} Nu_x \quad \text{and} \quad \bar{q} = \frac{kA\Delta T}{L}\int_0^L \sqrt{\frac{u_\infty}{\nu}}\frac{\sqrt{x}}{x}\,dx = 2\frac{kA\Delta T}{L}\sqrt{\frac{u_\infty L}{\nu}}$$

Then:

$$\overline{Nu_L} = \frac{\bar{q}L}{k\Delta T} = A Re_L^{1/2} = \frac{0.6774\,Pr^{1/3} Re_L^{1/2}}{[1+0.0468/Pr^{2/3}]^{1/4}}$$

6.11 Use an integral method to predict Nu_x for a flat plate with q_w = constant, and find ΔT at the leading edge of the plate.

$$\rho c u_\infty \frac{d}{dx}\Bigg[\delta_t \Delta T \underbrace{\int_0^1 \frac{u}{u_\infty}\left(\frac{T-T_\infty}{T_w-T_\infty}\right) d\left(\frac{y}{\delta_t}\right)}_{\frac{3}{20}\phi - \frac{3}{280}\phi^2 \simeq \frac{3}{20}\phi}\Bigg] = q_w$$

$$\rho c u_\infty (\delta_t \Delta T)\left(\frac{3}{20}\frac{\delta_t}{\delta}\right) = q_w x$$

$$\frac{3}{20}\frac{u_\infty}{\alpha}\,\delta\left(\frac{\delta_t}{\delta}\right)^2 = \frac{q_w x}{\Delta T\, k} = Nu_x$$

But $Nu_x = \dfrac{3}{2}\dfrac{x}{\delta}\left(\dfrac{\delta}{\delta_t}\right)$ so $\dfrac{3}{20}\dfrac{u_\infty}{\alpha}\left(\dfrac{4.64x}{\sqrt{Re_x}}\right)^2\left(\dfrac{\delta_t}{\delta}\right)^3 = \dfrac{3}{2}x$

$$\frac{\delta}{\delta_t} = \left(\frac{4.64^2}{10}\right)^{1/3} Pr^{1/3} = 1.291\,Pr^{1/3}$$

so

$$Nu_x = \frac{3}{2}\frac{1.291}{4.64} Pr^{1/3} Re_x^{1/2} = 0.417\,Pr^{1/3} Re_x^{1/2}$$

and, since $h \sim Re_x^{1/2}/x \sim 1/\sqrt{x}$

$$\Delta T = q_w/h \sim \sqrt{x}$$

we conclude that $\Delta T \to 0$ at the leading edge of the heater.

Problem 6.12 (a) Verify that eqn. (6.120) follows from eqn. (6.119). (b) Derive an equation for liquids that is analogous to eqn. (6.119).

Solution

a) Beginning with

$$\begin{aligned}\overline{h} &= \frac{1}{L\Delta T}\int_0^L q_w\,dx \\ &= \frac{1}{L}\left[\int_0^{x_l} h_{\text{laminar}}\,dx + \int_{x_l}^{x_u} h_{\text{trans}}\,dx + \int_{x_u}^{L} h_{\text{turbulent}}\,dx\right] \qquad (6.119)\end{aligned}$$

we may evaluate each integral separately. For a uniform temperature surface, the Nusselt numbers are given by these equations:

$$\text{Nu}_{\text{lam}} = 0.332\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3} \qquad (6.58)$$

$$\text{Nu}_{\text{trans}} = \text{Nu}_{\text{lam}}\big(\text{Re}_l, \text{Pr}\big)\left(\frac{\text{Re}_x}{\text{Re}_l}\right)^c \qquad (6.114\text{b})$$

$$\text{Nu}_{\text{turb}} = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{0.6} \quad \text{for gases} \qquad (6.112)$$

The three integrals are thus

$$\frac{1}{L}\int_0^{x_l} h_{\text{lam}}\,dx = \frac{0.332\,k\text{Pr}^{1/3}}{L}\int_0^{x_l}\sqrt{\frac{u_\infty}{\nu x}}\,dx = \frac{0.332\,k\text{Pr}^{1/3}}{L}\,2\sqrt{\frac{u_\infty x_l}{\nu}} = \frac{k}{L}\,0.664\,\text{Re}_l^{1/2}\,\text{Pr}$$

$$\begin{aligned}\frac{1}{L}\int_{x_l}^{x_u} h_{\text{trans}}\,dx &= \frac{k}{L}\frac{\text{Nu}_{\text{lam}}\big(\text{Re}_l,\text{Pr}\big)}{\text{Re}_l^c}\left(\frac{u_\infty}{\nu}\right)^c\int_{x_l}^{x_u} x^{c-1}\,dx = \frac{k}{L}\frac{\text{Nu}_{\text{lam}}\big(\text{Re}_l,\text{Pr}\big)}{\text{Re}_l^c}\left(\frac{u_\infty}{\nu}\right)^c\frac{1}{c}\big(x_u^c - x_l^c\big) \\ &= \frac{k}{L}\frac{\text{Nu}_{\text{lam}}\big(\text{Re}_l,\text{Pr}\big)}{\text{Re}_l^c}\frac{1}{c}\big(\text{Re}_u^c - \text{Re}_l^c\big) = \frac{k}{L}\frac{1}{c}\Big[\text{Nu}_{\text{turb}}\big(\text{Re}_u,\text{Pr}\big) - \text{Nu}_{\text{lam}}\big(\text{Re}_l,\text{Pr}\big)\Big]\end{aligned}$$

where the last step follows because eqn. (6.114b) intersects Nu_{turb} at Re_u, and

$$\begin{aligned}\frac{1}{L}\int_{x_u}^{L} h_{\text{turb}}\,dx &= \frac{0.0296\,k\text{Pr}^{0.6}}{L}\left(\frac{u_\infty}{\nu}\right)^{0.8}\int_{x_u}^{L} x^{-0.2}\,dx = \frac{0.0296\,k\text{Pr}^{0.6}}{(0.8)L}\Big(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\Big) \\ &= \frac{k}{L}0.037\,\text{Pr}^{0.6}\Big(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\Big)\end{aligned}$$

Collecting these terms, we find:

$$\begin{aligned}\overline{\text{Nu}}_L \equiv \frac{\overline{h}L}{k} &= 0.037\,\text{Pr}^{0.6}\Big(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\Big) + 0.664\,\text{Re}_l^{1/2}\text{Pr}^{1/3} \\ &\quad + \underbrace{\frac{1}{c}\Big(0.0296\,\text{Re}_u^{0.8}\text{Pr}^{0.6} - 0.332\,\text{Re}_l^{1/2}\text{Pr}^{1/3}\Big)}_{\text{contribution of transition region}} \quad \text{for gases} \qquad (6.120)\end{aligned}$$

b) For a liquid flow, the turbulent correlation should be eqn. (6.113):

$$\text{Nu}_{\text{turb}} = 0.032\,\text{Re}_x^{0.8}\,\text{Pr}^{0.43} \quad \text{for nonmetallic liquids} \qquad (6.113)$$

and the integral in the turbulent range changes to

$$\frac{1}{L}\int_{x_u}^{L} h_{\text{turb}}\,dx = \frac{0.032\,k\text{Pr}^{0.43}}{L}\left(\frac{u_\infty}{\nu}\right)^{0.8}\int_{x_u}^{L} x^{-0.2}\,dx = \frac{0.032\,k\text{Pr}^{0.43}}{(0.8)L}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right)$$

$$= \frac{k}{L} 0.040\,\text{Pr}^{0.43}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right)$$

Collecting these terms, we find:

$$\overline{\text{Nu}}_L \equiv \frac{\overline{h}L}{k} = 0.040\,\text{Pr}^{0.43}\left(\text{Re}_L^{0.8} - \text{Re}_u^{0.8}\right) + 0.664\,\text{Re}_l^{1/2}\text{Pr}^{1/3} + \underbrace{\frac{1}{c}\left(0.032\,\text{Re}_u^{0.8}\text{Pr}^{0.43} - 0.332\,\text{Re}_l^{1/2}\text{Pr}^{1/3}\right)}_{\text{contribution of transition region}} \quad \text{for nonmetallic liquids}$$

6.14 Do the differentiation in eqn. (6.24)

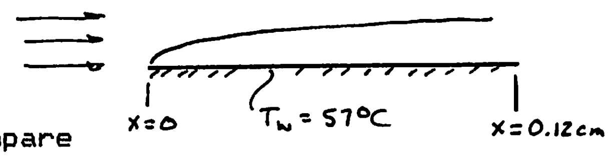
Leibnitz' rule says that: $\displaystyle \frac{d}{dx}\int_{a(x)}^{b(x)} f(\alpha,x)\,d\alpha = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f\,d\alpha + f(b,x)\frac{db}{dx} - f(a,x)\frac{da}{dx}$

Thus:

$$\underline{\underline{\frac{d}{dx}\int_0^{\delta(x)} u(u-u_\infty)\,dy = \int_0^{\delta(x)} \frac{\partial}{\partial x}\left[u(u-u_\infty)\right]dy}} + \underbrace{\underbrace{u(\delta,x)}_{u_\infty}\big(\underbrace{u(\delta,x)}_{u_\infty}-u_\infty\big)\frac{d\delta}{dx}}_{=0} - 0$$

and this gives eqn. above eqn. (6.24).

6.15 Glycerin or water flows over a flat plate at 2 m/s. $T_\infty = 23^\circ C$



Find: $h(x = 0.12)$ and compare the drag forces in each case.

Evaluating properties at $T_{avg} = (57 + 23)/2 = 40^\circ C$, we get:

water:
$k = 0.627\ W/m\text{-}^\circ C$
$\nu = 0.657\times10^{-6}\ m^2/s$
$\rho = 992\ kg/m^3$
$Pr = 4.36$
$Re_L = 0.12(2)/6.57(10)^{-7} = 365{,}300$
$Nu_L = 0.332\sqrt{365{,}300}\,(4.36)^{1/3} = 328$
$h(x=0.12) = \frac{k}{L} Nu_L = \underline{\underline{1715\ W/m^2\text{-}^\circ C}}$

glycerin:
$k = 0.285\ W/m\text{-}^\circ C$
$\nu = 0.000227\ m^2/s$
$\rho = 1249\ kg/m^3$
$Pr = 2541$
$Re_L = 0.12(2)/0.000227 = 1057$
$Nu_L = 0.332\sqrt{1057}\,(2451)^{1/3} = 146$
$h(x=0.12) = \frac{k}{L} Nu_L = \underline{\underline{346\ W/m^2\text{-}^\circ C}}$ ←

Thus the water is a significantly better coolant. Furthermore The ratio of drag forces is:

$$\frac{F_{D\,glyc}}{F_{D\,water}} = \frac{(\tau_w)_{gl} A}{(\tau_w)_{H_2O} A} = \frac{\rho_g \overline{C_{f_g}}}{\rho_w \overline{C_{f_w}}} = \frac{1249}{992}\sqrt{\frac{Re_{L\,w}}{Re_{L\,g}}} = \underline{\underline{23.4}} \leftarrow$$

So the glycerin exerts $\underline{23.4}$ $\underline{\text{times}}$ as much drag on the plate as the water does.

Problem 6.16 Air at $-10\,°\mathrm{C}$ flows over a smooth, sharp-edged, almost-flat, aerodynamic surface at 240 km/hr. The surface is at $10\,°\mathrm{C}$. Turbulent transition begins at $\mathrm{Re}_l = 140{,}000$ and ends at $\mathrm{Re}_u = 315{,}000$. Find: (a) the x-coordinates within which laminar-to-turbulent transition occurs; (b) $\overline{h}$ for a 2 m long surface; (c) h at the trailing edge for a 2 m surface; and (d) δ and h at x_l.

Solution

a) We evaluate physical properties at the film temperature, $T_f = (-10 + 10)/2 = 0\,°\mathrm{C}$: $\nu = 1.332 \times 10^{-5}$ m²/s, Pr = 0.711, and $k = 0.244$ W/m·K. Also, $u_\infty = 240(1000)/(3600) = 66.7$ m/s. Then:

$$x_l = \frac{\mathrm{Re}_l\,\nu}{u_\infty} = \frac{(140000)(1.332\times 10^{-5})}{(66.7)} = \underline{0.0280\text{ m}}$$

$$x_u = \frac{\mathrm{Re}_u\,\nu}{u_\infty} = \frac{(315000)(1.332\times 10^{-5})}{(66.7)} = \underline{0.0629\text{ m}}$$

Observe that the flow is fully turbulent over 1.937/2.00 = 96.9% of its length.

b) First, we need Re_L:

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{(66.7)(2)}{1.332\times 10^{-5}} = 1.00\times 10^7$$

Then we get c from eqn. (6.115):

$$c = 0.9922\ \log_{10}(140{,}000) - 3.013 = 2.09$$

Now we may use eqn. (6.120):

$$\begin{aligned}\overline{\mathrm{Nu}}_L &= 0.037(0.711)^{0.6}\left[(1.00\times 10^7)^{0.8} - (3.15\times 10^5)^{0.8}\right]\\ &\quad + 0.664\,(1.40\times 10^5)^{1/2}(0.711)^{1/3}\\ &\quad + \frac{1}{2.09}\left[0.0296(3.15\times 10^5)^{0.8}(0.711)^{0.6} - 0.332\,(1.40\times 10^5)^{1/2}(0.711)^{1/3}\right]\\ &= 11248.9 + 221.8 + 236.0 = 1.171\times 10^4\end{aligned}$$

Thus

$$\overline{h} = \frac{k}{L}\,\overline{\mathrm{Nu}}_L = \frac{(0.0244)(1.171\times 10^4)}{2} = \underline{143\text{ W/m}^2\text{K}}$$

c) With eqn. (6.112),

$$\mathrm{Nu}_L = 0.0296\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{0.6} = 0.0296\,(1.00\times 10^7)^{0.8}(0.711)^{0.6} = 9603$$

so

$$h(L) = \frac{k}{L}\,\mathrm{Nu}_L = \frac{(0.0244)(9603)}{2} = \underline{117\text{ W/m}^2\text{K}}$$

d) The flow is laminar here. From eqn (6.58):

$$\mathrm{Nu}_{x_l} = 0.332\,\mathrm{Re}_l^{1/2}\,\mathrm{Pr}^{1/3} = 0.332\,(1.40\times 10^5)^{1/2}(0.711)^{1/3} = 110.9$$

so

$$h(x_l) = \frac{k}{x_l}\,\mathrm{Nu}_{x_l} = \frac{(0.0244)(110.9)}{0.0280} = \underline{96.6\text{ W/m}^2\text{K}}$$

With eqn (6.2), we find that the boundary layer here is *very* thin:

$$\delta = \frac{4.92\, x_l}{\sqrt{\mathrm{Re}_{x_l}}} = \frac{4.92(0.0280)}{\sqrt{1.4 \times 10^5}} = 0.000368 \text{ m} = \underline{0.37 \text{ mm}}$$

Problem 6.17 Find $\overline{h}$ in Example 6.9 using eqn. (6.120) with $\mathrm{Re}_l = 80,000$. Compare with the value in the example and discuss the implication of your result. *Hint:* See Example 6.10.

Solution Equation (6.120) is

$$\overline{\mathrm{Nu}}_L \equiv \frac{\overline{h}L}{k} = 0.037\,\mathrm{Pr}^{0.6}\left(\mathrm{Re}_L^{0.8} - \mathrm{Re}_u^{0.8}\right) + 0.664\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3} + \frac{1}{c}\left(0.0296\,\mathrm{Re}_u^{0.8}\mathrm{Pr}^{0.6} - 0.332\,\mathrm{Re}_l^{1/2}\mathrm{Pr}^{1/3}\right) \qquad (6.120)$$

From Example 6.9, we have $\mathrm{Re}_L = 1.270 \times 10^6$ and $\mathrm{Pr} = 0.708$. We may find c from eqn. (6.115):

$$c = 0.9922\ \log_{10}(80,000) - 3.013 = 1.85$$

We also need Re_u, which we can find following Example 6.10:

$$\mathrm{Re}_u^{1.85-0.8} = \frac{0.0296(0.708)^{0.6}(80,000)^{1.85}}{0.332(80,000)^{1/2}(0.708)^{1/3}}$$

Solving, $\mathrm{Re}_u = 184,500$. Substituting all this into eqn. (6.120):

$$\overline{\mathrm{Nu}}_L = 0.037(0.708)^{0.6}\left[(1.270\times10^6)^{0.8} - (1.845\times10^5)^{0.8}\right] + 0.664\,(8.0\times10^4)^{1/2}(0.708)^{1/3} + \frac{1}{1.85}\left[0.0296(1.845\times10^5)^{0.8}(0.708)^{0.6} - 0.332\,(8.0\times10^4)^{1/2}(0.708)^{1/3}\right]$$

Evaluating, we find the contributions of the turbulent, laminar, and transition regions:

$$\overline{\mathrm{Nu}}_L = \underbrace{1806.6}_{\text{turb.}} + \underbrace{167.4}_{\text{lam.}} + \underbrace{167.1}_{\text{trans.}} = 2,141$$

The transition region contributes 7.8% of the total. The average heat transfer coefficient is

$$\overline{h} = \frac{2141(0.0264)}{2.0} = 28.26\ \mathrm{W/m^2K}$$

and the convective heat loss from the plate is

$$Q = (2.0)(1.0)(28.26)(310-290) = \underline{1130\ \mathrm{W}}$$

The earlier transition to turbulence increases the heat removal by $[(1130+22)/(756+22)-1]\times100 =$ 48%.

6.19 Mercury flows at 25°C and 0.7 m/s over a 4 cm long plate at 60°C. Find $\bar{h}$, $\overline{\tau_w}$, $h(x = 0.04\text{ m})$, and $\delta(x = 0.04\text{ m})$.

Solution Evaluate properties at (25+60)/2 = 42.5°C = 315.5°K

$\nu = 1.14(10)^{-7}\ \mathrm{m^2/s}$, $Pr = 0.0248$, $k = 7.39$, $Re_L = \dfrac{0.7(0.04)}{1.14(10)^{-7}} = 245{,}600$

so:

$$\overline{Nu_L} = 1.13\sqrt{245{,}600(0.0248)} = 88.2\ ; \quad \bar{h} = 88.2\,\frac{7.39}{0.04} = 16{,}293\ \frac{W}{m^2\text{-}°C}$$

$$\delta = \frac{4.92(0.04)}{\sqrt{245{,}600}} = \underline{\underline{0.004\ m = 0.4\ mm\ (\text{pretty thin})}}$$

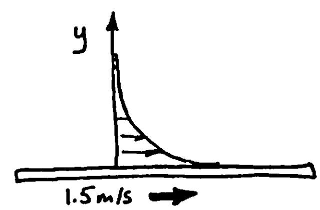
$$\underline{\underline{h_L = \tfrac{1}{2}\bar{h} = 8{,}147\ \frac{W}{m^2\text{-}°C}}}$$

$$\tau_w = \frac{1}{2}\rho u_\infty C_f = \frac{13{,}573(0.7)^2}{2}\,\frac{1.328}{\sqrt{245{,}600}} = \underline{\underline{8.91\ \frac{N}{m^2}}}$$

6.20 A plate is at rest in water at 15°C. It is suddenly translated parallel with itself at 1.5 m/s. Evaluate the liquid velocity, u, 0.015 from the plate at t = 1, 10, and 1000 s.

Pose the problem: $\dfrac{\partial^2 u}{\partial y^2} = \dfrac{1}{\nu}\dfrac{\partial u}{\partial t}$

with b.c.s: $u(y=0) = 1.5\ m/s\ ,\ t>0$
$u(t=0) = 0$



Compare this with the semi-infinite region solution in Sect 5.6

$$\frac{\partial^2 (T-T_i)}{\partial^2 x} = \frac{1}{\alpha}\frac{\partial (T-T_i)}{\partial t} \quad \text{with b.c.'s: } (T-T_i)_{x=0} = T_\infty - T_i\ ;\ (T-T_i)_{t=0} = 0$$

These problems are identical if: $T-T_i \Leftrightarrow u$, $x \Leftrightarrow y$, $\alpha \Leftrightarrow \nu$
and $(T_\infty - T_i) \Leftrightarrow 1.5$

Thus, its solution, $\dfrac{T-T_\infty}{T_i - T_\infty} = \mathrm{erf}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)$ is the solution to our problem once we make these changes. So:

$$\frac{u-1.5}{-1.5} = \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right) \qquad \text{where } \nu = 1.184\times10^{-6}\ \frac{m^2}{s}$$

Therefore: u at $y = 0.015\,m$ is given by

$$u = 1.5\left(1 - \mathrm{erf}\,\frac{6.89}{\sqrt{t}}\right)$$

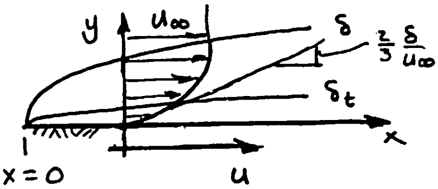
Then at $t = 1$ sec, $\underline{\underline{u \simeq 0\ m/s}}$

$t = 10$ sec, $\underline{\underline{u \simeq 0.003\ m/s}}$

$t = 1000$ sec, $\underline{\underline{u = 1.14\ m/s}}$

6.21 Use the fact that, when Pr is very large, $u/u_\infty = (3/2)(y/\delta)$ inside the thermal b.l., to create an expression for Nu_x during the flow of a High Pr fluid over over a flat isothermal plate.

We begin with the integrated energy equation in the form of eqn. (6.51).



$$u_\infty \Delta T \frac{d}{dx}\left[\delta_t \int_0^1 \underbrace{\frac{u}{u_\infty}}_{\frac{3}{2}\frac{y}{\delta} = \frac{3}{2}\phi\frac{y}{\delta_t}} \underbrace{\left(\frac{T-T_\infty}{\Delta T}\right)}_{1-\frac{3}{2}\frac{y}{\delta_t}+\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3} d\left(\frac{y}{\delta_t}\right)\right] = -\frac{\alpha \Delta T}{\delta_t}\underbrace{\left.\frac{d\,\frac{T-T_\infty}{\Delta T}}{d(y/\delta_t)}\right|_{\frac{y}{\delta_t}=0}}_{-\frac{3}{2}}$$

or

$$u_\infty \Delta T \frac{d}{dx}\left[\delta_t \underbrace{\int_0^1 \left(\frac{3}{2}\phi x - \frac{9}{4}\phi x^2 + \frac{3}{4}\phi x^4\right)dx}_{\left(\frac{3}{4}-\frac{3}{4}+\frac{3}{20}\right)\phi \,=\, \frac{3}{20}\phi}\right] = \frac{3}{2}\frac{\Delta T}{\delta_t}\alpha$$

or $\quad \dfrac{1}{10}\dfrac{1}{2}\dfrac{d\delta_t^2}{dx} = \dfrac{\alpha}{u_\infty \phi} \quad$ or $\quad \delta_t = \sqrt{\dfrac{20\alpha}{u_\infty\phi}x}$

Now we introduce $\delta_t = \delta\,\phi$: $\quad \delta = \sqrt{\dfrac{20\alpha x}{\phi^3 u_\infty}}$

Then using eqn. (6.31a) for δ: $\quad \delta = \sqrt{\dfrac{280}{13}\dfrac{\nu x}{u_\infty}}$

We equate the two expressions for δ and solve for ϕ:

$$\phi = \sqrt[3]{\frac{13}{14}\frac{1}{Pr}}$$

But, in accordance with eqn. (6.57), $\quad h = \dfrac{3}{2}\dfrac{k}{\delta_t} = \dfrac{3k}{2\delta\phi}$

Thus:

$$Nu_x = \frac{3}{2}\,\frac{x}{\frac{4.64x}{\sqrt{Re_x}}\left(\frac{13}{14}\right)^{1/3}\frac{1}{Pr^{1/3}}} = \underline{\underline{0.3314\, Re_x^{1/2}\, Pr^{1/3}}}$$

(The integral approx. in the text gives exactly this for $\phi \Rightarrow 0$)

6.23 Water at 7°C flows at 0.38 m/s across the top of a 0.207m long, thin copper plate. Methanol at 87°C flows across the bottom of the same plate, at the same speed, but in the opposite direction. Make the obvious first-guess as to the temperature at which to evaluate physical properties. Then plot the plate temperature as a function of position. Do not bother to correct the physical properties in this problem but note Problem 6.24.

We shall first guess that the plate is at the mean temperature of $(7+87)/2 = 47°C$. Evaluate meth. props. at $(47+87)/2 = 67°C$ -- water props. at $(7+47)/2 = 27°C$:

$$Re_{meth_{max.}} = \frac{0.38(0.207)}{0.44(10)^{-6}} = 178{,}773\ , \quad Re_{H_2O_{max.}} = \frac{0.38(0.207)}{0.826(10)^{-6}} = 95{,}230$$

Both are laminar. Use $h = 0.332\, k\, Pr^{1/3} Re_x^{1/2}/x$, so

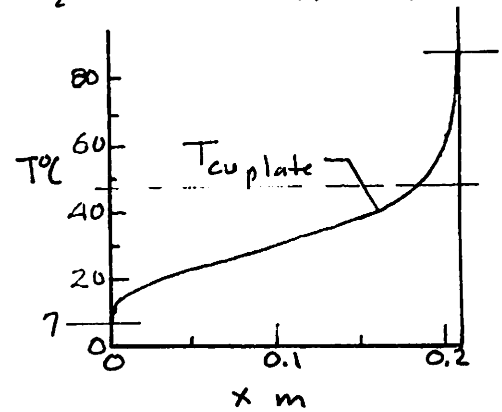
$$h_{meth} = 0.332(0.1908)(4.9)^{1/3}(0.38/0.44(10)^{-6})^{1/2}/\sqrt{x} = \frac{100}{\sqrt{x\ m}}$$

$$h_{H_2O} = 0.332(0.6084)(5.65)^{1/3}(0.38/0.826(10)^{-6})^{1/2}/\sqrt{0.207-x} = \frac{244}{\sqrt{.207-x}}$$

6.23 (continued)

$$U = 1\Big/\left[\frac{1}{h_{meth}} + \frac{1}{h_{H_2O}}\right] \quad \text{and} \quad U(T_{meth} - T_{H_2O}) = h_{meth}(T_{meth} - T_{cu})$$

x m	h_{meth} W/m²-°C	h_{H_2O} W/m²-°C	U W/m²-°C	$T_{cu} = 87 - \frac{U(87-7)}{h_{meth}}$
0	220	∞	220	7
0.05	252	1091	205	22
0.10	305	772	219	30
0.15	419	630	252	39
0.20	1195	546	375	62
0.207	∞	536	536	87



6.24 Work Problem 6.23 taking full account of property variations.

To do this, we regard the previous solution as a first iteration. Now we rework the problem using the plate temperatures above.

x_m	T_{plate} °C	$T_{film} = \frac{T_{flow}-T_{cu}}{2}$ meth	H_2O	ν $10^{-6}\frac{m^2}{s}$ meth	H_2O	k W/m-°C meth	H_2O	$Pr^{1/3}$ meth	H_2O	h W/m²-°C meth	H_2O	U $\frac{W}{m^2\text{-}°C}$	T_{cu} °C
0	7	47	7	0.6	1.922	0.1965	0.5818	1.847	2.173	211	∞	211	7
0.05	22	54.5	14.5	0.54	1.20	0.1944	0.5918	1.794	2.043	245	1010	197	22.7
0.10	30	58.5	18.5	0.51	1.08	0.1932	0.5911	1.765	1.967	299	724	212	30.3
0.15	39	63.0	23	0.47	0.95	0.1919	0.6031	1.730	1.873	415	612	247	39.4
0.20	62	74.5	34.5	0.90	0.729	0.1867	0.6190	1.672	1.700	1207	564	384	61.5
0.207	87	87	47	0.36	0.566	0.1851	0.6367	1.626	1.542	∞	587	587	87

No temperature changed more than 0.7°C. We can terminate the calculation. Furthermore, the plot above will stand. No point on it will move by more than a pencil-width.

Better property data have become available since we first worked this problem. Consequently, the numbers will change a bit. But the solution remains essentially correct.

6.25 If in Example 6.6 (with a constant q_w = 420 W/m^2) the wall temperature were instead held constant at its average value of 76°C, what would the average wall heat flux be?

$$\overline{Nu_L} = 0.664\, Re_L^{1/2} Pr^{1/3} = 0.664\sqrt{\frac{1.8(0.6)}{1.747(10)^{-5}}}\; 0.712^{1/3} = 147.4$$

where we have evaluated air properties at $(76+15)/2 = 45.5°C$.

Then:
$$\overline{h} = \overline{Nu_L}\,\frac{k}{L} = 147.4\,\frac{(0.02482)}{0.6} = 6.10\ \frac{W}{m^2\text{-}°C}$$

$$\overline{q}_w = \overline{h}\,\Delta T = 6.10\,(76-15) = \underline{\underline{372\ W/m^2}}$$

6.27 A two foot square slab of mild steel leaves a forging operation 0.25 in. thick at 1000°C. It is laid flat on an insulating bed and 27°C air is blown over it at 30 m/s. How long will it take to cool to 200°C. Assume the flow is laminar and state your assumptions about property evaluation.

Compute $\overline{h}$ based on air at $\frac{1}{2}\left[\frac{1000+27}{2} + \frac{200+27}{2}\right] = 313.5\,°C$. (This is pretty coarse. The value of ν drops by a factor of four over the range of film temperature involved.) Then:

$\nu = 4.96\times10^{-5}$, $Pr = 0.698$, $k = 0.0448$

And $\rho c_{steel} = 3.64(10)^6\ J/m^3\text{-}°C$, $k_{steel} = 35$ at $600°C$

$$Re_L = \frac{2(0.3048)30}{4.96\times10^{-5}} = 368{,}710$$

$$\overline{Nu_L} = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 357.7\ ;\ \overline{h} = 357.7\,\frac{0.0448}{2(0.3048)} = \underline{26.3\ \frac{W}{m^2\text{-}°C}}$$

Can we use lumped capacity? Check $Bi = \frac{\overline{h}L}{k_s} = \frac{26.3(0.0254\times1/4)}{35} = 0.048 \ll 1$

Lumped capacity is fine. Then: $T = \frac{\rho c\ \text{thickess}}{\overline{h}} = \frac{3.64(10)^6(0.0254/4)}{26.3} = \underline{879\ \text{sec.}}$

Finally:

$$\frac{T-T_\infty}{T_i-T_\infty} = e^{-t/T} = \frac{200-27}{1000-27} = 0.1778 = e^{-t/879}$$

So: $t = 1{,}519\ \text{sec} = \underline{\underline{25.3\ \text{minutes}}}$

This is a long time. Air is an ineffective coolant.

6.28 Do Problem 6.27 numerically, recalculating properties at successive points. If you did Problem 6.27, compare results.

First compute $\bar{h}$ at T_{plate} = 1000°C, 900°C, etc.

$$\overline{Nu_L} = 0.664\, Re_L^{1/2} P_r^{1/3}$$

$$\bar{h} = 0.664\left[\frac{\sqrt{30}}{\sqrt{0.6096}}\right]\frac{P_r^{1/3}}{\nu^{1/2}}k$$

$$\bar{h} = 4.658\, P_r^{1/3} k/\nu^{1/2}$$

T_{plate}	$\frac{1}{2}(T_p+27)+273$	ν $\frac{m^2}{s}$	P_r	k $\frac{W}{m°C}$	$\bar{h}$ W/m²-°C
1000°C	786.5°	8.04 (10)$^{-5}$	0.704	0.05614	25.95
900	736.5	7.22 "	0.703	0.05334	26.00
800	686.5	6.43 "	0.702	0.05052	26.08
700	636.5	5.68 "	0.700	0.04764	26.15
600	586.5	4.96 "	0.698	0.0448	26.28
500	536.5	4.28 "	0.698	0.04177	26.38
400	486.5	3.63 "	0.699	0.03865	26.52
300	436.5	3.02 "	0.702	0.0359	26.67

Now compute $T = \rho c$ thickness$/\bar{h}$ = 23,114 & use $\frac{T-27}{1000-27} = e^{-t/T}$ to advance 100°C per step. Thus

from previous T to	T sec	$\frac{T-27}{\text{Prior } T-27}$	$t = -T \ln\frac{T-27}{T+73}$	t_{total} sec
T = 900°C	891	0.897	96.6 sec	96.6
800	889	0.8855	108.2	204.8
700	886	0.8706	122.8	327.6
600	884	0.8514	142.2	469.8
500	880	0.8255	168.7	638.5
400	876	0.7886	208.1	846.6
300	872	0.7319	272.2	1118.8
200	867	0.6337	395.5	1514.3

We conclude that, since this is within 4.7 sec or 0.31 percent of the approximate result, the averaging that was used in Problem 6.27 is quite good in this case. The high variation of ν is compensated in its influence on $\bar{h}$ by the variation of k. Furthermore the initial under-estimate of **T** is compensated by the subsequent over-estimate of **T**.

6.31 A thin metal sheet separates air at 44°C, flowing at 48 m/s, from water at 4°C, flowing at 0.2 m/s. Both fluids start at a leading edge and move in the same direction. Plot T_{plate} and q as a function of x up to x = 0.1 m.

Make first calculation evaluating properties on the basis of $T_{plate} = 10°C$ (so $\overline{T}_{air} = 27°C$ and $\overline{T}_{H_2O} = 7°C$) Then:

$\nu_{air} = 1.566(10)^{-5}$ $\quad k_{air} = 0.02614$ $\quad Pr_{air} = 0.711$

$\nu_{H_2O} = 1.422(10)^{-6}$ $\quad k_{H_2O} = 0.6084$ $\quad Pr_{H_2O} = 10.26$

so: $Re_{air} = \dfrac{u_\infty}{\nu} x = \dfrac{48\ x}{1.566\times10^{-5}} = 3.066\times10^6 x$; $Re_{H_2O} = \dfrac{0.2\ x}{1.422\times10^{-6}} = 140{,}650x$

Thus the flows should be laminar: $h = \dfrac{k}{x} 0.322 Re_x^{1/2} Pr^{1/3}$.

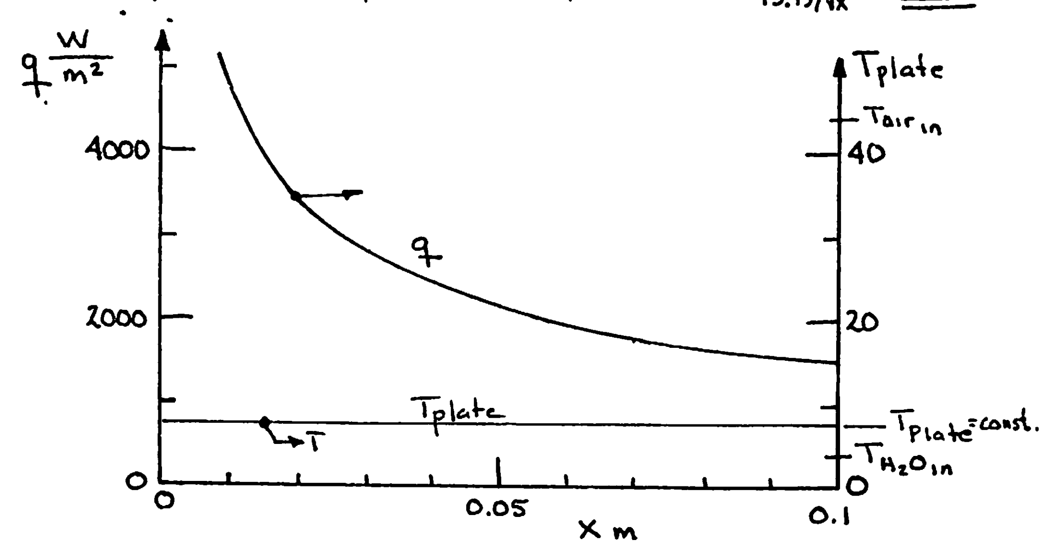
$$h_{air} = \frac{0.02614}{x} 0.322\left[3.066(10)^6 x\right]^{1/2} 0.711^{1/3} = \underline{13.15/\sqrt{x}}$$

$$h_{H_2O} = \frac{0.6084}{x} 0.322\left[140{,}650x\right]^{1/2} 10.26^{1/3} = \underline{160.0/\sqrt{x}}$$

Then: $$U = \frac{1}{\frac{1}{h_{air}} + \frac{1}{h_{H_2O}}} = \frac{1}{\frac{1}{13.15} + \frac{1}{160}}\left(\frac{1}{\sqrt{x}}\right) = \underline{12.15/\sqrt{x}}$$

$$q = U\Delta T = \frac{12.15}{\sqrt{x}}(44-4) = \underline{\underline{486/\sqrt{x}}}$$

and since $q = h_{air}(44 - T_{plate})$ so $T_{plate} = 44 - \dfrac{486/\sqrt{x}}{13.15/\sqrt{x}} = \underline{\underline{7.04°C}}$



6.32 A mixture of 60% glycerin and 40% water flows over a 1m long flat plate. The glycerin is at 20°C and the plate is at 40°. A thermocouple, 1mm above the trailing edge records 35°C. What is u_∞, and what u at the thermocouple?

Using the momentum integral result (eqn. 6.50) we have:

$$\frac{35-20}{40-20} = 1 - \frac{3}{2}\frac{y}{\delta_t} + \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

from which we obtain, by trial and error, $\frac{y}{\delta_t} = 0.1678$

Thus, at $y = 0.001\,m$, $\delta_t = 0.00596\,m$.

And (eqn. 7.55) $\quad \delta = Pr^{1/3}\delta_t = 49.3^{1/3}(0.00596) = 0.02185\,m$

Finally from eqn. (7.2) $\quad \frac{\delta}{x} = \frac{4.92}{Re_x^{1/2}}\,;\quad \delta = \frac{4.92(1)}{\sqrt{\frac{u_\infty}{6.89(10)^{-6}}}} = 0.02185\,m$

so $\underline{\underline{u_\infty = 0.349\ m/s}}$

(Is the flow really laminar? Check it: $Re_{x=L} = \frac{0.349(1)}{6.89(10)^{-6}} = 50{,}702$. Yes)

Then, at the thermocouple: $\frac{y}{\delta} = 0.0458$, so $\frac{u}{u_\infty} = \frac{3}{2}(0.0458) - \frac{1}{2}(0.0458)^3$

$= 0.0686$, $\underline{\underline{u = 0.0239\ m/s}}$

Thus the thermocouple is fairly deeply into the thermal b.l., and very deeply into the flow b.l.. Note that slightly greater accuracy would have resulted from the consistent use of Table 6.1 in place of the integral method approximations.

6.33 What is the maximum $\bar{h}$ that can be achieved in laminar flow over a 5m plate, based on data from Table A.3? What physical circumstances give this result?

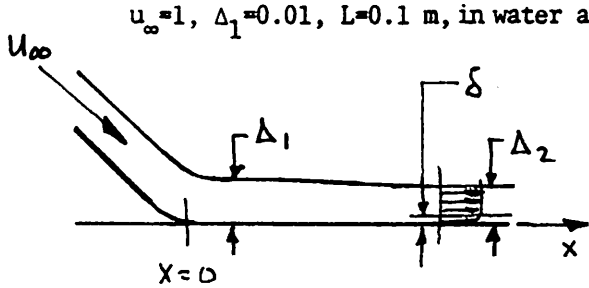
eqn. (6.68) $\bar{h} = 0.664\, k\, Pr^{1/3}\sqrt{\dfrac{u_\infty}{L\nu}}$, but $\dfrac{u_\infty L}{\nu} \leq 3.5(10)^5$ so:

$$\bar{h}_{max} = 0.664\, k\, Pr^{1/3}\sqrt{3.5(10)^5/5^2} = 78.56 k\, Pr^{1/3}$$

The largest $kPr^{1/3} = 12.3$, for glycerin at 0°C (a viscous oil might will be better if we had k data.) The gives: $h_{max} = 966\ W/m^2\text{-}°C$

This corresponds with $u_\infty = 3.5(10)^5(0.0083)/5 = 581\ m/s$. This looks high for a real system. It would require a Herculean pump.

6.34 A 17°C sheet of water, Δ_1m thick and moving at a constant speed u_∞m/s, impacts a horizontal plate at 45°, turns, and flows along it. Develop a dimensionless equation for the thickness Δ_2 at a distance L from the point of impact. Assume that $\delta << \Delta_2$. Evaluate the result for u_∞=1, Δ_1=0.01, L=0.1 m, in water at 27°C.



Mass balance:

$$\rho u_\infty \Delta_1 = \rho\int_0^\delta u\, dy + \rho\int_\delta^{\Delta_2} u_\infty dy$$

$$1 = \frac{\delta}{\Delta_1}\underbrace{\int_0^1\left(\tfrac{3}{2}\xi - \tfrac{1}{2}\xi^3\right)d\xi}_{\frac{3}{4}-\frac{1}{8}=\frac{5}{8}} + \frac{\Delta_2}{\Delta_1} - \frac{\delta}{\Delta_1}$$

so: $\dfrac{\Delta_2}{\Delta_1} = 1 + \dfrac{3}{8}\dfrac{\delta}{\Delta_1}$

The momentum eqn. is already summarized in: $\dfrac{\delta}{L} = \dfrac{4.92}{\sqrt{Re_L}}$

$$\therefore \quad \frac{\Delta_2}{\Delta_1} = 1 + \frac{3}{8}\frac{4.92}{\sqrt{Re_L}}\frac{L}{\Delta_1} = 1 + \frac{1.845}{\sqrt{Re_L}}\frac{L}{\Delta_1}$$

In the case in point: $Re_L = \dfrac{0.1 \times 1}{0.826(10)^{-6}} = 121{,}065$ so

$$\frac{\Delta_2}{\Delta_1} = 1 + \frac{1.845}{\sqrt{121{,}065}}\,10 = \underline{1.053}$$

(at this point $\delta = 0.001414$ m which is $<< \Delta_2$) Thus, contrary to the sketch above, the sheet swells to accommodate the reduced speed near the wall.)

6.35 A good approximation to the temperature dependence of μ in gases is given by the Sutherland formula: $\mu/\mu_{ref} = \left(\frac{T}{T_{ref}}\right)^{1.5} \frac{T_{ref}+S}{T+S}$, where the reference state can be chosen anywhere. Use data for air at two points to evaluate S for air. Use this vâlue to predict a third point. (T and T_{ref} are expressed in °K.)

Students might use any points from Table A-6. Let us do the problem for air using a value of $T_{ref} = 300^oK$ and a temperature, T, of interest equal to 500^oK.

First we calculate S based on the known values of

$$\mu(T = T_{ref} = 300^oK) = 1.857(10)^{-5}\ kg/m\text{-}s$$

$$\mu(T = 400^oK) = 2.310(10)^{-5}\ kg/m\text{-}s$$

Using these values in Sutherland's formula, we get

$$\underline{\underline{S = 120.7\,^oK}}$$

Then, using this S and $\mu_{ref}(T_{ref})$ in Sutherland's formula, we obtain:

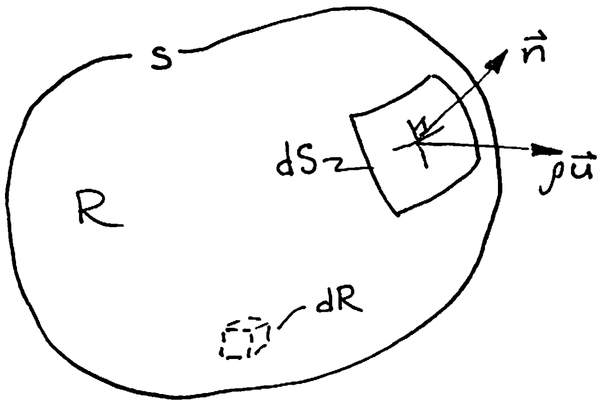
$$\mu(T = 500^oK) = \underline{\underline{2.71\ (10)^{-5}\ kg/m\text{-}s}}$$

which is exactly the tabled value to three decimal places.

6.36 We have derived a steady-state continuity equation in Section 6.3. Derive the time-dependent three-dimensional version of the equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

To do this, paraphrase the development of equation (2.14) requiring that mass be conserved instead of energy.



the mass into the control volume

$$= \frac{d}{dt}(\text{rate of mass storage})$$

or

$$-\int_S (\rho \vec{u}) \cdot \vec{n}\, dS = \frac{d}{dt}\int_R \rho\, dR$$

Now we use Gauss' theorem, $\int_S (\rho\vec{u}) \cdot \vec{n}\, dS = \int_R \vec{\nabla} \cdot \rho\vec{u}\, dR$ to get:

$$\int_R \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} \right) dR = 0$$

(where we have used Leibnitz' rule as we did in the context of eqn. (6.24.)

Finally, since the integral must vanish identically, we obtain:

$$\underline{\underline{\frac{\partial}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0}}$$

Notice that, since $\vec{\nabla} \cdot \rho\vec{u} = \rho \vec{\nabla} \cdot \vec{u} + \vec{u} \cdot \vec{\nabla}\rho$, this can be rewritten as:

$$\underline{\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot u = 0}$$

where $\frac{D}{Dt}$ is the substantial derivative, $\left(\frac{\partial \rho}{\partial t} + \vec{u} \cdot \vec{\nabla}\rho\right)$.

6.37 Various considerations show that the smallest scale motions in a turbulent flow have no prefered spatial orientation at large enough values of Re. Moreover, these small eddies are responsible for most of the viscous dissipation of kinetic energy. The dissipation rate, ϵ (W/kg), may be regarded as given information about the small scale motion, since it is set by the larger scale motion. Both ϵ and ν are governing parameters of the small scale motion.

a.) Find the characteristic length and velocity scales of the small scale motion. These are called the <u>Kolmogorov scales</u> of the flow.

b.) Compute Re for the <u>small scale</u> motion, and interpret the result.

c.) The Kolmogorov length scale characterizes the smallest motions found in a turbulent flow. If ϵ is 10 W/kg and the mean free path is $7(10)^{-6}$ m, for air at standard conditions, show that turbulent motion is a continuum phenomenon and it is thus properly governed by the equations of this chapter.

a.) Length & velocity scales, η & v, can be formed from $\epsilon \frac{W}{kg} = \epsilon \frac{m^2}{s^3}$ and $\nu \frac{m^2}{s}$. We get:

$$\text{length scale, } \eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

$$\text{velocity scale, } v = (\nu \epsilon)^{1/4}$$

b.) $Re = \frac{v\eta}{\nu} = \frac{1}{\nu}\left(\frac{\nu^3}{\epsilon}\right)^{1/4}(\nu\epsilon)^{1/4} = \underline{\underline{1}}$ since viscosity balances inertia, the small scales are extremely viscous.

c.) For air at 300°K, $\nu = 1.566(10)^{-5}\ m^2/s$. we get

$$\eta = \left(1.566\times10^{-5}\right)^{3/4}/10^{1/4} = 0.00014\ m$$

This is far larger than the mean free path. Therefore turbulent motion is a continuum phenomenon

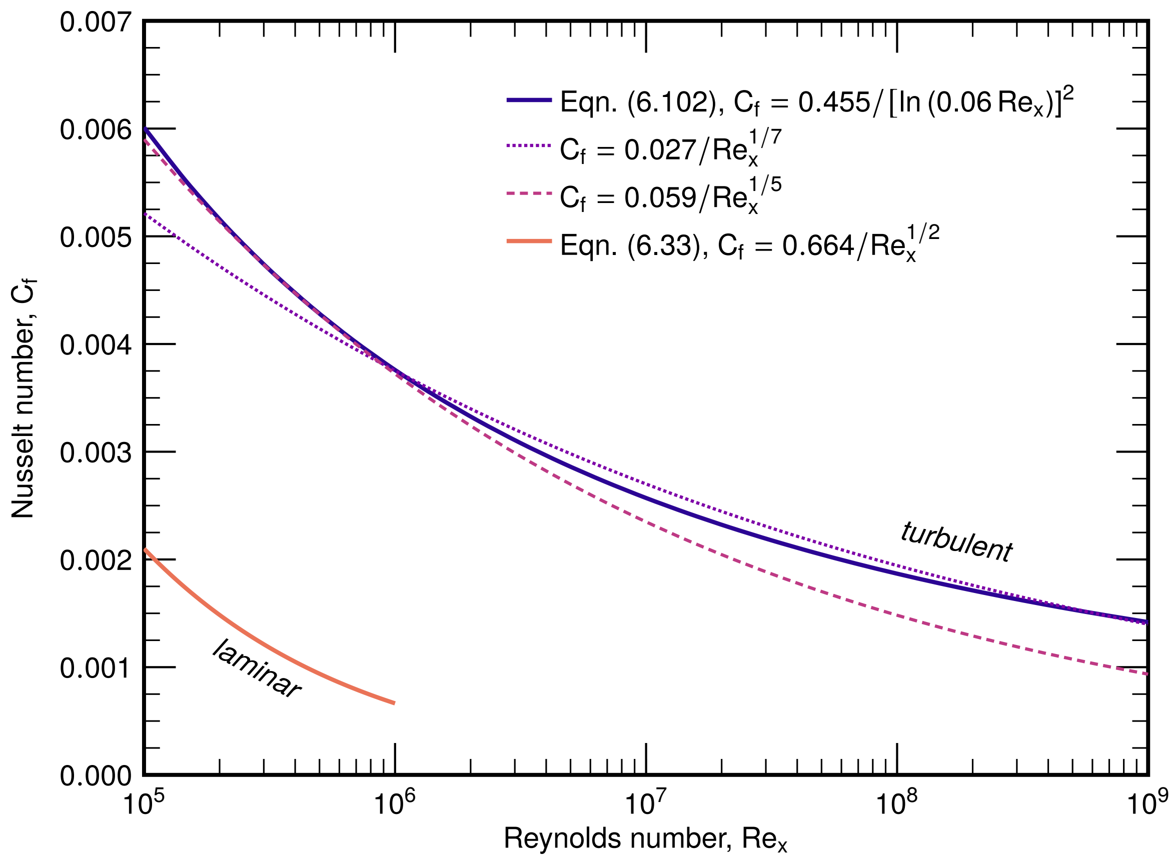
Problem 6.46 Two power laws are available for the skin friction coefficient in turbulent flow: $C_f(x) = 0.027\,\mathrm{Re}_x^{-1/7}$ and $C_f(x) = 0.059\,\mathrm{Re}_x^{-1/5}$. The former is due to White and the latter to Prandtl [6.4]. Equation (6.102) is more accurate and wide ranging than either. Plot all three expressions on semi-log coordinates for $10^5 \leqslant \mathrm{Re}_x \leqslant 10^9$. Over what range are the power laws in reasonable agreement with eqn. (6.102)? Also plot the laminar equation (6.33) on same graph for $\mathrm{Re}_x \leqslant 10^6$. Comment on all your results.

Solution The figure shows the two power laws and the mentioned turbulent and laminar expressions:

$$C_f = \frac{0.455}{\left[\ln(0.06\,\mathrm{Re}_x)\right]^2} \tag{6.102}$$

$$C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \tag{6.33}$$

The ⅐ power law is within 5% of eqn. (6.102) for $3.5 \times 10^5 \leqslant \mathrm{Re}_x \leqslant 10^9$, while the ⅕ power law is within 5% for $10^5 \leqslant \mathrm{Re}_x \leqslant 5 \times 10^7$. We also observe that skin friction in laminar flow is far less than in turbulent flow.



Problem 6.47 Reynolds et al. [6.27] provide the following measurements for air flowing over a flat plate at 127 ft/s with T_∞ = 86 °F and T_w = 63 °F. Plot these data on log-log coordinates as Nu_x vs. Re_x, and fit a power law to them. How does your fit compare to eqn. (6.112)?

$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$
0.255	2.73	1.353	2.01	2.44	1.74
0.423	2.41	1.507	1.85	2.60	1.75
0.580	2.13	1.661	1.79	2.75	1.72
0.736	2.11	1.823	1.84	2.90	1.68
0.889	2.06	1.970	1.78	3.05	1.73
1.045	2.02	2.13	1.79	3.18	1.67
1.196	1.97	2.28	1.73	3.36	1.54

Solution The film temperature is $T_f = (63 + 86)/2 = 74.5$ °F = 23.6 °C = 296.8 K. At this temperature, Table A.6 gives Pr = 0.707. We can convert the given data to $\mathrm{Nu}_x = \mathrm{St}\,\mathrm{Re}_x \mathrm{Pr}$ using a spreadsheet.

To make a fit, we must recognize that Pr does not vary. We have no basis for fitting a Pr exponent. So, we can fit to

$$\mathrm{Nu}_x = A\,\mathrm{Re}_x^b$$

This fit may be done by linear regression if we first take the logarithm:

$$\ln \mathrm{Nu}_x = \ln A + b \ln \mathrm{Re}_x$$

Using a spreadsheet, we can calculate the logarithms and perform the linear regression to find $A = 0.0187$ and $b = 0.814$ ($r^2 = 0.9978$), or

$$\boxed{\mathrm{Nu}_x = 0.0187\,\mathrm{Re}_x^{0.814}}$$

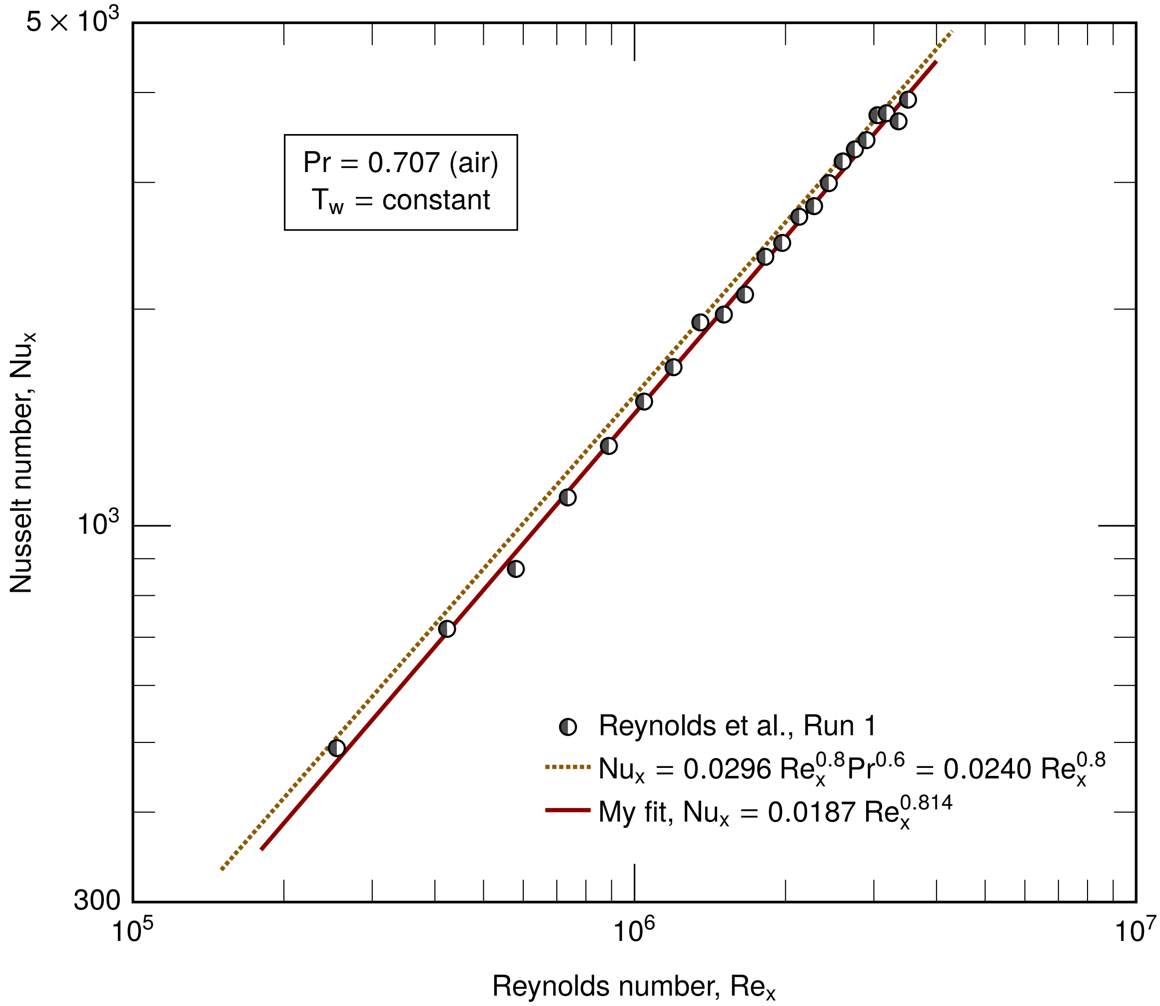
The fit is plotted with the equation, and the agreement is excellent.

With some additional effort, we may use the spreadsheet to find that the standard deviation of the data with respect to the fit is $s_x = 2.81\%$, which provides a 95% confidence interval (two-sided t-statistic for 21 points, $\pm 2.08 s_x$) of $\pm 5.8\%$.

Equation (6.112) for Pr = 0.712,

$$\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} = 0.0240\,\mathrm{Re}_x^{0.8} \qquad (6.112)$$

is also plotted in the figure, but it is systematically higher than this data set and our fit. (Reynolds et al. had 7 other data sets and reported an overall $s_x = 4.5\%$ for a $\pm 9\%$ uncertainty at 95% confidence.)



Pr = 0.707 (air)
T_w = constant
Reynolds et al., Run 1
$Nu_x = 0.0296\,Re_x^{0.8}Pr^{0.6} = 0.0240\,Re_x^{0.8}$
My fit, $Nu_x = 0.0187\,Re_x^{0.814}$
Nusselt number, Nu_x
Reynolds number, Re_x
5×10^3
10^3
300
10^5
10^6
10^7

Problem 6.48 Blair and Werle [6.36] reported the b.l. data below. Their experiment had a uniform wall heat flux with a 4.29 cm unheated starting length, $u_\infty = 30.2$ m/s, and $T_\infty = 20.5$°C.

a) Plot these data as Nu_x versus Re_x on log-log coordinates. Identify the regions likely to be laminar, transitional, and turbulent flow.
b) Plot the appropriate theoretical equation for Nu_x in laminar flow on this graph. Does the equation agree with the data?
c) Plot eqn. (6.112) for Nu_x in turbulent flow on this graph. How well do the data and the equation agree?
d) At what Re_x does transition begin? Find values of c and Re_l that fit eqn. (6.116b) to these data, and plot the fit on this graph.
e) Plot eqn. (6.117) through the entire range of Re_x.

$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$	$\mathrm{Re}_x \times 10^{-6}$	$\mathrm{St} \times 10^3$
0.112	2.94	0.362	1.07	1.27	2.09
0.137	2.23	0.411	1.05	1.46	2.02
0.162	1.96	0.460	1.01	1.67	1.96
0.183	1.68	0.505	1.05	2.06	1.84
0.212	1.56	0.561	1.07	2.32	1.86
0.237	1.45	0.665	1.34	2.97	1.74
0.262	1.33	0.767	1.74	3.54	1.66
0.289	1.23	0.865	1.99	4.23	1.65
0.312	1.17	0.961	2.15	4.60	1.62
0.338	1.14	1.06	2.24	4.83	1.62

Solution

a) Calculate the Nusselt number from the values of Stanton number using $\mathrm{Nu}_x = \mathrm{St\,Pr\,Re}_x$. This is easily done with software (or by hand if you are patient) using Pr = 0.71. The results are plotted on the next page. The regions can be identified from the changes in slope and curvature (part b makes the laminar regime more obvious).
b) The appropriate formula is eqn. (6.116) for a laminar b.l. with an unheated starting length:

$$\mathrm{Nu}_{\mathrm{lam}} = \frac{0.4587\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1-(x_0/x)^{3/4}\right]^{1/3}} \tag{6.116}$$

We have only Re_x, not x. However,

$$\frac{x_0}{x} = \frac{\mathrm{Re}_{x_0}}{\mathrm{Re}_x} \quad \text{and} \quad \mathrm{Re}_{x_0} = \frac{u_\infty x_0}{\nu} = \frac{(30.2)(0.0429)}{1.516\times10^{-5}} = 8.546\times10^4$$

With this, the expression can be plotted. The agreement is pretty good. (Equation (6.71) is shown for comparison.)
c) The equation,

$$\mathrm{Nu}_{\mathrm{turb}} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \tag{6.112}$$

is plotted in the figure, with excellent agreement.
d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, c. This slope can be determined iteratively if using

software, or by drawing the line if working by hand. The slope is well fit by $c = 2.5$. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by $\mathrm{Re}_l = 500{,}000$ and $\mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr}) = 321$. Hence,

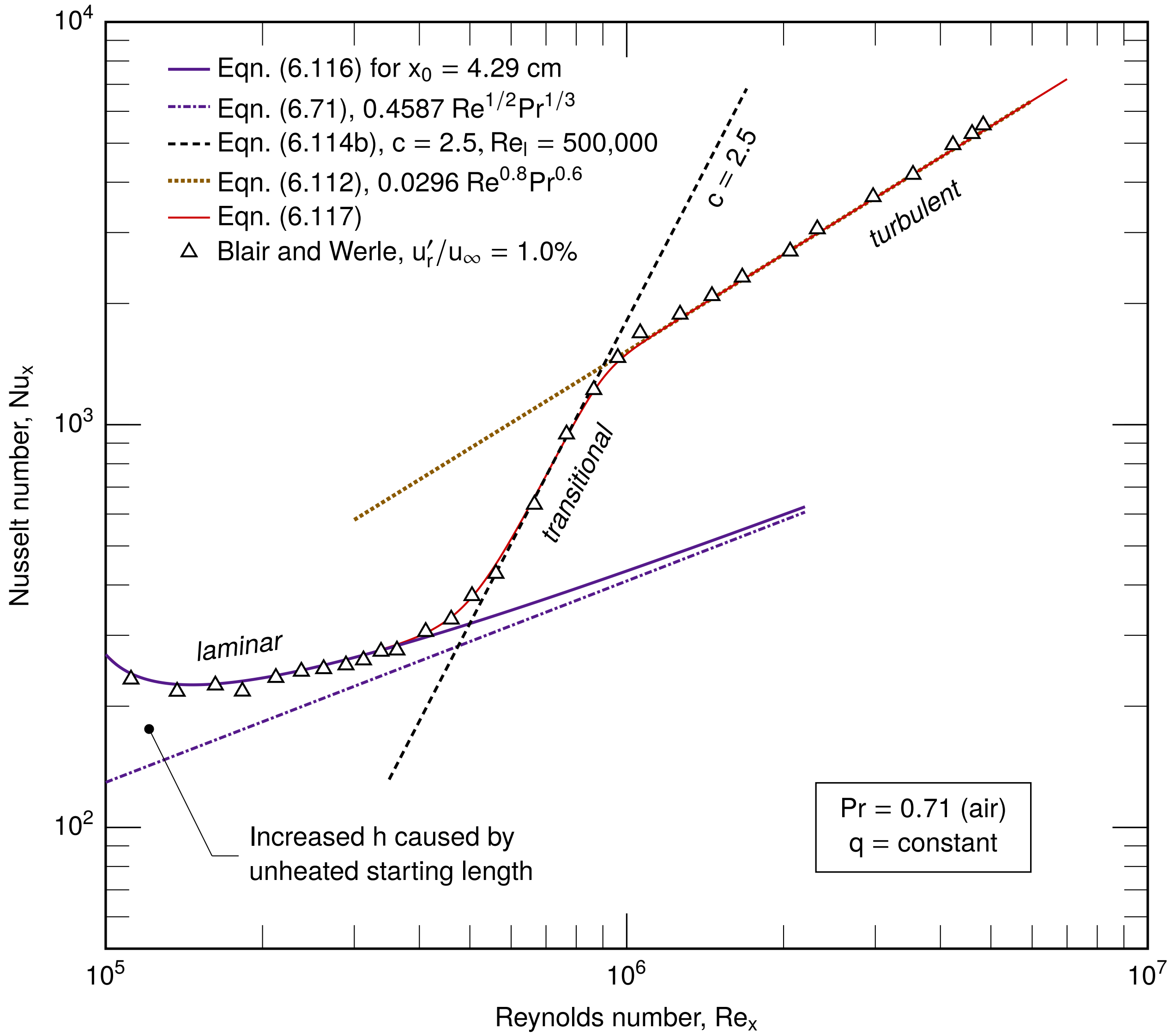
$$\mathrm{Nu}_{\mathrm{trans}} = \mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr})\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l}\right)^c = 321\left(\frac{\mathrm{Re}_x}{500,000}\right)^{2.5} \tag{6.114b}$$

This equation is plotted in the figure, with very good agreement. *Note that slightly different values of Re_l and Nu_{lam} may produce a good fit, if they lie on the same line. The best approach is to find Re_l and then* calculate *Nu_{lam} from eqn. (6.116).*

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

$$\mathrm{Nu}_x(\mathrm{Re}_x, \mathrm{Pr}) = \left[\mathrm{Nu}_{x,\mathrm{lam}}^5 + \left(\mathrm{Nu}_{x,\mathrm{trans}}^{-10} + \mathrm{Nu}_{x,\mathrm{turb}}^{-10}\right)^{-1/2}\right]^{1/5} \tag{6.117}$$

This equation is plotted in the figure as well, with very good agreement.



