Cautious "pre-Chinese" estimates predicted a doubling of the pre-industrial concentration of carbon dioxide in the atmosphere until 2050, i.e. an increase to 560 ppm. They also predicted an increase to 650 ppm by 2100 if business goes on as usual.[4] The more recent estimates published in the Stern Review[5] are more pessimistic. They suggest that a doubling of the pre-industrial CO_2 concentration could already take place up to 2035, and that by 2100 a value of about 900 ppm would be reached in a business-as-usual scenario. The estimated temperature increase, as measured from the pre-industrial level, resulting from the doubling of the pre-industrial concentration level is about $2^\circ C$ or more. A partial melting of glaciers and polar caps as well as the thermal expansion of the sea water would increase the sea level by about another 20 cm. The $5^\circ C$ increase that the Stern Review fears up to 2100 would increase the sea level by about one meter. If this does not sound much, note that a $5^\circ C$ increase is about the increase in the world temperature since the last ice age and that a one meter rise in the sea level would flood more than one fifth of Bangladesh.[6] There are further dangers including more powerful and devastating tropical storms, the elimination of a substantial fraction of the world's species, and droughts causing mass migrations toward more fertile countries and regions. Stern and his co-workers argue convincingly that temperature increases beyond $5^\circ C$ would "take humans into unknown territory".

Economists have challenged the Stern result that an increase by $5^\circ C$ could cost mankind up to 7 trillion dollars in present value terms,[7] but whatever the true value is, the developments are alarming by all means. It is understandable that the Stern Review calls the carbon dioxide problem the "greatest and widest-ranging market failure ever seen".

2. Carbon, carbon dioxide and public policy

Even before the Stern report fuelled a new public debate about the problem of global warming, most governments signing the Kyoto Protocol had taken action, subsidizing a wide variety of alternative technologies, including wind energy, water power stations, bio fuels, wood pellets, solar heating, photovoltaic panels and the like. High taxes on fuels have also given incentives to install better insulation of homes, mitigate the expansion of traffic and to build lighter cars empowered with hybrid engines or common-rail diesel engines. There is even a new interest in previously discarded nuclear technologies. The new EU system of

[4] See, e.g., Leggett, Pepper and Swart (1992).
[5] See Stern et al. (2006)
[6] See Muhtab (1998) and Houghton (2004, pp. 10 and 150-152 as well as figure 4.4.
[7] Nordhaus (2006), Tol and Yohe (2006), Byatt et al. (2006) and Carter et al. (2006).

CO_2 emissions trading has, moreover, induced business, in particular electricity producers and the chemical industry, to economize on their combustion processes.

All of this sounds encouraging in the efforts to overcome the world's greatest market failure and solve its largest public goods problem. The idea is that if one country or a group of countries cut their CO_2 emissions, aggregate emissions will be reduced by the same amount, and even if others do not follow, global warming will be mitigated at least somewhat. As described by the theory of privately provided public goods, the incentive to curtail emissions may not be enough from an efficiency perspective, but the situation is not hopeless.

Unfortunately, this view does not carry very far because it neglects the supply-side effects that result from the international and intertemporal linkages between the CO_2 emitters via the underlying energy markets. All the technological devices cited above are means to reduce the demand for fossil fuels. But what about the supply of energy? The public debate is silent about the supply side of the problem, and even the voluminous Stern Review mentions the energy markets only in passing (pp. 185, 318).

How the CO_2 concentration in the atmosphere changes depends on extraction, and extraction is the result of both demand and supply. Extracting the carbon from underground and accumulating it in the air as carbon dioxide is one economic act that cannot simply be separated. Ultimately, all the demand reducing measures will mitigate the problem of global warming only to the extent that they induce the oil sheiks and other owners of fossil fuel resources to keep the carbon underground.

Suppose for a moment the oil sheiks cannot be convinced, i.e. suppose the suppliers of carbon stubbornly follow their intended extraction plans whatever happens to the price of carbon. In this case, the demand reductions by one country or a group of countries will be useless. They will simply reduce the world energy price and induce other countries to increase their energy demand by exactly the same amount. The amount of carbon dioxide accumulated in the air will not change, and global warming will continue unchanged.

