6.6 Reasoning About Special Quadrilaterals

Goal
Identify special quadrilaterals based on limited information.
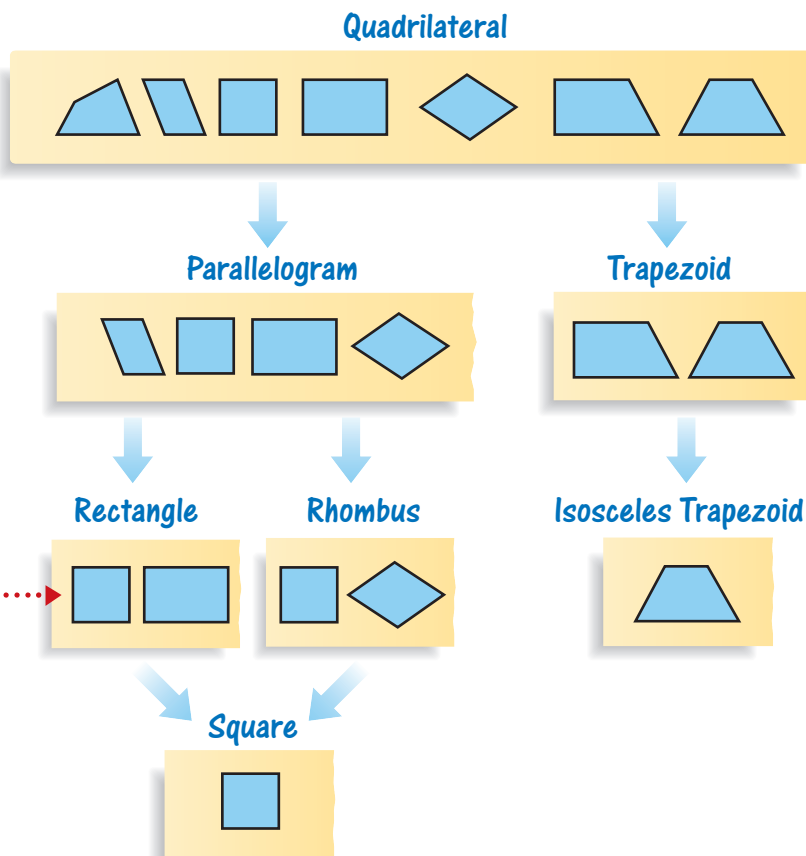
Key Words
- **parallelogram** p. 310
- **rectangle** p. 325
- **rhombus** p. 325
- **square** p. 325
- **trapezoid** p. 332
- **isosceles trapezoid** p. 332

In this chapter, you have studied six special types of quadrilaterals. The diagram below shows how these quadrilaterals are related. Each shape is a special example of the shape(s) listed above it.

Quadrilateral

Parallelogram   **Trapezoid**

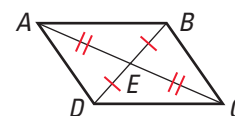**Rectangle**   **Rhombus**   **Isosceles Trapezoid**

Square

Student Help

> **STUDY TIP**
> The diagram shows that a rectangle is always a parallelogram and a quadrilateral, but it is not always a rhombus or a square.

EXAMPLE 1   **Use Properties of Quadrilaterals**

Determine whether the quadrilateral is a trapezoid, parallelogram, rectangle, rhombus, or square.

Solution
The diagram shows $\overline{CE} \cong \overline{EA}$ and $\overline{DE} \cong \overline{EB}$, so the diagonals of the quadrilateral bisect each other. By Theorem 6.9, you can conclude that the quadrilateral is a parallelogram.
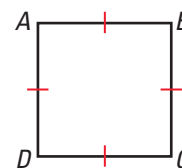
You *cannot* conclude that *ABCD* is a rectangle, rhombus, or square because no information about the sides or angles is given.
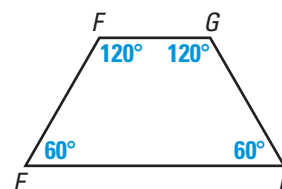
EXAMPLE 2 **Identify a Rhombus**

Are you given enough information in the diagram to conclude that *ABCD* is a square? Explain your reasoning.