Problem 6.49 Figure 6.21 shows a fit to the following air data from Kestin et al. [6.29] using eqn. (6.117). The plate temperature was 100 °C (over its entire length) and the free-stream temperature varied between 20 and 30 °C. Follow the steps used in Problem 6.48 to reproduce that fit and plot it with these data.

$Re_x \times 10^{-3}$	Nu_x	$Re_x \times 10^{-3}$	Nu_x	$Re_x \times 10^{-3}$	Nu_x
60.4	42.9	445.3	208.0	336.5	153.0
76.6	66.3	580.7	289.0	403.2	203.0
133.4	85.3	105.2	71.1	509.4	256.0
187.8	105.0	154.2	95.1	907.5	522.0
284.5	134.0	242.9	123.0		

Solution

a) The results are plotted on the next page. The regions can be identified from the changes in slope.

b) The appropriate formula is eqn. (6.58) for a laminar b.l. on a uniform temperature plate:

$$\mathrm{Nu}_{\mathrm{lam}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \tag{6.58}$$

The film temperature is between 60 and 65 °C, so Pr = 0.703. This equation is plotted on the figure. Only two data points touch the line, but they are in excellent agreement.

c) The appropriate equation,

$$\mathrm{Nu}_{\mathrm{turb}} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \tag{6.112}$$

is plotted in the figure, with very good agreement.

d) To use eqn. (6.114b), we can start by visualizing a straight line through the transitional data on the log-log plot to determine the slope, c. The slope is well fit by $c = 1.7$. Once the slope is found, we find the point at which this line intersects the laminar, unheated starting length curve. That point is well represented by $\mathrm{Re}_l = 60{,}000$ and $\mathrm{Nu}_{\mathrm{lam}}(\mathrm{Re}_l, \mathrm{Pr}) = 72.3$. Hence,

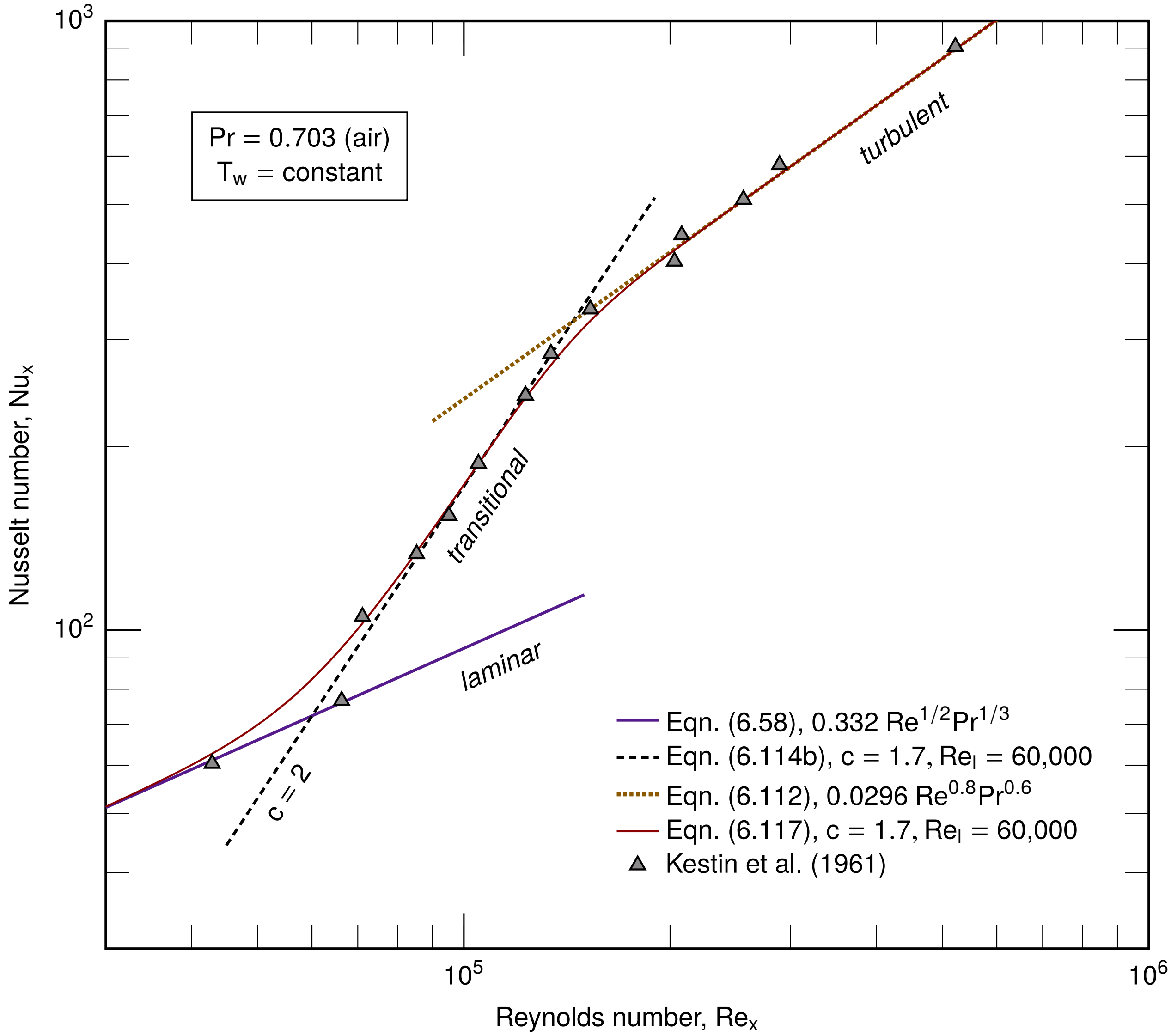
$$\mathrm{Nu}_{\mathrm{trans}} = \mathrm{Nu}_{\mathrm{lam}}\big(\mathrm{Re}_l, \mathrm{Pr}\big)\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_l}\right)^{c} = 72.3\left(\frac{\mathrm{Re}_x}{60000}\right)^{1.7} \tag{6.114b}$$

This equation is plotted in the figure, with good agreement. Note that the most consistent approach is to find Re_l and then *calculate* $\mathrm{Nu}_{\mathrm{lam}}$ from eqn. (6.58).

e) Equation (6.117) uses the laminar, transitional, and turbulent Nusselt numbers from parts (b), (c), and (d):

$$\mathrm{Nu}_x(\mathrm{Re}_x, \mathrm{Pr}) = \left[\mathrm{Nu}_{x,\mathrm{lam}}^5 + \left(\mathrm{Nu}_{x,\mathrm{trans}}^{-10} + \mathrm{Nu}_{x,\mathrm{turb}}^{-10}\right)^{-1/2}\right]^{1/5} \tag{6.117}$$

This equation is plotted in the figure as well, with very good agreement in the turbulent and transitional ranges. The laminar fit looks good with one data point, but not the other one. The data themselves make a sharp leap between Re_x of 66,300 and 85,300. (Kestin et al. varied the Reynolds number between these data by increasing the air speed, u_∞—these data are not from spatially sequential points (unlike the data of Blair in Problem 6.48). The onset of turbulence is an instability, and the change in flow conditions may well have affected the transition.)



10^3
10^2
10^5
10^6
Nusselt number, Nu_x
Reynolds number, Re_x
Pr = 0.703 (air)
T_w = constant
turbulent
transitional
laminar
c = 2
Eqn. (6.58), $0.332\,Re^{1/2}Pr^{1/3}$
Eqn. (6.114b), c = 1.7, Re_l = 60,000
Eqn. (6.112), $0.0296\,Re^{0.8}Pr^{0.6}$
Eqn. (6.117), c = 1.7, Re_l = 60,000
Kestin et al. (1961)

Problem 6.50 A study of the kinetic theory of gases shows that the mean free path of a molecule in air at one atmosphere and 20 °C is 67 nm and that its mean speed is 467 m/s. Use eqns. (6.45) obtain C_1 and C_2 from the known physical properties of air. We have asserted that these constants should be on the order of 1. Are they?

Solution We had found that

$$\mu = C_1\left(\rho\overline{C}\ell\right) \tag{6.45c}$$

and

$$k = C_2\left(\rho c_v\overline{C}\ell\right) \tag{6.45d}$$

We may interpolate the physical properties of air from Table A.6: $\mu = 1.82 \times 10^{-5}$ kg/m·s, $k = 0.0259$ W/m·K, $\rho = 1.21$ kg/m^3, and $c_p = 1006$ J/kg·K. In addition, the specific heat capacity ratio for air is $\gamma = c_p/c_v = 1.4$.

Rearranging:

$$C_1 = \frac{\mu}{\rho\overline{C}\ell} = \frac{1.82 \times 10^{-5}}{(1.21)(467)(67 \times 10^{-9})} = 0.481$$

and

$$C_2 = \frac{k\gamma}{\rho c_p\overline{C}\ell} = \frac{(0.0259)(1.4)}{(1.21)(1006)(467)(67 \times 10^{-9})} = 0.952$$

The constants are indeed $\mathcal{O}(1)$.

7.1 Relate u_{avg} to dp/dx in laminar pipe flow.

$$\rho u_{avg}\frac{\pi}{4}D^2 = \rho\int_0^{D/2} u(r)\,2\pi r\,dr = \frac{\rho\left(\frac{D}{2}\right)^2}{4\mu}\left(-\frac{dp}{dx}\right)2\pi\underbrace{\underbrace{\int_0^{D/2}\left[1-\left(\frac{2r}{D}\right)^2\right]r\,dr}_{\frac{1}{2}\left(\frac{D}{2}\right)^2-\left(\frac{2}{D}\right)^2\frac{1}{4}\left(\frac{D}{2}\right)^4}}_{+\frac{1}{4}\left(\frac{D}{2}\right)^2}$$

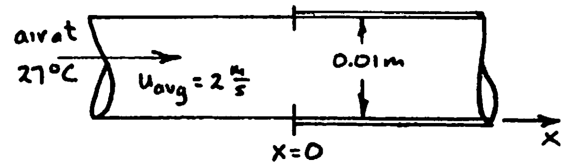
so

$$\underline{\underline{u_{avg} = -\frac{dp/dx}{32\mu}D^2 = -\frac{dp/dx}{8\mu}D^2}}$$

7.2 Consider the air flow shown:
After x = 0 either:

(a) T_w = 68.4°C

or (b) q_w = 378 W/m²



Plot T_w, q_w, and T_b vs. x in each case.

first evaluate $\underline{\frac{2}{Gz} = \frac{2x}{u_{avg}D^2/\alpha} = \frac{2x}{2(2)(0.01)^2/2.205(10)^{-5}} = 0.2205\,x}$

And $\underline{Nu_D = \frac{q_w D}{(T_w-T_b)k} = \frac{0.01}{0.02614}\frac{q_w}{\Delta T} = 0.383\,\frac{q_w}{\Delta T}}$

From eqn. (7.4) $$q_w = \frac{\rho c\, u_{avg} D}{4}\frac{dT_b}{dx} = \frac{1.193(1003)(2)(0.01)}{4}\frac{dT_b}{dx}$$

or $$\underline{q_w = 5.93\,\frac{dT_b}{dx}}$$

a) $T_w = 68.4°C$ = constant

$$\frac{dT_b}{dx} = \frac{T_{b_{i+1}}-T_{b_i}}{\Delta x} = 0.169\,q_w = 0.169\,Nu_D\,\frac{T_w - T_{b_{i+1}}}{0.383}$$

so $$T_{b_{i+1}} = T_{b_i} + 0.441\,\Delta x\,Nu_D\,(68.4 - T_{b_{i+1}})$$

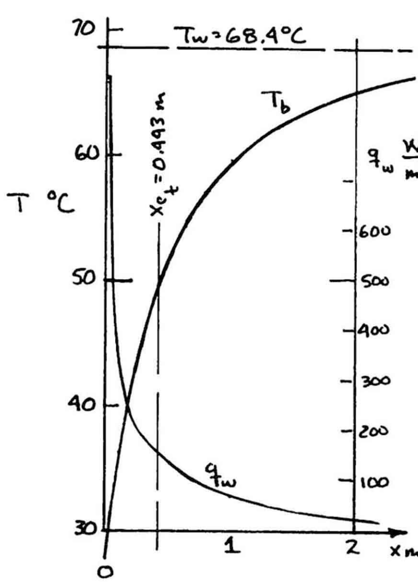
or $$\underline{T_{b_{i+1}} = \frac{T_{b_i} + 30.16\,\Delta x\,Nu_D}{1 + 0.441\,\Delta x\,Nu_D}}$$

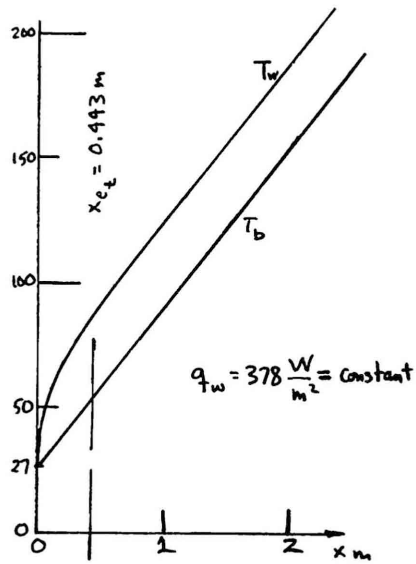
Now we pick an x = 0 where Nu_x will equal ∞. This gives $T_{b_{i+1}}$ = 27.0. Then we advance by Δx = 0.02, where we can get a new Nu_D from Fig. 8.4. Then we march forward as shown in the table below.

(b) $\frac{dT_b}{dx} = 0.169(378) = 63.88$ so $\underline{T_b = 27 + 63.88x}$

and $T_w = \frac{0.383}{Nu_D} 378 + T_b = \frac{145}{Nu_D} + 27 + 63.88x$ These are also tabled below.

Δx (m)	x (m)	$2/Gz$ $= 0.2205x$	Nu_D, Fig. 7.4 T_w = const.	q_w = const	$T_{b_{i+1}} = \frac{T_{b_i} + 30.16 Nu_D \Delta x}{1 + 0.441 Nu_D \Delta x}$	$q_w = 2.61 Nu_D \times (68.4 - T_{b_{i+1}})$	$T_w = \frac{145}{Nu_D} + 27 + 63.88x$ for q_w = const.
0.02	0	0	∞	∞	27 (given)	∞	27.0
↓	0.02	0.00441	7.7	9.5	29.63	779	43.5
↓	0.04	0.00882	6.15	7.7	31.63	590	48.4
↓	0.06	0.01323	5.55	6.8	33.34	420	52.2
↓	0.08	0.01762	5.0	6.5	34.82	363	54.4
0.1	0.10	0.02205	4.75	5.9	36.17	331	58.0
0.1	0.2	0.0441	4.1	5.05	41.11	242	68.5
0.1	0.3	0.0662	3.85	4.7	45.07	194	77.0
0.2	0.4	0.0882	3.75	4.55	48.38	162	84.4
0.2	0.6	0.1323	3.66	4.364	53.26	120	98.6
0.2	0.8	0.1762	↓	↓	56.96	90.9	111
1	1.0	0.2205	↓	↓	59.75	63.4	124
1	2	0.441	↓	↓	65.08	26.2	188
7	3	0.662	↓	↓	67.12	10.1	252
—	10	2.205	↓	↓	68.3	0.8	699





Problem 7.2: Added Note

Equation (7.30) expresses Nu_D as a function of the local Graetz Number, for a constant wall heat flux. We could thus use it, with the help of a spreadsheet, rather than reading from Fig. 7.4.

For a constant wall temperature, we can use equation (7.57) to find T_b. And equation (7.29) gives the overall heat transfer coefficient. We can then let L be the local value of x, and use these two equations to calculate T_b and the overall heat transfer coefficient at points along the tube. Once again, a spreadsheet would allow us to carry out the calculations and to plot the graph.

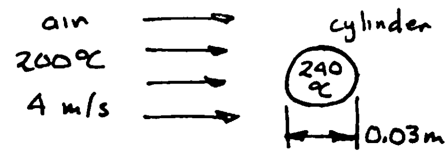
7.3 Prove that $C_f = 16/Re_D$ in laminar pipe flow.

$C_f = \dfrac{\tau_w}{\rho \bar{u}^2/2}$, but $\tau_w = \left| \mu \dfrac{\partial u}{\partial r}\Big|_{r=R} \right| = \mu \left| 2\bar{u}\left(0 - 2\dfrac{r}{R}\right)_{r=R} \right| = |-4\mu\bar{u}/R|$ (eqn. (8.8))

so: $C_f = \dfrac{2}{\rho \bar{u}^2}\left(\dfrac{4\mu\bar{u}}{R}\right) = \dfrac{8\nu}{\bar{u}R} = \dfrac{16}{\bar{u}D/\nu} = \underline{\underline{\dfrac{16}{Re_D}}}$

$\left[\text{\& } f = 4C_f = 64/Re_D\right]$

7.4 (a) Find $\bar{h}$ for the flow shown:



(b) If the flow were H_2O at 200°C, what velocity would give the same $\bar{h}$, the same $\overline{Nu_D}$, and same Re_D? Evaluate properties at 220°C

(c) Would it be possible to model this heat transfer situation using water at some other temperature. Discuss.

a) $Re_D = \dfrac{u_\infty D}{\nu} = \dfrac{4(0.03)}{3.106\times10^{-5}} = \underline{3863}$ so from Fig. 7.14 $\dfrac{\overline{Nu_D}-0.3}{Pr^{1/3}}\left[1+\left(\dfrac{0.4}{Pr}\right)^{2/3}\right]^{1/4} = 39$

but $Pr = 0.698$ so we calculate $\overline{Nu_D} = 30.64$

or: $\bar{h} = \dfrac{k\overline{Nu_D}}{D} = \dfrac{0.03595(30.64)}{0.03} = \underline{\underline{36.7\ \dfrac{W}{m^2 °C}}}$

b) For H_2O, $Re_D = \dfrac{u_\infty(0.03)}{1.444\times10^{-7}} = 3863$, so u_∞ = only $\underline{\underline{0.0186\ \frac{m}{s}}}$ for same Re_D

At $\overline{Nu_D} = 30.64$, $\dfrac{30.34}{(0.871)^{1/3}}\left[1+\left(\dfrac{0.4}{0.871}\right)^{2/3}\right]^{1/4} = 35.7$

where we read from Fig. 7.14 $Re_D \simeq 3250 = \dfrac{u_\infty(0.03)}{1.444\times10^{-7}}$

so $\underline{\underline{u_\infty = 0.0156\ \frac{m}{s}}}$ for same $\overline{Nu_D}$

If $\bar{h} = 36.7$: $\dfrac{\dfrac{36.7(0.03)}{0.691}-0.3}{(0.871)^{1/3}}\left[1+\left(\dfrac{0.4}{.871}\right)^{2/3}\right]^{1/4} = 1.67$

so we use eqn. (7.66) to calculate $Re = (1.67/0.62)^2 = 7.255$.

This gives $7.255 = \dfrac{u_\infty(0.03)}{1.444(10)^{-7}}$ so $\underline{\underline{u_\infty = 0.000035\ \frac{m}{s}}}$

which is absurdly slow!

c) $\overline{Nu_D} = fn(Re_D, Pr)$

In hot water, $Pr \Rightarrow Pr_{air}$ closely enough to make a good approximation.

This can be made equal to Re_D for an airflow, if u_∞ is kept very low.

Thus we <u>can</u> model the air flow approximately in water.

Problem 7.5 Compare the h value computed in Example 7.3 with values predicted by the Dittus-Boelter, Colburn, McAdams, and Sieder-Tate equations. Comment on this comparison.

Solution: Taking values of components from Example 7.3, we get:

$$h_{DB} = (k/D)(0.0243)(\Pr)^{0.4}(Re_D)^{0.8}$$

$$= (0.661/0.12)(0.0243)(3.61)^{0.4}(412{,}300)^{0.8} = \underline{\underline{6747\ \mathrm{W/m^2\text{-}K}}}$$

$$h_{Colburn} = (k/D)(0.023)(\Pr)^{1/3}(Re_D)^{0.8}$$

$$= (0.661/0.12)(0.023)(3.61)^{1/3}(412{,}300)^{0.8} = \underline{\underline{6193\ \mathrm{W/m^2\text{-}K}}}$$

$$h_{Mcdams} = (k/D)(0.0225)(\Pr)^{0.4}(Re_D)^{0.8} = (0.0225/0.0243)h_{DB}$$

$$= \underline{\underline{6247\ \mathrm{W/m^2\text{-}K}}}$$

$$h_{ST} = h_{Colburn}(\mu_b/\mu_w)^{0.14} = 6193(1.75)^{0.14} = 6193(1.081)$$

$$= \underline{\underline{6698\ \mathrm{W/m^2\text{-}K}}}$$

The more accurate Gnielinski equation gives h = 8400 W/m^2-K. Therefore, these old equations are low by roughly 20%, 26%, 26%, and 25%, respectively.

Why such consistently large deviations? It is because the old correlations represent much more limited data sets than Gnielinski's correlation. In this case, Re_D = 412,000 was a good deal higher than the Re_D values used to build the old correlations.

7.8 If u_∞ and T_w vary in Example 7.4, but all other conditions remain the same, plot u_∞ against T_w.

With reference to the Example, we write:

$$u_\infty = \frac{\nu}{D} Re_D = \frac{\nu}{D}\left[\frac{\frac{Q}{\pi D(T_w - T_\infty)}\frac{D}{k} - 0.3}{0.62(Pr)^{1/3}}\left[1+\left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}\right]^2$$

$$u_\infty = \frac{1.596\times 10^{-5}}{10^{-4}}\left[\frac{\frac{17.8}{\pi(T_w-20)(0.0299)} - 0.3}{0.62\,(0.71)^{1/3}}\left(1+\left(\frac{0.4}{0.71}\right)^{2/3}\right)^{1/4}\right]^2$$

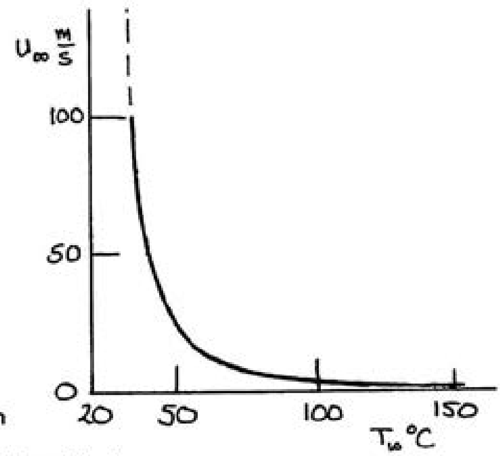
so

$$u_\infty = 0.677\left(\frac{189.4}{T_w - 20} - 0.3\right)^2$$

The speed is too high up to here

T_w °C	u_∞ $\frac{m}{s}$
20	∞
25	956
30	235
45	35.8
50	24.5

T_w °C	u_∞ $\frac{m}{s}$
60	13.3
80	5.52
100	2.89
150	0.91
200	0.38



(When $u_\infty \gtrsim 100$ m/s, the incompressible assumption breaks down and the prediction of q is more difficult.)

7.10 NAK flows full-developed at 8 m/s and 395°C in a 0.05 m I.D. tube. What is h if T_w is 403°C?

$k = 26.7\ \text{W/m-°C}$, $Pr = 0.0068$, $\nu = 2.67 \times 10^{-7}\ \text{m}^2/\text{s}$ at $\bar{T} = 399°\text{C}$.

$$Re_D = \frac{8(0.05)}{2.67(10)^{-7}} = 1.498 \times 10^6 \quad \text{so the flow is } \underline{\underline{\text{turbulent}}}.$$

Then eqn. (8.32) gives: $Nu_D = 0.625(1.498 \times 10^6 \times 0.0068)^{0.4} = 25.07$

$$\therefore\ h = \frac{26.7}{0.05}\, 25.07 = \underline{\underline{13{,}385\ \frac{\text{W}}{\text{m}^2\text{-°C}}}}$$

7.11 Water enters a 0.07 m diam., 73°C, pipe at 5°C. u_{avg} = 0.86 m/s. Plot T_b <u>vs.</u> x, neglecting entry conditions. Assume the pipe is smooth. Thus:

$$Nu_D = \frac{(f/8)\,Re_D\,Pr}{1.07 + 12.7\sqrt{\frac{f}{8}}\,(Pr^{2/3}-1)}\left(\frac{\mu_b}{\mu_w}\right)^{0.11} \quad ; \quad f = \frac{1}{(1.82\log_{10} Re_D - 1.64)^2}$$

If we move down the pipes in increments of δx,

$$T_{b_{x+\delta x}} = T_{b_x} + \frac{\bar{h}\,\pi D\,\delta x\,(T_w - T_{b_x})}{\rho_f c_p\, \pi\left(\frac{D}{2}\right)^2 u_{avg}} = T_{b_x} + \frac{4\bar{h}\,\delta x\,(T_w - T_{b_x})}{(\rho_f c_p)\,D\,u_{avg}}$$

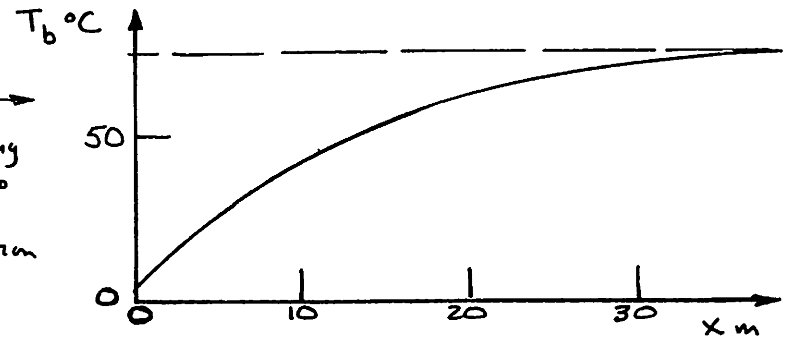
x(m)	T_{b_x} °C	$\frac{T_{b_x}+T_w}{2}$ °K	$\nu \times 10^6$ k $\rho c \times 10^{-6}$ Pr	$Re_D = \frac{0.0602}{\nu}$	$(\mu_b/\mu_w)^{0.11} \simeq \left(\frac{\nu(T_{b_x})}{0.393\times10^{-6}}\right)^{0.11}$	$\frac{f}{8}$	Nu_D	$h = \frac{k}{D}Nu_D$	$T_{b_{x+\delta x}}$ °C
0 (δx=2 m)	5	312	0.67 0.6253 4.14 4.48	85,851	1.157	0.0023	413	3689	13.0
2 (δx=2)	13	316	0.618 0.631 4.14 4.07	97,411	1.135	0.0023	453	4083	20.9
4 (δx=3)	20.9	320	0.566 0.6367 4.13 3.67	106,360	1.112	0.00222	508	4618	32.5
7 (δx=4)	32.5	326	0.532 0.6433 4.12 3.35	113,158	1.074	0.00219	494	4536	44.3
11 (δx=5)	44.3	331.6	0.482 0.6493 4.11 3.06	124,896	1.048	0.00214	498	4620	55.0
16 (δx=8)	55.0	337	0.440 0.6183 4.10 2.78	136,818	1.03	0.00210	512	4431	65.3
24 (δx=12)	65.3	342	0.411 0.6571 4.10 2.55	146,472	1.01	0.00207	497	4667	72.3

(continued ...)

7.11 (continued)

Plot the results ⟶

This might be converging too quickly owing to the large steps taken later in the calculation (cf Problem 7.13)



7.14 This problem can have hundreds of solutions. It has the value of putting the student in an active (attack) mode.

7.15 Water at 24°C flows at u_{avg} = 0.8 m/s in a 0.015 m smooth tube which is kept at 30°C. The flow could be either laminar or turbulent. Calculate $h_{turb.}/h_{laminar}$ if the flow is fully developed.

The properties at $T = \frac{24+30}{2} = 27°C$ or 300°K are:

$\nu = 0.826 \times 10^{-6}$
$k = 0.6084$
$\rho c = 4.161 \times 10^{6}$
$Pr = 5.65$

$$Re_D = \frac{u_{avg} D}{\nu} = \underline{14,528}$$

for laminar flow: $Nu_D = 3.658$, $h = \frac{0.6084}{0.015} 3.658 = \underline{\underline{148.4 \frac{W}{m^2\text{-}°C}}}$

for turbulent flow:

$$\frac{f}{8} = \frac{1}{8(1.82 \log_{10} 14,528 - 1.64)^2} = \underline{0.00355}$$

7.15 (continued)

$$Nu_D = \frac{\frac{f}{8} Re_D Pr}{1.07 + 12.7\sqrt{\frac{f}{8}}(Pr^{2/3}-1)} \left(\frac{\mu_b}{\mu_w}\right)^{0.11}$$

but $(\mu_b/\mu_w)^{0.11} \simeq (\nu_b/\nu_w)^{0.11} = \left(\frac{0.915}{0.787}\right)^{0.11} = 1.017$, so:

$$Nu_D = 109.2, \quad h = \frac{0.6084}{0.015} 109.2 = \underline{\underline{4429 \frac{W}{m^2 °C}}}$$

Thus:

$$h_{turbulent} / h_{laminar} = \frac{4429}{148.4} = \underline{\underline{29.8}}$$

Turbulent flow (in this case) gives 30 times the heat transfer in laminar flow.

7.16 Laboratory observations of heat transfer during the forced flow of air at 27°C over a bluff body, 12 cm wide, kept at 77°C, yield $q = 646\ W/m^2$ when the air moves 2 m/s and 3590 W/m^2 when it moves 18 m/s. In another test, everything else is the same, but now 17°C water flowing 0.4 m/s yields 131,000 W/m^2. The correlations in Chapt. 7 suggest that, with such limited data, we can probably create a fairly good correlation in the form: $\overline{Nu_L} = C Re^a Pr^b$. Estimate the constants C, a, and b, by cross-plotting the data on log-log paper.

for the air case: $Re_L = \frac{(2 \text{ or } 18)(0.12)}{1.809(10)^{-5}} = 13{,}267 \text{ or } 119{,}400$

and $Pr = 0.709$.

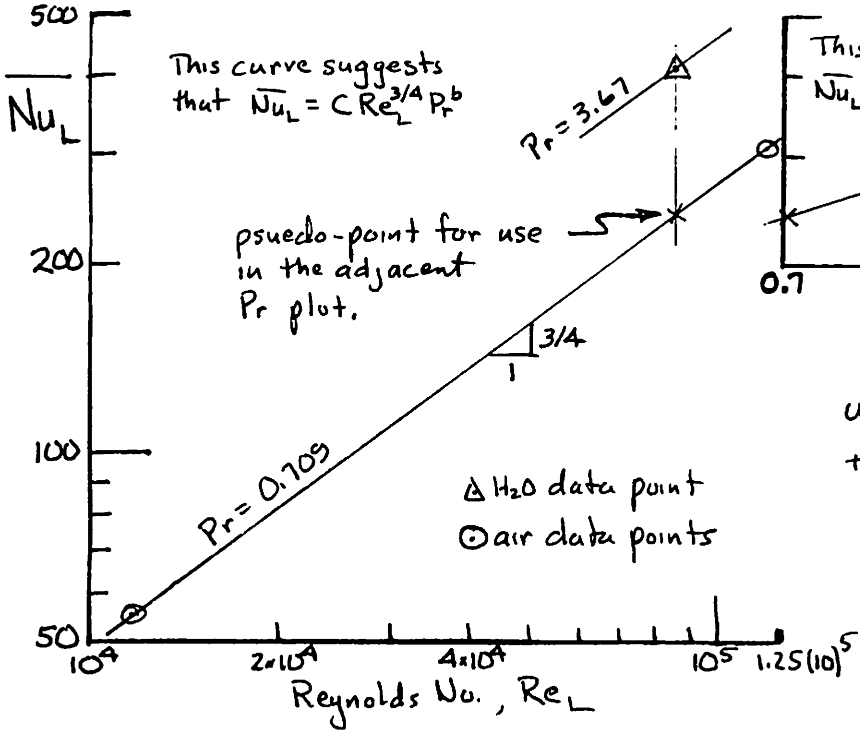
$$Nu_L = \frac{qL}{\Delta T\, k} = \frac{(646 \text{ or } 3590)(0.12)}{(77-27)(0.02792)} = \underline{55.5} \text{ or } \underline{308.6}$$

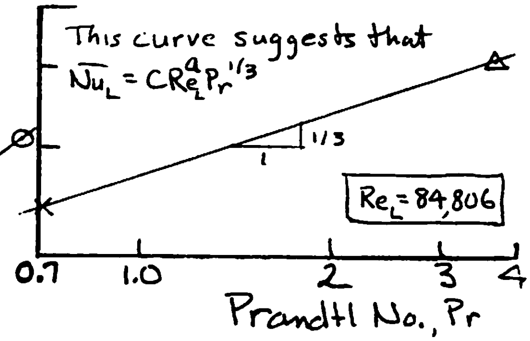
for water: $Re_L = \frac{0.4(0.12)}{0.566(10)^{-6}} = 84{,}806$, $Pr = 3.67$,

and $Nu_L = \frac{131{,}000\,(0.12)}{(77-17)(0.6367)} = \underline{411.5}$

(over)

7.16 (continued)





use $411.5 = C(84806)^{3/4}(3.67)^{1/3}$

to get $C = 0.0537$

Then:

$$\overline{Nu}_L = 0.0537\,Re_L^{3/4}\,Pr^{1/3}$$

Problem 7.17 Air at 1.38 MPa (200 psia) flows at 12 m/s in an 11 cm I.D. duct. At one location, the bulk temperature is 40 °C and the pipe wall is at 268 °C. Evaluate h if $\varepsilon/D = 0.002$.

Solution We evaluate the bulk properties at 40°C = 313.15 K. Since the pressure is elevated, we must use the ideal gas law to find the density of air with the universal gas constant, R°, and the molar mass of air, M:

$$\rho = \frac{pM}{R^\circ T} = \frac{(1.38 \times 10^6)(28.97)}{(8314.5)(313.15)} = 15.36 \text{ kg/m}^3$$

The dynamic viscosity, conductivity, and Prandtl number of a gas depend primarily upon temperature. At 313 K, $\mu = 1.917 \times 10^{-5}$ kg/m·s, $k = 0.0274$ W/m·K, and Pr = 0.706. Hence,

$$\text{Re}_D = \frac{\rho u_{\text{av}} D}{\mu} = \frac{(15.36)(12)(0.11)}{1.917 \times 10^{-5}} = 1.058 \times 10^6$$

The friction factor may be calculated with Haaland's equation, (7.50):

$$f = \left\{1.8\ \log_{10}\left[\frac{6.9}{1.058 \times 10^6} + \left(\frac{0.002}{3.7}\right)^{1.11}\right]\right\}^{-2} = 0.02362$$

We can see from Fig. 7.6 that this condition lies in the fully rough regime, as confirmed by eqns. (7.48):

$$\text{Re}_\varepsilon \equiv \frac{u^* \varepsilon}{\nu} = \text{Re}_D \frac{\varepsilon}{D}\sqrt{\frac{f}{8}} = (1.058 \times 10^6)(0.002)\sqrt{\frac{0.02362}{8}} = 114.9 > 70$$

Next, we may compute the Nusselt number from eqn. (7.49):

$$\begin{aligned}
\text{Nu}_D &= \frac{(f/8)\ \text{Re}_D\, \text{Pr}}{1 + \sqrt{f/8}\left(4.5\, \text{Re}_\varepsilon^{0.2} \text{Pr}^{0.5} - 8.48\right)} \\
&= \frac{(0.02362/8)\,(1.058 \times 10^6)(0.706)}{1 + \sqrt{0.02362/8}\left(4.5(114.9)^{0.2}(0.706)^{0.5} - 8.48\right)} \\
&= 2061
\end{aligned}$$

The temperature difference is quite large, so we should correct for variable properties using eqn. (7.45):

$$\text{Nu}_D = \text{Nu}_D\Big|_{T_b}\left(\frac{T_b}{T_w}\right)^{0.47} = (2061)\left(\frac{313.15}{541.15}\right)^{0.47} = 1594$$

Finally,

$$h = \frac{k}{D}\text{Nu}_D = \frac{0.0274}{0.11}(1594) = \underline{397 \text{ W/m}^2\text{K}}$$

7.18 How does $\bar{h}$ vary with the heater diameter during crossflow over a cylindrical heater when Re_D is very large.

From eqn. (8.34) $\displaystyle \lim_{\text{large } Re_D} \overline{Nu_D} = \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}} \left(\frac{Re_D}{282000}\right)^{\frac{5}{8}\frac{4}{5}} = fn(Pr)\, Re_D$

Therefore:

$\bar{h} = k\, fn(Pr) \frac{u_\infty}{\nu}$ which is independent of D

We encounter this size-independence again in natural convection when the size is large. See Problem 8.31.

7.20 Write Re_D in terms of $\dot{m}$ in pipe flow and explain why this repesentation could be particularly useful in dealing with compressible pipe flows.

$$Re_D = \frac{\rho \bar{u} D}{\mu} = \frac{\rho \bar{u} A}{\mu}\frac{4D}{\pi D^2} = \frac{4\dot{m}}{\pi \mu D}$$

$\dot{m}$ must remain constant in a compressible pipe flow while both ρ and u vary. In an isothermal gas flow with a pressure drop, we see that Re_D actually stays constant -- a fact that is not clear in the conventional form.

7.21 NAK at 394°C flows at 0.57 m/s across a 1.82 m length of 0.036 m O.D. tube. The tube is kept at 404°C. Find $\bar{h}$ and the heat removal rate from the tube.

Evaluate the properties a (94 + 404)/2 = 399°C:

$$Re_D = \frac{uD}{\nu} = \frac{0.57(0.036)}{2.67\times10^{-7}} = 78{,}854, \quad Pr = 0.0068\ , \quad Pe_b = Re_b Pr = 523$$

So we use eqn. (8.35)

$$\overline{Nu_D} = 0.3 + \frac{0.62(78{,}854)^{1/2}(0.0068)^{1/3}}{[1+(0.4/0.0068)^{2/3}]^{1/4}} = 16.55\ ; \quad \bar{h} = \overline{Nu_D}\frac{k}{D} = 16.55\frac{26.7}{0.036}$$

$$= \underline{\underline{12{,}275\ \frac{W}{m^2\text{-}°C}}}$$

And:

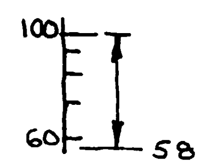
$$Q = \bar{h} A \Delta T = 12{,}275[1.82\pi(0.036)](404-394) = \underline{\underline{25{,}266 W}}$$

7.23 Check the value of h given in Example 7.3 by using Reynold's analogy <u>directly</u> to calculate it. Which h do you deem to be in error, and by what percent.

Direct use of Reynold's analogy yields the Colburn equation. We have already made this comparison in Problem 7.5. The resulting deviation from the far more accurate Gnielinski equation $\underline{\underline{\text{was 26\%}}}$.

7.26 Report the maximum percent scatter of data in Fig 7.14. What is happening in the fluid flow when the scatter is worst?

We identify the distance ↕ between the highest and lowest points at $Re_D \simeq 30{,}000$ and compare it with the log scale (as we see here:)



The error is such that $58(1+\text{scatter})(1+\text{scatter}) = 100$,

So: $\text{scatter} = \pm 0.31 = \underline{\underline{\pm 31\%}}$

The error, while not generally this bad, is still high in the range: $20{,}000 < Re_D < 300{,}000$. Figure 7.11 tells us that in this range, the conventional vortex street is gradually breaking down and becoming three-dimensional. When the b.l. on the cyl. finally becomes turbulent (and vortex shedding becomes unclear -- see Fig. 7.12), then the scatter reduces to about ±8%.

7.28 Freshly painted aluminum rods, 0.02 m in diameter are withdrawn from a drying oven at 150°C and cooled in a 3 m/s crossflow of air at 23°C. How long will it take to cool them to 40°C, so they can be handled?

We shall evaluate air properties at an average, average temperature of $\frac{1}{2}\left[\frac{150+23}{2}+\frac{40+23}{2}\right] = 57.25$ C = 330.4 K

$\nu = 1.982\,(10)^{-5}$, $k = 0.0282$, $Pr = 0.708$

for aluminum, $\rho c_p = 2707(905) = 2.45\times10^6\ \mathrm{J/m^3{\cdot}{}^\circ K}$, $k = 240\ \frac{\mathrm{W}}{\mathrm{m{\cdot}{}^\circ C}}$

Then: $Re_D = \dfrac{0.02\,(3)}{1.982\,(10)^{-5}} = 3027$ so we use eqn. (7.66)

$$\overline{Nu}_D = 0.3 + \frac{0.62\sqrt{3027}\ 0.708^{1/3}}{\left[1+(0.4/0.708)^{2/3}\right]^{1/4}} = 27.6$$

and $\bar{h} = \overline{Nu}_D \dfrac{k}{D} = 27.6\,(0.0282)/0.02 = \underline{39.2\ \frac{\mathrm{W}}{\mathrm{m^2{\cdot}{}^\circ C}}}$

Next, we calculate $Bi = \dfrac{39.2\,(0.02)}{240} = 0.0033 \ll 1$, so we can assume lumped capacity.

$$T = \frac{\rho c V}{\bar{h}A} = \frac{\rho c D}{4\bar{h}} = \frac{2.45(10)^6\,0.02}{4\,(39.2)} = \underline{308\ \mathrm{sec}}$$

Then: $$\left.\frac{T-T_\infty}{T_i-T_\infty}\right|_{T=50} = \frac{40-23}{150-23} = 0.134 = e^{-t/310}$$

Thus it will take $\underline{\underline{t = 623\text{s} = 10\text{min}, 23\text{s}}}$ to cool the rods.

7.29 At what speed, u_∞, must 20°C air flow across an insulated tube before the insulation on it will do any good? The tube is at 60°C and 6 mm in diameter. The insulation is 12 mm in diameter with k = 0.08 W/m-°C. (Notice that we do not ask for the u_∞ for which the insulation will do the most harm.)

With reference to Fig. 2.14, we require that the sum of the thermal resistances of the insulated tube must exceed the thermal resistance around the uninsulated tube. So:

$$R_{t_{ins}} + R_{t_{\substack{\text{conv. for}\\ \text{ins. tube}}}} \geq R_{t_{\substack{\text{conv. for}\\ \text{unins. tube}}}} \quad ; \quad \frac{\ln r_o/r_i}{2\pi k_{ins}} + \frac{1}{2\pi r_o \bar{h}_{ins.}} \geq \frac{1}{2\pi r_i \bar{h}_{unins.}}$$

or:
$$\frac{\ln 2}{2\pi(0.08)} + \frac{1}{2\pi(0.006)\bar{h}_{ins}} \geq \frac{1}{2\pi(0.003)\bar{h}_{unins}} \quad ; \quad 1.379 + \frac{26.53}{\bar{h}_{ins.}} \geq \frac{53.05}{\bar{h}_{unins.}}$$

To calculate $\bar{h}_{ins}$ we shall evaluate properties at T = 27°C ($T_{ins.} = 34°C$) and correct later if we must. ($\nu = 1.566\cdot10^{-5}$, $k = 0.02614$, $Pr = 0.711$)
To calculate $\bar{h}_{unins.}$ we evaluate at $(60+20)/2 = 40°C$ ($\nu = 1.69\cdot10^{-5}$, $k = 0.02707$, $Pr = 0.710$). Then using eqn. (7.68) we get:

$$\overline{Nu}_{D_{unins.}} = 0.3 + \frac{0.62\left(\frac{0.006}{1.69(10)^{-5}}\right)^{1/2} 0.710^{1/3}\sqrt{u_\infty}}{\left(1+(0.4/0.710)^{0.667}\right)^{1/4}}\left[1+\left(\frac{0.003}{1.69(0.282)}\right)^{1/2}\sqrt{u_\infty}\right]$$

$$= 0.3 + 9.15\sqrt{u_\infty}\left(1+0.0793\sqrt{u_\infty}\right)$$

&

$$\overline{Nu}_{D_{ins}} = 0.3 + \frac{0.62\left(\frac{0.012}{1.566\cdot10^{-5}}\right)^{1/2} 0.711^{1/3}\sqrt{u_\infty}}{\left(1+(0.4/0.711)^{0.667}\right)^{1/4}}\left[1+\left(\frac{0.012}{1.566(0.282)}\right)^{1/2}\sqrt{u_\infty}\right]$$

$$= 0.3 + 13.45\sqrt{u_\infty}\left(1+0.165\sqrt{u_\infty}\right)$$

Now solve by trial and error

u_∞ $\frac{m}{s}$	$\overline{Nu}_{D_{ins}}$	$\bar{h}_{ins} = \overline{Nu}_{D_i}\frac{0.02614}{0.012}$	$\overline{Nu}_{D_{unins}}$	$\bar{h}_{un} = \overline{Nu}_{D_u}\frac{0.02707}{0.006}$	$\frac{53.05}{\bar{h}_{unins.}}$	$1.379 + 26.53/\bar{h}_{ins.}$
1	15.97	34.79	10.18	45.9	1.156	2.142
0.5	10.92	23.79	7.133	32.18	1.648	2.494
0.25	7.58	16.51	5.056	22.81	2.32	2.986
0.2	6.76	14.72	4.537	20.47	2.59	3.181
0.1	4.78	10.40	3.266	14.74	3.60	3.929
0.05	3.42	7.45	2.382	10.75	4.94 =	4.94

Therefore, the velocity must be at least 5.0 cm/s if the insulation is to serve its function. (This gives $T_{outside}$ = 48.8°C so properties would better have been evaluated at (48.8 + 20)/2 = 34.4°C for $h_{insulation}$.

7.32 Evaluate $\overline{Nu}_D$ using Giedt's data for air flowing over a cylinder at Re_D = 140,000. Compare your result with the appropriate correlation, and with Figure 7.13

$$\overline{Nu_D} = \frac{1}{180}\int_0^{180} \frac{D}{k} h(\theta)\, d\theta = \frac{1}{180}\sum_i \left(\frac{Dh(\theta)}{k}\right)_i \delta\theta_i$$. Obtain data from Fig. 7.13

$$= \frac{1}{180}\Big[400(40) + 360(20) + 280(20) + 250(10) + 360(10) + 410(20) + 345(20) + 345(40)\Big]$$

$\overline{Nu_D} = 354$ from Giedt's data

The appropriate correlation is eqn. (7.68). We don't know the temp. of the air, but the only property we need to evaluate is Pr & it is very insensitive to temp. Use Pr = 0.711 (for T = 27°C).

Then:

$$\overline{Nu_D} = 0.3 + \frac{0.62(140{,}000)^{1/2}(0.711)^{1/3}}{\left[1 + (0.4/0.711)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{140000}{282000}\right)^{1/2}\right] = \underline{310}$$

(If one erroneously used eqn. (7.65), he'd get $\overline{Nu_D} = 271$ which is low.)

The correlation underpredicts the data by 12.4%.

From Fig. 7.13 we read: at Re_D = 140,000, Giedt's data point gives $\{--\}$ = 451 which is high but in the range of the other data. The correlation passes through $\{--\}$ = 395.

$$330 \le \left\{\left(\overline{Nu_D} - 0.3\right)\left[1 + \left(\frac{0.4}{0.711}\right)^{2/3}\right]^{1/4} \frac{1}{.711^{1/3}}\right\} \le 460$$

7.33 A 25 mph wind blows across a 0.25in. telephone line. What is the pitch of the hum that it emits?

we don't know T_{air}, but between 0 and 100°F, $1.6(10)^{-5} < \nu\ \frac{m^2}{s} < 2.9(10)^{-5}$

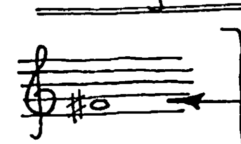
And 25 mph = 36.67 ft/s = 11.18 m/s, 0.25 in. = 0.0208 ft = 0.00635 m

Then $Re_D = \frac{u_\infty D}{\nu}$ so $2958 < Re_D < 4437$.

In this range (see Fig. 7.12) Str is virtually constant at 0.21.

Therefore: $0.21 = \frac{f_v D}{u_\infty}$; $f_v = 0.21(36.67)/0.0208 = \underline{370 \text{ cycle/sec}}$

[One half step in a tempered scale is a factor of $(2)^{1/12}$ or 1.0595 in frequency. We note that $440/(1.0595)^3 = 370$. Therefore we are 3 half tones below a concert A. The pitch is an f#.]



7.35 Consider the situation described in Problem 4.38 but suppose you do not know $\overline{h}$. Suppose instead that you know there is a 10 m/s crossflow of 27°C air over the rod. Then rework the problem.

With reference to the solution to Problem 4.38 we shall take the root temperature to be 122.4°C to evaluate properties. Then the average, average temp. on the rod is $\left(\frac{122.4+27}{2}+27\right) = 50.8°C$. Let's evaluate properties at 325°C for simplicity's sake:

$$\nu_{air} = 1.814(10)^{-5}, \quad k = 0.02792, \quad Pr = 0.709 \quad \text{and} \quad Re_D = \frac{0.005(10)}{1.814(10)^{-5}} = 2756$$

Then, with the help of eqn. (7.66)

$$\overline{Nu}_D = 0.3 + \frac{0.62\sqrt{2756}\,(0.709)^{1/3}}{\left[1+(0.4/0.709)^{2/3}\right]^{1/4}} = 25.8, \quad \overline{h} = 25.8\,\frac{0.02792}{0.005} = \underline{144\,\frac{W}{m^2\text{-}°C}}$$

This is within 4% of the original assumption of 150, so no reiteration is needed. Then in accordance with Problem 4.38, solution:

$$\Delta T = \frac{q_o}{km} = \frac{q_o}{km_{previous}}\sqrt{\frac{h_{previous}}{h_{new}}} = 95.4\sqrt{\frac{150}{144}} = \underline{97.4°C}$$

$$\text{Thus:} \quad T_{base} = 97.4 + 27 = \underline{\underline{124.4°C}}$$

7.36 A liquid whose properties are not known flows across a 40 cm diameter tube at 20 m/s. The measured heat transfer coefficient is 8000 W/m^2°C. We can be fairly confident that Re_D is very large indeed. What would $\overline{h}$ be if D were 53 cm? What would $\overline{h}$ be if u_∞ were 28 m/s?

At large Re_D eqn. (7.68) reduces to: $\overline{Nu}_D = fn\left(\begin{smallmatrix}\text{physical}\\ \text{properties}\end{smallmatrix}\right) Re_D$

or: $\overline{h} = fn(\text{physical properties})\, u_\infty$

Therefore: A change of diameter will not influence $\overline{h}$

And since $\overline{h} \sim u_\infty$, the new $\overline{h}$ will be:

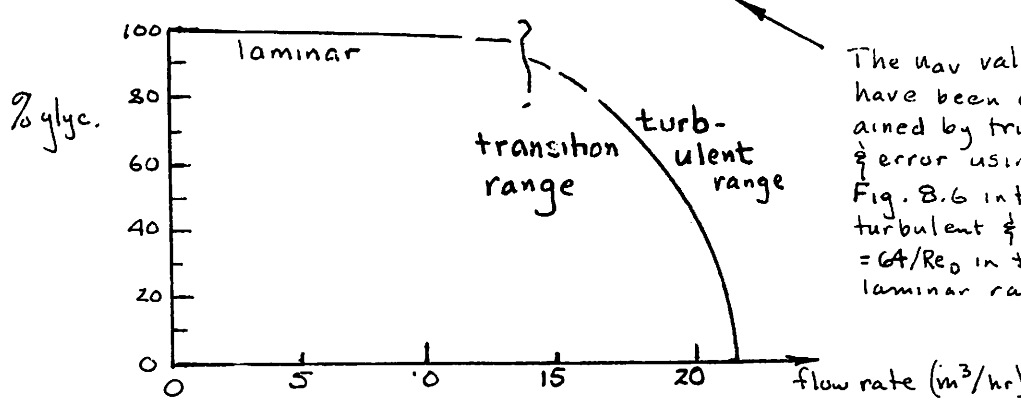
$$\underline{\underline{\overline{h} = 8000\,\frac{28}{20} = 11{,}200\,\frac{W}{m^2\text{-}°C}}}$$

7.38 Glycerin is added to water in a mixing tank at 20°C. The mixture discharges through a 4 m length of 0.04 m ID tubing under a constant 3 m head. Plot the discharge rate in m^3/hr as a function of composition.

Using eqn. (8.20),
$$f = \frac{3\,m - \frac{u_{av}^2}{2g}}{\frac{4}{0.04}\,\frac{u_{av}^2}{2g}} = \frac{3 - 0.051\,u_{av}^2}{5.10\,u_{av}^2} = \begin{cases} f(Re_D) \text{ from Fig. 7.6} \\ \dfrac{64}{Re_D} = \dfrac{1600\nu}{u_{av}} \end{cases}$$

(Note to instructors: How many students will forget to add the velocity head to 3m? Perhaps you should remind them.)

% glyc.	ν m^2/s	$\frac{u_{av}D}{\nu} = Re_D$	The u_{av} consist with eqn. above, & Fig 7.6		flow rate $\frac{\pi}{4}(.04)^2 u_{av}\,3600$
0	$1.035(10)^{-6}$	$38647\,u_{av}$	4.71	turbulent	21.31 m^3/hr
20	1.681 "	$23795\,u_{av}$	4.62	"	20.90
40	3.467 "	$11537\,u_{av}$	4.39	"	19.86
60	9.36 "	$4274\,u_{av}$	3.93	"	17.78
80	$4.97(10)^{-5}$	$805\,u_{av}$	(3.42 / 12.6)	(if turb. / if lam.)	(15.47 / 57.00)
100	0.00112	$35.7\,u_{av}$	0.0329	laminar	0.15



The u_{av} values have been obtained by trial & error using Fig. 8.6 in the turbulent & $f = 64/Re_D$ in the laminar range.

7.40 Rework Problem 5.40 without assuming the Bi to be very large.

we need $\bar{h}$. Since Bi should not be small we shall evaluate properties close to the gas temp. -- at 277°C or 550°K -- and use eqn. (7.68). Then: $\nu = 4.456(10)^{-5}$, $k = 0.0426$, $Pr = 0.698$

$Re_D = \dfrac{0.26(1)}{4.456(10)^{-5}} = 5835$ & $\overline{Nu}_D = 0.3 + \dfrac{0.62\sqrt{5835}(0.698)^{1/3}}{1.14}\left[1+\left(\dfrac{5835}{282000}\right)^{5/8}\right]^{4/5} = \underline{39.74}$

$\bar{h} = 39.74\dfrac{0.0426}{0.26} = 6.51$, $Bi^{-1} = \dfrac{0.68}{6.51(0.13)} = 0.803$ (where we use $k = k_{H_2O}$)

This gives $Fo = 0.3$ so $t = 0.3\,(0.13)^2/1.35(10)^{-7} = \underline{37{,}555\text{ sec}}$

Therefore the cooking time is considerably extended to $\underline{10.43\text{ hrs}}$. The cooking should actually commence at about $\underline{5{:}30\text{AM}}$

(When this is done in Utah, the pig is started around 7:00 or 8:00. It cooks more quickly than we predict because the flame also heats a bed of coals which radiate additional heat to the pig.)

7.41 Water enters a 0.5 cm ID pipe at 24°C. The pipe walls are held at 30°C. Plot T_b against distance from entry, if u_{av} is 0.27 m/s, neglecting entry behavior in your calculation. (Indicate the entry region on your graph, however).

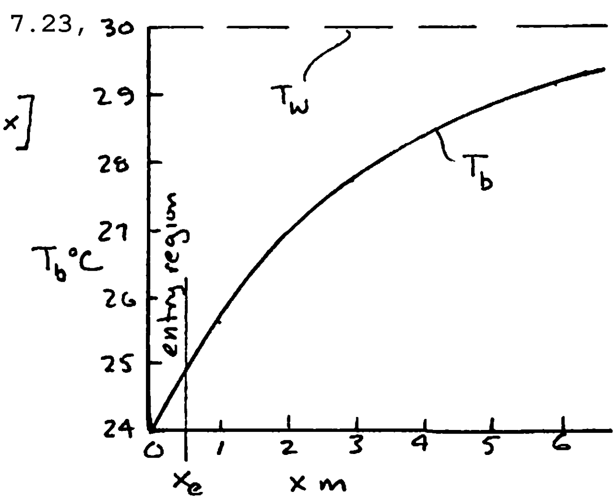
At 27°C: $\nu = 0.826(10)^{-6}$, $Pr = 5.65$, $Re_D = \dfrac{0.005(0.27)}{0.826(10)^{-6}} = 1634$ (laminar)

Then from eqns. 7.57, 7.58, and 7.23,

$$\frac{T_b - T_{b_{in}}}{T_w - T_{b_{in}}} = 1 - \exp\left[-\frac{Nu_D}{Pr\,Re_D}\frac{4}{D}x\right], \quad Nu_D = 3.658$$

$$\underline{T_b = 24 + 6\left[1 - \exp(-0.317x)\right]}$$

$x_e = 0.05 D\,Re_D = \underline{0.409\text{m}}$



7.42 Devise a numerical scheme that will allow you to be able to find the velocity distribution and friction factor in a square duct of side length a. Set up a square grid of size N by N and solve the difference equations by hand for N = 2,3 and 4. Hint: First show that the velocity distribution is given by the solution to the equation

$$\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = 1$$

where $\bar{u} = 0$ on the sides of the square and $\bar{u} = u \Big/ \frac{a^2}{\mu}\frac{dp}{dz}$, $\bar{x} = \frac{x}{a}$ and $\bar{y} = \frac{y}{a}$. Then show that the friction factor, f, [equation (8.21)], is given by

$$f = \frac{-2}{\frac{\rho u_{av} a}{\mu} \oint\!\!\oint \bar{u}\, d\bar{x}\, d\bar{y}}$$

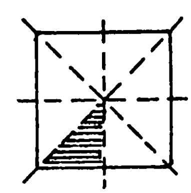
Note that the area integral can be evaluated as $\sum \bar{u}/N^2$.

Solution: By following the discussion preceding & following eqn. (8.6) we write the mom. eqn. for a square duct as:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dp}{dx} \quad \text{or, using } \bar{x} = \frac{x}{a},\ \bar{y} = \frac{y}{a},\ \&\ \bar{u} = \frac{u}{\frac{a^2}{\mu}\frac{dp}{dx}}$$

we get: $\underline{\dfrac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{u}}{\partial \bar{y}^2} = 1}$ with b.c.s $\bar{u} = 0$ on all sides.

Now, since the flow must be symmetrical about the diagonals & about the bisectors of the sides, we need only solve for flow in $(1/8)^{th}$ of the duct to know the entire flow field. However:

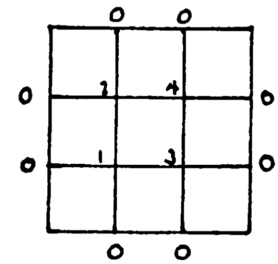


Laying out a square grid of size $\Delta\bar{x}$ & using central differences about a general point i,j, we get

$$\frac{\bar{u}_{i-1,j} + \bar{u}_{i+1,j} - 2\bar{u}_{i,j}}{\Delta\bar{x}^2} + \frac{\bar{u}_{i,j-1} + \bar{u}_{i,j+1} - 2u_{i,j}}{\Delta\bar{x}^2} = 1$$

7.42 (Continued)

Now for good accuracy $\Delta\bar{x}$ must be $\ll 1$. However, we only want to show the method here so we use a very coarse grid: $\Delta\bar{x} = 1/3$



$$\bar{u}_1 = \frac{\bar{u}_2 + \bar{u}_3}{4} - \frac{1}{4(9)}$$

$$\bar{u}_2 = \frac{\bar{u}_1 + \bar{u}_4}{4} - \frac{1}{4(9)}$$

etc.

By inspection we find that $\bar{u}_1 = \bar{u}_2 = \bar{u}_3 = \bar{u}_4 = -\frac{1}{18}$ in this case.