But is the link between the extraction of carbon and the production of carbon dioxide emissions really that strong? Would it not be possible for policy-makers to induce the production of technical devices that decouple the emission of CO_2 from the burning of carbon fuel by having more efficient combustion processes? Can't we continue to produce energy from burning carbon without pumping more CO_2 into the atmosphere? The answer is basically no, with only two exceptions, sequestration and afforestation, which will be discussed in section 6. The reason lies in the laws of chemistry. Fossil fuels basically consist of molecules that are composed of carbon and hydrogen. Oxidation generates usable energy, converting the carbon into carbon dioxide and the hydrogen into water. Coal consists predominantly of carbon.[8] In crude oils, every 5 to 9 carbon atoms bind one hydrogen atom. Methane has 4 hydrogen atoms for each carbon atom. Each hydrogen atom brings an energy of about 30% of the energy contained in a carbon atom.[9] Thus, for example, a molecule of methane generates 2.2 times the energy of a molecule of carbon while generating the same amount of carbon dioxide.[10] While the ratio of energy relative to carbon dioxide is best for methane and a bit better for oil than for coal, none of the fossil fuels can avoid the production of carbon dioxide. In fact, with all fossil fuels the ratio between the carbon burned and the amount of carbon dioxide produced is the same chemical constant.

There is of course the possibility of increasing the efficiency of combustion processes by avoiding a waste of oxidizable carbon or a waste of heat generated by oxidation, but this does not contradict this statement. The laws of chemistry imply that demand reducing measures will be unable to mitigate the greenhouse effect unless they succeed in also reducing carbon supply.

It is obvious what kind of reactions the demand reducing policies described above will have if the supply path for carbon remains unchanged. Genuine demand reducing measures such as insulating homes, building lighter cars, or reducing traffic will simply mean that domestic demand is replaced by foreign demand, which is stimulated through a decline in world energy prices relative to what they otherwise would be. Alternative methods of generating usable energy from wind, water, sunlight or biomasses may also depress the price of energy in the world markets and stimulate demand elsewhere, but if, as assumed, they do not affect the extraction path, the general equilibrium reaction of world energy markets must be such that the alternative energy produced simply is consumed in addition to the energy contained in fossil fuels. There is a contribution to economic growth and mankind’s well being, but not towards a mitigation of the greenhouse effect. The same is true for measures that avoid the waste of heat or brake energy (hybrid cars). They generate more useful energy, but cannot reduce the consumption of carbon. Even the energy provided by nuclear power stations will come on top of the fossil energy rather than replacing it. And ironically, measures that improve the technical efficiency of combustion processes by avoiding the

[8] Lignite coal consists to about 70% of carbon and about 5.5% of hydrogen, anthracite consists to about 93% of carbon, 3% of hydrogen. The rest is oxygen, nitrogen and sulphur. See Dubbel (1990).

[9] The figure cited refers to net calorific value, which is gross calorific value net of unavoidable loss of energy because of the vaporization of the water generated.

[10] One of the implications of this difference is that one tonne of methane generates 1.8 times the energy of one tonne of coal while generating even less carbon dioxide (2.75 versus 3.7 tonnes).

emission of unburned fossil fuel components through chimneys or exhaust pipes, such as the use of hotter combustion processes in power plants or the common rail diesel technology, would increase the world-wide output of CO_2 and exacerbate the problem of global warming.

How much carbon will end up in the air if all fossil fuels are burned? Are the stocks in the ground so limited that we do not have to be afraid or are they so big that measures to limit resource extraction are appropriate? A little back-of-the envelope calculation clarifies the dimensions of the problem. From the Industrial Revolution until the year 2000, humans burned about 300 Gt of carbon from fossil fuels.[11] The total reserves of oil, coal and methane that under present conditions seem worth extracting have been estimated to be in the range between 766 and 983 Gt of carbon, say about 900 Gt to take a number close to the average.[12] In the past, about 55% of the produced carbon dioxide was absorbed by land biomasses and the oceans (where 98% of carbon dioxide existing in the world is stored anyway).[13] Currently, (with the Stern figure of 380 ppm carbon dioxide) there are about 809 Gt of carbon in the atmosphere.[14] If the percentage of natural absorption is kept fixed, burning the reserves means that, roughly speaking, another 400 Gt of carbon will enter the atmosphere, which would be an increase by 49%, from 380 ppm to 566 ppm. According to the information given in the introduction, this would likely increase the world temperature by more than 2 $^\circ$C above the pre-industrial level.[15]