Solution

The diagram shows that all four sides are congruent. Therefore, you know that it is a rhombus. The diagram does not give any information about the angle measures, so you cannot conclude that *ABCD* is square.

EXAMPLE 3 **Identify a Trapezoid**

Are you given enough information in the diagram to conclude that *EFGH* is an isosceles trapezoid? Explain your reasoning.

Solution

1 *First* show that *EFGH* is a trapezoid. $\angle E$ and $\angle F$ are supplementary, so $\overline{FG}$ is parallel to $\overline{EH}$ by Theorem 3.10, the Same-Side Interior Angles Converse. So, *EFGH* has *at least* one pair of parallel sides.
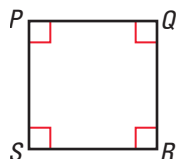
To show that *EFGH* is a trapezoid, you must show that it has *only one* pair of parallel sides. The opposite angles of *EFGH* are not congruent, so it cannot be a parallelogram. Therefore, *EFGH* is a trapezoid.

2 *Next* show that *EFGH* is isosceles. Because the base angles are congruent, *EFGH* is an isosceles trapezoid by Theorem 6.13.
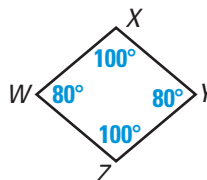
Checkpoint ✓ *Identify Quadrilaterals*

Are you given enough information to conclude that the figure is the given type of special quadrilateral? Explain your reasoning.
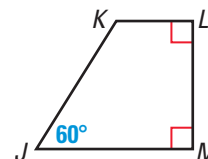
1. A square?

2. A rhombus?

3. A trapezoid?

6.6 Exercises

Guided Practice

Skill Check

Copy the chart. Put a ✓ mark in the box if the shape *always* has the given property.

	Property	▱	Rectangle	Rhombus	Square	Trapezoid
1.	Both pairs of opp. sides are ∥.	?	?	?	?	?
2.	Exactly 1 pair of opp. sides are ∥.	?	?	?	?	?
3.	Diagonals are perpendicular.	?	?	?	?	?
4.	Diagonals are congruent.	?	?	?	?	?

Practice and Applications

Extra Practice
See p. 686.

Properties of Quadrilaterals Copy the chart. Put a ✓ mark in the box if the shape *always* has the given property.

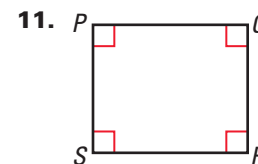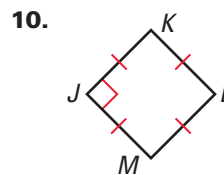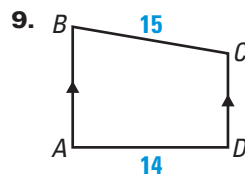|	Property	▱	Rectangle	Rhombus	Square	Trapezoid
5.	Both pairs of opp. sides are congruent.	?	?	?	?	?
6.	Diagonals bisect each other.	?	?	?	?	?
7.	Both pairs of opp. angles are congruent.	?	?	?	?	?
8.	All sides are congruent.	?	?	?	?	?

Using Properties of Quadrilaterals Determine whether the quadrilateral is a trapezoid, parallelogram, rectangle, rhombus, or square.
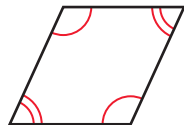
Homework Help

Example 1: Exs. 9–11, 18, 19
Example 2: Exs. 12–17
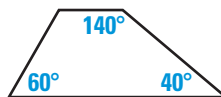**Example 3:** Exs. 12–17

9.

10.

11.

Identifying Quadrilaterals Are you given enough information to conclude that the figure is the given type of special quadrilateral? Explain your reasoning.
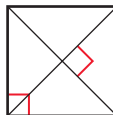
12. A rhombus?
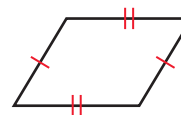
13. A trapezoid?

14. An isosceles trapezoid?

15. A rectangle?

16. A square?

17. A parallelogram?

Gem Cutting Use the diagrams and the following information.
There are different ways of cutting a gem to enhance its beauty. One of the cuts used for gems is called the *step cut*. Each face of a cut gem is called a *facet*.

