

## Theoretical analysis of high pressure phase transition and stability of LaTe and SmTe with NaCl-structure

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### Abstract

We have predicted the phase transition pressures and corresponding relative volume change of LaTe and SmTe having NaCl- type structure under high pressure using three body interaction potential (TBIP) approach and found better results with TBIP. In addition we have checked the conditions for relative stability in terms of modified Born criterion and also found that LaTe and SmTe follow the universal equation of state described by Vinet et al.

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### Introduction

Most of Rare Earth (RE) monochalcogenides show NaCl(B<sub>1</sub>)→CsCl(B<sub>2</sub>) structural phase transition and attracted both experimental and theoretical workers. These compounds have attracted attention due to its magnetic and electrical properties [1]. The presence of 4f- electron in these compounds is mainly responsible for these peculiar physical properties. They show metallic nature when the rare earth ion is in trivalent state and semi-conducting in case of divalent state. The pressure-volume relationship for divalent rare-earth chalcogenides have been extensively done by high-pressure X-ray diffraction technique [2-4].

As a proper reference material lanthanum monochalcogenides LaX (X= S, Se, Te),

the high pressure structural properties have been less studied experimentally. Out of these, only LaS Shows  $B_1 \rightarrow B_2$  phase transition at around 25GPa in a Silicon oil pressure medium [5] and no experimental studies of LaSe and LaTe are known at present. Hence, we refer to the self-consistent tight binding linear muffin tin orbital (TBLMTO) method [6]. Further, Lu et al [7] reported the local density approximation (LDA) results successfully for electronic structure of LaS and SmS. Furthermore, theoretical investigation of LaX (X= S, Se, Te) has been done by two-body interaction potential by Varshney et al. [8]. Among divalent rare-earth chalcogenides, SmTe shows  $B_1 \rightarrow B_2$  phase transition at around 11GPa [4]. The ground state configuration of SmTe is determined from total energy calculation using the self-interaction local-spin-density approximation[4].

Looking at the interesting properties of RETe compounds and the fact that no study has been done using three body interactions we thought it pertinent to apply three body interaction potential (TBIP) approach. The importance of three body interactions in potential model to improve results has also been emphasized by others like Sims et al. [9]. We have employed our three body interaction potential (TBIP) approach [10-12] to study high pressure behavior. We introduced our three body potential model which includes the long range Columbic, three body interaction, short range overlap repulsive interaction operative up to second neighbor ions within Hafemeister and Flygare approach [13] incorporated with van der Waals(vdW)attraction due to dipole-dipole and dipole-quadrupole interactions effects [14] and also with zero point energy effects [15,16]. The inclusion of short range (SR), vdW interactions and second neighbor ion (SNI) seems essential as their effects are considerably important as per the prediction of the relative stability of crystal structure. The importance of inclusion SR, vdW interaction and SNI has been established in our earlier work [15]. Also we have considered Zero point energy effects, which is the lowest possible energy that the compound may possess and is the ground state energy of the compound. The energy of the compound can be written as  $(\epsilon = (h\nu) / \{e^{(h\nu)/kt} - 1\} + (h\nu)/2)$ , here  $\nu$ ,  $t$  and  $k$  are the frequency, temperature and Boltzman constant of the compound and  $h$  is the Planck constant. It is clear from the above expression that even at absolute zero the energy of the compound cannot be zero but atleast  $1/2 (h\nu)$ . This term shows a small effects in gibbs free energy but cannot be ignored completely. The Potential model and method of calculation is given in section-I and result and discussion are given in section-III.