However, resources might be a better base for the calculation than reserves. Resources include stocks underground that under current energy prices and with current technologies are not worth extracting, but that could become profitable with higher prices. Estimates of the overall stocks of resources for oil, gas and coal in terms of carbon content range from 3,967 to

[11] Cf. World Energy Council (2000, p.149), between 1860 and 1998: 294 Gt of carbon. Marland et. al. (2005) between 1750 and 2004: 315 Gt. World Resource Institute (2005), between 1850 and 2000: 277 Gt of carbon. Note that the World Resource Institute reports CO_2 emissions which have to be multiplied by 12/44 to get carbon emissions (see IPCC 1996, p 1.8).

[12] World Resource Institute (2005): 862 Gt; World Energy Council (2000, p. 149): 983 Gt; calculations on basis of BP (2007, S. 6,22,32): 766 Gt; Calculations on basis of BGR (2007, S. 6 f..): 786 Gt. The carbon reserves consist to about 20–24 % of oil, 14-11 % of natural gas (methane) and 66–65 % of coal, calculated according to the proven reserves of BP (2007) and BGR (2007). Note that, for the reasons discussed above, the carbon shares cannot be equated with the energy shares.

[13] See Houghton (2004, p. 32).

[14]The stock of CO_2 in atmosphere is calculated using 5.137x10^18 kg as mass of the atmosphere, which translates to 1 ppm of CO_2 = 2.13 Gt of carbon (Trenberth 1981). For the early 1990s the UN Environmental Program (1998) estimated about 750 Gt Carbon in the atmosphere, for the year 2000 the CDIAC (2000) estimated 369 ppm and about 787 Gt of carbon in the atmosphere.

[15] Assuming that the other greenhouse gases remain constant, this would raise the concentration of GHG in the atmosphere to about 616 ppm. For this level of greenhouse gas concentration, the Stern Review assigns a chance of between 82% and 100% that the global temperature will increase by at least 2°C. See Stern et al. (2006, p. 195).

5,579 Gt.[16] If 45% of the lower of these two quantities enters the atmosphere, the stock of oxidized carbon existing there would increase from today's 809 Gt to 2,594 Gt, i.e. by 221%. The concentration of carbon dioxide in the atmosphere would accordingly increase from 380 ppm to about 1,220 ppm, far more than any model projections thus far have dared to predict.

The report of the Club of Rome (Meadows et al. 1972) and the oil crises of 1973/74 and 1982 once nourished public fears about the limits to growth resulting from the foreseeable resource scarcity. Market enthusiasts had countered these fears on the grounds that reserves tend to increase with exploration activities and that the explorable stocks underground would be much larger than Meadows et al. assumed. Ironically, these same enthusiasts now have to admit that their optimism is giving rise the environmental pessimism that results from the above calculations. The perils of global warming could be large enough to make everyone think back wishfully to the low estimates about remaining resources given by Meadows et al.

The calculations show that with regard to the use of fossil carbon, humans face an extremely difficult choice problem that involves the simultaneous reduction of the stock underground and accumulation of the stock above ground. The carbon problem is serious enough that the limited absorption capacity of the air may constrain resource extraction more than the scarcity of the resources itself. The economics of resource extraction may have to convert into an economics of waste accumulation.

From an economic perspective there are fundamental normative and positive aspects that center around the question to what extent market failures distort the extraction paths relative to the optimum and which policy instruments could possibly remedy them. The next two sections will go into this.

