The Dimensions Of Field Theory : From Particles To Strings

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31 August 1999

Abstract

This is an editorial summary of the contents of a Book comprising a set of Articles by acknowledged experts dealing with the impact of Field Theory on major areas of physics (from elementary particles through condensed matter to strings), arranged subjectwise under six broad heads. The Book which emphasizes the conceptual, logical and formal aspects of the state of the art in these respective fields, carries a Foreword by Freeman Dyson, and is to be published by the Indian National Science Academy on the occasion of the International Mathematical Year 2000. The authors and full titles of all the Articles (33) are listed sequentially (in the order of their first appearance in the narration) under the bibliography at the end of this Summary, while a few of the individual articles to appear in the Book are already available on the LANL internet.

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1 Birth, Decline And Rebirth Of Field Theory

If one must choose one single item of Twentieth Century Physics which stands out by the yardstick of most pervasive and decisive influence on its total development, Quantum Field Theory (QFT) certainly wins hands down. Historically, QFT was born out of the marriage of Relativity and Quantum Theory, at a hefty price of mathematical self-consistency underlying the celebrated Dirac Theory, whose full significance took several stages to unfold through the vissicitudes of logical deduction, going well beyond the immediate discovery of the positron. Indeed of far greater significance from the conceptual point of view, was the realization that the "sea of negative energy states" was already a tacit admission of the failure of relativistic quantum mechanics of a single particle, in favour of a collective many-particle, or $field$ description, a fact which was to be driven home by Dyson in his Cornell lectures of 1952. And once this realization dawned on the pioneers, the Klein-Gordon theory of scalar particles found a natural place in the new scenario, at the hands of Pauli-Weisskopf(1934) who now found little difficulty in quantizing these bosonic particles just as easily as the Dirac theory had done to fermions. Thus was born "Quantum Field Theory" (QFT) in its full glory, with Anti-matter playing a symmetrical role to Matter, irrespective of its fermionic or bosonic nature. [Feynman's brilliant positron theory was a bold attempt to resurrect the single particle quantum mechanics description via "zigzag" diagrams (negative time propagation of negative energy electrons), but the more universal language of Field Theory eventually carried the day].

QFT registered its first major success in the Covariant formulation of QED at the hands of Tomonaga and Schwinger on the one hand, and Feynman on the other, with Dyson playing the catalyst-role in synthesizing the two. This theory, in the course of circumventing unphysical infinities in the measurable quantities, gave rise to a new dogma of $Renormalizability$ which was to act as the yardstick of acceptability of theories to come. This dogma, together with the independent principle of "Gauge Invariance" (already in-built in QED), were to be two pillars of QFT in its march towards greater victories to come, especially in the formulation of strong interaction theories on analogous lines to QED. This led to the Yang-Mills theory (1954) of $SU(2)$ gauge bosons as the non-abelian counterpart of QED. Yet the path of QFT for strong interactions was strewn with so many thorns that for a long time it simply refused to move. Indeed at the meson-baryon level there was a (temporary) disenchantment with QFT, in favour of a new paradigm of Hadron Democracy (somewhat akin to the Mach Principle) based on a selection of items from the full QFT package viz., analyticity of S-matrix elements subject only to unitarity. This was the "Bootstrap" Philosophy of Chew which held its ground for a brief period, until the discovery of Quark substructures (1963-64) brought the Hadrons down from the pedestal of elementary particles to a (more modest) composite status, which told (by hindsight) the reason why the Yang-Mills gauge theory of strong interactions had not worked at the composite hadron level of gauge fields. But now the quarks and gluons offered a fresh basis for the application of the Gauge Principle at a deeper level of elementarity; thus was born the non-abelian QCD for strong interactions (1973).

1.1 QFT in Action

With the rebirth of QFT, three principal weapons in its arsenal that have stood the test of time may be listed as **Renormalizability**, **Gauge Principle**, and **Spontaneous**

Symmetry Breaking. Indeed these are the 3 pillars on which rests the grand edifice of Field Theory encompassing the diverse phenomena of Nature within an integrated structure. In this entire development, **Symmetry** has all along played the role of the "guiding light", with a conservation law (Noether) associated with a specific symmetry (invariance) type (Lorentz, Gauge, Chiral), making use of the powerful language of *Group Theory* and *Topology* at its command. And in reverse, Symmetry breaking (spontaneous or dynamical), with its universal appeal, has been a key element in the understanding of a whole range of phenomena from condensed matter to particle physics and early Universe cosmology, with definite experimental and observational support.

With the evolution of further unifying principles like *duality* and *Supersymmetry*, QFT has greatly consolidated its grip on Particle Physics, and extended its frontiers beyond its traditional domains, to the new heights of String Theory to "rein in" the formidable force of Gravity. These powerful weapons at the command of QFT have led to new insights which have helped reveal its hold on widely different branches of Physics (which had hitherto evolved on their own steam), a saga of victories in which the techniques of Path Integrals, Renormalization Group (RG) theory and Dirac/Bargmann constrained dynamics have played key roles. Especially noteworthy is the unification of the QFT language with those of Quantum Statistical Mechanics (QSM) and Condensed Matter Physics (CMP), leading all the way to Cosmology and Black Hole Physics (thanks to the unifying power of String Theory).

Progress has been far from uniform in the different sectors of QFT. In the Particle Physics domain which represents the Ultimate Laboratory for testing the most profound concepts emanating from QFT, the current state of the art is symbolized by the Standard Model (SM) designed to unify the three gauge sectors of strong, electromagnetic, and weak interactions. Of these, the last two sectors are unified by the Glashow-Salam-Weinberg (GSW) model of $SU(2) \otimes U(1)$ comprising photons and weak bosons, while a proper understanding of the QCD sector for $SU(3)$ strong interaction, still remains a distant goal. In this regard, a partial success has been achieved in the perturbative domain of asymptotic freedom, while the non-perturbative domain of $Confinement$ still remains elusive. This is reminiscent of the Churchillian phrase of "so much effort being spent towards so little effect" that was often used for the two-nucleon problem in the fifties, leading Hans Bethe to invoke his famous "Second Principle Theory" for effective nuclear interaction, which now seems to have shifted to the quark level. A part of the Book is devoted to the Strong Interaction problem of Quark Confinement from several different angles.

And yet the methodology and techniques QFT have shown a striking capacity to handle the problems of widely different sectors of physics well beyond particle physics, without changing the thematic framework, often with great success. Examples are QSM and Toda FT for non-linear systems.

Low dimensional Field Theories (2D, 3D) are often useful not only as soluble models designed to throw light on interesting features of QFT which often remain obscured in 4D form, but also as actual prototypes for systems moving in lower dimensions; (e.g., 2D conformally invariant field theory adequately describes the long range behaviour of systems undergoing second order phase transitions). A striking example of 2D QFT is the Schwinger Model (both chiral and non-chiral) which has been subjected to deep scrutiny from several angles, each with rich dividends. Similarly $(2+1)D$ QFT , the so-called Chern-Simons (CS) Theory, has found rich applications in Quantum Hall Effect in

Condensed Matter Physics. And most significantly from the point of view of formal QFT, Witten's demonstration of an exact connection of $(2+1)D$ QFT with Jones Polynomials did fulfil the long-cherished goal of an exact (non-perturbative) solution of a gauge field theory, at least in 3 dimensions for the first time.

1.2 Scope of The Book

This Book is an attempt to capture a cross section of the multifaceted flavour of QFT that has evolved over this Century, by putting together a collection (albeit subjective) of Articles by acknowledged experts in their respective fields. Admittedly, the selection is constrained by the accessibility of experts to *this* Editor within a relatively short span of time, and of necessity leaves out many important areas of QFT studies. The style of each Article varies from author to author, but the emphasis by and large is on conceptual and logical aspects of QFT formalism to the topic under study, designed to be instructive for a fairly wide class of readership, (with enough access to references for those who wish to pursue a particular line further), while the actual details of QFT methodology, or applications to phenomenology are outside the scope of the Book. Its theme is defined through a subjectwise classification of its contents under the following heads, in accordance with the specific sector/sectors of physics intended for exposition:
A): Basic Srtucture of QFT (*RG* Theory; *SM*; *SSB*; Confinement)
B): Topological Aspects of QFT
C): Formal Methods in QFT (QSM; Toda FT; LF and Constrained Dynamics)
D): Extension of QFT Frontiers (SUSY; CFT; String Theory)
E): QFT In $(2+1)D$: CS Theory and Applications
F): Methods of Strong Interaction in QFT
G): Conclusion (Concerning Foundations of Quantum Theory)
   In this Introduction to the Book, an attempt has been made to organize the contents under these categories, with a section devoted to each. The topics are arranged in descending order from 'classical' to 'evolving', with the former playing the background to the latter. The narrative draws freely from the perspective and language provided by the Authors concerned, often without quotes. The referencing, (except for a few special papers which are cited in the running text), is left to the Articles concerned, whose authorships are listed in the order of their appearance in this narrative, in the bibliography at the end.