## Potential Model and Method of calculations

It is well known that the application of pressure on crystals results in change in its volume which leads to an increased charge transfer (or three-body interaction effects) due to the deformation of the overlapping electron shells of the adjacent ions. The three body interaction arises during lattice vibrations when electron shells of neighboring ions overlap. This overlapping leads to the transfer of charge which interacts with other charges, many body interactions (MBI) take place, the dominant part of MBI is three body interactions [10-12]. This interaction becomes more

important to consider due to the decrease in inter ionic spacing of the lattice crystal when pressure gets increased and when anions experience sufficient overlap. Besides, enhance in overlap energy, the transferred charge due to the overlap in electron shells, modify the ionic charge which in turns modify the coulomb energy. The expression for the modified Coulombic energy due to three body interaction (TBI) is

$$\Phi_m(r_0) = \Phi^c + \Phi^T \quad (1)$$

$$\Phi_m(r_0) = [-\alpha_M Z^2 e^2 / r] [1 + (2n/Z) f(r_0)] \quad (2)$$

Here  $\alpha_M$  is the Madelung constant, which is 1.7476(1.7629) for NaCl (CsCl) structure solids and  $r_0$  is the equilibrium nearest neighbor(nn) ion separation,  $n$  is the number of nearest neighbor(nn) and  $f(r)$  is the TBI parameter and is dependent on the nearest neighbor distance ( $r$ ) as [12]

$$f(r) = f_0 \exp(-r/\rho) \quad (3)$$

These effects have been incorporated in the Gibbs free energy ( $G=U+PV-TS$ ) as a function of pressure ( $P$ ). Here  $U$  is the internal energy, which at  $T=0$  K is equivalent to the lattice energy,  $S$  in the vibrational entropy at absolute temperature  $T$ . Since the theoretical calculations are done at  $T=0$  K, hence the Gibbs's free energy is equivalent to enthalpy. At  $T=0$  K and pressure  $P$ , the Gibbs free energies for rock salt ( $B_1$ , real) and CsCl ( $B_2$ , hypothetical) structures are given by

$$G_{B_1}(r) = U_{B_1}(r) + PV_{B_1} \quad (4)$$

$$G_{B_2}(r') = U_{B_2}(r') + PV_{B_2} \quad (5)$$

With  $V_{B_1} (=2.00 r^3)$  and  $V_{B_2} (=1.54 r'^3)$  as the unit cell volumes for  $B_1$  and  $B_2$  phases respectively. The first terms in the energies (4) and (5) are lattice energies for  $B_1$  and  $B_2$  structures and they are expressed as:-

$$\begin{aligned} U_{B_1}(r) = & [ -(\alpha_m z^2 e^2) / r ] - [ (12\alpha_m z e^2 f(r)) / r ] + 6b \beta_{ij} \exp [(r_i + r_j - r) / \rho] + 6b \beta_{ii} \exp \\ & [(2r_i - 1.41r) / \rho] + 6b \beta_{jj} \exp [2r_j - 1.41r / \rho] - \{ (C_{ij} r^{-6} + D_{ij} r^{-8}) \} + (0.5) h \langle \omega^2 \rangle^{1/2}_{B_1} \quad (6) \\ U_{B_2}(r') = & [ -(\alpha'_m z^2 e^2 / r') ] - [ (16\alpha'_m z e^2 f(r')) / r' ] + 8b \beta_{ij} \exp [(r_i + r_j - r') / \rho] + 3b \beta_{ii} \\ & \exp [(2r_i - 1.154r') / \rho] + 3b \beta_{jj} \exp [2r_j - 1.154 r' / \rho] - \{ (C'_{ij} r'^{-6} + D'_{ij} r'^{-8}) \} + (0.5) h \langle \omega^2 \rangle^{1/2}_{B_2} \quad (7) \end{aligned}$$

Here,  $ze$  is the ionic charge,  $r_i$  ( $r_j$ ) is the ionic radii of  $i$ ( $j$ ) ions,  $\rho$  is the range parameter,  $b$  is the hardness parameter,  $C_{ij}$  and  $D_{ij}$  are the van der Waals coefficients from dipole-dipole and dipole-quadrupole interactions,  $r(r')$  is the inter ionic separation for  $B_1$  ( $B_2$ ) phases.  $\beta_{ij}$  is the Pauling coefficients defined as

$$\beta_{ij} = 1 + \frac{z_i}{n_i} + \frac{z_j}{n_j} \quad (8)$$

Where  $z_i$  ( $z_j$ ) and  $n_i$  ( $n_j$ ) as the valence and number of electrons in the outermost orbit of cations(anions).

