

Project Gutenberg's First Six Books of the Elements of Euclid,
by John Casey

This eBook is for the use of anyone anywhere at no cost and with
almost no restrictions whatsoever. You may copy it, give it away or
re-use it under the terms of the Project Gutenberg License included
with this eBook or online at www.gutenberg.net

Title: The First Six Books of the Elements of Euclid

Subtitle: And Propositions I.-XXI. of Book XI., and an
Appendix on the Cylinder, Sphere, Cone, etc.,

Author: John Casey

Author: Euclid

Release Date: April 14, 2007 [EBook #21076]

Language: English

Character set encoding: TeX

*** START OF THE PROJECT GUTENBERG EBOOK ELEMENTS OF EUCLID ***

Produced by Joshua Hutchinson, Keith Edkins and the
Online Distributed Proofreading Team at <http://www.pgdp.net>
(This file was produced from images from the Cornell University
Library: Historical Mathematics Monographs collection.)

Production Note

Cornell University Library produced this volume to replace the
irreparably deteriorated original. It was scanned using Xerox
software and equipment at 600 dots per inch resolution and com-
pressed prior to storage using CCITT Group 4 compression. The
digital data were used to create Cornell's replacement volume
on paper that meets the ANSI Standard Z39.48-1984. The produc-
tion of this volume was supported in part by the Commission on
Preservation and Access and the Xerox Corporation. Digital
file copyright by Cornell University Library 1991.

TRANSCRIBER'S NOTE: *The Index has been regenerated to fit the pagination of this edition. Despite the author's stated hope that "few misprints have escaped detection" there were several, which have here been corrected and noted at the end of the text.*

BOOK I.

THEORY OF ANGLES, TRIANGLES, PARALLEL LINES, AND PARALLELOGRAMS.

DEFINITIONS.

THE POINT.

I. A point is that which has position but not dimensions.

A geometrical magnitude which has three dimensions, that is, length, breadth, and thickness, is a solid; that which has two dimensions, such as length and breadth, is a surface; and that which has but one dimension is a line. But a point is neither a solid, nor a surface, nor a line; hence it has no dimensions—that is, it has neither length, breadth, nor thickness.

THE LINE.

II. A line is length without breadth.

A line is space of one dimension. If it had any breadth, no matter how small, it would be space of two dimensions; and if in addition it had any thickness it would be space of three dimensions; hence a line has neither breadth nor thickness.

III. The intersections of lines and their extremities are points.

IV. A line which lies evenly between its extreme **A**—————**B**
points is called a straight or right line, such as *AB*.

If a point move without changing its direction it will describe a right line. The direction in which a point moves is called its “*sense*.” If the moving point continually changes its direction it will describe a curve; hence it follows that only one right line can be drawn between two points. The following Illustration is due to Professor Henrici:—“If we suspend a weight by a string, the string becomes stretched, and we say it is straight, by which we mean to express that it has assumed a peculiar definite shape. If we mentally abstract from this string all thickness, we obtain the notion of the simplest of all lines, which we call a straight line.”

THE PLANE.

V. A surface is that which has length and breadth.

A surface is space of two dimensions. It has no thickness, for if it had any, however small, it would be space of three dimensions.

VI. When a surface is such that the right line joining any two arbitrary points in it lies wholly in the surface, it is called a *plane*.

A plane is perfectly flat and even, like the surface of still water, or of a smooth floor.—
NEWCOMB.

FIGURES.

VII. Any combination of points, of lines, or of points and lines in a plane, is called a *plane* figure. If a figure be formed of points only it is called a *stigmatic* figure; and if of right lines only, a *rectilineal* figure.

VIII. Points which lie on the same right line are called *collinear* points. A figure formed of collinear points is called a *row* of points.

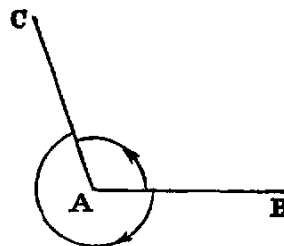
THE ANGLE.

