PHYSICS

FOR SCIENTISTS AND ENGINEERS **A STRATEGIC APPROACH** 4/E

Chapter 4 Lecture

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IN THIS CHAPTER, you will learn how to solve problems about motion in a plane.

How do objects accelerate in two dimensions?

An object accelerates when it changes velocity. In two dimensions, velocity can change by changing magnitude (speed) or by changing direction. These are represented by acceleration components tangent to and perpendicular to an object's trajectory.

« LOOKING BACK Section 1.5 Finding acceleration vectors on a motion diagram
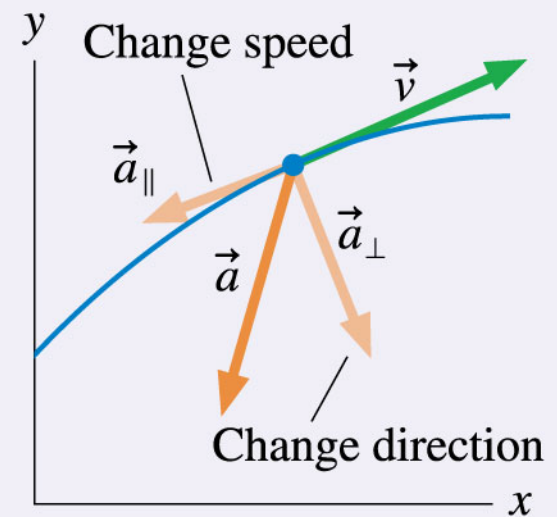
What is projectile motion?

Projectile motion is two-dimensional free-fall motion under the influence of only gravity. Projectile motion follows a **parabolic trajectory**. It has uniform motion in the horizontal direction and $a_y = -g$ in the vertical direction.
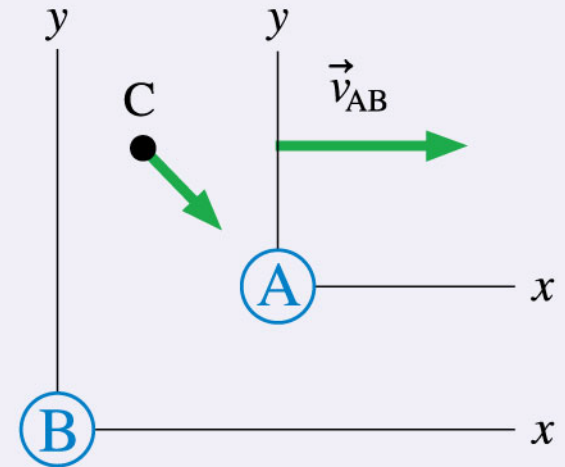
**« LOOKING BACK** Section 2.5 Free fall
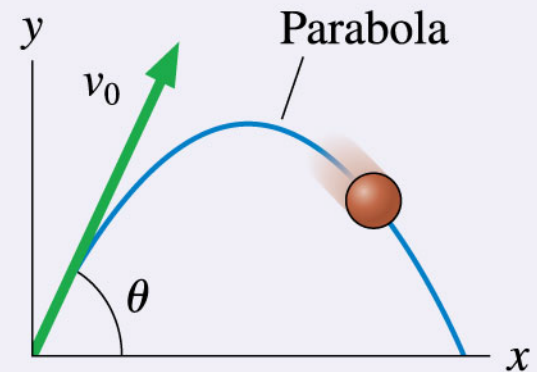
What is relative motion?

Coordinate systems that move relative to each other are called reference frames. If object C has velocity $\vec{v}_{CA}$ relative to a reference frame A, and if A moves with velocity $\vec{v}_{AB}$ relative to another reference frame B, then the velocity of C in reference frame B is $\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$.

What is circular motion?

An object moving in a circle (or rotating) has an *angular displacement* instead of a linear displacement. Circular motion is described by angular velocity ω (analogous to velocity v_s) and angular acceleration α (analogous to acceleration a_s). We'll study both uniform and accelerated circular motion.
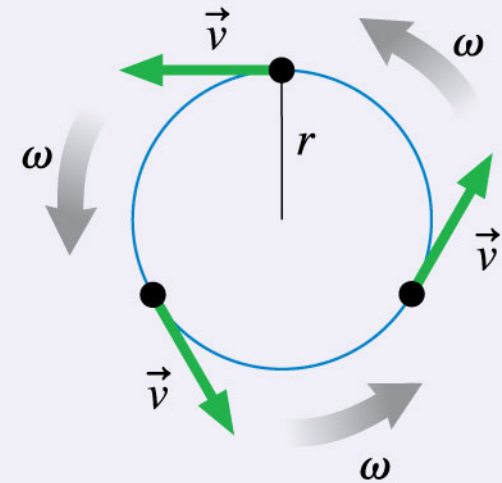
What is centripetal acceleration?

An object in circular motion is always changing direction. The acceleration of changing direction—called centripetal acceleration—points to the center of the circle. All circular motion has a centripetal acceleration. An object also has a *tangential acceleration* if it is changing speed.
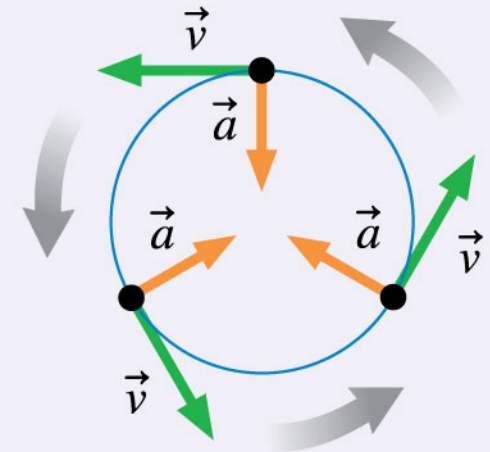
Where is two-dimensional motion used?

Linear motion allowed us to introduce the concepts of motion, but most real motion takes place in two or even three dimensions. Balls move along curved trajectories, cars turn corners, planets orbit the sun, and electrons spiral in the earth's magnetic field. Where is two-dimensional motion used? Everywhere!

Chapter 4 Reading Questions

A ball is thrown upward at a 45º angle. In the absence of air resistance, the ball follows a

A. Tangential curve.
B. Sine curve.
C. Parabolic curve.
D. Linear curve.

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B. The bullet hits the coconut.
C. The bullet passes beneath the coconut.
D. This wasn't discussed in Chapter 4.

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When discussing relative motion, the notation $\vec{v}_{AB}$ means

A. The absolute velocity.

B. The AB-component of the velocity.

C. The velocity of A relative to B.

D. The velocity of B relative to A.

E. The velocity of the object labeled AB.

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The quantity with the symbol ω is called

A.   The circular weight.
B.   The circular velocity.
C.   The angular velocity.
D.   The centripetal acceleration.
E.   The angular acceleration.

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B. The circular velocity.
C. The angular velocity.
D. The centripetal acceleration.
✔ **E. The angular acceleration.**

For uniform circular motion, the acceleration

A. Points toward the center of the circle.
B. Points away from the circle.
C. Is tangent to the circle.
D. Is zero.

For uniform circular motion, the acceleration

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D. Is zero.

Chapter 4 Content, Examples, and QuickCheck Questions

Motion in Two Dimensions

- The figure to the right shows a particle moving along a curved path—its *trajectory*—in the xy-plane.

- We can locate the particle in terms of its position vector $\vec{r}$.
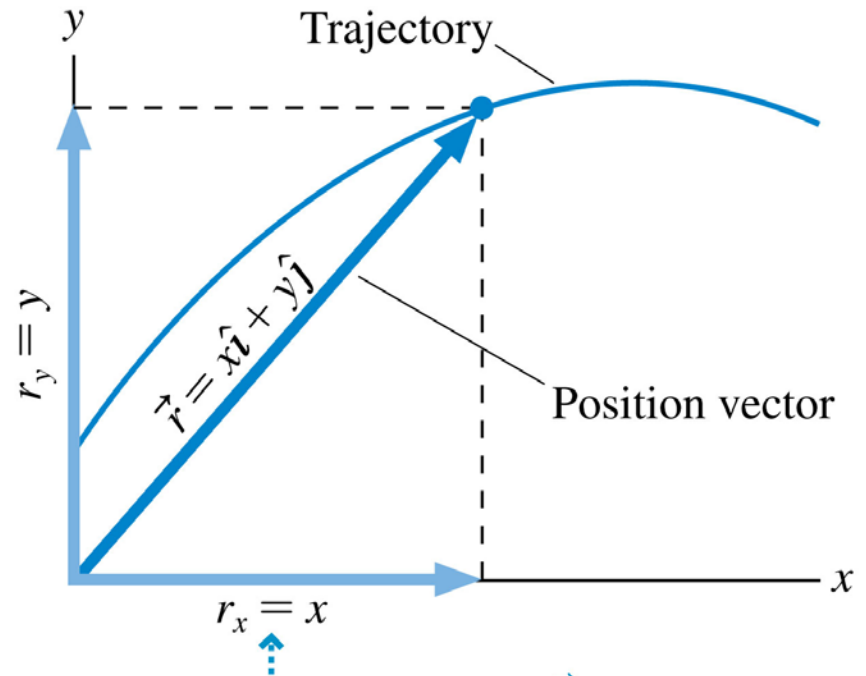
The x- and y-components of $\vec{r}$ are simply x and y.

- Like many of the graphs we'll use in this chapter, this is a graph of y versus x.

- It is an actual picture of the trajectory, not an abstract representation of the motion.

Motion in Two Dimensions

- This figure shows a particle moving from position $\vec{r}_1$ at time t_1 to position $\vec{r}_2$ at a later time t_2.

- The average velocity points in the direction $\Delta\vec{r}$ of the displacement and is

The displacement is $\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath}$

The average velocity points in the direction of $\Delta\vec{r}$.

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$
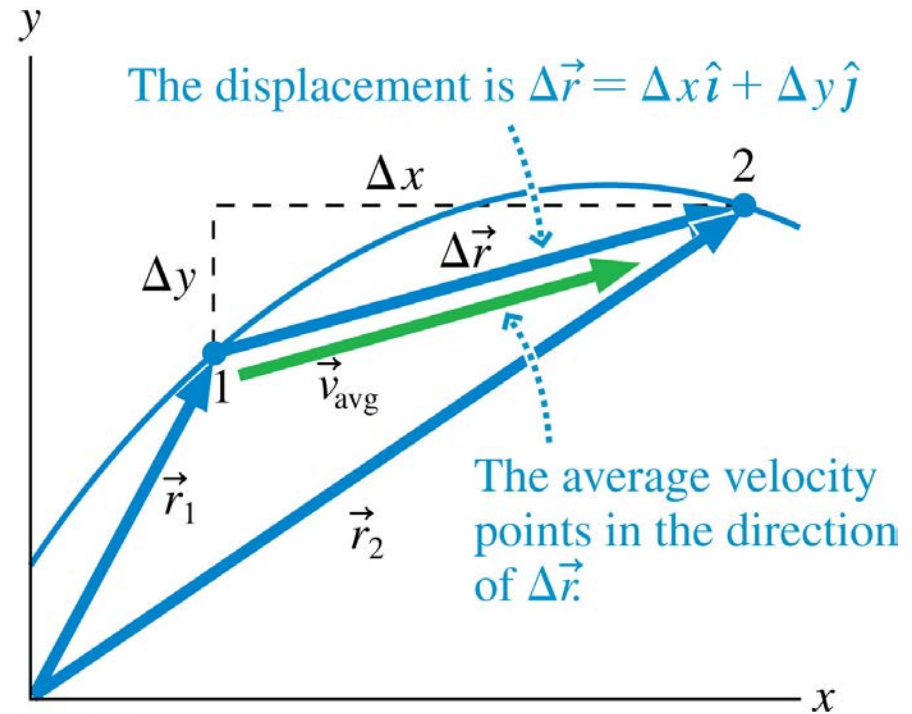
Motion in Two Dimensions

- The instantaneous velocity is the limit of $\vec{v}_{\text{avg}}$ as $\Delta t \to 0$.
- As shown, the instantaneous velocity vector is tangent to the trajectory.
- Mathematically:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

which can be written:

$$\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$$

where:

$$v_x = \frac{dx}{dt} \qquad \text{and} \qquad v_y = \frac{dy}{dt}$$

The instantaneous velocity $\vec{v}$ is tangent to the curve at 1.

As $\Delta t \to 0$, $\Delta \vec{r}$ becomes tangent to the curve at 1.
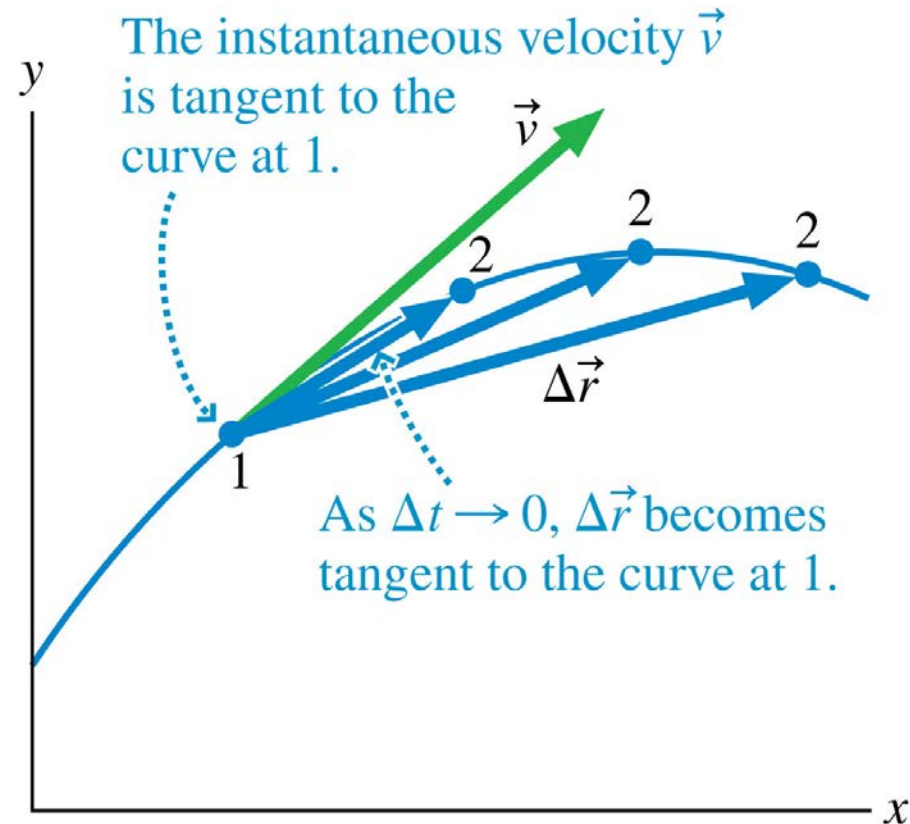
Motion in Two Dimensions

- If the velocity vector's angle θ is measured from the positive *x*-direction, the velocity components are:

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

where the particle's *speed* is

$$v = \sqrt{v_x^2 + v_y^2}$$

Angle θ describes the direction of motion.

- Conversely, if we know the velocity components, we can determine the direction of motion:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$
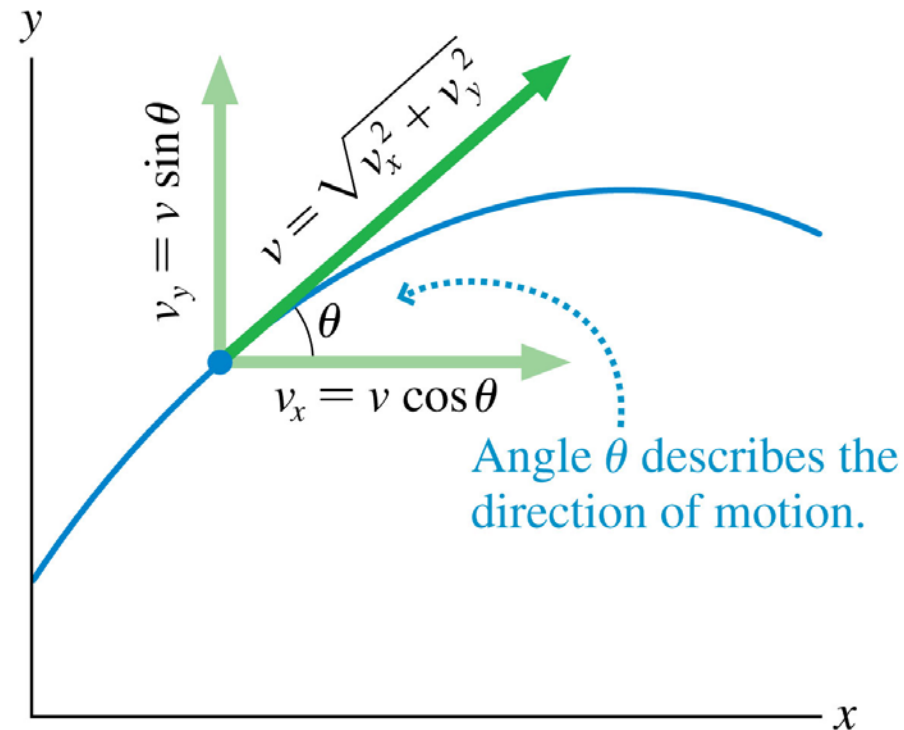
Acceleration Graphically

The *average acceleration* of a moving object is defined as the vector:

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$$

The acceleration $\vec{a}$ **points in the same direction as** $\Delta \vec{v}$, the change in velocity.

As an object moves, its velocity vector can change in two possible ways:

1. The magnitude of the velocity can change, indicating a change in speed, or

2. The direction of the velocity can change, indicating that the object has changed direction.

Tactics: Finding the Acceleration Vector
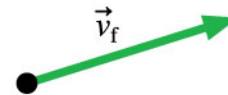
TACTICS BOX 4.1   MP

Finding the acceleration vector

To find the acceleration between velocity $\vec{v}_i$ and velocity $\vec{v}_f$:

❶ Draw the velocity vector $\vec{v}_f$.

❷ Draw $-\vec{v}_i$ at the tip of $\vec{v}_f$.

TACTICS BOX 4.1

③ Draw $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$
$= \vec{v}_f + (-\vec{v}_i)$
This is the direction of $\vec{a}$.
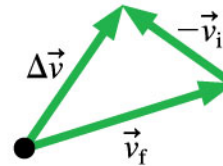
④ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration between $\vec{v}_i$ and $\vec{v}_f$.
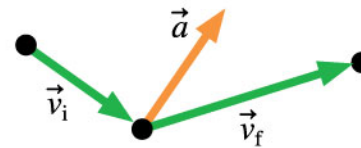
Exercises 1–4

Example 4.2 Through the Valley

EXAMPLE 4.2 | **Through the valley**

A ball rolls down a long hill, through the valley, and back up the other side. Draw a complete motion diagram of the ball.

MODEL Model the ball as a particle.

Example 4.2 Through the Valley

EXAMPLE 4.2 | Through the valley

VISUALIZE **FIGURE 4.4** is the motion diagram. Where the particle moves along a *straight line,* it speeds up if $\vec{a}$ and $\vec{v}$ point in the same direction and slows down if $\vec{a}$ and $\vec{v}$ point in opposite directions. This important idea was the basis for the one-dimensional kinematics we developed in Chapter 2. When the direction of $\vec{v}$ changes, as it does when the ball goes through the valley, we need to use vector subtraction to find the direction of $\Delta\vec{v}$ and thus of $\vec{a}$. The procedure is shown at two points in the motion diagram.

$\vec{a}$ is parallel to $\vec{v}$.
Only speed is changing.