These lattice energies consist of long-range Coulomb energy (first term), three body interactions corresponding to the nearest neighbor separation  $r$  ( $r'$ ) for  $B_1$ ( $B_2$ )

phases(second term), energy due to the overlap repulsion represented by Hafemeister and Flygare (HF) type potential (i, j)ions (third term) and extended up to the second neighbor ions (fourth and fifth terms), vdW interaction (sixth term) and last term indicates zero point effect term in eqns-6 and 7.

The effective inter-ionic potential described for NaCl [B<sub>1</sub>] and CsCl [B<sub>2</sub>] structures contain three model parameters [ $\rho$ , b, f(r)], namely range, hardness parameter and three body interaction parameter. To calculate these parameters we have employed the equilibrium conditions.

$$[d(U)/dr]_{r=r_0} = 0 \quad \text{and} \quad [d^2 U / dr^2] = 9k r_0 B_T \quad (9)$$

Now, in the last term  $\langle \omega^2 \rangle^{1/2}$  as the mean-square frequency related to the Debye temperature ( $\Theta_D$ ) as

$$\langle \omega^2 \rangle^{1/2} = k \Theta_D / h \quad (10)$$

Here  $\Theta_D$  can be expressed by well known Blackman's formula described in [16].

$$\Theta_D = (h/k) \sqrt{[(5r_0 B_t)/\mu]} \quad (11)$$

With  $B_t$  and  $\mu$  as the bulk modulus and reduced mass of the compounds

To understand elastic properties of these monotellurides we have calculated second order elastic constants (SOEC), ( $C_{11}$ ,  $C_{12}$  and  $C_{44}$ ) and their pressure derivatives at 0 K, since these elastic constants are functions of first and second order derivatives of short range potential, so their calculation will provide knowledge about the effect of short range forces on these materials following Jain et al.[17].The expression for SOE constants as

$$C_{11} = e^2/4r_0^4 [-5.112z \{z+12f(r)\} + A_1 + (A_2 + B_2)/2] + 9.30z (rdf/dr) \quad (12)$$

$$C_{12} = e^2/4r_0^4 [1.391z \{z + 12f(r)\} + (A_2 - B_2)/4 + 9.30z (rdf/dr)] \quad (13)$$

$$C_{44} = e^2/4r_0^4 [2.556z \{z+12f(r)\} + B_1 + (A_2 + 3B_2)/4] \quad (14)$$

In equation (12 to 14), first term represent the long-range coulomb interaction, second term represent the contribution due to TBIP and remaining contributes overlap repulsion expressed in terms of the short range parameters ( $A_1$ ,  $B_1$ ) and ( $A_2$ ,  $B_2$ ) for the nearest neighbor (nn) and next nearest neighbor (nnn) ions and these expression are expressed in our earlier paper [12].

## Result and Discussion

Using the measured values of the equilibrium lattice constant ( $r_0$ ) and isothermal bulk modulus ( $B_T$ ) with  $K=2$  for  $B_1$  phase, the model parameters thus calculated are given in Table I and are used to compute the results presented and discussed below. For simplicity, we have taken account of single set [ $\rho$ , b, f(r)], in  $B_1$  -phase and same set is used in  $B_2$  phase. The reason for this is that during phase transition from  $B_1 \rightarrow B_2$  the atomic distribution takes a different arrangement and they get arranged in CsCl structure after phase transition. Inter-ionic separation changes which are calculated by minimizing technique. The parameter f(r) is inter-ionic separation(r) dependent

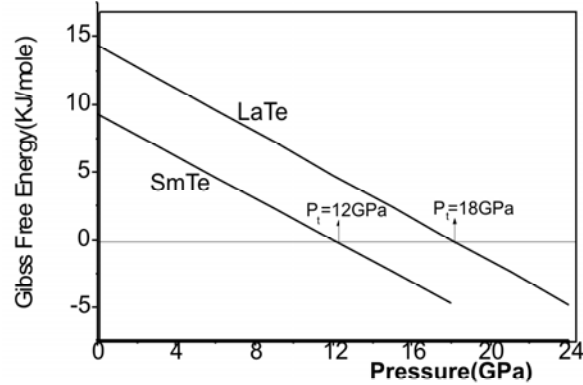
parameter so it is changed accordingly.