3. The nature of the market failure

If seen against the background of extracting fossil carbon from the ground, the market failure generated by CO_2 emissions has little in common with the static marginal externality model used in textbooks, which is also the conceptual center of the Stern report (2006, esp. pp. 24–28). To understand the market failure, an intertemporal analysis is needed that concentrates on the wealth society bequeaths to future generations. Society's bequest includes natural capital in the ground, man-made capital above ground and the industrial waste resulting from past extractions in the air. There are two basic choice problems involved. One is the optimal mix between man-made capital, the natural resource and the stock of waste. The other is the

[16] Cf. BGR (2005, p. 6 n.): 278 Gt of carbon from oil, 845 Gt of carbon from gas and 2,844 Gt of carbon from coal; World Energy Council (2000, p. 149): 426 Gt of carbon from oil, 534 Gt of carbon from gas and 4,618 Gt of carbon from coal.

overall wealth that society transfers to future generations. A crucial question is the extent to what market forces can be expected to find an appropriate solution to this double choice problem and, if markets fail, which kind of policy measures are appropriate to improve the intertemporal allocation of resources.

3.1 *Neoclassical optimism*

Let us approach this question stepwise and consider first the idealized neo-classical world of intertemporal resource allocation with exhaustible resources, abstracting from market failures in general and the problem of global warming in particular. Consider a representative resource owner who possesses a stock of the resource in situ, *S*, with different degrees of accessibility so that extraction costs can be written as $g(S)R$, $g'(S) < 0$, where $R = -\dot{S}$ is the current flow of extraction and *g* is the extraction cost per unit. The resource owner chooses his extraction path so as to maximize the present value of his cash flow $(P - g(S))R$ where P is the price of carbon and i the market rate of interest. If the resource owner extracts a unit today and invests the profit in the capital market he will earn a return of $i\,(P - g(S))$. If instead he postpones extraction, his return will be $\dot{P}$. Thus,

(1) $$i = \frac{\dot{P}}{P - g(S)}$$ (positive)

is a necessary condition for both an optimal extraction plan of the resource owner and a market equilibrium. In the special case where *g* = 0 this equation reduces to Hotelling's condition that the percentage rate of price increase equals the rate of interest.[17]

Because of the main theorem of welfare economics, the perfect market solution described by equation (1) must have its normative counterpart. Suppose output is given by the production function

(2) $$Y = f(K, R, t)$$

where *K* is the stock of man-made capital and *t* is calendar time. Output can be used for consumption of man-made goods *C*, investment of man-made goods $\dot{K}$, and resource

[17] Note also that the rule does not say that the price net of the marginal extraction cost rises at a rate equal to the market rate of interest, which would be the case with marginal extraction costs depending on the current flow of extraction rather than the stock not yet extracted. See Sinn (1981) for further details.

extraction:

(3) $$Y = C + \dot{K} + g(S)\,R \ .$$

Then, as shown in Sinn (1981), it is impossible to increase consumption in one period without decreasing it in another if, and only if,

(4) $$f_K = \frac{\dot{f}_R}{f_R - g(S)} \qquad \text{(normative; Pareto).}$$

Equation (4) is a generalization of the efficiency condition of Solow (1974a) and Stiglitz (1974) for the extraction of depletable economic resources to the case of stock-dependent extraction costs. The Solow-Stiglitz condition refers to the special case where $g = 0$ and says that the extraction path be chosen such that the growth rate of the marginal product of the resource be equal to the marginal product of capital. With extraction costs this condition is modified such that the increase in the marginal product of the resource relative to the marginal product net of the extraction cost be equal to the marginal product of capital. As competitive markets imply that $f_K = i$ and $f_R = P$, equation (4) obviously coincides with equation (1), demonstrating the efficiency of the market equilibrium.

While equations (1) and (4) describe an optimal portfolio mix between man-made and natural capital to be bequeathed to future generations, they do not address the problem of how much wealth should and will be bequeathed. Answering this question is more problematic as it involves difficult intergenerational welfare judgments specifying the altruistic weight present generations are willing to give future generations. A common utilitarian specification uses an additively separable utility function of the type

$$\int_0^\infty N(t)\, U\big(c(t)\big)\, \mathrm{e}^{-\rho t}\, \mathrm{d}t$$

where N is the number of people in a dynasty, $c(t) = C(t)/N(t)$ is per capita consumption, U instantaneous utility and ρ is the rate of utility discount across and within generations.

If individuals have the possibility of investing their wealth at the going market rate of interest, they allocate their consumption across the generations such that they equate their rate of time preference to the market rate of interest:

(5) $i = \rho + \eta\ \hat{c}$ (positive, utilitarian).