Both speed and direction are changing.
$\vec{a}$ has components parallel and perpendicular to $\vec{v}$.

$\vec{a}$ is perpendicular to $\vec{v}$.
Only direction is changing.
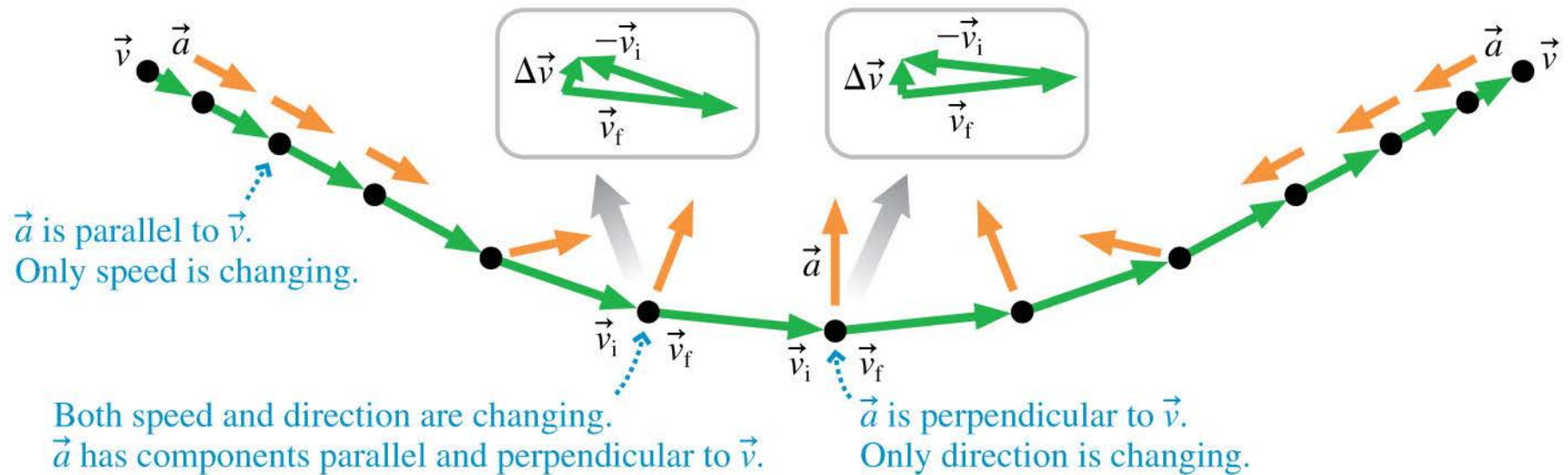
Analyzing the Acceleration Vector

- An object's acceleration can be decomposed into components parallel and perpendicular to the velocity.

- $\vec{a}_\parallel$ **is the piece of the acceleration that causes the object to change speed.**

- $\vec{a}_\perp$ **is the piece of the acceleration that causes the object to change direction.**

- An object changing direction *always* has a component of acceleration perpendicular to the direction of motion.

This component of $\vec{a}$ is changing the direction of motion.

This component of $\vec{a}$ is changing the speed of the motion.
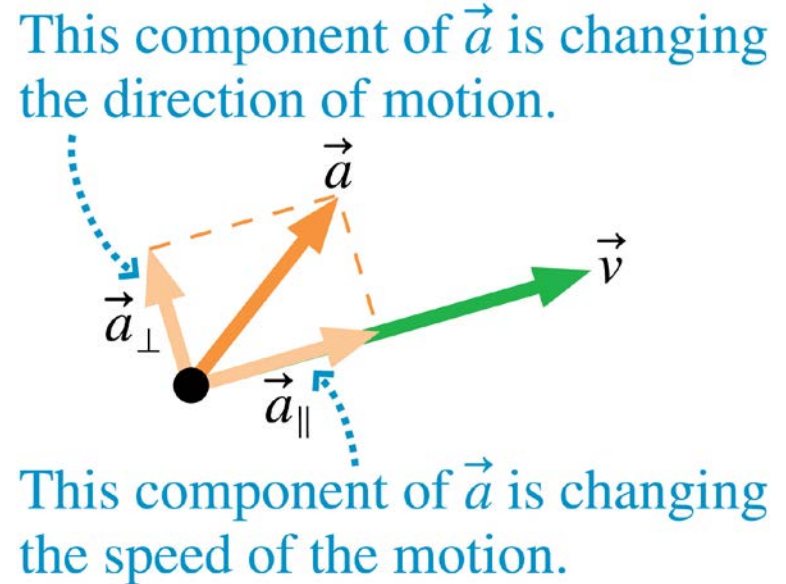
Acceleration Mathematically

- The average acceleration is found from two velocity vectors separated by the time interval Δt.

- If we let Δt get smaller and smaller, the two velocity vectors get closer and closer.

- In the limit $\Delta t \to 0$, we have the instantaneous acceleration $\vec{a}$ at the same point on the trajectory (and the same instant of time) as the instantaneous velocity $\vec{v}$.

Acceleration Mathematically

- The figure to the right shows the trajectory of a particle moving in the x-y plane.

- The acceleration $\vec{a}$ is decomposed into components $\vec{a}_\parallel$ and $\vec{a}_\perp$.

- $\vec{a}_\parallel$ is associated with a change in speed.

- $\vec{a}_\perp$ is associated with a change of direction.

- $\vec{a}_\perp$ always points toward the "inside" of the curve because that is the direction in which $\vec{v}$ is changing.

The parallel component is associated with a change of speed.

Instantaneous velocity

$\vec{v}$

y

$\vec{a}_\parallel$

$\vec{a}_\perp$

$\vec{a}$

Instantaneous acceleration

The perpendicular component is associated with a change of direction.
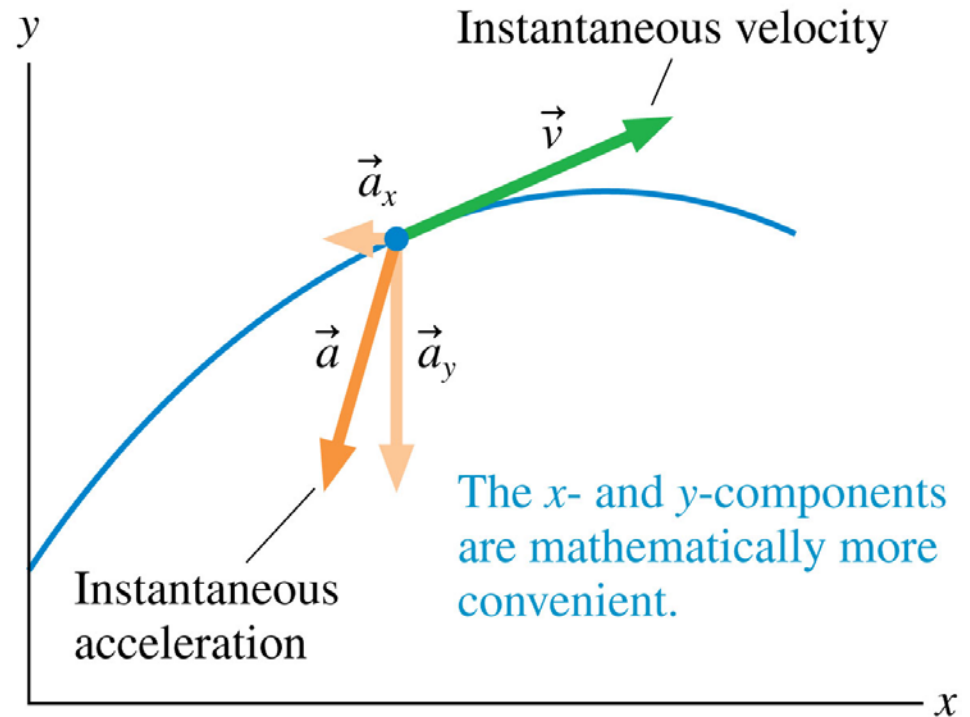
x
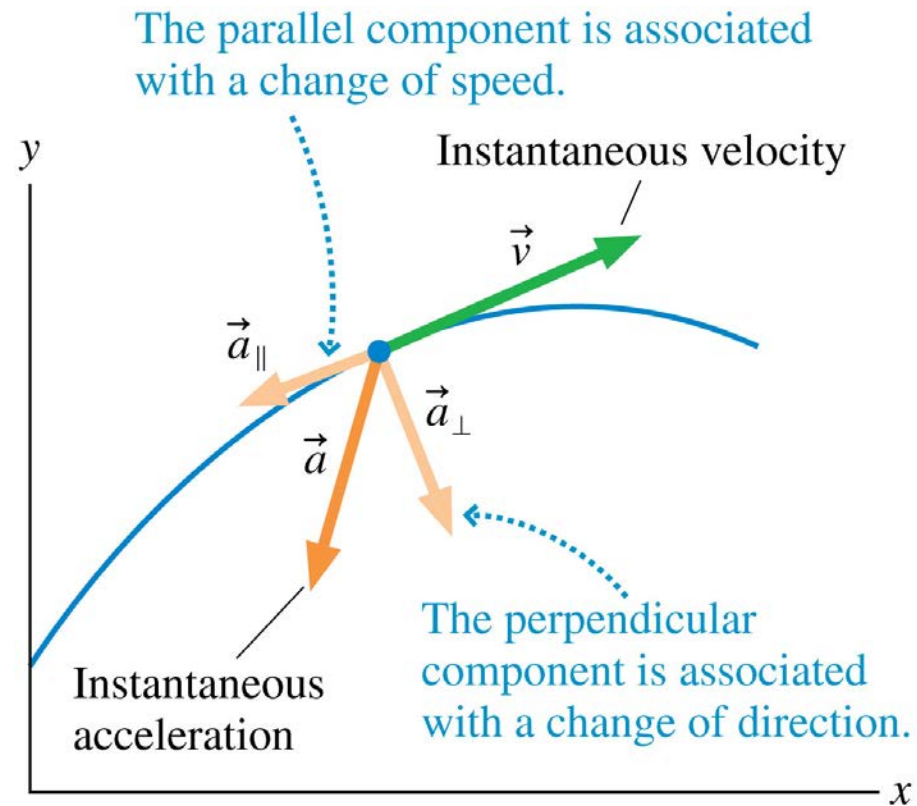
Decomposing Two-Dimensional Acceleration

- The figure to the right shows the trajectory of a particle moving in the x-y plane.

- The acceleration $\vec{a}$ is decomposed into components a_x and a_y.

Instantaneous velocity

$\vec{v}$

$\vec{a}_x$

$\vec{a}$ $\vec{a}_y$

Instantaneous acceleration

The x- and y-components are mathematically more convenient.

- If v_x and v_y are the x- and y- components of velocity, then

$$a_x = \frac{dv_x}{dt} \qquad \text{and} \qquad a_y = \frac{dv_y}{dt}$$

Constant Acceleration

- If the acceleration $\vec{a} = a_x \hat{i} + a_y \hat{j}$ is constant, then the two components a_x and a_y are both constant.

- In this case, everything from Chapter 2 about constant-acceleration kinematics applies to the *components*.

- The x-components and y-components of the motion can be treated independently.

- They remain connected through the fact that Δt must be the same for both.

$$x_f = x_i + v_{ix}\, \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \qquad y_f = y_i + v_{iy}\, \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\, \Delta t \qquad\qquad\quad v_{fy} = v_{iy} + a_y\, \Delta t$$

A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the object moves away from point 2?

A.   B.   C.   D.
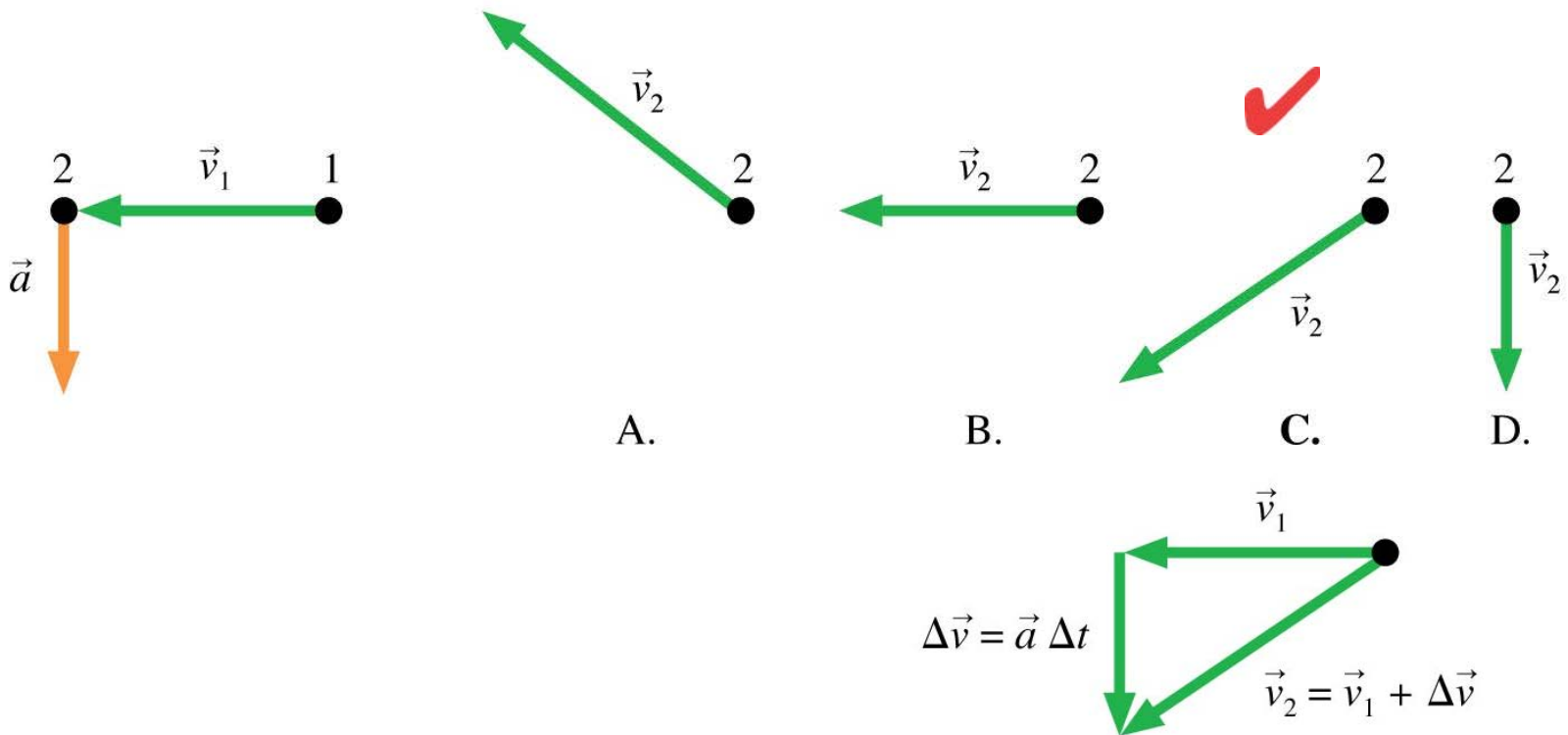
A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the object moves away from point 2?


A.

B.

C.

D.

$$\Delta\vec{v} = \vec{a}\,\Delta t$$

$$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$$
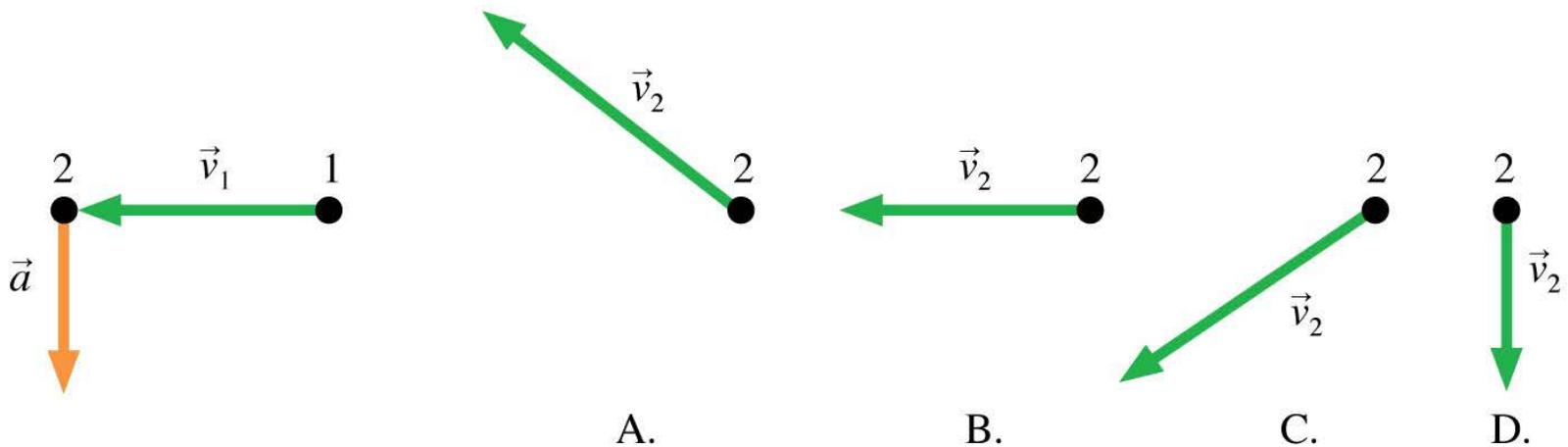
Projectile Motion

- Baseballs, tennis balls, and Olympic divers all exhibit *projectile motion*.

- A **projectile** is an object that moves in two dimensions under the influence of *only* gravity.

- Projectile motion extends the idea of free-fall motion to include a horizontal component of velocity.

- Air resistance is neglected.

- Projectiles in two dimensions follow a *parabolic trajectory* as shown in the photo.