**Table I :** Ionic radii  $r_i$  ,  $r_j$  , inter ionic separation  $r_0$  and range  $\rho$  are given in ( $\text{\AA}$ )

Compounds	Input parameters				Model parameters		
	$r_i$	$r_j$	$r_0$	$B_T$ (GPa)	$b(10^{-19}\text{J})$	$\rho$	$f(r)$
LaTe	1.04 <sup>c</sup>	2.21 <sup>c</sup>	3.215 <sup>a</sup>	52.8 <sup>a</sup>	58.21	0.38	0.091
SmTe	1.19 <sup>c</sup>	2.21 <sup>c</sup>	3.295 <sup>b</sup>	37.6 <sup>b</sup>	4.85	0.44	-0.008

<sup>a</sup> reference(8)   <sup>b</sup> reference(4)   <sup>c</sup> reference(35)

We have followed the technique of minimization of  $U_{B_1}(r)$  and  $U_{B_2}(r')$  at different pressures in order to obtain the inter ionic separations  $r$  and  $r'$  corresponding to  $B_1$  and  $B_2$  phases. Now we have evaluated the corresponding Gibbs free energies  $G_{B_1}(r)$  and  $G_{B_2}(r')$  and their respective differences  $\Delta G = (G_{B_2}(r) - G_{B_1}(r'))$ , then we have plotted  $\Delta G$  against pressure ( $P$ ) as shown in Fig.(1) for LaTe and SmTe. The phase transition pressure ( $P_t$ ) is the pressure at which  $\Delta G$  approaches zero.



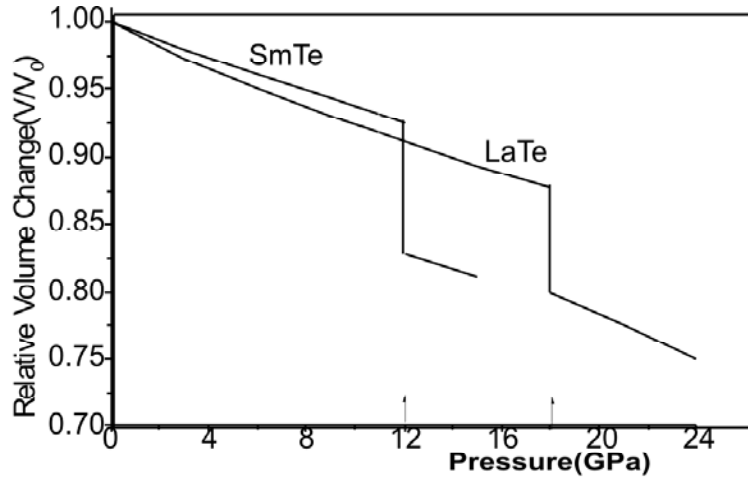
**Figure 1 :** Variation of Gibbs free energy differences  $\Delta G$ (KJ/mol) against pressure (GPa) for LaTe and SmTe.

We have also computed the relative volume changes  $V(P)/V(0)$  and plotted them against pressure as depicted in Fig-2 for LaTe and SmTe. It is clear from Fig.2 that phase transition  $B_1 \rightarrow B_2$  occur at about 18 GPa and 12 GPa for LaTe and SmTe respectively and calculated volume collapses from TBIP model is 8.6% for LaTe and, 9% for SmTe which are close to experimental value and other works. The phase transition pressures and relative volume change for SmTe is calculated by two-body approach as proceed by Varshney et al. [8] for LaTe and shown in Table-II .