IX. The inclination of two right lines extending out from one point in different directions is called a *rectilineal* angle.

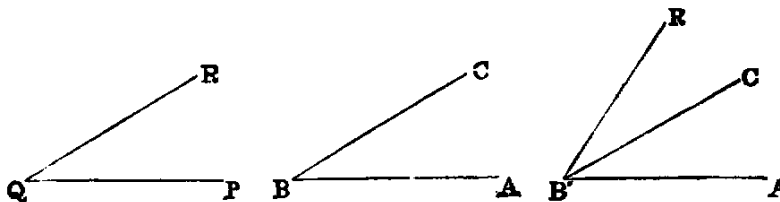
X. The two lines are called the *legs*, and the point the *vertex* of the angle.

A light line drawn from the vertex and turning about it in the plane of the angle, from the position of coincidence with one leg to that of coincidence with the other, is said to turn through the angle, and the angle is the greater as the quantity of turning is the greater. Again, since the line may turn from one position to the other in either of two ways, two angles are formed by two lines drawn from a point.

Thus if AB , AC be the legs, a line may turn from the position AB to the position AC in the two ways indicated by the arrows. The smaller of the angles thus formed is to be understood as the angle contained by the lines. The larger, called a *re-entrant* angle, seldom occurs in the "Elements."

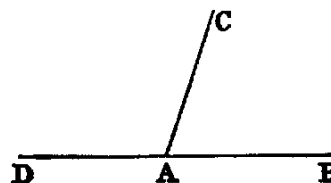


XI. *Designation of Angles.*—A particular angle in a figure is denoted by three letters, as BAC , of which the middle one, A , is at the vertex, and the other two along the legs. The angle is then read BAC .



XII. The angle formed by joining two or more angles together is called their *sum*. Thus the sum of the two angles ABC , PQR is the angle $AB'R$, formed by applying the side QP to the side BC , so that the vertex Q shall fall on the vertex B , and the side QR on the opposite side of BC from BA .

XIII. When the sum of two angles BAC , CAD is such that the legs BA , AD form one right line, they are called *supplements* of each other.

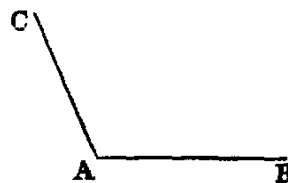
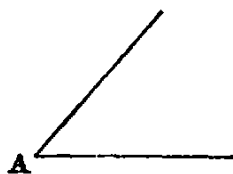
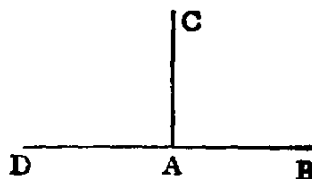


Hence, when one line stands on another, the two angles which it makes on the same side of that on which it stands are supplements of each other.

XIV. When one line stands on another, and makes the adjacent angles at both sides of itself equal, each of the angles is called a *right* angle, and the line which stands on the other is called a *perpendicular* to it.

Hence a right angle is equal to its supplement.

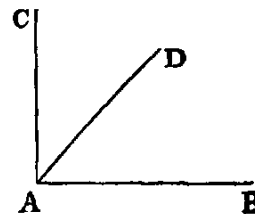
XV. An *acute* angle is one which is less than a right angle, as *A*.



XVI. An *obtuse* angle is one which is greater than a right angle, as *BAC*.

The supplement of an acute angle is obtuse, and conversely, the supplement of an obtuse angle is acute.

XVII. When the sum of two angles is a right angle, each is called the *complement* of the other. Thus, if the angle *BAC* be right, the angles *BAD*, *DAC* are complements of each other.



CONCURRENT LINES.

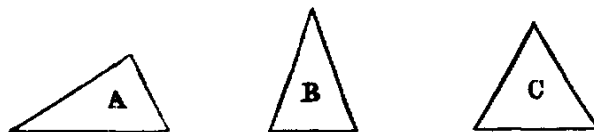
XVIII. Three or more right lines passing through the same point are called *concurrent* lines.