The ball's trajectory between bounces is a parabola.
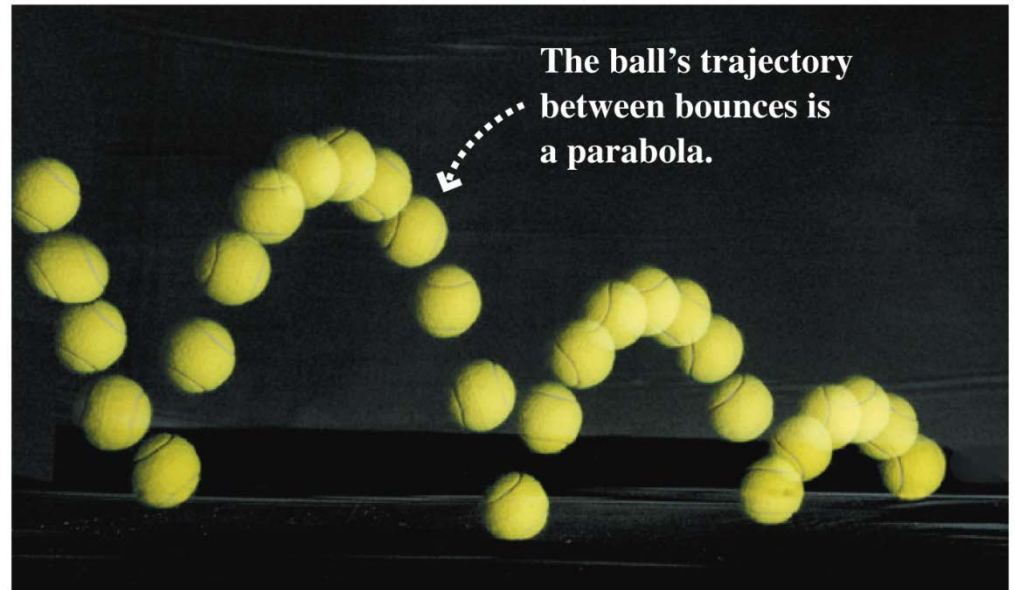
Projectile Motion

- The start of a projectile's motion is called the *launch*.
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**.
- The initial velocity vector can be broken into components.
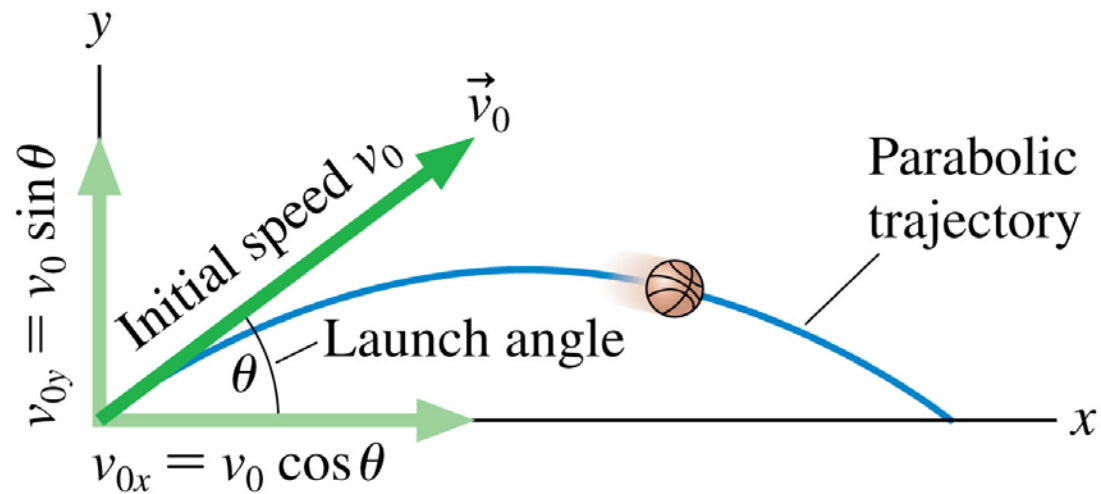
$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta$$

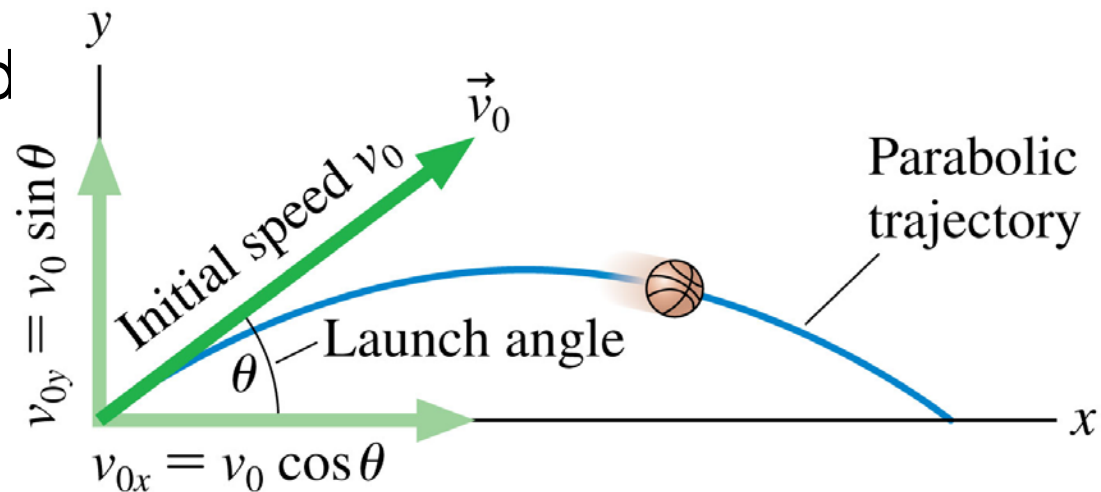where v_0 is the initial speed.

Projectile Motion

- Gravity acts downward
- Therefore, a projectile has no horizontal acceleration.
- Thus:

$$a_x = 0$$

(projectile motion)

$$a_y = -g$$

- The vertical component of acceleration a_y is ^-g of free fall.
- The horizontal component of a_x is zero.
- Projectiles are in free fall.

Projectile Motion

- The figure shows a projectile launched from the origin with initial velocity:

$$\vec{v}_0 = (9.8\hat{\imath} + 19.6\hat{\jmath}) \text{ m/s}$$

- The value of v_x never changes because there's no horizontal acceleration.

- v_y decreases by 9.8 m/s every second.

v_y decreases by 9.8 m/s every second.

v_x is constant throughout the motion.

9.8

9.8

9.8

9.8

9.8

9.8

$\vec{a}$

$a_y = -9.8$ m/s per s

19.6

9.8

-19.6

Velocity vectors are shown every 1 s. Values are in m/s.

When the particle returns to its initial height, v_y is opposite its initial value.

Example 4.4 Don't Try This at Home!

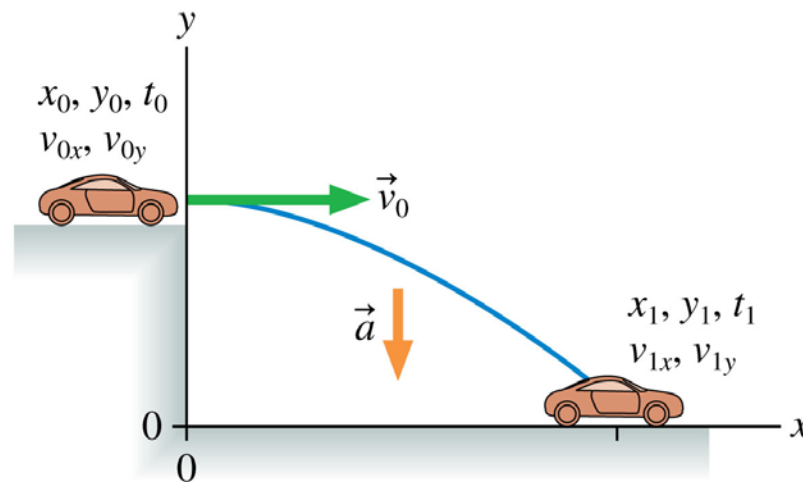**EXAMPLE 4.4** | Don't try this at home!

A stunt man drives a car off a 10.0-m-high cliff at a speed of 20.0 m/s. How far does the car land from the base of the cliff?

MODEL Model the car as a particle in free fall. Assume that the car is moving horizontally as it leaves the cliff.

Example 4.4 Don't Try This at Home!

EXAMPLE 4.4 | Don't try this at home!

VISUALIZE The pictorial representation, shown in **FIGURE 4.12**, is *very* important because the number of quantities to keep track of is quite large. We have chosen to put the origin at the base of the cliff. The assumption that the car is moving horizontally as it leaves the cliff leads to $v_{0x} = v_0$ and $v_{0y} = 0$ m/s.

Known			Find
$x_0 = 0$ m	$v_{0y} = 0$ m/s	$t_0 = 0$ s	x_1
$y_0 = 10.0$ m	$v_{0x} = v_0 = 20.0$ m/s		
$a_x = 0$ m/s^2	$a_y = -g$	$y_1 = 0$ m	

Example 4.4 Don't Try This at Home!

EXAMPLE 4.4 | Don't try this at home!

SOLVE Each point on the trajectory has x- and y-components of position, velocity, and acceleration but only *one* value of time. The time needed to move horizontally to x_1 is the *same* time needed to fall vertically through distance y_0. **Although the horizontal and vertical motions are independent, they are connected through the time t.** This is a critical observation for solving projectile motion problems. The kinematics equations with $a_x = 0$ and $a_y = -g$ are

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = v_0 t_1$$

$$y_1 = 0 = y_0 + v_{0y}(t_1 - t_0) - \tfrac{1}{2}g(t_1 - t_0)^2 = y_0 - \tfrac{1}{2}g t_1^2$$

We can use the vertical equation to determine the time t_1 needed to fall distance y_0:

$$t_1 = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.43\ \text{s}$$

We then insert this expression for t into the horizontal equation to find the distance traveled:

$$x_1 = v_0 t_1 = (20.0\ \text{m/s})(1.43\ \text{s}) = 28.6\ \text{m}$$

ASSESS The cliff height is $\approx 33\ \text{ft}$ and the initial speed is $v_0 \approx 40\ \text{mph}$. Traveling $x_1 = 29\ \text{m} \approx 95\ \text{ft}$ before hitting the ground seems reasonable.
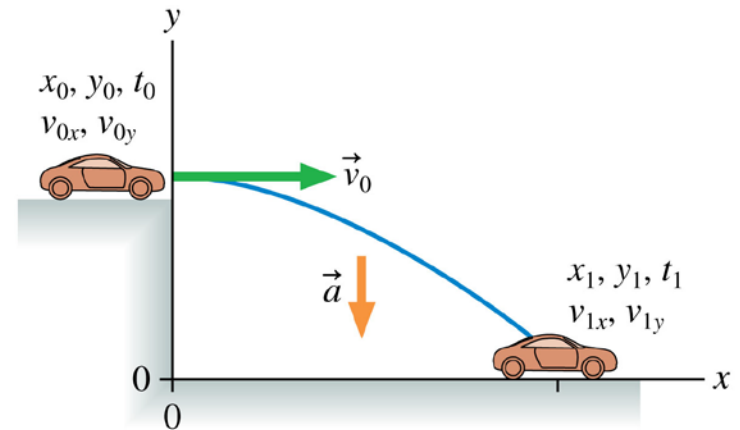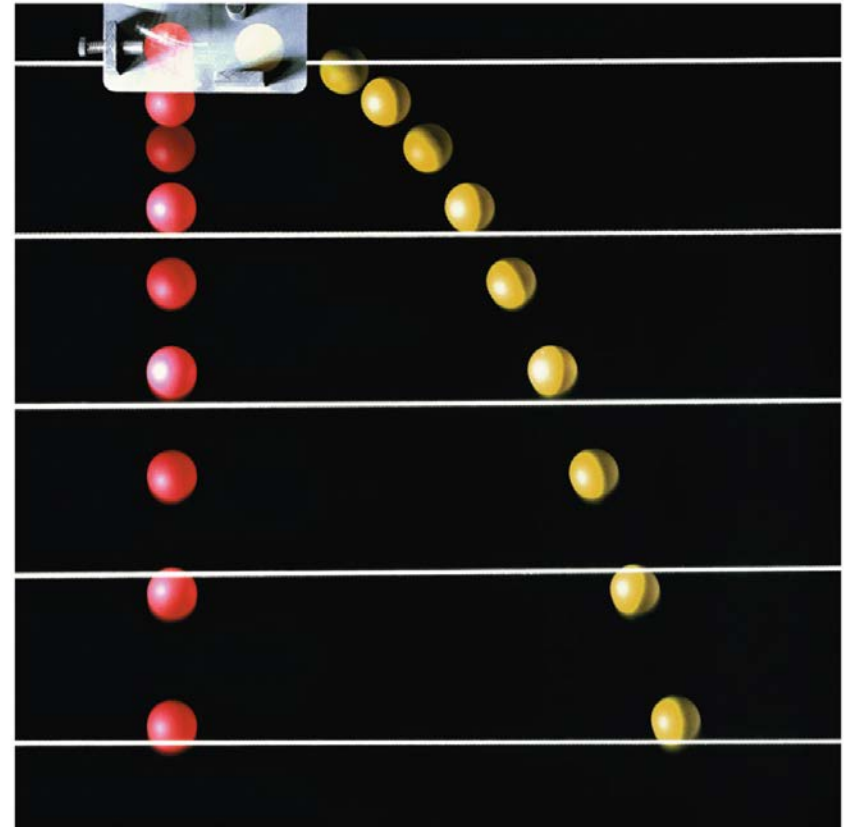

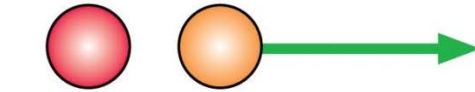| Known			Find
$x_0 = 0\ \text{m}$	$v_{0y} = 0\ \text{m/s}$	$t_0 = 0\ \text{s}$	x_1
$y_0 = 10.0\ \text{m}$	$v_{0x} = v_0 = 20.0\ \text{m/s}$		
$a_x = 0\ \text{m/s}^2$	$a_y = -g$	$y_1 = 0\ \text{m}$	

Reasoning About Projectile Motion

- Suppose a heavy ball is launched exactly horizontally at height h above a horizontal field.

- At the exact instant that the ball is launched, a second ball is simply dropped from height h.

- Which ball hits the ground first?

- If air resistance is neglected, the balls hit the ground *simultaneously.*

- They do so because the horizontal and vertical components of projectile motion are independent of each other.

A heavy red ball is released from rest 2.0 m above a flat, horizontal surface. At exactly the same instant, a yellow ball with the same mass is fired horizontally at 3.0 m/s. Which ball hits the ground first?

A. The red ball hits first.

B. The yellow ball hits first.

C. They hit at the same time.

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A. The red ball hits first.

B. The yellow ball hits first.

✔ **C. They hit at the same time.**

A 100 g ball rolls off a table and hits 2.0 m from the base of the table. A 200 g ball rolls off the same table with the same speed. It lands at distance

A. 1.0 m.

B. Between 1 m and 2 m.

C. 2.0 m.
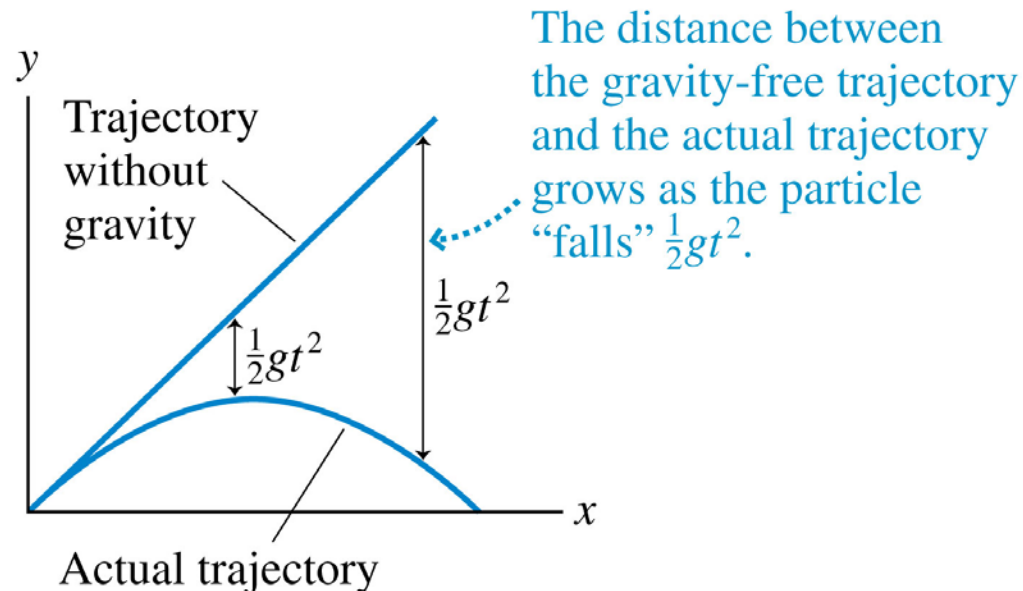
D. Between 2 m and 4 m.

E. 4.0 m.

A 100 g ball rolls off a table and hits 2.0 m from the base of the table. A 200 g ball rolls off the same table with the same speed. It lands at distance

A. 1.0 m.

B. Between 1 m and 2 m.

✔ **C. 2.0 m.**

D. Between 2 m and 4 m.

E. 4.0 m.

Reasoning About Projectile Motion

A hungry bow-and-arrow hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but as luck would have it, the coconut falls from the branch at the exact instant the hunter releases the string. Does the arrow hit the coconut?

- Without gravity, the arrow would follow a straight line.
- Because of gravity, the arrow at time t has "fallen" a distance $\frac{1}{2}gt^2$ below this line.
- The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.

The distance between the gravity-free trajectory and the actual trajectory grows as the particle "falls" $\frac{1}{2}gt^2$.

y

Trajectory without gravity

$\frac{1}{2}gt^2$

$\frac{1}{2}gt^2$

x

Actual trajectory

Reasoning About Projectile Motion

A hungry bow-and-arrow hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but as luck would have it, the coconut falls from the branch at the exact instant the hunter releases the string. Does the arrow hit the coconut?

- Had the coconut stayed on the tree, the arrow would have curved under its target as gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line.
- But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight.
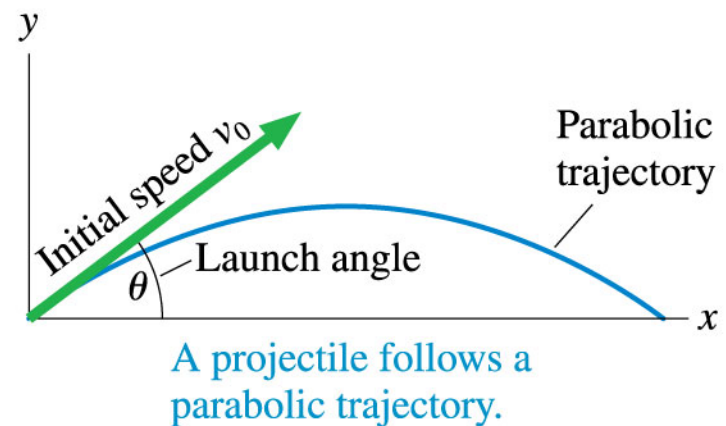- So *yes*, the arrow hits the coconut!
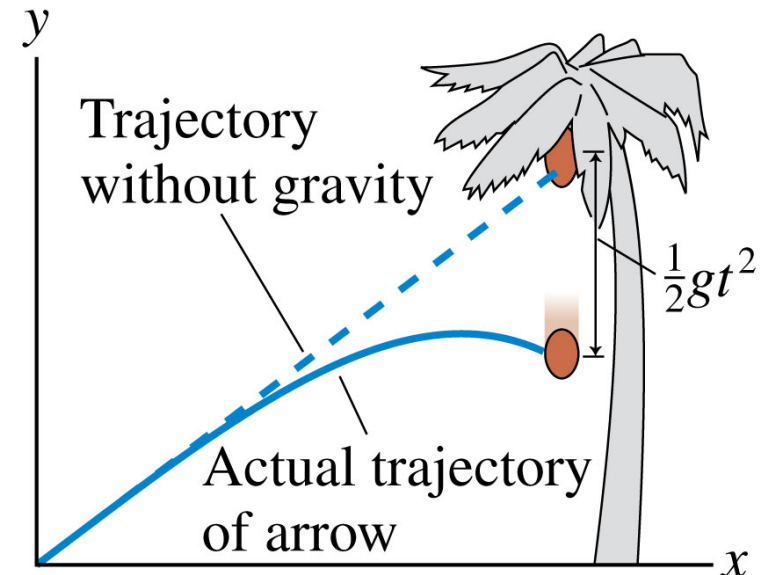
Projectile Motion

MODEL 4.1

Projectile motion

For motion under the influence of only gravity.

- Model the object as a particle launched with speed v_0 at angle θ:

- Mathematically:
  - **Uniform motion** in the horizontal direction with $v_x = v_0 \cos\theta$.
  - **Constant acceleration** in the vertical direction with $a_y = -g$.
  - Same Δt for both motions.

- Limitations: Model fails if air resistance is significant.

Initial speed v_0

Parabolic trajectory

Launch angle

θ

A projectile follows a parabolic trajectory.