2 Some Core Aspects Of QFT

In this Part we collect the Articles relating successively to RG Theory; Electroweak coupling in the Standard Model; the dynamics of Symmetry Breaking in different sectors of physics; and *two* novel mechanisms of Confinement in QCD, one proposed by **Gribov** and one by **Nishijima**.

2.1 RG Theory in QFT

Although the *Renormalization* strategy had originated in QED in the (limited) context of absorbing divergences in physical entities like mass and coupling constant (charge),

it turned out that the concept itself has a deeper meaning with much wider ramifications, which was later to get formalized as Renormalization Group (RG) Theory. The perturbative RG formulated by "Stuekelberg-Petermann in 1952-53 as a group of infinitesimal transformations, related to finite arbitrariness arising in the S-matrix elements upon elimination of the UV divergences" - (D.V.Shirkov [1]). In a parallel development, Gell-Mann-Low (1954) derived functional equations for the QED propagators in the UV limit, on the basis of Dyson's (1949) renormalization transformations, but missed the 'group character' implied in these equations. Finally, Bogoliubov-Shirkov (1955-56) put both aspects together and derived the "RG-equations" in a form which brings out the 'scaling' properties of the electron and photon propagators. Thus RG invariance boils down to the invariance of a solution w.r.t. the manner of its parametrization. These equations were further developed and made more rigorous with mathematicians and physicists working in tandem, so that renormalization became a well-developed method at the computational level. But the underlying physical concepts behind these equations took some more time to unfold until after Kadanov's, and especially Wilson's pioneering work on the understanding of the "critical indices" in phase transitions brought out the real physics behind the RG equations.

Wilson's work revealed the rich applicational potential of the RG ideas in various fields of physics, from 'critical phenomena' (spin lattices, polymer theory, turbulence) in condensed matter physics, to QCD parameters like the strong coupling constant α_s and the 'running mass' $m(p^2)$. In particular, the discovery of Asymptotic freedom in QCD allowed physicists to produce a logically consistent picture of renormalization, one in which the perturbative expansions at any high energy scale can be matched with one another, without any need to deal with intermediate expansions in powers of a large coupling constant. Another important aspect of these RG equations which has been emphasized by the Dubna School, is the concept of *functional self-similarity* in mathematical physics, which has led to applications like the study of strong non-linear regimes: asymptotic behaviour of systems described by non-linear partial differential equations; problem of generating higher harmonics in plasmas, and so on. The Book begins with a perspective Article by **Dmitri Shirkov**[1] on all aspects of the subject, from an introduction to RG in QFT to an overview of its methodology, together with applications of RG ideas in some important arenas of physics.

A relatively new approach to RG theory, termed "Similarity Renormalization Group" (SRG) was launched in this decade by Wilson and Glazek, as well as Wegner, and is based on the perception that divergences are in the first place due to the*locality* of the primary interactions. For a proper understanding of the features of the SRG theory, it is enough to consider only the non-relativistic quantum mechanics (the usual UV divergences of relativistic QFT are not relevant here!), where the locality condition on the potentials at all scales corresponds to taking only delta functions and their derivatives. The associated divergences can be regulated by introducing cut-offs whose effects may be removed by renormalization.

In the SRG, the transformations that explicitly "run" the cut-off parameter are developed. These similarity transformations are of course unitary, and constitute the group elements of SRG. They are characterized by a "running" cut-off on energy differences (not states). If the Hamiltonian is viewed as a large matrix, these cut-offs limit the off-diagonal matrix elements, and as they are gradually reduced, the Hamiltonian is forced towards the diagonal form. The perturbation expansion of the transformed Hamiltonians contains

no small energy denominators, so that the expansion does not break down unless the strengths of the interactions themselves are large. With the help of an associated concept of *coupling coherence*, SRG acquires respectability as a proper theory with the *same* number of parameters as the original (fundamental) theory. A review of the formalism and working of SRG is given by **R J Perry**[2], using as an example the exactly soluble case of a simple 2D delta function to act as a laboratory for testing the convergence of the SRG method in some detail.

2.2 Standard Model And Electroweak Coupling

The Gauge Principle, as a central ingredient of QFT, needed to be supplemented with fresh ideas and paradigms, within its broad framework, to extend its tentacles further. One such idea was based on the degenerate structure of the vacuum, dominated by vales and hills, which crystallized eventually as a new theme termed "Spontaneous Symmetry Breaking" (SSB), together with its companion "Dynamical Breaking of Chiral Symmetry" ($DB\chi S$), which would now enable gauge fields to acquire mass in a subtle but self-consistent manner. Armed with this paradigm, the Gauge Theory registered a signal success in the Weak interaction sector, culminating in the Glashow-Salam-Weinberg (GSW) Model of Electro-weak Interactions, which offered a unified view of weak and electromagnetic interactions in the form of an $SU(2) \otimes U(1)$ gauge theory. A more ambitious form of unification of the three principal gauge fields as a straightforward extension of the GSW, so as also to include the strong interaction (QCD) sector, under the umbrella of "Grand Unification Theory" (GUT), did not unfortunately bear fruit, so that, for the time being, the "Standard Model" (SM) has had to rest content with only a partial unification $SU(3) \otimes SU(2) \otimes U(1)$ of these gauge fields. Nevertheless this episode brings out a truism about the unpredictability of Nature, viz., its refusal to yield to a particular strategy for a second time, merely on the strength of its success on a previous occasion.

In a highly instructive and self-contained Article, **V Novikov** [3] gives a panoramic view of the conceptual and methodological framework of QFT (with the ingredients of gauge principle, renormalization group, and spontaneous symmetry breaking) that have been employed in the formulation of SM for elementary particle physics. He dwells in particular on the Higgs mechanism for the generation of the fermion masses for several generations, and brings out the powers of "loop corrections" in SM to predict accurate bounds on the masses of as yet undiscovered particles. This is vividly illustrated by the "correct" mass of the $t(op)$-quark *ahead* of its experimental discovery, stringent limits on the Higgs mass from the "Landau pole" structure of the running coupling constant, and the windows to the "physics beyond SM" that such analyses provide.

2.2.1 Discrete Symmetries in SM

An essential aspect of the Standard Model concerns the role of discrete symmetries P, C, T in determining the structure of the electroweak coupling. This subject has had a long history since the original Lee-Yang discovery of P-violation, going through successive phases of chiral symmetry (Landau-Salam), CP invariance (Lee-Oehme-Yang), its subsequent violation (Cronin-Fitch), and *ipso facto (?)* T-violation, a topic of intense experimental activity today. [This last is of course an immediate consequence of TCP-invariance (Pauli-Lueders Theorem), which puts the existence of antiparticles exactly on par with

particles]. A brief state- of-the-art review of the subject by **P K Kabir** [4] follows.

2.3 Dynamics Of Symmetry Breaking

Just as "Symmetry dictates interactions"- (C.N.Yang at the First Asia Pacific Conf, Singapore, 1983), the dynamical effects of its *breaking* (whether spontaneously or dynamically) during out-of-equilibrium phase transitions is equally at the root of a whole range of phenomena from condensed matter to particle physics, and so on, all the way to early universe cosmology. Indeed the dynamics of non-equilibrium phase transitions and the *orderingprocess* that occurs until the system reaches a broken symmetry equilibrium stage, have developed in tandem with controlled experimental techniques in many areas of condensed matter physics (binary fluids, ferromagnets, superfluids, liquid crystals), so as to provide a solid basis for describing the dynamics of phase ordering. In cosmology, measurements of Cosmic Microwave Background anisotropies, and the formation of large scale structures in the Universe, provide signatures for phase transitions during and after *inflation*. And at the accelerator energies (Brookhaven-RHIC or CERN-LHC), phase transitions predicted by QCD could occur out of equilibrium via pion condensates.

