PRINCIPLES OF ECONOMETRICS

5TH EDITION

ANSWERS TO ODD-NUMBERED EXERCISES IN APPENDIX A

EXERCISE A.1

Function (1): $Q = -3 + 2P$

(a) The slope is given by $\frac{dQ}{dP} = 2$ for all values of *P* including $P = 10$.

(b) A 1-unit increase in price leads to a 2-unit increase in quantity, suggesting a supply curve. The slope does not change for different values of *P* aand *Q*.

(c) The elasticity is given by $\frac{dQ}{dP}\frac{P}{Q} = 2\frac{P}{Q} = \frac{2P}{-3+2P}$. When $P = 10$, $\frac{dQ}{dP}\frac{P}{Q} = \frac{20}{17} = 1.17647$.

(d) At the point $P = 10$, $Q = 17$, a 1% increase in price leads to a 1.176% increase in quantity. The elasticity changes for different values of *P* and *Q*.

Function (2): $Q = 100 - 20P$

(a) The slope is given by $\frac{dQ}{dP} = -20$ for all values of *P* including $P = 4$.

(b) A 1-unit increase in price leads to a 20-unit decrease in quantity, suggesting a demand curve. The slope does not change for different values of *P* and *Q*.

(c) The elasticity is given by $\frac{dQ}{dP}\frac{P}{Q} = -20\frac{P}{Q} = -\frac{20P}{100-20P}$. When $P = 4$, $\frac{dQ}{dP}\frac{P}{Q} = -4$.

(d) At the point $P = 4$, $Q = 20$, a 1% increase in price leads to a 4% decrease in quantity. The elasticity changes for different values of *P* and *Q*.

Function (3): $Q = 50P^{-2}$

(a) The slope is given by $\frac{dQ}{dP} = -100P^{-3}$. When $P = 2$, $\frac{dQ}{dP} = -\frac{100}{8} = -12.5$.

(b) When $P = 2$, quantity decreases at the rate of 12.5 units per unit increase in price, suggesting a demand curve. The slope changes for different values of *P* aand *Q*.

(c) The elasticity is given by $\frac{dQ}{dP}\frac{P}{Q} = -\frac{100}{P^3} \times \frac{P}{50P^{-2}} = -2$ for all values of *P* and *Q*.

(d) A 1% increase in price leads to a 2% decrease in quantity. The elasticity does not change for different values of *P* and *Q*.

EXERCISE A.3

(a) A sketch of the curve $INF = -3 + 7/UNEMP$ for values of *UNEMP* from 1 to 10 appears below.

(b) The impact of a change in the unemployment rate on inflation is given by the slope of the function

$$\frac{d(INF)}{d(UNEMP)} = -\frac{7}{UNEMP^2}$$

The absolute value of this function is largest as *UNEMP* approaches zero and it is smallest as *UNEMP* approaches infinity. Thus, the impact is greatest as the rate of unemployment approaches zero and it is smallest as unemployment approaches infinity. This property is confirmed by examining Figure xr-A.3(a).

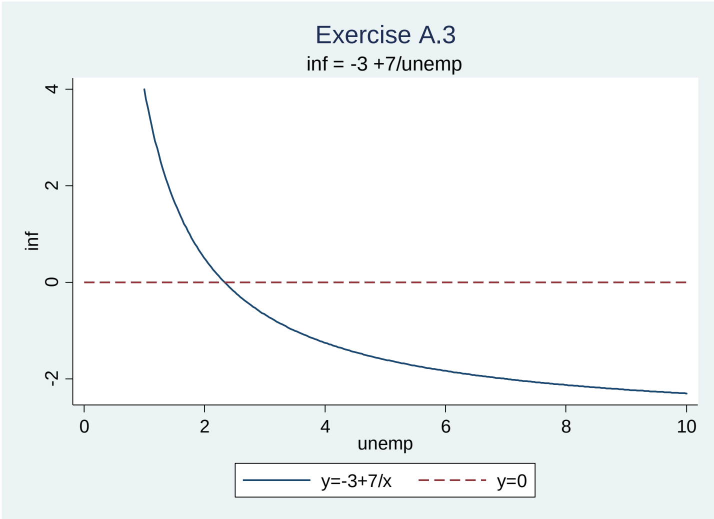

Figure xr-A.3(a) Curve relating inflation to unemployment

(c) The marginal effect of the unemployment rate on inflation when $UNEMP = 5$ is given by