Exercise 9

Projectile Motion Problems

PROBLEM-SOLVING STRATEGY 4.1 (MP)

Projectile motion problems

MODEL Is it reasonable to ignore air resistance? If so, use the projectile motion model.

VISUALIZE Establish a coordinate system with the x-axis horizontal and the y-axis vertical. Define symbols and identify what the problem is trying to find. For a launch at angle θ, the initial velocity components are $v_{ix} = v_0 \cos\theta$ and $v_{iy} = v_0 \sin\theta$.

SOLVE The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus the problem is one of two-dimensional kinematics. The kinematic equations are

Horizontal	Vertical
$x_f = x_i + v_{ix}\,\Delta t$	$y_f = y_i + v_{iy}\,\Delta t - \frac{1}{2}g(\Delta t)^2$
$v_{fx} = v_{ix} = \text{constant}$	$v_{fy} = v_{iy} - g\,\Delta t$

Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, then use that value for the other component.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.
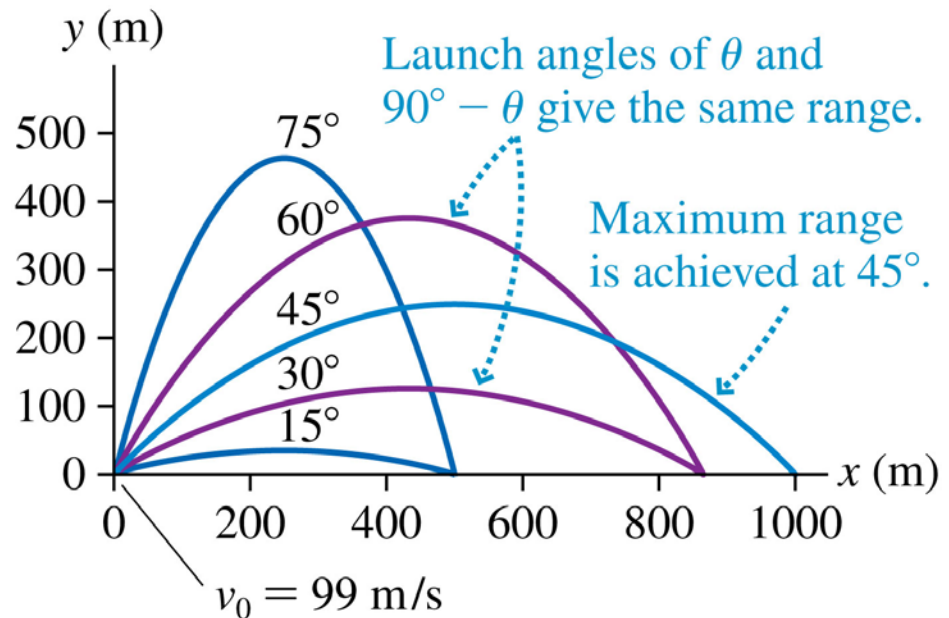
Range of a Projectile

- A projectile with initial speed v_0 has a launch angle of θ above the horizontal.
- How far does it travel over level ground before it returns to the same elevation from which it was launched?

- This distance is sometimes called the *range* of a projectile.
- Example 4.5 from your textbook shows:

$$\text{range} = \frac{v_0^2 \sin(2\theta)}{g}$$

Trajectories of a projectile launched at different angles with a speed of 99 m/s.

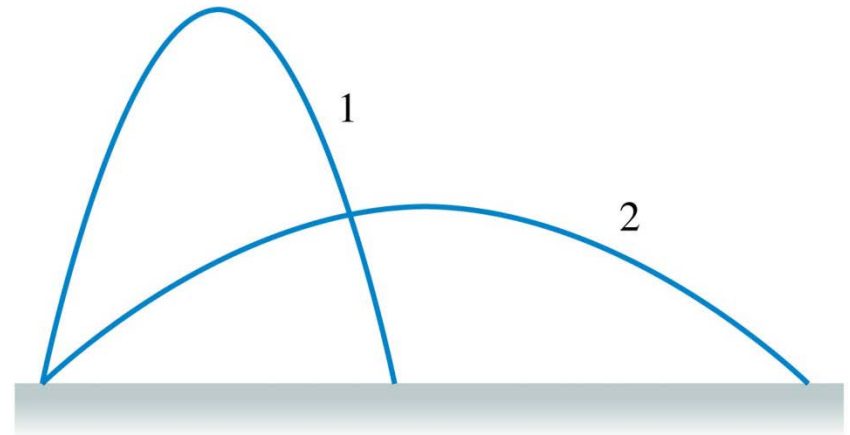- The maximum distance occurs for $\theta = 45°$.

Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.

A. Projectile 1 hits first.

B. Projectile 2 hits first.

C. They hit at the same time.

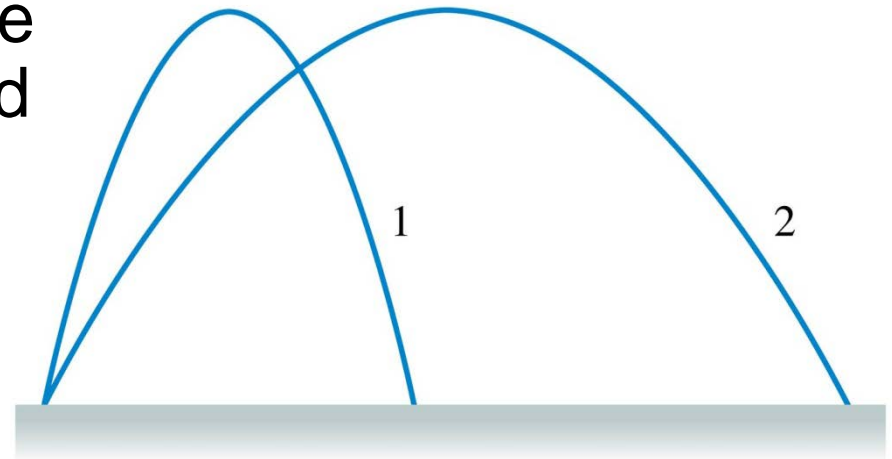D. There's not enough information to tell.

QuickCheck 4.4

Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.

A.   Projectile 1 hits first.

✔ **B.   Projectile 2 hits first.**

C.   They hit at the same time.

D.   There's not enough information to tell.

QuickCheck 4.5

Projectiles 1 and 2 are launched over level ground with different speeds. Both reach the same height. Which hits the ground first? Ignore air resistance.

A.   Projectile 1 hits first.

B.   Projectile 2 hits first.

C.   They hit at the same time.
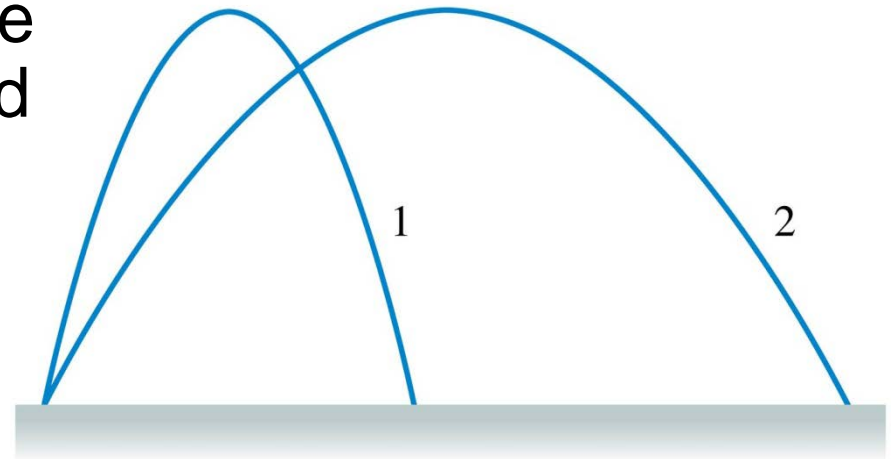
D.   There's not enough information to tell.

Projectiles 1 and 2 are launched over level ground with different speeds. Both reach the same height. Which hits the ground first? Ignore air resistance.

A.   Projectile 1 hits first.

B.   Projectile 2 hits first.

✔ **C.   They hit at the same time.**

D.   There's not enough information to tell.

Relative Motion

- The figure below shows Amy and Bill watching Carlos on his bicycle.

- According to Amy, Carlos's velocity is $(v_x)_{CA} = +5$ m/s.

- The CA subscript means "C relative to A."

- According to Bill, Carlos's velocity is $(v_x)_{CB} = -10$ m/s.

- Every velocity is measured *relative* to a certain observer.

- There is no "true" velocity.

Carlos

5 m/s

15 m/s

Bill

Amy
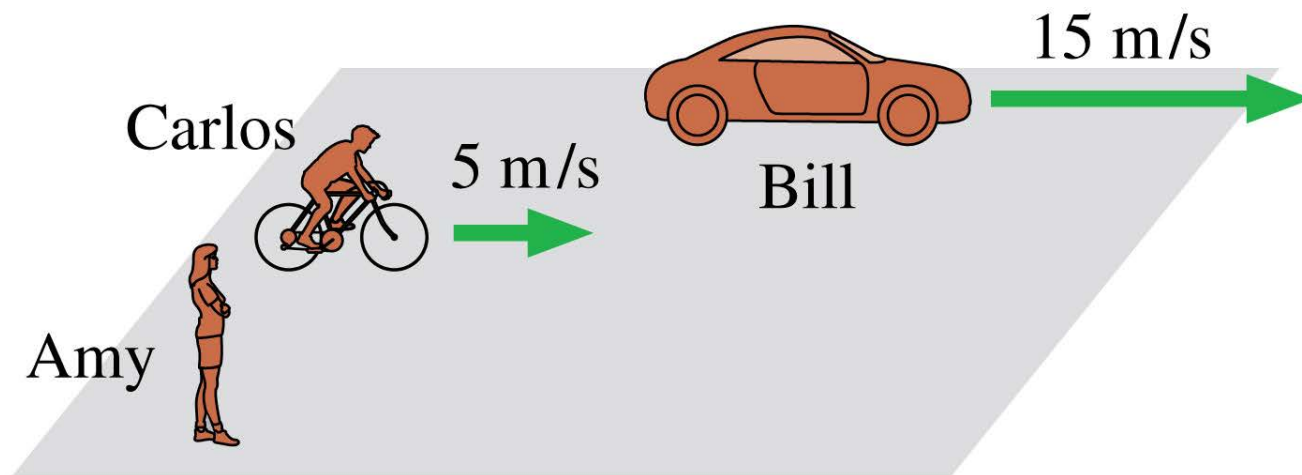
Relative Motion

- The velocity of C relative to B is the velocity of C relative to A *plus* the velocity of A relative to B.

The first subscript is the same on both sides.

The last subscript is the same on both sides.

$$(v_x)_{CB} = (v_x)_{CA} + (v_x)_{AB}$$

The inner subscripts "cancel."

- If B is moving to the right relative to A, then A is moving to the left relative to B.

- Therefore, $$(v_x)_{AB} = -(v_x)_{BA}$$

Example 4.6 A Speeding Bullet

EXAMPLE 4.6 | **A speeding bullet**

The police are chasing a bank robber. While driving at 50 m/s, they fire a bullet to shoot out a tire of his car. The police gun shoots bullets at 300 m/s. What is the bullet's speed as measured by a TV camera crew parked beside the road?

MODEL Assume that all motion is in the positive x-direction. The bullet is the object that is observed from both the police car and the ground.

SOLVE The bullet B's velocity relative to the gun G is $(v_x)_{BG} = 300$ m/s. The gun, inside the car, is traveling relative to the TV crew C at $(v_x)_{GC} = 50$ m/s. We can combine these values to find that the bullet's velocity relative to the TV crew on the ground is

$$(v_x)_{BC} = (v_x)_{BG} + (v_x)_{GC} = 300 \text{ m/s} + 50 \text{ m/s} = 350 \text{ m/s}$$

ASSESS It should be no surprise in this simple situation that we simply add the velocities.

Reference Frames

- A coordinate system in which an experimenter makes position measurements is called a **reference frame**.
- In the figure, Object C is measured in two different reference frames, A and B.
- $\vec{r}_{CA}$ is the position of C relative to the origin of A.
- $\vec{r}_{CB}$ is the position of C relative to the origin of B.
- $\vec{r}_{AB}$ is the position of the origin of A relative to the origin of B.

Object C can be located relative to A or to B.
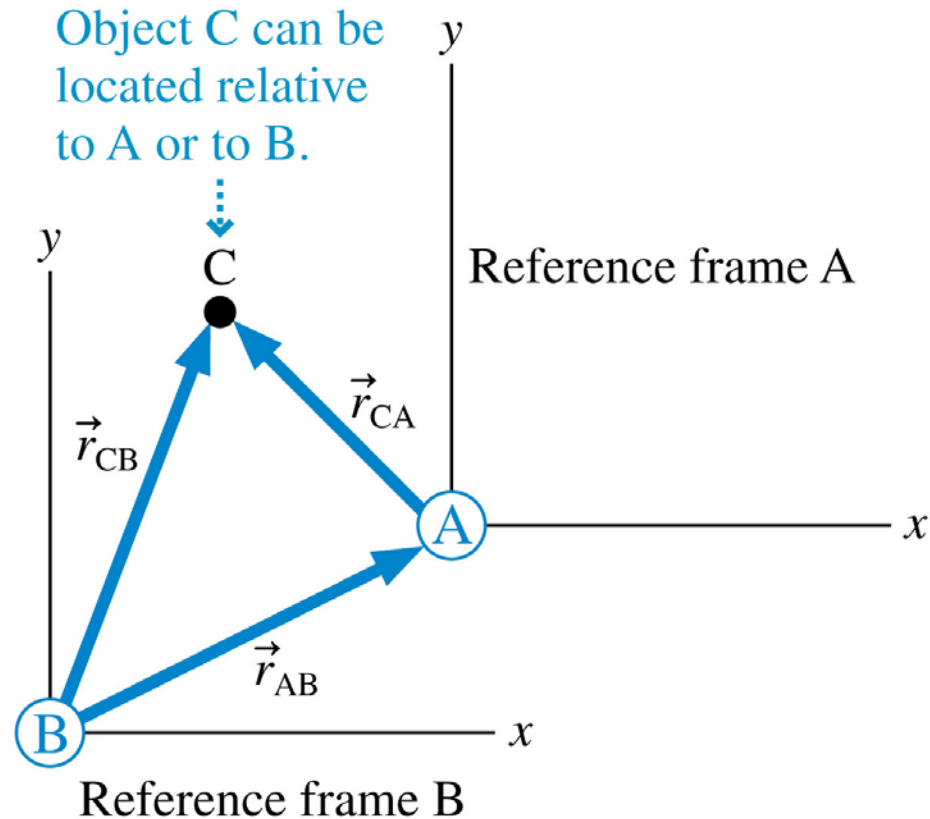
Reference frame A

Reference frame B

$$\vec{r}_{CB} = \vec{r}_{CA} + \vec{r}_{AB}$$
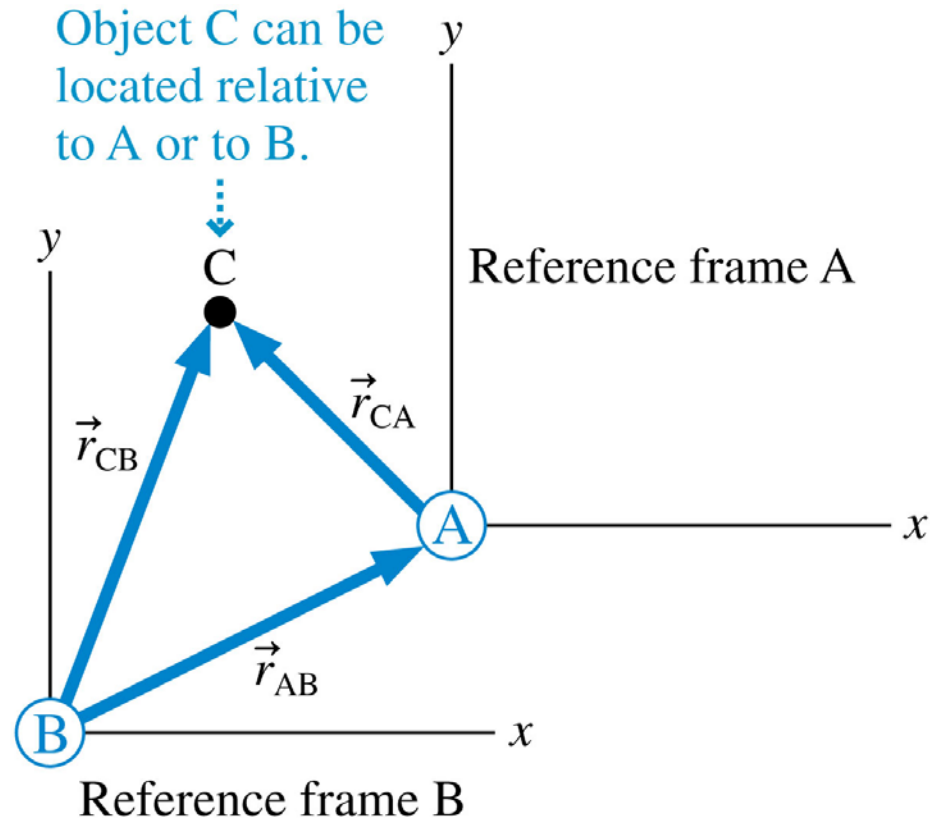
Reference Frames

- Relative velocities are found as the time derivative of the relative positions.
- $\vec{v}_{CA}$ is the velocity of C relative to A.
- $\vec{v}_{CB}$ is the velocity of C relative to B.
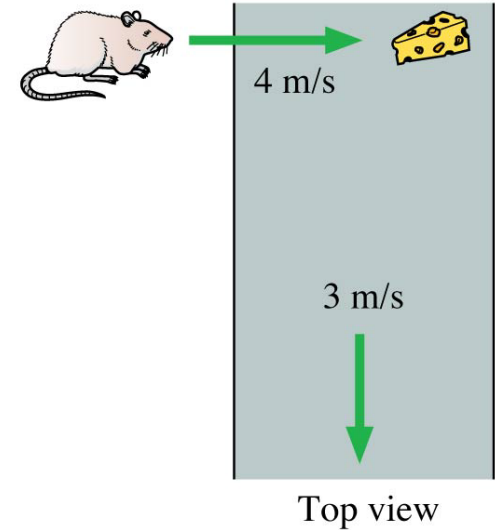- $\vec{v}_{AB}$ is the velocity of reference frame A relative to reference frame B.

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

Object C can be located relative to A or to B.

C

Reference frame A

$\vec{r}_{CB}$

$\vec{r}_{CA}$

A

$\vec{r}_{AB}$

B

Reference frame B

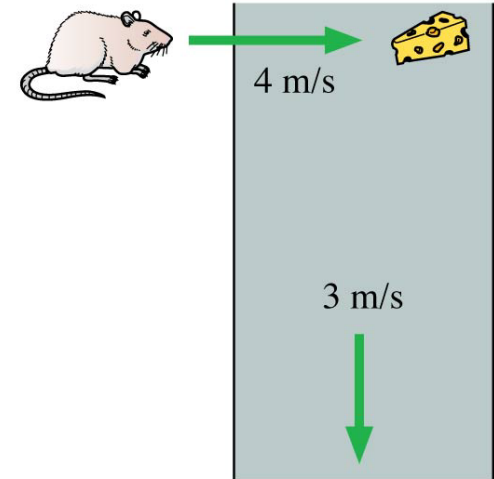- This is known as the **Galilean transformation of velocity.**

A factory conveyor belt rolls at 3 m/s. A mouse sees a piece of cheese directly across the belt and heads straight for the cheese at 4 m/s. What is the mouse's speed relative to the factory floor?