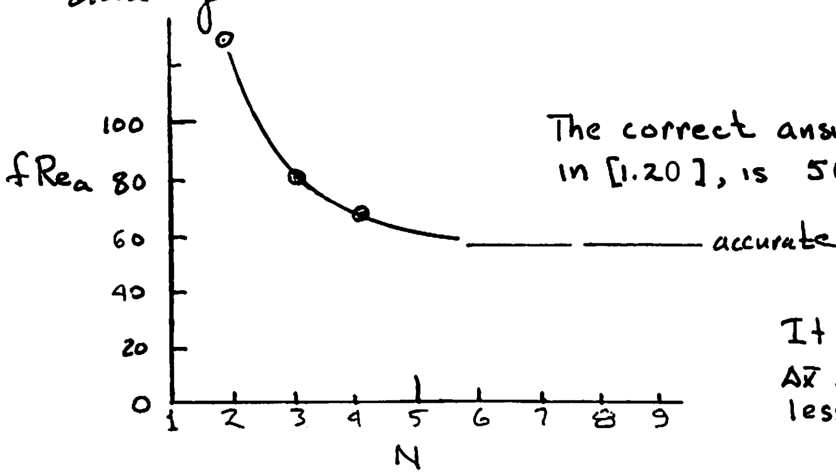
The friction factor (eqn. (7.34)) is $f = \frac{2a}{\rho u_{av}^2}\frac{dp}{dz}$

but $u_{av} = \frac{1}{A}\sum u\,\Delta x^2$ so:

$$f\,\frac{\rho u_{av} a}{\mu} = \frac{2N^2}{-\sum \bar{u}}$$

$$f\,Re_a = \frac{2(3)(3)}{4(\frac{1}{18})} = 81$$

This computation can be redone for larger N's (or smaller $\Delta\bar{x}$'s.) $\Delta\bar{x} = \frac{1}{4}$ can in fact still be done by hand with the result that $f\,Re = 69.424$.



The correct answer, given in [1.20], is 56.8.

It looks like $\Delta\bar{x}$ should be less than 1/5.

PROBLEM 7.51 Consider the water-cooled annular resistor of Problem 2.49 (Fig. 2.24). The resistor is 1 m long and dissipates 9.4 kW. Water enters the inner pipe at 47 °C with a mass flow rate of 0.39 kg/s. The water passes through the inner pipe, then reverses direction and flows through the outer annular passage, counter to the inside stream.

a) Determine the bulk temperature of water leaving the outer passage.
b) Solve Problem 2.49 if you have not already done so. Compare the thermal resistances between the resistor and each water stream, R_i and R_o.
c) Use the thermal resistances to form differential equations for the streamwise (x-direction) variation of the inside and outside bulk temperatures ($T_{b,o}$ and $T_{b,i}$) and an equation the local resistor temperature. Use your equations to obtain an equation for $T_{b,o} - T_{b,i}$ as a function of x.
d) Sketch qualitatively the distributions of bulk temperature for both passages and for the resistor. Discuss the size of: the difference between the resistor and the bulk temperatures; and overall temperature rise of each stream. Does the resistor temperature change much from one end to the other?
e) Your boss suggests roughening the inside surface of the pipe to an equivalent sand-grain roughness of 500 μm. Would this change lower the resistor temperature significantly?
f) If the outlet water pressure is 1 bar, will the water boil? *Hint:* See Problem 2.48.
g) Solve your equations from part (c) to find $T_{b,i}(x)$ and $T_r(x)$. Arrange your results in terms of $\mathrm{NTU}_o \equiv 1/(\dot{m}c_p R_o)$ and $\mathrm{NTU}_i \equiv 1/(\dot{m}c_p R_i)$. Considering the size of these parameters, assess the approximation that T_r is constant in x.

SOLUTION

a) The answer follows directly from the 1st Law, $Q = \dot{m}c_p\left(T_{b,\text{out}} - T_{b,\text{in}}\right)$:

$$\Delta T_b = Q/(\dot{m}c_p) = 9400/(0.39 \cdot 4180) = 5.77\ \text{°C}$$

so $T_{b,\text{out}} = 47 + 5.77 = \underline{52.8\ \text{°C}}$.

b) The inside thermal resistance, $R_i = 3.69 \times 10^{-2}$ K/W, is <u>23% greater</u> than the outside resistance, $R_o = 3.00 \times 10^{-2}$ K/W.

c) With eqn. (7.10), putting $(q_w P)_\text{inside} = (T_r - T_{b,i})/R_i L$ and $(q_w P)_\text{outside} = (T_r - T_{b,o})/R_o L$ where the tube length is $L = 1$ m:

$$\dot{m}c_p \frac{dT_{b,i}}{dx} = \frac{T_r - T_{b,i}}{R_i L} \tag{1}$$

$$-\dot{m}c_p \frac{dT_{b,o}}{dx} = \frac{T_r - T_{b,o}}{R_o L} \tag{2}$$

Recalling the solution of Problem 4.29, we can divide the resistance equation by L to obtain a local result (assuming that h is equal to $\overline{h}$ along the entire passage):

$$\frac{T_r - T_{b,i}}{R_i L} + \frac{T_r - T_{b,o}}{R_o L} = \frac{Q}{L} = \text{constant} \tag{3}$$

Each of $T_{b,i}$, $T_{b,o}$, and T_r are functions of x.

By adding eqn. (1) to eqn. (2), and then using eqn. (3),

$$-\dot{m}c_p \frac{d(T_{b,o} - T_{b,i})}{dx} = \frac{Q}{L}$$

and integrating (with $T_{b,o} = T_{b,i}$ at $x = L$), we find

$$\boxed{T_{b,o} - T_{b,i} = \frac{Q}{\dot{m}c_p}(1 - x/L)} \tag{4}$$

d) From working part (a) and Problem 2.49, we already know that the resistor will be much hotter than the water on either side (194 °C at the end where the water enters and exits). At any point, $T_r - T_b \gg T_{b,o} - T_{b,i}$, so that $T_r - T_{b,i} \simeq T_r - T_{b,o} \simeq$ constant, along the entire passage. From eqns. (1) and (2), then, the bulk temperature of each stream has a nearly straight line variation in x, but the outer passage temperature rises a bit faster because the thermal resistance on that side is lower. Similarly, eqn. (3) shows that the resistor temperature varies by no more than do the bulk temperatures.

e) Your solution to Problem 2.49 shows that the epoxy layers provide the dominant thermal resistance on each side. Roughness will make the convection resistance smaller, but convection resistance is only about 10% of the overall resistance. Your boss's idea will add cost and pressure drop, but it won't lower the resistor temperature much. (*Suggestion:* Find a diplomatic way to tell him that.)

f) The water will not boil if the highest temperature of the epoxy is below T_{sat}. The hottest point for the epoxy is in the outlet stream at the exit (where the bulk temperature is greatest). From the solution to Problem 2.49, using the voltage divider relation from Problem 2.48,

$$T_{\text{epoxy}} - T_{b,\text{outlet}} = (T_r - T_{b,\text{outlet}})\frac{R_{\text{conv}}}{R_{\text{outside}}} = (194 - 52.8)\frac{0.00307}{0.0300} = 14.4 \text{ K}$$

The water will not boil.

g) Rearranging eqn. (3) with eqn. (4):

$$T_r - T_{b,i} + (T_r - T_{b,i})\frac{R_i}{R_o} = QR_i - (T_{b,o} - T_{b,i})\frac{R_i}{R_o}$$

$$(T_r - T_{b,i})\left(1 + \frac{R_i}{R_o}\right) = QR_i - \frac{QR_i}{\dot{m}c_p R_o}(1 - x/L)$$

$$T_r - T_{b,i} = (QR_i)\left(\frac{R_o}{R_o + R_i}\right)\left[1 - \frac{1}{\dot{m}c_p R_o}(1 - x/L)\right] \tag{5}$$

From eqn. (3), we may estimate that $QR_i \approx (T_r - T_{b,i})/2$; thus, we can see that the second term on the right is very small and could be neglected entirely.

Upon substituting eqn. (5) into eqn. (1) we have:

$$\dot{m}c_p\frac{dT_{b,i}}{dx} = \frac{Q}{L}\left(\frac{R_o}{R_o + R_i}\right)\left[1 - \frac{1}{\dot{m}c_p R_o}(1 - x/L)\right]$$

Integration gives:

$$T_{b,i}(x) - T_{b,\text{in}} = \frac{Q}{\dot{m}c_p}\left(\frac{R_o}{R_o + R_i}\right)\left[\frac{x}{L} - \frac{1}{\dot{m}c_p R_o}\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]$$

Because the second term in the square brackets is small, we see that the bulk temperature has an essentially straight line variation.

More precisely, we may think of this arrangement as a heat exchanger, where $UA = 1/R_o$ so that

$$\text{NTU}_o = \frac{UA}{\dot{m}c_p} = \frac{1}{\dot{m}c_p R_o} = \frac{1}{(3.00\times 10^{-2})(0.39)(4180)} = 0.020 \ll 1$$

From Chapter 3, we recall that a heat exchanger with very low NTU causes very little change in the temperature of the streams, as is the case here. Putting our result in terms of the outside and inside NTUs:

$$\boxed{T_{b,i}(x) - T_{b,\text{in}} = (QR_i)\text{NTU}_i\left(\frac{R_o}{R_o+R_i}\right)\left[\frac{x}{L} - \text{NTU}_o\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]} \tag{6}$$

Substituting eqn. (6) into eqn. (5):

$$\boxed{T_r - T_{b,\text{in}} = (QR_i)\left(\frac{R_o}{R_o+R_i}\right)\left\{1 - \text{NTU}_o\left(1-\frac{x}{L}\right) - \text{NTU}_i\left[\frac{x}{L} - \text{NTU}_o\left(\frac{x}{L} - \frac{x^2}{2L^2}\right)\right]\right\}}$$

Since NTU_i has a similar value to NTU_o, the resistor temperature is indeed nearly constant, with variations on the order of $\text{NTU}_0 = 0.02$.

8.1 Show that Π_4 is equivalent to $PrRe^2/Ja$.

The velocity that the condensing film would reach in free fall through a characteristic length is $\sqrt{g(\rho_f-\rho_g)L/\rho_f}$. *Let's call this* u_{char}. *Then:*

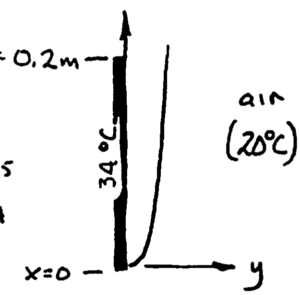
$$\Pi_4 = \frac{\rho_f(\rho_f-\rho_g)g\,h_{fg}L^3}{\mu k(T_{sat}-T_w)} = \frac{\rho_f u_c^2 h_{fg} L^2}{\nu k(T_{sat}-T_w)}$$

$$= \frac{u_c^2 L^2}{\nu^2}\,\frac{\nu}{\alpha}\,\frac{h_{fg}}{c_p(T_{sat}-T_w)} = \underline{\underline{\frac{Re^2 Pr}{Ja}}}$$

8.2 For the figure shown, Plot

δ and h vs. x

T and u vs. y

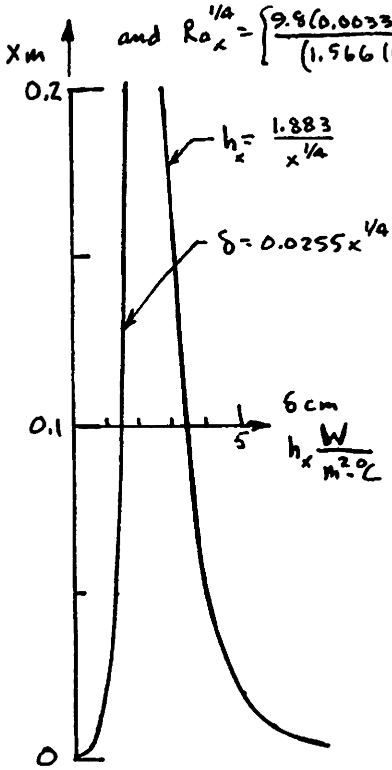


$T_{avg} = 27°C$
$Pr = 0.711$
$\nu = 1.566\times10^{-5}$
$k = 0.02614$
$\beta = \frac{1}{300} = 0.00333$

from eqns. (8.24) & (8.27)

$$\delta = 4.87\,x/Ra_x^{1/4}, \quad Nu_x = 0.3773\,Ra_x^{1/4}$$
$$h_x = 0.00986\,Ra_x^{1/4}/x$$

and $Ra_x^{1/4} = \left[\frac{9.8(0.00333)(14)}{(1.566(10)^{-5})^2}.711\right]^{1/4} x^{3/4} = 191\,x^{3/4}$



$$\frac{u}{u_c} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2$$

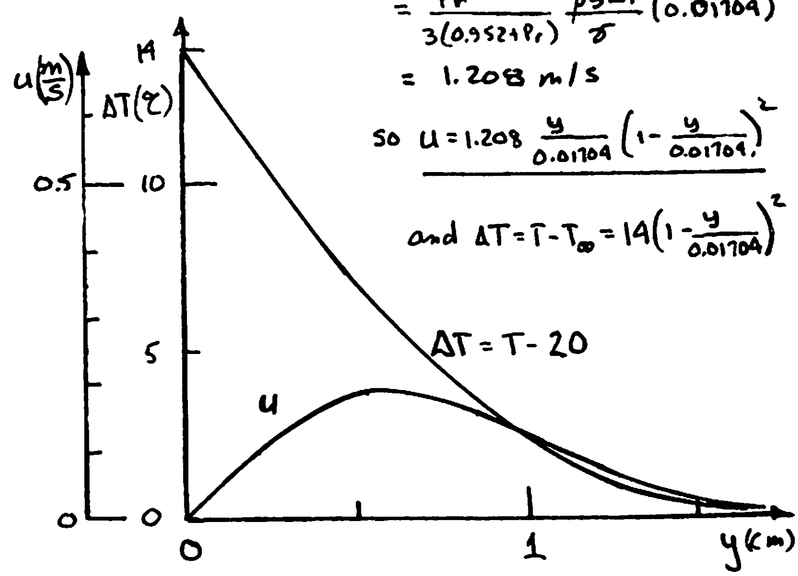
where $u_c = C_1\frac{\beta g\Delta T}{\nu}\delta^2$

$$= \frac{Pr}{3(0.952+Pr)}\,\frac{\beta g\Delta T}{\nu}(0.01704)^2$$

$$= 1.208\ m/s$$

so $u = 1.208\,\frac{y}{0.01704}\left(1-\frac{y}{0.01704}\right)^2$

and $\Delta T = T - T_\infty = 14\left(1-\frac{y}{0.01704}\right)^2$



8.3 Re-do the Squire-Eckert analysis neglecting inertia.

Omitting the inertial terms form the momentum equation, we reduce the equation after equation (8.20) to:

$$0 = \frac{1}{3} - C_1 \qquad \text{or} \qquad \underline{C_1 = 1/3}$$

eqn. (8.22) is unchanged, so we put this C_1 in it & get:

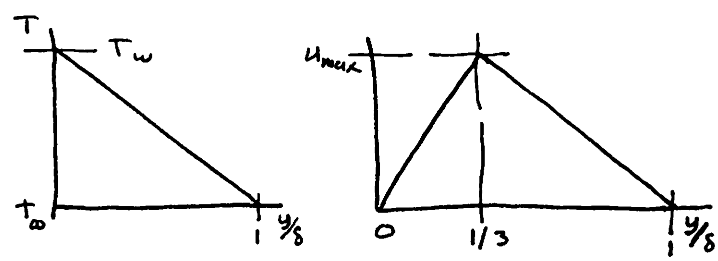
$$\delta^4 = \frac{240\,\nu^2}{\beta g\,\Delta T\,Pr}\,x \quad ; \quad \frac{\delta}{x} = 3.936\,Ra^{-1/4}$$

Then:

$$\underline{\underline{Nu_x = 2\frac{x}{\delta} = 0.508\,Ra_x^{1/4}}}$$

This is exactly the Squire-Eckert result for $Pr \gg 1$.

8.4 Predict Nu_x, using an integral method and the assumed profiles shown:



$$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{y}{\delta}$$

$$\frac{u}{u_{max}} = 3\cdot\frac{y}{\delta}\ , \quad \frac{y}{\delta} \le \frac{1}{3}$$
$$= 3\left(1 - \frac{y}{\delta}\right)\ ; \quad \frac{y}{\delta} \ge \frac{1}{3}$$

Thus:
$$\frac{d}{dx}\,\delta u_{max}^2 \underbrace{\left[\int_0^{1/3} 9\left(\frac{y}{\delta}\right)^2 d\left(\frac{y}{\delta}\right) + \int_{1/3}^{1} \frac{9}{4}\left(1 - \frac{y}{\delta}\right)^2 d\frac{y}{\delta}\right]}_{=1/3} = g\beta\Delta T\,\delta \int_0^1 \left(1 - \frac{y}{\delta}\right) d\frac{y}{\delta} - \frac{3\nu u_{max}}{\delta}$$

so:
$$\underline{\frac{1}{3}\frac{d}{dx}\left(\delta u_{max}^2\right) = \frac{g\beta\Delta T\,\delta}{2} - \frac{3\nu u_{max}}{\delta}} \qquad \text{mom. eqn.}$$

and the energy equation gives:

$$\frac{d}{dx}\,\delta u_{max}\Delta T \underbrace{\left[\int_0^{1/3} 3\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right) + \int_{1/3}^{1} \frac{3}{2}\left(1 - \frac{y}{\delta}\right)^2 d\frac{y}{\delta}\right]}_{=15/54} = +\frac{\alpha\,\Delta T}{\delta}$$

8.4 (continued)

or: $$\frac{5}{18}\frac{d(\delta u_{max})}{dx} = \frac{\alpha}{\delta} \qquad \text{energy eqn.}$$

But $u_{max} = C_1\delta^2$ so the mom. & en. eqns. become:

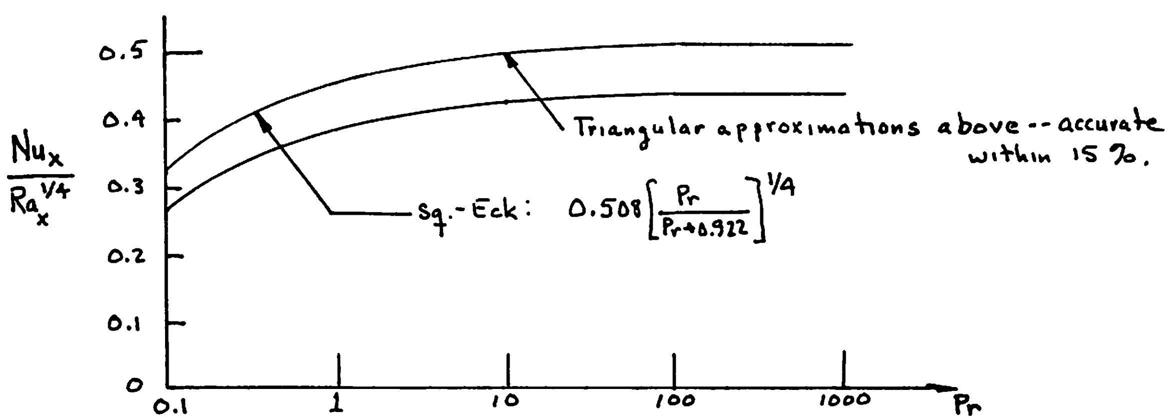
momentum: $$C_1^2\,\frac{1}{3}\,\frac{5}{4}\,\frac{d\delta^4}{dx} = \frac{g\beta\Delta T}{2} - 3\nu C_1 \qquad \text{or} \qquad \delta^4 = \left(\frac{6g\beta\Delta T}{5} - \frac{36\nu C_1}{5}\right)\frac{x}{C_1^2}$$

energy: $$C_1\,\frac{5}{8}\,\frac{3}{4}\,\frac{d\delta^4}{dx} = \alpha \qquad \text{or} \qquad \delta^4 = \frac{24}{5}\frac{\alpha x}{C_1}$$

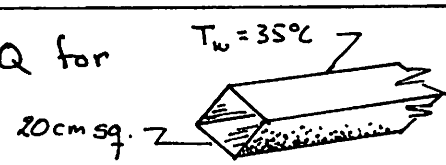
Equating these eqns. for δ^4, we get: $C_1 = \dfrac{g\beta\Delta T}{6\alpha(Pr + \frac{2}{3})}$, so:

$$\frac{\delta}{x} = \sqrt[4]{\frac{24\times 6}{5}\,\frac{\alpha\nu}{g\beta\Delta T x^3}\left(Pr + \frac{2}{3}\right)} = 2.317\,Ra_x^{-1/4}\left(\frac{Pr + 2/3}{Pr}\right)^{1/4}$$

and $$Nu_x = -\frac{\frac{dT}{dx}}{\Delta T}x = \frac{x}{\delta} = 0.432\left(\frac{Pr}{Pr + \frac{2}{3}}\right)^{1/4} Ra_x^{1/4}$$



8.5 Find $\bar{h}$ & Q for



$(T_{air} = 25°C)$

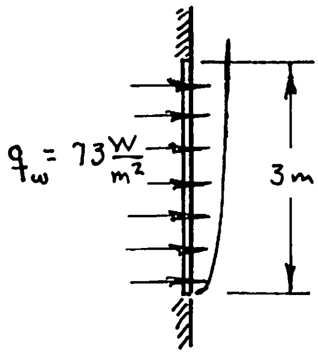
We know that $Nu_\tau = 0.52\,Ra_\tau^{1/4}$ where $\tau = 2(20)\,\text{cm} = 0.4\,\text{m}$. Thus (at $\bar{T} = 303°C$)

$$\bar{h} = 0.52\,\frac{k_{air}}{0.4\text{m}}\left(\frac{g\beta\Delta T(0.4)^3}{\nu\alpha}\right)^{1/4} = 0.52\,\frac{0.02635}{0.4}\left[\frac{9.8\frac{1}{25+273}(10)(0.4)^3}{(1.596)(2.246)}\right]^{1/4}$$

or $$\bar{h} = 3.00\ \frac{W}{m^2\text{-}°C}$$

and $$Q = \bar{h}A\Delta T = 3.00\,\frac{W}{m^2\text{-}°C}\,\frac{4(0.2)\,m^2}{m}\,10°C = 24.0\ \frac{W}{m}$$

8.6 Heat heat flux from a 3 m high electrically heated panel in a wall is 75 W/m^2 in an 18^{o}C room. What is the average temperature of the panel? What is the temperature at the top? -- at the bottom?



$$Ra^*_L = \frac{g\beta q_w L^4}{k\nu\alpha} = \frac{(9.8/(273+18))(75)(3^4)}{0.02614\,(15.66)(.2203)} 10^{10}$$

where we assume $(\overline{T_w}+T_\infty)/2 = 300$. Then $Ra^*_L = 2.2 \times 10^{13}$

$$\frac{0.67\, Ra^{*\,1/4}_L}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} = \frac{0.67\,(2182)}{\left(1+\left(\frac{0.492}{.708}\right)^{9/16}\right)^{4/9}} = 1122 = Nu_L^{5/4} - 0.68\, Nu_L^{1/4}$$

$$\underline{Nu_L = 276}$$

Then $\overline{\Delta T} = \frac{q_w}{Nu_L}\frac{L}{k} = \frac{75(3)}{276\,(.02614)} = \underline{31.2}$

This gives $\overline{T_w} = 319.5$°K and we assumed 309°C. If we accept this as close enough

$$\overline{T_w} = 18 + 31.2 = \underline{\underline{49.2\,^oC}}$$

and since this "average" is a mid-point value we go to Fig. 8.9 and write:

$$\frac{49.2}{C} = 0.8706 \quad , \quad C = 56.5$$

so $\qquad \Delta T = 56.5(x/L)^{1/5}$

at the leading edge $\Delta T = 56.5 \times 0 = 0$, $\underline{\underline{T_{x=0} = 18\,^oC}}$

at the top, $\quad \underline{\Delta T = 56.5\,^oC} \qquad \underline{\underline{T_{x=3} = 56.5\,^oC}}$

8.7 Find pipe diameters and wall temperatures for which the film condensation heat transfer coefficients given in Table 1.1 are valid.

We must make some assumptions here since there are many circumstances under which these values would be obtainable. Let us take p = 1 atm. Then:

for water:

$h_{fg} = 2{,}257{,}000$ J/kg $\qquad c_p = 4.219\times 10 \ \frac{kJ}{kg}$

$\rho_g = 0.577$ kg/m³

$\rho_f = 957.2$ "

$k_f = 0.6811$ W/m-°C

$\mu_f = 0.000278$ kg/m-s

for benzene:

$T_{sat} = 68°C$

$h_{fg} = 407000$

$\rho_g = 1.92$

$\rho_f = 827$

$k_f = 0.164$

$\mu_f = 0.000365$

$c_p = 1.74\times 10$

Note to instructor: These properties are not available in the text. The student must go to conventional data sources to obtain them.

$$\bar{h} = 0.729\, k_f\left[\frac{\rho_f(\rho_f-\rho_g)g\, h_{fg}}{\mu_f k_f}\right]^{1/4}\frac{1}{(D\Delta T)^{1/4}}\left(1+\frac{c_p}{h_{fg}}\Delta T\right)^{1/4}$$

$$15{,}000 = 8979\left(\frac{1+0.00187\Delta T}{D\Delta T}\right)^{1/4} \quad \text{for water}$$

$$1700 = 1746\left(\frac{1+0.0043\Delta T}{D\Delta T}\right)^{1/4} \quad \text{for benzene}$$

Some possible solutions:

water: $\Delta T = 3°C$, $D = 0.043$ m
$\Delta T = 6°C$, $D = 0.0216$ m
$\Delta T = 10°C$, $D = 0.013$ m

benzene: $\Delta T = 10°C$, $D = 0.116$ m
$\Delta T = 15°C$, $D = 0.079$ m
$\Delta T = 25°C$, $D = 0.049$ m

8.8 A 0.3 m high plate at 90°C condenses steam at 1 atm. Change the height or the temperature to values that will cause the laminar to turbulent transition to occur at the bottom.

From eqn. (8.72), turbulence occurs when:

$$\delta_{tu} = \sqrt[3]{[3\mu_f^2/\rho_f(\rho_f-\rho_g)g]450} \simeq \sqrt[3]{(3\nu_f^2/g)450}$$

$$= \sqrt[3]{[3(0.294)^2 10^{-12}/9.8]450} = \underline{0.0000228\ \text{m}}$$

Then, using eqn. (8.56) we get,

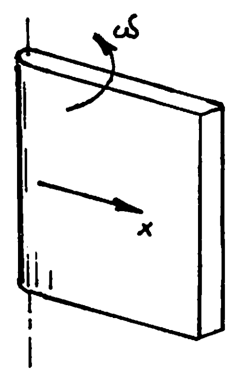
$$\frac{\delta_{tu}}{\delta_{30}} = \left(\frac{x_{tu}}{0.3\text{m}}\right)^{1/4} \quad ; \quad x_{turb} = 0.3\left(\frac{0.000228}{0.000103}\right)^4 = \underline{\underline{7.28\ \text{m}}}$$

or

$$\frac{\delta_{tu}}{\delta_{30_{cm}}} = \left(\frac{\Delta T_{turb}}{10°\text{C}}\right)^{1/4} \quad ; \quad \Delta T_{turb} = 10\left(\frac{0.000228}{0.000103}\right)^4 = \underline{\underline{240°\text{C}}}$$

We can't reach turbulence in a 0.3 cm length by cooling. The flow would freeze up first.

8.9 A cool plate spins in a synchronously rotating vapor, so $g(x) = \omega^2 x$. Find: Nu_L



$$g_{eff} = \frac{x\omega^{8/3} x^{4/3}}{x^{2/3}\int_0^x x^{1/3}dx} = \frac{4\omega^2}{3}x$$

so:

$$Nu_x = \left(\frac{\rho_f(\rho_f-\rho_g)4\omega^2 x^4 h_{fs}'}{4\mu k\Delta T\ 3}\right)^{1/4}$$

and

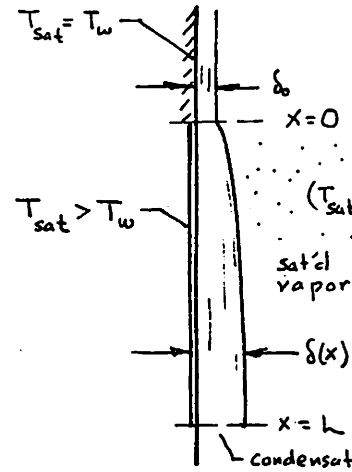
$$h = \left(\frac{\rho_f-\rho_g)\omega^2 k^3 h_{fs}'}{3\mu\Delta T}\right)^{1/4} = \text{constant} = \bar{h}$$

Thus

$$\underline{\underline{Nu_L = 0.760\left(\frac{\rho_f(\rho_f-\rho_g)\omega^2 h_{fg}' L^3}{\mu k\Delta T}\right)^{1/4}}}$$

8.10 For the flow shown, calculate

$\delta(x)$, Nu_x, and Nu_L



Eqn. (8.55) applies in this case. We rewrite it as follows:

$$\underbrace{\delta^3 \frac{d\delta}{dx}}_{\frac{1}{4}\frac{d\delta^4}{dx}} = \frac{k\mu\,\Delta T}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}}$$

And we integrate this using the b.c. $\delta(x=0) = \delta_o$:

$$\delta^4 - \delta_o^4 = \frac{k\mu\,\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}} \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\frac{4k\mu\,\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}} + \delta_o^4\right]^{1/4}}}$$

so :

$$\underline{\underline{Nu_x = \frac{x}{\delta} = \left[\frac{4k\mu\,\Delta T}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}\,x^3} + \left(\frac{\delta_o}{x}\right)^4\right]^{-1/4}}}$$

and

$$Nu_L = \frac{L}{k}\,\frac{1}{L}\int_0^L h(x)\,dx = \int_0^L \frac{dx}{\left[\frac{4k\mu\,\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}} + \delta_o^4\right]^{1/4}}$$

$$= \frac{4}{3}\,\frac{\rho_f(\rho_f-\rho_g)g\,h'_{fg}}{4k\mu\,\Delta T}\left[\left[\frac{4k\mu\,\Delta T\,x}{\rho_f(\rho_f-\rho_g)g\,h'_{fg}} + \delta_o^4\right]^{3/4}\right]_{x=0}^{x=L}$$

$$\underline{\underline{Nu_L = \frac{4}{3}\left[\left(\frac{\rho_f(\rho_f-\rho_g)g\,h'_{fg}\,L^3}{4k\mu\,\Delta T}\right)^{1/3} + \left(\frac{\rho_f(\rho_f-\rho_g)g\,h'_{fg}\,\delta_o^3}{4k\mu\,\Delta T}\right)^{4/3}\right]^{3/4} - \frac{4\rho_f(\rho_f-\rho_g)g\,h_{fg}\,\delta_o^3}{3k\mu\,\Delta T}}}$$

8.11 Prepare a table of formulas of the form:

$$\bar{h}\ \mathrm{W/m^2\text{-}^oC} = C[\Delta T\ {}^oC/L\ m]^{1/4}$$

for natural convection at normal gravity in air and in water at $T_\infty = 27^oC$. Assume that T_w is close to 27^oC. Your table should include results for vertical plates, horizontal cylinders, spheres, and possibly additional geometries. Do not include your calculations.

$$Ra_{H_2O} = \left(\frac{g\beta}{\alpha\nu}\right)\Delta T L^3 = \left(\frac{9.8(0.000275)}{1.462(0.826)\times 10^{-13}}\right)\Delta T L^3 = 2.232\times 10^{10}\ \Delta T L^3$$

$$Ra_{air} = \left(\frac{9.8\ \frac{1}{300}}{1.566(2.203)\times 10^{-10}}\right)\Delta T L^3 = 9.47\times 10^{7}\ \Delta T L^3$$

configuration	reference equation	simplified formula for $\bar{h}$: water	simplified formula for $\bar{h}$: air
vertical plate	eqn. (8.27)	$153(\Delta T/L)^{1/4}$	$1.414(\Delta T/L)^{1/4}$
horizontal cyl.	eqn. (8.28) (neglect lead const. restrict to larger values of Ra_D)	$109(\Delta T/D)^{1/4}$	$1.01(\Delta T/D)^{1/4}$
sphere	eqn. (8.32) (neglect lead const. restrict to larger Ra_D's.)	$101(\Delta T/D)^{1/4}$	$1.11(\Delta T/D)^{1/4}$
other situation where $\overline{Nu_D} = C\,Ra_D^{1/4}$		$C(235)\left(\frac{\Delta T}{D}\right)^{1/4}$	$C(2.58)\left(\frac{\Delta T}{D}\right)^{1/4}$

8.12 For what value of the Prandtl number is the condition:

$$\left.\frac{{}^2u}{y^2}\right|_{y=0} = \frac{\beta g\Delta T}{\nu}$$

satisfied exactly in the Squire-Eckert b.l. solution?

In the context of eqn. (8.19) we saw that C_1 must be 1/4; but eqn. (8.23) tells us that:

$$C_1 = \frac{Pr}{3(\frac{20}{21} + Pr)} = \frac{1}{4}$$

so:

$$\frac{20}{21} + Pr = \frac{4}{3}\,Pr$$

Solving this, we obtain: $\underline{\underline{Pr = 2.86}}$

PROBLEM 8.13 The side wall of a house is 10 m in height. The overall heat transfer coefficient between the interior air and the exterior surface is 2.5 W/m^{2}K. On a cold, still winter night $T_{\text{outside}} = -30$ °C and $T_{\text{inside air}} = 25$ °C. What is $\overline{h}_{\text{conv}}$ on the exterior wall of the house if $\varepsilon = 0.9$? Is external convection laminar or turbulent?

SOLUTION The exterior wall is cooled by both natural convection and thermal radiation. Both heat transfer coefficients depend on the wall temperature, which is unknown. We may solve iteratively, starting with a guess for T_w. We might assume (arbitrarily) that ⅔ of the temperature difference occurs across the wall and interior, with ⅓ outside, so that $T_w \approx (25 + 30)/3 - 30 = -11.7$ °C $= 261.45$ K. We may take properties of air at $T_f \approx 250$ K, to avoid interpolating Table A.6:

PROPERTIES OF AIR AT 250 K

thermal conductivity	k	0.0226	W/m·K
thermal diffusivity	α	1.59×10^{-5}	m^2/s
kinematic viscosity	ν	1.135×10^{-5}	m^2/s
Prandtl number	Pr	0.715	

The next step is to find the Rayleigh number so that we may determine whether to use a correlation for laminar or turbulent flow. With $\beta = 1/T_f = 1/(250)$ K^{-1}:

$$\text{Ra}_L = \frac{g\beta(T_w - T_{\text{outside}})L^3}{\nu\alpha} = \frac{(9.806)(-11.7 + 30)(10^3)}{(250)(1.59)(1.135)(10^{-10})} = 3.98 \times 10^{12}$$

Since, $\text{Ra}_L > 10^9$, we use eqn. (8.13b) to find $\overline{\text{Nu}}_L$:

$$\overline{\text{Nu}}_L = \left\{0.825 + \frac{0.387\ \text{Ra}_L^{1/6}}{\left[1 + (0.492/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(3.98 \times 10^{12})^{1/6}}{\left[1 + (0.492/0.715)^{9/16}\right]^{8/27}}\right\}^2 = 1738$$

Hence

$$\overline{h}_{\text{conv}} = (1738)\frac{0.0226}{10} = 3.927 \text{ W/m}^2\text{K}$$

The radiation heat transfer coefficient, for $T_m = (261.45 + 243.15)/2 = 252.30$ K, is

$$h_{\text{rad}} = 4\varepsilon\sigma T_m^3 = 4(0.9)(5.6704 \times 10^{-8})(252.30)^3 = 3.278 \text{ W/m}^2\text{K}$$

The revised estimate of the wall temperature is found by equating the heat loss through the wall to the heat loss by convection and radiation outside:

$$(2.5)(25 - T_w) = (3.927 + 3.278)(T_w + 30)$$

so that $T_w = -15.8$ °C, which is somewhat lower than our estimate. We may repeat the calculations with this new value (without changing the property data) finding $\text{Ra}_L = 3.09 \times 10^{12}$, $\overline{\text{Nu}}_L = 1799$, $\overline{h}_{\text{conv}} = 4.065$ W/m^{2}K, $T_m = 250.3$ K, and $h_{\text{rad}} = 3.201$ W/m^{2}K. Then

$$(2.5)(25 - T_w) = (4.065 + 3.201)(T_w + 30)$$

so that $T_w = -15.9$ °C. Further iteration is not needed. Since the film temperature is very close to 250 K, we do not need to update the property data.

To summarize the final answer, $\underline{\overline{h}_{\text{conv}} = 4.07 \text{ W/m}^2\text{K}}$ and <u>most of the boundary layer is turbulent.</u>

8.14 Plot T_{sheet} vs. time for Ex. 8.2, if the sheets are 1 % carbon, 2 m long and 6mm thick (w = 0.003 m). The bath is water at 60°C and the sheets are introduced at 18°C. Compare the result with exponential response.

With reference to Example 8.2, with properties evaluated at (60 + 80)/2, or 39°C:

$$\text{define } B \equiv 0.678 \frac{0.6253}{2}\left[\frac{4.96}{0.952+4.96}\right]^{1/4}\left[\frac{9.8(0.000371)2^3}{1.509(0.67)10^{-13}}\right]^{1/4} = \underline{148 \frac{W}{m^2\text{-}°C^{5/4}}}$$

Then:

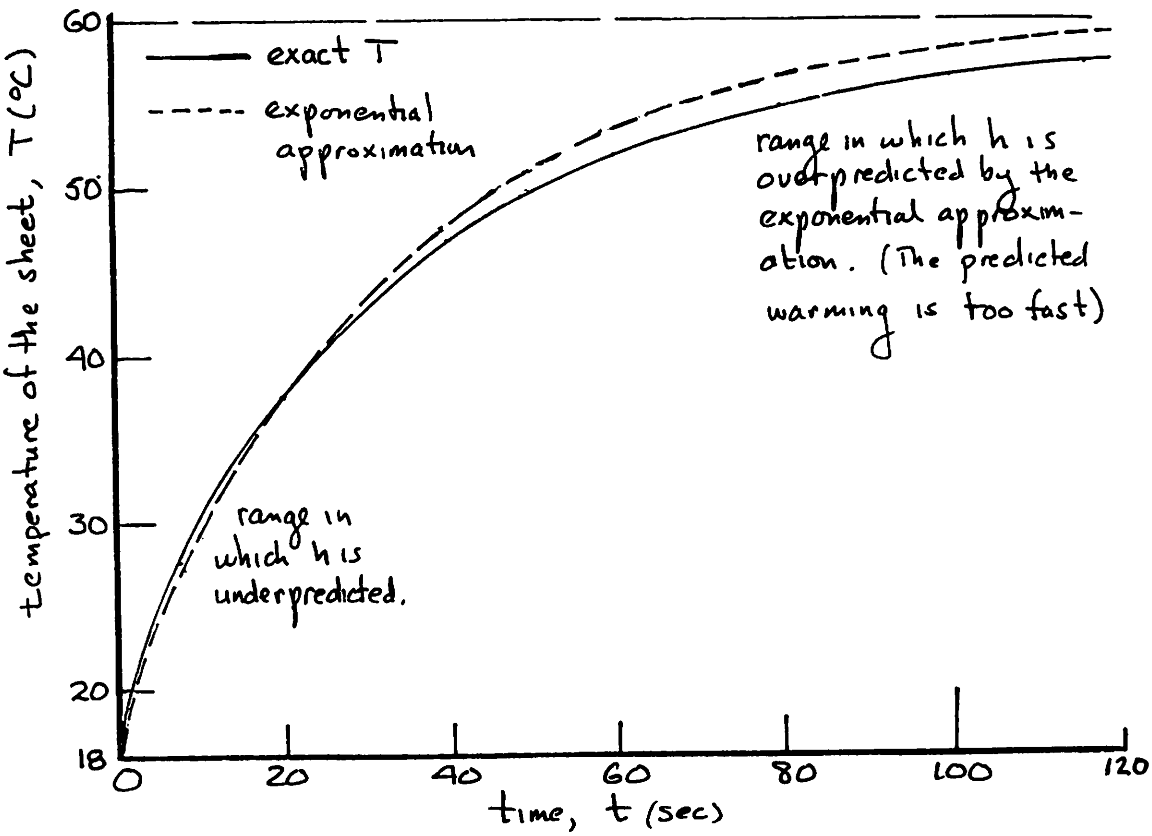
$$\underline{T = 60 - \left[\frac{1}{42^{1/4}} + \frac{B\ t}{4(7801)(473)(0.003)}\right]^{-4} = 60 - \frac{1}{[0.393+0.00334t]^4}}$$

The exponential response is: $\dfrac{T-T_\infty}{T_i-T_\infty} = e^{-t/\mathbf{T}}$

where $\mathbf{T} = \dfrac{\rho c V}{\bar{h} A} = \dfrac{7801(473)(w)}{B(39-18)^{1/4}} = 31.84\ \text{sec}$; $\underline{T = 60 - 42\ \exp(-0.0314\,t)}$

(Continued, next page)

8.14 (continued)

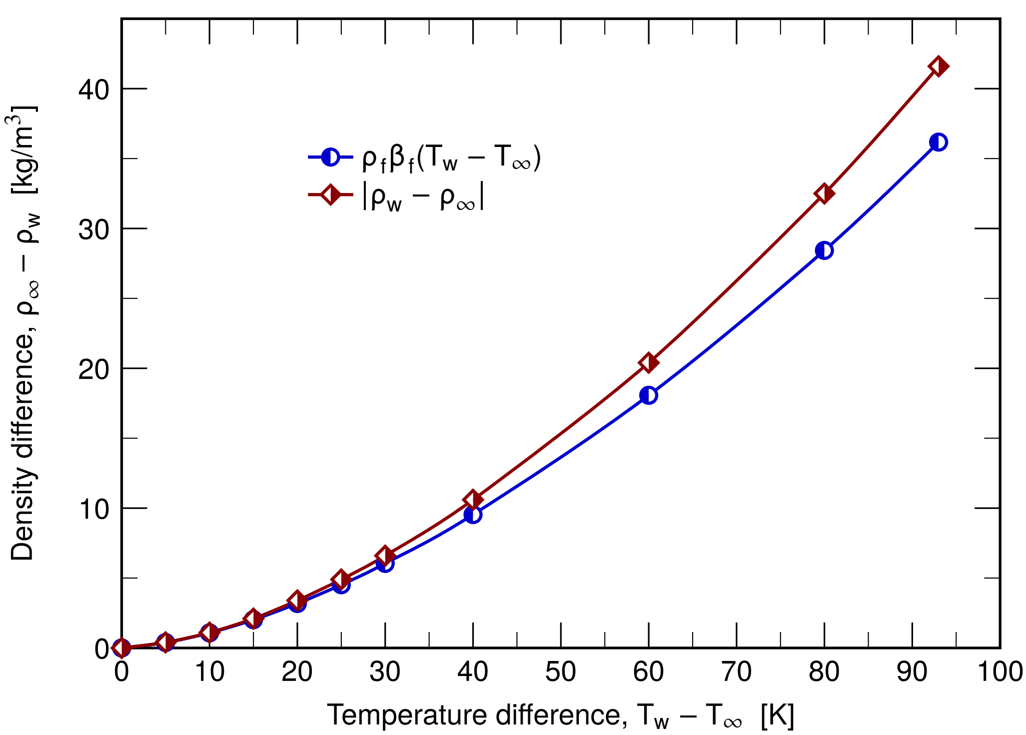


Problem 8.15 In eqn. (8.7), we linearized the temperature dependence of the density difference. Suppose that a wall at temperature T_w sits in water at $T_\infty = 7$ °C. Use the data in Table A.3 to plot $|\rho_w - \rho_\infty|$ and $|-\rho_f \beta_f (T_w - T_\infty)|$ for 7 °C $\leqslant T_w \leqslant$ 100 °C, where $(..)_f$ is a value at the film temperature. How well does the linearization work?

Solution With values from Table A.3, we may perform the indicated calculations and make the plot. The linearization is accurate to within 10% for temperature differences up to 40 °C, and within 13% over the entire range.

Properties of water from Table A.3

T [°C]	ρ [kg/m^3]	β [K^{-1}]	$(\rho_w - \rho_\infty)$	$-\rho_f \beta_f (T_w - T_\infty)$
7	999.9	0.0000436	0.0	0.000
12	999.5	0.000112	−0.4	−0.389
17	998.8	0.000172	−1.1	−1.08
22	997.8	0.000226	−2.1	−2.02
27	996.5	0.000275	−3.4	−3.18
32	995.0	0.000319	−4.9	−4.52
37	993.3	0.000361	−6.6	−6.05
47	989.3	0.000436	−10.6	−9.54
67	979.5	0.000565	−20.4	−18.1
87	967.4	0.000679	−32.5	−28.4
100	958.3	0.000751	−41.6	−36.2



8.16 A 77°C vertical wall heats air at 27°C. Find Ra_L, δ_{top}/L, and L, where the line in Fig. 8.3 ceases to be straight. Comment on the implications of your results.

The line in Fig. 8.3 begins to deviate from straightness, and flatten out, when:

$$Ra_L\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-1.7778} \simeq 10^3$$

But for $\overline{T} = 325°K$, $Pr = 0.7085$, so $\underline{\underline{Ra_L \simeq 2884}}$

(This result could reasonably range from 10^3 to 10^4.)

but

$$Ra_L = 2884 = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\left(\frac{1}{300}\right)50}{1.814(2.561)10^{-10}}L^3$$

$$\underline{\underline{L = 0.00936\,m = 0.936\,cm}}$$

Find $\frac{\delta_{top}}{L}$:

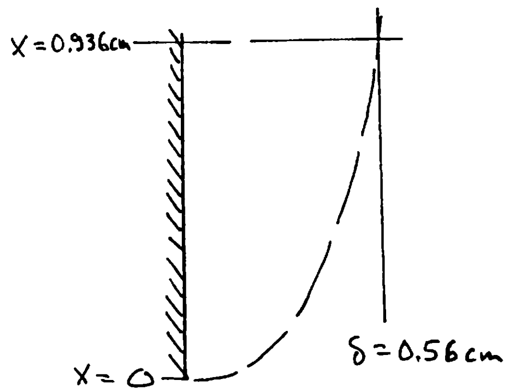
$$Nu_x\Big|_{x=L} = 2\frac{L}{\delta} = \frac{3}{4}\overline{Nu}_L$$

$$\frac{\delta_{top}}{L} = \frac{8}{3\overline{Nu}_L}$$

but $$\overline{Nu}_L = 0.68 + 0.67\frac{Ra_L^{1/4}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} = 0.68 + 0.67\left(10^3\right)^{1/4} = \underline{4.45}$$

8.16 (continued)

Thus;



$$\frac{\delta_{top}}{L} = \frac{8}{3(4.95)} = \underline{\underline{0.60}}$$

The b.l. looks something like this -- quite thick. Thus the deviation from the linear relationship reflects the breakdown of the b.l. assumptions.

8.17 A horizontal 0.08 m diameter pipe, at 150°C on the inside, has 85 % magnesia insulation with a 0.11 O.D. What is the heat loss if T_∞ = 17°C?

First we have to guess the outside temperature to evaluate properties. h on the outside should be low -- around 6 W/m^2-°C so we go to equation (2.25) and calculate

$$Q = \frac{\Delta T}{\frac{1}{2\pi \bar{h} r_o} + \frac{\ln r_o/r_i}{2\pi \underbrace{k}_{\text{for } 80\% \text{ mag.}}}} = \frac{150-17}{\frac{1}{2\pi(6)(0.055)} + \frac{\ln(0.055/0.04)}{2\pi(0.071)}} = 111 \frac{W}{m}$$

Therefore ΔT across $\bar{h} = \frac{Q}{\bar{h}(\pi D)} = \frac{111}{6(\pi)(0.11)} = 53.5$, $T_{\text{outside mag.}} = 70.5$ °C

we then evaluate properties at $\frac{70.5+17}{2} = 44$ °C or 317 °K

Then:

$$\frac{Ra_L^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = \frac{\left[\frac{9.8 \frac{1}{273+17}(70.5-17)(0.11)^3}{1.735(2.447)\,10^{-10}}\right]^{1/4}}{\left[1+\left(\frac{0.559}{0.710}\right)^{9/16}\right]^{4/9}} = 36.9$$

$$\overline{Nu}_D = 0.36 + 0.518(36.9) = 19.5 \quad , \quad \bar{h} = 19.5 \frac{0.02614}{0.11} = 4.63 \frac{W}{m^2\text{-}°C}$$

This gives an outside temp. of $\frac{99.3}{4.63\,\pi(0.11)} = 62$°C. It should suffice to correct the Rayleigh no. by a factor of $\left(\frac{62.0-17}{70.5-17}\right)^{1/4}$ or 0.958. Then $\bar{h}$ will become $\underline{4.44 \text{ W/m}^2\text{-}°\text{C}}$ which we shall use:

$$Q = \frac{150-17}{\frac{1}{4.44(2\pi)(0.055)} + \frac{\ln(0.055)/(0.04)}{2\pi(0.071)}} = \frac{133}{0.652+0.714} = \underline{\underline{97.3 \frac{W}{m}}}$$

8.18 How much heat is needed to keep a horizontal wire, 10^{-5} m in diam., at 40°C in 10°C water.

eqn. (8.28):
$$\overline{Nu}_D = 0.36 + \frac{0.518}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}}\Bigg[\underbrace{\frac{g\beta\Delta T D^3}{\nu\alpha}}_{\frac{9.8(0.00021)(30)10^{-15}}{1.454(0.885)\times10^{-13}}}\Bigg]^{1/4}$$

Pr: 6.11

where we evaluate properties at 298°K

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.6057}{0.00001}\,0.4337 = 26{,}269\ \frac{W}{m^2\text{-}°C}$$

So
$$Q = \bar{h}\pi D\Delta T = 26{,}269(\pi)10^{-5}(30) = \underline{\underline{24.8\ \frac{W}{m}}}$$

8.19 A 20 cm vertical run of 0.5 cm diam. tubing carries condensing vapor at 60°C. The air outside is at 27°C. What is the heat loss?

Neglect the resistance of the tubing and of $h_{cond.}$. They aren't specified, and they would have to be small.

First treat the tube like a vertical wall, π(0.005 m) wide and 0.2 m high. From equation (8.13a), using $\bar{T}$ = 316.5°K:

$$\overline{Nu}_L = 0.68 + 0.67\left[\frac{g\beta\Delta T L^3}{\nu\alpha}\right]^{1/4}\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-4/9}$$

$$= 0.68 + 0.67\left[\frac{9.8\left(\frac{1}{300}\right)33(0.2)^3}{1.402(2.44)\times10^{-10}}\right]^{1/4}\left[1+\left(\frac{0.492}{0.709}\right)^{0.5625}\right]^{-.444} = 37.11$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.02731}{0.2}\,37.11 = \underline{5.06\ \frac{W}{m^2\text{-}°C}}$$

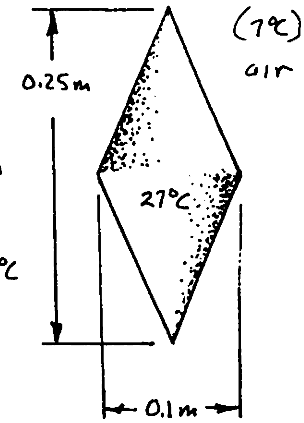
Now we must correct $\bar{h}$ for curvature using Fig. (8.7)

$$\frac{2\sqrt{2}}{(Ra_L/Pr)^{1/4}}\frac{L}{R} = \frac{2\sqrt{2}(0.2)}{77.22(0.0025)} = 2.93\text{ , so }\frac{\bar{h}_{actual}}{\bar{h}_{cyl.}} = 1.8$$

The actual value of $\bar{h}$ is $1.8(5.06) = \underline{9.11\ \frac{W}{m^2\text{-}°C}}$ and

the heat loss is: $\bar{h}A\Delta T = 9.11\left[\pi(0.005)(0.2)\right](33) = \underline{\underline{0.944\,W}}$

8.20 How much heat is removed from the body shown.



$$\tau = \frac{0.25\,m}{\cos(\tan^{-1}\frac{10}{25})} = 0.2693\,m$$

evaluate properties at 17°C

$$\overline{Nu}_\tau = 0.52\, Ra_\tau^{1/4}$$

$$= 0.52\left[\frac{9.8\frac{1}{290}20\,(0.2693)^3}{1.477(2.072)\cdot 10^{-10}}\right]^{1/4}$$

$$= 42.5$$

so: $\bar{h} = 42.5\,\dfrac{0.02536}{0.2693} = \underline{4.00\,\dfrac{W}{m^2\text{-}°C}}$

The area of the cone is $2\times(\frac{1}{2}$ perimeter of base)(lateral edge)

$$= \frac{0.2693}{2}\pi(0.1) = 0.0423\ m^2$$

so $\qquad Q = \bar{h} A \Delta T = 4(0.0423)(20) = \underline{\underline{3.38\,W}}$

PROBLEM 8.22 You are asked to design a vertical wall panel heater, 1.5 m high, for a dwelling. What should the heat flux be if no part of the wall should exceed 33 °C? How much heat goes to the room if the panel is 7 m wide with $\varepsilon = 0.7$? *Hint*: Natural convection removes only about 200 W depending on what room temperature you assume.

Assume $T_\infty = 23°C$ (73.4°F) as a maximum value. Then $\Delta T_{max} = 10°C$.
From Fig. 8.9, $\Delta T_{max} = 10°C$ gives $\overline{\Delta T} = 0.833 \times 10 = \underline{8.33°C}$.

To get q_w we must now solve eqn. (8.43a) by trial and error:

$$\left(\frac{q_w L}{k\overline{\Delta T}}\right)^{5/4} - 0.68\left(\frac{q_w L}{k\overline{\Delta T}}\right)^{1/4} = \frac{0.67\left[\dfrac{g\beta q_w L^4}{k\nu\alpha}\right]^{1/4}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

Evaluate properties at 28°C ≃ 300°K so:

$$\left(\frac{q_w\, 1.5}{0.02614(8.33)}\right)^{1.25} - 0.68\left(\frac{q_w\, 1.5}{0.02614(8.33)}\right)^{0.25} = \frac{0.67\left[\dfrac{9.8\frac{1}{296}1.5^4 q_w}{0.02614(1.566)(2.203)}10^{10}\right]^{1/4}}{\left\{1+\left(\dfrac{0.492}{0.711}\right)^{0.5625}\right\}^{0.444\ldots}}$$

or

$$11.16\, q_w^{1.25} - 1.102\, q_w^{0.25} = 190\, q_w^{0.25}$$

or

$$11.16\, q_w = 191.1 \qquad \text{so} \qquad \underline{q_w = 17.12\ \frac{W}{m^2}}$$

(This corresponds with $\bar{h} = \dfrac{17.12}{8.33} = 2.05\ \dfrac{W}{m^2\text{-}°C}$.)

so
$$Q = q_w A = 17.12(1.5)(7) = \underline{\underline{180\ W}}$$

Since the wall temperature is known, the radiation loss can be computed separately because T_m does not change much along the length of the wall:

$$T_m = (T_\infty + \overline{T}_{\text{wall}})/2 = (23 + 23 + 8.33)/2 + 273.15 = 300.3\ \text{K}$$

and then

$$q_{\text{rad}} = h_{\text{rad}} T_m^3 (\overline{T}_{\text{wall}} - T_\infty) = 4(0.7)(5.670\times10^{-8})(300.3)^3(8.33) = 35.8\ \text{W/m}^2$$

Thus, radiation carries an additional $(35.8)(1.5)(7) = 376$ W, for a total panel heating power of $376 + 180 = 556$ W.

8.23 A 0.14 cm high wall is heated by condensation of steam at one atm. What will happen to $\bar{h}$ and $\bar{Q}$ if the steam is replaced with an organic vapor?

$$\frac{\bar{h}_{organic}}{\bar{h}_{steam}} = \frac{k_o}{k_s}\left[\frac{\dfrac{\rho_o}{\rho_s}\left(\dfrac{\rho_o-\rho_{g_o}}{\rho_s-\rho_{g_s}}\right)\dfrac{h_{fg_o}}{h_{fg_s}}\ \dfrac{\left[1+\frac{c_p(T_s-T_w)}{h_{fg}}\right]_o}{\left[1+\frac{c_p(T_s-T_w)}{h_{fg}}\right]_s}}{\dfrac{\mu_o}{\mu_s}\dfrac{k_o}{k_s}\dfrac{T_{s_o}-T_w}{T_{s_s}-T_w}}\right]^{1/4}$$

We can probably neglect ρ_g at 1 atm and the $c_p(T_s-T_w)/h_{fg}$ terms will contribute little. Thus:

$$\underline{\underline{\frac{\bar{h}_o}{\bar{h}_s} \simeq \left(\frac{k_o}{k_s}\right)^{3/4}\left[\left(\frac{T_{s_s}-T_w}{T_{s_o}-T_w}\right)\frac{\nu_s}{\nu_o}\frac{\rho_o}{\rho_s}\frac{h_{fg_o}}{h_{fg_s}}\right]^{1/4}}}$$

8.23 (continued) Finally, since $Q = hA(T_s - T_w)$:

$$\underline{\underline{\frac{Q_o}{Q_s} \simeq \left(\frac{k_o}{k_s}\,\frac{T_{s_o}-T_w}{T_{s_s}-T_w}\right)^{3/4}\left(\frac{\nu_s}{\nu_o}\,\frac{\rho_o}{\rho_s}\,\frac{h_{fg\,o}}{h_{fg\,s}}\right)^{1/4}}}$$

These expressions give the factors by which $\bar{h}$ and Q will change, once the instructor specifies the particular fluid. The student should remember that T_{sat}, as well as the other thermal properties, will change when the fluid is changed.

8.24 A 0.01 m diam. tube, 0.27 m long, runs horizontally through saturated steam. Plot Q vs. T_{tube} for $50 < T_{tube} < 150°C$.

For natural convection, evaluating properties at $\frac{125+100}{2} = 113°C = 386$ K

$$Ra_D = \frac{9.8(0.0029)\,0.01^3}{(2.295)(2.201)10^{10}}\Delta T = 56.3\,\Delta T$$

so:
$$\overline{Nu}_D = 0.36 + \frac{0.518\,Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = \underline{0.36 + 1.12\Delta T^{1/4}}$$

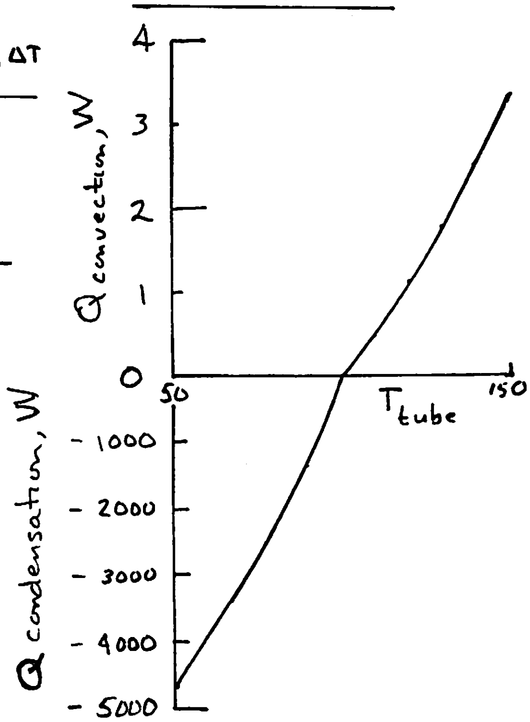
$\left[\ldots\right]^{4/9} = 1.02$

And for film condensation, $h'_{fg} = 2.257\times10^6\left[1+\left(0.683-\frac{0.228}{1.75}\right)Ja\right] = 2.257\times10^6(1-0.001\Delta T)$,
so we can use an average value (for $\Delta T = 25°C$) of 2,313,000

$$\overline{Nu}_D = 0.728\left[\frac{(\rho_f)\Delta\rho g\, h'_{fg} D^3}{\mu k\,(T_{sat}-T_w)}\right]^{1/4} = 0.728\left[\frac{958(9.8)(2.313)}{2.77\,(0.684)10^{-7}}\right]^{1/4}\frac{1}{\Delta T^{1/4}}$$

$$\underline{\overline{Nu}_D = \frac{426}{\Delta T^{1/4}}}$$

ΔT °C	$\overline{Nu}_D$: $0.36+1.12\Delta T^{1/4}$	$\overline{Nu}_D$: $\frac{426}{\Delta T^{1/4}}$		$Q = \frac{k}{D}\overline{Nu}_D A\Delta T = k_{water\ or\ steam}\,\pi(0.27)\overline{Nu}_D\Delta T$
50	3.34	—	$Q = 0.02\overline{Nu}_D\Delta T$	3.28 W
40	3.19	—		2.50
30	2.99	—		1.76
20	2.72	—		1.075
10	2.37	—		0.463
−10	—	240	$Q = 0.58\overline{Nu}_D\Delta T$	−1389 W
−20	—	201		−2337
−30	—	182		−3167
−40	—	169		−3930
−50	—	160		−4646



Notice that:

$Q_{cond.} \gg Q_{conv.}$

$Q_{cond.} \sim \Delta T^{3/4}$

$Q_{conv.} \sim \Delta T^{5/4}$

8.25 A plate, 2m high, condenses steam at 1 atm. Calculate ΔT at which: a) Nusselt's solution loses accuracy; b) The film becomes turbulent.

$$\Gamma_c = \frac{\rho_f(\rho_f-\rho_g)g\delta^3}{3\mu^2} \simeq \frac{g\delta^3}{3\nu^2} = 6 \text{ for (a) and } 450 \text{ for (b)}$$

but $\nu = 0.29 \times 10^{-6}$ so $\delta = \left(\frac{3\Gamma_c \nu^2}{g}\right)^{1/3} = 0.00008106$ m (a)

$= 0.000342$ m (b)

Now:

$$\delta = \left[\frac{4k\Delta T\mu L}{\rho_f(\rho_f-\rho_g)g\, h'_{fg}}\right]^{1/4} \simeq \left[\frac{4k\Delta T\nu L}{\rho_f g\, h_{fg}}\right]^{1/4}$$

We'll go back & use h'_{fg} if ΔT is large.

$$= \left[\frac{4(0.6811)(0.29)10^{-6}}{958(9.8)2257(10)^3}\right]^{1/4} \Delta T^{1/4} = 0.0000782\Delta T^{1/4}$$

(a) $0.00008106 = 0.0000782\Delta T^{1/4}$; $\Delta T = 1.15°C$

when Nusselt's solution losses accuracy.