**Table II :** Calculated transition pressures and volume collapses.

Comp.	Transition	Transition Pressure(GPa)			Volume Collapses		
		Present TBIP	Exp. and other	Two-body	Present	Exp. and other	Two-body
LaTe	B <sub>1</sub> → B <sub>2</sub>	18	16.5 <sup>a</sup>	(16.8) <sup>b</sup>	8.6%	(8.2%) <sup>a</sup>	(7.2%) <sup>b</sup>
SmTe	B <sub>1</sub> → B <sub>2</sub>	12	(11) <sup>c</sup>	13.5	9%	(9.1%) <sup>c</sup>	8.7%

<sup>a</sup> reference(8)    <sup>b</sup>reference(9)    <sup>c</sup>reference(4)

**Figure 2 :** Variation of relative volume with pressure for LaTe, and SmTe

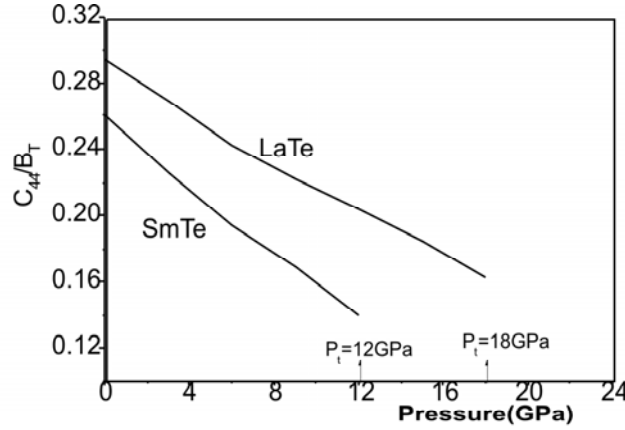
Furthermore, to test our TBIP model we have used the Vinet universal equation of state (EOS) for these compounds on the guidelines of Recio et al. [19]. The P-V data can be expressed by the equation.

$$\ln H = \ln B_0 + (3/2)(B_0' - 1)(1 - x) \quad (15)$$

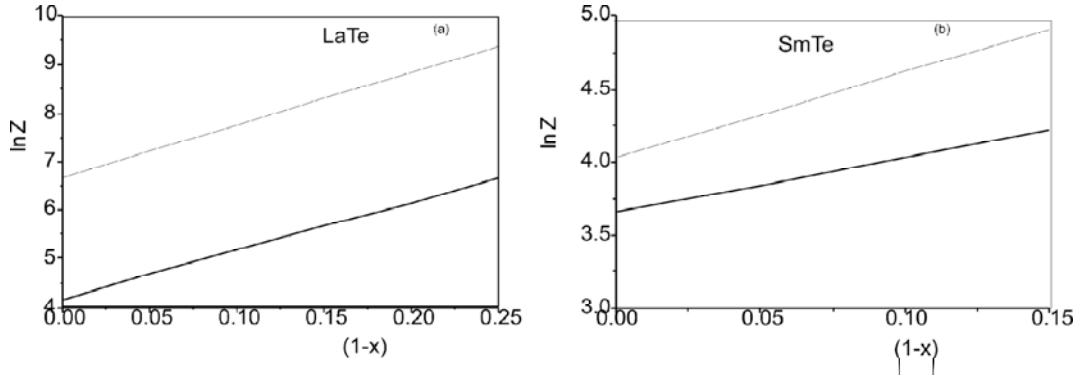
Here H and x defined as

$$H = Px^2 / (3(1 - x)) \quad \text{and} \quad x = V/V_0 \quad (16)$$

The curve representing B<sub>1</sub> (B<sub>2</sub>) phases computed from above equation (14) are shown in thick(thin) lines in Fig. 4. We have fitted the VEOS to our TBIP P-V data and in this way we have carried out comparison not directly with experiments but create consistency with the empirical behavior in many real solids.