18. In *ABCD*, ∠*A*, ∠*B*, ∠*C*, and ∠*D* are all right angles. What shape is *ABCD*?

19. $\overline{EF}$ is parallel to $\overline{DC}$; $\overline{ED}$ and $\overline{FC}$ are congruent, but not parallel. What shape is the facet labeled *EFCD*?
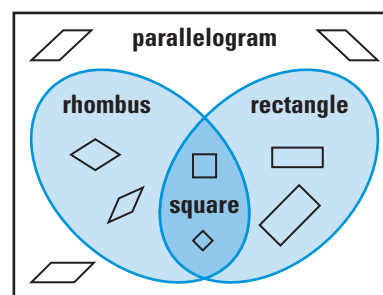
Using a Venn Diagram In Exercises 20–23, use the Venn diagram to decide whether the following statements are *true* or *false*.
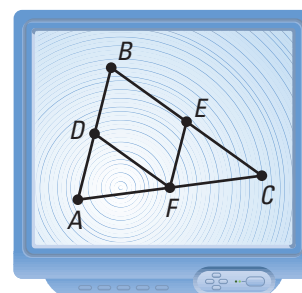
20. All rectangles are squares.

21. All squares are rectangles.

22. All squares are rhombuses.

23. All rhombuses are parallelograms.

24. Technology Use geometry software to draw a triangle. Construct the midpoint of each side and connect the midpoints as shown. What type of quadrilateral is *BEFD*? Explain.

25. Challenge What type of quadrilateral is *PQRS*, with vertices *P*(2, 5), *Q*(5, 5), *R*(6, 2), and *S*(1, 2)?

Standardized Test Practice

26. Multiple Choice Which of the following statements is *never* true?

Ⓐ A rectangle is a square.

Ⓑ A parallelogram is a trapezoid.

Ⓒ A rhombus is a parallelogram.

Ⓓ A parallelogram is a rectangle.

Mixed Review

Solving Proportions Solve the proportion. *(Skills Review, p. 660)*

27. $\dfrac{x}{3} = \dfrac{4}{12}$ **28.** $\dfrac{4}{7} = \dfrac{x}{21}$ **29.** $\dfrac{10}{x} = \dfrac{5}{8}$ **30.** $\dfrac{3}{10} = \dfrac{24}{x}$

31. $\dfrac{x}{24} = \dfrac{5}{12}$ **32.** $\dfrac{3}{5} = \dfrac{x}{20}$ **33.** $\dfrac{8}{x} = \dfrac{1}{2}$ **34.** $\dfrac{3}{7} = \dfrac{21}{x}$

Algebra Skills

Writing Decimals Write the fraction as a decimal. For repeating decimals, also round to the nearest hundredth for an approximation. *(Skills Review, p. 657)*
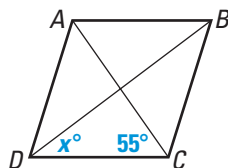
35. $\dfrac{1}{5}$ **36.** $\dfrac{3}{8}$ **37.** $\dfrac{5}{6}$ **38.** $\dfrac{7}{20}$
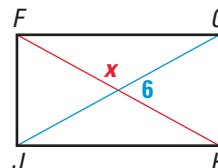
Quiz 2

Find the value of x. *(Lesson 6.4)*

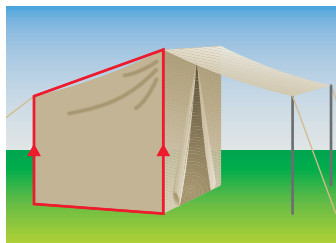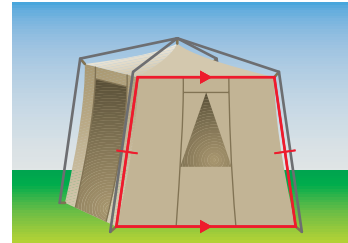**1.** rhombus *ABCD*   **2.** rectangle *FGHJ*   **3.** square *PQRS*

What kind of special quadrilateral is the red shape? *(Lesson 6.5)*

4.

5.

6. Which kinds of quadrilaterals can you form with four straws of the same length? You must attach the straws at their ends and cannot bend any of them. *(Lesson 6.6)*

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