Here the rate of time preference consists of the rate of utility discount ρ and the relative decline in marginal utility resulting from an increase in per capita consumption over time, $\eta\ \hat{c}$, where η is the absolute value of the elasticity of marginal utility.

The normative counterpart of equation (5) is

(6) $f_K = \rho + \eta\ \hat{c}$ (normative)

because a benevolent central planner who respects individual preferences would allocate consumption over time such that people's rate of time preference equals the return that a real investment is to be able to generate. Again, the market solution and the social planning solutions coincide.

3.2 *Nirvana ethics*

Many authors, notably Page (1977), Solow (1974 b), Anand and Sen (2000) as well as Stern et al. (2006, esp. annex to chapter 2) have argued that the market solution cannot be accepted on ethical grounds because discounting future utility means discriminating later generations relative to earlier ones. If anything, discounting could be justified by the probability of extinction for exogenous reasons, but the discount rate following from that argument is much smaller than the discount rates normally used, being in the order of one tenth of one percent.[18] Without discounting of utility, only technical progress that increases per capita consumption would in the long run be able to explain a positive rate of time preference from an ethical perspective, but as that rate would be much lower, equation (6) would imply a lower marginal product of capital. This would mean more capital accumulation and, because of (4), more resource conservation: The marginal product of the resource would have to rise at a lower speed, which requires a flatter extraction profile with a lower extraction volume in the present.

The argument is as old as the theory of interest. Eugen von Böhm-Bawerk (1888), who introduced the distinction between ρ and $\eta\ \hat{c}$ as the two main reasons for time preference, had already argued that people make a mistake when they underestimate future needs. Ramsey (1928, p. 543) and Pigou (1932, pp. 24–25) later repeated the argument.

[18] Stern et al. (2006, p. 47). The probability implies that mankind becomes extinct with a probability of 9.5% in one hundred years.

However, from the perspective of economic policy this argument leads nowhere, because it is not the philosophers who make collective policy decisions but the current generation of voters themselves. If the current generation discounts utility when they make their private intertemporal allocation decisions, they will elect politicians who do the same. These politicians will not find any mistakes in the intertemporal allocation pattern and will therefore not take countervailing policy actions.

Of course, one could counter from a philosophical perspective that it would nevertheless be wrong to follow the current generations' preferences, because these preferences are wrong. However, that would be a dubious position, to say the least, because it would imply that parents do not take the needs of their children and further descendants into account and that a benevolent dictator, presumably advised by philosophers, is needed to enforce the lacking altruism. As I see no indication that parents might be insufficiently altruistic towards their offspring and neither envisage future generations coming from Mars and thus lacking a proper representation among the people living today, I find the argument totally unconvincing. If economics adopted it, it would leave the firm ground of methodological individualism and get stuck in the moody waters of Nirvana ethics.

3.3 *Insecure property rights*

An argument that is not based on mistrust in people's preference is based on the fact that resource owners often face insecure property rights and might therefore overextract. It was developed by Long (1975) and extended by Konrad, Olson and Schöb (1994). Various papers by Chichilnisky (1994, 2004) also were written with a similar, yet more general message, although these papers were not focusing on the intertemporal dimension of the problem.

Think of an oil sheik. The sheik feels insecure as to how long his dynasty will possess the oil underground, because he fears the risk of revolt and subsequent expropriation by a rival. Let

$$e^{-\pi t},\ \pi = const. > 0,$$

be the probability of survival of his or his heirs' ownership until time *t*, where π is the instantaneous expropriation probability. For a resource owner who maximizes the expected present value of his cash flow from resource extraction this effectively means that he discounts with $i+\pi$ rather than *i* alone. Hence (1) changes to[19]

[19] When the resource owner extracts the resource immediately and invests the cash flow in the capital market he has a return $i\,(P - g(S))$ as before, but when he keeps the resource in the ground, the expected return now is $\dot{P} - \pi\,(P - g(S))$. Equating these two expressions gives (7).

(7) $$i + \pi = \frac{\dot{P}}{P - g(S)}$$ (positive, insecure property rights).