4 m/s

3 m/s

Top view

A.   1 m/s

B.   2 m/s

C.   3 m/s

D.   4 m/s

E.   5 m/s

A factory conveyor belt rolls at 3 m/s. A mouse sees a piece of cheese directly across the belt and heads straight for the cheese at 4 m/s. What is the mouse's speed relative to the factory floor?
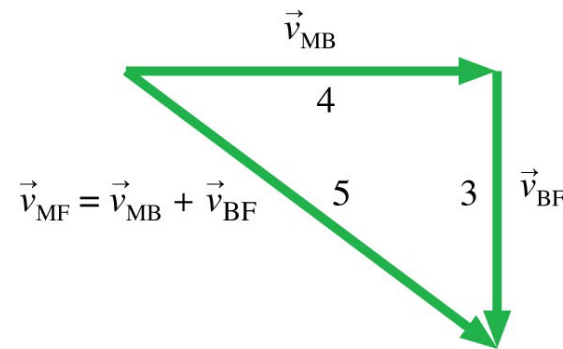
4 m/s

3 m/s

Top view

A.   1 m/s

B.   2 m/s

C.   3 m/s

D.   4 m/s

E.   **5 m/s**

$\vec{v}_{MB}$

4

$\vec{v}_{MF} = \vec{v}_{MB} + \vec{v}_{BF}$ 5   3 $\vec{v}_{BF}$

M = mouse
B = belt
F = floor

3-4-5 right triangle

EXAMPLE 4.7 Flying to Cleveland I

EXAMPLE 4.7 | Flying to Cleveland I

Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 50 mph. What is the plane's ground speed? Where is the plane 0.60 h later, when the pilot expects to land in Cleveland?

MODEL Establish a coordinate system with the *x*-axis pointing east and the *y*-axis north. The plane P flies in the air, so its velocity relative to the air A is $\vec{v}_{PA} = 500\hat{i}$ mph. Meanwhile, the air is moving relative to the ground G at $\vec{v}_{AG} = -50\hat{j}$ mph.

EXAMPLE 4.7 Flying to Cleveland I

EXAMPLE 4.7 | Flying to Cleveland I

SOLVE The velocity equation $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ is a vector-addition equation. **FIGURE 4.19** shows graphically what happens. Although the nose of the plane points east, the wind carries the plane in a direction somewhat south of east. The plane's velocity relative to the ground is

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} = (500\hat{\imath} - 50\hat{\jmath}) \text{ mph}$$
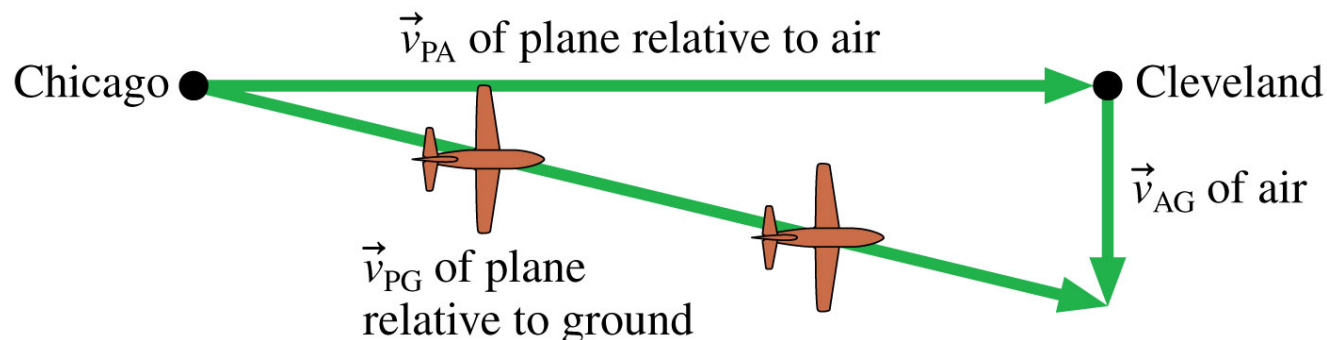
The plane's ground speed is

$$v = \sqrt{(v_x)_{PG}^2 + (v_y)_{PG}^2} = 502 \text{ mph}$$

After flying for 0.60 h at this velocity, the plane's location (relative to Chicago) is

$$x = (v_x)_{PG}t = (500 \text{ mph})(0.60 \text{ h}) = 300 \text{ mi}$$
$$y = (v_y)_{PG}t = (-50 \text{ mph})(0.60 \text{ h}) = -30 \text{ mi}$$

The plane is 30 mi due south of Cleveland! Although the pilot thought he was flying to the east, his actual heading has been $\tan^{-1}(50 \text{ mph}/500 \text{ mph}) = \tan^{-1}(0.10) = 5.71°$ south of east.

$\vec{v}_{PA}$ of plane relative to air

Chicago

Cleveland

$\vec{v}_{AG}$ of air

$\vec{v}_{PG}$ of plane relative to ground

EXAMPLE 4.8 Flying to Cleveland II

EXAMPLE 4.8 | **Flying to Cleveland II**

A wiser pilot flying from Chicago to Cleveland on the same day plots a course that will take her directly to Cleveland. In which direction does she fly the plane? How long does it take to reach Cleveland?

MODEL Establish a coordinate system with the x-axis pointing east and the y-axis north. The air is moving relative to the ground at $\vec{v}_{AG} = -50\hat{j}$ mph.

EXAMPLE 4.8 Flying to Cleveland II

EXAMPLE 4.8 | Flying to Cleveland II

SOLVE The objective of navigation is to move between two points on the earth's surface. The wiser pilot, who knows that the wind will affect her plane, draws the vector picture of **FIGURE 4.20**. She sees that she'll need $(v_y)_{PG} = 0$, in order to fly due east to Cleveland. This will require turning the nose of the plane at an angle θ north of east, making $\vec{v}_{PA} = (500\cos\theta\,\hat{\imath} + 500\sin\theta\,\hat{\jmath})$ mph.

The velocity equation is $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. The desired heading is found from setting the y-component of this equation to zero:
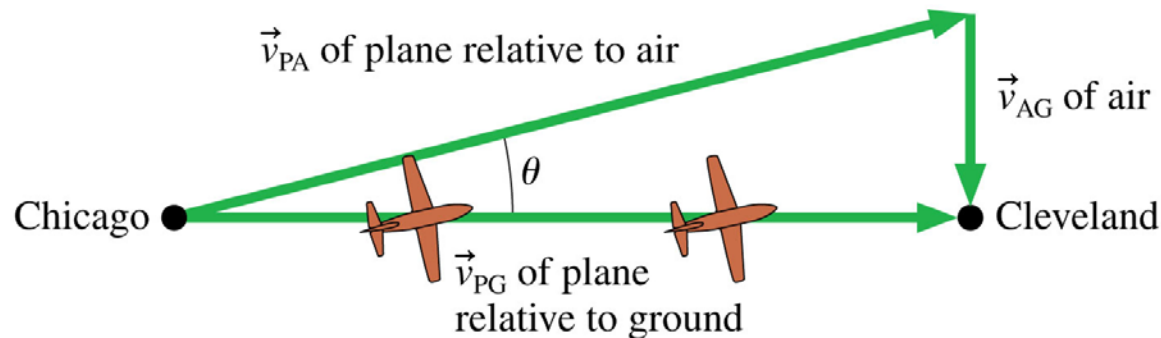
$$(v_y)_{PG} = (v_y)_{PA} + (v_y)_{AG} = (500\sin\theta - 50)\text{ mph} = 0\text{ mph}$$

$$\theta = \sin^{-1}\left|\frac{50\text{ mph}}{500\text{ mph}}\right| = 5.74°$$

The plane's velocity relative to the ground is then $\vec{v}_{PG} = (500\text{ mph}) \times \cos 5.74°\,\hat{\imath} = 497\,\hat{\imath}$ mph. This is slightly slower than the speed relative to the air. The time needed to fly to Cleveland at this speed is

$$t = \frac{300\text{ mi}}{497\text{ mph}} = 0.604\text{ h}$$

It takes $0.004\text{ h} = 14\text{ s}$ longer to reach Cleveland than it would on a day without wind.

$\vec{v}_{PA}$ of plane relative to air

$\vec{v}_{AG}$ of air

θ

Chicago

$\vec{v}_{PG}$ of plane relative to ground

Cleveland

EXAMPLE 4.8 Flying to Cleveland II

EXAMPLE 4.8 | Flying to Cleveland II

ASSESS A boat crossing a river or an ocean current faces the same difficulties. These are exactly the kinds of calculations performed by pilots of boats and planes as part of navigation.
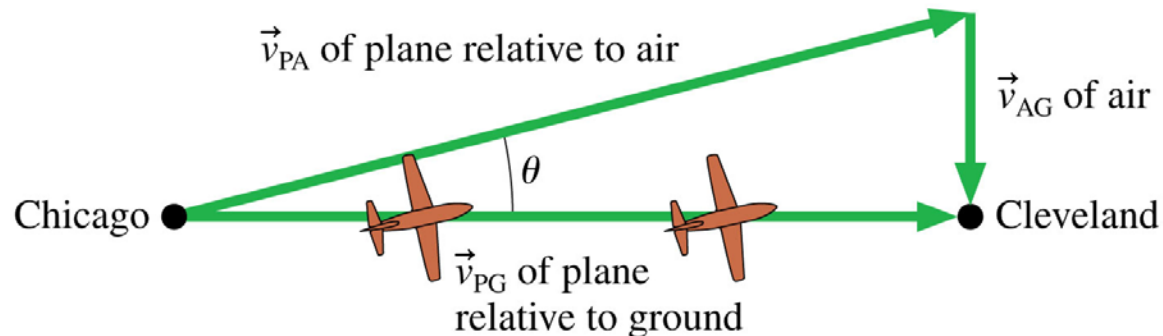
Circular Motion

- Consider a ball on a roulette wheel.

- It moves along a circular path of radius r.

- Other examples of circular motion are a satellite in an orbit or a ball on the end of a string.

- Circular motion is an example of two-dimensional motion in a plane.

Uniform Circular Motion

- To begin the study of circular motion, consider a particle that moves at *constant speed* around a circle of radius r.

- This is called **uniform circular motion**.

- The time interval to complete one revolution is called the period, T.

- The period T is related to the speed v:

The velocity is tangent to the circle.
The velocity vectors are all the same length.

$\vec{v}$

r

r

r

$\vec{v}$

$\vec{v}$

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$
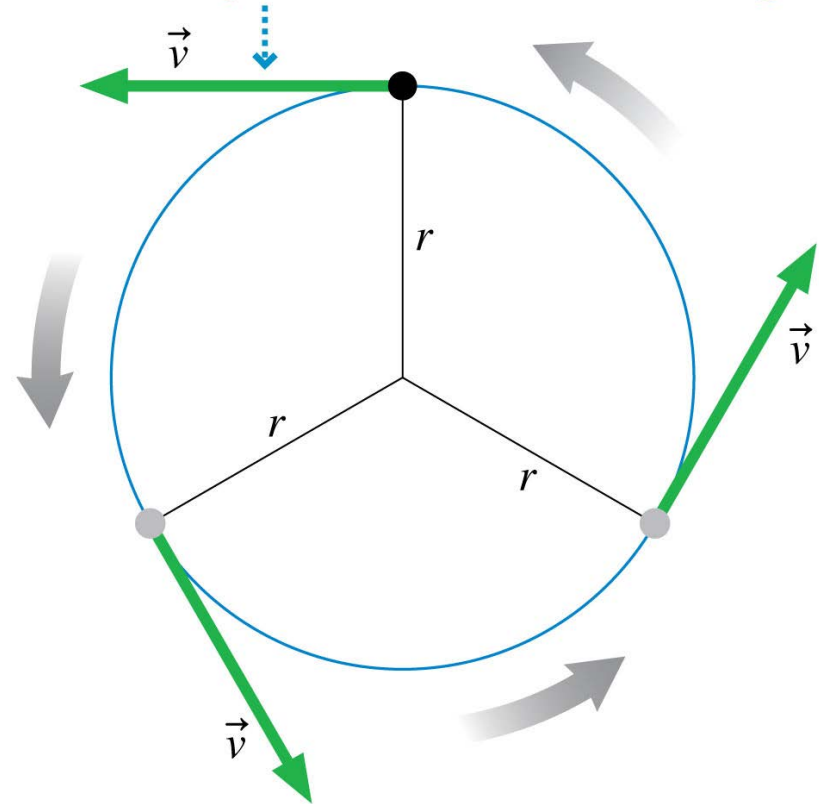
Example 4.9 A Rotating Crankshaft

EXAMPLE 4.9 | **A rotating crankshaft**

A 4.0-cm-diameter crankshaft turns at 2400 rpm (revolutions per minute). What is the speed of a point on the surface of the crankshaft?

SOLVE We need to determine the time it takes the crankshaft to make 1 rev. First, we convert 2400 rpm to revolutions per second:

$$\frac{2400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 40 \text{ rev/s}$$

If the crankshaft turns 40 times in 1 s, the time for 1 rev is

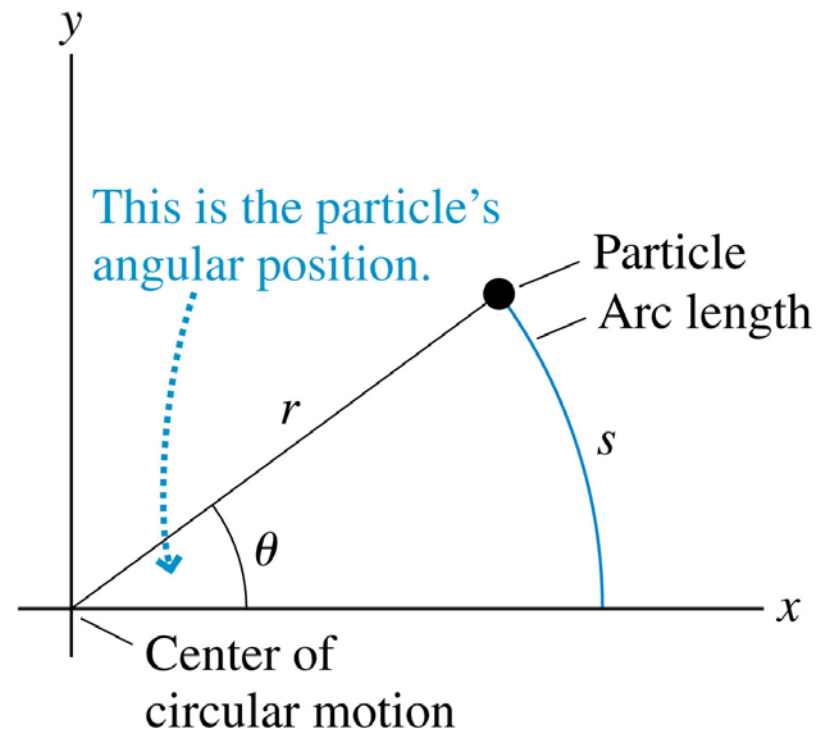$$T = \frac{1}{40} \text{ s} = 0.025 \text{ s}$$

Thus the speed of a point on the surface, where $r = 2.0$ cm $= 0.020$ m, is

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.020 \text{ m})}{0.025 \text{ s}} = 5.0 \text{ m/s}$$

Angular Position

- Consider a particle at a distance r from the origin, at an angle θ from the positive x-axis.
- The angle may be measured in degrees, revolutions (rev) or **radians** (rad), that are related by:

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

- If the angle is measured in radians, then there is a simple relation between θ and the **arc length** s that the particle travels along the edge of a circle of radius r:

$$s = r\theta \qquad (\text{with } \theta \text{ in rad})$$

This is the particle's angular position.

Particle

Arc length

y

r

s

θ

x

Center of circular motion

Angular Velocity

- A particle on a circular path moves through an **angular displacement** $\Delta\theta = \theta_f - \theta_i$ in a time interval $\Delta t = t_f - t_i$.

- In analogy with linear motion, we define

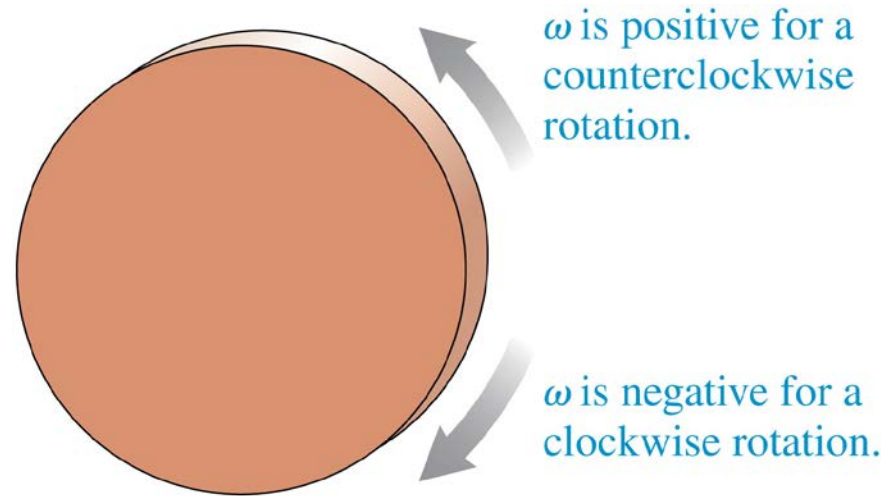$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t}$$

The particle has an angular displacement $\Delta\theta$.

- As the time interval Δt becomes very small, we arrive at the definition of instantaneous **angular velocity:**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(angular velocity)}$$
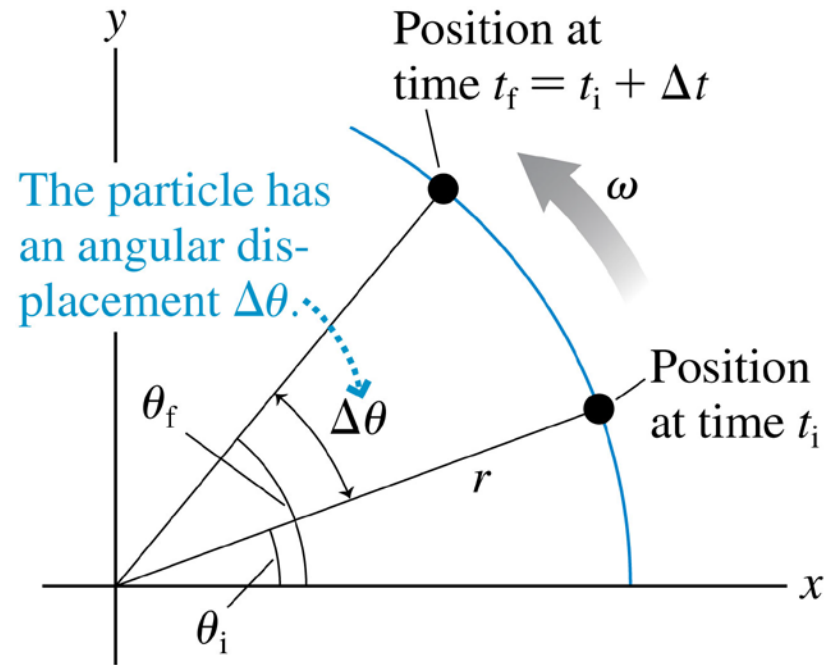
Angular Velocity

- Angular velocity ω is the *rate* at which a particle's angular position is changing.

- As shown in the figure, ω can be positive or negative, and this follows from our definition of θ.

ω is positive for a counterclockwise rotation.

