Research Questions

The absence of any iterature on the elationship between accountability procedures and the preceived legitimacy of the police and racial and ethnic relations in particular raises the flowing research questions.

- 24. Do accountability me chanisms have a positive impact on policecommunity relations and the per ceived legitimacy of the police?
 - 24a. Are citizens aware of the existence of specific accountability procedures (e.g., a revised use of force policy, an EIS?), and does that perception improve their attitudes toward the police?
 - 24b. Can citizen aw areness of accountability procedures be increase d through outreach programs on the part of a police department?
- 25. Are certain account ability me chanisms relatively more effective than others in enh ancing perceived leg itimacy?

CONCLUSION

Holding individual poice officers accountable for their conduct is an essential element of policing. It is directly and indirectly related to achieving the basic goals of policing: reducing the and disorder, enhancing the quality of neighborhood life, and providing fair, respectful and equal teatment for all people.

As this paper indicates, thestate of our knowledge about both itaditional and new accountability mechanisms is very limited. In many instances we donot even have basic descriptive dataon current pactices. With respect to effectiveness, in only a few instances does the existing literate meet the standards of evidence-based policy making. The research needs identified in the paper have direct implication for police policy.

Problem 3.1. Given a set of distinct nodes $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^3$ and a set of function values $\{f_i\}_{i=1}^N \subset \mathbb{R}$, find an interpolant $s : \mathbb{R}^3 \to \mathbb{R}$ such that

$$s(x_i) = f_i, \qquad i = 1, \dots, N.$$
 (2)

Note that we use the notation x = (x, y, z) for points $x \in \mathbb{R}^3$.

The interpolant will be chosen from $BL^{(2)}(\mathbb{R}^3)$, the Beppo-Levi space of distributions on \mathbb{R}^3 with square integrable second derivatives. This space is sufficiently large to have many solutions to Problem 3.1, and therefore we can define the affine space of interpolants:

$$S = \{s \in \mathrm{BL}^{(2)}(\mathbb{R}^3) : s(x_i) = f_i, \quad i = 1, \dots, N\}.$$
 (3)

The space $BL^{(2)}(\mathbb{R}^3)$ is equipped with the rotation invariant seminorm defined by

$$||s||^{2} = \int_{\mathbb{R}^{3}} \left(\frac{\partial^{2}s(x)}{\partial x^{2}}\right)^{2} + \left(\frac{\partial^{2}s(x)}{\partial y^{2}}\right)^{2} + \left(\frac{\partial^{2}s(x)}{\partial z^{2}}\right)^{2} + 2\left(\frac{\partial^{2}s(x)}{\partial x\partial y}\right)^{2} + 2\left(\frac{\partial^{2}s(x)}{\partial x\partial z}\right)^{2} + 2\left(\frac{\partial^{2}s(x)}{\partial y\partial z}\right)^{2} dx.$$
(4)

This semi-norm is a measure of the energy or "smoothness" of functions: functions with a small semi-norm are smoother than those with a large semi-norm. Duchon [12] showed that the smoothest interpolant, *i.e.*,

$$s^{\star} = \arg\min_{s\in S} \|s\|,$$

has the simple form

$$s^{\star}(x) = p(x) + \sum_{i=1}^{N} \lambda_i |x - x_i|,$$
 (5)

where *p* is a linear polynomial, the coefficients λ_i are real numbers and $|\cdot|$ is the Euclidean norm on \mathbb{R}^3 .

This function is a particular example of a *radial basis function* (RBF). In general, an RBF is a function of the form

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \phi(|x - x_i|),$$
 (6)

where *p* is a polynomial of low degree and the *basic function* ϕ is a real valued function on $[0,\infty)$, usually unbounded and of noncompact support (see, *e.g.*, Cheney & Light [11]). In this context the points x_i are referred to as the *centers* of the RBF.

