Solutions to Kinematics Problems

Amrita Sahu

August 2020

1 Solutions

Remember, a lot of these answers can be found in different ways, so don't worry if the method you chose isn't the same as the one outlined below. You should also consider watching the video walk-through for some of these problems (can be found on the Physics Team YouTube Channel). Specifically, problems 11, 12, and 16 can be a little tricky to solve at first.

These solutions use $g = 10$ m/s^2.

1. The correct answer is B - an easy way to solve this is to create a chart with the time and the distance traveled in one second (use the kinematic equations!)

2. The correct answer is $t = 2.4$ s and $h = 28.8$ m. First use $v_f = v_i + gt$ and substitute to find t, then plug that in to $h = \frac{t}{2}(v_f + v_i)$ to find h.

3. The correct answer is $2\sqrt{ad}$. This can be solved by setting up a system of equations where

$$x_A = d + \frac{1}{2}at^2 \text{ and } x_B = v_0 t - \frac{1}{2}at^2$$

The trains collide when $x_A = x_B$. This occurs when there is a real solution for t such that

$$t = \frac{v_0 + \sqrt{v_0^2 - 4ad}}{2a}$$

(Only the positive root matters). For those of you who are wondering about the edge case where $v_0 = 2\sqrt{ad}$, it implies the trains just touch (it becomes a matter of semantics whether you refer to that as a collision or not).

4. The correct answer is 226 m/s^2. Here we see that the stone falls 11 m away, so we can solve for the tangential velocity by first finding the time the stone is in the air (0.62 s) and then dividing the distance by the time. We should get $v_t = 17.8$ m/s. We then use that value in the equation for centripetal acceleration $a_c = \frac{v^2}{r} = 226$ m/s^2.

5. The correct answer is either piece of bread. To solve this problem, we can transform in to the reference frame of the water moving at 2.0 m/s. In this new frame, Nafi is stationary, the two pieces of bread are stationary, and you are moving at 2.0 m/s upstream. You can easily tell that both pieces of bread are equal distances apart from Nafi, so he should be able to swim to either one and reach it at the same time.

6. The correct answer to this problem is 3.3 m. This is another problem where you can simply plug in to one of our kinematics equations. First solve for the time it takes one object to reach the ground (3.35 s), then subtract the time difference $\Delta t = 0.1$ s, then finally calculate the distance one object travels in that time. If you subtract that distance (52.7 m) from the height of the tower, you should get your answer.

7. The correct answer is 200 s. If you draw a diagram, with the velocity vector for the current pointing east and the velocity vector for the boat pointing in the direction Connor will travel, you will see that they form two sides of a $3-4-5$ right triangle. This implies that the effective velocity east is 3.0 m/s and the time it takes to travel 600 m is $\frac{600\text{ m}}{3.0\text{ m/s}} = 200$ s.

8. The correct answer is 130 s. First we can calculate the distance Anya travels while in free fall (125 m), then subtract that from the total distance. Next we find the amount of time it takes to fall that remaining distance ($\frac{875\text{ m}}{7.00\text{ m/s}} = 125$ s), and add it to 5 s to get 130 s total.

9. The correct answer is 1.12 m/s. This is a straightforward calculation using the kinematics equations $y = \frac{1}{2}at^2$ and $v_x = x/t$.

10. The correct answer is 3290 m/s^2. We can convert the rpm of the machine to rps, and using the circumference we can find the distance traveled by a point in 1 s. This should be $v = 31.4$ m/s, which we can then plug in to the equation $a_c = \frac{v^2}{r} = 3290$ m/s^2.

11. The correct answers are 62 degrees and 32 degrees. When plugging in the known values to our kinematic equations, we get

$$h = v_0 \sin\theta - \frac{1}{2}gt^2 \text{ and } d = v_0 \cos\theta$$

Combining these together, we get

$$h = d\tan\theta - \frac{gd^2}{2v_0^2\cos^2\theta}$$

Using the trigonometric identity that $\sec^2\theta = 1 + \tan^2\theta$, we find that

$$0 = \frac{gd^2}{2v_0^2}\tan^2\theta - d\tan\theta + h + \frac{gd^2}{2v_0^2}$$

We can then let that nasty $\frac{gd^2}{2v_0^2}$ term equal some constant k, and solve this as a quadratic for $\tan\theta$. Then, plugging k back in we should get a positive root $\theta = \tan^- 1(1.87) = 62$ degrees and a negative root $\theta = \tan^- 1(0.625) = 32$ degrees.

