

Appendix A: Table of Standard Normal Distribution—Laplace’s Function

The values of Laplace’s function

t	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2752
0.8	0.2781	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3489	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3945	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4270	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767

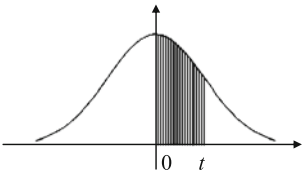
The values of a function
 $\Phi(t) = P(0 \leq U \leq t), t \geq 0$

(continued)

(continued)

<i>t</i>	0	1	2	3	4	5	6	7	8	9
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The values of a function
 $\Phi(t) = P(0 \leq U \leq t), t \geq 0$



2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4892	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998									

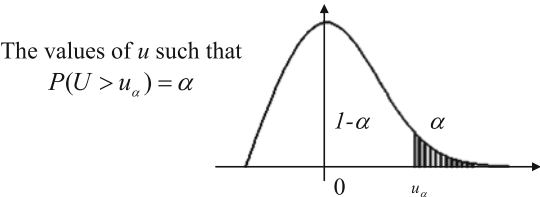
The relationship between the standard normal distribution $F_{N(0,1)}(t)$ and the Laplace’s function $\Phi(t)$

$$\begin{aligned} F_{N(0,1)}(t) &= P(T \leq t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-x^2} dx = 0.5 + \text{sign}(|t|) \frac{1}{\sqrt{2\pi}} \int_0^t e^{-x^2} dx \\ &= 0.5 + \text{sign}(|t|) P(0 \leq T \leq t) = 0.5 + \text{sign}(t) \Phi(t) \quad \text{for } t \in (-\infty, \infty) \end{aligned}$$

Appendix B: Table of Chi-Square Distribution

The χ^2 with n degrees of freedom distribution of a variable U

α	0.01	0.02	0.05	0.10	0.20	0.30	0.70	0.80	0.90	0.95	0.98	0.99
n												



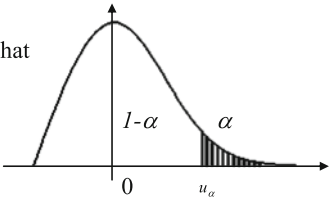
1	6.63	5.41	3.84	2.71	1.64	1.07	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.82	5.99	4.60	3.22	2.41	0.71	0.45	0.21	0.10	0.04	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	1.42	1.00	0.58	0.35	0.18	0.11
4	13.28	11.67	9.49	7.78	5.99	4.88	2.19	1.65	1.06	0.71	0.43	0.30
5	15.09	13.39	11.07	9.24	7.29	6.06	3.00	2.34	1.61	1.14	0.75	0.55
6	16.81	15.03	12.59	10.68	8.56	7.23	3.83	3.07	2.20	1.63	1.13	0.87
7	18.47	16.62	14.07	12.02	9.80	8.38	4.67	3.82	2.83	2.17	1.56	1.24
8	20.09	18.17	15.51	13.36	11.03	9.52	5.53	4.59	3.49	2.73	2.03	1.65
9	21.67	19.68	16.92	14.68	12.24	10.66	6.39	5.38	4.17	3.32	2.53	2.09
10	23.21	21.16	18.31	15.99	13.44	11.78	7.27	6.18	4.86	3.94	3.06	2.56
11	24.72	22.62	19.67	17.27	14.63	12.90	8.15	6.99	5.58	4.57	3.61	3.08
12	26.22	24.05	21.03	18.55	15.81	14.01	9.03	7.81	6.30	5.23	4.18	3.57
13	27.69	25.47	22.36	19.81	16.98	15.12	9.93	8.63	7.04	5.89	4.76	4.11
14	29.14	26.87	23.68	21.06	18.15	16.22	10.82	9.47	7.79	6.57	5.37	4.66
15	30.58	28.26	25.00	22.31	19.31	17.32	11.72	10.31	8.55	7.26	5.98	5.23
16	32.00	29.63	26.30	23.54	20.46	18.42	12.62	11.15	9.31	7.96	6.61	5.81
17	33.41	30.99	27.59	24.77	21.61	19.51	13.53	12.00	10.08	8.67	7.25	6.41
18	34.80	32.35	28.87	25.99	22.76	20.60	14.44	12.86	10.86	9.39	7.91	7.01
19	36.19	33.69	30.15	27.20	23.90	21.69	15.35	13.72	11.65	10.12	8.57	7.63
20	37.57	35.02	31.41	28.41	25.04	22.77	16.27	14.59	12.44	10.85	9.24	8.26
21	38.93	36.34	32.67	29.61	26.17	23.86	17.18	15.45	13.24	11.59	9.91	8.90
22	40.29	37.66	33.92	30.81	27.30	24.94	18.10	16.31	14.04	12.34	10.60	9.54