In an instructive review on this subject, **Boyanovsky and de Vega** [5] describe the relevant aspects of the dynamics of symmetry breaking in many areas of physics (from condensed matter to cosmology) vis-a-vis possible experimental signatures. In condensed matter, they address the dynamics of phase ordering, emergence of condensates, and dynamical scaling. In QCD, the possibility of disoriented chiral pion condensates arising from out-of-equilibrium phase transitions is considered. And in the early Universe, the dynamics of phase ordering in phase transitions, is described, especially the emergence of condensates and scaling in Friedman-Robertson-Walker cosmologies, within a QFT framework.

2.4 Confinement: Supercharged Nucleus

With the failure of GUT theories to take care of the strong interaction sector $SU(3)$ of the Standard Model, the central issue of Confinement, which has had a long history of approaches ranging from the fundamental to effective types, still remains an unsolved problem. There is a vast literature on the subject, from Lattice QCD to various analytical methods for non-perturbative QCD. Of these, 2 novel approaches to Confinement, which are fairly self-contained, and stand out from the more conventional ones, are included in Part A, leaving the rest for Part F. The first concerns an analogy to a super-charged nucleus, based on an old work of Pomeranchuk and Smorodinsky (1940), which offers the possibility of binding a particle in a small region of space. This method was elaborated in a set of THREE "Orsay Lectures" by the late **Vladimir Gribov** [6] during 1992-94. The basic idea is that if the charge Z in a nucleus N_Z is larger than a critical value $Z_c \approx 180$, then this nucleus will decay to an atom of charge $Z - 1$ and a positron: $N_Z \rightarrow A_{Z-1} + e^+$. If the product nucleus is unstable, the process gets repeated until the total charge of the final product is so small that further decay is impossible. Such a supercharged nucleus (a 'resonance') cannot exist freely, but only inside an atom, hence is reminiscent of a 'confined' state ! The region of stability of such a 'superbound' atomic state, (mainly due to the Pauli principle), works out as $r_0 << r < 1/m$, where r_0 is the radius of the nucleus, and m the electron mass. In these three lectures, which are reproduced

in this Book through the courtesy of his long term Associates Dokhshitzer, Ewarz and Nyiri, Gribov [6] gives a leisurely exposition of the detailed working of this mechanism on the confinement of heavy, followed by light, quarks. These ideas have since been extended by the Dokhshitzer Group in their subsequent publications hep-ph/9807224 and hep-ph/9902279, but these are outside the scope of this Book.

2.5 Confinement: BRST Mechanism

The second approach concerns a perspective on confinement due to Nishijima who relates its mechanism to that of an unbroken non-abelian gauge symmetry in QCD. The logic of this method which was mostly pioneered by Nishijima, may be illustrated for the case of abelian QED as follows. Quantization of of the e.m. field requires "gauge-fixing", say by a covariant (Fermi) gauge. This in turn requires introduction of the indefinite (Gupta-Bleuler) metric which, for the selection of physically observable states, must be eliminated by imposing the Lorentz condition on the state vector. There are now 4 kinds of photons (2 transverse, 1 longitudinal, and 1 scalar), of which the two 'scalar' photons must have negative norms, so as to ensure manifest covariance of the quantization in the Minkowski space.

   Now to project out the physical subspace, one introduces a subsidiary (Lorentz) condition (a 4-divergence of a vector field) which represents a *free, massless* field even under interactions. The photons involved in this operator (called a-photons) are special combinations of longitudinal and scalar photons with *zero norm*. A second (orthogonal) combination (called b-photons) also can be arranged to have zero norm. However the inner product of a- and b- photons is non-zero; they are 'metric partners' (somewhat akin to the 4-vectors $n_/mu$, $\tilde{n}_\mu$ defining a covariant null-plane: $n^2 = \tilde{n}^2 = 0$; $n.\tilde{n} = 1$). A physical state is defined as one that is annihilated by applying the positive frequency part of the Lorentz condition. And since the S-matrix in QED commutes with this 4-divergence, it transforms physical states into one another, without letting them out of this subspace which now includes only t (transverse) and a-photons, but *not* b-photons. However the inner product of a physical state with one a-photon, with another physical state (with or without an a-photon), vanishes identically. Thus a-photons give no contribution to observable quantities, and both a- and b-photons escape detection ! This is called *confinement* of longitudinal and scalar photons in QED, a *kinematical* phenomenon !

   In QCD, on the other hand, not only a- and b-gluons, but also the t-gluons are unobservable, giving a *dynamical* orientation to the confinement mechanism. While the basic logic and signature of confinement for non-abelian QCD remains the same as above for abelian QED, some extra ingredients of a highly technical nature are needed to bridge the gap. For not only the observable quantities now depend on the gauge parameter, but the 4-divergence of the gauge field is no longer a free field ! To eliminate the gauge-dependence of physical entities, Faddeev-Popov proposed to average the path integral over the manifold of gauge transformations, resulting in a new term in the Lagrangian (Faddeev-Popop ghost), involving a pair of anticommuting saclar fields whose violation of the Pauli theorem on spin-statistics connection requires introduction of the indefinite metric, as in QED. However, the operator analog of the Lorentz condition is more tricky in this case. It is facilitated by a novel symmetry found by Becchi-Rouet-Stora (BRS) which was originally used for renormalizing QCD. Nishijima successfully exploited this symmetry to construct the requisite operator, and obtained a formal proof of confinement

in the QCD case, as an extension of the logic employed for QED. A qualitative sketch of this proof appears in the Article by **K Nishijima and M. chaichian** [7].

3 Field Theory: Topological Aspects

An important sector of QFT that has come to occupy increasing importance in the last two decades, concerns its Topological aspects, as a powerful tool to probe the geometry and topology in *low* dimensions. This illustrates rather vividly the coming together of physicists and mathematicians, this time in building powerful links between quantum theory (through its path integral formulation) on the one hand, and the geometry and topology of low dimensional manifolds on the other. Indeed it appears that the properties of low dimensional manifolds can be nicely unravelled by relating them to infinite dimensional field manifolds, thus providing a powerful tool for studying these manifolds.

A unique characteristic of topological field theories is their independence of the metric of curved manifolds on which they are defined. This makes the expectation value of the energy-momentum tensor vanish. Since the only degrees of freedom are topological, there are no *local* propagating degrees of freedom. The operators are also metric independent. These features are addressed in some detail in a self-contained introductory Article by **Romesh Kaul** [8] on topological QFT regarded as a meeting ground for physicists and mathematicians.

3.1 CS Theory And Jones Polynomials

Quantum YM theories in $(2+1)D$ provide a field theoretic framework for the study of "knots and links" in a given 3-manifold, and illustrate the interplay of QFT and the topology of low dimensional manifolds. A striking result of this connection is that the famous "Jones Polynomials" of knot theory can be understood in 3D terms. This result was formally demonstrated by Edward Witten about a decade ago in a paper entitled "Quantum Field Theory And The Jones Polynomial", thus fulfilling a long-cherished goal of an exact (non-perturbative) solution of a gauge field theory, for the first time in 3 dimensions. Witten showed that the "Jones polynomial can be generalized from S^3 to arbitrary 3-manifolds, giving invariants that are computable from a surgery presentation". Witten further showed that these results shed new light on 2D conformal field theory. In view of the historical importance of this pioneering work in the context of this Book theme, we reproduce (with permission from Springer-Verlag) the celebrated **Witten paper** [9] (which had appeared in Commun.Math.Phys.**121** (1989) 351-399), in full.

3.2 Anomalies In QFT

An interesting pathology of QFT which has rich topological overtones is the problem of *anomalies* which originated in the famous ABJ (1969) paper to resolve the problem of $\pi^0 \Rightarrow \gamma\gamma$ decay whose hitherto standard explanation in terms of partial conservation of axial current ($PCAC$) used to fall far short of experiment. The ABJ paper finally resolved the issue by introducing an "anomalous" amplitude proportional to $F_\mu\nu\tilde{F}_\mu\nu$ in the $PCAC$ relation, whose interpretation brought into focus the pathology of $symmetry-breaking$ at the classical level through such "anomalies" at the QFT level. Such 'violation' of gauge

symmetry through 'anomalies' points to the need for their cancellation, which in turn constitutes an important constraint for physical gauge theories with *chiral* coupling to fermions. In this respect, "global chiral anomalies" play a key role in the understanding of physical effects associated with topologically non-trivial gauge-field configurations, via the celebrated Atiyah-Singer Theorem. This subject is briefly reviewed by **Haridas Banerjee** [10] in this Book.