XIX. A system of more than three concurrent lines is called a *pencil* of lines. Each line of a pencil is called a *ray*, and the common point through which the rays pass is called the *vertex*.

THE TRIANGLE.

XX. A *triangle* is a figure formed by three right lines joined end to end. The three lines are called its *sides*.

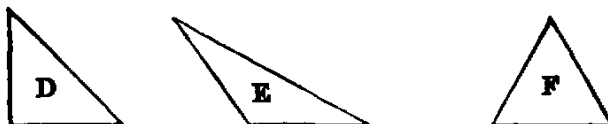
XXI. A triangle whose three sides are unequal is said to be *scalene*, as *A*; a triangle having two sides equal, to be *isosceles*, as *B*; and having all its sides equal, to be *equilateral*, as *C*.



XXII. A *right-angled* triangle is one that has one of its angles a right angle, as *D*. The side which subtends the right angle is called the *hypotenuse*.

XXIII. An *obtuse-angled* triangle is one that has one of its angles obtuse, as *E*.

XXIV. An *acute-angled* triangle is one that has its three angles acute, as *F*.



XXV. An *exterior* angle of a triangle is one that is formed by any side and the continuation of another side.

Hence a triangle has six exterior angles; and also each exterior angle is the supplement of the adjacent interior angle.

THE POLYGON.

XXVI. A *rectilineal* figure bounded by more than three right lines is usually called a polygon.

XXVII. A polygon is said to be *convex* when it has no re-entrant angle.

XXVIII. A polygon of four sides is called a *quadrilateral*.

XXIX. A quadrilateral whose four sides are equal is called a *lozenge*.

XXX. A lozenge which has a right angle is called a *square*.

XXXI. A polygon which has five sides is called a *pentagon*; one which has six sides, a *hexagon*, and so on.

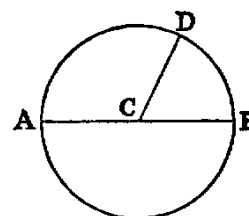
THE CIRCLE.

XXXII. A *circle* is a plane figure formed by a curved line called the *circumference*, and is such that all right lines drawn from a certain point within the figure to the circumference are equal to one another. This point is called the *centre*.

XXXIII. A *radius* of a circle is any right line drawn from the centre to the circumference, such as *CD*.

XXXIV. A *diameter* of a circle is a right line drawn through the centre and terminated both ways by the circumference, such as *AB*.

From the definition of a circle it follows at once that the path of a movable point in a plane which remains at a constant distance from a fixed point is a circle; also that any point *P* in the plane is inside, outside, or on the circumference of a circle according as its distance from the centre is *less* than, *greater* than, or *equal* to, the radius.



****The remaining part of the Book has been deleted****

Morning Song

Sawako Nakayasu

Every time, these days, it seems, an equation gets forced. Forged:

far cry

low rise

and every morning sticks, figure A, for alas, stick figures, it
figures that we awaken in the same rectangle at different points on the time
line, these every days the sum of all our

angles, a beyond-complementary
rate, exceeding three hundred sixty, then three hundred sixty-five, three
hundred seventy

days, and angles, a supersaturated moon. Also it is morning
and I am far

from and I cry.

The last ditch grows deeper and I stuff the
world into a quadratic of words, for example: But-I-love-you.
Place-in-the-box. Pass-the-god-damn-butter.
That's four against three. Far against which cry.

Calculations

"I don't know what to tell you.
Your daughter doesn't understand
math. Numbers trouble her, leave
her stuck on ground zero."

*Y fueron los mayas
quienes imaginaron el cero,
un signo para nada, para todo,
en sus gran calculaciones.*

Is zero the velvet swoop into
dream, the loop into plumes of our
breath?

"I suggest you encourage languages.
Already she knows a little Spanish,
and *you* can teach her more of that.
She lives for story time."

In the beginning there was nothing.
Then the green of quetzal wings.

*Las historias siguen
cambiando, sus verdades
vigorizadas
con cada narración
como $X \times X = X^2$*