(b) $0.000342\text{m} = 0.0000782\Delta T^{1/4}$; $\Delta T = 366°C$

The flow can never become turbulent. It will freeze first.

8.26 A reflux condenser has $\alpha = 18°$, d = 0.8, D = 6 cm. At 30°C it condenses steam at 1 atm. What is $\bar{h}$? (Evaluate properties at 65°C)

To use Fig. 8.14, compute: $B = \frac{\rho_f-\rho_g}{\rho_f}\frac{c_p\Delta T}{h_f}\frac{\tan^2\alpha}{Pr} = \frac{979.4-0.6}{979.4}$

$$= \frac{4186(70)\tan^2 18}{2,257,000(1.72)} = 0.008$$

$$d/D = 0.8/6 = 0.1333$$

So: $Nu_L = \left[\frac{(\rho_f-\rho_g)g\, h'_{fg}(d\cos\alpha)^3}{\nu k\Delta T}\right]^{1/4} \times 0.727$;

but $h'_{fg} = h_{fg}\left(1 + \left[0.683 + \frac{0.228}{Pr}\right]Ja\right) = h_{fg}\left(1+\left[0.683+\frac{0.228}{2.65}\right]Ja\right) = (1+0.77Ja)h_{fg}$

$$Nu_L = 0.727\left[\frac{978.8(9.8)2.257(10)^6\left(1+0.77\frac{4186(100-30)}{2,257,000}\right)(0.008\cos 18°)^3}{0.435(10)^{-6}(0.6585)(100-30)}\right]^{1/4}$$

$= 110$

Then: $\bar{h} = \frac{k}{d\cos\alpha}\overline{Nu}_L = \frac{0.6585(110)}{0.008\cos 18°} = 9512 \frac{W}{m^2\text{-}°C}$

8.27 A 0.05m helix of 0.005 m diam. tubing carries 15°C water through saturated steam at 1 atm. Specify α and the number of coils if 6 kg/hr of steam are to be condensed. $\bar{h}_{inside} = 600 W/m^2\text{-°C}$.

First establish an approximate T_{wall} assuming an ordinate from Fig. 8.14.

$$h_{cond}(T_{sat}-T_w) = \frac{k}{d\cos\alpha}\underbrace{0.729}_{\text{assumed}}\left[\frac{(\rho_f-\rho_g)h'_{fg}\,g(d\cos\alpha)^3}{\nu k(T_{sat}-T_w)}\right]^{1/4}(T_{sat}-T_w)$$

$$= \bar{h}_i(T_w-T_i)$$

For openers, call $h'_{fg} \simeq h_{fg}$ and $\cos\alpha \simeq 1$. Then:

$$\frac{0.6811}{0.005}(\cos\alpha)^{-1/4}\left[\frac{958(2{,}257{,}000)9.8(0.005)^3}{0.29\times10^{-6}(0.6811)}\right]^{1/4}(100-T_w)^{3/4} = 600(T_w-15)$$

$$77.23(100-T_w)^{3/4} = T_w-15\ ;\ \underline{T_w \simeq 98.88°C}$$

Now do an accurate computation based on this estimate. Base properties on $T_w \simeq 100°C$:

$$B = \frac{958.3-0.6}{958}\ \frac{4219(18)}{2{,}257{,}000}\ \frac{\tan^2\alpha}{1.72} = 0.002\tan^2\alpha \simeq 0, \text{and } \frac{d}{D} = 0.1$$

Then from Fig. 9.14 the lead const. = 0.727

So: $$\frac{0.6811}{0.005}\left[\frac{958(2{,}257{,}000)9.8(0.005)^3}{0.29(10)^{-6}(0.6811)}1.16^3\right]^{1/4}\cos^{-1/4}\alpha$$

$$= 600(83.88) \quad \text{(A)}$$

$$\cos^{-1/4}\alpha = 0.9997, \quad \text{not possible}$$

Pick $\underline{T_w = 1.07°C}$. Then:

$$\cos^{-1/4}\alpha = 0.9997\ \frac{83.93}{83.88}\left(\frac{1.116}{1.07}\right)^{3/4}, \ \underline{\underline{\alpha = 28°}}$$

So: $$\bar{h} = [\text{LHS of (A)}]\left(\frac{1.07}{1.116}\right)^{3/4} = 47{,}625\ \frac{W}{m^2\text{-°C}}$$

Then
$$\dot{m} = \frac{Q}{h_{fg}} = \bar{h}\ \frac{\text{length}(\pi d)\Delta T}{h_{fg}} = \frac{6}{3600}\ \frac{kg}{s}, \text{so } \underline{\text{length} = 0.783\ m}$$

Finally:
$$\text{no. of coils} = \frac{\text{length}}{\pi D/\cos\alpha} = \frac{0.783\cos 28°}{\pi(0.05)} = \underline{\underline{4.4}}$$

8.29 What is the maximum speed of air in the natural convection b.l., in Example 8.3?

first find where u maximizes in y/δ using eqn. (8.18)

$$\frac{d(u/u_c)}{d(y/\delta)} = 0 = 1 - 4\frac{y}{\delta} + 3\left(\frac{y}{\delta}\right)^2 \quad \text{or} \quad \left(\frac{y}{\delta}\right)^2 - \frac{4}{3}\left(\frac{y}{\delta}\right) + \frac{1}{3} = 0$$

Thus:

$$\frac{y}{\delta} = \frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{3}{9}} = 1 \text{ or } \frac{1}{3}\,; \qquad \frac{y}{\delta} = \frac{1}{3} \text{ gives } u_{max}.$$

Now using: $u_c(x) = C_1 \frac{\beta g \Delta T}{\nu} \delta^2$ and $C_1 = Pr/3\left(\frac{20}{21} + Pr\right)$

we get:

$$u_{max} = \frac{Pr}{3\left(\frac{20}{21} + Pr\right)} \frac{\beta g \Delta T}{\nu} \delta^2 \left[\frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)^2\right]_{\frac{y}{\delta} = \frac{1}{3}}$$

Using numbers from Example 8.3 we obtain:

$$u_{max} = \frac{0.711}{3(0.952 + 0.711)} \frac{0.00348(9.8)(40-14)}{1.566 \times 10^{-5}} (0.0172)^2 \left[\frac{1}{3}\left(\frac{2}{3}\right)^2\right]$$

$$\underline{\underline{= 0.354 \text{ m/s}}} \longleftarrow$$

8.31 A large industrial process requires that water be heated by a large cylindrical heater using natural convection. The water is at 27°C. The cylinder is 5 m in diameter and it is kept at 67°C. First find $\bar{h}$. Then suppose D is doubled (D=10 m). What is the new $\bar{h}$? Explain the similarity of these answers in the turbulent natural convection regime.

at 47°C:

$$Pr = 3.67 \quad \text{and} \quad Ra_L = \frac{9.8(0.000435)(67-27)5^3}{0.566(1.541)\times 10^{-6-7}} = 2.44\times 10^{14}$$

So we use equation (8.29) and obtain:

$$\overline{Nu_L} = \left[0.60 + 0.387\underbrace{\left[\frac{2.44(10)^{14}}{\left[1+\left(\frac{0.559}{3.67}\right)^{9/16}\right]^{16/9}}\right]^{1/6}}_{1.437\times 10^4}\right]^2 = 7951$$

$$\bar{h} = 7951\frac{k}{L} = 7951\frac{0.6367}{5} = \underline{\underline{1012\ W/m^2\text{-}°C}} \longleftarrow L=5m$$

If L is doubled we have

$$\overline{Nu_L} = \left[0.60 + 0.387(1.437\times 10^4 \times 2^3)^{1/6}\right]^2 = 15{,}840$$

$$h = 15{,}840\frac{0.6367}{10} = \underline{\underline{1009\ W/m^2\text{-}°C}} \longleftarrow L=10m$$

almost no change

We note that at high Ra, eqn. (8.29) reduces to:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \frac{0.387^2}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{16/9}} Ra_L^{1/3}$$

so

$$\bar{h} = fn(Pr)\times\left(\frac{g\beta\Delta T\, k}{\nu/\rho c_p}\right)^{1/3} \neq fn(L)$$

The 1/3 power dependence of $\overline{Nu_L}$ on Ra_L that occurs in <u>turbulent</u> natural convection causes $\bar{h}$ to be <u>independent</u> of length in this regime!

8.32 A vertical jet of liquid, of diameter, d, and moving at velocity, u_∞, impinges on a horizontal disc rotating ω rad/s. There is no heat transfer in the system. Develop an expression for $\delta(r)$, where r is the radial coordinate on the disc. Contrast the r dependence of δ with that of a condensing film on a rotating disc and explain the difference qualitatively.

Nusselt's expression for the mass flow rate in the film is valid:

$$\dot{m}\ \frac{kg}{m} = \frac{\rho_f(\rho_f-\rho_g)}{3\mu}\, g\, \delta^3(r)$$

However, in this case, ρ_f is the liquid density and ρ_g is the air density, ρ_{air}. The "gravity" is now $\omega^2 r$. Then the total mass flow is:

$$\dot{M} = \rho_f u_\infty \frac{\pi}{4} d^2 = \dot{m}(2\pi r) = \frac{2\pi}{3}\frac{\rho_f(\rho_f-\rho_{air})}{\mu}\omega^2 r^2 \delta^3(r)$$

we solve this for $\delta(r)$:

$$\delta(r) = \sqrt[3]{\frac{3}{8}\frac{\mu u_\infty d^2}{(\rho_f-\rho_{air})\omega^2}}\ \frac{1}{r^{2/3}}$$

The film thickness is uniform during condensation on a rotating disc (see discussion following eqn. (8.70).) because condensation causes the film to accrue liquid. Thus $\dot{m}$ increases as r^2, and this accretion just compensates the natural thinning that must occur as the sheet spreads.

But in this case, $\delta \sim r^{-2/3}$ because no fluid is added as the film spreads out.

8.33 We have seen that, if properties are constant, $h \sim \Delta T^{1/4}$ in natural convection. If we consider the variation of properties as T_w is increased over T_∞, will h depend more or less strongly on ΔT in air? -- in water?

We see that h in natural convection varies as $k/(\nu\alpha)^{1/4}$. We then find that this quantity increases strongly in water -- especially at lower values of T_∞ -- so h depends more strongly than as $\Delta T^{1/4}$ on ΔT. In the case of air $k/(\nu\alpha)^{1/4}$ is a constant within ± 13 % over the entire range of properties given in the book. It drops off only slightly with increasing temperature so the dependency of h on ΔT is only a little less strong than $\Delta T^{1/4}$. If T_w were less than T_∞, these trends would be reversed.

8.34 A film of liquid falls along a vertical plate. It is initially saturated and it is surrounded by saturated vapor. The film thickness at the top is δ_o. If wall temperature, T_w, is slightly above T_{sat}, derive expressions for $\delta(x)$, Nu_x, and x_f--the distance at which the plate becomes dry. Calculate x_f if the fluid is water at 1 atm., if T_w = 105°C, and δ_o = 0.1 mm.

Equation (8.54) still applies, but the sign is reversed, thus:

$$k\frac{T_w - T_{sat}}{\delta} = -h_{fg}\frac{d\dot{m}}{dx} = -\frac{h_{fg}(\rho_f - \rho_g)}{\nu}g\,\delta^2\frac{d\delta}{dx}$$

so:

$$\frac{4k\Delta T\nu}{h_{fg}'(\rho_f - \rho_g)g} = -\frac{d\delta^4}{dx}$$

Integrating from $\delta(x=0) = \delta_o$ to $\delta(x)$ we get:

$$\frac{4k\Delta T\nu x}{h_{fg}'(\rho_f - \rho_g)g} = \delta_o^4 - \delta^4(x) \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\delta_o^4 - \frac{4k\Delta T\nu x}{h_{fg}'(\rho_f - \rho_g)g}\right]^{1/4}}}$$

Then:

$$\underline{\underline{Nu_x = \frac{x}{\delta(x)} = \left[\left(\frac{\delta_o}{x}\right)^4 - \frac{4k\Delta T\nu}{h_{fg}'(\rho_f - \rho_g)gx^3}\right]^{-1/4}}}$$

and x_f is the value of x at which $\delta(x) = 0$

$$\underline{\underline{x_f = \frac{g(\rho_f - \rho_g)h_{fg}'\delta_o^4}{4k\nu\Delta T}}}$$

For the specified case we set $h_{fg} \simeq h_{fg}'$ and get

$$x_f = \frac{9.8(957.2 - 0.6)(2257{,}000)(0.0001)^4}{4(0.6811)(5)(0.290)10^{-6}}$$

the plate will dry out when $\underline{\underline{x_f = 0.5356\text{ m}}}$

8.35 In a particular solar collector, dyed water runs down a vertical plate in a laminar film, with thickness, δ_o, at the top. The sun's rays pass through parallel glass plates (see Section 11.6) and deposit q_s W/m^2 in the flowing water film. Assume the water to be saturated at the inlet and the plate behind it to be insulated. Develop an expression for $\delta(x)$ as the water evaporates. Develop an expression for the maximum length of wetted plate, and provide a criterion for the laminar solution to be valid.

Equation (8.54) applies to this problem, but we must replace $k(T_w - T_{sat})/\delta$ with $-q_w$. Thus:

$$-q_w = h_{fg}\frac{d\dot{m}}{dx} = \frac{h_{fg}(\rho_f-\rho_g)}{\nu}g\delta^2\frac{d\delta}{dx}$$

So:

$$\frac{3q_w\nu}{g\,h_{fg}(\rho_f-\rho_g)} = -\frac{d\delta^3}{dx}$$

Integrating from $\delta(x=0)=\delta_o$ to $\delta(x)$ we get

$$\frac{3q_w\nu x}{g\,h_{fg}(\rho_f-\rho_g)} = \delta_o^3 - \delta^3_{(x)} \quad \text{or} \quad \underline{\underline{\delta(x) = \left[\delta_o^3 - \frac{3q_w\nu x}{g\,h_{fg}(\rho_f-\rho_g)}\right]^{1/3}}}$$

The film will dry out at $x = x_f$ when $\delta(x_f)=0$.

$$\underline{\underline{x_f = \frac{g\,h_{fg}(\rho_f-\rho_g)}{3q_w\nu}\delta_o^3}}$$

and this solution will only be valid when $\Gamma_c \leqslant 450$. Thus we write at the top of the plate:

$$\Gamma_c = \frac{(\rho_f-\rho_g)g\,\delta_o^3}{3\nu} \leqslant 450\mu \quad \text{or} \quad \underline{\underline{\delta_o \leqslant \sqrt[3]{\frac{1350\nu\mu}{g(\rho_f-\rho_g)}}}}$$

8.36 What heat removal flux can be achieved at the surface of a horizontal 0.01 mm diameter electrical resistance wire in still 27°C air if its melting point is 927°C?

Evaluate β at 27°C and the other properties at $(927+27)/2$, or 477°C = 750°K: $\nu = 7.43(10)^{-5}$, $\alpha = 10.57(10)^{-5}$, $k = 0.054$, $Pr = 0.703$.

$$Ra_D = \frac{9.8(1/300)900(0.00001)^5}{7.43(10.57)10^{-5-5}} = 3.744(10)^{-6}$$

Using equation (8.29) -- applicable for $Ra_D \geq 10^{-6}$ -- we get:

$$\overline{Nu_D} = \left\{0.6 + 0.387\left[\frac{3.744(10)^{-6}}{\left(1+(0.559/0.703)^{0.5625}\right)^{1.778}}\right]^{1/6}\right\}^2 = 0.4096$$

Then: $\bar{h} = \overline{Nu_D}\frac{k}{D} = 0.4096\frac{0.054}{10^{-5}} = \underline{2212\ \frac{W}{m^2\text{-}°C}}$

Thus: $q = 2212(927-27) = \underline{\underline{1,990,700\ W/m^2}}$

This is an incredibly high heat flux. Natural convection, which normally inefficient, becomes remarkably effective when the diameter is very small.

Problem 8.37 A 0.03 m O.D. vertical pipe, 3 m in length with $\varepsilon = 0.7$, carries refrigerant through a 24°C room at low humidity. How much heat does it absorb from the room if the pipe wall is at 10°C?

Evaluate properties at $(10°C + 24°C)\frac{1}{2} = 17°C$. $\nu = 1.477(10)^{-5}$, $\alpha = 2.207(10)^{-5}$, $Pr = 0.713$, $k = 0.0254$

Then $\overline{Ra_L} = \frac{9.8(1/297)(24-10)3^3}{2.207(1.477)10^{-5-5}} = 3.826(10)^{10}$

eqn. (8.27)

$$\overline{Nu_L} = 0.68 + 0.67(3.826\times10^{10})^{1/4}\left[1+\left(\frac{0.492}{0.713}\right)^{0.5625}\right]^{-0.4444} = \underline{228}$$

$$\bar{h}_{flat\,plate} = 228\ \frac{0.0254}{3} = \underline{1.933\ W/m^2\text{-}°C}$$

Correct for curvature using Fig. 8.7: $\frac{2\sqrt{2}}{(Ra_L/Pr)^{1/4}}\frac{L}{R} = \frac{2\sqrt{2}}{\left(\frac{3.826(10)^{10}}{0.713}\right)^{1/4}}\frac{3}{0.015}$

so $\bar{h}_{cyl}/\bar{h}_{plate} = 1.37$ and $= 1.18$

$$\bar{h}_{cyl} = 1.37(1.933) = \underline{2.65 W/m^2\text{-}°C}$$

$$Q = \bar{h}_{cyl}A\Delta T = 2.65(3)(\pi)(0.03)(24-10) = \underline{\underline{10.5W}}$$

But $h_{rad} = 4\varepsilon\sigma T_m^3 = 4(0.7)(5.670\times10^{-8})(17+273)^3 = 3.88\ W/m^2K$

Adding the natural convection and thermal radiation heat transfer coefficients, we can compute Q:

$$Q = (\bar{h}_{cyl} + h_{rad})A\,\Delta T = (2.65+3.88)(3\pi)(0.03)(24-10) = \underline{25.8\ W}$$

8.38 A 1 cm OD tube at 50°C runs horizontally in 20°C air. What is the critical radius of 85% magnesium insulation on the tube?

From eqn. (2.27) we have: $r_{crit} = \dfrac{k_{mag}}{\bar{h}} = \dfrac{k_{mag}}{k_{air}} \dfrac{2r_c}{\overline{Nu_D}}$

& using eqn. (8.28):

$$\overline{Nu_D} = 0.36 + \frac{0.518\, Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{0.5625}\right]^{4/9}} = 2\sqrt{\frac{k_{mag}}{k_{air}}}$$

We'll evaluate properties at 27°C ($T_{wall} = 34$°C) & hope that we won't have to re-iterate.

$$0.36 + \frac{0.518}{\left[1+\left(\frac{0.559}{0.711}\right)^{0.5625}\right]^{4/9}} \left[\frac{9.8\,\frac{1}{293}}{1.566(2.203)} 10^{10}\right]^{1/4} \left[\Delta T (2r_c)^3\right]^{1/4} = 2\sqrt{\frac{0.067}{0.0264}}$$

$$0.36 + 38.89\left([2r_c]^3 \Delta T\right)^{1/4} = 3.202$$

So $\underline{2r_c = 0.03055/\Delta T^{1/3}}$.

Furthermore: $$q = \frac{k_{mag}(30-\Delta T)}{\ln(r_c/0.005)} = \bar{h}\,\Delta T$$

but: $$\bar{h} = \frac{k_{air}}{2r_c}\left(\overline{Nu_D}\right) = \frac{k_{air}}{2r_c}\left[2\sqrt{\frac{k_{mag}}{k_{air}}}\right] = \frac{1}{r_c}\sqrt{k_{mag}k_{air}}$$

so: $$\frac{30}{\Delta T} - 1 = \frac{\ln(r_c/0.005)}{r_c}\sqrt{\frac{k_{air}}{k_{mag}}} = \underline{\frac{30}{(0.03055)^3}(2r_c)^3 = \frac{\ln(r_c/0.005)}{r_c}0.625}$$

so we solve for r_c by trial & error: $\underline{\underline{r_c = 0.01745\text{m}}}$

This gives: $\Delta T = (0.03055/2[0.01745])^3 = \underline{0.67\text{°C}}$

Thus, we evaluated properties at a temperature 6-1/2°C above the right value. Further calculation would not be worth the trouble.

8.40 A horizontal electrical resistance heater, 1 mm in diameter, releases 100 W/m in water at 17°C. What is the wire temperature?

We modify eqn. (8.28), using $Ra_L = Ra^*_D / \overline{Nu}_D$, and get:

$$\overline{Nu}_D^{\,5/4} - 0.36\,\overline{Nu}_D^{\,1/4} = \frac{0.518}{\left[1+\left(\frac{0.559}{Pr}\right)^{0.5625}\right]^{4/9}} \left[\frac{g\beta q_w D^4}{k\nu\alpha}\right]^{1/4}$$

Guess $T_w = 37°C$. Then, at 27°C, $\nu = 0.826\times10^{-5}$, $\alpha = 1.462\times10^{-7}$, $Pr = 5.65$ & $k = 0.6084$. And $q_w = 100/\pi(.001) = 3183\ W/m^2$

Then: $$\overline{Nu}_D^{\,5/4} - 0.36\,\overline{Nu}_D^{\,1/4} = 0.4654\left[\frac{9.8(0.000275)(31830)10^{-12}}{0.826(1.462)10^{-12}\,0.6084}\right]^{1/4} = 1.53$$

By trial & error we get $\overline{Nu}_D = 1.70$, so

$$\bar{h} = 1.70\frac{k}{D} = 1.70\frac{0.6084}{0.001} = \underline{1034\ W/m^2\text{-}°C}$$

Then:

$$Q = \bar{h} A \Delta T, \quad 100 = 1034(\pi[0.001])\Delta T$$

$$\Delta T = 30.78\ °C$$

so $T_w = 17 + 30.78 = \underline{\underline{47.78\ °C}}$

The properties should have been evaluated at 32.4°C instead of at 27°C. This is not enough difference to warrant a recalculation. However, if we did the recalculation we'd get:

$T_w = \underline{\underline{46.64°C}}$

Which is less than 1°C improvement.

8.41 Solve Problem 5.39 using the correct formula for the heat transfer coefficient.

We shall evaluate the properties of water at $(47+27)/2 = 37°C$:

$\nu = 0.696(10)^{-6}$, $\alpha = 1.502(10)^{-7}$, $Pr = 4.66$, $k = 0.6226$, $\beta = 0.000355$

Then: $Ra_D = \dfrac{9.76(0.000355)(0.03)^3 \Delta T}{0.698(1.502)10^{-13}} = 69{,}400\ \Delta T$ so eqn. (8.29)

yields: $\bar{h} = \dfrac{k}{D}\overline{Nu}_D = \left[2.733 + 10.45\,\Delta T^{1/6}\right]^2$

This is exactly the value given in Problem 5.39. Therefore its solution applies here.

8.43 A 0.25 mm diameter platinum wire, 0.2 m long, is to be held horizontally at 1035°C. It is black. How much electric power is needed? Is it legitimate to treat it as a constant wall temperature heater, in calculating the convective part of the heat transfer? The surroundings are at 20°C and the surrounding room is virtually black.

$$Q_{rad} + Q_{conv} = \pi(0.00025)(0.2)\left[\sigma(1308^4 - 293^4) + \frac{k_{air}}{D}\overline{Nu}_D \Delta T\right]$$

evaluate properties at $(1035+20)/2 = 527.5°C \simeq 800°K$.

$\nu = 8.26(10)^{-5}$, $\alpha = 11.73(10)^{-5}$, $k = 0.0569$, $Pr = 0.704$

$$Ra_D = \frac{9.8\left(\frac{1}{800}\right)0.00025^3(1035-20)}{8.26(11.73)10^{-10}} = 0.0200$$

$$\overline{Nu}_D = 0.36 + 0.518(0.02)^{1/4}\Big/\left[1+\left(\frac{0.559}{.704}\right)^{0.5625}\right]^{0.4444} = \underline{0.507}$$

so:

$$Q = \left[165{,}546 + \underbrace{\frac{0.0569}{0.00025}0.507(1015)}_{117{,}124}\right]\pi(0.00025)(0.2) = \underline{\underline{44.4\,W}}$$

$$Bi_{conv} = \frac{\frac{117124}{1035-20}\,0.00025}{84} = 0.000343$$

$$Bi_{total} = \frac{117124+165546}{1035-20}\,0.00025/84 = 0.000829$$

In either case $Bi \ll 1$ so $T_w = $ const. is valid.

8.44 A vertical plate, 11.6 m long, condenses saturated steam at one atmosphere. We want to be sure that the film stays laminar. What is the lowest allowable plate temperature and what is $\overline{q}$ at this temperature?

Let us save work by adapting a result from Example 8.6:

$$\delta_{bottom} = 0.000138\,L^{1/4}\left(\frac{\Delta T}{10}\right)^{1/4} = 0.0001432\,\Delta T^{1/4} = \left(\underbrace{\frac{3\nu\mu}{(\rho_f-\rho_g)g}}_{\text{from eqn. (9.68)}}Re_c\right)^{1/3}$$

Then using $Re_c = 450$, we get:

$$\Delta T = \left[\frac{1}{14.32(10)^{-5}}\left(\frac{3\left[(0.290)10^{-6}\right]^2}{9.8}450\right)^{1/3}\right]^4 = \underline{6.23°C} \qquad T_{w_{lowest}} = \underline{\underline{93.77°C}}$$

$$\overline{q} = \frac{4}{3}\frac{k\Delta T}{\delta} = \frac{4}{3}0.681\frac{6.23^{3/4}}{0.0001432} = \underline{\underline{25{,}004\ W/m^2}}$$

8.45 a) Show that $\Theta_{\xi\xi} = m^2L^2\Theta^{5/4}$ for a straight fin in natural convection.
b) develop an iterative method to solve the equation assuming an insulated tip.
c) Solve the resulting difference equations.

a) With $Nu \sim Gr^{1/4}$, $\bar{h} = \bar{h}_o[(T-T_\infty)/(T_o-T_\infty)]^{5/4}$ where $\bar{h}_o$ is $\bar{h}$ at $T=T_o$. The given equation follows immediately from eqn. (4.30) and the subsequent non-dimensionalisation.

b) Using central differences for the second derivative, we have

$$\Theta_i^{k-1} - \left[2 + m^2L^2\,\delta\xi^2\left(\Theta_i^{k-1}\right)^{1/4}\right]\Theta_i^k + \Theta_i^{k-1} = 0$$

$$\Theta_o^k \equiv 1.0$$

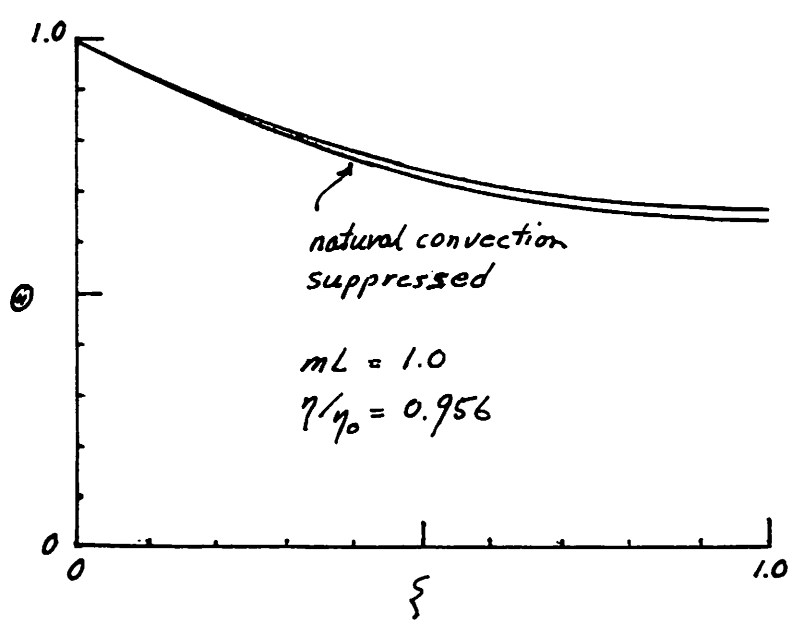
$$2\Theta_{n-1}^k - \left[2 + m^2L^2\,\delta\xi^2\left(\Theta_n^{k-1}\right)^{1/4}\right]\Theta_n^k = 0$$

(To suppress natural convection, we change the exponent $\frac{1}{4}$, to zero)

c) Sample output from a BASIC program that solves these equations is given below. The program itself is on the next page.

```
Fin with natural convection heat exchange
Natural convection suppressed

mL= 1
R= 0
Number of iterations is 2

Step size is X1/L= .02
Efficiency= .761625234908
```

```
Fin with natural convection heat exchange

mL= 1
R= .000218334308
Number of iterations is 5

Step size is X1/L= .02
Efficiency= .728340253762
```



8.45 (continued)

This old solution was carried out on an HP-85 calulator. Today we would certainly use more a more modern means of calculation.

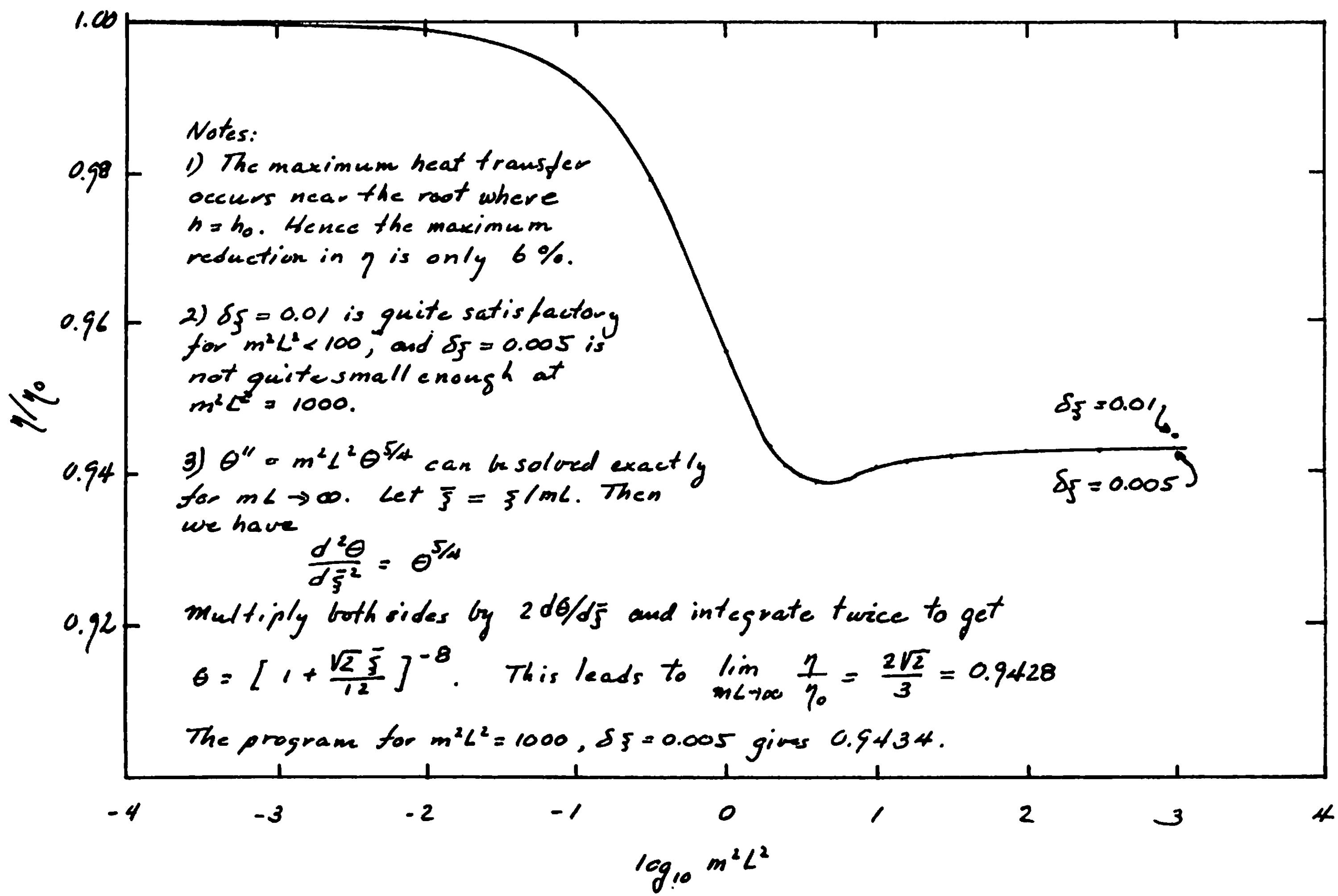
```
10 ! "FIN WITH NATURAL CONVECTION HEAT EXCHANGE"
20 ! T0=temp from previous iteration;T1=temp from current iteration;
30 ! A,B,C,R are coefficients in the difference equations
40 ! D is an intermediate value in the tridiagonal algorithm.
50 DIM T0(200),T1(200),B(200)
60 DIM A(200),C(200),D(200),R(200)
70 ! N is the number of spatial units
80 DISP "ENTER (ml),N"
90 INPUT A1,N
100 ! E1 is the min. sum of ABS(T1(I)-T0(I))
110 E1=.0001*N/10
120 X1=1/N
130 D=A1*X1^2
140 Z=0
150 FOR I=0 TO N
160 A(I)=1 @ B(I)=-(2+D) @ C(I)=1 @ R(I)=0
170 T1(I)=1 @ T0(I)=1
180 NEXT I
190 N1=0
200 A(1)=0 @ C(N)=0 @ A(N)=2
210 R(1)=-1
220 N1=N1+1
230 R(1)=-1
240 FOR I=2 TO N
250 R(I)=0
260 NEXT I
270 GOSUB 400
280 R=0
290 FOR I=1 TO N
300 R=R+ABS(T1(I)-T0(I))
310 ! Z=0 suppresses natural convection
320 T0(I)=T1(I)
330 IF Z=0 THEN GOTO 350
340 B(I)=-(2+A1*X1^2*SQR(SQR(T1(I))))
350 !
360 NEXT I
370 DISP "R=";R
380 IF R<E1 THEN DISP "R=";R @ GOSUB 580
390 GOTO 200
400 ! Tridiagonal algorithm
410 FOR I=0 TO N
420 T1(I)=T0(I)
430 NEXT I
440 N2=N-1
450 D(1)=C(1)/B(1)
460 R(1)=R(1)/B(1)
470 FOR I=2 TO N
480 D(I)=B(I)/A(I)-D(I-1)
490 R(I)=(R(I)/A(I)-R(I-1))/D(I)
500 D(I)=C(I)/A(I)/D(I)
510 NEXT I
520 T1(N)=R(N)
530 FOR I=2 TO N
540 I1=N+1-I
550 T1(I1)=R(I1)-D(I1)*T1(I1+1)
560 NEXT I
570 RETURN
580 PRINT
590 PRINT "Fin with natural convection heat exchange"
600 IF Z=0 THEN PRINT "Natural convection suppressed"
610 PRINT @ PRINT "mL=";A1
620 PRINT "R=";R
630 PRINT "Number of iterations is";N1
640 PRINT
650 PRINT "Step size is X1/L=";X1
660 F=0
670 FOR I=0 TO N
680 IF Z=1 THEN F=F+T1(I)^1.25
690 IF Z=0 THEN F=F+T1(I)
700 NEXT I
710 IF Z=1 THEN F=F-(T1(0)^1.25+T1(N)^1.25)/2
720 IF Z=0 THEN F=F-(T1(0)+T1(N))/2
730 F=F*X1
740 PRINT "Efficiency=";F
750 SCALE 0,1,0,1
760 XAXIS 0,.1
770 YAXIS 0,.1
780 MOVE 0,1
790 FOR I=1 TO N
800 IDRAW X1,T1(I)-T1(I-1)
810 NEXT I
820 GRAPH
830 IF Z=0 THEN Z=1 @ GOTO 150
835 COPY
840 END
```

Note:

The difference equations are written as

$$A_i \Theta_{i-1} + B_i \Theta_i + C_i \Theta_{i+1} = R_i$$

9.45 (continued)



Notes:

1) The maximum heat transfer occurs near the root where $h = h_0$. Hence the maximum reduction in η is only 6 %.

2) $\delta\xi = 0.01$ is quite satisfactory for $m^2L^2 < 100$, and $\delta\xi = 0.005$ is not quite small enough at $m^2L^2 = 1000$.

3) $\theta'' = m^2L^2\theta^{5/4}$ can be solved exactly for $mL \to \infty$. Let $\bar{\xi} = \xi/mL$. Then we have

$$\frac{d^2\theta}{d\bar{\xi}^2} = \theta^{5/4}$$

Multiply both sides by $2\,d\theta/d\bar{\xi}$ and integrate twice to get

$\theta = \left[1 + \frac{\sqrt{2}\,\bar{\xi}}{12}\right]^{-8}$. This leads to $\lim_{mL\to\infty} \frac{\eta}{\eta_0} = \frac{2\sqrt{2}}{3} = 0.9428$

The program for $m^2L^2 = 1000$, $\delta\xi = 0.005$ gives 0.9434.

8.46 Find the temperature of a black sphere in equilibrium with air at 20°C and surroundings at 1000°K.

Equation 8.32 gives

$$\bar{h} = \frac{k}{D}\left[2 + 0.43\, Ra^{1/4}\right]$$

The equation we need to solve is

$$T = T_\infty + \frac{\mathcal{F}\sigma}{\bar{h}}\left(T_s^4 - T^4\right)$$

We first guess that the properties in the expression for $\bar{h}$ can be evaluated at 500°K. This gives

$$\frac{g\beta D^3}{\nu\alpha} = \frac{9.8\,(293)^{-1}\,(2\times10^{-2})^3}{3.798\times10^{-5}\,(5.438\times10^{-5})} = 129.6\,(°C)^{-1}$$

Ra will be $< 10^5$ if $\Delta T < 772$ °C, or $T < 1065$°K.

This will always be the case, so

$$\bar{h} = 3.95 + 2.87\,(T - T_\infty)^{1/4}$$

or

$$T = 293 + \frac{0.56697\times10^{-8}\,(10^{12} - T^4)}{3.95 + 2.87\,(T-293)^{1/4}}$$

The solution to this equation is 601.43°K, so $T_{film} = 447$°K

If the properties are evaluated at 450°K, we have

$$\bar{h} = 3.63 + 3.14\,(T - T_\infty)^{1/4}$$

and $T = 591.35$°K. An additional iteration with $T_{film} = 440$°K gives $T = \underline{\underline{534.54\text{°K}}}$.

Note: The iteration process described in footnote 2 of chapter 6 diverges if the initial guess for T is too close to either 293 or 1000.

Problem 8.53 An inclined plate in a piece of process equipment is tilted 30° above horizontal and is 20 cm long in the inclined plane and 25 cm wide in the horizontal plane. The plate is held at 280 K by a stream of liquid flowing past its bottom side; the liquid is cooled by a refrigeration system capable of removing 12 W. If the heat transfer from the plate to the stream exceeds 12 W, the temperature of both the liquid and the plate will begin to rise. The upper surface of the plate is in contact with ammonia vapor at 300 K and a varying pressure. An engineer suggests that any rise in the bulk temperature of the liquid will signal that the pressure has exceeded a level of about $p_{\text{crit}} = 551$ kPa.

a) Explain why the gas's pressure will affect the heat transfer to the coolant.
b) Suppose that the pressure is 255.3 kPa. What is the heat transfer (in watts) from gas to the plate, if the plate temperature is $T_w = 280$ K? Will the coolant temperature rise?
c) Suppose that the pressure rises to 1062 kPa. What is the heat transfer to the plate if the plate is still at $T_w = 280$ K? Will the coolant temperature rise?

Solution

a) Sufficiently high pressures can cause condensation of the NH_3 vapor on the plate. In addition, before condensation occurs, pressure changes may cause significant properties variations in the NH_3 vapor.
b) At 255.3 kPa, the saturation temperature is $T\text{sat} = 260\text{ K} < 280\text{ K}$; condensation will not occur. Replacing g with an effective gravity $g \cos 60°$, the Rayleigh number is

$$\text{Ra}_L = \frac{g \cos 60° \beta \Delta T L^3}{\nu\alpha} = \frac{9.81 \times (1/2) \times 0.00345 \times 20 \times 0.2^3}{(5.242 \times 10^{-6})(5.690 \times 10^{-6})} \simeq 9.07 \times 10^7$$

The Nusselt number is

$$\begin{aligned}\overline{\text{Nu}}_L &= 0.68 + 0.67\text{Ra}_L^{1/4}\left[1 + \left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{-4/9}\\ &= 0.68 + 0.67 \times (9.07 \times 10^7)^{1/4}\left[1 + \left(\frac{0.492}{0.92}\right)^{9/16}\right]^{-4/9}\\ &\simeq 52.3\end{aligned}$$

Then,

$$h = \overline{\text{Nu}}_L \frac{k}{L} = 52.3 \times \frac{0.0244}{0.2} = 6.38\ \text{W/m}^2\text{K}$$

and the heat transfer is

$$Q = hA(T_\infty - T_w) = 6.38 \times 0.2 \times 0.25 \times (300 - 280) \simeq 6.38\text{ W} < 12\text{ W}$$

and the plate and liquid temperatures will not rise.
c) At a pressure of 1062 kPa, the saturation temperature is $T\text{sat} = 300\text{ K} > 280\text{ K}$; condensation occurs. The Nusselt number is

$$\overline{\text{Nu}}_L = 0.9428\left[\frac{\rho_f(\rho_f - \rho_g) g \cos 60° h'_{fg} L^3}{\mu k (T\text{sat} - T_w)}\right]^{1/4} = 1814$$

The heat transfer coefficient is

$$h = \overline{\mathrm{Nu}}_L \frac{k}{L} = 4353\ \mathrm{W/m^2K}$$

The heat transfer rate is

$$Q = hA(T\mathrm{sat} - T_w) = 4353\ \mathrm{W} \gg 12\ \mathrm{W}$$

and the plate and liquid temperatures will rise.

PROBLEM 8.54 A characteristic length scale for a falling liquid film is $\ell = (\nu^2/g)^{1/3}$. If the Nusselt number for a laminar film condensing on plane wall is written as $\mathrm{Nu}_\ell \equiv h\ell/k$, derive an expression for Nu_ℓ in terms of Re_c. Show that, when $\rho_f \gg \rho_g$, $\mathrm{Nu}_\ell = \left(3\mathrm{Re}_c\right)^{-1/3}$.

SOLUTION Starting with eqns. (8.58) and (8.72), we have

$$\mathrm{Nu}_x = \frac{hx}{k} = \frac{x}{\delta} \tag{8.58}$$

and

$$\mathrm{Re}_c = \frac{\rho_f\left(\rho_f - \rho_g\right) g\delta^3}{3\mu^2} = \frac{\rho_f \Delta\rho\, g\delta^3}{3\mu^2} \tag{8.72}$$

Then, by replacing x by ℓ

$$\mathrm{Nu}_\ell = \frac{h\ell}{k} = \frac{\ell}{\delta}$$

and, by rearranging Re_c,

$$\delta = \left(\frac{3\mu\nu}{g\Delta\rho}\,\mathrm{Re}_c\right)^{1/3}$$

So

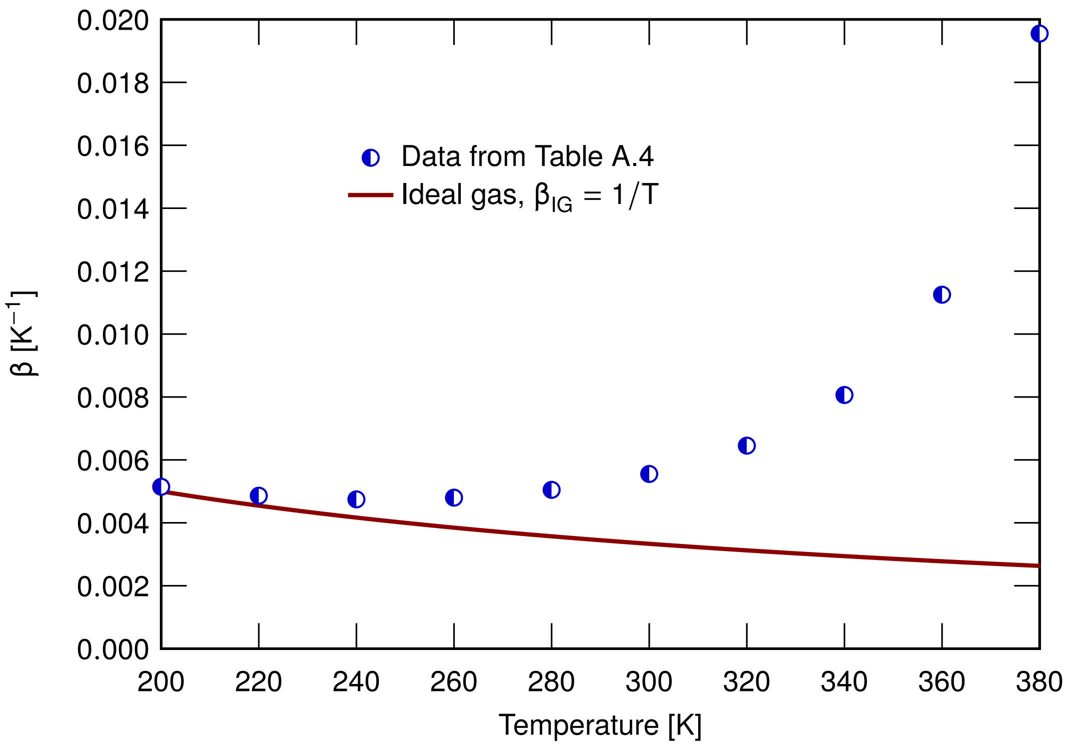
$$\boxed{\mathrm{Nu}_\ell = \left(\frac{\nu^2}{g}\right)^{1/3}\left(\frac{g\Delta\rho}{3\mu\nu}\right)^{1/3}\mathrm{Re}_c^{-1/3} = \left(\frac{\Delta\rho}{3\rho_f}\right)^{1/3}\mathrm{Re}_c^{-1/3}}$$

and when $\rho_f \gg \rho_g$, $\Delta\rho \simeq \rho_f$ so

$$\boxed{\mathrm{Nu}_\ell \simeq \left(3\mathrm{Re}_c\right)^{-1/3}} \quad \text{for } \rho_f \gg \rho_g$$

PROBLEM 8.59 Using data from Tables A.4 and A.5, plot β for saturated ammonia vapor for $200\ \text{K} \leqslant T \leqslant 380\ \text{K}$, together with the ideal gas expression $\beta_{IG} = 1/T$. Also calculate $Z = P/\rho RT$. Is ammonia vapor more like an ideal gas near the triple point or critical point temperature?

SOLUTION

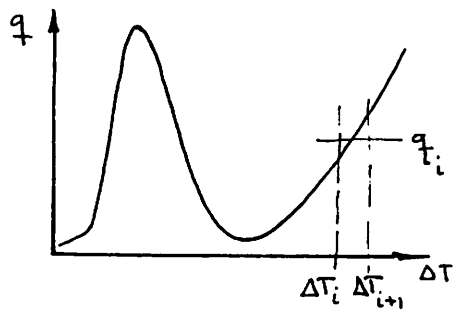


With p and ρ from Table A.5, and using $R = R^\circ/M_{NH_3} = 8314.5/17.031 = 488.2$ J/kg-K, we find Z as below. For an ideal gas, $Z = 1$.

T [°C]	Z	T [°C]	Z
200	0.9944	300	0.8788
220	0.9864	320	0.8263
240	0.9722	340	0.7606
260	0.9505	360	0.6784
280	0.9198	380	0.5716

Saturated ammonia vapor only behaves like an ideal gas for temperatures close the triple point temperature (195.5 K) and is highly non-ideal in the vicinity of the critical point temperature (405.4 K). This behavior underscores the importance of using data for β when dealing with vapors near saturation conditions.

9.1 Water boils, according to the graphical relation in Fig. 9.2, on a 1.27 cm thick copper slab which starts out at 650°C. Plot T_{slab} vs. time, indicating the regime of boiling and noting the temperature at which the cooling is most rapid.



$$Bi = \frac{hL}{k} \simeq \frac{0.0127\,h}{376} < 1 \quad \text{as long as } h \text{ is less than } 29{,}600\ \frac{W}{m^2\text{-}°C}$$

from eqn. (1.19)

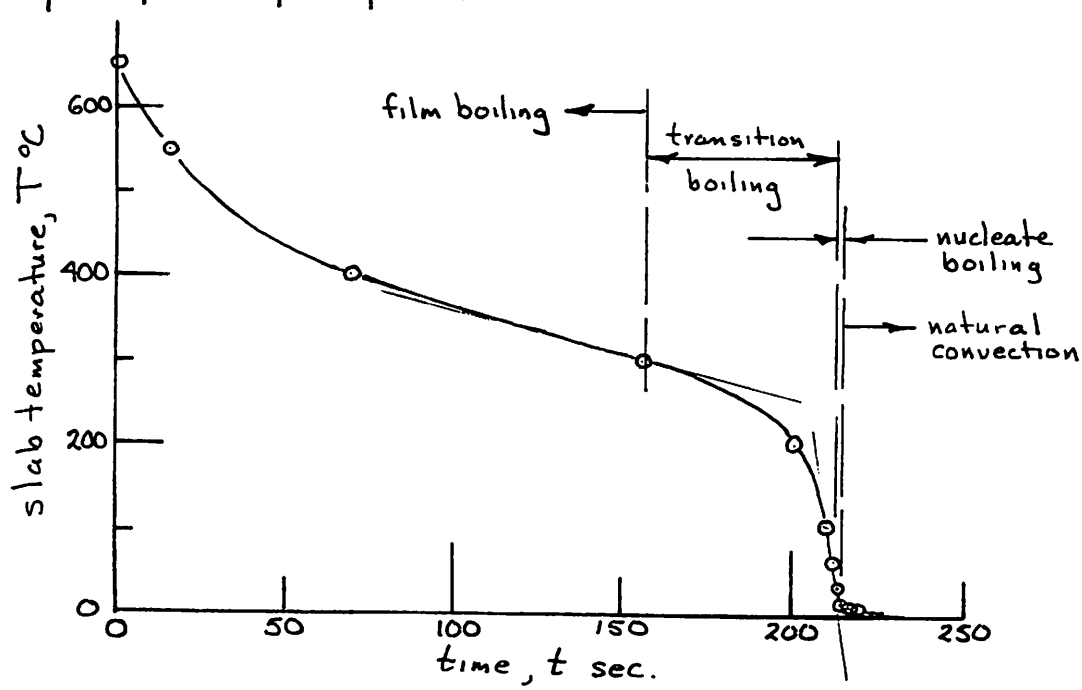
$$\frac{Q}{A} = q = \frac{\delta\left(\frac{\rho c V}{A}[T - T_{sat}]\right)}{\delta t}$$

or

$$\delta t = \rho c L\,\frac{\delta \Delta T}{q} = 43{,}667\,\frac{\delta \Delta T}{q}$$

Now we use this eqn & Fig 9.2 to calculate:

ΔT_{i+1} °C	ΔT_i °C	q_i $\frac{W}{m^2\text{-}°C}$	δt sec	$t_{i+1} = t_i + \delta t$	
550	400	4.5×10^5	14.56	14.56	
400	300	0.8 "	54.59	69.14	
300	200	0.5 "	87.33	156.5	q_{min}
200	150	" "	43.67	200.2	
150	100	2.8 "	7.8	208.	
100	60	7.5 "	2.3	210.3	
60	30	11.4 "	1.15	211.45	q_{max}
30	10	7.8 "	1.12	212.57	
10	4	2.3 "	1.14	213.71	$q_{\text{boiling inception}}$
4	2	0.3 "	2.9	216.62	
2	1	0.15 "	2.9	219.53	



9.2 Predict q_{max} for horizontal cylinders for the cases in Fig. 10.3b and indicate the fraction of q_{max} in each case.

(a) 0.0322 cm diam. in methanol with g = 98 m/s^2
(b) 0.164 cm diam. in benzene with g = 9.8 m/s^2

first find R': $R' = R\sqrt{g(\rho_f-\rho_g)/\sigma}$

	methanol	benzene
T_{sat}	64°C	80°C
σ	18.7 dyne/cm	21.3 dyne/cm
ρ_f	.814 gm/cm^3	.816 gm/cm^3
ρ_g	.00117 "	.00249 "
h_{fg}	1114 kJ/kg	391 kJ/kg

$$R'_{meth.} = \frac{.0322}{2}\sqrt{\frac{9800(0.813)}{18.7}} = \underline{0.332}$$

$$R'_{Benz.} = \frac{0.164}{2}\sqrt{\frac{980(0.814)}{21.3}} = \underline{0.502}$$

$$q_{max_{z_{meth}}} = \frac{\pi}{24}\left(1.17\,\frac{kg}{m^3}\right)^{1/2} 1{,}114{,}000\,\frac{J}{kg}\sqrt[4]{98\,\frac{m}{s^2}\,0.0187\,\frac{kg}{s^2}\left(813\,\frac{kg}{m^3}\right)} = \underline{980{,}000\,\frac{W}{m^2}}$$

$$q_{max_{z_b}} = \frac{\pi}{24}(2.44)^{1/2}\,391000\sqrt[4]{9.8(.0213)(814)} = \underline{288{,}600\,\frac{W}{m^2}}$$

From Fig. 9.13, upper left-hand corner, we read:

$$\left(\frac{q_{max}}{q_{max_z}}\right)_{\substack{meth.\\ R'=.332}} = 1.2 \qquad \text{so} \qquad q_{max_{Meth}} = \underline{\underline{1{,}176{,}000\ W/m^2}}$$

$$\left(\frac{q_{max}}{q_{max_z}}\right)_{\substack{Benz.\\ R'=0.502}} = 1.1 \qquad \text{so} \qquad q_{max_{Benz}} = \underline{\underline{318{,}000\ W/m^2}}$$

Fig. 9.3b shows methanol at 1,040,000 W/m^2 or 88.4 % of q_{max}.
Fig. 9.3b shows benzene at 350,000 W/m^2 or 90.7 % of q_{max}.

9.3 Water at 70°C is depressurized until it is subcooled 30°C. Find the pressure at this point and the diameter of the critical nucleus.

$p_{sat.}$ at 70°C = 31,170 N/m^2, $\underline{\underline{p_{sat}\text{ at }40^{\circ}C = 7375\ N/m^2}}$

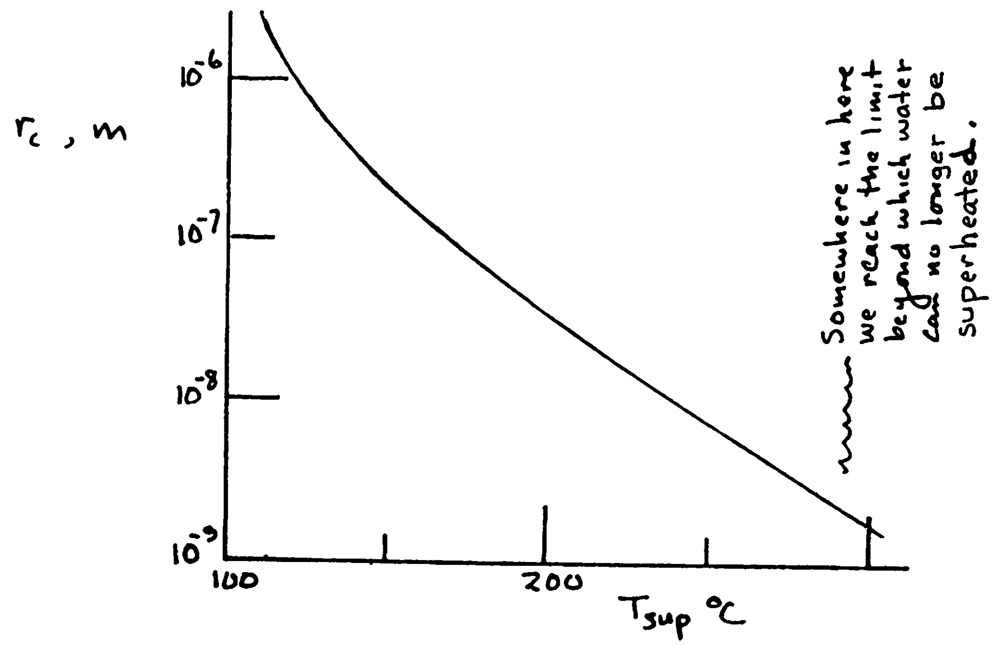
$$r_c = \frac{2\sigma_{at\ 70^{\circ}C}}{31{,}170 - 7375} = 2\,\frac{65.49\,\frac{dyne}{cm}\left(10^{-3}\,\frac{N/m}{dyne/cm}\right)}{23{,}795\ N/m^2}$$

diameter of nucleus = $2r_c$ = $\underline{\underline{1.1(10)^{-6}\ m}}$

9.4 Plot r_c vs. liquid superheat for water at 1 atm.

$$r_c = \frac{\sigma}{p_{sat\,at\,T_{sup}} - 10^5} \quad ; \quad \sigma = 0.2358\left(1 - \frac{T_{sup}}{647.2}\right)^{1.256}\left[1 - 0.625\left(\frac{T_{sup}}{647.2}\right)\right] \frac{N}{m}$$

T_{sup}, °C/°K	ΔT °C = $T_{sup} - 100$	σ $\frac{N}{m}$	$p_{sat\,at\,T_{sup}}$ (N/m²)	r_c m	r mm	r Å
110/383	10	0.0482	1.433×10^5	2.23×10^{-6}	0.00223	
130/403	30	0.0423	2.701 "	0.50 "	0.00050	
160/433	60	0.0342	6.18 "	0.132 "	0.000132	
200/473	100	0.0246	15.55 "	0.0338 "		338
250/523	150	0.0147	39.78 "	7.58×10^{-9}		76
300/573	200	0.00694	85.92 "	1.63×10^{-11}		16



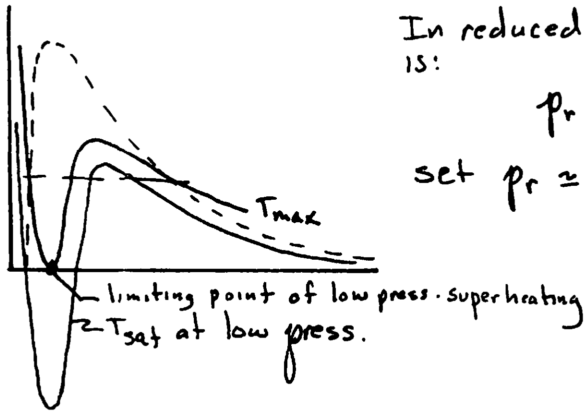
9.5 Why does bumping occur in a test tube, but not in a teakettle?

The test-tube is very smooth so $(r_c)_{test\text{-}tube} \ll (r_c)_{teakettle}$. It follows that, since $r_c = 2\sigma/(p_{sat.\,at\,T_{nucl.}} - p_{sat})$ is small, $p_{sat\,at\,T_{nucl.}}$ is large. Thus $T_{nucl.}$ is also much higher in the test-tube than in the tea-kettle.

It is beyond our scope here, but the thermodynamic availability is a measure of the damage that a superheated liquid can do when it nucleates. We can show [Jour. Ht. Transfer, Feb. 1981, Vol. 103, pp. 61-64] that the avalability rises as $(T_{sup} - T_{sat})^2$.

Thus Δa (and the possible damage) increase strongly with superheat.

9.6 Use van der Waals' equation to estimate how much superheat water can sustain at low pressure.



In reduced form the van der Waals equation is:

$$p_r = \frac{8T_r}{3(1-v_r)} - \frac{3}{v_r^2}$$

set $p_r \simeq 0$ and solve for $T_{r_{max}}$:

$$\underline{T_{r_{max}} = \frac{9}{8}\frac{1-v_r}{v_r^2}}$$

but at the limiting point: $\dfrac{\partial p_r}{\partial v_r} = 0 = -\dfrac{8T_r}{3(1-v_r)^2} + \dfrac{6}{v_r^3}$

substitute $T_{r_{max}}$: $\quad 0 = -\dfrac{8}{3}\dfrac{1}{(1-v_r)^2}\dfrac{9}{8}\dfrac{1-v_r}{v_r^2} + \dfrac{6}{v_r^3}$

or: $\quad 0 = -\dfrac{1}{1-v_r} + \dfrac{2}{v_r} \quad ; \quad v_r = \dfrac{2}{3}$

Thus: $\quad T_{r_{max}} = \dfrac{9}{8}\dfrac{1/3}{(2/3)^2} = \dfrac{27}{32}$

$$T_{max_{H_2O}} = \frac{27}{32}T_c = \frac{27}{32}647.2 = 546^\circ R$$

So, at 1 atm, the limiting superheat is $\Delta T = (546 - 373) = \underline{\underline{173^\circ C}}$

(The measured extremes are just a little greater than this.)

9.7 Find c in $n \sim \Delta T^c$ such that the result is consistent with Berenson's curves in Fig. 9.14 and Yamagata's equation:

$$q \sim n^{1/3}\Delta T^{1.2}$$

From the log-log plots in Fig. 9.14 we measure the slopes in the nucleate boiling range. Call this slope, d. The 5 values are 6, 5.7, 5.3, 4, 2.2. Then:

$$q \sim \Delta T^d \sim \Delta T^{c/3}\Delta T^{1.2} \quad \text{or} \quad c = 3d - 1.2$$

Thus the 5 values of c in $n = \Delta T^c$ are:

$\underline{\underline{c = 16.8}}$; $\underline{\underline{c = 15.9}}$; $\underline{\underline{c = 14.7}}$; $\underline{\underline{c = 10.8}}$; and $\underline{\underline{c = 5.4}}$

9.8 Suppose C_{sf} for a given surface is reported as being 50 % higher than is really is. How much error will this contribute to the calculated q?

from eqn. (9.4) we have: $q \sim (\Delta T/C_{sf})^3$

Then $q_{calculated} \sim \frac{1}{1.5^3}\left[\frac{\Delta T}{C_{sf_{correct}}}\right]^3$

or $q_{calculated} = 0.296\, q_{correct}$

Thus the calculation is low by 70%

9.9 Water at 100 atm boils on a nickel heater. $\Delta T = 6^oC$. Find q and h.

Properties at $T_{sat} = 310°C$: $\rho_f = 690$ kg/m³ $\quad h_{fg} = 1325000$ J/kg
$\rho_g = 54.7$ " $\quad c_p = 5600$ J/kg°C
$\mu_f = 0.0000875$ kg/m-s $\quad P_r = 1.02$
$\sigma = 0.0117$ kg/s² $\quad C_{sf} = 0.006$

Then, from eqn. (9.4)

$$\left(c_p \Delta T / h_{fg} P_r^{1}\right)^3 = \frac{C_{sf}}{\mu h_{fg}} \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}}\; q$$

So:

$$q = \left(\frac{5600(6)}{1325000(1.02)}\right)^3 \frac{0.0000875(1.325)10^6}{(0.006)^3} \Big/ \sqrt{\frac{0.0117}{9.8(635.3)}}$$

$$\underline{\underline{q = 6.017(10)^6 \frac{W}{m^2}}}$$

and. $\quad h = \frac{q}{\Delta T} = \frac{6.017(10)^6}{6} = \underline{\underline{1{,}003{,}000 \frac{W}{m^2\text{-}°C}}}$

This is very high.