Popular choices for the basic function ϕ include the thin-plate spline $\phi(r) = r^2 \log(r)$ (for fitting smooth functions of two variables), the Gaussian $\phi(r) = \exp(-cr^2)$ (mainly for neural networks), and the multiquadric $\phi(r) = \sqrt{r^2 + c^2}$ (for various applications, in particular fitting to topographical data). For fitting functions of three variables, good choices include the biharmonic ($\phi(r) = r$, *i.e.*, Equation (5)) and triharmonic ($\phi(r) = r^3$) splines.

RBFs are popular for interpolating scattered data as the associated system of linear equations is guaranteed to be invertible under very mild conditions on the locations of the data points [11, 18]. For example, the biharmonic spline of Equation (5) only requires that the data points are not co-planar, while the Gaussian and multiquadric place no restrictions on the locations of the points. In particular, RBFs do not require that the data lie on any sort of regular grid.

An arbitrary choice of coefficients λ_i in Equation (5) will yield a function s^* that is not a member of $BL^{(2)}(\mathbb{R}^3)$. The requirement that $s^* \in BL^{(2)}(\mathbb{R}^3)$ implies the orthogonality or side conditions

$$\sum_{i=1}^{N} \lambda_{i} = \sum_{i=1}^{N} \lambda_{i} x_{i} = \sum_{i=1}^{N} \lambda_{i} y_{i} = \sum_{i=1}^{N} \lambda_{i} z_{i} = 0.$$

More generally, if the polynomial in Equation (6) is of degree m then the side conditions imposed on the coefficients are

$$\sum_{i=1}^{N} \lambda_i q(x_i) = 0, \text{ for all polynomials } q \text{ of degree at most } m.$$
 (7)

These side conditions along with the interpolation conditions of Equation (2) lead to a linear system to solve for the coefficients that specify the RBF.

Let $\{p_1, \ldots, p_\ell\}$ be a basis for polynomials of degree at most *m* and let $c = (c_1, \ldots, c_\ell)$ be the coefficients that give *p* in terms of this basis. Then Equations (2) and (7) may be written in matrix form as

$$\begin{pmatrix} A & P \\ P^{\mathsf{T}} & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ c \end{pmatrix} = B \begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}, \tag{8}$$

where

$$A_{i,j} = \phi(|x_i - x_j|), \qquad i, j = 1, \dots, N, P_{i,j} = p_j(x_i), \qquad i = 1, \dots, N, \quad j = 1, \dots, \ell.$$

In the specific case of the biharmonic spline in 3D, if it is assumed that the polynomial part of the RBF in Equation (5) has the form $p(x) = c_1 + c_2x + c_3y + c_4z$, then

$$A_{i,j} = |x_i - x_j|, \qquad i, j = 1, \dots, N,$$

P is the matrix with *i*th row $(1, x_i, y_i, z_i)$, $\lambda = (\lambda_1, \dots, \lambda_N)^T$ and $c = (c_1, c_2, c_3, c_4)^T$.

Solving the linear system (8) determines λ and *c*, and hence s(x). However, the matrix *B* in Equation (8) typically has poor conditioning as the number of data points *N* gets larger. This means that substantial errors will easily creep into any standard numerical solution.

On initial inspection, the essentially local nature of the Gaussian, inverse multiquadric $(\phi(r) = (r^2 + c^2)^{-1/2})$ and compactly supported basic functions appear to lead to more desirable properties in the RBF. For example, the matrix B now has special structure (sparsity) which can be exploited by well-known methods and evaluation of Equation (6) only requires that the sum be over nearby centers instead of all *N* centers. However, non-compactly supported basic functions are better suited to extrapolation and interpolation of irregular, non-uniformly sampled data. Indeed, numerical experiments using Gaussian and compactly supported piecewise polynomials for fitting surfaces to point-clouds have shown that these basic functions yield surfaces with many undesirable artifacts in addition to the lack of extrapolation across holes.