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α	0.01	0.02	0.05	0.10	0.20	0.30	0.70	0.80	0.90	0.95	0.98	0.99
n												

The values of u such that
 $P(U > u_\alpha) = \alpha$



23	41.64	38.97	35.17	32.01	28.43	26.02	19.02	17.19	14.85	13.09	11.29	10.20
24	42.98	40.27	36.41	33.20	29.55	27.10	19.94	18.06	15.66	13.85	11.99	10.86
25	44.31	41.57	37.66	34.38	30.67	28.17	20.87	18.94	16.47	14.61	12.70	11.52
26	45.64	42.86	38.88	35.56	31.79	29.25	21.79	19.82	17.29	15.38	13.41	12.20
27	46.96	44.14	40.11	36.74	32.91	30.32	22.72	20.70	18.11	15.15	14.12	12.88
28	48.28	45.42	41.34	37.92	34.03	31.39	23.65	21.59	18.84	16.93	14.85	13.56
29	49.60	46.69	42.56	39.09	35.14	32.46	24.58	22.47	19.77	17.71	14.57	14.26
30	50.89	47.96	43.77	40.26	36.25	33.53	25.51	23.36	20.60	18.49	16.31	14.95

Appendix C: Table of λ -Kolmogorov Distribution

The values of $Q(\lambda)$ function

λ	$Q(\lambda)$	λ	$Q(\lambda)$	λ	$Q(\lambda)$	λ	$Q(\lambda)$
The values of a function $Q(\lambda) = \sum_{k=-\infty}^{\infty} (-1)^k \exp[-2k^2 \lambda^2]$							
0.32	0.0000	0.82	0.4880	1.32	0.9387	1.82	0.9973
0.33	0.0001	0.83	0.5038	1.33	0.9418	1.83	0.9975
0.34	0.0002	0.84	0.5194	1.34	0.9449	1.84	0.9977
0.35	0.0003	0.85	0.5347	1.35	0.9478	1.85	0.9979
0.36	0.0005	0.86	0.5497	1.36	0.9505	1.86	0.9980
0.37	0.0008	0.87	0.5645	1.37	0.9531	1.87	0.9981
0.38	0.0013	0.88	0.5791	1.38	0.9556	1.88	0.9983
0.39	0.0019	0.89	0.5933	1.39	0.9580	1.89	0.9984
0.40	0.0028	0.90	0.6073	1.40	0.9603	1.90	0.9985
0.41	0.0040	0.91	0.6209	1.41	0.9625	1.91	0.9986
0.42	0.0055	0.92	0.6343	1.42	0.9646	1.92	0.9987
0.43	0.0074	0.93	0.6473	1.43	0.9665	1.93	0.9988
0.44	0.0097	0.94	0.6601	1.44	0.9684	1.94	0.9989
0.45	0.0126	0.95	0.6725	1.45	0.9702	1.95	0.9990
0.46	0.0160	0.96	0.6846	1.46	0.9718	1.96	0.9991
0.47	0.0200	0.97	0.6964	1.47	0.9734	1.97	0.9991
0.48	0.0247	0.98	0.7079	1.48	0.9750	1.98	0.9992
0.49	0.0300	0.99	0.7191	1.49	0.9764	1.99	0.9993
0.50	0.0361	1.00	0.7300	1.50	0.9778	2.00	0.9993
0.51	0.0428	1.01	0.7406	1.51	0.9791	2.01	0.9994
0.52	0.0503	1.02	0.7508	1.52	0.9803	2.02	0.9994
0.53	0.0585	1.03	0.7608	1.53	0.9815	2.03	0.9995
0.54	0.0675	1.04	0.7704	1.54	0.9826	2.04	0.9995
0.55	0.0772	1.05	0.7798	1.55	0.9836	2.05	0.9996
0.56	0.0876	1.06	0.7889	1.56	0.9846	2.06	0.9996
0.57	0.0987	1.07	0.7976	1.57	0.9855	2.07	0.9996
0.58	0.1104	1.08	0.8061	1.58	0.9864	2.08	0.9996