9.10 Compute q_{max} for saturated water at 1 atm on a flat plate -- very large in extent -- at g/g_e = 1/6 and 10^{-4}.

At earth-normal gravity: $q_{max} = 1{,}260{,}000$ W/m² (Example 10.5)

Thus, at $\frac{g}{g_e} = \frac{1}{6}$

$$q_{max} = 1{,}260{,}000 \sqrt[4]{\frac{1}{6}} = \underline{\underline{805{,}000 \frac{W}{m^2}}}$$

And at $\frac{g}{g_e} = 10^{-4}$

$$q_{max} = 1{,}260{,}000 \sqrt[4]{\frac{1}{10^{-4}}} = \underline{\underline{126\,000 \frac{W}{m^2}}}$$

Since, in accordance with eqn. (9.11)

$$q_{max\,flat\,plate} \sim g^{1/4}$$

9.11 Water boils on a 0.001 m radius copper wire. Plot as much of the boiling curve as you can, for this case.

We go through the regimes of the boiling curve, one at a time, starting with natural convection.

$$\overline{Nu_D} = 0.36 + \frac{0.518 Ra_D^{1/4}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{4/9}} = 0.36 + \underbrace{\frac{0.518\left[\frac{9.8(0.00072)(0.002)^3}{1.653(0.292)10^{-13}}\right]^{1/4}}{\left[1+\left(\frac{0.559}{1.74}\right)^{9/16}\right]^{4/9}}}\Delta T^{1/4}$$

$$q = \overline{Nu_D}\frac{k}{D}\Delta T = 122\,\Delta T + 844\,\Delta T^{5/4}$$

ΔT °C	q $\frac{W}{m^2}$
1	966
2	2251
3	3332
4	5262
5	6920

The nucleate boiling heat flux is given by eqn. (9.4)

$$q = \left[\frac{c_p \Delta T}{h_{fg} Pr\, C_{sf}}\right]^3 \mu h_{fg}\sqrt{\frac{g\Delta\rho}{\sigma}} = \left[\frac{4218}{2.286(10)^6 1.74(0.013)}\right]^3 0.265(10)^{-6}\, 2.286(10)^6\sqrt{\frac{9.8(958.5)}{0.0589}}\,\Delta T^3$$

$$q = 124\,\Delta T^3$$

ΔT °C	q $\frac{W}{m^2}$
3	3358
4	7958
5	15,544
10	124,000
20	994,900

The peak heat flux at $R' = R\sqrt{\frac{9.8(958)}{0.0589}} = 0.399$ is given by eqn. (9.20)

$$q_{max} = \left[\frac{1}{1.14}\underbrace{q_{max_F}}_{\text{get from Example 10.5}}\right]\frac{0.94}{(R')^{1/4}} = \frac{1.26}{1.14}(10)^6\frac{0.94}{(0.399)^{1/4}} = 1,307,000\ \frac{W}{m^2}$$

The minimum heat flux for $R' = 0.399$ is

$$q_{min} = 0.515\left[\frac{18}{R'^2(2R'^2+1)}\right]^{1/4}\left[0.09\rho_g h_{fg}\sqrt[4]{\frac{\sigma g(\rho_f-\rho_g)}{(\rho_f+\rho_g)^2}}\right]$$

$$= 0.0464\left[\frac{18}{0.399^2(2[0.399]^2+1)}\right]^{1/4}(0.597)(2,257,000)\sqrt[4]{\frac{0.0589(9.8)(958)}{959^2}}$$

$$q_{min} = 29,800\ W/m^2$$

In the film boiling regime $h = h_{f.b.} + \frac{3}{4}h_{rad}$. Thus:

$$q = \frac{k\Delta T}{D}\left\{\left(0.661+\frac{0.243}{R'}\right)R'^{1/4}\, 0.62\left[\frac{(\rho_f-\rho_g)g h'_{fg}D^3}{\nu_g k_g \Delta T}\right]^{1/4}\right\} + \frac{3}{4}\sigma\left(T_w^4 - T_{sat}^4\right)$$

where we assume $\epsilon = 1$ and in which:

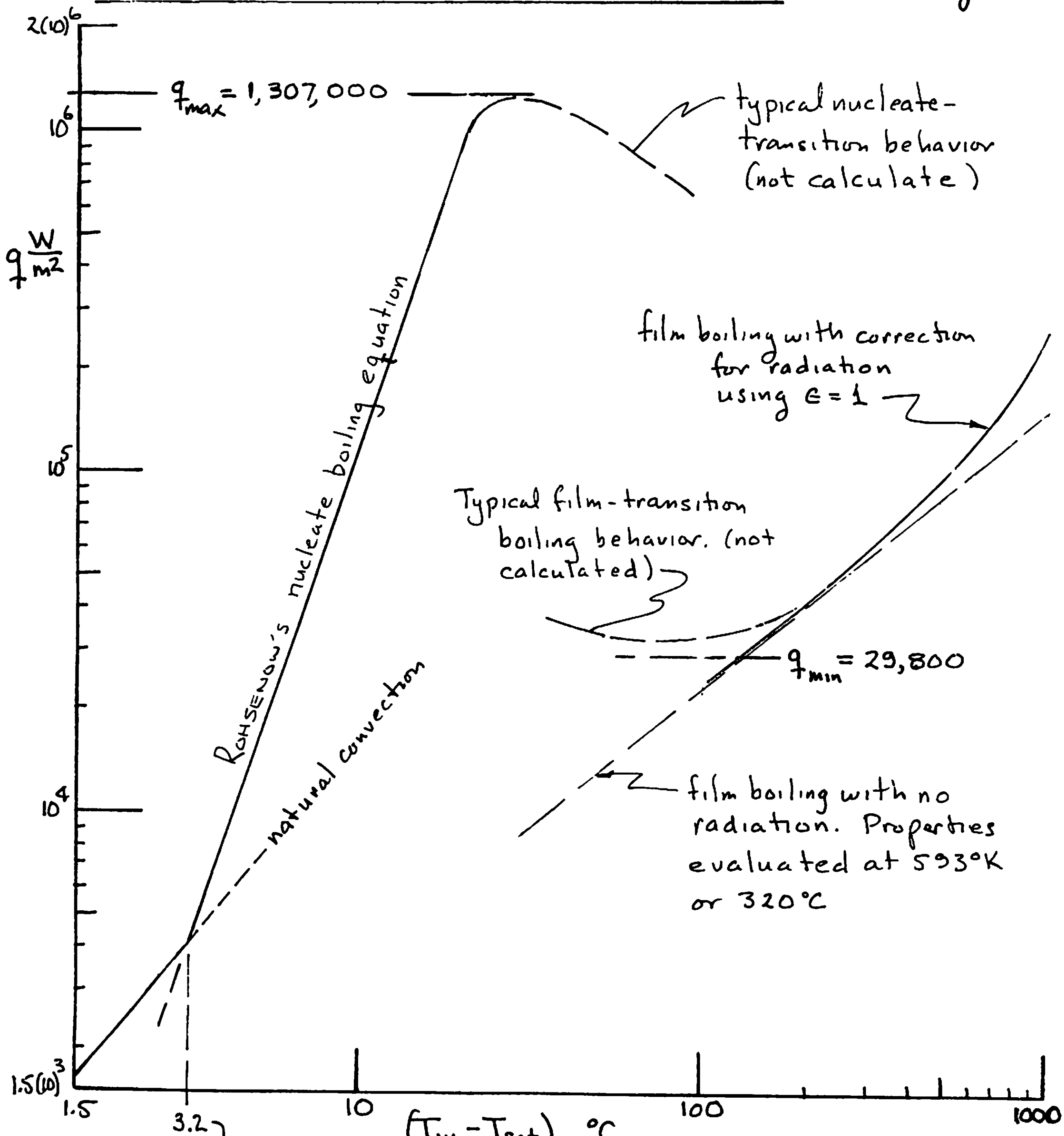
$$h'_{fg} = h_{fg}\left(1+\left[0.968-\frac{0.163}{Pr_g}\right]Ja\right) = 2,257,000\left(1+\left[0.968-\frac{0.163}{1.052}\right]\frac{2030\,\Delta T}{2,257,000}\right)$$

$$= 2,257,000(1+0.00075\,\Delta T)$$

9.11 (continued)

$$q = \frac{0.0237}{0.002}\left(0.661+\frac{0.243}{0.399}\right)0.399^{1/4}(0.62)\left[\frac{958(9.8)2.257(10)^6(1+0.00075\Delta T)0.002^3}{5884\times 10^{-5}(0.04)}\right]^{1/4}\Delta T^{3/4}$$
$$+0.75(5.67)10^{-8}\left(T_w^4-1.936(10)^{10}\right)$$

$$\underline{\underline{q = 683(1+0.00075\Delta T)^{1/4}\Delta T^{3/4}+0.4253(10)^{-8}\left(T_w^4-1.936(10)^{10}\right)}} \quad \text{for film boiling}$$



$\Delta T_{inception}$ occurs where $(122\Delta T + 844\Delta T^{5/4}) = 124\,\Delta T^3$

Trial & error gives $\Delta T_{incept.} = 3.2°C$ or $5.8°F$.

9.12 This problem is the same as 9.11 with the following exceptions: Since the heater is a sphere, we use equation (8.32) instead of eqn. (8.28) for natural convection. The peak heat flux is given by eqn. (9.22) instead of (9.20). Film boiling is still given by $h = h_{f.b} + 0.75h_{rad.}$, and $h_{rad.}$ is still the same. But we no longer need include a curvature correction in calculating $h_{f.b.}$ because a 0.03 m sphere is sufficiently large not to need it.) There is no reliable eqn. for q_{min} in this case.

9.13 Predict q_{max} for a small flat plate with only one jet on it.

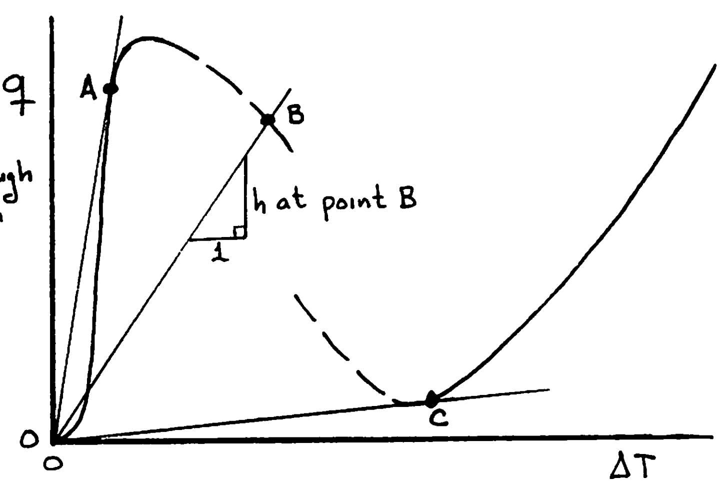
$$q_{max} = \underbrace{(1.14\, q_{max_Z})}_{q_{max}\text{ for a flat plate}} \frac{(A_{heater})_{actual}}{(A_{heater})_{ideal}}$$

but $(A_{heater})_{ideal} = \lambda_{d_1}^2$, therefore $\underline{\underline{\dfrac{q_{max}}{q_{max_Z}} = \dfrac{1.14}{\lambda_{d_1}^2} A_{heater}}}$

9.14 Show how to locate points of miximum and minumum h during pool boiling.

Use Fig. 9.2

$h = \dfrac{q}{\Delta T}$ = slope of lines which pass through the origin & touch the curve



(Notice that this can only be done on linear coordinates. Fig. 9.2 is semi-logarithmic. Therefor we must locate points A & C by trial & error in Fig. 9.2) we get:

$$h_{max} = \frac{960{,}000}{2} = \underline{\underline{48{,}000\ \frac{W}{m^2\text{-}^\circ C}}} \qquad h_{min} = \frac{170{,}000}{250} = \underline{\underline{680\ \frac{W}{m^2\text{-}^\circ C}}}$$

9.15 A 0.002 m diam. jet of saturated water flows normal to a 0.015 m diam. disc, at 1 m/s. How much energy can the disc dissipate?

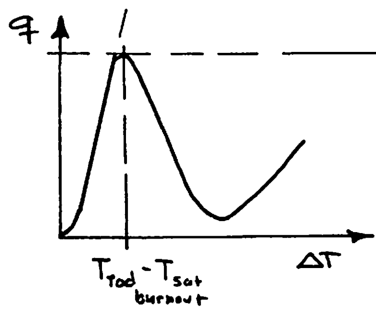
$\dfrac{\rho_f}{\rho_g} = \dfrac{958.3}{0.597} = 1605$ so eqn. (9.41) gives $A = 0.329$. Then eqn. (9.40) gives:

$$q_{max} = 2.939 \rho_g h_{fg} u_{jet} \left(\frac{0.002}{0.015}\right)^{1/3} \left(\frac{1000[1605]}{\rho_f u_j D/\sigma}\right)^A$$

$$= 2.939(0.597)(2.257\cdot 10^6)1\,(0.1333)^{1/3}\left(\frac{1{,}605{,}000}{958.3(1)(0.015)/0.0589}\right)^{0.329} = \underline{3.65\times 10^7\ \frac{W}{m^2}}$$

So the maximum heat dissipation is $Q_{max} = q_{max}\frac{\pi}{4}D^2 = \underline{\underline{6{,}448\ W}}$

9.16 Saturated water at 1 atm. boils on a 0.005 m diam. rod of platinum. What is T_{rod} at burnout?



$$q_{max} = \left(0.94/R'^{1/4}\right) q_{max_z} \quad \text{(eqn. 9.20))}$$

$$R' = \sqrt{\frac{g(\rho_f - \rho_g)}{\sigma}}R = \sqrt{\frac{9.8(957.6)}{0.0589}}(0.0025) = 0.9919$$

$$q_{max_z} = \frac{\pi}{24}(0.597)^{1/2}(2257000)\sqrt[4]{0.0589(9.8)(957.6)} = 1{,}107{,}000 \frac{W}{m^2}$$

so $q_{max} = (0.94/R'^{1/4})\, q_{max_z} = \underline{1{,}041{,}000 \frac{W}{m^2}}$

from eqn. (9.4)

$$T_{rod_{burnout}} = T_{sat} + \frac{C_{sf} Pr\, h_{fg}^{2/3}}{c_p\, \mu^{1/3}} \left(\frac{\sigma}{g(\rho_f - \rho_g)}\right)^{1/6} q_{max}^{1/3}$$

$$= 100 + \frac{0.013(1.72)(2{,}257{,}000)^{2/3}}{4219(0.29 \times 10^{-6} \times 957.2)^{1/3}} \left(\frac{0.0589}{9.8(957.6)}\right)^{1/6} (1.04 \times 10^6)^{1/3} = \underline{\underline{119.2^\circ C}}$$

9.19 Verify the form of eqn.(9.8) using dimensional anlysis.

$u_g = fn(\delta, \rho_g, \lambda_H)$, 4 variables in m,s, and kg. Thus we look for 4-3, or 1 π-group. Let's write that like a Weber number:

$$\pi = \frac{\rho_g u_g^2 \lambda_H}{\sigma} = \text{const.} \quad \text{or} \quad \underline{\underline{u_g = C\sqrt{\frac{\sigma}{\rho_g \lambda_H}}}}$$

eqn.(9.8) is of this form with $C \equiv \sqrt{2\pi}$

9.20 Compare the value of q_{max} implied by data for pool boiling from a 1 in. diam. sphere in Problem 5.6, with the appropriate prediction.

The measured value of q_{max} can be obtained using the expression derived in the solution of Problem 5.6.

$$q_{max} = \bar{h}\Delta T_{sat} = 0.712 \frac{\text{Btu}}{\text{ft}^2\text{-°F}} \left(\frac{dT}{dt} \frac{\text{°F}}{\text{s}} \right)_{max}$$

From the figure associated with Problem 5.6 we read

$$\left.\frac{dT}{dt}\right|_{max} = 102 \frac{\text{°F}}{\text{s}} \quad \text{so} \quad q_{max} = 72.62 \frac{\text{Btu}}{\text{ft}^2\text{-s}} = \underline{\underline{261{,}446 \frac{\text{Btu}}{\text{ft}^2\text{-hr}}}}$$

9.20 (continued)

Now for this sphere, $R' = \dfrac{0.0254 \text{ m}/2}{\sqrt{\dfrac{0.0589}{9.8(958.2-0.6)}} \text{ m}} = \underline{5.07}$

Therefore we use eqn. (9.21):

$$q_{max} = 0.84\, q_{max_Z} = 0.84\, \frac{q_{max_F}}{1.14} = 0.737\, q_{max_F}$$

where q_{max_F} is given in Example 9.5 as 1,260,000 $\dfrac{W}{m^2}$

So:

$$q_{max} = 0.737(1,260,000)/3.154 = \underline{\underline{294,426 \ \frac{Btu}{ft^2\text{-hr}}}}$$

In this case the measurement is 11% below the prediction.

9.22 Verify equation (9.53) which gives δ for a condensing film subject to a shear stress, τ_δ.

We first integrate eqn. (8.50) twice and get:

$$\frac{du}{dy} = \frac{\rho_f-\rho_g}{\rho_f}\,gy + C_1 \quad \text{and} \quad u = -\frac{\rho_f-\rho_g}{\rho_f}\,\frac{y^2}{2} + C_1y + C_2$$

The first b.c., $u(y=0) = 0$ gives C_2 & the second $\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = \frac{\tau_\delta}{\mu}$

gives $C_1 = \frac{\tau_\delta}{\mu} + \frac{\rho_f-\rho_g}{\rho_f}\,g\delta$. Thus

$$u = \frac{(\rho_f-\rho_g)g\delta^2}{2\mu}\left[2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right] + \frac{\tau_\delta}{\mu}y$$

Then equation (8.53) gives

$$\dot{m} = \int_0^\delta \rho_f u\,dy = \frac{\rho_f-\rho_g}{3\nu}\,g\,\delta^3 + \frac{\tau_\delta}{2\nu}\,\delta$$

and equation (8.54) becomes:

$$\frac{k\Delta T}{h'_{fg}\delta} = \frac{d\dot{m}}{dx} = \left[\frac{\rho_f-\rho_g}{\nu}\,g\delta^2 + \frac{\tau_\delta}{\nu}\,\delta\right]\frac{d\delta}{dx}$$

which we integrate, subject to $\delta(x=0) = 0$:

$$\frac{2k\Delta T\nu}{h'_{fg}}\,dx = (\rho_f-\rho_g)g\,\delta^2 d\delta^2 + \tau_\delta\sqrt{\delta^2}\,d\delta^2$$

or

$$\underline{\underline{\frac{4k\Delta T\nu x}{g(\rho_f-\rho_g)h'_{fg}} = \delta^4 + \frac{4}{3}\,\frac{\tau_\delta}{g(\rho_f-\rho_g)}\,\delta^3}} \longleftarrow \text{eqn. (9.53)}$$

Now if $\tau_\delta = -\frac{3g(\rho_f-\rho_g)}{4}\,\delta$ equation (9.53) reduces to:

$$\frac{4k\Delta T\nu x}{g(\rho_f-\rho_g)h'_{fg}} = 0$$

Which means that only $\Delta T = 0$ will work (otherwise the film must grow to a larger value of δ so τ_δ no longer equals $-(3g(\rho_f-\rho_g)/4)\delta$.)

9.23 A 0.07 m O.D. pipe is at 40 °C. Saturated steam at 80 °C blows across it. Plot $\bar{h}_{cond.}$ for $0 \le Re_D \le 10^6$.

We are given the following expression for flow over a cylinder, (where we evaluate μ_f and k_f at 60 °C, and the other properties at 80 °C):

$$\bar{h} = 0.64\frac{k_f}{D}\sqrt{\frac{\rho_g u_\infty D}{\mu_f}\left[1+\left(1+1.69\,\frac{g\,h'_{fg}\,\mu_f D}{u_\infty^2 k_f (T_{sat}-T_w)}\right)^{1/2}\right]}\ ,\qquad \text{note: } u_\infty \equiv Re_D\frac{\nu_f}{D}$$

$$\text{where}\quad h'_{fg} = 2{,}308{,}000\left(1+\left[0.683+\frac{0.228}{4.2}\right]\frac{2063(40)}{2{,}308{,}000}\right) = 2{,}369{,}000$$

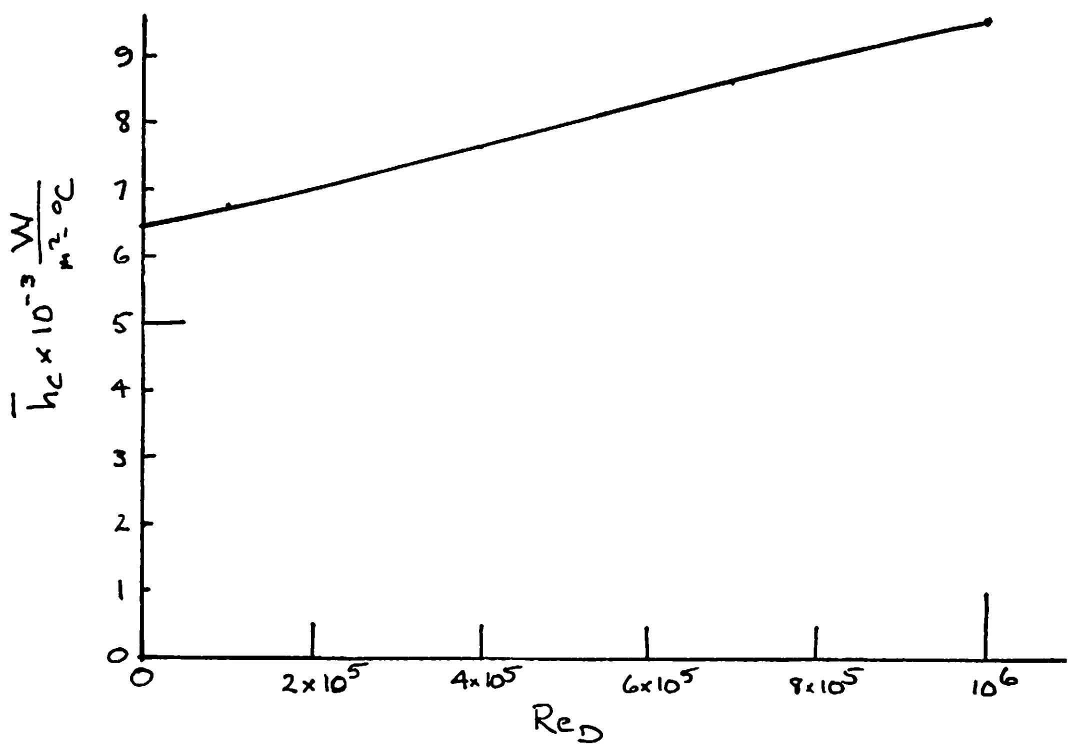
so:

$$\bar{h} = 0.64\,\frac{0.651}{0.07}\sqrt{1+\left(1+1.69\,\frac{9.8(2{,}369{,}000)(0.000355)}{Re_D^2\,\frac{(3.61\times10^{-7})^2}{0.07^3}\,0.651(40)}\right)}\ Re_D^{\,2}$$

$$= 5.952\sqrt{1+\left(1+\frac{1.408\times10^{12}}{Re_D^{2}}\right)^{1/2}}\ Re_D^{1/2}$$

When $Re_D \Rightarrow 0$, $\bar{h} \Rightarrow 6480\ \mathrm{W/m^2\text{-}^\circ C}$, or eqn. (9.55) reduces to: $\bar{h} = 0.7297\,\dfrac{k_g}{D}\sqrt[4]{\dfrac{g\,h'_{fg}\,\rho_f D^3}{\nu_f\, k_f\,\Delta T}}$

which is virtually the same as equation (8.67) for static condensation.



9.24 a) Suppose you have pits of roughly 0.002mm diameter in a metallic heater surface. At about what temperature might you expect water to boil on that surface, if the pressure is 20 atm.

b) Measurements have shown that water at atmospheric pressure can be superheated about 200°C above its normal boiling point. Roughly how large an embryonic bubble would be needed to trigger nucleation in water in such a state.

a) $T_{sat} = 213°C = 486.2°K$; $T_{reduced} = \dfrac{486.2}{647.2} = 0.7512$

$$\sigma = 235.8(1-.7512)^{1.256}(1-.625(1-.7512)) = \underline{34.70\ dyne/cm} = \underline{34.7\frac{mN}{m}}$$

$$\text{then } \Delta p = \frac{2(\sigma)}{R} = \frac{2(34.7)\frac{mN}{m}}{.000001\ m} = 69{,}400\ \frac{N}{m^2} = \underline{10.064 psi}$$

T_{sat} at $(20(14.7) + 10.064)$psia is $\underline{419.8°F}$

$\Delta T = \underline{\underline{4.60°F}}$ or $\underline{\underline{2.56°C}}$

much less ΔT is needed to drive boiling at elevated pressures

b) $p_{sat}(300°C) = p_{sat}(572°F) = \underline{1246.6 psia}$

$R = \dfrac{2\sigma}{\Delta p}$ but what is σ? Probably it should be evaluated at 300°C or

$$T_r = \frac{300+273}{647.2} = 0.885$$

$$\sigma = 235.8(1-.885)^{1.256}(1-.625(.115)) = 15.57\frac{dyne}{cm}$$

$$= \underline{0.001067\frac{lbf}{ft}}$$

$$R = \frac{2(0.001067)}{(1246.6-14.7)(144)} = 1.203\times10^{-8} ft$$

$$= 1.443\times10^{-7} ft$$

$$= 3.666\times10^{-6} mm$$

$$= \underline{\underline{36.6 Å}}$$

And that is very small indeed.

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9.25 Obtain the dimensionless functional form of the pool boiling q_{max} equation, and the q_{max} equation for flow boiing on external surfaces, using dimensional analysis.

The pool boiling result is worked out fully in the solution of Problem 4.28. It takes the form:

$$\underbrace{\frac{\pi}{24}\frac{q_{max}}{q_{max_Z}}}_{\text{this called the "Kutateladze" No.}} = f(L')$$

All solutions for q_{max} in Table 9.3 take this form.

For external flows we have:

$$\underset{J/m^2\cdot s}{q_{max}} = fn\left(\underset{kg/m^3}{\rho_g}, \underset{kg/m^3}{\rho_f}, \underset{J/kg}{h_{fg}}, \underset{kg/s^2}{\sigma}, \underset{m}{L}, \underset{m/s}{u_\infty}\right)$$

There are 7 variables in J, m, kg, & s or 7-4 = 3 Π-groups:

$$\Pi_1 = \frac{q_{max}}{\rho_g h_{fg} u_\infty} \qquad \Pi_2 = \frac{\rho_f}{\rho_g} \qquad \Pi_3 \equiv We_L = \frac{\rho_g u_\infty^2 L}{\sigma}$$

Thus:

$$\frac{q_{max}}{\rho_g h_{fg} u_\infty} = fn\left(\frac{\rho_f}{\rho_g}, We_L\right)$$

We see that the flow boiling burnout expressions in the text take this form unless there is an additional characteristic length in the problem. (See the expression for q_{max} when a jet of diameter, d, impinges on a disc of diameter, D. This introduces an additional group d/D.) (See also the Katto flow boiling burnout correlation form.)

9.26 A (magical?) additive to water increases σ tenfold at 1 atm. By what factor will it improve q_{max} during pool boiling on: (a) infinite flat plates and (b) small horizontal cylinders; and (c) when a jet impinges on a disc.

a) from eqn. (9.11) $\dfrac{q_{max}(\sigma_{high})}{q_{max}(\sigma_{low})} = \left(\dfrac{\sigma_{high}}{\sigma_{low}}\right)^{1/4} = (10)^{1/4} = \underline{\underline{1.78}}$

b) from eqn. (9.20) $\dfrac{q_{max}(\sigma_h)}{q_{max}(\sigma_1)} = \left(\dfrac{\sigma_h}{\sigma_1}\right)^{1/4}\left(\dfrac{R'(\sigma_1)}{R'(\sigma_h)}\right)^{1/4} = (10)^{1/4}\left(\dfrac{\sigma_h}{\sigma_1}\right)^{1/8} = 10^{3/8} = \underline{\underline{2.37}}$

c) from eqn. (9.40) $\dfrac{q_{max}(\sigma_h)}{q_{max}(\sigma_1)} = \left(\dfrac{We_D(\sigma_1)}{We_D(\sigma_h)}\right)^A = \left(\dfrac{\sigma_h}{\sigma_1}\right)^A$

and from eqn. (9.41) we get, for $\rho_f/\rho_g = 957.2/0.597 = 1603 \equiv r$, $\underline{A = 0.329}$. Thus:

$$\frac{q_{max}(\sigma_h)}{q_{max}(\sigma_1)} = 10^A = 10^{0.329} = \underline{\underline{2.133}}$$

9.27 Steam a 1 atm. is blown at 26 m/s over a 1 cm OD cylinder at 90°C. What is $\bar{h}$? Suggest a physical process within the cylinder that could sustain this temperature in this flow.

$$h'_{fg} = 2{,}257{,}000\left(1+\left[0.683+\frac{0.228}{1.72}\right]\frac{4219(10)}{2{,}257{,}000}\right) = 2{,}291{,}408$$

$$\overline{Nu}_D = 0.64\left\{\frac{26(0.01)}{0.29(10)^{-6}}\left[1+\left(1+1.69\,\frac{9.8(2.291)10^6(0.0002776)(0.01)}{26^2(0.6811)(100-90)}\right)^{1/2}\right]\right\}^{1/2}$$

$= 858.4$, so $\bar{h} = \overline{Nu}_D\, k/D = 858.4(0.6811)/0.01$

$= \underline{\underline{58{,}466\ W/m^2\text{-}°C}}$

This means that we need a powerfully effective heat removal process in the cylinder -- enough to carry q = 584,660 W/m^2 away from the surface. Nucleate boiling to water at less than 1 atm. could do it, especially at high velocity. The right liquid -- one that is very cold and moves at high velocity -- might be made to do it.

9.28 The water shown in Fig. 9.17 is at one atmosphere and the nichrome heater can be approximated as nickel. What is $T_w - T_{sat}$?

$q_{FC} = \frac{\Delta T k}{L}\overline{Nu}_L$ where we scale L=18 cm from the photo, & use eqn. (6.68) for $\overline{Nu}_L$.

$$q_{FC} = \frac{\Delta T(0.6817)}{0.18}\,0.664\left[\frac{0.18(0.52)}{0.29(10)^{-6}}\right]^{1/2} 1.72^{1/3} = \underline{1712\,\Delta T}$$

And from eqn. (9.4):

$$q_B = \frac{\Delta T^3}{C_{sf}^3}\frac{\mu c_p^3}{h_{fg}^2 Pr^3}\sqrt{\frac{g\Delta\rho}{\sigma}} = \frac{\Delta T^3}{0.006^3}\,\frac{957.2(0.29)10^{-6}\,4219^3}{2.257^2\,10^{12}\;1.72^3}\sqrt{\frac{9.8(957.2-0.6)}{0.0589}}$$

$q_B = \underline{1485\,\Delta T^3}$

Then, noting that q is high we use the limiting form of eqn. (9.37), namely $q = \sqrt{q_B q_{FC}}$:

$480000 = \sqrt{1712(1485)}\,\Delta T^2$, $\underline{\underline{\Delta T = 17.55\,°C}}$

This is quite low. It gives $\bar{h}$ = 27,700 W/m^2-°C. The process is very efficient.

9.29 For film boiling on horizontal cylinders, eqn. (9.6a) is modified with Fig. 9.3d to: $\lambda_d = 2\pi\sqrt{3}\left([g(\rho_f-\rho_g)/\sigma] + 2/(\text{diam.})^2\right)^{-1/2}$. If ρ_f is 748 kg/m^3 for saturated acetone, compare this λ_d, and the flat plate value,

22 gage ⇒ 0.000643 m
$T_{sat_{acetone}} = 56$ °C
so $\sigma = 0.020$ kg/s^2

so $\lambda_d = \dfrac{2\pi\sqrt{3}}{\sqrt{\dfrac{9.8(748)}{0.020} + \dfrac{2}{0.000643^2}}} = \underline{\underline{0.00477 m}}$

$= \underline{\underline{0.477\ cm}}$

(over)

In Fig. 9.3d we find 4 wavelengths in 2.92 cm so

$\lambda_{d\,\text{expt'l.}} = \underline{\underline{0.73\text{ cm}}}$

$$\text{deviation from theory} = \frac{0.73-0.977}{0.73} = \underline{35\%}$$

$$\lambda_{d\,\text{plane}} = \frac{2\pi\sqrt{3}}{\sqrt{\dfrac{9.8(793)}{0.020}}} = 0.018\text{m} = \underline{\underline{1.8\text{cm}}}$$

which is 147% above the expt'l. value & 277% above the data.

9.30 Water at 47°C flows through a 13 cm diameter thin-walled tube at 8 m/s. Saturated water vapor, at one atmosphere, flows across the tube at 50 m/s. Evaluate T_{tube}, U, and q.

Guess $T_{tube} = 67°C$ for property evaluation. Then $\overline{T}_{H_2O} = 57°C$ & $\overline{T}_{film} = 83.5°C$

in the pipe $\nu = 0.493\times10^{-6}$, $k = 0.6477$, $Pr = 3.14$, $\mu_b = 0.00056$, $\mu_w = 0.000411$

in the film $\nu = 0.346\times10^{-6}$, $k = 0.6743$

(This guess is already a second iteration. we don't present the first one here.)

Now we use eqn. (7.41) $Re_D = \dfrac{8(0.13)}{0.493(10)^{-6}} = 2.11(10)^6$

$$f/8 = 1/8[1.82\log_{10}Re_D - 1.64]^2 = 0.001283$$

$$\overline{Nu}_D = \frac{0.001283(2.11\times10^6)3.14}{1.07+12.7(0.03582)(3.14^{2/3}-1)}\left(\frac{0.00056}{0.000411}\right)^{0.25} = 5774$$

$$\overline{h}_{H_2O\text{ in tube}} = 5774(0.6477)/0.13 = \underline{28,768\text{ W/m}^2\text{-°C}}$$

& for condensation outside, use eqn. (9.56) -- use of eqn. (9.55) would be more accurate; but only by about 1% at this very high Re_D (see below)

$$Re_D = \frac{0.13(50)}{0.346(10)^{-6}} = 1.879(10)^7$$

$$\overline{Nu}_{D_{film}} = 0.64\sqrt{2(1.879)10^7} = 3,923\;,\quad \overline{h}_{film} = 3923\frac{0.6743}{0.13} = \underline{\underline{20,348\frac{\text{W}}{\text{m}^2\text{°C}}}}$$

Then:

$$U = \frac{1}{\dfrac{1}{28,768}+\dfrac{1}{20,348}} = \underline{\underline{11,918\text{ W/m}^2\text{-°C}}}$$

&

$$q = U\Delta T = 11,918(100-47) = \underline{\underline{631,663\text{ W/m}^2}}$$

&

$$q = h_{film}(100-T_{tube})\;;\quad T_{tube} = 100 - \frac{631,663}{20,348} = \underline{\underline{69°C}}$$

This temperature is within 2°C. Further iteration is not needed.

9.31 A 1 cm diameter, thin-walled tube carries liquid metal through saturated water at one atmosphere. The throughflow of metal is increased until burnout occurs. At that point the metal temperature is 250°C and h inside the tube is 9600 W/m^2-°C. What is the wall temperature at burnout?

$R' = R/\sqrt{\sigma/g(\rho_f-\rho_g)} = 0.005/\sqrt{0.0589/9.8(958.2-0.6)} = 1.996$, so we use eqn. (9.19) for burnout.

$$q_{max} = 0.9\, q_{max_{\text{flat plate}}} = 0.9\underbrace{(1{,}260{,}000)}_{\text{See Example 9.5}} = 1{,}134{,}000 \text{ W/m}^2$$

Then: $1{,}134{,}000 = 9600(250 - T_{wall})$ so $\underline{\underline{T_w = 132°C}}$

9.32 At about what velocity of liquid metal flow does burnout occur in Problem 9.31 if the metal is mercury?

The Nusselt no. at q_{max} is $\dfrac{9600(0.01)}{6.12} = 15.69$ so reading from Fig. 7.9,

$$Pe_D = 2200 = u_\infty D/\alpha = u_\infty(0.01)/5.49\times10^{-6} \text{ so } \underline{\underline{u_\infty = 1.2 \text{ m/s}}}$$

9.33 Explain, in physical terms, why equations (9.23) and (9.25) instead of differing by a factor of two, are almost equal. How do these equations change when H´ is large?

In both cases, burnout occurs when enough vapor is generated to cause Helmholtz instability to occur -- it does not matter whether from one side or two. Thus, when H′ is large, q_{max} is equal to 0.9 q_{max_Z} -- the <u>same</u> value in <u>both</u> cases.

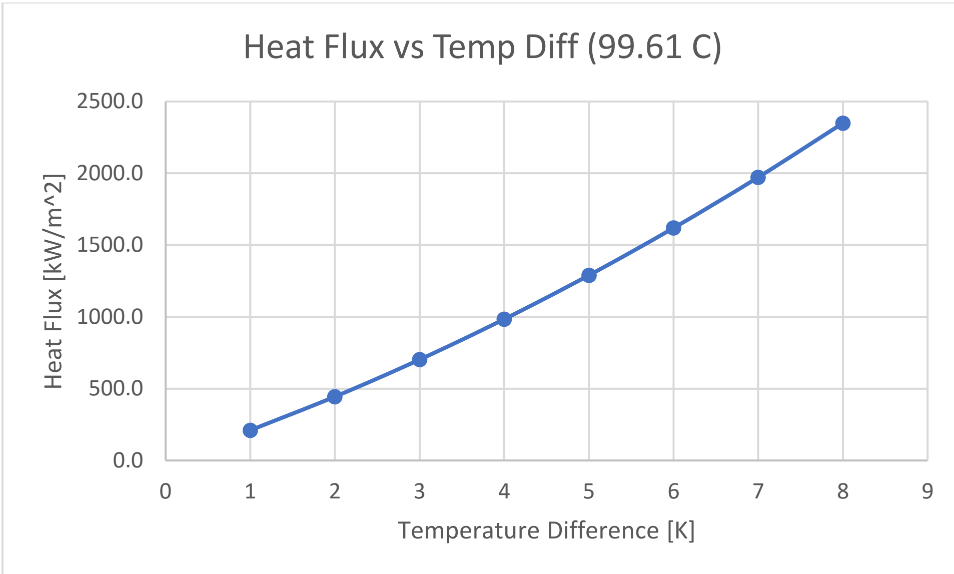
Indeed, if we have the same vapor volume at q_{max} in both cases, and if both q_{max} values are the same, then H′ must be twice as large in the insulated case. That is why the constant, 1.4, in eqn. (9.24) is exactly $2^{1/4}$ times the constant, 1.18, in eqn. (9.23). i.e.:

$$\frac{1.18}{H'^{1/4}} \equiv \frac{1.4}{(2H')^{1/4}}$$

Problem 9.37

P_vap (Pa)	T_sat (K)	T_sat (C)
1000	280.12	6.97
10000	318.96	45.81
100000	372.76	99.61

t (C)	Delta T (K)	q (kW/m^2)	h (kW/m^2K)
6.97	1	25.1	25.1
	2	52.9	26.5
	3	83.7	27.9
	4	117.2	29.3
	5	153.6	30.7
	6	192.9	32.1
	7	234.9	33.6
	8	279.8	35.0
45.81	1	113.0	113.0
	2	238.8	119.4
	3	377.3	125.8
	4	528.7	132.2
	5	692.9	138.6
	6	869.8	145.0
	7	1059.6	151.4
	8	1262.1	157.8
99.61	1	210.3	210.3
	2	444.5	222.2
	3	702.5	234.2
	4	984.2	246.1
	5	1289.8	258.0
	6	1619.2	269.9
	7	1972.4	281.8
	8	2349.4	293.7


Heat Flux vs Temp Diff (99.61 C)
Heat Flux [kW/m^2]
2500.0
2000.0
1500.0
1000.0
500.0
0.0
0
1
2
3
4
5
6
7
8
9
Temperature Difference [K]

Problem 9.38

Surface at 100 C

Delta T	P_0	Delta P	rho_0	factor	mdot (kg/m^2s)	q (MW/m^2)
0	101420.0	0	0.59817	0.000345552	0.0	0
1	105090.0	3670	0.61841	0.000345231	1.3	3
2	108870.0	7450	0.6392	0.000344907	2.6	6
3	112770.0	11350	0.66056	0.000344564	3.9	9
4	116780.0	15360	0.6825	0.000344241	5.3	12
5	120900.0	19480	0.70503	0.000343941	6.7	15
6	125150.0	23730	0.72816	0.000343615	8.2	18
7	129520.0	28100	0.7519	0.000343299	9.6	22
8	134010.0	32590	0.77627	0.000343	11.2	25
9	138630.0	37210	0.80127	0.000342697	12.8	29
10	143380.0	41960	0.82693	0.000342402	14.4	32

T_0	373.15	
p_0	101420	
R	461.404	
coef	1.6678	
sigma	0.31	
factor1	3.0329914	
hfg	2246000	treat as constant

Surface at 40 C

Delta T	P_0	Delta P	rho_0	factor	mdot (kg/m^2s)	q (MW/m^2)
0	7384.9	0	0.051242	0.000372416	0.0	0.0
1	7787.8	402.9	0.053871	0.00037186	0.1	0.3
2	8209.6	824.7	0.056614	0.000371306	0.3	0.7
3	8650.8	1265.9	0.059474	0.000370756	0.5	1.1
4	9112.4	1727.5	0.062457	0.000370213	0.6	1.5
5	9595	2210.1	0.065565	0.00036967	0.8	1.9
6	10099	2714.1	0.068803	0.000369146	1.0	2.3
7	10627	3242.1	0.072176	0.000368579	1.2	2.8
8	11177	3792.1	0.075688	0.000368067	1.4	3.2
9	11752	4367.1	0.079343	0.000367534	1.6	3.7
10	12353	4968.1	0.083147	0.000366984	1.8	4.2

40 C

T_0	313.15	
p_0	7384.9	
R	461.403996	
coef	1.6678	
sigma	0.31	
factor1	3.0329914	
hfg	2306000	treat as constant

10.1 What will be the apparent values of $\epsilon_{\lambda=0.2\mu m}$ and $\epsilon_{\lambda=0.65\mu m}$ for the sun as viewed from the earth's surface.

From Fig. 10.2 we scale

$$\epsilon_{\lambda=0.2} = \left.\frac{e}{e_b}\right|_{\lambda=0.2} = \underline{\underline{0}}$$

$$\epsilon_{\lambda=0.65} = \left.\frac{e}{e_b}\right|_{\lambda=0.65} = \underline{\underline{0.77}}$$

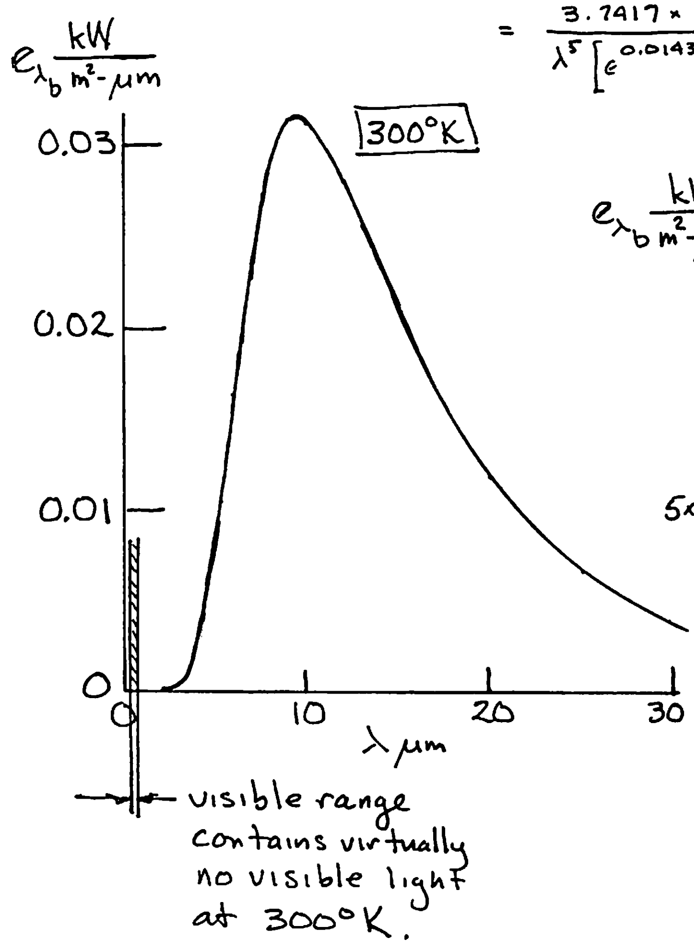
These are low. They show energy has been removed by the earth's atmosphere. (The sun itself is virtually black.)

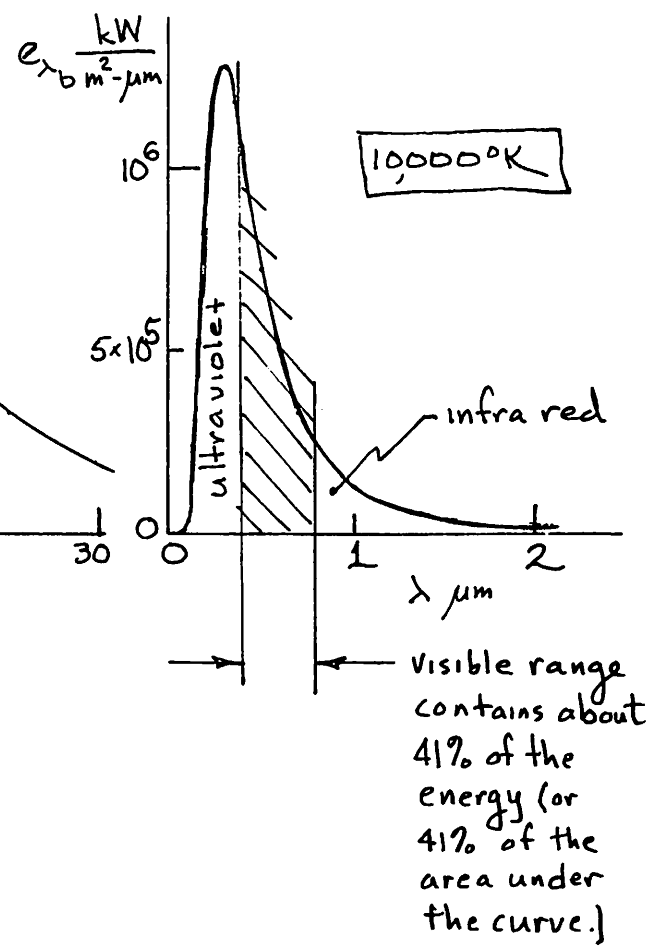
10.2 Plot $e_{\lambda,b}$ vs. T for T = 300°K and 10,000°K. What portion of the total energy is radiated in the visible range.

equation (1.30):
$$e_{\lambda_b} = \frac{2\pi(6.6256\times10^{-34})(2.998\times10^{8})^2 \ \frac{J\,m^2}{s}}{\lambda^5\left[e^{6.6256(2.998)10^{-26}/1.3805(10)^{-23}T\lambda}-1\right] m^5}$$

$$= \frac{3.7417\times10^{-25}}{\lambda^5\left[e^{0.01439/T\lambda}-1\right]} \ \frac{kW}{m^2\text{-}\mu m}$$

where λ is in μm and T is in °K





10.3 A 0.0006 m diam. wire ($\epsilon = 0.85$) at 950°C is on the center of a 0.07 m diam. thin metal tube ($\epsilon = 0.09$). The tube is horizontal in air at 25°C. Find T_{tube}.

First guess $\bar{h}_{conv.} = 6$ W/m²-°C. Then from eqn. (10.14)

$$Q_{1-2} = \mathcal{F}_{1-2}\sigma A_1\left(T_1^4 - T_2^4\right) = \frac{\sigma\,\pi D_1\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{D_1}{D_2}\left(\frac{1}{\epsilon_2}+1\right)} = \frac{5.67(10)^{-8}\pi(0.0006)\left(1223^4 - T_2^4\right)}{\frac{1}{0.85} + \frac{0.0006}{0.07}\left(\frac{1}{0.09}-1\right)}$$

$$Q = 8.46\times 10^{-11}\left(2.237[10]^{12} - T_2^4\right) = \bar{h}A\Delta T = 6\pi(0.07)(T_2 - 298)$$

Trial and error gives: $T_2 = 437°K = 167°C$

Then, evaluating all properties but β at $\frac{167+25}{2} = 96°C = 369°K$

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8\,\frac{1}{298}(167-25)\,0.07^3}{2.266\,(3.215)\,10^{-10}} = 2.20\times 10^6$$

so:

$$\overline{Nu_D} = 0.36 + \frac{0.518\,(2.20\times 10^6)^{1/4}}{\left[1+\left[\frac{0.559}{0.707}\right]^{9/16}\right]^{4/9}} = 15.1$$

and

$$h = 15.1\,\frac{0.03097}{0.07} = 6.68\ \frac{W}{m^2\,°C}$$

using 6.68 instead of 6 in the heat balance equation above, we get $T_2 = 422°K = 149°C$. That gives $h = 6.46\ \frac{W}{m^2\,°C}$. Then T_2 will drop in proportion:

$$Q = 6.68(149-25) \simeq 6.46(T_2 - 25)$$

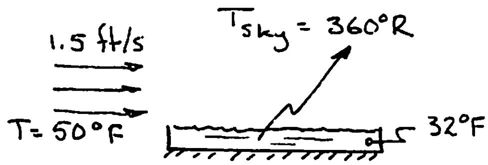
$$T_2 = T_{shield} = \underline{\underline{153\ °C}}$$

$$h_{rad_{wire}} = \mathcal{F}_{w-s}\,\sigma\,\frac{T_w^4 - T_s^4}{\Delta T} = 124\ ,\quad h_{conv._{wire}} \text{ should be less than this.}$$

$$h_{rad_s} = \epsilon_w\,\sigma\,\frac{T_s^4 - T_\infty^4}{\Delta T} \simeq 1.0,\quad \text{this is about } (1/6)^{th}\ h_{conv.}$$

Thus the present assumptions are not bad, but a refined calculation would account for convection inside and radiation outside.

10.4 A 1 ft^2 shallow pan with adiabatic sides is filled to the brim with water at 32^oF. It radiates to the night sky whose temperature is 360^oR while a 50^oF breeze blows over it at 1.5 ft/s. Will the water freeze or warm up?



$\epsilon_{H_2O} = 0.96$

• Find q_{conv} using $\overline{Nu}_L = 0.664\, Pr^{1/3} Re_L^{1/2}$ (Evaluate properties at $\frac{50+32}{2} = 41°F = 5°C$.)

so: $$\bar{h} = \frac{0.02493 \frac{W}{m\text{-}°C}}{0.3048\,m}\, 0.664(0.717)^{1/3}\left(\frac{1.5(0.3048)(0.3048)}{1.371\times10^{-5}}\right)^{1/2} = 4.802 \frac{W}{m^2\text{-}°C}$$

$$q_{conv} = 4.802\times\left[(50-32)/1.8\right] = \underline{48.02\ W/m^2}$$

• Find $$q_{rad} = \frac{1}{\frac{1}{\epsilon_1} + \underbrace{\frac{A_1}{A_2}}_{=0}\underbrace{\left(\frac{1}{\epsilon_2}-1\right)}_{=0}}\,\sigma\left(T_{water}^4 - T_{sky}^4\right).$$

$$= \epsilon_1\sigma\left(T_w^4 - T_s^4\right) = 0.96(5.67\times10^{-8})(273^4-200^4) = \underline{\underline{215\ \frac{W}{m^2}}}$$

Thus about four times as much heat radiates away as flows into the water by convection. It is, in fact, possible to freeze water in the desert in this way, on warmish nights.

10.5 Find the temperature, T_T, of a thermometer in 10°C air and 27°C walls if it and the room are black.

Let's treat the thermometer bulb as a vertical wall, 0.01m in height; and evaluate properties at 291.5°K & β at 283°K.

Then, using the simple Squire-Eckert equation (8.27)

$$\bar{h} = \frac{0.0255}{0.01}\, 0.678\left[\frac{0.713}{0.952+0.713}\right]^{1/4}\left[\frac{9.8(1/283)(T_T-10)(0.01)^3}{1.490(2.092)10^{-10}}\right]^{1/4}$$

$$\underline{q = 4.54(T_T-10)^{5/4}} = \underline{4.54[(T°R)-283]}$$

And: $$q_{rad} = -\sigma(T_T^4 - 300^4) = \underline{-5.67\text{x}10^{-8}(T_T^4 - 8.1\text{x}10^9)}$$

Setting these equations equal to one another and solving them simultaneously for T_T, we get

$$T_T = 292.6°K = \underline{\underline{19.6°C}}$$

Notice that we should have evaluated properties at (19.6+10)/2 = 14.8°C = 287.8°K. That's only 3°C off the mark so we let the calculation stand.

10.6 What will T_T be in Problem 11.5 if $\varepsilon_T = 0.94$ and $\varepsilon_w = 0.92$ to 0.58.

$$q_{rad} \text{ is now } = \frac{-1}{\frac{1}{\varepsilon_T}+\underbrace{\frac{AT}{Aw}\frac{1}{\varepsilon_w}-1}_{\text{neglect}}}\,\sigma(T_T^4-300^4) = \underline{5.33\text{x}10^{-8}(T_T^4-300^4)}$$

The trial and error calculation now gives: $\underline{\underline{T_T = 19.4°C}}$

The black body assumption was very good in this case.

10.7 Two thin aluminum sheets, one polished (ε=0.05) and the other painted black (ε=0.9) are placed horizontally outdoors in 10°C air. $\bar{h} \simeq 5W/m^2$-°C on both the top and bottom. The top is irradiated with 750W/m^2 and it re-radiates to a sky which is at 170°K. The earth beneath each plate is black at 10°C. Find the temperature of each plate. (Assume the sky to be black).

In either case we can write:

$$\underbrace{q_{sun}}_{=750\alpha_{pl.} \; =750\varepsilon_{pl.}} + \varepsilon_{plate}\,\sigma(\underbrace{T^4_{sky}}_{170^4} - T^4_{pl}) + \varepsilon_{pl}\,\sigma(\underbrace{T^4_{earth}}_{283^4} - T^4_{pl}$$

$$= 2\bar{h}(T_{pl} - \underbrace{T_{air}}_{283})$$

Put in the known numerical values for each of the two plates and get the following two equations.

$$\underbrace{37.5 + 20.55 + 2830}_{2888} = 5.67\text{x}10^{-9}\,T^4_{pl_1} + 10T_{pl_1}$$

and

$$\underbrace{675 + 270 + 2830}_{3875} = 1.0206\text{x}10^{-7}\,T^4_{pl_2} + 10T_{pl_2}$$

Trial and error using a pocket calculator gives

$T_{pl_1} = 285°K = \underline{\underline{12°C}}$

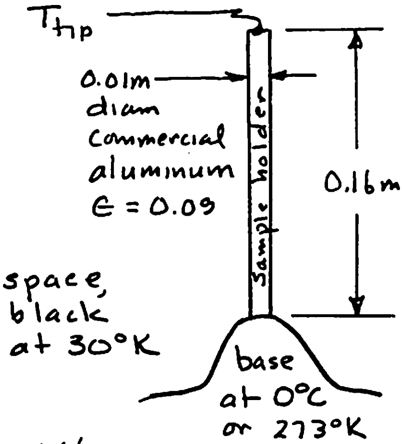
$T_{pl_2} = 302.3°K = \underline{\underline{29.3°C}}$

Notice that the highly reflecting surface is, as we would anticipate, almost unaffected by solar radiation. For this reason, unpainted aluminum rooting is sometimes used in sunny climates.

10.8 Find the tip temperature of the sample holder shown. Assume

$F_{\text{sample holder-sky}} \cong 1$

and take the holder to be a finite fin.



$$h_r = \frac{\sigma \overset{=\epsilon}{\widetilde{\mathcal{F}_{h-s}}} (T_h^4 - T_s^4)}{T_h - T_s} \quad \text{where } T_h \simeq 0°C$$

$$= \frac{5.67 \cdot 10^{-8}(0.09)(273^4 - 30^4)}{(273-30)} = \underline{0.1166 \frac{W}{m^2\text{-}°C}}$$

Now for the fin: $mL = \sqrt{\dfrac{0.1166(\pi \times 0.01)}{204(\pi \times 0.005^2)}}\,(0.16) = 0.0765$

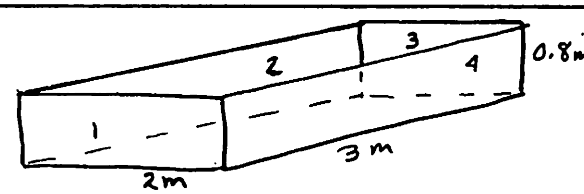
Then $\dfrac{T_{tip} - T_\infty}{T_{root} - T_\infty} = \dfrac{1}{\cosh mL} = \dfrac{2}{e^{0.0765} + e^{-0.0765}} = 0.9971$

so $T_{tip} = 0.9971(273-30)+30 = 272.3\ °K$

or $\underline{\underline{T_{tip} = -0.7\,°C}}$

(Note that with so little temperature drop, it is justifiable to base h_r on a constant fin temperature.)

10.9 Find the percentages of leaving the bottom of the box that reach sides 1, 2, 3, 4, and the top.



These percentages are equal to $F_{\text{bottom-1}}$, F_{b-2}, F_{b-3}, F_{b-4}, and $F_{b\text{-top}}$.

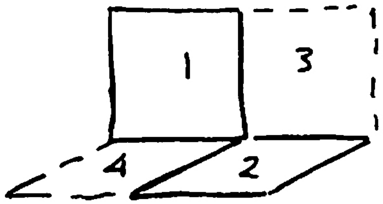
From Fig.10.9, $a/c = \frac{2}{0.8} = 2.5$, $b/c = \frac{3}{0.8} = 3.75$, $\underline{\underline{F_{b-t} = 0.53}}$

From Fig. 10.9, $h/\ell = \frac{0.8}{3} = 0.267$, $w/\ell = \frac{2}{3} = 0.667$, $\underline{\underline{F_{b-2} = F_{b-4} = 0.14}}$

From Fig. 10.9, $h/\ell = \frac{0.8}{2} = 0.4$, $w/\ell = \frac{3}{2} = 1.5$, $\underline{\underline{F_{b-1} = F_{b-3} = 0.095}}$

Check the result $\sum_{n=0}^{5} F_{b-n} = 0.53 + 2(.14) + 2(0.095) = 1.00$

PROBLEM 10.10 Consider Fig. (10.11). Find $F_{1-(2+4)}$ and $F_{(2+4)-1}$.



SOLUTION First note that $F_{1\text{-}(24)} = F_{1\text{-}4}$. We use shape factor algebra to break this down.

$$F_{4\text{-}(123)} = F_{4\text{-}1} + F_{4\text{-}2} + F_{4\text{-}3}$$
$$= 2F_{4\text{-}1} + F_{4\text{-}2}$$
$$\frac{A_{(123)}}{A_4} F_{(123)\text{-}4} = 2\frac{A_1}{A_4} F_{1\text{-}4} + \frac{A_2}{A_4} F_{2\text{-}4}$$

Divide through by $2A_1/A_4$ and rearrange:

$$F_{1\text{-}4} = \frac{A_{(123)}}{2A_1} F_{(123)\text{-}4} - \frac{A_2}{2A_1} F_{2\text{-}4}$$

$F_{2\text{-}4}$ is the subject of Problem 10.11, and the answer is $F_{2\text{-}4} = 0.255$. The other shape factor may be found using Fig. 10.9, letting surface 4 be the h surface:

$$F_{(123)\text{-}4} = 0.23 \quad \text{with} \begin{cases} h/l = 0.5/1.2 = 0.42 \\ w/l = 0.6/1.2 = 0.5 \end{cases}$$

Hence

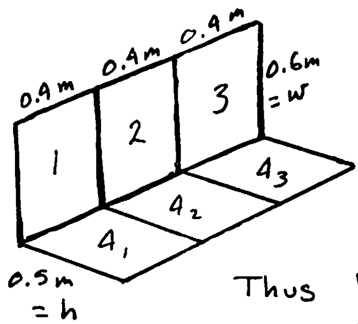
$$F_{1\text{-}4} = \frac{(1.2)(0.6)}{2(0.4)(0.6)}(0.23) - \frac{1}{2}(0.255) = 0.218$$

Then,

$$F_{(24)\text{-}1} = F_{(4)\text{-}1} = \frac{A_1}{A_4} F_{1\text{-}4} = \frac{(0.4)(0.6)}{(1.2)(0.5)}(0.218) = 0.087$$

10.11 Find F_{2-4} for the situation shown.

To solve this problem, we make use of Fig. 10.10. But notice that the upright and horizontal surfaces are inverted here.