$$\frac{d(INF)}{d(UNEMP)} = -\frac{7}{UNEMP^2} = -\frac{7}{5^2} = -0.28$$

EXERCISE A.5

(a) (i) Maldives *GDP* \$3,142,812,004 = 3.142812004×10^9

(ii) Nicaragua *GDP* \$12,692,562,187 = $1.2692562187 \times 10^{10}$

(iii) Ecuador *GDP* \$100,871,770,000 = 1.0087177×10^{11}

(iv) New Zealand *GDP* \$173,754,075,210 = $1.7375407521 \times 10^{11}$

(v) India *GDP* \$2,073,542,978,208 = $2.073542978208 \times 10^{12}$

(vi) United States *GDP* \$17,946,996,000,000 = 1.7946996×10^{13}

(b) (i) $$\frac{17,946,996,000,000}{3,142,812,004} = \frac{1.7946996 \times 10^{13}}{3.142812004 \times 10^{9}} = 0.571049 \times 10^4 = 5710.49$$

(ii) $$\frac{17,946,996,000,000}{100,871,770,000} = \frac{1.7946996 \times 10^{13}}{1.0087177 \times 10^{11}} = 1.779189 \times 10^2 = 177.9189$$

(c) Per capita income in New Zealand in 2015 is 37.814×10^3.

(d) Per capita income in St. Lucia in 2015 is 7.764×10^3.

(e) The sum of US and New Zealand GDP is given by 18.12075×10^{12}.

EXERCISE A.7

(a) Plot of the curve $WAGE = 10 + 200AGE - 2AGE^2$ for AGE between $AGE = 20$ and $AGE = 70$.

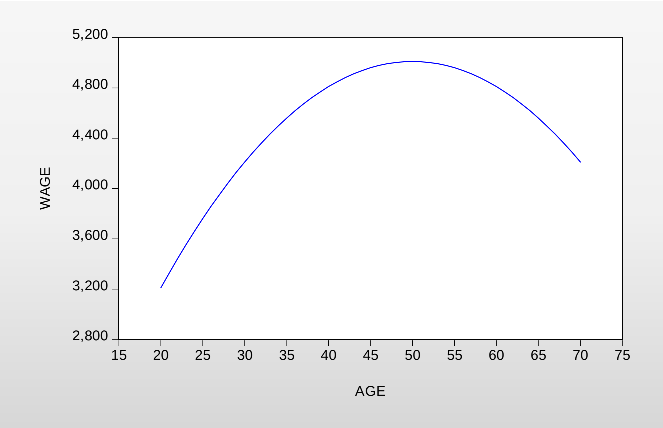

Figure xr-A.7(a) Relationship between *WAGE* and *AGE*

(b) The derivative $dWAGE/dAGE$ is given by $\dfrac{dWAGE}{dAGE} = 200 - 4WAGE$

When $AGE = 30$, $\dfrac{dWAGE}{dAGE} = 200 - 4 \times 30 = 80$.

When $AGE = 50$, $\dfrac{dWAGE}{dAGE} = 200 - 4 \times 50 = 0$.

When $AGE = 60$, $\dfrac{dWAGE}{dAGE} = 200 - 4 \times 60 = -40$.