ω is negative for a clockwise rotation.

- A particle moves with uniform circular motion if ω *is* constant.

- ω and θ are related graphically:

ω = slope of the θ-versus-t graph at time t

$\theta_f = \theta_i +$ area under the ω-versus-t curve between t_i and t_f

$\qquad = \theta_i + \omega \Delta t$

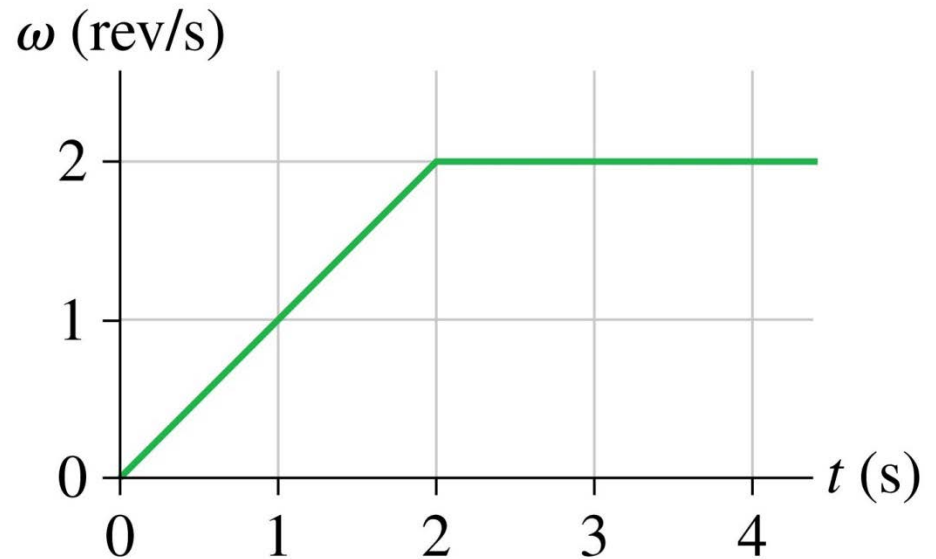This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?

A. 1

B. 2

C. 4

D. 6

E. 8

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?

A. 1

B. 2

C. 4
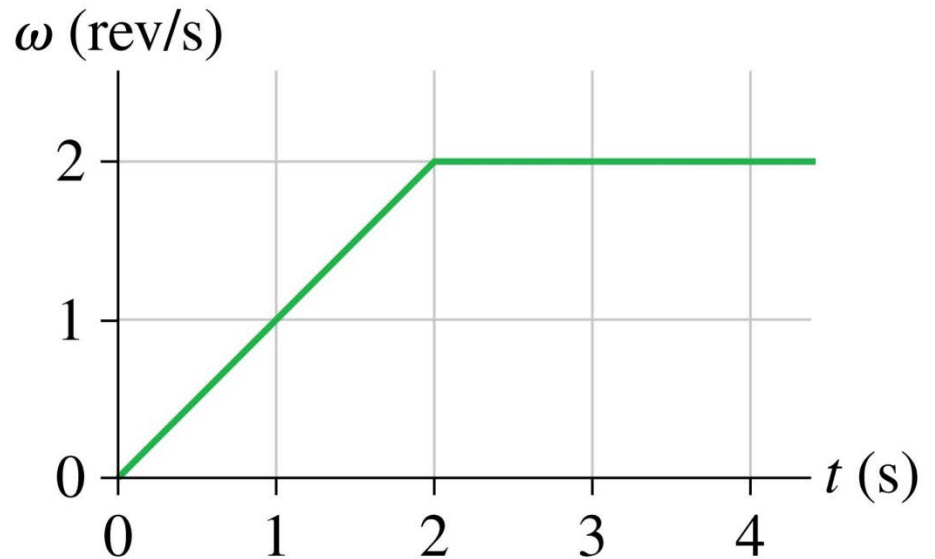
✔ **D. 6** $\Delta\theta$ = area under the angular velocity curve

E. 8

Angular Velocity in Uniform Circular Motion

- When angular velocity ω is constant, this is uniform circular motion.

- In this case, as the particle goes around a circle one time, its angular displacement is $\Delta\theta = 2\pi$ during one period $\Delta t = T$.

- The absolute value of the constant angular velocity is related to the period of the motion by

$$|\omega| = \frac{2\pi \ \text{rad}}{T} \qquad \text{or} \qquad T = \frac{2\pi \ \text{rad}}{|\omega|}$$

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

A. $\frac{1}{2}$ s

B. 1 s

C. 2 s

D. $\frac{1}{2\pi}$ s

E. $\frac{1}{4\pi}$ s

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

✔ A. $\dfrac{1}{2}$ s $\qquad T = \dfrac{2\pi}{\omega}$

B. 1 s

C. 2 s

D. $\dfrac{1}{2\pi}$ s

E. $\dfrac{1}{4\pi}$ s

Example 4.11 At the Roulette Wheel

EXAMPLE 4.11 | **At the roulette wheel**

A small steel roulette ball rolls ccw around the inside of a 30-cm-diameter roulette wheel. The ball completes 2.0 rev in 1.20 s.

a. What is the ball's angular velocity?

b. What is the ball's position at $t = 2.0$ s? Assume $\theta_i = 0$.

MODEL Model the ball as a particle in uniform circular motion.

Example 4.11 At the Roulette Wheel

EXAMPLE 4.11 | **At the roulette wheel**

SOLVE a. The period of the ball's motion, the time for 1 rev, is $T = 0.60$ s. Angular velocity is positive for ccw motion, so

$$\omega = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.60 \text{ s}} = 10.47 \text{ rad/s}$$

b. The ball starts at $\theta_i = 0$ rad. After $\Delta t = 2.0$ s, its position is

$$\theta_f = 0 \text{ rad} + (10.47 \text{ rad/s})(2.0 \text{ s}) = 20.94 \text{ rad}$$

where we've kept an extra significant figure to avoid round-off error. Although this is a mathematically acceptable answer, an observer would say that the ball is always located somewhere between 0° and 360°. Thus it is common practice to subtract an integer number of 2π rad, representing the completed revolutions. Because $20.94/2\pi = 3.333$, we can write

$$\theta_f = 20.94 \text{ rad} = 3.333 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 0.333 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 2.09 \text{ rad}$$

In other words, at $t = 2.0$ s the ball has completed 3 rev and is 2.09 rad = 120° into its fourth revolution. An observer would say that the ball's position is $\theta_f = 120°$.
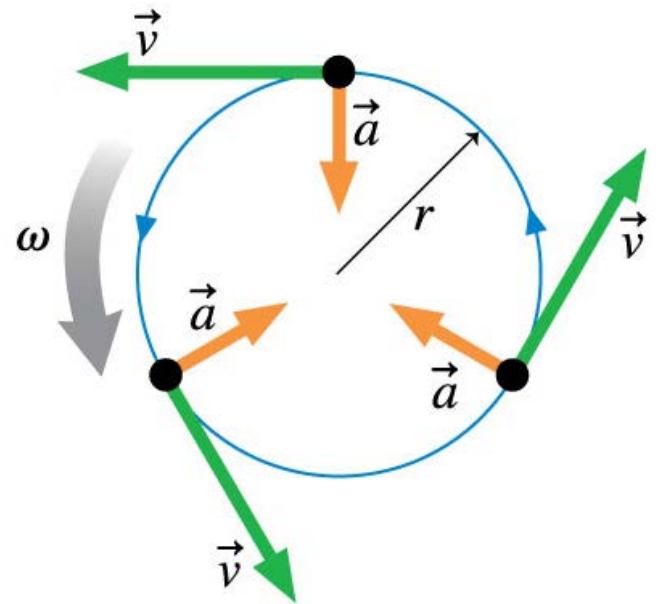
Tangential Velocity

- The tangential velocity component v_t is the rate ds/dt at which the particle moves around the circle, where s is the arc length.

- The tangential velocity and the angular velocity are related by

$$v_t = \omega r \qquad (\text{with } \omega \text{ in rad/s})$$

- In this equation, the units of v_t are m/s, the units of ω are rad/s, and the units of r are m.
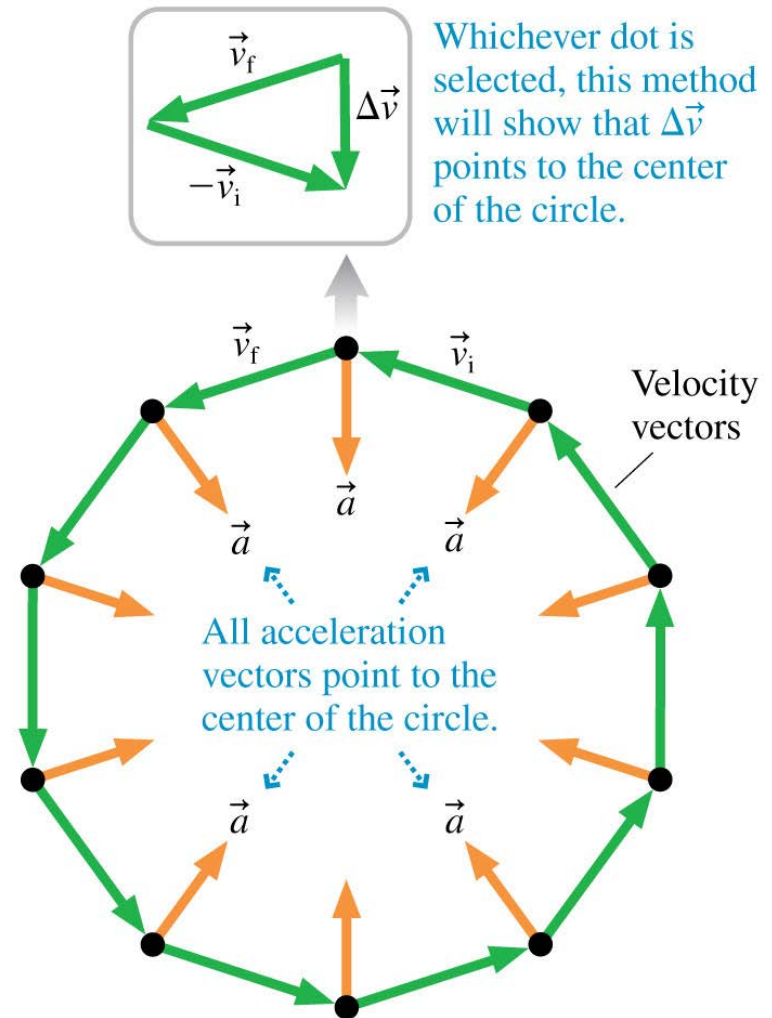
Centripetal Acceleration

- In uniform circular motion, although the speed is constant, there is an acceleration because the *direction* of the velocity vector is always changing.

- The acceleration of uniform circular motion is called **centripetal acceleration**.

- The direction of the centripetal acceleration is toward the center of the circle.

- The magnitude of the centripetal acceleration is constant for uniform circular motion.

The velocity is tangent to the circle.
The acceleration points to the center.

Centripetal Acceleration

- The figure to the right shows a motion diagram of Maria riding a Ferris wheel.

- Maria has constant speed but *not* constant velocity, so she is accelerating.

- For every pair of adjacent velocity vectors, we can subtract them to find the average acceleration near that point.

Whichever dot is selected, this method will show that $\Delta \vec{v}$ points to the center of the circle.

$\vec{v}_\text{f}$
$\Delta \vec{v}$
$-\vec{v}_\text{i}$

$\vec{v}_\text{f}$ $\vec{v}_\text{i}$

Velocity vectors

$\vec{a}$

All acceleration vectors point to the center of the circle.

$\vec{a}$ $\vec{a}$

$\vec{a}$ $\vec{a}$

$\vec{a}$

Maria's acceleration is an acceleration of changing direction, not of changing speed.

Slide 4-87

Centripetal Acceleration

- At every point Maria's acceleration points toward the center of the circle.

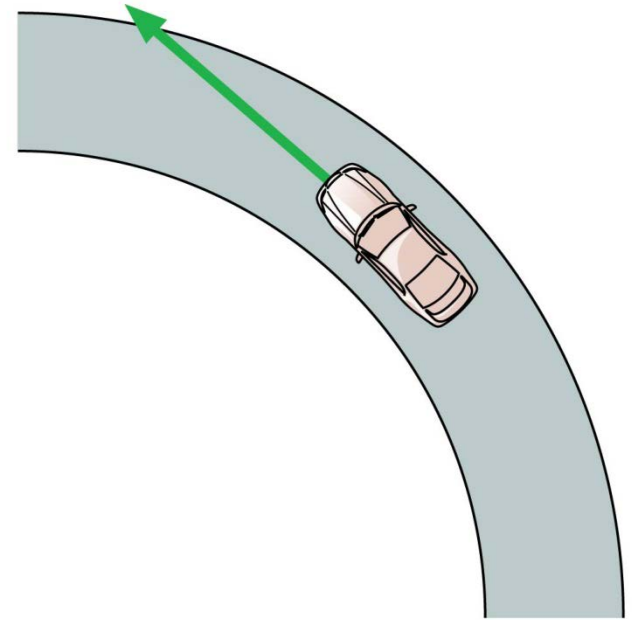- This is an acceleration due to changing direction, not to changing speed.

Whichever dot is selected, this method will show that $\Delta \vec{v}$ points to the center of the circle.

Velocity vectors

All acceleration vectors point to the center of the circle.

Maria's acceleration is an acceleration of changing direction, not of changing speed.

A car is traveling around a curve at a steady 45 mph. Is the car accelerating?

A. Yes

B. No

A car is traveling around a curve at a steady 45 mph. Is the car accelerating?
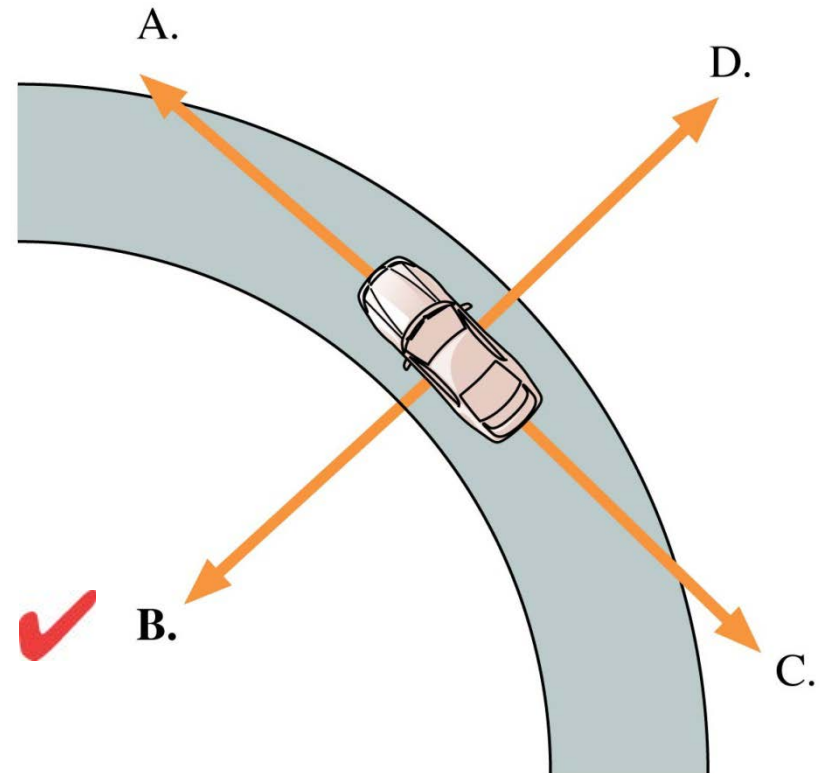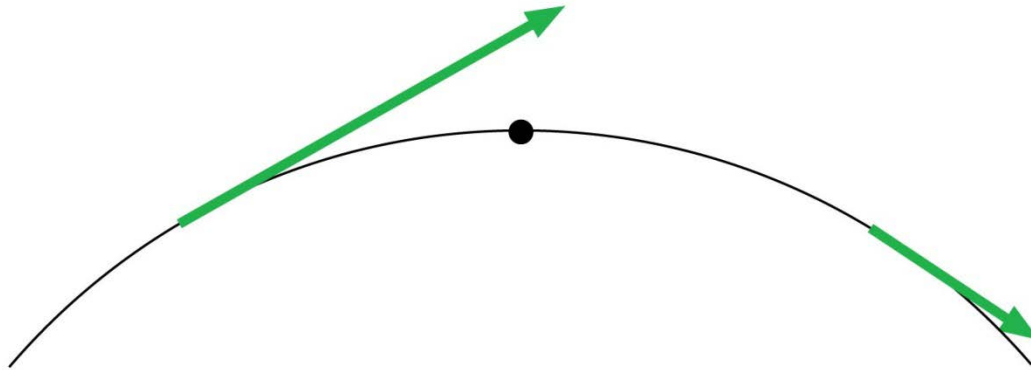
✔ **A. Yes**

B. No

A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?

A.

D.

B.

C.

E. The acceleration is zero.

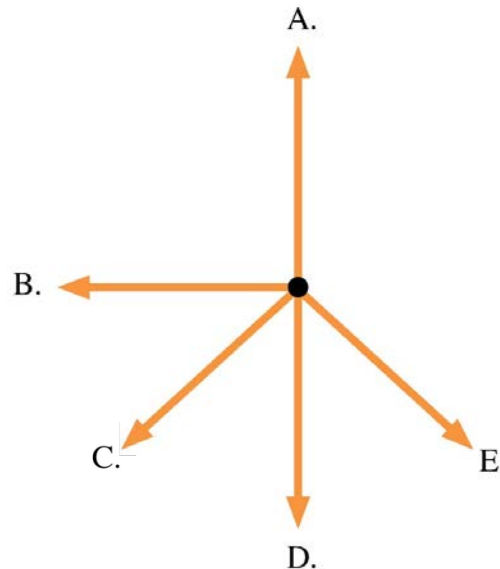A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?

A.

D.

B. ✔

C.

E. The acceleration is zero.

A car is slowing down as it drives over a circular hill.
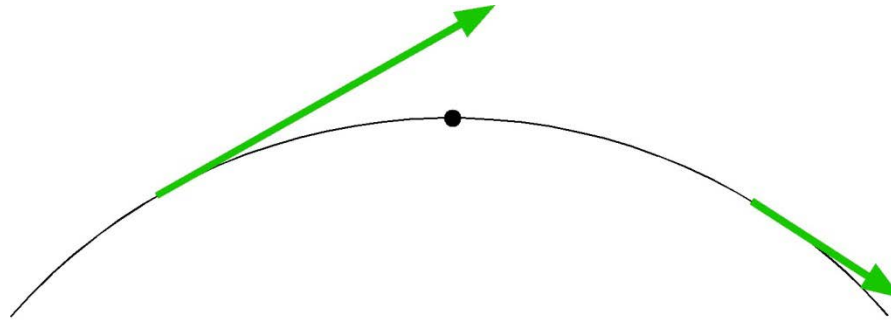

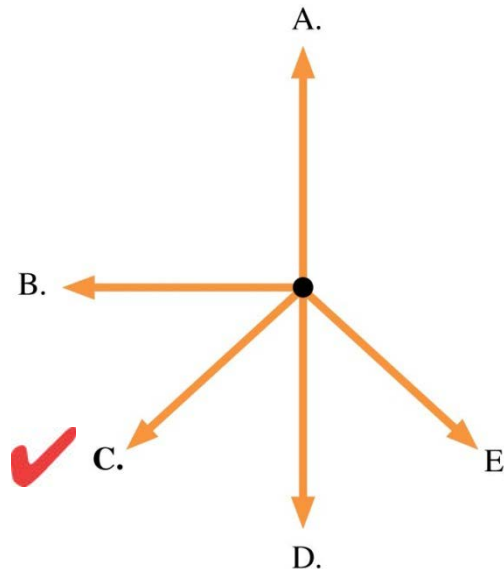Which of these is the acceleration vector at the highest point?