First find F_{2-4_1} : $2F_{12-4_14_2} = \underbrace{F_{1-4_1} + F_{2-4_2}}_{=2F_{2-4_2}} + \underbrace{F_{2-4_1} + F_{1-4_2}}_{=2F_{2-4_1}}$

but from Fig.10.10②, $\left.\begin{array}{l} h/\ell = \frac{0.5}{0.4} = 1.25 \\ w/\ell = \frac{0.6}{0.4} = 1.5 \end{array}\right\} F_{2-4_2} = 0.165$

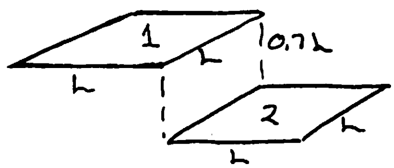
and : $\left.h/\ell = \frac{0.5}{0.8} = 0.625,\ w/\ell = \frac{0.6}{0.8} = 0.75\right\} F_{12-4_14_2} = 0.21$

Thus $F_{2-4_1} = 0.21 - 0.165 = \underline{0.045}$

Then $F_{2-4} = F_{2-4_14_24_3} = F_{2-4_1} + F_{2-4} + F_{2-4_3} = 0.045 + 0.165 + 0.045$

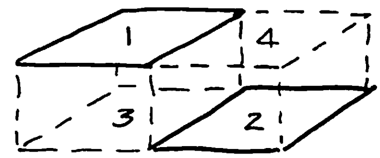
$= \underline{\underline{0.255}}$

10.12 Find F_{1-2} for the configuration shown.



Find F_{1-2}

Set up fictitious planes thus:



Therefore: $$A_{14} F_{14-32} = \underbrace{A_1}_{A_4} \left(\underbrace{F_{1-3}}_{F_{4-2}} + \underbrace{F_{1-2}}_{F_{4-3}} + F_{4-3} + F_{4-2} \right)$$

So: $$2F_{1-3} + 2F_{1-2} = 2F_{14-32}$$

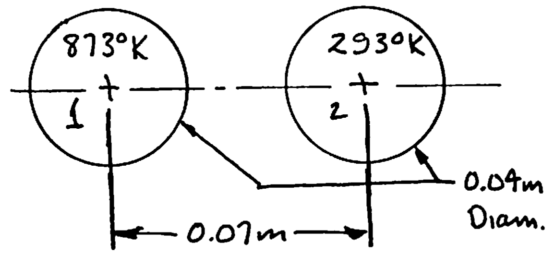
From Fig. 10.9 we then read ①:

$$\left.\begin{array}{l} \frac{a}{c} = \frac{1}{0.7} = 1.43 \\ \frac{b}{c} = \frac{1}{0.7} = 1.43 \end{array}\right\} \text{so } F_{1-3} = 0.3$$

$$\left.\begin{array}{l} \frac{a}{c} = \frac{2}{0.7} = 2.86 \\ \frac{b}{c} = \frac{1}{0.7} = 1.43 \end{array}\right\} \text{so } F_{14-32} = 0.38$$

It follows that: $F_{1-2} = \dfrac{2F_{14-32} - 2F_{1-3}}{2} = \underline{\underline{0.08}}$

10.13 Compute the net heat transfer between the black cylinders shown.



First find F_{1-2}. From Table 10.2 we read:

we read:

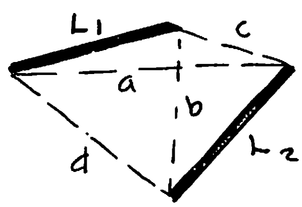
$$F_{1-2} = \frac{1}{\pi}\left[\sqrt{\left(1+\frac{0.03}{0.04}\right)^2 - 1} + \sin^{-1}\frac{1}{1+\frac{0.03}{0.04}} - 1 - \frac{0.03}{0.04}\right]$$

$$\underline{F_{1-2} = 0.0937}$$

so: $Q = (\pi D)(0.0937)\sigma\left(T_1^4 - T_2^4\right) = \pi(0.04)(0.0937)5.67\left(8.73^4 - 2.93^4\right)$

$$= 383 \frac{W}{m \text{ of length}}$$

10.14 Develop the string method for evaluating F_{1-2} between two-dimensional surfaces.



Noting that areas are proportional to the distances shown we write view factors for the two triangles, L_1-a-c, and L_1-b-d, using case 4 in Table 10.2.

Thus: $F_{1-c} = \dfrac{L_1 + c - a}{2L_1}$ and $F_{1-d} = \dfrac{L_1 + d - b}{2L_1}$

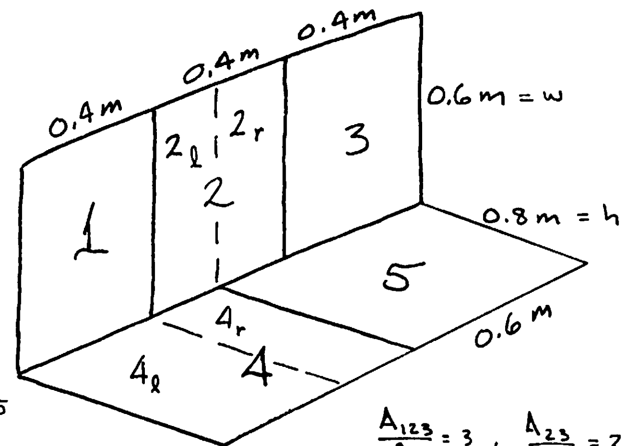
Then, since $F_{1-2} = 1 - F_{1-c} - F_{1-d}$,

$$F_{1-2} = \frac{\cancel{2L_1}}{2L_1} - \frac{\cancel{L_1} + c - a + \cancel{L_1} + d - b}{2L_1} = \frac{(a+b) - (c+d)}{2L_1}$$

So it would be possible to obtain F_{1-2} by comparing the difference between the lengths of the crossed strings, (a + b), and the edge strings, (c + d), with $2L_1$. Hottel and Sarofim [10.15] show that this will also work if L_1 and L_2 are curved in complicated ways.

(Note: If the student is not clever in attacking this problem, he can easily embark on some pretty complicated, albeit correct, strategies.)

10.15 Find F_{1-5} for the configuration shown:



We shall write:

$$F_{1-5} = F_{1-4_r5} - F_{1-4_r}$$

Then:

$$A_1F_{1-4_r5} = A_{123}F_{123-45} - A_{23}F_{23-4_r5} - \underbrace{A_{23}F_{23-4\ell}}_{=\underbrace{A_{4\ell}}_{=A_1}\underbrace{F_{4\ell-23}}_{=F_{1-4_r5}}} - A_1F_{1-4\ell}$$

$$\frac{A_{123}}{A_1} = 3, \quad \frac{A_{23}}{A_1} = 2$$

$$\frac{A_{12\ell}}{A_1} = \frac{3}{2}, \quad \frac{A_{2\ell}}{A_1} = \frac{1}{2}$$

so $\underline{F_{1-4_r5} = \frac{3}{2}F_{123-45} - F_{23-4_r5} - \frac{1}{2}F_{1-4_\ell}}$

Likewise

$$A_1F_{1-4_r} = A_{12_\ell}F_{12_\ell-4} - A_1F_{1-4_\ell} - \underbrace{A_{2\ell}F_{2\ell-4\ell}}_{A_{4\ell}F_{1-4_r}} - A_{2\ell}F_{2_\ell-4_r}$$

so $\underline{F_{1-4_r} = \frac{3}{4}F_{12_\ell-4} - \frac{1}{2}F_{1-4\ell} - \frac{1}{4}F_{2_\ell-4_r}}$

Finally:

$$F_{1-5} = F_{1-4_r5} - F_{1-4_r} = \underline{\frac{3}{2}F_{123-45} - F_{23-4_r5} - \frac{3}{4}F_{12_\ell-4} + \frac{1}{4}F_{2\ell-4_r}}$$

now from Fig. 10.9 ②

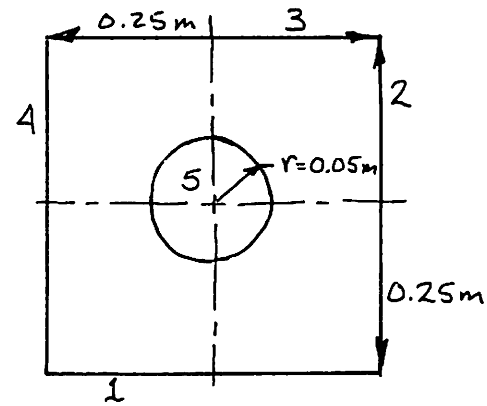
- at $h/\ell = \frac{0.8}{1.2} = 0.67$, $w/\ell = \frac{0.6}{1.2} = 0.5$, $F_{123-45} = 0.265$
- " " $= \frac{0.8}{0.8} = 1.0$, " $= \frac{0.6}{0.8} = 0.75$, $F_{23-4_r5} = 0.23$
- " " $= \frac{0.8}{0.6} = 1.333$, " $= \frac{0.6}{0.6} = 1.0$, $F_{12_\ell-4} = 0.218$
- " " $= \frac{0.8}{0.2} = 4$, " $= \frac{0.6}{0.2} = 3$, $F_{2_\ell-4_r} = 0.125$

Thus:

$$F_{1-5} = \frac{3}{2}(0.265) - 0.23 - \frac{3}{4}(0.218) + \frac{1}{4}(0.125) = \underline{\underline{0.035}}$$

This result could easily suffer 10 or 20 % error from accumulative inaccuracy of graph reading. Notice, too, that without recognizing some tricks in manipulating F's, one could have a hard time solving this one.

10.16 Find $F_{1-2,3,4}$ for the configuration shown.



$$F_{1-2,3,4} = F_{1-2,3,4}\Big|_{\text{with no blockage by the cyl. 5}} - F_{1-5}$$

However: (equation (10.12))

$$F_{1-2,3,4} = F_{1-2} + F_{1-3} + F_{1-4} = 1$$

To get F_{1-5} we use eqn. (5) in Table 10.2, with

$r = 0.05\,\text{m}$, $b = 0.125\,\text{m}$, $c = 0.125\,\text{m}$

and $a = -0.125$ (notice the important inclusion of a minus sign in front of a.)

SO:

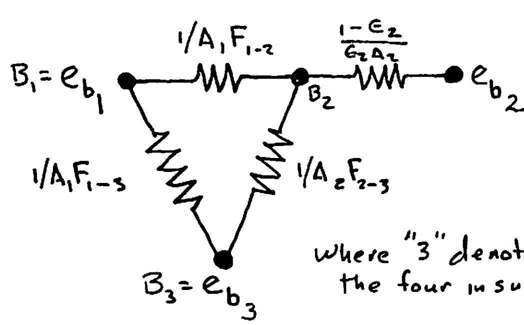
$$F_{1-5} = \frac{0.05}{0.125-(-0.125)}\left[\tan^{-1}\frac{0.125}{0.125} - \tan^{-1}\frac{(-0.125)}{0.125}\right] = 0.3142$$

Then:

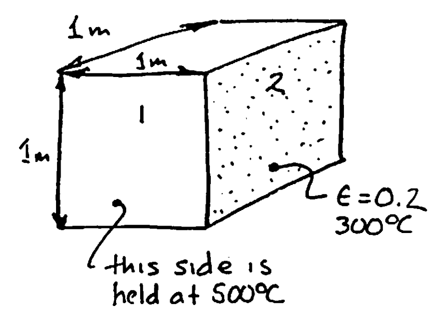
$$F_{1-2,3,4} = 1 - F_{1-5} = \underline{\underline{0.6858}}$$

(Some students will use Table 11.2 to calculate the unblocked values of $F_{1-2} = F_{1-4} = 0.293$ and $F_{1-3} = 0.4142$ and only then discover that $0.293 + 0.293 + 0.414 = 1$.)

10.17 All sides of the box, except 1 and 2 are insulated. Find Q_{1-2}.



where "3" denotes the four insulated sides.



From eqn. (11.31):

$$Q_{1-2} = \frac{e_{b_1} - e_{b_2}}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1} + \dfrac{1}{\dfrac{1}{\dfrac{1}{A_1F_{1-3}} + \dfrac{1}{A_2F_{2-3}}} + \dfrac{1}{\dfrac{1}{A_1F_{1-2}}}} + \dfrac{1-\epsilon_2}{\epsilon_2 A_2}}$$

but $\epsilon_1 = 1$ so $\dfrac{1-\epsilon_1}{\epsilon_1 A_1} = 0$ and from Fig. 10.9, case (2) we read

10.17 (continued)

for $a/c = b/c = 1$, $F_{1-2} = 0.2$. It follows that $F_{1-3} = 0.8$ and, by symmetry, $F_{2-3} = 0.8$. Then, since $A_1 = A_2 = 1$ and $A_3 = 4$, the equation gives:

$$Q_{1-2} = \frac{5.67(10)^{-8}\left(773^4 - 573^4\right)}{\dfrac{1}{\dfrac{1}{\dfrac{1}{1(0.8)} + \dfrac{1}{1(0.8)}} + \dfrac{1}{\dfrac{1}{1(0.2)}}} + \dfrac{1-0.2}{0.2(1)}} = \underline{\underline{2494\text{ W}}}$$

Note: One could, alternatively, write the three nodal equations:

Node 1 : $e_{b_1} A_1 = \dfrac{5.67\times10^{-8}(773^4) - B_2}{\dfrac{1}{1(0.2)}} + \dfrac{5.67\times10^{-8}(773^4) - B_3}{\dfrac{1}{1(0.8)}}$

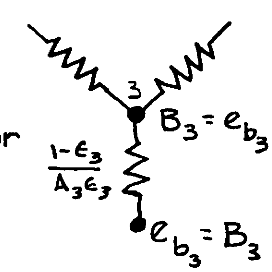
Node 2 : $\dfrac{5.67\times10^{-8}(573^4) - B_2}{\dfrac{1-0.2}{1(0.2)}} = \dfrac{B_2 - 5.67\times10^{-8}(773^4)}{\dfrac{1}{1(0.2)}} + \dfrac{B_2 - B_3}{\dfrac{1}{1(0.8)}}$

Node 3 : B_2 and B_3 are already specified. This equation is redundant.

solve for $B_2 = 16{,}088$ & $B_3 = 18{,}166$ Then $Q = \dfrac{B_1 - B_2}{\dfrac{1}{1(0.2)}} + \dfrac{B_3 - B_2}{\dfrac{1}{1(0.8)}} = \underline{\underline{2494\text{ W}}}$

10.18 Find Q_{1-2} and $T_{ins.-walls}$ for Problem 10.17 is $\epsilon_{ins.\ wall}$ is 0.6, and if it is 1.0.

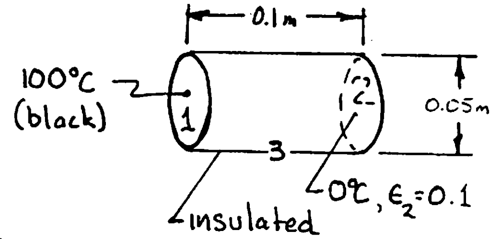
Note that, since node 3 is at an insulated wall, there is no heat flow across the thermal resistance, $(1 - \epsilon_3)/A_3\epsilon_3$. Thus $e_{b,3} = B_3$ and ϵ_3 is irrelevant to the determination of either Q_{1-2} or $T_{ins.-wall}$. With reference to the solution of Problem 11.17, we can immediately write, for $\epsilon_{ins.-wall}$ equal to either 1.0 or 0.6:



$$Q_{1-2} = \underline{\underline{2494\text{ W}}}$$

$$T_{ins.\,wall} = \sqrt[4]{\frac{e_{b_3}}{\sigma}} = \sqrt[4]{\frac{B_3}{\sigma}} = \sqrt[4]{\frac{18{,}166}{5.67(10)^{-8}}} = \underline{\underline{752\,\text{K} = 479^\circ\text{C}}}$$

10.19 Find F_{1-3} within the insulated cylinder shown.

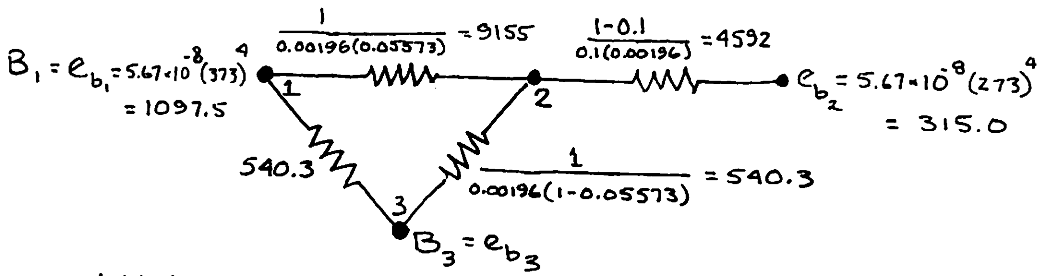


First find F_{1-2}:

$R_1 = \frac{2.5}{10} = R_2 = 0.25$, $X = 1 + \frac{1+0.25^2}{0.25^2} = 18$

so $F_{1-2} = \frac{1}{2}\left[18 - \sqrt{18^2 - 4}\right] = \underline{0.05573}$

Furthermore: $A_1 = A_2 = \frac{\pi}{4}(0.05)^2 = 0.00196\,m^2$, $A_3 = \pi(0.05)(0.1) = 0.0157\,m^2$



nodal balances:

$$1: \quad 0 = \frac{1097.5 - B_2}{9155} + \frac{1097.5 - B_3}{540.3}$$

$$2: \quad \frac{315 - B_2}{4592} = \frac{B_2 - 1097.5}{9155} + \frac{B_2 - B_3}{540.3}$$

$$3: \quad 0 = \frac{B_3 - 1097.5}{540.3} + \frac{B_3 - B_2}{540.3}$$

so we use the second two of these three equations:

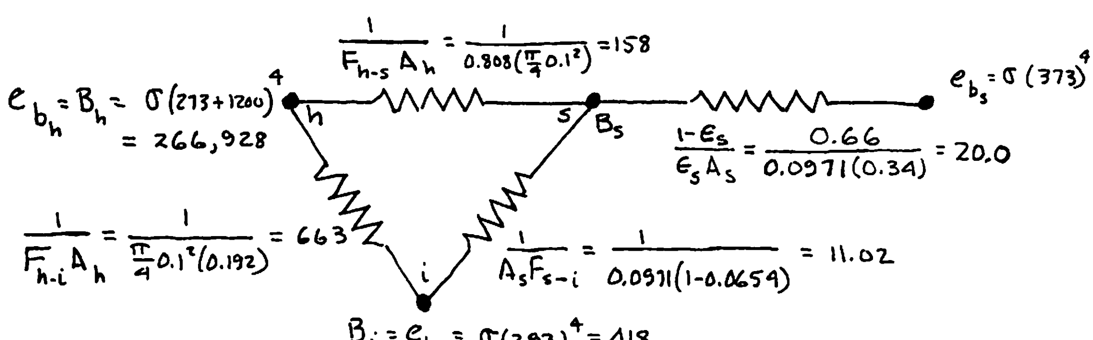
$B_3 = 548.8 + B_2/2$ & $37.06 - 0.1171B_2 - 0.05902B_2 + 64.77 - B_2 + 548.8 + B_2/2 = 0$

so. $B_2 = 961.45$, $B_3 = e_{b_3} = 1029.5$

and $T_{cylinder} = \sqrt[4]{e_{b_3}/\sigma} = \underline{\underline{367.1\,°K = 94\,°C}}$

and: $Q_{net} = \frac{B_1 - B_2}{9155} + \frac{B_3 - B_2}{540.3} = \underline{\underline{0.1408\,W}}$

10.20 Rework Example 10.3 if $\epsilon_{shield} = 0.34$. (Refer to the text for the sketch and numbers.)



Since neither h or i is adiabatic, we can only write a nodal energy balance a node s:

10.20 (continued)

$$0 = \frac{B_s - 1098}{20} + \frac{B_s - 418}{11.02} + \frac{B_s - 266{,}928}{158}$$

or

$$0 = 0.14707\, B_s - 1782.25; \qquad \underline{B_s = 12{,}118}$$

Then:

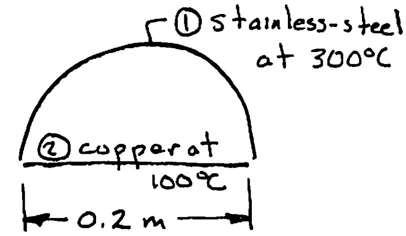
$$Q_{h-s} = \frac{B_h - B_s}{\frac{1}{A_h F_{h-s}}} = \frac{266{,}928 - 12{,}118}{158} = \underline{\underline{1613 \text{ W}}}$$

$$Q_{h-i} = \frac{B_h - B_i}{\frac{1}{A_h F_{h-i}}} = \frac{266{,}928 - 418}{663} = \underline{402 \text{ W}}$$

$$Q_{s-i} = \frac{B_s - B_i}{\frac{1}{A_s F_{s-i}}} = \frac{12{,}118 - 418}{11.02} = \underline{1062 \text{ W}}$$

Thus the net cooling of the shield must be $Q_{h-s} - Q_{s-i} = \underline{\underline{551 \text{ W}}}$

10.23 Calculate the heat transfer between the some shown and its base, by radiation. ($\epsilon_{s.s} = 0.4$ and $\epsilon_{cu} = 0.15$)



$A_1 = 2\pi R^2$, $A_2 = \pi R^2$, $A_2/A_1 = 1/2$

$$Q = \frac{\epsilon_1 A_1}{1-\epsilon_1}(\sigma T_1^4 - B_1)$$

so we need B_1. To get it, write:

$$B_1 = \epsilon_1 \sigma T_1^4 + (1-\epsilon_1)[B_1 F_{1-1} + B_2 F_{1-2}]$$
$$B_2 = \epsilon_2 \sigma T_2^4 + (1-\epsilon_2)[B_2 F_{2-2} + B_1 F_{2-1}]$$

$F_{1-1} + F_{1-2} = 1$

$F_{2-1} + \underbrace{F_{2-2}}_{=0} = 1 \Rightarrow F_{2-1} = 1$

$A_1 F_{1-2} = A_2 F_{2-1}$, $F_{1-2} = \frac{A_2}{A_1} F_{2-1}$

$F_{1-2} = \frac{1}{2}$

$F_{1-1} = 1 - F_{1-2} = \frac{1}{2}$

so:

$$B_1[1-(1-\epsilon_1)F_{1-1}] + B_2[-F_{1-2}(1-\epsilon_1)] = \epsilon_1 \sigma T_1^4$$
$$B_1[-F_{2-1}(1-\epsilon_2)] + B_2[1-(1-\epsilon_2)F_{2-2}] = \epsilon_2 \sigma T_2^4$$

or:

$$0.7B_1 - 0.3B_2 = 0.4\sigma T_1^4$$
$$-0.85B_1 + 1 \cdot B_2 = 0.15\sigma T_2^4$$

Therefore:

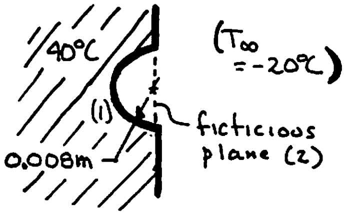
$$B_1 = \frac{\begin{vmatrix} 0.4\sigma T_1^4 & -0.3 \\ 0.15\sigma T_2^4 & 1 \end{vmatrix}}{0.7 - 0.3(0.85)} = \frac{0.4\sigma T_1^4 - 0.045\sigma T_2^4}{0.445}$$

so

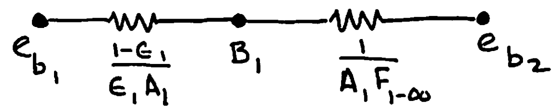
$$Q_1 = 2\pi(0.1)^2 \frac{0.4}{1-0.4} 5.67 \cdot 10^{-8}\left[(573)^4\left(1-\frac{0.4}{0.445}\right) - \frac{0.045}{0.445}(373)^4\right]$$

$$\underline{\underline{Q_1 = 21.24\ \mathrm{W}}}$$

10.24 A hemispherical indentation in a smooth wrought iron plate has a 0.008 m radius. How much heat radiates from the 40°C dent to the −20°C surroundings?



$A_1F_{1-2} = A_2\underbrace{F_{2-1}}_{=1}$; $F_{1-2} = F_{1-\infty}$; $\underline{F_{1-\infty} = \dfrac{A_2}{A_1}}$



so :

$$Q = \frac{e_{b_1} - e_{b_2}}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1} + \dfrac{1}{A_1F_{1-\infty}}} = \frac{1}{(1-\epsilon_1)\frac{A_2}{A_1}+\epsilon_1}\underbrace{\epsilon_1 A_2 \sigma\left(T_1^4 - T_\infty^4\right)}_{\text{heat flow without a dent}}$$

$$= \underbrace{\frac{1}{(1-0.35)\dfrac{\pi(0.008)^2}{2\pi(0.008)^2}+0.35}}_{\text{1.48 times the heat flow that would occur with no dent.}} 0.35(\pi[0.008]^2)5.67(10)^{-8} \times [313^4 - 253^4]$$

so :

$$\underline{\underline{Q = 0.0325\ \text{W}}}$$

10.25 A conical hole in a block of metal, for which ϵ = 0.5, is 5 cm in diameter at the surface and 5 cm deep. By what factor will the radiation from the area of the hole be changed by the presence of the hole?

(This following solution breaks down if the cone is very deep and slender since the the apex recieves little and we cannot use the network analogy.)

From the solution to problem 10.24 we find that

$$\frac{Q\ \text{with hole}}{Q\ \text{no hole}} = \frac{1}{(1-\epsilon_1)\frac{A_2}{A_1}+\epsilon_1} = \frac{1}{0.5\dfrac{\pi(2.5)^2}{43.9}+0.5} = \underline{\underline{1.382}}$$

where the area of the cone is the product of its average circumference, 2.5π, and its slant height, 5/cos(tan^{-1}2.5/5) = 43.9 cm^2.

10.26 A single-pane window in a large room is 4 ft wide and 6 ft high. The room is kept at 70°F but the pane is at 67°F owing to heat loss to the colder outdoor air. Find: a) the heat transfer by radiation to the window; b) the heat transfer by natural convection to the window; and c) the fraction of heat transferred to the window by radiation.

a) $$\mathcal{F}_{\text{window to room}} = \frac{1}{\dfrac{1}{\epsilon_{\text{window}}} + \underbrace{\dfrac{A_{\text{window}}}{A_{\text{room}}}}_{\simeq 0}\left(\dfrac{1}{\epsilon_{\text{room}}} - 1\right)} = \epsilon_{\text{window}} = \underline{0.94}$$

$$Q_{rad_{\text{window to room}}} = \epsilon_{\text{window}}\,\sigma\,A_{\text{window}}\left(T^4_{\text{window}} - T^4_{\text{room}}\right)$$
$$= 0.94(5.67)10^{-8}(4\times 6)(0.3048)^2\left[292.59^4 - 294.26^4\right]$$

$$Q_{rad_{\text{room to window}}} = -Q_{rad_{\text{window to room}}} = \underline{\underline{20.06\text{ W} = 68.43\ \frac{\text{Btu}}{\text{hr}}}}$$

b) Use eqn. (8.13a) with $Ra_L = \dfrac{9.8(1/294.26)(1.67)(6\times 0.3048)^3}{(1.508\times 10^{-5})(2.117\times 10^{-5})} = \underline{1.066\times 10^9}$,

and $Pr = \underline{0.713}$:

$$\overline{Nu_L} = 0.68 + 0.67\,Ra_L^{1/4}\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{-4/9} = 93.64$$

so
$$\bar{h} = \overline{Nu_L}\,k/L = 93.64(0.02562)/6(0.3048) = 1.31\ \frac{\text{W}}{\text{m}^2\text{-}^\circ\text{C}}$$

(this is a <u>very</u> low $\bar{h}$.)

Then: $$Q_{conv.} = \bar{h}A(\Delta T) = 1.31(6\times 4)(0.3048)^2(1.67)$$
$$= \underline{\underline{4.88\text{ W} = 16.64\ \frac{\text{Btu}}{\text{hr}}}}$$

c) % of heat transfer by radiation $= \dfrac{20.06}{20.06+4.88} = 0.804 = \underline{\underline{80.4\%}}$

10.27 Suppose the window-pane temperature is unknown in Problem 10.26. The outdoor air is at 40°F and h = 62 W/m²-°C on the outside. It is night and the effective T_{sky} = 15°C. Assume $F_{window-sky}$ = 0.5 and the other surroundings are at 40°C. Evaluate T_{window} and draw the analogous electric circuit evaluating the thermal resistances. (The window is opaque to infra-red radiation but it offers little resistance to conduction so T_{window} is approximately uniform.)

$$(Q_{rad.} + Q_{nat'l.\,conv.})_{indoors} = (Q_{conv.} + Q_{rad.\,sky} + Q_{rad\,\substack{other\\surr.}})_{outdoors}$$

$$\epsilon_{glass} A_w \sigma(T_i^4 - T_w^4) + h_i A_w (T_i - T_w) = \bar{h}_o A_w (T_w - T_o) + \epsilon_g F_{w-s} \sigma A_w (T_w^4 - T_s^4) + \epsilon_g F_{w-o.s.} \sigma A_w (T_w^4 + T_s^4)$$

Let's cut through a potentially terrible lot of trial and error computation by noting that natural convection is not going to be very important on the inside. (We take our cue in this from the solution of Problem 10.26. Therefore we'll guess that h_i is 3 W/m²-°C and correct this assumption later if we must. Then divide that equation above by $\epsilon_g \sigma A_w$ and get (in S.I. units):

$$294.24^4 - T_w^4 + \frac{3(10)^8}{0.94(5.67)}(294.4 - T_w) = \frac{62(10)^8}{0.94(5.67)}(T_w - 277.6) + \frac{1}{2}(2T_w^4 - 277.6^4 - 263.7^4)$$

or

$$7.496(10)^9 - T_w^4 + 339.5(10)^9 - 1.22(10)^9 T_w = T_w^4 - 5.387(10)^9$$

so:

$$T_w^4 + 0.61(10)^9 T_w = 176.2(10)^9$$

solving by trial and error we get: $T_w = 279.0°K$

$= 42.53°F$

Now check $\bar{h}_{nat'l\,conv.}$ using eqn. (8.13a). (Details are given in solution to Problem 10.26.)

$$Ra_L = \underbrace{1.066 \times 10^9}_{\text{when } \Delta T = 1.67°C} \frac{294.24 - 279.0}{1.67} = 9.728(10)^9$$

so:

$$\overline{Nu}_L = 0.68 + 0.67\, Ra_L^{1/4}\left[1 + \left(\frac{0.492}{.713}\right)^{0.5625}\right]^{-0.4444} = 162.3$$

and $\bar{h} = 162.3(0.0256)/6(0.3048) = \underline{2.27\ W/m^2\text{-}°C}$

Going back through the trial and error solution based on this value of $\bar{h}$ we get 42.35°F -- almost no change.

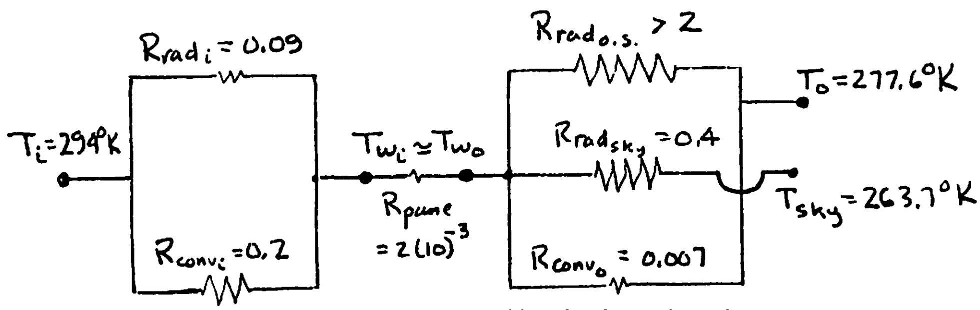
so $\underline{\underline{T_w = 42.35°F}}$

10.27 (continued)

Next calculate resistances: $R_{rad_i} = \frac{1}{h_{rad_i} A_w} = \frac{\Delta T}{Q_{rad_i}} = \frac{15.2}{170.7} = \underline{\underline{0.0893 \frac{°C}{W}}}$

$R_{rad_{sky}} = \frac{30.54}{72.71} = \underline{\underline{0.420}}$; $R_{rad_{o.s.}} = \frac{16.64}{7.17} = \underline{\underline{2.32}}$; $R_{conv_o} = \frac{1}{h_o A_w} = \underline{\underline{0.00723}}$,

the hitherto neglected $R_{pane} = \frac{\text{thickness}}{k A_w} = \underline{\underline{0.001725}}$, & $R_{conv_i} = \underline{\underline{0.1975}}$.



Heat is basically short-circuited to the window by radiation.

Heat is short-circuited to the outdoors by convection.

So convection is irrelevant inside. Radiation is irrelevant outside. And conduction through the glass causes no ΔT.

10.28 An effective low-temperature insulation is made by evacuating the space between metal sheets. Calculate q between 150°K and 100°K for: (a) two sheets of highly polished al., (b) three sheets of highly polished al., and (b) three sheets of rolled sheet steel.

In all cases, $\mathcal{F}_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$ or $\frac{1}{\frac{1}{\varepsilon_1} + 1\left(\frac{1}{\varepsilon_2} - 1\right)} = \frac{1}{\frac{2}{\varepsilon} - 1}$

Case a.) $\mathcal{F}_{1-2} = \frac{1}{\frac{2}{0.04} - 1} = 0.02041$ (where we use ε_{al} for a higher temp.)

$$q = \mathcal{F}_{1-2}\,\sigma\left(T_1^4 - T_2^4\right) = \underbrace{0.02041(5.67)10^{-8}}_{1.157(10)^{-9}}\left(150^4 - 100^4\right) = \underline{\underline{0.470 \frac{W}{m^2}}}$$

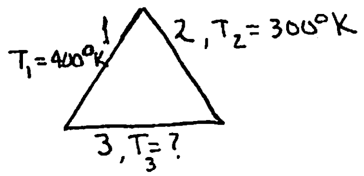
b.) $q = 1.157(10)^{-9}\left[150^4 - T_{middle}^4\right] = 1.157(10)^{-9}\left[T_{middle}^4 - 100^4\right]$

$$T_{middle} = \left[\left(150^4 + 100^4\right)/2\right]^{1/4} = \underline{131.95°K}$$

$$q = 1.157(10)^{-9}\left[150^4 - 131.95^4\right] = \underline{\underline{0.235 \frac{W}{m^2}}}$$

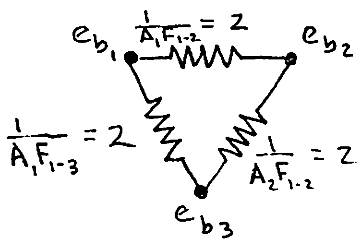
c.) $\mathcal{F}_{1-2} = \frac{1}{\frac{2}{0.66} - 1} = 0.4925$, $q = 0.4925\,\sigma\left[131.95^4 - 100^4\right] = \underline{\underline{5.67 \frac{W}{m^2}}}$

10.29 Three parallel black walls, 1m wide, form an equilateral triangle. One wall is held at 400°K, one at 300°K, and the third is insulated. Find Q W/m and the temperature of the third wall.



By symmetry, we see: $F_{1-2} = F_{1-3} = F_{2-1} = F_{2-3} = \frac{1}{2}$

$$\& \quad e_{b_1} = \sigma 400^4 = 1452 \text{ W/m}^2$$
$$e_{b_2} = \sigma 300^3 = 459.3 \text{ W/m}^2$$



Using equation (10.35) we get:

$$Q = \frac{1452 - 459.3}{\left(\frac{1}{2+2} + \frac{1}{2}\right)^{-1}} = \underline{\underline{744 \text{ W/m}}}$$

And to get e_{b_3} we write for the node #3;

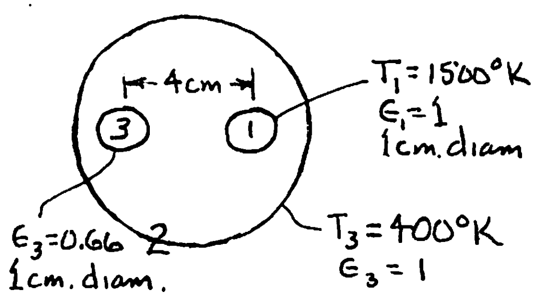
$$0 = \frac{e_{b_1} - e_{b_3}}{2} + \frac{e_{b_2} - e_{b_3}}{2}$$

or

$$e_{b_3} = (e_{b_1} + e_{b_2})/2 = 955.7 = \sigma T_3^4$$

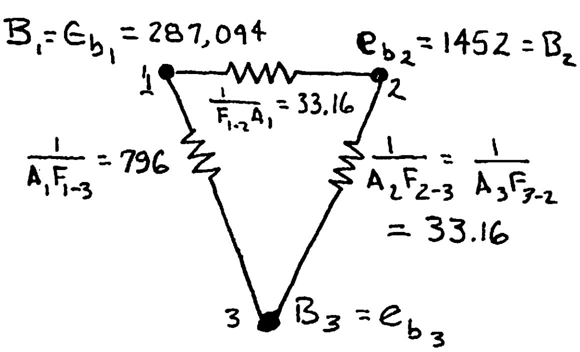
so $\underline{\underline{T_3 = 360.3\,°K}}$

10.30 Two 1cm diameter rods run parallel with centers 4 cm apart. One is at 1500°K and black. The other is unheated and ε = 0.66. They are both encircled by a cylindrical black radiation shield at 400°K. Evaluate Q W/m and the temperature of the unheated rod.



From Table 10.2 #6, $X = 1 + \frac{3}{1} = 4$

$$F_{1-3} = \frac{1}{\pi}\left[\sqrt{15} + \sin^{-1}\frac{1}{4} - 4\right] = \underline{0.0400}$$
$$= F_{3-1}$$
$$F_{1-2} = 1 - F_{1-3} = \underline{0.960} = F_{3-2}$$
$$A_1 = A_3 = 0.031416 \text{ m}^2\text{/m}$$



Using equation (10.35) we get

$$q_{1-2} = \frac{287{,}044 - 1452}{\dfrac{1}{\frac{1}{796+33.16} + \frac{1}{33.16}}} = \underline{8957 \frac{\text{W}}{\text{m}}}$$

$$Q = 0.031416(8957) = \underline{\underline{281.4 \frac{\text{W}}{\text{m}}}}$$

Now at Node #1, $q = -8957 = \frac{B_2 - B_1}{33.16} + \frac{B_3 - B_1}{796}$

so $B_3 = 12{,}858 = \sigma T_3^4$

$\underline{\underline{T_3 = 690°K}}$

(We note that ϵ_3 does not enter the problem.)

10.31 A small diameter heater is centered in a large cylindrical shield. Discuss the relative improtance of the emittance of the shield during specular and diffuse radiation.

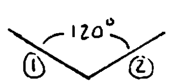
In this case $A_{inside}/A_{outside}$ is very small so (see eqn. (10.30):

$$\mathcal{F}_{1-2}\Big|_{diffuse} \Rightarrow \frac{1}{\frac{1}{\varepsilon_1}+0} = \varepsilon_1 \; ; \quad \mathcal{F}_{1-2}\Big|_{specular} = \frac{1}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

In the pure diffuse limit, ϵ_2 is irrelevant, while in the pure specular limit, ϵ_1 and ϵ_2 are equally important.

10.32 Two, 1m wide, commercial aluminum sheets are joined at a 120° angle along one edge. The back (or 240°-angle)side is insulated. The plates are both held at 120°C. The 20°C surroundings are distant. What is the net radiant heat transfer from the left hand plate: to the right hand side, and to the surroundings.

The business about heat transfer from left to right is a red-herring. We see at once that symmetry requires this to be zero. We may therefore treat the plates a common heater, calculate Q from both, and then divide it by two.



$\varepsilon_1 = \varepsilon_2 = 0.09$

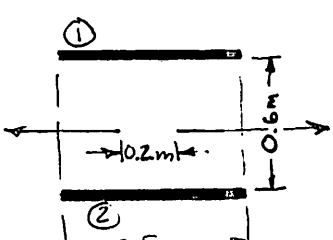
$$A_{1\&2}F_{1\&2-s} = 2A_1F_{1-s} = 2(1-F_{1-2}) = 1.732$$

$$= 1-\sin\frac{120}{2} = 0.134, \text{ from Tab. 10.2, \#2}$$

Then eqn. (11.24) gives:

$$Q = \frac{1}{2}\left[\frac{(5.67)10^{-8}\left([120+273]^4-[20+273]^4\right)}{\frac{1-0.09}{0.09(2)}+\frac{1}{1.732}+\frac{1}{\infty}}\right] = \underline{\underline{82.9\,6W/m}}$$

10.33 Two parallel discs of diameter equal to 0.5m are separated by an infinite parallel plate, midway between them, with a 0.2m diameter hole in it. What is the view factor between the two discs, if they are 0.6 m apart.



$$F_{1-2} = F_{1-hole} = \frac{1}{2}\left(X-\sqrt{X^2-4(R_{hole}/R_1)^2}\right)$$ Table 10.3 No. 3

$$X = 1+\frac{\left(1+\left[\frac{R_{hole}}{h}\right]^2\right)}{(R_1/h)^2} = 1+\frac{1+(0.2/0.6)^2}{(0.5/0.6)^2} = 2.6$$

$$F_{1-hole} = \frac{1}{2}\left(2.6-\sqrt{2.6^2-4(0.2/0.6)^2}\right)$$

$$= \underline{\underline{0.0435}}$$

10.34 An evacuated spherical cavity, 0.3 m in diameter in a zero-gravity environment, is kept at 300°C. Saturated steam at 1 atmosphere is then placed in the cavity. a) What is the initial flux of radiant heat transfer to the steam? b) Determine how long it will take for $q_{conduction}$ to become less than $q_{radiation}$. (Correct for the rising steam temperature if it is necessary to do so.)

In this case: $pL = 0.3$ so at 373°K, $\varepsilon_g = f_1 f_2 = 0.275(1.21) = \underline{0.333}$

$\therefore\ pL\frac{573}{373} = 0.46$ at 573°K, $\alpha_g = \underbrace{\varepsilon_g}_{0.325(1.21)}\left(\frac{373}{573}\right)^{0.45} = \underline{0.324}$

Then $q_{net} = q_{w-g} - q_{g-w} = 0.333\,\sigma(573)^4 - 0.324\,\sigma(373)^4 = \underline{\underline{1680\ W/m^2}}$

and from eqn. (5.54) $\quad q_{cond} = \dfrac{k_{stm}\Delta T}{\sqrt{\pi\alpha_{stm}t}} = \dfrac{0.0237(200)}{\sqrt{\pi\, 2.032\times10^{-5}\, t}}$

Thus q_{cond} will equal 1680 after $\underline{\underline{t = 0.353\ sec}}$

Should we have accounted for temperature rise? It would appear not, but let's check:

Heat capacity of steam $= \rho c_p Vol = 0.597(2030)\frac{4\pi}{3}(0.15)^3$
$= 17.13\ W/°C$

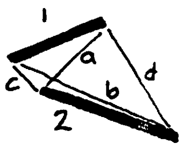
$Q_{rad}\, t = 1680(4\pi(0.15)^2)\, 0.353 = 167.7\ W$

$\int Q_{cond}\, dt = A\underbrace{q(t)}_{1680}\, 2t = 2(167.7)\ W$

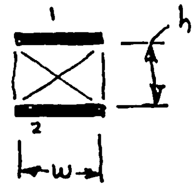
so $\quad \Delta T = \dfrac{Q_{total}}{\text{Heat cap}} = \dfrac{3(167.7)}{17.13} = \underline{29.4°C}$

That's a lot more ΔT than one might first expect, but it's still a small number. We can probably ignore it.

10.35 Verify cases 1, 2, and 3 in Table 10.2 using the "string method" described in Problem 10.14.



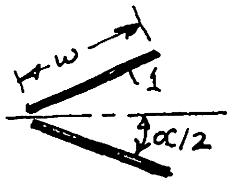
$$F_{1-2} = \frac{(a+b)-(c+d)}{2L_1}$$

1) 

$$c = d = h$$

$$a = b = \sqrt{h^2+w^2}$$

$$F_{1-2} = \frac{(a+b)-(c+d)}{2w} = \underline{\underline{\sqrt{1+\left(\frac{h}{w}\right)^2} - \frac{h}{w}}}$$

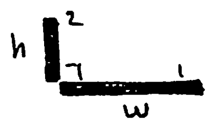
2) 

$$a = b = w$$

$$c = 0$$

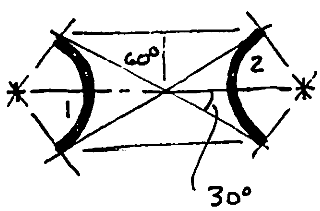
$$d = 2w\sin\frac{\alpha}{2}$$

$$F_{1-2} = \underline{\underline{1 - \sin\alpha/2}}$$

3) 

$$F = \frac{(h+w)-\left(0+\sqrt{h^2+w^2}\right)}{2w} = \underline{\underline{\frac{1}{2}\left[1+\frac{h}{w} - \sqrt{\left(\frac{h}{w}\right)^2+1}\right]}}$$

10.36 Two long parallel heaters consist of 120° segments of 10 cm diameter parallel cylinders, whose centers are 20 cm apart. The segments are those nearest each other, symetrically placed on the line connecting their centers. Find F_{1-2} using the "string method" described in Problem 10.14



$$F_{1-2} = \frac{(a+b)-(c+d)}{2L_1}$$

(see general sketch in solution above.)

$$a = 20\sin 60° = b$$

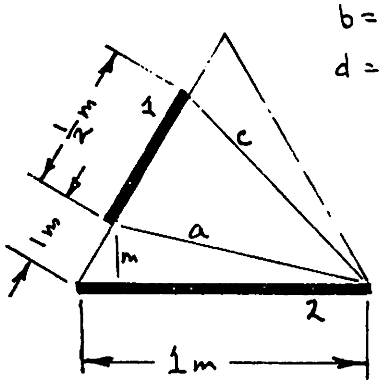
$$c = (20\sin 60°)\sin 60° = d$$

$$L_1 = \frac{1}{3}(\pi \times 10)$$

so: $$F_{1-2} = \frac{40\sin 60° - 40\sin 60° \cdot \sin 60°}{20\pi/3} = \frac{6}{\pi}\sin 60°(1-\sin 60°)$$

$$\underline{\underline{F_{1-2} = 0.2216}}$$

10.37 Two long parallel strips of rolled steel sheet lie along sides of an imaginary 1 m equilateral triangular cylinder. One piece is 1 m wide and kept at 20°C. The other is (1/2) m wide, centered in an adjacent leg, and kept at 400°C. The surroundings are distant and they are insulated. Find Q. (You will need a shape factor. It can be found using the method described in Problem 10.14.)



$b = \frac{3}{4}$ m

$d = \frac{1}{4}$ m

$m = \underbrace{d \sin 60°}_{\frac{1}{4}\,0.866} = \sqrt{a^2 - \underbrace{(1 - \underbrace{d\cos 60°}_{1/8})^2}_{49/64}} = 0.2165$

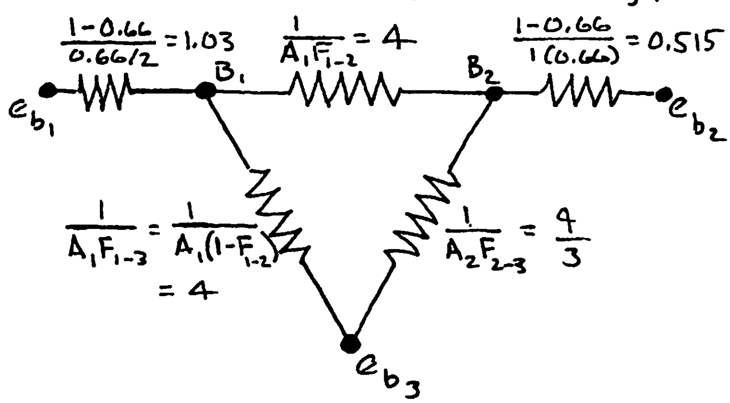
so $a = 0.9014$

& $c = 0.9014$ by symmetry

Then: $F_{1-2} = \dfrac{(a+b)-(c+d)}{2h_1} = \dfrac{0.9014 + 0.75 - 0.9014 - 0.25}{2(0.5)}$

$= \dfrac{0.5}{2(0.5)} = \underline{0.5}$

using $\varepsilon_1 = 0.66 = \varepsilon_2$, & calling the surroundings, 3 :



$A_1 F_{1-2} = A_2 F_{2-1}$

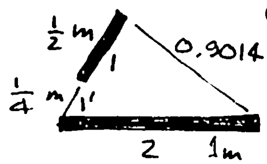
but $F_{2-1} = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$

so $\underline{F_{2-3} = 1 - F_{2-1} = \frac{3}{4}}$

Then eqn. (10.35) gives:

$$Q = \frac{\sigma(673^4 - 293^4)}{1.03 + \dfrac{1}{\dfrac{1}{4+\frac{4}{3}} + \dfrac{1}{4}} + 0.515} = \underline{\underline{2927\ W}}$$

10.38 Find the shape factor from the hot to the cold strip in Problem 11.37 using, not the string method, but Table 10.2. If your instructor asks you to do so, complete Problem 10.37 when you have F_{1-2}.



(see details of dimensions in Problem 10.37 solution.)

Now: $A_{1,1'}F_{1,1'-2} = A_1F_{1-2} + A_{1'}F_{1'-2}$

But from Table 10.2, #4: $F_{1,1'-2} = \dfrac{3/4 + 1 - 0.9014}{2(\frac{3}{4})} = 0.5657$

$F_{1'-2} = \dfrac{1/4 + 1 - 0.9014}{2(1/4)} = 0.6972$

so:

$$\frac{3}{4}(0.5657) = \frac{1}{2}F_{1-2} + \frac{1}{4}(0.6972) \qquad \text{so} \qquad \underline{\underline{F_{1-2} = 0.5}}$$

(From this point forward, details of solution are given in Prob 10.37.)

11.39 Prove that, as the figure becomes very long, the view factor for the 2nd case in Table 10.3 reduces to that given for the 3rd case in Table 10.2.

In this case $H \to 0$, $W \to 0$, $H/W \to h/w$, so the equation reduces to

$$F_{1-2} \to \frac{1}{\pi}\left(\tan^{-1}\infty + \frac{h}{w}\tan^{-1}\infty - \sqrt{\left(\frac{h}{w}\right)^2+1}\ \tan^{-1}\infty + \frac{1}{4}\left(\ln 1 + 0\ln 1 + 0\ln 1\right)\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{h}{w}\frac{\pi}{2} - \sqrt{\left(\frac{h}{w}\right)^2+1}\ \frac{\pi}{2} + 0\right) = \underline{\underline{\frac{1}{2}\left(1 + \frac{h}{w} - \sqrt{1+\left(\frac{h}{w}\right)^2}\right)}}$$

This is given in Table 10.2.

10.40 Show that F_{1-2} for the first case in Table 10.3 reduces to the expected result when plates 1 and 2 are extended to infinity.

X and $Y \to \infty$ so

$$F_{1-2} \to \frac{2}{\pi}\left[\underbrace{\frac{\ln \frac{XY}{\sqrt{X^2+Y^2}}}{XY}}_{\to 0} + \underbrace{1\tan^{-1}\frac{X}{Y} + 1\tan^{-1}\frac{Y}{X}}_{\text{but } \tan^{-1}a + \tan^{-1}\frac{1}{a} = \frac{\pi}{2}} - \underbrace{\frac{\tan^{-1}X}{Y}}_{\to 0} - \underbrace{\frac{\tan^{-1}Y}{X}}_{\to 0}\right]$$

so $\underline{\underline{F_{1-2} = 1}}$ which is correct for two infinite facing plates

10.42 In problem 2.26 you were asked to neglect radiation in showing that q was equal to 8227 W/m^2 as the result of conduction alone. Discuss the validity of the assumption, quantitatively.

In this case we have: $q_{rad} = \dfrac{\sigma}{\frac{2}{\epsilon}-1}\left([1000+273]^4 - [200+273]^4\right)$

where ϵ = the emittance of <u>both</u> plates. When $q_{cond} = q_{rad}$ = 8227, $\epsilon = 0.1066$. This is a reasonable value for polished metal, but it doesn't give a <u>negligible</u> value of q_{rad}. If q_{rad} = only 10% q_{cond} = 822.7 then <u>$\epsilon = 0.0112$</u>. This would be hard to achieve. It would require, for example, a very highly polished silver.

10.42 A 100°C sphere with ε = 0.86 is centered within a second sphere at 300°C with ε = 0.47. The outer diameter is 0.3 m and the inner diameter is 0.1 m. What is the radiant heat flux?

e_{b_1} — B_1 — B_2 — e_{b_2}

$$\frac{1-0.47}{\pi(0.3)^2\,0.47} = 3.988 \qquad \frac{1}{A_1F_{12}} = \frac{1}{\pi(0.3)^2\left(\frac{0.05}{0.15}\right)^2} = 31.83 \qquad \frac{1-0.86}{\pi(0.1)^2\,0.86} = 5.182$$

(We get F_{1-2} from Table 10.3, case 4.)

Then, using eqn. (10.35), we get:

$$Q = \frac{\sigma(573)^4 - \sigma(373)^4}{3.988 + 31.83 + 5.182} = \underline{122.3\ W}$$

$$q = \frac{Q}{A} = \frac{122.3}{\pi(0.3)^2} = \underline{\underline{432.6\ \frac{W}{m^2}}}$$

Problem 10.52: The fraction of blackbody radiation between wavelengths of 0 and λ is

$$f = \frac{1}{\sigma T^4}\int_0^\lambda e_{\lambda,b}\,d\lambda \tag{11}$$

a) Work Problem 10.51.
b) Show that

$$f(\lambda T) = \frac{15}{\pi^4}\int_{c_2/\lambda T}^\infty \frac{t^3}{e^t - 1}\,dt \tag{12}$$

where c_2 is the second radiation constant, hc/k_B, equal to 1438.8 μm·K.
c) Use the software of your choice to plot $f(\lambda T)$ and check that your results match Table 10.7.

Solution. Following the solution to Problem 10.51:

$$\begin{aligned}
f &= \frac{1}{\sigma T^4}\int_0^\lambda e_{\lambda,b}\,d\lambda &(13)\\
&= \frac{1}{\sigma T^4}\int_0^\lambda \frac{2\pi h c_o^2}{\lambda^5\left[\exp(hc_o/k_B T\lambda) - 1\right]}\,d\lambda &(14)\\
&= \frac{1}{\sigma T^4}\int_{c_o/\lambda}^\infty \frac{2\pi h \nu^3}{c_o^2\left[\exp(h\nu/k_B T) - 1\right]}\,d\nu &(15)\\
&= \frac{1}{\sigma T^4}\frac{2\pi k_B^4 T^4}{h^3 c_o^2}\int_{c_2/\lambda T}^\infty \frac{t^3}{e^t - 1}\,dt &(16)\\
&= \frac{15}{\pi^4}\int_{c_2/\lambda T}^\infty \frac{x^3}{e^x - 1}\,dx &(17)\\
&= \frac{15}{\pi^4}\int_0^\infty \frac{x^3}{e^x - 1}\,dx - \frac{15}{\pi^4}\int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1}\,dx &(18)\\
&= 1 - \frac{15}{\pi^4}\int_0^{c_2/\lambda T} \frac{x^3}{e^x - 1}\,dx &(19)
\end{aligned}$$

The numerical integration can be done in various ways, depending on the software available. (On a sophisticated level, the last integral can be written in terms of the Debye function which is available in the Gnu Scientific Library.) This equation is plotted in Fig. 1.

Problem 10.53: Read Problem 10.52. Then find the central range of wavelengths that includes 80% of the energy emitted by blackbodies at room temperature (300 K) and at the solar temperature (5777 K).

Solution. From Table 10.7, $f = 0.10$ at $\lambda T = 2195$ μm·K and $f = 0.90$ at $\lambda T = 9376$ μm·K. Dividing by the absolute temperatures gives:

T [K]	$\lambda_{0.1}$ [μm]	$\lambda_{0.9}$ [μm]
300	7.317	31.25
5777	0.380	1.62

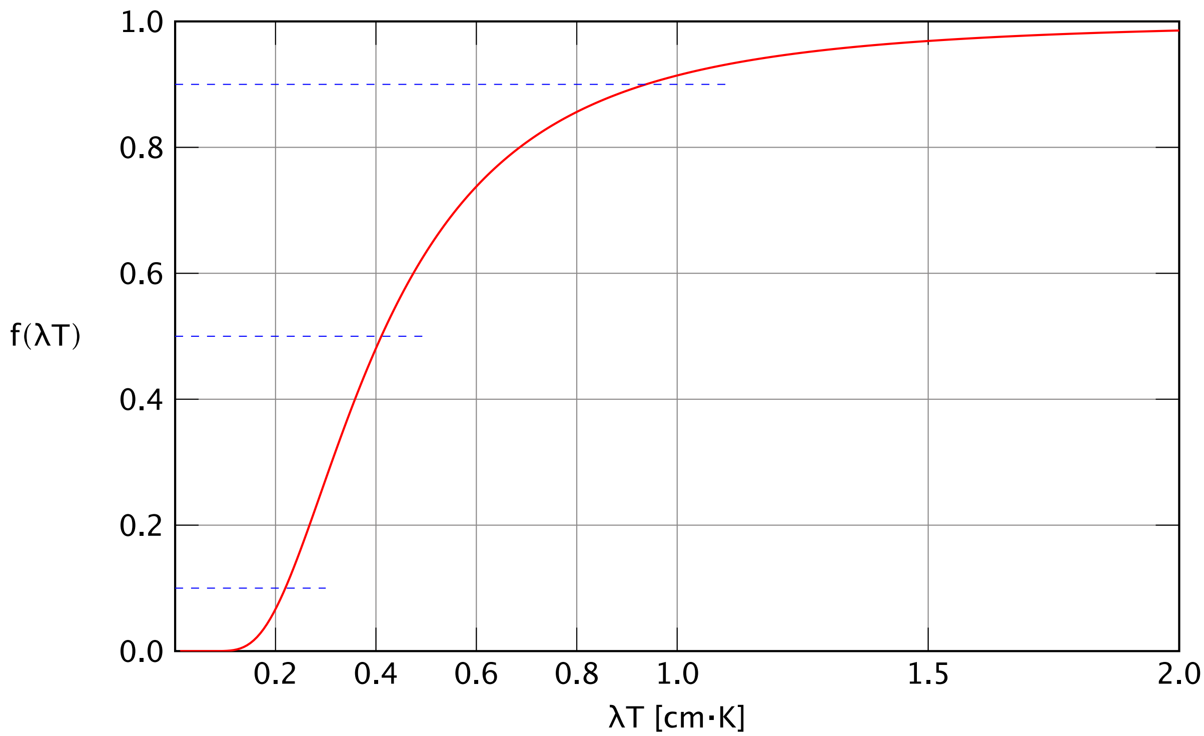


FIGURE 1. The radiation fractional function

PROBLEM 10.54: Read Problem 10.52. A crystalline silicon solar cell can convert photons to conducting electrons if the photons have a wavelength less than $\lambda_{\text{band}} = 1.11$ μm, the *bandgap* wavelength. Longer wavelengths do not produce an electric current, but simply get absorbed and heat the silicon. For a solar cell at 320 K, make a rough estimate of the fraction of solar radiation on wavelengths below the bandgap? Why is this important?

SOLUTION. The relevant temperature is that of the sun, 5777 K, not that of the solar cell. We approximate the sun as a blackbody at 5777 K, ignoring atmospheric absorption bands.

$$\lambda_{\text{band}} T = (1.11)(5777)\ \mu\text{m} \cdot \text{K} = 6412\ \mu\text{m} \cdot \text{K}$$

Referring to Table 10.7, a bit less than 80% of solar energy is on these shorter wavelengths (with a more exact table, 77%). This is significant because the solar cell can convert less than 80% of the solar energy to electricity; additional considerations lower the theoretical efficiency still further, to less than 50%.

Problem 10.55 Two stainless steel blocks have surface roughness of about 10 μm and $\varepsilon \approx 0.5$. They are brought into contact, and their interface is near 300 K. Ignore the points of direct contact and make a rough estimate of the conductance across the air-filled gaps, approximating them as two flat plates. How important is thermal radiation? Compare your result with Table 2.1 and comment on the relative importance of the direct contact that we ignored.

Solution The gaps are very thin, so little circulation will occur in the air. Heat transfer through the air will be by conduction. Radiation and conduction act in parallel across the gap. The temperature difference across the gap will likely be small, so we may use a radiation thermal resistance. The conductance is the reciprocal of the thermal resistance, per unit area, so $h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}}$.

Letting the gap width be $\delta = 10$ μm and taking $k_{\text{air}} = 0.0264$ W/m·K, we can estimate

$$h_{\text{cond}} \approx \frac{k}{\delta} = \frac{0.0264}{10 \times 10^{-6}} = 2,640 \text{ W/m}^2\text{K}$$

With eqns. (2.29) and (10.25):

$$\mathcal{F}_{1\text{–}2} = \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)^{-1} = \left(\frac{2}{0.5} - 1\right)^{-1} = \frac{1}{3}$$

$$h_{\text{rad}} = 4\sigma T_m^3 \mathcal{F}_{1\text{–}2} = 4(5.67 \times 10^{-8})(300)^3(0.3333) = 2.041 \text{ W/m}^2\text{K}$$

Then

$$h_{\text{gap}} = h_{\text{cond}} + h_{\text{rad}} = 2640 + 2.041 = 2,642 \text{ W/m}^2\text{K}$$

This conductance is on the lower end of the range of given in Table 2.1. Conduction through contacting points will add significantly to the heat transfer, although it will be highly multidimensional and not easily calculated. Thermal radiation, however, is negligible.

11.1 Derive equations 11.9

$$m_i = \frac{\rho_i}{\rho} = \frac{M_i c_i}{M c} = \frac{x_i M_i}{\sum_k x_k M_k} \quad \text{by eqns. (11.2,4,6,8)}$$

$$x_i = \frac{c_i}{c} = \frac{\rho_i}{\rho M_i} \cdot M = \frac{(m_i/M_i)}{\sum_k (m_k/M_k)}$$

11.2 A 1000 liter cylinder a 300°K contains a gaseous mixture comprised of 0.10 kgmole of NH_3, 0.04 kgmole of CO_2, and 0.06 kgmole of He. a.) Find the mass fraction each species and the pressure in the cylinder. b.) If the cylinder is heated to 600°K, what are the new mole fractions, mass fractions, and molar concentrations. c.) The cylinder is now compressed isothemally to a volume of 600 liters. What are the molar concentrations, mass fractions, and partial densities? d.) If 0.40 kg of gaseous N_2 is injected into the cylinder while the temperature remains at 600°K, find the mole fractions, mass fractions, and molar concentrations.