3.3   Coherent States In QFT

Still another sector of QFT with topological (geometric) features, is the subject of *Coherent States* which has grown rapidly since its birth 36 years ago at the hands of Glauber and Sudarshan [R.J.Glauber, Phys.Rev.**130**, 2529 (1963); E.C.G.Sudarshan, Phys.Rev. Lett.**10**, 277 (1963)], although the basic idea dates back to the founder of Quantum Mechanics himself [Erwin Schroedinger:Naturwissenshaften, **14**, 644 (1926)] in connection with the quantum states of a harmonic oscillator, i.e., almost immediately after the birth of quantum mechanics. Coherent States have 3 main properties: coherence, over-completeness and intrinsic geometrization, all of which play a fundamental role in QFT. These include the calculation of physical processes involving infinite number of virtual particles; the derivation of functional integrals and various effective field theories; and last not least, the exploration of the origins of topologically non-trivial gauge fields and the associated (gauge) degrees of freedom. All these topics are addressed systematically in a perspective, self-contained review by **Wei-Min Zhang** [11].

3.4   Pancharatnam-Bargmann-Berry Phase

An outstanding example of a topological aspect in quantum mechanics (which may be termed 'field theory with a finite number of degrees of freedom'), is provided by the existence of a "geometric phase" in quantum theory which had remained obscured from public view until rather recently when M.Berry (1984) drew attention to it under the term "quantum adiabatic anholonomy". Historically, however, the existence of this pathology in physics had first been noted by S.Pancharatnam (1956) in the regime of classical polarization optics, but this important work had somehow gone by default. A similar fate befell a second attempt by V.Bargmann (1964) to resurrect this idea in the context of Wigner's theorem on the representation of symmetry operators in quantum mechanics. It was only after the work of Berry that its full implications were appreciated within the physics community, but its connection with the Pancharatnam and Bargmann phases was left unattended. In an instructive Article, **N Mukunda** [12] describes these developments in a proper perspective by emphasizing the mutual connections among these ideas. He also describes the subsequent developments to date, by relating these phases to the presence of a complex vector space and the effect of group action among them. He then goes on to show that the geometric phase is the simplest invariant expression under certain groups of transformation acting on curves in Hilbert space.

3.5   Skyrmion Model for Confinement

A confinement mechanism with topological overtones is offered by the large N_c limit of QCD which has played a crucial role in unifying its premises with a solitonic, hadron-based

approach that is known as the *Skyrmemodel* which was discovered by Skyrme (1961), just before quarks (1964) were born. Skyrme's novelty was to provide a model in which the fundamental fields consisted only of *pions*, wherein the nucleon was obtained as a certain classical configuration of pion fields. The apparent contradiction of making Fermi fields out of Bose fields was resolved by demanding a non-zero "winding number" for this (classical) field configuration, thus giving the "Skyrmion" the status of a topological soliton, which is a solution of a classical field equation with localized energy density.

On the face of it, the Skyrme scenario looked so different from the conventional picture of nucleons as a 'white' composite of 3 'colored' quarks bound together by their interactions with $U(3)$ gauge fields, that a reconciliation between the two pictures appeared rather remote. It turned out however that the Skyrme model could be a plausible approximation to the orthodox QCD picture, one in which a key role is played by the large N_c limit of the latter. The logic goes roughly as follows.

Despite the increasing strength of QCD at low energies, it is plausible that the pseudoscalar mesons as $q\bar{q}$ *composites*, could still interact relatively weakly with each other, thus permitting the formulation of some *effective* Lagrangian for the pions, subject of course to the correct symmetries of the underlying gauge theory, which includes a (spontaneously broken) chiral $SU(N_f) \otimes SU(N_f)$ flavour (N_f) symmetry that allows 'massless' pseudoscalars to co-exist with massive scalars. An effective Lagrangian on these lines may be obtained from "a non-linear realization of chiral symmetry", without the explicit appearance of scalars, a structure which has an uncanny resemblance to the very Lagrangian obtained by Skyrme (1961).

How about the baryons in this QCD-motivated "chiral perturbation theory" picture ? It is here that t'Hooft's (1974) large N_c limit comes into play, with the proportionality to N_c for the baryon mass being the signal that the baryon state under study is a soliton of the effective meson theory initiated by Skyrme. In a perspective review of the Skyrme model approach, **Joseph Schechter and Herbert Weigel** [13] trace its connection with QCD in the large N_c limit, and discuss the properties of light baryons treated as solitons, within the framework of an effective Lagrangian of QCD containing only meson degrees of freedom.

4 Formal Methods In QFT: Selected Topics

The universal language of QFT and its powerful techniques broke fresh ground through the establishment of the equivalence of its tenets with those of Statistical Mechanics which had traditionally been developed on entirely 'classical' lines. In the words of A.M.Tsvelik (QFT in CMP, Camb.Univ Press 1995), this equivalence may be succinctly expressed by the following statement: " QFT of a D-dimensional system can be formulated as a statistical mechanics of a $(D+1)$-dimensional system. This equivalence .... allows one to get rid of non-commuting operators and to forget about time ordering, which seem to be the characteristic properties of quantum mechanics....". The Path Integral formulation of QFT which is the key element in dispensing with the problem of non-commuting operators in QFT, has had a crucial role in bringing about this vital correspondence of QFT with the partition function in quantum statistical mechanics (QSM). Armed with the powerful techniques of Renormalization Group Theory (RGT), this new approach has opened up a whole vista of applications to new emerging areas like critical phenomena in condensed

matter physics.

4.1 Unified View of QFT and QSM

An important outcome of a unified view of QFT and Quantum Statistical Mechanics has been the emergence of two new areas: Euclidean Field Theory, and Finite Temperature Field Theory. Actually the origins of the former date back to the Fifties at the hands of Wick ("Wick rotation" for the Bethe-Salpeter equation) and Schwinger (as a possible direction for the evolution of QFT), wherein the transition from Minkowski to Euclidean space (via analytic continuation from real to imaginary "time") was perceived as a means of curing many ills in QFT, such as positivity and finiteness of norms in the computation of physical quantities. In more recent times, the Euclidean formulation of QFT has led to an interesting relationship between "stochastic mechanics" (Nelson) and the Feynman-Kac formulae for Green's functions expressed as path integrals. In a crisp Article in this Book, **R.Ramanathan** [14] provides a formulation of QFT in Euclidean space-time, to bring out the basic ideas of the Euclidean formulation, as well as the above relationship between the Nelson and Feynman-Kac formulations.

Finite Temperature Field Theory on the other hand, (in contrast to zero temperature for Euclidean QFT), provides access to a much wider class of complicated quantum mechanical systems, and addresses questions like thermal averages in QFT, symmetry restoration in theories with spontaneous symmetry breaking, and indeed the evolution of the universe at early times (from the high temperature phase). More recently, chiral symmetry-breaking phase transitions, especially the "confinement-deconfinement" phase transitions in QCD leading to quark-gluon plasmas (QGP), have acquired great interest in view of planned experiments on heavy ion collisions to detect (QGP). A few selected topics in Finite Temperature Field Theory are treated in an informative Article by **Ashoke Das** [15] in this Book.

4.2 Integrable Systems: Toda FT

Although most approaches to QFT have been traditionally associated with linear partial differential equations, (e.g., Schroedinger, Klein-Gordon, Dirac, Proca), *non-linear* equations, (i.e., equations where the potential term is non-linear in the field ϕ), have also been known for some time. Among the earliest non-linear wave equations known in physics are the *Liouville* and *Sine − Gordon* equations. The Liouville equation in 2D arose in the context of a search for a manifold with constant curvature, something like covering the surface with a fishing net whose arc length is constant (knots do not move!), while the 'threads' in the net correspond to a local coordinate system on the surface. The "field" ϕ in the Liouville equation is the phase space density ρ satisfying the equation $\partial_x \partial_y \rho = \exp \rho$, where x, y are the local orthogonal coordinates. The Sine-Gordon (SG) equation has a similar structure, with exp replaced by sin on the RHS. Variants of these equations, e.g., adding a 'mass' term $m^2 \phi$ on the LHS, and/or the hyperbolic replacement of sin by sinh, etc, give rise to several more varieties of similar types. A third type of non-linear equation which has received much attention, is the so-called KdV equation $u_t - 6uu_x + u_{xxx} = 0$, with interesting properties like an infinite number of conservation laws. The corresponding conserved quantities can be used as Hamiltonians for an integrable system (KdV hierarchy). A striking feature of such non-linear equations is an

infinite number of conserved quantities, which imply that the solutions of these systems must be infinitely restricted. This results in such solutions being quite stable structures (*solitons*) which retain their shapes even after collisions.