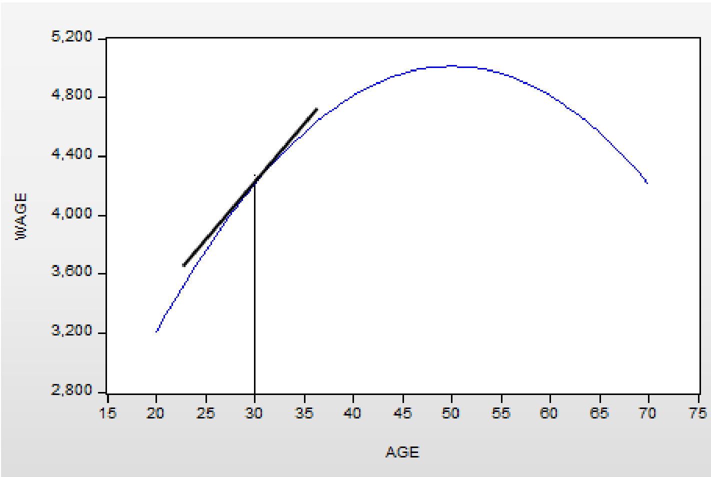

Figure xr-A.7(b) Tangent at *AGE* = 30

(c) Since $\frac{dWAGE}{dAGE} = 0$ when $AGE = 50$, and $\frac{d^2WAGE}{dAGE^2} = -4 < 0$, *WAGE* is maximized at $AGE = 50$.

(d) The required values are given by

$$WAGE_1 = 10 + 200 \times 29.99 - 2 \times 29.99^2 = 4209.1998$$

$$WAGE_2 = 10 + 200 \times 30.01 - 2 \times 30.01^2 = 4210.7998$$

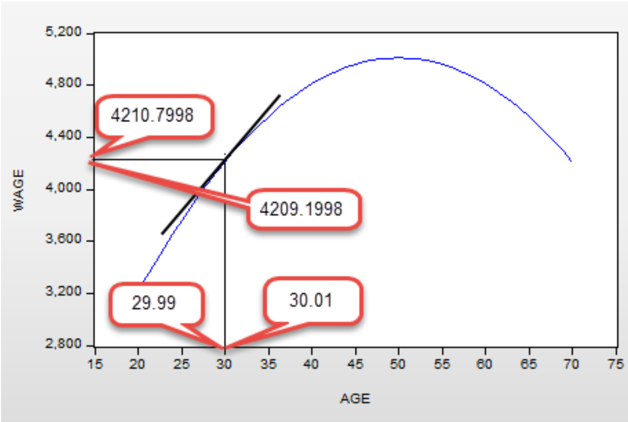

Figure xr-A.7(d) Approximate location of *AGE* and *WAGE* values

(e) The numerical derivative is

$$m = \frac{4210.7998 - 4209.1998}{0.02} = \frac{1.6}{0.02} = 80$$

which is identical to the derivative computed in part (b). The values should be close because the slope of the tangent (the derivative) is approximately equal to the "rise" divided by the "run" for a triangle centered at $AGE = 30$.

EXERCISE A.9

(a) The area under $f(y) = 1/100$ within the interval $30 < y < 50$ is a rectangle of width 20 and height $1/100$. Thus, the area is equal to $1/100 \times 20 = 0.2$.

(b) Using integration to find the area, we have $\int_{30}^{50} \frac{1}{100} dy = \frac{y}{100}\Big|_{30}^{50} = \frac{1}{100}(50 - 30) = 0.2$

(c) A general expression for the area under $f(y)$ over the interval $[a, b]$ is $\text{Area} = \frac{b-a}{100}$

(d) The required integral is $\int_{0}^{100} \frac{y}{100} dy = \frac{y^2}{200}\Big|_{0}^{100} = \frac{1}{200} \times 100^2 = 50$

EXERCISE A.11

The answers to parts (a) and (b) are given in the following table

y_1	$100\dfrac{y_1 - y_0}{y_0}$	$100\left[\ln(y_1) - \ln(y_0)\right]$	Percent error
1.01	1	0.99503	0.497
1.05	5	4.87902	2.420
1.10	10	9.53102	4.690
1.15	15	13.9762	6.825
1.20	20	18.2322	8.839
1.25	25	22.3144	10.743

(c) The approximation works well when the difference $y_1 - y_0$ is small but deteriorates, both in absolute and relative terms, as the difference $y_1 - y_0$ increases.

EXERCISE A.13

(a) At $x = 1.5$, $f\left(1.5\right) = 3\times 1.5^2 - 5\times 1.5 + 1 = 0.25$, and at $x = 2$, $f\left(2\right) = 3\times 2^2 - 5\times 2 + 1 = 3$.