The energy minimisation properties of biharmonic splines make them well suited to the representation of 3D objects. Since the corresponding basic function $\phi(r) = r$ is not compactly supported and grows arbitrarily large as r tends to infinity, the corresponding matrix B of Equation (8) is not sparse and, except for symmetry, has no obvious structure that can be exploited in solving the system. Storing the lower triangle of matrix B requires space for N(N+1)/2 real numbers. Solution via a symmetric solver will require $N^3/6 + O(N^2)$ flops. For a problem with 20,000 data points this is a requirement for approximately 1.6×10^9 bytes (1.5GB) of core memory, and 10^{13} flops, which is impractical. Furthermore, ill-conditioning of the matrix B is likely to make any solution one gets from such a direct computation highly unreliable. Thus, it is clear that direct methods are inappropriate for problems with $N \gtrsim 2,000$. Moreover, a single direct evaluation of Equation (6) requires O(N) operations. These factors have led many authors to conclude that, although RBFs are often the interpolant of choice, they are only suitable for problems with at most a few thousand points [13, 14, 20]. The fast methods described in the following section demonstrate that this is no longer the case.



Figure 6: A greedy algorithm iteratively fits an RBF to a point cloud resulting in fewer centers in the final function. In this case the 544,000 point cloud is represented by 80,000 centres to a relative accuracy of 5×10^{-4} in the final frame.



Figure 5: Illustration of the fast fitting and evaluation parameters.

	Direct methods	Fast methods		
Fitting storage required	N(N+1)/2	O(N)		
flops to solve system	$N^3/6 + O(N^2)$	$O(N \log N)$		
Evaluation flops per evaluation	O(N)	$O(1)$ after $O(N \log N)$ setup		

Table 1: Comparison of direct and fast methods.

4 Fast methods

Fast evaluation of RBF's is performed via the Fast Multipole Method (FMM) of Greengard & Rokhlin [15]. The FMM was designed for the fast evaluation of potentials (harmonic RBF's) in two and three dimensions. However Beatson *et al.* [7] have adapted the expansion and translation theory for the potential to higher order polyharmonic RBFs. Note that polyharmonic RBFs include the biharmonic spline of Equation (5). The FMM may also be used with polyharmonic splines in two [5] and four dimensions [3].

A full description of the FMM is beyond the scope of this paper and the interested reader is directed towards the introductory short course [4] for an expository treatment of the FMM. However, we give a brief outline of the method.

The FMM makes use of the simple fact that when computations are performed, infinite precision is neither required nor expected. Once this is realised, the use of approximations is allowed. For the evaluation of an RBF, the approximations of choice are far- and near-field expansions. With the centers clustered in a hierarchical manner, far- and near-field expansions are used to generate an approximation to that part of the RBF due to the centers in a particular cluster. A judicious use of approximate evaluation for clusters "far" from an evaluation point and direct evaluation for clusters "near" to an evaluation point allows the RBF to be computed to any predetermined accuracy and with a significant decrease in computation time compared with direct evaluation.

These fast evaluation methods, when used together with fitting methods particular to RBFs [2, 6], greatly reduce the storage and computational costs of using RBFs. They reduce the cost of evaluating s(x) at M points from O(MN) to $O(M + N \log N)$ operations. The cost of simultaneously computing the gradient $\nabla s(x)$ with s(x) is approximately twice that of computing s(x) alone. Table 1 summarises the gains of fast methods over direct methods.

Fig. 5 illustrates two parameters introduced by fast methods: a *fitting accuracy* and an *evaluation accuracy*. The fitting accuracy specifies the maximum allowed deviation of the fitted RBF value from the specified value at the interpolation nodes. If desired, a different fitting accuracy may be specified at each data point, as illustrated by the varying error bars in Fig. 5. The evaluation accuracy specifies the precision with which the fitted RBF is then evaluated. Fig. 5 shows an RBF evaluated at regular intervals lying between the dashed evaluation accuracy bands either side of the actual function. It is sensible to choose the evaluation accuracy to be numerically smaller than the fitting accuracy. The resulting dotted curve is an approximation to the smooth and continuous RBF.