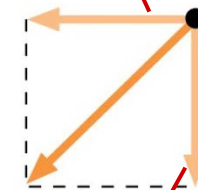A car is slowing down as it drives over a circular hill.

Which of these is the acceleration vector at the highest point?

A.

B.

C. ✔

D.

E.

Acceleration of changing speed

Acceleration of changing direction

Centripetal Acceleration

$d\vec{v}$ is the arc of a circle with arc length $dv = v\,d\theta$.

- The figure shows the velocity $\vec{v}_i$ at one instant and the velocity $\vec{v}_f$ an infinitesimal amount of time dt later.

- By definition, $\vec{a} = d\vec{v}/dt$.

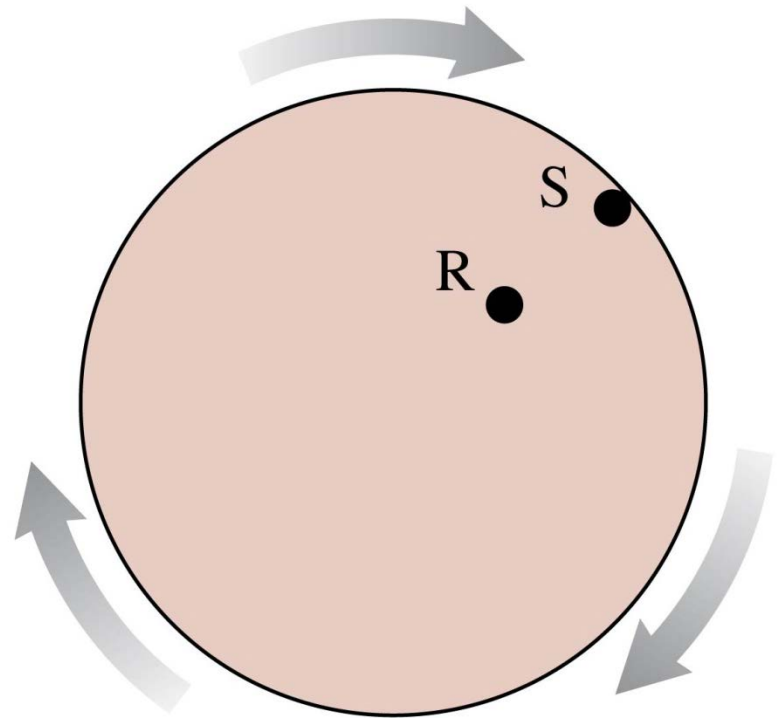- By analyzing the isosceles triangle of velocity vectors, we can show that:

These are the velocities at times t and $t + dt$.
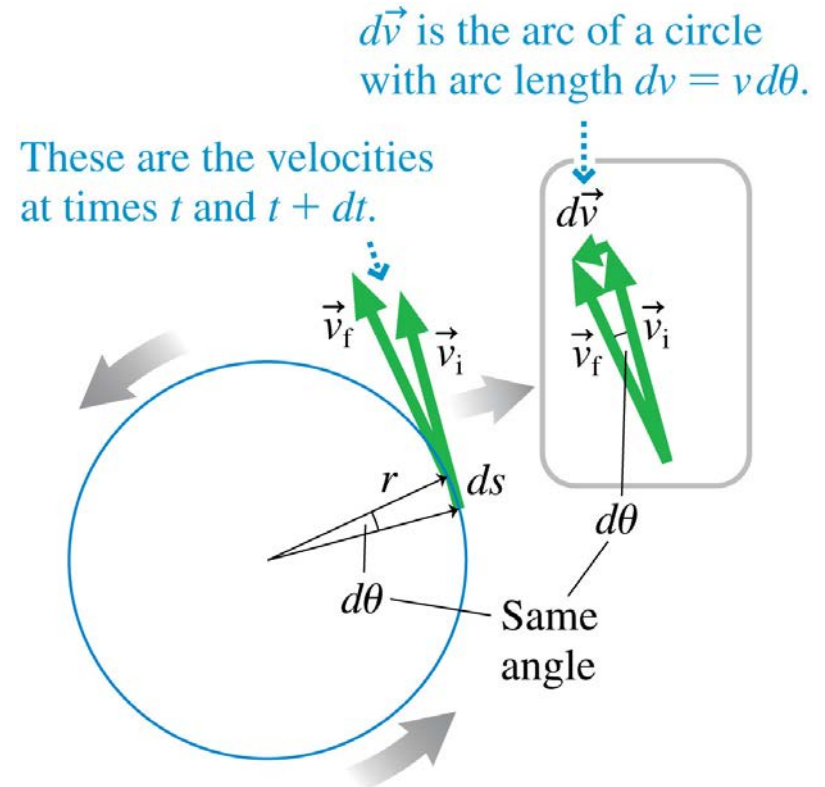
$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle} \right) \qquad \text{(centripetal acceleration)}$$

which can be written in terms of angular velocity as $a = \omega^2 r$.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _____ that of Rasheed.

A.   half

B.   the same as

C.   twice

D.   four times
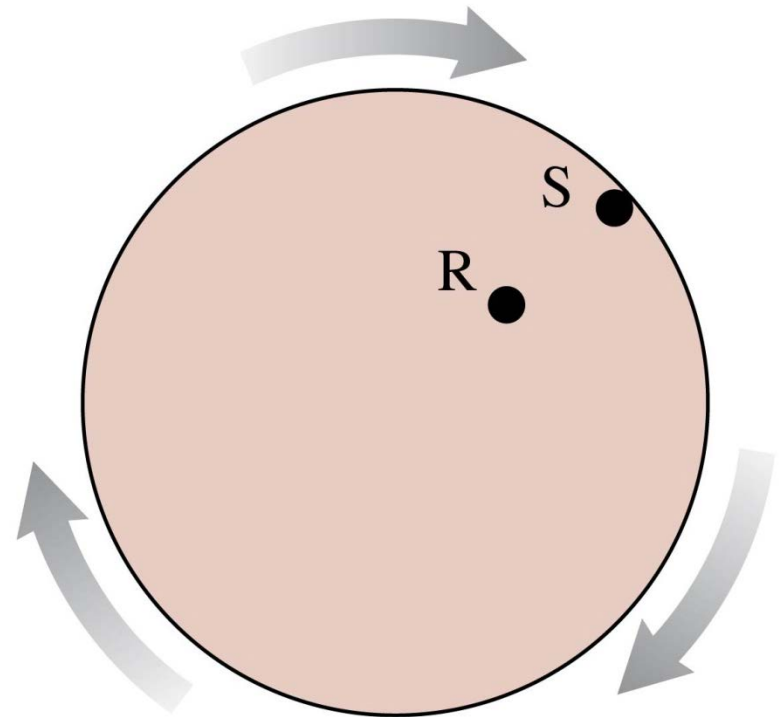
E.   We can't say without knowing their radii.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _____ that of Rasheed.
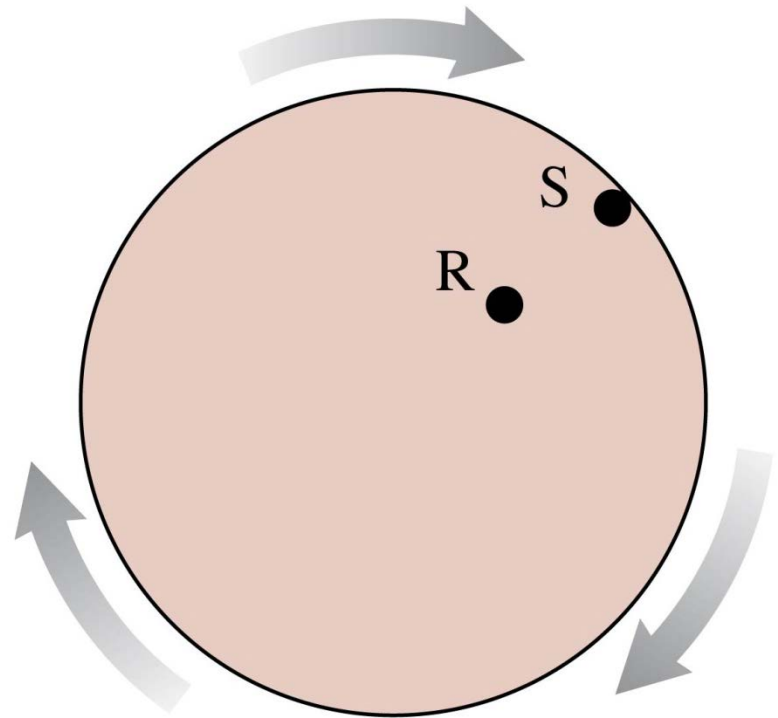
A. half

✔ **B. the same as**

C. twice

D. four times

E. We can't say without knowing their radii.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _____ that of Rasheed.

A. half

B. the same as

C. twice

D. four times

E. We can't say without knowing their radii.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _____ that of Rasheed.

A.   half

B.   the same as

✔ **C.   twice** $v = \omega r$

D.   four times

E.   We can't say without knowing their radii.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's acceleration is _____ that of Rasheed.

A.   half

B.   the same as

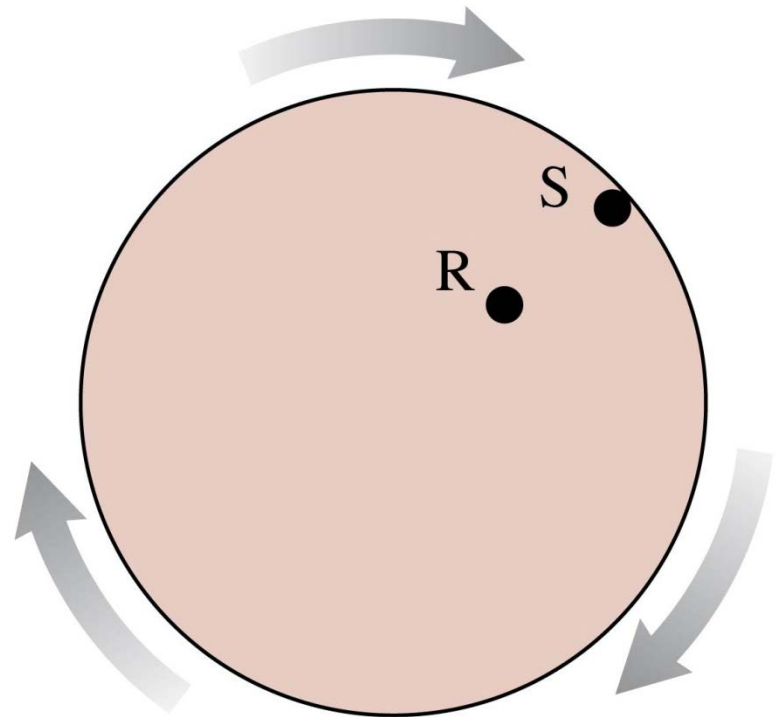C.   twice

D.   four times

E.   We can't say without knowing their radii.
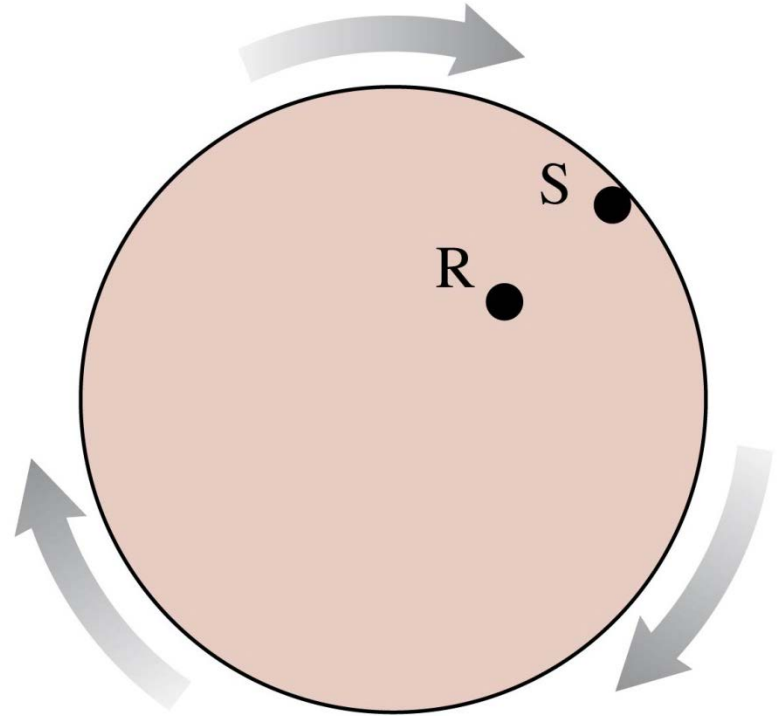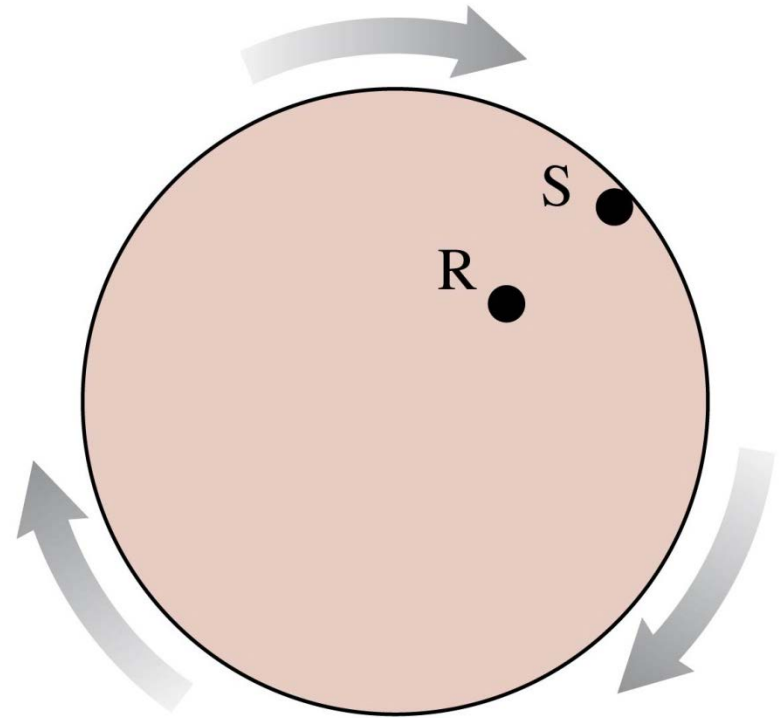
Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's acceleration is _____ that of Rasheed.

A. half

B. the same as

C. twice Centripetal acceleration $a = \dfrac{v^2}{r} = \omega^2 r$

D. four times

E. We can't say without knowing their radii.

MODEL 4.2

Uniform circular motion

For motion with constant angular velocity ω.

- Applies to a particle moving along a circular trajectory at constant speed or to points on a solid object rotating at a steady rate.

- Mathematically:
  - The tangential velocity is $v_t = \omega r$.
  - The centripetal acceleration is v^2/r or $\omega^2 r$.
  - ω and v_t are positive for ccw rotation, negative for cw rotation.

- Limitations: Model fails if rotation isn't steady.
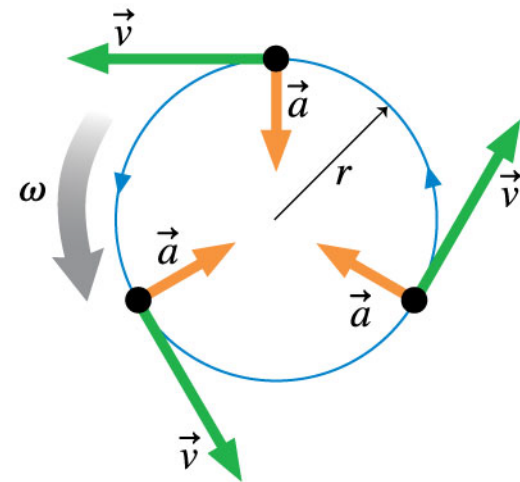
The velocity is tangent to the circle.
The acceleration points to the center.

Exercise 20

Example 4.12 The Acceleration of a Ferris Wheel

EXAMPLE 4.12 | **The acceleration of a Ferris wheel**

A typical carnival Ferris wheel has a radius of 9.0 m and rotates 4.0 times per minute. What speed and acceleration do the riders experience?

MODEL Model the rider as a particle in uniform circular motion.

SOLVE The period is $T = \frac{1}{4}$ min $= 15$ s. From Equation 4.21, a rider's speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(9.0\text{ m})}{15\text{ s}} = 3.77\text{ m/s}$$

Consequently, the centripetal acceleration has magnitude

$$a = \frac{v^2}{r} = \frac{(3.77\text{ m/s})^2}{9.0\text{ m}} = 1.6\text{ m/s}^2$$

ASSESS This was not intended to be a profound problem, merely to illustrate how centripetal acceleration is computed. The acceleration is enough to be noticed and make the ride interesting, but not enough to be scary.

Nonuniform Circular Motion

- The figure shows a point speeding up as it moves around a circle.

- This motion has *changing angular velocity*.

The angular velocity is changing.

ω

ω

- We define the **angular acceleration** α (Greek alpha) of a rotating object, or a point on the object, to be
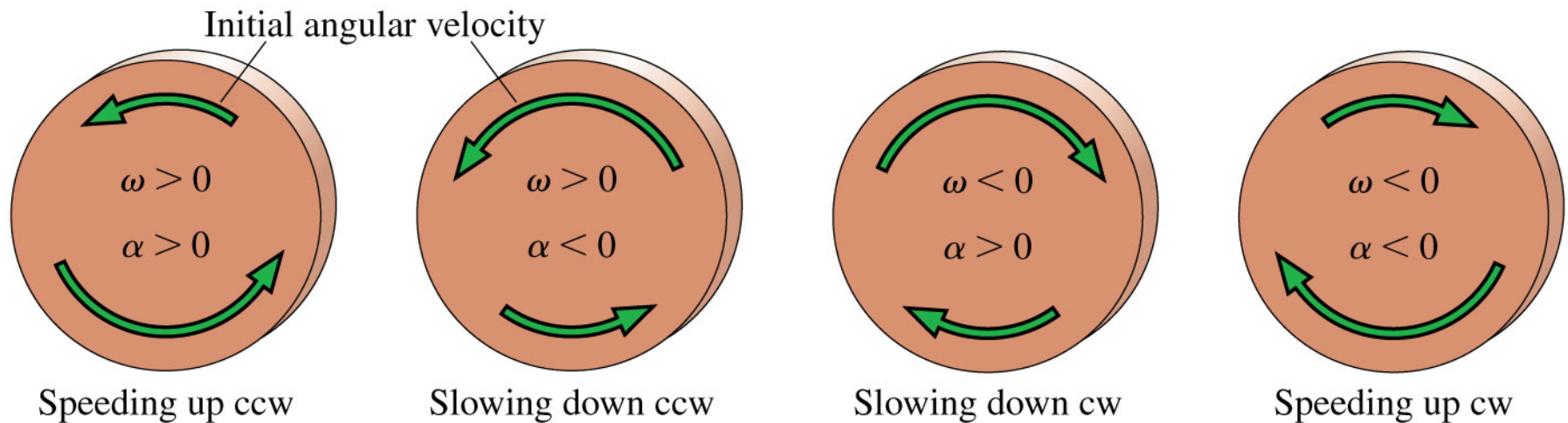
$$\alpha \equiv \frac{d\omega}{dt} \qquad \text{(angular acceleration)}$$

- The units of angular acceleration are rad/s^2.
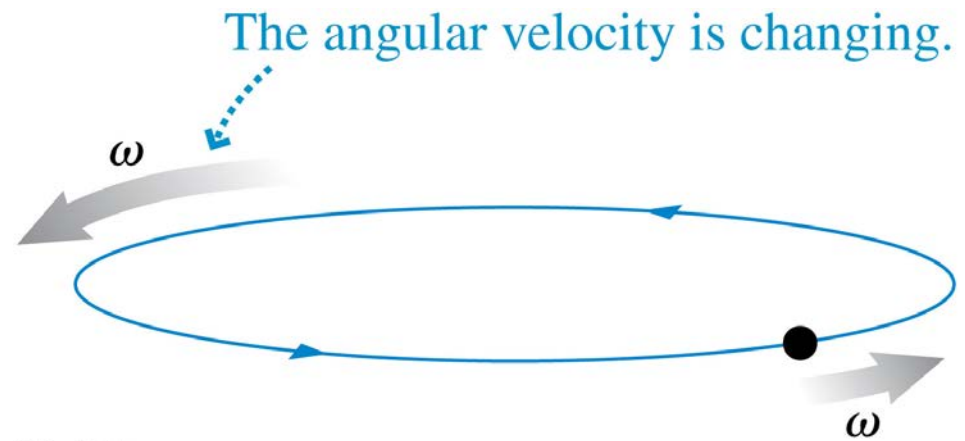
The Sign of Angular Acceleration

- α is positive if $|\omega|$ is increasing and ω is counter-clockwise.