As the probability of being expropriated denotes a private, but not a social damage, the welfare optimum continues to be given by (4) and (6). As $i = f_K$ as before, equation (7) shows that for any given *P* the price path becomes steeper, which indicates overextraction and is a legitimization for conservative policy actions.

There is a similar implication for the extraction path if the property rights are improperly defined insofar as a multitude of firms extract from the same pool of oil or gas underground. The literature, including Khalatbari (1977), Kemp and Long (1980), McMillan and Sinn (1984) as well as Sinn (1982, 1984a), has demonstrated why the common pool problem implies overextraction and has discussed the possible policy remedies. The common pool problem was of major importance in the early years when the farmers of Texas detected they were sitting on a common pool of oil, and it therefore bears some responsibility for today's CO_2 problem. However, it seems that it has been largely solved by consolidating the oil fields or sharing arrangements between extracting firms.[20]

Unfortunately, the problem of insecure property rights has not gone away over time, and indeed it could be substantial, in particular in the case of oil and gas extraction. Think of Venezuela, the Arab countries, Iran or the former Soviet Union, where the political situation has been extremely insecure over the last decades and is likely to remain so in the future. It is estimated that in these countries there are between 70% and 80% of the world's oil and about three quarters of the world's gas reserves.[21] Thus people like Hugo Chávez, Saddam Hussein, Muammar al-Gaddafi, Mahmud Ahmadinejad, Mikhail Khodorkovsky or Roman Abramovich are or were the custodians of substantial parts of mankind's fossil fuel resources (and as it turns out now, also of the world's atmosphere). If such people feel insecure about for how long they, their descendants or members of their clans will be able to extract the resources they currently own, they better hurry up, extract the resources now and safeguard the proceeds on Swiss bank accounts.

How exactly political risk affects resource extraction is still subject to debate. On the basis

[20] The problem has regained its importance in the case of fossil water pools such as the Ogalalla aquifer beneath many Great Plain states in the US.

[21] BP (2007) reports that in 2006 Venezuela, the former Soviet Union and the Middle East (i.e. Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, Yemen and others) owned 79% of proven world oil reserves and 74% of proven gas reserves. For the same group of countries, EIA (2007) reports figures of 70% and 75%, respectively.

of a careful and extensive empirical study Bohn and Deacon (2000) showed that political risk may actually slow down extraction because it reduces the incentive to invest in exploration of new fields and in extraction technology. The authors construct a political risk index that explains ordinary investment well and then show that there is a negative correlation between this index and the speed of oil extraction. Interestingly enough, however, upon decomposition of the effects, Bohn and Deacon (pp. 476–477) also find that dictators tend to conserve the oil more than democracies do, while frequent coups or constitutional changes tend to speed up extraction. One interpretation of this result is that, while democracies offer more safety for outside investors and hence attract direct investment, they at the same time tend to challenge the property rights of the countries' existing clans, who would not have carried out ordinary investment but own the countries' natural resources. Democracy for these clans is a serious ownership risk, which gives them every reason to speed up extraction in a similar way as increasing political turmoil does. If this interpretation is correct, the result of Bohn and Deacon fully supports the view that increased ownership risk leads to overextraction.

3.4 *Global warming*

Let us now turn to global warming, the theme of this paper. What is the exact way in which this type of externality enters the positive and normative equations describing intertemporal allocation of resources? The answer to the first part of this question is obvious, as, by its very nature, the externality does not affect the conditions that characterize market behavior. Equations (2) or (6) respectively remain valid. The emissions of carbon dioxide are an externality par excellence as they distribute evenly around the globe, damaging air quality, the world's most precious public good.

The real question is how the normative conditions are affected. Assume, in line with what was discussed above, that the temperature on Earth is a monotonically increasing function of the stock of carbon dioxide in the air, that the stock of carbon dioxide in the air is a monotonically increasing function of stock emitted, and that the stock emitted is proportional to the stock of carbon extracted. To the extent that the temperature deviates from the pre-industrial level, it creates damages in terms of costs of dislocation, dyke building, air conditioning, reconstruction of buildings, agricultural damages and the like. As the damage or the necessary repair activities can be described as a loss of output, a reduced form of the aggregate production function with the damage from global warming is

(8) $$Y = f(K, R, S, t),$$

where the resource in situ, *S*, stands in for the environmental quality in the sense of carbon being absent from the air. With $f_S > 0,\ f_{SS} < 0,$ the normal properties of a production function can be assumed, which then also imply positive and increasing marginal damage from cumulative resource extraction. As shown in Sinn (2007) it follows from (8) and (3) that it is impossible to make one generation better off without making another one worse off, if and only if,

(9) $$f_K = \frac{\dot{f}_R + f_S}{f_R - g(S)}$$ (normative, with greenhouse effect).