An interesting class of coupled non-linear equations was introduced by M.Toda (1967) to describe a 1D crystal with non-linear coupling between nearest neighbour atoms. These (lattice) models also admit *soliton* solutions which reduce to the KdV equation in the continuum limit. At the 'field' level, such models (with exponential 'potentials') simulate a general class of non-linear equations–called Toda Field Theory–which include the Liouville and Sine-Gordon equations as special cases. For the solution of these equations, a general method of "inverse scattering" was proposed by Gelfand-Levitan. The logic of this method is to convert, via a suitable transformation, the original non-linear equation to an equivalent *linear* equation, and study the evolution of the latter, more or less according to standard methods already developed for them (including group-theoretic, Lie-algebraic, etc methods). The inverse scattering method paved the way to connections with other known models of QFT, such as conformally invariant FT and the Hamiltonian reduction of Wess-Zumino-Witten model. Similarly the KdV equation is related to the 4D Yang-mills theories, thus providing a connection of the latter with 2D integrable models. In an instructive, self-contained article on this subject, **Bani Sodermark** [16] gives a perspective view of integrable systems with special reference to the Toda Lattice hierarchy, and reveals the connections of such non-linear field theories with other sectors of QFT.

4.3 Light-Front Dynamics

Dirac laid the foundations of QFT, not only through his famous Equation, but at least with 2 more seminal contributions within a year's gap from each other: a) light-front (LF) quantization [Rev.Mod.Phys.**21**, 392 (1949)]; b) constrained dynamics [Can.J.Math.**2**, 129 (1950)]. In the former, he suggested that a relativistically invariant Hamiltonian theory can be based on different classes of initial surfaces: instant form ($x_0 = const$); light-front (LF) form ($x_0 + x_3 = 0$); hyperboloid form ($x^2 + a^2 < 0$). The structure of the theory is strongly dependent on these 3 surface forms. In particular, the " LF form" remains invariant under 7 generators of the Poincare' group, while the other two are invariant only under 6 of them. Thus the LF form has the maximum number (7) of "kinematical" generators (their representations are independent of the dynamics of the system), leaving only 3 "hamiltonians" for the dynamics.

Dirac's LF dynamics got a boost after Weinberg's discovery of the $P_z = \inf$ frame which greatly simplified the structure of current algebra. The Bjorken scaling in deep inelastic scattering, supported by Feynman's parton picture, brought out the equivalence of LF dynamics with the $P_z = \inf$ frame. The LF language was developed systematically within the QFT framework by Kogut-Soper (1970), Leutwyler-Stern (1978), Srivastava (1998) and others. The time ordering in LF-QFT is in the variable $\tau = x_0 + x_3$, instead of $t = x_0$ in the instant form. And despite certain technicalities, the LF dynamics often turns out to be simpler and more transparent than the instant form, without giving up on the net physical content. This is borne out from comparative studies: of spontaneous symmetry breaking on the LF ; of degenerate vacuum in certain $(1 + 1)D$ QFT which are exactly soluble and renormalizable (e.g., the Schwinger model and its chiral version); of chiral boson theories; and of QCD in covariant gauges. Indeed, the LF quantization

of QCD in the Hamiltonian form bids fair to be a viable alternative to the lattice gauge theory for calculating non-perturbative quantities. Removal of constraints by the Dirac method gives fewer independent dynamical variables in the LF formalism than in the instant form; for this reason, LF variables have found applications even in String and M-theories. In an instructive self-contained review (with a rich collection of references), **Prem Srivastava**[17] gives a detailed review of most of these topics in a leisurely and systematic manner, and leads the interested reader all the way to the frontier with several new results.

4.3.1 $2D$ Field Theory

$2D$ models in QFT have also been of great interest in the contemporary literature. Such theories reveal some remarkable features, such as fermion-boson equivalence, which facilitates the solution of fermion-FT in terms of its bosonized version. This concept of bosonization in turn has been useful in the understanding of $4D$ phenomena that can be described by an effective $2D$ FT, such as the demonstration of quark confinement in exactly soluble $2D$ models [Casher-Kogut-Susskind (1973)]. Another important discovery in $2D$ FT concerns an "anomaly-generated" mass [Jackiw-Rajaraman (1985)] for the gauge boson in the Chiral Schwinger model. (This mechanism may be contrasted to the standard Higgs mechanism for generating the vector boson mass via spontaneous symmetry breaking). The "anomaly" here stands for the loss of the conservation property due to quantum corrections involved in the quantization of the gauge theory. This disease in turn needs Dirac's second weapon for cure: Constrained dynamics. In a short perspective article in this Book, **Dayashankar Kulshreshtha** [18] reviews the constrained dynamics and local gauge invariance of several $2D$ FT models, in both Instant and LF forms, and in so doing, brings out the detailed working of the BRST formalism as applied to such $2D$ models.

4.4   Constrained Dynamics

To recall the essential elements of a constrained dynamical system, which includes most systems of physical interest (e.g., QED, QCD, Electroweak and Gravity theories), it is characterized by an $over-determined$ set of coordinates. These are best kept track of within a Hamiltonian formulation, which has a natural place for all the coordinates (canonical and redundant), so that the complete set of constraints emerges easily. The nature of these constraints in turn is determined by the structure of the matrix of Poisson brackets (PB) of the constraints of the theory, which also carries the signature of whether or not the underlying theory is gauge invariant (GI). Thus if this PB matrix is singular, then the set of constraints is $firstclass$, and the theory is GI. On the other hand, if this matrix is non- singular, then the set of constraints of the theory is $secondclass$, and the theory is non-GI. (Indeed this is often taken as a criterion for distinguishing a GI from a non-GI system). These GI systems are then quantized under some appropriate gauge choices, or "gauge fixing" (GF). Now in the usual Hamiltonian formulations of a GI theory under some GF's, one necessarily destroys the gauge invariance, since the GF corresponds to converting the first class constraints to second class constraints. To quantize a GI theory by maintaining gauge invariance despite GF, one needs the more general BRST (1974) formulation, wherein the theory is rewritten as a quantum system with generalized GI,

called BRST invariance. This in turn requires enlarging the Hilbert space, and replacing the gauge transformation by a BRST transformation which involves the introduction of (anti-commuting) Faddeev-Popov *ghostfields*. This amounts to embedding the GI system into a BRST invariant system (but isomorphic to the former), whose unitarity is guaranteed by the conservation and nilpotency of the BRST charge.

Thus the Dirac[Can.J.Math.**2**, 129 (1950)]-Bergmann [Phys.Rev.**83**, 1018 (1951)] theory of Constraints lies at the root of (Hamiltonian) description of interactions in QFT based on Action principles which, due to the requirements of Lorentz, local gauge, (and/or diffeomorphism) invariances, must employ singular Lagrangians. This is generally adequate for the study of simple gauge theories (controlled by some Lie groups acting on some internal spacein Minkowski space-time), via the covariant approach based on BRST symmetry which, at least for infinitesimal gauge transformations, allows a regularization and renormalization of the relevant theories within the local QFT framework. On the other hand, the gauge freedom of theories that are invariant under diffeomorphism groups of the underlying space-time (e.g., in general relativity or string theory) is encumbered by the arbitrariness for the observer in the "definitory properties" of space-time and/or the measuring apparatus;[see **L.Lusanna**-this Book]. Such ambiguities affect bigger issues like: the understanding of *finite* gauge transformations; the Gribov ambiguity in the choice of function space for the fields; proper definition of relativistic bound states vis-a-vis quark confinement; and last not least the conceptual and practical problems posed by gravity. These require a fresh look at the foundations of QFT to know if we: i) understand the physical degrees of freedom hidden behind gauge and/or general covariance; ii) can meanfully reformulate the physics (both classical and quantum) in terms of them. Logically this would amount to abandoning local QFT for non-perturbative interactions, and a reformulation of relativistic theories to allow natural coupling to Gravity. These and allied issues are addressed in a state of the art review by **Luca Lusanna** [19], aimed at a unified reformulation of the 4 basic interactions in terms of Dirac-Bergmann observables, with emphasis on the open problems–mathematical, physical and interpretational.