- α is positive if $|\omega|$ is decreasing and ω is clockwise.

- α is negative if $|\omega|$ is increasing and ω is clockwise.

- α is negative if $|\omega|$ is decreasing and ω is counter-clockwise.

Initial angular velocity

$\omega > 0$
$\alpha > 0$
Speeding up ccw

$\omega > 0$
$\alpha < 0$
Slowing down ccw

$\omega < 0$
$\alpha > 0$
Slowing down cw

$\omega < 0$
$\alpha < 0$
Speeding up cw

The fan blade is slowing down. What are the signs of ω and α?

A. ω is positive and α is positive.

B. ω is positive and α is negative.

C. ω is negative and α is positive.

D. ω is negative and α is negative.

E. ω is positive and α is zero.
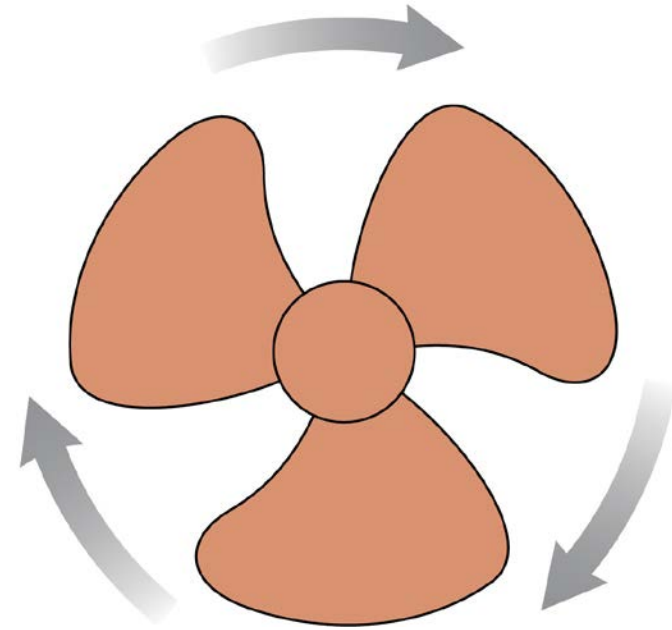
The fan blade is slowing down.
What are the signs of ω and α?

A. ω is positive and α is positive.

B. ω is positive and α is negative.

✔ **C. ω is negative and α is positive.**

D. ω is negative and α is negative.

E. ω is positive and α is zero.

"Slowing down" means that ω and α have opposite signs, not that α is negative
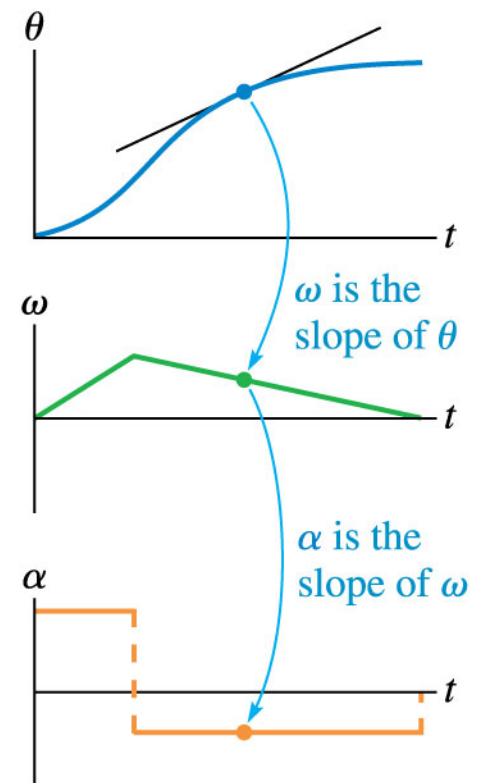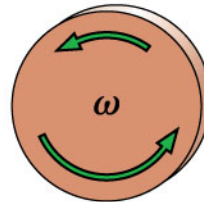
Constant Angular Acceleration

MODEL 4.3

Constant angular acceleration

For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.

- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
  - Analogs: $s \rightarrow \theta \quad v_s \rightarrow \omega \quad a_s \rightarrow \alpha$

Rotational kinematics	**Linear kinematics**
$\omega_f = \omega_i + \alpha \, \Delta t$	$v_{fs} = v_{is} + a_s \, \Delta t$
$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha \, \Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s \, \Delta s$

ω is the slope of θ

α is the slope of ω

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time $2t$?

A. 25 rad

B. 50 rad

C. 75 rad

D. 100 rad

E. 200 rad

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time $2t$?

A. 25 rad

B. 50 rad

C. 75 rad

✔ **D. 100 rad** $\Delta \theta \propto (\Delta t)^2$

E. 200 rad

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time $2t$?

A.   25 rpm

B.   50 rpm

C.   75 rpm

D.   100 rpm

E.   200 rpm

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time $2t$?

A.   25 rpm

✔ **B.   50 rpm** $\Delta\omega \propto \Delta t$

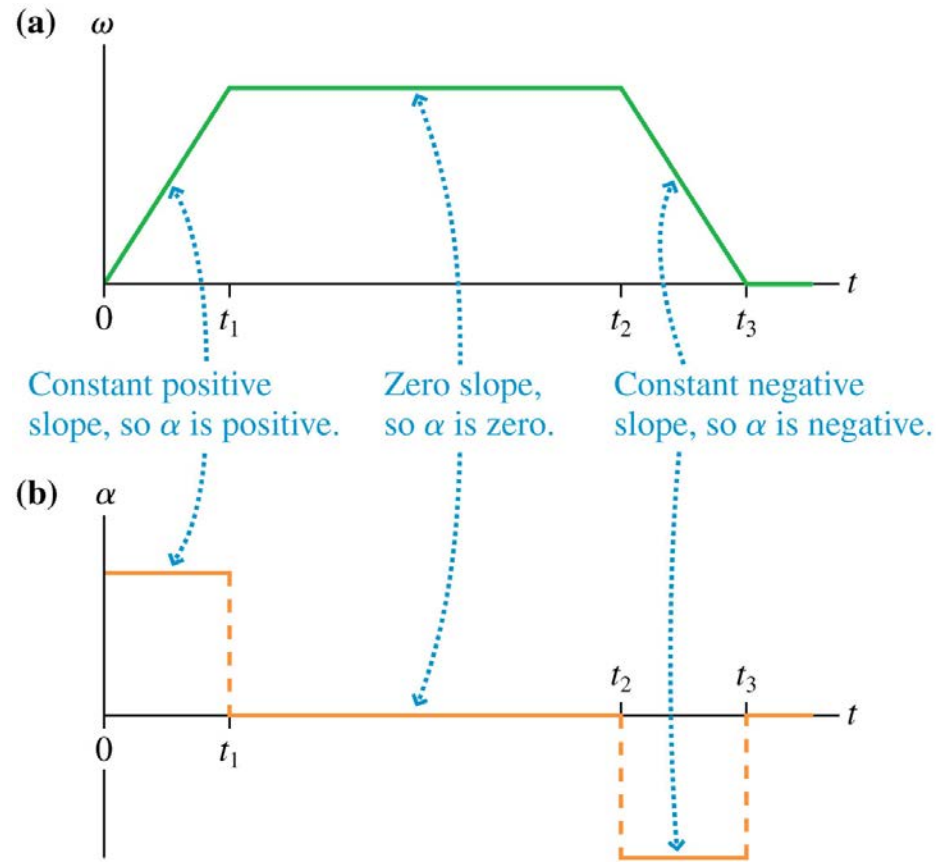C.   75 rpm

D.   100 rpm

E.   200 rpm

Example 4.13 A Rotating Wheel

EXAMPLE 4.13 | A rotating wheel

FIGURE 4.31a is a graph of angular velocity versus time for a rotating wheel. Describe the motion and draw a graph of angular acceleration versus time.

SOLVE This is a wheel that starts from rest, gradually speeds up *counterclockwise* until reaching top speed at t_1, maintains a constant angular velocity until t_2, then gradually slows down until stopping at t_3. The motion is always ccw because ω is always positive. The angular acceleration graph of **FIGURE 4.32b** is based on the fact that α is the slope of the ω-versus-t graph.

Conversely, the initial linear increase of ω can be seen as the increasing area under the α-versus-t graph as t increases from 0 to t_1. The angular velocity doesn't change from t_1 to t_2 when the area under the α-versus-t is zero.

(a) ω

Constant positive slope, so α is positive.

Zero slope, so α is zero.

Constant negative slope, so α is negative.

(b) α

Example 4.14 A Slowing Fan

EXAMPLE 4.14 | **A slowing fan**

A ceiling fan spinning at 60 rpm coasts to a stop 25 s after being turned off. How many revolutions does it make while stopping?

MODEL Model the fan as a rotating object with constant angular acceleration.

Example 4.14 A Slowing Fan

EXAMPLE 4.14 | A slowing fan

SOLVE We don't know which direction the fan is rotating, but the fact that the rotation is slowing tells us that ω and α have opposite signs. We'll assume that ω is positive. We need to convert the initial angular velocity to SI units:

$$\omega_i = 60 \, \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 6.28 \text{ rad/s}$$

We can use the first rotational kinematics equation in Model 4.3 to find the angular acceleration:

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 6.28 \text{ rad/s}}{25 \text{ s}} = -0.25 \text{ rad/s}^2$$

Then, from the second rotational kinematic equation, the angular displacement during these 25 s is

$$\Delta\theta = \omega_i \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$= (6.28 \text{ rad/s})(25 \text{ s}) + \tfrac{1}{2}(-0.25 \text{ rad/s}^2)(25 \text{ s})^2$$

$$= 78.9 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 13 \text{ rev}$$
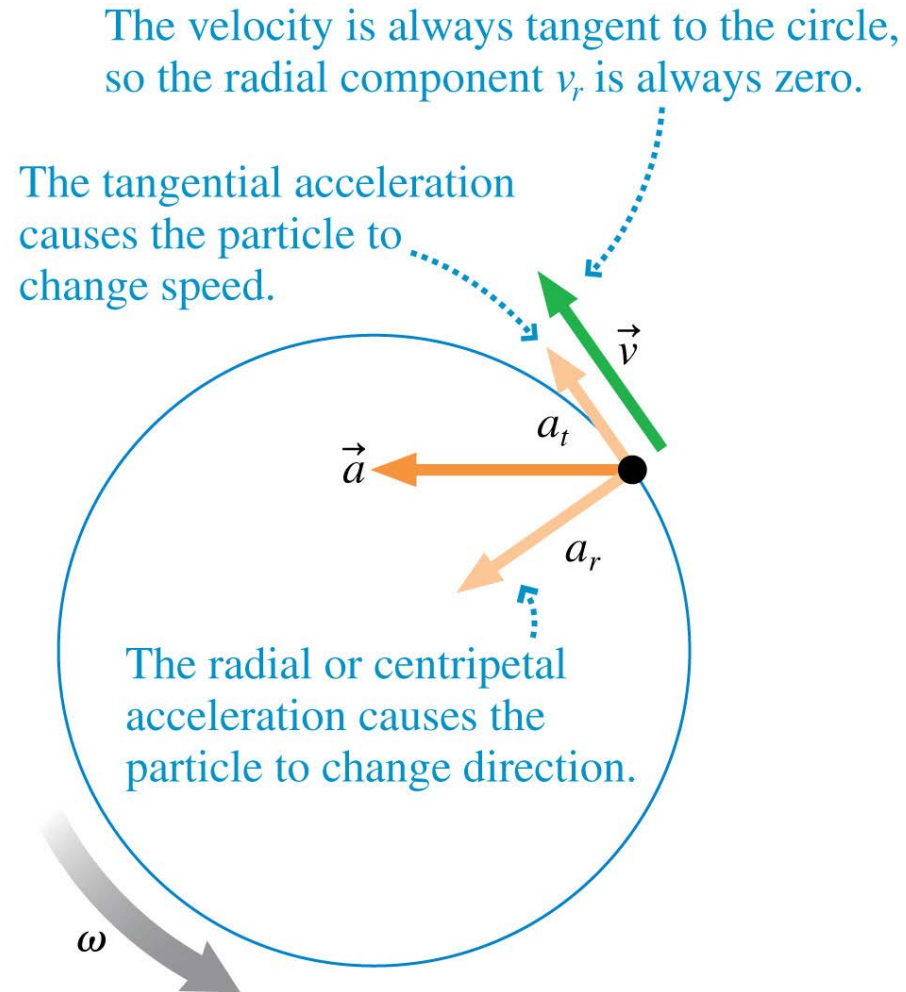
The kinematic equation returns an angle in rad, but the question asks for revolutions, so the last step was a unit conversion.

ASSESS Turning through 13 rev in 25 s while stopping seems reasonable. Notice that the problem is solved just like the linear kinematics problems you learned to solve in Chapter 2.

Tangential Acceleration

- The particle in the figure is moving along a circle and is speeding up.

- The centripetal acceleration is $a_r = v_t^2/r$, where v_t is the tangential speed.

- There is also a tangential acceleration a_t, which is always tangent to the circle.

- The magnitude of the total acceleration is

$$a = \sqrt{a_r^2 + a_t^2}$$

The velocity is always tangent to the circle, so the radial component v_r is always zero.

The tangential acceleration causes the particle to change speed.

$\vec{v}$

a_t

$\vec{a}$

a_r

The radial or centripetal acceleration causes the particle to change direction.

ω

Tangential Acceleration

- Tangential acceleration is the rate at which the tangential velocity changes, $a_t = dv_t /dt$.

- We already know that the tangential velocity is related to the angular velocity by $v_t = \omega r$, so it follows that

$$a_t = \frac{dv_t}{dt} = \frac{d(\omega r)}{dt} = \frac{d\omega}{dt}r = \alpha r$$

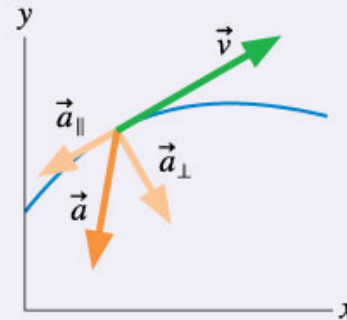# Chapter 4 Summary Slides

The **instantaneous velocity**

$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The **instantaneous acceleration** is

$$\vec{a} = d\vec{v}/dt$$

$\vec{a}_\parallel$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_\perp$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.
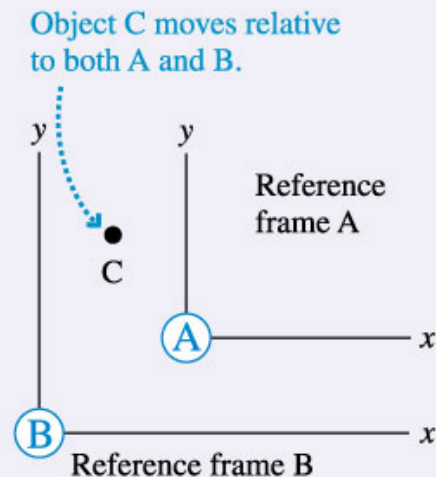
Relative Motion

If object C moves relative to reference frame A with velocity $\vec{v}_{CA}$, then it moves relative to a different reference frame B with velocity

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

where $\vec{v}_{AB}$ is the velocity of A relative to B. This is the Galilean transformation of velocity.

Object C moves relative to both A and B.

C

Reference frame A

A
x

B
Reference frame B
x

Important Concepts

Uniform Circular Motion

Angular velocity $\omega = d\theta/dt$.

v_t and ω are constant:
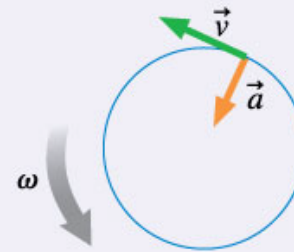
$$v_t = \omega r$$

The centripetal acceleration points toward the center of the circle:

$$a = \frac{v^2}{r} = \omega^2 r$$

It changes the particle's direction but not its speed.

Important Concepts

Nonuniform Circular Motion

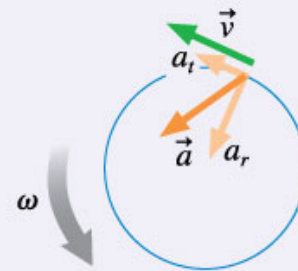**Angular acceleration** $\alpha = d\omega/dt$.
The radial acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

changes the particle's direction. The tangential component

$$a_t = \alpha r$$

changes the particle's speed.

Kinematics in two dimensions

If $\vec{a}$ is constant, then the x- and y-components of motion are independent of each other.

$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\,\Delta t$$

$$v_{fy} = v_{iy} + a_y\,\Delta t$$

Applications

Projectile motion is motion under the influence of only gravity.

MODEL Model as a particle launched with speed v_0 at angle θ.

VISUALIZE Use coordinates with the x-axis horizontal and the y-axis vertical.

The trajectory is a parabola.

SOLVE The horizontal motion is uniform with $v_x = v_0 \cos\theta$. The vertical motion is free fall with $a_y = -g$. The x and y kinematic equations have the *same* value for Δt.
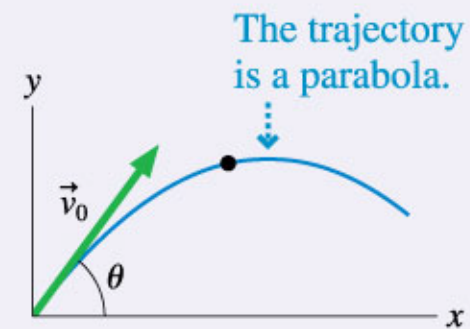
Applications

Circular motion kinematics

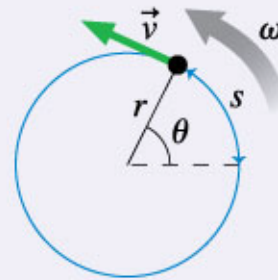Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

Angular position $\theta = \dfrac{s}{r}$

Constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$
$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

Applications

Circular motion kinematics

Circular motion graphs and kinematics are analogous to linear motion with constant acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.