5 RBF center reduction

Conventionally, an RBF approximation uses all the input data points (the x_i 's in Equation (2)) as nodes of interpolation, and as centers of the RBF. However, the same input data may be able to be approximated to the desired accuracy using significantly fewer



Figure 7: Illustration of center reduction.

centers, as illustrated in Fig. 7. A greedy algorithm can therefore be used to iteratively fit an RBF to within the desired *fitting accuracy*. A simple greedy algorithm consists of the following steps:

- 1. Choose a subset from the interpolation nodes *x_i* and fit an RBF only to these.
- 2. Evaluate the residual, $\varepsilon_i = f_i s(x_i)$, at all nodes.
- 3. If $\max\{|\varepsilon_i|\} < fitting accuracy$ then stop.
- 4. Else append new centers where ε_i is large.
- 5. Re-fit RBF and goto 2.

If a different accuracy δ_i is specified at each point, then the condition in step 3 may be replaced by $|\varepsilon_i| < \delta_i$.

Center reduction is not essential when using the fast methods described in Section 4. For example, no reduction was used when fitting to the LIDAR example of Fig. 8. However, reducing the number of RBF centers results in smaller memory requirements and faster evaluation times, without a loss in accuracy. Fig. 6 illustrates the fitting process with center reduction. As more centers are added to the RBF, the zero-surface more closely approximates the entire set of data points. In this case, a laser scan of a Buddha figurine, consisting of 544,000 points, has been approximated by an RBF with 80,000 centers to a relative accuracy of 1.4×10^{-4} (achieved at all the data points).

The greedy algorithm often results in a net faster fitting time, even with a moderate reduction in the number of centers. This is due to the efficiencies associated with solving and evaluating a similar system at each iteration and the fact that initial iterations involve solving much smaller problems. The results presented in Section 8 are typical of our experience and show that dense laser scan data can be represented by significantly fewer centers than the total number of data points.

6 RBF approximation of noisy data

In Section 3 we looked for an interpolant that minimized a measure of smoothness. However, if there is noise in the data, the interpolation conditions of Equation (2) are too strict and we would prefer to place more emphasis on finding a smooth function, where smoothness is measured by Equation (4). Thus, consider the problem

$$\min_{s \in \mathrm{BL}^{(2)}(\mathbb{R}^3)} \rho \|s\|^2 + \frac{1}{N} \sum_{i=1}^N \left(s(x_i) - f_i \right)^2, \tag{9}$$

where $\rho \ge 0$ and $\|\cdot\|$ is defined in Equation (4). The parameter ρ balances smoothness against fidelity to the data. It can be shown [24] that the solution s^* to this problem also has the form of

Equation (5), but now the coefficient vector $(\lambda^{\mathsf{T}}, c^{\mathsf{T}})^{\mathsf{T}}$ is the solution to

$$\begin{pmatrix} A - 8N\pi\rho I & P \\ P^{\mathsf{T}} & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}, \tag{10}$$

where the matrices *A* and *P* are as in Equation (8). The parameter ρ can be thought of as the stiffness of the RBF *s*(*x*). The system (10) can also be solved using fast methods.

In Fig. 8 we illustrate RBF approximation (also known as spline smoothing) in the context of reconstructing a surface from 3D LI-DAR data. LIDAR data is often noisy and irregular due to limited range resolution and misregistration between scans taken at different scanner positions. Restricted physical viewing angles for the scanner mean increased susceptibility to occlusion and therefore regions of incomplete data. Elsewhere, due to overlapping scans, the data may contain redundancy. Consequently, LIDAR scans represent one of the more difficult surface fitting problems. In this example a smooth surface has been automatically fitted to LIDAR scans of a fountain in Santa Barbara using an RBF approximation. The statue is approximately 2m×5m and was scanned using a CYRAX 2400 scanner. The data set consists of 350,000 points imaged from several viewpoints in front of the statue. Three spheres apparent on top of and beside the statue correspond to landmarks added to the scene in order to align the scans. Fig. 8(a) is the point-cloud taken from several scanner positions in front of the statue. Note the large occluded regions where no data has been recorded. Fig. 8(b) is the automatically fitted RBF surface. Fig. 8(c) is a detailed view of the fitted surface which illustrates how the smoothness constraint inherent in the biharmonic RBF has correctly preserved the gap between the arm and the rest of the statue, despite having no data in these regions. Along the cut-away planes in this figure we show the magnitude of the RBF. Lighter colored points have smaller magnitudes, *i.e.*, they are "closer" to the zero-surface than darker points.