5   Extension Of QFT Frontiers

A long term ambition of QFT has been the dream of unificication of all the gauge fields with the Gravitation Field whose quantization has all along posed a big challenge in its own right. [A major difficulty in the way of unification of this sector with the other three, as was once succinctly put by Abdus Salam, lies in the "spin mismatch" of their respective fields (vector vs tensor), which would militate against a common strategy]. Nevertheless such a unification was to come about from an entirely new paradigm which envisaged extension of the original tenets of Field Theory based on a point particle description to one with *Strings*. In this Section we offer a panoramic view of some major theoretical developments from seemingly unrelated angles, which, apart from their impact on Physics in their own right, have provided some key ingredients converging towards the emergence of modern *StringTheory*. These developments which may be termed *Supersymmetry* (SUSY), *ConformalFieldTheory* (CFT), and *Duality*, are outlined next.

5.1 SUSY In Field Theory

In its march towards Unification, Field Theory has continued to break new ground in several directions. An important step in Unification was marked by the discovery of Supersymmetry ($SUSY$), introduced in the early seventies by a galaxy of authors in the context of 2D QFT (Gervais-Sakata) as well as in 4D QFT (Golfand, Likhtman, Akulov, Volkov, Wess and Zumino), for a unified understanding of the two known forms of matter–bosons (integral spins) and fermions(half integral spins)–hitherto regarded as two distinct field types, with commuting and anticommuting properties respectively. The new symmetry between bosons and fermions may be incorporated within the definition of a single "Superfield", with transformations inter-relating the two constituents, so that $SUSY$ becomes a part of space-time symmetry implied by relativistic invariance. The Gauge principle too admits of a corresponding extension to unify both these sectors.

What are the motivations for such a lavish extension of space-time symmetry ? Apart from its aesthetic appeal, there are some theoretical considerations of a more concrete nature which are dwelt on in this Book through two complementary reviews of $SUSY$ in Field Theory, (with special reference to Particle Physics), by two leading experts in the field: **Rabi Mohapatra** [20] and **Norisuke Sakai** [21] respectively. According to Sakai [21], the most important motivation for $SUSY$ is the *Gaugehierarchyproblem* showing up via the vastly different mass scales of the electroweak (M_W) vs the "GUT-theoretic" (M_G): $M_W^2/M_G^2 \approx 10^{-28}$. A similar gap exists between the "GUT" vs Planck (gravity) mass scales: $M_G^2/M_{pl}^2 \approx 10^{-6}$.

To account for this phenomenon, it is necessary to invoke a suitable *Symmetry* reason which may be precisely formulated by the so-called "naturalness" hypothesis (t'Hooft 1979) which demands that a system acquires a higher symmetry as a certain (small) parameter goes to zero, e.g., chiral symmetry occurs when a (small) fermion mass goes to zero; or a local gauge symmetry corresponds to the vanishing of a vector boson mass. Now the mass scale M_W of weak bosons arises from the vacuum expectation value $< \phi >_0 \equiv v \neq 0$, related to the mass M_H of the Higgs scalar field ϕ. So to regard the gauge hierarchy problem as the result of some symmetry breaking, we must give a *Symmetry* reason to make the Higgs scalar mass vanishingly small. Classically a vanishing scalar mass corresponds to a symmetry called scale invariance, which however cannot be maintained quantum mechanically. In a perspective review on this subject, **Norisuke Sakai** [21] argues for "Supersymmetry" between the Higgs scalar and a spinor partner as a good option: Chiral symmetry gives zero mass to the latter, while $SUSY$ makes the former massless (through a cancellation of the respective contributions to the self-energy loops).

In a complementary perspective review on the same subject, **Rabi Mohapatra** [20] stresses the versatility of $SUSY$ as a tool for understanding many unsolved problems of physics: a) improvement in the singularity structure of local fields for understanding the disparate scales of Nature (e.g., Electroweak vs Gravity); b) possibility of unifying Gravity with the other forces by making $SUSY$ local instead of global; c) prospects of understanding $non-perturbative$ properties of field theories, hitherto considered 'impossible' in non-$SUSY$ form.

As to the manifestations of $SUSY$ in a real world, this "Bose-Fermi" symmetry is supposed to be badly broken, so that any search for superpartners (bosons vs 'bosinos'; fermions vs 's-fermions') has so far yielded zero dividends. On the other hand the formulation of Supersymmetry in non-relativistic quantum mechanics is relatively free from

constraints. Indeed, since Schroedinger (1940) noticed the existence of well-defined "supersymmetric partners" for the energy levels of a given quantum mechanical system, many applications to such systems (including nuclear and condensed matter physics), have kept pace with the rapid strides of SUSY in field theory in recent years. Indeed, the existence of SUSY partners in the energy levels of (appropriately chosen) even vs odd nuclei have been systematically established by group theoretic methods (interacting boson models, etc). Similarly, in solid state physics, an interesting correspondence has been observed between the critical behaviour of a 'spin system' in random magnetic fields in d dimensions, and that of the spin system without the random magnetic field in $d-2$ dimensions. This "dimensional reduction" may be traced to an underlying $SYSY$ for the spin system in random magnetic fields (see **N Sakai**–this Book).

In the absence of discovery of $SUSY$ partners in Field Theory, the benefits from $SYSY$ have so far been purely theoretical, varying from reduction of the degrees of divergence arising from various loop integrals in standard field theory (by at least two orders), to a heavy reduction in the number of dimensions (from 26 to 10) needed for self-consistency in a string theoretic formulation. The Articles by Mohapatra [20] and Sakai [21] between them provide quite a complementary description of the $SUSY$ formalism in QFT, together with an glimpse of the recent developments. And apart from its applications in particle physics, this formalism also serves as a background to the vast field of supersymmetric string theory.

5.2   CFT

An independent insight into the origin of String Theory comes from the role of $Conformal$ $Field$ $Theory$ (CFT), viz., conformally invariant QFT in 2D(imensions), not only as as a vital ingredient of its anatomy, but also with firm hold on other disciplines like condensed matter physics. The CFT route to the evolution of String Theory is sketched in this Book as part of a bigger (historical) survey by **Werner Nahm** [22], tracing a whole sequence of developments in QFT right from its (Dirac) beginning, and encompassing in the process several other areas of physics on which CFT has had a decisive impact. In this saga, the interplay of physical intuition and mathematical rigour has brought together the practitioners of these respective disciplines, though not necessarily working in tandem. On the one hand, the beauty and transparency of CFT have made for a rich variety of intellectual exercises in abstract mathematics (with new emerging areas like automorphic groups, Kähler-Einstein metrics, etc), and on the other, facilitated the study of intensely practical physical systems such as continuous phase transitions in condensed matter physics.

The impact of CFT on string theory has had its origin in several theoretical developments: the Thirring model in 2D; Skyrme's idea of the equivalence between Fermions and Bosons; Coleman's equivalence theorem on the Thirring Model versus the Sine-Gordon equation (despite their apparent dissimilarity); and the role of conformal invariance in the structure of Wilson's Renormalization Group equations. To recall the essentials of Conformal invariance, this symmetry is satisfied in the absence of any 'scale' dimension. Examples are Maxwell's Equations in free space; Dirac equation for massless fermions which satisfy conformal invariance. The 2D Thirring model, which may be regarded as a basic ingredient of string theory, also has this property due to absence of a scale dimension. Using this mathematical picture, the string may be regarded as a 1D object in

space spanning a world sheet (a Riemann surface) embedded in 2D space-time, where a point on the string is represented by $X^\mu(\sigma, \tau)$; σ, τ being the 2 world sheet coordinates.