(a) by eqn. (11.6): $x_{NH_3} = 0.1/0.2 = 0.5$, $x_{CO_2} = 0.04/0.2 = 0.2$, $x_{He} = 0.06/0.2 = 0.3$. By eq'n (11.8): $M = (0.5)(17.03) + (0.20)\cdot(44.01) + (0.3)(4.003) = 18.52$ kg/kgmole. Thus, from eqn. (11.9):

$$m_{NH_3} = (0.5)(17.03)/18.52 = \underline{0.460}$$

$$m_{CO_2} = (0.2)(44.01)/18.52 = \underline{0.475}$$

$$m_{He} = (0.3)(4.003)/18.52 = \underline{0.0648}$$

The pressure is: $p = c R^\circ T = (0.2\ \text{kgmole}/1\ \text{m}^3)(8314.3\ \text{J/kgmole°K})\cdot(300°\text{K}) = 4.99 \times 10^5\ \text{Pa} = \underline{4.92\ \text{atm}}$

(b) The mass fractions are unchanged. The total pressure, like the temperature, is doubled: $p = 2(4.92) = 9.84$ atm.

By eqn. (11.17):

$$p_{NH_3} = (0.5)(9.84\ \text{atm}) = \underline{4.92\ \text{atm}}$$

$$p_{CO_2} = (0.2)(9.84\ \text{atm}) = \underline{1.97\ \text{atm}}$$

$$p_{He} = (0.3)(9.84\ \text{atm}) = \underline{2.95\ \text{atm}}$$

(c) The mass fractions are still unchanged. The molar concentrations are:

$$c_{NH_3} = (0.1\ \text{kgmole}/0.6\ \text{m}^3) = \underline{0.167\ \text{kgmole/m}^3}$$

$$c_{CO_2} = (0.04\ \text{kgmole}/0.6\ \text{m}^3) = \underline{0.0667\ \text{kgmole/m}^3}$$

$$c_{He} = (0.06\ \text{kgmole}/0.6\ \text{m}^3) = \underline{0.100\ \text{kgmole/m}^3}$$

By eqn. (11.4), the partial densities are:

$$\rho_{NH_3} = (0.167)(17.03) = \underline{2.84\ kg/m^3} \longleftarrow$$
$$\rho_{CO_2} = (0.0667)(44.01) = \underline{2.94\ kg/m^3} \longleftarrow$$
$$\rho_{He} = (0.100)(4.003) = \underline{0.400\ kg/m^3} \longleftarrow$$

(d) We have added (0.4 kg)/(28.01 kg/kgmole) = 0.0143 kgmole N_2.

$$x_{NH_3} = 0.1/0.2143 = \underline{0.467} \longleftarrow$$
$$x_{CO_2} = 0.04/0.2143 = \underline{0.187} \longleftarrow$$
$$x_{He} = 0.06/0.2143 = \underline{0.280} \longleftarrow$$
$$x_{N_2} = 0.0143/0.2143 = \underline{0.0667} \longleftarrow$$

The molecular weight of the mixture is: $M = (0.467)(17.03) + (0.187)(44.01) + (0.280)(4.003) + (0.0667)(28.01) = 19.17$ kg/kgmole.

Thus,
$$m_{NH_3} = (0.467)(17.03)/(19.17) = \underline{0.415} \longleftarrow$$
$$m_{CO_2} = (0.187)(44.01)/(19.17) = \underline{0.429} \longleftarrow$$
$$m_{He} = (0.280)(4.003)/(19.17) = \underline{0.0585} \longleftarrow$$
$$m_{N_2} = (0.0667)(28.01)/(19.17) = \underline{0.0975} \longleftarrow$$

The molar concentrations of part (c) are unchanged.

$$c_{N_2} = 0.0143\ kgmole/0.6\ m^3 = \underline{0.0238\ kgmole/m^3} \longleftarrow$$

11.3 Planetary atmospheres show significant variations of temperature and pressure in the vertical direction. Observations suggest that the atmosphere of Jupiter has the following composition at the tropopause level:

number	density	(molecules/m^3)	of	H_2	$= 5.7 \times 10^{21}$
"	"	"	"	He	$= 7.2 \times 10^{20}$
"	"	"	"	CH_4	$= 6.5 \times 10^{18}$
"	"	"	"	NH_3	$= 1.3 \times 10^{18}$

Find the mole fraction and partial density of each species at this level if p = 0.1 atm and T = 113°K. Estimate the number densities at the level where p = 10 atm and T = 400°K, deeper within the Jovian troposphere. (Surface pressures on Jupiter may actually be as high as 10^5 atm.)

The number density $\mathcal{N} = \sum_i \mathcal{N}_i = 6.428 \times 10^{21}$ molecules/m^3. The molar concentration is: $c = \mathcal{N}/N_A = (6.428 \times 10^{21})/(6.0225 \times 10^6)$

$= 1.067\times10^{-5}$ kgmole/m³, by eq'n (12.10). The partial molar concentrations are: $c_{H_2} = (5.7\times10^{21})/(6.0225\times10^{6}) = 9.465\times10^{-6}\ \frac{\text{kgmole}}{\text{m}^3}$ $c_{He} = (7.2\times10^{20})/(6.0225\times10^{6}) = 1.196\times10^{-6}$ kgmole/m³, $c_{CH_4} = 1.079\times10^{-8}$ kgmole/m³, $c_{NH_3} = 2.159\times10^{-9}$ kgmole/m³. From eq'n (12.6), we find:

$$x_{H_2} = 9.464\times10^{-6}/1.067\times10^{-5} = \underline{0.887}$$
$$x_{He} = 1.196\times10^{-6}/1.067\times10^{-5} = \underline{0.112}$$
$$x_{CH_4} = 1.079\times10^{-8}/1.067\times10^{-5} = \underline{0.001}$$
$$x_{NH_3} = 2.159\times10^{-9}/1.067\times10^{-5} = \underline{0.0002}$$

From eq'n (12.4), the partial densities are $\rho_i = c_i M_i$:

$$\rho_{H_2} = (9.465)(2.016)\times10^{-6} = \underline{1.9\times10^{-5}\ \text{kg/m}^3}$$
$$\rho_{He} = (1.196)(4.003)\times10^{-6} = \underline{4.8\times10^{-6}\ \text{kg/m}^3}$$
$$\rho_{CH_4} = (1.079)(16.04)\times10^{-8} = \underline{1.7\times10^{-7}\ \text{kg/m}^3}$$
$$\rho_{NH_3} = (2.159)(17.03)\times10^{-9} = \underline{3.7\times10^{-8}\ \text{kg/m}^3}$$

To estimate the number densities at lower levels, assume that relative amounts of each species don't change, i.e. the mole fractions are the same. We have: $\mathcal{N}_i = c_i N_A = \frac{x_i N_A}{c}$

From eq'n(12.15): $c = p/R^\circ T = 10(101325)/(8314.3)(400) = 0.305$ kgmole/m³

So: $\mathcal{N}_{H_2} = \underline{1.8\times10^{27}\ \text{molecules/m}^3}$ $\quad \mathcal{N}_{CH_4} = \underline{2.0\times10^{24}\ \text{molecules/m}^3}$

$\mathcal{N}_{He} = \underline{2.2\times10^{26}\ \text{molecules/m}^3}$ $\quad \mathcal{N}_{NH_3} = \underline{3.9\times10^{23}\ \text{molecules/m}^3}$

11.4 Using the definitions of the fluxes, velocities, and concentrations, derive equation (11.29) from equation (11.26) for binary diffusion

From eq'ns (12.21, 27, 31, 38): $\vec{j}_1 = -\rho \mathcal{D}_{12}\nabla m_1 = \rho_1(\vec{v}_1 - \vec{v})$

$\vec{J}_1^{\,*} = -c\mathcal{D}_{12}\nabla x_1 = c_1(\vec{v}_1 - \vec{v}^{\,*})$. With (12.18, 24), (12.3, 7):

$$\vec{v}_1 - \vec{v}^{\,*} = \vec{v}_1 - (x_1\vec{v}_1 + x_2\vec{v}_2) = x_2(\vec{v}_1 - \vec{v}_2)$$
$$\vec{v}_1 - \vec{v} = \vec{v}_1 - (m_1\vec{v}_1 + m_2\vec{v}_2) = m_2(\vec{v}_1 - \vec{v}_2)$$

Thus: $\vec{J}_1^* = c_1 x_2(\vec{v}_1 - \vec{v}_2) = c_1(m_2 M/M_2)(\vec{v}_1 - \vec{v}_2) = \dfrac{M}{M_1 M_2}\vec{j}_1$

where we use the equation before (11.25). It also gives us

$$\nabla m_1 = \nabla\left(\frac{x_1 M_1}{M}\right) = \frac{M_1}{M}\nabla x_1 - \frac{x_1 M_1}{M^2}\left\{M_1 \nabla x_1 + M_2 \nabla x_2\right\}$$

$$= \frac{M_1}{M}\left\{1 - \frac{x_1}{M}M_1 + \frac{x_1}{M}M_2\right\}\nabla x_1 = \frac{M_2 M_1}{M^2}\nabla x_1$$

(The relation $x_1 + x_2 = 1$ and eq'n (11.8) were used here, also)

Hence, $\vec{J}_1^* = \dfrac{M}{M_1 M_2}(-\rho \mathcal{D}_{12}\nabla m_1) = -c\mathcal{D}_{12}\left(\dfrac{M^2}{M_1 M_2}\cdot\dfrac{M_2 M_1}{M^2}\cdot\nabla x_1\right)$

$$\boxed{\vec{J}_1^* = -c\mathcal{D}_{12}\nabla x_1}$$

11.5 Show that $\mathcal{D}_{12} = \mathcal{D}_{21}$ in a binary mixture.

From eqns. (11.25, 26): $\vec{j}_1 + \vec{j}_2 = 0 \Rightarrow \mathcal{D}_{12}\nabla m_1 = -\mathcal{D}_{21}\nabla m_2$

But: $\nabla m_1 = \nabla(1 - m_2) = -\nabla m_2$. Thus: $\mathcal{D}_{12}\nabla m_1 = -\mathcal{D}_{21}\nabla m_2$

$= -\mathcal{D}_{21}(-\nabla m_1) = \mathcal{D}_{21}\nabla m_1$. Therefore: $\boxed{\mathcal{D}_{12} = \mathcal{D}_{21}}$

11.6 Fill in the details involved in obtaining equation (11.32) from equation 11.31.

The Taylor expansions are: $\left.\dfrac{\mathcal{N}_A}{\mathcal{N}}\right|_{x_0 \pm a\ell} = \left.\dfrac{\mathcal{N}_A}{\mathcal{N}}\right|_{x_0} + \left.\dfrac{d}{dx}\left(\dfrac{\mathcal{N}_A}{\mathcal{N}}\right)\right|_{x_0}\cdot(\pm a\ell)$

$+ O(\ell^2)$, so that we obtain

$$\left\{\left.\left(\frac{\mathcal{N}_A}{\mathcal{N}}\right)\right|_{x_0 - a\ell} - \left.\left(\frac{\mathcal{N}_A}{\mathcal{N}}\right)\right|_{x_0 + a\ell}\right\} \simeq -2a\ell \left.\frac{d}{dx}\left(\frac{\mathcal{N}_A}{\mathcal{N}}\right)\right|_{x_0}$$

to an error $O(\ell^3)$ (not ℓ^2). Thus, the flux through the plane $x = x_0$ is: $j_A = -2\eta a(\mathcal{N}\bar{C}\ell)\dfrac{M_A}{N_A}\dfrac{d}{dx}\left(\dfrac{\mathcal{N}_A}{\mathcal{N}}\right)$

Now:

$$\frac{d}{dx}\left(\frac{\mathcal{N}_A}{\mathcal{N}}\right) = \frac{d}{dx}\left(\frac{\rho_A}{\rho}\frac{M}{M_A}\frac{N_A}{N_A}\right) = \frac{d}{dx}\left(m_A\frac{M}{M_A}\right) = \frac{M}{M_A}\frac{dm_A}{dx} +$$

$$\frac{m_A}{M_A}\frac{d}{dx}M = \frac{M}{M_A}\frac{dm_A}{dx} + \frac{m_A}{M_A}\frac{d}{dx}\left(x_A M_A + x_{A'} M_{A'}\right)$$

$$= (M_A - M_{A'})\frac{d}{dx}x_A = 0$$

because we assume identical molecules. Eqn. (11.34) follows.

11.7 Batteries commonly contain an aqueous solution of sulfuric acid with lead plates as electrodes. Current is generated by the reaction of the electrolyte with the electrode material. At the negative electrode, the reaction is

$$Pb(s) + SO_4^{2-} \rightleftarrows PbSO_4 + 2e^-$$

where the (s) denotes a solid phase component and the charge of an electron is 1.609×10^{-19} Coulombs. If the current density at such an electrode is i = 5 milliamperes/cm^2, what is the mole flux of SO_4^{2-} to the electrode? (1 Amp = 1 Coulomb/s.) What is the mass flux of SO_4^{2-}? At what mass rate is $PbSO_4$ produced? If the electrolyte is to remain electrically neutral, at what rate does H^+ flow toward the electrode? Hydrogen does not react at the negative electrode.

The current is: i = (charge per ion)(no. ions per kgmole)·(flux of ions at electrode surface, kgmole/m^2·s) from which

$$(5\times10^{-3}\ \text{A})/(10^{-4}\ \text{m}^2) = i = 2(1.609\times10^{-19}\ \text{C})(N_A)(N_{SO_4^{2-},s})$$

Solving, we have:

$$N_{SO_4^{2-},s} = \frac{(5\times10^{1})\ \text{kgmole/m}^2\cdot\text{s}}{2(1.609\times10^{-19})(6.0225\times10^{26})} = \underline{2.580\times10^{-7}\ \frac{\text{kgmole}}{\text{m}^2\cdot\text{s}}}$$

The mass flux is:

$$n_{SO_4^{2-},s} = M_{SO_4^{2-}}\cdot N_{SO_4^{2-},s} = (96.06)(2.58\times10^{-7})$$

$$= \underline{2.478\times10^{-5}\ \text{kg/m}^2\cdot\text{s}}$$

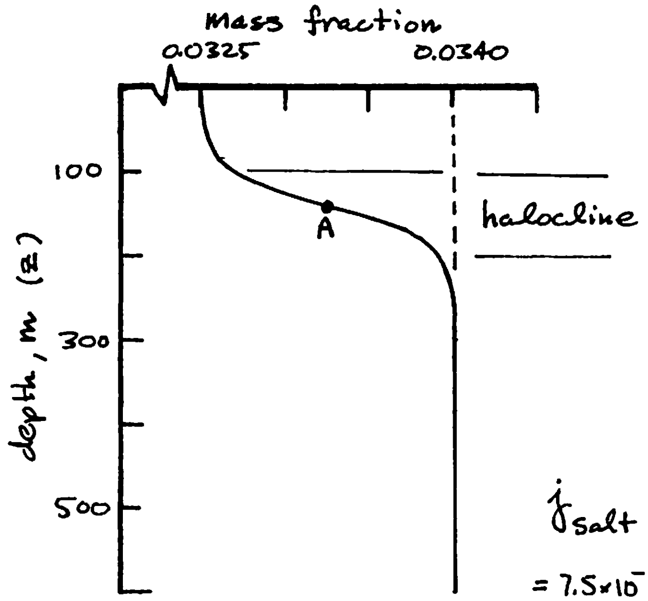
One mole of $PbSO_4$ is formed for every mole of SO_4^{2-} reaching the electrode. Thus:

$$\dot{m}''_{PbSO_4} = M_{PbSO_4}\cdot N_{SO_4^{2-},s} = (303.3)(2.58\times10^{-7})$$

$$= \underline{7.825\times10^{-5}\ \text{kg/m}^2\cdot\text{s}}$$

In steady state, the hydrogen ions will be effectively stationary near the negative electrode. H^+ doesn't react and SO_4^{2-} diffuses past it. Thus, $\underline{N_{H^+,s} = 0}$

11.8 The salt concentration in the ocean increases with increasing depth, z. A model for the concentration distribution in the upper ocean is $S = 33.25 + 0.75 \tanh(0.026z - 3.7)$ where S is the salinity in grams of salt per kg of ocean water and z is the distance below the surface in m. a.) Plot the mass fraction of salt as a function of z. (the region of rapid transition of $m_{salt}(z)$ is called the <u>halocline</u>.) b.) Ignoring the effects of oceanic motion, compute $j_{salt}(z)$. Use a value of $\mathcal{D}_{salt,\ water} = 1.5 \times 10^{-5}$ cm²/s. Indicate the position of maximum diffusion on your plot of the salt concentration. c.) The upper region of the ocean is well-mixed by wind-driven waves and turbulence while the lower region and halocline tend to be calmer. Using $j_{salt}(z)$ from b.), make a simple estimate of the amount of salt carried upward in one week in a 5 km² horizontal area of the sea.



(a) Salinity, S = 1000 × m_{salt}

(b) Under this assumption, just differentiate $m_{salt}(z)$ and using Fick's Law, i.e. assume pure diffusion.

$$j_{salt}(z) = -\rho \mathcal{D}_{12} \frac{d}{dz} m_{salt}$$

$$= 7.5\times10^{-4}(0.026)\,\text{sech}^2(0.026z - 3.7)(-\rho\mathcal{D}_{12})$$

The density of salt-water may be approximated as having a uniform value:

$$\rho = \rho + \rho_{H_2O} = \rho_{H_2O}\left(1 + \frac{m_{salt}}{1 + m_{salt}}\right) \simeq \rho_{H_2O}(1 + m_{salt})$$

We make the additional approximation that the partial density, ρ_{H_2O}, is the same as the density of pure water $\simeq 999$ kg/m³.

so: $\rho \simeq 999(1 + 0.03325) = \underline{1032 \text{ kg/m}^3}$.

Then: $j_{salt}(z) = -3.02\times10 \frac{kg}{m^2\text{-}s}\,\text{sech}^2(0.026z - 3.7)$

The flux is vertically <u>upward</u>. The point of maximum j_{salt} is the maximum slope point of m_{salt} (pt. A): $0 = 0.026 z_A - 3.7$

$\underline{z_A = 142 \text{ m}.}$

(c) The flat form of the salt concentration profile in the upper region may be attributed to turbulent mixing. (The fresher water near the surface originates from rainfall and nearby land masses.) We may reason that salt diffuses slowly from lower water into the fresher water above at a rate given approximately by $j_{salt,max}$: transport is primarily diffusive at this level and the upward flux shouldn't exceed this value. Then:

$$\text{mass flow upward per 5 sq. km per week} = j_{max} \cdot \text{sq km} \cdot 1\,\text{wk}$$
$$= (3.02\times10^{-11}\,\text{kg/m}^2\text{s})(1000\,\text{m})^2(3600\cdot24\cdot7)\,5 = \underline{91.3\,\text{kg}}$$

This flux complicates oceanographic modeling.

11.9 Butane reacts with hydrogen on the surface of a nickel catalyst to form methane and propane. This heterogeneous reaction, referred to as "hydrogenolysis," is

$$C_4H_{10} + H_2 \xrightarrow{Ni} C_3H_8 + CH_4$$

The molar rate of consumption of C_4H_{10} per unit area in the reaction is $\dot{R}_{C_4H_{10}} = A\,(e^{-\Delta E/R^oT})\,p_{C_4H_{10}}\,p_{H_2}^{-2.4}$, where $A = 6.3\times10^{10}$ kgmole/m^2-s and $\Delta E = 1.9\times10^8$ J/kgmole. (a) If $p_{C_4H_{10},s} = p_{C_3H_8,s} = 0.2$ atm, $p_{CH_4,s} = 0.17$ atm, and $p_{H_2,s} = 0.3$ atm at a nickel surface with conditions of 440^{o}C and 0.87 atm total pressure, what is the total rate of consumption of butane? (b) What are the mole fluxes of butane and hydrogen to the surface? What are the mass fluxes of propane and ethane away from the surface? (c) What is $\dot{m}''$? What are v, v^*, and $v_{C_4H_{10}}$? (d) What is the diffusional mole flux of butane? What is the diffusional mass flux of propane? What is the flux of Ni?

(a) $$\dot{R}_{C_4H_{10}} = (6.3\times10^{10})\exp\left\{-\frac{(1.9\times10^8)}{(8314.3)(440+273.15)}\right\}(0.2)(0.3)^{-2.4}$$
$$= \underline{2.746\times10^{-3}\ \text{kgmole/m}^2\cdot\text{s}}$$

(b) From stoichiometry: $\underline{\dot{R}_{C_4H_{10}} = -N_{C_4H_{10},s} = -N_{H_2,s}}$

and $N_{C_3H_8,s} = N_{CH_4,s} = \dot{R}_{C_4H_{10}}$. Signs reflect fluxes of C_4H_{10}, H_2 toward the surface and C_3H_8, CH_4 away. Because $n_i = M_iN_i$ and $M_{C_3H_8} = 44.09$ kg/kgmole, $M_{CH_4} = 16.04$ kg/kgmole, we have

$$n_{C_3H_8,s} = (44.09)(2.746\times10^{-3}) = \underline{0.1211\ \text{kg/m}^2\text{s}}$$
$$n_{CH_4,s} = (16.04)(2.746\times10^{-3}) = \underline{0.04405\ \text{kg/m}^2\cdot\text{s}}$$

(c) There is no net flux through the surface: $\underline{\dot{m}''=0}$,

$\underline{V = \dot{m}''/\rho = 0}$. There is no net mole flux through the surface, either: $\underline{v^* = N/c = 0}$

$v_{C_4H_{10}} = n_{C_4H_{10}}/\rho_{C_4H_{10}} = N_{C_4H_{10}}/c_{C_4H_{10}}$; $c_{C_4H_{10}} = p_{C_4H_{10}}/R^\circ T$

$= (101325)(0.2)/(8314.3)(753.15) = 3.236\times10^{-3}$ kgmole/m³. $v_{C_4H_{10},s} =$

$v_{C_4H_{10},s} = (-2.746\times10^{-3})/(3.236\times10^{-3}) = \underline{-0.8485 \text{ m/s}}$ toward the surface.

(d) $J^*_{C_4H_{10}} = c_{C_4H_{10}}(v_{C_4H_{10}} - \cancel{v^*}_0) = N_{C_4H_{10},s} = \underline{-2.746\times10^{-3} \text{ kgmole/m}^2\cdot\text{s}}$

$j_{C_3H_8} = \rho_{C_3H_8}(v_{C_3H_8} - \cancel{v}_0) = n_{C_3H_8} = \underline{0.1211 \text{ kg/m}^2\cdot\text{s}}$

Nickel does not have a net reaction. All fluxes of Ni are zero.

11.10 Consider two chambers held at temperatures T_1 and T_2, respectively, and joined by a small insulated tube. The chambers are filled with a binary gas mixture, with the tube open, and allowed to come to steady state. If the Soret effect is taken into account, what is the concentration difference between the two chambers? Assume an effective mean value of the thermal diffusion ratio is known.

In steady state, there is no mass flow between the chambers. Assuming the tube is too small to allow convection (recirculation) Eqn. (11.36) gives:

$$\vec{j}_1 = 0 = -\rho\mathcal{D}_{12}\left[\frac{d}{dx}m_1 + \frac{M_1M_2}{M^2}k_T\frac{d}{dx}\ln T\right]$$

so

$$\frac{dm_1}{dx} = -\frac{M_1M_2}{M_2}k_T\frac{d}{dx}\ln T$$

Integrate from $T_1 = T(x_1)$ to $T_2 = T(x_2)$

$$m_1(x_2) - m_1(x_1) = \frac{M_1M_2}{M^2}\int_{x_1}^{x_2} k_T\frac{d\ln T}{dx}\cdot dx = \frac{M_1M_2}{M^2}\overline{k_T}\ln\left(\frac{T_2}{T_1}\right)$$

for a mean value $\overline{k_T}$. The difference in concentration between the chambers is:

$$\boxed{m_1(x_2) - m_1(x_1) = \frac{M_1M_2}{M^2}\overline{k_T}\ln\left(\frac{T_2}{T_1}\right)}$$

11.11 Compute $\mathcal{D}_{12}$ for oxygen gas diffusing through nitrogen gas at p = 1 atm using equations (11.113) and (11.116) for T = 200°K, 500°K, and 1000°K. Observe that (11.113) shows large deviations from equation (11.116) even for such similar molecules.

	O_2	N_2	
σ	3.467	3.798	Å
ϵ/k	106.7	71.4	°K
M	32.00	28.01	kg/kgmole

$\sigma_{O_2-N_2} = \frac{1}{2}(3.467 + 3.798) = 3.633$ Å

$(\epsilon/k)_{O_2-N_2} = (106.7 \cdot 71.4)^{1/2} = 87.28$°K

T	kT/ϵ	Ω_D
200	2.291	1.027
500	5.729	0.8205
1000	11.46	0.7310

For eq'n (11.113), use $d = \sigma_{O_2-N_2}$ and $M = 30.0$; convert p to N/m² and d to m, rather than Å. At 200K, we have:

$$\mathcal{D}_{AA'} = \frac{(1.38\times10^{-23}/\pi)^{3/2}}{(3.633\times10^{-10})^2}\left(\frac{6.0225\times10^{26}}{30.0}\right)^{1/2}\frac{(200)^{3/2}}{(101325)} = 8.724\times10^{-6}\ \text{m}^2/\text{s}$$

$$\mathcal{D}_{AB} = \frac{(1.8583\times10^{-7})(200)^{3/2}}{(1)(3.633)^2(1.027)}\left(\frac{1}{28.01} + \frac{1}{32.00}\right)^{1/2} = 1.003\times10^{-5}\ \text{m}^2/\text{s}$$

T	(11.113)	(11.116)	$\frac{(11.113)}{(11.116)}$
200	8.724×10^{-6}	1.003×10^{-5}	0.87
500	3.440×10^{-5}	4.963×10^{-5}	0.69
1000	9.754×10^{-5}	1.575×10^{-4}	0.62

$\mathcal{D}_{O_2-N_2}(T)$ in m²/s from (11.113) & (11.116)

11.12 a.) Compute the binary diffusivity of each of the noble gases when they are individually mixed with nitrogen gas as 1 atm and 300°K. Plot the results as a function of the molecular weight of the noble gas. What do you conclude? b.) Consider the addition of a small amount of heluim (x_{He} = 0.04) to a mixture of nitrogen (x_{N_2} = 0.48) and argon (x_{Ar} = 0.48). Compute $\mathcal{D}_{He,m}$ and compare it with $\mathcal{D}_{Ar,m}$ Note that the higher concentration of argon does not improve its ability to diffuse through the mixture.

a)

	Ar	He	Kr	Ne	Xe	N_2
σ (Å)	3.542	2.551	3.655	2.820	4.047	3.798
ϵ/k (°K)	93.3	10.22	178.9	32.8	231.0	71.4
M (kg/kgmole)	39.95	4.003	83.80	20.18	131.1	28.01

Compute σ_{AB}, $(\epsilon/k)_{AB}$ for each gas in collision with N_2 from eqns. on page 676, read Ω_D from Table 11.3 and compute $\mathcal{D}_{ab}$:

a:	Ar	He	Kr	Ne	Xe
σ_{aN_2} (Å)	3.670	3.175	3.727	3.309	3.923
$(\epsilon/k)_{aN_2}$ (°K)	81.62	27.01	113.0	48.39	128.4
(kT/ϵ)	3.676	11.11	2.654	6.199	2.336
Ω_D	0.9017	0.7337	0.9819	0.8079	1.021
$\mathcal{D}_{a,N_2} \times 10^5 \left(\frac{m^2}{s}\right)$	1.959	6.978	1.546	3.187	1.279

Lighter gases diffuse more easily.

b) From eqn. (11.116), $\mathcal{D}_{He,Ar} = 7.310 \times 10^{-5}\ m^2/s$.

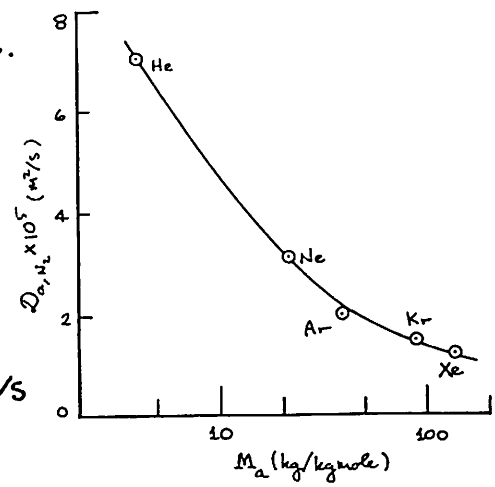
From eqn. (11.118):

$$\mathcal{D}_{He,m} = \left\{\frac{0.48}{7.310} + \frac{0.48}{6.978}\right\}^{-1} \times 10^{-5}\ m^2/s$$

$$= \underline{7.438 \times 10^{-5}\ m^2/s}$$

Since helium is a trace gas, eq'n (11.119) gives: $\mathcal{D}_{Ar,m} \cong \mathcal{D}_{Ar,N_2} = \underline{1.959 \times 10^{-5}}\ m^2/s$

The heavier Ar still diffuses more slowly than He.



11.13 a.) One particular correlation shows that gas phase diffusion coefficients vary as $T^{1.81}$ and p^{-1}. If an experimental value of $\mathcal{D}_{12}$ is known at T_1 and p_1, develop an equation to predict $\mathcal{D}_{12}$ at T_2 and p_2. b.) The diffusivity of water vapor (1) in air (2) was measured to be $2.39 \times 10^{-5}\ m^2/s$ at 8°C and 1 atm. Provide a formula for $\mathcal{D}_{12}(T,p)$.

(a) $\mathcal{D}_{12} \propto p^{-1} \cdot T^{1.81}$ so $\mathcal{D}_{12}(p,T) = C\, p^{-1} T^{1.81}$ for a constant C. Thus, $\mathcal{D}_{12}(p,T) / \mathcal{D}_{12}(p_{ref}, T_{ref}) = (p/p_{ref})^{-1} (T/T_{ref})^{1.81}$,

or:

$$\boxed{\mathcal{D}_{12}(p,T) = \mathcal{D}_{12}(p_{ref}, T_{ref}) \left(\frac{p_{ref}}{p}\right)\left(\frac{T}{T_{ref}}\right)^{1.81}}$$

(b) $\mathcal{D}_{12}(p,T) = (2.39 \times 10^{-5}\ m^2/s)\left(\frac{1\ atm}{p}\right)\left(\frac{T}{281\ °K}\right)^{1.81}$

11.14 Kinetic arguments lead to the Stefan-Maxwell equation for a dilute-gas mixture:

$$\nabla x_i = \sum_{j=1}^{n} \frac{c_i c_j}{c^2 \mathcal{D}_{ij}} \left(\frac{\vec{J}_j^*}{c_j} - \frac{\vec{J}_i^*}{c_i} \right)$$

a.) Derive equation (11.118) from this, making the appropriate assumptions. b.) Show that if $\mathcal{D}_{ij}$ has the same value for each pair of species, then $\mathcal{D}_{im} = \mathcal{D}_{ij}$.

(a) $\vec{J}_j^* \simeq 0$ for $j \neq i$ so: $\nabla x_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{c_i c_j}{c^2 \mathcal{D}_{ij}} \left(-\frac{\vec{J}_i^*}{c_i} \right) = -\frac{\vec{J}_i^*}{c^2} \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{c_j}{\mathcal{D}_{ij}} \right)$

or, with Fick's Law:

$$\nabla x_i = \left(\frac{c \mathcal{D}_{im} \nabla x_i}{c^2} \right) \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{c_j}{\mathcal{D}_{ij}} \right); \quad \boxed{\mathcal{D}_{im} = \left\{ \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{x_j}{\mathcal{D}_{ij}} \right) \right\}^{-1}}$$

(b) In this case, $\mathcal{D}_{ij}$ is independent of i and j, so:

$$\nabla x_i = \frac{1}{c^2 \mathcal{D}_{ij}} \left\{ c_i \underbrace{\sum_{j=1}^{n} \vec{J}_j^*}_{=0,\ \text{eq'n } (12.30)} - \vec{J}_i^* \underbrace{\sum_{j=1} c_j}_{=c} \right\} = -\frac{\vec{J}_i^*}{c \mathcal{D}_{ij}}$$

With Fick's Law, $\vec{J}_i^* = -c\mathcal{D}_{im} \nabla x_i$ and $\boxed{\mathcal{D}_{ij} = \mathcal{D}_{im}}$

11.15 Compute the diffusivity of methane in air using a.) equation (11.118) and b.) Blanc's Law. For part b.) ignore argon; use $x_{methane} = 0.05$, $T = 420^oF$, and $p = 10$ psia.

a) $\sigma_{CH_4\text{-}air} = 3.735\text{Å}$, $(\epsilon/k)_{CH_4\text{-}air} = 108.1$ K from eqns. (11.114/5).

$kT/\epsilon = 4.523$ so, from Table 12.2, $\Omega_D = 0.8601$. The pressure is $p = (10/14.696) = 0.6805$ atm. Eq'n (12.43) gives:

$$\mathcal{D}_{CH_4\text{-}air} = \frac{(1.8583\times 10^{-7})(488.9)^{3/2}}{(3.735)^2(0.6805)(0.8601)} \sqrt{\frac{1}{28.96} + \frac{1}{16.04}}$$

$$= \underline{7.658 \times 10^{-5}\ \text{m}^2/\text{s}}$$

b) We need $\mathcal{D}_{O_2\text{-}CH_4}$, $\mathcal{D}_{N_2\text{-}CH_4}$, x_{O_2}, and x_{N_2} for eqn. (11.118).

$O_2\text{-}CH_4$: $\sigma = 3.613\text{Å}$, $\epsilon/k = 125.9$ K, $\Omega_D = 0.8897$: $\mathcal{D} = 7.776\times 10^{-5}\ \text{m}^2/\text{s}$

$N_2\text{-}CH_4$: $\sigma = 3.778\text{Å}$, $\epsilon/k = 103.0$ K, $\Omega_D = 0.8512$: $\mathcal{D} = 7.608\times 10^{-5}\ \text{m}^2/\text{s}$

$x_{air} = 0.95$, so that: $x_{N_2} \simeq (0.78)(0.95) = 0.74$, $x_{O_2} \simeq (0.21)(0.95) = 0.20$

Thus, eqn. (11.118) yields:

$$\mathcal{D}_{CH_4,air} = \left\{\frac{0.74}{7.608} + \frac{0.20}{7.776}\right\}^{-1} \times 10^{-5} = \underline{8.131 \times 10^{-5}\ m^2/s} \longleftarrow$$

11.16 Diffusion of solutes in liquids is driven by the chemical potential, μ. Work is required to move a mole of solute A from a region of low chemical potential to a region of high chemical potential, i.e.

$$dW = d\mu_a = \frac{d\mu_A}{dx}dx$$

under isothermal, isobaric conditions. For an ideal (very dilute) solute, μ_A is given by

$$\mu_A = \mu_o + R^oT\ln(c_A)$$

where μ_o is a constant. Using an elementary principle of mechanics, derive the Nernst-Einstein equation. Note that the solution must be assumed to be very dilute.

From mechanics, if F_m is the force resisting the motion, $dW = -F_m\,dx$. Thus, $F_m = -d\mu_A/dx$ is the force per mole resulting from the potential gradient. F_m is opposed by the drag on the solute. If F_A is drag per molecule, $F_m = -F_A \cdot N_A$, N_A = Avagadro. Thus, $F_A = -(R^oT/c_A N_A)\,dc_A/dx$. From Sections 11.2&3 $J_A^* = -c\mathcal{D}_{AB}\frac{d}{dx}\left(\frac{c_A}{c}\right) = c_A(v_A - v^*)$. Since the solution is very dilute: (i) $c \simeq c_B$ = constant, (ii) $v^* = x_A v_A + x_B v_B \simeq v_B = v$, the mass average velocity. Collecting these results, we have

$$(v_A - v^*) = \underbrace{(v_A - v)}_{\text{steady average velocity of solute relative to solvent } = V_A \text{ in (12.49)}} = -\frac{c\mathcal{D}_{AB}}{c_A}\frac{1}{c}\frac{d}{dx} = -\mathcal{D}_{AB}\frac{1}{c_A}\frac{dc_A}{dx} = \frac{\mathcal{D}_{AB}F_A N_A}{R^oT}$$

Thus, $\boxed{\mathcal{D}_{AB} = k_B T\left(\frac{V_A - V}{F_A}\right)}$

11.17 A dilute aqueous solution at 300°K contains potassium ions, K^+. If the velocity of aqueous K^+ ions is 6.61×10^{-4} cm^2/s-V per unit electric field (1 V/cm), estimate the effective radius of K^+ ions in an aqueous solution. Criticize this estimate. The charge of an electron is -1.609×10^{-19} C and V = J/C.

The force on a K^+ ion in a 1 V/cm electric field is $F = qE = (1.609\times10^{-19}\ C)(1\ V/cm) = 1.609 \times 10^{-17}\ N$ so that the ✓ mobility of a K^+ ion is

11.18 a.) Obtain the following diffusion coefficients: (i) for dilute CCl_4 diffusing through liquid methanol at 340°K, (ii) for dilute benzene diffusing through water at 290°K, (iii) for dilute ethyl alcohol diffusing through water at 350°K, and (iv) for dilute acetone diffusing through methanol at 370°K. b.) Estimate the effective radius of a methanol molecule in a dilute aqueous solution.

a) Use eq'n (11.124) and App. A. (i) $\mu_{\text{Methanol}} = 3.3\times10^{-4}$ kg/m·s

$$\mathcal{D}_{CCl_4\text{-meth.}} = \frac{(340)}{(3.3\times10^{-4})}(4.4\times10^{-15})\left(\frac{0.042}{0.102}\right)^{1/6}\left(\frac{35.53}{29.93}\right)^{1/2}$$

$$= \underline{4.3\times10^{-9}\ \mathrm{m^2/s}}$$

(ii) $\mu_{H_2O} = 1.123\times10^{-3}$ kg/m·s. Eq'n (11.124) gives: $\mathcal{D}_{\text{benz-}H_2O} = \underline{9.94\times10^{-10}\ \mathrm{m^2/s}}$

(iii) $\mu_{H_2O} = 3.65\times10^{-4}$ kg/m·s. Eq'n (11.124) gives: $\mathcal{D}_{\text{ethyl-}H_2O} = \underline{3.49\times10^{-9}\ \frac{\mathrm{m^2}}{\mathrm{s}}}$

(iv) $\mu_{\text{meth}} = 2.4\times10^{-4}$ kg/m·s. Using eq'n (11.124) we have:

$$\mathcal{D}_{\text{acet.-meth.}} = \frac{(370)}{(2.4\times10^{-4})}(4.4\times10^{-15})\left(\frac{0.042}{0.074}\right)^{1/6}\left(\frac{35.53}{28.90}\right)^{1/2} = \underline{6.8\times10^{-9}\ \frac{\mathrm{m^2}}{\mathrm{s}}}$$

b) Use eq'ns (11.123b) and (11.124). From (11.124), $\mathcal{D}\mu/T = 4.11\times10^{-15}$ kg·m/K·s². Solving (11.123b) for R_{meth}, we have:

$$R_{\text{meth}} = \frac{k_B}{4\pi}\left(\frac{\mathcal{D}_{\text{meth-}H_2O}\,\mu_{H_2O}}{T}\right)^{-1} = \frac{(1.3805\times10^{-23})}{4\pi(4.11\times10^{-15})} = 2.67\times10^{-10}\ \mathrm{m}$$

$$= \underline{2.67\ \text{Å}} \longleftarrow$$

11.20 a.) Show that $k = (5/2)\mu c_v$ for a monatomic gas. b.) Obtain Euken's formula for the Prandtl number of a dilute gas

$$Pr = 4\gamma/(9\gamma - 5)$$

c.) Recall that for an ideal gas $\gamma \cong (D + 2)/D$ where D is the number of modes of energy storage of its molecules. Obtain an expression for Pr as a function of D and describe what it means. d.) Use Eucken's formula to compute Pr for gaseous Ar, N_2, and H_2O. Compare the result to data in Appendix A over the range of temperatures. Explain the results obtained for steam as opposed to Ar and N_2. (Note that for each mode of vibration there are two modes of energy storage, but that vibration is normally inactive until T is very high.)

(a) The simplest approach is to divide eqn. (11.125) into (11.126)

$$k = \left(\frac{0.083228}{2.6693\times10^{-6}}\right)\left(\frac{\mu}{M}\right) = (3.118\times10^4\ \mathrm{J/kg\cdot K})\left(\frac{\mu}{M}\right)$$

For a monatomic gas

$$c_v = \frac{3}{2}(R^\circ/M) \quad \text{and} \quad \frac{5}{2}\left(\frac{3}{2}R^\circ\right) = 3.118\times10^4\ \mathrm{J/kg\cdot K}$$

Therefore, $\underline{k = \frac{5}{2}\mu c_v}$ ←

(b) The Euken correction (11.127) gives

$$k = \left(\frac{9\gamma-5}{4\gamma}\right)\mu c_p$$

and $Pr = (\mu c_p/k)$ so: $\underline{Pr = \left(\frac{4\gamma}{9\gamma-5}\right)}$ ←

(c) Substituting the expression stated into Euken's formula for Pr, we have:

$$Pr = \frac{4(D+2)}{9(D+2)-5D} = \underline{2\left(\frac{D+2}{2D+9}\right)} \leftarrow$$

Pr tends to unity as D becomes large, as for complex molecules and at high temperature when vibrational modes become more important.

(d) The value of γ must be specified. For monatomic Ar, D = 3 (3 translational modes) so $\gamma = 5/3$ and $Pr = 2/3$. For diatomic N_2 we add two rotational modes. D = 5 and $\gamma = 7/5$ so $Pr = 28/38 = 14/19$. For triatomic H_2O, we have an additional rotational mode, so D is at least 6. and

$Pr = 16/21$. H_2O may also be more suseptable to vibrational modes.

	Ar		N_2		H_2O
Euken Pr:	0.666		0.737		0.762
Data:					
100°K	0.700	100°K	0.767		
311 K	0.676	300	0.715	373°K	1.052
533	0.664	500	0.696	453	1.027
811	0.660	800	0.713	513	1.039
1089	0.663	1100	0.723	613	1.067
1500	0.671	1500	0.704		

The agreement for Ar and N_2 is within about 5% over the whole range. The agreement for H_2O is poor. The underprediction may result from active vibrational modes.

11.21 A student is studying the combustion of a premixed gaseous fuel with the molar composition: 10.3 percent methane, 15.4 percent ethane, and 74.3 percent oxygen. She passes 0.006 ft^3/s of the mixture (at 70°F and 18 psia) through a smooth 3/8 inch ID tube, 47 inches long. a.) What is the pressure drop? b.) The student's advisor recommends pre-heating the fuel mixture, using a nichrome strip heater wrapped around the last 5 inches of the duct. If the heater produces 0.8 W/inch, what is the wall temperature at the outlet of the duct? Let c_{p,CH_4} = 2280 J/kg-°K, γ_{CH_4} = 1.3, c_{p,C_2H_6} = 1730 J/kg-°K, $\gamma_{C_2H_6}$ = 1.2, and approximate the properties at the inlet conditions.

First determine the properties of the gas mixture at (70°F, 18 psi) = (294.4 K, 1.225 atm). Use (11.128,129) for the mixture. From Appendix A, $\mu_{O_2} = 2.032 \times 10^{-5}$ kg/ms, $k_{O_2} = 0.02629$ W/mK. For CH_4, C_2H_6 use (11.125,127):

	σ (Å)	ϵ/k (K)	$M\left(\frac{kg}{kgmole}\right)$	kT/ϵ	Ω_μ	μ (kg/ms) (11.125)
CH_4	3.758	148.6	16.04	1.981	1.179	1.102×10^{-5}
C_2H_6	4.443	215.7	30.07	1.365	1.368	9.300×10^{-6}

and from the given data: $k_{CH_4} = 0.03237$ W/mK, $k_{C_2H_6} = 0.01944 \frac{W}{mK}$ by (11.127). To compute ϕ_{ij}, let $O_2 = 1$, $CH_4 = 2$, $C_2H_6 = 3$. The results are: $\phi_{12} = 0.9378$, $\phi_{13} = 1.484$, $\phi_{23} = 1.476$,

$\phi_{21} = 1.015$, $\phi_{31} = 0.7226$, $\phi_{32} = 0.6645$

Representing the sums in the denominators of (11.128-9) as $\Sigma_i \equiv \sum_{j=1}^{3} x_j \phi_{ij}$, we have: $\Sigma_1 = 1.068$, $\Sigma_2 = 1.084$, $\Sigma_3 = 0.7593$.

Thus,
$$\mu_m = \left\{ \frac{(0.743)(2.032)}{1.068} + \frac{(0.103)(1.102)}{1.084} + \frac{(0.154)(0.930)}{0.7593} \right\} \times 10^{-5}$$
$$= \underline{1.71 \times 10^{-5}\ \text{kg/ms}}$$
$$k_m = \underline{0.0253\ \ \text{W/mK}}$$

(a) Use eqn. (3.25): $\Delta p = (L/D)(\rho u_{av}^2/2) f$. $u_{av} = \dot{m}/\rho A$ where $\dot{m}/\rho = 0.006\ \text{ft}^3/\text{s} = 1.70 \times 10^{-4}\ \text{m}^3/\text{s}$ and $A = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 \text{in}^2 = 7.13 \times 10^{-5}\ \text{m}^2$, so that $u_{av} = 2.38\ \text{m/s}$. The density of the mixture is, by (12.8) and the ideal gas law: $M = 30.06$ kg/kgmole,
$\rho = (101325)(1.225)(30.06)/(8314.3)(294.4) = 1.524\ \text{kg/m}^3$. The Reynolds number is: $Re_D = (1.524)(2.38)(3/8)(0.0254)/(1.71 \times 10^{-5})$ $= 2020$, barely laminar. From Fig. 7.6, $f = 64/Re_D = 0.0317$

so: $\Delta p = \left(\frac{47}{3/8}\right)\left(\frac{(1.524)(2.38)^2}{2}\right)(0.0317) = \underline{17.2\ \text{N/m}^2 = 2.49 \times 10^{-3}\ \text{psi}}$

The pressure drop is entirely negligible.

(b) At the end of the tube we should have fully developed laminar flow: $Nu = 4.368$, eqn. (7.21). From Ex. 7.1 and eqn. (7.21)
$$T_w(L) = 294.4\,°\text{K} + \frac{4 q_w \alpha}{u_{av} D k} L + \frac{11}{48} \frac{q_w D}{k}, \quad L = 5''.$$
Now, $\alpha/k = k/\rho c_p k = 1/\rho c_p$. From App. A, $c_{p_{O_2}} = 919.8$ J/kg K and $c_{p_m} = \sum_i m_i c_{p_i} = \frac{1}{M} \sum_i x_i c_{p_i} M_i = 1119$ J/kg K. Thus:
$$T_w(\text{exit}) = 294.4\,°\text{K} + q_w \left\{ \frac{11}{48} \frac{3/8\,(0.0254)}{(0.0253)} + \frac{4(5'')}{(2.38)(3/8'')(1.524)(1119)} \right\}$$
$$= 294.4\,°\text{K} + 0.09942\, q_w$$
The wall heat flux is $q_w = (0.8\ \text{W/in})/\pi D = (0.8)/\pi\left(\frac{3}{8}\right)(0.0254)^2$ $= 1053\ \text{W/m}^2$, so that
$$T_w(\text{exit}) = \underline{399.1\ \text{K}}$$

11.22 a.) Work Problem 6.36. b.) A fluid is said to be incompressible if the density of a fluid particle does not change as it moves about in the flow (i.e., if $D\rho/Dt = 0$). Show that an incompressible flow satisfies $\vec{\nabla}\cdot\vec{u} = 0$. c.) How does the condition of incompressibility differ from that of constant density?" Describe a flow that is incompressible but which does not have "constant density."

(b) From Problem (6.36):

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = \left(\frac{\partial \rho}{\partial t} + \vec{u}\cdot\nabla\rho\right) + \rho\,\nabla\cdot\vec{u} = 0$$

or

$$\frac{D\rho}{Dt} = -\rho\,\nabla\cdot\vec{u}$$ For an incompressible fluid, we clearly have $\underline{\nabla\cdot\vec{u} = 0}$

(c) "Constant density" is generally taken to mean that ρ is spatially uniform and constant in time. Thus, all fluid particles would have the same density. In an incompressible flow each particle may have a different, but unchanging, density. Thus, the flow may have spatial gradients of density, i.e. it may be stratified. The most significant example of an incompressible, stratified fluid is the ocean, which is colder and saltier at greater depths and thus shows increasing density as one moves downward from the surface.

11.23 Carefully derive equation (11.43). Note that ρ is not assumed constant in (11.43).

Eqns. (11.39) and (11.41) give:

$$\frac{\partial \rho_i}{\partial t} + \vec{v}\cdot\nabla\rho_i = -\nabla\cdot\vec{j}_i + \dot{r}_i \quad (*)$$ (2nd term in incompressible form)

Introduce Fick's Law and assume $\rho\mathcal{D}_{im}$ spatially uniform:

$$\frac{\partial \rho_i}{\partial t} + \vec{v}\cdot\nabla\rho_i = -\nabla\cdot(-\rho\mathcal{D}_{im}\nabla m_i) + \dot{r}_i = +\rho\mathcal{D}_{im}\nabla^2 m_i + \dot{r}_i$$

Now: $$\frac{D\rho_i}{Dt} = \rho\frac{Dm_i}{Dt} + m_i\frac{D\rho}{Dt} = \rho\frac{Dm_i}{Dt}$$, for incompressible flow

Thus:

$$\boxed{\frac{Dm_i}{Dt} = \mathcal{D}_{im}\nabla^2 m_i + \dot{r}_i/\rho_i} \quad (11.43)$$

To get (12.66) let ρ_i, $\mathcal{D}_{im}$ separately be independent of (x,y,z): $-\nabla\cdot\vec{j}_i = +\mathcal{D}_{im}\nabla(\rho\nabla m_i) = \mathcal{D}_{im}\nabla^2\rho_i$

Substitute into (*) above to get:

$$\boxed{\frac{D\rho_i}{Dt} = \mathcal{D}_{im}\nabla^2\rho_i + \dot{r}_i}$$

11.24 Derive the equation of species conservation on a molar basis, using c_i rather than ρ_i. Also obtain an equation in c_i alone, analogous to equation (11.43), but without introducing incompressibility. What assumptions must be made to obtain the latter result?

$$\frac{d}{dt}\int_R c_i\,dR = -\int_S \vec{N}_i\cdot d\vec{S} + \int_R \dot{R}_i\,dR \qquad \left(\dot{R}_i = \text{molar rate of creation of species } i \text{ kgmole/m}^3\cdot\text{s}\right)$$

$$= -\int_S c_i\vec{v}^*\cdot d\vec{S} - \int_S \vec{J}_i^*\cdot d\vec{S} + \int_R \dot{R}_i\,dR$$

using (11.24) and (11.22). In the first line, the first term is the rate of storage, the second is the rate of inflow, and the third the rate of creation of species i in the region R. By Gauss' theorem and the usual arguments, this leads to:

$$\boxed{\frac{\partial c_i}{\partial t} + \nabla\cdot(c_i V^*) = -\nabla\cdot\vec{J}_i^* + \dot{R}_i} \quad \text{molar analog of (11.38).}$$

Now use Fick's Law, $\vec{J}_i^* = -c\mathcal{D}_{im}\nabla x_i$ and assume both c and $\mathcal{D}_{im}$ are independent of (x,y,z). The boxed eq'n becomes:

$$\boxed{\frac{\partial c_i}{\partial t} + \nabla\cdot(c_i V^*) = \mathcal{D}_{im}\nabla^2 c_i + \dot{R}_i} \quad \text{closest analog of (11.38)}$$

The first two terms are <u>not</u> $\frac{D}{Dt}$. The eq'n may be recast to give D/Dt, but the result is rather awkward.

11.25 Find the following concentrations: a) The mole fraction of air in solution with water at 5°C and 1 atm, exposed to to air at the same conditions. $H = 4.88 \times 10^4$ atm. b.) The mole fraction of ammonia in air above an aqueous solution with $x_{NH_3} = 0.05$ at 0.9 atm. and 40°C, and H = 1522 mm hg. c.) the mole fraction of SO_2 in an aqueous solution at 15°C and 1 atm., if $p_{SO_2} = 28.0$ mm hg and $H = 1.42 \times 10^4$ mm Hg. d.) the partial pressure of ethylene over an aqueous solution at 25°C and 1 atm. with $x_{C_2H_4} = 1.75 \times 10^{-5}$ and $H = 11.4 \times 10^3$ atm.

(a) Under these conditions, the air is essentially dry and $p_{air} \simeq 1$ atm. Thus, $x_{air} = (1 \text{ atm})/(4.88 \times 10^4 \text{ atm}) = \underline{2.05 \times 10^{-5}}$ by Henry's Law, (11.45).

(b) From Henry's Law, $p_{NH_3} = (1522 \text{ mm Hg})(0.05) = 76.1$ mmHg. The gas-side mole fraction is $x_{NH_3} = p_{NH_3}/0.9 \text{ atm} = \underline{0.111}$

(c) Again use Henry's Law: $x_{SO_2} = p_{SO_2}/H = 28.0/1.42 \times 10^4$
$= \underline{1.97 \times 10^{-3}}$

(d) $p_{C_2H_4} = (11.4 \times 10^3 \text{ atm})(1.75 \times 10^{-5}) = \underline{0.200}$

11.26 Use a steam table to estimate: a.) The mass fraction of water vapor in air over water at 1 atm and 20°C, 50°C, 70°C, and 90°C. b.) The partial pressure of water over a 3 percent by weight aqueous solution of HCl at 50°C. c.) The boiling point at 1 atm of salt water with a mass fraction $m_{NaCl} = 0.18$.

(a) Using Raoult's law and the reasoning of example 11.3, $p_{H_2O,s} = p_{sat,H_2O}(T_s)$. With a steam table, eqn. (11.15), (11.8), and (11.9), we have:

T (°C)	p_{sat} (kPa)	$x_{H_2O,s}$	M_s (kg/kgmole)	$m_{H_2O,s}$
20	2.339	0.02308	28.71	0.01449
50	12.349	0.1219	27.63	0.07951
70	31.19	0.3078	25.59	0.2167
90	70.14	0.6922	21.39	0.5832

(b) $M_{HCl} = 36.46$ kg/kgmole. By (11.8), with $m_{HCl} = 0.03$, $M_{soln} = 18.30$. By (11.9), $x_{HCl,soln} = 0.0151$. Raoult's law then yields:

$$p_{H_2O,s} = p_{sat,H_2O}(1-0.0151) = (12.349\ \text{kPa})(0.9849)$$
$$= \underline{12.16\ \text{kPa}}$$

(c) $M_{NaCl} = 58.44$ kg/kgmole. The molecular weight of the soln is 20.58 kg/kgmole, and $x_{NaCl,soln} = 0.06340$. The solution's boiling point is the temperature at which $p_{H_2O,s}$ = 1 atm:

$$101325\ \text{Pa} = p_{sat,H_2O}(T_{B.P.})(1-0.06340)$$

or $p_{sat,H_2O}(T_{B.P.}) = 108{,}184$ Pa. From a steam table, we find

$$T_{B.P.} \doteq \underline{101.8^\circ\text{C}}$$

11.28 a.) Write eqn. (11.43) and the b.c.'s in terms of a nondimensional mass fraction, Ψ, analogous to the dimensionless temperature in eqn. (7.42). b.) For $\nu = \mathcal{D}_{im}$, relate Ψ to the Blasius function, f, for flow over a flat plate. c.) Note the similar roles of Pr and Sc in the two boundary later transport processes. Infer the mass concentration analog of eqn. (6.55) and sketch the concentration and momentum b.l. profiles for Sc << 1, Sc = 1, and Sc >> 1.

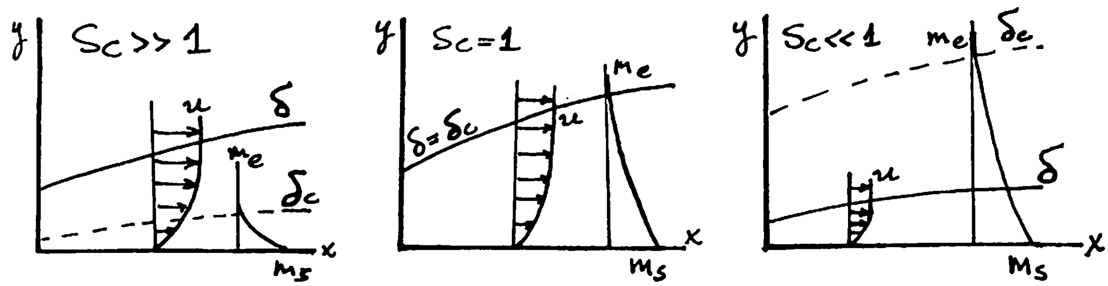
(a) The appropriate nondimensionalization is: $\psi = \left(\dfrac{m_i - m_{i,s}}{m_{i,e} - m_{i,s}}\right)$

The analog to (6.42) is then:

$$u\frac{\partial}{\partial x}\psi + v\frac{\partial}{\partial y}\psi = \mathcal{D}_{i,m}\frac{\partial^2}{\partial y^2}\psi\,; \quad \psi(y=0)=0\,,\ \psi(y\to\infty)=1\,,$$
$$\frac{\partial}{\partial y}\psi\Big|_{y\to\infty}=0$$

(b) By eqns (6.21),(6.41) when $\nu = \mathcal{D}_{i,m}$, the distributions of concentration and velocity are identical: $\underline{\psi = f'}$

(c) The similarity of heat transfer and low-rate mass transfer shows that we need only replace δ_t by δ_c and α by $\mathcal{D}_{i,m}$. Therefore: $\boxed{\dfrac{\delta_c}{\delta} = Sc^{-1/3}}$ for $0.6 \le Sc \le 50$ and low rates



11.29 When Sc is large, momentum diffuses more easily than mass, and the concentration b.l. thickness, δ_c, is much less than the momentum b.l. thicknesss, δ. On a flat plate, the small part of the velocity profile within the concentration b.l. is approximately $u/U_e = 3y/2\delta$. Compute $Nu_{m,x}$ based on this velocity profile, assuming a constant wall concentration. (Hint: Use the mass transfer analogs of eqn. (6.47) and (6.50) and note that $q_w/\rho c_p$ becomes $j_{i,s}/\rho$.)

The analog of (6.47) is: $\dfrac{d}{dx}\displaystyle\int_0^{\delta_c} u(m_i - m_{i,e})\,dy = j_{i,s}/\rho$

and the analog of (6.47) is: $\left(\dfrac{m_i - m_{i,e}}{m_{i,s}-m_{i,e}}\right) = 1 - \dfrac{3}{2}\dfrac{y}{\delta_c} + \dfrac{1}{2}\left(\dfrac{y}{\delta_c}\right)^3$

Now, $j_{i,s}/\rho = -\mathcal{D}_{i,m}\dfrac{\partial}{\partial y}(m_i - m_{i,e})\Big|_{y=0} = (m_{i,s}-m_{i,e})\mathcal{D}_{i,m}\cdot\dfrac{3}{2}\dfrac{1}{\delta_c}$

And:
$$\frac{d}{dx}\int_0^{\delta_c} u(m_i - m_{i,e})\,dy = U_e(m_{i,s}-m_{i,e})\frac{d}{dx}\int_0^{\delta_c}\left(\frac{3}{2}\frac{y}{\delta}\right)\left(1-\frac{3}{2}\frac{y}{\delta_c}+\frac{1}{2}\left(\frac{y}{\delta_c}\right)^3\right)dy$$

$$= U_e(m_{i,s}-m_{i,e})\frac{d}{dx}\left(\frac{3}{20}\frac{\delta_c^2}{\delta}\right) = U_e(m_{i,s}-m_{i,e})\cdot\frac{3}{20}\left(\frac{\delta_c}{\delta}\right)\frac{d\delta_c}{dx}$$

since $\delta_c/\delta = fn(Sc \text{ only})$ by analogy to eq'n (6.46).

Combining the above equations, we obtain a d.e. for $\delta_c(x)$:

$$\mathcal{D}_{i,m}\frac{3}{2}\frac{1}{\delta_c} = U_e\frac{3}{20}\left(\frac{\delta_c}{\delta}\right)\frac{d\delta_c}{dx} \qquad \text{where } \frac{\delta_c}{\delta} \neq fn(x)$$

$$\left(\frac{10\,\mathcal{D}_{i,m}}{U_e}\right)\left(\frac{\delta}{\delta_c}\right) = \frac{d}{dx}\left(\frac{\delta_c^2}{2}\right) \longleftarrow \text{integrate with } \delta_c = 0 \text{ at } x = 0$$

$$\underline{\delta_c = \sqrt{(20\,\mathcal{D}_{i,m}/U_e)(\delta/\delta_c)\cdot x}} \longleftarrow$$

Next introduce eq'n (6.2) for δ and rearrange to find

$$\left(\frac{\delta_c}{\delta}\right)^{3/2} = \sqrt{\frac{20\,\mathcal{D}_{i,m}\cdot x}{U_e}}\cdot\sqrt{Re_x}\Big/(4.64x), \text{ or } \quad \underline{\left(\frac{\delta_c}{\delta}\right) = \frac{1}{1.025\,Sc^{1/3}}} \longleftarrow$$

The mass transfer coefficient is:

$$g^*_{m,i} = \frac{j_{i,s}}{(m_{i,s}-m_{i,e})} = \rho\mathcal{D}_{i,m}\cdot\frac{3}{2}\frac{1}{\delta_c}$$

and the Nusselt number for mass transfer is:

$$Nu_{m,x} = g^*_{m,i} \cdot x/\rho \mathcal{D}_{i,m} = \frac{3}{2}\frac{x}{\delta_c} = \frac{3}{2\sqrt{20}}\sqrt{\frac{u_e x}{\nu}}\sqrt{\frac{\nu}{\mathcal{D}_{i,m}} \cdot \frac{\delta_c^2}{\delta}}$$

$$= \frac{3}{2\sqrt{20}} Re_x^{1/2} \cdot \frac{Sc^{1/3}}{(1.025)^{1/2}}; \quad \boxed{Nu_{m,x} = 0.331 \; Re_x^{1/2} Sc^{1/3}}$$

This may be compared to problem (6.21) and eq'n (6.59).