Fig. 9 illustrates the affect of varying ρ on a detailed portion of the surface in Fig. 8. Fig. 9(a) is an exact fit to the raw data ($\rho = 0$), Fig. 9(b) is the value of ρ corresponding to Fig. 8(b) and Fig. 9(c) illustrates increased smoothing with a larger value for ρ .

In this example a global value for ρ has been chosen, however, by dividing Equation (9) by ρ it is possible to specify a particular smoothing parameter ρ_i for each data point or group of points.

7 Surface evaluation

An RBF fitted to a set of surface data forms a *solid* model of an object. The surface of the object is the locus of points where the RBF is zero. This surface can be visualised directly using an implicit ray-tracer [9], or an intermediate explicit representation, such as a mesh of polygons, can be extracted. In the latter case, well-known iso-surfacing algorithms such as Marching Cubes [17] can be used to polygonize the surface. However, conventional implementations are optimised for visualising a complete volume of data sampled on a regular voxel grid. The cost associated with evaluating an RBF means that an efficient surface-following algorithm is desirable.

In this paper a marching tetrahedra variant, optimised for surface following, has been used to polygonize surfaces. A mesh optimisation scheme adapted from [21] results in fewer triangles with better aspect ratios, *i.e.*, thin and elongated triangles are avoided. A typical output mesh from this algorithm is illustrated in Fig. 10(b). Wavefronts of facets spread out from seed points across the surface until they meet or intersect the bounding box. For clarity, a wavefront from a single seed is shown in red in Fig. 10(a) spreading over the surface of the Buddha figurine during iso-surfacing. Surfacefollowing is initiated from seed points corresponding to RBF centers. By design, many centers will lie on, or very near, the surface. In the case of off-surface centers, the RBF gradient is used to search for the nearest zero-crossing. Convergence is rapid because the



Figure 8: RBF approximation of noisy LIDAR data. (a) 350,000 point-cloud, (b) the smooth RBF surface approximates the original pointcloud data, (c) cut-away view illustrating the RBF distance field and the preservation of the gap between the arm and the torso.



Figure 9: (a) Exact fit, (b) medium amount of smoothing applied (the RBF approximates at data points), (c) increased smoothing.

gradient is approximately constant near the surface. Local minima have not been observed in our experience, but this is not surprising since we deliberately constructed the function in Section 2 to have these properties. In any case, only a small subset of centers is required to seed the surface, one for each distinct surface section. The surface-following strategy avoids the conventional requirement for a 3D array of sample points and therefore minimises the number of RBF evaluations. Consequently, the computational cost increases with the square of the resolution, rather than the cube, as it would if a complete volume were sampled. Memory overhead is also minimised since it is only necessary to retain the sample vertices associated with the advancing wavefronts. Quadrilateral meshes can also be produced in this manner. Surfacing ambiguities can be resolved easily due to the ability to analytically evaluate the gradient of the RBF.

8 Results

Table 2 quantifies the fitting and evaluation times for the figures presented in this paper. In all cases the biharmonic spline was fit-

ted. Two off-surface points were generated for every second point in the original surface data, hence the number of interpolation nodes to which an RBF is fitted is approximately twice the number of surface points. Center reduction was used throughout, except in the LIDAR example, where the number of RBF centers consequently equals the number of interpolation nodes. Fitting and evaluation was performed on a 550MHz PIII Pentium with 512MB RAM. Figs. 1(a), 3, 6, 12, 13 and 14 illustrate fitting surfaces to pointclouds while Figs. 1(b) and 11 illustrate fitting to partial meshes. Fig. 8 demonstrates approximation with an RBF in the context of fitting a smooth surface to noisy LIDAR data.