The impact of CFT has been no less impressive in the domain of condensed matter physics (CMP) where there exist a rich class of QFT's exhibiting the structure of conformally invariant fields, such as in 2D surface coatings. Thus at a critical temperature (T_c), the long range fluctuations of arbitrary scales make irrelevant the details of molecular structure, and the theory approaches a *continuumlimit*, with no visible scale dimension to keep track of. Indeed in this limit, the correlation functions behave like the Euclidean n-point functions of standard QFT with conformal invariance properties. Nahm [22] discusses an interesting correspondence between the Ising model in CMP and Thirring model in QFT. The equation satisfied by the spin waves of the Ising model is formally identical to the 2D Dirac equation for massless fermions. Indeed condensed matter physics provides a more stable and economical background for testing these ideas than the expensive HEP laboratories !

5.3   String Theory Via Duality

Perhaps the most startling "revolution" in Physics to date which had its origin in QFT, has been the String Theory, and its successive "Avatars" (incarnations), aimed at unifying all the forces of Nature (from orthodox gauge theories of strong, e.m. and weak interactions, all the way to gravity). An orthodox route to its evolution may be attributed to the strong interaction problem in QFT, which has had wide ramifications from vastly different angles, each providing an independent insight into its mysteries. A very promising approach to strong interactions came from the *Duality* Principle which has had a long history (prehaps traceable to the Bootstrap hypothesis), based on the equivalence of the direct channel (resonances) and crossed channel Regge poles with a universal slope $\alpha' \approx 1 GeV^{-2}$. An explicit realization of this idea was achieved via the Veneziano representation for 4-point amplitudes satisfying the requirements of duality and crossing symmetry, which was soon generalized to N-point amplitudes satisfying the same properties. Through a path integral representation of such amplitudes, Nambu, Nielsen and Susskind recognized that these amplitudes describe a 1D (string-like) object moving in space, with the inverse of the universal Regge slope identified as the "string" tension T. The "string" interpretation was further reinforced by a subsequent representation due to Virasoro, with very similar properties. And its promise of relevance to particle physics (despite stiff competition from QCD!) got a boost from the Scherk-Schwarz (1974) observation that such a "string theory" could serve as a candidate for incorporating gravity in its ambit, on the ground that the massless spin-2 particle appears naturally in the closed string spectrum. To that end the string tension T needed to be increased by 19 orders of magnitude (up to the Planck scale!) to qualify for a viable theory of gravity. The conceptual gap was finally bridged by the seminal work of Green-Schwarz (1984) who succeeded in constructing a consistent $10D$ super Yang-Mills theory coupled to supergravity which is free from anomalies only for certain gauge groups ($SO(32)$ or $E_8 \times E_8$). This work, perhaps for the first time, showed real prospects for unifying the fundamental forces.

The String Theory has grown by leaps and bounds during the past decade, and its vast ramifications have grown to such formidable literature over the past decade, that a minimal justice to it would itself require several volumes of review. Nevertheless, after a short overview by the Master, **John Schwarz** [23] of the subject, a panoramic account

of the major developments in this exciting field (together with an exhaustive set of references) is given in a perspective Article by **Jnanadeva Maharana** [24]. Schwarz [23] views the different superstring theories (and an extension called $M-theory$) as different facets of a unique underlying theory going beyond ordinary QFT's. However, recent duality conjectures suggest that a more complete definition of these theories may come from the large N limits of suitably chosen $U(N)$ gauge theories; (see **L Bonora** [25] below). The Maharana Article [24] leads the interested reader through several stages of its development, from i) perturbative aspects of ST; successively through ii) $Duality Symmetries$ as a characteristic of String Theories (ST); iii) M-theory as a unified view of the $five$ perturbatively consistent $ST's$; iv) microscopic understanding of Black Holes, and so on, all the way to the frontiers of the field.

Attempting to cover the later stages of development in this rapidly growing field, **Loriano Bonora** [25] reviews some advances in the study of the relation between Yang-Mills (YM) theory and strings, based on the classical YM-theory solutions ($Riemannian instantons$) which are 2D solutions describing Riemann surfaces in the strong coupling limit. Strictly, such relations historically date back all the way to 't Hooft ('74) through his famous $1/N_c$ expansion for large N_c, wherein the dominant Feynman amplitudes correspond to the 2D Riemann surfaces. This 'natural connection' with strings was subsequently upgraded to a concrete shape via studies of 2D QCD (for string-like properties), which was further generalized to a connection between conformal super-YM and super-string theory of type IIB, in the large N_c limit. The **Bonora** Article reveals, among other things, a direct link between String Theory and non-abelian YM theory, through the emergence in the latter of classical solutions modelled over Riemann surfaces, leading to a "string" interpretation. Historically, this came about only after the proposal of the $Matrix Theory$, which in the large N_c limit converges to the (non-perturbative) $M-Theory$.

6 CS Field Theory And Condensed Matter Physics

While the dominant concern in Field Theory has been in the traditional domain of particle physics, its powerful language and tecnniques have found profitable employment over a much wider domain, which comprises topics in Condensed Matter Physics, and newly emerging fields like Quantum Hall Effect, fractional statistics and $Anyons$. These phenomena lend themselves to QFT treatment in $(2+1)$ dimensions, where the celebrated "Chern-Simon" (CS) term plays a key role (see also Part B on Topological Field Theories).

What are the special features of QFT in $(2+1)D$, and what specific role does the CS term play in this reduced space-time continuum ? Perhaps the most striking feature is the appearance of $fractional statistics$! For, whereas in 3 (or higher) space dimensions, all particles must either be bosons (integral spin) or fermions (half-integral spin), in 2 space dimensions, the particles can have any fractional spin/statistics with impunity ! Such particles are called $Anyons$. Now since the usual spin-statistics relation follows from the premises of the standard 4D relativistic QFT, it is natural to ask if $Anyons$ can be understood from the corresponding 3D QFT. The question goes beyond mere academic interest since lower dimensions can be effectively realized in the physical world through the "freezing" out of certain degrees of freedom, (e.g., in a strongly confined potential, or at low enough temperatures), so that these 'quasi-particles' may well exhibit anyon-like properties. And indeed experiments on Quantum Hall Effect (QHE) have revealed the

existence of fractionally charged excitations (thus implying anyons).

A critical discussion on the question of anyons and fractional statistics in (2+1) dimensions, with particular reference to the role of the Chern-Simons (CS) term in 3D QFT, is given by **Avinash Khare** [26] in a perspective Article on the subject in this Book. To that end, Khare clarifies the definition of "quantum statistics" which relates to the "phase" picked up by a wave function when two identical particles are *adiabaticallyexchanged*, as distinct from the usual definition of permutation symmetry for two identical particles. [While both definitions coincide for 3 and higher dimensions, they differ in 2 dimensions]. He then discusses in detail the main properties of the CS term, especially its role as a gauge field mass term, in whose presence anyons can appear in one of two different ways: i) as a soliton of the corresponding QFT ; or ii) as fundamental quanta carrying fractional statistics. So far, the state of the art is based on non-relativistic QFT, wherein the CS term provides an effective cushion against a non-local formulation of anyon fields, thus facilitating a 'local' formulation. However a full-fledged relativistic QFT formulation is not yet feasible.

Perhaps the most tangible success from CS fields so far is a natural understanding of the Quantum Hall (QH) Effect. A state-of-the-art review by **R Rajaraman** [27] puts this topical subject in perspective. We summarize some essential features of a QH system, from his own account. A QH system which is defined as "quasi 2D layers of electrons trapped in the interface of semi-conductors, at very high magnetic fields and very low temperatures, has revealed many remarkable features ". Particularly interesting is the presence of certain states characterized by the so-called "filling fractions" (ν) which are either integers, or certain *odddenominator* fractions; $\nu = hc\bar{\rho}/eB$, where $\bar{\rho}$ is the mean electron density, and B the applied field. The special states corresponding to these ν-values show extremely flat plateaus in Hall conductivity which (in units of e^2/h) are exactly equal these values to within an accuracy of 1 in 10^7 ! These features are very universal inasmuch as the details of the material seem irrelevant. It was earlier recognized that the electrons in these QH states form an incompressible fluid, described by "Laughlin wave functions" (which are reminiscent of Jastrow-type correlations in nuclear wave functions). A more analytical study of these empirical functions suggested a Landau-Ginsberg type scenario for the QHE in terms of an order parameter field (subsequently to be identified with a Chern-Simons field), thus formally bringing this subject within a 3D QFT network.