12. The correct answers are 10 and 5 mph. Let the cliff have some height h. Then the time for the car to accelerate down to the ground will obey

$$h = \frac{1}{2}gt^2 \text{ or solving for } t,\ t_{\text{car}} = \sqrt{\frac{2h}{g}}$$

The toy car falls to the ground in a time

$$t_{\text{model}} = \sqrt{\frac{2h}{100g}}$$

Comparing these equations, we see $t_{\text{car}} = 10t_{\text{model}}$, so the video should be slowed down by a factor of 10. The model car is $\frac{1}{100}$ times as large as a real car, so if the film were shown in real time, it should travel $\frac{1}{100}$ times as fast as a real car, or 0.5 mph. However, the film is being shown slowed down by a factor of 10 so the car will have to travel 10 times as fast as that, or 5 mph to appear correct in the film.

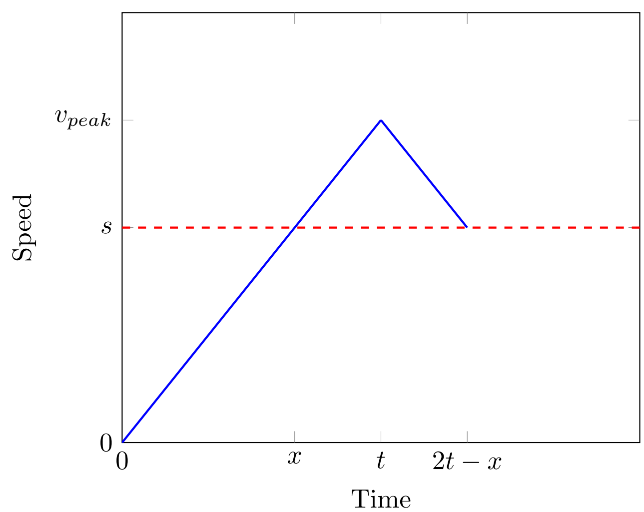
13. The correct answer is $\frac{L\sqrt{3}}{2}$. We know that max range will occur when the projectile is aimed at 45 degrees, or $\frac{\pi}{4}$ above the horizontal. Plugging this in to the kinematics equations, we find that $L = \frac{v_0^2}{g}$. Then, substituting the $\frac{\pi}{6}$ in the range equation gives us

$$d = \frac{v_0^2 \sin\frac{\pi}{3}}{g} = L\sin\frac{\pi}{3} = \frac{L\sqrt{3}}{2}$$

14. The correct answers are $t = \frac{2\sqrt{2gh}}{g}$ and $L = 4\sqrt{2}h$. We first calculate the speed of the ball after bouncing off P1 by using $v_f^2 = v_i^2 + 2ad = 2gh$, which gives us $v_f = \sqrt{2gh}$. We then let d be the horizontal and vertical displacement of the ball to reach P2 (remember, the plane is at 45 degrees). We know $d = v_f t = \sqrt{2gh}t$ and $d = \frac{1}{2}gt^2$. Setting those equal to each other, we calculate $t = \frac{2\sqrt{2gh}}{g}$. We can then find $L = d\sqrt{2} = 2\sqrt{gh}t = 4\sqrt{2}h$.

15. The correct answer is 1 s. At any given time, $a_{||} + a_{\perp} = g$, which means that the horizontal and vertical velocities are equal and the trajectory is 45 degrees below the horizontal. Since we know $v_x = 10$ m/s, we simply need to find when $v_y = 10$ m/s. This happens at $t = v_x/g = 1$ s.

16. The correct answer is $a = \frac{s}{t}(1 + \frac{1}{\sqrt{2}})$. The best way to solve this is to draw a graph! Let's visualize what Bryan's speed (shown in red) and the robocop's speed (shown in blue) will look like with respect to time.



As you can see, the robocop and Bryan have the same speed at some time after t. Additionally, we know that their displacements must be equal. This implies that the area under the blue and the area under the red from 0 to $2t - x$ must be equal. This means that the area under the top triangle and the area of the leftmost triangle are equal. Mathematically,

$$\frac{sx}{2} = (v_{peak} - s)(t - x)$$

We also know $v_{peak} = at$ and $s = ax$, so rewriting that equation in terms of only s, t and a we get

$$\frac{s^2}{2a} = (at - s)(t - \frac{s}{a})$$

Multiplying through by a and taking the square root we get

$$\frac{s}{\sqrt{2}} = (at - s)$$

Finally, solving for a we get $a = \frac{s}{t}(1 + \frac{1}{\sqrt{2}})$.

17. The correct answer is 0 m/s. You can solve this using kinematics (try to work it out!) but it can be a little long and involved compared to using a principle we will cover in the future - Conservation of Energy :)