The dragon in Fig. 1(a), the Buddha figurine (Fig. 6) and the skeleton hand (Fig. 12) demonstrate the ability of fast methods to model large complicated data sets to high accuracy and the compact nature of the RBF representation. The dragon data was derived from a mesh consisting of 438,000 vertices and 871,000 faces, the Buddha from 544,000 vertices and 1,087,000 faces. Normals were computed at vertices from the adjacent faces. Direct methods would inevitably fail on these problems. For example, a direct approach to fitting the Buddha data would require 4,700GB of storage just to store the matrix of the interpolation system (8). The peak core



Figure 10: Iso-surfacing an RBF. (a) Surface-following from a single seed, (b) example of an optimised mesh.

Figure	Number of	umber of Number of inter-		Peak RAM	Fitting	Surfacing	Relative
	surface points	-polation nodes	RBF centers	(MB)	time	time	accuracy
Face	14,806	29,074	3,564	29	68s	27s	7×10^{-4}
Hand	13,348	26,696	4,299	29	97s	32s	1×10^{-3}
Dragon	437,645	872,487	72,461	306	2:51:09	0:04:40	8×10^{-4}
Buddha	543,652	1,086,194	80,518	291	4:03:26	0:04:07	5×10^{-4}
Cherub statue	331,135	662,269	83,293	187	3:09:06	0:06:41	4×10^{-4}
Skeleton hand	327,323	654,645	85,468	188	3:08:44	0:04:04	3×10^{-4}
LIDAR statue	345,910	518,864	518,864	390	3:08:21	0:25:39	6×10^{-3}

Table 2: Comparison of RBF fitting and evaluation times on a 550MHz PIII with 512MB RAM.

memory requirements of the new fast methods in Table 2 are particularly noteworthy in this respect.

Fig. 1(b) and Fig. 11 illustrate the application of RBFs to mesh repair. In Fig. 1(b) an RBF is fitted to a partial mesh obtained from a laser scanner. This figure demonstrates the ability of the biharmonic spline to smoothly interpolate across large irregular holes, for example under the chin, and smoothly extrapolate a surface far from the data. In Fig. 11 an RBF has been fitted to a large statue data set. Although carefully scanned, the statue contains many small holes and larger holes corresponding to occluded regions between the embracing figures. The fitted RBF has automatically filled all the holes and generated a water-tight model of the statue without the user having to specify any parameters other than the desired fitting accuracy. Note how the fragment of shoulder data on the right figurine has been extrapolated to reconstruct the missing chest data.

Fig. 13 illustrates the reconstruction of the asteroid Eros from scattered range data. This is a good example of non-uniformly distributed data, often difficult to reconstruct using other methods. Furthermore, in this example the accuracy associated with the range measurements varies at each point.

Table 3 compares the size of the original meshes in the dragon, Buddha and skeletal hand data sets with the size of the equivalent RBF representation. The uncompressed mesh sizes were derived by assigning 3 floats (12 bytes) to each vertex and 3 integers (12 bytes) to each triangular facet. The uncompressed RBF file size corresponds to representing each center with 3 floats (12 bytes) and each coefficient (λ_i) with a double precision number (8 bytes). It is likely that single precision would be adequate, which would result in more compression, but we have not yet quantified the effect of coefficient precision on the evaluated surface. This table demonstrates the significant compression of both point-cloud and mesh data that an RBF representation offers. We have also listed the size of the meshes derived from evaluating the RBF at a comparable resolution to the original data. It appears from initial results that a promising application of RBFs is the remeshing of existing meshes. Fitting an RBF not only fills holes, but a more uniform (and hence more compact) mesh can be derived from it.