The analogy of the order parameter field in QHE to that obtaining in superconductivity of the Landau-Ginsberg description, is of course not a literal one since there are no bosonic Cooper pairs in QHE. Indeed in this 3D QFT scenario, the "anyons" (Chern Simons fields) have an intermediate status between bosons and fermions. However for the special case when the anyon angle is an odd multiple of π, a composite of the electron with an *odd* number of flux tubes, effectively amounts to constructing a "bosonic" analogue of Cooper pairs from out of "fermions" which now provides the desired order-parameter (CS) field operating in the plateau of the QH system. Rajaraman reviews a formal QFT procedure for constructing such CS gauge fields, as well as the formulation of their dynamics at the 3D QFT level. As to the connection of the CS gauge fields at the first quantized level, these are of course expressible in terms of the "phase angles" involved in the exchange of electrons in an N- electron wave function in 2D (see also Khare [26] in this Book).

7 QCD-Motivated Strategies For Strong Interactions

Turning now to the strong interaction problem in the standard field theoretic picture, its prime candidate, QCD, has since its birth been beset with problems of reliable calculational techniques to deliver results. An introductory overview of several approaches [symmetries, effective Lagrangians and Wilson expansions] to deduce hadron properties from QCD is sketched in the Article by **Olivier Pene** [28], aimed at establishing a link between perturbative and non-perturbative QCD via lattice methods. We now go into more specific details of a few principal QCD-based methods.

7.1 QCD Sum Rules

To recall the main signatures of the prime candidate, QCD, which it shares with any non-abelian gauge theory, are expressed by a two-fold pattern: i) decreasing coupling strength at shorter distances (Asymptotic Freedom); and ii) increasing coupling strength at longer distances (*confinement*). The former is fairly well understood, and provides a perturbative basis for calculating QCD effects in high energy processes. In particular, the powerful method of "QCD Sum Rules", based on Wilson's Operator Product Expansion (OPE), was developed by Shifman- Vanstein-Zakharov for the study of non-perturbative QCD in a large variety of applications from hadronic masses (with two-point functions), coupling constants, form factors (with three-point functions), and reactions (four point functions). The basic philosophy is one of a *duality* between two ways of representing a correlator: i) OPE with various "twist" terms (vacuum condensates, treated as free parameters of the theory) representing successive non-perturbative corrections to an otherwise perturbative expansion; ii) a dispersion formula saturated by certain low-lying hadron resonances. Equating the two amounts to evaluating hadronic parameters in terms of the quark-level condensates. Despite certain conceptual problems of "microscopic causality" encountered in the "matching" of two sides of the equation, this method (QCD-SR) has proved very popular among a wide class of high energy phenomenologists, and has been continually refined over the years. A leisurely review of the state of the QCD-SR art on the quark structure of hardons, as well as its working on the problem of hadrons in nuclear matter (at finite temperature) is given by **Leonard Kisslinger** [29] in this Book.

7.2 Non-Perturbative Methods With QCD Features

The state of the art in this field is so diffuse that a more organized exposition is needed for such methods. To that end the attempts at addressing the strong interaction problem in QCD may be divided into two broad categories: i) soluble models designed to shed light on its general features through exact calculations; and ii) effective Lagrangian methods for 4-fermion interactions, somewhat reminiscent of the Bethe "Second Principle" Theory of effective nucleon-nucleon interactions of the Fifties. Srivastava [17], as well as Kulshreshtha [18], in Part C of this Book, have already provided a flavour of the results to be expected from type (i) theories, using the method of LF-QFT.

Type (ii) which deals with more realistic situations, albeit at the cost of some phenomenology, has a much wider literature to choose from. To do a semblance of justice to this field, this Book includes *two* articles of this type, reviewing the methodology and working of such QFT-based approaches. The first one, by **Vladimir Karmanov** [30],

gives an in-depth review of covariant light-front (LF) dynamics, with applications to field theory and relativistic wave functions. The formalism is effectively 3D in content, which can be obtained by projecting the (4D) Bethe-Salpeter amplitudes on the light-front plane, and although a reversal of steps is not possible to reconstruct the 4D BS amplitude, the LF formalism still represents a powerful alternative for solving QFT problems. Karmanov [30] also discusses some typical applications.

7.2.1 Markov-Yukawa Transversality Principle

The second article by **Asoke Mitra** [31] offers a comparative view of the state of the art in several QFT approaches based on effective 4-fermion interactions (including QCD features), of both 3D and 4D types (Tamm-Dancoff, Bethe-Salpeter, Salpeter, quasi potentials, light-front). In this context, attention is focussed on an important but somewhat less known principle called "Markov-Yukawa Transversality" ($MYTP$) which decrees that the interaction between the two (quark) constituents be *transverse* to the composite (hadron) 4-momentum, by virtue of which the BSE kernel has an effective (albeit covariant) 3D support. As a result of this "Covariant Instantaneity" the starting 4D BSE is exactly reducible to a 3D form, and conversely the steps can be reversed so as to allow an *exactreconstruction* of the original 4D BSE in terms of 3D ingredients ! Thus $MYTP$ allows an exact interlinkage between the 3D and 4D BSE forms, so that both forms can be used interchangeably, unlike most other approaches in the literature which employ either a 4D or a 3D form of the BS dynamics, but not both simultaneously.

It might be of some historical interest to note that the Salpeter equation has a 3D structure stemming from its (instantaneous) kernel with a 3D support, and therefore its original 4D form can be recovered a la $MYTP$ by reversing the steps, but this possibility had never been explored. This gap is now filled by $MYTP$ which provides a formally covariant basis for the instantaneous approximation. The same principle ($MYTP$) can also be generalized from covariant instantaneity to the covariant light-front.

A fall-out of the 3D-4D interlinkage provided by $MYTP$ is that it gives a $two-tier$ description: the 3D form for the hadron spectra which are $O(3)$-like; and the 4D form to address the transition amplitudes as 4D loop integrals using standard (4D) Feynman rules. This Principle can be easily incorporated in the usual framework of coupled Bethe-Salpeter and Schwinger-Dyson equations (BSE-SDE) stemming from a (chirally invariant) 4-fermion Lagrangian with current quarks interacting via the full gluon propagator, so that the quark mass is acquired via the NJL-mechanism. And the generalization from covariant instantaneity to the covariant light-front helps remove certain problems of Lorentz mismatch of vertex functions that arise in a 4D loop integral under the covariant instantaneity ansatz. These and other details are reviewed in the article by Mitra [31] which also stresses a parallelism of treatment of $q\bar{q}$ and qqq systems.

7.3 The Harmonic Oscillator: A Powerful Bridge In QFT

No amount of literature on the impact of QFT in Physics would be complete without an exposure of the role of the Harmonic Oscillator (HO) in shaping Quantum Theory, as an integral part of this Book theme. It was therefore a matter of great satisfaction when **Marcos Moshinsky** [32], who may be regarded as the "Father of the Harmonic Oscillator in Physics", agreed to contribute a perspective article on the HO

theme. The only obstacle against a regular format for his Article was that he had only recently written a comprehensive book on the subject [M. Moshinsky and Yu.F.Smirnov, $The Harmonic Oscillator In Modern Physics$, (Harwood Academic Press, the Netherlands, 1996)]. Nevertheless in his Article, he has provided a comprehensive list of contents of his HO-book, which already offers a glimpse of the depth and range of physical problems (from the simplest quantum mechanical ones to the n-body Relativistic Oscillator) that are amenable to the amazing powers of HO techniques in tandem with the standard methods of Group Theory. In addition he has reviewed some recent work of his on relativistic particles of arbitrary spin in a $confining$ HO potential, with applications to Spectroscopy.

8   Conclusion: Foundations Of Quantum Theory

We conclude this Introduction to the Book with an Article by **Dipankar Home**[33] on the modern perspectives on the foundations of quantum mechanics (the predecessor of QFT), which are increasingly being scrutinized by relating them to precise experimental studies. In this Article, Home [33] picks two main issues: i) quantum measurement problem; ii) quantum non-locality, for a detailed exposition in a theory vs experiment scenario. He concludes with a quotation from John Bell: "It seems to me possible that the continuing anxiety about what quantum mechanics means or entails will lead to still more tricky experiments which will eventually find some soft spot." Translated to the QFT level, this looks like an appropriate conclusion for this Book as well.

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