(continued)

(continued)

λ	$Q(\lambda)$	λ	$Q(\lambda)$	λ	$Q(\lambda)$	λ	$Q(\lambda)$
The values of a function $Q(\lambda) = \sum_{k=-\infty}^{\infty} (-1)^k \exp[-2k^2 \lambda^2]$							
0.59	0.1228	1.09	0.8143	1.59	0.9873	2.09	0.9997
0.60	0.1357	1.10	0.8223	1.60	0.9880	2.10	0.9997
0.61	0.1492	1.11	0.8299	1.61	0.9888	2.11	0.9997
0.62	0.1632	1.12	0.8374	1.62	0.9895	2.12	0.9997
0.63	0.1778	1.13	0.8445	1.63	0.9902	2.13	0.9998
0.64	0.1927	1.14	0.8514	1.64	0.9908	2.14	0.9998
0.65	0.2080	1.15	0.8580	1.65	0.9914	2.15	0.9998
0.66	0.2236	1.16	0.8644	1.66	0.9919	2.16	0.9998
0.67	0.2396	1.17	0.8706	1.67	0.9924	2.17	0.9998
0.68	0.2558	1.18	0.8765	1.68	0.9929	2.18	0.9999
0.69	0.2722	1.19	0.8823	1.69	0.9934	2.19	0.9999
0.70	0.2888	1.20	0.8877	1.70	0.9938	2.20	0.9999
0.71	0.3055	1.21	0.8930	1.71	0.9942	2.21	0.9999
0.72	0.3223	1.22	0.8981	1.72	0.9946	2.22	0.9999
0.73	0.3391	1.23	0.9030	1.73	0.9950	2.23	0.9999
0.74	0.3560	1.24	0.9076	1.74	0.9953	2.24	0.9999
0.75	0.3728	1.25	0.9121	1.75	0.9956	2.25	0.9999
0.76	0.3896	1.26	0.9164	1.76	0.9959	2.26	0.9999
0.77	0.4064	1.27	0.9206	1.77	0.9962	2.27	0.9999
0.78	0.4230	1.28	0.9245	1.78	0.9965	2.28	0.9999
0.79	0.4395	1.29	0.9283	1.79	0.9967	2.29	0.9999
0.80	0.4559	1.30	0.9319	1.80	0.9969	2.30	0.9999
0.81	0.4720	1.31	0.9354	1.81	0.9971	2.31	1.0000

Appendix D: Table of Gamma Function

The values of a gamma function

p	$\Gamma(p)$
1.00	1.00000
1.05	0.97350
1.10	0.95135
1.15	0.93304
1.20	0.91817
1.25	0.90640
1.30	0.89747
1.35	0.89115
1.40	0.88726
1.45	0.88566
1.50	0.88623
1.55	0.88887
1.60	0.89352
1.65	0.90012
1.70	0.90864
1.75	0.91906
1.80	0.93138
1.85	0.94561
1.90	0.96177
1.95	0.97988
2.00	1.00000

The gamma function properties

$$\Gamma(1) = 1,$$

$$\Gamma(p+1) = p\Gamma(p), \quad p > 0,$$

$$\Gamma(n+1) = n!, \quad n \in N.$$