11.30 Consider a 1-dimensional, binary gaseous diffusion process in which species 1 and 2 diffuse in opposite directions along the z-axis at equal molar rates. This process is known as equimolar counter diffusion. a.) What are the relations between N_1, N_2, J_1^*, and J_2^*? b.) If steady state prevails and conditions are isothermal and isobaric, what is the concentration of species 1 as a function of z? c.) Write the mole flux in terms of the difference in partial pressure of species 1 between locations z_1 and z_2.

(a) By assumption, $N_1 = -N_2$. Therefore $N = N_1 + N_2 = 0$ and we have by (11.24)

$$\underline{N_1 = -N_2 = J_1^* = -J_2^*}$$

(b) N_1 = constant in steady state, by eq'n (11.49) with $_1 = M_1 N_1$. With Fick's Law: $N_1 = -c\mathcal{D}_{12}\frac{dx_1}{dz}$ = constant. Integrating this we obtain:

$$x_1(z) = -\frac{N_1}{c\mathcal{D}_{12}} z + x_1(z=0)$$

since $c\mathcal{D}_{12}$ is constant under isothermal, isobaric conditions.

(c) Use the above result: $x_1(z_1) - x_1(z_2) = \frac{N_1}{c\mathcal{D}_{12}}(z_2 - z_1)$

$$N_1 = \frac{c\mathcal{D}_{12}}{(z_2 - z_1)}(x_1(z_1) - x_1(z_2))$$

Now recall that $x_1 = p_1/p$ and $c/p = 1/R^o T$. Then obtains

$$\boxed{N_1 = \frac{(p_1 - p_2)}{R^o T (z_2 - z_1)}}$$

11.31 Consider steady mass diffusion from a small sphere. When convection is negligible, the mass flux in the radial direction is $n_{r,i} = j_{r,i} = -\rho \mathcal{D}_{i,m} dm_i/dr$. For the case in which the concentration is $m_{i,\infty}$ far from the sphere and $m_{i,s}$ at its surface, use a mass balance to obtain the surface mass flux in terms of the overall concentration difference (assuming that $\rho\mathcal{D}_{i,m}$ is constant.) Then apply the definitions **(11.77)** and **(11.58)** to show that $Nu_{m,D} = 2$ for this situation.

In steady state, the mass flow through any spherical shell of radius r about the sphere is the same. Thus,

$$4\pi r^2 \cdot n_{r,i} = \text{constant with respect to } r = C_1$$

Use the given information to obtain a differential eq'n:

$$C_1 = -\rho\mathcal{D}_{i,m} \cdot 4\pi r^2 \cdot \frac{dm_i}{dr}$$

Integrate:

$$\underbrace{\int_{m_{i,s}}^{m_{i,\infty}} dm_i}_{m_{i,\infty}-m_{i,s}} = -\frac{C_1}{\rho\mathcal{D}_{i,m}} \frac{1}{4\pi} \underbrace{\int_{R_s}^{\infty} \frac{dr}{r^2}}_{=1/R_s}$$

where R_s is the radius of the sphere

Rearrange to find:

$$j_{i,s} = n_{i,s} = \frac{C_1}{4\pi R_s^2} = \frac{\rho\mathcal{D}_{i,m}}{R_s}(m_{i,s} - m_{i,\infty})$$

Using **(11.77)** and **(11.58)**, with $g_{m,i} = g^*_{m,i}$ for this low-rate process:

$$Nu_{m,D} = \frac{2R_s}{\rho\mathcal{D}_{i,m}} \cdot \frac{j_{i,s}}{(m_{i,s} - m_{i,\infty})} = \underline{\underline{2}}$$

11.32 An experimental Stefan tube is 6 cm in diameter and 30 cm from the liquid surface to the top. It is held at 10°C and 8.0×10^4 Pa. Pure argon flows over the top and liquid CCl_4 is at the bottom. The pool level is maintained while 0.69 ml of CCL_4 evaporates during a period of 8 hrs. What is the diffusivity of carbon tetrachloride in argon measured under these conditions? The specific gravity of liquid CCl_4 is 1.59 and its vapor pressure is $\log_{10} p_v = 8.004 - 1771/T$, where p_v is expressed in mm hg and T in °K.

Use equation **(11.71)** after calculating the mole flux. First calculate the number of moles evaporated:

$$\rho_{ccl_4}(\ell) = (1.59)\,\rho_{H_2O}(\ell) = (1.59)(999.4) = 1589 \text{ kg/m}^3$$

$$c_{ccl_4}(\ell) = \rho/M = (1589)/(153.8) = 10.33 \text{ kgmole/m}^3$$

The number of moles evaporated $= (0.69\times10^{-6}\text{ m}^3)(10.33\text{ kgmole/m}^3)$ $= 7.13\times10^{-6}$ kgmole. The mole flux is then:

$$N_s = \frac{(\text{kgmoles evaporated})}{(\pi D^2/4)(\text{time})} = \frac{(7.13\times10^{-6})}{(0.06)^2\pi/4(8)(3600)} = 8.76\times10^{-8}\ \frac{\text{kgmole}}{\text{m}^2\cdot\text{s}}$$

The mole concentration of gas in the tube is $c = p/R^\circ T =$ $(8.0\times10^4)/(8314.3)(283.15) = 0.0340$ kgmole/m³. Using the given eq'n, the vapor pressure of CCl_4 is $p_v = 56.15$ mm Hg $= 7.49\times10^3$ Pa; the mole concentration of CCl_4 at the liquid surface is $x_{ccl_4,s} = p_v/p = 0.0936$. Now apply (11.72) as:

$$\mathcal{D}_{12} = \left(\frac{N_s\cdot L}{c}\right)\left(\ln\left(1+\frac{x_{1,e}-x_{1,s}}{x_{1,s}-1}\right)\right)^{-1} = \frac{(8.76\times10^{-8})(0.3)}{\ln\left(1+\frac{0-0.0936}{.0936-1}\right)(0.034)}$$

$$\underline{\mathcal{D}_{12} = 7.87\times10^{-6}\text{ m}^2/\text{s}}$$

11.33 Repeat the analysis given in Section 12.6 on the basis of <u>mass</u> fluxes, assuming that $\rho\mathcal{D}_{im}$ is constant and neglecting any buoyancy driven convection. Obtain the analog of eqn. (11.71).

From eq'n (11.49), we find: $n_1 = n_{1,s}$, $n_2 = n_{2,s}$. From (11.20)

$$n_{1,s} = m_1 n - \rho\mathcal{D}_{12}\frac{dm_1}{dy}.$$

However, n is constant ($n_s = n_{1,s}+n_{2,s}$), so we may integrate this d.e. as before, assuming $(\rho\mathcal{D}_{12})$ is a constant:

$$\rho\mathcal{D}_{12}\frac{dm_1}{dy} = m_1 n_s - n_{1,s} \Rightarrow \left(\frac{n_s y}{\rho\mathcal{D}_{12}}\right) = \ln(n_s\cdot m_1 - n_{1,s}) + \text{const.}$$

The b.c. at $y=0$ is $m_1 = m_{1,s}$ from which the constant is $-\ln(n_s\cdot m_{1,s} - n_{1,s})$. The boundary condition at $y=L$ is $m_1 = m_{1,e}$ and the result is:

$$\boxed{N_s = \left(\frac{\rho\mathcal{D}_{12}}{L}\right)\ln\left(1+\frac{m_{1,e}-m_{1,s}}{m_{1,s}-n_{1,s}/n}\right)}$$

11.34 In sectios 11.4 & 11.7, it was assumed at points that $c\mathcal{D}_{12}$ or $\rho\mathcal{D}_{12}$ was independent of position. a.) If the mixture composition varies in space this asumption may be poor. Using eqn.(11,116) and the definitions from Section 1 .2, examine the composition-dependence of these two groups. For what type of mixture is $\rho\mathcal{D}_{12}$ most sensitive to composition? What does this indicate about molar versus mass-based analysis? b.) How do each of these groups depend on pressure and temperature? Is the analysis of Section(11.7)really limited to isobaric conditions? c.) Do the Prandtl and Schmidt mumbers depend on composition, temperature, or pressure?

(a) From eq'n (11.116), $\mathcal{D}_{12} \propto \dfrac{T^{3/2}}{p\,\Omega_{\mathcal{D}}}$ (molecular parameters).

The molecular parameters depend only upon which two species are involved and not on the relative amounts of each in the mixture. Now, $c = p/R^\circ T$ and $\rho = pM/R^\circ T$.

Thus,
$$c\mathcal{D}_{12} \propto \left(\frac{T^{1/2}}{\Omega_{\mathcal{D}}}\right)(\text{molecular parameters})$$

$$\rho\mathcal{D}_{12} \propto \left(\frac{T^{1/2}}{\Omega_{\mathcal{D}}}\right)\cdot M \cdot (\text{molecular parameters})$$

We see that $c\mathcal{D}_{12}$ is <u>independent</u> of spatial varitions of the mixture composition. $\rho\mathcal{D}_{12}$ is <u>not</u> because of its dependence on M:
$$M = x_1 M_1 + x_2 M_2 = M_1\left\{1 + x_2\left(\frac{M_2}{M_1} - 1\right)\right\}$$

$\rho\mathcal{D}_{12}$ is most sensitive to variations of composition (x_2) when M_2 and M_1 differ greatly. When $M_2 \simeq M_1$, the composition variation is small. For mixtures with greatly differing molecular weights, molar analysis assuming constant $c\mathcal{D}_{12}$ will yield better results than mass based analysis assuming constant $\rho\mathcal{D}_{12}$.

(b) Both groups are <u>independent</u> of p and depend on T as $(T^{1/2}/\Omega_{\mathcal{D}})$. The isobaric assumption in is <u>unnecessary</u>.

(c) $Pr = \mu c_p/k$. Make reference to eqns. (11.125-129). The Prandtl number is independent of pressure and depends on mixture composition. The $\sqrt{T}$ factors in μ, k will cancel out of the mixture rules, but the complicated dependence of Pr on the temperature dependent Ω_μ, Ω_k may introduce a mild dependence of Pr on T in mixtures. $Sc = \mu/\rho \mathcal{D}_{12}$. The Schmidt number is independent of p, depends on composition, and has some dependence on T.

11.35 A Stefan tube contains liquid bromine at 320°K and 1.2 atm. Carbon dioxide flows over the top and is also bubbled up through the liquid at the rate of 40 ml/hr. If the distance from the liquid surface to the top is 16 cm and the diameter is 3 cm, what is the evaporation rate of Br_2? ($p_{sat,Br}$ = 0.680 bar at 320°K.)

From eq'n (12.43), compute $\mathcal{D}_{12} = 6.229 \times 10^{-6}$ m²/s. From (11.13) compute $c = 0.04570$ kgmole/m³. From Raoult's law, assuming a negligible solubility of CO_2 in Br_2 liquid, and (11.15) find $x_{1,s} = p_{sat,Br_2}/p = 0.680/(1.01325)(1.2) = 0.559$. The flux of CO_2 through the liquid surface is:

$$N_{CO_2,s} = (\text{volume flow rate}) \cdot c/(\text{area of tube})$$
$$= (40 \times 10^{-6}\ \text{m}^3/3600\ \text{s})(0.04570\ \text{kgmole/m}^3)/\left(\tfrac{\pi}{4}(0.03)^2\right)$$
$$= 7.18 \times 10^{-7}\ \text{kgmole/m}^2\cdot\text{s}$$

Now apply (11.73) and solve iteratively:

$$N_{Br_2,s} + (7.18\times10^{-7}) = \frac{(0.04570)(6.229\times10^{-6})}{(0.16)} \ln\left[1 + \frac{-0.559}{0.559 - \dfrac{N_{Br_2,s}}{N_{Br_2,s}+7.18\times10^{-7}}}\right]$$

Guess $N_{Br_2,s}$, substitute into the ln term and solve for implied $N_{Br_2,s}$ on L.H.S.:

Guess $N_{Br_2,s}$	Get $N_{Br_2,s}$
1.5×10^{-6} kgmole/m^{2}s	2.40×10^{-6} kgmole/m^{2}s
2.40×10^{-6}	1.59×10^{-6}
1.59×10^{-6}	2.25×10^{-6}
2.25×10^{-6}	1.66×10^{-6}
1.95×10^{-6}	1.86×10^{-6}
1.90×10^{-6}	1.90×10^{-6}

$\underline{N_{Br_2,s} = 1.90\times10^{-6} \text{ kgmole/m}^2\cdot\text{s}}$ ←

11.36 **Show that $g_{m,1} = g_{m,2}$ and $B_{m,1} = B_{m,2}$ in a binary mixture.**

From (11.77), (11.25), and (11.3), we have:

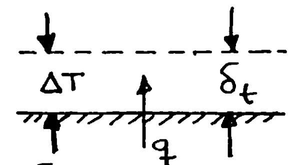
$$g_{m,1} = \frac{j_{1,s}}{(m_{1,s} - m_{1,e})} = \frac{-j_{2,s}}{(1 - m_{2,s} - 1 + m_{2,e})} = \frac{j_{2,s}}{(m_{2,s} - m_{2,e})} = g_{m,2}$$

From (12.91), (12.20), and (12.3), we have:

$$B_{m,1} = \frac{m_{1,e} - m_{1,s}}{m_{1,s} - n_{1,s}/\dot{m}''} = \frac{-(m_{2,e} - m_{2,s})}{1 - m_{2,s} - (\dot{m}'' - n_{2,s})/\dot{m}''}$$

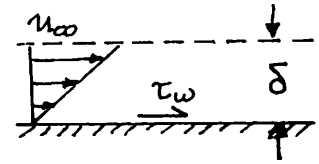
$$= \frac{m_{2,e} - m_{2,s}}{m_{2,s} - n_{2,s}/\dot{m}''} = B_{m,2}$$

11.37 **Demonstrate that stagnant film models of the momentum and thermal boundary layers reproduce the proper dependence of $C_{f,x}$ and Nu_x on Re_x and Pr. Using eqns. (6.31b) and (6.55) to obtain the dependence of δ and δ_t on Re_x and Pr, show that stagnant film models give eqns. (6.33b) and (6.55) within a constant on the order of unity. (The constants in these results will differ from the exact results because the effective b.l. thicknesses of the stagnant film model are not the same as the exact values -- see eqn. (6.57).)**



Heat transfer across a stagnant fluid layer having no horizontal gradients must be by conduction in the vertical direction: $q = k\Delta T\ \delta_t$.
The heat transfer coefficient is then $h = k/\delta_t$ and the Nussett number is $Nu_x = hx/k = x/\delta_t$.

To model momentum transfer, use the <u>absence of horizontal gradients</u> to infer a linear velocity distribution, $u = (y/\delta)u_\infty$, in the film. (This is the "Couette flow" encountered in your basic fluid mechanics course.) The linear velocity distribution here is like the linear temperature distribution assumed above. Then, $\tau_w = \mu\frac{du}{dy} = \mu u_\infty/\delta$ and $C_{f,x} = \tau_w/\frac{1}{2}\rho u_\infty^2 = 2x/\delta \cdot Re_x$.



Now introduce (6.33) and (6.55): $\delta/x = 4.64/\sqrt{Re}$, $\delta/\delta_t = Pr^{1/3}$.

$Nu_x = x/\delta_t = (\sqrt{Re_x}/4.64)\,Pr^{1/3} = \underline{0.216\ Re^{1/2}\ Pr^{1/3}}$ ⟵

$C_{f,x} = 2x/\delta \cdot Re_x = (2/4.64)\cdot Re_x^{-1/2} = \underline{0.431\ Re_x^{-1/2}}$ ⟵

The constants obtained this way differ by a factor of 1.54 from those in (6.33) and (6.55). Eq'n (6.57) shows that the appropriate effective boundary layer thickness is not that used here, but $2/3\ \delta_t$.

11.38 (a) What is the largest value of the mass transfer driving force when one species is transferred? What is the smallest value? (b) Plot the blowing factor as a function of $B_{m,i}$ for one species transferred. Indicate on your graph the regions of blowing, suction, and low-rate mass transfer. (c) Verify the two limits used to show that $g^*_{m,i} = \rho \mathcal{D}_{i,m}/\delta_c$.

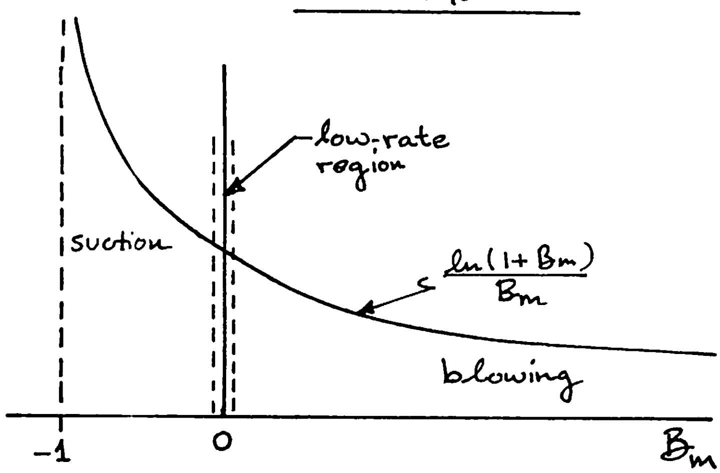
(a) $\mathcal{B}_{m,i} = \dfrac{m_{i,e} - m_{i,s}}{m_{i,s} - 1}$. For fixed $m_{i,e}$ take the limits:

$$\lim_{m_{i,s}\to 0} \mathcal{B}_{m,i} = -m_{i,e} \geq -1 \quad ; \quad \lim_{m_{i,s}\to 1} \mathcal{B}_{m,i} = \begin{cases} +\infty & m_{i,e} < 1 \\ -1 & m_{i,e} = 1 \end{cases}$$

(Note that when $m_{i,e} = 1$, the driving force is -1 for any $m_{i,s}$.)

We see that: $\underline{-1 \leq \mathcal{B}_{m,i} < +\infty}$ ⟵

(b)



(c) First argue that $\dot{m}'' \to 0$ is identical to $\mathcal{B}_{m,i} \to 0$: Recall that $\dot{m}'' = g_{m,i} \cdot \mathcal{B}_{m,i}$. From our knowledge of transport coefficients, we know that $g_{m,i}$ is essentially independent of $\mathcal{B}_{m,i}$ in forced convection and varies as $\mathcal{B}_{m,i}^{1/4}$ in natural convection, when $\mathcal{B}_{m,i}$ is small. $g_{m,i}$ is either constant or goes to zero as $\mathcal{B}_{m,i} \to 0$; therefore $\dot{m}''$ always goes to zero as $\mathcal{B}_{m,i} \to 0$.

prove by L'Hospital's Rule

Thus:

$$g^*_{m,i} = \lim_{\mathcal{B}_{m,i}\to 0} g_{m,i} = \left(\frac{\rho \mathcal{D}_{i,m}}{\delta_c}\right) \lim_{\mathcal{B}_{m,i}\to 0} \frac{\ln(1+\mathcal{B}_m)}{\mathcal{B}_m} = \left(\frac{\rho \mathcal{D}_{i,m}}{\delta_c}\right)$$

11.39 Nitrous oxide is bled through the surface of a porous 3/8 in. OD tube at 0.025 liter/s per meter of tube length. Air flows over the tube at 25 ft/s. Both the air and tube are at 18°C and the ambient pressure is 1 atm. Estimate the mean concentration of N_2O at the tube surface.

At 1 atm, 18°C eq'n (11.116) gives: $\mathcal{D}_{air,N_2O} = 1.483\times10^{-5}$ m²/s. As a first estimate, take all properties as those of pure air and assume low-rate mass transfer — both assumptions are motivated by the low flow rate of N_2O from the tube.

At 18°C, 1 atm: $\rho_{air} = 1.212$ kg/m³, $\nu_{air} = 1.488\times10^{-5}$ m²/s

$$Sc = \nu/\mathcal{D} = 1.003$$

The velocity of air and tube diameter are $U = 7.62$ m/s, $D = 0.009525$ m, and $Re_D = 4878$. This is a uniform wall flux problem and eq'n (7.68) is appropriate:

$$\overline{Nu}_{m,D} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Sc^{1/3}}{[1+(0.4/Sc)^{2/3}]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 41.64$$

$$\overline{g^*_{m,N_2O}} = \overline{Nu}_{m,D}\cdot \rho\mathcal{D}_{air,N_2O}/D = (41.64)(1.212)(1.483\times10^{-5})/0.009525$$

$$= 0.07858 \text{ kg/m}^2\text{s}$$

The mass flow rate of N_2O is, with $\rho_{N_2O} = 1.842$ kg/m³ (ideal gas),

$$n_{N_2O,s} = (0.025\times10^{-3}\text{ m}^3/\text{s}\cdot\text{m})(1.842\text{ kg/m}^3)/\pi(0.009525\text{ m})$$

$$= 1.539\times10^{-3}\text{ kg/m}^2\text{s}$$

At low rates, $n_{N_2O,s} = \overline{g^*_{m,N_2O}}\cdot\overline{\mathcal{B}_{m,N_2O}}$ from which $\overline{\mathcal{B}_{m,N_2O}} = 0.01958$.

With $m_{N_2O,e} = 0$, $\mathcal{B}_{m,N_2O} = (0 - m_{N_2O,s})/(m_{N_2O,s} - 1)$. Solving,

$$\underline{\overline{m_{N_2O,s}} = 0.0192} \longleftarrow$$

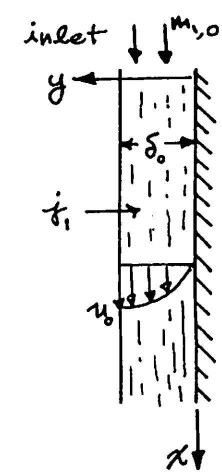
This result justifies the use of air properties and low-rates.

11.40 **Gases** are sometimes absorbed into liquids through "film absorbtion." A thin film of liquid is run down the inside of a vertical tube, through which flows the gas to be absorbed. Analyze this process under the following assumptions: The film flow is laminar and of constant thickness, δ_o, with a velocity profile given by eqn. (8.41) The gas is only slightly soluble in the liquid, so the liquid properties are unaffected by it and it does not penetrate far beyond the liquid surface. The gas concentration at the s and u-surfaces does not vary along the length of the tube. The inlet concentration of gas in the liquid is $m_{i,o}$. Show that the mass transfer is given by

$$\mathrm{Nu}_{m,x} = (u_o x/\pi \mathcal{D}_{12})^{1/2}, \qquad u_o = \frac{(\rho_f - \rho_g) g \delta_o^2}{2\mu_f}$$

$m_{1,0}$

The mass transfer coefficient here is based on the concentration difference between the u-surface and the bulk liquid at $m_{1,o}$. (<u>Hint</u>: The small penetration assumption can be used to reduce the species equation for the film to the diffusion equation.)



Because the liquid film is thin, we ignore the curvature of the tube. As in Chapter 9, we may apply boundary layer assumptions and the appropriate form of the species conservation equation (11.43) becomes:

$$u\frac{\partial m_1}{\partial x} + v\frac{\partial m_1}{\partial y} = \mathcal{D}_{1,m}\frac{\partial^2 m_1}{\partial y^2}$$

The velocity profile is (8.51):

$$u = \underbrace{\frac{(\rho_f - \rho_g) g \delta_o^2}{2\mu}}_{u_o}\left[2\left(\frac{y}{\delta_o}\right) - \left(\frac{y}{\delta_o}\right)^2\right] \quad , \quad v = 0$$

Because the gas does not penetrate far into the film, we may approximate u in the species eq'n by its surface value, u_o. This is the important feature of the small penetration assumption. The species eq'n reduces to the diffusion eq'n:

$$u_o\frac{\partial m_1}{\partial x} = \mathcal{D}_{1,m}\frac{\partial^2 m_1}{\partial y^2} \quad ; \quad \frac{\partial m_1}{\partial x} = \left(\frac{\mathcal{D}_{1,m}}{u_o}\right)\frac{\partial^2 m_1}{\partial y^2}$$

The boundary and initial conditions in the liquid film are:

$$m_1(y=\delta_0, x) = m_{1,u} \; ; \; m_1(x=0,y) = m_{1,0} \; ; \; m_1(y \to -\infty, x) = m_{1,0}$$

In the last condition, we use $y \to -\infty$ rather than $y=0$ because the small penetration approximation essentially removes all effects of the wall. We may cast this problem into the form (11.52) by transforming the y-coordinate to $\bar{y} = \delta_0 - y$; the d.e. is unaffected and the b.c.'s are

$$m_1(\bar{y}=0, x) = m_{1,u} \; ; \; m_1(x=0, y) = m_{1,0} \; ; \; m_1(y \to \infty, x) = m_{1,0}$$

From example 11.8, the solution is

$$\left(\frac{m_1 - m_{1,u}}{m_{1,0} - m_{1,u}}\right) = \operatorname{erf}\left(\frac{\bar{y}}{2\sqrt{\mathcal{D}_{1,m} x/u_0}}\right)$$

To find the mass transfer coefficient, compute $\dot{j}_1$:

$$\dot{j}_1 = -\rho \mathcal{D}_{1,m} \left.\frac{\partial m_1}{\partial \bar{y}}\right|_{\bar{y}=0} = \rho \mathcal{D}_{1,m} (m_{1,u} - m_{1,0}) \cdot \frac{2}{\sqrt{\pi}} \, \frac{1}{2} \left(\frac{u_0}{\mathcal{D}_{1,m} x}\right)^{1/2}$$

(see eqn 5.54). The mass transfer coeff., $g^*_{m,1} = \dot{j}_1 / (m_{1,u} - m_{1,0})$ is

$$g^*_{m,1} = \rho \left(\frac{u_0 \mathcal{D}_{1,m}}{\pi x}\right)^{1/2} \quad \text{so} \quad \boxed{Nu_{m,x} = \left(\frac{u_0 x}{\pi \mathcal{D}_{1,m}}\right)^{1/2}}$$

11.41 Benzene vapor flows through a 3 cm ID vertical tube. A thin film of initially pure water runs down the inside wall of the tube at a flow rate of 0.3 ℓ/s. If the tube is 0.5 m long and 40°C, estimate the rate (in kg/s) at which benzene is absorbed into water over the entire length of the tube. The mass fraction of benzene at the u-surface is 0.206. (<u>Hint</u>: Use the result stated in Prob. 11.40. Obtain δ_0 from the results in Chapter 8.)

The properties of water at 40°C are: $\mu = 6.501 \times 10^{-4}$ kg/m·s, $\rho = 991.8$ kg/m³. The gas density will be small compared to ρ_{H_2O}.

The diffusion coefficient is calculated from eqn. (11.124):

$$\mathcal{D}_{C_6H_6-H_2O} = \left(\frac{313.15}{6.501\times10^{-4}}\right)(4.4\times10^{-15})\left(\frac{0.0187}{0.096}\right)^{1/6}\left(\frac{40.62}{30.76}\right)^{1/2}$$

$= 1.85 \times 10^{-9}$ m²/s. Next obtain the film thickness: the volume flow rate $= 0.3\,\ell/s = 0.0003$ m³/s $= \pi D\left(\dfrac{\dot m}{\rho_f}\right) = \pi D \dfrac{(\rho_f - \rho_g)}{3\mu_f} g \delta_0^3$ by eq'n (9.45). Thus:

$$\delta_0^3 = (0.0003)(3\mu_f)/[\pi D g(\rho_f - \underset{\text{neglect}}{\cancel{\rho_g}})] \; ; \quad \delta_0 = 0.861 \text{ mm}$$

Using the results of problem 11.40, $u_0 = (\rho_f - \underset{\text{neglect}}{\cancel{\rho_g}}) g \delta_0^2 / 2\mu_f = 5.545$ m/s, and $Nu_{m,x} = (u_0 x/\pi \mathcal{D})^{1/2}$.

To get the overall rate of absorption, we need to integrate:

$$\text{rate of absorption} = \pi D \int_0^L n_{C_6H_6,s}\, dx = \pi D (m_{C_6H_6,u} - m_{C_6H_6,0}) \int_0^L g_m\, dx$$

$$= \rho_f (u_0 \mathcal{D}/\pi)^{1/2} \cdot \pi D (m_u - m_0) \underbrace{\int_0^L x^{-1/2}\, dx}_{= 2\cdot L^{1/2}}$$

$$= (991.8)((5.545)(1.85\times10^{-9})\pi)^{1/2}(0.03)(0.206)2(0.5)^{1/2}$$

$$= 1.556 \times 10^{-3} \text{ kg/s} = \underline{1.56 \text{ g/s}}$$

We can check the small penetration assumption with eq'n (5.51):

penetration depth $= 3.65\sqrt{\mathcal{D} t} \simeq 3.65\sqrt{\mathcal{D} L/u_0} = 13\ \mu m \ll \delta_0$.

11.44 Consider the process of "hydrogenolysis", described in Problem **11.9**, in which the reactants diffuse toward a catalyst surface and the products diffuse away from it. (a) What is $\dot{m}''$? (b) Reaction rates in catalysis are of the form (see Problem **11.9**):

$$\dot{R}_{reactant} = A\ e^{-\Delta E/R^oT}(p_{reactant})^n(p_{product})^m\ \text{kgmole/m}^2\text{-s}$$

for the rate of consumption of a reactant per unit surface area. The p's are partial pressures and A, ΔE, n and m are constants. Suppose n = 1 and m = 0 for the reaction B + C → D. Approximate the reaction rate, in terms of mass, as

$$\dot{r}_B = A' e^{-\Delta E/R^oT}\rho_{B,s}\ \text{kg/m}^2\text{-s}$$

and find the rate of consumption of B in terms of $m_{B,e}$ and the mass transfer coefficient for the geometry in question. (c) The ratio $Da \equiv \rho A' e^{-\Delta E/R^oT}/g_m^*$ is called the Damkohler number. Explain its significance in catalysis. What features dominate the process when Da approaches 0 or ∞? What temperature range characterizes each?

(a) $\underline{\dot{m}'' = 0}$: nothing passes through the u-surface since the catalyst undergoes no net reaction. The net mass transfer through the s-surface must be zero.

(b) Write $\dot r_B = \underbrace{\left(A' e^{-\Delta E/R^\circ T}\right)}_{=k''} \rho_{B,s} = k''\cdot\rho\cdot m_{B,s}$ for convenience. Now form a mass balance on B at the s-surface: $j_{B,s} = \dot r_B$, that is, rate of consumption equals rate at which B reaches the surface. This is

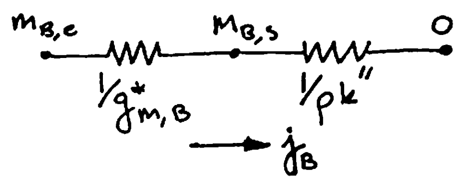
$$g^*_{m,B}\left(m_{B,e} - m_{B,s}\right) = k''\cdot\rho\cdot m_{B,s} \qquad (*)$$

Note that in this process $\dot m'' = 0$ with <u>non</u> zero net fluxes of B, C, and D; thus $n_{B,s} = j_{B,s} = g^*_{m,B}(m_{B,e} - m_{B,s})$.
We may rearrange (*) as: $m_{B,s} = \dfrac{m_{B,e}}{1/g^*_{m,B} + 1/\rho k''}\cdot\dfrac{1}{\rho k''}$

Substitute this into our expression for $\dot r_B$:

$$j_{B,s} = \dot r_B = k''\cdot\rho\cdot m_{B,s} = \underline{\left(\frac{m_{B,e}}{1/g^*_{m,B} + 1/\rho k''}\right)} \longleftarrow$$

We may view this as a resistance network for j_B:



(c) From the result of (b), we see that $Da = \dfrac{\text{diffusion resistance}}{\text{reaction resistance}}$ or (reaction rate)/(diffusion rate). When $Da \to \infty$, the rate of consumption of B is limited by the diffusion resistance; the surface reaction is comparatively fast, as at <u>high</u> temperature. When $Da \to 0$, the process is limited by the reaction rate; this corresponds to <u>low</u> temperatures.

11.45 One typical kind of mass exchanger is a fixed bed catalytic reactor. A flow chamber of length L is packed with a catalyst bed. A gas mixture containing some species i to be consumed by the catalytic reaction flows through the bed at a rate, $\dot{m}$. The effectiveness of such a exchanger is (cf. Chapter 3)

$$\epsilon = 1 - e^{-NTU}, \quad \text{where} \quad NTU = g_{m,oa}PL/\dot{m}$$

where $g_{m,oa}$ is the overall mass transfer coefficient for the catalytic packing. In testing a 0.5 m catalytic reactor for the removal of ethane, it is found that the ethane concentration drops from a mass fraction of 0.36 to 0.05 at a flow rate of 0.05 kg/s. The packing is known to have a surface area of 11 m^2. What is the overall mass transfer coefficient in this bed? What is the exchanger effectiveness? P = surface area per unit length. ϵ is defined in terms of mass fractions.

By comparison to Chapter 3 (eq'n 3.16), the definition of mass exchanger effectiveness must be

$$\varepsilon \equiv \frac{\text{actual mass transfered}}{\text{maximum mass transfer possible}} = \frac{\Delta m_{C_2H_6}}{m_{C_2H_6,in}}$$

$$= \frac{0.36-0.05}{0.36} = \underline{0.861}$$

From this we calculate NTU = 1.97, so that, with $PL = 11\ m^2$, we have

$$g_{m,oa} = (0.05\ kg/s)(1.97)/(11\ m^2) = \underline{0.00895\ \frac{kg}{m^2 \cdot s}}$$

11.46 (a) Perform the integration to obtain eqn. (11.92). Then take the derivative and the limit needed to get eqns. (11.93) and (11.94). (b) What is the general form of eqn. (11.95) when more than one species is transferred?

(a) We have $\frac{d}{dy}\left(-k\frac{dT}{dy} + n_{i,s}c_{p,i}T\right) = 0$ which can be integrated once immediately:

$$-k\frac{dT}{dy} + n_{i,s}c_{p,i}T = C \text{, an unknown constant}$$

Rearrange and integrate:

$$\int_0^y dy = y = \int_{T_s}^{T} \frac{(-k)\,dT}{C - n_{i,s}c_{p,i}T} = \frac{k}{n_{i,s}c_{p,i}} \ln\left(\frac{C - n_{i,s}c_{p,i}T}{C - n_{i,s}c_{p,i}T_s}\right)$$

or

$$(*) \quad \exp\left(\frac{y\, n_{i,s}c_{p,i}}{k}\right) = \frac{\mu - T}{\mu - T_s},$$ where the constant μ is $C/n_{i,s}\cdot c_{p,i}$ for convenience

Now apply the b.c. $T(y=\delta_t) = T_e$:

$$\frac{\mu - T_e}{\mu - T_s} = \exp\left(\frac{\delta_t\, n_{i,s}c_{p,i}}{k}\right) = \eta$$, for convenience define η as shown.

Thus, $\mu = (T_e - T_s\eta)/(1-\eta)$ and we have found the unknown constant μ (i.e. C). Substitute for μ in (*) and rearrange:

$$\frac{(T_e - T_s\eta) - T(1-\eta)}{(T_e - T_s\eta) - T_s(1-\eta)} = \frac{(T - T_s)\eta - (T - T_e)}{(T_e - T_s)}$$

$$= \frac{(T-T_s)(\eta - 1)}{(T_e - T_s)} + 1 = \exp\left(\frac{y \cdot n_{i,s}c_{p,i}}{k}\right)$$

Now substitute for η and rearrange to obtain (11.92):

$$\boxed{\left(\frac{T - T_s}{T_e - T_s}\right) = \frac{\exp\left(\frac{y\, n_{i,s}c_{pi}}{k}\right) - 1}{\exp\left(\frac{\delta_t\, n_{i,s}c_{pi}}{k}\right) - 1}}$$

To get h, write:

$$h = +k \frac{d}{dy}\left(\frac{T-T_s}{T_e-T_s}\right)\Bigg|_{y=0} = \frac{n_{i,s} c_{p_i} \cdot \cancel{k/k}^{\,1}}{\exp\left(\frac{\delta_t n_{i,s} c_{p_i}}{k}\right)-1}.$$

Then use L'Hospital's rule to get h^*:

$$h^* = \lim_{n_{i,s}\to 0} h = \lim_{n_{i,s}\to 0} \frac{c_{p,i}}{(\delta_t c_{p_i})/k \cdot \exp(\delta_t n_{i,s} c_{p_i}/k)} = \frac{k}{\delta_t}$$

(b) This form may be obtained with a minimum of work by recognizing that the sum in (11.90) becomes:

$$\sum_i \rho_i h_i \vec{v}_i = \sum_i n_i h_i = \left(\sum_i n_i c_{p_i}\right)(T - T_{ref})$$

provided we select a common T_{ref} for each species (even if we don't, d/dy will eliminate the T_{ref}'s below.) Now we just integrate $\frac{d}{dy}\left(-k\frac{dT}{dy} + \left(\sum_i n_{i,s} c_{p_i}\right)T\right) = 0$ as before. Thus, the result is:

$$\boxed{\frac{T-T_s}{T_e-T_s} = \frac{\exp\left[\frac{y\left(\sum_i n_{i,s} c_{p_i}\right)}{k}\right]-1}{\exp\left[\frac{\delta_t\left(\sum_i n_{i,s} c_{p_i}\right)}{k}\right]-1}}$$

11.47 (a) Derive eqn. (11.105) from eqn. (11.104). (b) Suppose that 1.5 m2 of the wing of a spacecraft re-entering the earth's atmosphere is to be cooled by transpiration. 900 kg of the vehicle's weight is allocated for this purpose. The low rate heat transfer coefficient is about 1800 W/m^2-^oK in the region of interest and the hottest portion of re-entry is expected to last 3 min. If the air behind the shock wave ahead of the wing is at 2500^oC, which of these gases H_2, He, and N_2 -- keeps the surface coolest? (Of course, the result for H_2 is marred by fact that it would burn under these conditions.) The reservoir is at 5°C.

(a) Substitute for h in (11.104) with (11.95) and divide through by $n_{i,s} c_{pi}$:

$$T_s = \frac{T_e + T_r\left(\exp(n_{i,s} c_{pi}/h^*) - 1\right)}{1 + \left(\exp(n_{i,s} c_{pi}/h^*) - 1\right)}$$

$$= T_r + (T_e - T_r)\exp(-n_{i,s} c_{pi}/h^*)$$

(b) If the entire gas reservoir is to be used at an even rate during the 3 min. of hottest reentry, the mass flux is:

$$n_{i,s} = \frac{(900\ \text{kg})}{(1.5\ \text{m}^2)(4\ \text{min})(3600\ \text{s/min})} = 0.0417\ \frac{\text{kg}}{\text{m}^2\cdot\text{s}}$$

The specific heat c_p is not strongly dependent on p or T so, using App. A, we estimate

$$c_{p_{H_2}} \cong 17\times 10^3\ \text{J/kg K}, \quad c_{p_{He}} = 5200\ \text{J/kg K},$$
$$c_{p_{N_2}} \cong 1.4\times 10^3\ \text{J/kg K}$$

Substitute all this into (11.105), with $T_r = 5°C$ and $T_e - T_r = 2495°C$ and $h^* = 1800\ \text{W/m}^2\cdot\text{K}$, to obtain:

$$T_s = \begin{cases} 1688\ °C & \text{for } H_2 \\ 2120\ °C & \text{"}\ He \\ 2420\ °C & \text{"}\ N_2 \end{cases}$$

H_2 is the coolest. Even when it burns, it may still be the coolest.

11.48: We do not presently have a solution for this problem. However, we'd previously solved it for a much lower terminal velocity -- one that might have occured with a drag-inducing streamer attached to the small sphere. The present terminal velocity is more appropriate for a free-fall of the sphere. The previous solution, below, outlines the solution method.

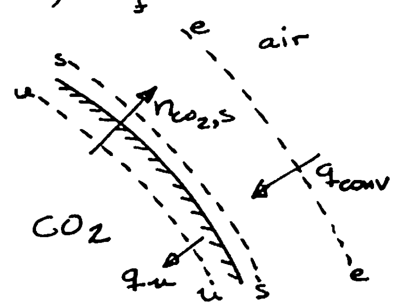
Dry ice (solid CO_2) is used to cool medical supplies transported by a small plane in Alaska. A roughly spherical chunk of dry ice, roughly 5 cm in diameter, falls from the plane through air at 5°C with a terminal velocity of 15 m/s. If steady state is reached quickly, what are the temperature and sublimation rate of the dry ice? The latent heat of fusion is 574,000 J/kg, ρ_s = 1550 kg/m^3, and $\log_{10} p$ (mm Hg) = 9.9082 − 1367.3/T°K. The temperature will be well below the sublimation point of CO_2 which is −78.6°C. Use the heat transfer relation $\overline{Nu}_D = 2 + 0.3\, Re_D^{0.6} Pr^{1/3}$. (<u>Hint</u>: first estimate the surface temperature using properties for pure air, then correct the properties as necessary.)

We know that $T_s \leq -78.6°C$ because solid CO_2 does not exist at higher temps. To make a first estimate of the surface temperature, use properties of pure air at $T_f = -45°C$, which corresponds to a guess of $T_s = -95°C$. For air:

$\rho_f = 1.55$ kg/m^3, $\mu_f = 1.49 \times 10^{-5}$ kg/m-s, $k_f = 0.0205$ W/m·K

$Pr_f = 0.729$; $\mathcal{D}_{CO_2\text{-air}} = 0.9458 \times 10^{-5}$ m^2/s, $Sc_f = 1.02$

Now construct the energy balance:



$q_u = 0$ for wet-bulb conditions

$$\underbrace{n_{CO_2,s} \cdot h_{sg}}_{\text{heat absorbed in phase change}} = q_{conv} = \underbrace{h\,(T_e - T_s)}_{\text{heat convected to the surface}}$$

Use this balance in conjunction with (11.95) for h and $n_{CO_2,s} = \overline{g^*_{m,CO_2}} \cdot \ln(1 + B_{m,CO_2})$. The Reynolds number is $Re_D = 7.80 \times 10^4$; compute $\overline{g^*_m}$ and $\overline{h^*}$ from the given eq'n:

$$\overline{g^*_m} = \frac{\rho \mathcal{D}_{CO_2\text{-air}}}{D} \left\{ 2 + 0.3\, Re_D^{0.6}\, Sc^{1/3} \right\} = 0.07688 \text{ kg/m}^2\text{-s}$$

$$\overline{h^*} = \frac{k}{D} \left\{ 2 + 0.3 \cdot Pr^{1/3} \cdot Re_D^{0.6} \right\} = 96.20 \text{ W/m}^2\text{-K}$$

To estimate the blowing factor use a guess of $T_s = -95°C$ and compute $p_v = 171.1$ mmHg from given eqn.; this gives $x_{CO_2,s} = 0.2251$ and $m_{CO_2,s} = 0.3063$. The mass transfer driving force is $\mathcal{B}_{m,CO_2} = (0 - 0.3063)/(0.3063 - 1) = 2.265$. This yields $n_{CO_2,s} = (0.07688)\ln(1 + 2.265) = 0.09097$ kg/m²·s. With $c_{p_{CO_2}} = 789$ J/kg-K, (11.95) yields $h = 64.73$ W/m²·K. Solve for T_s from the energy balance: $(0.09097)(5.74 \times 10^5) = (64.73)(5°C - T_s)$ or $T_s = -802°C$. Our guess is too high!

Leave properties alone, make a new guess for T_s in calculating p_v. Then get a new $\mathcal{B}_m \Rightarrow n_{CO_2,s} \Rightarrow h \Rightarrow T_s$: Guess $T_s = -100°C$; $p_v = 102.7$ mmHg; $m_{CO_2,s} = 0.192$; $\mathcal{B}_m = 0.237$; $n_{CO_2,s} = 0.01638$ kg/m²-s; $h = 89.88$ W/m²-K; $\underline{T_s = -99.6\,°C}$.

Note that the calculation of T_s is <u>very</u> sensitive to the initial guess.

Now repeat the calculation using updated property values. Take $T_f = \frac{1}{2}(-100 + 5) = -47.5°C$. For h^*, use $m_{CO_2,f} = 0.0$:

$k_h = 0.0202$ W/m·K, $\mu_h = 1.47 \times 10^{-5}$ kg/m-s, $\rho_h = 1.56$ kg/m³

$Pr_h = 0.730$, $Re_D = 7.98 \times 10^4$

The result is $\overline{h^*} = 96.12$. For $\overline{g_m^*}$, the appropriate reference state is $T_f = -47.5°C$ and $m_{CO_2,f} = 0.09$ ($x_{CO_2,f} = 0.061$):

$M_f = 29.88$ kg/kgmole, $\rho_f = 1.61$ kg/m³, $c_{p_f} = 983.7$ J/kg·K

$\mathcal{D}_{CO_2-air} = 9.266 \times 10^{-6}$ m²/s

Now compute the mixture viscosity, using (11.128):

$\mu_{air_{(1)}} = 1.47 \times 10^{-5}$ kg/m·s, $\mu_{CO_{2(2)}} = 1.14 \times 10^{-5}$ kg/m·s

$\phi_{12} = 1.403$, $\phi_{21} = 0.7162$; $\mu_m = 1.44 \times 10^{-5}$ kg/m·s

We have $Re_D = 5.9 \times 10^4$, $Sc = 0.965$ so $\overline{g_m^*} = 0.0802$ kg/m²·s
We now repeat the guessing process for T_s:

Guess $T_s = -100°C$ and $B_m = 0.237$ from above; $n_{CO_2,s} = 0.0171$ kg/m²·s; $h = 89.6$ W/m²K; Solve to find $T_s = -104.5°C$.

Guess $T_s = -101°C$; $x_{CO_2,s} = 0.122$; $B_m = 0.210$; $n_{CO_2,s} = 0.0153$ kg/m²·s; $h = 90.2$ W/m²·K; Solve to find $T_s = -92.4°C$.

It appears that the surface temperature is slightly less than -100°C and the sublimation rate is about 0.016 kg/m²·s. Additional iteration is probably meaningless because of limited accuracy of property data.

11.49 The following data were taken at a weather station over a period of several months:

Date	$T_{dry-bulb}$	$T_{wet-bulb}$
3/15	15.5°C	11.0°C
4/21	22.0	16.8
5/13	27.3	25.8
5/31	32.7	20.0
7/4	39.0	31.2

Use eqn. (12.126) to find the mass fraction of water in the air at each date. Compare these values to ones obtained using a psychrometric chart.

Use eq'n (11.63): $B_{m,H_2O} = (T_e - T_{w.b.})\, c_p / h_{fg}(T_{w.b.}) \cdot Le^{2/3}$

Set $Le = 1.18$ and find $m_{H_2O,s}$ and $h_{fg}(T_{w.b.})$ using a steam table. To get c_p, note $c_{p_{air}} = 1003$ J/kg in the range of interest, estimate $c_{p_{H_2O}} = 2030$ J/kg and use $c_p = \sum_i m_i c_{p_i}$. Evaluate c_p at $m_{H_2O,s}$ and correct if necessary:

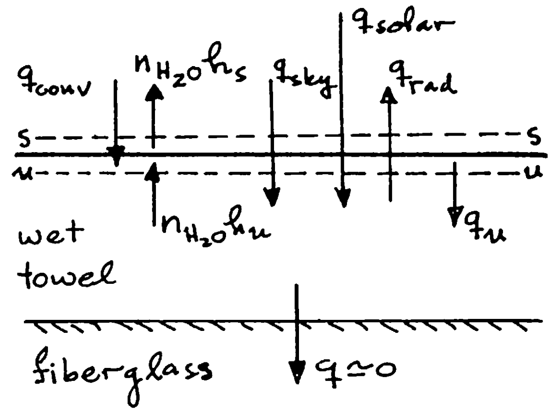
$T_{W.B.}$ (°C)	$h_{fg}(T_{W.B.})$ (kJ/kg)	$m_{H_2O,s}$	c_p (J/kg)	B_{m,H_2O}	$m_{H_2O,e}$
11.0	2475.4	0.00810	1011	0.00165	0.00647
16.8	2461.6	0.01184	1015	0.00192	0.00994
25.8	2440.4	0.02067	1024	0.00056	0.02012
20.0	2454.1	0.01449	1018	0.00472	0.00984
			$c_{p_g} = 1015$	0.00471	0.00985
31.2	2427.6	0.02841	1032	0.00297	0.02553

Psychrometric charts give γ (kg H_2O/kg air). To convert to m_{H_2O} note: $m_{H_2O} = \rho_{H_2O}/\rho = (1+\rho_{air}/\rho_{H_2O})^{-1} = (1+1/\gamma)^{-1}$. We can now use a psychrometric chart to prepare another table:

Date	γ from chart	$m_{H_2O,e}$ from chart	$m_{H_2O,e}$ calculated	% difference
3/15	0.0064	0.00636	0.00647	1.7 %
4/21	0.0098	0.00970	0.00994	2.4
5/13	0.0206	0.0202	0.02012	0.3
5/31	0.0095	0.00941	0.00985	4.6
7/4	0.0260	0.0253	0.02553	0.7

11.50 Biff Harwell has taken Deb sailing. Deb and Biff's towel fall into the harbor. Biff rescues them both from a passing dolphin , and then spreads his wet towel out to dry on the fiberglass foredeck of the boat. The incident solar radiation is 1050 W/m^2; the ambient air is at 31°C with m_{H_2O} = 0.017; the wind speed is 8 knots relative to the boat (1 knot = 1.151 mph); $\epsilon_{towel} \cong \alpha_{towel} \cong 1$; and the sky has the properties of a black body at 280 K. The towel is 3 ft in the windward direction and 2 ft wide. Help Biff to figure out how rapidly (in kg/s) water evaporates from the towel. (Help Biff to understand why Deb won't date him anymore.)

Construct an energy balance on the towel:



<u>s-u surfaces:</u> $q_{conv} = n_{H_2O} h_{fg} + q_u$

<u>u and below:</u> $q_{solar} + q_{sky} + q_u = q_{rad}$

Fiberglass is a poor conductor, so the heat conducted from the bottom of the towel is $\simeq 0$.

Combining the two balances, we have:

$$q_{solar} + q_{sky} - q_{rad} + q_{conv} = n_{H_2O,s} h_{fg}$$

In this eq'n:

$$q_{solar} = 1050 \text{ W/m}^2, \quad q_{sky} = \sigma T_{sky}^4, \quad q_{rad} = \sigma T_s^4$$

and we have taken $\varepsilon = \alpha = 1$ for the towel.

To begin the calculations, let's assume low-rates and properties of pure air at a film temperature $T_f = 36^\circ$C (i.e. $T_s = 41^\circ$C): $\rho_f = 1.142$ kg/m^3, $k_f = 0.02680$ W/m·K, $\mu_f = 1.895 \times 10^{-5}$ kg/m·s, $Pr_f = 0.710$, $\mathcal{D}_{air\text{-}water} = 2.697 \times 10^{-5}$ m^2/s, $Sc_f = 0.615$

The wind speed is $U = 8(1.151)(5280)(0.3048)/(3600) = 4.116$ m/s. The Reynolds number is $Re_L = 2.27 \times 10^5$ (laminar) so $\overline{Nu_L} = 0.664\, Re_L^{1/2} Pr^{1/3}$. From this, $\overline{Nu_L} = 282.2$ and $\overline{Nu_{m,L}} = 269.0$; Solving, we have $\bar{h}^* = 8.271$ W/m^{2}K and

$\overline{g_m^*} = 0.009061$ kg/m²·s.

In terms of the unknown, T_s, the low rate energy balance is

$$(1398\ W/m^2) - \sigma T_s^4 + \overline{h^*}(T_e - T_s) = \overline{g_m^*}\, \mathcal{B}_m\, h_{fg}$$

where $T_e = 304.2$°K and $\mathcal{B}_m$ and h_{fg} depend on T_s. Solve by iteration on T_s:

$$\overline{h^*}(T_s - 304.2) = 1398 - \sigma T_s^4 - \overline{g_m^*}\, \mathcal{B}_m\, h_{fg}$$

Guess T_s, evaluate the RHS with a steam table, and solve for the value of T_s implied on the LHS:

Guess T_s	$m_{H_2O,s}$	h_{fg} (kJ/kg)	$\mathcal{B}_m$	Implied T_s
314.2°K	0.04924	2404.3	0.03391	317.1 °K
315.2	0.05200	2401.9	0.03692	308.5
314.7	0.05062	2403.1	0.03541	312.8

The surface temperature is slightly higher than 41.0°C and slightly less than 41.5°C. Evaluate the evaporation rate at 41.3°C : $m_{H_2O,s} = 0.05007$, $h_{fg} = 2403.6$ kJ/kg, $\mathcal{B}_m = 0.03481$; implied $T_s = 41.3$°C. $n_{H_2O,s} = (0.009061)\cdot(0.03481) = 0.0003154$ kg/m²·s.

Check the assumptions: $\ln(1+\mathcal{B}_m)/\mathcal{B}_m = 0.983$, low rates alright; $m_{H_2O,f} = 0.0335$ – only changes will be for ρ, μ, Sc but since both μ and ρ decrease the effect on ν is small: the present property values are adequate for engineering purposes. The overall evaporation rate is $\dot{m} = 3(0.3048)(2)(0.3048)(n_{H_2O,s}) = \underline{1.76 \times 10^{-4}\ \text{kg/s}}$

μ and k from (11.125,127) with $c_p \simeq 2040$ J/kg·K:

$\mu_{steam} = 1.127 \times 10^{-5}$ kg/m-s , $k_{steam} = 0.02299$ W/m-K $(r \doteq 1)$

From the ideal gas law, $\rho_{steam} = 0.05593$ kg/m^3. Take $Pr = 1$. The Reynolds number is $Re_L = 9925$: laminar.

Calculate $\overline{h^*}$ from equation (6.68): $\overline{h^*} = 6.083$ W/m K.

Finally, calculate $\overline{g^*_m}$. Evaluate properties at $T_f = 310$ K and $m_{air,f} = 0.30$, as a guess. Use (11.128) for μ: $\mu_{air_{①}} = 1.899 \times 10^{-5}$ kg/m-s , $\phi_{12} = 1.015$, $\phi_{21} = 0.9680$, $x_1 = 0.2105$, $x_2 = 0.7895$: $\mu_f = 1.291 \times 10^{-5}$ kg/m-s. From (12.43), $\mathcal{D}_{air,steam} = 3.433 \times 10^{-4}$ m^2/s. $\rho_f = 0.06288$ kg/m^3 and $Sc_f = 0.5981$. Again using (6.68) find $\overline{g^*_m} = 0.004768$ kg/m^2-s with $Re_L = 9741$.

The operating equations are:

$$\overline{h}\,(318.2 - \overline{T}_s) - \overline{n}_{H_2O,s}\,(2.438 \times 10^6) = 1.197 \times 10^4 (\overline{T}_s - 298.2)^{3/4}$$

$$\overline{n}_{H_2O,s} = 0.004768 \ln(1 + B_m)$$

$$c_{p_{H_2O}} \simeq 2040 \frac{J}{kg \cdot K}$$

$$\overline{h} = (\overline{n}_{H_2O,s}\, c_{p_{H_2O}}) / [\exp(\overline{n}_{H_2O,s}\, c_{p_{H_2O}} / 6.083) - 1]$$

+ Steam table for $m_{H_2O,s}(T_s)$. We solve iteratively for T_s:

T_s (°C)	$m_{H_2O,s}$	B_{m,H_2O}	$\overline{n_{H_2O,s}}$ (kg/m^2-s)	$\overline{h}$ (W/m^2-K)	$\overline{T_s}$ (°C)
30.0	0.4131	−0.9318	−0.01281	26.49	28.7
29.0	0.3845	−0.9350	−0.01303	26.93	28.8
28.9	0.3818	−0.9353	−0.01305	26.97	28.8

Appears that $T_s \simeq 28.8°C$. Check our assumptions now:

$T_f = \frac{1}{2}(45+28.8)°C = 310.1\ K$ for the vapor, $T_f = \frac{1}{2}(25+28.8)°C = 300.1\ °K$ for the liquid; $m_{air,f} = \frac{1}{2}(0.04+0.618) = 0.329$ for the mass transfer coeff. and $m_{air} = m_{air,e} = 0.04$ for the heat transfer coeff.. Our guesses for the temperature reference states were right on target. Our choices of composition reference state were close enough that additional iteration is probably unnecessary.

Condensation rate, $\underline{-n_{H_2O,s} = 0.01305\ \frac{kg}{m^2\text{-}s}}$ ◄

(b) Without air, the liquid surface will be at T_{sat}(8000 Pa $= 41.51\ °C$. The film temperature is $T_f = 306°K$, close enough to our previous calculation:

$$\dot{m}'' = (h_{fg})^{-1}\,\bar{h}_{Nusselt}\cdot(T_{sat} - T_w)$$

$$= (2438.0\times10^3)^{-1}(1.197\times10^4)(41.51-25.0)^{3/4}$$

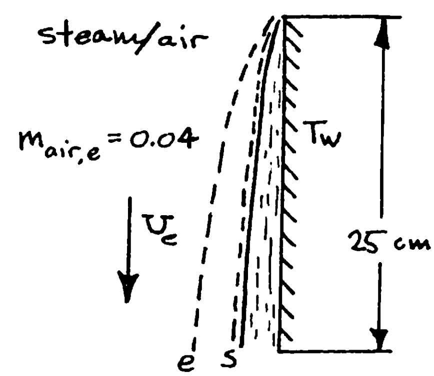
$$= \underline{0.04021\ kg/m^2\text{-}s} \longleftarrow$$

This small amount of air cuts condensation by 68%! Air leaks are a very serious problem for condensers.

11.51 Steam condenses on a 25-cm-high, cold vertical wall in a low-pressure condenser unit. The wall is isothermal at 25°C, and the ambient pressure is 8000 Pa. Air has leaked into the unit and reached a mass fraction of 0.04. The steam-air mixture is at 45°C and is blown downward past the wall at 8 m/s. (a) Estimate the rate of condensation on the wall. (b) Compare the result of part (a) to condensation without air in the steam. What do you conclude?

(a)

The energy balance for the wall is:



$$\bar{h}(T_e - \bar{T}_s) - \overline{n_{H_2O,s}}\, h_{fg} = \left(\overline{\frac{k}{\delta}}\right)(\bar{T}_s - T_w)$$

where $\overline{n_{H_2O,s}} = \overline{g_{m,H_2O}} \cdot \overline{B_{m,H_2O}} < 0$

and $\overline{(k/\delta)} = \bar{h}_{Nusselt}$ from eq'n (8.62b)

This condensation problem will be characterized by high rates of suction which will tend to concentrate air near the liquid surface.

First evaluate $\bar{h}_{Nusselt}$. Guess a property reference temperature of 300 K for the liquid: $\rho_f = 996.6$ kg/m³, $k = 0.6084$ W/m-K, $\mu = 8.23 \times 10^{-4}$ kg/m-s, $h_{fg} = 2438.0$ kJ/kg. Approximate: $\rho_f - \rho_g \simeq \rho_f$, $h'_{fg} \simeq h_{fg}$ since ρ_g and ΔT are small. Substitute into eq'n (8.62b)

$$\overline{Nu_L} = 4.919 \times 10^3 (T_s - T_w)^{-1/4}$$

or $$\bar{h}_{Nus.} = 1.197 \times 10^4 (T_s - T_w)^{-1/4}$$

Next evaluate $\bar{h}^*$ for the vapor flow. Guess a property reference temperature of 310 K and take the composition reference state as pure steam; calculate

$u_{film} \sim 0.03$ m/s

μ and k from (12.54, 56) with $c_p \simeq 2040$ J/kg-K:

$\mu_{steam} = 1.127 \times 10^{-5}$ kg/m-s , $k_{steam} = 0.02299$ W/m-K $(\gamma \doteq 1)$

From the ideal gas law, $\rho_{steam} = 0.05593$ kg/m^3. Take $Pr = 1$. The Reynolds number is $Re_L = 9925$: laminar.

Calculate $\overline{h^*}$ from equation (7.68): $\overline{h^*} = 6.083$ W/m K.

Finally, calculate $\overline{g_m^*}$. Evaluate properties at $T_f = 310$ K and $m_{air,f} = 0.30$, as a guess. Use (12.57) for μ: $\mu_{air_{①}} = 1.899 \times 10^{-5}$ kg/m-s , $\phi_{12} = 1.015$, $\phi_{21} = 0.9680$, $x_1 = 0.2105$, $x_2 = 0.7895$: $\mu_f = 1.291 \times 10^{-5}$ kg/m-s. From (12.43), $\mathcal{D}_{air,steam} = 3.433 \times 10^{-4}$ m^2/s. $\rho_f = 0.06288$ kg/m^3 and $Sc_f = 0.5981$. Again using (7.68) find $\overline{g_m^*} = 0.004768$ kg/m^2-s with $Re_L = 9741$.

The operating equations are:

$$\overline{h}\,(318.2 - \overline{T}_s) - \overline{n}_{H_2O,s}\,(2.438 \times 10^6) = 1.197 \times 10^4 (\overline{T}_s - 298.2)^{3/4}$$

$$\overline{n}_{H_2O,s} = 0.004768 \ln(1 + B_m)$$

$$c_{p_{H_2O}} \simeq 2040 \frac{J}{kg \cdot K}$$

$$\overline{h} = (\overline{n}_{H_2O,s}\, c_{p_{H_2O}}) / [\exp(\overline{n}_{H_2O,s}\, c_{p_{H_2O}}/6.083) - 1]$$

+ Steam table for $m_{H_2O,s}(T_s)$. We solve iteratively for T_s:

T_s (°C)	$m_{H_2O,s}$	B_{m,H_2O}	$\overline{n_{H_2O,s}}$ (kg/m^2-s)	$\overline{h}$ (W/m^2-K)	$\overline{T_s}$ (°C)
30.0	0.4131	−0.9318	−0.01281	26.49	28.7
29.0	0.3845	−0.9350	−0.01303	26.93	28.8
28.9	0.3818	−0.9353	−0.01305	26.97	28.8

Appears that $T_s \simeq 28.8°C$. Check our assumptions now: $T_f = \frac{1}{2}(45+28.8)°C = 310.1\ K$ for the vapor, $T_f = \frac{1}{2}(25+28.8)°C = 300.1°K$ for the liquid; $m_{air,f} = \frac{1}{2}(0.04+0.618) = 0.329$ for the mass transfer coeff. and $m_{air} = m_{air,e} = 0.04$ for the heat transfer coeff.. Our guesses for the temperature reference states were right on target. Our choices of composition reference state were close enough that additional iteration is probably unnecessary.

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