**Figure 3 :** Variation of  $C_{44}/B_T$  with pressure for LaTe, and SmTe



**Figure 4 :** Thick and thin solid curves represent Vinet equation of state (VEOS) (a) for LaTe (b) for SmTe.

As the study of elastic constant and their combinations is important to understand the elastic properties and their inter-atomic behavior. We have computed the second-order elastic constants (SOEC) and their combinations which are shown in Table-III. The quantities  $C_{44}$  and  $C_S$  are the shear and tetragonal moduli of a Cubic crystal. It is necessary for lattice to satisfy Born criterion for mechanically stable state. For this purpose, the elastic energy density must be a positive definite quadratic function of strain. In order to fulfill the above criterion the principal minor (eigen values) of the elastic constant matrix should all be positive. Since the tetragonal modulus  $C_S = (C_{11} - C_{12})/2$  for PbTe and SnTe increases linearly with pressure and the pressure-induced change in  $C_{44}$  decreases slightly for PbTe and SnTe. Also the pressure dependence of the corresponding stiffness  $1/3 (C_{11} + C_{12} + 2C_{44})$  is linear for PbTe and SnTe. And hence TBIP confirmed that the features of LaTe and SmTe show the same trends as reported for pressure dependence elastic stiffness for PbTe and SnTe possessing the NaCl structure [19]. Vukceovich [20] also stated the high pressure stability criterion for ionic crystal, the stable phase of a crystal is one in which the shear elastic constant  $C_{44}$  is non-zero (for mechanical stability) and which has the lowest potential energy

among the mechanically stable lattices. Thus the stability of NaCl-type structure in terms of elastic constants should satisfy the following conditions

**Table III :** Calculated Elastic Constants and their combinations. (GPa)

Comp.	$C_{11}$	$C_{12}$	$C_{44}$	$C_{44}/B_T$	$C_S = \frac{1}{2}(C_{11} - C_{12})$	$C_L = \frac{1}{2}(C_{11} + C_{12} + 2C_{44})$
LaTe	135	24	16.2	0.294	55.5	95.7
SmTe	92.4	12.4	9.8	0.14	39.9	57.6

$$B_T = 1/3 (C_{11} + 2C_{12}) > 0, \quad C_{44} > 0 \text{ and } C_S = (C_{11} - C_{12})/2 > 0 \quad (17)$$

The estimated shear moduli are  $C_{44}=16.2$  GPa and  $C_{44}= 9.8$  GPa and tetragonal moduli are  $C_S = 55.5$  and  $C_S =39.9$ GPa for LaTe and SmTe which are well suited with the above elastic stability criterion for RETe compounds. From Table-III, our estimated  $C_{44}$  for LaTe and SmTe are positive and hence the above stability criterion is satisfied for ionic crystal. Musgrave and Pople [21] pointed that if either  $C_{44}$  or  $C_S = (C_{11} - C_{12})/2$  go to zero as the pressure is increased, then the structure becomes unstable and the crystal transform by a spontaneous shear. Also in case of B1-structure,  $C_{44}$  decreases up to  $P_1$  but the phase transition to the B2 –structure takes place before the  $C_{44}$  reaches zero. Demarest et al. [22] proposed a slight modification of the Born stability criterion that a phase transition take place when the ratio  $C_{44} / B_T$  reaches a critical value in the neighborhood of 0.14 and 0.2. On the same guide lines we have obtained exactly the same trends from TBIP and are tabulated in Table-III and shown in Fig-3. The deviation from Cauchy relations between second order elastic constants as well as the third order elastic constants can be more significantly expressed in terms of TBIP approach.

Finally, we have noticed that during the crystallographic transition from NaCl to CsCl, the volume discontinuity in pressure-volume phase diagram identifies the same trends as the experimental data and these results are well fitted with Vinet (EOS) . In order to judge stability of these compounds we have computed elastic constants and plotted  $C_{44}/B_T$  with different pressures and hence the trends shown from TBIP are consistent with the requisites of first-order phase transition. On the basis of above work, it is concluded that the TBIP approach is adequately suitable for the prediction of B1→B<sub>2</sub> phase transition pressures and associated volume collapses in RETe compounds.

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