Thus, (9) is a condition for intertemporal Pareto efficiency in the extraction of fossil fuels with stock-dependant damages from global warming, the analogue of equation (4) above.

Equation (9) shows that with global warming and hence $f_S > 0$, $\dot{f}_R$ must be smaller for any given time and any given values of *K*, *S* and *R*. Thus it demands a flatter extraction path with less extraction in the present, but a lower decline thereafter. The larger the damage from global warming is, the wiser it is to shift extraction to the future.

If compared with the market equation (7) two aspects are worth noting. One the one hand, because of global warming, $f_S > 0$, the relative increase in the cash flow per unit extracted resulting from postponing extraction should be less than the rate of interest:

$$i > \frac{\dot{P}}{P - g(S)}$$ (normative).

On the other, because of the risk of expropriation, $\pi > 0$, the relative increase in the cash flow per unit extracted resulting from postponing extraction is even greater than the rate of interest:

$$i < \frac{\dot{P}}{P - g(S)}$$ (positive).

The resource owners take a risk into account that they should not take into account, and they neglect a peril they should not neglect. For both reasons there is overextraction.

This result is in itself not surprising because it confirms the common belief that, because of global warming, the emissions of carbon dioxide should be reduced. Note, however, that it does not involve a value judgment that derives from considerations of inter-generation equity, fairness or sustainability, but follows merely from economic efficiency considerations.

Equation (9) describes an optimal composition in the wealth portfolio consisting of man-made capital, fossil fuels in situ and carbon waste in the atmosphere that society should bequeath to future generations whatever the size of the bequest is. Unfortunately, however, society does not obey this equation, leaving future generations too little fossil fuels relative to the capital it provides.

4. A simplified interpretation

To summarize the discussion up to this point a graphical presentation that uses a somewhat simplified version of the neo-classical production function may be useful. Assume that $F(K,R,S,t) = i\,K + \phi(R) + \psi(S)$ with $i = \text{const.}$ and otherwise the properties assumed above, i.e. $\phi' > 0,\ \phi'' < 0$ and $\psi' > 0,\ \psi'' < 0$. Let $P(R) = \phi'(R)$ denote the inverse demand function for carbon implied by this specification and assume that the price elasticity of demand, $-\varepsilon$, is a constant.

Let us demonstrate the extraction path in *R,S* space, following a method developed in Sinn (1982). The slope of the possible time paths in *R,S* space is given by

$$\text{(10)} \qquad \frac{\mathrm{d}\,R}{\mathrm{d}\,S} = \varepsilon\,\hat{P}$$

as $\mathrm{d}\,R/\mathrm{d}\,S = \dot{R}/\dot{S} = -\dot{R}/R = -(\hat{R}/\hat{P})\hat{P}$ and $\varepsilon = \hat{R}/\hat{P}$ by definition. Rearranging (7) and using (10) gives

$$\text{(11)} \qquad \frac{\mathrm{d}\,R}{\mathrm{d}\,S} = \varepsilon\,(i+\pi)\left(1 - \frac{g(S)}{P(R)}\right) \qquad \text{(positive).}$$

Equation (11) uniquely defines a slope for each point in *R,S* space and thus the set of possible paths compatible with the marginal conditions derived. Assume that $g(S)$ and $\psi'(S)$ are differentiable and bounded from above so that they cannot go to infinity as *S* goes to zero while, by the assumption of a constant ε, the price is unbounded as *R* goes to zero. As is shown in the appendix this ensures that the extraction paths will lead to the origin. Thus, in fact, (11) uniquely defines the equilibrium path itself.

Figure 1 depicts the equilibrium paths for three alternative specifications. The middle path is an example of a path that characterizes a market equilibrium where $\pi = 0$. As illustrated by