Using a Taylor's series expansion,

$$f(1.5) \cong f(1) + (6x-5)\big|_{x=1}(1.5-1) = -1 + 1\times 0.5 = -0.5$$

$$f(2) \cong f(1) + (6x-5)\big|_{x=1}(2-1) = -1 + 1\times 1 = 0$$

In these two cases the approximation error is very large. The percentage errors are:

For $x = 1.5$, the percentage error is $100\dfrac{-0.5-0.25}{0.25} = -300\%$

For $x = 2$, the percentage error is $100\dfrac{0-3}{3} = -100\%$

(b) At $x = 1.5$, $f\left(1.5\right) = \ln(2\times 1.5) = 1.098612$, and at $x = 2$, $f\left(2\right) = \ln(2\times 2) = 1.386294$.

Using a Taylor's series expansion,

$$f(1.5) \cong f(1) + x^{-1}\big|_{x=1}(1.5-1) = 0.693147 + 1\times 0.5 = 1.193147$$

$$f(2) \cong f(1) + x^{-1}\big|_{x=1}(2-1) = 0.693147 + 1\times 1 = 1.693147$$

In these two cases the percentage errors are:

For $x = 1.5$, the percentage error is $100\dfrac{1.193147 - 1.098612}{1.098612} = 8.605\%$

For $x = 2$, the percentage error is $100\dfrac{1.693147 - 1.386294}{1.386294} = 22.13\%$

(c) At $x = 1.5$, $f(1.5) = e^{2\times 1.5} = 20.085537$, and at $x = 2$, $f(2) = e^{2\times 2} = 54.598150$.

Using a Taylor's series expansion,

$$f(1.5) \cong f(1) + 2e^{2x}\Big|_{x=1}(1.5-1) = 7.389056 + 14.778112 \times 0.5 = 14.778112$$

$$f(2) \cong f(1) + 2e^{2x}\Big|_{x=1}(2-1) = 7.389056 + 14.778112 \times 1 = 22.167168$$

In these two cases the percentage errors are:

For $x = 1.5$, the percentage error is $100\dfrac{14.778112 - 20.085537}{20.085537} = -26.42\%$

For $x = 2$, the percentage error is $100\dfrac{22.167168 - 54.598150}{54.598150} = -59.40\%$

EXERCISE A.15

(a) When y changes value from $y_0 = 4$ to $y_1 = 4.6$, its relative change is 0.15.

(b) When y changes value from $y_0 = 4$ to $y_1 = 4.6$, its percentage change is 15%

(c) If $y = 4$ and it increases by 18%, its new value is $y_1 = 4 \times 1.18 = 4.72$.

EXERCISE A.17

(a) GDP_B = \$54,608,962,634.99 = $\$5.460896263499 \times 10^{10}$

(b) GDP_P = \$474,783,393,022.95 = $\$4.7478339302295 \times 10^{11}$

$$\frac{GDP_P}{GDP_B} = \frac{4.7478339302295 \times 10^{11}}{5.460896263499 \times 10^{10}} = 0.869424 \times 10 = 8.69424$$

(c) $\ln(GDP_P) = \ln(474{,}783{,}393{,}022.95) = 26.88612452228621$

(d) $\ln(GDP_B) = \ln(54{,}608{,}962{,}634.99) = 24.72346385704999$

$\ln(GDP_P) - \ln(GDP_B) = 26.88612452228621 - 24.72346385704999 = 2.162660665236217$

$\exp\left[\ln(GDP_P) - \ln(GDP_B)\right] = \exp\left[2.162660665236217\right] = 8.69424 = 0.869424 \times 10$

EXERCISE A.19

The value of *EXPER* that maximizes *WAGE* is found by setting $\partial WAGE/\partial EXPER$ equal to zero.

$$\frac{\partial WAGE}{\partial EXPER} = 0.55 - 0.014EXPER = 0 \quad \Rightarrow \quad EXPER = 39.3$$

The maximizing value does not depend on years of education. The value $EXPER = 39.3$ gives a maximum not a minimum because $\partial^2 WAGE/\partial EXPER^2 = -0.014 < 0$.