Finally, for the doubting reader, we demonstrate in Figure 14 that a single RBF can model an extremely complicated surface with a high genus. In this case 594,000 centers were required to model the turbine blade to 10^{-4} accuracy. The fast evaluation methods described in Section 4 are essential for working with RBFs of this size. The RBF has reproduced intricate internal structure along with surface detail present in the original data. The point-cloud data were only available as a Marching Cubes (MC) mesh derived from X-ray CT data. However, if the CT data were available, then we would fit an RBF directly to the CT values. Specifically, we would fit to a subset of the data with values in the vicinity of the threshold used by Marching Cubes to iso-surface the data. This would result in a smaller fitting problem since the MC mesh contains extraneous vertices and the construction of a signed-distance function, which requires adding off-surface points, would be avoided. Smooth RBF interpolation of the CT data would also be preferable to the linear interpolation used in the MC algorithm, particularly between CT slices, which are usually spaced further apart relative to the pixel resolution within a slice. An approximating spline could also be used to reduce noise in the CT data.

	Original mesh		New mesh			RBF representation		
Figure	# vertices	# facets	Storage	# vertices	# facets	Storage	RBF centers	Storage
Dragon	437,645	847,414	15.4MB	126,998	254,016	4.5MB	72,461	1.4MB
Buddha	543,652	1,086,798	19.6MB	96,766	193,604	3.5MB	80,518	1.6MB
Skeleton hand	327,323	654,666	11.8MB	81,829	163,698	2.9MB	85,468	1.7MB

Table 3: Comparison of storage requirements for RBF representations and derived meshes.



Figure 11: An RBF has automatically filled small holes and extrapolated across occluded regions in the scan data (left), to produce a closed, water-tight model (right). The complex topology of the statue has been preserved.



Figure 12: With sufficient sampling, complicated objects can be represented with RBFs.

9 Summary and future work

Fast methods make it computationally possible to represent complicated objects of arbitrary topology by RBFs. The scaleindependent, "smoothest interpolator" characterisation of polyharmonic splines make RBFs particularly suited to fitting surfaces to non-uniformly sampled point-clouds and partial meshes that contain large irregular holes. The smoothest surface, most consistent with the input data, is produced. Details are resolved, provided they are adequately sampled. The functional nature of the RBF representation offers new possibilities for surface registration algorithms, mesh simplification, compression and smoothing algorithms.

Within a Constructive Solid Geometry (CSG) framework the RBF offers a new way of modeling real-world (scanned) objects since it is inherently a solid model. It can be manipulated through



Figure 13: RBF reconstruction of the asteroid Eros from nonuniformly distributed range data (top). Photograph and model from a similar view (bottom).

a series of Boolean unions and intersections with other objects in a manner similar to how simpler geometric primitives are currently used to construct more complicated objects.

RBFs are also relevant to the problem of visualising volume data acquired on an irregular grid, as can occur in medical imaging and geophysical data. Here an RBF can be fitted to the sampled data in order to approximate the underlying scalar distribution. Vector fields can be modeled easily if the components are independent. In that case, an RBF can be fitted to each field component. This does not require much extra computation time since the matrix in Equation (8) depends only on the location of the interpolation nodes and is therefore common to each component RBF.

Planned future work includes improving the performance of the reduction algorithm presented in Section 5. We have already achieved better reductions in the number of centers than those given in Table 2 but at the cost of slower fitting times. For example, the dragon has been modeled by as few as 32,000 centers and the Buddha by 42,000 centers, to the same fitting accuracy. This improvement was achieved by simply adding fewer centers to the RBF at each iteration. The global nature of the RBF representation implies RBFs inherit the drawbacks of any global model when manipulation of part of that model or ray-tracing is required. However, we believe that it should be straightforward to decompose a global RBF description into a piecewise mesh of implicit surface patches in the spirit of [1] to facilitate local manipulation and ray-tracing. Future speed improvements in computing and evaluating RBFs are likely to come from the inherent suitability of the algorithms to parallel processing and by tailoring to particular applications. In particular, we believe that significant speed improvements are likely if the data lie on a grid, albeit an incompletely sampled grid.



Figure 14: Solid and semi-transparent renderings of an RBF model of a turbine blade containing intricate internal structure. The RBF has 594,000 centers.

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